The Contactless Measurement of the Electrical Resistivity

by

Frank Schippan

A Master Thesis submitted in partial fulfilment of
the requirements for the award of
Master of Philosophy of the Loughbourough University

October 1997

Loughborough University

Acknowledgements

First of all I wish thank my parents for their support and patience during the time of my study in Berlin and Loughborough.

Many thanks to the **Friedrich-Naumann Stiftung** for the uncomplicated financial support during the year at Loughborough.

There are many people in the Department of Physics at Loughborough University who have made this time such an enjoyable and scientifically fruitful one. I would like to thank all of them for their encouragement, ideas and enthusiasm:
My supervisor, **Dr. Klaus-Ulrich Neumann**, and my director of research, **Prof. Kurt R. Ziebeck**. From the very beginning they have been a powerful supporting team allowing me to drive the project to completion. At the same time I was encouraged to maintain the highest standards.
Special kudos to **Bryan Dennis** for his ideas and technical expertise for the experimental implementation and design. I have really enjoyed the discussions concerning technical things and tricks. As a further "user" of the contactless resistivity measurement apparatus I am sure that Bryan will obtain valuable scientific results. Many thanks also for teaching me the proper art of playing golf.

I am grateful for having had the opportunity of learning another language In doing so I have made new friends and enjoyed the company of old ones. In particular I would like to thank **Irina Gorbunowa** for her help during the first weeks and for her support in the process of mastering the British dental-health service.

I thank the other PhD students of the Solid State research group: **Tim Smith**, **Mark Parson**, **John Tailor** and **Achmed Bargawi**. They have been helpful and friendly colleagues.

Special thanks for **Dr. Feodor Kusmartsev** who has allowed me a glimpse into the higher spheres of theoretical physics and **Dr. David E. Williams** who was the centre of calmness in times of excitement and worry.

Special thanks to **Werner Schiffers** from the optical group for the introduction to the world of cavitation and nonlinear ODE's.

Finally I would like to thank all great people at the Condensed Matter group who made it a stimulating, entertaining and friendly place to work.

I also thank my new colleagues **Prof. Dr. Klaus Ploog** and **Dr. Lutz Däweritz** from the Paul Drude Institute for their patience and understanding during the final stages of this project.

Preface

Physicists are interested in understanding the processes of nature. Within the field of Solid State Physics the characterisation of materials and the measurement of their properties is the first step towards identifiying new and interesting areas of scientific activities.

The electrical resistivity of conductive materials is an important property which provides information about the electronic behaviour of the material. An elegant method to determine this characteristic is the measurement without using electrical contacts. Such a method avoids a whole set of experimental problems connected with the physics of electrical contacts to the sample.

This Master-thesis gives an introduction into this experimental technique. A detailed theoretical description is developed. The experimental activity has involved the design, construction and testing of the apparatus. In the process of testing the method novel aspect emerged : The measurement at resonance point. These measurements can yield separate values for two different physical quantities: the electrical resistivity ϱ and the magnetic susceptibility χ.

The innovation of this project is the simultaneous characterisation of both values for the material under investigation.

The report starts by giving the theoretical background within which the first part of the detailed theoretical predictions are discussed. The second part contains experiments and a description of the experimental set-up. This design is the result of a long period of optimisation and testing. The working of the apparatus is demonstrated by the measurement of some samples. ⋄

Contents

Part I

Overview and Theory

Chapter 1

Introduction

1.1 Overview

The investigation and characterisation of materials forms an essential part of Solid State Physics. There are different experimental methods to characterise the properties of a material, e.g. X-ray experiments are used for the structural investigation of a crystal, heat capacity measurements for the thermal properties or electrical conductivity measurements for the determination of the electronic behaviour of a material.
The measurement of the electrical resistivity as a function of temperature is a standard method which is employed for the detailed investigation of different metallic materials.
According to Ohm's law [21], the magnitude of a current I flowing through a wire is proportional to the potential difference $V = \Phi_2 - \Phi_1$ along the wire. Where Φ_1 and Φ_2 are the electrical potential at the ends of the wire. The factor of proportionality is the resistance of the wire:

$$V = IR \tag{1.1}$$

The electrical resistance R of a wire or a sample depends on its dimensions, but it is independent of the size of the current or the potential difference. The resistance itself is not a material constant because it depends on the dimensions of the sample.
The material constant which can be extracted from the resistance measurements is ϱ the specific electrical resistivity. In most cases it is denoted as the electrical resistivity. It does not depend on the geometry of the sample. Within the Drude model the electrical resistivity ϱ, as a material-parameter, is defined to be the factor of proportionality between the electric field $\vec{E}$ at a point $\vec{r}$ in the metallic sample and the current density $\vec{j}$:

$$\vec{E}(\vec{r}) = \varrho\, \vec{j}(\vec{r}) \tag{1.2}$$

If $\vec{E} \parallel \vec{j}$ for all directions of $\vec{E}$ then the resistivity ϱ is a scalar. Otherwise ϱ is given by a matrix which reflects the directional dependence of the current. The current density is a vector-field. Its magnitude corresponds to the amount of charge (per unit time) crossing a unit area which is oriented perpendicular to the direction of the flow. For an uniform current density which flows through an isotropic wire characterised by a constant cross-section A, the current density is given as:

$$j = \frac{I}{A} \tag{1.3}$$

where I is the total current. The potential difference along the wire of length l is given by:

$$V = El \tag{1.4}$$

Thus eq. (1.2) is rewritten as:

$$V = \frac{\varrho l}{A} I \tag{1.5}$$

Hence the resistance of a wire is obtained as:

$$R = \frac{\varrho l}{A} \tag{1.6}$$

In summary the resistance R of a metallic material depends on the geometry of the sample as shown in (1.6). The specific resistivity ϱ is a material constant characterising its electrical properties.

1.2 Measurement-Methods

Various examples of the experimental procedures of electrical resistivity measurements are discussed in this section. The conventional technique for measuring the electrical resistivity of a sample is to cast or shape the sample into a rod and attach current and voltage leads. The principal setup is shown in figure 1.1.

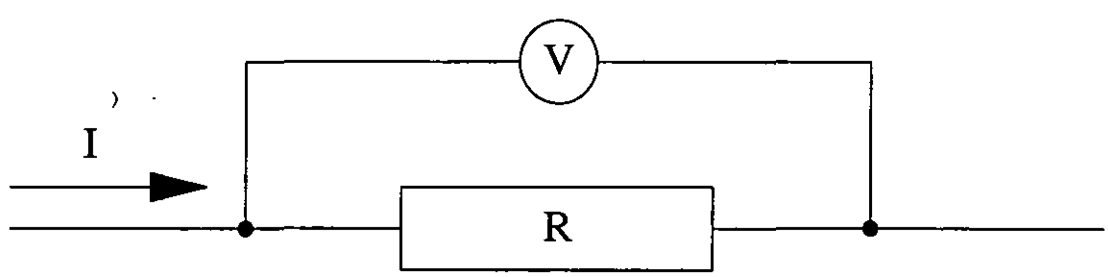


Figure 1.1: *The basic set-up for resistivity measurement*

If the geometry of the specimen is known the electrical resistivity ϱ can be calculated using (1.6)[1].
This method is applicable if the sample resistance is much higher than the resistance of the electrical connections.

[1] assuming a constant cross-section A along the length of the sample

The next two examples are the two-point and the four-point probe methods. They are also based on the measurement principle of applying a current and measuring the potential difference between two points. For the case of the two point method the contacts for current and voltage are applied at two points. The resulting set-up is shown at the top of figure 1.2(a).

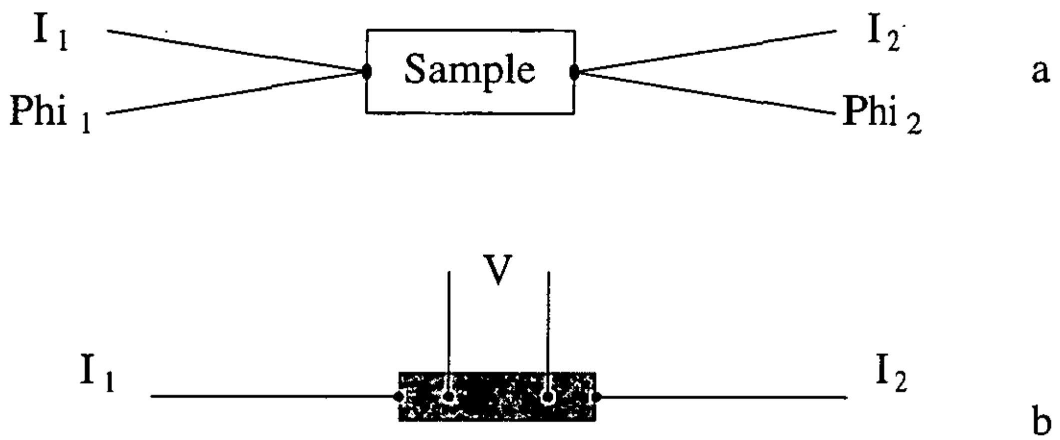


Figure 1.2: *Two point (a) method and the four-point (b)*

A more sophisticated method is the four-point method. The electrical set-up is shown in figure 1.2b. The four-terminal method is required for low resistivity materials [2]. There are different methods for calculating the resistivity ϱ, e.g. for a thin sample the resistivity is given by [17]:

$$\varrho = tF\frac{V}{I} \tag{1.7}$$

where t is the sample thickness, V is the measured voltage, I is the applied current and F is a geometric correction factor. For the case of a regular sample geometry e.g. a rectangular parallelepiped F is a function of the length and width of the sample, e.g. the average of the effective cross-section area.

$$F \sim \left\langle \frac{1}{A} \right\rangle \tag{1.8}$$

1.2.1 Experimental Challenges

For lowest contact resistances mechanically well fixed or soldered contacts are required for using any of these methods. However the use of electrical contacts may present problems. The first difficulty arises due to contact-potentials. This occurs if different materials are used for the sample and the electrodes. This might be neglected with a correction, e.g. a subtraction of the contact-potentials. Another difficulty can originate from a change of state (e.g. fluid $\leftrightarrow$ solid) of the sample during a measurement of ϱ against temperature. For low resistivity materials e.g. for pure metals the measurement of the potential difference must be done very accurately. At low temperatures, however, the resistivity decreases substantially [3], and these methods may not work reliably. In order to carry out a sensitive measurement of the electrical resistance of a high conductivity metal at low temperatures, several methods are available to overcome these experimental problems. The obvious methods are a reduction in the cross-section or an increase of the applied current. However as pointed out by Delaney and Pippard [3] for some materials and especially brittle ones it is impossible to manufacture fine wires. The alternative of an increase of current results in sample heating and hence temperature problems.

In general the mounting of electrical electrodes implies a non-homogeneous distribution of the electrical potential which results in a non-homogeneous current flow through the sample. The effect is illustrated using a thin rectangular sample geometry of an infinite plane.

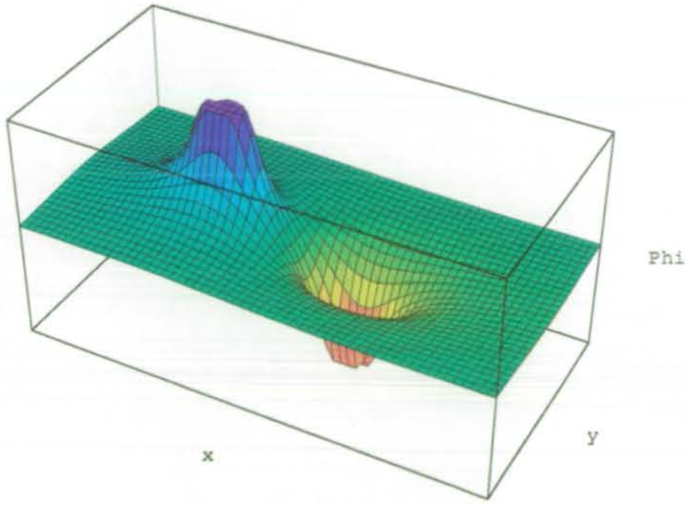


Figure 1.3: *Electrical Potential for two point contacts*

The two point method is used here. The voltage and currents leads are connected to the material at two points. The resulting electrical potential is shown in figure 1.3.
The corresponding electrical field lines in the infinite plane and the equipotential (black) lines are shown in figure 1.4. The electrical current flows along the electrical field lines. It is apparent that the vector field is not homogenous as indicated by the grey lines.

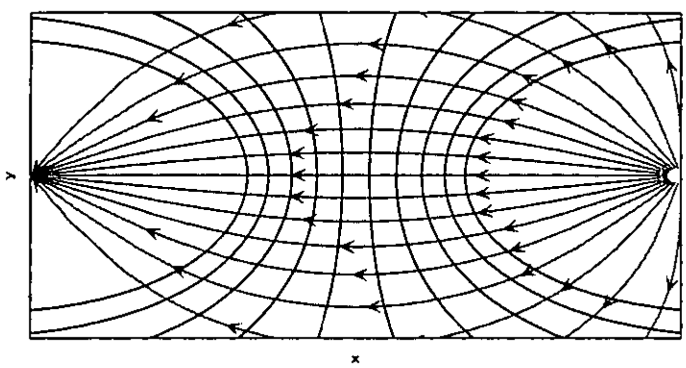


Figure 1.4: *Field lines and equipotential lines in a infinite plane*

Furthermore a comparison of electrical resistivities for different materials requires the measurement to be carried out on an absolute scale. In order to achieve such an absolute measurement the geometry of the sample must be known exactly and with small error-bars. As discussed above the manufacturing of some materials in the form of a regular structure can be very difficult. Thus measurements on an absolute scale are a challenging experimental problem.
The next section introduces an alternative method which was originally developed by Bean et al. [1].

1.2.2 An alternative determination of ϱ

In the late fifties a new method for measuring the resistivity was developed and published by Bean et al. [1]. The basic idea is the following:

If a magnetic field is applied to a sample (either switched on or off) eddy currents are induced inside the sample material. A magnetic field is induced by these eddy currents. Due to Lenz's rule the eddy currents flow in such a manner that a magnetic field is induced in the opposite direction compared to the direction of the applied field. Hence the applied field is not able to immediately penetrate into the whole of the sample. Rather the penetration is delayed due to the induced magnetic field. Due to the electrical resistivity of the sample the attenuation of eddy currents results in a decrease of the induced field within a short period of time. As a result the applied magnetic field *diffuses* into the material. It will be shown in chapter 2 that the time dependence of the magnetic field can be described by a diffusion equation [1].

Diffusion phenomena are generally represented by parabolic differential equations [18, 22, 28]. The diffusion of a magnetic field into a material is described here as an isothermal process[2] which implies $\nabla T = 0$. Here all effects which are related to the heating by eddy currents are neglected[3]. The material under investigation is assumed to be both isotropic and homogeneous. With these assumptions it will be shown below that the diffusion equation takes the form:

$$\mathbf{\Delta \vec{B}} - \mathbf{D}\frac{\partial}{\partial \mathbf{t}}\mathbf{\vec{B}} = 0 \tag{1.9}$$

Equation (1.9) is a linear parabolic equation which is characterised by a second derivative in space and a first derivative in time [18, 22]. D is the diffusion constant which will be derived explicitly later in terms of material constants. The diffusion constant D determines the speed of the diffusion process. In more general cases D can be a function of $\vec{\mathbf{B}}$ and x. Here however, D is a constant.

There are some important general properties of the diffusion processes which are worth mentioning. In general, diffusion implies the transport of a physical quantity e.g. heat or material. Without any sources as for example in electrodynamics where one has $div\vec{\mathbf{B}} = 0$, and in equilibrium the distribution of a physical quantity is homogenous. This state is reached in the limit $t \to \infty$:

$$\lim_{t\to\infty}\left(\frac{\partial}{\partial t}\vec{\mathbf{B}}(\vec{x},t)\right) = 0 \tag{1.10}$$

In this limit the diffusion equation reduces to the Laplace equation.

[2] a small heating up speed in the set-up

[3] low field strength of the applied field

In general diffusion problems are Initial Boundary Value Problems (IBVP). In order to solve such problems initial and boundary conditions are needed. In the next chapter the problem will be analysed in detail and a complete solution will be presented.

The value of the electrical resistivity ϱ of a material can be experimentally obtained by determining the penetration rate of the magnetic field [1, 2, 3]. In experiments it is realized by a pick up coil tightly wound around the sample [1]. No electrical contacts to the sample are required. Thus all experimental problems due to the use of contacts are eliminated. This contact-less method has been evaluated for various sample geometries. Some applications are described in [3]. Special applications include the measurement of ϱ for semiconductors [10] or the measurement of the thickness of a conducting shell [5]. In this report the contact-less measurement of electrical resistivity ϱ is discussed for samples with a cylindrical geometry.

Chapter 2

Theory

2.1 Mathematical Description

Within this section the mathematical background is presented which is needed for the understanding and experimental optimisation of the contact-less measurement of ϱ. A qualitative description was given in the last chapter of how an applied magnetic field diffuses into a material. The reason for the diffusion to occur is the non-equilibrium distribution of the magnetic field inside and outside of the sample. The diffusion equation is discussed and solved for the sample geometry (long cylinder) which is used in our experiments.
In general, the macroscopic electromagnetic properties of matter are described by using the following set of Maxwell equations[1]:

$$\begin{aligned} div\vec{B} &= 0 \\ curl\vec{E} + \dot{\vec{B}} &= 0 \\ div\vec{D} &= \rho \\ curl\vec{H} - \dot{\vec{D}} &= \vec{j} \\ \mu_0(\vec{H} + \vec{M}) &= \vec{B} \\ \epsilon_0\vec{E} + \vec{P} &= \vec{D} \end{aligned} \tag{2.1}$$

The theory is built on the assumption that the medium is linear:

$$\begin{aligned} \vec{B} &= \mu_0\mu_r\vec{H} \\ \vec{D} &= \epsilon_0\epsilon_r\vec{E} \end{aligned}$$

In the following derivation it is assumed that

1. $\vec{P} = 0$ and

[1]SI units are used here

2. $\dot{\vec{D}} \simeq 0$.

The first assumption states that the medium or the material is linear and that there is no electrical polarisation. The second assumption, $\dot{\vec{D}} = 0$, states that the displacement current density $\dot{\vec{D}}$ can be neglected (the inclusion of radiation corrections is not required). This is equivalent to $\dot{\rho} = 0$ where ρ is the charge density. This implies the use of low frequencies in the experimental set-up. The conductivity σ and the permeability μ_r are scalars which are independent of frequency within the frequency range covered in these experiments[2]. For σ the DC-conductivity is used. Using Ohm's law:

$$\vec{j} = \sigma \vec{E} \tag{2.2}$$

and combined with the vector-operator identity:

$$curl(curl) = grad(div) - \Delta \tag{2.3}$$

the equations in (2.1) can be written as:

$$curl \vec{H} = \vec{j} \tag{2.4}$$

$$\frac{1}{\mu_0 \mu_r} curl \vec{B} = \vec{j} \tag{2.5}$$

Applying the *curl* operator:

$$-\Delta \vec{B} = \mu_0 \mu_r \; curl(\vec{j}) \tag{2.6}$$

combined with (2.2)

$$-\Delta \vec{B} = \mu_0 \mu_r \sigma \; curl(\vec{E}) \tag{2.7}$$

Using $curl \vec{E} = -\dot{\vec{B}}$

$$\Delta \vec{B} = \mu_0 \mu_r \sigma \dot{\vec{B}} \tag{2.8}$$

Finally, using $\sigma = 1/\varrho$ one obtains:

$$\boxed{\Delta \vec{B} - \frac{\mu_0 \mu_r}{\varrho} \frac{\partial}{\partial t} \vec{B} = 0} \tag{2.9}$$

where ϱ is the electrical resistivity of the material and μ_r the permittivity. This is a **diffusion** equation for the magnetic field $\vec{B}$. It is a description of the diffusion of a magnetic field into the metallic sample as discussed above.
This is the basic equation for the contact-less measurement of the electrical resistivity. This model is applicable for macroscopic systems. Before the solution is presented for a given geometry, some important general properties of diffusion equations are discussed.

[2] $1\, kHz$

2.1.1 Diffusion

Diffusion and (simple) transport processes are described by a *parabolic* Partial Differential Equation (PDE), such as (2.9). Parabolic differential equations are characterised by a second derivative in space and a first derivative in time [28]. For example heat conduction processes are also described by parabolic PDE's. The heat and diffusion equations, which are treated in this report, are mathematically equivalent.
In the previous section a diffusion process was obtained for the $\vec{B}$ field (2.9),

$$\Delta\vec{\mathbf{B}} - \frac{\mu_0\mu_r}{\varrho}\frac{\partial}{\partial t}\vec{\mathbf{B}} = 0 \tag{2.10}$$

with the diffusion constant:

$$D = \frac{\mu_0\mu_r}{\varrho} \tag{2.11}$$

The magnetic field is diffusing into or out of a volume V (e.g. a sample), as a result of the law of induction. The speed of diffusion is expressed by the diffusion constant D, which is directly proportional to the electrical resistivity:

$$D \sim \frac{1}{\varrho} \tag{2.12}$$

Thus the electrical resistivity determines the time constant of diffusion of magnetic flux. There are two important and significant properties of parabolic PDE's which will be needed for its solution: the **Maximum-Minimum Principle** and the **Steady-State-solution**.

The Maximum-Minimum Principle

When applied to a diffusion of a magnetic field into a metallic sample this principle results in the following statement

> The maximum or minimum of the magnetic field inside the sample occurs either initially or at the boundary of the sample.

The general proof of this statement can be found in [18, 22].

Characterisation of the Steady State

The steady state case, as discussed above, is reached in the limit $t \to \infty$. It is characterised by:

$$\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{2.13}$$

the diffusion equation reduces to the *steady state* form:

$$\frac{1}{D}\boldsymbol{\Delta}\mathbf{B} = 0 \tag{2.14}$$

This is the Laplace equation. Consequently, in the absence of sources (in electrodynamics $div\vec{\mathbf{B}} = 0$), the steady state magnetic field distribution in a homogeneous, isotropic medium is governed by the Laplace equation [18, 22, 28].

$$\Delta\mathbf{B} = 0 \tag{2.15}$$

Summary

The diffusion process of the magnetic field was given in (2.9) with the result that the applied magnetic field penetrates into a material with a time constant which depends on the electrical resistivity ϱ. This behaviour is attributed to eddy currents. The decay time of eddy currents contains the information about the attenuation. Therefore a measure of the time constant provides direct information about ϱ, the electrical resistivity of the sample.
In the next section the complete analytical solution is presented for an infinite cylinder. $\diamond$

2.2 Solution for an infinite Cylinder

The diffusion process of the magnetic field will be discussed for an infinite cylinder. For the experiment cylindrical samples are made using a flop cast method. This technique is standardly used within the Department of Physics at Loughborough University. For the calculation, a cylinder with an infinitive length is chosen because this eliminates end effects. An infinitely long cylinder (denoted by Ω) with radius R is given by the parametrisation:

$$\Omega := (r) \times (\phi) \times (z) = (0, R) \times (-\pi, \pi) \times (-\infty, \infty) \tag{2.16}$$

Expressed in Cartesian coordinates:

$$\begin{aligned} x &= r\cos(\phi) \\ y &= r\sin(\phi) \\ z &= z \end{aligned} \tag{2.17}$$

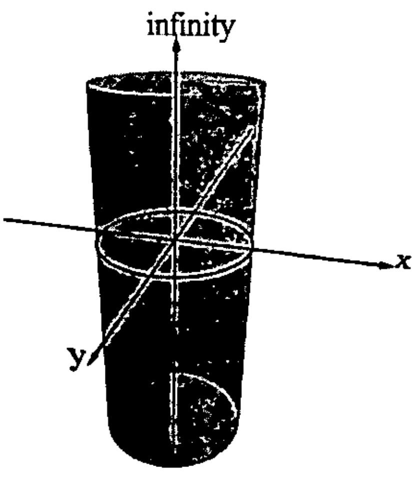


Figure 2.1: *Infinite cylinder*

As an experimental configuration the applied $\vec{\mathbf{B}}$-field is chosen parallel to the cylinder axis (z-axis). Thereby the experimental set-up has rotational symmetry around the z-axis and translational symmetry along the z-axis. The diffusion equation (2.9) is rewritten in cylindrical coordinates as:

$$\Delta\vec{\mathbf{B}}(\mathrm{r}, \phi, \mathrm{z}, \mathrm{t}) - \frac{\mu_0\mu_\mathrm{r}}{\varrho}\frac{\partial}{\partial \mathrm{t}}\vec{\mathbf{B}}(\mathrm{r}, \phi, \mathrm{z}, \mathrm{t}) = 0 \tag{2.18}$$

The assumption that the material is homogenous (the $\vec{B}$ field does not depend on a translation along the z-axis) implies that:

$$\frac{\partial}{\partial \mathbf{z}}\vec{\mathbf{B}}(\mathbf{r}, \phi, \mathbf{z}, \mathbf{t}) = 0 \tag{2.19}$$

The assumption of an isotropic material requires:

$$\frac{\partial}{\partial \phi}\vec{\mathbf{B}}(\mathbf{r}, \phi, \mathbf{z}, \mathbf{t}) = 0 \tag{2.20}$$

The vector field $\vec{B}$ has a unique direction oriented parallel to the z-axis. Therefore it is possible to reduce the vector equation to a scalar one.

$$\Delta B(r,t) - \frac{\mu_0 \mu_r}{\varrho}\frac{\partial}{\partial t}B(r,t) = 0 \tag{2.21}$$

This is a diffusion equation of a magnetic field in one dimension. The problem will be solved for the application of a magnetic field B_0 at $t = t_0 = 0$. Which is taken as the initial condition for the IBVP [3]. The time dependence of the field is shown in figure 2.2.

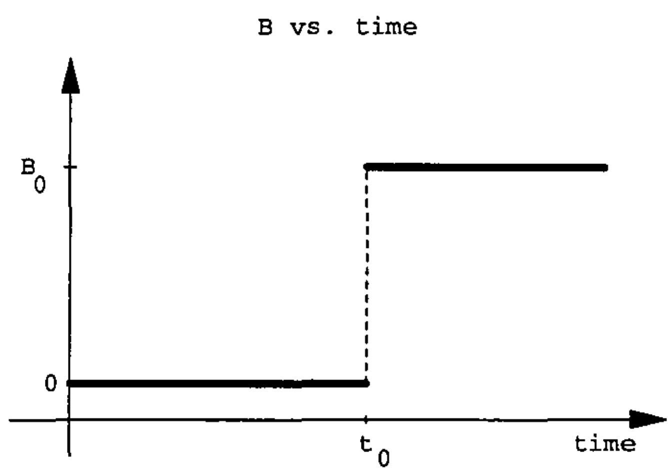


Figure 2.2: *Applied field B_0 as a function of t*

The various stages can be described as:

[3]Initial Boundary Value Problem

1. $t < t_0$ zero magnetic field in the cylinder Ω
2. at $t = t_0$ a field B_0 is applied, diffusion has not yet start
3. $t > t_0$ diffusion of magnetic field, with $\dot{B} \sim \frac{1}{\varrho}$
4. $t \to \infty$ a steady state is obtained

First the steady state case at $t \to \infty$ will be studied. It has been shown above that due to $div\vec{B} = 0$ and for $t \to \infty$ the solution is homogeneous. The condition of continuity of the parallel component of the $\vec{\mathrm{H}}$-field at the cylinder surface requires

$$B(r, t \to \infty) = \begin{cases} \mu_r B_0 & \text{for } r < R \\ B_0 & \text{for } r \geq R \end{cases} \tag{2.22}$$

The applied field strength is denoted by B_0. This solution is obtained by a straightforward application of electromagnetic boundary conditions.
The general time dependent structure of the solution is given by:

$$B(r, t) = \begin{cases} \mu_r B_0 - b(r, t) & \text{for } r < R \\ B_0 & \text{for } r \geq R \end{cases} \tag{2.23}$$

The time dependence of the magnetic field is governed by $b(r, t)$. The diffusion i.e. the decay of eddy currents is described by $b(r, t)$. The field $b(r, t)$ is located in the sample cylinder and it is created by eddy currents. These currents are induced due to the applied magnetic field at $t = 0$. The field $b(r, t)$ will vanish for $t \to \infty$. The time dependence of $b(r, t)$ yields direct information about the resistivity ϱ of the sample. A measurement of this field is the experimental aim of the contact-less measurement.
Substituting (2.23) into (2.21) the diffusion equation for the interior of the cylinder is:

$$-\Delta \mathrm{b}(\mathrm{r}, \mathrm{t}) + \frac{\mu_0 \mu_r}{\varrho} \frac{\partial}{\partial \mathrm{t}} \mathrm{b}(\mathrm{r}, \mathrm{t}) = 0 \tag{2.24}$$

With the conditions:

$$\begin{array}{lll} 1.) & b(R, t) = 0 & \text{boundary at the radius} \\ 2.) & |b(0, t)| < \infty & \text{condition for non singular solutions} \\ 3.) & b(r, 0) = \mu_r B_0 & \text{initial condition} \end{array} \tag{2.25}$$

The justification of these boundary and initial conditions is given due to the following: It was shown that $B(r = R, t) = B_0$ at the boundary, hence

$$B(R, t) = B_0 - b(R, t) = B_0 \tag{2.26}$$

The second condition ensures that the solution is smooth and analytic[4]. The third statement, i.e. the initial condition, results from the induced field at $t = 0$:

$$B(r, 0) = 0 \tag{2.27}$$

2.2.1 Separation of Variables

For PDE's there does not exist a complete theory as for Ordinary Differential Equations (ODE's). There are some standard methods available for reducing PDE's to ODE's, namely integral-transformations e.g. Hankel-, Laplace-, or Euler transformations [28] or the method of conformal mappings [28]. One method of finding solutions of partial differential equations is by *Separation of Variables.* For the solution of the diffusion problem only the interior domain is of interest. The Laplace operator is transformed to cylindrical coordinates.

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \Longrightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \tag{2.28}$$

It reduces, due to symmetry, to:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{2.29}$$

Due to the independence of ϕ and z as assumed by the rotational and translational symmetry of the problem 2.24) is expressed as:

$$\frac{\partial^2}{\partial r^2}b(r,t) + \frac{1}{r}\frac{\partial}{\partial r}b(r,t) = \frac{\mu_0\mu_r}{\varrho}\frac{\partial}{\partial t}b(r,t) \tag{2.30}$$

with the conditions for $r = 0$, $r = R$ and $t = 0$:

$$\begin{aligned} &b(R,t) = 0 \\ &b(0,t) < \infty \\ &|b(r,0)| = B_0 \end{aligned} \tag{2.31}$$

Using the product ansatz [28]:

$$b(r,t) = R(r)T(t)$$

The separation of variables results in:

$$\frac{R''(r) + \frac{1}{r}R'(r)}{R(r)} = \frac{\mu_0\mu_r}{\varrho}\frac{T'(t)}{T(t)} \tag{2.32}$$

[4]linear PDE

A set of ordinary differential equations is obtained. These will be solved in the next subsection.

$$R''(r) + \frac{1}{r}R'(r) = -\eta R(r) \tag{2.33}$$

$$\frac{\mu_0 \mu_r}{\varrho}T'(t) = -\eta T(t) \tag{2.34}$$

Here η is a constant.

2.2.2 Solution for $T(t)$

The time dependent part of $b(r,t)$ has to be a solution of the following differential equation:

$$T'(t) + \frac{\eta\varrho}{\mu_0\mu_r}T(t) = 0 \tag{2.35}$$

As $b(r, t \to \infty) = 0$ one requires that $T(t \to \infty) = 0$. A solution is found by standard methods yielding:

$$T(t) = Ae^{-\frac{\eta\varrho}{\mu_0\mu_r}t} \tag{2.36}$$

The steady state solution is obtained in (2.36), for $t \to \infty$ and

$$\lim_{t\to\infty} b(r,t) = 0 \tag{2.37}$$

2.2.3 Solution for $R(r)$

The solution is obtained here for the radial part of $b(r,t)$ of the induced field. The domain in which (2.30) has to be solved forms a circle of radius R.

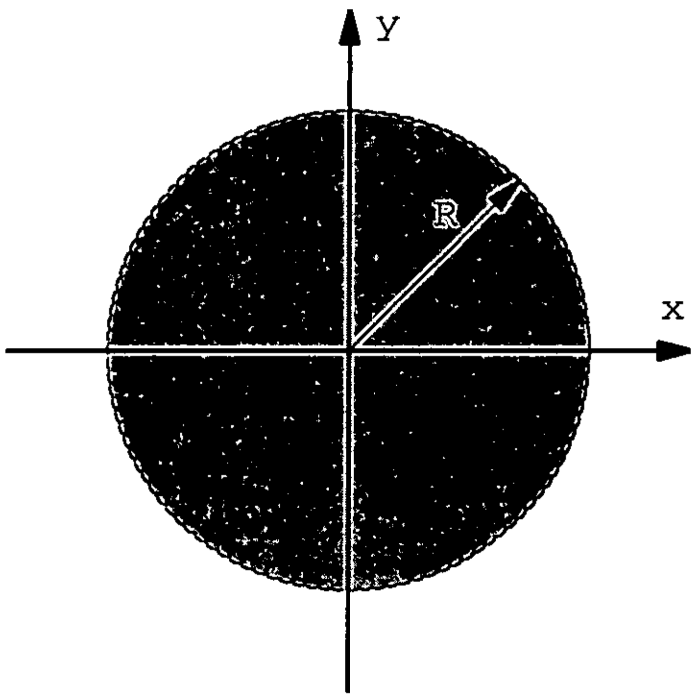


Figure 2.3: *circle of radius R*

The second order differential equation for the radial part $R(r)$ is given by:

$$\begin{aligned} R''(r) + \frac{1}{r}R'(r) + \eta R(r) &= 0 \\ r^2 R''(r) + rR'(r) + \eta r^2 R(r) &= 0 \end{aligned} \tag{2.38}$$

Solutions are required within the grey area indicated in figure 2.3 and subject to two conditions:

$$\begin{aligned} R(R,t) &= 0 \\ |R(0,t)| &< \infty \end{aligned} \tag{2.39}$$

The eigenvalue problem in (2.38) is similar to the Bessel-equation of 0th order. The equation will be transformed into a Bessel-equation. However, before presenting this transformation the Bessel-functions are discussed first.

Bessel functions

Bessel functions of order m correspond to the general solution of the Bessel equation of m-th order:

$$r^2 R''(r) + rR'(r) + (r^2 - m^2)R(r) = 0 \tag{2.40}$$

The general solution has the form :

$$R(r) = c_1 J_m(r) + c_2 Y_m(r) \tag{2.41}$$

The Bessel function can be presented in different ways[5], either integral or power series presentations. For J_m the power series expansion is given as:

$$J_m(z) = \frac{1}{\Gamma(\frac{1}{2})}\left(\frac{z}{2}\right)^m \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!} z^{2n} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+m+1)} \tag{2.42}$$

Here Γ is the gamma-function as defined by:

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma(n+\frac{1}{2}) = \frac{(2n)!}{2^{2n}n!}\Gamma(\frac{1}{2})$$

resulting in:

$$J_m(z) = \left(\frac{z}{2}\right)^m \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{1}{\Gamma(n+m+1)} \left(\frac{z}{2}\right)^{2n} \tag{2.43}$$

If m is not an integer then none of the coefficients $1/\Gamma(n+m+1)$ vanishes. However, if m is an integer, then:

$$\Gamma(n+1) = n!$$

$$\frac{1}{\Gamma(n+m+1)} = \begin{cases} 0 & \text{for } n+m+1 \leq 0 \\ \frac{1}{(n+m)!} & \text{for } n+m+1 > 0 \end{cases} \tag{2.44}$$

For the case of $m = 0$ in (2.43) the series expansion is simply:

$$J_0(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{1}{\Gamma(n+1)} \left(\frac{z}{2}\right)^{2n} \tag{2.45}$$

The first few terms have the explicit form:

$$1 - \frac{z^2}{4} + \frac{z^4}{64} - \frac{z^6}{2304} + \frac{z^8}{147456} - \frac{z^{10}}{14745600} + \ \ldots$$

[5]Courant Hilbert, Methods of Mathematical Physics, Volume I and II

The solution of a Bessel-equation of 0-th order:

$$r^2R''(r) + rR'(r) + r^2R(r) = 0 \tag{2.46}$$

is given by:

$$R(r) = c_1J_0(r) + c_2Y_0(r) \tag{2.47}$$

The coefficients are determined by the boundary conditions. The functions $J_0(r)$ and $Y_0(r)$ are shown in figure 2.4.

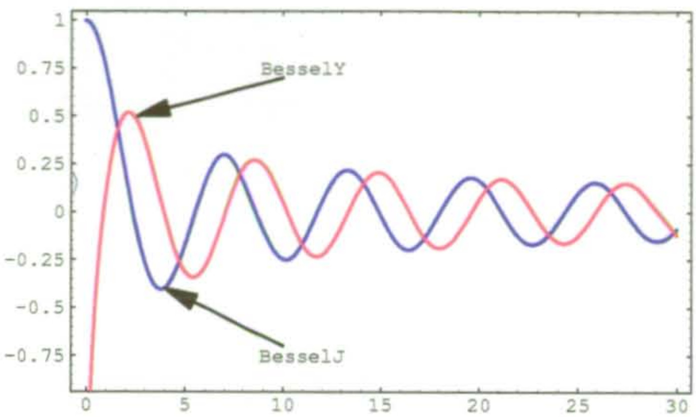


Figure 2.4: *Bessel functions of* 0-*th order*

As $r \to 0$ the limiting behaviour of $J_0(r)$ is:

$$\lim_{r\to 0} J_0(r) = 1 \tag{2.48}$$

while for $Y_0(r)$:

$$\lim_{r\to 0} Y_0(r) = -\infty \tag{2.49}$$

The zeros x_n of the Bessel-function are important for finding the solution of the boundary problem. If at the boundary the solution has to be zero then

$$J_0\left(\frac{r}{R}x_n\right)$$

is a solution which satisfies the boundary condition at $r = R$:

$$J_0\left(\frac{R}{R}x_n\right) = 0$$

The zeros can be found with a computer algebra system e.g *Mathematica*[6]. Some zeros of $J_0(r)$ are:

$$
\begin{aligned}
x_0 &= 2.404825558 \\
x_1 &= 5.520078110 \\
x_2 &= 8.653727913 \\
x_{299} &= 941.6925307 \\
x_{4560} &= 14328.0187 \\
x_{782313} &= 2457711.1298
\end{aligned}
\tag{2.50}
$$

In general the Bessel functions are of great importance to applied mathematics, because every boundary value problem for a circle, or any problem which can be transformed to a circle, is governed by Bessel functions.

Radial solution

After the brief overview of the properties of the Bessel function the radial diffusion problem of the magnetic field into an infinity long cylinder can be solved analytically. It is described by:

$$r^2R''(r) + rR'(r) + \eta r^2 R(r) = 0 \tag{2.51}$$

This eigenvalue equation can be transformed into a Bessel function of 0-th order. Setting:

$$\rho = r\sqrt{\eta} \tag{2.52}$$

and using:

$$\frac{d}{dr} = \frac{d\rho}{dr}\frac{d}{d\rho} = \sqrt{\eta}\frac{d}{d\rho} \tag{2.53}$$

and

$$\frac{d^2}{dr^2} = \eta\frac{d^2}{d\rho^2} \tag{2.54}$$

Substituting (2.53) and (2.54) into (2.51) and combined with

$$r = \frac{\rho}{\sqrt{\eta}}$$

[6]for the first hundred zeros with a precision of 32 digits only a few seconds were needed

one obtains:

$$\rho^2 R''(\rho) + \rho R'(\rho) + \rho^2 R(\rho) = 0 \tag{2.55}$$

with:

$$\begin{array}{lll} 1.) & R(R\sqrt{\eta}, t) = 0 & \text{boundary at the radius} \\ 2.) & |R(0,t)| < \infty & \text{condition for none singular solutions} \\ 3.) & R(\rho, 0) = \mu_r B_0 & \text{initial condition} \end{array} \tag{2.56}$$

This is the Bessel equation of 0-th order, as discussed above. The general solution is:

$$R(\rho) = a_1 J_0(\rho) + a_2 Y_0(\rho) \tag{2.57}$$

with

$$a_2 = 0$$

because $Y_0(0) \to -\infty$. Due to the boundary condition it is found that:

$$J_0(R\sqrt{\eta}) = 0$$

so it follows that $R\sqrt{\eta}$ must be equal to a zero x_n of the Bessel-function:

$$R\sqrt{\eta} = x_n \quad \Rightarrow \quad \eta = \frac{x_n^2}{R^2}$$

Thus

$$R_n(\rho) = R_n(r\sqrt{\eta}) = a_n J_0\left(\frac{r}{R}x_n\right)$$

$R(r)$ can be represented as a series:

$$R(r) = \sum_{n=0}^{\infty} R_n = \sum_{n=0}^{\infty} a_n J_0\left(\frac{r}{R}x_n\right)$$

The coefficients a_n are determined by the geometry as demonstrated in the next section.

2.2.4 The Complete Solution $R(r)T(t)$

After the separation of time and radial variables two solutions were obtained for every n:

Radial part:

$$R_n(r) = a_n J_0\left(\frac{r}{R}x_n\right)$$

Time part:

$$T_n(t) = Ae^{-\frac{x_n^2 \varrho}{R^2 \mu_0 \mu_r}t} \tag{2.58}$$

Combining the two yields the general solution of the diffusion equation:

$$b(r,t) = \sum_{n=0}^{\infty} R_n T_n = \sum_{n=0}^{\infty} a_n J_0\left(\frac{r}{R}x_n\right) e^{-\frac{x_n^2 \varrho}{R^2 \mu_0 \mu_r}t} \tag{2.59}$$

With the initial condition

$$b(r,0) = \mu_r B_0 \tag{2.60}$$

it is possible to calculate every coefficient in (2.59).

$$\mu_r B_0 = \sum_{n=0}^{\infty} a_n J_0\left(\frac{r}{R}x_n\right) \tag{2.61}$$

According the theory of Fourier series the coefficients are given by:

$$a_n = \frac{2}{R^2 J_1^2(x_n)} \int_0^R \mu_r B_0 J_0\left(\frac{r}{R}x_n\right) r\, dr \tag{2.62}$$

The ortho-normalisation is given with:

$$\int_0^R J_0\left(\frac{r}{R}x_n\right)^2 r\, dr = \frac{R^2}{2}\left(J_0'(x_n)\right)^2 \tag{2.63}$$

and

$$\int_0^R J_0\left(\frac{r}{R}x_n\right) J_0\left(\frac{r}{R}x_m\right) r\, dr = \frac{1}{\|R_n(r)\|}\delta_{n,m} \tag{2.64}$$

Thus the exact analytical solution of the diffusion problem for a magnetic field in an infinite cylinder with radius R was obtained as:

diffusion equation

$$\Delta \mathrm{B}(\mathrm{r},\mathrm{t}) - \frac{\mu_0 \mu_r}{\varrho} \frac{\partial}{\partial \mathrm{t}} \mathrm{B}(\mathrm{r},\mathrm{t}) = 0 \tag{2.65}$$

solution

$$B(r,t) = \begin{cases} \mu_r B_0 - b(r,t) & \text{for } r < R \\ B_0 & \text{for } r \geq R \end{cases} \tag{2.66}$$

and

$$b(r,t) = \sum_{n=0}^{\infty} a_n J_0\left(\frac{r}{R} x_n\right) e^{-\frac{x_n^2 \varrho}{R^2 \mu_0 \mu_r} t} \tag{2.67}$$

Discussion of the Solution

The solution of a diffusion problem of a magnetic field yields a Fourier-series solution with a spatial and a time-dependent part. The time dependent part describes the decay of eddy currents. This inner field $b(r,t)$ is damped by the resistivity of the material [1]. The induced currents are damped by the finite resistance of the sample. The decay constant is determined by ϱ and μ_r.

Limits

There are two limits. If the resistivity is infinite the magnetic field penetrates instantaneously into the material. If a superconducting sample is under investigation the eddy currents are persistent currents (Meisser-Ochsenfeld). Therefore the critical temperature can be estimated.
Between these two limits the resistivity determines the decay time of the diffusion of the magnetic field.

Chapter 3

Theoretical Studies

After solving the diffusion equation some numerical examples will be given in this section. Equation (2.67) is completely implemented in a Mathematica algorithm[1].These calculations are approximations because the Fourier series is infinite. However, the characteristic behaviour can be demonstrated very clearly with a finite summation of terms in equation (2.67)[2].
The dependence of ϱ is shown with different values. The magnetic field inside the sample is represented. First the behaviour is investigated at the limit $t = 0$. The influence of the electrical resistivity on the diffusion of a magnetic field is discussed for different cases.

For the next examples an infinite cylinder with a radius of $R = 3mm$ is used and the applied magnetic field is taken as $B_0 = 1mT$.

[1]The complete source code is available from the author
[2]magnitude $\sim 10 - max.\ 1000$

3.0.5 The B-field for $t = 0$

At first the system is studied for $t = 0$, and also for a zero magnetic field inside the sample. Every combination of ϱ and μ_r is identical for $t = 0$.

$$\left[e^{-\frac{x_n^2 \varrho}{R^2 \mu_0 \mu_r} t} \right]_{t=0} = 1 \tag{3.1}$$

The first example shows a copper cylinder at $t = 0$ in figure 3.1, the B-field is plotted along the x-axis.

$$B(r, 0) = \widehat{B}_0 - b(r, 0) = 0 \tag{3.2}$$

For the red curve the summation was extended to $n = 4$ while for the blue one the limit was set to $n = 80$. The red curve is given by:

$$b_{red}(r, 0) = \sum_{n=0}^{4} a_n J_0 \left(\frac{r}{R} x_n \right) \tag{3.3}$$

and the blue one by:

$$b_{blue}(r, 0) = \sum_{n=0}^{80} a_n J_0 \left(\frac{r}{R} x_n \right) \tag{3.4}$$

A typical property, the Gibbs phenomenon, of Fourier series is demonstrated in figure 3.1. The oscillations decrease with increasing summation index n. This phenomenon was first found by Gibbs[3].

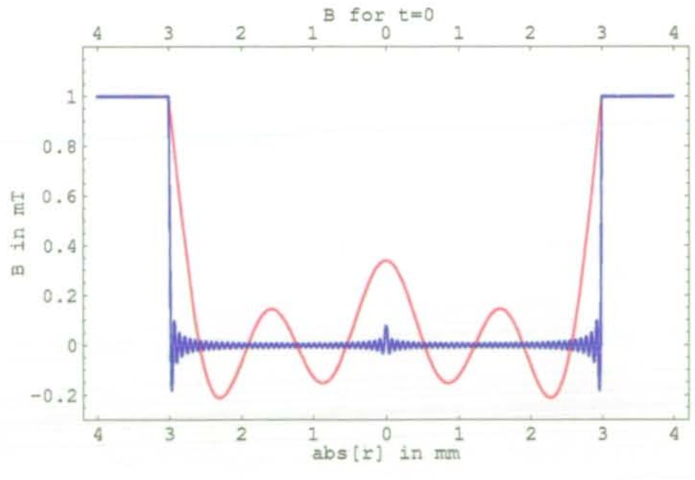


Figure 3.1: *The magnetic field*

The physical interpretation follows from the qualitative derivation as given in chapter 2. At $t = 0$ the inner field $b(r, t)$ is induced instantly, thus it has the

[3]Fourier's series, Nature Vol. 59 1898/90

same magnitude as the outer applied field. This results in a zero magnetic field inside the cylinder.

3.0.6 The B-field for $t > 0$

For $t > 0$ the magnetic field is shown in figure 3.2. In this case the diffusion has already started, which implies that the eddy currents are being attenuated due to the resistivity of the material. The parameters were chosen as:

ϱ	$1.67\ 10^{-8}\ \Omega m$
summation index n	30

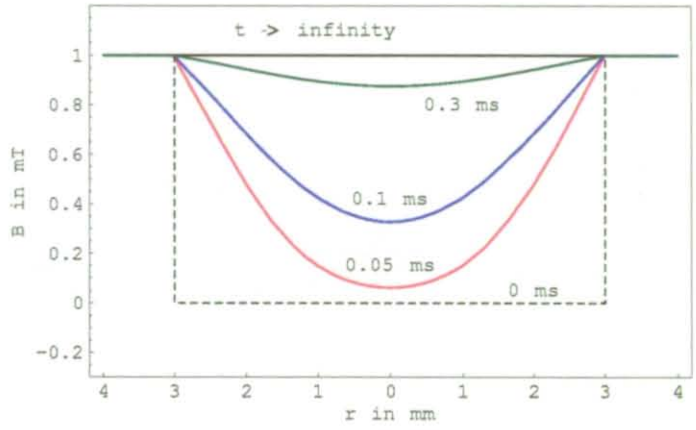


Figure 3.2: *The magnetic field for $t > 0$*

The time-dependence shows how the outer field penetrates into the cylinder.

3.0.7 Resistivity and Decay

The influence of the resistivity is investigated for three different values of the resistivity. The dependence of the induced field was given by (2.67)

$$B(r,t) \sim b(r,t) \tag{3.5}$$

and

$$b(r,t) \sim e^{-\frac{x_n^2 \varrho}{R^2 \mu_0 \mu_r} t} \tag{3.6}$$

For $\mu_r \sim 1$ the exponential decay of $b(r,t)$ is solely governed by the resistivity. Three examples are shown in figure 3.3. The parameters are:

μ_r	1
ϱ_{red}	$1\ 1.67\ 10^{-8}\ \Omega m$
ϱ_{blue}	$2\ 1.67\ 10^{-8}\ \Omega m$
ϱ_{green}	$4\ 1.67\ 10^{-8}\ \Omega m$
r	$1\ mm$
summation index n	100

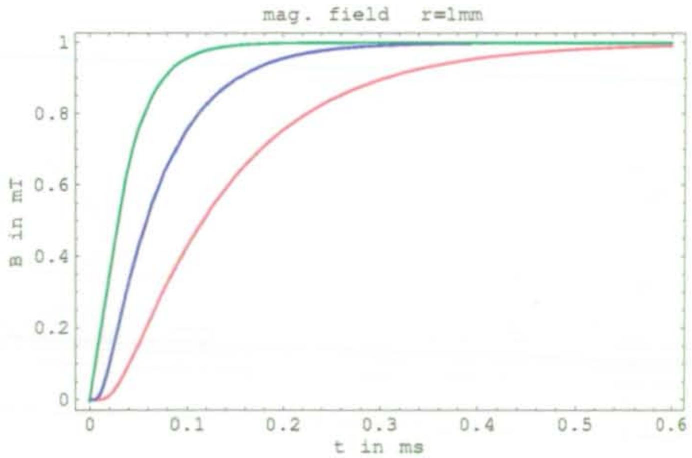


Figure 3.3: *Different resistivities*

Finally, the time and radius dependence of the magnetic field $B(r, t)$ for a Cu-cylinder in shown in figure 3.4.

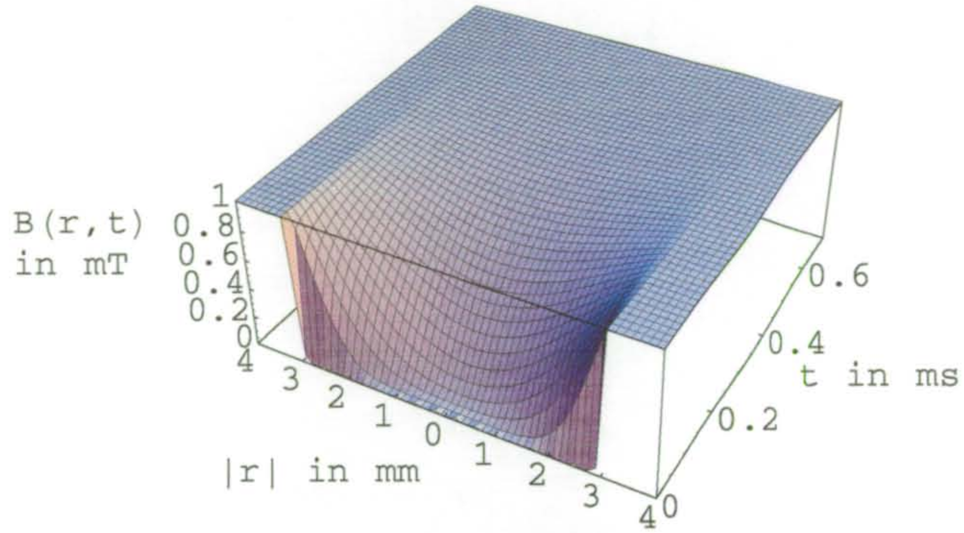


Figure 3.4: *Copper cylinder*

The next chapter describes in detail the experimental implementation and set-up.

Part II

Experiments and Results

Chapter 4

Set-up

4.1 Overview

The next chapter describes the experimental set-up for the contactless measurement of the electrical resistivity and discusses experiments which were carried out with the help of the apparatus.
In general the implementation of a theory or an idea requires a well prepared design. The contactless measurement of resistivity is based on an applied magnetic field and the notation of a change of flux inside the sample material. Therefore two basic coil systems are needed, one for the applied homogenous field and a second coil to measure the change of flux inside the sample. This pick up coil should have a very small influence on the whole system. The basic set-up is similar to the experiments carried out before [1]. In order to determine the temperature dependent resistivity ϱ of a material a temperature measurement is needed, as well as a means for cooling down or heating up. All these devices are discussed in this chapter.

A new advantage is added to the contactless resistivity measurement method. The pick up coil, which records the response of the sample, is an oscillating system. Thus a new data analysis is used for the signal processing. Due to this algorithm it is possible to simultaneously measure electric and magnetic properties of the sample under investigation.
The set-up is discussed first by scatching the design. Thereafter every part will be explained in detail.

4.1.1 Experimental Arrangement

At first the principle experimental set-up is shown.

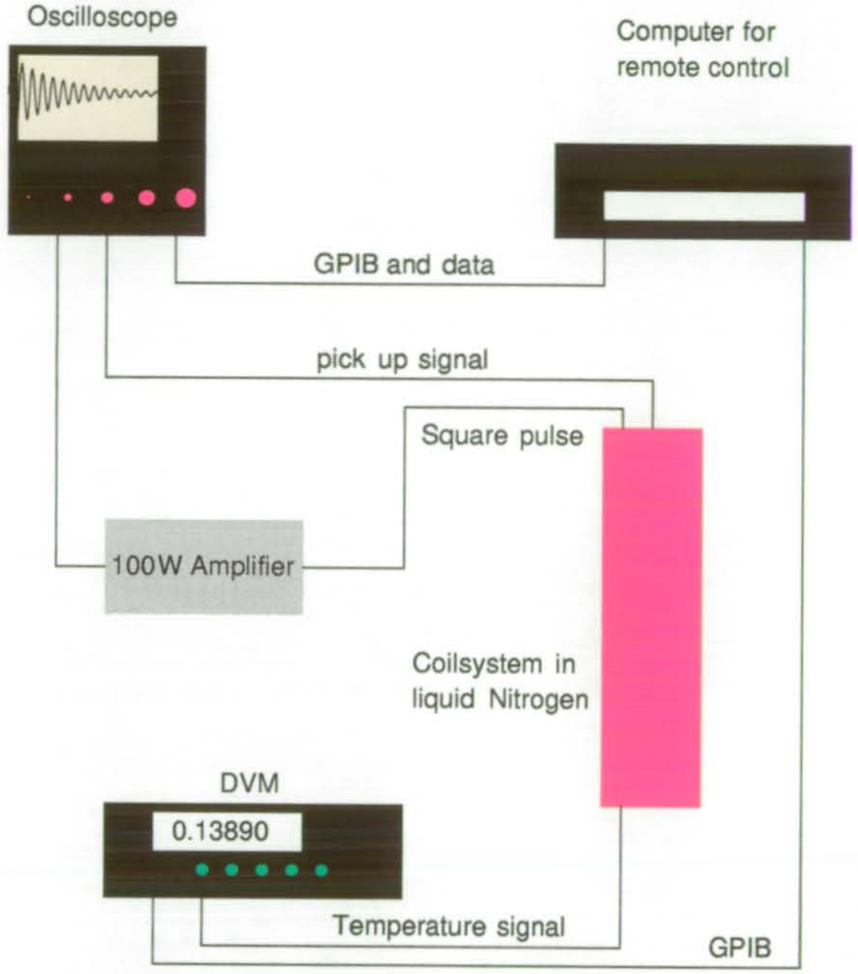


Figure 4.1: *The setup in diagram form*

The whole system can be divided into four main sections:

1. A coil system with driver and pick up coil
2. Temperature measurement
3. Data recording
4. Data analysis and software

Before the detailed set-up is discussed the basic method is explained.

4.1.2 The Method

As pointed out in the theory chapter an applied magnetic field diffuses into a material. This applied (outer) field is provided by a driver coil which is driven by a square pulse[1]. The change of the magnetic field inside the sample is measured with a pick up coil which is tightly wound around the sample. A magnetic field, $b(r,t)$ is induced inside the material. If $\dot{b}(r,t) \neq 0$ a voltage is induced in the pick-up coil, with a dependence given by:

$$U \sim -\dot{b}(r,t) \tag{4.1}$$

According to eq.(2.67) the long time behaviour is obtained as:

$$U \sim e^{-\frac{\varrho x_0^2}{R^2 \mu_0 \mu_r} t} \tag{4.2}$$

The time dependence of the signal of the pick up coil is determined by the time dependence of the penetrating magnetic field. The long time dependence is given by an exponential decay. For $\mu_r \sim 1$ the time constant of this decay immediately yields a value of ϱ.

[1] following the initial condition of the diffusion equation

Chapter 5

The experimental Set-up in Detail

As part of the project the experimental arrangements had to be designed, optimised and tested. The detailed description as given in this chapter contains the relevant information which is needed for the actual construction. The following description is based on a substantial amount of practical experience gained during the testing and construction period of the device.
The data recording and transfer is controlled by a computer and due to the design high resolution measurements against temperature are possible (ten measurements per Kelvin).
First the driving coil system is explained. Details of the pick up coil system are explained at the end of this chapter. The parameters set are given stating the range for the best experimental conditions.

5.1 The Coil System

The arrangement of the coil system is shown in figure 5.1. The coil holder of the driver coil is removed and the number of turns is reduced. The coil holder is constructed using a non-magnetic plastic[1] material. For the coil copper wire is used with a diameter of 0.5 mm.
Two pick up coils are used. This gives the possibility to compare two samples or to measure two samples in one measurement.

5.1.1 The Driver coil

The magnetic field of the driver coil is given by:

$$\widehat{B}_0 = 4\pi\mu\mu_r n I \tag{5.1}$$

[1] Darvic

$$n = \frac{N}{l} \tag{5.2}$$

Where N is the number of turns and l the length. Typical values are 1 - 10 mT The coil is driven by an amplifier with a square pulse signal. The signal for the amplifier is taken from the calibration signal of the oscilloscope which is also used for triggering.

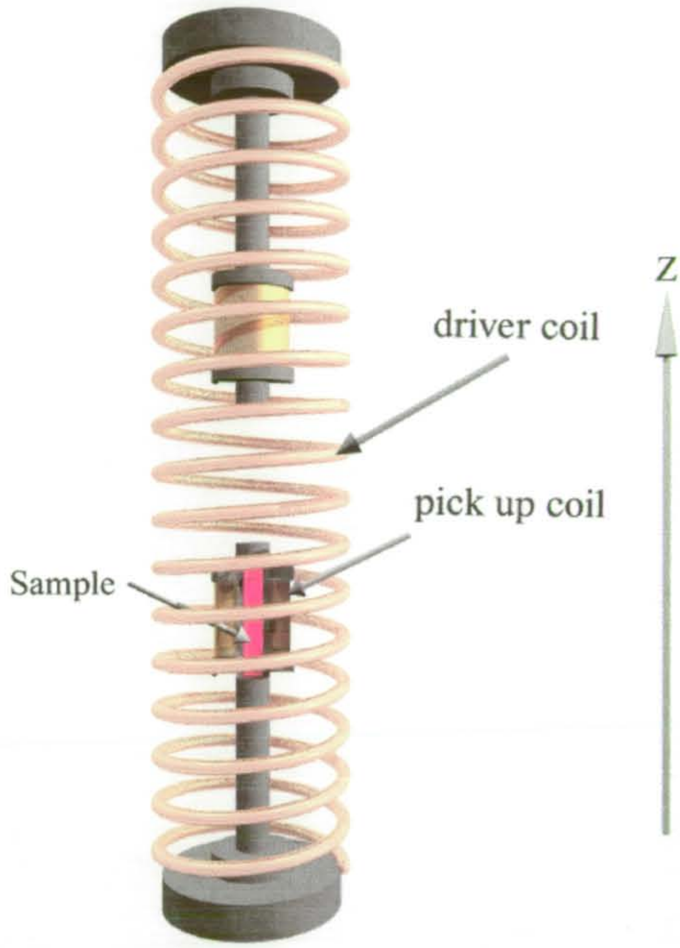


Figure 5.1: *The coil system*

Parameter for the driver coil

Number of turns	$N = 370$
Length	$l = 205\ mm$
Max. current	$I_{max} = 0,7A$

5.2 Temperature Measurement

For the temperature measurement a silicon diode is used in the four point configuration. This set-up compensates for the resistivity of the wires. The diode is driven with an applied voltage in forward direction using a constant current of $I_{diode} = 10\mu A$. For these conditions the functional dependence voltage of vs. temperature is plotted in figure 5.2. A fit of a third degree polynomial is used for both calibration and measurement.

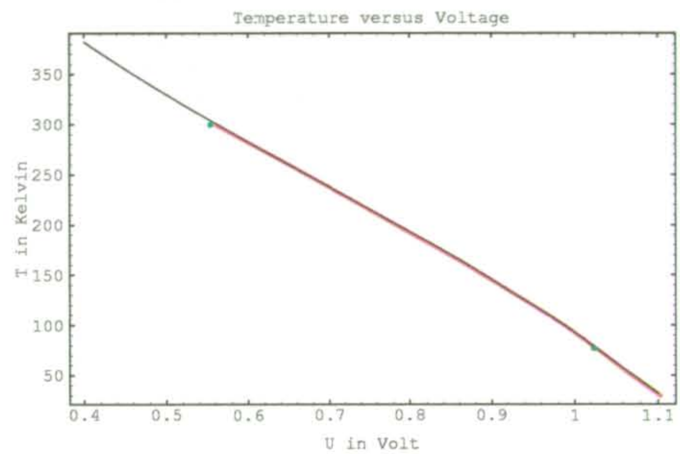


Figure 5.2: *Silicon diode*

The red points in figure 5.2 are the data and the black curve is the fit. The fit for temperature (in Kelvin) as a function of the voltage (in Volts) is given as:

$$T(U) = 697.63684 - 1087.9679U + 925.56713U^2 - 442.36497U^3 \tag{5.3}$$

The green dots in figure 5.2 are two fix points, one for 77.4K and the other one for the melting point of ice. The sensor is located at the top of the pick up coil close to the sample. The voltage is measured by a digital nanovoltmeter (DVM) which is controlled by the computer via a GPIB. Using (5.3) the control program (written in C) converts the voltage into absolute temperature following the fit of figure 5.2. The temperature can be measured five times per second. Finally the absolute temperature is determined to an accuracy of :

$$\Delta T = \pm 1K \tag{5.4}$$

Parameter of the temperature control

Method	silicon diode (forward bias)
I_{max}	$10\ \mu A$

5.2.1 The Heater

The change of temperature has to be an approximately adiabatic process which means that the system changes from an equilibrium state to another equilibrium state via a sequence of equilibrium states. In this experiment the temperature ranges from 77.4 K to room temperature. The coil system is covered by a copper tube with heating wires. There are three heating coils which are drawn yellow in figure 5.3. This copper tube is thermally insulated from a bigger copper tube which is closed airtight. For a good thermal insulation the whole set-up is put under vacuum. The long tubes on the top are for the electrical connections and for the connection to the vacuum pump.

Figure 5.3: *Heating system*

Parameters of Heating System

max. heating up speed	$1K\ min^{-1}$
heating voltage	$U = 20 - 40V$

5.2.2 Data Recording

An oscilloscope is used in this experiments to measure the signal of the pick-up coil and a digital voltmeter (DVM) to measure the voltage of the temperature sensor. Both devices are remotely controlled by a computer via the GPIB[2]-interface technique [24].

5.3 The Pick up Coil

The functioning of the pick up coil is illustrated using an equivalent circuit as shown in figure 5.4. Electrical connections and cables are characterised by a capacitance C, a resistance R of the wire and an inductance L of the empty coil system.
The configuration as shown in figure 5.4 is that of an oscillating circuit.

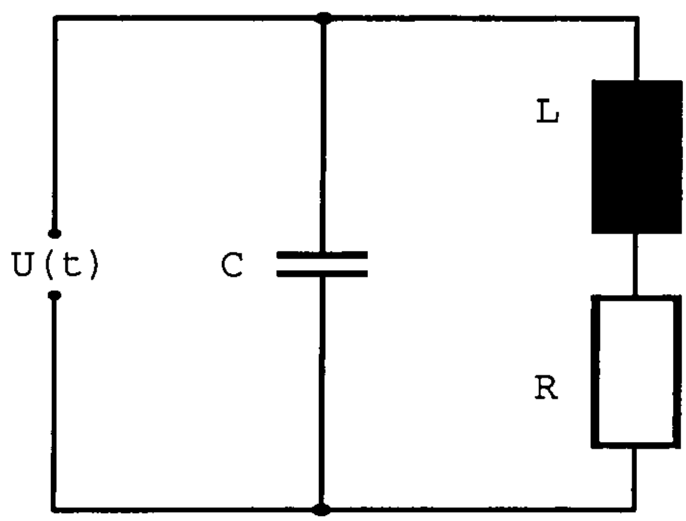


Figure 5.4: *The pick up coil system*

The voltage $U(t)$ is recorded by an oscilloscope. The input-resistance of this oscilloscope is around $1M\Omega$ and therefore its influence on the circuit can be neglected. The following derivation yields the time-dependent voltage $U(t)$.

[2]General Purpose Interface Bus

The system is explained by using as the variable the charge on the capacitance [20]. The differential equation 5.5 describing the circuit shown in figure 5.4.

$$L\ddot{Q} + R\dot{Q} + \frac{Q}{C} = 0 \tag{5.5}$$

With the well known substitutions for this kind of ODE,

$$2\beta = \frac{R}{L} \quad \text{Damping-constant} \tag{5.6}$$

$$\omega_0^2 = \frac{1}{LC} \quad \text{Eigenfrequency} \tag{5.7}$$

the differential equation takes the form:

$$\ddot{Q} + 2\beta\dot{Q} + \omega_0^2 Q = 0 \tag{5.8}$$

The initial conditions are defined by the experimental set-up. At $t = 0$ the magnetic field is applied by the driver coil. Therefore, in the pick up coil a voltage U_0 is induced. This voltage charges up the capacitance C via the resistance R ($\dot{Q} \neq 0$)
Hence:

$$Q(0) = 0 \tag{5.9}$$

and

$$\dot{Q}(0) = \frac{U_0}{R} \tag{5.10}$$

It will be solved using the ansatz:

$$Q(t) = Ce^{i\,\bar{\omega}t} \tag{5.11}$$

where $\bar{\omega}$ is a complex frequency. Substituting (5.11) into (5.8) one obtains:

$$\bar{\omega}^2 - 2\,i\,\beta\bar{\omega} - \omega_0^2 = 0 \tag{5.12}$$

This is solved by two complex frequencies:

$$\bar{\omega}_{1,2} = i\beta \pm \omega \tag{5.13}$$

where ω is given as:

$$\omega = \sqrt{\omega_0^2 - \beta^2} \tag{5.14}$$

The combination is given as:

$$Q(t) = c_1 e^{i\omega_1 t} + c_2 e^{i\omega_2 t} \tag{5.15}$$

Using the initial conditions one obtains:

$$Q(0) = c_1 + c_2 = 0 \leftrightarrow \quad c_1 = -c_2 \tag{5.16}$$

Hence:

$$Q(t) = c_1 e^{-\beta t} \left(e^{i\omega t} - e^{-i\omega t}\right) = c_1 2ie^{-\beta t} \sin(\omega t) \tag{5.17}$$

The second initial condition yields:

$$\dot{Q}(0) = 2i \; c_1 \omega = \frac{U_0}{R} \tag{5.18}$$

and c_1 is given as:

$$c_1 = \frac{U_0}{R\omega} \frac{1}{2i} \tag{5.19}$$

And finally the time dependent charge is obtained as:

$$Q(t) = e^{-\beta t} \frac{U_0}{R\omega} \sin(\omega t) \tag{5.20}$$

Using

$$U_C(t) = \frac{1}{C} Q(t) \tag{5.21}$$

the recorded voltage by the oscilloscope is given as:

$$U_C(t) = e^{-\beta t} \frac{U_0}{C \; R \, \omega} \sin(\omega t) \tag{5.22}$$

or as:

$$U_C(t) = e^{-\beta t} \frac{U_0 \omega_0^2}{2\beta\omega} \sin(\omega t) \tag{5.23}$$

The pure signal of the pick up coil is an exponentially damped oscillation as shown in figure 5.5,

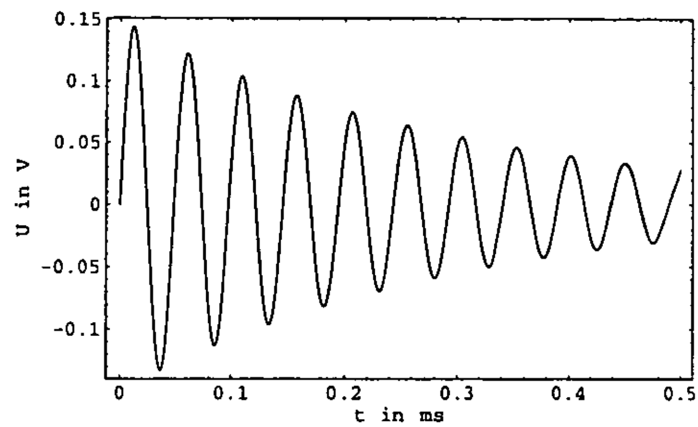


Figure 5.5: *Pick up signal*

In figure 5.5 the parameters were taken as $\beta = 3333s^{-1}$ and $\omega_0 = 129099s^{-1}$. The oscillations are characterised by an oscillation frequency ω and the decay time $1/\beta$. If a magnetic field is applied by the driver coil with a square pulse a voltage is induced in the pick up coil. Thus the pick up coil starts to oscillate as shown above with the real frequency ω:

$$\omega = \sqrt{\omega_0^2 - \beta^2} \tag{5.24}$$

For the empty coil the attenuation constant $1/\beta$ yields the value of the damping constant of the secondary coil.

Discussion

In the past all experiments [1] which used the contactless measurement method for the determination of the electrical resistivity were made for determining $1/\beta$ and for the case for which the secondary coil system was not driven at resonance. For non-magnetic materials with $\mu_r \sim 1$ the technique was successfully implemented [1] and analysed [2]. For magnetic sample materials, however, the analysis is more complicated. For long time the decay is now proportional to:

$$decay \quad \sim e^{-\frac{\varrho x_0^2}{\mu_r \mu_0 R^2} t} \tag{5.25}$$

If the secondary coil system is not driven as a harmonic oscillator the signal is a simple exponential decay [1] which depends on ϱ and μ_r. Therefore in earlier

measurements either μ_r was determined if ϱ is known or, alternatively, ϱ with a known value of μ_r.
The advantage of using the pick up coil as an oscillating system is the simultaneous determination of the resonant frequency

$$\omega_0 \sim \frac{1}{\sqrt{\mu_r}} \tag{5.26}$$

and the decay or damping constant β of the system. With (5.25) the resistivity can be extracted. The method for determining these two properties is discussed in the next section.

Spectral Analysis

In our experiments the oscillations are investigated using the discrete Fourier transform. Before experimental data are analysed the spectrum is investigated with the analytical Fourier transform which is given by:

$$S(\omega) = abs\left(\mathcal{F}_{cos}(U_C(t)) + i\mathcal{F}_{sin}(U_c(t))\right) \tag{5.27}$$

Where:

$$\mathcal{F}_{cos}(U_c(t)) = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \Theta(t)\, U_C(t) \cos(\omega t)\ dt \tag{5.28}$$

and

$$\mathcal{F}_{sin}(U_c(t)) = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \Theta(t)\, U_C(t) \sin(\omega t)\ dt \tag{5.29}$$

The result is plotted in figure 5.6 for different values of the damping constant β. A set of Lorenz curves is obtained.
The spectral function $S(\omega)$ is obtained as:

$$S(\omega) = \frac{1}{2\beta} \frac{U_0 \omega_0^2}{\sqrt{4\beta^2\omega^2 + (\omega^2 - \omega_0^2)^2}} \tag{5.30}$$

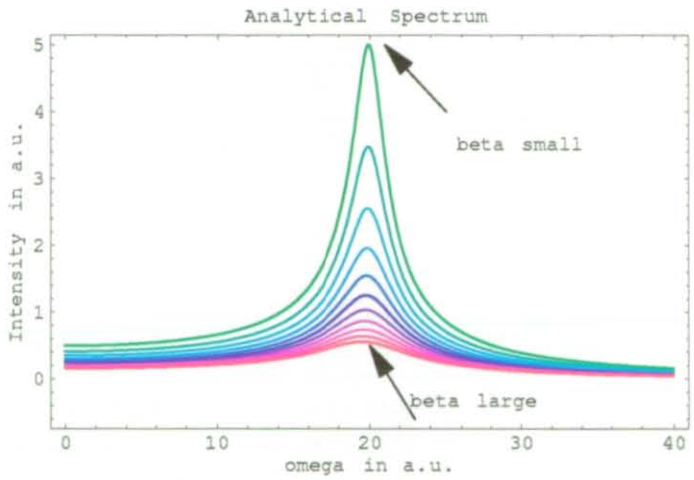


Figure 5.6: *Frequency-spectrum*

A larger damping constant implies an increased width, combined with a decreased intensity.

If a sample is inserted inside the pick up coil and a field is suddenly applied it starts to oscillate. Due to the resistivity of the sample material the damping inside the pick up coil is increased which results in a shorter decay time. There are two classes of materials which will be discussed separately.

5.3.1 Weakly Magnetic Samples

For weakly magnetic materials ($\mu_r \sim 1$). The resonant frequency ω_0 is similar to the frequency of the empty coil-system[3] without a sample. The induced eddy currents in the sample are damped due to the electrical resistivity of the material. This is expressed by an additional damping of the oscillations of the pick-up coil. It can be written mathematically as an exponential time factor as in eq.(5.23).

$$U_C(t) = \frac{U_0 \omega_0^2}{2\beta\omega} e^{-(\beta+d)t} sin(\tilde{\omega}t) \tag{5.31}$$

Where d is directly proportional to the resistivity ϱ and the new resonance frequency of $\tilde{\omega}$ is obtained as:

$$\tilde{\omega} = \sqrt{\omega_0^2 - (\beta + d)^2} \tag{5.32}$$

5.3.2 Magnetic Samples

If a magnetic material is fixed in the pick-up coil the situation is different compared to weakly magnetic materials. Due to the resistivity of the material the damping is also changed. The inductance is changed because:

$$L \sim \mu_r \tag{5.33}$$

Therefore the resonant frequency is different:

$$\omega_0 \longrightarrow \omega_0^{sample} \sim \frac{1}{\sqrt{\mu_r}} \tag{5.34}$$

Thus the pick up signal is different as compared to the signal of a non-magnetic sample:

$$U_C(t) = \frac{U_0 \omega_0^2}{2\beta\omega} e^{-(\beta+d)t} sin(\tilde{\omega}t) \tag{5.35}$$

The frequency of oscillation is given as:

$$\tilde{\omega} = \sqrt{(\omega_0^{Sample})^2 - (\beta + d)^2} \tag{5.36}$$

The separate determination of the damping constant as well as the 'new' resonance frequency allows for the simultaneous determination two values: Thus a measurement at the resonance point may yield a value of the electrical resistivity of the sample as well as a determination of the magnetic susceptibility χ or μ_r of the material under investigation. While the above discussion is restricted to a ferromagnetic material the technique is also applicable for paramagnetic or weakly magnetic samples.

[3] $\omega_0^2 > \beta^2$

Chapter 6

Data Analysis

The data analysis for the contactless measurement of resistivity is an important part in the process of obtaining proper results. The signal of the pick up coil is a damped oscillation with frequency $\tilde{\omega}$ and damping constant β. For analysing the signal the method of Fourier transform (FT) is commonly used. The advantage of this method is the possibility of simultaneously estimating the damping constant and the resonance frequency. This method is a very sensitive one, as demonstrated below. At first a short introduction and an example is given for this important method.
Consider a set of equidistant measurement values as presented by a vector $\vec{a}$. The discrete Fourier transform of a vector $\vec{a}$ into a vector $\vec{b}$

$$\vec{a} = (a_1, a_2, ..., a_n) \longrightarrow \vec{b} = (b_1, b_2, ..., b_n) \tag{6.1}$$

is given by:

$$b_s = \frac{1}{\sqrt{n}} \sum_{r=1}^{n} a_r e^{\frac{2\pi(r-1)(s-1)}{n}} \tag{6.2}$$

As a result of a Fourier transform of an oscillating signal measured at equidistant time intervals a power spectrum is obtained. The intensity distribution of the power spectrum yields the intensity of all frequencies which are contained within the signal. In the definition of the Fourier transform as given in (6.2) it is important to note that the frequency with the value zero appears at the index number one.

In the experiments the oscillations are recorded and stored by the oscilloscope. The driver program calculates the discrete FT. The analytical Fourier spectrum in (5.30) is used as a fit function for the resulting spectrum. To explain the data analysis method two calculated examples are given in the next section.

6.1 Signal Analysis - an Example

The effect of a discrete Fourier transform on a damped harmonic oscillation is investigated. The signal is modelled such that it is similar to the output of the oscilloscope which is used for the experiments. The oscilloscope has a resolution of 512 points. Therefore the discrete signal was chosen as a vector of 512 entries with two different values for the damping constant:

$$U_c(t)_{example} = U_c(t) + \; \textit{Random noise} \tag{6.3}$$

with t the time in steps of one in arbitrary units:

$$t = 0...511 \tag{6.4}$$

The noise is set to 3 % of the total intensity.

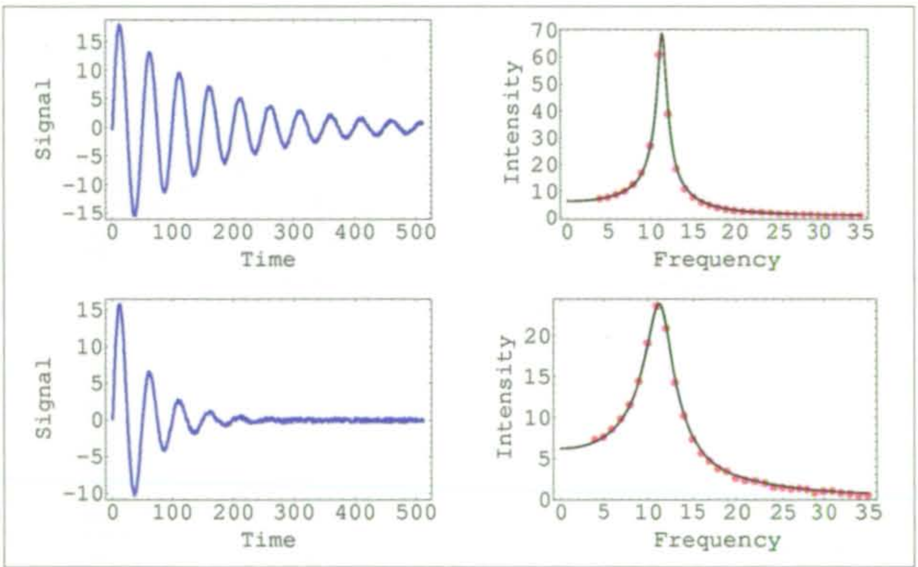
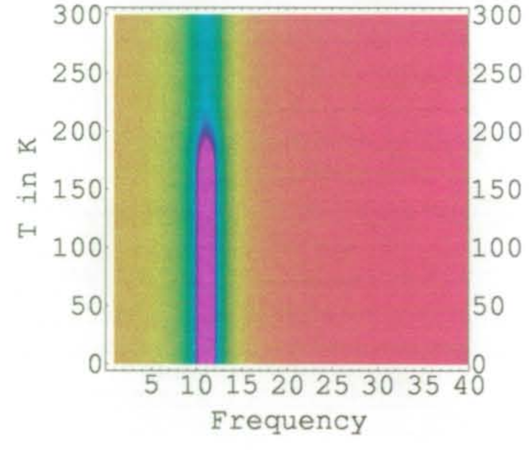


Figure 6.1: *Signal and Spectrum*

The signal at the top simulates a low resistivity sample while the signal at the bottom is typical of a high resistivity conductor. The magnetic field which diffuses into the material, encounters little resistance due to weak damping. Thus the FT spectrum is sharply peaked. The spectrum for the high resistivity sample is found to be substantially decreased in intensity.

6.1.1 Non-magnetic Samples

The data analysis for a non-magnetic sample is modelled here. It is assumed to be a conductive material with a change of resistivity at approximately $T_C = 200\ K$ in such a manner that the resistivity is decreasing around $200\ K$ to a constant value. The oscillations were generated with a noise of 3 % at every temperature. The corresponding Fourier spectrum is calculated for all temperatures and stored in an array. The result is shown in figure 6.2, the right hand side shows a density plot of the spectrum which is used for further data presentations.



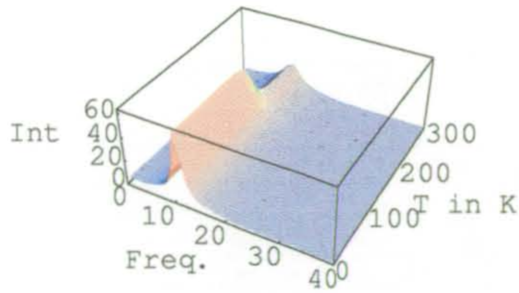


Figure 6.2: *The spectrum for a non-magnetic Sample*

Every spectrum is analysed using a non-linear fit algorithm [19, 27] employing:

$$S(\omega) = \frac{1}{2\beta} \frac{U_0 \omega_0^2}{\sqrt{4\beta^2\omega^2 + (\omega^2 - \omega_0^2)^2}} \tag{6.5}$$

With such a fitting procedure the value of β is obtained as a function of temperature.

The plot is shown in figure 6.3. Due to the high resistivity at temperatures above 200 K the spectrum is flatter. Therefore the error bars, as indicated by the scatter of data points of b, are larger.

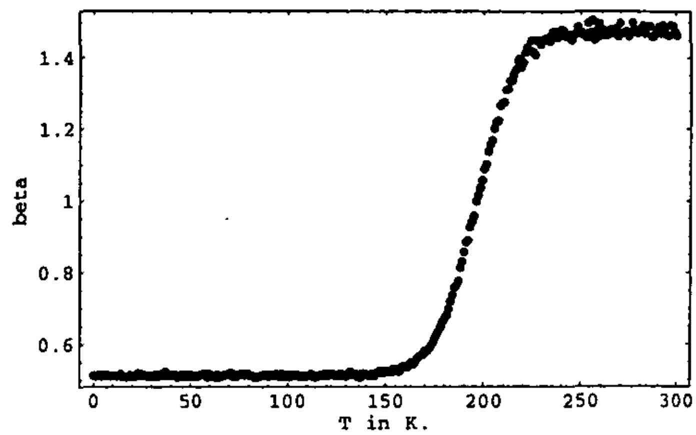


Figure 6.3: *A non-magnetic Sample*

6.1.2 Magnetic Samples

A similar resistivity behaviour discussed as before is modelled here but with an additional change of magnetic behaviour at the same temperature. The resulting spectrum (top left), the damping constant (bottom left), the change of inductance (which means μ_r, top left) and the resonant frequency (bottom right) are shown in the next graph:

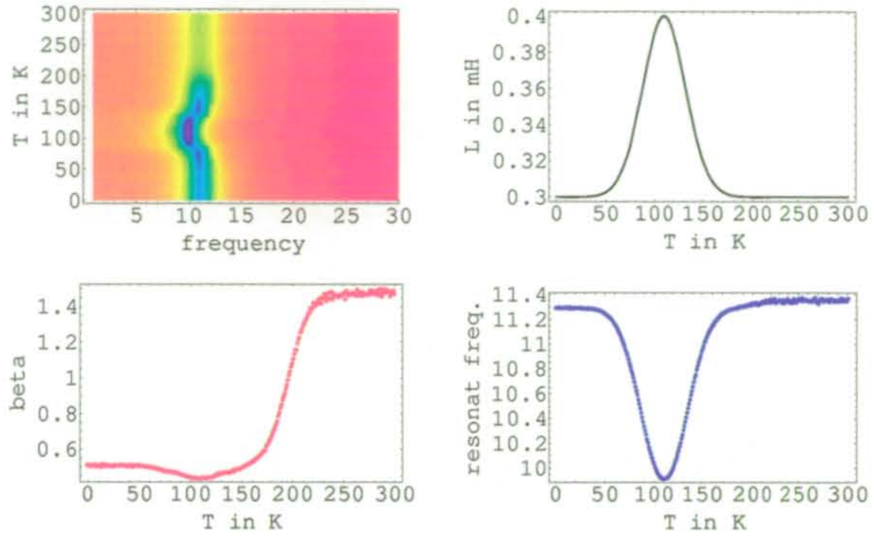


Figure 6.4: *Signal and Spectrum for magnetic Samples*

The Fourier analysis of the oscillations of the pick-up coil provides two pieces information: The width of the Lorentz peak which is used for the determination of the electrical resistivity ϱ of the material and the position of the peak which is additional information and which is related to μ_r of the sample material.

Chapter 7

Results

7.1 Non-Magnetic Samples

This section shows the experimental results for non-magnetic samples. The time step for the measurements of the oscilloscope is given by the time window of the oscilloscope divided by the number of points (512). This results in a time step $\sim 10^{-6}$ seconds per point. This value translates to a step of 10^4 Hz in ω-space. An estimate of the slope of the curve in figure 7.1 can be made by only using the first zero of the series. This results in a β-value (and assuming $\mu_r = 1$):

$$\beta \sim \frac{x_0^2}{R^2 \mu_0} \varrho \tag{7.1}$$

As non-magnetic samples Al, Cu and Zn were measured. The samples were investigated at room temperature. For every spectrum the damping constant was obtained by averaging 30 runs. The result is shown in figure 7.1.

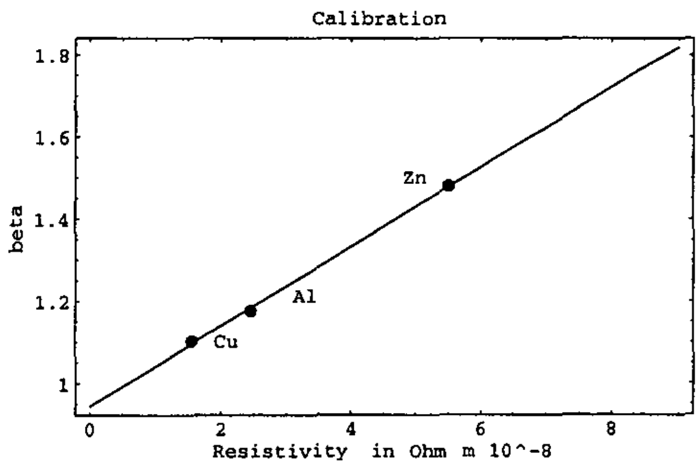


Figure 7.1: *Non-mag. samples Cu, Al and Zn*

The damping constant depends linearly on the electrical resistivity of the material. The values for the damping constant were measured as:

- $\beta_{Cu}^{osci} = 1.102$
- $\beta_{Al}^{osci} = 1.176$
- $\beta_{Zn}^{osci} = 1.481$

These values were measured in oscilloscope units[1]. After fitting one obtains:

$$\beta^{osci} = 0.944 + 0.0971 * \varrho[10^{-8}\Omega\ m] \tag{7.2}$$

applying the fit for copper yields:

$$\beta_{Cu}^{osci} = 0.944 + 0.0971 * 1.6 = 1.099 \tag{7.3}$$

The experimental value for Cu can be estimated (up to factor of order 1) using only the first zero of the Bessel function as ($x_0 \sim 2.404$, $R = 3\ mm$, $\mu_r \sim 1$ and $\mu_0 = 4\pi 10^{-7}$).

$$\beta_{Cu} \sim \frac{x_0^2}{R^2 \mu_0} \varrho_{Cu} = 5 * 10^{11} * 1.6 * 10^{-8} = 8 * 10^3\ s^{-1} \tag{7.4}$$

This value has to be multiplied with the unit of the Fourier transform (this amounts to a factor of $10^{-4}s$). This yields in oscilloscope units:

$$\beta_{Cu} \sim 0.8 \tag{7.5}$$

A small shift of β is obtained due to the damping of the empty pick up coil. The calibration procedure corrects for this shift.
The result of the measurement series for non-magnetic samples is used for the calibration of the system. This calibration allows the measurement of the electrical resistivity on an absolute scale.

$$\varrho[10^{-8}\Omega\ m] = -9.71 + 10.28 * \beta^{osci} \tag{7.6}$$

The resistivity which is plotted on the x-axis is the one obtained by using different experimental techniques [21].

[1]The values for β and ω_0 in oscilloscope units are obtained by using the fit function in (5.30)

Summary

The contactless measurement of electrical resistivity with an oscillating pick up coil system works reliably for non-magnetic samples. With the help of the discrete Fourier transform the data analysis of these oscillations yields a direct measurement of the electrical resistivity.
It is important to note that the obtained values for β and the resulting linear dependence for ϱ are suitable for metallic samples. As pointed above the basic mechanism is the induction of eddy currents inside the sample. If the material is an non-conducting material no eddy currents are induced at all. Therefore the oscillating system pick-up coil is not damped.

Limits

There are two important limits for conductive materials. The first limit is given by e.g. a superconducting material $\varrho = 0$ the eddy currents are persistent currents. Thus the damping due to the sample is equal to zero. The other limit is defined by a conductive material with a high resistivity, the maximum value for ϱ is derived below.
Using the pick-up coil as an oscillating circuit requires $\beta < \omega_0$, which means that the frequency $\omega = \sqrt{\omega_0^2 - \beta^2}$ is a real number. The oscillating frequency of the pick-up coil is obtained in a range of $20 - 25$ in oscilloscope units and therefore the measurable resistivity range is given by:

$$\varrho = 0....200\ 10^{-8}\Omega m \tag{7.7}$$

This range covers most of the metallic samples and alloys. A measurement above the maximum value is not possible. Due to the complex frequency the system does not oscillate and therefore the Fourier analysis as presented above can not be used in a meanigful manner.

7.2 Magnetic Samples

7.2.1 Ni_2MnGa

The first magnetic sample under investigation was the material Ni_2MnGa. It orders ferro-magnetically with a Curie temperature of $T_C = 376\ K$. A martensitic phase transition to a complex tetragonal structure occurs on cooling below 210 K. Previous neutron scattering measurements and magnetisation investigations have shown a change of properties around 210 K [15]. These data were used for comparison with results obtained by using the contactless apparatus measurement of the resistivity.

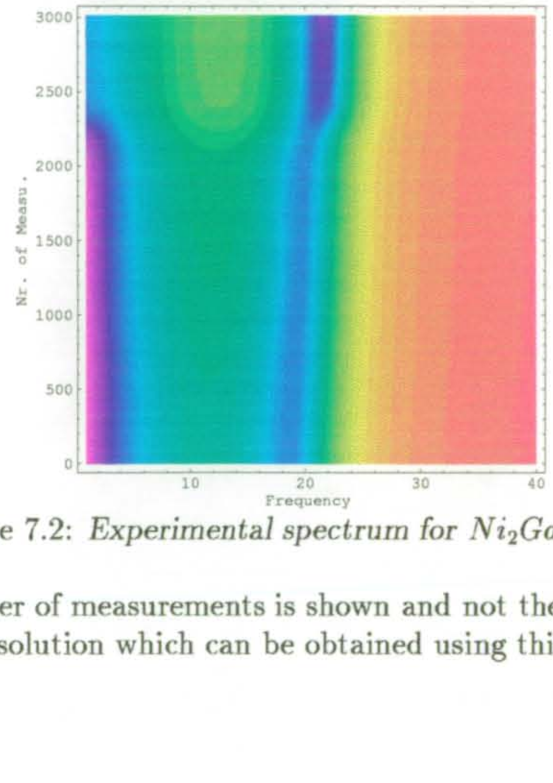


Figure 7.2: *Experimental spectrum for Ni_2GaMn*

On y-axis the number of measurements is shown and not the temperature. This demonstrates the resolution which can be obtained using this set-up.

The analysis of the spectrum and using the nonlinear fit algorithm yields the following results:
The electrical resistivity:

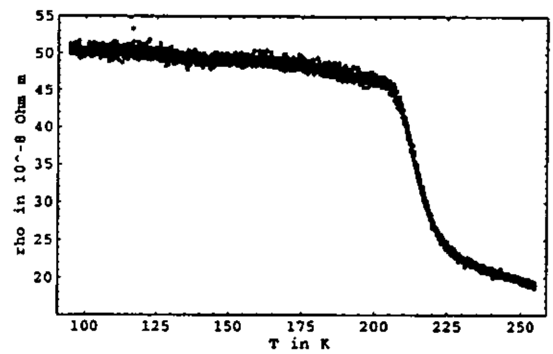


Figure 7.3: *ϱ and for Ni_2MnGa*

and the resonant frequency (in a.u.):

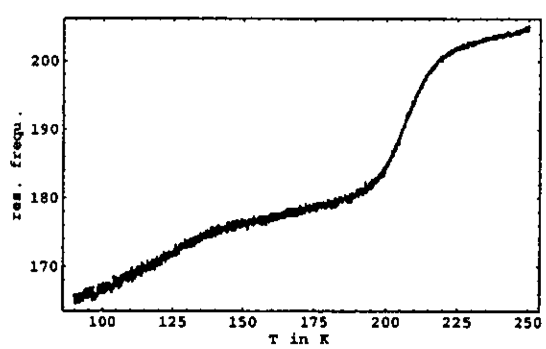


Figure 7.4: *ω_0 for Ni_2MnGa*

7.3 Conclusions and Outlook

As shown above the contactless measurement of the electrical resistivity is understood in theory and experiment.

The experimental arrangement as idealised in chapter 3 is able to capture the essential physics of the process. The time dependence of the pick up coil system is adequately described by (5.35). The analysis of the time dependence of the signal by discrete Fourier transform allows to separate the electronic and magnetic response of the system to the applied magnetic field. Irregularities in the production of the coil system will have a non-analytic influence on the data analysis. However, these effects are small. They may be evidenced in a weak coupling of β and ω_0. Such a coupling is not fully treated in the analysis as presented here. The effect of this coupling always shifts the damping constant by only a small amount even if μ_r changes substantially (figure 6.4).

Experimentally varying the length of the sample ($\pm 5\%$) did not change the values of the resistivity. This demonstrates that end effects due to the finite length of the sample stick are not important.

A number of applications are possible using the set-up which is described in this project. Powdered samples or bars of a different shape can be used to compare the results of different experimental methods. In the Department of Physics at Loughborough University rectangular sample sticks were investigated. These bars were originally made for a four point electrical resistivity set-up. Due to the increased sensitivity of the contactless resistivity method interest has been focused on a measurement series of these sample sticks. The result of a series of measurements has shown that relative changes of electrical resistivity are readily observable. However, due to the different size and shape of these samples the absolute values of the electrical resistivity were found to be too low as compared to the value expected for cylindrical samples. With a new calibration for this geometry it is also possible to investigate such samples on an absolute scale.

In conclusion the experimental idea was successfully implemented. This technique is now well established as a new standard measurement method within the Department of Physics.

Future developments may include the extension of this method to the investigation static magnetic field effects on the electrical resistivity. This can be achieved by a superposition of the time dependent field by a static magnetic field.

Part III

Appendix and Programs

Appendix A

The Source Code

```
/******************************** INFO *************************************
* This program is a C routine for the contactless measurement
* first some information about global variables
*   -------------------------------------------------------------
*   |   TYPE      |        NAME             |   USED IN OR WHAT    |
*   -------------------------------------------------------------
*   | double      |        xyarray[513][2}  |   InAndOut           |
*   | double      |        y[513]           |   FourierTransform   |
*   | double      |        T                |   the temperature    |
*   |             |                         |   in kelvin          |
*   | int         |        Tmin, Tmax       |   range of T         |
*   | int         |        scope, dvm       |   the two devices    |
*   | double      |        Tmin, Tmax       |   range of T         |
*   |-------------------------------------------------------------|
*
*    This program was written in the classical rule based style.
*    just a few pointers
*
*
*
*    regards         Frank Schippan
*                                          (schippan@physik.hu-berlin.de)
**************************************************************************/

#include <math.h>
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#include <string.h>
#include <conio.h>
#include <decl.h>
#include <complex.h>
#include <dos.h>


/* Name of the device TEKTRONIX 7854 OSZI as configured in CBCONF.EXE */
#define DEV_NAME1 "scope"
#define DEV_NAME2 "dvm"

/* Size of the ibrd buffer for the size of data there will be read*/
#define arraylength 513
```

```
/******************************* Greetings **********************************
*       Name      : Greetings
*       Arguments : none
*       returns   : two temperature values Tmax and Tmin
*       Warnings  : none
*       something about the program, it is not a manual !!
*
*****************************************************************************/
int Tmin, Tmax;
void
Greetings()
{
FILE *f1;
system("cls");
printf("------------------------------------------------------------------\n");
printf("---------------------- COOLING ALGORITHM -------------------------\n");
printf("This is a program for the contactless measurement of resistivity\n");
printf("The minimum temperature is about 77 Kelvin\n");
printf("Define the temperature range as whole numbers in the next step\n");
printf("Make sure that your old data are already saved, \n
otherwise stop it with CTRL+C\n");
printf("Press ENTER and go on ... \n");
getchar();
f1 = fopen("D:/Log/Sp.dat", "w+");
fclose(f1);
printf(" -- message: old data are removed -- !\ntoo late\n");
printf("\n\n\n");
printf("--------------------------------------------------------------------\n");
printf("Temperature chooser:\n");
printf("---- START VALUE IN KELVIN -----\n");
printf("The start temperature (Tmax) = ");
scanf("%d", &Tmax);


printf("-- Here is Tmax = %d Kelvin\n\n\n", Tmax);

printf("---- STOP VALUE IN KELVIN -----\n");
printf("The stop temperature (Tmin) = ");
scanf("%d", &Tmin);

printf("-- Here is Tmax = %d Kelvin\n\n\n", Tmin);

printf("The data will be recorded from %d K down to %d K.\n
If you are happy with that press ENTER and go on ... \n", Tmax, Tmin);
getchar();
getchar();

printf("program started ... \n");
printf("Have a cup of tea or coffee ...\n");


return;
}

/************************* CheckGPIBError **********************************
*       Name      : CheckGPIBError
*       Arguments : none
*       returns   : none
*       Warnings  : none
*       it looks for the GPIB error status byts in just in case a little
*       help
*
*****************************************************************************/
struct ValToName
   {
```

```
    int  value;
    char *name;
    };

struct ValToName StatBits[] =
    {
    ERR,  "ERR",      TIMO, "TIMO",     END,  "END",      SRQI, "SRQI",
    RQS,  "RQS",      CMPL, "CMPL",     LOK,  "LOK",      REM,  "REM",
    CIC,  "CIC",      ATN,  "ATN",      TACS, "TACS",     LACS, "LACS",
    DTAS, "DTAS",     DCAS, "DCAS",     -1,   NULL
    };

struct ValToName  ErrVals[] =
    {
    EDVR, "EDVR",     ECIC, "ECIC",     ENOL, "ENOL",     EADR, "EADR",
    EARG, "EARG",     ESAC, "ESAC",     EABO, "EABO",     ENEB, "ENEB",
    EOIP, "EOIP",     ECAP, "ECAP",     EFSO, "EFSO",     EBUS, "EBUS",
    ESTB, "ESTB",     ESRQ, "ESRQ",     -1,   NULL
    };


void CheckGPIBError(void)
{
if(ibsta & ERR)
{       printf(" -- ERROR --\n");
switch (iberr)
  {
    case EDVR:
printf("EDVR=DOS Error\n");
    case ECIC:
printf("ECIC=Board Error\n");
    case ENOL:
printf("ENOL=No listener\n");
    case EADR:
printf("EADR=Address Error\n");
    case EARG:
printf("EARG=Invalid argument was passed to library\n");
    case ESAC:
printf("ESAC=Board is not system controller\n");
    case EABO:
printf("EABO=I/O operation terminated\n");
    case ENEB:
printf("ENEB=No GPIB Board installed do that!!\n");
    case EOIP:
printf("EOIP=Background I/O already in
progress no multitasking!\n");
    case ECAP:
printf("ECAO=Board missing required capability\n");
    case EFSO:
printf("EFSO=File system error!\n");
    case EBUS:
printf("EBUS=Command error\n");
    case ESTB:
printf("ESTB=Status byte lost\n");
    case ESRQ:
printf("ESRQ=SQR line is stuck on\n");
  }
  if(iberr == EDVR)
printf("DOS Error Code=%d\n", iberr);
  if(ibsta & TIMO)
printf("GPIB device timed out sorry\n");
        }
   return;
}
```

```
/****************************** InitScope ******************************
*
*       Name        :       Init the TEKTRONIX 7854
*       Arguments :         DevName - must be defined with CBCONF.EXE
*       Returns     :       handle to open device
*       Warnings    :       none
*       What        :       Open the TEKTRONIX 7854. If there are a ghastly
*                           error in the opening process you get a message
*                           and an exit.
*                           If you are sucessfull the timeout will be set
*                           to 1000 seconds.
***********************************************************************/
int scope;
int
InitScope (char *DevName1)
{       scope = ibfind (DevName1);
if (scope < 0)
      {
printf("    IBFIND coudn't find the  %s\n ", DEV_NAME1);
exit(1);
}
ibtmo (scope, T1000s);
return (scope);}

/************************ TakeDvmMeasurement ****************************
*       Name        : TakeDvmMeasurement
*       Arguments : none
*       returns     : a float datatype (this is the voltage in Volts)
*       Warnings    : none
*       the DVM will be found and some commands will be sent
*       the result is a char datatype,
*       convert in a double float datatype:
*                       result will be written in a file as a char
*                       the file will be closed and open again
*                       and now the data can read as a float
*
*************************************************************************/

int dvm;
double TakeDvmMeasurement()
{
int dvm,i;
char rd[20];
int  rdint[15];
float voltage;
FILE *dvmfile;

if ((dvm = ibfind("dvm")) < 0 ){        printf("oh oh oh\n");
CheckGPIBError(); }

i=0;
ibtrg(dvm);
// for the command description look in the manual
ibwrt(dvm, "TON1H102Q1G",11);
ibrd(dvm, rd, 15);
// write ion a file close and open again
dvmfile = fopen("D:/Tmp/Tmp.dat", "r+");
while(i<=14) {
fprintf(dvmfile, "%c", rd[i]);

i++;}

fclose(dvmfile);
dvmfile = fopen("D:/Tmp/Tmp.dat", "r");
```

```
fscanf(dvmfile, "%E", &voltage);
voltage;
fclose(dvmfile);

return(voltage); }

/***************************** Temperature ***********************************
*       Name      : Temperature
*       Arguments : none
*       returns   : a float datatype this is the voltage in Volt
*       Warnings  : none
*
*       the float result of DVM Measurement is the argument for this
*       function. The voltage will be converted in Kelvin
*       For futher information and errors lock in a manual
*
*
*****************************************************************************/
double Temperature( double V)
{
double Temp;
Temp = 697.63684 - V*1087.9679 + V*V*925.56713 - V*V*V*442.36497 - 3.0 ;

return Temp;
}


/************************** Write Command **********************************
*       Name      : WriteCommand
*       Arguments : device - for the opened scope
*       returns   : --
*
*       send a command to the GPIB device all commands in the
*       language from the Scope (Tek...)
*       For information about the Osci. look in the users manual of the osci
*
*****************************************************************************/

void
WriteCommand (int device, char *cmd)
{
int cmdlength;
cmdlength = strlen (cmd);
/* ibwrt writes some commands in the device */
ibwrt (device, cmd, cmdlength);

return;
}



/************************* TakeScopeMeasurement *****************************
*       Name      : TakeScopeMeasurement
*       Arguments : device - for the opened device
*       returns   :
*       Warnings  : the file name like this "C:/data/ghastly.dat"
*                   do not use this "\"
*                   the command "cls" produces an error (see the red LED)
*                   but without this command it does not work !!
*       sends some commands to the TEKTRONIX 7854 and writes
*       the data in a file as a char. In this file is the whole curve
*       information. The file will be opened again for read the datas,
*       the scaling factors and so on.
*
*
```

```
*******************************************************************************/
void
TakeScopeMeasurement (int device, char *file)
       {

/* here some commands for the TEKTRONIX 7854 */
WriteCommand (device, "cls");
WriteCommand (device, "stored");
WriteCommand (device, "avg100");
WriteCommand (device, "sendx");

/* this lines write the X-register from the TEKTRONIX 7854*/
/* in a file in ASCII-format */
ibrdf (device, file);

}

/****************************** InAndOut ***********************************
*       Name      : InAndOut
*       Arguments : none
*       returns   : xyarray
*       Warnings  : none
*
*       The data were written in "C:/Tmp/Test.dat" this file will be opened
*       a pointer "goes" along the file. The pointer shows on some values.
*       This values will be read in the program and will be written in
*       some files, like Ydata.dat in /Tmp. Finally an array "xyarray"
*       is returned. The entries are {time, voltage}.
*       This subprogram is not very excellent, but it works !
*       If you do some corrections and write a better code........
*
*******************************************************************************/
double xyarray[arraylength][2];
double y[arraylength];
void
InAndOut()
 {
FILE  *datainf, *xinrcf, *yfactorf, *ydataf;

int   device, i, j;
float xinrc, yfactor, yi;

char  xinrcfile[]   = "D:/Tmp/Xinrc.dat"  ;
char  yfactorfile[] = "D:/Tmp/Y.dat";
char  ydatafile[]   = "D:/Tmp/Ydata.dat"  ;
char  c                                   ;

  // printf("InAndOut()\n");
/* open some files for read in and read out */
datainf  = fopen("D:/Tmp/Test.dat", "r+");
if (datainf == NULL){printf("the file test1.dat is not open..\n");}

xinrcf   = fopen(xinrcfile,   "w+");
if (datainf == NULL){printf("the file xinrc.dat is not open..\n");}

yfactorf = fopen(yfactorfile, "w+");
if (datainf == NULL){printf("the file y.dat is not open..\n");}

ydataf   = fopen(ydatafile,   "w+");
if (datainf == NULL){printf("the file ydata.dat is not open..\n");}


/* all files are open, now some values from the orignal file will be read
 * and some values will be written in some files                        */
```

```
 /* the xinrc will be read */

 fseek (datainf, 51, 0);
 i=0;
 while ((c = getc(datainf)) != EOF && c != ',')
       {
 fprintf (xinrcf, "%c", c); i++ ;
       };
        fprintf (xinrcf, "\nthis file contains the xinrc");

        i = ftell(xinrcf);
        fseek(xinrcf, -i, 1);
        fscanf  (xinrcf, "%E", &xinrc);

 /* the y scale factor (yfactor) will be read */
      i=fseek (datainf, 83, 0);

 i=0;
 while ((c = getc(datainf)) != EOF && c != ',')
       {
 fprintf (yfactorf, "%c", c); i++ ;
       };
        fprintf (yfactorf, "\nthis file contains the yfactor");

        i = ftell(yfactorf);
        fseek(yfactorf, -i, 1);
        fscanf  (yfactorf, "%E", &yfactor);


 /* read the datas*/
 fseek (datainf, 15, 1);
 while ( (c=getc(datainf)) != EOF)
  {    fseek (datainf, -1, 1);
       while (( c = getc (datainf)) != EOF && c != ',')
       {
 fprintf (ydataf, "%c", c);
       }
    fprintf (ydataf, "\n");

 }

 fseek(ydataf, -ftell(ydataf), 1);

 i=0;
 while (i <= 511)
  {
   fscanf (ydataf, "%E", &yi);
   xyarray[i][1] = xinrc * i;
   xyarray[i][2] = yi * yfactor;

   fseek (ydataf, 0, 1);
   i++;
  }

 i = 0;
 while(i<=511)
  {
   y[i] = xyarray[i][2] - xyarray[511][2];

   //printf("%E\t%E\n", xyarray[i][1], y[i]);
   i++;

  }

 /* close all streams*/
```

```
fclose(datainf );
fclose(xinrcf  );
fclose(yfactorf);
fclose(ydataf  );

return;

}
/*************************** ArraySort *************************************
* Name     : ArraySort
*       Arguments : None, the global array y[512] is used
*       returns   : None, the array is reordered
* Warnings  : Nothing known :-)
*
* This subprogram gives the y -array a ordering for the
* Fourier transfrom. Why? If the first y-values of the array are
* a constant (the reason is in the triggering) like a line
* then the FT esp. the Imaginary part is going wrong.
* This source code is not manual so look in the manual!
******************************************************************************/
double ft[512];
void
ArraySort()
{
int i, l;
//printf("ArraySort()\n");
i=2;
while(i<=100)
{
      if(       y[i] > y[i-1]&&
y[i] > y[i+1]&&
y[i] > y[i+2]&&
y[i] > y[i-2]
)
{ l = i;
i = 511; }
 i++;
}
      // printf("%d\n", l);

i = 0;
while(i<=511)
{ if(l<=511)
ft[i] = y[l];
else    ft[i] = 0.0;

//test
// printf("%E\t%E\n", ft[i], y[i]);
i++; l++;
}
/*  the first points (20) a were set to y[0]..y[19] == 0 */
return;
}
/****************************** FourierTransform ***************************
*       Name      : FourierTransform
*       Arguments : vector for the y-values
*       returns   : nothing
*       Warnings  : this is FT without a filter (aliasing!)
*
*       it is a discrete Fourier transform for the input vector invec[512]
*       the output is a vector b[512] where b[j] (j=0...511)
*
*
*                          511
*                          ---          2Pi I k j (1/n)
```

```
*       b[j]  = 1/sqrt(n)   \     a[k] Exp
*                           /--
*                           k=0
*
*      where are n = 512 the length
*      in this transformation every point of the curve is used for the
*      b[j]'s, the spectrum. this means 512 * 512 loops per curve.
*      The main (range of interest) part of the spectrum is at the
*      beginning. j = 2...70,
*      the peaks at the end 450...512 are the same as at the beginning.
*      This is an effect of alaising, skip it! It saves CPU time (expensive!)
*      For further information look in a book!!
*
******************************************************************************/
FourierTransform(double invec[512])
{
complex I(0,1), b[512];
FILE *spectrumfile;
double T, Volts, spectrum[100];
int i=0, k=0, n=511, m=40;

Volts     = TakeDvmMeasurement();
T         = Temperature(Volts);
spectrumfile  = fopen("D:/Log/Sp.dat", "a+");
fprintf(spectrumfile, "%E\n", T);

for(i=0; i < m; i++)
{
b[i]  = 0;
for(k=0; k < n; k++)
{
 b[i] += invec[k] * exp(2*3.1415927*k*i*I/n);

 }
b[i] /= sqrt(n);
spectrum[i]  = abs( b[i] );
fprintf(spectrumfile, "%E\n", spectrum[i]);
}

fclose(spectrumfile);
return 0;}

/*************************** END OF SUBPROGRAMS ******************************/
/*#########################################################################*/

/******************************** MAIN *************************************
*       here is the main routine. What's going on?
*       1.) it makes an array for temperature [min,........ max]
*       with a stepsize of your choice
*       then it looks for the temperature or volts
*       if the value in the array+-tolerance then the measurement will be run
*       and that's it !!!
*       every value will be written in a log file
*       see the manual for more usefull information
*
****************************************************************************/
double T;
main()
{
FILE *f1;
double Volts, deltat;
int i=1, scope, j;
time_t t1, t2;
```

```
Greetings();
t1  = time(NULL);
f1 = fopen("D:/Log/Tst.dat", "w+");
while(i<2)
{       Volts   = TakeDvmMeasurement();
T       = Temperature(Volts);

if(T < Tmax)
{
scope   = InitScope(DEV_NAME1);

ibclr(scope);
t2 = time(NULL);
deltat = (t2 - t1)/60.00;
TakeScopeMeasurement(scope, "D:/Tmp/Test.dat");
InAndOut();
ArraySort();
FourierTransform(ft);
printf("measurement at T = %E Kelvin\n", T);
printf("time = %E min\n", deltat);
fprintf(f1, "%E\t%E\n", deltat, T);}
else    printf("Waiting for cooling down .. T =%E K\n", T);

if(T < Tmin) i = 2;}

ibloc(scope);
fclose(f1);
printf("\n\nprogram finished\n");
printf("How was the tea/coffee?\n\n");


return i;}

/************************* HERE IS THE END !!!! **************************/
/****************************** REALLY *********************************/
```

Appendix B

The Analysis Program

The Fourier spectra are analysed using a program which is written *Mathematica*. Working with data graphical presentation are is convenient with this computer algebra system.
As worked out above the spectra are Lorentz curves:

$$S(\omega) = \frac{U_0\omega_0}{\sqrt{4\beta^2\omega^2 + \left(\omega^2 - \omega_0^2\right)^2}} \tag{B.1}$$

This is implemented in a *Mathematica* program. The data are written in one row by the measurement computer:

$$\begin{pmatrix} T_1 \\ a_1^1 \\ a_2^1 \\ \vdots \\ a_{40}^1 \\ T_2 \\ a_1^2 \\ \vdots \\ a_{40}^2 \end{pmatrix} \tag{B.2}$$

Where T_1 is the temperature for the first measurement and $a_1^1 ... a_{40}^1$ is the Fourier spectrum for this temperature. Due to the use of low frequencies the main peak is at the beginning of the spectrum. Therefore the whole spectrum is not needed.

B.1 The Code

```
<<Statistics`NonlinearFit`
DampingFitNew[ A_String] := Module[{ T, dat, fitdat, model, sol, sollist},

dat          = ReadList[A, Table[ Number, {41}]];
```

```
freqlist     = {};
dampinglist  = {};
intelist     = {};
fitdatlist   = {};
Do[
fitdat = Table[ {j, dat[[i]][[j]]}, {j, 5, 35}];
AppendTo[fitdatlist, fitdat];

model =U0 *x0/Sqrt[4 b^2 x^2 + (x^2-x0^2)^2];
sol =NonlinearRegress[ fitdat, model, x,
{{U0,11.2}, {b, 0.4},{x0, 10.0}}];

        T = dat[[i, 1]];
AppendTo[ freqlist,    {T ,sol[[1,2, 3, 2]]}];
AppendTo[ intelist,    {T, sol[[1,2, 1, 2]]}];
AppendTo[ dampinglist, {T,  sol[[1, 2, 2, 2]]}],
{i, 1, Length[ dat], 1 }];

Return[ {dampinglist, freqlist, fitdatlist}] ]
```

B.2 Code for Chapter 2

Within this section the code and a few exapmles for diffusion of magnetic field os given.

```
(* some functions *)

muCu   = 1 - 9.63 10^-6;
muAl   = 1 + 2.08 10^-5;
Bz[t_]:= 1;
innerBField[r_, R_, sigma_, mr_, time_, limit_] =  Module[ {xn, an, coeff, a},

xn = Read[ "/home/schippan/Work/Lboro/Contactless/Data/Zeros.dat"];
an = Read[ "/home/schippan/Work/Lboro/Contactless/Data/Coeff.dat"];
     Close[ "/home/schippan/Work/Lboro/Contactless/Data/Zeros.dat"];
     Close[ "/home/schippan/Work/Lboro/Contactless/Data/Coeff.dat"];

a = If[ Abs[r] < R,
mr Bz[time] - Sum[ an[[i]] BesselJ[0, xn[[i]] r/R]
 Bz[time]*Exp[ - xn[[i]]^2 /((R 0.001)^2 mr 4 Pi 10^-7 sigma) time] ,
{i, 1, limit}], Bz[time] ];
         a];

BField[r_, R_, sigma_, mr_, time_, limit_] =  Module[ {xn, an, coeff, a},

xn = Read[ "/home/schippan/Work/Lboro/Contactless/Data/Zeros.dat"];
an = Read[ "/home/schippan/Work/Lboro/Contactless/Data/Coeff.dat"];
     Close[ "/home/schippan/Work/Lboro/Contactless/Data/Zeros.dat"];
     Close[ "/home/schippan/Work/Lboro/Contactless/Data/Coeff.dat"];

a  = If[ Abs[r] < R,
 Sum[ an[[i]] BesselJ[0, xn[[i]] r/R] mr Bz[time] *
            Exp[ - xn[[i]]^2 /((R 0.001)^2 mr 4 Pi 10^-7 sigma)*time] ,
      {i, 1, limit}], Bz[time] ];

a];
 (*********************************)
time    = 0.0;
pic1 = Plot[ innerBField[r, 3, 5.99 10^7, muCu, time , 4] 1000,
```

```
{r, -4, 4},
PlotStyle->Red, DisplayFunction->Identity];

pic2 = Plot[ innerBField[r, 3,  5.99 10^-7, muCu, time, 80] 1000,
{r, -4, 4},
PlotStyle->Blue, PlotPoints->300,
                        DisplayFunction->Identity];


pic4 = Show[pic1, pic2,
PlotLabel-> "B for t=0 ",
FrameLabel->{"abs[r] in mm", "B in mT"},
FrameTicks->{Table[{i, Abs[i]}, {i, -4, 4, 1}], Automatic},
PlotRange->{All, {-0.3, 1.2}}, DisplayFunction->$DisplayFunction,
DefaultFont->{"Courier", 14}];
```

Bibliography

[1] ***Bean, DeBlois and Nesbitt***
Eddy-current method for measuring the resistivity of metals
J. Appl. Phys **30**, 1976 (1959) pp 1976-1979

[2] ***R.C.Callarotti and M.Alfonzo***
Measurement of the Conductivity of Metallic Cylinders by Means of an Inductive Method
J.Appl.Phys.,**43** 1972 pp 3040-3046

[3] ***J.A. Delaney and A.B. Pippard***
Electrode-less method for conductivity measurements in metals
Rep. Prog. Phys. **35** 1975 pp 677-715

[4] ***J.A.Garcia, S.Hull, S.Messoloras and R.J.Stewart***
Rapid and accurate electrical resistivity measurements of bulk and alloy samples
J. Phys E: Sci. Instrum. **21** 1988 pp 466-470

[5] ***H. Arque, R.C. Callarotti and P.Schmidt***
Theory of the measurement of thickness and conductivity of cylindrical shells by an inductive Method
J. Appl. Phys, **43** 1972 pp 3952-3958

[6] ***R.G. Chambers and J.G. Park***
Measurement of electrical resistivity by a mutual inductance method
British Journal of applied physics **12** 1961 pp 507-510

[7] ***R.C. Callarotti***
Measurement of conductivity by induction
J.Appl. Phys, **43** 1972 pp 3949-3951

[8] ***W. Wejgaard and V.S. Tomar***
Contact-less measurement of electrical resistivity by eddy current methods: discussion and a new technique

[9] ***J. LePage, A. Bernalte and D.A. Lindholm***
Analysis of resistivity measurements by the eddy currents method
Rev. Sci. Instrum. 39 1968 pp 1019-1026

[10] ***M.C. Chen***
Sensitive contact-less eddy current conductivity measurements on Si and HgCdTe
Rev. Sci. Instrum. 60 (6), 1989

[11] ***A.E. Drake and A.C. Lynch***
AC conductivity standards for the calibration of eddy-currents conductivity meters
Phys. E: Sci. Instrum, (20) 1987

[12] ***E.J. McRae, J.F. Mareche and A. Herold***
Contactless resistivity measurements a technique adapted to graphite intercalation compounds
J. Phys.E: Sci. Instrum. 13 1980 pp.241-243

[13] ***E.D. Wheeler, J.L. Boone and J.L. Drewniak***
A contactless method for measuring the bulk resistance of II-VI compound semiconductrs
Rev. Sci. Instrum. 65 (12) 1994 pp. 3844-3847

[14] ***T. Ogawa***
Measurement of the electrical resistivity and dielectric constant without electrodes
J. Appl. Phys. 32 1961 pp.583-587

[15] ***P.J. Webster, K.R.A. Ziebeck, S.L. Town and M.S. Peak***
Magnetic order and phase transform in Ni_2MnGa
Philo. Magazine B 1984 40 pp.295-310

[16] ***W.P.Smythe***
Static and Dynamic Electricity
McGraw-Hill Book Company

[17] ***Keith A. Ross and Hyung-Teak Kim***
A method of measuring the resistivity of copper from 298 to 77K
Rev. Sci. Instrum. 61 (12), 1990

[18] ***John L. Troutman***
Boundary Value Problems of Applied Mathematics
PWS Publishing Company

[19] ***Stephen Wolfram***
Mathematica a System for doing Mathematics by Computer
Addison-Wesley Publishing Company

[20] ***W. Nolting***
Theoretische Physik Band 3 Elektrodynamic
Verlag Zimmermann Neufang

[21] ***Ashcroft/Mermin***
Solid State Physics

[22] ***R.Kent Nagle / Edward B.Saff***
Fundamentals of Differential Equations
Addison-Wesley Publishing Company

[23] ***Leslie Lamport***
Das LaTeX- Handbuch
Addison-Wesley Publishing Company

[24] ***Computer Boards Inc.***
Software Manual for DRVR-PC2A (GPIB-Card)

[25] ***Textronics***
The User Manual for the 7854 Oscilloscope

[26] ***Borland***
Programmer's Guide for Borland C/C++ 3.1

[27] ***Willam H. Press, Saul A. Teulosky, Willam T. Vetterlind and Brian P. Flannery***
Numerical Recipes in C *The Art of Scientific Computing*
Cambrigde University Press

[28] ***Ferdinad Cap***
Wie l"oßt man Randwertprobleme in Physik und Technik
Walter de Gruyter

About this Report

The thesis was produced with the typesetting system LaTeX.
The source code contains

- 1498 strings
- 21245 string characters
- 69741 words
- 3855 multi-letter control sequences
- 12791 words of font info for 42 fonts

The major part of all figures and graphics was produced with the help *Mathematica 3.0* and imported as POSTSCRIPT. The coil system, the heater figure and the infinite cylinder were rendered with the ray-tracing program PovRay 3.0. The colour output was printed on a EPSON Stylus Color .

List of Figures

Index