

This item was submitted to Loughborough University as a PhD thesis by the author and is made available in the Institutional Repository (<u>https://dspace.lboro.ac.uk/</u>) under the following Creative Commons Licence conditions.

COMMONS DEED
Attribution-NonCommercial-NoDerivs 2.5
You are free:
<ul> <li>to copy, distribute, display, and perform the work</li> </ul>
Under the following conditions:
<b>Attribution</b> . You must attribute the work in the manner specified by the author or licensor.
Noncommercial. You may not use this work for commercial purposes.
No Derivative Works. You may not alter, transform, or build upon this work.
<ul> <li>For any reuse or distribution, you must make clear to others the license terms of this work</li> </ul>
<ul> <li>Any of these conditions can be waived if you get permission from the copyright holder.</li> </ul>
Your fair use and other rights are in no way affected by the above.
This is a human-readable summary of the Legal Code (the full license).
Disclaimer 🖵

For the full text of this licence, please go to: <u>http://creativecommons.org/licenses/by-nc-nd/2.5/</u>

### Mathematical Optimization Techniques for Demand Management in Smart Grids

by

### Ziming Zhu

A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy (PhD), at Loughborough University.

May 2014

Advanced Signal Processing Group, School of Electronic, Electrical and Systems Engineering, Loughborough University, Leicestershire, LE11 3TU, UK.

© by Ziming Zhu, 2014

For the love of Jue

#### Abstract

The electricity supply industry has been facing significant challenges in terms of meeting the projected demand for energy, environmental issues, security, reliability and integration of renewable energy. Currently, most of the power grids are based on many decades old vertical hierarchical infrastructures where the electric power flows in one direction from the power generators to the consumer side and the grid monitoring information is handled only at the operation side. It is generally believed that a fundamental evolution in electric power generation and supply system is required to make the grids more reliable, secure and efficient. This is generally recognised as the development of smart grids.

Demand management is the key to the operational efficiency and reliability of smart grids. Facilitated by the two-way information flow and various optimization mechanisms, operators benefit from real time dynamic load monitoring and control while consumers benefit from optimised use of energy.

In this thesis, various mathematical optimization techniques and game theoretic frameworks have been proposed for demand management in order to achieve efficient home energy consumption scheduling and optimal electric vehicle (EV) charging. A consumption scheduling technique is proposed to minimise the peak consumption load. The proposed technique is able to schedule the optimal operation time for appliances according to the power consumption patterns of the individual appliances. A game theoretic consumption optimization framework is proposed to manage the scheduling of appliances of multiple residential consumers in a decentralised manner, with the aim of achieving minimum cost of energy for consumers. The optimization incorporates integration of locally generated and stored renewable energy in order to minimise dependency on conventional energy. In addition to the appliance scheduling, a mean field game theoretic optimization framework is proposed for electric vehicles to manage their charging. In particular, the optimization considers a charging station where a large number of EVs are charged simultaneously during a flexible period of time. The proposed technique provides the EVs an optimal charging strategy in order to minimise the cost of charging. The performances of all these new proposed techniques have been demonstrated using Matlab based simulation studies.

### Contents

1	INT	rod	UCTION	1
	1.1	The E	volution of Smart Grids	1
	1.2	Dema	nd Management in Smart Grids	5
	1.3	Thesis	5 Outline	6
<b>2</b>	MA	THEN	IATICAL OPTIMIZATION AND GAME THEO-	
	$\mathbf{RE}'$	TIC M	ODELING TECHNIQUES	9
	2.1	Mathe	ematical Optimization	9
		2.1.1	Formulation of an optimization problem	10
		2.1.2	Optimal solutions	11
		2.1.3	Linear programming	12
		2.1.4	Quadratic programming	13
		2.1.5	Integer programming	14
	2.2 Game Theoretic Modeling Techniques		17	
		2.2.1	Basis of game theory	17
		2.2.2	Nash equilibrium: the solution concept	21
		2.2.3	Variations of Nash equilibrium	24
		2.2.4	Existence, uniqueness and efficiency of Nash equilibrium	26
		2.2.5	Obtaining Nash equilibrium	30
		2.2.6	Game theoretic optimization for communications and	
			smart grids	30

	GR	GRIDS AND ENABLING WIRELESS COMMUNICATION			
	TECHNOLOGIES				
	3.1 Introduction			35	
	3.2	Overv	iew of Demand Management in Smart Grids	36	
		3.2.1	Features	36	
		3.2.2	Challenges	38	
	3.3 Wireless infrastructure		ess infrastructure	39	
		3.3.1	Neighbourhood Area Networks	41	
		3.3.2	Home Area Networks	42	
	3.4	Devel	opments of Demand Management Approaches and Pro-		
	posed Mechanisms			44	
		3.4.1	Demand side consumption optimization	44	
		3.4.2	Dynamic pricing	46	
		3.4.3	Distributed energy resource (DER) management	48	
	3.5	Summ	lary	49	
<b>4</b>	4 A MIXED INTEGER LINEAR PROGRAMMING BASED			D	
	CO	NSUM	IPTION SCHEDULING TECHNIQUE	50	
	4.1	Introd	luction	50	
	4.2	MILP	Based Home Consumption Scheduling Optimization	53	
		4.2.1	Classification of appliances	53	
		4.2.2	The optimization formulation	55	
	4.3	Simul	ation Results	59	
		4.3.1	Simulation parameters	59	
		4.3.2	Results and discussion	61	

**3 OVERVIEW OF DEMAND MANAGEMENT IN SMART** 

### 5 A GAME THEORETIC OPTIMIZATION FRAMEWORK INCORPORATING LOCALLY GENERATED ENERGY 67

4.4 Summary

5.1	Introduction		67
5.2	The MIP Scheduling Optimization Framework		
	5.2.1	The optimization objective	70
	5.2.2	Constraints formulation	72
	5.2.3	The local scheduling optimization problem	73
5.3	Game	Theoretic Scheduling Approach	74
	5.3.1	Game components	74
	5.3.2	Equilibrium solution	76
	5.3.3	Potential game	77
	5.3.4	Game procedure and iterative convergence	78
	5.3.5	Efficiency, complexity and privacy	80
	5.3.6	Behaviour of players and their participation in the game	82
5.4	Simulations and Performance Evaluation		
	5.4.1	System set up	83
	5.4.2	Performance evaluation	84
5.5	Summ	ary	89

### **6** A MEAN FIELD GAME THEORETIC APPROACH FOR

ELECTRIC VEHICLES CHARGING 9			90
6.	1 Int	Introduction	
6.5	2 The	Game Theoretic Optimization Framework	93
	6.2	Optimization costs	94
	6.2	2 Dynamic EV charging process	96
	6.2	K-person game theoretic formulation	98
	6.2	Mean field game representation	100
6.3 Analysis of the Mean Field Game		lysis of the Mean Field Game	102
	6.3	The coupled stochastic partial differential equations	103
	6.3	2 Existence and uniqueness of the MFE	104
	6.3	3 Mean field game versus $K$ -person game	106

	6.4	Numerical Results and Performance Evaluation	107	
		6.4.1 System set up	107	
		6.4.2 Performance evaluation	107	
	6.5	5 Summary		
7	7 CONCLUSIONS AND FUTURE WORK			
	7.1	Conclusions	116	
	7.2	Future Work	119	

iv

### Statement of Originality

The contributions of this thesis are mainly on the study of demand management in smart grids and its enabling information and communications technology. Various mathematical optimization techniques and game theoretic frameworks for demand management are proposed. The following aspects of this thesis are believed to be originals:

- There are significant challenges as well as great opportunities for research at both policy and technology levels on the efficient use of energy. Based on the knowledge and the research on the topic of demand management in smart grids, an overview of the features of demand management with a particular focus on the necessary enabling wireless technologies is presented in Chapter 3. Various optimal demand management mechanisms using these wireless technologies were also reviewed. Co-authoring with the researchers from Toshiba Research Europe Telecommunications Research Laboratory, a comprehensive overview of the research perspectives, including communications and information technologies infrastructure, demand management, security, and standardization for smart grids has been completed with two journal publications as listed at the end of this statement [1,2].
- In Chapter 4, a mixed integer linear programming based consumption scheduling optimization technique is proposed to schedule the consumption of different types of home appliances in a centralised manner. The mechanism is able to optimise the consumption profile of the household and schedule the energy consumption over the scheduling period to reduce the peak load of the power grid. As electric vehicle (EV) becomes popular, it will be increasingly important to shift the power consumption of EVs from peak times since an EV may have the potential to consume as much power as an average home and so could

increase the peak load considerably. Furthermore, if integrated properly into the electrical grid, EV represents a huge opportunity, as they can assume the role of a dispatchable energy resource. Hence, a sizable number of EVs are incorporated into the proposed optimization framework [3,4].

- The improvement of proposed optimization technique has been completed by incorporating the scheduling of the use of locally generated and stored energy in Chapter 5. The optimization framework is aimed to minimise the consumption cost as well as the dependency on conventional energy. In particular, a game theoretic modelling framework is developed to schedule the consumption requirements for multiple households in a decentralised manner. The scheduling game is played sequentially by each consumer in a round robin process to manage their consumption and to achieve lowest consumption cost. Given a pricing plan for the group of consumers, the game theoretic algorithm has the ability to converge to a stable point. This means the consumption cost is able to achieve its minimum and therefore all the participating consumers form an effective consumption scheduling coordination. Theoretical analysis of the game theoretic framework in terms of the properties of Nash equilibrium and its convergence procedure based on the concepts of potential games are presented. Numerical simulations demonstrate the scheduling technique and show the effectiveness [5,6].
- In addition to home area consumption optimization techniques, in Chapter 6, a dynamic game theoretic optimization framework is developed for modelling a scenario of multiple EVs charging simultaneously in a charging station. The proposed technique provides every individual EV an optimal charging strategy to proactively control its charging rate (speed) in order to minimise the charging costs. The optimization

is based on a specific differential game theoretic modelling technique, known as the stochastic mean field game. Theoretical analysis of the game in terms of its formulation, solution, and the comparison of the mean field game with the traditional game theoretic techniques are provided. Numerical results are also presented to demonstrate the proposed framework [7,8].

The novelty of the contributions is supported by the following international journal and conference paper publications.

### Publications

- Z. Zhu, S. Lambotharan, W. H. Chin, and Z. Fan, "Overview of Demand Management in Smart Grid and Enabling Wireless Communication Technologies," IEEE Wireless Communications Magazine, vol.19, no.3, pp.48-56, June 2012.
- Z. Fan, P. Kulkarni, S. Gormus, C. Efthymiou, G. Kalogridis, M. Sooriyabandara, Z. Zhu, S. Lambotharan, and W.H. Chin, "Smart Grid Communications: Overview of Research Challenges, Solutions, and Standardization Activities," IEEE Communications Surveys & Tutorials, vol.15, no.1, pp. 21-38, 2013.
- 3. Z. Zhu, J. Tang, S. Lambotharan, W.H. Chin and Z. Fan, "An integer linear programming based optimization for home demand-side management in smart grid," in 2012 IEEE PES Innovative Smart Grid Technologies (ISGT), pp.1-5, Jan. 2012.
- 4. H. Sherif, Z. Zhu, and S. Lambotharan, "An Optimization Framework for Smart Grid Demand Side Management Incorporating Electric Vehicles," Accepted in the ISGT-ASIA 2014.
- 5. Z. Zhu, J. Tang, S. Lambotharan, W.H. Chin and Z. Fan, "An Inte-

ger Linear Programming and Game Theory Based Optimization for Demand-side Management in Smart Grid," in IEEE GLOBECOM, Workshop on Communications Technologies for Secure, Reliable, and Sustainable Smart Grids (CT-Smart Grid), Houston USA, pp.1205-1210, Dec.2011.

- 6. Z. Zhu, S. Lambotharan, R. Kalawsky, W. H. Chin, and Z. Fan, "A Game Theoretic Optimization Framework for Home Demand Management Incorporating Locally Generated Energy," submitted to IEEE Transactions on Industrial Informatics.
- Z. Zhu, S. Lambotharan, W. H. Chin, and Z. Fan, "A Stochastic Optimization approach to Aggregated Electric Vehicles Charging in Smart Grids," Accepted in the ISGT-ASIA 2014.
- Z. Zhu, S. Lambotharan, W. H. Chin, and Z. Fan, "A Mean Field Game Theoretic Approach for Electric Vehicles Charging," to be submitted to IEEE Transactions on Signal Processing.

### Acknowledgements

I AM DEEPLY INDEBTED to Professor Sangarapillai Lambotharan, my esteemed supervisor, for his kind interest, generous support and inspiring guidance throughout the past three and a half years. I have benefitted tremendously from his profound knowledge, boundless patience and rigorous scholarship. It is my very great privilege to be his student.

I am extremely thankful to my industrial supervisors Dr. Zhong Fan and Dr. Woon Hau Chin of Toshiba Research Europe Ltd., Telecommunications Research Laboratory for their valuable comments and advice on every part of my research. I would also like to thank Professor Roy Kalawsky for his supervisory support. I am very grateful to the School of Electronic, Electrical and Systems Engineering, Loughborough University and Toshiba Research Europe Ltd. for funding this work.

I would like to express my appreciations to many members from the ASPG, who have generously helped me to develop my knowledge through wide-ranging discussions.

I wish to take this opportunity to thank my parents, Shouyu Zhu and Xueyan Xie, and my wife, Jue Liu, for their persistent moral support throughout my studies in the UK.

> Ziming Zhu May, 2014

# List of Acronyms

AMI	Advanced Metering Infrastructure
CPP	Critical Peak Pricing
CSMA	Carrier Sense Multiple Access
DAP	Data Aggregation Points
DER	Distributed Energy Resources
DG	Distributed Generation
DSM	Demand Side Management
EMU	Energy Management Unit
EV	Electric Vehicles
FPK	Fokker-Planck-Kolmogorov
G2V	Grid to Vehicle
HAN	Home Area Network
HJB	Hamilton-Jacobi-Bellman
ICT	Information and Communication Technology
ILP	Integer Linear Programming
IP	Integer Programming

LP	Linear Programming
LTE	Long Term Evolution
MFE	Mean Field Equilibrium
MILP	mixed integer linear programming
MIMO	Multiple Input Multiple Output
MIP	Mixed Integer Programming
NAN	Neighbourhood Area Network
OFDMA	Orthogonal Frequency Division Multiple Access
PDE	Partial Differential Equations
PHEV	Plug-in Hybrid Electric Vehicle
PLC	Powerline Communication
PSO	Particle Swarm Optimization
PV	Photovoltaic
QCQP	Quadratically Constrained Quadratic Programming
QoS	Quality of Service
QP	Quadratic Programming
RTP	Real Time Pricing
SM	Smart Meter
SPDE	Stochastic Partial Differential Equations
SPE	Subgame Perfect Equilibrium
TBBP	Time Block Based Pricing

TDMA	Time Division Multiple Access
ToU	Time-of-Use
V2G	Vehicle to Grid
WAN	Wide Area Network
WiMAX	Worldwide Interoperability for Microwave Access
WSN	Wireless Sensor Networks

# List of Symbols

Scalar variables are denoted by plain lower-case letters, (i.e., x), vectors by bold-face lower-case letters, (i.e.,  $\mathbf{x}$ ), and matrices by upper-case bold-face letters, (i.e.,  $\mathbf{X}$ ). Some frequently used notations are as follows:

- $E\{\cdot\}$  Statistical expectation
- $(\cdot)^T$  Transpose
- ||.|| Euclidean norm
- $(.)^{-1}$  Matrix inverse

# **List of Figures**

1.1	A typical smart grid architecture.	3
2.1	A possible tree structure of the branch and bound method.	16
2.2	A normal form game representation.	19
2.3	Prisoner's dilemma game.	20
2.4	An extensive form game representation.	21
2.5	Rock-Paper-Scissors game.	24
2.6	Traffic game.	25
2.7	Illustration of fixed points for a function.	27
2.8	Illustration of a fixed point for a correspondence mapping.	28
3.1	AMI wireless communication architecture	40
4.1	System components.	52
4.2	Illustration of different types of operations of appliances.	54
4.3	Hourly power consumption schedule.	60
4.4	Optimal schedule for individual appliances.	63
4.5	Hourly power consumption schedule for multiple households.	63
4.6	Appliances' total energy consumption profile when zero EV	
	present.	64
		xiv

4.7	Appliance energy consumption when 30 EVs are modeled.	64
4.8	Appliance energy consumption when 90 EVs are modeled.	65
5.1	Convergence of the scheduling game (played sequentially by	
	each player), e.g., iteration 20 means each player played twice.	84
5.2	Scheduled accumulated consumption load over the day.	87
5.3	Comparison of scheduled consumption for five players.	87
5.4	Scheduled consumption from local and main supply for five	
	players.	88
6.1	System model: aggregated EV charging at a station.	93
6.2	Optimal charging rates for a single EV $(T = 55)$ .	108
6.3	Optimal trajectory of the charging battery capacity for a sin-	
	gle EV $(T = 55)$ .	109
6.4	Optimal trajectory of the cost-to-go value for a single EV	
	(T = 55).	109
6.5	Optimal charging rates for one particular EV in a mean field	
	game theoretic scenario $(T^* = 59)$ .	110
6.6	Optimal trajectory of the charging battery capacity for one	
	particular EV in a mean field game theoretic scenario $(T^* = 59)$	.110
6.7	Optimal trajectory of the cost-to-go value for one particular	
	EV in a mean field game theoretic scenario $(T^* = 59)$ .	111
6.8	Optimal battery capacity dynamics $(T^* = 59)$ .	112
6.9	Distributions of battery capacity over charging duration.	112
6.10	Total energy consumption of 100 EVs $(T^* = 59)$ .	114
6.11	Optimal batteries capacity dynamics for 100 charging EVs	
	$(T^* = 47).$	114

# List of Tables

3.1	Comparison of candidate NAN and HAN wireless technologies	43
3.2	Dynamic pricing schemes	47
4.1	Appliances and power consumption patterns	60
5.1	Comparison of the consumption cost of consumers (pence)	85
5.2	Local energy usage of the consumers	86

# INTRODUCTION

### 1.1 The Evolution of Smart Grids

The electricity supply industry is facing significant challenges in terms of meeting and addressing the demand for energy, environmental concerns, security, and reliability.

Currently, most of the power generation and delivery grids are based on many decades old vertical hierarchical broadcast infrastructures (i.e. fewto-many distribution), where a few central power generators (i.e. power stations) provide all the electricity production in a country or region, and dispatch this electricity to consumers via a large network of cables and transformers. The electric power flows in one direction from the power generators to the consumers and the monitoring information is handled only at the operation side. Based on load forecasting models developed over time, the utility providers generally over-provision for the demand (considering peak load conditions). If the demand increases above the average, they may have to turn on the peaker plants which use non-renewable sources of energy (e.g. coal) to generate additional supply of energy to cope with the demand. The provisioning for peak load approach is wasteful when the average demand is much lower than the peak because electricity, once produced, has to be consumed as energy storage is normally very expensive [1]. Moreover, setting up and maintaining peaker plants are environmentally unfriendly. Also, given the increasing and locally dynamic demand for energy, it may be difficult, perhaps impossible in the longer run, to match the supply to this peak demand.

It is attractive in such a situation to match the demand to the available supply by using two way communications between the grid and the customers and providing incentives (e.g. through variable pricing) to encourage the consumers to shift (reschedule) the consumption load to off peak demand period so as to improve utilization of the available capacity. This necessitates the flow of metering information from the customer premises to the grid to analyse the demand, and the flow of control information (e.g. pricing information) in the opposite direction to encourage the customers to manage their demand. The bi-directional flow of information will provide the utility operators the full visibility of the grids and will help them making informed decisions on the energy supply. It also provides opportunities for consumers to participate in the energy demand management to reduce the cost of their energy consumption. In particular, the utility operators and consumers can communicate and cooperate in order to achieve bi-directional load control and efficient consumption management.

Renewable energy sources offer a key solution to the environmental problem. However, their integration into existing power grids comes with a whole new set of barriers, such as the intermittency of generation, the high level of distribution of the energy sources and the lack of proven control algorithms to manage such a highly distributed generation. In addition to the seamlessly access of renewable energy, building a sustainable power network for future generations also requires monitoring and control technology for reducing central generation emission and transmission loss.

In order to support the aforementioned functionalities, it is believed that a fundamental evolution in electric power generation and supply system, generally recognised as the Smart Grid, is needed. There are various definitions and visions for smart grids. However, the core of such evolution is enabling



Figure 1.1. A typical smart grid architecture.

the future generation electricity network smart and intelligent by integrating bi-directional information and communication technology (ICT) with power grids.

Figure 1.1<sup>1</sup> illustrates a possible overall smart grid architecture. It is highly integrated and complex, yet flexible and reliable network with various centralized and distributed energy sources. The power flow direction is no longer just downhill from the bulk power plants to consumers. Instead, dynamic flows can start from any generation sources and could end up anywhere in the grids. Distributed generation (DG) from solar, small wind turbine, biomass and other renewable sources, mid/low voltage distribution and storage systems will be integrated into the conventional centralised bulk generation and high voltage transmission systems. Energy can be stored and released back to the grids even at household level. DG enables local electricity generation using all kinds of energy sources to get access to the power grid, which can reduce the high demand for central fossil-fuel plants. Besides, mobile energy storage devices, e.g. plug-in electric vehicles (EVs),

<sup>&</sup>lt;sup>1</sup>Elements of this figure are from google images. http://www.google.co.uk/

can easily access the grid as supportive power sources.

The integration of ICT enables real-time monitoring of the operational conditions of every part of the grid. Based on the collected information, the operation control center is able to detect, analyse and respond fast to emerging problems. The ICT infrastructure enables not only the grid operator to make informed decisions and optimise the energy flow, but it also provides opportunities for consumers to participate in the energy demand management. Development of this complex new system requires national and even international efforts in technology development, standards, and regulatory activities.

National governments and various relevant stakeholders have already launched massive investments on smart grid research projects and made significant progress. The US Department of Energy (DoE) states that a smart grid uses digital technology to improve reliability, security, and efficiency of the electricity system [2]. Smart grid is a vital component of President Obama's comprehensive energy plan: the American Recovery and Investment Act includes 11 billion in investments to "jump start the transformation to a bigger, better, smarter grid". One of the key elements behind the current intensive work program towards smart grid in the United States is tightly linked with the need to modernize their power system. In particular, the lack of electricity distribution network reliability under stress conditions was born out of under-investment in the infrastructure combined with growing energy demand. This was emphasized in a series of major supply disruption events (e.g. the North-East blackout), widely seen as a wake-up call to address network stability by increasing inter-connectivity, local and wide area control. There are growing expectations on the integration of a wide range of renewable energy sources with the power grid. Therefore the US DoE Smart Grid Research and Development Program has set the following performance targets for 2030: 20% reduction in the nation's peak energy demand; 100% availability to serve all critical loads at all times and a range of reliability services for other loads; 40% improvement in system efficiency and asset utilization to achieve a load factor of 70%; 20% of electricity capacity from distributed and renewable energy sources (200 GW) [2]. There are also a number of huge industrial research projects currently underway, for example, the IBM GridWise project [3] and the smart grid trial in New Mexico [4].

Europe, by contrast, presents a highly interconnected, mesh distribution network exhibiting more robustness than the US system. The main highlight of the EU definition is that a smart grid is an electricity network that can intelligently integrate the behaviour and actions of all users to ensure sustainable, economic, and secure electricity supply [5]. The biggest challenge in the Europe is the integration of renewable power generation to meet the 2020 targets for reduction of carbon emissions from fossil power generation. The important role of smart grids is mentioned in the European Commission's 2020 strategy document [6], in the EU Smart Grids Technology Platform [5], and also highlighted in the new initiative on Future Internet research as a key application [7]. The EU, through the technology development platform, has established a carefully planned approach to the implementation of smart grid technologies in the medium to long term. Establishing work on standardization, research projects involving academia with industries (utilities and manufacturers), and demonstration/pilot projects are the current priority.

#### 1.2 Demand Management in Smart Grids

Demand management in the electricity industry mainly consists of load monitoring, analysis and response. Facilitated by the two-way information flow and various optimization mechanisms, operators benefit from real time dynamic load monitoring and control while consumers benefit from optimized use of energy. Effective demand management provides significant benefits to the operational efficiency and reliability of smart grid. For example, it has been reported in [8] that in Europe, five to eight percent of installed capacity is used only one percent of the time. By deferring the peak demand to off peak times, the capacity and transmission cost could be reduced up to 67 billion euros in Europe. The work in [9] states that even a conservative estimate of potential saving due to grid modernization is 40 billion dollars per year in the US. In addition to the direct savings, there are many important economical and societal benefits such as reduction of carbon emissions, integration of renewable energy, elimination of regional blackouts and reduced operational costs via for example automated meter readings.

A comprehensive overview of demand management in smart grids, including its key features, enabling ICT and proposed approaches are provided in Chapter 3.

### 1.3 Thesis Outline

The work in this thesis is mainly focused on developing optimization frameworks for home consumption scheduling and electric vehicle charging using mathematical optimization and game theoretic modeling approaches. The contents of the thesis are outlined as follows.

Chapter 2 provides a literature review on mathematical optimization and game theoretic modeling, with a focus on their utilizations in electrical and electronic engineering, in particular for smart grids. These are the two main approaches for the optimization frameworks proposed in this thesis.

The novel contributions of this thesis are in Chapters 3, 4, 5 and 6. Chapter 3 presents a comprehensive overview of demand management with a particular focus on the necessary enabling information and communication technologies. Various mechanisms for the optimal demand management in smart grids using these wireless technologies are reviewed.

Chapter 4 studies the consumption optimization problem for home area demand management. A consumption scheduling optimization technique based on integer linear programming (ILP) is proposed. The aim of the proposed scheduling is to minimise the peak hourly load in order to achieve an optimal (balanced) daily load schedule. The proposed mechanism is able to schedule the optimal operation time for each of the home appliances according to their power consumption patterns. The penetration of EVs is also considered in the optimization framework. Matlab based simulation results on home and neighbourhood area consumption scheduling are presented to demonstrate the effectiveness of the proposed technique.

The proposed optimization technique is further improved in Chapter 5. Given the pricing information, the optimization framework is aimed to minimise dependency on conventional energy and the consumption cost of the residential consumers. Considering that the consumers will have the flexibility to consume energy from various sources and make the best use of locally generated/stored energy in the smart grids, local energy is also included in the formulation of the optimization framework. In particular, a game theoretic model is proposed to coordinatively manage the scheduling of appliances of the consumers. Theoretical analysis is presented to show that the proposed game theoretic algorithm admits Nash equilibrium, which means a stable solution to the optimization, exists. The scheduling optimization converges to the equilibrium where all consumers can benefit from participating in. Matlab based simulation results are presented to demonstrate the proposed approach and the benefits of successful home demand management.

Electric vehicle is considered to be an important component of distributed energy storage and supply devices in smart grids. In addition to home area consumption optimizations, which have the ability to optimally schedule the use of EVs at home, Chapter 6 focuses on the topic of aggregated EVs' optimal charging. A dynamic game theoretic optimization framework is proposed to formulate the EV charging problem. In particular, the optimization considers a charging station where a large number of EVs can be charged simultaneously during a specific but flexible period of time. The proposed technique provides every individual EV an optimal charging strategy to proactively control its charging rate (speed) in order to minimise the cost of charging. The dynamic optimization is based on a specific differential game theoretic modeling technique, known as the mean field game. Theoretical analysis of the game in terms of its formulation and solution is provided with a comparison of mean field game with the traditional game theoretic techniques. Numerical results are presented to demonstrate the performance of the proposed framework.

Conclusions are drawn in Chapter 7. A brief summary of possible potential future directions are also outlined.

# MATHEMATICAL OPTIMIZATION AND GAME THEORETIC MODELING TECHNIQUES

In this chapter, a comprehensive literature review of the basics of mathematical optimization and game theory is presented. Their formulation or modeling, as well as various approaches for obtaining optimal solutions are discussed. Various combinations of these techniques are the main ingredients for the optimization frameworks proposed in this thesis, which promise efficient solutions to the smart grid demand management.

### 2.1 Mathematical Optimization

Optimization is one of the most pervasive words for researchers who value system performance, efficiency and cost effectiveness. Mathematical Optimization, also called as mathematical programming is the foundation of the development of various mechanisms and algorithms. The use of mathematical optimization techniques plays a crucial role in communication engineering and advanced signal processing [10]. Techniques such as the well known least squares and linear programming have been widely applied in real world problems [11, 12]. Solving the optimization problems provide useful solutions and references for the design of system parameters. Mathematical optimizations are valuable and have been widely applied in research areas such as automatic control systems, communication networks, economics and finance [13–15].

### 2.1.1 Formulation of an optimization problem

The basic concept of mathematical optimization is to search for optimal solutions for the optimization parameters under specific conditions, in order to achieve certain criteria of satisfaction. It can hence be seen that the formulation of particular optimization problem consists of three basic components, namely *optimization variable*, *objective function* and *constraint*.

A basic and classic mathematical representation of an abstract optimization problem has the following form [10]:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & f_0(\mathbf{x}) \\ \text{subject to} & f_i(\mathbf{x}) \leq 0 \qquad i = 1, \dots, m \\ & h_i(\mathbf{x}) = 0 \qquad i = 1, \dots, p \end{array}$$
(2.1.1)

where the vector  $\mathbf{x} = [x_0, ..., x_n]^T \in \mathbb{R}^n$  is the optimization variable of the problem. The function  $f_0 : \mathbb{R}^n \mapsto \mathbb{R}$  is the objective function or cost function represents the cost of choosing  $\mathbf{x}$ . It can also be considered that  $-f_0(\mathbf{x})$ represents the level of satisfaction, or utility, of choosing  $\mathbf{x}$ . The functions  $f_i(\mathbf{x}) : \mathbb{R}^n \mapsto \mathbb{R}$  and  $h_i(\mathbf{x}) : \mathbb{R}^n \mapsto \mathbb{R}$  are called the inequality and equality constraints, which represent the requirements or specifications of choosing  $\mathbf{x}$ . Particular optimizations can be classified as unconstrained problems if there is no constraint. The *domain* of the optimization problem denoted by D is the set of points where the objective function and the constraints are defined,

$$D = \bigcap_{i=0}^{m} \operatorname{dom} f_i \cap \bigcap_{i=1}^{p} \operatorname{dom} h_i.$$
(2.1.2)

The problem is *feasible* if there exists a subset of points  $\mathbf{x} \in D$  which satisfies all the constraints. The *optimal solution* of the optimization exists only when the problem is feasible and is obtained at the point  $\mathbf{x}^*$  if and only if

$$f_0(\mathbf{x}^{\star}) \le f_0(\mathbf{x}) \quad \forall \mathbf{x} \in D.$$
 (2.1.3)

Problem (2.1.1) describes the process that minimises the value of  $f_0$  (obtain minimum cost or maximum utility) by selecting the best possible choice  $\mathbf{x}$  subject to all the constraints. One practical interpretation of such formulation can be be considered as a process of seeking the best way to invest some capital in a set of assets, i.e., portfolio optimization [10]. The variable  $\mathbf{x}$  describes the portfolio allocation across the set of assets. Each element in  $\mathbf{x}$  represents the investment in a particular asset. The constraints might consist of a limit on the budget, the requirement of minimum investments, and a minimum acceptable value of expected return for the whole investment. The optimization (2.1.1) chooses a portfolio profile that minimises risk, among all possible constrained allocations [10].

#### 2.1.2 Optimal solutions

A solution method for a mathematical optimization problems is an algorithm that computes a (global or local) optimal solution of the problem to some given accuracy [10]. Significant efforts have been on developing algorithms for solving various classes of optimizations. The effectiveness of these algorithms, i.e., the ability to solve the optimization problems, depends on the particular optimization problem. For example, the type of the objective and constraint functions, the number of variables and constraints, and special structures.

A general optimization problem which has smooth objective and constraint functions, e.g., polynomials, and even small number of variables and constraints, can still be difficult to solve. Solving some of the optimizations might require very long computation time, or even has possibility of not be able to obtain the solution. For example, problems requiring non-polynomial computation time.

However, there are a few classes of optimization problems for which effective and reliable solutions exist, even when the size of the problem is significantly large, with hundreds or thousands of variables and constraints. Two classic and well investigated examples are least squares problems and linear programs [11,12]. Convex optimization is another well known class of optimization problems which has very effective and efficient solutions [10]. Various practical problems in communication engineering and signal processing can be appropriately formulated into convex problems and hence reliable solutions are guaranteed, as seen in [16] and [17]. In fact, linear programming problems as discussed in the following section, falls into this class of convex optimization.

#### 2.1.3 Linear programming

Linear programming (LP) is an optimization problem where the objective and all constraints are linear, as follows [10, 11],

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \mathbf{c}^{T}\mathbf{x} + d \\ \text{subject to} & \mathbf{G}\mathbf{x} \leq \mathbf{h}, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{array} \tag{2.1.4}$$

where the vectors  $\mathbf{c} \in \mathbb{R}^n$ ,  $\mathbf{h} \in \mathbb{R}^m$ ,  $\mathbf{b} \in \mathbb{R}^p$ , scalar  $d \in \mathbb{R}$  and matrices  $\mathbf{G} \in \mathbb{R}^{m \times n}$  and  $\mathbf{A} \in \mathbb{R}^{p \times n}$ , specify the objective and constraint functions.

There is no standard analytical formula for obtaining the optimal solution for linear programming, however, various effective methods exist, such as the Dantzig's simplex method and the well known interior-point methods [18]. The interior-point algorithms cannot provide the exact number of arithmetic operations required to solve a linear program, to a given accuracy, however it can establish rigorous bounds on the computation time. In practice, the complexity of these algorithms has a order of  $n^2(m + p)$  (assuming  $(m + p) \ge n$ ) [10]. These algorithms are quite reliable and efficient. General linear programs, with thousands of variables and constraints, can be solved in seconds by using particular solvers embedded computer applications. For example, the CVX toolbox for Matlab [19].

It is still a challenge to solve extremely large scale linear programming, or when having real-time computing requirements. However, it can be considered that solving (most) linear programs is a mature technology. Using linear programming, some applications lead directly to the form as in (2.1.4). In many other cases, although the original optimization problems do not seem to have a standard form, they can be transformed or reduced to equivalent linear programs, and hence can be efficiently solved.

#### 2.1.4 Quadratic programming

As an extension of LP, quadratic programming (QP) optimization problem has a quadratic objective function and affine constraint functions. It has the following standard form:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \frac{1}{2}\mathbf{x}^{T}\mathbf{P}_{0}\mathbf{x} + \mathbf{q}_{0}^{T}\mathbf{x} + r_{0}\\ \text{subject to} & \mathbf{G}\mathbf{x} \preceq \mathbf{h},\\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{array}$$
(2.1.5)

where  $\mathbf{P}_0$  is a symmetric  $n \times n$  full rank matrix and  $r_0 \in \mathbb{R}$  is a constant. A QP falls into convex optimization when the objective is a convex quadratic function, i.e.,  $\mathbf{P}_0$  is positive semidefinite. A convex quadratic programming can also be solved efficiently. It can be seen in (2.1.5) that a QP can be reduced to a LP by setting  $\mathbf{P}_0 = 0$  in the objective function.

Another related class of optimization problems, quadratically constrained quadratic programming (QCQP), can be introduced by adding quadratic constraints in (2.1.5), as follows:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \frac{1}{2}\mathbf{x}^{T}\mathbf{P}_{0}\mathbf{x} + \mathbf{q}_{0}^{T}\mathbf{x} + r_{0}\\ \text{subject to} & \mathbf{x}^{T}\mathbf{P}_{i}\mathbf{x} + \mathbf{q}_{i}^{T}\mathbf{x} + r_{i} \leq 0, \qquad i = 1, 2, \dots, m,\\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{array}$$

$$(2.1.6)$$

where  $\mathbf{P}_i$  is a symmetric full rank matrix and  $r_i \in \mathbb{R}$ , i = 1, 2, ..., m. A QCQP is a convex problem when the objective function and all constraint functions are convex. In (2.1.6), by setting  $\mathbf{P}_i = 0$ , i = 0, ..., m, the problem reduced to an LP. Convex QP and QCQP are important in economics and finical analysis because people usually mathematically model the cost and revenue into convex quadratic forms, whose optimal solutions can be efficiently obtained [14].

### 2.1.5 Integer programming

Apart from aforementioned optimizations defined for real-valued variables, many practical optimization problems require the optimization parameters to be integer or binary variables. Those optimizations are common in discrete analysis and decision making. Such problems fall into a particular type of optimization, namely integer programming (IP). Problems with real variables as well as integer variables are classified as mixed integer programming (MIP) [12]. (Mixed) integer programming has a similar form as in (2.1.1), however the domain of the problem is no longer a continuous domain but an intersection of integer domains (and real domains). Recall the formulation of LP, one can easily transform a linear programming problem to an integer linear programming (ILP) by setting the optimization variable as  $\mathbf{x} \in \mathbb{Z}$ . Formulation of integer QP and integer QCQP can also be seen in various discrete/binary problems as been discussed in [20].

Integer (and mixed integer) programming have very wide applications in practice. For example, a warehouse location problem where a manager must decide which of the n potential warehouses are needed to be built/operated for meeting the demands of shipping goods to the m customers, with an objective of minimum operational costs [21].

Scheduling is another important class of integer problems. Consider the scheduling of students and classrooms, for example, the *i*th student is scheduled for the *j*th class during the *k*th time period or not. Such a variable is either zero or one. There are constraints on the number and size of classrooms available at any one time, and the students' preferences for particular schedules [22,23]. These are applicable to communications research as well. In [24], the problem of radio resource allocation in orthogonal frequency division multiple access (OFDMA) system is formulated as a MIP. MIP based scheduling optimization is used in the proposed demand management techniques later in this thesis.

Computing the global solution for a (mixed) integer programming problem is relatively more difficult than that of a continuous valued problem. Because IP/MIP is generally non-convex, which means the reliable and efficient solution algorithms designed for continuous valued convex optimization are no longer valid. One may solve an IP by enumerating all of the possible solutions and choose the best one. This can actually work for problems with very limited number of variables, however the computation effort grows very rapidly (exponentially) and such method becomes unworkable for even



Figure 2.1. A possible tree structure of the branch and bound method. medium size problems with tens/hundreds of variables, let alone large scale industrial problems [20].

Branch and bound algorithm is one of the well known effective methods for solving non-convex optimization problems [25]. It is a basic technique for solving integer and discrete problems. The method is based on the observation that the enumeration of integer solutions has a tree structure, as illustrated in Figure 2.1. The main idea in branch and bound is to avoid growing the whole tree as much as possible [25]. Instead it grows the tree in stages, and grows only the most promising branch (solid lines in Figure 2.1). It determines which node is the most promising by estimating a bound on the best value of the objective function that can be obtained by growing that node to later stages, while permanently discards (prunes) nodes and their descendants which will never be neither feasible nor optimal (dotted lines in Figure 2.1) [20]. It maintains an upper and lower bound on the (globally) optimal value and finally terminates with a certificate proving that the suboptimal point found is  $\epsilon$ -optimal [26].
Branch and bound is a very general framework. It is necessary to develop completed algorithms which include the policies of node selection, pruning, and the specification of termination point. Computer toolbox for modeling and solving integer problems based on the use of branch and bound are available [27]. It is worth to mention that these algorithms are not guaranteed and often slow, in the worst case they require effort that grows exponentially with problem size. However in majority of small-medium cases, they converge to optimal solutions in a reasonable number of iterations.

# 2.2 Game Theoretic Modeling Techniques

Game theory is a powerful mechanism for understanding and modeling mathematically the interaction of various rational decision makers. Examples of games in the real world include conventional games of strategy such as chess and poker, as well as daily decision-making situations such as deciding what movie to see with your family or friends. Game theoretic methods have been widely applied in resource competing and social welfare optimization scenarios [28].

#### 2.2.1 Basis of game theory

Game theory provides tools for evaluating the outcome when agents (decision makers, usually with conflicting interests) take certain actions in a strategic situation, where the outcome of an agent taking a particular action depends not only on the action itself but also the actions of all other agents. Game theory can help these agents in making optimal decisions. The earliest investigations of probability, games of chance, and even strategic choices in warfare might be considered as game theory. The work by Cournot, Zermelo, and Borel in the last two centuries built the foundation for modern game theory, and proposed its applications mainly in economics [29]. Tremendous work on the development of both noncooperative (strategic) game and cooperative game theory was done by mathematician John Nash, in papers [30] and [31]. The most important contribution was probably the proof of the existence of equilibrium points in noncooperative games.

One classical game formulation of an interaction consists of three main components, namely players, strategies and payoffs [32]. Every decision maker is treated as a player of the game. In noncooperative games, players are usually a finite number of interacting individuals. However, groups of decision makers (coalitions) with shareable value can be treated as one player in cooperative games. The decisions of action are called the *strategies* of the players. There are usually many possible actions for each player to choose. All the possible action choices of a player form a strategy space. Players are assumed to be rational. This means when given opportunity to play the game (to make an action), every player will try to choose the best strategy from its available strategy space. Based on the understanding of the game status, the result of playing the selected strategy must maximise the player's own satisfaction. The game status is usually determined according to the knowledge of the types of other players and their preferences (known as information completeness), and the actions they have made (known as information perfectness). However complete and perfect information are not a promise in every game. Obtainable information depends on individual settings. The level of satisfaction of choosing a particular strategy, under certain game situation, is usually quantified by a payoff value. A payoff function can be used to describe the relationships between different payoff values and strategies. Detailed mathematical notations in relation to these game components will be clearly defined in relevant parts later in this thesis. However, it is useful to firstly define two general game representations, namely normal form game and extensive form game [29].

In game theory, a normal form (strategic form) is a way of describing a



Figure 2.2. A normal form game representation.

game using a matrix [29]. For example, a normal form representation for a simple two by two game scenario is depicted in Figure 2.2. The two players are put on the row side and the column side of the matrix. Each strategy they can make are marked (A and B) in different rows and columns. Players' payoff values for particular strategy profiles occupy the corresponding cells of the matrix, representing all the possible outcomes of the game. Such representation allows us to quickly analyse each possible outcome of a game. As seen in Figure 2.2, when both players choose A, the game results in the top left cell, with player 1 getting a payoff of  $p_{1AA}$  and player 2 getting a payoff of  $p_{2AA}$ .

The strategic form is usually the right description for simultaneous games, where players choose their strategies simultaneously. The term 'simultaneous' is not in the sense of 'time' but 'information'. If players make decisions sequentially however without knowing what others have done, the game can be treated as a simultaneous game. On the other hand, if a player knows what its opponents are going to do, even all players are meant to make decisions simultaneously, the game is already a sequential move game. Therefore, by using the normal form, it is implied that the game has complete

	PRISONER 2		
	confess	defect	
confess	-1,-1	-10,0	
defect	0,-10	-3 , -3	

Figure 2.3. Prisoner's dilemma game.

but imperfect information. A well-known example of simultaneous games described using the strategic form is the prisoner's dilemma, where two separated locked prisoners are asked to make a decision between confess and defect. One prisoner does not know what the other would do however their payoffs are known. Figure 2.3 shows a typical prisoner's dilemma game.

While simultaneous games are usually described using the strategic form, sequential move games are better described using the extensive form. The extensive representation has a tree structure. It's simply a diagram that shows that all possible choices can be made by the players at different times (corresponding to each tree node), with specific payoffs for all possible outcomes (at the end nodes).

Figure 2.4 illustrates one possible extensive form game. As seen, player 1 plays first and then player 2 makes a decision after observing player 1's move. The game finishes with a payoff profile when both players have made their moves. For example, if player 1 chooses strategy A and player 2 chooses strategy B, the resulting payoffs are  $p_{1AB}$  and  $p_{2AB}$ . The extensive form clearly shows the actions at different moments. In such representation, each



Figure 2.4. An extensive form game representation.

player knows exactly what game status he/she is in and observes all the previous decisions made by all players so far. The detailed payoffs of the players are also common knowledge. Therefore the extensive form is normally used to describe games with complete and perfect information. A well known example of such games is chess.

The above two representation forms are helpful for the proper design of a game theoretic model. It is even sufficient for analysing the player's behaviours and the properties of equilibrium outcomes in simple scenarios. For games formulated having neither incomplete nor imperfect information, it is valuable to reconstruct such a complex game as a number of simple games and lay out their basic forms for more intuitive and better understanding.

# 2.2.2 Nash equilibrium: the solution concept

Having the basic structure of a game theoretic model, it is important to analyse the players' behaviours and the properties of all the possible outcomes of the game. Nash equilibrium, named after its inventor John Nash, is probably the most well known solution concept in (noncooperative) game theory [32]. It designates the ultimate way that the rational players should play the game. That is, by playing the Nash equilibrium strategy, no player has the incentive to further deviate to any another strategy. Nash equilibrium is critical to noncooperative games. Its existence and possibly uniqueness are generally used to evaluate the effectiveness of a game theoretic model.

Before stating the definition of Nash equilibrium, it is useful to firstly describe the basic logic of playing the game rationally, which leads the players to the Nash equilibrium.

#### Best response

In a simultaneous game, a player typically does not know about the game situation i.e., what the other players have done or intend to do. However, the player can guess about the opponents' strategy choices and then respond to this guess rationally by playing the strategy that maximises his/her payoff. This particular strategy (set) is defined as the best response:

Definition 2.2.1 (best response): The best response  $\mathfrak{B}_n(\mathbf{s}_{-n})$  of player n to the strategies  $\mathbf{s}_{-n}$ , where  $\mathbf{s}_{-n} = \{s_1, ..., s_{n-1}, s_{n+1}, ...\}$  represents the strategy choices of players other than n, is given by

$$\mathfrak{B}_{n}(\mathbf{s}_{-n}) = \operatorname{argmax}_{s_{n} \in \mathbb{S}_{n}} \mathfrak{u}_{n}(s_{n}, \mathbf{s}_{-n}), \qquad (2.2.1)$$

where  $\mathbb{S}_n$  here represents player *n*'s strategy space and  $\mathfrak{u}_n(s_n, \mathbf{s}_{-n})$  is the payoff function for player *n* for choosing strategy  $s_n$  when all other players have chosen  $\mathbf{s}_{-n}$ .

It can be seen that, provided the other players do play the strategies  $\mathbf{s}_{-n}$ , player *n* should choose a strategy from the set of best responses  $S_n$  to maximise the payoff. However, suppose every player in the game has got the correct guess of  $\mathbf{s}_{-n}$  and they all follow the method of playing best response. In this case, no one would have any reason to do anything else even given another chance to reconsider. It can be claimed that the game has resulted

in a Nash equilibrium point.

Similar logic is also suitable for playing a sequential move game, however given the perfect information of all players' exact moves, every player has the absolute confident of playing the best response without guessing when it is his/her turn to play.

#### Nash equilibrium

Definition 2.2.2 (Nash equilibrium): The Nash equilibrium of an N-person noncooperative game is a joint strategy profile where no player can achieve further gain in payoff by unilaterally deviating. In pure strategies, that is  $\mathbf{s}^* = \{s_1^*, s_2^*, ..., s_N^*\}, \ \mathbf{s}^* \in \mathbb{S}$ , and satisfies for all n,

$$\mathfrak{u}_n(\mathbf{s}^*) \ge \mathfrak{u}_n(s_n, \mathbf{s}^*_{-n}), \forall s_n \in \mathbb{S}_n, s_n \neq s_n^*,$$
(2.2.2)

where  $\mathbb{S} = \prod_{n=1}^{N} \mathbb{S}_n$  is the joint strategy space of all players and  $\mathbf{s}_{-n}^* = \{s_1^*, ..., s_{n-1}^*, s_{n+1}^*, ..., s_N^*\}$  represents the pure strategies (actual action choices) of all players other than n which consistent with the equilibrium. It can be observed in (2.2.2) that, given the equilibrium strategy choices of all other players  $\mathbf{s}_{-n}^*$ , player n is able to gain a highest payoff by choosing the strategy  $s_n^* \in \mathbf{s}^*$ . As the property is true for all players, it is claimed that no player has any incentive to deviate to any another strategy from the joint strategy profile  $\mathbf{s}^*$ . Therefore,  $\mathbf{s}^*$  is the Nash equilibrium in pure strategy. The relationship shown in (2.2.2) implies that players are indeed playing their best responses to each other by choosing  $\mathbf{s}^*$ . Nash equilibrium can be alternatively defined as a set of best response strategies  $\{s_1^*, s_2^*, ..., s_N^*\}$ , where

$$\mathbf{s}_n^* \in \mathfrak{B}_n(\mathbf{s}_{-n}^*), \forall n.$$
 (2.2.3)

For example, it can be claimed that the strategy profile  $\{defect, defect\}$  is the pure strategy Nash equilibrium of the prisoner's dilemma game as

ę

		Player 2		
		Rock	Paper	Scissors
	Rock	0, 0	-1, 1	1, -1
Player 1	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

Figure 2.5. Rock-Paper-Scissors game.

shown in Figure 2.3. This is because by choosing these actions, both players best respond to each other and neither of them has any other deviation with higher payoff.

# 2.2.3 Variations of Nash equilibrium

In addition to pure strategy scenarios, Nash equilibrium also exists in mixed strategies where players play the game by mixing a number of available actual actions, each assigned with certain probability. A mixed strategy is defined as the probability distribution over the pure strategies [29]. Mixed strategies are widely considered in games with no equilibrium solution in pure strategies. For example, the simple two player 'Rock-Paper-Scissors' game, which is described in a strategic form in Figure 2.5.

As observed in Figure 2.5, playing pure strategies results in a 'cycle' of best responses. That is, for any of the nine strategy profiles, one of the players would have the incentive to deviate to another strategy. Hence, the game admits no Nash equilibrium in pure strategies. However, it can be considered that the mixed strategy profile  $\{(1/3, 1/3, 1/3), (1/3, 1/3, 1/3)\}$ , i.e., both player randomise the three strategies with the probability distri-



Figure 2.6. Traffic game.

bution (1/3, 1/3, 1/3) is the mixed strategy Nash equilibrium of this game. In the equilibrium, the game reaches all of its nine possible results, each with a probability of 1/9. The expected payoff for either player will hence be  $1/9 \times 1 + 1/9 \times 0 + 1/9 \times (-1) = 0$ .

A further relaxation of the concept of Nash equilibrium, namely correlated equilibrium was introduced in [33]. Consider the following 'traffic' game. Two players drive their cars to the same intersection at the same time. If both players choose to cross, the result is a crash, with a payoff of -100 for each player. If one of them choose to cross while the other chooses to stop, the player crossing successfully gets a payoff of 1. The game is illustrated using a strategic form in Figure 2.6.

It can be claimed that the game has two Nash equilibria in pure strategies, i.e., {cross, stop} and {stop, cross}, and one in mixed strategies, i.e., cross with a probability of 1/101 and both get the expected payoff of approximately 0.00001. However, considering that players choose their strategies independently and simultaneously, there is no guarantee that crash can be avoided. Therefore it is useful to introduce a 'third party' who is considered as a regulator to the game. The 'third party' announces a rule of game play for both players as a reference strategy. For this traffic game, the 'third party' can be seen as 'a set of traffic lights', and the reference strategy can be 'cross when green light is given, stop when red light is given'. If both players follow this rule, crash will be successfully avoided. Moreover, it can be claimed that players are playing the best responses to each other. Neither of them would have an incentive to change from following the reference because otherwise a crash will occur. This reference strategy given by the 'third party' is considered as the correlated Nash equilibrium of the game.

Various other extensions of Nash equilibrium for analysing different types of games, for example subgame perfect equilibrium for games with perfect information, as well as other types of game solutions, for example the concept of core in cooperative games, are also discussed in [32].

# 2.2.4 Existence, uniqueness and efficiency of Nash equilibrium

It is crucial that a game admits a solution. However, the existence of Nash equilibrium is not generally guaranteed in every game. Besides, games might have sole or multiple equilibrium points, each of which yields a different outcome in players' payoffs. The consideration of whether the game has unique equilibrium, as well as players' preferences for a particular equilibria will consequently arise. Therefore, for any game theoretic formulation, it is necessary to discuss the existence, uniqueness and efficiency of the Nash equilibrium.

It has been showen in [30] that every game with finite number of players and finite strategy spaces admits at least one Nash equilibrium in mixed strategies. This strong statement means that every interaction situation that can be formulated as a finite game has a solution. The author of [34] further studied the properties of Nash equilibrium for the class of concave games, with continuous (infinite) and compact strategy space. These achievements have laid out a firm foundation for the analysis of Nash equilibrium in games.



Figure 2.7. Illustration of fixed points for a function.

In a broader sense, since the Nash equilibrium can be described as the mutual information of the players' best responses, showing the Nash equilibrium of a game is equivalent to determining the fixed points of the mathematical mapping of best responses [35]. Theory on fixed points is very useful for analysing the property of Nash equilibrium in a widespread classes of games, such as the dynamic game proposed in Chapter 6 in this thesis.

A fixed point of a function is an element of the function's domain that is mapped to itself by the function, i.e., the particular points of  $x \in \text{dom} f(x)$ which satisfy f(x) = x [36]. Figure 2.7 illustrates a typical function which admits fixed points. This definition can be generalised to apply in game theory. Consider a best response mapping  $\mathfrak{B} : \mathbb{S} \to \mathbb{S}$  such that for all the joint strategy profiles  $\mathbf{s} \in \mathbb{S}$ ,

$$\mathfrak{B}(\mathbf{s}) = [\mathfrak{B}_n(\mathbf{s}_{-n})]_{n \in \mathbb{N}}.$$
(2.2.4)



Figure 2.8. Illustration of a fixed point for a correspondence mapping.

A fixed point of the best response mapping  $\mathbf{s}^*$  satisfying  $\mathbf{s}^* = \mathfrak{B}(\mathbf{s}^*)$  means that  $\mathbf{s}_*$  is a best response to itself. This reflects the definition of Nash equilibrium of everyone's strategy is a best response to the others'. Figure 2.8 illustrates the fixed point property for the best response correspondence.

The fixed points of a function (mapping) f(x) can be located numerically by adopting the fixed point iteration process. This process uses iterated functions to obtain a sequence of  $\{x_0, x_1, x_2, ...\}$ , where

$$x_{n+1} = f(x_n), n = 0, 1, 2, \dots$$
(2.2.5)

Starting with a random  $x_0$  in the domain of the function, the sequence is to convergence at the fixed point of f(x), if one exists. This mechanism is also useful for obtaining the fixed point of the best response mapping, i.e., the Nash equilibrium.

It should be stated that playing the Nash equilibrium of the game does not necessarily mean that the best result for everyone is obtained. The payoff corresponding to the Nash equilibrium can be low, even though it is the result of rational play. The performance of Nash equilibrium in overall payoffs allocation is recognised as its efficiency, also known as the Pareto optimality [32]. At a Pareto optimal allocation, it is impossible for any individual to gain higher payoff without making the payoff of at least one of others' lower. Therefore, a Pareto optimal outcome of the game is considered as socially best for the players. Consider games with multiple equilibrium points, each yields different payoff. It is natural players would prefer to play the Pareto optimal equilibrium strategies to get highest payoff. In such cases, game procedure should be more carefully designed to ensure the game converges to the most efficient Nash equilibrium.

For games with unique equilibrium, as well as those games with multiple equilibria however all yielding identical payoff, it is still valuable to consider the issue of efficiency. This is because players might play the game under consideration of coordination rather than playing the best responses. They might be willing to choose non-equilibrium strategies, provided such strategies result in a higher payoff for everyone. In particular, this situation can happen when the game admits inefficient Nash equilibrium. For example, in the prisoner's dilemma game as shown in Figure 2.3, it can be observed that the only Nash equilibrium is both player playing *defect*, which results in a payoff of  $\{-3, -3\}$ . However, players could have achieved a better result with a higher payoff of  $\{-1, -1\}$  by both playing *confess*. Therefore, it can be claimed that the only NE of the game is inefficient, also known as being Pareto dominated by the strategy of playing both *confess* [32].

Unfortunately, a Pareto efficient result may not be stable since it is not necessarily a Nash equilibrium. In games such as prisoner's dilemma, achieving the efficient result requires certain agreement (e.g., a contract or absolute trust) to force the player to coordinate. Otherwise, at least one of the players would just deviate to its best response for highest payoff when given opportunity, which ultimately leads the game back to the stable point, i.e., the Nash equilibrium. In brief, Pareto efficiency provides a view of social performance of the outcomes of games. It also provides guidance for the design of certain games, where games are desired to have a stable and efficient solution.

# 2.2.5 Obtaining Nash equilibrium

Generally, the design of game procedure is based on the consideration of playing the best responses. However, the development of algorithms for obtaining the Nash equilibrium can be much more difficult. This is true even for games with limited number of players. Besides, it can be imagined that any change of action by any of the players at any time before reaching the equilibrium, has impact on all other players' actions. They will have to be acknowledged and responded accordingly. This results in significant computational complexity, as well as information and communication overhead. Therefore, the development of workable convergence process and algorithms is crucial for effective game theoretic modeling. One of the widely applied approaches is through iterative best response dynamic process [37]. Detailed description of such process is presented in Chapter 5.

# 2.2.6 Game theoretic optimization for communications and smart grids

Game theory has been widely applied in electrical and electronic engineering, in particular in wireless communications and machine to machine communications. It provides an alternative, sometimes an even better understanding of interactive optimization situations, for example see [38] for applications in communication systems. In a wireless network, each distributed node could make its own decisions on transmitting power, forwarding packets, back off timing etc., (possibly relying on information from other nodes). These decisions may have constraints imposed by regulations or protocols. In some cases, these nodes value the network as a whole and are willing to work coordinatively. In other cases, nodes can be selfish and care only about their own interests. Regardless of the types, these nodes can be seen as autonomous players of a game. Game theory can help them to analyse the strategic situation and enable the nodes to make optimal actions in order to maximise benefits.

There are various challenges for using game theory. The first and most important one is the formulation of system parameters into abstract mathematical models. The basic three components of a game, namely the players, the strategies, and the payoffs must be clearly defined. The game must be designed in correct form. Theoretical assumptions have to be considered carefully. Various practical limitations in relations to the strategy choices should be considered. In addition to the formulation, the effectiveness of the game model, in terms of the properties of Nash equilibrium must be evaluated. Finally, workable algorithms for achieving the desirable equilibrium must be developed.

Applying game theoretic formulation to various optimization problems in communication and networks has become fashionable and very productive in recent years. Applications can be found in topics such as power control, routing and distributed protocols for cellular networks and ad-hoc networks [39–43]. The work in [41] studied the uplink power control problem where each mobile wants to maximise its throughput which depends on the transmission powers of all mobiles. A finite number of choices of power levels are available to each mobile but has a constraint on the average power consumption. In [42], by defining the quality of service (QoS) of a wireless terminal as the payoff, a distributed power control technique for many interacting terminals has been proposed using a noncooperative game framework. The authors introduced pricing for transmission powers to alter the behaviour of selfish players. This improves the Pareto efficiency of the Nash equilibrium of the game.

In multiple input and multiple output (MIMO) systems, the work in [44] proposed a strategic game where each greedy base station determines its optimal downlink beamformer in a distributed manner but without any coordination between themselves. Compared to a fully coordinated design where the optimal beamformers are jointly designed, the scheme in [44] provided benefits in terms of system complexity.

The authors of [45] proposed an interference management technique based on the theory of potential games in orthogonal frequency division multiple access (OFDMA) systems. The particular potential game formulation guaranteed a pure strategy Nash equilibrium solution, and the convergence via sequential best response dynamics. In [46], a coalitional game was successfully formulated based on a mixed integer optimization framework for artificial intelligence research. Dynamic differential game models, in which players play the game continuously for a period of time, have also been considered in topics such as optimal control of system parameters in wireless networks [37,47].

It is expected that the information and communication network for smart grids will consist of both the wired and the wireless technologies [48–50]. The aforementioned game theoretic techniques have the potential applications in the communication layer of smart grids as well. A network formation game was proposed in [51] for a multihop powerline communication (PLC) system. Based on the consideration of capacity maximization in a low reliability environment, a number of players (distributed data access points) were able to determine the best formation of a network among themselves.

Game theory is also very suitable for analysing the interaction of consumers and/or utility operators in energy demand management. Game theoretic optimization framework has the potential to provide very efficient consumer incentive based distributed consumption scheduling [52]. An energy scheduling game based on convex optimization technique was proposed in [53] to schedule energy consumption of various appliances. Constraints such as minimum standby power and maximum operating power of the appliances were formulated using a convex optimization framework. The framework has been extended to multiple household scenarios and a game was formulated to enable consumers to distributively respond to energy price information. Based on the concave game settings, the participating users have the potential to quickly move towards the Nash Equilibrium at which the consumption cost can be optimized. In [54], a two-layer optimization framework was established. At the lower level, appliances are scheduled for energy consumption for each household. At the upper level, a dynamic differential game was used to capture the interaction among different households in their demand responses through the market price. The authors of [55] proposed a leader-follower Stackelberg game between utility companies, who are seen as the leaders, and the following end consumers to maximise the revenue of each utility company and the payoff of consumers. A distributed algorithm was developed which converged to the Nash equilibrium with only local information available.

In the above discussed works, time-varying energy pricing has been considered as a method to provide economic incentives for consumers to participate in demand management. In [56], a sequential game-theoretic approach was proposed for making optimal pricing plans. It proposed models of costs to utility companies arising from consumer demand fluctuations, and models of consumer satisfaction with the difference between the nominal demand and the actual consumption. The Nash equilibrium and the optimal pricing plan were obtained by using backward induction. The work in [57] proposed a game theoretic technique by which optimal time-varying prices can be achieved. In particular, the pricing aligned individual optimality with Pareto optimality, i.e., by using such pricing plans, households automatically maximise the social welfare by selfishly optimise their own benefits.

Incorporating locally generated energy and the use of electric vehicle (EV), the authors of [58] proposed a game theoretic mechanism for analysing the energy demand of households with a photovoltaic (PV) power system and weather forecast information. According to the availability of information to predict the amount of PV power generation, differential game models were used for decentralized optimal control of energy consumption. The work in [59] also considered that the utility company is able to procure electricity from both the traditional and the renewable energy sources. The residential consumers have their plug-in EVs so that they could either consume power or supply power to the grid through the battery storage. The retailer can decide the amount of electricity purchased from the renewable and the traditional energy sources, and the corresponding price. The consumers can respond by adjusting their energy demand. The optimal solution of the game was provided through finding the subgame perfect equilibrium (SPE) of all the consumers using a backward induction method.

In [60], the problem of optimal energy distribution was studied using cooperative game theory. The dynamically changing coalition consists of one microgrid and several customers. The microgrid acts as one of the players, determines the size of the coalition for utilising the generated energy optimally which assures an efficient power distribution for consumers.

The original contributions presented from Chapter 3 to Chapter 6 in this thesis have been built based on the use of mathematical optimization and game theoretic modeling techniques explained in this chapter.

# OVERVIEW OF DEMAND MANAGEMENT IN SMART GRIDS AND ENABLING WIRELESS COMMUNICATION TECHNOLOGIES

This chapter provides an overview of demand management with a particular focus on the associated enabling wireless technologies. Various mechanisms for the optimal demand management in smart grids using these wireless technologies are also provided.

# 3.1 Introduction

There have been significant interests globally at technology level and policy level on the efficient use of energy. More intelligent smart grids are required as the demand for energy increases and more emphasizes are placed on the supply of renewal energy. The main ingredient of smart grids is the integration of information and communication technology (ICT) into the grids to monitor and control power generation and demand.

In this chapter, the features of demand management in smart grid are studied, followed by a discussion on its associated research challenges. Major characteristics of various candidate wireless technologies required to facilitate demand management is then described. The features of demand management enabling mechanisms including consumption scheduling, real-time response and load balancing are also presented.

# 3.2 Overview of Demand Management in Smart Grids

# 3.2.1 Features

Demand management mainly consists of load monitoring, analysis and response. In conventional power grids, the two sides of the electricity demand and supply system are basically disconnected, as such demand management is performed exclusively by the utility operators using mainly the raw data based local operation monitoring and state estimation. These approaches have significant drawbacks in terms of high response time (delay) and inaccuracy. The development of smart grid provides demand management with advanced features to enable many new essential functions and applications as follows:

#### **Bi-directional coordination**

In smart grid, demand management is expected to be a combination of centralized and distributed schemes. Monitoring and control activities will not only be based at the operation centers but can also be distributed across the whole network. Every node at the demand side of the network will be able to manage its own demand and consumption optimally according to the current supply condition. These activities will be acknowledged by the supply side utility operators via effective bi-directional information exchanges. Taking advantage of the full visibility of the demand condition of the grids, operators can alter their supply policies such as price rates dynamically. Both sides of the electricity market can participate in the demand management and achieve a bi-directional coordination to fulfil the requirements of the consumers while responding to the current circumstances of the grid. This will reduce the management cost of the grid operators and will potentially lead to a win-win situation for the utility operators and the consumers.

#### Data gathering and information processing

The advanced instrumentation technology enabled by real time sensing and data communication will be the most important interface of the power grids for monitoring the demand and supply. For this purpose, the advanced metering infrastructures (AMI) have been proposed to gather and convey real time raw measurement data. Advanced signal processing techniques spanning from data compression, data mining and optimizations will become important tools to extract useful information from the raw data and to generate appropriate demand and supply control messages. Monitoring specific performance parameters such as potential demand and local back-up supply capability will enable grid operators to conduct more effective and accurate demand management. The communication architecture will facilitate the data processing and analysis to be performed either locally or distributively to reduce the workload of transmission and central controls.

#### Real-time and online processing

Considering the highly dynamic nature of the energy supply from, for example renewable resources, in the electricity grids and the huge impact that can be caused by possible control delays, it is important to handle the dynamics of the supply and demand in a timely manner. As for bi-directional management in smart grid, effective communications is of paramount importance. Modern communication and Internet technologies will ensure prompt and transparent exchange of information in the network. For example, after detection of a potential outage, both the consumers and control authorities in the impacting area will be notified immediately. Early actions can be taken before further disturbances are spread. Local area data processing and demand assessment is subject to a minor delay of seconds so that associated control can respond effectively. These activities are expected to be performed online using various user interfaces. Every participant of the activities will be responded and acknowledged transparently.

#### Proactiveness

The success of the smart grid lies in the full participation of the consumers. The smart grid should enable everyone to have access and participate in demand management. Importantly, the consumers should be given incentives for participating proactively and coordinating with the operators and other stake holders in the grid. To achieve this, an efficient and transparent exchange of information system facilitated by advanced communication architecture and attractive electricity consumption and price plans are required. Proactive participation of the demand side provides the operators not only the opportunity to respond in real time to the supply and demand, but also to predict the future demand more accurately and devise appropriate actions on the generation and supply of energy.

#### 3.2.2 Challenges

There are various perceived challenges spanning from policy level to technology level including social and behavioral aspects. The policy level challenges include capital investment, enforcement rules on grid operators to provide considerable incentives to consumers, standardization of electrical appliances and third party engagement of consumer raw data. The social and behavioral aspects include trust and engagement of consumers in the demand management. The technological level challenge mainly spans the integration of high quality and low delay two-way communication infrastructure with the power grids. There are various state of the art communication technologies available, however, it is the choice of the most appropriate technology and the integration of all the components of the smart grids that will form the important challenge. To balance the supply and demand, similar techniques as used in communication networks for managing capacity of the network and the resources can be used. For example, optimization techniques using distributed strategies and game theory can be developed, as discussed in the subsequent sections.

#### 3.3 Wireless infrastructure

The information and communication network built in the smart grids is essential to facilitate the aforementioned demand management. It consists of a Home Area Network (HAN) which is formed by appliances and devices within a home to support different distributed applications (e.g. smart metering and energy scheduling in the consumer premises); a Neighbourhood Area Network (NAN) that collects data from multiple local HANs and deliver the data to a data concentrator; and a Wide Area Network (WAN), which can have a radius of tens of kilometers, is the data transport network that carries metering data to central control centers. Suitable technologies must be chosen to address various requirements in the different parts of the network. High capacity systems (a few hundred Mbps to a few Gbps) are required for WAN. Technologies such as the Long Term Evolution (LTE) wireless network, fiber optic links and the power line communications (PLC) built directly onto the power transmission network are



Figure 3.1. AMI wireless communication architecture

the available solutions [61,62]. The NANs and HANs of advanced metering infrastructure (AMI) communications are particularly suitable for wireless deployment, largely due to the ease and low cost of adopting wireless instead of wired solutions. The backhaul network connecting the AMI headend and the data aggregation points (DAPs) can either be wireless or wired. The AMI communication architecture is illustrated in Figure 3.1.

The link between the DAPs and consumers requires NANs with a coverage in the range of thousands of meters. Each DAP can connect to hundreds of smart meters (SMs). As a result, a key requirement of candidate wireless solutions is coverage of wide area, which can also be achieved through a mesh network architecture or relay stations. Additionally, the wireless network must be able to provide a certain level of reliability as well as low enough latency not only to satisfy demand side management (DSM) requirements but also to serve all other AMI applications. According to communication requirements from OpenSG [63], this translates to a minimum reliability figure of 99.5% and a latency requirement of less than 1 second, which is a relatively relaxed figure as compared to the commercial broadband requirements.

On the other hand, HANs which facilitate energy management and planning within customer premises require a relatively smaller coverage area. The requirements are also relatively less stringent as there are less control messages and information exchange between the smart meter and smart appliances (and plug-in hybrid electric vehicle (PHEV)). In general, the HAN requires a minimum reliability figure of 99.5% and a latency requirement of less than 5 seconds [63].

#### 3.3.1 Neighbourhood Area Networks

Candidate technologies for NAN have to provide coverage radius of over a thousand meter. Reliability of communication channels between the DAP and the smart meters dictates that the spectrum used will have to be exclusive or interference free. Consequently, the most suitable candidates need to be licensed or leased wireless solutions. A comparison of the characteristics of different NAN technologies can be found in Table 3.1.

#### WiMAX

Implementations of IEEE's 802.16 standard for metropolitan networks [64], commonly referred to as WiMAX (worldwide interoperability for microwave access), is a leading candidate for providing connectivity between DAPs and SMs. WiMAX is based on orthogonal division multiplexing access (OFDMA), which assigns slices of the frequency spectrum to different users [65], avoiding interference among the users and increasing the spectral efficiency of the system. Although WiMAX is not being widely adopted as a wireless broadband platform, it does not diminish its chance of being a candidate as some utilities are expected to set up dedicated DAPs. As a result, WiMAX is more attractive in the sense that its structure is much less sophisticated as compared to rival cellular standards such as 3GPP Release 8 (commonly known as Long Term Evolution (LTE)). Additionally, amendment j to the standard added multihop relay capabilities [66], which can enlarge the coverage area using low cost relay stations.

#### UMTS/LTE Cellular

Current cellular technologies such as UMTS and LTE [67] also provide attractive solutions for providing NAN coverage. Relaying functionality had also been incorporated in 3GPP Release 10 (commonly known as LTE Advanced) [68], which will allow extended coverage using relay/repeater stations. However, the utilities have to be willing to overlay DAP-SM communications over existing communication infrastructure. Although the advantage of overlaying is a lower setup cost since the existing infrastructure can be used, the utility operator will have to work with the telecommunication operators to set up the network which can be contentious due to security and privacy concerns.

#### **IEEE 802.22**

An alternative candidate to mainstream broadband wireless is the IEEE 802.22 wireless regional area network [69], which uses white spaces in the television spectrum. The IEEE 802.22 standard proposes to use cognitive radio technologies to exploit unused spectrum in the frequency spectrum allocated to television broadcast. As the spectrum used is not dedicated, the latency in data transmission could be higher as compared to other solutions mentioned earlier.

### 3.3.2 Home Area Networks

Wireless solutions for HANs have a slightly different set of requirements, which are not as stringent as those for NANs. In general, the message arrival rate within a customer premise is not as high as that between SMs

Coverage	Technology	Range	Latency	Reliability	Cost & Ease of Deployment
	WiMAX	$30 \mathrm{km}$	Low	High	Medium/Medium
NAN	UMTS/LTE	$30 \mathrm{km}$	Low	High	Medium/Low
	802.22	$30 \mathrm{km}$	Medium	Medium	High/Medium
	Wifi	200m	Medium-High	Low-Medium	Low
HAN	ZigBee	100m	Low-Medium	Medium	Low
	Bluetooth	100m	Low	Medium	Low

and DAPs. Additionally, the data volume is also much lower. A comparison of various wireless candidate technologies for HAN is provided in Table 3.1.

 Table 3.1. Comparison of candidate NAN and HAN wireless technologies

#### Wifi

IEEE's suite of standards for wireless local area networks, IEEE 802.11 or Wifi, is the most commonly deployed wireless standard within homes. As such, Wifi devices and chips are relatively cheap, making it an attractive solution. Amendment s of the standard also incorporates mesh networking capability.

#### Zigbee

Zigbee is one of the leading candidate technologies for networking of devices in HANs. The specification builds upon the IEEE 802.15.4 standard, and is tailored for mesh networking. Zigbee also has various profiles to support different applications, such as Smart Energy. Zigbee Smart Energy 2.0 profile, which adds many more features such as the support for PHEV, will be ratified by the end of 2011.

#### Bluetooth

The Bluetooth specification was designed for personal area networks. The specification supports functions such as mesh networking. Furthermore, the

specification ensures less latency as compared to the two previously mentioned standards through the use of a time division multiple access (TDMA) like medium access scheme. Both Wifi and Zigbee uses contention based carrier sense multiple access (CSMA) which can result in large latency if many devices are in operation.

# 3.4 Developments of Demand Management Approaches and Proposed Mechanisms

The success of smart grid lies in the design of flexible and robust demand management techniques underpinned by the deployment of ICT infrastructure mentioned earlier. Apart from improving the legacy load control approaches, the main contributions of recent research have been in the demand side consumption scheduling, dynamic pricing and load balancing using distributed energy resources (DER).

### 3.4.1 Demand side consumption optimization

Demand side consumption optimization is an important feature to manage the (peak) demand on the main grid and to maintain system reliability and stability. It has been an active research topic for many years. For example, some approaches in terms of peak clipping and flexible load shape shifting and related management mechanisms have been outlined in [70]. However, it is the recent advancement of communication technology that has facilitated an entirely new set of approaches and methods to perform demand side management on a real time basis. The operators could apply direct response and control mechanisms through local or remote control systems which directly control the energy usage of different appliances in the customer premises either coarsely by ON/OFF switching, or by changing operational parameters such as the temperature of hot water tank or heating system. For indirect approaches, incentive based management and social interaction can be adopted. The latter approach provides more management flexibility and enables proactive consumption optimization by the distributed consumers, which might turn out to be more cost-effective and efficient. It suits better for managing real-time/daily consumption and reducing peak loads. However, it has stricter requirements in terms of metering technologies (to support local analysis and computation) and communication security. The centralized direct management schemes are more suitable for emergency response to prevent outages.

As an analogy to the design of hierarchical topology based Internet routing, finding the most suitable system architecture of consumption management system in smart grid is an important topic. For example, a three step optimization methodology using a decision tree structure was considered in [71]. A root node acts as a global planner which tries to achieve an overall control of the load profiles. The root node can be the control centers at the utility side. It decomposes the profiles in subparts and assigns them over its follower nodes in the hierarchical structure. Follower nodes will take the responsibility to plan its part of the consumption using similar optimization techniques as the root node and will further decompose the work into the leaf nodes. The leaf nodes are directly linked to the controllers, e.g. smart meters, located at the consumers' terminals.

Communications between all the nodes are essential to support the networked coordination. As shown in Figure 3.1, the AMI architecture suits very well for the management structure. The cost-effective and short-range Zigbee/Bluetooth based wireless sensor networks can be deployed at the demand side to support exchange of information between the leaf nodes and the appliances. In the middle level which represents NAN, the low latency and high reliability 3G/4G wireless solutions can be adopted. High capacity wired technologies are suitable to handle mass data flow at the top level between DAP and AMI headend. Besides, the hierarchical decomposition supports scalability because the deployment of leaf nodes is reasonably independent of other components of the tree.

The tree structure was designed primarily for implementing global optimization to regulate consumption. For decentralized and consumer oriented mechanisms such as those based on game theory, a mesh structure is more suitable. The utility operators will only responsible for issuing interactive signals, such as dynamic pricing. It is the distributed nodes at the demand side to take full control of their consumption accordingly. In this case, highly interconnected AMI architecture especially in the NANs is desired, while also challenging.

The scheduling of appliances may introduce discomfort to the consumers mainly due to possible delays introduced by shifting the operation of the appliances. A successful scheduling should therefore ensure that the appliances are scheduled according to certain user preference. User preference can be formulated into the consumption optimization problem using additional constraints. Also, considering the discomfort as a cost of inconvenience, this can be factored into the overall optimization cost. However for operator's direct global optimization, this increases the system complexity. Therefore, a better strategy might be to provide incentives to the consumers and encourage them to participate in the demand management.

# 3.4.2 Dynamic pricing

Generally, issuing dynamic pricing policies as user incentive is the most effective way to achieve indirect demand management for the grid side operators and controllers. Dynamic pricing mainly consists of *time-of-use* (ToU) pricing, critical peak pricing (CPP), time block based pricing (TBBP), and real-time pricing (RTP), as listed in Table 3.2.

ToU and CPP rates have already been included in many utility con-

tracts/tariffs in the current electricity markets for load control purposes. In order to account for the dynamic demand in smart grid, pricing policies should be updated frequently. It is believed that a combination of time block based pricing and ToU/CPP rates could be a possible solution for the early stage of smart grid development with limited ICT deployments. Some economically driven consumption optimization algorithms, such as that in [53], consider this kind of pricing schemes. However, the performance of such pricing mechanism will highly depend on the accuracy of the demand estimation/prediction and risk assessment. Self-learning algorithms can be used for demand prediction. Various risk control mechanisms used in business research can also be adopted in the design of pricing policies.

Pricing	Characteristics	Cost & ease of
policy		deployment
Time-of-use	One-off issuing rates depending on the time of use	Load estimation
pricing	Limited performance for dynamic demand control	Low ICT requirements
Critical peak	One-off issuing rates depending on particular events	Hard to define critical events
pricing	Critical rate for pre-defined peak times (or loads)	Low ICT requirements
	Limited performance for dynamic demand control	
Time block based	Monthly/weekly/daily updating rates	Load prediction and risk control
pricing	ToU rates or load-sensitive rates	Non-real-time ICT required
	Enhanced performance for dynamic demand control	
Real-time	(Near) real-time updating rates	Advanced real-time ICT required
pricing	Advanced performance for dynamic demand control	High communications overhead

#### Table 3.2. Dynamic pricing schemes

RTP is believed to be robust in terms of responding to the dynamics even when there is unpredictable energy demand in the grid. The main challenge of implementing RTP is the expectation of a high quality communication infrastructure for real-time monitoring purposes. Latency will be the primary concern in choosing the communications solution for RTP. In order to support continuous and mass flow of data, the throughput of the communication network should also be very high. Finally, the power consumption of the communication infrastructure itself has to be managed optimally.

# 3.4.3 Distributed energy resource (DER) management

In addition to balancing the supply and demand, the smart grid when integrated with the distributed energy sources will enable the consumers to choose different type of energy sources and suppliers as well as to optimally use and sell back the locally generated and stored energy. The optimal use of available energy at different times can help reducing the dependency on the central supply. The grid side utilities can balance the load by choosing the supply from different generation systems especially at peak demand periods. For example, the work in [72] discusses how various energy supplies can be aggregated and dispatched. The idea of load-based services can bring true benefit for the access of varying energy generation from green resources (such as wind and solar) and smart charging of PHEVs. Residential DER management based on instantaneous supply conditions (both from the central energy source and the distributed energy sources) is also an important research topic. For example, various households in a neighbourhood area can share locally generated power and draw power from the central energy source only when it is required. The authors of [73] developed a decision-support algorithm using particle swarm optimization (PSO) to support this kind of schemes. Accurate estimation of system frequency is a prerequisite for the integration of distributed energy resource. In [74], approaches to adaptive frequency estimation were introduced in a three-phase system with balanced and unbalanced operational conditions. This work also proposed solutions for system fault identification and also mismatched generation and load.

### 3.5 Summary

One of the main functions of the smart grids is to perform demand management to reduce peak loads. This requires acquisition of real time data from various points in the grid and optimization of the power supply and demand. The smart meters and sensors will be deployed in various parts of the grid, starting from the generation, through distribution, and all the way to the household level. These will be interconnected through both wired and wireless connections. Wireless solutions are preferred at the NAN and HAN levels and wired connections could be used for backhaul networks. In order for the demand management to be successful, consumers should be given adequate incentives for full participation. This chapter covered various candidate communication technologies and mechanisms to enable demand management, in particular for home and neighbourhood areas. As communication is an underpinning technology for the success of smart grid, it can be envisaged that smart grids will be an exciting research area in communication engineering.

# A MIXED INTEGER LINEAR PROGRAMMING BASED CONSUMPTION SCHEDULING TECHNIQUE

In this chapter, a consumption scheduling technique for home area demand management using mixed integer linear programming (MILP) is proposed. The aim of the proposed scheduling is to minimise the peak hourly consumption in order to achieve an optimal (balanced) daily load schedule. The proposed mechanism is able to schedule the optimal operation time for all appliances according to their power consumption patterns. Simulation results based on home and neighbourhood scheduling scenarios are presented to demonstrate the effectiveness of the proposed technique.

# 4.1 Introduction

Consumption scheduling is one of the important fundamental approaches for grid operators to achieve centralised peak load control. Combined with a home energy management unit, the smart meters not only provide power to every household appliance but also serve to collect information on that appliance's consumption pattern and globally optimize the total power consumption. Recently researchers have explored the use of optimization algorithms to achieve end-consumer energy management [75–77]. In [53], a powerful convex optimization (linear programming) technique was proposed to schedule the power of individual appliances. However, the optimization framework in [53] might not be suitable for all appliances in practice. This is because some appliances have a fixed power consumption pattern, which means that once the appliance is scheduled for operation, it has to operate according to its own power consumption pattern until the task is finished. In this case, only the starting time can be optimised, but the power consumption during the operation of the appliance is not under the control of the optimizer, hence it is not an optimization parameter.

In this chapter, a consumption scheduling technique based on mixed integer linear programming is proposed. The proposed technique will be able to optimally schedule the daily operation of home appliances in order to minimise the peak hourly consumption and satisfy both the user preference and specific requirements of all individual appliances. The optimization technique is also suitable for the scheduling of the energy consumption expected from grid connected electric vehicles (EVs), known as grid to vehicle (G2V). In addition, the situation of the connected EVs acting as energy resources that provide power to the grid, known as vehicle to grid (V2G), is also considered in the optimization.

The scheduling technique can be applied to home area energy management. Figure 4.1 depicts the overall structure of a home energy management system. It can be seen that the energy management unit (EMU) is the key component. It connects with the user interface to collect the user's own power consumption plan and preference and display the scheduling information. On the other side, it connects with all the home appliances, not only to provide electricity for the appliances but also to determine the total require-



Figure 4.1. System components.

ments and power consumption patterns of all individual appliances. Based on all the collected information, the meter will globally optimize the hourly consumption and schedule all appliances. For non-shiftable appliances, the optimization will ensure the supply of power. The scheduling optimization will be carried out mainly for the shiftable appliances for which the EMU will be able to make an optimal scheduling.

The system can be further extended to scenarios in which many EMUs are connected together and they agree to achieve a global scheduling. The central control node will take the overall responsibility of scheduling the whole network and assigning individual EMUs their corresponding tasks. Effective communication networks are required for the system. Wireless sensor networks (WSN) combines sensing and communications together, and provides low-cost and low-power information gathering, processing and communication with flexible self-organising network deployments [78, 79]. WSN will be one of the promising technologies for communications between appliances and the EMU. For communications beyond the home environment, the obvious candidates are wireless cellular technologies and various broadband solutions as discussed in Chapter 3.
The optimization framework can be further applied to distributed demand management where individual consumers participate proactively in consumption scheduling, as will be discussed in Chapter 5.

This rest of this chapter is organised as follows. The detailed mathematical formulation of the home area appliances scheduling optimization is presented, followed by a set of numerical simulations to demonstrate the proposed approach. Conclusions are drawn at the end of this chapter.

#### 4.2 MILP Based Home Consumption Scheduling Optimization

#### 4.2.1 Classification of appliances

Home appliances are the objects of the scheduling optimization. They are classified into two groups namely non-shiftable and shiftable appliances.

#### Non-shiftable appliances

Appliances for which scheduling is not possible are defined as non-shiftable appliances. For example, a fridge normally operates continuously throughout the day, and a central heating system needs to be in operation whenever it is required by the consumer. The operations of these appliances are strictly dominated by user comfort and convenience. Shifting operations of these appliances can bring considerable discomfort to consumers, hence not allowed. Alternative approaches for managing user preferences and benefits can be referred to [80] and [57].

#### Shiftable appliances

The second class of appliances is called as shiftable appliances whose operations can be scheduled during certain predefined periods. Appliances such as washing machines, storage heating systems belong to this class. The use of plug-in EVs can also be viewed as shiftable operation. The consumers can



Figure 4.2. Illustration of different types of operations of appliances.

tolerate the shift (postponing) of the operations of these appliances as long as the required operations will be finished within a preferred time period.

For certain shiftable appliances, the scheduling optimization should allocate power according to the appliance's own power consumption pattern during the operation period while ensuring the fulfilment of the daily consumption requirement. To this extend, it is necessary to distinguish the elastic and inelastic demands within the class of shiftable appliances. The elastic demand means that there is a flexibility to change the power consumption pattern of the appliances, for example, storage heating system, and preemptive scheduling is possible for these appliances. Inelastic demand means the power consumption pattern of the appliance cannot be changed during the operation of the appliance, for example a washing machine can be considered within this class and the scheduling for these appliances are non-preemptive [81].

For appliances whose operation can be performed non-continuously as in preemptive tasks, the whole operation time can be divided into several non-continuous time slots. The power consumption for each slot can be individually specified. For example, for a water tank boiler, the operation can be broken into several heating tasks throughout the day and certain level of power is required to complete each task. Mathematically, the scheduling mechanism will treat every task as an individual appliance that can be scheduled to operate at different times. However, some appliances, such as washing machines or dish washers have inelastic demand, i.e. should have a pre-programmed operation, which requires continuous and non-preemptive power consumption. This means when the appliance starts its operation, it will need to draw power according to its own consumption pattern and cannot be changed until the operation is finished. In this case, the total operation should be scheduled as a whole with the power supply according to the appliance's consumption patterns. Fig. 4.2 illustrates typical operations and power consumptions of various classes of appliances.

### 4.2.2 The optimization formulation

The consumption scheduling mechanism can be described as a optimization problem with a objective function as shown below

$$\min_{\substack{\Gamma, x_{n,a,t} \in \mathbb{R} \\ \text{s.t.}}} \Gamma$$

$$\sup_{a \in \mathbb{A}_n} x_{n,a,t} \leq \Gamma, \ \forall t \in \{1, ..., T\},$$

$$\sum_{\substack{a \in \mathbb{A}_n \\ \sum_{t=t_{n,a,s}}^{t_{n,a,f}}} x_{n,a,t} = l_{n,a}, \ \forall a \in \mathbb{A}_n.$$

Consider consumer n has a set of home appliances  $\mathbb{A}_n$ . An appliance  $a \in \mathbb{A}_n$  has a total daily energy consumption requirement of  $l_{n,a}$ . The vector  $\mathbf{x}_{n,a} = [x_{n,a,1}, x_{n,a,2}, \dots, x_{n,a,T}]^T$  is used to denote the scheduled energy consumption over the day for the appliance a. The parameter  $x_{n,a,t}$  denotes the energy consumed by the appliance a of user n at time t. t is the time-of-use parameter which is also the time slot indicator for the scheduling optimiza-

tion. T accounts for the time resolution, for example, T = 24 and T = 1440represent respectively the hourly and minute based scheduling. A time resolution of one hour for hourly scheduling is used in this thesis, hence  $\mathbf{x}_a$ contains 24 elements. The variable  $\Gamma$  denotes the peak accumulated hourly consumption which should be greater than or equal to the sum of the scheduled power for all appliances in an hour. Suppose the appliance a is required to operate between the preferred time instants  $t_{n,a,s}$  and  $t_{n,a,f}$ , its total energy requirement should be ensured by the constraint  $\sum_{t=t_{n,a,s}}^{t_{n,a,f}} x_{n,a,t} = l_{n,a}$ .

The above optimization problem is aimed to minimise  $\Gamma$ , i.e., to minimise the peak accumulated hourly consumption. It has the effect of suppressing the peak consumption at a particular time to reduce peak load and balance the consumption over time. For the complement of the scheduling optimization, the next step is to formulate the requirements for all the individual appliances and the user preferences as additional constraints to be included into the optimization problem.

#### Scheduling constraints for appliances

The consumption requirements of various appliances are formulated as various constraints in the optimization problem. A shiftable appliance  $a \in \mathbb{A}_{n,s} \subset \mathbb{A}_n$  can have a predefined power consumption pattern which can be written as  $\mathbf{p}_{n,a} = [p_{n,a,1}, p_{n,a,2}, \dots, p_{n,a,T}]^T$ . In this case only the optimal starting time can be scheduled. The scheduling result  $\mathbf{x}_{n,a}$  can be viewed as one of the cyclic shifts of the pattern  $\mathbf{p}_{n,a}$ . All possible shifts for the vector  $\mathbf{p}_{n,a}$  can be put together in a matrix form as

$$\mathbf{P}_{n,a} = \begin{bmatrix} p_{n,a,1} & p_{n,a,T} & \dots & p_{n,a,3} & p_{n,a,2} \\ p_{n,a,2} & p_{n,a,1} & \dots & p_{n,a,4} & p_{n,a,3} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{n,a,T} & p_{n,a,T-1} \dots & p_{n,a,2} & p_{n,a,1} \end{bmatrix}, \ \forall a \in \mathbb{A}_{n,s}.$$
(4.2.1)

A binary integer vector  $\mathbf{s}_{n,a} = [s_{n,a,1}, s_{n,a,2}, \ldots, s_{n,a,T}]^T$ ,  $s_{n,a,t} \in \{0, 1\}$ is defined as the switch control of the power consumption from the main supply for the shiftable appliance  $a \in \mathbb{A}_{n,s}$ . There is only one non-zero element in the vector  $\mathbf{s}_{n,a}$  which is equal to one. Hence the vector  $\mathbf{s}_{n,a}$  is an optimization parameter which chooses appropriate column from  $\mathbf{P}_{n,a}$  to optimise the energy consumption, i.e.,

$$\mathbf{x}_{n,a} = \mathbf{P}_{n,a} \mathbf{s}_{n,a}, \quad \sum_{t} s_{n,a,t} = 1, \ \forall a \in \mathbb{A}_{n,s}.$$
(4.2.2)

For a non-shiftable appliance with a strictly inflexible operation requirement, the consumption scheduling should be fixed as required by the consumer. Suppose there is a non-shiftable appliance  $a \in \mathbb{A}_{n,f}$ ,  $\mathbb{A}_{n,f} \subset \mathbb{A}_n$ , with the power consumption pattern  $\mathbf{p}_{n,a}$ . Since both the value and the position of the elements in  $\mathbf{p}_{n,a}$  cannot be changed, the scheduling constraint can be written as

$$\mathbf{x}_{n,a} = \mathbf{p}_{n,a}, \ \forall a \in \mathbb{A}_{n,f}. \tag{4.2.3}$$

As discussed before, the proposed mechanism treats every part of a breakable operation as an individual appliance operating in different time slots. Suppose the operation of an appliance can be decomposed into  $\mathbb{K}_a$  scheduling tasks (appliances),  $\mathbb{K}_a = \{a_1, ..., a_k\}$ . The total consumption scheduling can be denoted as the sum of all individual tasks, i.e.,  $\mathbf{x}_{n,a} = \mathbf{x}_{n,a_1} + \mathbf{x}_{n,a_2} + \cdots + \mathbf{x}_{n,a_k}$ . Each task has its consumption requirement of  $l_{n,a_k}$  and a power profile  $\mathbf{p}_{n,a_k}$  which can be decomposed from those of the original appliance. Switch variables also used to schedule each of the consumption tasks  $\mathbf{x}_{n,a_k}$  and formulate them into the following set of constraints

as similar to (4.2.2),

$$\mathbf{x}_{n,a_k} = \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k}, \quad \sum_t s_{n,a_k,t} = 1, \tag{4.2.4}$$

$$\sum_{k \in \mathbb{K}_a} \mathbf{s}_{n, a_k} \le \mathbf{1}, \forall k \in \mathbb{K}_a, \forall a \in \mathbb{A}_{n, s}.$$
(4.2.5)

where constraint (4.2.5) ensures that the divided tasks are operating in different time slots and  $\mathbf{1} = [1, ..., 1]^T$ . Actually, an appliance with non-breakable operation can be viewed as a particular appliance with only one scheduleable task.

Based on the above formulations, the optimization problem for the individual consumer n is modeled as a minimization of peak hourly load through optimum scheduling of consumption, subject to the consumption requirements of all appliances, as follows:

$$\min_{\substack{\Gamma, x_{n,a_k,t} \in \mathbb{R}^+, \\ \mathbf{s}_{n,a_k} \in \mathbb{Z}^{+^{T \times 1}}}} \Gamma \\
\text{s.t.} \quad \sum_{a \in \mathbb{A}_n} x_{n,a,t} \leq \Gamma, \ \forall t \in \{1, ..., T\}, \\
\sum_{k \in \mathbb{K}_a} \sum_{t=t_{n,a_k,s}}^{t_{n,a_k,f}} x_{n,a_k,t} = l_{n,a}, \ \forall a \in \mathbb{A}_n, \\
\mathbf{x}_{n,a} = \mathbf{p}_{n,a}, \ \forall a \in \mathbb{A}_{n,f}, \\
\mathbf{x}_{n,a_k} = \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k}, \sum_t s_{n,a_k,t} = 1, \\
\sum_{k \in \mathbb{K}_a} \mathbf{s}_{n,a_k} \leq \mathbf{1}, \forall k \in \mathbb{K}_a, \forall a \in \mathbb{A}_{n,s}. \quad (4.2.6)$$

The optimization problem above is a mixed integer linear programming (MILP) which contains both integer variables and non-integer variables. It can be solved using Branch and Bound method [12]. The method divides the large problem into smaller ones based on the enumeration of the integer solutions. It uses linear programming relaxation to estimate how good a solution it can get for each smaller subproblems by dividing the subproblem further using a tree structure, until a problem with the optimal solution is reached. Note that if there are more than one global optimal solution (with equal cost value) to the problem, the method will provide one of them.

The mechanism can be applied to consumption optimization of individual household or centralised scheduling systems where all appliances of the households are included in one optimization. However, due to the nature of using Branch and Bound for integer problems, the computational complexity can grow exponential with the problem size, i.e., the number of appliances.

## 4.3 Simulation Results

A set of system simulations have been carried out using Matlab to implement the proposed scheduling mechanism in a single home area scenario and multiple household scenarios. Scheduling simulations incorporating the use of EVs are also carried out in a neighbourhood setting, in order to illustrate the effect of EV penetration in residential demand management.

#### 4.3.1 Simulation parameters

In the simulation, a set of home appliances and their individual daily consumption requirements are defined as listed in Table 4.1. The selection of home appliances and their individual consumption patterns are according to the guidance as can be found in [82] and [83].

#### Modeling electric vehicles

It is important that the scheduling of EV charging does not exacerbate the peak load problem by creating additional peaks at times that previously would have had weak demand. One strategy that reduces this possibility is to charge the EV during lunch and nighttime and avoid time intervals that has already experienced large peak loads [84].

Name	Туре	User preference and	
		power requirement	
1.Hob	Non-	Operating period:	
and oven	shiftable	$7 \mathrm{pm}$ - $8 \mathrm{pm}$	
		Hourly consumption: 1kWh	
2.Heater	Non-	Operating period:	
	shiftable	9pm-10pm, $3$ am- $5$ am	
		Hourly consumption: 1kWh	
3.Fridge	Non-	Operating 24hrs	
and freezer	shiftable	Hourly consumption: 0.12kWh	
4.Water boiler	Shiftable	Hourly consumption: 0-1.5kWh	
	preemptive	Daily requirement: 3kWh	
5.Electric vehicle	Shiftable	Plug-in period:	
(15 miles daily	preemptive	8pm-8am	
driving [85])		Charging power: 0.1kW-3kW	
		Daily requirement: 5kWh	
6.Washing	Shiftable	Operating 2hrs, once per day	
machine	non-preemptive	1kWh for the 1st hr	
		0.5kWh for the 2nd hr	
7.Dish washer	Shiftable	Power: 0.8kWh for 1 hour	
	non-preemptive	Daily requirement: 0.8kWh	

 Table 4.1. Appliances and power consumption patterns



Figure 4.3. Hourly power consumption schedule.

In addition, the process of charging and discharging battery power, known as cycling, can considerably reduce the lifetime of EV batteries. This cycling cost needs to be considered when embarking on a successful EV scheduling [86]. A maximum of two cycles per day are allowed in the simulations. Round-trip energy loss is another optimization parameter that needs to be factored into any successful V2G strategy. This occurs when energy from the grid is used to charge EV batteries or in the reverse case when EV energy is used to power the grid. For instance, charging a battery from the grid results in a 20% loss whilst sending energy from the battery to the grid, which involves inverting DC power incurs an additional 10% loss [86].

## 4.3.2 Results and discussion

After incorporating all the corresponding power requirements and consumption patterns as optimization constraints, optimal scheduling can be achieved by solving the MILP problem in (4.2.6). Figure 4.3 depicts the scheduled hourly load. It can be seen that the maximum hourly load of 1.22kWh appears during the hours of 3-5, 14 and 19-22. The minimum load of 0.3kWh appears during the hours of 9-13 and 17-18. From the appliances consumption requirement, it can seen that when the heater is turned on between hours of 21 and 22, and considering the power of the fridge and the charging of electric vehicle battery, the required load is 1+0.12+0.1=1.22 (kWh). For any time of the day, the scheduled load is less than or equal to 1.22kWh. Hence it can be claimed that the optimal hourly scheduling with a minimum possible peak is achieved. When the number of appliances is increased, a more balanced hourly power consumption schedule can be expected.

Figure 4.4 depicts the scheduling result for the individual appliances. Clearly, it can be seen that during 9-10pm, the heater (app2) is consuming fixed 1kWh and the fridge (app3) is consuming 0.12kWh. The water boiler (app4) has been totally shut down and only 0.1kWh which is the minimum charging power requirement for the electric vehicle (app5) is provided. Timeshiftable appliances were not scheduled to operate during the period. Indeed, the washing machine (app6) and the dish washer (app7) are both scheduled in the day time between hours of 14 and 17 where there is lower demand from other appliances.

A simulation for the scenario of a small neighbourhood area with four households is conducted. Assume the total daily consumption requirement for the neighbourhood is 43 units. Each house has similar appliances as in the previous case but with different consumer preferences and power patterns for some of the appliances. All the required information is collected from the EMU of each household. Now a centralised scheduling which will meet all the requirements is achievable. Figure 4.5 depicts the overall scheduled hourly load. As seen, the optimized hourly peak load is 2.14kWh which is just around one unit higher than that of the previous case. Besides, the overall load allocation is more balanced over the 24 hours. The peak to average load ratio is 2.14/(43/24) = 1.19, which means the peak load is just 19% higher than the daily average. It can be observed that the scheduling mechanism is able to reduce the maximum load and improve the performance and the reliability of the power grid due to reducing the peak to average load.

In order to understand how the consumption scheduling is affected by the introduction of EV, simulations are conducted with respect to a neighborhood of 30 households and with the number of EV varying from 0 up to 90. The home appliances are assumed to be operated mainly in the latter half of the day. For each simulation the appliances' constraints are kept constant. The use of EV is randomized to allow for natural variation in user driving and parking habits, but still followed a general V2G/G2V profile. During the early morning, majority of the EVs are charged to an upper limit in keeping with battery requirements. Between approximately the hours of 6 and 9 the EVs are disconnected from the grid to simulate a typical work commute



Figure 4.4. Optimal schedule for individual appliances.



Figure 4.5. Hourly power consumption schedule for multiple households.



**Figure 4.6.** Appliances' total energy consumption profile when zero EV present.



Figure 4.7. Appliance energy consumption when 30 EVs are modeled.

and parking schedule. When the EV returned home in the afternoon they are forced into a charge cycle to ensure that they had adequate charge for any potential evening journeys and/or to provide dispatchable energy to the grid.

Figure 4.6 depicts the initial scheduling with zero EV penetration. As



Figure 4.8. Appliance energy consumption when 90 EVs are modeled.

can be seen, home appliances create a consumption peak toward the latter half of the day at approximately 56kWh. The peak to average ratio is approximately 1.40. The optimization reduces the peak that would have been created from the appliances between the hours of 18 and 22 by offsetting their consumptions.

In Figure 4.7 and Figure 4.8, it is observed that the total energy consumption has been balanced over 24 hours and reached a small peak to average ratio of 1.02. As the number of EV is increased from 30 to 90 the overall energy consumption also increases but the balanced load profile remains. In both figures, it is observed that the EV energy consumption offsets the shiftable devices. The use of V2G mode in the evening time reduces the peak load that would have been created by the home appliances.

## 4.4 Summary

In this chapter, a mixed integer linear programming based consumption scheduling optimization technique is proposed. The proposed mechanism is able to schedule the optimal operation time for appliances according to user preference and the power consumption patterns of the individual appliances. Appliances' consumption requirements were classified into different groups and formulated into the optimization. Simulation results of home and neighbourhood demand management scenarios demonstrated the effectiveness of the mechanism in terms of reducing the peak to average ratio. When multiple households participate in the scheduling as well as EVs are intergraded, a more balanced hourly load profile is achieved.

# A GAME THEORETIC OPTIMIZATION FRAMEWORK INCORPORATING LOCALLY GENERATED ENERGY

In addition to the proposed centralised consumption scheduling technique, this chapter discusses the approach of consumer incentive based indirect demand management. A game theoretic consumption scheduling optimization framework is proposed with the aim of achieving minimum energy cost for individual consumers. It is shown that the game theoretic algorithm converges to an equilibrium where all consumers can benefit from participating in. Simulation results are presented to demonstrate the proposed approach and the benefits of home demand management.

# 5.1 Introduction

As micro level local renewable energy generation such as rooftop solar cells and the use of hybrid electric vehicles become popular, electricity can be generated and stored by consumers and can be released to the grid when necessary [5]. In the future, consumers will have the flexibility to consume energy from various sources and make the best use of locally generated energy. Demand management for smart grids needs to be efficient in terms of optimizing energy demand and supply. Recent advances in information and communication technologies (ICT) have enabled real time monitoring and control of the grid's operational conditions. In particular, the utility operators and consumers can communicate and cooperate in order to facilitate bi-directional load control and consumption optimizations.

Indirect demand management system operates through incentives, such as pricing, energy trading/brokering and even social interaction [87]. Issuing attractive price plans containing changeable rates, for example block based time-of-use (ToU) pricing for different times and dynamic/real-time pricing schemes [88], provides consumers economic incentives to manage their energy consumption efficiently and to reap financial benefits. Candidate models for electricity market have been studied in [89–92]. Considering that households are directly responsible for actual energy consumption and management of local energy generation, consumer oriented proactive and distributed consumption scheduling is very attractive. A number of households in a neighbourhood area could participate locally to reduce the peak load based on very minimum level of instruction from utility operators. Hence the indirect demand management can benefit from low delay and low data traffic for controlling the appliances.

The development of appropriate optimization algorithms for energy consumption is essential for facilitating demand management. The optimization technique proposed in the previous chapter is able to provide an optimal consumption plan for individual appliances, with an objective of reducing the peak load. In addition to the mathematical programming, game theoretic modeling is very suitable in particular for consumer oriented consumption optimization, which is claimed to have the advantage of reduced computational complexity.

In this chapter, a decentralised consumption scheduling optimization framework is proposed with the aim of achieving minimum energy cost for individual consumers. The framework is based on the MIP optimization technique proposed in the previous chapter and game theory. In particular, the optimization incorporates the use of locally generated renewable energy in order to minimise dependency on conventional energy and the consumption cost.

This rest of this chapter is organised as follows. The detailed formulation of the consumption optimization for individual consumers is firstly proposed, followed by the formulation of the scheduling game among these consumers. The existence of Nash equilibrium, which promises the game a stable solution, the convergence algorithm and consumers' behaviour will be analysed. Numerical simulation results are presented to demonstrate the proposed approach. Conclusions are drawn at the end of this chapter.

# 5.2 The MIP Scheduling Optimization Framework

Based on the design of the household level consumption scheduling optimization as proposed in Chapter 4, an optimization technique for scheduling energy consumption of appliances with the aim of minimising the cost of energy is proposed. In particular, it is considered that the household consumers are able to draw power from both the main grid under a given pricing plan as well as from locally generated renewable energy sources. The optimization is also expected to maximise the usage of locally generated energy while drawing energy from the main grids optimally whenever required.

#### 5.2.1 The optimization objective

Consider a daily pricing scenario where the cost of energy is determined as a function of time and energy load generated by all subscribing users in the billing area. The total cost of the energy consumption of all users is represented by a vector  $\mathbf{C} = [C_1, C_2, \ldots, C_t, \ldots, C_T]^T$ , where t is the timeof-use parameter. The cost of energy  $C_t$  at time t, is written as a function of the accumulated load of all consumers  $L_t$  as follows,

$$C_t = \omega_t L_t^2 + \omega_t L_t + \phi_t, \qquad (5.2.1)$$

where  $\omega_t$  is the basic unit rate which can take different values for different time instances and  $\phi_t$  is an independent standard charge at time t, e.g., additional fees for critical peak events [93]. We denote the scheduled daily consumption result for individual consumer  $n \in \mathbb{N}$  as  $\mathbf{L}_n = [L_{n,1}, L_{n,2}, \ldots, L_{n,T}]^T \in \mathbb{R}^{T \times 1}$ , where  $L_{n,t}$  is the energy consumption of consumer n at time t, and  $\mathbb{N}$  denotes the set of N consumers. Hence  $L_t = \sum_{n \in \mathbb{N}} L_{n,t}$ . We also denote  $\mathbf{L} = \sum_{n \in \mathbb{N}} \mathbf{L}_n \in \mathbb{R}^{T \times 1}$  as the consumption scheduling profile of the area considered in the optimization. The energy consumption cost of the day can be calculated as

$$C(\mathbf{L}) = \sum_{t} C_{t} = \sum_{t} (\omega_{t} (\sum_{n \in \mathbb{N}} L_{n,t})^{2} + \omega_{t} (\sum_{n \in \mathbb{N}} L_{n,t}) + \phi_{t}).$$
(5.2.2)

The pricing plan can be viewed as a continuous function approximating the existing stepwise (multi-step) pricing models adopted in the current electricity market [94] and [95]. However, most of these models were designed to charge individual consumers at different rates according to their monthly/yearly accumulated energy consumption in a bid to encourage them to save energy. These models have limited leverage on customers to reduce the instantaneous peak load of the area at various times. By issuing ToU pricing, utility operators can benefit from reducing peak loads by enforcing high basic rates during peak demand periods. The consumers will be motivated to shift their consumption as much as possible from the peak ToU periods due to reduced cost. In particular, it can be observed in (5.2.2) that the cost is not only proportional to the ToU rate  $\omega_t$  but also increases quadratically with the total instantaneous load  $L_t$ . Therefore the actual cost per unit of consumption (assumed  $\phi_t = 0$ ) at time t is  $C_t/L_t = \omega_t L_t + \omega_t$ , which is a function of both the basic ToU rates and the sum of the instantaneous loads. The cost of energy charged to the consumer n at time t is

$$C_{n,t} = (\omega_t L_t + \omega_t) L_{n,t}.$$
(5.2.3)

This relationship implies that the consumption cost of every consumer depends on the current total load dynamics and the consumer's use of energy. Hence, consumers have incentive to participate in reducing the total instantaneous load as this will in turn reduce individual energy cost. As all consumers are assumed to be rational, constructive participation of consumers to reduce overall peak load is possible. Utility operators also have the flexibility to control the peak accumulated load at various times.

Having provided this type of pricing scheme, consumers are encouraged to manage the demand to achieve low cost by shifting the consumption from peak ToU periods. Consumers have the incentives to further reduce the cost by shifting their consumptions in terms of balancing the load among each other to avoid overlapping loads at various times. These activities are modeled using a constrained game in the next section.

For a given aggregated load profile  $\tilde{L}_{n,t} = \sum_{i \in \mathbb{N}, i \neq n} L_{i,t}$  of all consumers other than n, the consumer n aims to optimise the following,

$$C_{(n)}(\mathbf{L}) = \sum_{t} (\omega_t (L_{n,t} + \tilde{L}_{n,t})^2 + \omega_t (L_{n,t} + \tilde{L}_{n,t}) + \phi_t).$$
(5.2.4)

It is needed to highlight again that by reducing the total cost C in (5.2.2), each consumer aims to reduce the cost of his/her energy use as in (5.2.3). The subscript (n) is used to explicitly indicate consumer n's contribution of the optimization of cost.

In the proposed mechanism, it is assumed that the energy consumption from locally generated sources and the storage elements results into zero cost to the consumer's electricity bill. Therefore the consumers will attempt to make full use of local energy supply to minimise dependency on conventional energy and to optimise the consumption cost charged by the utility operator. Consumers will reserve any unused/surplus local energy for future use or may be able to release it to the main grid and generate revenue [96]. However this is not considered in the proposed optimization framework.

## 5.2.2 Constraints formulation

The classification of appliances and the methods of formulating constraints for the appliances are consistent with those described in the previous chapter. That is, for a non-shiftable appliance with a strictly inflexible operation requirement, the scheduling constraint can be written as

$$\mathbf{x}_{n,a} = \mathbf{p}_{n,a}, \ \forall a \in \mathbb{A}_{n,f},\tag{5.2.5}$$

Constraints for a shiftable appliance, which may be further decomposed as multiple scheduling tasks, are formulated as

$$\mathbf{x}_{n,a_k} = \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k}, \quad \sum_t s_{n,a_k,t} = 1, \tag{5.2.6}$$

$$\sum_{k \in \mathbb{K}_a} \mathbf{s}_{n, a_k} \le \mathbf{1}, \forall k \in \mathbb{K}_a, \forall a \in \mathbb{A}_{n, s}.$$
(5.2.7)

In particular, for the scheduler to be able to incorporate local energy, i.e., draw power optimally from either the main grid or local energy sources for every appliance, a separate vector  $\mathbf{s}_{n,a}^{l}$  to indicate the switching parameter for the local energy consumption for appliance *a* is introduced. Now the appliance has two switching parameters to determine its operation time and the source of energy. The scheduling constraints for a shiftable appliance

can be formulated as follows,

$$\mathbf{s}_{n,a_k} + \mathbf{s}_{n,a_k}^l \le \mathbf{1},\tag{5.2.8}$$

$$\mathbf{x}_{n,a_k} = \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k} + \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k}^l, \tag{5.2.9}$$

$$\sum_{k \in \mathbb{K}_a} \mathbf{s}_{n, a_k} \le \mathbf{1}, \sum_{k \in \mathbb{K}_a} \mathbf{s}_{n, a_k}^l \le \mathbf{1}, \forall k \in \mathbb{K}_a, \forall a \in \mathbb{A}_{n, s}.$$
 (5.2.10)

Considering that local energy resources, such as wind and solar, could be intermittent, local energy should be scheduled only when its available capacity is sufficient for supplying power during the appliance's operation period. Therefore, it is needed to ensure that the scheduled power consumption from local energy supply for all appliances must not exceed the consumer's local generation and storage capacity  $y_n$ , i.e.,

$$\sum_{a \in \mathbb{A}_n} \sum_{k \in \mathbb{K}_a} \mathbf{P}_{n, a_k} \mathbf{s}_{n, a_k}^l \le y_n.$$
(5.2.11)

#### 5.2.3 The local scheduling optimization problem

Based on the above formulations, the optimization problem for the individual consumer n as minimization of utility cost as defined in (5.2.4) through optimum scheduling of consumption load, subject to the consumption requirements of all appliances and the capacity of local energy supply, is formulated as follows:

 $\mathbf{s}$ 

$$\min_{\substack{x_{n,a_k}, t \in \mathbb{R}^+, \\ n,a_k, s_{n,a_k}^l \in \mathbb{Z}^{+T \times 1}}} C_{(n)}$$
s.t. 
$$\sum_{a \in \mathbb{A}_n} \sum_{k \in \mathbb{K}_a} x_{n,a_k,t} = L_{n,t},$$

$$\sum_{k \in \mathbb{K}_a} \sum_{t=t_{n,a_k,s}}^{t_{n,a_k,f}} x_{n,a_k,t} = l_{n,a}, \quad \forall a \in \mathbb{A}_n,$$

$$\mathbf{x}_{n,a} = \mathbf{p}_{n,a}, \quad \forall a \in \mathbb{A}_{n,f},$$

$$\mathbf{x}_{n,a_k} = \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k} + \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k}^l,$$

$$\mathbf{s}_{n,a_k} + \mathbf{s}_{n,a_k}^l \leq \mathbf{1}, \sum_t s_{n,a_k,t} = 1, \sum_t s_{n,a_k,t}^l = 1,$$

$$\sum_{k \in \mathbb{K}_a} \mathbf{s}_{n,a_k} \leq \mathbf{1}, \sum_{k \in \mathbb{K}_a} \mathbf{s}_{n,a_k}^l \leq \mathbf{1},$$

$$\sum_{a \in \mathbb{A}_n} \sum_{k \in \mathbb{K}_a} \mathbf{P}_{n,a_k} \mathbf{s}_{n,a_k}^l \leq y_n, \forall k \in \mathbb{K}_a, \forall a \in \mathbb{A}_{n,s}.$$
(5.2.12)

The mixed integer optimization in (5.2.12) can be solved using Branch and Bound method [12]. Note the optimum scheduling requires  $\tilde{L}_{n,t}$  which are sent by other consumers within a neighbourhood area. Hence the optimization requires interaction among consumers. In this chapter, the consumers' scheduling actions is analysed as a strategic game.

## 5.3 Game Theoretic Scheduling Approach

#### 5.3.1 Game components

In this section, a game theoretic model is presented for the optimal consumption scheduling of various consumers under a pricing plan as described in (5.2.2). In game theory, a non-cooperative game is defined as a strategic interaction of rational players consisting of three main components, namely players, strategies and payoffs [32]. The constrained consumption scheduling optimization game components are defined as follows:

**Players:** The set of N distributed energy consumers  $\mathbb{N}$  in the same billing area.

**Strategies:** The daily consumption scheduling plan that each player n chooses to play the game, i.e.,  $\mathbf{L}_n \in \mathbb{S}_n$  is used to represent the strategy chosen by player n, where  $\mathbb{S}_n$  denotes the strategy space of player n. The strategy space of all players is defined as  $\mathbb{S} = \prod_{n \in \mathbb{N}} \mathbb{S}_n$ .

**Payoffs:** The payoff of each player is the negative of the energy consumption cost charged by the utility company. The function  $\mathfrak{u}_n(\mathbf{L}_n, \mathbf{L}_{-n}), \mathfrak{u}_n : \mathbb{S} \mapsto \mathbb{R}$  is used to represent the payoff for a chosen strategy of player n, given the strategy choices  $\mathbf{L}_{-n} \in \mathbb{S}_{-n}$  of all other players (i.e., the load profiles of all other consumers), where  $\mathbb{S}_{-n} = \prod_{i \in \mathbb{N}, i \neq n} \mathbb{S}_i$ .

Every consumer will want to minimise only his/her energy cost and will be tempted to select a consumption schedule that maximises his/her payoff  $\mathfrak{u}_n$  as the best strategy in response to the price plan and other players' chosen strategies. Since the consumption charge has been designed to depend on the total load of all consumers, consumers may not be able to achieve the lowest possible cost (i.e., maximal payoff) unless they participate in the game to distribute the load as much as possible. Suppose all players schedule operation of appliances in isolation to respond to the price, then it will lead all players scheduling their consumption of energy when the ToU rate is low. This will increase the chances of all consumers operate the appliances at the same time and will result into high instantaneous accumulated load. This will in turn increase the cost of energy to every consumer as the price increases quadratically with the peak load as shown in (5.2.2). Hence, consumers will attempt to schedule their consumption to minimise the total group cost C using the optimization framework in (5.2.12). Individual's optimal scheduling is achieved when the cost of all consumers has reached its minimal. However, each consumer will need to pay only their share of the cost, as in (5.2.3), and the total cost is used only for the purpose of defining the utility for the optimization. This is an analogy to network utility maximization situation where individuals benefit the most when the whole network is optimised [97]. In this case, the payoff function for all players can be represented by the consumption cost as defined in (5.2.4), i.e.,  $\mathfrak{u}_n(\mathbf{L}_n, \mathbf{L}_{-n}) = -C_{(n)}, \forall n \in \mathbb{N}$ . The game is expected to provide a more balanced scheduling result and reduced cost, as demonstrated in the simulation section.

Note that in the scheduling game defined above, players must solve constrained optimizations to obtain their optimal payoffs and strategies. Therefore, it is necessary to define  $\mathbb{T}_n$  as the set of constraints in the optimization (5.2.12) for player n, and  $\mathbb{T} = {\mathbb{T}_n, \forall n}$ . Players' optimal strategies and the payoff values should therefore be  $\mathbb{T}$ -feasible solutions of the optimization problem [46].

#### 5.3.2 Equilibrium solution

The Nash equilibrium of the proposed consumption scheduling game can be defined as a joint strategy profile  $\mathbf{L}^* = {\mathbf{L}_1^*, \mathbf{L}_2^*, ..., \mathbf{L}_N^*}, \mathbf{L}^* \in \mathbb{S}$ , where  $\mathbf{u}_n(\mathbf{L}_n^*, \mathbf{L}_{-n}^*) \geq \mathbf{u}_n(\mathbf{L}_n, \mathbf{L}_{-n}^*), \forall \mathbf{L}_n \in \mathbb{S}_n, \forall n \in \mathbb{N}$ , i.e., given the equilibrium strategy choices of other players  $\mathbf{L}_{-n}^*$ , player n has no incentive to change his/her own strategy from  $\mathbf{L}_n^*$  unilaterally [32]. Nash equilibrium is critical to the non-cooperative game theoretic modeling because, if it exists, it guarantees a stable solution where every player plays the best response to the strategic choices of all others and the players have no incentive to deviate from this equilibrium.

As for the particular game involving constrained consumption optimization, the Nash equilibrium will be the strategy profile which has the above property and also T-feasible. Its existence will ensure that the scheduling process will be able to provide every consumer an optimal consumption scheduling. Different from the constrained games as discussed in [34] which will always guarantee a Nash equilibrium in pure strategies, the constrained solution is no longer in a continuous space due to the integer nature of the optimization of the payoff function. It is useful to adopt potential game approach to establish Nash equilibrium solution for the scheduling game.

# 5.3.3 Potential game

For a game with a set of players  $\mathbb{N}$ , feasible strategy space  $\mathbb{S}$  and payoff functions  $\mathfrak{u}_n(\mathbf{L}_n, \mathbf{L}_{-n})$ , a function  $\mathfrak{U} : \mathbb{S} \to \mathbb{R}$  is called an *exact potential* function, if the following holds:

$$\mathfrak{u}_{n}(\mathbf{L}_{n},\mathbf{L}_{-n}) - \mathfrak{u}_{n}(\mathbf{L}_{n}',\mathbf{L}_{-n}) = \mathfrak{U}(\mathbf{L}_{n},\mathbf{L}_{-n}) - \mathfrak{U}(\mathbf{L}_{n}',\mathbf{L}_{-n}),$$
  
$$\forall n \in \mathbb{N}, \forall \mathbf{L}_{n}, \mathbf{L}_{n}' \in \mathbb{S}_{n}, \forall \mathbf{L}_{-n} \in \mathbb{S}_{-n}, \mathbf{L}_{n} \neq \mathbf{L}_{n}'.$$
 (5.3.1)

If a game admits an exact potential function which reflects the changes in the strategy chosen by any of the players, it falls into a specific class of strategic games called *exact potential games* [98]. It is observed that in the proposed scheduling game, although individual players are responsible for optimising only their own consumptions, the goal for all players is the minimization of the total cost. Any changes in the scheduling  $\mathbf{L}_n$  will result in a change in the total cost, i.e.,

$$\mathfrak{u}_{n}(\mathbf{L}_{n},\mathbf{L}_{-n}) - \mathfrak{u}_{n}(\mathbf{L}_{n}',\mathbf{L}_{-n}) = -(C(\mathbf{L}) - C(\mathbf{L}')),$$
$$\forall n \in \mathbb{N}, \forall \mathbf{L}_{n}, \mathbf{L}_{n}' \in \mathbb{S}_{n}, \forall \mathbf{L}_{-n} \in \mathbb{S}_{-n}, \mathbf{L}_{n} \neq \mathbf{L}_{n}',$$
(5.3.2)

where  $C(\mathbf{L})$  and  $C(\mathbf{L}')$  are the costs for the load profiles  $(\mathbf{L}_n, \mathbf{L}_{-n})$  and  $(\mathbf{L}'_n, \mathbf{L}_{-n})$  respectively. Therefore  $\mathfrak{U} = -C(\mathbf{L})$  is used as the exact potential function and the scheduling game is an exact potential game.

The theorem proposed in [98], which is very important for establishing

Nash equilibrium for potential games, states that the potential game admits a pure strategy Nash equilibrium  $\mathbf{L}^*$  if and only if  $\mathbf{L}^*$  is a maximiser of the potential function. In other words, establishing Nash equilibrium of the scheduling game is equivalent to determining the solution of the constrained maximization of the potential function. The maximum is derived through an iterative best response convergence process.

## 5.3.4 Game procedure and iterative convergence

A best response iterative process is a dynamic process that players update their actions by choosing the strategies that maximise their payoffs, given other players' current strategies remain fixed [99]. The best response  $\mathfrak{B}_{n}^{m}(\mathbf{L}_{-n}^{m})$  of player *n* to the strategies  $\mathbf{L}_{-n}^{m}$ , where *m* is the game iteration indicator starting from 0, is given by

$$\mathfrak{B}_{n}^{m}(\mathbf{L}_{-n}^{m}) = \operatorname{argmax}_{\mathbf{L}_{n} \in \mathbb{S}_{n}} \mathfrak{u}_{n}(\mathbf{L}_{n}, \mathbf{L}_{-n}^{m}).$$
(5.3.3)

Player n will update its strategy to a new strategy  $\mathbf{L}_n^{m+1} \in \mathfrak{B}_n^m$  if and only if the new strategy gains an improvement to the payoff, i.e.,

$$\mathfrak{u}_n(\mathbf{L}_n^{m+1}, \mathbf{L}_{-n}^m) > \mathfrak{u}_n(\mathbf{L}_n^m, \mathbf{L}_{-n}^m).$$
(5.3.4)

For the scheduling potential game, players will be able to carry out this process in a round robin manner. At each play, the player will have the opportunity to revise his/her scheduling with the aim of reducing the consumption cost, which is equivalent to increasing the potential payoff. If the revised payoff is higher than the payoff obtained from the previous play, the player will play the revised new strategy, otherwise, the old strategy will be retained. Observing this move, the player at the next turn will optimise his/her strategy with the aim of further increasing the potential payoff. At every game iteration, the value of the potential function satisfies

$$\mathfrak{U}(\mathbf{L}^{m+1}) \ge \mathfrak{U}(\mathbf{L}^m). \tag{5.3.5}$$

As the players keep optimising their strategies, the best response dynamics will result into a non-decreasing sequence of changes in the potential payoff { $\mathfrak{U}(\mathbf{L}^0), \mathfrak{U}(\mathbf{L}^1), \mathfrak{U}(\mathbf{L}^2), \ldots$ }. This is called 'improvement path' which will finish at a point where no player will see any improvement in the payoff. At this point, the potential function  $\mathfrak{U}$  will have converged to the maximum, which is the Nash equilibrium of the game. Since the consumption cost is bounded above zero, i.e., it is non-negative, and its value will be changing non-increasingly within the game process, the convergence of the sequential game is guaranteed. In conclusion, the consumption scheduling game admits a T-feasible Nash equilibrium in pure strategies.

The scheduling game is expected to start every time a new pricing plan is issued. In order to play the best strategy  $\mathbf{L}_n$  and to obtain maximal payoff, a player needs information which informs the player the current game status and the chosen strategies of other players  $\mathbf{L}_{-n}$  (i.e., their load profiles). A home energy management unit is responsible for collecting and scheduling the consumption requirements. It serves as a data access point for scheduling information exchange during the game process. ICT infrastructure as in neighborhood area networks (NAN) and local area networks (LAN) can be used in smart grids to enable efficient and reliable communications among players. Candidate solutions include wireless 3G/LTE cellular and the emerging IEEE 802.22 which uses cognitive radio technologies in the white spaces of the television spectrum. At the beginning of the game, every player should initialize a consumption schedule according to his/her own preference and announce it through the communication network. Acknowledging this information, the players will start to adjust their energy consumption plan using the best response process. The game theoretic algorithm for scheduling has been summarised in Algorithm 5.1.

Algorithm 5.1: Game procedure Initialization: Each player generates its intended consumption schedule according to its preferences as initial strategy and broadcasts it to other players. On detection of a new pricing signal, execute: **For** n = 1 : NPlayer n solves optimization in (6.2.6) and obtains the consumption cost  $C_{(n)}(\mathbf{L})$  and scheduling  $\mathbf{L}_n$ . If the optimised scheduling  $\mathbf{L}_n$  is different from the previous scheduling strategy Broadcast the new strategy  $\mathbf{L}_n$  to other players. else Remain silent, i.e., no need for broadcasting  $\mathbf{L}_n$ . End If End For Repeat For until no further improvement to all players.

#### 5.3.5 Efficiency, complexity and privacy

The efficiency of the equilibrium solution of the game theoretic algorithm is often measured by Pareto optimality. Particularly for this game, a weaker version called constrained Pareto optimality is considered, because all outcomes must be  $\mathbb{T}$ -feasible [100]. It can be claimed that the outcome of Algorithm 5.1 is automatically constrained Pareto optimal since it maximises the potential payoff which reflects the payoff for every player.

One of the important benefits of the proposed distributed approaches is that the computational complexity can be distributed among various home energy management units by decomposing the large scale centralised optimization using a decentralised game theoretic method. In terms of exchange of information, the distributed algorithms can also be efficient as compared to centralised algorithms. For example the distributed design approach in wireless networks enables coordinated beamforming without the need of explicit inter-base-station information exchange as in [101], and the game theoretic approach as in [44]. For the demand management to be performed centrally, each household needs to inform the centralised scheduling processor the type of each appliance, its power consumption pattern, consumer's preferred time of use, the use of electric vehicle and the availability of local energy, etc. This may require extensive amount of explicit data exchange between households and the centralised processor. For the proposed game theoretic approach, details on individual appliances are not required to be exchanged. As can be observed in Algorithm 5.1, the optimizations of detailed appliance consumption are done locally at every iteration, only the overall consumption profile  $\mathbf{L}_n$  is required to be exchanged among players. This significantly reduces the amount of information exchange. In addition, the distributed approach has the benefit of enhanced privacy because the exact details of appliances and the information of individual household's locally generated energy are not exchanged. Only the aggregated use of energy is exchanged which has relatively lower private information. However, such information needs to be communicated among all players repeatedly during the iterative updating process. This may turn out to increase the communications overhead, especially for large number of consumers. Moreover, a centralised processor may possibly have more computational capacity to perform complex optimizations, hence centralised processer may be advantageous if there is any limitations on the computation and communication

82

capability of the distributed EMUs. These tradeoffs need to be considered carefully in the choice between centralised and decentralised optimization approaches.

## 5.3.6 Behaviour of players and their participation in the game

The capacity of local energy supply is critical to the players in order to decide whether to participate in the scheduling game or not. Consider a particular consumer whose local capacity exceeds his daily consumption requirement. The consumer has the ability to self-supply his own energy demand and will achieve zero energy cost charged by the utility company even without any scheduling. In this case, as there is no impact in terms of pricing, this consumer has no incentive to participate in the scheduling game until there is a need to draw power from the main grid again. It can be considered that these players need not to play the game because scheduling does not provide any benefit to them. By observing the capacity of local energy supply, consumers should be able to decide whether to participate in the game immediately.

In non-cooperative games, there is a possibility that certain players may be untruthful during the game process by providing false information to the scheduling results ( $\mathbf{L}_n$ ). This will make the optimizations invalid and unable to reach the optimal result. Since the consumption optimization promises a lowest group cost and an optimal scheduling at the Nash equilibrium for every player in the game, there should be no incentive for players to cheat. However cheating could still occur when there are malicious players who always cheat intentionally to hurt others. Considering that the demand management activities will be carried out repeatedly on a daily basis, cheating players can be punished in future plays using various mechanisms as proposed for repeated games as in [32]. Developing workable mechanisms for detecting and preventing untruthful players is an open research topic. A possible direction is based on the work of [102] where cheating detection algorithms have been proposed for client-server information revealing.

## 5.4 Simulations and Performance Evaluation

#### 5.4.1 System set up

This section presents an illustrative simulation for the daily consumption scheduling optimization using the proposed game theoretic approach. Consider a small residential area consisting of N = 10 individual households. Each household has a set of appliances such as a 24 hour operational fridge & freezer (hourly consumption of 0.12kWh) and electric heating (daily requirement of 4kWh) with multiple non-shiftable operations, washing machine with one shiftable operation program (daily requirement of 1.2kWh) and a water tank boiler (daily consumption of 1.6kWh) with multiple shiftable tasks over the day. The consumption requirements for these appliances have been obtained from [82] and [83]. Assume that the size of each household is different. The consumption requirements of the appliances and the total daily demands could vary. The daily demand of the 10 households is set at 256kWh. User preferences for particular appliances also vary individually. In order to study the performance of the proposed consumption scheduling, the capacity of the local energy supply is assumed to be a small fraction chosen randomly between 15 percent and 25 percent of the daily requirement of each consumer so that every one will have incentive to participate in the game.

The pricing plan used in the simulation is based on a basic hourly ToU rate, hence the scheduling time resolution is one hour (T = 24). The unit rate  $\omega_t$  is assumed to have the lowest value of one pence during the night period while the daytime rate is two pence and the peak rate of five pence appears in the morning between 9am and 10am and in the evening between



Figure 5.1. Convergence of the scheduling game (played sequentially by each player), e.g., iteration 20 means each player played twice.

6pm and 8pm. Assume there was zero standard admission charge  $\phi_t$ .

## 5.4.2 Performance evaluation

The convergence of the consumption cost against iteration of the game process is shown in Figure 5.1. It is observed that the consumption cost has dropped considerably fast during the first 10 iterations, i.e., when all the players finished their moves in the first round. The cost continued to reduce gradually and then became steady at 3374 pence after 20 plays. As the game went on for another two rounds, no player was able to reduce the cost further (improve the payoff). Therefore, after 30 iterations of individual consumers' scheduling of daily consumption, the game process is considered as finished and reached the Nash equilibrium point. Compared to rather high initial cost of 6586 pence, the proposed mechanism offered a significant reduction of nearly 50 percent of the total cost. As seen, the convergence of the scheduling game is reasonably fast. As for this simulation scenario, it can be claimed that only three rounds of play per player is adequate for the game to reach stable NE point.

In order to evaluate the benefits for individual consumers, the actual consumption cost for each consumer has been provided in Table 5.1. Compared to the cost without scheduling, every consumer gains a cost reduction of around 50 percent by participating in the optimization game, which is fairly identical to the overall cost reduction. In other words, the overall benefits have been fairly distributed to all players who participated in the game.

	Without	Game	Isolated
	scheduling	result	scheduling
Consumer 1	433.80	227.83	258.03
Consumer 2	595.57	258.01	294.83
Consumer 3	609.21	307.26	323.70
Consumer 4	689.55	342.26	375.44
Consumer 5	515.02	283.71	305.93
Consumer 6	694.08	412.96	444.00
Consumer 7	839.80	389.16	445.40
Consumer 8	780.08	400.42	461.41
Consumer 9	879.62	499.94	502.80
Consumer 10	548.25	253.05	292.32
Total	6584.96	3374.60	3703.85

Table 5.1. Comparison of the consumption cost of consumers (pence)

Table 5.1 compares the cost with the result of an isolated scheduling scheme where players are assumed to have the same appliances and user preferences as in the game above, however they schedule their consumption independently according to the price plan, without knowing others' scheduling information. As seen, an average cost reduction of 35 pence has been achieved for each game participant. Therefore, the proposed game theoretic framework has the ability to reduce the cost by encouraging consumers to participate. The difference in performance is expected to become considerably high when a large number of subscribers are involved.

Figure 5.2 depicts the scheduled optimal consumption for all the participating households after the game has converged. It is observed that the consumptions remain reasonably low during the day time and the majority of the demand has been shifted to the night time corresponding to lower price. Most of the local energies were drawn to further reduce the potential high demand, in particular at peak times in the morning and in the evening. The consumption optimization performed effectively in both integrating local power supply optimally and also scheduling possible consumption away from the peak ToU periods. It should be noted that even though the terminology of balanced scheduling is used, it is not expected to achieve equal load distribution throughout the day. This is because the aim is not to distribute the load equally, but to fit it to the expectation of the utility operator according to the price profile, i.e., to move most of energy consumptions to off peak period as seen in Figure 5.2. The reason for this is that the demand might be high, for example due to industrial use of electricity in the day time, and the aim of utility operator is to balance the overall load consisting of both the industrial loads and the household loads, by issuing appropriate pricing plans. The game theoretic method proposed helps to ensure the consumers do not operate all the appliances at the same time, but distribute over the time as much as possible, as seen in the results.

Table 5.2 lists the usage of local energy for each player. The majority of local capacity has been scheduled to be used. However, it appears that there is still a small portion of energy that was not utilised. This is because this energy was not being able to be served to any of the appliance on the current date of scheduling. This unused energy can be stored for future use.

	Usage		Usage
Consumer 1	87%	Consumer 6	96%
Consumer 2	96%	Consumer 7	85%
Consumer 3	87%	Consumer 8	90%
Consumer 4	93%	Consumer 9	86%
Consumer 5	90%	Consumer 10	95%

Table 5.2. Local energy usage of the consumers



Figure 5.2. Scheduled accumulated consumption load over the day.



Figure 5.3. Comparison of scheduled consumption for five players.



Figure 5.4. Scheduled consumption from local and main supply for five players.

Figure 5.3 compares the consumption schedules of five individual consumers. As seen, the peak energy consumption of consumers occurs at different time slots. This implies the shiftable operations of the appliances have been well distributed with minimal overlapping with each other. However, considerable overlapping still appeares, e.g., in the hours between 7pm and 8pm. This is caused due to the non-shiftable requirements during the peak hours in the evening. As depicted in Figure 5.4, the local energy supplies were mainly scheduled at these periods in order to reduce the dependency on the main grid to prevent high accumulated load and cost. The chargeable consumption has decreased by approximately 70 percent between the hours of 7pm and 8pm, i.e., the effect of overlapping was significantly reduced. In summary, through the scheduling game, consumers are able to optimally manage their daily consumption and integrate local generated/stored energy.

It is worth to point out that the detailed system parameters, such as the pricing plan and the capacity of local generation used in the above simula-
tions are artificial. The simulations have demonstrated the effectiveness of the proposed optimization framework. However, the performance in terms of peak consumption reduction and cost reduction may require further justification when considering more realistic settings. Statistics and analysis of real consumption data of the consumers have indicated similar benefits of performing residential demand management based on the understanding and scheduling of consumption incorporating locally generated energy [8], [103].

#### 5.5 Summary

This chapter presented a game theoretic consumption scheduling framework based on the MILP optimization technique to schedule the energy consumption at household level. The framework has the ability to achieve minimum energy cost for the consumers. In addition, locally generated renewable energy has been integrated into the consumption scheduling optimization to further reduce the demand on conventional energy. The proposed game admits a Nash equilibrium where the scheduling optimization process finds a stable solution at which every consumer benefits from low consumption cost. The simulations demonstrated the convergence of the algorithm and the benefits to the consumers and the grid operators in terms of the cost of energy and load balancing.

# A MEAN FIELD GAME THEORETIC APPROACH FOR ELECTRIC VEHICLES CHARGING

Smart grid demand management should consider carefully the inclusion of EVs. One critical challenge in the consumption optimization for EVs is the management of battery charging. This chapter proposes a dynamic game theoretic optimization framework to formulate the optimal charging problem. The framework considers a charging station where a large number of EVs can be charged simultaneously during the flexible permitted charging time. The optimization will provide every individual EV an optimal charging strategy to proactively control their charging speed in order to minimise the cost of charging. The optimization is based on stochastic mean field game theory. Numerical results are presented to demonstrate the performance of the proposed framework.

#### 6.1 Introduction

Electric vehicles (EVs) are expected to be one of the main components of distributed energy consumption, storage and supply system in smart grids. EVs can serve as a distributed mobile energy resource in the electricity market. Enabled by the advanced information and communications technologies (ICT), EVs can be optimally scheduled and dispatched to meet the dynamic demand on energy and to respond fast to emergency situations [104]. The storage and transportation of energy from one geographical area to another as supportive supply enhances the overall flexibility of the grid [105]. As EVs will eventually be employed at household level, as alternative to traditional petrol cars, they should be included in home electricity demand management and consumption optimization [106].

EV is a major electricity consumer and draws significant amount of power in order to retain sufficient battery capacity. For the grid operators, such high loads attached to the grid will have to be managed carefully. EV owners should consider the best charging times and charging rates (speed) to reduce the cost of energy consumption. Optimal charging becomes one of the critical challenges in the attachment of EVs in smart grids. Recent research on developing scheduling optimization algorithms for EV charging in both the centralised and the distributed manners can be found in [107], [108], [109] and [110]. In the centralised approach, a scheduling agent is responsible for handling all EVs connected to the grid and to optimise the charging schedules globally. Optimal charging can also be achieved via distributed algorithms where each EV schedules its own charging according to the current demand and operational information, such as the pricing signal of the grids. Consumer oriented fast response and near real-time consumption scheduling can be achieved.

It is useful to apply game theoretic methods for EV consumption opti-

mizations. In [110], a mean field game theoretic framework was developed to minimise the consumption cost for the EVs within a predefined period of time. The energy consumption behaviour of a large number (tends to infinity) of EVs, including charging power from the grid (grid to vehicle) and releasing power to the grid (vehicle to grid), was modeled as a stochastic optimization. Individual player (EV) chooses its optimal strategy (the amount of energy charging/discharging at any particular time) according to the statistical behaviour of the total group of players, which is known as the mean filed. The mean field game is a novel differential game theoretic modeling mechanism which was first proposed in [111]. It provides a powerful mathematical modeling based on the formulation of two coupled backward-forward partial differential equations (PDE) for problems with a large number of indistinguishable players. The optimal game solution, which is claimed as the Nash-mean field equilibrium (Nash-MFE) is derived by solving the coupled equations [111]. Successful applications of mean field game theory can also be found in other research areas such as control engineering [112].

In this chapter, a stochastic mean field game theoretic framework is proposed for a charging scenario where individual EVs manage their charging at an aggregated charging station. The EV charging station has the capability of charging a large number of EVs during a period of time. This specific period of time is determined in advance according to the general demand management of the electricity operator to prevent undesired peak load on the grid. However, there is a degree of flexibility assumed in terms of the finishing time. Acknowledging this charging time, recharging EVs should arrive to the station on time and charge their battery within the defined charging period. The optimization framework enables the EVs to have the ability to proactively control their charging in order to minimise the cost of charging.

The rest of this chapter is organised as follows. The dynamic game the-



Figure 6.1. System model: aggregated EV charging at a station.

oretic optimization framework is formulated in Section 6.2, followed by a detailed discussion on the issue of Nash-mean field equilibrium and methodology for obtaining the optimal solution. Numerical simulation results are presented in the subsequent section to demonstrate the performance of the proposed method. Conclusions are drawn at the end of the chapter.

#### 6.2 The Game Theoretic Optimization Framework

The system consists of a charging station where a large number of EVs can be charged simultaneously. The EVs are aggregated at the station and recharge their batteries in a predefined period of time. This period of time can be modestly extended, i.e., the charging station can tolerate certain delays after the scheduled finishing time. The actual time of completing the charging will depend on the dynamics of the charging EVs, e.g., at the point when a certain quorum of fully charged EVs is reached. Compared to fixed schedules, this setting provides a practical flexibility.

The station charges an EV according to its instant power consumption, i.e., the EV's charging rate (speed), and the charging time. Detailed description of cost of charging is provided in the subsequent section. Given the pricing information, individual EV is encouraged to optimise and control its charging in order to minimise the cost of charging. Due to the non-deterministic charging time, it is necessary to formulate the charging optimization framework as a dynamic control process, based on the knowledge of the current charging status of all EVs in the station. The proposed optimization framework aims to provide an optimal control strategy that minimises the cost of charging for every individual EV, i.e., a profile that defines the dynamics of the charging rates for the whole charging duration. The system is illustrated in Figure  $6.1^1$ .

#### 6.2.1 Optimization costs

The minimization of cost of charging is the objective of the optimization for the EVs. The cost consists of the energy consumption cost during charging and the endpoint costs that are related to the finishing time of charging. Consider a finite set of aggregated charging EVs  $\mathbb{K} = \{1, ..., K\}$ . The charging station's pricing policy for any EV k is defined as a continuous function  $u_t^{(k)}$  of the charging speed  $a_t^{(k)}$ , i.e., its instantaneous energy consumption at time t,

$$u_t^{(k)} = \frac{1}{2} (a_t^{(k)})^2.$$
(6.2.1)

This pricing uses a simplified quadratic representation which is widely used in the area of economics to formulate both the production cost function

<sup>&</sup>lt;sup>1</sup>Elements of this figure are from google images. http://www.google.co.uk/

and the revenue function [113]. This pricing policy implies that in order to achieve low cost anytime during the whole charging duration, an EV should maintain slow charging. Such pricing benefits the grid in terms of reducing the accumulated instant peak load when considering multiple EVs charging at the same time.

The issue of charging time is now described. Denote the station's scheduled operation period as  $[0, \hat{t}], \hat{t} > 0$ . All connected EVs are expected to start their charging at time 0. They should be willing to maintain lowest possible charging rates until their batteries are fully charged at time  $\hat{t}$ , when the station is scheduled to terminate its charging service. However in reality the actual charging duration of any particular EV can vary from the expected time. This variation is mainly due to charging efficiency (loss), degree of degradation of individual battery. Denote an EV's actual finishing time as  $\tau^{(k)}$ . As mentioned above, the charging station will tolerate a modest delay in terms of the finishing time, up to a maximum of  $t_{max}$ . The actual charging finishing time, denoted by T, between  $[\hat{t}, t_{max}]$  will depend on the dynamics  $\{\tau^{(k)}, \forall k\}$  of all EVs.

In the following, several endpoint costs are defined as functions of the charging finishing times  $\hat{t}, \tau$  and T. Firstly, a punctuality cost is set. It can be viewed as a price paid for lateness, payable to the charging station, for the charging EV k:

$$c_1^{(k)}(\hat{t},\tau^{(k)}) = f_1([\tau^{(k)} - \hat{t}]_+).$$
(6.2.2)

The charging station can issue such lateness penalty to regulate the punctuality behaviour of charging EVs. The selection of this cost function  $f_1(\cdot)$  will have influence on the result of charging optimization directly, as discussed in Section IV.

Secondly, a cost of the lateness is defined in terms of the actual finish-

ing time of charging. This reflects the loss of incomplete battery recharge because the charging station will have to stop power supply after this time:

$$c_2^{(k)}(T,\tau^{(k)}) = f_2([\tau^{(k)} - T]_+).$$
 (6.2.3)

This represents the cost of inefficiency for the charging EV. Although not actual financial cost, it is included in the optimization. There is a possibility that some EVs may opt for fast charging. The priority for them is charging time rather than financial costs. However this type of charging is not considered in this chapter.

Finally,  $C_T^{(k)} = c_1^{(k)} + c_2^{(k)}$  is used to represent the cost at the finishing time of the charging. Individual EV would want to minimise this cost along with the charging expense during the whole charging duration. Assume that all these cost functions are continuous and twice differentiable. The total cost function  $J^{(k)} : a^{(k)} \to \mathbb{R}$  of the optimal charging is therefore

$$J^{(k)} = \int_0^{\tau^{(k)}} u^{(k)}(t)dt + C_T^{(k)}(\hat{t}, \tau^{(k)}, T).$$
 (6.2.4)

#### 6.2.2 Dynamic EV charging process

Let us suppose that an EV's charging is represented by its battery capacity  $X^{(k)} \in [0, 1]$  moving from an initial state  $X_0^{(k)} > 0$  (battery capacity when charging starts) towards the fully charged point of 1. This movement is described using a dynamic process, written as a stochastic differential equation

$$dX_t^{(k)} = \eta^{(k)} a_t^{(k)} dt + \sigma_t dW_t^{(k)} + dN_t^{(k)}, \qquad (6.2.5)$$

where the charging rate  $a_t^{(k)}$  is a controlled drift at time t in return for a cost as defined in (6.2.1) and  $\eta^{(k)}$  represents the measurable charger efficiency for the EV, which is assumed to be 1 for simplicity.  $W_t^{(k)}$  is an independent Brownian motion (Wiener process) with a diffusion coefficient  $\sigma_t$ . It's differentiation should follow the rules of Itō calculus [114]. The choice of  $W_t^{(k)}$ represents the adjustment (uncertainty in power loss) added to the charging which indicates that the charging process is independent among the EVs at different times, due to different battery characteristics and individual EV's minor operational consumption during the charging time. The term  $N_t^{(k)}$  is a reflective noise which ensures that the value of  $X^{(k)}$  remains in (0, 1]. The reader is referred to [115] for more details on Brownian motion.

At any particular point in time during the charging process, the EV will be able to obtain the information of the charging status, i.e., the current battery capacity  $\{X_t^{(1)}, ..., X_t^{(K)}\}$  and the instantaneous charging rates  $\{a_t^{(1)}, ..., a_t^{(K)}\}$  of all charging EVs via communications through the ICT infrastructure. An estimation of actual finishing time T is obtained based on this gathered information. Mechanisms for the estimation of T can vary depending on the particular algorithms. This chapter considers the mean field game theoretic method. Due to the dynamic nature of the optimization, such information must be exchanged continuously and in real-time during the charging period. However for each EV, the amount of data overhead required for exchanging information at any time is limited. Considering that the EVs are aggregated in the station, data communications take place at a short distance through wireless sensor networks embedded in the EVs.

Having included the information of estimated T into the cost function, the EV is able to optimise its own charging process. The optimization is described as a stochastic control:

$$\min_{a(t)^{(k)}} E\left[\frac{1}{2} \int_0^{\tau^{(k)}} a^2(t)^{(k)} dt + C_T^{(k)}(\hat{t}, \tau^{(k)}, T)\right],$$
(6.2.6)

subject to the dynamic  $dX^{(k)} = a(t)^{(k)}dt + \sigma dW^{(k)} + dN^{(k)}$  and an initial state  $X_0^{(k)}$ . The optimization will aim to find an optimal law  $\gamma^*(t, X^{(k)*})$ 

which defines the optimal charging strategy of the control trajectory  $a(t)^{(k)*}$ and hence the movement of  $X^{(k)*}$  for the particular EV. Note that in practice, the charging rate of an EV is usually valued in a range of  $[a_{min}, a_{max}]$ . This should be included as an additional constraint in the optimization problem.

#### 6.2.3 *K*-person game theoretic formulation

Based on the above formulation for a single EV, the optimization of the total K EVs at the charging station is now formulated as a game theoretic framework.

One classical formulation is the K-person dynamic differential game where every EV is treated as an individual player, hence K represents set of players. They are assumed to be rational meaning that they will play the best strategies, i.e., at every time instant t, player k optimises its instantaneous charging control  $a_t^{(k)}$  based on the understanding of the game situation in order to maximise its own utility. The situation of the game at time t is determined by the charging status of every individual EV in the station. Here  $\Omega_t^{(k)} = (X_t^{(1)},...,X_t^{(K)},a_t^{(1)},...,a_t^{(K)})$  denotes the set of information available to the player k at time t. It is assumed that players are memoryless as they do not have this status information of previous time instants. The level of satisfaction (utility) under certain game situation is represented by a payoff. In this particular optimal charging game, a player's payoff can apparently be measured by its cost of charging. Therefore, the objective of every player is to determine a dynamic trajectory of charging rates  $a(t)^{(k)}$  that maximise its payoff, i.e., achieving minimum cost by conducting the optimization as defined in (6.2.6).

Define a mapping  $\mathfrak{B}_t^{(k)}: \Omega_t^{(k)} \mapsto \mathbb{S}_t^{(k)}$ , to represent the choice of strategy for player k at time t, with  $\mathbb{S}_t^{(k)}$  the set of all possible controls  $a_t^{(k)}$  for the player. In particular,  $\mathfrak{B}_t^{(k)}$  yields a best response control that maximises the payoff. The optimal charging action of player k at time t is therefore a (own-state) feedback strategy determined by

$$a_t^{(k)} = \mathfrak{B}^{(k)}(\Omega_t^{(k)}), \ 0 \le t \le T.$$
(6.2.7)

Referring to the game theoretic interpretations,  $\mathbf{a}_t^{(-k)} \in \mathbb{S}_t^{(-k)}$ , where  $\mathbb{S}_t^{(-k)} = \prod_{i \in \mathbb{K}, i \neq k} \mathbb{S}_t^{(i)}$ , denotes the joint strategy choices of all players other than k at time t. For player k, the choice of strategy  $a_t^{(k)}$  is a best response to the current game status and the strategies chosen by all the players  $(a_t^{(k)}, \mathbf{a}_t^{(-k)})$ .

The formulation is completed by defining the strategy set over the total charging period for player k as  $\mathbb{S}^{(k)} = {\mathbb{S}^{(k)}_t, 0 \leq t \leq T}$ , and the overall strategy space for all players as  $\mathbb{S} = \prod_{k \in \mathbb{K}} \mathbb{S}^{(k)}$ . Having the above formulation, the game can be viewed as a dynamic optimization process where every individual player (EV k) chooses best strategy (optimal charging rate  $a(t)^{(k)}$ ) to maximise its payoff (minimise cost  $J^{(k)}$ ), for the whole charging duration. The solution of the game is considered as a feedback Nash equilibrium (NE), as defined below:

Definition 1: The feedback Nash equilibrium of the K-person charging optimization game is a joint strategy profile  $\mathbf{a}^* = \{a^{(1)*}, a^{(2)*}, ..., a^{(K)*}\}, \mathbf{a}^* \in$  $\mathbb{S}$ , where  $a^{(k)*} = \{a_t^{(k)*} = \mathfrak{B}^{(k)}(\Omega_t^{(k)*}), 0 \leq t \leq T\}$ , and satisfies for all  $k \in \mathbb{K}$ ,

$$J^{(k)}(\mathbf{a}^*) \le J^{(k)}(a^{(k)}, \mathbf{a}^{(-k)*}), \forall a^{(k)} \in \mathbb{S}^{(k)}, a^{(k)} \ne a^{(k)*}.$$
(6.2.8)

This definition states that given the equilibrium strategy choices of other players  $\mathbf{a}^{(-k)*}$ , player k has no incentive to change its own strategy from  $a^{(k)*}$ unilaterally. Nash equilibrium is critical because, if exists, it guarantees a stable game situation where every player plays the best strategy responding to the strategic choices of all other players. For the particular charging game, obtaining the NE point is equivalent to achieving an optimal charging result for every EV in the system. Analysing the NE in terms of showing its existence and uniqueness is never obvious [32, 34]. Even if NE does exist, it may be difficult to develop convergence algorithms to exploit. This is more complex in the case of K-person games, where K can be considerably large. Various mechanisms for analysing multi-player differential games can be found in [54] and [47]. Nevertheless, it can be claimed that under the proposed game formulation, any change of any player at any time, i.e., changes in  $\Omega_t$ , has impact on all players' payoffs. They will have to be acknowledged and respond accordingly. This results in significant computation complexity and increased ICT overhead. In order to resolve these potential issues, it is necessary to modify

### 6.2.4 Mean field game representation

Mean field game theory is powerful in modeling and analysing games with numerous players. For the simultaneous charging scenario involving a large number of EVs, it is possible to formulate a statistical performance of the whole population to represent the mean field, and every player optimises its charging strategy accordingly.

the formulation and propose the following mean field game approach.

In order to model the above discussed charging optimization scenario as a mean field game, two additional assumptions in relation to the players are required. Firstly, the total number of players is very large so that they can be viewed as a continuum instead of individuals. In other words, we now consider the charging of infinite EVs. Having this assumption in place, we are able to analyse the charging status based on a statistical distribution of the population, without the need of detailed observation of individual EVs. This condition is justified later. The second assumption is that the players are indistinguishable. This implies that all EVs have similar type of batteries and charging control abilities (however still their initial battery states and the efficiency loss, etc., may vary). They are modeled mathematically identical.

Now the notations k and X used in the classical formulation can be removed. The charging process can be analysed using a state variable  $x_t \in [0, 1]$  representing the battery status of the continuum at time  $t \in [0, T]$ , according to a continuous statistical distribution m(t, x(t)). The distribution is described using the limiting distribution of an empirical distribution function, defined as

$$m(t, x_t) = \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} \mathbf{1}(X_t^{(k)} = x_t),$$
(6.2.9)

where **1** is an indicator function which has a value of 1 only when  $X_t^{(k)} = x_t$ , otherwise 0. The distribution is defined in the compact domain of  $[0,T] \times [0,1]$  and has a compact support. The charging of the continuum moves from initial state  $m(0,x_0) = m_0$  towards  $m(\cdot, 1)$  indicating the state of fully charged (if time permits). The movement of  $x_t$  is indicated using a differential equation

$$dx_t = a_t dt + \sigma_t dW_t + dN_t, \tag{6.2.10}$$

which is similar to the one in (6.2.5) however without the player index k. The choice of  $W_t$  now represents the added uncertainty to indicate the independency of  $x_t$  at different times.

In this context, the time when the dynamic flow reaching 1, can be seen as  $\tilde{t} \mapsto \partial_x m(\tilde{t}, 1)$ . The cumulative distribution function (CDF) F of finishing times can be defined by

$$F(\tilde{t}) = \int_0^{\tilde{t}} \partial_x m(s, 1) ds.$$
(6.2.11)

The actual finishing time T of the charging can be defined by this information of the dynamics of EVs. For example, T is fixed by a quorum rule of  $\theta = 85\%$ , which means 85 percent of the EVs have finished their charging,

$$T = \begin{cases} \hat{t}, & \text{if } F^{-1}(\theta) \leq \hat{t} \\ t_{max}, & \text{if } F(t_{max}) \leq \theta \\ F^{-1}(\theta), & \text{otherwise} \end{cases}$$
(6.2.12)

Under the above formulation, the charging optimization game is played from the viewpoint of an 'average' player. The player minimises his cost of charging with the optimal control  $a(t, x) \in \overline{\mathbb{S}}$  for the trajectory of the battery state x according to the statistical behaviour m which defines T.  $\overline{S}$  denotes the space of all controls for the state dynamics. The actual players (the individual EVs) will be argued into the optimal strategy a. This means that the EV which is at the state of  $X_t^{(k)} = x_t$  will choose the optimal strategy according to  $a(t, x_t)$ . Having the participation of all the EVs, their strategy repartition will lead to the trajectory of distribution m, which is claimed to be the optimal behaviour of the continuum. This will in return feeds back to the cost optimization to determine the optimal charging strategy. The game can be considered as a coupled system of cost minimization and the optimal behaviour of the statistical trajectory. Solving the dynamic optimization system will naturally lead to the optimal solution of the game [116]. In this way, the optimization of the cost of charging no longer requires information  $\Omega_t$  of all individuals however it knows the status  $m_t$ . System complexity and communications overhead are therefore reduced.

#### 6.3 Analysis of the Mean Field Game

The solution of the mean field game theoretic optimal charging framework, known as the Nash-mean field equilibrium (Nash-MFE) [111], is discussed. As the mean field game is transformed from a classical K-person game, the definition of feedback Nash-MFE is stated based on *Definition 1*, as follows.

103

Definition 2: The Nash-mean field equilibrium in feedback strategies of the charging optimization game is a control  $a^* \in \overline{\mathbb{S}}$ , consistent with the distribution  $m^*$  of the charging dynamics for a given initial state of  $m_0$ , and satisfies

$$J(a^*, m^*) \le J(a, m^*), \forall a \in \overline{\mathbb{S}}, a \ne a^*.$$
(6.3.1)

Having followed the equilibrium strategy of  $a^*$ , individual players of the game (EVs) have no incentive to deviate from  $a^*$ . Hence the dynamics of battery states will be according to  $m^*$ . Therefore it is claimed that their optimal charging processes with an actual finishing time of  $T^*$  are determined.

#### 6.3.1 The coupled stochastic partial differential equations

Based on the work in [111], a mathematical scheme is formulated using coupled stochastic partial differential equations (SPDE) in order to obtain cost minimization and the optimal behaviour of the statistical trajectory, and to generate the MFE of the game.

Firstly, consider the cost minimization problem which has the objective as in (6.2.6), however without the index k as now only the mean field 'average' player is considered. At any particular time t, the player will obtain a T fixed by the observation of  $m_t$ , and the agent is looking for the optimal control  $a^*$  for the minimum cost-to-go. The cost-to-go value function  $U(t,x): [0,\tau] \times [0,1] \mapsto \mathbb{R}$  has the following form:

$$U(t,x) = \min_{a(t'), t \le t' \le \tau} \left( \frac{1}{2} \int_t^\tau a^2(t) dt + C_T(\hat{t},\tau,T) \right), \tag{6.3.2}$$

subject to the dynamics of x as defined in (6.2.10).

The optimal solution of the cost minimization is the value function U which satisfies the backward Hamilton-Jacobi-Bellman (HJB) equation:

$$\partial_t U + \min_a \left(\frac{1}{2}a^2 + a\partial_x U\right) + \frac{\sigma^2}{2}\partial_{xx}^2 U = 0.$$
 (6.3.3)

Solving the minimization part by using the optimal control term  $a^* = -\partial_x U$ , this equation is formulated as:

$$\partial_t U - \frac{1}{2} (\partial_x U)^2 + \frac{\sigma^2}{2} \partial_{xx}^2 U = 0,$$
 (6.3.4)

with the boundary conditions  $U(\tau, 1) = C_T(\hat{t}, \tau, T)$  corresponding to the endpoint cost when fully charged, and  $U(t_{max}, x) = C_T(\hat{t}, t_{max}, t_{max})$  corresponding to the endpoint cost defined in terms of the maximum allowed charging delay time. Provided an optimal  $m_t^*$ , the HJB equation will determine the function U and hence indicate the optimal  $a^*(t)$  of the player.

The optimal movement of  $m_t^*$ , for a given  $m_0$ , is determined by the following forward Fokker-Planck-Kolmogorov (FPK) equation,

$$\partial_t m + \partial_x (a^* m) - \frac{\sigma^2}{2} \partial_{xx}^2 m = 0, \qquad (6.3.5)$$

with the compact boundary conditions of  $m(\cdot, 0) = 0$  and  $m(\cdot, 1) = 0$ . It is observed in the FPK equation that  $a^*$  is exactly the optimal control strategy results from the HJB equation. Solving the two coupled SPDEs will determine the MFE solution, if exists.

#### 6.3.2 Existence and uniqueness of the MFE

The justification of the above mathematical scheme stems from proving the existence and uniqueness of a MFE solution. Similar to classical games, Brouwer fixed point theorem is used for establishing the equilibrium point from the best responses mapping. For the proposed optimal charging mean field game, the mapping is between the optimal control  $a^*$  and  $m^*$  consisting all players' controls. It is discovered that, one chooses best strategy  $a^*$  by solving the HJB equation corresponding to a given T. T is determined by the dynamics of flow  $m^*$  which is given by the FPK equation. Hence it is useful to investigate the time T coherent with the rational behaviours of the

players. The MFE is eventually a matter of locating the fixed point of the mapping  $T \mapsto T$ .

Consider the following representation of the coupled SPDE scheme:

$$T \mapsto C_T \mapsto U \mapsto -\partial_x U \mapsto m \mapsto \partial_x m(\cdot, 1) \mapsto T, \tag{6.3.6}$$

It can be seen that the scheme is from  $[\hat{t}, t_{max}]$  to  $[\hat{t}, t_{max}]$  itself. In order to obtain a fixed point result for the mapping, it is needed only to show the scheme is continuous [35].

The first part of the scheme,  $C_T(\hat{t}, \tau, T)$  is assumed to be a  $C^2$  continuous function. Following the second part, it can be observed that function U is continuous in  $C_T$ . It is further stated that the HJB equation provides a solution of  $U \in C^2$  with  $-\partial_x U$  is Lipschitz continuous according to [116]. Also, the solution m of the FPK equation is  $C^1$  continuous and  $\partial_x m(\cdot, 1) \in$  $C^0$  admits a positive lower bound for any  $T \in [\hat{t}, t_{max}]$  [116]. Now the final mapping of the scheme is considered, which is  $\Gamma : \partial_x m(\cdot, 1) \mapsto T$ . Define  $\gamma_1$  and  $\gamma_2$  to represent the two different flows of dynamics reaching 1. They are both bounded by a common  $\epsilon$ . Assuming  $T_1 = \Gamma(\gamma_1)$  and  $T_2 = \Gamma(\gamma_2), \hat{t} \leq T_1 < T_2 \leq t_{max}$ , it has

$$\int_{0}^{T_2} \gamma_2 \le \int_{0}^{T_1} \gamma_1 \Rightarrow \int_{T_1}^{T_2} \gamma_2 \le \int_{0}^{T_1} (\gamma_1 - \gamma_2).$$
(6.3.7)

The left term  $\int_{T_1}^{T_2} \gamma_2$  is bounded by  $\epsilon(T_2 - T_1)$  while the value of  $\int_0^{T_1} (\gamma_1 - \gamma_2)$  is below  $t_{max}(||\gamma_1 - \gamma_2||_{\infty})$ . Thus,

$$(T_2 - T_1) \le \frac{t_{max}}{\epsilon} (||\gamma_1 - \gamma_2||_{\infty}),$$
 (6.3.8)

which satisfies the Lipschitz condition. Therefore the mapping  $\Gamma$  is  $C^0$  continuous. The overall scheme is a continuous mapping of  $T \mapsto T$ , which admits a fixed point  $T^*$  coherent with the behaviours  $a^*$  and  $m^*$ . Hence, the existence of a MFE solution for the charging optimization game is established.

In terms of the uniqueness of the MFE solution, it is stated in [117] that, in order to produce a unique MFE, the game theoretic formulation requires additional monotonicity conditions in relation to the cost optimization in the HJB equation. It can be argued that these conditions are subject to individual game modeling, and they are not necessarily general premises in EV charging scenarios.

#### 6.3.3 Mean field game versus K-person game

The formulation of mean field game introduces a generalization approach by which the interaction among large populations can be analysed, based on the assumptions that players are treated indistinguishable and continuum. The settings of mean field players have advantages in the sense of increased computational efficiency [111]. By formulating players into a continuum, it enables the use of powerful differential calculus and statistics for analysing the optimal behaviours of the players. As they take actions based only on the statistical state of the total mass, information exchange in terms of their exact game play can be omitted. This reduces the system ICT overhead while enhancing privacy. Moreover, comparing to K-person game where players are sensitive to the changes of the others, changes of particular players in a mean field game has little impact on the performance of the total mass. Therefore the optimal strategy choice of every player can remain. This enhances the efficiency and the stability of the optimization system. However, the mean field players result in less sophisticated than that of Kperson games, because players are able to observe and respond to the exact moves of all others in a K-person game.

In addition, the mean field solutions can be considered as the limit approximation of K-person games as  $K \to \infty$ . It is claimed that a corrective

term in the order of 1/K is sufficient to describe the precision of the approximation [116]. Thus, the efficient mean field game approach can be applied to a wider range of practical applications including those with limited dimension (small K), for example in oil production, and in the case of EV charging.

#### 6.4 Numerical Results and Performance Evaluation

#### 6.4.1 System set up

Consider a charging station with the total ability of charging 100 EVs simultaneously. The scheduled time length of charging is  $\hat{t} = 40$  minutes, with the allowed maximum extension to  $t_{max} = 60$  minutes. A quorum rule of  $\theta = 85\%$  is used to determine T. The station's pricing policy has been defined as in (6.2.1). Two terminal costs for all EVs will be determined by the following linear functions:  $c_1(\hat{t}, \tau) = 4([\tau - \hat{t}]_+)$ , and  $c_2(T, \tau) = 4([\tau - T]_+)$ . The battery capacity of the EVs, as well as the charging status parameter x, are represented by percentage values in between 0 and 100. Assume that each EV has a full battery capacity of 5kWh. However they have been assigned with different initial charging states, i.e., a battery capacity value randomly chosen between 10 and 15 percent of its full capacity. The minimum charging rate of the EVs is assumed to be  $a_{min} = 1$  percent per minute while the maximum is  $a_{max} = 4$  percent per minute.

#### 6.4.2 Performance evaluation

First, the dynamic charging optimization is demonstrated for a single EV scenario. Consider 100 EVs are charging in the station however only one EV is optimising its charging according to the charging status of all other EVs. In this scenario, it is assumed that all the other EVs randomly change their charging rates without optimization.



Figure 6.2. Optimal charging rates for a single EV (T = 55).

Figure 6.2 depicts the dynamics of the optimal charging rate over time for the single EV considered. The trajectories of the charging battery capacity and the cost-to-go function are depicted in Figure 6.3 and Figure 6.4 respectively. As seen, the EV uses different charging speed with an average of two percent per minute during its charging. The charging finishes with 100 percent battery capacity at the elapse of 55 minutes, which is exactly T, the actual finishing time of this charging event for the 100 EVs. In this way, the lateness cost at the endpoint has been successfully minimised. The cost-to-go during the charging process is generally decreasing with a final cost of 60 which is only due to the penalty as defined in  $c_1$ . Therefore it can be claimed that the optimization has efficiently made the full utilization of the permitted time and obtained the optimal strategy for the EV.

Now it is considered that all charging EVs are participating in the proposed mean field game to optimise their cost of charging. Figure 6.5, Figure 6.6 and Figure 6.7 depict the results in terms of charging rates, battery ca-



Figure 6.3. Optimal trajectory of the charging battery capacity for a single EV (T = 55).



Figure 6.4. Optimal trajectory of the cost-to-go value for a single EV (T = 55).



Figure 6.5. Optimal charging rates for one particular EV in a mean field game theoretic scenario  $(T^* = 59)$ .



Figure 6.6. Optimal trajectory of the charging battery capacity for one particular EV in a mean field game theoretic scenario  $(T^* = 59)$ .



Figure 6.7. Optimal trajectory of the cost-to-go value for one particular EV in a mean field game theoretic scenario  $(T^* = 59)$ .

pacities and the costs for a randomly chosen EV (index=8) from the 100 participants. Similar to the discussion in the above scenario, the EV is able to finish its charging at time  $\tau = 58$  that is the closest possible time to the actual finishing time  $T^* = 59$  which is determined by the MFE. The average of charging rates is similar to the previous case, however less dynamics appear. This reflects the fact that the the players in a mean field game setting have less clear vision of the charging system and they only take average actions which result in less dynamics in time. Also, it can be deduced that their charging behaviours will potentially be coincident due to the similarity and rationality of the players. Such reduced variation of the participants results into a more stable charging trajectory for every EV, which is beneficial to the demand management of the charging station and the batteries of the EVs themselves.

Figure 6.8 depicts the dynamics of optimal battery capacity for all 100



Figure 6.8. Optimal battery capacity dynamics  $(T^* = 59)$ .



Figure 6.9. Distributions of battery capacity over charging duration.

EVs during the charging period. It can be observed that all EVs indeed behaved reasonably similar in terms of the trajectories of battery capacity. The distributions of battery capacity value of the EVs, over the charging period, have a fairly low standard deviation that is between 0.8 and 2.3.

Detailed distributions of the battery capacity of all the EVs over the charging period is depicted in Figure 6.9. The distributions are shown in 10 different time instants from the start of charging towards the finish, with 15 minutes intervals during the first 45 minutes and one minute intervals in the final period from 55 to 60 minutes. As seen, the variation of batteries capacity remains in a range of approximately 10 percent of full capacity. As the charging moved near to the finish point, the variation became smaller. Majority of the EVs finish their charging after 55 minutes. At time  $T^* = 59$  when the charging really stopped, 14 EVs did not obtain full recharge. This reflects the setting of the quorum rule. However, considering their batteries have already reached 97 percent of full capacity, the results should be acceptable by these EVs.

From the demand management perspective, the charging is optimised in the sense of balancing the consumption over time to avoid accumulated peak loads at the charging station. Figure 6.10 depicts the accumulated consumption of the station over the charging time. The consumption profile seems fairly flat without any high peaks. A total of 100 charging EVs consume an average of 10kWh of electricity per minute. Such consumption can be claimed to be reasonably low while also stable, which is beneficial to the reliability of the grid. However such result may have limitations in terms of realistic requirements. For example, the energy required for recharging can be higher than 5kWh. Certain types of EV batteries cannot maintain normal operation at low capacity level, therefore a starting point of 15 percent is not tolerant. Besides, the charging station may not have sufficient capacity to support full charge within 60 minutes.



Figure 6.10. Total energy consumption of 100 EVs  $(T^* = 59)$ .



Figure 6.11. Optimal batteries capacity dynamics for 100 charging EVs ( $T^* = 47$ ).

Finally, the effectiveness in terms of the punctuality regulation is evaluated by changing the endpoint costs in the game. In the previous scenario, the charging finished with a 19 minute delay to the scheduled finishing time. Although it is tolerable according to the settings of the charging station, such delay can still reduce the accuracy of load planning and hence the reliability of demand management. The station can issue more critical punctuality costs to regulate the charging EVs and urge them to finish sooner. Figure 6.11 shows the dynamics of all EVs with a slightly increased terminal cost of  $c_1(\hat{t}, \tau) = 5([\tau - \hat{t}]_+)$ , while other settings remain the same. As seen, the charging of the batteries became quicker. The charging finished at  $T^* = 47$ , which is an improvement by 12 minutes compared to the previous scenario.

#### 6.5 Summary

This chapter proposed a dynamic game theoretic optimization framework based on stochastic mean field game approach for charging electric vehicles in smart grids. It is designed for an optimal charging scenarios where a large number of EVs charge simultaneously in an aggregated charging station. Given the pricing policy of the charging station and the statistical charging status of all EVs, the game theoretic framework provides an optimal solution for for every individual EV to proactively control their charging rate in order to minimise the cost of charging. Numerical results have been presented to demonstrate the performance of the proposed framework.

## CONCLUSIONS AND FUTURE WORK

The thesis has four contributing chapters and each chapter is summarised below, followed by a discussion on future works.

#### 7.1 Conclusions

This thesis has investigated demand management in smart grids, with a focus on the development of various mathematical optimization techniques and game theoretic frameworks for home consumption scheduling and electric vehicle charging.

Chapter 3 provided an overview of demand management with a particular focus on the associated enabling wireless technologies. In order to perform demand management to reduce peak loads in the smart grid, the acquisition of real time data from various points in the grid and optimization of the power supply and demand are required. The smart meters and sensors will be deployed in various parts of the grid, starting from the generation, through distribution, and all the way to the household level. These will be interconnected through both wired and wireless connections. Wireless solutions are preferred at the NAN and HAN levels and wired connections could be used for backhaul networks. This chapter covered various candidate communication technologies and mechanisms to enable demand management, in particular for home and neighbourhood areas. As communication is an underpinning technology for the success of smart grid, it is envisaged that smart grids will be an exciting research area for communication engineers. In addition, potential approaches for the optimal demand management in smart grids using these wireless technologies were reviewed in this chapter. Direct and indirect demand management mechanisms including consumption scheduling optimization, dynamic pricing have been discussed. Distributed energy resource, such as locally generated and stored energy, should also be carefully considered. In particular, for the demand management to be successful and efficient, consumers should be given adequate incentives for full participation.

Chapter 4 studied the consumption optimization problem for home area demand management. A consumption scheduling mechanism for residential demand management using mixed integer linear programming (MILP) technique was proposed. The aim of the proposed scheduling was to minimise the peak hourly load in order to achieve an optimal (balanced) daily load schedule. The proposed technique was able to schedule the optimal operation time for appliances according to the power consumption patterns of the individual appliances. The penetration of EVs was considered in the optimization framework. Matlab based simulation results of home and neighbourhood area consumption scheduling scenarios have been presented to demonstrate the effectiveness of the proposed technique in terms of reducing the peak to average consumption ratio. When multiple households participate in the scheduling as well as EVs are intergraded, a more balanced hourly load profile was achieved.

In addition to the centralised consumption scheduling optimization technique, Chapter 5 discussed the approach of consumer incentive based indirect demand management. Given the pricing information, a decentralised consumption scheduling optimization framework was designed to coordinatively manage the scheduling of appliances of multiple residential consumers with the aim of achieving minimum energy cost. The framework was based on the MILP optimization technique proposed in the previous chapter and game theory. In particular, the optimization incorporates integration of locally generated and stored renewable energy in order to minimise the dependency on conventional energy. One of the important benefits of the proposed approach was that the computational complexity can be distributed among the individual home demand management units by decomposing the large scale centralised optimization using a decentralised game theoretic method. Theoretical analysis was presented to show that the proposed game theoretic algorithm admits Nash equilibrium, i.e., the stable solution of the optimization framework. Also, the scheduling optimization converged to the equilibrium where all consumers can benefit from participating in. Simulation results were presented to demonstrate the proposed approach and the benefits of home demand management in terms of reduced energy cost and more balanced consumption profile over the scheduling period.

Electric vehicle is considered to be an important component of demand management in smart grids. In addition to home area consumption optimizations, which could already have the ability to optimally schedule the use of limited number of EVs at home areas, Chapter 6 focused on the topic of optimal charging for aggregated and numerous EVs. This chapter proposed a dynamic mean field game theoretic optimization framework for the players, i.e., the EVs, to manage their charging according to the statistical performance of all the players. In particular, the optimization considered a charging station where a large number of EVs can be charged simultaneously during a flexible permitted period of time. The proposed technique provides every individual EV an optimal charging strategy to proactively and dynamically control its charging rates in order to minimise the charging costs. Theoretical analysis of the game in terms of its formulation, solution, and a comparison of mean field game with the traditional game theoretic techniques were provided. The mean field game theoretic formulation benefited from reduced computational complexity, information and communications overhead, and enhanced privacy. However, players in the mean field games were considered to be less sophisticated than that of classical K-person games, because players are not able to observe and respond to the exact actions taken by all the other players in a mean field game. Numerical results demonstrated the performance of the proposed framework.

All the mathematical optimization techniques proposed in the thesis facilitate optimal use of energy. They have the merits in terms of reducing the peak consumption load and the consumers' energy costs.

#### 7.2 Future Work

The potential directions for future research are outlined as follows.

For integer programming based algorithms, the complexity increases exponentially with the problem size. Therefore, complexity reduction for the technique proposed in Chapter 4 needs to be investigated. One possibility is to consider the formulation of the consumption optimization in continuous time domain, so that the problem can be solved using convex algorithms. However, various manipulations may be required to the formulation so that the solution is approximately equivalent to the optimal solution. In addition to the cycling effect and round-trip loss of the EVs as considered in the proposed framework, the efficiency of energy consumption for various appliances can also be included.

The scheduling optimization is based on the idea of advanced consumption planning. It is useful to extend the optimization framework to (near) real time scheduling scenarios where operators and consumers have more flexibility to respond to the dynamic demand conditions. The successful deployment of such scheduling optimization requires the availability of real time pricing and the sophisticated ICT solutions.

The game theoretic optimization framework proposed in Chapter 5 for appliances' consumption scheduling was formulated based on consumers' coordination. A further research can be directed under the consideration of competitive situations, where consumers are able to sell back their locally generated energy to the grid in order to generate additional profits in a competitive pricing manner. Bayesian games with imperfect and incomplete information of consumers' consumption behaviour can be included. Investigation on how user privacy impacts on the effectiveness of demand management is also required.

Mean field game is a novel research topic in game theory. The perfection of its theoretical basis and its potential applications are currently open research topics. The study of EV charging optimization in Chapter 6 can be extended to include practical conditions such as reasonable selection of pricing functions and justification of Brownian motion representation. The efficiency of the optimal MFE solution should be investigated.

Finally, the application of the powerful mathematical optimization techniques, in particular game theoretic modeling is not limited to appliances' consumption scheduling and EV charging scenarios. Applications such as autonomous systems and self configuring and learning systems based on game theoretic methods can be used in both the smart grid communications and demand management. The development of games at the regulation level of the demand-supply chain, such as robust auction games against cheating are also important. Although the roadmap of worldwide smart grid deployment is still indeterminate, it is almost certain that the future intelligent energy network will bring substantial benefits and fundamental changes in way similar to the Internet. Surely the smart grids is an exciting research area for electrical and electronic engineers for many years to come.

### References

- D. Lindley, "Smart grids: The energy storage problem," *Nature*, vol. 463, pp. 18–20, Jan. 2010.
- [2] US DoE, "Smart Grid Research & Development: Multi-Year Program Plan(MYPP) 2010-2014," Available http://www.oe.energy.gov.
- [3] O. Sundstrom, D. Gantenbein, and C. Binding, "Adding gridwise information to ecogrid deliverable d1.1," *IBM*, *Tech. Rep.*, 2011.
- [4] New Mexico Business Weekly, "Japan signs smart grid accords with New Mexico," March 2010.
- [5] European Commission, "European Smart Grids Technology Platform
   Vision and Strategy for Europes Electricity Networks of the Future," Directorate-General for Research – Sustainable Energy Systems, 2006, Available http://www.smartgrids.eu.
- [6] European Commission, "Commission Working Document Consultation on the future EU 2020 strategy," 2009.
- [7] European Commission, "Future Internet for future European economies and societies," 2010.
- [8] A. Faruqui, D. Harris, and R. Hledik, "Unlocking the 53 billion euros savings from smart meters in the EU: How increasing the adoption of dynamic tariffs could make or break the EU's smart grid investments," *Energy Policy*, vol. 38, no. 10, 2010.

- [9] National Energy Technology Laboratory for the U.S. Department of Energy, Office of Electricity Delivery and Energy Reliability, "The NETL Modern Grid Initiative - Powering our 21st-Century Economy," *Modern Grid Benefits*, Aug. 2007.
- [10] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, UK: Cambridge University Press, 2004.
- [11] D. G. Luenberger, *Linear and Nonlinear Programming*. Reading, Massachusetts: Addison-Wesley Publishing Co., 1984.
- [12] G. L. Nemhauser and L. A. Wolsey, Integer and combinatorial optimization. John Wiley and Sons, New York, 1998.
- [13] W. H. Fleming and R. W. Rishel, Deterministic and Stochastic Optimal Control. Springer, 1975.
- [14] A. K. Dixit, Optimization in Economic Theory. Oxford, UK: Oxford University Press, 1990.
- [15] K. Schmedders, "Numerical optimization methods in economics," The New Palgrave Dictionary of Economics, 2008.
- [16] B. Krongold, K. Ramchandran, and D. Jones, "Computationally efficient optimal power allocation algorithms for multicarrier communication systems," *IEEE Transactions on Communications*, vol. 48, pp. 23–27, Jan. 2000.
- [17] D. Palomar, J. Cioffi, and M. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, Sept. 2003.
- [18] Y. Nesterov and A. Nemirovski, "Interior-point polynomial algorithms in convex programming," SIAM Stud. Appl. Math., vol. 13, 1994.

- [19] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," *Optimization Methods and Software*, 2007.
- [20] J. W. Chinneck, *Practical optimization: A gentle introduction*. Available online at http://www.sce.carleton.ca/faculty/chinneck/po.html, 2012.
- [21] W. Wiesemann, *Integer Programming*. Available online at http://www.doc.ic.ac.uk/ br/berc/IPlecture1.pdf, 2009.
- [22] D. S. Chen, R. G. Batson, and Y. Dang, Applied Integer Programming: Modeling and Solution. John Wiley and Sons, 2010.
- [23] S. P. Bradley, A. C. Hax, and T. L. Magnanti, Applied Mathematical Programming. Addison-Wesley, 1977.
- [24] Z. Mao and X. Wang, "Efficient optimal and suboptimal radio resource allocation in ofdma system," *IEEE Transactions on Wireless Communications*, vol. 7, pp. 440–445, Feb. 2008.
- [25] E. L. Lawler and D. E. Wood, "Branch-and-bound methods: A survey," Operations Research, vol. 14, p. 699, 1966.
- [26] R. E. Moore, "Global optimization to prescribed accuracy.," Computers and Mathematics with Applications, vol. 21(6/7), p. 25, 1991.
- [27] J. Lofberg, "Yalmip: A toolbox for modelling and optimization in matlab," in *CACSD*, 2004.
- [28] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for analyzing equilibria in wireless games," *IEEE Signal Processing Magazine*, vol. 26, pp. 41–52, Sept. 2009.
- [29] P. K.Dutta, Strategies and games: Theory and practice. MIT Press, 1999.
- [30] J. Nash, "Equilibrium points in n-person games," Proceedings of the National Academy of Sciences, vol. 36, no. 1, pp. 48–49, 1950.

- [31] J. Nash, "Non-cooperative games," The Annals of Mathematics, vol. 54, no. 2, pp. 286–295, 1951.
- [32] M. J. Osborne, An introduction to game theory. Oxford University Press, 2004.
- [33] R. J. Aumann, "Acceptable points in general cooperative n-person games," Contributions to the Theory of Games IV, Annals of Mathematics Study 40, pp. 287–324, 1959.
- [34] J. Rosen, "Existence and uniqueness of equilibrium points for concave n-person games," *Econometrica*, vol. 33, no. 3, pp. 520–534, 1965.
- [35] K. C. Border, Fixed Point Theorems with Applications to Economics and Game Theory. Cambridge University Press, 1985.
- [36] T. S. Morrison, The Banach and Brouwer Fixed Point Theorems and Applications. ProQuest, 2008.
- [37] Z. Han, D. Niyato, W. Saad, T. Basar, and A. Hjorungnes, Game Theory in Wireless and Communication Networks: Theory, Models and Applications. Cambridge University Press, UK, 2011.
- [38] A. B. MacKenzie and L. A. DaSilva, Game Theory for Wireless Engineers. Morgan & Claypool, 2006.
- [39] H. Ji and C. Huang, "non-cooperative uplink power control in cellular radio systems," *Wireless Networks*, vol. 4, no. 3, pp. 233–240, 1998.
- [40] T. Alpcan, T. Basar, R. Srikant, and E. Altman, "CDMA uplink power control as a noncooperative game," in *Proceedings IEEE Conference on Decision and Control*, pp. 197–202, Feb. 2001.
- [41] E. Altman, K. Avrachenkov, I. Menache, G. Miller, B. Prabhu, and A. Shwartz, "Dynamic discrete power control in cellular networks," *IEEE Transactions on Automatic Control*, vol. 54, no. 10, pp. 2328–2340, 2009.
- [42] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "efficient power control via pricing in wireless data networks," *IEEE Transactions on Communications*, vol. 50, pp. 291–303, Feb. 2002.
- [43] V. Srivastava, J. Neel, A. B. MacKenzie, R. Menon, L. A. DaSilva, J. E. Hicks, J. H. Reed, and R. P. Gilles, "Using game theory to analyze wireless ad hoc networks," *IEEE Communication Surveys and Tutorials*, 2005.
- [44] D. H. N. Nguyen and T. Le-Ngoc, "Multiuser downlink beamforming in multicell wireless systems: A game theoretical approach," *IEEE Transactions on Signal Processing*, vol. 59, no. 7, pp. 3326–3338, 2011.
- [45] Q. D. La, Y. H. Chew, and B. H. Soong, "An interference-minimization potential game for OFDMA-based distributed spectrum sharing systems," *IEEE Transactions on vehicular technology*, vol. 60, pp. 3374–3385, Sept. 2011.
- [46] G. Greco, E. Malizia, L. Palopoli, and F. Scarcello, "Non-transferable utility coalitional games via mixed-integer linear constraints," *Journal of Artificial Intelligence Research*, no. 38, pp. 633–685, 2010.
- [47] T. Basar and G. Olsder, Dynamic non-cooperative game theory. SIAM Series in Classics in Applied Mathematics, Society for Industrial and Applied Mathematics, 2nd ed., 1999.
- [48] F. Li, et al., "Smart transmission grid: Vision and framework," *IEEE transactions on Smart Grid*, vol. 1, no. 2, pp. 168–177, Sep.2010.
- [49] C. Wietfeld, C. Müller, J. Schmutzler, S. Fries, A. Heidenreich, and H. J. Hof, "ICT reference architecture design based on requirements for future energy marketplaces," in *Proceedings IEEE International Conference* on Smart Grid Communications (SmartGridComm), pp. 315–320, Oct 2010.

- [50] Z. Fan, G. Kalogridis, C. Efthymiou, M. Sooriyabandara, M. Serizawa, and J. McGeehan, "The new frontier of communications research: Smart grid and smart metering," in *Proceedings of the ACM 1st International Conference on Energy-Efficient Computing and Networking*, pp. 115–118, 2010.
- [51] W. Saad, Z. Han, and H. V. Poor, "A game theoretic approach for multihop power line communications," in *Proceedings International Conference* on Game Theory for Networks (GameNets), Apr. 2011.
- [52] G. Scutari, D. Palomar, F. Facchinei, and J. Pang, "Convex optimization, game theory, and variational inequality theory," *IEEE Signal Processing Magazine*, vol. 27, pp. 35–49, May 2010.
- [53] A. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Transactions on Smart Grid*, vol. 1, no. 3, pp. 320–331, Dec. 2010.
- [54] Q. Zhu, Z. Han, and T. Basar, "A differential game approach to distributed demand side management in smart grid," in *Communications* (ICC), 2012 IEEE International Conference on, pp. 3345 –3350, june 2012.
- [55] S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing, and T. Basar, "Dependable demand response management in the smart grid: A stackelberg game approach," *IEEE Transactions on Smart Grid*, vol. 4, pp. 120–132, March 2013.
- [56] P. Yang, G. Tang, and A. Nehorai, "A game-theoretic approach for optimal time-of-use electricity pricing," *IEEE Transactions on Power Systems*, vol. 28, pp. 884–892, May 2013.
- [57] N. Li, L. Chen, and S. Low, "Optimal demand response based on utility maximization in power networks," in *Proceedings IEEE Power and Energy Society General Meeting*, pp. 1–8, July 2011.

- [58] R. Arai, K. Yamamoto, and M. Morikura, "Differential game-theoretic framework for a demand-side energy management system," in *Proceedings IEEE International Conference on Smart Grid Communications (Smart-GridComm)*, pp. 768–773, Oct 2013.
- [59] Y. Li, Y. Wang, S. Nazarian, and M. Pedram, "A nested game-based optimization framework for electricity retailers in the smart grid with residential users and pevs," in *Proceedings IEEE Online Conference on Green Communications (GreenCom)*, pp. 157–162, Oct 2013.
- [60] A. Mondal and S. Misra, "Dynamic coalition formation in a smart grid: A game theoretic approach," in *Proceedings IEEE International Conference* on Communications Workshops (ICC), pp. 1067–1071, June 2013.
- [61] A. Bose, "Smart Transmission Grid Applications and Their Supporting Infrastructure," *IEEE trans. on Smart Grid*, vol. 1, no. 1, pp. 11–19, Jun. 2010.
- [62] T. Sauter and M. Lobashov, "End-to-end communication architecture for smart grids," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 4, pp. 1218 –1228, Apr. 2011.
- [63] OpenSG, "SG Network Systems Requirement Specification v5.0 (draft)".Aug. 2011, Available http://osgug.ucaiug.org.
- [64] "IEEE Standard for Local and metropolitan area networks Part 16: Air Interface for Broadband Wireless Access Systems," *IEEE Std 802.16-2009* (*Revision of IEEE Std 802.16-2004*), pp. C1–2004, May 2009.
- [65] H. Zhu and J. Wang, "Chunk-based resource allocation in OFDMA systems - part I: chunk allocation," *IEEE Transactions on Communications*, vol. 57, pp. 2734–2744, Sept. 2009.

- [66] "IEEE Standard for Local and metropolitan area networks Part 16: Air Interface for Broadband Wireless Access Systems Amendment 1: Multiple Relay Specification," *IEEE Std 802.16j-2009 (Amendment to IEEE Std 802.16-2009)*, pp. C1–290, Dec. 2009.
- [67] 3GPP TSG RAN, "Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 8)," TS 36.211, V8.9.0, Dec. 2009.
- [68] 3GPP TSG RAN, "Evolved Universal Terrestrial Radio Access (E-UTRA); Physical Channels and Modulation (Release 10)," TS 36.211, V10.4.0, Dec. 2011.
- [69] "IEEE Standard for Information Technology–Telecommunications and information exchange between systems Wireless Regional Area Networks (WRAN)–Specific requirements Part 22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Policies and Procedures for Operation in the TV Bands," *IEEE Std 802.22-2011*, pp. 1–680, Jan. 2011.
- [70] C. Gellings and W. Smith, "Integrating demand-side management into utility planning," *Proceedings of the IEEE*, vol. 77, no. 6, pp. 908–918, 1989.
- [71] V. Bakker, M. Bosman, A. Molderink, J. Hurink, and G. Smit, "Demand side load management using a three step optimization methodology," in *Pro*ceedings 1st IEEE International Conference on Smart Grid Communications (SmartGridComm), pp. 431–436, Gaithersburg, Maryland, USA, Oct. 2010.
- [72] A. Brooks, E. Lu, D. Reicher, C. Spirakis, and B. Weihl, "Demand dispatch," *IEEE Power and Energy Magazine*, vol. 8, no. 7, pp. 20–29, Jun. 2010.
- [73] M. Pedrasa, T. Spooner, and I. MacGill, "Coordinated scheduling of residential distributed energy resources to optimize smart home energy ser-

vices," *IEEE Transactions on Smart Grid*, vol. 1, no. 2, pp. 134–143, Sept. 2010.

- [74] Y. Xia, S. Douglas, and D. Mandic, "Adaptive frequency estimation in smart grid applications: Exploiting noncircularity and widely linear adaptive estimators," *IEEE Signal Processing Magazine*, vol. 29, pp. 44–54, Sept 2012.
- [75] G. Koutitas and L. Tassiulas, "A delay based optimization scheme for peak load reduction in the smart grid," in *Proceedings Future Energy Systems: Where Energy, Computing and Communication Meet (e-Energy),* 2012 Third International Conference on, pp. 1–4, may 2012.
- [76] J. Medina, N. Muller, and I. Roytelman, "Demand response and distribution grid operations: Opportunities and challenges," *IEEE Transactions* on Smart Grid, vol. 1, no. 2, pp. 193–198, Sep. 2010.
- [77] K. Hamilton and N.Gulhar, "Taking demand response to the next level," *IEEE Power and Energy Magazine*, vol. 8, no. 3, pp. 60–65, Jun. 2010.
- [78] I.F. Akyildiz, et al., "A survey on sensor networks," *IEEE Communica*tions Magazine, vol. 40, no. 8, pp. 102–114, Aug. 2002.
- [79] V.C. Gugor, et al., "Opportunities and challenges of wireless sensor networks in smart grid," *IEEE trans. on Industrial Electronics*, vol. 57, no. 10, pp. 3557–3564, Oct. 2010.
- [80] C. Wang and M. de Groot, "Managing end-user preferences in the smart grid," in *Proceedings ACM International Conference on Energy-Efficient Computing and Networking*, pp. 357–363, Apr. 2010.
- [81] I. Koutsopoulos and L. Tassiulas, "Control and optimization meet the smart power grid-scheduling of power demands for optimal energy management," in *Proceedings of 2nd International Conference on Energy-Efficient Computing and Networking (E-Energy)*, 2011.

- [82] EERE. US Department of Energy. http://www.eere.energy.gov/.
- [83] DECC. UK Department of Energy & Climate Change. http://www.decc.gov.uk/.
- [84] F. Koyanagi and Y. Uriu, "A strategy of load leveling by charging and discharging time control of electric vehicles," *IEEE Transactions on Power Systems*, vol. 13, no. 3, pp. 1179–1184, 1998.
- [85] Renault. Fluence Z.E. Specifications, Available http://www.renault-ze.com/.
- [86] T. Morgan, "Smart grids and electric vehicles: made for each other?," in International Transport Forum, Discussion Paper, no. 2012-02. 2012.
- [87] H. Zareipour, A. Janjani, H. Leung, A. Motamedi, and A. Schellenberg,
  "Electricity price thresholding and classification," in *Proceedings 2011 IEEE* Power and Energy Society General Meeting, pp. 1–7, July 2011.
- [88] H. Chao, "Competitive electricity markets with consumer subscription service in a smart grid," *Journal of Regulatory Economics*, vol. 41, no. 1, pp. 155–180, 2012.
- [89] E. Celebi and J. D. Fuller, "A model for efficient consumer pricing schemes in electricity markets," *IEEE Transactions on Power Systems*, vol. 22, pp. 60–67, Feb. 2007.
- [90] M. Albadi and F. El-Saadany, "Demand Response in Electricity Markets: An Overview," in *IEEE Power Engineering Society General Meeting*, 2007.
- [91] R. Weron, Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach. New York: Wiley, 2006.
- [92] T. Lui, W. Stirling, and H. Marcy, "Get smart," *IEEE Power and Energy Magazine*, vol. 8, pp. 66–78, May-June 2010.

- [93] Z. Wang and F. Li, "Critical peak pricing tariff design for mass consumers in great britain," in *Proceedings 2011 IEEE Power and Energy Society Gen*eral Meeting, pp. 1–6, July 2011.
- [94] C. Li, S. Tang, Y. Cao, Y. Xu, Y. Li, J. Li, and R. Zhang, "A new stepwise power tariff model and its application for residential consumers in regulated electricity markets," *IEEE Transactions on Power Systems*, vol. PP, no. 99, p. 1, 2012.
- [95] F. Li, "Continuous locational marginal pricing (CLMP)," IEEE Transactions on Power Systems, vol. 22, pp. 1638 –1646, Nov. 2007.
- [96] B. Li, S. Gangadhar, S. Cheng, and P. Verma, "Maximize user rewards in distributed generation environments using reinforcement learning," in *Proceedings IEEE Energytech*, pp. 1–6, may 2011.
- [97] N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani, Algorithmic Game Theory. UK: Cambridge University Press, 2007.
- [98] D. Monderer and L. S. Shapley, "Potential games," Games and Economic Behavior, vol. 14, pp. 124–143, 1996.
- [99] M. Voorneveld, "Economic letters," Best-response potential games, vol. 66, pp. 289–295, 2000.
- [100] D. L. Brito, J. H. Hamilton, S. M. Slutsky, and J. E. Stiglitz, "Pareto efficient tax structures," *Oxford Economic Papers*, vol. 42, pp. 61–77, 1990.
- [101] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," Wireless Communications, IEEE Transactions on, vol. 9, no. 5, pp. 1748–1759, 2010.
- [102] S. Yeung, J. Lui, J. Liu, and J. Yan, "Detecting cheaters for multiplayer games: theory, design and implementation[1]," in *Proceedings 3rd IEEE*

Consumer Communications and Networking Conference, vol. 2, pp. 1178–1182, 2006.

- [103] J. Palmer, N. Terry, and T. Kane, "Early Findings: Demand side management," *Further Analysis of the Household Electricity Survey*, vol. https://www.gov.uk/government/uploads/system/uploads /attachment\_data/file/275483/early\_findings\_revised.pdf, Nov. 2013.
- [104] M. Erol-Kantarci, J. Sarker, and H. Mouftah, "Communication-based plug-in hybrid electrical vehicle load management in the smart grid," in *Proceedings IEEE Symposium on Computers and Communications (ISCC)*, pp. 404–409, 2011.
- [105] G. Venayagamoorthy, P. Mitra, K. Corzine, and C. Huston, "Real-time modeling of distributed plug-in vehicles for v2g transactions," in *Proceedings IEEE Energy Conversion Congress and Exposition*, pp. 3937–3941, 2009.
- [106] B. Kim, S. Ren, M. van der Schaar, and J.-W. Lee, "Bidirectional energy trading and residential load scheduling with electric vehicles in the smart grid," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 7, pp. 1219–1234, 2013.
- [107] S. Sojoudi and S. Low, "Optimal charging of plug-in hybrid electric vehicles in smart grids," in *Proceedings 2011 IEEE Power and Energy Society General Meeting*, pp. 1–6, 2011.
- [108] S. Shao, M. Pipattanasomporn, and S. Rahman, "Challenges of phev penetration to the residential distribution network," in *Proceedings Power Energy Society General Meeting*, 2009. PES '09. IEEE, pp. 1–8, 2009.
- [109] H. Han, H. Xu, and Z. Yuan, "Research of interactive charging strategy for electrical vehicles in smart grids," in *Proceedings International Conference on Electrical Machines and Systems (ICEMS)*, pp. 1–6, 2011.

- [110] R. Couillet, S. Perlaza, H. Tembine, and M. Debbah, "Electrical vehicles in the smart grid: A mean field game analysis," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 6, pp. 1086–1096, 2012.
- [111] J.-M. Lasry and P.-L. Lions, "Mean field games," Jpn. J. Math., vol. 2, no. 1, pp. 229–260, 2007.
- [112] M. Nourian, P. Caines, R. Malhame, and M. Huang, "Nash, social and centralized solutions to consensus problems via mean field control theory," *Automatic Control, IEEE Transactions on*, vol. 58, pp. 639–653, March 2013.
- [113] J. E. Stiglitz and C. E. Walsh, *Economics*. New York London : W. W. Norton, 4th ed., 2006.
- [114] K. Itō, "Multiple wiener integral," J. Math. Soc. Japan, vol. 3, no. 1, pp. 157–169, 1951.
- [115] I. Gihman and A. Skorohod, Stochastic Differential Equations. Springer, Berlin, 1972.
- [116] O. Guéant, P.-L. Lions, and J.-M. Lasry, Mean Field Games and Applications, Paris-Princeton Lectures on Mathematical Finance 2010. Springer. Berlin., 2011.
- [117] P.-L. Lions, *Mean Field Games Courses at Collège de France.* www.college-de-france.fr., 2010.