On Mixed-Mode Fracture in Layered Materials

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Abstract

This paper reports the authors' recent work on partition theories of energy release rate (ERR) for 1D fracture in fiber-reinforced laminated composite beams and plates. A novel and powerful methodology is created to partition the total ERR based on beam and 2D elasticity theories.

1. Introduction

Although delamination in real laminated composites is typically irregularly shaped, 1D fracture can provide an ideal focus for research to gain fundamental mechanical understanding. 1D fracture has only mode I and II action. A double cantilever beam (DCB) is the simplest case. Despite the apparent simplicity, the partition of its energy release rate (ERR) into mode I and II contributions has caused much confusion. The authors have created a novel and powerful method to partition the ERR of 1D fractures in layered composite beams and plates [1–6]. Some of the main results are presented here.

2. Theory of brittle interfacial fracture for laminated composite beams

Figure 1 shows a composite DCB. The crack tip is at B. The furthest extent of the crack's influence is at A. Only axial forces and bending moments at the crack tip are considered.

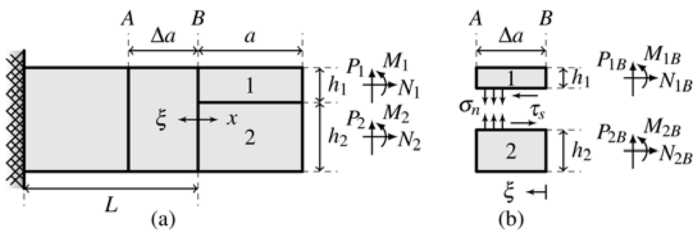

Figure 1. A laminated composite DCB and its loading condition

The total ERR, based on both beam and 2D elasticity theories, can be written as

$$G=\frac{1}{2b^2}\left(\frac{M_{1B}^{\;2}}{D_1^*}+\frac{M_{2B}^{\;2}}{D_2^*}-\frac{M_B^{\;2}}{D^*}+\frac{N_{1B}^{\;2}}{A_1^*}+\frac{N_{2B}^{\;2}}{A_2^*}-\frac{N_B^{\;2}}{A^*}-\frac{2B_1M_{1B}N_{1B}}{B_1^*}-\frac{2B_2M_{2B}N_{2B}}{B_2^*}+\frac{2BM_BN_B}{B^*}\right) \tag{1}$$

In Eq. (1), $A_i^*=A_i-B_i^2/D_i$, $B_i^*=B_i^{\;2}-A_iD_i$ and $D_i^*=D_i-B_i^2/A_i$ with $i=1,2$ for the beams above and below the crack respectively. A_i, B_i and D_i are the extensional, coupling and bending stiffness respectively of beam i, and b is the beam width.

2.1. Classical and first-order shear-deformable beam partition theories

Using classical thin beam theory, the total ERR in Eq. (1) can be partitioned into G_I and G_{II} as follows [1–4]:

$$G_{IE} = c_{IE}\left(M_{1B} - M_{2B}/\beta_1 - N_{1B}/\beta_2 - N_{2B}/\beta_3\right)\times\left(M_{1B} - M_{2B}/\beta_1' - N_{1B}/\beta_2' - N_{2B}/\beta_3'\right) \quad (2)$$

$$G_{IE} = c_{IE}\left(M_{1B} - M_{2B}/\theta_1 - N_{1B}/\theta_2 - N_{2B}/\theta_3\right)\times\left(M_{1B} - M_{2B}/\theta_1' - N_{1B}/\theta_2' - N_{2B}/\theta_3'\right) \quad (3)$$

$$c_{IE} = G_{\theta_1}\left[\left(1-\theta_1/\beta_1\right)\left(1-\theta_1/\beta_1'\right)\right]^{-1} \quad \text{and} \quad c_{IIE} = G_{\beta_1}\left[\left(1-\beta_1/\theta_1\right)\left(1-\beta_1/\theta_1'\right)\right]^{-1} \quad (4)$$

$$G_{\theta_1} = \left[1/D_1^* + \theta_1^2/D_2^* - \left(1+\theta_1\right)^2/D^*\right]/\left(2b^2\right) \quad \text{and} \quad G_{\beta_1} = \left[1/D_1^* + \beta_1^2/D_2^* - \left(1+\beta_1\right)^2/D^*\right]/\left(2b^2\right) \quad (5)$$

$$\theta_1 = \left[\left(B_2^2 - A_2D_2\right)\left(B_1 + h_1A_1/2\right)\right]/\left[\left(B_1^2 - A_1D_1\right)\left(B_2 - h_2A_2/2\right)\right] \quad \text{and} \quad \theta_1' = -1 \quad (6)$$

where θ_1, θ_2, θ_3, β_1, β_2 and β_3 are the orthogonal pure modes of the first set $\{\theta, \beta\}$, and θ_1', θ_2', θ_3', β_1', β_2' and β_3' are the orthogonal pure modes of the second set $\{\theta', \beta'\}$ [1–6]. From Eq. (6), all of the pure modes can be calculated from the orthogonality condition, e.g. β_2 and β_2' can be calculated by the following where $[C]$ is the coefficient matrix of the quadratic form of Eq. (1).

$$\{1 \quad \theta_1 \quad 0 \quad 0\}[C]\{1 \quad 0 \quad \beta_2 \quad 0\}^T = 0 \quad \text{and} \quad \{1 \quad \theta_1' \quad 0 \quad 0\}[C]\{1 \quad 0 \quad \beta_2' \quad 0\}^T = 0 \quad (7)$$

In the first-order shear-deformable beam theory, the two sets of pure modes coincide on the first set.

2.2. 2D elasticity partition theory

The ERR partitions for a 2D elastic laminated unidirectional composite beam are [5]

$$G_{I-2D} = c_{I-2D}\left(M_{1B} - M_{2B}/\beta_{1-2D} - N_{1Be}/\beta_{2-2D}\right)^2 \quad \text{where} \quad N_{1Be} = N_{1B} - N_{2B}/\gamma \quad (8)$$

$$G_{II-2D} = c_{II-2D}\left(M_{1B} - M_{2B}/\theta_{1-2D} - N_{1Be}/\theta_{2-2D}\right)^2 \quad (9)$$

$$c_{I-2D} = G_{\theta_{12D}}\left(1-\theta_{1-2D}/\beta_{1-2D}\right)^{-2} \quad \text{and} \quad G_{\theta_{1-2D}} = \left[1/I_1 + \theta_{1-2D}^2/I_2 - \left(1+\theta_{1-2D}\right)^2/I\right]/\left(2bE\right) \quad (10)$$

$$c_{II-2D} = G_{\beta_{1-2D}}\left(1-\beta_{1-2D}/\theta_{1-2D}\right)^{-2} \quad \text{and} \quad G_{\beta_{1-2D}} = \left[1/I_1 + \beta_{1-2D}^2/I_2 - \left(1+\beta_{1-2D}\right)^2/I\right]/\left(2bE\right) \quad (11)$$

where θ_{1-2D}, θ_{2-2D}, β_{1-2D} and β_{2-2D} are the orthogonal pure modes which are functions of the beam thickness ratio $\gamma = h_2/h_1$. Approximate formulae for θ_{1-2D} are [5]

$$\theta_{1-2D} = -\gamma^2\left[\left(1+\gamma\right)^2 + c_\theta\right]/\left[\left(1+\gamma\right)^2 + c_\theta\gamma^2\right], \quad c_\theta\left(\gamma\right) = \bar{c}_\theta^{\left[\hat{c}_\beta^{1/2}(1) - \hat{c}_\beta^{1/2}(\gamma)\right]} \quad \text{and} \quad \hat{c}_\beta\left(\gamma\right) = \left(1+\gamma\right)^3/\left(1+\gamma^3\right) \quad (12)$$

where $\bar{c}_\theta \approx 6/5$. The other θ_i and β_i pure modes can be obtained by using the orthogonality condition, similar to in Eq. (7).

2.3. Local versus global partitions

The above partition theories are local. 'Local' means that the ERR partition is calculated at the crack tip B. If the ERR partition is instead calculated over the entire region mechanically affected by the crack tip (the region AB in Figure 1), then the same partition theory is obtained as from classical thin beam theory, regardless of whether the calculation is based on 2D elasticity, classical or first-order shear-deformable beams. The classical thin beam partition theory therefore unifies all the partition theories through its global nature [1–4].

2.4. Experimental validation

From symmetric DCB fracture tests ($h_1 = h_2$), the failure locus for glass/epoxy material, is known to be given very closely by the linear failure locus [7] where $(G_I / G_{Ic}) + (G_{II} / G_{IIc}) = 1$. Fracture tests with asymmetric specimens ($h_1 \neq h_2$) should produce this same linear failure locus if the partition theory that is used to partition the total ERR is correct. Mixed-mode fracture test data from Ref. [7] is repartitioned using the authors' partition theories and compared in Figure 2 against Williams' [8] and Hutchinson and Suo's [9] partition theories.

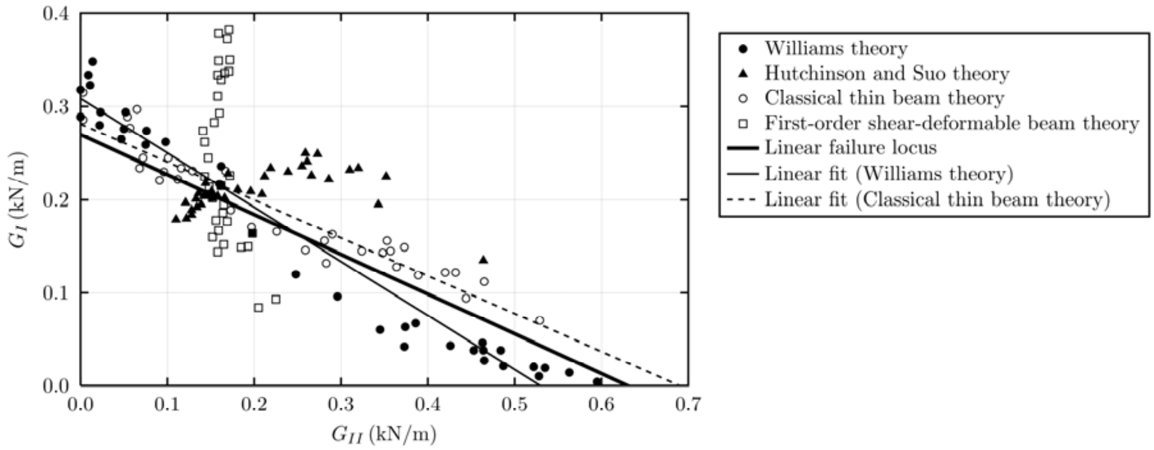

Figure 2. Experimental assessment of various partition theories

It was expected that the theories in Refs. [5] and [9], based on 2D elasticity, should give the best agreement with the linear failure locus; however Figure 2 shows that the authors' classical thin beam partition theory [1–4] performs the best, followed by Williams' [8] theory. One possible reason for this could be to do with the global nature of the classical thin beam partition theory.

3. Theory of non-rigid interfacial fracture

3.1. 2D elasticity partition theory

In 2D elasticity, the ERR partitions G_I and G_{II} are still given by Eqs. (8) to (9), however the θ_i and β_i pure modes are now different and are functions of both the beam thickness ratio $\gamma = h_2 / h_1$ and the interface stiffness-to-modulus ratio $k_{er} = k_\sigma / E = k_\tau / E$. Numerical simulations give the following approximate formulae for θ_{1-2D}. The orthogonality condition can be used to determine the other pure modes.

$$\theta_{1-2D}(\gamma, k_{er}) = \theta_a + 1/2(\theta_a - \theta_c)\log k_{er} + 1/2(\theta_a - 2\theta_b + \theta_c)(\log k_{er})^2 \tag{13}$$

$$\theta_a(\gamma) = (\theta_1 + 3\theta_1')/4 \quad \text{and} \quad \theta_b(\gamma) = (\theta_a + \theta_c)/2 \tag{14}$$

$\theta_c(\gamma)$ is obtained by using the orthogonality condition with respect to $\beta_c(\gamma)$ as follows:

$$\{1 \quad \theta_c \quad 0\}[C]\{1 \quad \beta_c \quad 0\}^T = 0 \quad \text{where} \quad \beta_c(\gamma) = (\beta_1 + 3\beta_1')/4 \tag{15}$$

The other quantities used above are

$$\theta_1 = -\gamma^2 \;\; , \;\; \theta_1' = -1 \;\; , \;\; \beta_1 = \gamma^2(3+\gamma)/(1+3\gamma) \quad \text{and} \quad \beta_1' = \gamma^3 \tag{16}$$

3.2. Finite element method tests

Consider a layered isotropic DCB with its geometry defined as in Figure 1. The intact length is $L = 100\,\mathrm{mm}$, the crack length is $a = 10\,\mathrm{mm}$, the beam width is $b = 10\,\mathrm{mm}$, and the total thickness is $h = 2\,\mathrm{mm}$. A bending moment is applied to the tip of beam 1, $M_1 = 1\,\mathrm{Nm}$. The Young's modulus is $E = 1.0\,\mathrm{GPa}$ and the interface constitutive law is linear elastic and non-rigid. Table 1 shows the 2D finite element method (FEM) results and the calculated results using the 2D elasticity partition theory described above [6]. The comparison is very good.

Table 1. Comparisons between analytical and numerical values of G_I (kN/mm) and G_I/G (%)

		Analytical (×10^6 N/m)			FEM (×10^6 N/m)		
γ	k_{er} (1/m)	0.1	0.5	1	0.1	0.5	1
1	G_I	3.000	3.000	3.000	3.029	3.029	3.028
	G_I/G	57.14	57.14	57.14	57.66	57.60	57.79
3	G_I	45.30	43.75	42.99	45.12	44.09	43.48
	G_I/G	95.87	92.59	90.98	94.72	92.71	91.56
5	G_I	159.7	154.9	152.0	159.0	156.4	154.7
	G_I/G	99.05	96.06	94.24	97.80	96.39	95.50

4. Conclusion

A novel and powerful methodology has been created for the mixed-mode partition of 1D fractures in layered composite beams and plates [1–6]. It has strong capabilities in all the fracture cases investigated so far. Furthermore, it is expected to also work well in even more complex mixed-mode fracture problems.

References

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