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# STRATEGIES FOR TEACHING ENGINEERING MATHEMATICS 

## By

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A doctoral thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of the Loughborough University of Technology, November 1988.


## DEDICATION

This thesis is dedicated to my mother and to the memory of my father and my grandmother.

# ABSTRACT <br> Strategies for Teaching Engineering Mathematics 

by L R Mustoe<br>Department of Mathematical Sciences<br>Loughborough University of Technology

This thesis is an account of experiments into the teaching of mathematics to engineering undergraduates which have been conducted over twenty years against a background of changing intake ability, varying output requirements and increasing restrictions on the formal contact time available.

The aim has been to improve the efficiency of the teaching-learning process.

The main areas of experimentation have been the integration in the syllabus of numerical and analytical methods, the incorporation of case studies into the curriculum and the use of micro-based software to enhance the teaching process.

Special attention is paid to courses in Mathematical Engineering and their position in the spectrum of engineering disciplines.

A core curriculum in mathematics for undergraduate engineers is proposed and details are provided of its implementation. The roles of case studies and micro-based software are highlighted. The provision of a mathematics learning resource centre is considered a necessary feature of the implementation of the proposed course. Finally, suggestions for further research are made.

Key words: Mathematical Education, Engineering Mathematics, Case Studies, Microcomputers, Mathematical Modelling

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## Chapter 1

## Introduction

### 1.1 Background

The theme running through this thesis is that of integration. This integration has been sought in two contexts: the integration of topics within the mathematics syllabus, and the integration of the mathematics course within the engineering curriculum. The author has striven to make the mathematics course that he teaches to engineering undergraduates an engineering course taking its place alongside structures, dynamics, fluid mechanics, etc.

Too many engineering students graduate with a dislike of mathematics, having seen the course that they received as being largely irrelevant to their interests and requirements - a necessary evil appended to their studies, and necessary only in the sense that they were examined in it. Too often the only contact with the mathematics lecturer was in the formal setting of the lecture theatre, where perhaps they shared the lecture with students from other engineering departments. No attempt was made by the lecturer to show them where the mathematics that they were being taught was relevant to their engineering subjects.

The author was fortunate in being appointed to a department which took seriously the teaching of mathematics to engineering undergraduates. He has been privileged to learn from the experience of colleagues whose dedication and enthusiasm set him an example of the high standards to be maintained in his teaching. Their encouragement and cooperation over the years has been invaluable. Also, many of the author's engineering colleagues have provided both helpful advice and the facilities to carry out his research.

The last twenty years have seen considerable changes in the field of mathematics education at primary, secondary and tertiary levels. Today's freshmen were not born when the author started his teaching career and they are entirely the product of the changes that have taken place in that period of time.

The schools have witnessed much upheaval since the late 1960's. 'Modern' mathematics has pervaded all age groups with sometimes alarming effects on the mathematical skills found in those undergraduates who have/brought up via that approach. There has more recently been a move back to . more traditional syllabuses but there is no question that the average engineering freshman of 1988 is less well-versed in standard mathematical techniques than was his predecessor in 1968.

Despite the avowed intention of the 'A' Level examining boards to agree a core curriculum in mathematics (and other subjects) there is sufficient disparity in the syllabuses to ensure that a lecture group of freshman engineers will be quite heterogeneous in their knowledge of mathematics, let alone in their mathematical ability. Add to the 'A' level entrants those who have a BTEC qualification and the element of commonality becomes relatively small. The situation has not been static, as was indicated in the previous paragraph, and therefore the mathematics lecturer has been confronted with a system which has a time-varying heterogeneous input. The rapid changes in computer technology have added another variable to the teaching task. As the mainframe computer which had punched card input and punched card output was superseded in turn by the terminal link and the microcomputer and as the slide rule gave way to the pocket electronic calculator, the importance of numerical methods relative to their analytical cousins has increased. Fortunately, Loughborough has stressed the value of numerical methods for many years and has therefore been better placed than several institutions to adapt to these changes; even so, it has required considerable effort to keep up with the pace of change.

The needs of industry as regards the knowledge required by its graduate
recruits have not been restricted to an increased awareness of, and experience in using, numerical methods. New areas have entered the arena, notably discrete mathematics, and this has posed further problems for the lecturer. In order to treat these areas satisfactorily room must be found in an already crowded syllabus; what can be omitted without leaving the student deficient in some important topic? The output from the system is therefore time-varying. It is also heterogeneous since the specific needs of the mechanical engineer do not coincide with those of the electrical engineer, the production engineer or the civil engineer.

There are further problems, too, in that the engineering lecturers will require particular mathematics topics to be covered at certain times in order that they can draw upon them in their teaching. These requirements are often incompatible with each other, in addition to making it almost impossible to satisfy them whilst running a coherent mathematics course. Also, there is increasing pressure on mathematics lecturers to reduce contact hours whilst retaining the syllabus in full.

Against these difficulties can be set the growing need for engineers to be trained more thoroughly to cope with the increasingly complex engineering systems with which they will work in their professional careers. As these systems become more complex the need for increased mathematical awareness, knowledge and skill is more acute. The engineer of tomorrow must be more of a mathematician than the engineer of today and far more a mathematician than the engineer of yesterday. There is an urgent need, therefore, to develop a system of teaching mathematics to engineering undergraduates which can meet the requirements of today and which can cope with the changes that are likely to occur in the next decade or so.

### 1.2 The Author's Involvement

The author's earliest experience of teaching engineering undergraduates was in taking problem classes whilst a postgraduate student. Having taken a first
degree in Pure Mathematics he was soon made aware of the different needs and outlook of engineers from mathematicians. He was aware, in particular, of the preferred approach of starting with concrete examples before proceeding to a general formulation. He was made painfully aware of the engineers' desire to see the relevance of the mathematics that they were studying, painful because he was usually unable to give them a satisfactory response. There was a general resentment that the lecture class comprised engineers, scientists, mathematicians and social scientists: it was totally impersonal. The author remembered how in his undergraduate days his engineering contemporaries would complain that the mathematics course was designed by the mathematics staff on a 'take it or leave it' basis with no attempt whatsoever to introduce relevant examples. There was no contact outside the lecture room and the problems classes were conducted by postgraduate students who were mostly not willing to give up additional time to helping with the difficulties of the students in understanding the lecture material.

When the author took up his lecturing appointment in Loughborough in September 1969 he was assigned the task of lecturing to the first year Civil Engineering students. He knew that to be successful he had to be seen by these students as a member of the teaching team and not as an outsider. He had to set about the task of learning the Civil Engineering subjects so that he could relate the mathematics topics to the work done by the other lecturers. The fact that he was prepared to make this effort won him the respect not only of his students but also of his engineering colleagues who gave freely of their time to help him in his task. At that time the Mathematics Department was the other side of the campus from the Civil Engineering Department and the latter provided the author with an office which he could use between lectures to be available to his students for help outside the formal contact hours. (In those days, it was accepted that students were important members of the University community).

Strong links with schools have been maintained through regular lectures given to groups of sixth-formers, whilst liaison with industry has provided case study material which has been used in lectures and tutorials. Most recently, the
author has been involved with European colleagues in endeavouring to draw up a common core of mathematics which will be taught to all engineering undergraduates in Europe, particularly in the countries which form the European Economic Community.

After twenty years of teaching engineering students and striving to improve that teaching the author was in a position to take stock of his contribution to the field of mathematical education of engineers. It was time to look back on the ground which had been covered and to look forward to what might be achieved in the next phase of his career. The writing of a thesis provides an excellent discipline in this stock-taking.

### 1.3 Outline of the Thesis

Chapter 2 of this thesis presents a historical account of the research carried out in the area of mathematical education of engineers. The account begins in 1948, when speakers at the British Association meeting expressed disquiet at the quality of mathematics teaching to engineering students. There was a plea for more relevance and the case was argued for numerical methods to play a more important role in the syllabus.

A key factor in the history was the publication in 1966 of a report by the Organisation for Economic Cooperation and Development; this report set ambitious targets for a core mathematics syllabus for all engineering students and provided a reference point for developments which have taken place subsequently. It strongly advocated a greater training in computer-based methods of solution. This report is discussed in Section 2.2.

The following section considers the work which was conducted in the ten years following the publication of the Report. During this period the author began to make his contribution to the field of mathematical education of engineers and throughout the chapter reference to his work is made in context. Section 2.4
reviews the progress which had been made by the mid 1970's. It begins by discussing two papers of which Mustoe was a joint author; the first of these papers took a critical look at the current state of teaching and highlighted the perceived shortcomings whilst the second paper pointed a possible way forward. A further review of the situation was provided by a conference entitled 'The Mathematical Education of Engineers - Where Next?' Many attempts were being made to enhance the teaching, in particular via the involvement of the computer.

Section 2.5 looks at the research carried out in the last twelve years. Another review of progress since the 1966 Report had been conducted and there was a gloomy picture which emerged. In the United Kingdom, contact hours were well short of the more modest of the core curriculum hours suggested by the Report. In many institutions in Europe the emphasis on computing was lower than had been advocated and little real progress seemed to have taken place. Section 2.6 considers the attempts being made to establish a core curriculum for all engineering undergraduates in Europe. Drawing on the lessons to be learned from the 1966 Report, the targets are less ambitious.

Section 2.7 examines four specific aspects of recent activity. First, the problems associated with lack of mathematical skills which have been found in freshmen on entry to tertiary education are discussed. Then the impact of the computer on the curriculum is briefly treated and an account is given of work in the area of computer-based learning. The inclusion of modelling in the syllabus, including the role of case studies is reviewed and the expectations of industry with regard to the mathematical skills of its graduate recruits is mentioned.

Chapter 3 is devoted to the pioneering work carried out at Loughborough into the integration of numerical and analytical methods in the syllabus. Section 3.1 discusses a seminal paper on the 'integrated approach' which showed the way forward using the topic of ordinary differential equations as an example. The following section describes the author's work in developing a two-year undergraduate programme based on the integrated approach. Section 3.3 is
concerned with the text books written by the author and two colleagues which developed from this programme. An example of the integrated approach in action in a tutorial discussion is presented in Section 3.4. The next section treats the later developments to the two-year programme including the introduction of computing coursework. Section 3.6 is concerned with the knowledge and ability of freshmen entrants and discusses the results of a questionnaire and a test paper which are given to the students during their first week at Loughborough. Finally, Section 3.7 takes a critical look at the integrated approach and asks 'does it work?'

Chapter 4 is an account of the changes brought about by the technological developments in computers and pocket calculators. The first section reflects on the dangers inherent in the proliferation of the pocket calculator without an awareness of the limitations of operating on imprecise data. Section 4.2 describes the benefits to engineering students resulting from the introduction of terminals on the campus. The following section describes the way in which the author implemented the concept of a computer laboratory in his teaching, in particular the use of prepared programs in carrying out assignments on particular numerical techniques. When the microcomputer made its way onto the market it offered more flexibility to the teacher and an account is given in Section 4.4 of the work of several researchers into the use of the micro in the classroom or in a purpose-built laboratory. The final section considers the impact of the computer on the teaching of mathematics to engineers and laments the lack of good quality software currently available.

Chapter 5 describes the work carried out by the author and his colleagues in the area of computer enhanced learning. In the early 1980's there was very little micro-based software for mathematics at the school/university interface level and the Micros in Mathematics Education Project, of which the author was a founder member, was established at Loughborough in an attempt to fill the gap. Section 5.1 outlines the setting up of the project and the early decisions which were made, in particular the choice of mechanics as a first area to tackle. The second section gives an account of the software unit on Projectile Motion which was allocated to the
author and was the first to be completed. Two parts of the unit are described in detail to illustrate the author's thinking. In Section 5.3 the problems associated with presenting a statics topic are discussed. Other units are described in Section 5.4. The testing and evaluation of the units is considered in the following section and the project is reviewed critically.

In 1985 the project turned its attention to the production of software units to cover topics in first and second year engineering mathematics, with the hope and expectation that the material might also prove useful to science and mathematics undergraduates. Section 5.6 describes the early units which were produced and how the approach differed from that of the mechanics units. The three following sections discuss the use of three of the later units in the lecture, in the tutorial and in individual student usage respectively. Section 5.10 describes the evaluation of the engineering mathematics units and assesses their usefulness in teaching.

Case Studies in the Curriculum is the subject-matter of Chapter 6. The opening section demonstrates the author's commitment to models and modelling in his course and describes the discussion of mathematical models that he conducts in his first lecture. Section 6.2 examines some workers' views on modelling in the curriculum; a distinction is drawn between the use of models to illustrate the applications of mathematics and the development of modelling skills. For some years a shared lecture has been given with an engineering colleague to first year students which attempts to show the inter-relationship between mathematics and engineering in the modelling process; this lecture forms the basis of Section 6.3. The following section contains a description of a tutorial session in which two models for an engineering system are developed and contrasted. Section 6.5 is concerned with an extended modelling exercise which is conducted with first year mathematical engineers. In each of these three sections a report is given on the student reaction. Finally, Section 6.6 debates the issue of the place of modelling in the curriculum.

For the last eleven years there has been an undergraduate degree course in

Mathematical Engineering in the Loughborough portfolio. Chapter 7 is an account of the establishment and development of that course, together with a comparative exposition of other courses of a similar nature. Section 7.1 again makes reference to the Organisation for Economic Cooperation and Development Report of 1966 in which the case is argued for courses in the area of mathematical engineering to be established. The second section describes the establishment of the course at Loughborough whilst the third section details the major revisions to the course that have taken place and relates these changes to the aims of the course when it was first proposed. Particular attention is paid to the roles of modelling and project work. The growth of the demand for an Industrial Year, the position of Engineering Applications (as defined by the Finiston Report) and the employment of graduates from the course are also examined. In Section 7.4 the courses in the area of mathematics combined with engineering which are offered at the universities of Nottingham, Bristol and Eindhoven are described and compared both with each other and with the Loughborough course. Finally, in Section 7.5 the contribution of these courses to the spectrum of engineering education is assessed.

Chapter 8 presents a number of topics which are related to the main areas of work described in this thesis. First, a method of teaching the difficult subject of partial differential equations is outlined; then, Section 8.2 shows how a tutorial is used to underpin the lecture material on Fourier series. The next section takes a critical look at some features of written examinations whilst Section 8.4 examines the role of television and video in teaching. Finally, aspects of distance learning are discussed in Section 8.5 with one eye on future developments.

Chapter 9 draws together the threads that have been running through the previous chapters. It contains the author's proposed teaching model to take the teaching of engineering mathematics into the 1990's. Successive sections consider what mathematics should form the core curriculum, the extra needs of the various engineering specialisms, who should do the teaching, how and when it should be taught and what teaching aids should be employed. The chapter
concludes by discussing the implementation of the proposed models.

Chapter 10 looks both backward and forward. It summarises the research that has been conducted and suggests further work which needs to be carried out.

## Chapter 2

## Historical Perspective

### 2.1 Early Years

In 1948 the British Association held its annual meeting in Brighton; most of the final morning was devoted to a discussion on "Applicable Mathematics". The first speaker, D.N. de G. Allen (1) stated that there were four obstacles that the engineer had to overcome in solving his problems, having formulated them in engineering terms. First, the engineering problem had to be translated into mathematical terms, then the mathematical problem had to be set up. He argued that the engineer must learn to recognise whether a problem was properly posed before attempting a solution. It was more important that the engineer should be able to formulate his mathematical problem than to be able to solve it. The third obstacle was to find the mathematical solution and here there was a need to teach those techniques which were of general application; the value of numerical methods was emphasised. The fourth stage was to translate the mathematical solution back into an engineering solution.

The second speaker, J.A. Pope (2), emphasised the value to the engineer of keeping in view the physical importance of the mathematics and advocated a greater use of graphical methods. Significantly, he referred to the difference in outlook between mathematicians and engineers: the former were "masters of abstract thought" whilst the latter were "servants of practical necessity". He asked why engineering students behaved "barbarously" in their mathematics lectures and why their failure rate in mathematics was greater than that in their other subjects. He suggested that the lecture material was either too difficult or had not been properly selected and that the mathematics lecturer needed a feel for engineering. In a university with an engineering school there should be mathematicians who made a special study of mathematics for engineers. In
summary, he saw the problem as being not only what to teach, but how to teach it.

This last point was reiterated by W.G. Bickley (3). In addition he criticised the attitude of some engineering text-books and lecturers that mathematics was to be "avoided like the plague", rather than regarded as a useful tool which could be employed to increase efficiency. Risking the scorn of his highbrow mathematical colleagues, he stated his teaching aims as:
(i) to present the fundamental concepts and techniques of "elementary" mathematics which arose naturally in an attempt to describe, explain and predict the quantitative behaviour of engineering systems,
(ii) to show the mathematics in action,
(iii) to encourage the acquisition of sufficient knowledge and techniques to allow the students to cope with their text-books and design problems. His plea, for an Institute of Applicable Mathematics, scorned by a subsequent editorial (4), was to be realised sixteen years later.

In other articles relating to the Brighton meeting, (5) and (6), reference was made to an attitude espoused by Perry at the turn of the century that a watch may be very useful to a person who does not understand how it works; Perry allowed his students to perform Fourier analyses without knowing the mathematical proofs of their work. Before the First World War, Bouasse in France had advocated a shift in the emphasis of teaching mathematics to engineers towards applications. Since proofs were most often devised after the truth of a proposition had been established, they had an air of artificiality. It is worth noting that even at this time, numerical and graphical solutions were being advocated as being of equal importance to analytical methods.

However, in 1948 attitudes to teaching mathematics to engineers were not always enlightened. At the then Loughborough College, mathematics was taught at the first lecture of the day and the majority of those lecturing were not mathematicians. Banks (7) recalled that his non-mathematical colleagues would
'compare notes' in the following coffee-break. One of these remarked that he had just told his class that $\sin ^{2} x-\cos ^{2} x=1$ and asked if this were true, eliciting the response: "not often". This story is as poignant as it is humourous. In the early 1960s, when the author was an undergraduate, the task of lecturing the engineers was often given to the academic staff who had blotted their copy-book with their Head of Department. The courses provided were devised with little or no consultation with engineering staff. At that time the computer had yet to make an impact on mathematics courses and the teaching of numerical analysis was relegated to a secondary role, if indeed it ever occurred.

In a book compounded from lectures given under the heading 'The Engineer in the University', Christopherson (8) devoted a chapter to 'Mathematics - Friend or Foe'. He asked what sort of mathematics was needed by engineers and argued that the main objective was to give students an understanding of what mathematics is and can do, not to teach those parts which have (or are assumed to have) direct relevance to engineering. He warned that specific knowledge could have a transient life, whereas general principles had a more lasting existence. Rather than ask mathematicians for specific topics to be taught, engineering staff should leave the former to teach what they thought would give the students a full development of their mathematical ability. Computers had made certain techniques redundant from the syllabus, yet if more were asked from the student in the way of deeper understanding of principles then the examination papers would have to be less formidable.

However, Hart \& Wood (9) had carried out a survey of mathematicians in industry which asked for a selection of topics that they reckoned to be useful. They were more concerned with usefulness than mathematical understanding and the selection bore this out. Interestingly, there was no mention of numerical methods.

In his inaugural lecture as Professor of Mathematics applied to Engineering at Imperial College, Jones (10) compared the reports which had recently appeared
on undergraduate programmes in engineering mathematics. The first of these by the American Committee on the Undergraduate Program in Mathematics (11) was prepared by mathematicians who were able to state in detail how to provide the mathematics that their invesigations had indicated was needed. They argued for a core of 'beginning analysis', linear algebra, and probability and statistics; this core could be supplemented by courses in functions of several variables, further ordinary differential equations and functions of a complex variable for those intending to go into research and development and by courses in partial differential equations and real variable theory for those capable of graduate study. Computational methods should be integrated with associated analytical methods as appropriate. The second report (12) was the outcome of a seminar organised by the European Organisation for Economic Cooperation and Development and was prepared by engineers and scientists who were dissatisfied with the mathematical education of engineers then currently provided. Jones (10) compared, for both civil engineers and electrical engineers, the proportion of time allocated to undergraduates at the Technological University of Delft, Milan Polytechnic, ETH Zürich, M.I.T., Cal. Tech., Bristol, and Imperial College. The UK universities showed up poorly in comparison with the other institutions. Jones commended the practice of teaching numerical methods 'as one goes along' and suggested an 'express route' through some of the first year mathematics for the more able student. He outlined three methods of providing the mathematics: by engineers as it was needed, by a special group of applied mathematicians within the engineering faculty and by the department of mathematics itself. His inclination was to the second of these methods. He emphasised the importance of the formulation of mathematical models as part of the course. Finally, he was in favour of courses in mathematical engineering being established along the lines of that at Delft and Nottingham's Theoretical Mechanics.

### 2.2 The O.E.C.D. Report of 1966

In May 1963, at a meeting in Rome, the seeds were sown for the Organisation for Economic Cooperation and Development (O.E.C.D.) to make its major
contribution to the teaching of mathematics for engineers (13). Following the meeting, several working groups were established to prepare reports for a Working Seminar which was held in Paris in the first half of January 1965. The outcome of this Seminar was a substantial report entitled 'Mathematical Education of Engineers' (14). It has to be said that this report is not well-structured and quite hard going to read; however, it is a recognised landmark in the subject and has often been cited as a datum against which to compare subsequent progress.

It is worth quoting in full the reasons that the report gives for the importance of mathematics in the training of engineers.
"(i) It provides a training in rational thinking and justifies confidence in such thinking;
(ii) It is the principal tool for the derivation of quantitative information about natural systems.
(iii) It is the "second language" of human discourse and parallels natural language by providing a means of communication for ideas, as evidenced by the contents of technical papers.
(iv) It facilitates the analysis of natural phenomena.
(v) It is important in assisting the engineer to generalise from experience.
(vi) It trains the imagination and 'inquisitiveness' of the student if properly taught.
(vii) It is a training for adaptation to the future." (14)

Among reasons advanced to back up their assertions, the authors of the report mention the completeness of a mathematical description of an engineering system, its efficiency in assisting the prediction of how that system will perform under given conditions and the means by which to optimise that performance. The authors were convinced that the engineers of the future would need to be trained to a much greater depth than was the current state and that the rôle of the computer in engineering practice would increase, thus freeing the engineer to concentrate on more creative work.

After examining the various branches of engineering, the report continued by considering the changes taking place in secondary school mathematics before turning to the tertiary level. In comparing the current situation in engineering courses in 17 of the countries participating in the Seminar, two tables were produced. The first showed that the minimum length of university undergraduate courses varied from 3 years in the United Kingdom to 6 years in Portugal. The total number of hours typically devoted to mathematics in Civil, Electrical and Mechanical Engineering courses varied from 200 in the United Kingdom to 800 in Scandinavian countries. The second table compared the countries as regards whether they taught each of 25 selected topics. These comparisons were based on overall views of each country. The table indicated a lack of teaching of numerical methods and digital computing in Eire, the United Kingdom and Belgium. Whereas most countries taught a core of about half the topics cited, the United Kingdom rated poorly on the more advanced subjects.

In 1966 Scott et al (15) had published the results of their survey on the use of mathematics in the electrical industry based on a questionnaire sent out in 1963. The O.E.C.D. report reproduced that questionnaire and its findings. Among their conclusions were
(i) Many respondents felt that post-graduate training in mathematics would be beneficial to them.
(ii) Mathematics was essential in model-building.
(iii) The formulation of mathematical models was the most difficult stage of the modelling process.
(iv) Programming in a universal language and numerical analysis should be taught in the undergraduate syllabus.

The O.E.C.D. report proposed two core syllabuses ; the longer one was to cater for research and development engineers and the shorter one for all engineers. The hours suggested for the short core totalled 345, the breakdown into core topics being shown in Table 2.1. The long core syllabus required a further 280 hours for

Algebra and Analysis. The detailed syllabuses for the short core are shown in Appendix 1.

Table 2.1
OECD Short Core Syllabus Hours versus UK Hours

| Subject Area | OECD | UK |
| :--- | ---: | ---: |
| Analysis | 180 | $\}$ |
| Algebra | 40 | $\}$ |
| Digital Computation | 21 | 27 |
| Analogue | 4 | 8 |
| Numerical Analysis | 40 | 18 |
| Statistics/Probability | $\underline{40+20}$ | $\underline{245}$ |
|  | $\underline{268}$ |  |

In a chapter on teaching methods the following recommendations were made.
(i) The teaching should be done by mathematicians sympathetic to the needs of engineers.
(ii) At the introduction of a new topic motivation should be provided by illustrative applications.
(iii) Engineering departments should help in preparing tutorial problems.
(iv) The students should be taught in departmental groups.
(v) An introductory course in computing should be taught as early as practicable.
(vi) Numerical methods should be taught so that the student saw them in relation to analytical methods; the example of differential equations was cited.
(vii) Additionally, statistics and probability should be taught in the core.
(viii) The proportion of time allocated to mathematics should be sufficient to cover the core syllabuses.

Further chapters dealt with advanced elective courses, special consideration for different engineering disciplines and the use of computers. Finally the Report
proposed further follow-up work in the various aspects it covered.

Shortly after the publication of the O.E.C.D. report, Noble (16) published a book commissioned by the C.U.P.M. on Applications of Undergraduate Methods in Engineering. He had written to engineering and other university departments and to industrial concerns. The book contained a selection of 45 problems from those submitted, the level of mathematics varying from elementary algebra to sophisticated methods of advanced calculus.

### 2.3 The Next Ten Years

If the authors of the O.E.C.D. report had expected a ready acceptance of their proposals, they were to be disappointed. Quite recently, James (17) expressed the view that a major reason for the lack of positive response by engineering staff was the feeling that the Report was too ambitious in its recommendations. Just five years after the Rome seminar, a conference on 'The Teaching of Mathematics for Engineers' was held at Loughborough. Kerr (18), who was in the chair at Rome, warned of the effect of 'new mathematics'; he re-iterated concern about the lack of time that some U.K. courses were allowing for mathematics and he suggested that there seemed to be too little progress in incorporating computers into the curriculum.

Kerr had been involved in the establishment of a Committee by the then Council of Engineering Institutions (C.E.I.) and the Joint Mathematical Council which was aiming to exert pressure on U.K. institutions of higher and further education. In particular, he aimed to stimulate the closer integration of computational and analytical approaches, to encourage a greater amount of time allocated to mathematics, and to establish a continuing dialogue between mathematics and engineering lecturers and with school colleagues. The mathematics syllabuses for the C.E.I. Part I and Part II examinations had a close similarity to the O.E.C.D. core curriculum.

Davies (19) suggested that some time could be saved by cutting down the hours devoted to analysis and this could be replaced by courses in computational mathematics and experimental analysis, the latter to include probability and statistics. He further advocated the placing of linear algebra at the head of the pecking order.

Bajpai had founded the Centre for Advancement of Mathematical Education in Technology (CAMET) in 1966 and had been engaged in many of the activities desired by Kerr; he had recently founded the International Journal of Mathematical Education in Science and Technology. His contribution to the conference (20) posed the questions: What? For how long? Who should teach it? How? What relation should there be between the mathematics and engineering? How should the curricula be reviewed? He felt that the O.E.C.D. short core was a basis for the syllabus to be taught but it demanded much more time than was likely to be available. He suggested the inclusion of some lectures on Mathematical Models in Engineering on a team teaching arrangement and argued that the main syllabus should be taught by a sympathetic mathematician who, if not someone who had been in industry, was in close liaison with engineering colleagues. To help alleviate the problem of shortage of time he suggested the use of audio-visual aids and programmed texts. He raised the question of how to deal with a mixed entry standard. Finally, he quoted a paragraph from the C.E.I. Part II syllabus in mathematics which argued for the integration of analytical and numerical methods and the early introduction of computer programming.

Bajpai had championed the cause of integrating analytical and numerical methods at many conferences, seminars and private meetings and he believed that the way forward was to provide a concrete example of how it should be done. He solicited the help of two departmental colleagues to put the finishing touches to his ideas and in 1970 Bajpai, Calus and Simpson (21) wrote a seminal paper entitled "An approach to the teaching of ordinary differential equations". They outlined the usual approach which consisted of an introduction including some
motivational examples, followed by methods of analytical solution and then in a separate course, perhaps over a year later, a selection of numerical techniques. They argued for an integrated approach which combined analytical and numerical methods in a single package of lectures. They first showed how this approach could be used on Newton's Law of Cooling and then on a vibration problem which gave rise to a linear second order equation with constant coefficients. The analogue computer played a prominent rôle, demonstrating qualitatively how the solution varied when the parameters in it were altered. It is difficult eighteen years later to appreciate how revolutionary this 'integrated approach' (now re-christened the 'Bajpai approach') was at the time. The author of this thesis was the first to take this approach and work it into a first-year course with a group of engineers at Loughborough; a fuller account is found in Chapter 3.

At a conference on the Teaching of Mathematics in Universities and Polytechnics held in London in January 1971, Bajpai (22) again expressed concern over the wide range of mathematical ability of freshman engineering undergraduates. In addition to espousing the integrated approach, he argued for motivational lectures at the introduction of a new topic and for applications to be provided both during and after the set of lectures. He described how Mustoe had introduced first year Civil Engineers at Loughborough to computer programming in their first week and had made use of their skills during the rest of their course by providing relevant problems. He described the success in teaching ONC/OND students separately from their ' A ' Level contemporaries. He cited a paper by Elton (23) which listed several sets of aims for teaching engineers amongst which were
(i) the student should be able to understand the teaching and solve problems in his other engineering subjects which relied on mathematics,
(ii) he should know and understand sufficient "modern" mathematics to cope with later developments in engineering,
(iii) he should be able to formulate problems mathematically,
(iv) he must be able to manipulate mathematics safely.

Bajpai went on to champion more emphasis on the modelling process which he
saw as comprising five steps
(i) the phrasing of a problem in mathematical terms
(ii) the formulation of a mathematical model, incorporating any assumptions made
(iii) the mathematical solution, using analytical, numerical, graphical or other techniques
(iv) the modification of the model until the solution is satisfactory
(v) the analysis and interpretation of results.

Finally, he referred to the work being done at Loughborough on the use of programmed texts and integrated systems combining these texts with audio-tapes, slides and videotapes.

Bajpai (24) returned to this last theme at a conference on the 'Teaching of Mathematics to Non-specialists' held later in the year at Loughborough. He demonstrated two videotapes based on scripts written by Mustoe who was also the presenter in the tape sequences. Further examples of audio-visual material were given. Later in the conference, Mustoe (25) presented an interim report on the course he was giving to Civil Engineering students. Scott (26) was concerned with objectives in teaching and pointed out the conflicts between the mathematics lecturer striving for advancement of mathematical understanding and engineering staff and students seeking "relevant" techniques of solution. Lighthill (27) suggested that the mathematics lecturer needed to absorb much of the engineering that his students were learning and the head of his department should encourage this and recognise the effort required.

In 1970 Bajpai and Francis (28) published the results of a survey carried out on behalf of the Committee referred to by Kerr (18); this survey was conducted via a questionnaire in an endeavour to discover how much time was being spent on mathematics in engineering degree courses in the U.K. The results were disappointing to those who saw the O.E.C.D. short core syllabus as a datum.

As Table 2.1 on page 17 shows, the mean hours of mathematics fell well short of this target, the most noticeable shortfalls being in numerical analysis and statistics. It seemed that on the whole the U.K. was still teaching the mathematics courses of twenty years earlier. There were wide variations between different courses: the total hours committed varied from under 150 hours in two cases to over 400 hours in five cases; five courses taught no numerical work at all, three ignored digital computing, seven analogue computing, and four statistics and probability. As regards the percentage of total teaching time set aside for mathematics the average figures for three year courses were: Year 1, 19\%, Year II, $15 \%$; Final Year, $7 \%$; for four year courses the figures were $20 \%, 19 \%, 14 \%, 11 \%$. The sponsoring Committee welcomed the fact that most courses had some numerical/computing content, regretted that so little time was being spent on statistics and probability, asked for more courses in operations research to be started and urged that there be a strong liaison between mathematics lecturers and their engineering colleagues.

Francis (29) conducted a further survey on the coverage of differential equations in engineering courses. Again there seemed to be little emphasis on numerical solutions. As might be expected, there was overwhelming use of second order linear equations with constant coefficients, followed in popularity by first order linear equations and first order separable equations, with other types proving less popular. Some idea was given of the applications of differential equations in various engineering disciplines.

In the early 1970s much innovatory work was taking place on the mathematical education of engineers. At Lancaster, Tagg (30) had established his 'accelerated teaching' project for freshmen with poor entry qualifications. Knowles (31) was among those endeavouring to bring real-life applications into the school curriculum. Cornelius (32), (33) was concerned with the transition from school to university. He wondered whether the "right" topics were dealt with at school and suggested that schools should concentrate less on clever techniques and more on appreciation and understanding. Flegg (34) felt that the
service courses at university should be scrapped in favour of integrated courses on mathematical modelling taught by a joint team of mathematics and engineering lecturers. The theme of enlivening applied mathematics by real-life examples was continued by Davies (35) and Sida (36).

The problem of weak students was tackled at Kings College, London by Baker et al (37) who had introduced a crash course in calculus to first year undergraduates. Using programmed texts, small group discussions, talks and films they attempted for the first six weeks on one day each week to motivate the students in addition to revising basic material. Post-tests indicated a satisfactory improvement in skills.

McLone (38) argued that traditional methods of assessment tested the acquisition of basic techniques and theory and the use of techniques to solve standard problems. The abililty to devise new techniques when existing ones proved inadequate, how to formulate problems in suitable terms and the skills of communicating ideas in written form and orally were generally not tested. He suggested that use should be made of essays, extended problems, prescribed reading, class-timed tests and practical work. He stated that it should first be established what are the aims/objectives of teaching and what student qualities should be developed; then it should be decided which of these qualities needed to be assessed; finally, the question of how to assess them should be settled.

At the end of this period, Pollak (39) presented the case from industry's point of view. He asserted that students should understand their mathematics and when, how and why it works. Open-ended problems were commonly met in industry and students needed exposure to that type of question. There was a need for computing and probability and statistics to be taught. A balance should be struck between technique and understanding: his experience was that there was not enough technique taught in linear algebra and not enough understanding imparted in the study of partial differential equations.

### 2.4 Ten Years On: Progress or Failure?

Ten years after the O.E.C.D. report there was a feeling that little progress had been made in the United Kingdom towards the implementation of its recommendations. In that sense, the Report could be regarded as a failure. However, the author and his colleagues believed that there was much to be commended in the Report and they felt that it was necessary to re-awaken interest in it by stimulating a debate on the then current state of the teaching of engineering mathematics. In a wide-ranging paper Bajpai, Mustoe and Walker (40) reviewed the progress that had been made towards achieving the stated objectives. They began by examining the questions: who does the teaching?, to whom?, what is taught?, when is mathematics taught?, how is it taught?, and how is it assessed? Then they set down a mixed list of aims and objectives which they felt were of fundamental importance.

The first of these was the appreciation of the concept of a mathematical model and the methods of obtaining solutions to the model: Figure 2.1, which is reproduced from their paper, is their flow chart for the modelling process. The second requirement was for the student to develop a suitable level of competence and the third was that he should be aware of the need for rigour. The remaining two fundamental requirements were for the student to see mathematics as an integrated part of his engineeering discipline and to develop a logical approach to the formulaion and solution of problems. They were definite in stating two 'non-aims'; a 'cook-book' approach was to be avoided and the students were not to be overwhelmed by rigour.

The authors gave ten major criticisms of the current system from the engineers' point of view.
(i) The teaching of mathematics to engineers was often uninspired.
(ii) The servicing department often failed to liaise with the engineers.


Figure 2.1
(iii) Some courses concentrated on a 'cook-book' approach.
(iv) Other courses concentrated on rigour for its own sake.
(v) Students were often given too little help outside lectures.
(vi) Examination questions failed to encourage students' interest.
(vii) Symbols were used without understanding of their meaning.
(viii) Numerical techniques were undervalued.
(ix) Too much of the teaching was compartmentalised.
(x) Mathematics was directed towards the research and design engineer.

The mathematics lecturer could add his own list of criticisms.
(i) Mathematics was often relegated by engineering lecturers to a minor rôle.
(ii) Students did not like departures from standard note-taking exercises.
(iii) Few suitable text-books were available.
(iv) Mathematics was seen as an appendix to the engineering.
(v) The ability of the students in mathematics covered a wide range.
(vi) Lecturers received no incentive to learn the relevant engineering discipline.

The authors then suggested improvements to the system and described the courses developed at Loughborough based on that first given by Mustoe, a fuller description of which is found in the next chapter. Particular emphasis was given to motivation, the integration of numerical and analytical methods, modelling and the use of project work as part of the assessment.

In a follow-up paper, Bajpai, Mustoe and Walker (41) provided further details of their course proposals. Examples were given of the way in which the integrated approach could be employed; the analytical methods often allowing the student to understand a system and the numerical techniques helping him to solve practical problems. An example of a case study from electrical engineering was presented: this showed how several mathematical techniques were called into play in the solution of a problem. Some examples were provided of mini-projects of varying degrees of difficulty which could be given to the students as coursework. After discussing problems in implementing such a course, not the least of which was the effort required for the mathematics lecturer to adjust to the new approach, the authors looked to the future and suggested the establshment of liaison committees between the mathematics department and its engineering counterparts. In conclusion, they demanded that the student should find his mathematics course stimulating, relevant and useful. They warned that if engineering departments were dissatisfied with the service given to their students, they might mount their own mathematics courses.

One of the comments made by lecturers following the paper by Bajpai et al (21) was the lack of suitable text-book material to allow them to teach the suggested approach with confidence, particularly as many of them were unfamiliar with numerical methods and computer programming.

In consequence, Bajpai, Mustoe and Walker wrote a set of text-books (42) which was tested widely by students and lecturers prior to publication. Received comment indicates that the material has been instrumental in moving syllabuses towards a more integrated approach. It must be recognised that the weaker students need additional support and that self-instructional material based on a programmed learning style has a rôle to play. Bajpai et al (43), (44) and (45) wrote a series of such books to fill this need. One of the authors, Calus, subsequently gave an account of the advantages of the programmed text approach (46). She saw it as bridging the gap between the lecturer demonstrating the solution of a problem and the student solving one on his own. She admitted that one drawback was the fact that the student had no summary to refer back to during revision. Godfrey (47) described how the texts were used at Warwick University in the form of weekly assigned reading supplemented by supplementary notes and quizzes which are interspersed by plenary sessions and tutorials. Stroud (48) has written two books along similar lines; their popularity confirms that such books do fill a need on the part of a substantial number of students. These students, on the whole, are those at the lower end of the spectrum of ability. Bajpai, Mustoe and Walker deliberately avoided the 'cook-book' approach, preferring via motivational case studies and harder examples, to widen the students' experience of applying mathematics.

At a SEFI conference on 'Essential Elements in Engineering Education', Mustoe (49) re-emphasised the points made in the two papers previously cited (40), (41). He returned to the theme of providing a suitable service at an I.M.A. conference on 'The Mathematical Education of Engineers - Where Next?' and argued the case for a three-way collaboration between the mathematics lecturer, the engineering lecturers and the students (50). He took the message to Civil

Engineers in America in a third paper (51). It is to be remembered that these ideas were regarded as somewhat revolutionary at that time. There was a certain amount of resistance from those mathematics lecturers who had little or no knowledge of computer methods.

The opening address at the I.M.A conference was given by Professor J.A. Pope (52) who had spoken at the British Association meeting thirty years earlier. He referred first to the concern with the output from schools due to
(i) the wide variety of ' A ' Level courses leading to a smaller common core of knowledge than might be hoped,
(ii) the introduction of a conceptual approach which has weakened manipulative skills and had led to the omission of traditional subjects which are useful to engineers,
(iii) an inadequate number and quality of mathematics teachers.

He felt that three-dimensional geometry was a valuable subject for potential engineers to study and regretted its decline in schools.

Having stated that design should form a more important part of university engineering courses, Pope reminded his audience that to engineering students mathematics was primarily a tool. He felt that the application of a particular topic in mathematics should be introduced prior to a treatment of the mathematics itself. This required an effort on the part of the mathematics lecturer to understand the relevant engineering; it also required the engineering lecturer to modify his approach to teaching. The two should combine their efforts in working to a common goal.

Shercliff (53) argued for mathematics not to be taught as a separate course, but rather to be intermixed with the applications which give rise to the reasons for studying it. In this way he believed that mathematics could, and should, form the heart of the engineering curriculum. He stated that modelling was a theme which had to become more prominent. He said that to an engineer, infinity was a value
so big that its actual size did not matter; this kind of thinking was important and should be developed. By placing mathematics in a central rôle, he reasoned that the engineering curriculum could be unified and telescoped and analogies between different applications of a mathematical idea could be highlighted.

Wakely (54) believed that every engineer should be capable of:
(i) understanding the formulation of an engineering problem,
(ii) recognising the limitations of the model,
(iii) following critically the mathematical arguments,
(iv) interpreting the resulting answers in engineering terms.

His 'compleat engineer' would be "mathematically sure-footed, even though he did not know all the steps of the dance". A solid foundation was more important than precisely what was taught and approximately one quarter of the entire engineering course should be devoted to mathematics. A cook-book approach would not equip the graduate with the ability to cope with technological changes. He also begged for the mathematics to be made relevant to engineers and suggested that a project which gave the student an exposure to the activity of mathematical modelling was essential.

Other papers tackled specific modes of teaching. Hunter (55) illustrated the use of computer-assisted learning (CAL) in his teaching. Blackburn (56) described the Open University course TM 281 'Modelling by Mathematics', which had been written with the adult student in mind. Clements (57) presented details of simulations/case studies that he had used with students of engineering mathematics. Craggs (58) was concerned with the place of rigour in courses to engineers; he said that a suitable structure was
(i) accuracy in manipulation,
(ii) enunciation of theorems under strong conditions, with a rigorous proofif it was simple and a heuristic justification where it was not,
(iii) illustration of the possible gain and loss if one were to work outside the domain of the theorems,
(iv) appreciation of the rôle of a fully trained mathematician who could move safely in territory unknown to the engineer.

At the suggestion of the author representatives from both junior school and secondary school were invited to participate; their contributions provided some surprises for delegates. Brighouse (59) and Gibbons (60) defended the changes in school mathematics which were moving away from 'traditional' computational skills towards "understanding". It was a defence not too well received.

Lighthill added his considerable standing to the debate in a paper (61) which argued that an education in applied mathematics was a fundamental part of an engineering education. Modern engineering needed a special kind of mathematics which was concerned with the art of making and using mathematical models. These, however imperfect, needed to be well-posed and well-conditioned. The lecturer concerned with the mathematics course should have a wide and deep knowledge of mathematics allied to an extensive knowledge of engineering applications. He wanted universities which taught engineering to have strong research schools in applied mathematics. Similar ideas have been advanced by Gnedenko and Khalil (62) and Lund and Christiansen (63).

### 2.5 Recent Developments

In the last dozen years or so there has been a marked growth in the interest shown in the mathematical education of engineers. It would be impossible to attempt a strict historical appraisal of the work done; rather, this section will be completed by describing a number of general articles and subsequent sections will tackle specific themes.

Heard (64) questioned over 4,800 students in nearly 50 engineering departments in a survey of their mathematical backgrounds and needs and the methods universities adopted to meet those needs. He also contacted university
lecturers for their views. Not surprisingly, this group were concerned at the diversity of the students' backgrounds and attainments and specifically at a general lack of confidence in processes of algebra, trigonometry and calculus. Also mentioned several times were lack of understanding of a mathematical argument, lack of stamina in following a lengthy calculation, difficulty in converting from a physical system to an appropriate mathematical model and especial weakness in mechanics. Of the students surveyed, $44 \%$ had taken double mathematics at Advanced Level, although the indications were that this percentage was falling; there was clear evidence that this extra mathematical exposure resulted in better performance in first year examinations.

The problems of ONC/OND/HNC entrants were highlighted: it seemed that a disproportionate number of them failed to complete their university courses.

Many of the mathematical topics taken for granted had, in fact not been covered thoroughly prior to university in a sizeable proportion of cases. As a consequence students often felt that their courses were too fast and contained too much material for them to absorb. There was a widespread complaint that courses were too theoretical and were not relevant to engineering and there was a demand for more tutorials with smaller group sizes. In instances where self-paced courses or programmed texts and audio-visual material were used, the response was generally favourable. The report emphasised the need for liaison between the mathematics and engineering lecturers.

Barrett, James and Steele (65) examined the mathematical content of first year engineering courses in British universities and polytechnics. They confirmed Heard's view that the number of entrants with double mathematics at Advanced Level was falling. There was a large proportion of entrants with a $D$ or $E$ in single mathematics or ONC/OND qualifications; therefore, the common core of knowledge at entry was smaller than previously. The time allocated for mathematics ranged between three and six hours per week with four being the norm, i.e. somewhat less than 120 hours in the session. The first year course
comprised few topics outside the envelope of Advanced Level syllabuses but the treatment was more rigorous than that at school.

The teaching of a high level programming language was a common feature of many first year courses but, whilst the teaching emphasis might have changed as a result of increased computing facilities, the feeling was that the content had probably not changed. Only a small minority of courses seemed to treat mathematics as a subject in its own right, the bulk of them being content to provide the techniques required to solve specific problems. Assessment was almost entirely by examination, the most popular being a single 3 -hour test. The authors (65) gave a 'typical' first year syllabus for Electrical, Civil and Mechanical Engineers and a 'typical' examination paper.

Petroski (66) gave some general views on the value of mathematics in engineering education. He wanted more emphasis to be placed on the understanding of the limitations of a theory and argued that a mathematical training in the proof of theorems would lead naturally to an appreciation of the limits of their applicability. With the increasing use of computers there was a growing need to instil in students the requirement to know whether answers and output were reasonable. A particular pitfall was lack of awareness of the domain of a function. He remarked that students acquired mathematics attitudes in addition to skills; whereas skills could be sharpened later, attitudes seldom could be changed.

James reviewed progress since the O.E.C.D. report and pointed a way forward in two articles (67), (68). He was unhappy with the slow response towards achieving the O.E.C.D. objectives and he was disappointed with the Finniston report on "Engineering Our Future" (69) in that it suggested a reduction in the mathematics taught. He believed that the first two years of an undergraduate course should build on understanding of mathematical processes and their relevance rather than concentrate on competence; standard models should be solved with the help of suitable software. In the second year, an introduction to
modelling skills assessed by short assignments would be a preliminary step to a modelling course in the third year of study.

Bajpai and James (70) joined forces to reinforce their arguments. They suggested that the importance of mathematics in the curriculum needed to be recognised more fully by validating bodies and professional organisations. They urged a re-appraisal of the position of mathematics in engineering courses so that future graduates would be equipped to cope with the demands of the technology of tomorrow. The pair carried out a short study-visit to ten institutions in Denmark, the Netherlands and Belgium and found a general lack of computing in the courses; modelling seemed also to be neglected (71).

The International Commission on Mathematical Instruction and The International Council of Scientific Unions' Committee on the Teaching of Science mounted a joint study to look at all aspects of mathematics as a service subject (72). They were concerned with why mathematics was taught, what should be taught and how should it be taught, re-echoing the themes espoused by Bajpai some fifteen years earlier (20).

Shannon \& Sleet (73) conducted a survey in Australia amongst students and staff in engineering and other disciplines. One of their findings was that although mathematics lecturers felt that it was important for the students to enjoy their mathematics course, this view was not shared by the engineering staff. Whereas the latter group did not see relevance as an essential ingredient in the mathematics teaching, their students emphatically did. The students rated very highly the development of logical thinking and problem-solving ability. A later survey by Sekhon and Shannon (74) amongst graduate engineers and their employers revealed that matrix theory, numerical analysis, regression analysis and simulation techniques were regarded as important and used most frequently. Surprisingly, perhaps, less than $40 \%$ used ordinary differential equations often. Employers were looking first and foremost for effectiveness in problem-solving and modelling; the ability to handle real-life problems with all their complexity
and vague definition was found to be less widespread than was felt desirable. Sekhon (75) argued the case, in view of rapidly changing technology, for post-graduate education to be expanded and for the availability of a range of continuing education. His aims for this training were: acquiring the engineering philosophy, developing critical abilities, updating on mathematical methods and encouraging self-development.

Bajpai (76) gave a historical perspective of the twenty years since the O.E.C.D. report. He drew particular attention to the increasing availability and power of microcomputers and to the work being done on teaching strategies. He wanted a serious review of undergraduate curricula and asked once more the questions
(i) who should teach the mathematics?
(ii) what mathematics should be taught?
(iii) how and when should it be taught?
(iv) what aids to teaching should be employed?

Whilst the 'integrated approach' had found wide acceptance there was still much to be done to keep the teaching relevant and up-to-date in view of increasing constraints. Scanlan (77) presented an engineer's view: he wanted the students to understand the mathematics they were using and gave an example from Fourier transforms where a lack of understanding had led to a false result. Whilst a continuing dialogue should exist between engineers and mathematicians he did not like either departments of engineering mathematics or courses in mathematical engineering. He suggested that demonstrators from engineering departments could be involved in mathematics but the mathematics lecturer should not per se concern himself with the relevance of the topics that he taught. He saw dangers in the mathematics lecturer using an engineering model to motivate the students since they would probably not understand the engineering, hence any motivation would evaporate.

The Conference of Professors of Applied Mathematics carried out a survey on service teaching in 1987; their findings made somewhat gloomy reading (78).

The total number of compulsory contact hours of mathematics, comprising lectures, tutorials and problems classes showed the following ranges:
(ii) Mechanical/Aeronautical Engineering

70 to 250 hours
(ii) Civil Engineering
(iii) Electrical,Electronic and Control Engineering
(iv) Chemical Engineering

30 to 250 hours
70 to 250 hours
90 to 250 hours
(only 3 courses over 180 hours)

The amount of compulsory mathematics taught in the third year was typically zero, with a few courses providing over 50 hours; even fewer courses provided fourth year compulsory mathematics. In most cases the service teaching was carried out by mathematics departments; in five instances there was a separate Department of Engineering Mathematics and in three cases the teaching took place entirely within Engineering. Fourteen institutions reported that they taught separate courses in statistics. In seven institutions the serviced department did some of the teaching even when the bulk was done by a mathematics department; in some instances, this teaching penetrated down to the second year of a course.

In most cases the fraction of contact mathematics time per student which was spent in lectures was over two-thirds. As regards the provision of optional mathematics courses offered in addition to the compulsory core, only one Chemical Engineering course had any, and that was taken by less than five percent of those students able to do so. Mechanical/Aeronautical and Electrical, Electronic and Control Engineering had more instances of optional courses being made available and a much higher percentage take-up.

In response to a question on topics taught, almost all institutions offered Calculus, Ordinary Differential Equations and Linear Algebra. Partial Differential Equations, Vector Analysis, Fourier Series, Complex Variable, Statistics and Numerical Analysis were close in popularity. Only 10 were making Discrete Mathematics available but this represented an increasing trend. However, there
seemed to be a growing tendency for a reduction in time available for mathematics and for engineering departments to teach their own computing courses.

About half the respondents indicated the use of computer-aided learning, programmed texts and project work in their teaching, with a handful incorporating video tapes. Encouragingly, there was a degree of consultation between servicing and serviced departments in almost all cases where syllabuses were under review; in some institutions, formal joint committees existed. The amount of mathematics was generally determined by the engineering department, but in one half of institutions there was no representation of the mathematics department at accreditation visits in engineering departments. Usually, the person chosen to do the service teaching had some experience in or empathy with the relevant engineering discipline and in many instances the mathematics department contributed to engineering departments' postgraduate programmes.

Clements (79) saw a clear need to adapt the curriculum in view of the increasing availability of computers. He believed that with packages already on the market it was desirable to provide students with a greater understanding of general concepts: for example, rather than simply teach the fourth order Runge-Kutta methods it was important to recognise a stiff system of equations and to choose a suitable method of solution. Methods could be included in a course if they illustrated the difficulties which might arise in practical problems.

Clements envisaged a much reduced rôle for complex variables and suggested computational geometry as a replacement. Whereas Laplace transforms had been considered essential, Z-transforms now had an equal demand on inclusion in the syllabus. He argued for a balanced approach to teaching, lying between that of the 'cook-book' school and that of the 'rigour above all' advocates. His own experience of the pressure to reduce teaching time - by less rigour and more relevance - suggested the construction of a 'tree' of knowledge, from a disorganised heap of leaves of fragments of mathematical understanding to a twig of underlying structure and then from twigs through branches to trunk; at each
stage, when enough material was available, unifying concepts would enable the next stage to be reached. The problem was that the educator who saw the whole tree tended to describe the branch first, then the twig and finally the leaf. The student preferred the process to occur in reverse. Clements concluded that the latter approach was preferable, provided that the unifying concepts were emphasised at appropriate points in the course.

### 2.6 The European Dimension

In 1982 SEFI established a Mathematics Working Group under the joint chairmanship of James and Spies (from the Federal Replublic of Germany). A pilot exercise was conducted amongst some institutions in Denmark, the Federal Republic of Germany, Sweden, Switzerland and the United Kingdom as to student knowledge of mathematics assumed on entry and mathematics curriculum hours, together with content and depth of treatment. The limited findings did suggest that the mathematics curriculum had not evolved in line with developments in computer technology and that the treatment of numerical methods and statistics varied from exclusion through to specific provision. Accordingly the Group decided to adopt a 'bottom up' approach, first establishing what rôle mathematics should play in modern mechanical, electrical and civil engineering courses and then identifying an appropriate core curriculum. The Working Group initiated a programme of European seminars to encourage a wider debate, and has to date held five seminars on an annual basis.

The first was at Kassel in 1984 on the theme of "Developments and Innovation' and was mostly concerned with the impact of the increased availability of computers, especially microcomputers, on curriculum content and mode of presentation (80).

Three main issues were identified:
(i) Software packages should be used with understanding - a 'black box' approach was to be avoided.
(ii) Certain mathematical topics were becoming less important - special functions provided ingenious methods of obtaining approximate theoretical solutions, but packages had become available which would handle the exact geometry of boundaries.
(iii) Mathematical modelling should be promoted via an interdisciplinary course with input from both engineering and mathematics staff (cf the comments made seven years earlier by Mustoe (49)).

The Kassel seminar was anxious to see a shift in emphasis away from the mastery of solution techniques towards developing an understanding of concepts and principles. However, it was recognised that a high degree of manipulative ability was a prerequisite for the understanding sought: it was a question of achieving the correct balance.

Also in 1984, an Anglo-Swedish conference on 'The Teaching of Mathematics to Science and Engineering Students' was held near Stockholm (81). Themes covered included the impact of computers on teaching, statistics, and the needs of industry; in addition, a modelling workshop was held.

The second SEFI seminar was held in Denmark, at Lyngby, on the theme "Impact of Computers' (82). It was felt that the computer could be used to enhance the learning process both by illustration or animation and by the ability to analyse more complex, realistic systems. It was felt advantageous to integrate numerical methods with their analytical counterparts. The growing importance of discrete mathematics was recognised and it was proposed that to support the traditional numerical methods two specific branches should be incorporated into the curriculum:
(i) discrete analytical methods to cope with digital data etc.
(ii) discrete structures and combinatorial mathematics.

It was also recognised that probability and statistics had to be given more prominence and that the use of computers in teaching the subjects was essential.

At the third seminar in Turin there were three themes: computer alegbra, discrete mathematics, and probability and statistics (83). The latter two areas were concerned with consolidating progress made at the previous meeting but the new feature was the treatment of computer algebra. There was concern about how such systems as muMATH should be used in the curriculum and what were the implications on the curriculum as a whole. In 1987 at Gothenberg for the fourth seminar, the themes chosen were linear algebra, statistics and probability, discrete mathematics, and the rôle of computers in mathematics teaching (84). One feature of the linear algebra discussions was the debate on the degree of rigour to be adopted. At one extreme it was suggested that a formal presentation with definitions, theorems and proofs was wanted by the engineers, whilst at the other extreme the emphasis was to be on applications and problem solving. Again, it was stressed that numerical aspects should be integrated within the topics.

This year, at Plymouth, the details of a proposed core curriculum for all engineers were taken a stage further. The working subgroup on discrete mathematics had presented its report and efforts were concentrated on analysis/calculus, linear algebra, and statistics and probability. Concerns were expressed at the changing nature of secondary school education in the United Kingdom and at the pressures on the time allocated to mathematics within the engineering curriculum. It was emphasised that the student should be provided with a coverage of the mathematical ideas and techniques which were currently directly applicable and essential to support his curriculum and to give him the foundation to update his knowledge after graduation. Delegates were asked to consider additional topics that were essential for the various engineering disciplines which now included control engineering, manufacturing systems engineering and software engineering.

### 2.7 Teaching Strategies and the Curriculum

### 2.7.1 The Entry Problem

Reference has already been made to the concern expressed by mathematics lecturers over the varied mathematical ability of the engineering undergraduates, for example Bajpai (20). In the United States, Fowler et al (85) gave a multiple-choice test of sixty items to four groups of freshmen in an attempt to determine how well they were able to apply basic algebra, geometry and trigonometry to the solution of problems. It was hoped that the test would be a diagnostic tool to aid the student and his instructor in devising a review programme of study. Snyder and Meriam (86) reported on a nation-wide multi-choice test to be carried out in class; the test was of 45 minutes duration and comprised 25 questions on topics from trigonometry, coordinate geometry, areas of plane figures, similar triangles, hydrostatics, weight, vectors and elementary calculus. Each question was accompanied by five suggested answers, the last of which was 'none of the above'. Over 9,500 students participated in the test and achieved an average score of $51 \%$. The questions which attracted the worst answers were those on vectors and calculus. The authors expressed concern about the low level of ability of the students in dealing with 'the most elementary tools of mathematics'. Questions which caused especial difficulty were the calculation of the length of the side of a triangle via the cosine rule, the area of a half-annulus and the slope of the curve $y=\cos 2 \pi x$.

At the same time, Mustoe (87) was conducting a test with freshmen Civil Engineers to discover their pre-knowledge of elementary pure and applied mathematics. The 12 questions were set from a pool contributed by Mustoe and the Civil Engineering lecturers and occupied a period of 1.5 hours. A copy of the test paper is given in Appendix 2. The questions which were attempted most successfully were those on rearrangement of a formula, the double integration of the beam equation and the calculation of the resultant force. Less than $8 \%$ of the students were able to tackle successfully the questions on hydrostatics, which is
more likely to be a problem of omission from school physics syllabuses, but even fewer could determine the centroid of an area bounded by a parabola and the coordinate axes, or could find the forces in a loaded truss. The test was carried out for three consecutive years and the results showed no significant change from year to year. One consolation was that the Civil Engineering lecturers appreciated the task facing their mathematical colleague.

In response to the results of the test, the core material which occupied three hours per week of lectures was supplemented by an additional hour each week during the first term. The topics were chosen from ' A ' Level syllabuses and the students knew well in advance which topics were lectured on in which week; those who felt that they needed to attend a particular lecture could do so.

Fyfe (88) conducted a similar test at Kingston Polytechnic. He found that trigonometric ability was very poor and lamented the students who evaluated ( $\sin ^{2} 28^{\circ}+\cos ^{2} 28^{\circ}$ ) on a calculator and presented answers of 0.99999999 . He found that the Technician Education Certificate students who had superceded the Ordinary National Certificate entry were in particular difficulties.

Elton (89) had carried out tests on freshmen at Surrey University and had asked the students why their performance was not high. They cited
(a) they had forgotten the relevant material
(b) they had not covered the relevant material in earlier studies
(c) they had made careless mistakes.

Unfamiliarity with notation or lack of understanding of what concepts were conveyed by notation such as $y=y(x)$ were held to be key sources of difficulty.

González - León (90), (91) described the test he had conducted with freshmen engineers and reminded first year lecturers in mathematics that their audience would have forgotten some of their assumed knowledge, those with ' A ' Level grades below $C$ were not totally $a u$ fait with the subject and there were
considerable variations in 'A' Level syllabuses. He had also surveyed engineering lecturers' views on the basic mathematics knowledge and skills needed in their first year courses. The lecturers were given a list of topics and invited to provide examples of the level of difficulty they expected. He hoped that the results of his survey would be of use to mathematics lecturers and he suggested that a test based on the examples should be given to engineering freshmen and suitable remedial work provided.

Other workers reporting on similar tests include Howarth and Smith (92), Hubbard (93) and Kurz (94).

School mathematics in the United Kingdom has undergone many changes in the last twenty-five years or so. The conversion from 'traditional' to 'modern' syllabuses led by the School Mathematics Project (SMP) caused many university lecturers to bemoan the lack of manipulative skills in those who had followed the latter approach. However, Turner and Mustoe (95) surveyed students in the fourth term of residence to solicit their views on the usefulness of modern mathematics topics that they had studied at secondary level. The students' judgement was that mapping notation, matrix algebra and statistics were the most useful and that vector algebra and topology were the least useful. There appeared to be no hard evidence that following a modern syllabus had disadvantaged students by the end of their first year at university; what they lacked in a tool-kit of techniques they soon acquired and their greater understanding of concepts provided the necessary breathing-space.

Walkden and Scott (96) expressed concern with the attitudes shown by their students; they found the following shortcomings
(i) The students expected to assimilate new ideas without mental effort.
(ii) They were reluctant to devote time to study and to practising skills.
(iii) They lacked persistence to tackle non-standard problems.

Walkden and Scott attributed these shortcomings to current teaching methods in schools. They believed that ' O ' Levels were tending towards breadth and
superficiality; this provided a poor foundation for ' $A$ ' Level studies and once again depth was sacrificed for breadth. This poor foundation had reduced the level of conceptual understanding; when allied to the possibly $25-50 \%$ knowledge of the syllabus which could engender a reasonable pass at ' A ' Level, the consequences could be dire. Since their students either did not know or did not understand enough factual material and they did not have the capacity to learn or understand new material quickly enough, the prospects appeared bleak.

Power (97) was involved with teaching technician engineers; he said that their mathematics should ignore relevance and should concentrate on conceptual understanding. Since the sequence of mathematics topics was usually out of phase with their engineering applications, attempting to motivate students via applications was often a waste of time. Instead, it should be the task of the engineering lecturer to explain when he is using mathematical algorithms.

More recently, there was an attempt to broaden sixth form studies by replacing ' $A$ ' Levels by ' $N$ ' and ' $F$ ' Levels (98), but this came to nothing. Currently, a revised proposal to introduce Advanced Supplementary ('AS') Levels to supplement 'A' Levels has been implemented in some schools (99). Lower down, the Certificate of Secondary Education (CSE) and ' O ' Levels are being replaced by the General Certificate of Secondary Education (GCSE) (100). The mathematics syllabuses being adopted owe much to the Cockcroft Report on the Teaching of Mathematics (101). The institutions of higher education have yet to feel the effects of these changes; it is not clear how well they are prepared for these effects.

### 2.7.2 The Impact of the Computer

Perhaps the greatest impetus for change has come from the advances in computer technology; these changes have taken place with such rapidity that mathematics educators have yet to come to terms with them. In Chapter 4 a fuller account of the impact of these changes is given, but it is pertinent here to pin-point
a few of the key contributions to teaching over the years.

Reference has already been made to some early advocations of a greater rôle for numerical methods as a result of the improving computing power available. In 1970, Schey et al (102) had described the use of a computer laboratory for teaching calculus. At this time terminals were beginning to become more widely available as a replacement for batch processing; Bajpai and Mustoe (103) showed how the use of terminals could enhance aspects of the mathematics teaching. These ideas were expanded by Mustoe (104) at a one-day conference organised by the IMA at Loughborough. He felt that there were five drawbacks to the traditional teaching approach:
(i) There was tedium in too much calculation.
(ii) The problems given to students were often artificial.
(iii) An in-depth study of a topic was often impracticable.
(iv) Batch-processing led to delays which lowered motivation.
(v) Results often had to be presented as a fait accompli.

The terminal offered five enhancements to teaching:
(i) The speed of its response was very high
(ii) It was possible to make use of stored programs.
(iii) It relieved the tedium of calculation.
(iv) It allowed the user the opportunity to vary model parameters.
(v) It gave the user the chance of making decisions.

Mustoe warned that there were five dangers associated with careless use of terminals:
(i) The terminals will be used 'because they are there'.
(ii) The students will be spoonfed.
(iii) The students may take a passive rôle and learn little from a session at the terminal.
(iv) The students may program at the terminal instead of thinking out the program beforehand.
(v) The students lose the habit of performing 'hand' calculations.

He suggested that a suitable format for use of a terminal laboratory was for a
tutor-led discussion to follow the demonstration of a stored program in action and then for a hands-on session to reinforce the ideas. Initially, students would use stored programs and then, as their programming skills improved, they would write their own. Whilst VDUs were useful for graphical displays they did not have the facility for providing a permanent record of program listing and output.

Mustoe wanted the students to write reports on their session at the terminal; these reports would be assessed coursework and this would bring mathematics into line with other engineering subjects. English (105) gave demonstrations of a typical session based on these ideas.

Leach and Hampton (106) had developed a Computer-Aided Learning (CAL) system called MATLAB which could be used in a variety of ways: as a calculating aid, as a means of extending the range of problems available to the student, as a means of illustrating mathematical theory by allowing experimental calculations and as an aid for project work. Projects on CAL were now beginning to mushroom and a biennial conference on 'Computers in Higher Education' initiated by Bajpai was held alternately at Loughborough and Lancaster. Typical of the work being carried out was that by Hundhausen (107) on the teaching of ordinary differential equations in an engineering environment.

The impact of the computer on teaching was considered by several authors, for example Eriksson (108), Winkelmann (109), Rowe (110), Canelos and Carney (111), and Murakami and Hata (112). CAL had now become CBL (Computer-Based Learning) and was soon to spawn CEL (Computer Enhanced Learning).

If the terminal was one revolutionary step forward, the arrival of the micro was a greater one. Among early workers in this field was Harding who was to develop a mathematical tool-kit and produce computer-illustrated texts (113), (114). Jacques and Judd (115) showed how the micro could be used in teaching a numerical methods course while James and Wilson (116) concentrated on the rôle of micros in mathematical modelling courses.

In 1983 Bajpai formed the Micros In Mathematics Education (MIME) team of which the author was a member. In a series of papers the team described its experiences in the early years (117), (118), (119) and (120). The first 13 software units prepared were designed to cover ' A ' Level mechanics; the next 5 were in the area of statistics. With the help of a grant from the University Grants Committee and the Computer Board further units were written on topics selected from first and second year syllabuses in engineering mathematics. Descriptions of these units and their use in the classroom are given in several papers (121), (122), (123) and (124). A full account of this work is found in Chapter 5.

### 2.7.3 Modelling and other Strategies

There has been an explosion of activity in the area of mathematical modelling in education in recent years. The IMA has a journal on Teaching Mathematics and its Applications and a number of books have appeared at a level suitable for undergraduates (125), (126), (127) and (128).

Amongst the many authors who have tackled the rôle of modelling in the engineering undergraduate curriculum mention may be made of Heaton (129), Ford and Hall (130), McDonald (131), McLone (132), Sekhon et al (133), Burley and Trowbridge (134) and Beckett (135). Hall (136) and Gadian et al (137) considered the assessment of modelling skills by project work.

Robson (138) examined the rôle of computer simulations in the teaching of modelling whilst Clements (139), (140), (141) and (142) has developed a set of case studies/simulations that he uses with his students. Andrie (143) made a reasoned case for an applications-orientated approach to the teaching of civil engineering students and gave examples of the case studies and models he had used in Germany. He insisted that such an approach was essential to provide motivation for those students who had no professional experience.

Crilly, Kropholler and Mustoe (144) gave a demonstration, in the form of a
dialogue between a mathematics educator and a chemical engineer, of how the process of modelling batch distillation employed many mathematical concepts and techniques in its development. The different attitudes and expectations of the protagonists were highlighted.

More details on the case study approach will be found in Chapter 6.

A variety of approaches to the teaching have appeared in the literature. Mustoe (41) had reported on his experiences of sharing lectures with Civil Engineering staff and Sharp (145) gave further examples of such sharing. MacNab et al (146) described two teaching strategies they had employed as an alternative to lectures alone:
(i) lectures, course booklet, tutorials
(ii) lectures, course booklet, computer exercises.

The latter approach reduced the number of lectures given by $60 \%$ and used computer-assisted learning to provide drill and practice examples with a branching facility. The two approaches had been tried on a 15 hour module on complex numbers and the results, whilst not too definitive, indicated that modest improvements had been achieved by low entry students in comparison with the lecture-only method.

Clark had employed a Keller plan mode of teaching weaker students selected by a pre-test on particular topics (147); in an attempt to combat shortage of resources, Cuming and McIntosh (148) had employed a Personalised System of Instruction on third year students with assessment based on a set of tests marked with the student present.

Searl (149) had encountered difficulties in teaching partial differential equations; his approach was to concentrate first on three points:
(i) recognising the type of equation
(ii) recognising the type of boundary conditions and their physical meaning
(iii) knowing the special properties of the solution.

He took specific examples of the wave equation, Laplace's equation and the diffusion equation to illustrate these points. Students were encouraged to use engineering commonsense and to interpret the mathematical solutions physically. Searl then turned his attention to making tutorial classes more effective (150). He described the system adopted at Edinburgh University to enliven tutorials and make the students take an active rôle. He had employed tape/booklet sequences and videotaped programmes; the accompanying tutorial sheets had been constructed to make them more interesting for the students.

### 2.7.4 Industry Expects....

The expectation must be that most graduate engineers will ply their trade in industry and the occasional reminder from industrialists as to the mathematical needs of their employees is to be welcomed. Lawes (151), however, believed that educators should not ask what industry requires, but that they should ask "what education today could do to influence industry tomorrow". He suggested that there was a great need for the engineer to be aware that a mathematical technique was available, to know where it might be useful and to know to whom to turn for help.

Einarsson (152) wondered whether the average engineer was being taught too much mathematics. However, he did want the engineer to be trained more in rigour; he had read too many engineering papers which demonstrated a lack of rigour. He was unsure whether the engineer nowadays needed to know how to program. Hennig (153) and Sundström (154) both recognised the availability of good software and wanted a solid, general mathematical foundation to be laid; extra skills could be acquired in the work environment. Wilkinson (155) was concerned that not many university staff had much industrial experience, which he believed to be important. A general concern was the lack of understanding of the physical significance of the mathematics: the example of eigenvalues was a case in point.

### 2.8 Coda

The author was able recently to span the last forty years in a possibly unique way. Sir Joseph Pope, who had spoken at the Brighton meeting in 1948, is now the chairman of a firm, based in the East Midlands, which manufactures technical equipment for use in teaching. He is still showing his concern for the education of engineers by taking demonstration experiments into local schools to excite the interest of sixth-formers in engineering problems.

His view was that most engineers were not interested in mathematics as a subject in its own right. They wanted to design and build things, and mathematics was merely a tool - albeit an important one. He welcomed the increasing availability of computers, provided that they were used as an aid to engineering thinking. It was crucial to make the mathematics relevant, and demonstrations using simple models which would give the student an engineering feel for the problem were highly desirable. At all stages of the mathematical solution a model, the physical interpretation should be elicited. Above all the mathematics taught to students should give them added freedom in their work, and not act as a restricting agent.

Pope was apprehensive about the changing nature of school syllabuses and feared the outcome on the numbers of pupils who would wish to follow a career in engineering. Pope concluded that much had been achieved in the mathematical education of engineers, but much remained to be achieved.

## Chapter 3

## Implementing the Integrated Approach

### 3.1 The Path to Integration

The author's first direct experience of teaching mathematics to engineering undergraduates was with Civil Engineering freshmen in the session 1969-70. At that time, the engineering mathematics syllabuses at Loughborough were arranged in modular form; they are reproduced in Appendix 3. In essence, the Part A modules were: Mathematical Methods I, occupying the first academic term (of ten weeks) and two weeks of the second term, with 36 lectures and 12 tutorials; Numerical Methods I and Computer Programming, which required 24 lectures and 8 tutorials during the second and third academic terms; Statistics I, comprising 12 lectures and 4 tutorials in the third term. Written examinations of 1.5 hours duration were held after week 2 of the second term and at the end of the third term, each contributing one half of the overall assessment. The only exception to this pattern was that Electrical Engineering students took a module in Vector Field Theory instead of Statistics I.

To allow individual departments to choose mathematical topics more suited to their particular needs, seven modules were offered at part B. (It is worth mentioning here that some departments operated a thick sandwich scheme whilst others ran a thin sandwich arrangement and this severely complicated the timetabling of courses). The modules Mathematical Methods II and Numerical Methods II were universally chosen, but the selection of other modules was tailored to individual needs: for example, Civil Engineers opted for Statistics II and III whilst Electrical Engineers followed Mathematical Methods III and Functions of a Complex Variable. Assessment was by a single three-hour written examination at the end of the third term. Eight modules of ten hours were offered at Part C; the take-up varied from department to department and within a department. The
courses were often optional and usually non-examinable.

One of the major problems confronting the author was the high proportion (16 out of 68) of Civil Engineering freshmen who did not possess 'A' Level mathematics; in the previous session nearly all such students had failed the mathematics component of their course. Clearly such a situation could not be allowed to recur. An added complication was that it had been agreed to conduct an experiment whereby the first three days of the session were to be given over entirely to an intensive introductory course in Fortran programming, consisting of lectures, tutorials, and practical sessions on the University's IBM 1620 computer.

There was a third hurdle to overcome: at that time the author did not know much about Civil Engineering. He horrified his engineering colleagues by asking whether there was a difference between the prestressing and the reinforcing of concrete! With willing cooperation from the Civil Engineering staff he set himself the task of learning as much about their subjects as he could, in particular where, when and how mathematics was applied. There was little in the published mathematics literature to help and it meant that Civil Engineering text-books had to be read thoroughly. Fortunately, this effort was encouraged by Professor Bajpai who was then responsible for the service teaching of engineers.

The intensive Fortran course went well and the students' enthusiasm was manifested partly in the high attendance at voluntary practice sessions in the evenings and over the weekend which were led by the author. The nature of the computer, which was a hands-on machine, allied to the fact that for almost all of the students it was their first experience of computing helped to sustain this enthusiasm. As a means of measuring their newly-acquired skills a set of eight programming problems were given with the requirement of completing the first two successfully and as many of the remainder as they were able (or willing). The average number of problems tackled successfully was about 4.5 with all but a handful completing more than 3.

Having introduced the students to computer programming it was important to make use of it at intervals during their course. Accordingly, the author would give them problems which encouraged the writing of a program; for example using the Maclaurin series for $\sin \times$ and $\exp \times$ to decide how many terms were needed to achieve a particular degree of accuracy or how, for a fixed number of terms, accuracy varied with the value of x . When the module on Numerical Methods was being taught, it was easier to find suitable problems to program.

Although the students, including the non-'A' Level entrants, performed well in the half-year examination compared with the other groups sitting the same paper, it became clear during the first term that the concentration on analytical and algebraic material was hard going for the weaker students. It took many hours of extra tuition, sometimes on an individual basis, to keep them in touch mathematically with their more able colleagues. It seemed curious too, that having dealt with calculus methods of integration in early November, the numerical methods were not taught until the February following; if the lecturer thought that it was a peculiar practice to separate what are in fact two means of attack on the same problem, then the students could be forgiven for thinking similarly.

During this period the seminal paper by Bajpai, Calus and Simpson (21) was being written to demonstrate the way in which the topic of ordinary differential equations could be taught via the integration of analytical, digital computing and analogue computing approaches. Their argument, that students taught by the traditional, compartmentalised approach 'were often unable to appreciate that on many occasions a combination of techniques is used in solving problems arising in real-life or industry', coincided with the author's experience and the integrated approach seemed to him to offer a more realistic way of teaching his students.

### 3.1.1 The 1970 Paper

Eighteen years on it is difficult to appreciate how radical were the
suggestions made in the paper by Bajpai et al. It is necessary to remember that most of those who were then teaching mathematics to engineers had received no formal training in numerical methods or computer programming and few were even using such techniques in their own work. It was not unexpected, therefore, that the former group in particular were reluctant to take up the challenge of teaching via the new approach.

The paper began by considering traditional approaches to the teaching of differential equations; each approach was associated with a list of disadvantages. In general, students had little idea of the significance of the structure of a differential equation and the roles of the various parts of its solution. Furthermore, the formulation of a differential equation from a physical statement of a problem was either neglected or relegated to a minor place.

The paper then suggested a possible strategy employing the integrated approach. First, suitable motivational examples should be discussed and for each one the derivation of the appropriate differential equation and attendant boundary or initial conditions should be covered. Then, one of the first-order examples, say Newton's Law of Cooling, is to be studied in depth. The analytical solution is obtained and its graph compared with the experimental results. Next, the analogue computer is used to demonstrate how the nature of the solution depends upon the parameters in the model, via a graphical display. Finally, a simple step-by step method is used on a digital computer to obtain a table of results and a comparison is made between the three approaches.

A mass-spring-dashpot system is modelled via a linear second-order ordinary differential equation with constant coefficients. An analogue computer demonstration is given in order to show the three cases of simple harmonic motion, light damping and heavy damping. Each case can be discussed via its analytical solution - which can be derived at a later stage. A step-by-step method can be used to obtain appropriate digital computer results which can be compared with the other solutions. At a suitable point the case of critical damping can be
discussed. This stage of the teaching concludes with a consideration of what happens when the system is subjected to a disturbing force; the analytical solution is presented and the complementary function and particular integral are given physical interpretations.

The authors of the paper recommended that such an introductory coverage should be followed by a more detailed treatment of the relevant techniques of solution and then some examples should be given of differential equations which must be solved numerically. A further suggestion was that use could be made of video-tape to record experimental demonstrations and computer results as they were produced. The author of this thesis was one of an in-house team which prepared a tape on ordinary differential equations along the lines of the teaching strategy discussed above. The tape was shown to several groups of students and the reception was generally favourable. At about this time the Open University was making its first programmes and the range of technical facilities at its disposal far exceeded those at Loughborough. A few more tapes were made, the author being involved with one on mathematical models in engineering and one on accuracy and error which were to appear in written form in Chapter 1 of 'Engineering Mathematics' (42).

### 3.2 Teaching the Integrated Approach: Early Attempts

The author was aware of one overriding problem confronting those, including himself, who wished to teach the integrated approach: there was no literature on how to put the ideas of the approach into practice, other than with regard to ordinary differential equations. Certainly, no text book was suitable. It was felt important that the students should have more than a set of weekly hand-outs; if they saw a text-book they could feel confident that the teaching material had been thought through and was not merely proceeding from week to week with the lecturer keeping one step ahead of them.

Accordingly, the author produced three booklets for his students: the first,
entitled 'Functions and Their Behaviour', comprised 111 pages and covered the topics shown in Figure 3.1, which is reproduced from the booklet; the second, entitled 'Differential Equations', comprised 63 pages and covered the topics shown in Figure 3.2; the third, entitled 'Basic Statistics' covered the topics shown in Figure 3.3 and comprised 105 pages.

As in the previous year the course commenced with an intensive three day introduction to Fortran programming. It is scarcely a revelation to admit that many students found this early shock to the system hard going; however, with the aid of some sympathetic second year students taking the tutorials and the offer of help outside formal teaching hours, everyone was able to survive and eventually to program successfully.

It was thought to be important to set the mathematics course in context and the first lecture was devoted to explaining how mathematics was used to help solve engineering problems, both by analytical methods and by computer-based techniques. In addition, the formulation, including assumptions made, solution and validation of a mathematical model of beam bending were discussed. The difference between easily-solved problems (such as forces in the members of a truss) and ones which were easily posed but hard to solve (the time taken for a polluted stream to become potable) was highlighted. As an aid to students new to extensive note-taking the author provided the class with a hand-out giving his summary of this first lecture in order that they could compare it with their own notes; this procedure was repeated for two further lectures.

Most ' $A$ ' Level entrants had met four-figure tables as their calculating aid whilst the ONC/OND entrants had already used a slide rule - which was then standard equipment for engineering students. The second lecture, which has become increasingly important as pocket calculators have become more universal and more powerful, was devoted to a discussion of accuracy and errors, stressing in particular the expected accuracy of the result of a lengthy calculation based on experimental data.




Figure 3.3

Although there was a machines laboratory available - containing a large number of electro-mechanical calculators - in practice there was no intermediary between the slide rule and the main frame computer. When the student was asked to carry out a lengthy calculation it had to be a worthwhile exercise to justify the effort expended. Emphasis was placed on developing flow charts for numerical methods and results from pre-written programs which were presented for discussion. However, to capitalise on the students' programming skills, programming exercises were set on a regular basis.

Wherever possible a topic was introduced via a practical problem: the need to fit a least squares straight line to data led to the study of partial differentiation; the construction of transition curves was used as an introduction to curvature. The author is aware of a school of thought which holds that motivational case studies can be confusing for students and counter-productive to their performance in mathematics; he does not accept that view. Properly handled and sensibly introduced, such case studies are an essential part of any mathematics course for engineers. Relevance engenders motivation and motivation encourages effort.

Part of the function of the teaching is to bring students out of the protected atmosphere of school mathematics, where equations have simple roots and all integrations can be performed, into the real world of inaccurate data, approximate models and uncertain answers. This can be a culture-shock to many students and needs careful treatment. Those analytical techniques with which the students entered tertiary education are not to be discarded as worthless; rather they are to be used with more care and discrimination. In the right circumstances they are unrivalled, but there are situations in which they have to yield pride of place to their numerical cousins.

Often, a numerical cousin could cast the light of understanding on a related analytical method, long a weapon in the student's armoury, the workings of which had never been fully comprehended. For example, the method of direct search to locate the minimum of a function whose values can only be sampled
was introduced via the problem of locating the nearest point to the earth's surface of a layer containing a material to be extracted. A discussion took place on suitable search strategies and this led to a comparison with differentiation-based methods of locating optima of functions specified by formulae - and that in turn led to a realisation of the difference between local and global optima.

The idea of splitting the examination into one 90 minute paper after each half-session was abandoned in favour of a three hour end-of-session paper; this conformed with the practice that had been maintained at second year level. Nonetheless, a mid-sessional examination was still set and sat; although it no longer formed part of the students' assessment, it did serve as an indicator of their progress, which was important to a lecturer with nearly one hundred students in his class.

Many students had not met differential equations prior to this course and the decision was made to include integration problems as examples of first order differential equations. The remaining material on first order equations and that on second order equations was an expansion of the ideas in the 1970 paper. After discussion with the Civil Engineering staff the decision was taken to teach both the trial solution method of finding particular integrals and the Laplace transform approach, but not to include D-operators.

Taking a realistic view of student attitudes it was recognised that the end-of-session examination ought to reflect the way in which the course had been taught; if it did not, the students might have felt cheated.

The examination paper set in 1971 is reproduced in Appendix 4.

At this juncture the author wishes to make a comment which he believes to be of vital importance. The teaching experiments he has conducted over the last twenty years would not have been so successful had he not gained and maintained the willing cooperation of his students. Certainly, he was fortunate in having the
continued encouragement of his Head of Section and, subsequently Head of Department. In addition, the Head of the Department of Civil Engineering, Professor Brock, and his staff were generous with their time in discussing the position of mathematics in their subjects as taught and as practised and were helpful and encouraging at all times.

That the staff came to regard the author as an integral part of the teaching team was a valuable attitude which permeated to the students. However it is the students who are to be the beneficiaries of the whole exercise and it was necessary that they should not regard themselves as guinea-pigs in an academic exercise; rather they were partners in an enterprise which would result, hopefully, in their increased understanding of, and proficiency in, mathematics applied to engineering. It was gratifying that at the latest visit in 1987 by the accreditation team from the Institution of Civil Engineers, the author was asked especially to attend the final plenary session; the Chairman of the team commented that not only was Loughborough the exception in that the students made no complaint about the mathematics teaching, but also that they were positive in their appreciation of the efforts made to link mathematics to their other engineering subjects.

The author has always believed that it is important that each student, particularly in a class of about 100 , should feel able to ask for help, if needed, outside the classroom. He feels that there was a beneficial spin-off to his experiments in that the students would be prepared to discuss their feelings about the course, the manner of presentation and their progress. In this way the author was able to make early corrections and modifications before the class suffered too much. Fortunately, the author has managed to maintain good relations with his students and has received a continuous feedback from them.

### 3.2.1 Statistics and Probability

Of all parts of the first year course, that which gave rise to the greatest
difficulty to those who had not met it before was the area of elementary probability and statistics. The actual mathematical manipulation is straightforward: almost entirely arithmetic with a smidgen of algebra. The difficulty lies elsewhere. In the case of probability, it is recognising the essence of the problem and devising a suitable method of solution: there seems to be a different method for each problem and this does force the student to think more carefully about the nature of the problem itself.

With regard to the statistical inference component the students found it difficult to listen to a verbal argument, however well constructed. They seemed to have particular problems in understanding the logic behind the test of hypotheses concerning a sample mean. Perhaps the fact that the statistics section of the syllabus was the last to be taught added to their difficulties; they were tired mathematically at the end of a long and varied course and the assimilation time for new ideas was short.

However, those students who, despite being new to these topics, made a determined effort to comprehend the ideas involved soon found themselves able to cope. One likened it to learning to drive: "In the early stages you stall when waiting at traffic lights and despair of ever coping successfully; then one day you don't stall and you never have any more trouble. It's all a question of confidence".

The author has subsequently moved part of the component covering probability theory and the binomial and Poisson models to the middle of the second term, leaving the normal distribution and large sampling theory to be dealt with at the start of the third term. In addition, many more students have studied statistics at school, albeit at the expense of statics and dynamics, and this has helped to alleviate the problem to an extent. The students who have not covered statistics at school still have an initial hurdle of confidence to surmount, however.

In this context, it is interesting to refer to work carried out by Green on probability concepts in children (156).

### 3.2.2 The Second Year Course

In the same session as the experiments on teaching the integrated approach to the first year students were being conducted, the author was also lecturing to those students who had been taught via the traditional approach in the previous session and who were now in their second year. It was not sensible to embark on a wholesale revision of this syllabus until the first year modifications had run through at least once. Accordingly, only a few instances of integrating the analytical and numerical methods were attempted.

One example was the topic of eigenvalues and eigenvectors. Having introduced the topic via a structural vibration problem, in broad terms, the algebraic method of determining eigenvalues and their associated eigenvectors was developed. It took but a slight change in the parameters of the model to produce a characteristic equation which did not factorise easily. After a debate on whether it was better to attempt an iterative solution to this equation or set up a numerical method to determine the eigenvalues, it was decided to follow the latter approach. As the method was being developed it became clear that it was, in fact, a method for determining the "dominant" eigenvector with the associated eigenvalue appearing as a by-product. Although the method appeared unattractive, it was apparent that a system which gave rise to a matrix with many rows (or columns) would effectively have to be handled this way.

Once again, the numerical method took a different view of the problem from that taken by the analytical technique. In the first year course, for example, the students had seen that Euler's method regarded a differential equation as a provider of information about the slope on a solution curve rather than as an intrinsic source of information about the dependent variable. This juxtaposition of different views adds to the understanding of both.

### 3.2.3 Project Work

It was recognised that the further computer programming taught in the second year needed more application than was currently provided by the other engineering lecturers, many of whom were strangers to programming themselves. Accordingly, a project was set on numerical optimisation which is reproduced below. The students handed in for marking a report which contained a listing of their program, sample output and comments on their methods.

## Project

Show by partial differentiation that the function

$$
f(x, y)=100\left(y-x^{2}\right)^{2}+(1-x)^{2}
$$

has a stationary point at $(1,1)$ and use common sense to show that it is a local minimum and, indeed, the overall minimum of the function.

Use the computer to help produce contours of $f(x, y)$.
Write a computer program using the method of Hooke and Jeeves to locate the minimum starting from $(2,1)$ and $(-1,0)$.

Comment on your results.

Although the marks did not form part of the end-of-year assessment, the students tackled the project with enthusiasm and considerable success.

### 3.3 The Need for Suitable Textbooks

In the autumn of 1971, the author (25) presented an interim report on his experiment to a conference on the Teaching of Mathematics to Non-Specialists. Since the publication of the 1970 paper the question being asked was: 'Where is a suitable text-book to be found?'. Many lecturers, whilst sympathetic to the integrated approach, did not have the time or perhaps the expertise to assemble suitable teaching material for themselves. Obviously, it was of little value to teach differential equations alone in this way. The answer was clear: a series of text-books had to be written by those who had championed the approach. Early in

1972, Bajpai, Mustoe and Walker took up the challenge. After an initial period it was decided that although draft material might be produced by any of the authors, one should be responsible for writing the chapters in final form; Mustoe took on the task. Some of the material grew from the booklets that he had prepared in the previous session and was refining for the current crop of students. However, these booklets had been written specifically for Civil Engineers and it was necessary to select examples from the whole range of engineering disciplines.

In order to ensure that the material passed the crucial test of being suitable for the students at whom it was aimed, the draft chapters were issued to a combined group of Civil Engineering and Chemical Engineering freshmen in the 1972-3 session. In addition to receiving general feedback in tutorials and casual conversation, the author selected three students to act as detailed reviewers. Each week the three would discuss with the textbook authors the previous week's material, offering suggestions as to where more explanation was needed or where better examples were required or where the problems did not tie in too well with the text. Two of the students were Civil Engineers to reflect approximately the ratio in the group as a whole; of these one had an ONC entry qualification.

In addition to these discussions, feedback was obtained from colleagues in other institutions who had read the draft chapters and had class-tested them; in particular, W. Ted Martin of M.I.T. agreed to be a collaborating author. In Spring 1974 'Engineering Mathematics' was published and gained steadily in acceptance. Three years later 'Advanced Engineering Mathematics' appeared; since it sought to cover most second year engineering course syllabuses in mathematics it was always recognised that each student would find a non-trivial part of the book surplus to requirements. The book, whilst popular, did not enjoy the success of its predecessor. In 1980 the final book in the series, 'Specialist Techniques in Engineering Mathematics', was published. A number of mathematical topics had become necessary for the study of modern branches of engineering and science but were available only in advanced textbooks. Many students found these books difficult to read and had expressed a wish to have an introduction which was easy
to understand. Having appreciated the basic principles of such a topic the student would be better equipped to tackle the advanced volumes. The philosophy of "Specialist Techniques' was to provide that readable introduction. The topics covered were: system models, linear systems, stability of systems, optimal control, random processes, cartesian tensors, the finite element method, design of experiments and functional analysis.

As was mentioned in the previous chapter, there was always the problem of the entry being heterogeneous. It was decided to include in 'Engineering Mathematics' some work which was covered in Advanced Level syllabuses; whilst this would be revision for some students, it would be new to others and it was included for the sake of completeness.

The author was soon able to teach using 'Engineering Mathematics' as a required text-book. This enabled him to prepare a hand-out issued to the students at the start of the session detailing which topics were to be covered when, which pages of the book related to which lecture and which problems in the book to attempt. Problems were designated 'A' or 'B'; the former were to be attempted by everyone and a successful understanding of the material and techniques that they entailed would guarantee a knowledge sufficient to do very well in the sessional examinations. Problems ' $B$ ' could be used either as revision practice or to extend the student beyond the basic level.

Those students without 'A' Level mathematics were provided with an extra lecture each week in the Autumn Term. Since the topics were given in the initial hand-out, any students who felt that they would benefit by attending were welcome to do so. Seldom did the attendance for these optional lectures fall below $50 \%$ of those for whom it was not primarily intended.

### 3.3.1 The Rôle of Worked Examples

The author has, for several years, been an examiner for the Engineering

Council (formerly the Council of Engineering Institutions), in common with his co-authors Bajpai and Walker. By their emphasis and style of examination questions it has been possible to influence a wider audience and introduce them to the benefits of the 'integrated approach'. His experience with these examinations allied to that gained with his own students convinced him that a world-wide failure of engineering students (and, no doubt, others) is how to get started on a problem. Consequently, he has written two books (157), (158) to give students guidance through the solution of real examination problems. This has freed him in his own tutorials to concentrate more on reinforcing conceptual understanding. The feedback from his own students indicates that they have found these additional books of benefit.

### 3.4 Example of the Integrated Approach: Year I

The author had the tutorial sessions so arranged that he saw each student once a fortnight. If the alternate tutor was not as well versed in computing and numerical methods as was desirable then he could be left to concentrate on the analytical solutions; many of the author's departmental colleagues were still unused to computer programming at this time. The author could then draw the threads together in his tutorial sessions.

An example of how the integrated approach was emphasised is provided below. This example would be introduced when the topic of integration was being covered in lectures; the topic of iterative solution of non-linear equations had been encountered earlier.

## Problem

A sphere of density $\rho$ and radius $r$ has mass

$$
\frac{4}{3} \pi r^{3} \rho
$$

It floats on water of density 1 , submerged to a depth $h$. Show that the volume of
water displaced by the sphere is

$$
\frac{\pi}{3}\left(3 r h^{2}-h^{3}\right)
$$

If the density of the sphere is 0.4 , find the ratio $\alpha=h / r$ to $2 \mathrm{~d} . \mathrm{p}$. using the Newton-Raphson method with an initial approximation $\alpha_{0}=1$. Why is $\alpha_{0}=1$ suggested?

The following is an edited transcript of a good tutorial session on this problem; T represents the tutor, while $\mathrm{S} 1, \mathrm{~S} 2$ etc. are the students.

T : What is the first thing we should do?
S1: Use Integration to find the volume of water displaced.
T: That's jumping the gun a little. Even before we do any calculation. . .
S2: Draw a diagram.
T: That's a good start - draw one then.
S2: (Draws Figure 3.4)
T: No. I meant going back to the floating sphere.
S3: (Draws Figure 3.5)
T: That's it. Now, what results from your Water Engineering course can you use.

S2: There's Archimedes Principle . . . There's an upthrust on the sphere that equals the weight of water displaced.
T : What does weight of water displaced mean?
(A discussion ensues with long pauses broken by prompting by T . Eventually. . .)

S4: There's a part of the sphere below the water-line. Its volume is the volume of water displaced. The weight of that volume of water is the one we want.
T: Right, but we haven't used the fact that the sphere is floating. How does that take us forward?


Figure 3.4


Figure 3.5

S3: The upthrust must equal the weight of the sphere or else the sphere would be moving.
T: So if those forces are equal, the sphere must be stationary. Is that what you are saying?
S3: Yes.
S5: Not true. All that means is that the sphere isn't accelerating. It could still be moving with a constant speed.
T: Well, how could we make sure that it isn't?
S4: If we held the sphere stationary and then released it . . .
T: Fine. Now, how much of the sphere do you think lies below the water-line when $\rho=0.4$; I mean what value of $\alpha$ do you estimate?
(Long pause)
All right, let's take several possible values for $\rho$.
(Writes down $0,0.4,0.5,0.8,1,1.2,2$ ).
What about $\rho=2$ ?
S1: The sphere would sink.
$\mathrm{T}: \quad$ And $\rho=1.2$ ?
S1: The same.
T: $\quad$ And $\rho=1$ ?
S6: The sphere would lie with its top just on the water-level.
$\mathrm{T}: \quad$ What value of $\alpha$ is that then?
S6: $\quad \alpha=1$.
( T looks unimpressed).
S4: $\quad$ Surely $\alpha=2$ since $h=2 r$.
T: Right. Now what about $\rho=0.8,0.5,0.4$ ?
S2: Each time a little bit less is below the water-line so that $\alpha$ will be decreasing, say $1.6,1,0.8$.

T: Finally, if $\rho=0$ ?
S2: Don't be daft. There won't be any sphere.

T: Now we can go back to the integration.
(Talks through the derivation of the volume with occasional contributions from the students).

Let's just check our result as far as we can - we might not have been given the formula. I'm going to look at three special cases: $h=0, r, 2 r$. If I don't get the correct answer in all three cases I know that I've made a mistake; if I do, it's no guarantee that my formula is correct, of course.
(Class verifies the answers).
Now where do we go?
S1: You know that the upthrust is equal to

$$
\frac{\pi}{3}\left(3 r h^{2}-h^{3}\right) \text { times the density of water times } g .
$$

S3: So that means the weight of the sphere is also

$$
\frac{\pi}{3}\left(3 r h^{2}-h^{3}\right) g
$$

T: Don't we already know the mass of the sphere by another formula? So what do we do with the two values for weight?
S3: Equate them ... Oh,

$$
\frac{\pi}{3}\left(3 r h^{2}-h^{3}\right) g=\frac{4}{3} \pi r^{3} \rho g
$$

$\mathrm{T}: \quad$ Go on then.
S3: Cancelling $\pi / 3$ and $g$ we get $3 r h^{2}-h^{3}=1.6 r^{3}$, since $\rho=0.4$.
T : How does $\alpha$ come into it?
S5: You've got no way of knowing $h$ as a number so you have to express it as a fraction of $r$ - that's $\alpha$.

T: Yes, but how do I get $\alpha$ into that equation?
(Pause).
S3: If you divide by $r^{3}$ you get

$$
3 \frac{h^{2}}{r^{2}}-\frac{h^{3}}{r^{3}}=1.6
$$

and that's $3 \alpha^{2}-\alpha^{3}=1.6$.
T: If I write that as
$\alpha^{3}-3 \alpha^{2}+1.6=0$
how can we use Newton-Raphson?
What's $\mathrm{f}(\mathrm{x})$ ?
S4: $\quad \alpha^{3}-3 \alpha^{2}+1.6$
T: And $f^{\prime}(x)$ ?
S4: $\quad 3 \alpha^{2}-6 \alpha$
T: Write down the Newton-Raphson formula for our equation.
S3:

$$
\alpha_{n+1}=\alpha_{n}-\frac{\left(\alpha_{n}^{3}-3 \alpha_{n}^{2}+1.6\right)}{\left(3 \alpha_{n}^{2}-6 \alpha_{n}\right)}
$$

T: $\quad$ If $\alpha_{0}=1$ what is $\alpha_{1}$ ?
S2: . 0.867 to 3dp.
T : And what does that make $\alpha_{2}$ ?
S2: 0.869 to 3dp.
T: Keep on until you can be sure that you have got $\alpha$ correct to 2 dp .
S2: $\quad 0.87$.
T : That's correct. Why did we suggest $\alpha_{\mathrm{O}}=1$ ?
S3: That means the sphere is half-submerged, and that's a simple first approximation.
T: What I'd like you to notice is how we tackled that problem. First we thought about it in engineering terms, then we used principles of hydrostatics to set up our equation. Next we employed calculus to calculate the volume submerged. Finally we used a numerical technique to get an approximation to $\alpha$. As a mathematician, I'm interested in the equation

$$
\alpha^{3}-3 \alpha^{2}+4 \rho=0
$$

It could have one real root or three. But I know from our sphere problem
that we only get sensible answers from $0<\alpha<2$. I wonder how that relates to the equation? But that's another story.

### 3.5 Later Developments

The most significant factor which has affected the teaching of engineering mathematics over the last two decades has undoubtedly been the development of computing facilities. Few would have imagined twenty years ago that every student would have a cheap pocket calculator which had such computing power, that the main form of communication with the mainframe computer would be the terminal and that the micro would have found its way onto so many desks. The story of that impact is told in the next chapter. It is relevant here to point out that after the hands-on computer at Loughborough was scrapped, the computing facility available to students was a cafeteria-service on a batch processing mode with back up from the data preparation section of the Computer Centre and a four-times-a-day delivery sevice to departments. However, the remoteness of this system caused students' interests to wane noticeably. The arrival of terminals only partly restored that interest since the mainframe seemed almost always to be 'down' just when it was needed and even when it was 'up' the waiting time could be so long as to dishearten all but the keenest. Not until the micro arrived in large numbers on campus did the old level of interest and enthusiasm reappear.

The author has always interpreted 'the integrated approach' as meaning more than the interweaving of related analytical and numerical techniques. He believes in integrating the mathematics into the engineering curriculum via case studies, joint projects, shared lectures and so on. That aspect of the teaching is discussed in Chapter 6.

The author carried the integrated approach to other engineering groups and several variations on the combining of groups tried; the extreme case was the assembling of a class of Mechanical, Automotive, Aeronautical and Production Engineers - some 270 in all - with the lecturing being shared with a departmental
colleague. This was unsuccessful, partly because the sheer size of the class changed the nature of the lectures, partly because the four disciplines, although similar in outlook, identified themselves as different groups coming together for administrative convenience.

Another experiment initiated by the author was to change the format of the examination. Instead of 'do 6 out of 9 ', the paper was divided into 3 sections: Section A comprised two double length questions, Section B comprised five questions and Section $C$ contained two questions on probability and statistics. The candidates were required to answer one question from Section A, one from Section $C$ and three others from Sections $B$ and $C$. The aims were to encourage the students to make an effort to understand the statistics component and to test them in depth on a topic. This format also gave the examiner a flexibility in the questions that he could set.

Bajpai has for many years been the Moderator for the Mathematics Part I and Part II examinations of the Engineering Council (formerly the Council of Engineering Institutions). In an endeavour to encourage candidates to take seriously the study of numerical methods and statistics, he had arranged for the Part II examination paper to be in two sections of five questions each: one on analytical methods and one on numerical methods and statistics. Candidates were instructed to attempt five questions, including one from the first section and one from the second. In recent years it became evident that candidates were accepting numerical methods as an integral part of their course; consequently the paper currently is not sectionalised and candidates can attempt any five questions.

In the last three years, the author has adopted the format of 'do 5 from 9', with all questions equally weighted. There is no evidence to suggest that any of the variations has made any significant difference in student performance.

With the arrival of the micro came a request from the Civil Engineering staff to abandon the three-day crash course in programming and give the necessary
lectures within the mathematics allocation; the language was to be Basic. The staff wished to show their belief in the importance of computing and agreed to man practical sessions in the first half of the Autumn Term during which the students would familiarise themselves with the micro in additon to writing and running some simple programs. The author took the opportunity to bring mathematics more in line with the other engineering courses by introducing a coursework element in Year 1. In the second half of each of the first and second terms the students are given a programming problem from a list of 8 available. In each case a member of the Civil Engineering staff acts as tutor, providing an initial briefing and being available for guidance for the next five weeks. He will mark the students' reports and the author acts as overall moderator. Each of the two items counts $10 \%$ towards the final assessment in mathematics. The problems were suggested by the engineers and refined in discussion with the author. An example of the problems and the specification are provided in Appendix 5.

In Year II the students are taught to program in Fortran 77 and a sample programming problem is shown in Appendix 6. It is hoped shortly to introduce this as coursework.

### 3.6 How Much Does the Freshman Know?

Reference has already been made in Section 2.7.1 to the problems associated with the variation in knowledge and ability at entry to tertiary education. In recent years the author has handed out to his freshman class during his first lecture a questionnaire and a test paper. These are to be handed back a week later.

The questionnaire is reproduced in Appendix 7; it seeks to discover the areas in which students have gaps or weaknesses. If, for example, hardly any of the class had covered or understood complex numbers - ... whereas most were $a u$ fait with elementary vector algebra then an appropriate adjustment could be made to the time spent on these topics during lectures. As mentioned in the questionnaire, the first list on which the students are asked to comment with
regard to prior coverage and understanding was taken from a proposed Common Core in mathematics for all ' $A$ ' Level Boards (159). The second list, taken from the same publication, comprises those topics that the Physics panel expected to be covered in all mathematics syllabuses; the third list was that used by Heard in his project (64).

The questionnaire was given to students over the last four years; the results are shown in abbreviated form after the questionnaire in Table A1.

The test paper is shown in Appendix 7. The number of attempts and percentages of correct responses are shown following the test paper. At this point some of the wrong responses will be discussed.

Question 1. The only real difficulty here was that many students did not recognise that $x^{3}+a^{3} \equiv(x+a)\left(x^{2}-a x+a^{2}\right)$.

Question 2. Several responses opted for

$$
\frac{B}{x^{2}+4} \text { or } \frac{C}{(x-5)^{2}}
$$

but no "extra" functions like

$$
\frac{D x}{x^{2}+4} \text { or } \frac{E}{x-5}
$$

Question 3. Many students went straight to a calculator to produce the numerical value; some never quoted the accuracy of the value they wrote down. Of those who obtained $4 / \sqrt{ } 5$ before using a calculator, many explained their unwillingness to leave the answer in that form or as $4 \sqrt{5} / 5$ by stating that to them "value" implied a decimal answer correct to a "reasonable" number of decimal places.

Question 4. The word "sketch", although underlined for emphasis, seemed alien to many respondents. Far too many, seeing the words "maximum or minimum value" immediately plunged into differentiation. They expressed ignorance of the fact that the curve was a parabola and became wide-eyed when told that the optimum lay half-way between the zeros. One or two did point out that if the parabola did not cross the $x$-axis they would have to use differentiation anyway, so why bother looking for a special method. When they were shown the method of completing the square, they reluctantly accepted that perhaps it was worthwhile as an alternative to differentiating.

Question 5. It was quite clear that the notation baffled the majority of students who attempted this question. To many the nth term was $a(1+r)^{n}$. On reflection, the question was too clever for its purpose.

Question 6. The question of a binomial expansion having a restricted range of validity had clearly never crossed the minds of some students.

Question 7. The form of $y=x^{1 / 2}$ was not correctly sketched by a large minority of respondents - perhaps they would have been more successful in plotting the curve as some persisted in doing. The relationship between $x^{2}$ and $x^{4}$ was not too well appreciated. When those who sketched the three curves on the same axes failed to have them all pass through the point $(1,1)$ it led the author to speculate how many of their colleagues who opted for three separate axes would have fared better.

Question 8. It is clear that the notation $f(x)$ is very badly understood, even by those who claimed to comprehend the idea of a function thoroughly. Most people coped with $2 f(x)$ and $f(x)+4$ but, as anticipated, produced $f(x+4)$ instead of $f(x-4)$ and $f(x / 2)$ instead of $f(2 x)$, if they made any attempt at all. Post-test questioning of these wrong attempts or no attempt respondents revealed that they did not try putting particular values for $x$ to see what the outcome was; rather, they
tried to reason it out from the notation directly.

Question 9. The first step taken by many to simplify $(y+7)(y-2)>0$ was to expand the left-hand side! For most of these students, it was also the last step. Again, few recognised immediately that the curve of the relationship $z=(y+7)(y-2)$ is a parabola. Some, having expanded the right-hand side then used the formula approach to find the zeros of the resulting quadratic!

Question 10. This question was generally answered without any difficulty as one would hope, but there were disappointments.

Question 11. Ignoring the plotters, it was noticeable that very few students either knew or bothered to show in their sketches that the gradient of $\tan x$ at $x=0$ is 1 and not zero. It does seem that there is almost no approach occupying the ground between a careful plot and a vague sketch which shows a few salient features accurately.

Question 12. This question was simply too hard for or totally alien to the overwhelming majority of the students who sat the test.

Question 13. No particularly common errors - usually just slips.

Question 14. The main stumbling block was to calculate $\sin 2 B$ via calculating $\cos B$ first. Some were unaware of how to expand $\sin (A+C)$.

Question 15. Either this type of equation had been met before or it had not. Other than arithmetic slips, that seemed to be the main differentiating factor between respondents.

Question 16. Of those who twigged that $\cos 2 x \approx 1-2 x^{2} \operatorname{since} \sin x \approx x$ for the value of $x$ given, a sizeable number did not appreciate that $x$ should be in radians,
hence the relevance of the information $\pi^{2} \approx 10$ eluded them.

Question 17. The main differentiating factor here was, as in question 15, whether the student had previously encountered the topic at school or college or not.

Question 18. It was not anticipated that so many students would have difficulty in sketching $y=2 x^{3}+5$. Fortunately, some of them redeemed themselves by sketching correctly the inverse function to their function. To several students "inverse" was interpreted as "one over". Not enough used the line $y=x$ as a reflector.

Question 19. Even if a respondent obtained $2^{X}$ it was an occasion for rejoicing if he then gave a correct sketch. However, $e^{x}+2$ and $2 e^{x}$ were not uncommon "simplifications".

Question 20. A few claimed not to have encountered the term "derived function", but most of these people interpreted it correctly. Several used the quotient rule on $\tan 2 x$ in the form $\sin 2 x / \cos 2 x$. It was not unknown to see the coupling of answers

$$
\sec ^{2} 2 x \text { and } \frac{1}{2} e^{\frac{x}{2}}
$$

this was never satisfactorily explained.

Question 21. Too many students obtained the equation $\tan x=-x$ and gave up; they did not appreciate that $\mathrm{x}=0$ is $a$ root of this equation and would suffice to answer the question.

Question 22. No common error was apparent, although some eliminated $t$ from the problem or from the answer.

Question 23. There was the expected crop of students who apparently could not distinguish between "integrate" and "differentiate". The integral of $\sin 2 x$ proved a pitfall to many; they had something along the right lines, but had not checked that they were correct by differentiating back.

Question 24. Usually only arithmetic slips prevented a correct answer although the occasional respondent did not know or did not understand the method of integration by parts.

Question 25. Apart from those who saw $x^{2}$ and produced $\int \pi x^{2} d x$, there was a significant number who, having written down the correct definite integral would insist on evaluating it, either as a multiple of $\pi$ or as a decimal.

The author has for many years been an examiner for Part 1 Mathematics examinations of the Engineering Council (formerly the Council of Engineering Institutions). A few years ago the Chief Examinership changed hands and the new incumbent decided to test the candidates' knowledge more thoroughly at more fundamental levels. It soon became evident that the main reason why candidates had done badly in previous years on more searching questions was their lack of ability or knowledge at these more fundamental levels. Certainly, the author's findings indicate that much of the knowledge one might be tempted to assume on entry is not there or is only partly there. This is obviously an important consideration in planning a lecture course to first year engineers. If anything, the trend is towards a slight worsening of the situation, although nothing significant has been detected. What can be said with some certainty is that the situation is not improving.

Generally, the students' perception of their understanding as provided by their questionnaire responses was borne out by their performances in the test.

### 3.7 Does the Integrated Approach Work?

As has been mentioned earlier, it is all too easy to forget how radical the proposals for the integrated approach were in the days when they were first propounded. Many mathematics lecturers had a fear of computers or even a revulsion for them; in many institutions a separate department was responsible for teaching numerical analysis. It was recognised therefore that it would be an uphill struggle to convert colleagues to the philosophy of the integrated approach. The author has always believed, however, that engineering students use mathematics to help them solve their engineering problems and that is the only reason which most of them can see for studying it. That student attitude has not changed over twenty years and it might as well be accepted. Consequently, as with a car mechanic attempting to effect a repair, the engineer will search his tool box for the appropriate tools for solution be they analytical, numerical, statistical or a combination. He should therefore be taught by an approach which parallels such a problem-solving philosophy and that is the approach that the author believes he has followed and has been beneficial to his students.

As one means of testing the belief that students benefit from the integrated approach the author recently conducted two experiments. In the first he taught a group of students on the Education and Mathematics degree course who had previously received separate modules on analysis and on numerical methods. He taught them eigenvalues via the integrated approach as outlined earlier. Although these students had been introduced to eigenvalues in their first year the overwhelming majority admitted to understanding them properly for the first time and agreed that the juxtaposition of analytical and numerical methods enhanced their understanding of both. They wanted to know why they had been taught numerical methods for solving ordinary differential equations in a separate course from the analytical solution techniques. One student said that the integrated approach seemed "more natural" and this was agreed by her fellows.

The second experiment was, in a sense the converse. The bulk of the course
given to the first year Civil Engineers was integrated, but the numerical methods for tackling definite integrals were taught a term after the analytical methods. Several students asked the author why they were not taught in the same package of lectures; one even remarked that he thought that it was "silly" to separate them. When put to the class, this view was generally supported.

Tutorial help at Loughborough is often provided by post-graduate students; those whose first degree was from a university which did not practise our 'integrated' approach found the approach somewhat strange at first, especially if they had to learn numerical methods themselves. Almost without exception they expressed the opinion that the integrated approach was a more enlightening strategy of teaching. This is a state of affairs in which the author can find some satisfaction.

Although it may be agreed that there is little quantitative appraisal of the integrated approach, the collective experience of teaching it over 18 years both at Loughborough and at other institutions has led to the conclusion that the students are better motivated, are more aware mathematically and are generally better practitioners of the art of applying mathematics to engineering than those who followed the previously accepted approach.

## Chapter 4

From Slide Rule to Micro

### 4.1 The Arrival of the Pocket Calculator

Unquestionably the greatest single technological influence on the teaching of mathematics over the last twenty years has been the advent and development of the pocket electronic calculator. Those calculations which took several minutes of slide rule time to produce an approximately correct answer can now be performed in seconds to obtain a precise and wrong answer. It may be regarded as cynical to say that the impact of the calculator has been more detrimental than beneficial (especially since the author was persuaded by his engineering students to forsake the four-figure tables with which he grew up for a slide rule at precisely the time that cheap pocket calculators were launched onto the market). However, what was envisaged as being a liberating agent, freeing the student from hours of monotonous drudgery has had the effect of robbing many owners of the ability and the will to think carefully about the calculations which they are carrying out. At least the student using a slide rule had to know where to put the decimal point; at least he had a feel for the approximate value which would result from his calculations; at least he could multiply by 3 in his head. Lest it be thought that this is a die-hard view, it has been the author's experience that the student of today has lost an instinctive 'feel' for the calculations that he performs.

Of course, most of the calculations that the engineering undergraduate is required to undertake are in subjects other than mathematics. At first, the tutorial sheets did not change to take account of the new calculating power at the students' disposal. The latter would often complain to their lecturers when their answers did not agree with those given at the bottom of the tutorial sheets. After a few years explaining to the students that the answers provided had been obtained using a slide rule, the engineering staff finally amended the numerical values
concerned. However, given that there was no longer a need to concoct input data so that relatively straightforward calculations could be performed, there was, in general, a slow response to the demise of the slide rule.

The author was concerned from the outset that this new aid to calculation, in untutored hands, would be used unwisely. Accordingly, the second lecture in his course to his freshman engineers is devoted to warning them of the dangers of working with inexact data. Entitled " $2+2=3.99$ ", it has also been given to many groups of sixth formers (160). The first manifestation of trouble is that of "digit diarrhoea".

No matter how imprecise the numbers on which he operates, the student displays his answers to 11 figures, since that is what his calculator output provides. The foolishness of this approach is highlighted via the example of calculating the third side of a triangle given two sides and the included angle, which have been obtained by measurement.

The lecture continues with some examples of ill-conditioning and examines the spread of errors by arithmetic processes. The exercises accompanying this lecture are designed to encourage the students in the sensible use of calculators by providing suitable examples which require an a priori estimate of the answer and a statement of both the accuracy expected and the justification for the precision quoted.

It is sad to reflect that after fifteen years this lecture is still as necessary today as it was when it was first given. Despite using a pocket calculator for several years at school far too many students arrive at the tertiary stage without, apparently, an awareness of the need to produce only as many significant figures as are merited by the information provided in each problem.

A thorn in the flesh of the examiner has been the proliferation of different makes of pocket calculator: some have a long-term memory, some are
programmable, some have hyperbolic functions and some have statistical features, to quote a few examples. Legislation as to which calculators can be permitted in examinations has proved to be a nightmare and, if Loughborough is typical, the question has still not been resolved satisfactorily. "Can't you set an examination which does not involve arithmetic calculation?" was the suggestion of one harassed Examinations Officer. The author's response has been to ensure that the necessary calculations can be carried out on the cheaper calculators just as easily as on their more expensive counterparts. He feels that it would be quite unrealistic to set an examination paper which did not involve any calculation element. After all, mathematics, and especially mathematics for engineers, is not a spectator sport; it is a practical subject and the student needs to "get his hands dirty". The written examination should reflect that practical approach. Naturally, coursework can be set to test the students' ability on more complex problems.

In consultation with his engineering colleagues the author selected a simple calculator - the Casio fx 100 - which they believed was sufficient for virtually all the undergraduates' needs. In the information sent out to new students before they arrive the recommendation of this calculator is included. Of course, there is no compulsion on the freshmen to purchase the one recommended but it is a useful guideline for them.

### 4.2 Going On-line at Loughborough

In the late summer of 1971 the author was informed that the University's IBM 1620 was to be taken out of commission; undergraduates would in future have to use the ICL 1904A. This proposed change filled the author with misgivings because he believed that the hands-on facility offered by the older machine was a key factor in maintaining student enthusiasm. For many years, sixth form pupils from a wide catchment area had spent three days at the University on a short course in Fortran programming and the enthusiasm shown by these school-children was necessary to the success of the course.

Although the author had no wish to be a computer King Canute, he made strenuous representations to keep the older machine for a while and managed to delay its demise for a few months; the time thus gained was used to obtain the best deal possible for the undergraduates. It was agreed that the Data Preparation Section of the University Computer Centre would prepare a deck of punched cards from the coding forms submitted by the students. Any subsequent corrections would normally be punched by the students themselves. If required, the output from the computer run could be collected from the Computer Centre, but the norm was to make use of the four-times-a-day delivery service provided to several points on the campus. The intention was to cut down the time that a student would have to spend in getting a program to run successfully.

Despite all these efforts, the lack of direct involvement with the computer led to a decline in enthusiasm for computer programming, as had been expected. Having written a program and submitted the coding form there might easily be a day's wait before the cards were produced. There was encouragement from the Computer Centre to have the program compiled but not run on a first pass through the system: understandably, they did not want a program which might require graphics and other facilities to fail on compilation and abort the run. However, this had an obvious drawback in that it provided a further delay to a program which compiled successfully first time and that delay could be more than half a day if the student's timetable did not permit him to check the next delivery.

If the compilation was unsuccessful then the student could punch the required corrections on to new cards, re-assemble the deck and re-submit the job. This process would continue until compilation was successful and, hopefully, the run was also successful. Some students preferred to use the 'cafeteria service' available at the Computer Centre so that they could achieve several passes through the system at one visit. There was a Program Advisory Service operating there at certain times of the day but the queues were often long.

The author agreed to check the coding forms prior to submission to try to
eradicate "obvious" errors and to help seek out the sources of compilation errors. Looking back, however, it does seem that our expectations of the system exceeded its ability to "deliver the goods". The batch processing era was a dark age in the computing facility available to undergraduates.

In 1974 the University installed a Modular 1 computer which had a limited number of teletypes on-line. In 1978 two Prime Computers were installed with a larger number of terminals available across the campus. However, batch processing on the mainframe was still the order of the day for most students, even though several departments had purchased their own mini-computers, and it was not until the early 1980's that there was a large terminal facility for undergraduates organised under the auspices of the Computer Centre.

A difficulty arose in some engineering departments in that the in-house computing facilities which they offered were often varied and not necessarily compatible with those provided by the Computer Centre. This caused frustration for students in that they had yet another variable with which to cope as they started to learn programming. It is important to realise the obstacles placed in the way of a beginner in his endeavours to run successfully his first program. The effort is not commensurate with the outcome and he needs considerable encouragement to maintain interest and enthusiasm until he becomes more proficient.

The question of which programming language should be taught to the engineering undergraduates is a long-running one. The wishes of the serviced department have to be borne in mind; since it would be unfair to ask the students to cope with a second language just as the first one was being mastered, it is likely that the engineering staff's preference would be taken up.

In 1969 at Loughborough there was no choice: Fortran II was the order of the day. Similarly, when the ICL 1904A was the undergraduates' computer there was one language that was to be taught and that was Fortran IV. Not only was it
the most widely-used scientific language but there was a great deal of supporting software, including the Numerical Algorithms Group (NAG) library.

In general, most students did not find the language too difficult to master, with one exception. Both for the undergraduate engineers and for the sixth-formers who attended the three-day programming course it was the input/output statements which caused the chief problem, especially the correct use of FORMAT statements. The majority of compiling errors lay in that category and an attempt to redress the balance was made by a colleague who produced a simplified input/output routine; the attempt was only partially successful and many students found the subsequent transition to the more usual format requirements equally as difficult as their predecessors had done.

A major stumbling-block for the novice programmer is that unless the program is grammatically correct it will not compile successfully: probably the only aspect of the undergraduate course where correctness is essential for further progress. This is very discouraging for the learner: it is difficult enough to write programs in the early stages without the constant irritation provided by all the niggling and trivial errors that seem to occur.

Undoubtedly, the highest incidence of errors at compilation occurs in the making of elementary mistakes - mis-spellings, omitted commas, unmatched parentheses, and so on. This is true of many mistakes in mathematics generally: where the student is having to concentrate, errors are less likely, but where he is carrying out a routine, straightforward task then he is at his most vulnerable. How often has an attempt at an examination question come to grief because a minus sign has been lost or a line of working has been copied wrongly from one page to the next? Of course, with a respectable editor and when typing in a program at a terminal on a line-by-line basis such mistakes are more easily detected and corrected. However, there is a tendency amongst some students to let the system do the error-checking and for them not to think too carefully before they type in each line. In addition, there is the time-wasting approach of 'thinking
at the terminal' which Mustoe (104) had warned about.

There is little doubt that when students were taught Basic as a first language, these mistakes diminished, perhaps because they had less to remember in the early stages of the learning process. Basic was a more natural language to learn, being more closely allied to written English and Mathematics.

Under threat from Basic, the Fortran advocates responded by developing Fortran 77; among the new features was a simple input/output option as an alternative to the standard FORMAT construction. It was now possible to input via READ *, list in a fashion similar to the Basic INPUT list.

Output, also, could be achieved via PRINT*, list which mirrored PRINT list in Basic. Another beneficial modification was the introduction of the "block IF" construction which allowed a greater flexibility when dealing with branching in a program.

In the late 1970's a number of engineering departments at Loughborough began to feel that Fortran was too difficult as a first language for their students and, when terminals became more common across the campus, asked that Basic should be taught in the first year instead. Most still wanted Fortran to be taught in the second year, however. Accordingly, the author acceded to their wishes despite a personal belief that those students who might have found exposure to Fortran as their first language to be a painful process would find difficulty in the newer approach. He remembered only too well the lessons of the experiment with the Initial Teaching Alphabet. Personal experience indicates that roughly the same proportion of students find programming hard going now as was the case under the former approach.

There is a school of thought which argues strongly that Fortran is moribund and Basic is out-dated; indeed, Pascal is the language to teach. That may well be true in the future, but it is not a proven case for engineering undergraduates at the
moment (161). If it was really Mrs. Thatcher's hope that her policy of a micro in every school would lead to an upsurge in computer literacy it does not seem to have been realised in many cases - at least with regard to experience of programming prior to entry to tertiary education. There may well be a few micros in every school, but very few freshmen that are taught by the author seem to have had much practical experience of using them. Many of those who have such experience own a micro themselves and have pursued their interest out of school. A colleague who tried teaching his students Pascal as a first language found the experiment unsuccessful, mainly because of their inexperience. In any event, there is a dearth of texts on Pascal currently available which are suitable for a novice programmer and at Loughborough the support offered by the Computer Centre at the moment is not satisfactory.

On another front, there are those engineering lecturers who believe that it is unnecessary for their students to learn any programming language. After all, they will probably use only packages during their working lives and it will suffice for them to know how to operate them successfully. Would such lecturers subscribe to the view that a Civil Engineering undergraduate need never construct a model structure and test its load-bearing capacity or that an Aeronautical Engineering undergraduate need never carry out experiments in a wind tunnel?

The question of who should teach programming is another item for debate. There are those who would argue that the teaching of computer programming should be carried out by the Computer Centre or by a Computing Science Department, or even by the engineering departments themselves. The author strongly disagrees with these views. The objections to the first two categories are based on his experience that the more involved in computers and computing a person is, the less able he is to appreciate the difficulties of the novice programmer. Courses given by these experts have assumed far too much familiarity with the process of programming and have gone far too quickly over the fundamentals, (which, admittedly, can be boring to teach). The type of person best qualified to teach the elements of computer programming in, say, Basic or

Fortran is a user of the computer service provided. He will know the system from the customers' point of view and can cushion the student through the frustrations and delays which cloud the problems of learning the language itself. Why not let the engineering staff teach the programming, then? The author believes that, since the integrated approach incorporates computer-based methods of solving problems, it is natural that the mathematics lecturer should be able to introduce programming as one of the items in the engineer's mathematical tool-kit. The optimum involvement of the engineering staff is in the setting and marking of suitable exercises for coursework assessment, as explained in Section 3.5. These members of staff can set programming exercises as part of their own lecture courses if they so wish, of course. However, a necessary safeguard is to check that there is no duplication of work and at Loughborough in the Civil Engineering Department this rôle is assumed by the appropriate year tutor who also ensures a reasonably uniform loading over the session.

### 4.3 The Computer Terminal Laboratory

When terminals were installed across the Loughborough campus, most were sited within departments in twos and threes. Two laboratories were set up, however, one in the Computer Centre and one on the other side of the campus in the area where most of the engineering departments are located. There were, in the main, two kinds of terminal: Trend teletypes and Newbury VDU's. It was decided to stock the "engineering" terminal laboratory with the former kind; this allowed hard-copy to be obtained on the paper roll output, but it did not encourage the graphical display of results and this was a concern.

Schey et al (102) had described how they had designed and tested a laboratory, computer and calculus-based course in mathematics where the rôle of computer programs was to test mathematical models against experimentally-obtained results. However, experimental work tends to take a disproportionate amount of time to organise and carry out and the present author was envisaging a more modest approach in Loughborough - the use of a computer
terminal laboratory to assist in the teaching of numerical methods; reference has already been made in Chapter 2 to his ideas (104).

Harding (162) described his Computer-Aided Teaching of Applied Mathematics (CATAM) project six years after its inception in 1968. His system comprised a Modular One Computer and a DEC TSS/8 System linked by specially designed hardware. There were 15 teletypes, a hard copy unit and two television monitors. The aim of the project was to supplement the teaching of analytical and numerical techniques both in the lecture room via class demonstrations and in the laboratory via individual usage. To allow the rapid assimilation of results, graphical output was emphasised. In the 8 -week course given to second year students the 16 one-hour lectures were supplemented by 8 two-hour optional practical class sessions.

The first two practical sessions were devoted to learning how to use the system; these were followed by five exercise sessions, the last week being set aside for catching up. Each exercise was assessed by a written report which included copies of programs and the results obtained. The five exercises were:
solution of equations,
quadrature,
ordinary differential equations I and II,
Laplace's equation.
In addition to the items listed earlier, each report had to contain answers to specific questions asked in the hand-out accompanying each of the exercises.

A follow-up course was available to final year students: this also comprised an optional laboratory component. In his paper (162), Harding gave an indication that these optional sessions were enjoyed by his students and were a valuable addition to the teaching.

Reference was made in Section 2.7.2 to the author's hopes for a terminal laboratory at Loughborough. The reality was much more prosaic, however. The
room provided was small and narrow with no real facility to use it as a teaching area. Sometimes the mainframe computer was down, often not all of the ten terminals would be working, more than occasionally there was no reserve stock of paper rolls. These may seem to be trivial points, but anyone who has experienced such annoying set-backs will know the harmful effects that they have on the morale and the enthusiasm of the students (and the lecturer). It is embarrassing when the students complain that they had better facilities at school, as was the case on three occasions.

The author envisaged two modes of use for the laboratory: as a room where the students would attempt to run the programs that they had written and could correct them under supervision and as a place where investigations could be carried out using pre-written programs which the students could call up. In this latter mode five assignments were to be tackled; the topics chosen were
numerical integration
solution of simultaneous linear equations
solution of non-linear equations
approximation of functions and data
ordinary differential equations.
Due to the group size and the number of terminals available, each student attended the laboratory for one afternoon on a fortnightly basis, although he could use the laboratory at the Computer Centre outside timetabled hours if he required extra time.

For each of the five assignments, the following format was adopted. Before each session the students would be given a sheet comprising about five problems on a particular topic. If any preliminary analytical work was required then the students were expected to carry it out before the practical session. The problems themselves were designed to go beyond the simple use of a black box. Having used the prepared programs to obtain results either in numerical form or as coarse graphs, a short report was to be written on the relative merits of each method used and on any difficulties encountered with each problem. The assignment sheet on
numerical integration is shown on page 95. (Programs were available for carrying out Simpson, Trapezoidal and Gauss-Legendre integration).

## Question 1

Here the student is expected to see that for the same computational effort Simpson's rule gives a more accurate result than the trapezoidal rule. In each case, the accuracy improves as the number of strips increases. By using the formula for the maximum predicted error the student can obtain the number of strips required to ensure that the quoted accuracy is achieved. It is to be hoped that he will remember that the Simpson rule demands an even number of strips. The number of points chosen for the Gauss-Legendre method is a matter for him to decide, and justify.

## Question 2

It was not anticipated originally that some students would transform the integral analytically so that it was rendered finite. The problem here is to decide how to break down the infinite interval into a succession of sub-intervals. How many? How wide? How accurately should the value of each of these sub-integrals be obtained? Can any of the later intervals be ignored?

## Question 3

The difficulty here is that an analytical solution is not available. The hope is that the collection of estimates obtained using the step sizes suggested will allow the student to decide how accurately he can quote his answer. It should also suggest the dangers inherent in using just one step size when evaluating an integral numerically; in any event, what should that single step size be?

Question 4
At first sight this question may seem futile. However, an inquisitive student might be expected to investigate the problem to see how many strips

## ASSIGNMENT TWO

1 Using (a) the Trapezoidal rule (b) Simpson's rule estimate the value of

$$
I=\int_{0}^{1} \frac{2}{1+x} d x
$$

With each method try the following values of $h$ : $1,0.5,0.25$.

Try to get answers correct to 5 dp .
Finally, make a further estimate using Gauss' method.
2 Estimate via Simpson's rule, then Trapezoidal rule the value of

$$
I=\int_{1}^{\infty} \frac{1}{x^{2}} d x .
$$

3 Using Simpson's rule with $h=0.25,0.1,0.05,0.02,0.01$ evaluate

$$
I=\int_{-1}^{1} \frac{x^{7} \sqrt{1-x^{2}}}{(2-x)^{6 \cdot 5}} d x
$$

4 Estimate $\mathrm{J}=\int_{0}^{10} \mathrm{e}^{-\mathrm{x}} \mathrm{dx}$ by Trapezoidal and Simpson's rules; use $\mathrm{h}=1$.
5 The Debye function, encountered in thermodynamics, is

$$
D(x)=3 x^{-3} \int_{0}^{x} \frac{y^{3}}{e^{y}-1} d y
$$

Evaluate $\mathrm{D}(\mathrm{x})$ using (a) Simpson's rule and (b) Gauss' method for $\mathrm{x}=0.5,10,50,100$.
would be necessary to achieve a specified accuracy. A comment about the difficulty of representing reasonably accurately an exponentially decaying function by straight line or parabolic segments would be an obvious conclusion.

## Question 5

The first hurdle to overcome is to cope with the non-standard notation; it is unusual for an integral to have a variable upper limit and yet to be evaluated by a numerical technique. The second hurdle is that $\mathrm{e}^{y}$ becomes too large for the computer to handle when $y=100$ and this will occur with the Simpson rule which uses the upper limit of the integral as one of its ordinates; further, if too high an order of Gauss-Legendre formula is used, the same problem will be encountered.

Generally, the students performed better on this form of investigation and report-writing assignment than might have been expected from their overall level of attainment. There was, however, a very wide range of quality of responses and this made marking difficult. The course of which these assignments formed a part has shifted emphasis in assessment from $50 \%$ examination, $50 \%$ coursework to $100 \%$ coursework. The danger with coursework assessed in pieces is the the marker can lean towards the lenient and a good student can obtain a very high mark or the marker can fight shy of this approach and give students an unwarranted low mark. At the other end of the spectrum it is a worry how low to set one's base: how bad is a poor submission? The author grades each item of coursework as it is submitted and then collects all the coursework back and assesses each student on his complete submission.

The five basic items of coursework were augmented by one on a set of Basic programs, one on a set of Fortran programs, one on using the NAG library and one on using the graphics library of GINO and GINOSURF.

### 4.4 Enter the Micro

In the early 1980's the micro began to make its presence felt in educational circles. The Government's "pound for pound" policy to aid schools in the purchase of the BBC micro was backed by the Microelectronics Education Programme, by articles in journals encouraging teachers to get involved and by a series of programmes on television. Several engineering departments at Loughborough decided to purchase micros for their own students to use, but there was not always a clear policy within a department, let alone between departments.

The Civil Engineering department, for example, bought a mixture of PET and Sharp computers and this meant that the early tutorial sessions to back up the lectures in Basic had to be arranged so that a student was able to use the same machine each time. As the decade advanced some students arrived with their own micros, yet many had never used a computer in their lives and this meant that the range in computational experience was far greater than some twenty years earlier.

However, the central facilities were limited and the computing assignments continued to be centred on the terminal laboratory. Not until the summer vacation of 1986 was the enlightened step taken to make available a microcomputer room on the "engineering" side of campus. This room was spacious, light and well laid out in complete contrast to the terminal laboratory. It contained thirty BBC B micros linked via an Econet level III facility.

The first task required was to re-write the prepared programs in BBC Basic and to take the opportunity of utilising the graphics facility of the micro.

The contrast between the last two sessions and the ones which preceded them has been marked. The students have responded to the more favourable surroundings by displaying an enthusiasm much greater than their counterparts. Although the room is booked for three consecutive hours in the afternoon, the
first two only are timetabled; however, the majority of students stay for the third hour. After the first three weeks, attendance is programmed on a fortnightly basis, the group being divided into two approximately equal subsets. Many students attend on their "unscheduled" weeks in order to make use of the facility.

From the author's point of view there was no need to worry about the mainframe being "down" and there were no paper rolls to worry about. There were no frustrations due to the system having several competing users. It was possible to concentrate on the purpose of the laboratory session.

The opportunity offered by the micro was taken up in a different way. Since 1983 the author had been a member of the MIME team at Loughborough which was concerned with Computer Enhanced Learning of Mathematics. In addition to the work described earlier, software was used in lectures and in tutorial sessions. The details of the MIME project appear in the next chapter.

Several authors have described the work that they have carried out in an endeavour to involve the computer in their teaching programmes. Here we review a personal selection of their views which gives an awareness of the wide range of different approaches.

Haggett and Le Masurier (163) gave details of the course in computer appreciation given to first year Mechanical Engineering students at Brighton Polytechnic. They took advantage of their Induction Week to introduce the students to the VAX system and to the elements of Basic programming. Each student receives four lectures and has four hours of terminal time allocated. During the year he is given exercises of increasing difficulty each of which requires the writing and running of a computer program for its solution. Following the summer examinations a period is spent on Engineering Applications EA1 as required by the Engineering Council. During the first two weeks of this period students are given a number of small group projects of an open-ended nature.

The authors have three stated aims: to improve their students' computing skills; to help them appreciate the use of the mathematics they have learned; to make them appreciate the relevance of computing in an engineering context. They prefer to use the VAX system to a micro computer environment, citing as one advantage of the former the simplicity with which data or programs can be transferred from one user to another.

Jacques and Judd (115) have developed micro-based software to support courses in numerical mathematics given to science and engineering undergraduates at Coventry (Lanchester) Polytechnic. The topics covered included the solution of simultaneous linear equations, numerical integration, curve fitting, eigenvalues and eigenvectors, and the solution of ordinary and partial differential equations. The programs were designed to allow the students to investigate in an experimental mode the properties of the various numerical methods covered in the lectures.

In a typical tutorial session the student will run a stored program to solve a simple example which can also be solved using a pocket calculator. Then the student progresses to more difficult examples designed to test the robustness and behaviour of each method. Some examples are chosen deliberately in order to see the method breaking down. The student is required to write a report on his investigations. Jacques and Judd prefer a micro for their tutorial sessions, not least because of its more friendly interface with the user.

Beilby (164), faced with a reduction in lecturing time allowed by engineering departments supplements his formal lectures with a series of laboratory sessions held in a microcomputer room. The BBC machines are linked by an Econet facility. Each week the students are given an examples sheet comprising a set of problems on a common theme. Stored programs are available as indicated on the sheet. Sometimes the use of these programs is as computational aid - for example a Newton-Raphson search program may be used to find the roots of the auxiliary equation of a third-order linear ordinary differential equation. On other examples
the problem is heavily couched in engineering terms and the stored program is to be used in an investigative manner to study the behaviour of the underlying engineering system.

The student responses to the exercises are collected in on given days, marked with comments, and returned. The overall performance on these exercises forms part of the mathematics assessment.

Mackie (165) had followed the advice given by Mustoe and his colleagues (118) when designing a computer-based package, NODES, to solve ordinary differential equations. The package was used by both first and second year students to carry out investigative assignments in a manner similar to that of Beilby.

Hundhausen (107) describes how she used the computer to enhance the teaching of ordinary differential equations as part of an analysis course. A program which can be run on a DEC-10 computer can "solve" a system of up to 18 simultaneous first-order ordinary differential equations. The student user supplies the system of equations programmed in Fortran and the input parameters are step-size, maximum number of iterations and initial values of dependent and independent variables. The output of the program is in graphical form; for each dependent variable the graph of the variable itself is produced alongside the graph of its derivative. In addition, a phase plane plot is available. The students taught by this approach showed better comprehension of the behaviour of the systems studied than their counterparts taught by a traditional approach.

Tall (166) discussed the relative merits of two ways of using the computer to enhance the learning of mathematics:
(i) using prepared software which was error-protected and designed specially for demonstration and investigation
(ii) the students modifying short programs and writing their own programs. He believed that a sensible mix of both approaches was the ideal. Programming demanded a substantial overhead in time and effort, but it did give initiative to
the student. Good prepared software helped the student gain greater insight into mathematical processes and concepts.

Jaen Gallego et al (167) describe a computer-assisted system of teaching linear programming. They show in detail how the teaching of the simplex method can be enhanced via a micro-based program. The successive simplex tableaux are displayed together with suitable messages on screen. The program also offers the student the opportunity to carry out a full post-optimality analysis.

### 4.5 The Impact of the Computer

Several authors have tried to assess the impact that the computer has had, or might have on the teaching of mathematics. Again, a personal selection of papers is presented to give a flavour of what is currently being debated.

Morris (168) believed that it was of crucial importance to emphasise the algorithmic approach to the solution of problems. He listed six steps to be followed in such an approach:
(i) the problem must be specified
(ii) the algorithm must be designed
(iii) there must be a verification that the algorithm does indeed meet the specification
(iv) the demands in time and storage space to carry out the computation must be acceptable
(v) the algorithm must be implemented and tested
(vi) the algorithm should be documented.

Whilst the mathematics needed by a civil engineer would differ from that of a chip designer, the requirement for both to have a foundation of algorithmic training was increasing.

Winkelmann (109) was concerned with the impact of the computer on the teaching of analysis. The dominant position of analysis in the curriculum was
threatened since calculations such as the location of extreme values could easily be programmed and since many engineering applications of mathematics use discrete methods. He believed that the teaching of analysis should be modified to incorporate numerical methods and the process of modelling. Analysis should guide and direct the use of appropriate numerical methods rather than be contorted into producing artificial numerical results. There was scope for including symbolic differentiation by program in an analysis course.

Eriksson (108) felt that the computer was not making as beneficial an impact as it should. The main contributory factor was the lack of good software. He feared that the alternative to using other people's software which did not quite fit the bill was to write one's own and thereby commit the bulk of one's working hours to that activity. Because attention had been diverted towards computers, mathematics was suffering and the impact of the computer had if anything been harmful. He expected that it would be some considerable time before the computer found its rightful place as an indispensable tool of an applied mathematician.

Robson (138), (169) argued the case for computer simulation models. He expected that the development of 'expert' mathematical systems would lead to a thorough reappraisal of mathematics curricula for engineers and, as a consequence, computer simulation would assume a more dominant rôle.

Rowe (110) stated that all real design problems would, in the future, be handled numerically; consequently, he believed that the need for teaching advanced mathematical techniques would diminish. He was strongly opposed to the idea that all design teaching should be computer-orientated, since this would lead to a black-box approach whence the basic principles would be lost. Sufficient mathematics, especially calculus and trigonometry, should be taught to allow the analysis of those basic principles; he quoted examples from the area of mechanics of solids to reinforce his arguments. An inter-departmental approach to the teaching was a key feature of his scheme.

The concern that the author had felt about the lack of suitable computing facilities for teaching was shared by many other academic staff. In June 1982 the Computer Board for Universities and Research Councils responded to this concern by setting up a Working Party to assess the type and level of facilities that should be provided. In December 1983, the Working Party, which was chaired by Dorothy Nelson, produced its report (170) which was intended as a discussion document.

The Report came out strongly in favour of a massive upgrading of facilities; it was unequivocal in stating that "existing computer facilities are inadequate, both in quality and quantity, for the genuine needs of students on undergraduate and postgraduate taught courses". Computer Centres would have to be strengthened in order to provide the support services necessary to cope with the expected increase in demand. There was concern that the universities compared unfavourably with the polytechnics with regard to spending on computer facilities for teaching. The Computer Board would need to increase its spending from less than $£ 1 \mathrm{M}$ per annum to over $£ 5 \mathrm{M}$ per annum.

The Report commented that there was currently an average of 5 micros per secondary school in the UK. The general increase in computing awareness had led students to expect a range of good computer facilities at university level; the danger was that this expectation could often not be met. As an example of the shortcomings they estimated that the average allocation of central file store was 0.25 M byte per student, whereas they envisaged a future requirement of at least four times that amount.

The Report declared that a good university provision by 1992 would include a workstation in every study-bedroom and on every library desk. Fast high-quality printers would be located in most university buildings. New tutorial software would be produced by small teams of lecturers, programmers and educational technologists. All students would regularly send and receive electronic mail for a variety of uses including writing essays and answering test question.

Did the Working Party have a collective tongue-in-cheek attitude when they wrote their recommendations? In 1988, more than half-way to their target date of 1992, there is little indication that many universities are on schedule to realising the status of "good"!

However, one outcome of the Report was a joint initiative by the Computer Board and the University Grants Committee to provide support to universities for the use of computing facilities for teaching. The aim was to stimulate new teaching methods in all subject disciplines by "pump priming" a number of projects. Each project had to develop the use of computer facilities in teaching in a specific subject area, to assess the hardware requirements, to decide how the educational potential of the new technology could best be achieved and to produce the appropriate software tools and enabling software.

In all, 129 projects from individual universities and 10 joint projects were supported by the initiative. Two were in the area of engineering mathematics: at Heriot-Watt University and Loughborough University. The latter project, proposed by Professor Bajpai and the author, will be described in the next chapter.

The CALM project at Heriot-Watt University was established to help combat the problems of large tutorial classes: Beevers et al (171). A networked laboratory of 32 Research Machines Nimbus microcomputers was established and software units were written in the area of first year calculus. The language chosen was Pascal and each unit was designed with three major components: a theory section, a worked examples section and a test section. In all, 25 units were prepared, one for each week of the course. In an endeavour to evaluate the effectiveness of each unit, seven students, chosen at random, met with a project member each week to give their opinions from a user's point of view. Early results indicated that some of the students with good entry grades had benefitted from the computer-assisted approach.

At Canterbury in 1986, a conference was held on the rôle of computers in
the teaching of mathematical sciences in higher education. One feature of this conference was the inclusion of a demonstration of the computer algebra system REDUCE. A year later at Edinburgh, Hodgkinson (172) demonstrated the merits of REDUCE, MACSYMA, MAPLE, muMATH and SMP. He had used algebraic computing to reinforce and consolidate material taught in traditional style in lectures. Fugard (173) described his experiences in using MACSYMA as part of a solution procedure. He was of the firm opinion that the computer algebra package had a useful rôle to play in releasing the student from time-consuming manipulation to allow more thought on the nature of a problem and the concepts involved in its solution. It was also important that a student be able to write simple programs which incorporated the facilities of the computer algebra package. The author of this thesis has a somewhat less than enthusiastic view of the rôle of such packages in engineering mathematics education at the present time. Considerable effort has to be expended to learn how to use the packages sensibly and until they are widely available in industry it does not seem a worthwhile investment.

Over the last twenty years the author has witnessed enormous changes in the computing power available to his students; from slide rule and four-figure tables to the pocket electronic calculator and from the IBM 1620 (with the need to input punched cards by hand) to terminals and microcomputers. What have been the effects on the teaching of mathematics to engineers as a result of these changes and similar changes in other institutions?

The author fears that the overall response to such changes has not been as positive as it might have been. The increase in computing power has not been matched by a comparable rise in the relative importance of numerical and computer-based methods. In some cases the only evidence of a move towards recognition of the computer age has been the appearance of a token question on numerical methods in the examination paper. Part of the problem is that in several institutions numerical methods and computer programming are taught by different departments or are taught in separate courses given by the same
department. Furthermore, there has been a reluctance to make extensive use of computer packages.

There was a considerable time-lag before the importance to engineers of graphical output was recognised by those in computing circles and suitable hardware and software was provided. The sheer scale of the task of interpreting extensive amounts of tabulated data was daunting and the unsatisfactory nature of graph plots emanating from the line printer detracted from the use of the computer as a natural aid to the processing of experimental results; rather, the activity was seen at best as a necessary evil.

The microcomputer has been around for many years now but it is under-used to an extent which gives cause for concern. Where is the software to allow this new teaching aid to be exploited fully? Unfortunately there is not sufficient quality software available to permit the involvement of the micro in the teaching process to a level anywhere near that which might have been expected a few years ago.

The clear view coming from all the workers in the area of computer-assisted learning is that a laboratory-based approach which uses good software with well-prepared assignments is of definite benefit to most students. The stumbling-block of the lack of good-quality software is a situation which is deteriorating as advances in hardware make it more difficult for the software authors to keep pace. The story is taken up in the next chapter.

## Chapter 5

## Computer Enhanced Learning of Mathematics

### 5.1 The MIME Project

At the start of the 1980's the microcomputer was being seen as an exciting new tool for teaching. Software was being written, but mainly for the primary school and the early years of the secondary stage. Relatively little seemed to be taking place at the level of the school/university interface and above and there was a feeling abroad that this situation needed rectifying. It was suggested that Loughborough, as a leading innovator in mathematical education, notably in the mathematical education of engineers, should 'blaze the trail' in this area of activity. Whereas individuals were writing software which they would use in their own teaching, the time scale was long and the finished product had somewhat naive screen displays. It was clear that if real progress was to be made then several individuals with a common interest and purpose would have to work together, with suitable back-up from a number of programmers.

Early in 1983 the author and Dr D. Walker were invited by Professor A. C. Bajpai to discuss with him the possibility of forming a project team to explore the use of the microcomputer in mathematics teaching. These discussions centred on the kind of material to be produced, which syllabus to cover and the form in which the material would appear.

The early examples of microcomputer software for mathematics education seen by the author were, on the whole, a disappointment. In some cases the sole concession to user-friendliness was an initial request to the user to type in his name; thereafter, at suitable places in the text appearing on the screen the appropriate name would be reproduced. This approach smacked of the advertising campaigns of firms who send out regular mail-shots. In other cases the screen
displays appeared to constitute a programmed text which had been transferred onto the micro. In yet other cases the screen displays were somewhat simplistic and paled into insignificance when compared with screen displays of the profusion of software games then available.

The author had in mind the comments of school teacher friends who had tried out software in their classes only to be confronted with a "crashed" program which left them with a blank screen and no idea of how to return to the point immediately prior to the disaster. Others had expressed the view that some of the commercially available software was too rigid in its construction, leaving the teacher in a passive role. The comment was also made that some of the screen displays looked amateurish.

It was decided early on in the tripartite discussions that the software to be produced would constitute a computer enhanced learning package, that is to say the software would be written in as flexible a form as possible, placing the minimum of restriction on the teacher in terms of his style of teaching, his order of dealing with the relevant topics, and the subject development. A menu-driven suite of programs was an obvious essential. The software was to be supplementary to the teacher, used either by him in the classroom or by individual students in a self-teaching mode. Our underlying philosophy was emphatically that the teacher was to be the key figure in the learning process, involving the computer as and when he required. Our software units would differ in style from conventional Computer-Aided Learning and Computer-Aided Instruction packages.

It was felt that the area of mechanics at the 'A' Level/first year university interface was appropriate as a first area to tackle for the following reasons.
(i) This area of study appeared not to have attracted much attention from the authors of educational software.
(ii) There was experience amongst the author and his colleagues in teaching the material at the appropriate level.
(iii) The dynamics part of the syllabus offered a natural medium for graphical displays.
(iv) Many engineering students arrived at tertiary level institutions with a poor, or sometimes non-existent, knowledge of mechanics.
(v) There was a wider potential market if sixth form students could be included in the target user population.

At this stage, three further colleagues were recruited to the team: one was currently teaching mechanics on our undergraduate Education and Mathematics course and was a co-author of several programmed texts; one had experience of applying the principles of mechanics to industrial problems; the third was well versed in the workings and capabilities of microcomputers.

The next decision to be made was on which micro to base the software; the best information to hand suggested that the BBC B micro would allow us to reach the widest group of users. Bajpai and Downend (122) quoted the results of a survey conducted early in 1987 which indicated that the BBC micro still had a share of the UK market of over $70 \%$ in universities and polytechnics, about $90 \%$ in further education and $93 \%$ in secondary education.

Two programmers, who had both graduated from our course in Mathematical Engineering, were recruited to the team. Both had spent some time in industry before embarking on the course and had an ideal combination of knowledge and skills to help us achieve our aims. It was decided that a typical software unit would comprise one or two discs, a user guide, a teacher's guide and a set of work cards which could be used by the student when employing the unit in a self-paced mode. The discs were 40 -track, single-sided, double density of $51 / \mathbf{4}^{\prime \prime}$ diameter.

The project was entitled Micros in Mathematics Education, to be known as MIME, and was launched at Loughborough under the directorship of Professor Bajpai. The experiences of the MIME team in the early years have been well
documented: (117), (118), (119), and (120).

The first unit to be completed was on projectile motion; it is perhaps not in the form and style which would have been the case had it been tackled later in the series of units. It was considered essential to have one unit ready for public viewing as early as practicable so that the work being done by the MIME team could be demonstrated and comments received which could be fed back into the production process.

Mustoe was the author of this unit and one of the programmers was allocated to the unit on a full-time basis; the other programmer was involved in many of the discussions, since knowledge shared could only be of benefit to all the team, especially in the early stages of the project.

The author, although by no means an experienced user of the BBC micro, was well aware of the limitations likely to be imposed on his lofty ideas by the practicalities of the machine's capabilities. He realised that it was important to draw up his plans before consulting the programmer, as even his limited experience had taught him that the likely first response of the programmer was "We can't do that ". At a later stage of the project one of the team acted as software advisor, being an intermediary between author and programmer.

Laurillard (174) has given a list of the minimum steps that she considers necessary to carry out a "real" evaluation of micro-based software. In essence, these are as follows:
(i) Specify aims and objectives and any assumed pre-knowledge.
(ii) Check that the program can conform to the above.
(iii) Design pre- and post-test questions to check both the pre-knowledge and the attainment of objectives.
(iv) Monitor and record student performance on the programs.
(v) Analyse the data and decide on modifications to the software.

The author, in an informal manner, effectively followed this recommended path. The team had decided on a set of general aims and objectives. These were (117):
(i) To aid understanding of the subject matter.
(ii) To add interest to the study of the subject matter.
(iii) To be interactive.
(iv) To be 'user friendly'.
(v) To be 'idiot-proof'.

In addition, the following specific objectives were set.
(i) Show that the motion of a projectile under the influence of gravity alone is the resultant of two separate motions - vertical motion under gravity and a uniform horizontal velocity.
(ii) Show that the horizontal range is proportional to the horizontal velocity.
(iii) Show that the sum of kinetic and potential energies of the projectile is constant during the motion.
(iv) Demonstrate the general properties of the parabolic trajectory of the projectile.
(v) Introduce the concept of a parabola of safety.
(vi) Show the effects of landing on an inclined plane.
(vii) Demonstrate the effects of impact with a vertical wall or on an inclined plane.
(viii) Show qualitatively the effects of air resistance and of variation of air density with height, in particular upon the angle of projection which would achieve the greatest horizontal range.

The author was quite convinced that it was vital to achieve certain further aims.

First, he was determined that the users would be able to discover relationships and properties; they would be motivated to use analytical
mathematics to verify (or refute) the suggestions. Unlike traditional experimenters, the laboratory to be used would be the computer. The strategy of experiment followed by mathematical analysis is surely how mechanics should be taught: as a living experimentally-based science, not as a dull series of examples which provide the student with some equations on which to exercise his pure mathematical skills. In this context it is pertinent to mention the project carried out from Leeds University which included the use of simple experimental apparatus as an aid to understanding the fundamental concepts of mechanics. The students who used the apparatus were well motivated and achieved better results than their predecessors (175).

The second aim was to take the user past the point where his pure mathematics could support him into areas where he would be able to see what was happening in a qualitative way only. Later in his mathematical career he might acquire the skills required and then he would be able to derive the quantitative relationships that were previously denied him. It does seem that far too many engineering freshmen have their initial enthusiasm dampened by the way in which their first year engineering subjects are taught. Why is it necessary for the lecturers to involve mathematics from week 1? This approach kills any 'feel' that the student should have for his engineering subjects and reduces too much of the subject matter to an exercise in applying mathematics. The book on structural mechanics by Morgan (176) is an excellent example of what Mustoe has in mind. The principles of structural analysis are developed from simple diagrams of men sitting on see-saws to the construction of Gothic cathedrals. Every student to whom this author has introduced the book has been enthusiastic about its approach. It was precisely that kind of enthusiasm which he sought to engender in his unit on projectile motion.

The software advisor in the team constructed a shell for each unit; this acted as a frame into which a unit could be woven. It provided the management functions necessary for the successful running of the unit and allowed each new program which formed part of the unit to be incorporated into the unit as soon as
it had been written. In this way the production process could be streamlined for greater speed. Given that good quality software (which needs to be flexible, robust, easy to use and both clear and exciting to see), takes much time to produce, the greater the speed that can be achieved, the better. The shell provides the means whereby the topics comprising the unit can be selected from a main menu and it controls the input to the programs. It also introduces a standardisation into the series of units planned. The user, once familiar with one unit would immediately feel at home with any other.

The procedure adopted for the production of this first unit was to be followed in subsequent units. First the relevant subject-matter would be researched; this included applications, preferably of a more practical kind than were to be found in the majority of text-books. Then a rough story-line was produced which provided an outline of a unit with sufficient detail being included to indicate the difficulties which might arise in programming. At this stage, the software advisor was able to pick up and iron out any obvious hazards. Regular meetings were held during the development of the unit between any two of the programmer, author and software advisor, with the occasional meeting of all three.

From the story-line a more detailed script was prepared for the programmer. This varied in the level of details specified, depending on the nature of the topic. However, it always included the animated sequences which were required, the formulae to be displayed on the screen and the text to be shown. Examples of a story-line and its associated script are shown in Appendix 8.

At about this time, routines were being developed to facilitate the operation of the suite of programs by the user: input was always checked to make sure that it conformed to the specification; at any stage the BREAK key would return the user to the start of a section and the unit could be started up by simultaneous pressing of the SHIFT and the BREAK keys.

The advantages of the two programmers' backgrounds were greater than at first realised. They had both been trained in draughtmanship whilst technical apprentices and these skills helped them to produce attractive and clear screen displays. Their knowledge of mathematics and of the engineering applications of mechanics made them very useful and it was decided to involve them closely in the planning of the story-line and the script.

The mechanics syllabuses which the team studied suggested a series of thirteen software units. The titles were
(i) Projectile Motion
(ii) Momentum and Impacts
(iii) Friction
(iv) Linear Motion
(v) Equilibrium
(vi) Relative Motion
(vii) Newton's Laws of Motion
(viii) Angular Motion
(ix) Vectors
(x) Centres of Gravity
(xi) Simple Harmonic Motion
(xii) Circular Motion
(xiii) Work, Energy and Power

Mustoe was allocated titles (i), (iii), (v), (vii) and (xi). These units will be described in the next three sections.

### 5.2 Projectile Motion

The unit comprised nine Parts, with most Parts being subdivided into Sections. These are listed below.

## Part 0 Inexperienced User Guide

## Part 1 Horizontal Launch from a Cliff

## Part 2 Launch from Level Ground

### 2.1 Fixed Initial Angle, Varying Initial Speed

2.2 Varying Initial Angle, Fixed Initial Speed
2.3 Varying Initial Angle or Varying Initial Speed

## Part 3 Hit a Target

### 3.1 Elevated Target

3.2 Ground Target
3.3 Ball Games

Part 4 Launch from a Cliff
4.1 General Angle of Projection
4.2 Hit a Target below Cliff

Part 5 Landing on Inclined Planes
5.1 Launch up the Plane
5.2 Launch down the Plane
5.3 Find Maximum Range

Part 6 Motion after Impact
6.1 Impact on Level Ground
6.2 Impact with Vertical Wall
6.3 Impact on Inclined Plane

Part 7 Resisted Motion
7.1 Examples
7.2 Variable Coefficient and Index
7.3 Variation of Air Density with Height.

## Part 8 Exit from Unit.

Part 0 and the final Part were common to all units. Since this particular unit was the first to be completed it is not fair to quote the actual length of time spent in its production. However, later units were taking of the order of 3 to 4 full-time programmer months to complete. The projectile motion unit was not untypical in comprising over 40 separate programs; in many Sections there were a number
of programs chained together. It will be seen therefore that the mechanics software units represent a considerable effort in man-hours.

A copy of the user guide and teacher's notes for this unit appears as Appendix 9. In the next two subsections, Parts 2 and 7 are described in detail in order to explain the thinking behind their construction.

### 5.2.1 Launch from Level Ground

This Part comprises three Sections.

The first Section considers the trajectory of a particle fired from ground level with initial speed $u$ at an angle $\alpha$ to the horizontal. A typical trajectory is shown and then $H$, the greatest height reached, and $R$, the horizontal range, are marked on the diagram. The user is now able to vary the initial speed $u$ whilst keeping the angle $\alpha$ fixed. It is suggested that $\alpha=60^{\circ}$ is a suitable choice, since this gives a reasonably-sized screen display. The user is recommended to try $u=100$ and to note the greatest height reached, the horizontal range, the time of flight and the time to maximum height. He is next invited to choose two further values for u (50 and 75 are recommended) and to note the same features of the trajectory. He is asked to deduce the relationship between each of these features and the initial speed $u$. If required, further initial speeds can be tried. Having made the suggested deductions, the user can be shown by the teacher the mathematical derivations of the relationships. Alternatively, if the unit is being used in self-paced mode, the student can read the appropriate accompanying work cards.

The second Section is concerned with the effect of varying the angle of projection on the horizontal range. The user is recommended to input an initial speed of 100 to make full use of the graphics window; he can then input the angle $\alpha$ in sets of three. For each angle, the trajectory is graphed and the range achieved
is displayed. It is suggested that $\alpha=30^{\circ}, 40^{\circ}, 50^{\circ}$ is a possible set of values or, as an alternative, $30^{\circ}, 60^{\circ}, 70^{\circ}$. The hope is that the user would have noticed that if two angles are complementary then they lead to equal ranges; that fact could suggest that the maximum range is achieved with an initial angle of projection of $45^{\circ}$. On the other hand, the first set of angles would imply an optimum angle between $40^{\circ}$ and $50^{\circ}$ and further angles in that range would help to tie down the value more accurately. At this stage, the relevant mathematics can be applied to obtain the formula for the range and from that can be derived the optimum angle and the maximum range.

The final Section allows the user to vary either the initial speed or the angle of projection to discover other features of the motion. Before leaving this section the user will be shown the potential, kinetic and total energies of the particle at several stages during its motion. It should be readily apparent that the total energy remains constant, the balance shifting between potential and kinetic. Once again, this conjecture can be verified mathematically. The user has also got the opportunity to ask what assumptions are made in the model which he is employing.

### 5.2.2 Resisted Motion

When the author was taught Applied Mathematics at school he was told that the case of projectile motion under air resistance could not be tackled at that stage because "your pure mathematics isn't up to it". Only when he was a first year undergraduate was resisted motion discussed and even then it was merely a case of deriving the analytical solution to the governing differential equation. When he was preparing the unit on projectile motion he was determined that the users would not be prevented from gaining a qualitative understanding of the effects of air resistance; while they were absorbing the results of the simulation, they could have the opportunity to delve into the topic of ballistics by seeing the effects of allowing the air density to vary with height.

The first Section begins with a display of the trajectory of a launch on horizontal ground with no resistance present; this acts as a datum. Then, for the case of resistance proportional to velocity several trajectories are shown using different values for the coefficient of resistance. Figure 5.1 shows a typical screen display. The user should be able to appreciate that the effects of air resistance reduce both the maximum height reached and the horizontal range and in addition render the trajectory unsymmetrical. The sequence is repeated, but this time the case considered is that of resistance proportional to the square of the velocity; the values of the coefficient of resistance are much smaller than for the previous case but even so the effects are seen to be much more dramatic: see Figure 5.2. Hence the student is able to see very vividly the differences in effect between the two proposed "laws of resistance". To show the power of the computing approach, the case of resistance proportional to velocity to the power 1.5 is considered. (The trajectories have been produced by using a Runge-Kutta fourth-order method which gives sufficient accuracy for display purposes.)

The second Section allows the user to choose, in effect, a variety of resistance laws of the form $k v^{n}$ where $k$ is the coefficient of resistance which can take one of a continuous range of values, $v$ is the velocity of the particle and $n$ is an index which can take values between 1 and 2 inclusive. In this investigative mode the user should get a feel for the contributions of $k$ and $n$ to the resulting trajectory.

The final Section is concerned with the effects of air density varying with height. The sequence opens by displaying a graph of this variation. Using the case of constant air density as a datum, the case of variable air density is displayed and the contrast is again shown graphically. Then the user can see how well he has understood the changes in trajectory brought about by relaxing the assumptions of no air resistance and constant air density by attempting to find the initial angle of projection which gives maximum horizontal range. In World War I, artillery shells failed to reach their target because the formulae being used by the gunners had not taken account of varying air density.


Figure 5.1


Figure 5.2

Experience in using this part of the Unit with sixth-forms has indicated that they are able to appreciate the qualitative nature of the effects of air resistance and varying air density and to understand better the nature of the process of mathematical modelling in this context.

### 5.3 Enlivening Statics: Friction

The lessons learned in the writing and production of the unit on Projectile Motion were applied to future units. However, there was a new challenge to be met in this second unit: how to make a topic in statics come alive. Projectiles, being dynamic in nature, had an obvious link with an animated screen display; friction, on the other hand, did not.

The first idea which was suggested in the quest for a new approach was in connection with the standard problem of a man on a ladder. The problem as usually stated in text-books is of a ladder resting in rough contact on a vertical wall with its lower end in rough contact with a horizontal floor. A man stands a certain distance up the ladder. Knowing the mass of the man, the mass of the ladder, the angle of inclination of the ladder to the horizontal and the limiting coefficients of friction at the points of contact with both wall and floor, the question is whether the system is in (stable) equilibrium. The answer required is "yes" or "no" and there the matter rests.

It was decided to employ a gradual build-up to this problem. The first stage was to have no man on the ladder, rough contact with the floor and smooth contact with the wall. For a specified mass of the ladder and given limiting coefficient of friction the user was invited to input a range of angles of the ladder to the horizontal in order to discover the least angle at which the ladder could remain at rest. This result could be verified by the necessary analytical mathematics. Then the user was allowed to vary the mass of the ladder and the limiting coefficient of friction in an endeavour to determine their effects on the critical angle. It is expected that a good teacher will take the opportunity to get his
pupils to use their intuition by asking them to predict such effects before carrying out the "experiment".

The second stage was a repeat of the first, but this time the contact at the wall was assumed to be rough. The third stage placed a man on the ladder; after a fixed example, the user was given the opportunity to see how far up the ladder the man could walk in safety. Then, the opportunity to vary the parameters of the problem was provided: what were the effects on how far up the ladder the man could go safely of increasing or decreasing his mass, the limiting coefficients of friction and the angle of inclination of the ladder to the horizontal?

In this way, the students could bring back the experimental aspect into mechanics and a problem which had long been a feature of applied mathematics text-books could be enhanced and extended. More could be learned about the effects of friction and the difference between actual friction force and limiting friction force.

The contents of the unit, excluding Parts 0 and 8 are as follows

## Part 1 The Frictional Force <br> 1.1 The Effect of Friction <br> 1.2 Coefficient of Friction <br> 1.3 Direction of the Frictional Force

## Part 2 Angle of Friction

2.1 Relationship to Coefficient of Friction
2.2 Least Force to Move a Block on a Horizontal Plane

## Part 3 Block on Inclined Plane

### 3.1 Supporting the Block

### 3.2 Moving the Block down the Plane

### 3.3 Moving the Block up the Plane

## Part 4 Slide or Topple?

## Part 5 Ladders

### 5.1 Smooth Wall, Rough Floor

### 5.2 Rough Wall, Rough Floor

5.3 Man on Ladder

Part 6 Wedges
6.1 Theory

### 6.2 Examples

## Part 7 Belt Friction

### 5.3.1 Outline of the Unit

Part 1 is concerned with the nature of the frictional force. The first sequence shows a block at rest on a horizontal table; it is attached to a weight which hangs freely via a string which passes over a pulley at which frictional forces can be ignored. The weight is increased in value until the block begins to move; the value at which the block starts its motion is noted. It is demonstrated that once the block is moving the value of the force exerted by friction falls slightly. The user is then given a definition of coefficient of friction and some typical values of both static and kinetic coefficients. Then the static coefficient can be estimated by using the simulation of the block-on-table experiment. Finally, the direction of the friction force is illustrated in a number of situations so that the user can see that it is always in opposition to the direction of motion.

Part 2 is designed to illustrate the concept of angle of friction using two standard examples. In the first, a crate of weight $W$ is at rest on an inclined plane. The angle of inclination of the plane is increased until the crate begins to slide down the plane; the user is invited to consider the nature of the relationship between this critical angle and the coefficient of friction, $\mu$. To help him in this task, the sequence is repeated in stages and at the end of each stage the values of the angle $\alpha, \tan \alpha$, the friction force $F=W \sin \alpha$, the normal reaction $N=W \cos \alpha$ and the ratio $F / N$ are displayed. The user should be able to infer
that so long as $F / N$ is less than $\mu$ the block does not slide but when $\tan \alpha=\mu$ sliding begins to occur. The critical value of $\alpha$ is denoted $\lambda$ - the angle of friction. The user can repeat the simulated experiment using different input values of $\mu$. It is also demonstrated graphically that $\lambda$ is the angle made with the vertical by the total reaction force $S$.

The second example is of a sled being towed along a rough horizontal plane by a force applied to a horizontal rope. At first, the sled is at rest; an increasing force $P$ is applied to the rope and the angle made by the combined reaction force $S$ to the vertical is shown. At the point when the sled begins to move this angle is seen to be the angle of friction. A second sequence shows the force $P$ applied upwards via a rope inclined at an angle $\theta$ to the horizontal; again, the sled moves when the ratio $F / N$ attains the value of $\mu$. Next a set of different values of $\theta$ is provided and at each value the least force necessary to move the sled is calculated and displayed. It should come as no surprise to observe that this least force occurs at the value $\theta=\lambda$. The final sequence shows a downwards force applied to the sled. This time, as the angle of this force to the horizontal increases so does the least force necessary to move the sled; therefore the least overall force for this kind of configuration is when the force is horizontal. A good teacher would ask his students to explain these results and draw from them the fact that pulling upwards on the sled reduces the normal reaction $N$, whereas pushing down on the sled increases $N$. In this latter case the maximum friction force can take a larger value and hence a larger force $P$ is necessary to overcome it. It is understandable, then, why increasing the angle to the horizontal at which the downward force is applied causes this least force to be larger. Why however should the optimum angle for an upward pull be $\lambda$ ?

Part 3 takes a comprehensive look at the system of a block in rough contact with an inclined plane. The author was particularly concerned that the user
should be able to appreciate the difference between holding the block at rest and actually moving it up the plane at constant speed. The first sequence is concerned with holding the block at rest. It seems likely that the optimum angle at which the supporting force is applied is again $\lambda$ and this can readily be verified. The second sequence shows the block being held in place by a pull from above; it might be expected that the value of the least force increases as the angle of the pull to the direction parallel to the plane increases. The user is invited to conclude this section by making a decision as to whether it is more effective to pull or to push. In all these cases the user is able to use the "experimental" results to make conjectures which can be verified by calculus, algebra and trigonometry.

The second and third Sections of this Part consider respectively the cases of moving the block down the plane and moving it up the plane. In the former case it is to be hoped that the user will realise that the problem is only meaningful if the angle of inclination of the plane to the horizontal is less than the angle of friction. In each case the force is applied either as a push or a pull and the least force necessary is calculated for a variety of different angles of application. It is not suggested that the teacher make each pupil work through all the various cases covered but, rather, a selection.

Part 4 is concerned with the situation of a tall block at rest on a horizontal table, acted on by a horizontal force which is increasing. The question is whether it will topple before it slides. After a fixed example is worked through, the user can vary the dimensions of the block, the coefficient of friction and how high up the block the applied force acts. He should then be able to get a feel for how these parameters affect the outcome of the question. It was envisaged this part would form the basis of a possible project in the school environment. Many such possibilities are scattered throughout the units.

Part 5 has already been described.

Parts 6 and 7 constitute an attempt to add a practical dimension by
considering two topics which are not usually dealt with at school level but which feature in a number of engineering mechanics texts, namely wedges and belt friction. In the former case, a wedge is to be driven horizontally under a load which rests against a vertical wall in an attempt to lift the load. A fixed example is worked through on screen, using a build-up of the forces on the free bodies; this leads to the calculation of the least force needed to be applied to the wedge. Then the user is allowed to vary the model parameters; in particular, he can place rollers between any two surfaces to reduce the coefficient of friction there to effectively zero. Between which two surfaces he should place the rollers to gain maximum benefit is a question asked. Once more intuition and insight should precede experimental verification.

The effect of friction at a pulley is considered in Part 7 via the example of a wide flat belt passing over a drum. The least force necessary to hold a given weight stationary and the least force necessary to raise it are displayed. The relationship between these two forces and the value of weight is teased out of the user. Then the effect of wrapping the belt through a full turn and one-and-a-half turns is shown; as expected, the more half-turns the less the force needed to hold the weight, but the greater the force needed to raise it. Finally the variation of the two forces with angle of wrap is demonstrated via graphs of supposed experiments.

### 5.4 Equilibrium, Newton's Laws and Simple Harmonic Motion

In this section the remaining three units which were written by the author are described in outline.

The contents of the unit on Equilibrium, excepting the standard "top and tail" are as follows.

## Part 1 Forces and Moments

### 1.1 Types of Force

### 1.2 Moment of a Force

## Part 2 Free Body Diagrams

## Part 3 Concurrent Forces in the Plane

### 3.1 Closing the Polygon

### 3.2 Mooring a Boat

3.3 Jib Crane

## Part 4 Three Concurrent Forces

### 4.1 Triangle of Forces

### 4.2 Lami's Theorem

4.3 At the Docks
4.4 Derrick

## Part 5 Parallel Forces

### 5.1 Maintaining Equilibrium

5.2 Moving Train on a Bridge
5.3 Couples
5.4 Force and Couple

### 5.5 Example

In Part 1, four basic examples are provided of the actions of a force, viz. tension, compression, smooth contact and rough contact. Then a framework is shown with those members in tension and those in compression highlighted in turn. Next, the definition of the moment of a force is illustrated via a spanner tightening a nut; the applied force is shown being resolved into perpendicular components, one of which passes through the axis of rotation. Finally, a lever pinned at one end is acted on by a force whose direction can be varied. The effect on the resulting moment as the direction of the force charges is shown.

Part 2 is devoted to free body diagrams as a response to comments received from teachers who emphasised both the importance of the topic and the lack of ability of their students in this area. Six examples of systems in equilibrium are
shown. In each instance the user is expected to decide which forces are acting and to build up the force diagram step by step.

Part 3 is used to amplify the methods of solving systems of coplanar forces, viz. polygon of forces, and resolving in two perpendicular directions. Attention is paid to the usefulness of the polygon approach in deducing conditions for maximum forces. The special case of three concurrent forces is treated in Part 4. The method of triangle of forces and its restatement as Lami's theorem can be seen separately and in comparison. These can be contrasted with the method of resolution of forces.

Attention is moved to parallel force systems in Part 5. The first Section gives the user the opportunity to get a feel for the principle of moments by being asked to hold a number of systems in equilibrium. The second Section shows how the reactions at the supports of a bridge vary as a train moves over the bridge. In the third Section the concept of a couple is examined via the example of a car steering wheel. The two final Sections show how a force-couple system can be reduced to a single force and vice-versa.

The contents of the unit on Newton's Laws of Motion are as follows:

## Part 1 Newton's Laws Defined

### 1.1 The three Laws of Motion

### 1.2 Mass and Weight

Part 2 Newton's Laws Applied

### 2.1 Man in Lift

2.2 When in doubt, throw Ballast out
2.3 Engine with a Tender behind
2.4 Engine with coaches

## Part 3 Connected Masses

### 3.1 Single fixed Pulley

3.2 Three Masses, moveable heavy Pulley
3.3 Connected Blocks on inclined Planes

Part 4 Variable Mass Systems
4.1 When the Balloon goes up

### 4.2 Single-stage Rocket-no gravity force

4.3 Single-stage Rocket-gravity included
4.4 Two-stage Rocket under gravity

Following a statement of the three laws, Part 1 shows a mass of 1 kg being weighed at the earth's surface and at various heights above the surface to see how the weight changes. In Part 2, the first sequence treats the case of a man travelling in a lift. By displaying the graphs of acceleration, velocity and reaction of the man on the lift floor against time the user is invited to discover the relationship between acceleration and reaction. The cases of the lift ascending and descending are both considered. In the second Section the user has to release ballast in the form of 1 kg bags in order to stop the balloon's descent or to achieve a specified upward acceleration. The third Section is designed to show the effect on the tension in the coupling and the tractive force required as the acceleration of the engine is varied. The final Section allows the user to "build" his own train from three types of coach available and to repeat the previous Section.

Part 3 first examines the system of two masses hanging from the ends of an inextensible string which passes smoothly over a fixed pulley; the user can input values for the masses and predict the direction of motion. The second Section deals with a more complicated system involving four masses and allows several possibilities for experimentation. Once again, a good teacher would allow his pupils to predict the outcome before carrying out the simulation. The final Section is concerned with the study of two blocks resting on inclined planes, connected by an inextensible string which passes smoothly over a fixed pulley. By varying the masses and the limiting coefficients of friction the user can discover
what range of values leave the masses at rest.

Part 4 begins by repeating the balloon problem, but this time the ballast is in the form of sand which can be continuously released. The second Section deals with the motion of a single-stage rocket with the force of gravity being ignored. By varying the fraction of the mass of the rocket which is fuel the user can see how the velocity and height at the end of the 'burn' can be altered. The next Section shows how these parameters are affected when gravity is taken into account in the model. Finally, a rocket comprising a satellite as a payload and two stages containing fuel is studied. The user can design his own rocket by selecting the mass of the satellite, the mass of each of the fuel stages, the fraction of fuel in each of these stages, the rate at which fuel is burned and the exhaust velocity of the gases produced relative to the rocket. The question to be answered is whether a two-stage rocket is more efficient than a single-stage one.

The contents of the unit on Simple Harmonic Motion are

## Part 1 Examples of SHM

## Part 2 Horizontal Spring

2.1 Spring released from rest
2.2 Spring given initial velocity
2.3 Energy Considerations

Part 3 Associated Circular Motion
Part 4 Vertical Springs and Strings
4.1 Vertical Spring
4.2 String-Incomplete SHM

## Part 5 Damped Oscillations

Part 6 Applied Force

Four examples of SHM are shown in Part 1: a particle on an oscillating spring, liquid oscillating in a U-tube, a simple pendulum and a cork oscillating in a tank of liquid. The examples are shown together and then considered separately,
with a graph of displacement against time being shown in each case.

In the first Section of Part 2, a mass attached to a light spring is released from rest after being displaced from its equilibrium position. Graphs and tables are provided to help the user determine the relationships between acceleration and displacement and between velocity and displacement. The effect of the variation of initial displacement on the period and amplitude of the oscillations can be studied. In the second Section the mass is given an initial velocity when at its equilibrium position so that a comparison with the previous case can be made.

In the final Section the kinetic and potential energies of the mass are displayed at certain stages in the oscillation so that the balance between the two is seen to change as the motion proceeds, whilst their sum remains constant.

In Part 3 a particle moves in a circular path. The "horizontal" and "vertical" components of velocity are each seen to follow SHM.

Part 4 first considers a mass attached to the bottom of a light spring which hangs vertically from a fixed point. The mass is displaced downwards from its equilibrium position and the subsequent motion is displayed. The user can investigate the changes in that motion induced by varying in turn each of the parameters: mass, modulus of elasticity of the spring, initial displacement and natural length of the spring. The second Section replaces the spring by an elastic string and the differences between the two types of motion are examined. The user should be aware of the consequences of the string going slack.

Part 5 incorporates a damping force which is proportional to the velocity of the mass and examines the effects on the amplitude and period of the oscillations when the damping coefficient is varied. Then the spring constant is altered and the resulting changes considered. Part 6 starts with an undamped system subject to a force Fcos $\omega$. After noticing the effects of superimposing the applied force and the undamped oscillations the user is asked to consider the consequence of putting
$\omega=0$ to create a constant applied force. Then by putting $\omega=2$ and varying $F$ the relationship between $F$ and the amplitude of the resulting oscillations can be studied. Finally, the investigations can be extended by including a damping term in the model. The teacher can explain the ideas of transient and steady-state motions and discuss the phenomenon of resonance.

### 5.5 Evaluation of the Project

Testing of the units was an essential part of the production phase. This ranged from informal trials carried out by colleagues in the MIME team, through use by local teachers in schools and colleges to controlled experiments conducted by these teachers with their students. Two of the main purposes of these early trials were to discover whether the units were sufficiently robust for use in class and to pick up any features which were unpopular.

There was general agreement that the quality of the software was excellent: the screen displays were well-planned, the units were easy to operate and the menu-driven nature of the programs was a great help to teachers who wished to omit some Sections or to present the material in a different order. Some teachers did ask specifically for the teacher's notes to include the unit author's suggestions for a possible path through the topics. Whilst some teachers were content to pick and choose from items in a unit to suit their own needs, others clearly required more guidance preferring to make as little input as possible to the use of the software in class.

Other comments received included the criticism that some screen displays contained too much information to be absorbed at one time and recommended that part of this information should be banished temporarily from the screen in order to allow the user to concentrate his attention on that part of the original display which was relevant. The author of this thesis had been aware of this possibility and had taken pains to ensure that his units were free from this
particular criticism; the comments indicated that he had been successful in this regard.

It had also been remarked that at some stages in some units it was possible for the user to "work" through sequences without really playing an interactive rôle; it had been noticed that pupils who were not well motivated could spend their time unprofitably. As far as was reasonable, steps were taken to remedy this shortcoming.

Whenever teachers undertook to use the units with their pupils it was emphasised to them that they should familiarise themselves with the software, probably in a self-paced mode. This aspect cannot be stressed too strongly. One sympathises with teachers who are pressed for time, but to use a unit with a class when one has not worked systematically through it is a recipe for trouble.

There was a plea for smaller units to be produced. Teachers looked wistfully at our software, desperately keen to incorporate them in their teaching but only too aware that to purchase one of the 13 units would reduce the whole school annual budget for software by a third.

The author personally conducted several trials of the software units mainly but not exclusively, those for which he was primarily responsible. Since many freshmen arrived at Loughborough with little knowledge of or exposure to statics and dynamics it was possible to test the units both with students who had met the relevant topics at school and with those who had not. In addition, the author exploited his rôle as Admissions Tutor for the Mathematical Engineering degree course to test the software with applicants to the course when they attended for interview; again, some of these had not met the topics before, whilst others had, or were in the process of so doing.

Both via informal comments and from replies to short questionnaires the author was able to gauge the reaction of the students both in general terms and
with regard to specific points. Because the early trials had taken place when the first units were being written, it was possible to incorporate some of the changes suggested (directly or indirectly) into the software.

Many of the general comments were similar to those to which reference has already been made. Specific points led to revisions of particular animated sequences where it was felt that things happened too quickly or too slowly or some aspect was not as clear as it should be. Sometimes it was suggested that it was not necessary to see a fixed example before trying out one with user-provided data. Sometimes it was felt to be desirable to be able to repeat a fixed example.

Those who had not covered a topic before using the appropriate unit seemed to understand the concepts and principles well enough. The questionnaire would attempt to ascertain whether a concept had been grasped either by asking for an explanation of the concept 'in the student's own words' or by providing a physical problem and requiring an outline of how a principle could be applied, without carrying out any arithmetic or algebraic manipulation. Those who had met the topic earlier in their studies were generally enthusiastic about the software; a typical comment was 'The subject comes alive when you study it this way'. Statics in particular was held to benefit from the style of the units. On a number of occasions an applicant has written to say that his experience in using the software on a topic currently being studied at school had helped him to cope more effectively on his return from interview.

It is, of course, not claimed that such a student would be able to solve the problems set on that topic with a greater rate of success, since many other factors are involved (for example algebraic skill), but there is strong evidence that the units have enhanced the students' learning of mechanics.

Some teachers have asked the MIME team to write a text-book to accompany the units. The author is well aware that such a book might have its merits but the principal aim of the Project was to write software to enhance the learning process
and to be an aid which the teacher could use as and when he or she felt appropriate. So far, the team has not acceded to the requests.

Reviews in the journals have been generally favourable; odd criticisms and adverse comments have been made, but the clear message is that the units have set a high standard for future software. Regrettably the disruption in schools in the United Kingdom came at just the time when the units were being launched on the market and this gave the momentum of the launch a severe set-back.

The next problem to beset the Project was lack of financial support for educational software. Both Government and industry seemed to regard the provision of quality software as a low priority. Manufacturers of microcomputers were wrapped up in the need to produce continual technological improvements in the hardware. The MIME team suggested that for every $£ 1$ that was spent on technnical developments, $£ 3$ should be spent on producing educational software (118). They feared that failure to invest in software development would imply that the exciting possibilities in education opened up by the arrival of the micro would not be fully exploited.

At the stage when the last few units were being produced, the BBC Master series of microcomputer was introduced. A considerable amount of re-programming had to be undertaken to make the units Master compatible; at the same time the units were revised to allow them to be networked on the newly available Econet system.

As an experiment, a programmer well used to the IBM personal computers was commissioned to convert the unit 'Momentum and Impacts' to a form which would be IBM compatible. However, the task was more time-consuming and more difficult than was first envisaged. The MIME programmers had been exercised by the limited memory of the BBC B microcomputer and had intruded into areas of the memory space that were really never intended to be breached. (This had caused difficulties for the software house which had been hired by the
publishers to protect the units). The complexity of the task of converting the one unit, combined with a lack of funding, caused that part of the project to be abandoned. Our very forte - the quality of the software and, in particular, the extensive use of the graphics facility - had been our stumbling block.

Was the Project a success? In many aspects it has proved disappointing. Certainly, many potential users have praised the quality of the software and acknowledged that it has achieved most of the stated aims and objectives, whilst colleagues in other institutions have been impressed by it. However, the bitter pill that has had to be swallowed is that funding organisations and the computer industry, who have also lauded the software, have not been prepared to put their money where their mouths are. They seem content to allow the production of educational software to be left to individuals, working in isolation. The author believes that their attitude is short-sighted and will be regretted in years to come.

### 5.6 Undergraduate Engineering Mathematics

In 1984 the Computer Board for Universities and Research Councils launched an initiative to provide financial support to universities for the use of computing facilities in teaching. The following year saw the involvement of the University Grants Committee in the continuance of the funding. Professor Bajpai and the author were awarded a grant of $£ 36,000$ in the third tranche of support to allow the continued employment of one of the senior programmers on the MIME Project to help write units for first and second year engineering undergraduate mathematics. It was anticipated that use could be made of these units by science and mathematics undergraduates also.

The first unit to be written was on the topic of complex transformations. The author's teaching experience had led him to believe that this was an area in which understanding would be enhanced by the student being able to vary the parameters of each transformation. As an experiment, it was decided to provide the facility for dumping the screen display at pre-selected stages in the program.

This facility would allow the teacher, for example, to make transparencies of certain screen displays which would help him illustrate a lecture on a particular transformation. It was decided that the unit would comprise five programs:

## Simple Transformations

## Inversion

Joukowski Transformations

## Streamlines

Schwartz-Christoffel Transformations.

Discussions took place throughout the development phase with lecturers from the author's department and from other engineering departments as regards content and presentation. All the programs were tested by colleagues, who had a range of expertise with micros from the buff down to the novice level.

The software was written originally in two versions:
(i) A user interface which required complete alpha-numeric input and demanded a familiarity with the keyboard.
(ii) A user interface which was restricted to a maximum of five keys only. The input could be made via
(a) the keyboard using the cursor control keys together with the return key
(b) a joystick connected to the analogue input socket
(c) a custom-made five-key pad connected to the user port.

Early reactions from staff suggested a preference for (ii) (b) \& (c) for the lecture environment. However, the joystick generally required two hands to operate it and it was rejected in favour of the pad; even those who were not confident with micros found that they could work comfortably with the pad. Students working in a self-paced mode preferred (ii) (a) to (i). The software was written in the two preferred modes.

There was a price to be paid for limiting input to five keys only; the moving
bar type of menu was required to replace alphanumeric selection. Further, the inputting of numerical values caused some problems; although increasing and decreasing with the up and down keys and confirming with the RETURN key worked well, it was a slower process than ordinary keyboard entry if the input number had a wide range of possible values.

Most of the programming was carried out in the resident Basic, although assembly language sub-routines were employed to speed up or, in some cases, re-organise the graphics displays. The limited memory space of the BBC B micro did present problems for the programmer; longer programs had to be split into two or three parts to remain viable. There was a temptation to overcome this problem by writing only for the Master 128 series of BBC micros but this would have reduced the user potential by a considerable amount.

Other early comments from colleagues led to the decisions to make some of the screen displays less crowded and not to produce accompanying workcards. The unit was successfully tested on Econet Level III.

In all, nine further units were produced under the aegis of the project, the first eight under the authorship of Mustoe. The titles are:

Poles and Residues<br>Numerical Solution of Linear Equation<br>Numerical Integration<br>Numerical Solution of Non-Linear Equations<br>Cubic Splines<br>Numerical Solution of ODE's<br>The Water Tank<br>Fourier Series<br>ANOVA.

The units were produced, as were the mechanics units, on 40-track, single-sided, double density $51 / 4^{\prime \prime}$ discs. In the next three sections the use of three
of these units in a lecture, in a tutorial and in self-paced mode, respectively, are described. The final section discusses the evaluation of the units and looks to the future.

### 5.7 Enhancing a Lecture: Simultaneous Linear Equations

The unit on simultaneous linear equations was designed to cover the methods of Gauss elimination, Gauss-Jordan, Gauss-Seidel and LU decomposition, in addition to matrix inversion. The user notes are presented in Appendix 10.

A group of 18 students was assembled in a lecture/tutorial room into which had been wheeled a trolley on which were a large television monitor and a micro with a disc drive. The students had already been introduced briefly to the idea of the solution of linear equation systems via elimination methods in their linear algebra lecture earlier in the week. The class was provided at the outset with a four-page lecture summary showing all the matrices on display, together with the row operations connecting them. In this way the students could concentrate on the lecture, annotating the summary as and when they felt necessary.

The lecture began by quoting examples where simultaneous equations arise in engineering. The case of finding the forces in a truss was discussed, asking whether there was any difference of approach needed if the number of members in the truss was 25 rather than 5 . This part of the lecture was presented via blackboard and overhead projector.

Then the system

$$
\begin{array}{r}
x_{1}+2 x_{2}+3 x_{3}=11 \\
2 x_{1}+3 x_{2}+4 x_{3}=15 \\
x_{1}+5 x_{2}+7 x_{3}=28
\end{array}
$$

of solving large sets of equations it was explained that a $3 \times 3$ system provided a balance between ease of computational simplicity and being able to describe the essential features of the solution process.

There was a remote control provided which allowed the author to move away from the micro and hence to be able to see the same screen display as the students. He was able to point to the screen to emphasise certain aspects of the computations and therefore was really using the screen as an electronic blackboard. The first program in the unit was run to show the step-by-step development of the Gauss elimination process. First, the augmented matrix for the system of equations was created; then a block diagram which indicated the overall structure of the process was displayed. Next, each reduction of the matrix was effected with the calculation that was being carried out being shown on the right of the two matrices relating to that stage of the process; Figure 5.3 shows a typical screen display. At the stage immediately prior to back substitution the reduced system of equations was shown; then the back substitution was carried out to obtain the solution to the original system. Finally, the whole process was repeated more quickly, without detailed explanation at each stage.

At this point the author moved attention away from the screen to the blackboard. His previous sessions where the micro was used in a teaching environment had convinced him that after watching a monitor for more than 5 to 10 minutes the students would become restless. It was desirable to give them a break from concentrating on the small screen and, in general, the lecture consisted of a mixture of screen display, blackboard, overhead projector and talk.

An exposition on the problem of inaccuracies in the solution was given. Since a simple check for a proposed solution of a small system was to substitute the values back into the original equations, the question was raised as to whether a reasonably good balance between left-hand and right-hand sides implied that the proposed solution was reasonably accurate. In this somewhat informal manner, the idea of an ill-conditioned system was introduced. Some simple examples were


Figure 5.3
shown on the blackboard and the need to make the Gauss elimination process as accurate as possible was stressed. This led to the idea of partial pivotting. The micro was brought back into play to demonstrate the extended process. It is pertinent here to mention that the program included the facility to scroll back the display to review the previous stages; this was felt to be important for the user in self-paced mode. .The Gauss-Jordan variation was mentioned and the method was illustrated on the monitor. Whilst it was acknowledged that it was easier to read off the solutions in this fashion it was recognised that the variation required more computational effort, with the attendant worry of increased round-off error.

The final part of the lecture was concerned with iterative methods of solution. The unit gave only fixed examples and these were obviously the ones to be covered in the lectures. On the blackboard it was shown how the system

$$
\begin{aligned}
& 5 x+3 y=6 \\
& 4 x+7 y=8
\end{aligned}
$$

could be re-organised to

$$
\begin{gathered}
x=1.2-0.6 y \\
y=1.14-0.57 x
\end{gathered}
$$

and thence to the Jacobi scheme

$$
\begin{gathered}
x^{(n+1)}=1.2-0.6 y^{(n)} \\
y^{(n+1)}=1.14-0.57 x^{(n)}
\end{gathered}
$$

or to the Gauss-Seidel scheme

$$
\begin{gathered}
x^{(n+1)}=1.2-0.6 y^{(n)} \\
y^{(n+1)}=1.14-0.57 x^{(n+1)}
\end{gathered}
$$

The two methods were worked through on the micro and the results compared. Then the rearrangement

$$
\begin{aligned}
& x=2-1.75 y \\
& y=2-1.67 x
\end{aligned}
$$

was tackled via the Jacobi and Gauss-Seidel methods and seen to lead to non-convergence. The class was asked to think about the reasons for this and to
come to the next session with their suggested answers.

A subsequent session took place with a group of over 80 students from a different course in a lecture room in which four television screens were placed along the sides. Apart from administrative problems in arranging for the televisions to be unlocked, and so on, the author noticed one severe drawback. Whereas on the previous occasion it had been possible to focus the students' attention on one point and hence always to keep that attention, at this session the students' attention was divided between the four screens and it proved difficult to re-focus their attention on one place. During a micro demonstration it was more awkward to explain what was happening than when all the students were looking at one point. A further worry was that some of the numerical detail was not clearly visible to all the students in the room, even after using double height characters. Add to this the fact that only a few rooms had the facility of being used for micro demonstrations in this way (and then they were not always available when required), and it is easy to see why many lecturers fight shy of using micros in their lectures.

### 5.8 Computer Enhanced Tutorial: Numerical Integration

The user guide for the unit on numerical integration is shown in Appendix 11. It explains the philosophy behind the unit and how the teacher might want to use it.

The group sizes chosen for the tutorial were either six or seven. For ease of administration the first session was held in the programmer's office with the programmer conducting the tutorial and the normal monitor being used. The students had been told before the session that they would be having a tutorial in which the software on numerical integration would be used as forum for discussion. Lest it be thought that the author was opting out of his responsibility by not conducting the tutorial himself, it should be pointed out that he wanted to observe someone else demonstrating the software and he wanted the programmer
to see at first hand the reactions to his software.

The first session proved something of a disappointment. This was the first occasion on which the students had been exposed to this kind of teaching and they were a little reluctant to answer questions unless asked as individuals. Only once did a student ask to see a sequence a second time and the tutor was therefore controlling the pace and content of presentation.

The software was not as amenable to this mode of use as had been expected. The change-over from one example to another was too slow, especially when a different function was selected. The lack of a suitable blackboard facility also hindered the presentation; consequently, for the second session a number of changes were made. The room used was a designated tutorial room which was larger and had a blackboard. The hardware was transported to the room on a purpose-built trolley. A larger TV screen was used and this made for a more relaxed environment, since the students did not need to sit so close together or so next to the screen. It also allowed the tutor to point to parts of the screen display without blocking the students' view! The opening examples for both Trapezoidal and Simpson's rules were simplified. The functions chosen were: for Trapezoidal, $\sin x$ with one strip and for Simpson, $1 /\left(x^{2}+1\right)$ with two strips. These examples were found to give a more instant picture of the methods than the functions used previously. Refer to Figure 5.4 on the following page.

The session was conducted by alternating between informal discussion using the micro - where this line originated, what that area represents etc - and a more formal blackboard derivation of formulae. The student response was better.


Figure 5.4

A typical coverage of topics was
(i) Estimation of area: upper and lower sums, mid-ordinate rule, the effects of varying $a, b$ and $n$ (or $h$ ).
(ii) Trapezoidal rule: derivation of the formula, examples, errors.
(iii) Simpson's rule: derivation of the formula, examples, errors.
(iv) Mention of Romberg integration and Gauss-Legendre quadrature.

### 5.9 Individual Usage: Cubic Splines

The user guide for the unit on cubic splines is shown in Appendix 12.

The students had received a package of lectures on the approximation of functions. This began with an outline of the tangent and quadratic
approximations and led in the direction of Maclaurin and Taylor series; these would be covered in detail in a separate course. Then the drawbacks of such approximations were pointed out and the students given an assignment on Maclaurin's series for $\exp x$ and $\sin x$. This had two objectives: to see how for a fixed number of terms the accuracy of approximation varied as $X$ was moved away from zero and to see how for a fixed value of $x$ the accuracy varied with the number of terms.

Then the concept of Fourier Series was introduced by employing the unit on that topic. The students were able to see how the approximation improved as successive terms were added on and were introduced to the concept of orthogonal functions in a very informal manner. No attempt was made to derive any Fourier series expansions.

Attention shifted to the approximation of data, first via least squares and then by interpolation methods. This, then, was the background to the lecture on cubic splines. The lecture started by describing the draughtsman's spline: a flexible metal strip to which weights could be attached so that the strip passed over those points on a drawing through which the curve to be constructed was required to pass. The spline then took up a suitable shape for, say a road which was being planned; by moving the weights, the strip could be made to take up different shapes. The aim of the mathematical approach was to model this draughting tool.

A set of 6 data points was shown and the class asked to fit a cubic curve to the first 4 and a second cubic curve to the last 4 . It was seen, via a pre-programmed routine for solving simultaneous equations, that at the third and fourth points the slope of the cubics did not agree. It was then explained that the method of cubic spline fitting would fit 5 cubic segments to the six points, each segment passing through a consecutive pair of points. A cubic curve had just enough flexibility to make a useful tool, whilst retaining relative simplicity in subsequent calculations. However, since a cubic curve contains four parameters in its complete definition, it was clear that the 20 parameters involved in the set of segments would require

20 conditions for their evaluation.

It was also clear that since each segment had to pass through 2 of the data points there were 10 conditions immediately obtainable. It was suggested that to allow a smooth journey along the segments the slopes of two successive segments should be equal at the "hand-over point" and that also the curvatures would have to be equal also.

However, these conditions between them provided a further $4 \times 2=8$ equations which left a shortfall of $20-10-8=2$ equations. It was then explained that the practice was to use the set of second derivative of each cubic segment at the end-points of its existence as the variables around which the equations were defined; these variables were denoted $S_{1}, S_{2}, S_{3}, \ldots S_{n}$. In the example of six data points we had $S_{1}, S_{2}, . ., S_{6}$. A popular choice of two extra conditions were the so-called "natural spline conditions", viz. $S_{1}=S_{6}=0$ which corresponded to the metal spline being allowed to take up its natural shape at the end points. Then it was explained how the coefficients of each cubic equation can be derived from the set $S_{1}, \ldots, S_{6}$. (A mathematical derivation of the equations is shown in Appendix 12). The students were provided with the user guide prior to the laboratory session. Refer to Figure 5.5.

The tutorial sheet is attached to the user guide. It is clear from the reports submitted that not only did the students understand more about the nature of cubic spline approximations than their predecessors who had used a black box routine to obtain numerical results for a given set of data points, but also they gained more personal satisfaction from this type of guided investigation.

It was envisaged that the software would be employed in three modes of teaching: the lecture, the small group tutorial and the laboratory. For the lecture environment a particular type of software was required, namely that which is fixed


Figure 5.5
in style. The ideal is to have pre-programmed examples that have been selected in order to make specific points. In this way a minimal amount of user interaction is needed and the continuity of the lecture is maintained. The teacher can use the software as an animated equivalent to a set of overhead transparencies.

On the other hand, the software to be used in small group tutorials needs to be flexible. The author had found that when using the MIME mechanics software with his students there was an advantage in being able to follow up a 'what if' question from a student using the software. Finally, laboratory exercises require the student to work with the software under a minimum of supervision. Worksheets would be provided to guide the students through the software by asking them to carry out a sequence of guided experiments and draw appropriate conclusions. This method of individual guided discovery makes especial demands for both the software and the documentation.

These various modes of teaching necessitated very carefully designed software. Ease of use was a top priority and the MIME shell would allow for this requirement. One result of the comments received on the MIME mechanics software was to aim to produce smaller units with less ambitious animations. Apart from the need to use the precious resource of programmer time to the maximum effect, it was felt that at the tertiary level students would need less exciting displays to motivate them.

### 5.10 Evaluation and Assessment of the UGC Project

The units produced under the UGC funding have been tested by colleagues at Loughborough and elsewhere. Nine departments in other universities agreed to participate in testing and evaluating the software; they comprised six departments of mathematics or mathematical sciences, one computer science and mathematics, one physics and one mechanical engineering. At the time of writing this thesis, replies had been received from seven of these departments. In addition, one final year student, who was carrying out a project on mathematics software undertook to test the units as part of that project. One department invited a project member to conduct a workshop for interested staff.

The evaluation was somewhat informal for two reasons.
(i) The units were designed as a resource for enhancement of the learning process and therefore intended to be used flexibly together with other resources in a variety of teaching styles: no two teachers would be expected to use them in precisely the same way.
(ii) For the most part the material has been tested by teachers with their students in a way which is not typical of how they envisaged using the material in their courses.

A simple questionnaire - one version for teachers, one for students - was included with the units to provide a framework around which comments and criticisms might be concentrated. These versions are presented in Appendix 13.

Student reaction was mixed: whilst many responses were favourable, indicating that the units were interesting and helpful, there were some who reported lack of interest and said that they learned very little from using some units. Staff comments were analysed under five headings.

## (i) General

Whilst the screen displays attracted much praise and the ease of use was welcomed, there was a feeling that the restriction of parameters to a set of fixed incremental values was a drawback. More example functions were requested in some units and some respondents bemoaned the absence of the facility to input their own functions.
(ii) Running the programs

Occasional difficulties were experienced in running the software in a large networked system. On reflection it would have been preferable to have given the user more information on the files in each unit. There is often a problem in knowing which system an intending user will be operating and it may well have been possible to anticipate potential pitfalls by holding discussions with that user prior to him attempting to use the software.

## (iii) Particular units

Comments were received on each unit and were generally complimentary. There was the odd criticism but this tended to reflect the respondent's particular preferences, and sometimes the fact that the lecturer wanted to sit back and let the software do all the work despite being told at the outset that the keyword was enhancement. The unit on simultaneous linear equations was well received because of the way in which the processes were animated and row interchanges were illustrated dynamically. The unit on the watertank, as an example of a system modelled by a first order differential equation, was rated a useful addition to the teaching because of the simultaneous display of the falling water level and the
graph of the variation of water level with time, especially as steady state was approached. Some reservations were expressed as to the usefulness of the units on complex variables but the unit on cubic spline approximation was highly praised, all respondents expressing a firm intention to incorporate it into their teaching.

## (iv) Documentation

There was a strong feeling that the documentation to accompany the software discs should be in three versions:
(a) for a lecture demonstration,
(b) to support course notes given in class,
(c) "teach yourself" notes.

It was clear that colleagues were, in the main, unwilling to try out the software and then produce documentation to suit their own needs. On reflection, this should have been foreseen; it is being taken in hand.

## (v) Hardware

Whilst the BBC B microcomputer is still very popular in tertiary institutions, there is a shift towards IBM compatible machines and future units will need to be written for them, even though the BBC micro is a more suitable vehicle for educational software.

Despite the difficulties in selecting a suitable hardware and operating system which will not be superseded during the lifetime of a project, it is likely that the IBM compatibles will be the most favoured for at least the next few years.

There is no doubt that the project has been a worthwhile exercise. The respondents have been keen to use some of the units in their teaching and would encourage their colleagues to do likewise. It is clear that whereas certain topics lend themselves readily to presentation via such software (for example cubic
splines) others are not so easily accepted (for example, complex transformations).

Full documentation and notes for usage will need to be provided, the attitude being to allow lecturers to opt out of the material provided rather than opt in with their own.

In conclusion, the author looks back on the whole MIME project with a mixture of emotions. There is pleasure at the successes and the plaudits but there is disappointment that after five years it is still a tiresome logistic exercise to use micro-based software in lectures and tutorials. There is sadness, too, that many colleagues are unwilling to take software and work with it to obtain a package to suit their needs and preferences. They seem to want to play a passive rôle in its use in their teaching programme. On the other side of the coin there are those who appear to regard the software as perhaps bearing too much of the software author's personality. Whereas a text-book is a fairly low-key teaching resource, a piece of micro-based software will have something of the intimacy of a teacher-student relationship which a different lecturer may find difficult to reconcile with his own attitudes.

The author believes that with sensible use in lecture room, tutorial room and microcomputer laboratory, micro-based software can truly enhance the learning process. Clearly, more research needs to be undertaken into the effective use of software in the lecture room and in tutorials. However, there needs to be much more provision of resources to allow the successful implementation of the software and to encourage more software production. At the moment, the hardware has raised expectations which the software cannot meet. It is going to take longer than first envisaged before computer enhanced learning becomes widely regarded as the norm.

It seems appropriate to end this chapter with an extract from the CTISS project report of Bajpai and Mustoe (177)

## "Lessons for the Future

To have a long-term future the software must be supported by suitable facilities in the form of purpose-equipped lecture rooms each with a large screen in situ, tutorial rooms with permanently-installed hardware and several micro laboratories.

The quality of software desirable to maintain interest requires experienced programmers whose expertise will demand high salaries.

Having pump-primed several projects across universities and other institutions it would be unwise to expect any large-scale continuation of these projects. External funding for educational software is scarce and without such funding it is hard to see how much of the work will continue at other than a low level.

It must be the aim of projects of this kind under the umbrella of CTISS to achieve a long-term impact. Strong support is needed to fulfill this through considerable funding of a small group of institutions most capable of performing the task. The selected institutions should have expertise and established reputation in the subject areas. Collaboration between these institutions is a 'must' to avoid duplication of effort and dissipation of scarce resources and energy. The time is here when concentration should supersede dispersion."

## Chapter 6

## Case Studies in the Curriculum

### 6.1 Modelling with Mathematics

In the 1960's pure mathematicians began a radical overhaul of mathematics syllabuses at secondary school level; the most influential of these was the School Mathematics Project (SMP). There was a mood abroad to reduce the time spent on drill and practice exercises and to increase the emphasis on structural and conceptual understanding. The author felt at the time, partly from his experience as an examiner at Advanced Level GCE, that too much was being expected of less able pupils and that the mathematical toolkit was less well stocked than was desirable for engineering undergraduates. Unabated, the change in philosophy spread down into the primary school level and up into undergraduate mathematics degree courses. More recently, there has been a slight swing back to the 'traditional' approach, which has shown that common sense can have an influence even if it cannot prevail.

At the end of the 1960's, those teaching applied mathematics began to show concern for the way in which the number of candidates for Advanced Level Applied Mathematics was beginning to decline. They realised that they, too, had to carry out a radical reappraisal of their teaching. The problems tackled, not only at school but also at undergraduate level bore little or no resemblance to real, practical problems. Furthermore, the formulation of the problems in mathematical terms was neglected since they were already posed in a way which merely awaited immediate mathematical solution.

Among influential voices raised in favour of a change was that of Pollak (180) who wanted an open-ended approach typified by the statements "Here is a situation. Think about it. Find out what the problem should be, or what the
theorem is that you ought to be trying to prove." This contrasted sharply with the traditional approach of "Here is a Problem. Solve it." In the twenty years which have elapsed since that paper, the literature on mathematical models and modelling has mushroomed. Most of it has been directed towards school mathematics and mathematics degree courses, but the principles are relevant to the teaching of mathematics to engineering undergraduates. A selection of papers of interest is provided in references (181) to (190).

There is now a clear distinction drawn between a model and modelling. It is only in the last fifteen years or so that much attention has been paid in textbooks and journals to the construction of a model; hitherto, even in textbooks which included the word "modelling" in their titles, most of the space was occupied by deriving the solution of the mathematical model. Although many expositions of the modelling process exist it is widely accepted that the activity comprises the four stages of formulation, solution, interpretation and validation.

In the fomulation stage, a real-life situation is analysed to identify a particular problem (or set of problems) and the problem is then posed in mathematical terms. The solution stage is where attempts are made to solve the mathematical problem and the interpretation stage relates the mathematical solution obtained to the original problem. Finally, the validation stage checks the predictions of the model against a wide range of circumstances in the original context; this stage may involve the collection of data by observation or experiment.

The definition of a mathematical model is not so readily agreed. The author has found the definition due to Andrews and McLone (191) as helpful as any; they declare a model to be ". . the representation of our so-called 'real world' in mathematical terms so that we may gain a more precise understanding of its significant properties in order to allow some form of prediction of future events". Oke (192) has made a more compact attempt: "a simplified and solvable mathematical representation of an aspect of a practical problem". He argues that his definition emphasises that the model is an imperfect representation because
simplification necessarily ignores some details of the problem and because to make the mathematical representation tractable often requires further simplification.

The term 'case studies' is commonly used to describe the presentation of a model and its solution in which the formulation stage has been ignored or sketched over very briefly. For the purposes of this thesis the author will use the term to describe the presentation of a model (and its solution) which has actually been used to solve a real problem. Contrary to the advice offered by Beckett (135) experience has shown that engineering students will not be impressed by the use of mathematics to solve problems related to leisure pursuits; that approach smacks too much of the mathematics looking for an application. In any event, the engineering student seldom studies mathematics for its own sake. By choosing examples where the mathematical model has made a real contribution to the understanding of and solution of a practical problem the lecturer is more likely to persuade his students of the usefulness and relevance of mathematics in engineering. Whether the presentation of the case study includes the formulation stage and whether the students are guided through that stage with a greater or lesser degree of help from the lecturer is a matter of choice.

As has been mentioned in Chapter 3 the author has always emphasised the role of mathematics in modelling engineering systems at the outset of his course of lectures. A good idea of the philosophy that is expounded to the students is to be found in Chapter 1 of Engineering Mathematics (42) from which is taken Figure 6.1 on page 156, which was shown previously as Figure 2.1 on page 25.

As part of their induction week the Civil Engineering freshmen at Loughborough have a set of mini-lectures from the members of staff responsible for their teaching, each giving an overview of his subject. The author in his 'spot' explains to the freshmen that they have learned certain mathematical skills akin to an apprentice joiner having been trained to make dovetail joints, to chamfer and so on. Now they are going to make a cabinet applying those skills. But the
mathematics will get messy and compromises and approximations will often be necessary.

It is pointed out that in their structures course they will be told that for a homogeneous, uniform thin horizontal beam of length I simply supported at its ends and experiencing a uniformly distributed load of $w /$ unit length the deflection of the mid-point is $5 w^{4} / 384 \mathrm{El}$. The students should ask how thin is a 'thin' beam and how small is a 'small' deflection; further, what would happen if the beam were not homogeneous or uniform. Before such a formula could be derived, a mathematical model for the deflection of the beam had to be constructed; certain simplifying assumptions had to be made and any predictions or deductions from that model were only as reliable as those assumptions. Having constructed the mathematical model, it could be solved (hopefully) by the techniques that the students had already acquired. If not, then they had to acquire new techniques or make further simplifying assumptions.


Figure 6.1

Similarly, in their hydraulics course they will be told that as part of the study of the action of a reciprocating pump they will consider the laminar flow of water between two parallel plates. A formula that can be easily remembered is

$$
v=t w i t / 12 \mu
$$

where $v$ is the mean velocity of flow, $t$ is the distance between the plates, $w=\rho g$ where $\rho$ is the density of the fluid, $\mu$ is the dynamic viscosity of the fluid and $i$ is the hydraulic gradient given by

$$
i=h_{f} / l,
$$

$h_{f}$ being the head loss and I the length of the flow path.

Mathematics at this level, then, is more than simply the acquisition of new analytical or numerical techniques. It is an integral part of the understanding of the behaviour or engineering systems and it is in this spirit that the mathematics course is to be taught.

In the tutorial session following the lecture the students are given a hand-out entitled Models in Engineering which is shown on pages 158 and 159. They are encouraged to discuss their reactions to each problem: whether they consider the problems straightforward, whether they have enough information (or too much), how they would set about constructing a model, and so on.

Example 1 is not usually regarded by the students as involving a model; these are equations to be found by resolving forces at each joint both vertically and horizontally and by taking moments about particular points an the truss. Then the students are asked what they understand by a point load or a point support and complacency begins to wane.

With regard to Example 2, many students have not really met much statistical analysis at school and the question of handling imprecise data is a

## MODELS IN ENGINEERING

1 In the plane truss shown below, all angles are $60^{\circ}$. The applied loads are in kN . All joints are pin joints. Find the forces in each member.


Would your method of solution differ if the truss contained twice as many members?
2 Mass-produced spars are claimed to have a breaking strength which is on average $28 \mathrm{~N} / \mathrm{m}^{2}$.
What does this mean?
How can we check whether the claim is valid?
3 The measurements shown in the survey are correct to the last figure quoted.
What is the length of the side AB ?


4 A uniform cantilever of length 1 has an end-load $P$.
Find the deflected profile of the cantilever, the deflection may not be small.
5 A metal specimen is tested in an extensiometer to find Young's modulus for the metal. How can we calculate this modulus reliably?

6 A beam of length 1 is freely supported at its ends and carries a load of $w$ per unit length. It rests on elastic foundations.

7 A cylindrical tank of volume $1000 \mathrm{~m}^{3}$ is to contain hot liquid. What dimensions should be chosen to minimise heat loss through its surface?

8 The profile of an impeller blade is bounded by lines $x=0 \cdot 1, y=2 x, y=e^{-x}, x=1$ and the $x$-axis. Find the volume of the blade if its thickness is given by $t=(1 \cdot 1-x) r$ where $r$ is a constant. Find its second moment of area about the $x$-axis in the case of uniform thickness.

9 Measurements of the cross-section area of a hole are taken at equally-spaced depths. Estimate the volume of the hole. How accurate is your estimate?

10 A reservoir has been contaminated by effluent. The capacity of the reservoir is $10^{6}$ litres. The degree of contamination is $0.02 \%$ by weight. The average daily rate of consumption of water for non-drinking purposes is $2 \times 10^{4}$ litres and this is continuously replaced by pure water. How long will it be before the concentration of contaminant drops to the safe level of $10^{-5} \%$ ?

How do your results differ if the incoming water contains $10^{-6} \%$ of contaminant?
11 How does soil drain excess water?
12 How can we forecast the next in a sequence of inflation indices?
13 How can we determine whether the river Dee will flood after a severe rainstorm?
14 (Term exercise).
Road repairs on a two-lane road reduce it to half-width, allowing only single file in one direction at one time. How would you design a temporary traffic light system?

L R Mustoe
novelty. Those who have are hard put to convince their fellows that testing of statistical hypotheses is a valid process.

Example 3 is easily understood; it is readily accepted that carrying out the calculations using the values given is merely to produce an approximation to the correct answer, but : how to estimate the error in the number quoted is a puzzle. Sometimes it is suggested that the largest possible value for the length and the smallest possible value could be quoted but it is agreed by the proponents this is heavy work for the kind of answer required; perhaps a quick, rough method would be preferable.

Example 4 is a stumbling block for most students; they protest that they have no equation to solve. When the author asks how to set about obtaining this equation there is usually a stony silence. Eventually it is agreed that a mathematical model for the deflection of a cantilever is the first step and this requires a knowledge of structural mechanics principles that they do not yet possess.

Example 5 is intended to remind the students that experimental measurements are subject to error. If, as is often the case, the appropriate mathematical model is founded on the assumption of uniform cross-sectional area of the specimen, what are we to do if our measurements indicate that the assumption is not borne out in practice?

Example 6 is introduced as a first step in predicting the behaviour of railway track under operational conditions. The students can usually recognise that the full problem is extremely difficult to analyse and that it is a reasonable first step to consider the rail in the absence of the loading of a train (and a moving load at that!). Even the problem as posed raises a number of questions about the support given to the rail by sleepers, ballast. etc. A short discussion takes place on the nature of the elastic foundations, including the meaning of the adjective 'elastic' in this context.

Example 7 restores a little of the students' confidence. This is the sort of problem they can recognise as mathematics, even if they do not possess the necessary mathematical skills to solve it.

Example 8 is definitely not a model - it is merely an application of integration is the usual response. Then what would happen if the boundary of the blade were specified by a drawing or a set of coordinates? is the riposte which leads to a discussion of the approximation of data. Should the approximation be in manufacturing equations for the boundary curves or in the integration process itself?

Example 9 is unfair. How widely-spaced are the cross-section areas -how many of them? When the students are told that this decision is up to them they are most disconcerted: how can we possibly say? is the response, followed by a plaintive it depends on the hole. The thought that they have to make such a decision is anathema to them since they have always had it made for them in the past.

The usual reaction to Example 10 is that the students have no idea where to begin. The author tells them that an early consideration is the information provided in the statement of the example. Is it helpful? What does 'the degree of contamination is $0.02 \%$ by weight mean? Are we talking about an average value, a maximum value or a minimum value? If the figure quoted is an average then this means presumably that our answer assumes a (reasonably) uniform distribution of pollutant in the reservoir.

Examples 11 to 13 are beyond the pale. There is virtually nothing to work on. It is pointed out that in the case of example 11 there is a need to understand the mechanism of water transport in soil and from the physical principles and/or experimental observations it should be possible to build a mathematical model of some kind.

The students are promised that they will return to these examples at various points in the mathematics course and are invited to look back at the hand-out from time to time in order to decide whether their mathematical skills and knowledge are currently sufficient to make a start on solving each problem. They are told to think about Example 14, discuss it amongst themselves and be prepared to re-consider it during the last tutorial of term.

In Section 6.2, some ideas on teaching and assessing modelling proposed by other authors are discussed. Section 6.3 describes a shared lecture which aims to illustrate the interaction between engineering and mathematics that takes place in the development of a model. Section 6.4 is devoted to a typical tutorial session which considers Example 13, whilst Section 6.5 is a representation of a longer-term tutorial discussion on a problem from the steel industry. Finally, Section 6.6 debates the rôle of modelling in an undergraduate engineering mathematics course, given the constraints that exist and are likely to exist.

### 6.2 Some Views on Modelling

One of the major influences on the teaching of university mathematics in the last two decades has been the Open University. Among the courses which emphasised the modelling aspect of mathematics was the second level course TM 281 'Modelling by Mathematics', which was introduced in 1977. Blackburn (56) described his view of the thinking that went into the design of this course. He, too, was critical of the artificiality of traditional applied mathematics teaching, citing the examination questions that were set as typical villains of the piece; they provided key words and conventions to allow the candidates to set off on a well-signposted path to the solution of the problem. The course TM 281 was designed to develop skills other than sustained analysis, for example the process of selecting relevant information from that available and the maintaining of an awareness of alternatives and the possible consequences of these alternatives.

Blackburn believed that problems should be set which had no unique solution. Suppositions would have to be made and seen not as truths but as reasonable statements about a situation. Should they lead down a false trail, then some other set of suppositions would have to be tried. He drew a distinction between "suppositions", which were the statements of a mathematical model, and "assumptions" which related to the real world and carried the possibility of truth. Assumptions were excluded therefore from the modelling process. He acknowledged that, in order to make time for the development of modelling skills the students had to be provided with a handbook containing standard formulae, thus eliminating the need to learn the derivation of basic results.

In 1985 the course was revised as TM 282 - 'Modelling With Mathematics. An Introduction'. It comprises 16 units whose titles are

1 Modelling with linear models
2 Non-linear models
3 Modelling position
4 Modelling motion
5 Rates of change
6 Growth and decay
7 Circular Motion
8 Modelling with rates of change
9 Adding things up
10 Using integration
11 Differential equations and integration
12 Modelling with integration
13 Modelling with differential equations
14 Growth, decay and oscillation 1
15 Growth, decay and oscillation 2
16 Revision: modelling heat

An earlier course, MST 281, featured a unit on distillation which the author had helped to make. This aimed to show how mathematics helped one to understand
the behaviour of a distillation column, leading to the design of a column to perform a specified separation of two components. The ideas behind that unit appear in expanded form in an article by Crilly, Kropholler and Mustoe (144).

Another course which merits mention here is the second-level inter-faculty course MST 204 which began in 1982. It is entitled 'Mathematical Models and Methods' and is divided between the development of mathematical methods and mathematical modelling. Berry and O'Shea (193) described the assessment procedures that were used to grade student performance on the modelling exercises that formed part of the course. Hall (56) also reports on the difficulties of applying a formal marking scheme to modelling exercises.

Murthy and Page (194) presented details of the 'Mathematical Modelling' course that they had introduced into the undergraduate curriculum in Mechanical Engineering at the University of Queensland. They stressed the need for a studio approach to teaching in addition to the traditional lecture. The lecture programme included examples of different mathematical formulations as well as highlighting the various stages of the modelling process. Five assignments of increasing complexity were set in parallel with the lectures; the fourth was designed to examine a range of physical systems which enjoyed the same kind of mathematical formulation for example parabolic partial differential equations, while in the last assignment the students were asked to conduct a critical examination of different models of the same physical system, for instance pollution in a river. Assessment was based on reports on the assignments and on participation in tutorials and seminars. The students believed that they had improved their critical thinking, enhanced their confidence to venture into self study, gained more idea of the use of mathematical models and made more use of library facilities as a result of following the course.

James and Wilson (195) described a course of 90 hours duration which they had operated in the second year of an undergraduate mathematics degree programme at Lanchester Polytechnic. They highlighted the difficulties which they
had experienced in operating the course and suggested ways to overcome these problems. They were adamant that such a course was not the place to introduce new mathematical concepts and stressed the need for using models which could be studied with the current mathematical knowledge of the student.

The introductory section of their course was concerned with identifying the variables to be used and the processes which connected them, representing the situation as a model and appraising (briefly) the suitability of the model. The major stumbling block was usually the identification of a process and its representation in mathematical form. They believed that it was necessary at the outset of a modelling course to describe a number of processes for which the students are required to produce mathematical expressions; then the problem of fitting the processes/expressions to form a mathematical model should be considered.

Shortly afterwards, James (68) advocated the inclusion of mathematical modelling in the engineering undergraduate curricula. He recalled that the OECD report of 1966 had found two roles for mathematics in the education of engineers, namely the provision of the necessary skills for the determination of quantitive information about natural phenomena and the provision of an experience in rational and logical thinking. James had been a co-organiser of a series of national workshops on the teaching of mathematical modelling and he believed that every potential teacher of modelling should attend such a workshop.

He fully recognised the time constraints imposed on the lecturer in mathematics for engineers. He called upon engineering staff to recognise the need for their students to become competent mathematically and to develop modelling skills. These staff should emphasise modelling in the teaching of their own subjects. He commended the comment of Murthy and Page (194) that it was essential to equip new engineers with the tools necessary to tackle ever more complex systems and that one such tool, with its intrinsic flexibility and economy was mathematical modelling. James warned against concentrating on passive
models at the expense of the more difficult modelling approach.

He suggested that in the first two years of the engineering courses continued use should be made of standard models to reinforce the role of mathematics in engineering. User-friendly software should be introduced and used by the students in practical sessions. During the first year any deficiencies that students had in the understanding of the fundamental principles of algebra and calculus should be remedied. In the second year an introduction to modelling skills should be provided with short assignments being carried out on a group basis. Simulation packages could play a useful part at this stage.

James made a plea for the inclusion of a course on mathematical modelling of engineering systems to be part of the third year curriculum. This course should be jointly taught by mathematics and engineering staff. (It is comforting to note that James endorsed the teaching methods that had been employed by the author for several years.)

James and Wilson (116) had incorporated the use of microcomputers into their modelling course. Like the author of this thesis they had found the mainframe unsuitable for their purposes, but they were disappointed by the lack of multi-purpose packages for the micro. However, more recently they had adopted the package TUTSIM with the Apple II microcomputer; this is a package for simulating continuous dynamical systems which are modelled using block diagrams or bond graphs. They found that it was very easy to use, totally interactive and had full numeric or graphic output to both screen and printer; it was also relatively inexpensive. The package had allowed James and Wilson to develop modelling exercises within the classroom in a genuine interactive manner.

Their students could use the package when investigating systems either individually or in small groups and this removed the need to write their own programs. They spent their time more effectively extracting information from the
model, generating results and assessing their validity. This philosophy is akin to that expounded in Chapters 4 and 5. James and Wilson warned that students may not appreciate that the package is there merely to give them a powerful number cruncher; they must be told that the package should not be seen as a replacement for the processes of model formulation, consideration and enhancement. Partly for this reason they advised postponing the introduction of such a package until the later stages of a modelling course.

Oke and Bajpai (196) were among those who reported on the difficulty of teaching the formulation stage of modelling. Rubin (197), referring to the stage as 'system realisation', suggested the following procedure.

1 Identify the basic components of the modelling problem, viz. information, questions, evaluation criteria.
2 Formulate objectives.
3 Produce a list of variables used in stating the objective.
4 Determine what types of information are required. Introduce new variables as necessary.

5 Identify the components which are described by the variables.
6 Simulate the phenomenon in a diagram, adding new variables as necessary.

7 Continue the previous step.
8 Examine the list of variables for inconsistencies and redundancies.
9 Remove the inconsistencies and redundancies.
10 Eliminate inconsistent or redundant information.

It is difficult to believe that these 'ten commandments' can be carried out in the order suggested, but the list does give the teacher a feel for the difficulties likely to be encountered.

Morris (198) offered some general advice to the would-be modeller.
1 Factor the problem given into simpler problems.
2 Establish a clear statement of the objectives.

3 Seek analogies.
4 Consider a specific numerical instance of the problem.
5 Establish some symbols.
6 Write down the obvious.
7 If a tractable model is obtained, enrich it; otherwise simplify it.

Should simplification be necessary at stages 1 or 7 Morris provides a list of possible actions:
(i) Make some variables into constants.
(ii) Eliminate some of the variables.
(iii) Use linear relations.
(iv) Add stronger assumptions and restrictions.
(v) Suppress randomness (in a stochastic problem).

### 6.3 Rainfall and Runoff: A Shared Lecture

In the second term of the mathematics course given to first year Civil Engineering undergraduates the methods of separation of variables and integrating factor are covered as part of the package on ordinary differential equations. In the same week a shared lecture is given in the form of a dialogue between the author and one of his Civil Engineering colleagues. What follows is an abridged version of the lecture: the author is denoted by $M$ and his partner by C.
(i) The hydrological background
(C) One of my jobs in industry was to help design a flood control scheme. The idea was to relate the rainfall which fell on a catchment area to the subsequent runoff so that we could predict the outcome of a storm and take any appropriate action before flooding occurred.
(M) Could you please explain the terms 'catchment area' and 'runoff'?


Figure 6.2
(C) By 'catchment area', I mean the land and water surface area which contributes its rainfall to the outflow at a particular point on a river or stream. The further we move downstream the larger the catchment area becomes (See Figure 6.2).

Some of the rainfall may be interrupted by vegetation or by roofs, pavements and other artificial surfaces; some of the rainfall will evaporate; some will infiltrate the soil; some may be stored in topographical depressions. The remainder of the rainfall flows over the surface to the nearest stream; this surface runoff can be very important, especially during violent storms when it becomes large and may cause flooding. It is clear that there is some cause-and-effect relationship between rainfall and runoff, but the complexity of the problem makes the nature of the relationship such that a physical representation is very difficult. There are catchment size, shape, slope, altitude and drainage pattern to be taken into account; the type of vegetation cover of the area is important - and this may depend on the season. Then the condition of the catchment is important - whether it is wet or dry when the rainfall occurs. And then . . .
(M) Hold on. If the problem is so complicated why don't you simply measure the runoff directly? After all, even if you measure rainfall it's only at one point and you would really need to measure it at several points in the catchment area to get a representative picture of rainfall over the area as a whole.
(C) It's mainly a question of cost. To purchase and install a recording rain gauge costs about $£ 250$, whereas the cost for a river gauge to measure runoff on a large river could well be of the order or $£ 40,000$. And then you must take into the fact that in this country we have very good rainfall records; by comparison, runoff records are very sparse. So we need a way of relating rainfall to runoff. There are several graphical or empirical methods but they're not really satisfactory. You're a mathematician; can't you help?
(ii) A simple model
(M) I'll have a go. If we can formulate a mathematical model for the system we might be able to make some useful predictions. We could test the model with some known rainfall and resulting runoff data and, providing that we find it to be satisfactory, we can 'calibrate' it. Then it will be ready for us to use.
(C) I've read something about such models. In essence, they regard the catchment as a storage area; the rainfall is the input and somewhat later this water, in part, appears as runoff. Some of the water appears relatively soon after it has fallen and some, with a longer journey to make, appears later. What I really want from you is a means of determining what runoff will result from a particular storm rainfall.
(M) I want to start with a very simple model. It might do the trick and that will save us a lot of unnecessary work. If not, then we can see where its shortcomings are and make appropriate adjustments. Can you give me some physical principles which I can put into mathematical terms?
(C) We can use the hydrologist's storage model which states that the input rate less the output rate is equal to the rate of change of storage in the catchment. Also, we can assume that the volume rate of outflow is proportional to the amount of water currently in storage.
(M) My first attempt at a model for the catchment is a tank of constant cross-section area which has an outlet at the bottom. If I assume an initial head of water of $h_{0}$ then I can predict how the head varies with time after the outlet is opened. Let . . .
(C) It doesn't seem very realistic to me. On a real catchment the rainfall would be of variable intensity over a finite time. What you're doing in effect is to drop a lump of water on the catchment all at once.
(M) I did explain that I was going to start with a simple model; too many complications at the outset could lead us up a blind alley. Let us consider the tank shown in Figure 6.3. It is cylindrical with cross-sectional area $A$. The input to the tank measured as a volume rate is $I$ and the volume rate of outflow is Q. Both these rates, in general, will be functions of time. (I am setting up a general model, even though I shall start with $I=0$.) Let the amount of water in store at time $t$ be $S(t)$ and the head at that time be $h(t)$.

Then we have the relationships

$$
\begin{equation*}
S=A h \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
1-Q=\frac{d S}{d t} \tag{2}
\end{equation*}
$$

Your assumption that the rate of outflow is proportional to storage can be written

$$
\begin{equation*}
Q=k S \tag{3}
\end{equation*}
$$

where $k$ is a constant for the tank.

In our model we have one independent variable, $t$, and three dependent variables, $h, S, Q$; in addition we have two parameters $k$ and $A$. We have a fourth dependent variable, $I$, but we intend to put $I=0$ at the first stage of our analysis.




Figure 6.3

I assume that $h=h_{0}$ at $t=0$ and that there is no subsequent input to the system. Then using (1) we obtain

$$
\frac{d S}{d t}=A \frac{d h}{d t}
$$

and via (2) we arrive at the differential equation for the head, namely,

$$
\begin{equation*}
\frac{d h}{d t}=-k h \tag{4}
\end{equation*}
$$

We can solve (4) via separation of variables to obtain the formulation

$$
\begin{equation*}
h=h_{0} e^{-k t} \tag{5}
\end{equation*}
$$

The graph of this relationship is shown in Figure 6.3.
(C) I'm afraid that this is no use to me. The shape of the outflow that I had expected is shown in Figure 6.3. Since $Q=k S=k A h$ we see that $Q$ is proportional to $h$ and your graph should be of a similar shape.
(iii) Revisions to the model
(M) I should have been very surprised if our simple model was adequate. We must not give in so easily. Perhaps we need a more sophisticated way of modelling the delays in water reaching the point of interest. I don't want to abandon the tank idea just yet. Suppose we let the output from the tank become the input to a second, identical tank and study the outflow from this latter tank. In effect, we are providing an extended storage. Since the outflow from the first tank, viz.

$$
\begin{equation*}
Q_{1}=k A h_{0} e^{-k t} \tag{6}
\end{equation*}
$$

is the inflow to the second tank then (2) applied to the latter takes the form

$$
\begin{align*}
& k A h_{0} e^{-k t}-k A h=a \frac{d h}{d t} \\
& i e \frac{d h}{d t}+k h=k h_{0} e^{-k t} \tag{7}
\end{align*}
$$


(INTEGRATING FACTOR)

$$
Q_{2}=Q_{0} k t e^{-k t}
$$

Figure 6.4

We can no longer use separation of variables to solve this equation and we resort to the integrating factor technique. For (7) the integrating factor is $e^{k t}$; applying the technique and noting that at $t=0, h=0$ we obtain the formula for the head in the second tank:

$$
\begin{equation*}
h=h_{0} k t e^{-k t} \tag{8}
\end{equation*}
$$

A graph of this relationship is shown in Figure 6.5. The peak value occurs at $t=\frac{1}{k}$.
(C) That's better, but it's not quite right yet. Could we pursue this idea further? What would happen if the outflow from the second tank was allowed to leak into a third, identical tank? (Why do the tanks in your example have to be identical, anyway?) Could you work out for me the outflow from the third tank?
(M) I should have remarked that from (8) I can deduce the outflow from the second tank by multiplying the formula for $h$ by kA. Incidentally, we use identical tanks so that we are dealing with two parameters only, viz. $k$ and A. As for a third tank, it is straightforward to show that the head is governed by the equation

$$
\frac{d h}{d t}+k h=h_{0} k^{2} t e^{-k t}
$$

with the initial condition that $h=0$ at $t=0$. The integrating factor method can be used on this equation to obtain the solution

$$
\begin{equation*}
\omega_{s} h=h k^{2} \frac{t^{2}}{2} \cdot e^{-k t} \tag{9}
\end{equation*}
$$

Whilst we are on this tack we might as well produce the general formula for the head in the last tank of a series (or cascade) of $n$ tanks. Using the principle of mathematical induction we can show this to be

$$
\begin{equation*}
h=h_{0} k^{n-1} \frac{t^{n-1}}{(n-1)!} e^{-k t} \tag{10}
\end{equation*}
$$



$$
Q_{n}=Q_{0} \frac{(k t)^{n-1}}{(n-1)!} e^{-k t}
$$

## WHAT ARE VALUES FOR $n \& k$ ?

Figure 6.5

Since you want the outflow from this tank we multiply (10) by $k A$ to obtain

$$
\begin{equation*}
Q_{n}=h_{0} A k^{n} \frac{t^{n-1}}{(n-1)!} \tag{11}
\end{equation*}
$$

This has a shape similar to that of Figure 6.5. The peak occurs at $t=(n-1) / k$. As $n$ is increased the peak moves to the right and becomes flatter.
(C) This is looking more promising. I suppose if I choose the values of n and k to suit my catchment area I could get a reasonable model. But how can I choose them? I will need two equations from which to determine $n$ and $k$, won't I? Where are they coming from?
(M) Well, we could take the first and second moments of area of both your curve and my curve about the vertical axis and equate them. But first I want to standardise matters. If we consider the outflow from the $n^{\text {th }}$ tank due to a unit amount of storage in the first tank (that is, $A h_{0}=1$ ) then we obtain the formula

$$
\begin{equation*}
Q_{n}=k^{n} \frac{t^{n-1}}{(n-1)!} e^{-k t} \tag{11a}
\end{equation*}
$$

(C) We call this arrangement an Instantaneous Unit Hydrograph (IUH) - I've seen it in the literature.
(M) You see, if we had an initial storage of I in the first tank we simply scale the IUH by a factor of I . Now I can show by simple calculus that the first moment of the IUH about the vertical axis is

$$
M_{1}=n / k
$$

and the second moment is

$$
M_{2}=n(n+1) / k^{2}
$$

All I used was integration by parts and a simple reduction formula. Then we can work out the first and second moments of the "observed" curve using numerical methods and we have the necessary two equations.
(C) Just a moment. In a real rainstorm we have a variable intensity over a finite time, not a lump of rain falling at one instant. We'd better examine the assumptions you've made. If you remember, we said at the outset that not all the rainfall that occurred ended up as surface runoff; if we are comparing theory with reality we must make sure that my experimental graph does relate only to runoff. Now we really can't take one storm as typical and I would need some average over a number of storms. Also, the characteristics of the catchment may vary with the seasons and that will affect your choice of $n$ and $k$. The storm itself has an effect on the catchment response: a high intensity storm of short duration will produce a greater fraction of surface runoff. And then there's the fact that the storm is a variable intensity, finite duration affair. That really bothers me.
(M) I accept that we've got some difficulties to iron out. However, no model is perfect, and to expect to get a near-perfect model in under an hour is asking a lot. There is a division of labour at this stage. As a hydrologist, you should collect more data. If for each catchment area and season and type of storm you can give me some observed rainfall and runoff, I can supply the best IUH under my assumptions. It's my job now to explain your big worry, namely the storm of finite duration and variable intensity. One thing I should point out here: the data you collect would be most suitable if the storm were of reasonably constant intensity and uniformity over the catchment area.
(C) That would mean restricting our observations to areas of less than 2000 square miles.
(M) Look at Figure 6.6(a); it represents the intensity of a particular rainstorm.


Figure 6.6

Now consider that portion of rain which falls in the very small time interval $[\tau, \tau+\delta \tau]$. Obviously, it cannot contribute to the runoff until time $t>\tau$. If we break up the intensity-time graph into a set of narrow strips then each strip makes its contribution starting from different times. Each "strip of rain" can be considered to be instantaneously dumped on the system. The volume of rain represented by the strip can be combined with the IUH for the catchment to give its contribution to the runoff; however that IUH will be shifted to the right so that it begins at $t=\tau$ as shown in Figure 6.6(b). Then at any time $t$, after the start of the storm, the runoff is the sum of all such contributions for those parts of the storm which occurred at $t_{1} \geq \tau$; see Figure 6.6(c).

The contribution of the rainfall element in $[\tau, \tau+\delta \tau]$, which has volume $\mathrm{I}(\tau) \delta \tau$, to the outflow is $\mathrm{I}(\tau) . \mathrm{u}(\mathrm{t}-\tau) \delta \tau$. Hence, total outflow is given by

$$
\begin{equation*}
Q(t)=\int_{0}^{t_{1}} I(\tau) \cdot u(t-\tau) d \tau \tag{12}
\end{equation*}
$$

(C) Wait. You've lost me. Could you go over that again?
(M) Sorry. The IUH which starts at $t=0$ has equation $u(t)$. Now the one which starts at $t=\tau$ has equation $u(t-\tau)$; you see, at $t=\tau$ this latter curve has value $u(0)$ etc. Hence the contribution to the outflow is $I(\tau) \cdot u(t-\tau) d \tau$ and since we are dealing with continuous functions we sum the individual contributions by integration to obtain formula (12).
(C) That's interesting; when I worked in industry we did something like that, but we didn't use integration. We broke the $l(t)$ and $u(t)$ graphs into strips and combined them by a tabular method.
(M) You were really using a similar method to me, but summing discrete lumps instead of integrating continuous functions. There's nothing wrong with a tabular method if its accuracy is suitable for your purposes but you were using it without really understanding what you were doing. Now you know the background you can can use the tabular method more intelligently.

## (iv) A practical application of the model

(C) I've been a little unfair on you. Engineers who were designing a flood control scheme for the River Dee did use your leaking tanks idea as a basis for a mathematical model they built. Here is a schematic diagram for their model: (Figure 6.7). It's somewhat more complicated than our model.
(M) That's not surprising. I see that there are several linked subsystems and that the tanks can leak from the sides as well as the bottom. How do they use the model?
(C) When a severe rainstorm falls on the catchment area the relevant rainfall data is fed into the computer model. It can then produce predictions of the maximum depth of the river and its tributaries at several critical points. If any of these are greater than pre-determined values, flood alleviation schemes are put into operation. Since these schemes cost ratepayers' money to implement they are not put into operation unless there is a real need.
(M) So mathematics has been used in practice. We can't go into the full details of the River Dee model, but we can at least appreciate that the model has been founded on a set of assumptions and is only as good as those assumptions. A realistic model will take many man-hours to construct and undergo many revisions before being used in practice. Mistakes at that stage are very expensive.


Figure 6.7
(C) We hope that this dialogue has given you an insight into the interplay between mathematics and engineering that takes place when we seek to solve an engineering problem by constructing a mathematical model.

## (v) Student reaction

Feedback from the students in the form of verbal comments to the author and responses to a short questionnaire indicates that the exercise was worthwhile. They found that the dialogue approach retained their attention and were interested in the real-life example at the end of the lecture. They had grasped the essential structure of the modelling process but were definitely of the opinion that they could not have carried out the formulation themselves. 'We could never have thought of using a leaking tank as the basis of our model' was the tenor of the responses.

### 6.4 Reservoir Contamination: A Tutorial Session

As has been mentioned in the previous section, first year Civil Engineers study the method of separation of variables for the solution of first order ordinary differential equations during their second term. In the tutorial session immediately following that topic they are invited to consider the following problem.

A reservoir of capacity $10^{6}$ litres has been contaminated by effluent. The degree of contamination is $0.02 \%$ by weight. The average daily rate of consumption of water is $2 \times 10^{4}$ litres and this is continuously replaced by pure water. How long will it be before the concentration of contaminant drops to the safe level of $10^{-5} \%$ ?

What follows is a description of an imaginary session distilled from the many that have taken place. Bearing in mind that this is the first such session the students have had, the author took more of a lead in the discussion than he would
expect to do later in the course.

Having posed the problem which was left in view on the blackboard throughout the session, the author then asked the students to comment on the information provided. Was it sufficient? Was it in a suitable form? Was any of it redundant? What did the degree of contamination mean: average, maximum or some other statistic? If it was an average figure, how useful is the value quoted? What does 'continuously replaced' mean?

At some stage one of the students will usually complain that the problem cannot be solved. Then he will be asked to explain what "solved" means and hopefully he or another student will propose that we could at least suggest an order of magnitude for the answer requested. If there is still discontent then the students must be reminded that they are training to be engineers who have to work in the real world and an order-of-magnitude answer is better than no answer at all. As a first step the class is invited to make a guess as to the answer expected. The next task is to talk about the way we might set about solving the problem. If mathematics is to be used then we shall need some variables and some relationships connecting those variables. It is suggested by the students that the "obvious" variables are concentration of contaminant, $c$, and time, t. Having made clear the distinction between dependent and independent variables the students are then asked to identify the parameters they will use: capacity (or volume) of the reservoir, $V$; average daily rate of consumption of water, $r$. Other information in the problem is the initial concentration of contaminant, $c_{0}$ and the required safe level, $\mathrm{C}_{\mathrm{f}}$. 'What about the replacement "pure" water?' asks the author to be told that "obviously" it contains no contaminant.

How then to find relationships connecting the variables and the parameters? The author draws a box to represent the reservoir and asks the input to it and the output from it (Figure 6.8(a)). It is agreed that the input has zero concentration of contaminant but what about the concentration of the

(a)

(b)

Figure 6.8
output? One student will volunteer the suggestion that it will be the same as currently in the reservoir as a whole; another will ask why, a third will say that it will not be the same. On a good day the proponent of the suggestion will round on his critics and ask them to put forward an alternative.

As no such alternative is forthcoming the group settles for the original suggestion; the diagram is completed as in Figure 6.8(b). Now the students are asked whether the choice of concentration as the dependent variable was a wise one in view of the next stage, namely finding a relationship to connect the variables. The alternative selection of mass of contaminant is usually suggested fairly readily.

The students will have met the idea that for such an input - output situation a useful starting-point is the relationship
rate of input - rate of output $=$ rate of increase of storage

Almost without exception, the fact that the students are currently covering differential equations in lectures leads to the search for a model based on a differential equation. At time $t$ the mass of contaminant in the reservoir is $V . c(t)$ and therefore the rate of increase of that mass is

$$
\begin{equation*}
\frac{d}{d t}(V c(t)) \equiv V \frac{d c}{d t} \tag{2}
\end{equation*}
$$

There is a potential pitfall here: in the terms of the problem given the mass of contaminant clearly decreases as time elapses. However, it is emphasised at this point in the development of the model that equation (1) has been stated in such a way that the right-hand side can be replaced by the formula in (2).

It has already been agreed that the concentration of contaminant in the water leaving the reservoir at time $t$ is also $c(t)$, but it is not obvious what should
be the units of the rate of output. The information provided in the statement of the problem gives a global change over one day, whereas the use of $\mathrm{dc} / \mathrm{dt}$ is firmly rooted in the students' minds as representing a continuous change. The class agrees to consider the changes that take place in one day, in effect taking $\delta t=1$ day. The proviso is made that if the answer to the given problem is a few hundred days then this approach is perhaps acceptable, but not if the answer is an order of magnitude smaller.

In the course of one day the fraction $2 \times 10^{4}+10^{6}$, ie a fiftieth, of the total volume of the reservoir is removed and replaced. Under the assumptions made so far the mass of contaminant removed is $(1 / 50) \mathrm{V} . \mathrm{c}(\mathrm{t})$; since the incoming water is free from contaminant, no mass is input. Hence equation (1) becomes

$$
\begin{align*}
& 0-\frac{1}{50} V c(t)=V \frac{d c}{d t}  \tag{3}\\
& \text { ie } \frac{d c}{d t}=-\frac{c}{50}
\end{align*}
$$

The general solution of this equation is

$$
\begin{equation*}
c=c_{0} e^{-\frac{1}{50}} \tag{4}
\end{equation*}
$$

where $c_{0}$ is the initial concentration of contaminant. We are interested in how long it takes the concentration to fall to a specified level, $\mathrm{c}_{\mathrm{f}}$. Then the time is obtained from

$$
\begin{gather*}
c_{f}=c_{0} e^{-\frac{t}{50}} \\
\text { ie } t=50 \ln \left(\frac{c_{0}}{c_{f}}\right) \tag{5}
\end{gather*}
$$

At this stage the attention of the students is drawn to the fact that the time depends on the ratio of the initial to the "final" concentration. Hence, if it takes $n$ days to halve the concentration it will take $2 n$ days to reduce it by a factor of four,
and so on.

Substituting the data values into (5) gives $t=380$ to the nearest day. (It is quite usual for two or more different answers to be given by the students.)

The students generally accept that the selection of $\delta t=1$ day was justified. The author asks what would happen if, instead, $\delta t=0.5$ day. Then, during the course of that half-day $1 / 100$ of the current mass of contaminant is removed so that equation (1) becomes

$$
\frac{d c}{d t}=-\frac{c}{100}
$$

and hence we obtain

$$
c=c_{0} e^{-t / 100}
$$

Substituting the data values we obtain $t=760$ to the nearest integer. Since our units of time are half-days, we recover $t=380$ days, as before.

The students are then asked what would happen if $\delta t$ were chosen as 0.1 day and it is accepted that this would have no effect on the final answer; indeed, as the model has been constructed, any $\delta t$ less than one day leads to the same result; we cannot increase the accuracy of the answer by reducing $\delta t$. This comes as a surprise to most of the students who have been used to the concept of reducing the step size to increase the accuracy.

Before any complacency can set in, the students are reminded that equation (3) was something of a fudge since it was a continuous model applied to a non-continuous form of data. It is proposed that it might at the least be a reasonable alternative to take a day-by-day approach throughout and let $m_{n}$ be the mass of contaminant in the reservoir at the end of day $n$. What then can be said about $m_{n+1}$ ? A long silence is usually the response and therefore the author
proposes a modification to equation (1) in the form
daily input - daily output $=$ change of storage in a day

Now the amount of contaminant removed during day $n$ is

$$
\begin{align*}
& \frac{1}{50} m_{n-1} \text { so that (7) becomes } \\
& -\frac{1}{50} m_{n-1}=m_{n}-m_{n-1} \\
& \text { or } m_{n}=\frac{49}{50} m_{n-1} \tag{8}
\end{align*}
$$

It is pointed out that if one part in fifty of the contaminant is removed then $49 / 50$ will remain, and this is what ( 8 ) is saying. This is a difference equation, which is a new concept for the students.

It is usually necessary to ask for the relationship between $m_{1}$ and $m_{0}$, viz.

$$
m_{1}=\frac{49}{50} m_{0} \text { and then }
$$

between $m_{2}$ and $m_{0}$ via

$$
m_{2}=\frac{49}{50} m_{1}=\left(\frac{49}{50}\right)^{2} m_{0}
$$

before the solution to (8) is obtained as

$$
\begin{equation*}
m_{n}=\left(\frac{49}{50}\right)^{n} m_{0} \tag{9}
\end{equation*}
$$

Hence

$$
\frac{m_{0}}{m_{n}}=\left(\frac{50}{49}\right)^{n}
$$

and

$$
n=\frac{\ln \left(\frac{m_{0}}{m_{n}}\right)}{\ln \left(\frac{50}{49}\right)}
$$

(This last equation takes some teasing out of the students.)

What is to be done with $m_{0} / m_{n}$ ? It is usually quickly suggested that

$$
\frac{m_{0}}{m_{n}}=\frac{c_{0}}{c_{f}}=300
$$

and n is obtained, not without difficulty, as 376 , to the nearest integer.

This value compares favourably with the value 380 obtained using the differential equation model. Then the students are asked the question as to which of the two models - that employing a differential equation and that based on a difference equation - is the more appropriate. Indeed, is one model an approximation to the other or are both equally approximate statements of the truth? The students are reminded of all the assumptions that were made in formulating either model, and, therefore, how accurate the answer is likely to be.

Whilst the students are pondering the answer, they are asked to suppose that the problem was set by their supervisor at work who wanted some advice to help him to decide what action to take. What, then, should the advice be? Generally, the students combine to suggest that the reservoir will take over a year to recover, even under the very favourable assumptions that have been made in formulating the model. The author then comments that this might be considered an inordinately long time and consequently other, more immediate action would have to be taken. Therefore the model has been used to provide background information to someone who has other considerations to take into account.

## Student reaction

It is evident that many students do not like this kind of tutorial session. There is a feeling amongst many of them that they come to tutorials to get the lecturer to do some work for them by helping with the difficulties they have encountered on the problems set by the lecturer. They resent to some extent having to do the work themselves in the way outlined earlier.

### 6.5 Furnace Bumper Problem: A Modelling Exercise

In the third term of the first year course in Modelling and Simulation given to Mathematical Engineering undergraduates the following exercise in modelling is conducted. The students are divided into two groups of about 9 and are taken through the exercise over a period of two weeks, spending about six timetabled hours, in addition to their own time. In this section the problem will first be described; then an outline solution will be given, with points of interest from a teaching standpoint being highlighted. Next, a typical run-through will be described and, to conclude, an assessment of the exercise will be carried out.

## (i) A description of the problem

Part of the process of producing plate steel consists of re-heating steel slabs in a furnace and discharging them onto a roller table which takes them to a plate mill. It is necessary to slow down the slab after it leaves the furnace so that it can be conveyed in a direction at right angles to its direction of discharge. One such scheme that was used in practice was to allow the slab, of typical mass 18 tons ( $\approx 18,300 \mathrm{~kg}$ ), to hit a horizontal bumper, of typical mass 2 tons; the bumper was supported by six steel cables attached to two weights of mass $30-60$ tons which rested on slopes inclined at $45^{\circ}$ to the horizontal. The cables passed through slits in the surface along which the slab and bumper moved. The system is shown schematically in Figure 6.9. When the speed of the slab had been slowed sufficiently, the rollers were activated and the slab moved off to the mill.


Figure 6.9

However, the steel cables broke earlier in their working lives than was expected. The requirement is to model the system and to suggest what modifications, if any, might alleviate the problem of early failure of the cables.

## (ii) Outline analysis of the problem

Before embarking on a mathematical analysis, it is wise to think qualitatively about the stages of motion. There is a danger that in this early stage of the modelling process, the initial phase (before the slab hits the bumper) is ignored. At time $t=0$ the slab will make contact and the combined masses will move, stretching the cables; in this phase the masses on the inclined planes remain stationary. At the end of this phase, the tension in the cables will be increased to the stage where the static friction force is overcome and the masses on the inclined planes are on the point of moving.

In the next phase the masses move up their planes until either they or the combined mass of slab and bumper comes to rest. The mass at rest may begin to move again or remain stationary, depending upon the magnitude of the tension in the cables.

If the slab and bumper move back through the equilibrium position the slab will separate from the bumper, and will move back towards the furnace under the retardation of friction.

It is worthwhile examining the equations of motion at a general stage in the process when all the masses are moving. Because of the symmetry in the problem we can consider that half of the system which is depicted in Figure 6.10, bearing in mind that the masses of the slab and bumper are, in effect, halved; however, we shall consider the full system when formulating the equations. If we apply Newton's second law of motion to the combined mass we obtain the equation

$$
\begin{equation*}
\bar{M} \ddot{x}=-2 N T \sin \alpha-\mu_{x} \bar{M} g \tag{1}
\end{equation*}
$$



Figure 6.10
where $N$ is the number of cables connecting each mass on the inclined planes to the combined mass of slab and bumper, $\mu_{X}$ is the limiting coefficient of friction and

$$
\begin{equation*}
\alpha=\tan ^{-1}\left(\frac{2 x}{L}\right) \tag{1a}
\end{equation*}
$$

Similarly, for the mass on the plane

$$
\begin{equation*}
M \ddot{y}=N T-M g \frac{1}{\sqrt{2}}\left(1+\mu_{y}\right) \tag{2}
\end{equation*}
$$

We assume the stress-strain relationship

$$
\begin{equation*}
\sigma=E \varepsilon \tag{3}
\end{equation*}
$$

where $\sigma$ is the stress in a cable, $\varepsilon$ is the strain produced and $E$ is Young's modulus of elasticity for the material. Now, from Figure 6.10 we can see that the strain is given by

$$
\begin{equation*}
\varepsilon_{1}=\frac{\left[\frac{1}{2} L(\sec \alpha-1)-y\right]}{L} \tag{4}
\end{equation*}
$$

However, the cable cannot be compressed (it will go slack) and we wish to avoid negative values being provided by (4). Accordingly we define the actual strain as

$$
\begin{equation*}
\varepsilon=\frac{1}{2}\left(\varepsilon_{1}+\left|\varepsilon_{1}\right|\right) \tag{5}
\end{equation*}
$$

Then, combining (3), (4) and (5) together with the assumption of constant cross-section area $A_{0}$ for each cable we obtain

$$
\begin{equation*}
T=\frac{1}{2}\left(T_{1}+\left|T_{1}\right|\right) \tag{6}
\end{equation*}
$$

where

$$
\begin{equation*}
T_{1}=K\left(\frac{L}{2}(\sec \alpha-1)-y\right) \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
K=\frac{E A_{0}}{L} \tag{7a}
\end{equation*}
$$

In order to allow the use of these equations in the general situation we need to specify the coefficients of friction, assumed to be equal, via

$$
\mu_{x}=\left\{\begin{array}{cc}
\mu, & \dot{x}>0  \tag{8a}\\
-\mu, & \dot{x}<0
\end{array}\right.
$$

and

$$
\mu_{y}=\left\{\begin{align*}
\mu, & \dot{y}>0  \tag{8b}\\
-\mu, & \dot{y}<0
\end{align*}\right.
$$

It is further assumed that the limiting value of the coefficient of static friction is equal to the value of the coefficient of dynamic friction.

The system is determined by equations (1), (2), (6), (7), (7a), (8a), (8b), with appropriate initial conditions.

## (iii) Stage by stage analysis

## Initial phase

Figure 6.11 shows the system prior to the slab making an impact with the bumper. If we resolve the forces acting on the mass $M$ in the direction up the slope we obtain

$$
N T_{0}+\mu M g \frac{1}{\sqrt{2}}=M g \frac{1}{\sqrt{2}}
$$

where $T_{0}$ is the initial tension in each cable which just prevents the mass $M$ from sliding down the slope.
Then

$$
T_{0}=\frac{M g}{\sqrt{2} N}(1-\mu)
$$



Figure 6.11

The strain, $\varepsilon$, is given by the formula

$$
\varepsilon=\frac{\left[\left(\frac{1}{2} L-y_{0}+\frac{1}{2} L\right)\right]}{L}=\frac{-y_{0}}{L}
$$

so that

$$
T_{0}=-\frac{A_{0} E y_{0}}{L}
$$

therefore

$$
\begin{equation*}
y_{0}=-\frac{T_{0}}{K} \tag{9}
\end{equation*}
$$

## Phase I

At time $t=0$ the slab hits the bumper. Via conservation of momentum and a knowledge of the velocity of the slab immediately prior to impact, we can calculate the common velocity $x_{0}$ after impact. During this phase of the motion the mass $M$ remains at rest. Referring to Figure 6.12 we deduce that the frictional force increases from

$$
-\frac{\mu M g}{\sqrt{2}} \text { to }+\frac{\mu M g}{\sqrt{2}}
$$

as the tension in each cable increases from $T_{0}$. At the end of this phase the tension has increased to the value where static friction has just been overcome. At this stage, let the values of $\varepsilon, x, \alpha, T$ be $\varepsilon_{1}, x_{1}, \alpha_{1}, T_{1}$ respectively. Noting that $\dot{y}=0$ we can eventually obtain the differential equation

$$
\begin{equation*}
\ddot{x}=\frac{-2 N K}{\bar{M}} x+\frac{2 N K}{\bar{M}}\left(L+2 y_{0}\right) \frac{x}{\sqrt{4 x^{2}+L^{2}}}-\mu g \tag{10}
\end{equation*}
$$

To simplify the resulting formulae, we introduce the variables

$$
A_{1}=\frac{-2 N K}{\bar{M} g}, \quad B_{1}=L+2 y_{0}
$$



Figure 6.12

Multiplying (10) by $\dot{x}$ and integrating with respect to $t$ we obtain

$$
\begin{equation*}
\frac{1}{2} \dot{x}^{2}=A_{1} \frac{x^{2}}{2} g-\frac{A_{1} B_{1}}{4}\left[\sqrt{4 x^{2}+L^{2}}\right] g-\mu x g+C \tag{11}
\end{equation*}
$$

where $C$ can be found from

$$
\begin{equation*}
\frac{1}{2} \dot{x}_{0}^{2}=-\frac{A_{1} B_{1}}{4} L g+C \tag{11a}
\end{equation*}
$$

Rewriting (11) as $\dot{x}^{2}=f(x)$, we finally obtain the formula

$$
\begin{equation*}
t_{1}=\int_{0}^{x_{1}} \frac{1}{\sqrt{f(x)}} d x \tag{12}
\end{equation*}
$$

for the time taken to complete phase $I$.
Since

$$
T_{1}=K\left[\frac{1}{2} L\left(\sec \alpha_{1}-1\right)-y_{0}\right]
$$

and

$$
T_{1}=\frac{M g}{\sqrt{2} N}(1+\mu)
$$

we can use the formula

$$
\alpha_{1}=\tan ^{-1}\left(\frac{2 x_{1}}{L}\right)
$$

to obtain $x_{1}$.
Hence we can calculate $t_{1}$ from (12) by employing, say, Simpson's rule.

## Phase II

Now both the masses are moving and $\dot{x}, \dot{y}>0$. If we introduce state variables

$$
z_{1}=x, z_{2}=\dot{x}, z_{3}=y, z_{4}=\dot{y}
$$

and constants

$$
\begin{equation*}
A=-\frac{2 N}{\bar{M} g}, B=\frac{N}{M g} \tag{13}
\end{equation*}
$$

we have the general system of equations

$$
\begin{aligned}
& \alpha=\tan ^{-1}\left(\frac{2 z_{1}}{L}\right) \\
& T_{u}=K\left[\frac{1}{2} L(\sec \alpha-1)-z_{3}\right] \\
& T=\frac{1}{2}\left(T_{u}+\left|T_{u}\right|\right)
\end{aligned}
$$

$$
z_{1}=z_{2}
$$

$$
\begin{equation*}
z_{2}=\left(A T \sin \alpha-\mu_{x}\right) g \tag{14}
\end{equation*}
$$

$$
z_{3} \quad=z_{4}
$$

$$
z_{4}=\left(B T-\frac{\left(1+\mu_{y}\right)}{\sqrt{2}}\right) g
$$

$$
\mu_{x}=\left\{\begin{array}{rl}
\mu & , \\
-\mu & z_{2}>0 \\
-\mu & z_{2}<0
\end{array} \quad \mu_{y}=\left\{\begin{aligned}
& \mu, \\
&-\mu, z_{4}^{4}<0 \\
&
\end{aligned}\right.\right.
$$

The differential equations can be solved using a Runge-Kutta fourth order scheme. The initial conditions are $t=t_{1}, z_{1}=x_{1}, z_{2}=\dot{x}_{1}, z_{3}=y_{0}, z_{4}=0$ and the step-by-step solution is continued until one of the masses $M, \bar{M}$ comes to rest.

It is useful to note that the program which is written to carry out the solution can be checked by modifying the equations to simulate phase I and compared with the results obtained via Simpson's rule.

## Subsequent phases of motion

At the end of phase II several motions are possible. If the mass $M$ has come to rest then it will move up the plane if

$$
\begin{equation*}
T>\frac{M q}{\sqrt{2} N}(1+\mu) \tag{15a}
\end{equation*}
$$

and down the plane if

$$
\begin{equation*}
T<\frac{M g}{\sqrt{2} N}(1-\mu) \tag{15b}
\end{equation*}
$$

otherwise it will remain at rest. Similarly if the mass $\bar{M}$ has come to rest it will move back towards the equilibrium position $(x=0)$ if

$$
\begin{equation*}
T>\frac{\mu \bar{M} g}{2 N|\sin \alpha|} \tag{16}
\end{equation*}
$$

otherwise it will remain at rest. It should be pointed out that the mass $\bar{M}$ cannot move further away from the equilibrium position since this would require that

$$
T<\frac{-\mu \bar{M} g}{2 N|\sin \alpha|}
$$

and since $T \geq 0$ this cannot occur.

Hence, if a mass comes to rest and the current value of T is outside the relevant bounds it will begin to move and the appropriate coefficient of friction will take that sign which causes the friction force to oppose the motion. Then a stage similar to phase II takes place. However, if the value of $T$ is within the bounds the mass will remain at rest while a stage similar to phase II occurs.

## Separation of slab from bumper

If the slab/bumper combination returns to $x=0$ then separation will occur: Figure 6.13. Applying Newton's second law to the bwmper' . we obtain

$$
M_{1} \ddot{x}=-2 N T \sin \alpha-\mu_{x} M_{1} g
$$



Figure 6.13

After separation, the value of $\dot{z}_{2}$ is

$$
\begin{aligned}
& \quad \dot{z}_{2}=\left(A_{0} T \sin \alpha-\mu_{x}\right) g \\
& \text { where } A_{0}=-\frac{2 N}{M_{1} g}
\end{aligned}
$$

The slab will move back in the direction of the furnace under the action of retardation due to friction.

## (iv) Overall numerical solution

In order that all the stages of motion can be covered by one set of equations it is necessary to make some refinements. When a mass is at rest then modifications to the two differential equations governing its motion are required. For example when the mass $M$ is at rest we will replace the relevant equations by

$$
\dot{z}_{3}=0, \quad \dot{z}_{4}=0
$$

One way of incorporating this possibility into the system is to introduce flags $G_{x}$, $G_{y}$ which take only integer values 0 or 1 . The modified system to be solved is

$$
\begin{aligned}
& \alpha=\tan ^{-1}\left(\frac{2 z_{1}}{L}\right) \\
& T_{u}=K\left[\frac{1}{2} L(\sec \alpha-1)-z_{3}\right] \\
& T=\frac{1}{2}\left(T_{u}+\left|T_{u}\right|\right) \\
& \dot{z}_{1}=G_{x} z_{2} \\
& \dot{z}_{2}=G_{x}\left(A T \sin \alpha-\mu_{x}\right) g \\
& \dot{z}_{3}=G_{y} z_{4} \\
& \dot{z}_{4}=G_{y}
\end{aligned}
$$

where

$$
\mu_{x}=\left\{\begin{array}{rl}
\mu, & z_{2}>0 \\
-\mu, & z_{2}^{2}<0
\end{array} \quad \mu_{y}=\left\{\begin{aligned}
\mu, & z_{4}>0 \\
-\mu, & z_{4}^{4}<0
\end{aligned}\right.\right.
$$

and

$$
A=-\frac{2 N}{\overline{M g}} \quad, \quad B=\frac{N}{M g}
$$

When the slab and bumper separate, the constant $A$ is replaced by the constant

$$
A_{0}=-\frac{2 N}{M_{1} g}
$$

A computer program to carry out the calculations was written and was available to the students to allow them to see the effects of varying certain parameters of the system. Refer to Figures 6.14 to 6.17. One conclusion that was drawn in practice was that it would be better to use one thick cable instead of the six originally used.

## (v) A typical run-through

In an exercise which is constrained by time it is tempting for the tutor to hurry the students along in order to make progress; the author must confess to having succumbed to that temptation on more than one occasion. Paradoxically, the first session was spent restraining the students from embarking on an immediate mathematical analysis before thinking about the problem in a qualitative manner. Once this had been done the next step was to produce some equations to describe the motion. It is suggested that the students first consider the situation in which all the masses are moving; they can attempt to formulate the relevant equations, and by so doing will be able to 'get a feel' for the problems which they might encounter when analysing the motion stage by stage. There is a general feeling that, after this analysis has been carried out it will probably be necessary to write a computer program to obtain the results required.




Time: (:me)


It is a well-rooted idea in most students that a diagram is almost always a good bet in starting the process of solving an "applied" mathematics problem. Therefore, a diagram is suggested as a first step. The students are asked what they will mark on the diagram: will they mark every quantity that seems useful or only those that seem necessary, adding others later if required. Responses are fairly evenly split, some students maintaining that it is easier to select from an over-provision of constants and variables. It is rare to find that half the full diagram is produced and they will be asked after the equations have been formulated whether, for the sake of simplicity and clarity, it would have been preferable to have drawn, say, the upper half of the diagram and appealed to symmetry when deriving the equations of motion. Sometimes there is reluctant agreement, but sometimes there is a protest that it is more difficult to use only a half-diagram since there is a tendency to forget the lower cables when considering the forces on the slab/bumper combination.

There is hardly ever much difficulty in deciding that the sensible starting-point is the application of Newton's second law of motion to each massive object, viz. a block on an inclined plane and the combined slab and bumper. There have been occasions when equations of motion for both blocks have been derived; given that these blocks have equal masses it is quickly seen that one of these equations is redundant. The author cannot resist the temptation to point out that the redundant equation need not have been produced had more thought been given to the symmetry in the problem, as exemplified by drawing only the top half of the full diagram.

Students show a reluctance to use two simple expressions when one complicated expression will do. For example, instead of using $\sin \alpha$ in equation (1) and defining $\alpha$ by equation (1a) the students opt to find the expression
$\frac{x}{\sqrt{x^{2}+\frac{L^{2}}{4}}}$
for $\sin \alpha$. It is suggested that this is a cumbersome expression to carry along during the formulation stage; sometimes the students can be persuaded that they should adopt the two-stage approach.

There is some hesitation before an expression for strain is obtained; it may well be the first time that the students have had to find a formula for strain from first principles. They have had to be prodded to think whether their expression could take negative values and what the implication would be if it could. Sometimes the inquiry will be delayed until a formula for the tension in the cables has been obtained. Clearly, there is no such thing as negative tension; clearly, the cable will go slack; however, how to build this into the formulation is another matter. The author has always had to explain the idea of replacing $T$ by

$$
\frac{1}{2}(T+|T|)
$$

but he does so in the spirit of providing another useful item for the students to add to their mathematical tool-kits.

Once again, it is necessary to advocate the usefulness of simplicity in the expressions and equations being developed. Is it really necessary to keep the expression $E A_{0} / L$ or is it more economical to replace it by a single symbol, $K$, writing it in its full form only if required? Problems occur when it is realised that the actual written form of the equations of motion depends on the direction of motion. There is general agreement that two equations will be have to be written for each mass to take account of this. The students are invited to look ahead to the computer program that is to be written at a later stage of the exercise and are asked whether it would be more economical to produce one equation with a parameter that can take two different values, e.g.

$$
\mu_{x}=\left\{\begin{aligned}
\mu, & \dot{x}>0 \\
-\mu & , \dot{x}<0
\end{aligned}\right.
$$

This artifice is acknowledged as being acceptable in the context of a
computer program but there is something of a struggle before equation (10) is derived and help has usually been necessary. Again, a plea is made for simplifying the coefficients to facilitate subsequent analysis. Sometimes a student knows the trick of multiplying (10) by $\dot{x}$ in order to allow its integration, but most often he has to be told.

It would be desirable to allow the students to continue to develop the modelling of subsequent stages of the motion in much the same way. However the constraints of time usually begin to tell and the students find it hard to sustain their concentration. For phases II and subsequent phases the author takes a more prominent rôle.

The students have met the idea of state variables earlier in the course and the development of equations (14) does not provide much discomfort. A discussion of how to approach the subsequent phases of motion leads to the idea of building various checks into the program to pick up the relevant values of parameters for a particular phase.

Then the program that had been written earlier is given to the students and they are invited to use it to obtain some results on the tension in the cables as a function of time. At a later stage in the term their results are considered. A discussion takes place as to whether the students have made sensible use of the program and obtained the most useful set of results for their investigations. Finally, the author gives a resumé of the exercise and explains the results that were obtained and conclusions reached when the modelling was originally carried out in industry.

## (vi) Assessment of the exercise

After teething troubles with the first run-through, subsequent occasions proved to be a less bumpy ride. It was deliberately arranged that each group
contained the more able and the less able in the class in order to assist the momentum of progress. The reservoir contamination model had been developed with the class in the earlier part of the year so that the concept of modelling was not new.

Comments received from the students were surprisingly mature. Whilst most had found it tough going, the majority accepted that learning to model was a difficult task but a crucial one for their development as mathematical engineers. They accepted the need at this stage for occasional help and prodding and did not believe that it had been overdone. Asked whether they were disappointed that they had not been able to complete the exercise under their own steam the vast majority said that they were not.

The students strongly emphasised how important it was that they had been dealing with a model which had been used to tackle a problem that was a real-life industrial example. Whilst they knew that they would not have been able to tackle it in an unguided environment they felt that they had learned a considerable amount about the modelling process. Some students commented that they were surprised that the model had been formulated and solved, by and large, without using "advanced" mathematical techniques. The author asked whether the students would have preferred to have been given additional mathematical techniques, for example the use of

$$
\frac{1}{2}(x+|x|)
$$

to ensure that the expression does not take negative values, in advance of the modelling exercise. No one stated this preference and some suggested that such an approach was unrealistic, since in practice they might well expect to have to read up on certain techniques before making progress.

Having run the exercise for a number of years, the author is reasonably satisfied that it is serving its purpose of putting the students into a realistic example of the modelling process. They are more mathematically able than the
mainstream engineers at Loughborough and towards the end of their first year of study they are sufficiently mathematically mature to tackle the exercise profitably.

### 6.6 Modelling: Its Place in the Curriculum

The view expressed by Murthy and Page (194), that mathematical modelling is an increasingly important item in the engineer's tool-kit as the systems he tackles become ever more complex, has been expressed with increasing frequency over recent years. Recently, the author participated in the 15th annual conference of SEFI and took the opportunity to have a discussion at length with the Vice-President of SEFI and the Joint Chairman of their Mathematics Working Group (199). Their firmly-held belief was that engineering undergraduates in the United Kingdom suffer relative to their counterparts in mainland Europe because of the lack of time given over to mathematics. The consequence is that a compromise has to be made between devoting sufficient time to developing modelling skills and devoting sufficient time to teaching mathematical techniques; that compromise is usually made by spending too little time on the former, sometimes by neglecting it altogether.

Mathematics is the cement which holds the fabric of engineering together and it is right and proper that the mathematics course should be the home of the modelling activity. The Vice-President and the Joint Chairman endorsed the approach to teaching the modelling process as outlined in this chapter. It was agreed that the earliest possible opportunity should be taken to emphasise the modelling role of mathematics to the freshman engineer and that wherever possible case studies should be used to enhance the teaching. Before holding any modelling exercises it is necessary to conduct a session where the students can see how the modelling process is conducted. After that, modelling exercises of increasing complexity should be undertaken with the amount of direction and guidance provided by the lecturer being gradually reduced.

One of the problems facing the author is the scale of the task in conducting
these modelling exercises with a group of, say, eighty students. Ideally, the sub-group size for an exercise such as the furnace bumper problem should be between four and six. This means that there would be between 14 and 20 sub-groups and effectively rules out conducting the exercise as might be wished. Furthermore, it would be preferable to have some of the engineering staff leading some of the sub-groups; however, pressures on university academics militate against them devoting more time to teaching activities, however desirable these might be from the point of view of the educational benefits to their students.

A further difficulty placed in the way of the author's aims is the restriction of mathematics to the first two years of the undergraduate engineering courses at Loughborough. The final year is the ideal time to tackle modelling exercises seriously, since the students have attained greater maturity and the mathematical ideas which they have met in the first two years have had time to percolate and be assimilated.

Given the limited amount of time available for the modelling activity for mainstream engineers, the author has not assessed it explicitly. After all, the students must not be rewarded for every exercise by marks which count towards their final assessment: they are supposed to be receiving an education not collecting stickers to put in a book which can be traded in for a degree.

In the Civil Engineering course at Loughborough one of the final year options is in Civil Engineering Design and this is assessed continuously. The students carry out the design exercises in groups of four and each student is awarded a mark comprised of a group mark plus an individual modifier. The two members of staff responsible for the option have expressed dissatisfaction with the resulting marks awarded to the students since they find it hard to accept that one student can be about twice as competent as another. All too often the mark range is considerably narrower than for other options and this has attracted criticism from their colleagues. The author has decided that for the small effect on the final assessment in mathematics of the marks awarded for the modelling exercise the
effort is not worthwhile. On two consecutive occasions a compulsory question was set in the first year examination which related to the traffic lights problem, Example 14 of section 6.1. The students were told in advance that they would be asked a question on the exercise that had been conducted in their first term and that they would be able to refer to the notes which they had taken. The notes were collected from each student and handed out with the examination paper. The question asked for an account of how they had set about the modelling of the problem and what features of the problem had been incorporated into the model, what assumptions had been made and how useful they rated the resulting model. This was a way that the students could be rewarded for their efforts during the exercise earlier in the year. The author was not happy with this form of "reward" and the practice was abandoned.

When the furnace bumper exercise is conducted with the Mathematical Engineering students they are required to hand in a short report on the exercise, discussing the models they constructed and the difficulties that they encountered in its construction. This is one of the 12 assignments which are assessed during the year to provide the mark for that component of their course.

In summary, the author believes firmly that mathematical modelling has a crucial role in the education of the engineering undergraduate and that as the bottom line the teaching should be undertaken by the mathematics lecturer. Perhaps more enlightened times will arrive and then sufficient time will be provided to mount a full-scale course in modelling lectured by both engineering and mathematics staff. However, in the current climate that does not seem likely. Meanwhile, hours will have to be made available by the lecturer without sacrificing too much of the teaching of techniques and concepts which are also a vital part of the undergraduate engineer's training.

## Chapter 7

## Mathematical Engineering

### 7.1 Introduction

Undergraduate courses variously entitled "Mathematics with Engineering ", "Engineering Mathematics " and "Mathematical Engineering " are available at a few universities in England. Their aim is to produce a graduate with a sound understanding of engineering principles together with a deeper knowledge of mathematics than a standard engineering course will provide.

The courses are no longer than current undergraduate courses in mathematics or in engineering; it could therefore be argued that since they must contain less mathematics than the former and less engineering than the latter they are in danger of producing a jack-of-both-trades. What is the justification for producing graduates who know some mathematics, but less than a mathematics graduate, and some engineering but less than a general engineering graduate (and much less in a particular engineering discipline than a specialist engineer)? Advocates of mathematical engineering will counter by arguing that they are producing graduates who know more engineering than a mathematics graduate who may wish to enter the engineering industry, and more mathematics than an average engineering graduate. Further, there is a need for such graduates, who can readily occupy a place in the spectrum between "pure" Mathematics and engineering practice; this place has been hitherto occupied, equally unsatisfactorily, by a product of either a traditional mathematics course or a traditional engineering course.

Those responsible for designing and running these hybrid courses saw their graduates as engineering "trouble-shooters", employed in small numbers by firms which traditionally had recruited many engineers to their staff. These
trouble-shooters, with their ability to appreciate the engineering implications of mathematical procedures would be seen as people to whom would be taken those engineering problems which demanded a mathematical skill and knowledge beyond that usually acquired (and retained) by an engineer.

There have been courses at the Universities of Aston and Warwick, which have since ceased to function, which come under the general umbrella of mathematical engineering. In addition there are courses in "Engineering Mathematics with Computer Methods " at University College, Swansea and in "Engineering Mathematics " at Queen Mary College, London which are on the fringe of the umbrella; both attract only small numbers of applicants and the latter course is only one of a large number facilitated by the modular structure of mathematics courses at Queen Mary College. This chapter will concentrate on the courses at Bristol, Nottingham and Loughborough universities.

The OECD Report of 1966 (16) highlighted mathematical engineering as a discipline which should receive special attention. It stated that "Technological developments and the rapidly increasing availability of large-scale computing facilities in the various branches of engineering and in the management of industrial organisations and government agencies, require, in an ever increasing measure, better mathematical methods, and greatly increased numbers of highly trained scientists capable of handling these methods and/or using the computers. . ... The title "Mathematical Engineer" seems suitable ...
. . . The word "engineer" is justified since the mathematical engineer can contribute much to engineering development, either in solving engineering problems - often in cooperation with other engineers -, or in dealing with problems specifically related to computers. The qualification "Mathematical" refers both to the methods he uses and to his specific ability in this line."

The Report remarked that the adjective "mathematical" was not to be seen alongside the adjectives "electrical", mechanical" etc. as denoting a subdivision of engineering according to the physical principles involved. Rather, mathematical
engineering should penetrate into all the traditional branches of engineering. The distinction was between goals and means. It was further remarked that it would be of immense benefit to have in a university a mathematical department specifically devoted to the development of mathematical engineering. It was emphasised that there was a growing number of problems of an inter-disciplinary nature resulting from the technological developments taking place in areas such as systems analysis, industrial engineering and operational research. The last of these areas was necessary to the construction of the mathematical models required by the increasingly complex problems of industry.

The Report set goals for the education of mathematical engineers:
"(i) To provide adequate knowledge of those branches of mathematics that have, or are likely to have, important applications in engineering;
(ii) to provide adequate knowledge of numerical methods, probability, statistics and other topics important in operations research work, and to develop the ability to use this knowledge in engineering problems;
(iii) to develop a good insight into physics and into integrating elements in engineering such as engineering mechanics, control engineering, and industrial organisation;
(iv) to provide a sound working knowledge of - and experience in - the use of modern computing machinery, for the mathematical engineer must be skilled in both numerical and simulation techniques."

The Report gave the aims of the distinct approaches to mathematical engineering courses in the Netherlands, France and the United States of America and provided detailed syllabuses. In the Netherlands, the established courses at Delft Technological University (started 1956) and at the Technological University of Eindhoven (1961) each required the student to complete the first and second year curriculum in one of the engineering departments before embarking on the specialist programme. In France the picture was similar, the first two years of study being devoted to a general scientific education in mathematics, physics and chemistry followed by three years of specialised training at an engineering school
or at a university. The USA had required many of the mainstream engineering students to take substantially more mathematics in their courses than hitherto and the CUPM had proposed an option which would be taken by mathematics undergraduates who wished to be employed in computing work.

The next section of this chapter examines the establishment of the course in Mathematical Engineering at Loughborough and discusses its first curriculum. The course was established in 1977. Section 7.3 reviews the changes in content and approach that have taken place since the first year of operation and assesses the effects of those changes. Section 7.4 is devoted to a consideration of the courses at Nottingham (established in 1964), Bristol (1977) and Eindhoven (1961) and compares them with the Loughborough course. Finally, section 7.5 appraises the contribution of mathematical engineering programmes to the needs of industry and examines the future of these courses.

### 7.2 Establishing the Course at Loughborough

Seven years after the OECD Report, Richards, who was then Vice-Chancellor at Loughborough, argued the case for the introduction of courses in Mathematical Engineering (200). He considered that many courses then in existence were, to varying degrees, 'mathematics first, engineering second'. He provided an outline syllabus for a course in mathematical engineering (Appendix 14) which he admitted was somewhat biased by his background as an aeronautical engineer, albeit one whose first degree was in mathematics. He saw that a course in mathematical engineering "must be recognised as a challenging course in mathematics but constrained to mathematical and numerical methods which are usable in subsequent careers. It would need to be based on a collection of the mathematical and numerical methods now being used in engineering practice, the emphasis being placed on the establishment of mathematical models to real physical processes, with laboratory experiments or demonstrations designed to illustrate the validity of these mathematical or computer models rather than to build up experimental prowess in the individual."

Richards believed that such courses would free existing engineering schools to concentrate on the 'design' context which was necessary, given that industry seemed unable to train its own designers. He cited the example of the common mathematical link between the electric potential in a water tank occupying various blockages, the seepage of water through a slightly porous dam and the flow of strain in an elastic solid, to illustrate the approach which a mathematical engineer would follow.

In 1970 it had been proposed that the Engineering School at Loughborough should develop an undergraduate course in some area of engineering science. Subsequent upon Richards' paper the Engineering School set up a Working Party to consider the feasibility of establishing a course in the area of Mathematical Modelling and Systems Engineering. The Working Party comprised representatives from each of the Departments in the Engineering School, viz. Mechanical Engineering, Civil Engineering, Electrical Engineering, Production Engineering and Transport Technology; the chairman was a Professor of Surface Transport who had been trained as a mathematician. During the lifetime of the Working Party, the Mathematics Department at Loughborough had been reorganised into three separate Departments, one of Computer Studies, and a rump Mathematics Department both of which remained within the School of Pure and Applied Science, and a Department of Engineering Mathematics which was sited in the School of Engineering. Professor Bajpai, as Head of the Department of Engineering Mathematics, was invited to join the Working Party. A report was prepared for the Engineering School and presented in September 1974 (201).

The Working Party was convinced that there was room in the spectrum of courses at Loughborough for a broadly-based theoretical engineering programme. There was a distinction to be drawn between Systems Engineering, and Mathematical Modelling and Engineering Systems: the former was seen as dealing with the design of complex systems so that they achieve their overall objectives properly and effectively, whereas the latter was concerned with "the
mathematical and theoretical background of the design, production, development, behaviour and performance of engineering goods". It was the latter approach which the Working Party chose as the one to be followed.

Naturally, the hope was that the proposed course would attract new applicants to the University, rather than 'poach' some from the established engineering courses. Further, it was hoped that the new course would lie within the Loughborough pattern of engineering education; it had to produce graduates who were recognisably engineers. These graduates were to have considerable flexibility in terms of a future career; they should not feel constrained to a particular industry and they were to be regarded as professional problem-solvers.

The Working Party had in mind the need for the entrants to the new course to be proficient mathematically and suggested an entry requirement, at appropriate grades, of three 'A' Levels, two of them in mathematics. Significantly, there was no requirement for ' $A$ ' Level Physics; this was a deliberate move to help ensure that applicants would not be drawn from the pool of the other engineering courses. There was a warning that such applicants would therefore require special provision in the teaching of engineering/physical principles in the early stages of the course.

The theme of the course was to be the solution of engineering problems by the application of appropriate theoretical and numerical techniques. The course was to be based on three main subject areas:

1 Theory of Engineering Systems, which involves mathematical modelling as a primary technique.

2 Mathematical Methods.
3 Engineering System Design.
It was suggested that the emphasis given to these three areas be approximately in the ratio 5:5:2.

It was felt to be important that the course should be accredited by the

Council of Engineering Institutions (now the Engineering Council). It should be available either as a straight three year programme or with an industrial training year interspersed between the second and final academic years.

The area 'Theory of Engineering Systems' would contain the principal subjects of System Dynamics, Solid Mechanics and Field Theory. 'Mathematical Methods' would embrace Computing and Numerical Analysis, Statistics, Analysis, and Function Theory, Algebra, and Differential Equations. The third area mentioned above would consist of Engineering Drawing, Design, Production Processes, Management Science, Experimentation and Instrumentation, and Economics and Engineering.

It was suggested that subject specialisations available in the final year should include Structures, Dynamics, Fluid Dynamics, Propulsion, Production Engineering, and Transport Systems Engineering.

An outline structure for this course was proposed and is shown as Table 7.1. To illustrate their thinking, the Working Party considered the subject of System Dynamics and prepared a detailed syllabus as part of their report.

The Working Party had examined the course proposed by Richards (200) and those described by the OECD Report (16) but were of the opinion that these were significantly closer to applied mathematics than their own objectives would suggest. They were anxious that their course would fall clearly within the concept of engineering education acceptable to the School of Engineering at Loughborough. They had recommended that a new department be created to develop the new course and that existing departments cooperate actively in its running. As has already been mentioned, the Department of Engineering Mathematics was created during the lifetime of the Working Party. It was the belief of the Working Party that this department should be augmented by new appointments with an engineering background.

## Table 7.1

| PART A |  | Hours/week |  |
| :---: | :---: | :---: | :---: |
|  |  | Lectures | Tutorials |
| Theory of Engineering Systems | $\left\{\begin{array}{l} \text { Mechanics } \\ \text { Fluids/Thermodynamics } \\ \text { Electricity } \\ \text { Strength of Materials } \end{array}\right.$ | 1 $\frac{1}{2}$ | $\frac{1}{2}$ |
|  |  | $1 \frac{1}{2}$ | $\frac{1}{2}$ |
|  |  | 1 | $\frac{1}{2}$ |
|  |  | 1 | $\frac{1}{2}$ |
| Mathematical Methods | $\left\{\begin{array}{l} \text { Analysis } \\ \text { Vectors } \\ \text { Comp \& Num Analysis } \\ \text { Algebra } \\ \text { Statistics } \end{array}\right.$ | 1 $\frac{1}{2}$ | $\left\{\frac{1}{2}\right.$ |
|  |  | $1 \frac{1}{2}$ | $\frac{1}{2}$ |
|  |  | 1 | $\frac{1}{2}$ |
|  |  | 1 | $\frac{1}{2}$ |
| Engineering Design | $\left\{\begin{array}{l} \text { Eng Drawing and Design } \\ \text { Production Processes } \end{array}\right.$ | 1 | 0 |
|  |  | 1 | 0 |
| PART B |  |  |  |
| Theory of Engineering Systems | $\left\{\begin{array}{l} \text { System Dynamics } \\ \text { Field Theory } \\ \text { Strength of Materials } \end{array}\right.$ | 2 | 1 |
|  |  | $1 \frac{1}{2}$ | $\frac{1}{2}$ |
|  |  | $1 \frac{1}{2}$ | $\frac{1}{2}$ |
| Mathematical Methods | $\left\{\begin{array}{l} \text { Differential Equations \& } \\ \text { Function Theory } \\ \text { Signal Analysis } \\ \text { Computing } \\ \text { Statistics } \end{array}\right.$ | 2 | 1 |
|  |  | 1 | $\frac{1}{2}$ |
|  |  | 1 | $\frac{1}{2}$ |
|  |  | 1 | $\frac{1}{2}$ |
| Engineering Design | Experimentation and Instrumentation | 1 | 1 (lab) |

## Table 7.1/contd.



Two options to be taken from each of the three core areas together with one further option from either of the first two core areas.

There was concern that if the Department of Engineering Mathematics were to act as the host department for the new course then there would be a fragmentation of its nature due to service teaching being shared amongst a number of other engineering departments.

The author believes that the majority of members of the Working Party were unhappy at the creation of the Department of Engineering Mathematics; their vision of a new department was not one in which mathematicians were predominant. They probably believed that the type of course that had originally been proposed was unlikely to see the light of day.

In November 1974 the Department of Engineering Mathematics officially came into being and it took over the responsibility for presenting proposals for an undergraduate course in Mathematical Engineering to the Board of Studies of the School of Engineering, and thence to the University Senate. It was decided to form a new Working Party comprising staff from the new Department (one of whom was the author) and six engineering staff, three of whom had been on the original Working Party.

The engineering staff were given the task of drawing up syllabuses for their own subject areas and these were discussed during plenary sessions of the Working Party. The course structure as presented to the Board of Studies is shown in Table 7.2; by comparing this with Table 7.1 it will be seen that several changes had already been made. Many of these changes were necessitated by the current University directive for new courses to make as much use of existing modules as possible. It was feared that the hours suggested by the original Working Party were unrealistic, a fear which became realised as detailed syllabuses were being prepared. To some extent, the structure of the course and the syllabuses reflected the particular enthusiasms of the members of the Working Party; one particular course which comes to mind is Heat and Mass Transfer.

Table 7.2

| SUBJECT | Lecture Hrs/Week |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Term |  |  |  |  |  |
| Year I | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | Subject <br> Exists | Department <br> Responsible |
| Circuit and Field Theory | 2 | 2 | 2 | Yes | Elec Eng |
| Introduction to Engineering Design $\dagger$ | - | 2 | 2 | No | Eng Prod |
| and Production Processes | $1 \frac{1}{2}$ | $1 \frac{1}{2}$ | $1 \frac{1}{2}$ | Yes | Mech Eng |
| Dynamics of Systems | 6 | 6 | 6 | Yes | Eng Maths |
| Engineering Mathematics | 2 | 2 | 2 | Yes | Trans Tech |
| Fluid Dynamics | 2 | 2 | 2 | Yes | Trans Tech |

Tutorials to be arranged to a maximum of 5 hours/week.
A course on computer programming will be given at the beginning of theyear. $\dagger$ Assessed by coursework only.

Year II

| Automatic Control | 2 | 2 | 2 | Yes | Trans Tech |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Circuit \& Field Theory | 2 | 2 | 2 | Yes | Elec Eng |
| Engineering Mathematics | 5 | 5 | 5 | Parts | Eng Maths |
| Experimentation | 2 | - | - | No | Mech Eng |
| Fluid Dynamics | 2 | 2 | 2 | Yes | Trans Tech |
| Heat and Mass Transfer | - | 2 | 2 | No | Mech Eng |
| Mechanics of Structures <br> and Dynamics | 2 | 2 | 2 | No | Trans Tech |
| Laboratory Work | 25 hours in total |  | No | Various |  |

## Table 7.2/cont

| SUBJECT | Lecture Hrs/Week Term |  |  | Subject Exists | Department <br> Responsible |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Year III | 1 |  | 3 |  |  |
| Advanced Engineering Mathematics | 2 | 2 | 2 | No | Eng Maths |
| Operations Research/ Engineering Statistics | 2 | 2 | 2 | Part | Eng Maths |
| State Space Methods | 2 | 2 | 2 | No | Elec Eng |
| Options |  |  |  |  |  |
| Communications, Signal Processing $\dagger$ \& Acoustics | 2 | 2 | 2 | Yes | Elec Eng |
| Structural Mechanics $\dagger$ | 2 | 2 | 2 | No | Trans Tech |
| Thermodynamics and Fluid Mechanics $\dagger$ | 2 | 2 | 2 | No | Mech Eng |
| Engineer in Society | 1 | 1 | 1 | Yes | Trans Tech |

$\dagger$ One of these options will be taken. In addition, credit will be given for anyone taking the option 'Engineer in Society'.
A project will also be undertaken.
Tutorials are provided in most subjects.

The author and his departmental colleagues were adamant from the outset that they wanted to devise an engineering degree; indeed, it was the intention that at some stage a submission would be made to the Engineers Registration Board of the Council of Engineering Institutions in order to seek accreditation. This was deemed especially important, since such official recognition was far more valuable than any proclamation by the Department. A key feature of the course was that for "engineering" subjects our students would attend lectures in common with other engineering students. It was vital that they should rub shoulders with these mainstream engineers at this formative stage of their careers. Whilst it was recognised that problems might happen when these lecture courses were revised, it was believed strongly that the benefits would far outweigh any likely drawbacks.

An important element of any engineering degree course is the Final Year project; accordingly, it was agreed to include a project as a compulsory component of the Final Year with an insistence that each project would have a clear engineering content. The author's experience of supervising projects in engineering departments led him to argue successfully for the inclusion of a smaller-scale project at Second Year level. This minor project would give the students a feel for this project activity without incurring too much investment of time (or marks); any shortcomings could be improved upon before the more important Final Year project.

It was decided to offer the course in two forms: a straight-through version and a thick sandwich version with an Industrial Year being taken between the Second and Final Academic Years.

The author was in no doubt that when the first students were admitted to the course the syllabuses would be revised regularly during their stay; however, to get a course accepted by the various University committees requires something of a tongue-in-cheek approach. Once the course was approved, the mathematics component was handled quite differently from that suggested by the syllabuses, which were heavily based on courses already given to the mainstream
engineering students.
It was decided to modify the original proposals for entry requirements, working on the principle that in the early years of the course it would be sensible to cast the net wide. Accordingly, candidates offering GCE 'A' level qualifications had to include a Mathematics subject, preferably with a second Mathematics subject or Statistics or Physics or Nuffield Physical Sciences or Engineering Sciences. Candidates whose 'A' level passes did not include Physics would need to offer 'O' level Physics at an acceptable grade. Candidates who offered Ordinary or Higher National Certificates or Diplomas in appropriate subjects would also be considered.

It was seen as a positive move to allow the Mathematical Engineering students to share some of their engineering subjects with mainstream engineers and to carry out coursework in these subjects. After all, if they were to be accepted as engineers by the Council of Engineering Institutions, they had to study some engineering subjects in the same way as their mainstream counterparts.

The planned intake was 15 for the sessions 1977-78 and 1978-79, through 20 in 1979-80, to 25 in 1980-81. Whilst it proved feasible to attract sufficient applicants to recruit about 20 good quality students each year, the target of 25 was never realised.

The author undertook to visit several industrial organisations including British Gas, British Nuclear Fuels Limited, Pilkingtons Limited and Rolls-Royce Limited in an endeavour both to obtain feedback on the usefulness of the proposed course and to make tentative enquiries as to the possibility of placing students for their industrial year. In all, fifteen organisations were visited and the overall response was most favourable. Many mathematicians to whom the author spoke expressed their regret that they had not followed such a course themselves. Some suggestions as to change in content or emphasis were made but these were accompanied by the qualification that they were only minor and would no doubt be contradicted by others to whom the author would be speaking (or had
already spoken); this proved to be the case.

With one exception, all the organisations said that, subject to their usual recruitment procedure, they would be happy to take undergraduates from the course for their industrial training year. (It should be mentioned at this stage that the course led to the award of a Bachelor of Science degree, together with a Diploma in Industrial Studies for those who completed the industrial training satisfactorily).

The University Senate did not give formal approval to the course until 1977 and consequently it was not advertised in the UCCA handbook for 1977 entrants.

In the event, twelve students were admitted to the course in October 1977 (remember that the quota was 15). The quality was far below that hoped for and it was disappointing to find that only six students proceeded to the second year. One subsequently withdrew and one opted for the Industrial Year, leaving four to graduate in July 1980. The cold wind of reality had blown into the warm room of early optimism. Clearly, a long struggle lay ahead to attract good quality students in sufficient numbers and, having attracted them keep them on the course.

### 7.3 Development of the Course

The author has been deeply involved with the Mathematical Engineering course at Loughborough since the earliest days of the second Working Party. He has been a member of every Departmental group set up to assess the course and suggest revisions to it; he has been First Year Tutor and, for the last seven years, Admissions Tutor. He has, therefore, been in an ideal position to study the development of the course in the eleven years of its existence.

As he had forecast, changes to the course were in operation from the outset. After one year of operation it was decided that the first year topic of Design should be interchanged with the second year topic of Experimentation; it was believed, not
unreasonably, that there was little point in studying the principles of design in advance of the principles of engineering subjects. At the same time, it was decided to introduce a new final year option in Electromagnetic Theory to fill a gap in the spectrum of options on offer.

The mathematics components of Years I and II underwent gradual refinement. In the First Year there were separate modules in Algebraic Structures, Differential Equations, and Statistics whilst analytical and numerical methods were not taught in the 'integrated' style. This was partly the result of other teaching demands on the staff concerned and partly because it was felt that the combined module would have been too large; instead, constant cross-references were made in each course to the other.

In the Second Year the modules were: Differential Equations and Waves, Transform Methods, Stochastic Processes, Numerical Linear Algebra, and Functions of a Complex Variable. For the purposes of examination, Statistics and Differential Equations in Year I were lumped together and in Year II Transform Methods and Functions of a Complex Variable were combined with Field Theory whilst Stochastic Processes was tagged on to Automatic Control. The worry of setting too many examination papers was a continual one, as was the high number of contact hours, especially in the Second Year of the course.

Throughout the lifetime of the course the author has been responsible for the teaching of computing and numerical methods to the first year students. Whilst he was a staunch advocate of the integrated approach, he was increasingly concerned that the needs of the mathematical engineer in the area of computational techniques were so much greater than those of his mainstream counterparts that a special provision was necessary. Accordingly, he persuaded his colleagues that it was desirable to have a separate module in Computing and Numerical Methods which would include a laboratory element of practical programming. Some amount of 'integration' could still take place in the mathematical methods course, but the in-depth study of numerical methods
would be left to the new course. At the same time, some of the statistics material was transferred to Year II; this had merit also in that First Year students new to the subject found it too much to swallow in one year.

At the Year II level, the courses in Heat and Mass Transfer, and Circuit Theory were removed. Some of this material was covered in other modules and the rest was deemed to be of less importance than would be merited by its retention. In addition to the advanced statistics which had now been transferred into Year II, room was made for a short course on the use of Finite Element methods for solving structures problems with particular emphasis being placed on the computational aspect. Finally, the project was given an increased weighting and the norm was set that it would involve a substantial element of computing. A cohesive course in Computing and Numerical Methods was created from existing material; whereas the first year counterpart had a coursework component in its associated assessment, this module had a coursework component indirectly in the project assessment.

In 1980 a reappraisal was prompted by the Finniston Report 'Engineering our Future' (69). The Engineering School at Loughborough established a Working Party to examine the implications of the Report on its courses. The author who was a member of this Working Party was concerned that the proposals of the Report would require so much modification to the Mathematical Engineering course for it to be acceptable in 'Finniston terms' that it would depart significantly from its essential philosophy. In the event, the main effect of the Finniston Report was the inclusion of explicit coverage of Engineering Applications; this is discussed in Section 7.3.4.

Shortly after the course commenced, a visiting Australian academic spent a year in the Department of Engineering Mathematics. Part of his study period was devoted to examining the courses in the area of Mathematical Engineering currently on offer in England with a view to setting up such a course in his home country. In discussions with the author (202) he foresaw twin dangers of
over-reaction by mathematicians responsible for these courses regarding their service teaching as second class work, and of resentment by the engineers of a service role to which they were not accustomed. He was concerned that the nature of such courses demanded a greater amount of phasing of topics between the various constituent subjects than was usually the case; he advocated the use of team teaching in this context. Whilst he accepted that recruitment within the Loughborough spectrum was widened by the introduction of the Mathematical Engineering course, he believed that the number of optional subjects available in the Final Year could produce a graduate little different from, say, a graduate from a joint mathematics/electronics course, rather than one who occupied the middle ground between a mathematician and a general engineer. He hoped that the final year options, given that they demanded a relatively deep knowledge of particular fields of engineering, would have as a prime requirement the development of a problem-solving capacity which lay outside traditional engineering theory or practice. His views were to prove pertinent in later years.

In 1982, following an earlier correspondence with the Institution of Electrical Engineers, it was decided to make an application to the Engineers Registration Board of the Council of Engineering Institutions for exemption from their Part I and Part II examinations. The application was successful. The course was approved by both the Institution of Production Engineers and by the Institution of Electronic and Radio Engineers for exemption from their academic requirements. Of all similar courses, the Loughborough one had now achieved a unique status which remains unique at the time of writing. It is worth quoting the philosophy and aims of the course as stated in the submission to the Engineers Registration Board (202).
"The theme of the course is the mathematical solution of engineering problems. It has been developed by the Department of Engineering Mathematics with co-operation and assistance of staff from other departments in the School of Engineering. This interaction is both desirable and necessary for a course which aims to produce chartered engineers with a good mathematical background.

The aims of the first two years of the course are to provide a working knowledge of the basic engineering disciplines, to lay the foundations of a good mathematical technique and to introduce the student to the process of mathematical modelling. The engineering components of the course are taken mostly with other engineering students in order to foster an understanding of the approach to problems of engineers from various disciplines. The emphasis in mathematics is in that which is applicable, taught in the context of the mathematical modelling of engineering systems. The first year course in modelling and simulation epitomises this emphasis in that students learn Basic and Fortran by writing programs to solve engineering problems, these programs forming part of a coursework assessment. They also carry out 'experiments' in the terminal laboratory with pre-written programs which allow the student to compare the effectiveness of numerical methods. In line with modern thinking many such laboratory sessions replace some of the engineering laboratories normally undertaken by the engineering student.

An important element in the course is individual project work: two projects are set, one in each of the second and final years. The aims of the projects are to give the student a chance to work by himself with guidance from a supervisor on the application of the theoretical methods encountered to a realistic engineering problem. In each case the student is required to write a report and to give a short talk on his work thereby improving his communication skills.

In the final year, the subjects studied fully develop the student's mathematical expertise, extend his knowledge of engineering and the student is able to weld together his mathematics and engineering to become an effective mathematical model builder and solver. We have recently introduced 'The Engineer in Society' as a compulsory topic at Final Year level.

Students opting for the four year sandwich course spend the third year in practical training either with an industrial concern or a research establishment, contact being maintained in this period by means of regular visits from members
of staff. The Diploma in Industrial Studies (DIS) is awarded to those who satisfactorily complete this period of training in addition to the satisfactory completion of the academic part of the course."

The course structure and content had undergone further modification. In Year I the Computing and Numerical Methods module was renamed Modelling and Simulation to emphasise the change in approach; this will be discussed in section 7.3.1. Circuit Theory had been replaced by a more general Electrical Studies component which was designed to fit in with the suggestions of the Finniston Report. In Year II the Computing and Numerical Methods underwent a change similar to its first year counterpart whilst Field Theory was replaced by a longer course in Engineering Electromagnetics to provide a continuity of electrically-orientated modules in the first two years. The final year course in Control was renamed Control of Industrial Processes to reflect the new lecturer's concern with practical applications of his subject. The module in Advanced Engineering Mathematics had focussed on Finite Difference and Finite Element Methods and had changed its name accordingly. The list of possible final year options had lengthened. It now comprised Digital Data Transmission (this was offered by the Electronic and Electrical Engineering Department who had re-vamped their option in Signal Processing), Solid Mechanics, Fluid Mechanics, Electromagnetic Fields and Waves, Operations Research and Engineering Statistics, Computer-Aided Design, and Microcomputer Systems and Simulation. Apart from the first one of these, the options were run from within the Department of Engineering Mathematics. In anticipation of the application for accreditation the module 'Engineer in Society', now a longer one provided by the Department of Mechanical Engineering, had been made a compulsory element of the course.

There was general satisfaction expressed by the joint accreditation team fielded by the two Institutions; in particular, it was seen as advantageous that our students shared their engineering lecture courses with their counterparts in other engineering departments; however, a number of helpful suggestions had been
made. There was a feeling that the Final Year project was underweighted in the Schedule of Assessment and its relative weighting was subsequently increased by $50 \%$. To respond to the call for more design work in the course the second year subject was extended by a follow-up module in the final year.

It was decided to include Second Year marks in the degree assessment; now the classification of the degree was based on $25 \%$ of the Second Year marks and 75\% of the Final Year marks. Previously, the assessment had been based entirely on Final Year marks.

The following session saw the introduction of a practical training element to cover certain aspects of Engineering Applications EA1 and EA2 as specified by the Engineering Council. The Spring Term in Year I was extended by two weeks to allow students to attend a practical training course in the University's Centre for Industrial Studies. This course was weighted into the Schedule of Assessment and was examined by continuous assessment.

In 1985 the author had become concerned at the lack of first class honours graduates emerging from the course in comparison with those from the courses at Bristol and Nottingham. He carried out an investigation which resulted in the lowering of grade boundaries in the degree classification to bring them into line with these other universities.

There were other minor changes carried out in the following session and the course structure current at the time of writing is shown in Table 7.3. It is instructive to compare this table with Table 7.2 and, indeed with Table 7.1.

In 1987 the course was re-accredited by the Institution of Electronic and Radio Engineers and the Institution of Production Engineers. They had been particularly pleased with the change of the Second Year project from an individual exercise to one conducted in small groups; this activity is discussed in Section 7.3.2. Following their recommendation, the course was restyled as leading to the degree

Table 7.3

| SUBJECT | Lecture Hrs/Week <br> Term <br> $\mathbf{2}$ |  |  |  | Relative <br> Weighting |
| :--- | :---: | :---: | :---: | :---: | :--- | Examination

$\dagger$ Modelling and Simulation contact hours include $2 \mathrm{hrs} /$ week computer laboratory.
*Materials includes laboratory sessions and $20 \%$ of the total mark for this subject derives from coursework.

| Year II | 3 | 3 | 3 | 2 | 3 hrs written |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Differential Equations | 3 | 3 | 3 | 2 | 3 hrs written |
| Engineering Mathematics | 2 | 2 | 5 | 2 | 3 hrs written |
| Modelling and Simulation $\dagger^{*}$ | 2 | 3 | 2 | 2 | 3 hrs written |
| Signal and Systems Analysis | 3 | 3 | 3 | 2 | 3 hrs written |
| Automatic Control $\dagger$ | 3 | 3 | 3 | 2 | 3 hrs written |
| Fluid Dynamics $\dagger$ | 3 | 3 | 3 | 2 | 3 hrs written |
| Mechanics of Solids $\dagger$ | - | 2 | - | 1 | 3 hrs written |
| Comprehensive Design $\dagger \dagger$ | 2 | 2 | - | $\frac{2}{17}$ | Continuous Ass. |
| Project |  |  |  |  |  |

[^0]| SUBJECT | Table 7.3/cont |  |  |  | Examination |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lecture Hrs/Week Term |  |  | Relative Weighting |  |
| Final Year | 1 | 2 | 3 |  |  |
| Finite Difference/Finite | 3 | 3 |  | 2 | 3 hrs written |
|  |  |  |  |  |  |
| Control of Industrial Processes | 3 | 3 | - | 2 | 3 hrs written |
| Engineer in Society | 3 | 4 | - | 2 | 3 hrs written |
| Comprehensive Design | - | 2 | - | 1 | Continuous Ass |
| Project | 5 | 5 | - | 3 |  |
| Optional Subjects (Two from) |  |  |  | 10 |  |
| Digital Data Transmission Systems | 2 | 2 | - | 2 | $2 \times 2 \mathrm{hrs}$ writte: |
| Solid Mechanics | 3 | 3 | - | 2 | 3 hrs written |
| Fluid Mechanics | 3 | 3 | - | 2 | 3 hrs written |
| Electromagnetic Fields and Waves | 3 | 3 | - | 2 | 3 hrs written |
| Operations Research/Engineering Statistics | 3 | 3 | - | 2 | 3 hrs written |
| Computer-Aided Design | 3 | 3 | - | 2 | 3 hrs written |
| Microcomputer Systems and Statistics | 3 | 3 | - | 2 | 3 hrs written |
| Boundary Element Methods | 3 | 3 | - | 2 | 3 hrs written |
|  | 9/20 | $9 / 2$ |  | 14 |  |

Other options may be offered, subject to approval by the Heads of Department concerned
of BEng rather than BSc, thus emphasising further the commitment of the teaching staff to the idea of the course as an engineering course.

It has to be emphasised that this course was designed specifically not to be a Joint Honours course in Mathematics and Engineering. Furthermore, the author and his colleagues at Loughborough have always drawn a clear distinction between Mathematical Engineering and Engineering Mathematics. In the former discipline the over-riding aim is the solution of engineering problems using mathematics whereas in the latter the engineering problem is merely a source of a mathematical problem: although in many cases the engineering problem is solved, it is the mathematics which provides the interest (and, indeed, the motivation).

There have been many alterations to the academic programme in eleven years, some caused by a need to update the course in line with developments in industry, some to keep in line with the perceived idea of an engineering education and some to take account of the increased quality of the intake to the course over the last few years. How have these alterations, in total, affected the nature of the course; how does the graduate of 1988 compare with the graduate of 1980 ?

Five aspects of the course are examined in order to help identify the answers to these questions. They are: the Modelling and Simulation module in the First Year of the course, project work, industrial placement, Engineering Applications and the career choices of graduates from the course. Each of these aspects is examined in turn and an attempt is made to capture the essence of the changes which have taken place.

### 7.3.1 Modelling and Simulation

Already in this thesis a number of references have been made to the course module in Modelling and Simulation which has been given to freshman Mathematical Engineers, in one form or another, since the course began. In that
time the module has assumed a greater importance in the First Year of the course and it is worthwhile spending a little space to reflect upon its growth and its current status; the author believes that the changes which the module has undergone mirror to a large extent those changes which have taken place in the course itself.

In the early years of the lifetime of the course there was no real modelling content in the First Year. There was an attempt to encourage all the lecturers of First Year modules to emphasise the modelling theme in their subjects but this was not particularly successful, especially in the engineering modules; these modules were taught to combined groups of our own students and traditional engineers, and the needs of the latter took precedence. Certainly the Numerical Methods and Computer Programming module was given little enough time to cover the syllabus let alone delve into modelling aspects, however desirable that was. The Second Year module in Numerical and Linear Algebra did contain some lectures on modelling, with illustrations coming from the areas of control theory, eigenvalue problems and linear programming, but it was not a central feature of that module.

The author was unhappy at this state of affairs since he wanted modelling to be the focal point of the course, especially in the First Year. Consequently, he put forward proposals to enlarge the numerical/computing component in Year I to include a modelling aspect. This proposal received general support from his colleagues and was in place by the time the Department was visited by the Accreditation Party from the Council of Engineering Institutions in 1982. Whereas the Numerical Methods and Computer Programming module had been assessed by written examination only (and then as a mere half of a paper), the new Modelling and Simulation module was to be assessed entirely by coursework. Since its introduction, the new module has undergone minor modification, but it has remained essentially the same.

Basic; most of the students have done no programming before, let alone in that language. In addition to two hours of classroom time there is one afternoon in the computing laboratory for each week throughout the session. The students use the computer laboratory which contains 30 BBC microcomputers linked by an Econet Level III facility. They are required to submit 8 programs which relate to specific problems in engineering.

The remainder of the first term is spent tackling assignments on Numerical Integration, Solution of Simultaneous Linear Equations, and Iteration. Standard programs are available and these can be modified by the students should they so wish. A report on each assignment must be submitted for assessment. The assignment sheet for Numerical Integration has already been discussed in Section 4.3 and is shown on page 95.

In the second term the students are introduced to Fortran 77 and are set 8 programming problems which are to be completed by the end of the session. An example of these problems is shown in Appendix 15. In addition, the students are required to undertake assignments on Approximation of Functions, Interpolation and Approximation of Data, Ordinary Differential Equations, and the use of the NAG and GINO libraries.

In the third term an experiment is carried out on the use of the analogue computer in the modelling of a dynamical system via a second-order ordinary differential equation.

During the module the students are introduced to several case studies and carry out a number of modelling exercises, culminating in the furnace bumper problem which was discussed in Section 6.5. They also study a simple computer-based simulation of a reservoir, using input data of monthly rainfall and monthly demand. (In the Second Year of the course the students use a continuous simulation language and a draughting package for CAD and study further aspects of the modelling process including sensitivity, validity, refinement
and reformulation of models.)

It will be seen from the above discussion that the changes described have brought the course more into line with the aims of the original Engineering School Working Party (201) and with the ideas of Richards (200). Students now see themselves as model-builders and model-solvers much more than their predecessors did.

### 7.3.2 Project Work

As mentioned in Section 7.2, it was decided to set a project in each of the Second and Final Years to be conducted on an individual basis. For the latter project the following procedure was adopted. Students would choose, in order of preference, three projects from a list prepared by the academic staff; as far as possible they would be allocated their highest choices and this would be made known to them in the summer term of their Second Year (or twelve months later in the case of students who spent a year in industry prior to the Final Year). A designated member of staff would act as project supervisor and he would draw up the project specification; the student would be expected to liaise regularly with his supervisor throughout the duration of the project, seeking advice or help, or merely reporting on the progress made since the previous meeting.

In the last week of the Autumn Term each student would give a 15 minute report on what had been achieved and what future work was being planned; the audience comprised other Final Year students, supervisors and Second Year students who would be given an early inkling of what to expect when their turn arrived.

In the first week of the Summer Term the students were required to submit a written report on their project; three weeks later they submitted themselves to an oral examination with their supervisor and the overall moderator. This moderator was a member of staff who had not been supervisor to any of the
projects. The assessment for the project was based on marks for the two oral presentations, the quality and content of the written report and the student's interest and application during the year. With the exception of the last category which was the responsibility of the supervisor alone, all marks were agreed between supervisor and moderator.

The first talk was not a common feature in other engineering courses at Loughborough. However, the author and his colleagues were of the opinion that it was very important for the student to take stock at approximately the half-way stage, especially if his progress was too slow, and the knowledge that the mark awarded would count towards his project assessment would act as an incentive to take the exercise seriously.

In general, the students tackled their projects enthusiastically and competently. Interestingly enough, no one so far has seriously exceeded or fallen short of the effort and time which might have been expected to be the norm; this has not always been the case in other engineering departments.

The difficulties associated with assessment of projects soon made themselves felt, even with an overall moderator supposedly ensuring uniformity of standard. The age-old questions of how highly to mark an outstanding project and how low a mark to award to a very poor project remain to be resolved to mutual satisfaction. The view has been expressed that project marks ought not to be higher on average than for other subjects but the author does not subscribe to that view: many students will put considerable effort into the project and that effort should be rewarded. Problems have arisen in other departments when students take options and it is believed that one option has been marked too generously or too harshly. In an attempt to remove these perceived inequities, complex procedures have been devised which have sometimes resulted in substantial differences between a candidate's new mark and his 'adjusted' mark, eliciting serious concern from external examiners. These are artificial procedures and have as little merit as the attitude which seems to accept project marks in the
range $40 \%$ to $80 \%$ only.

One of the factors contributing to the student's pleasing performance at the Final Year project has been the progression from assignments in Year I Modelling and Simulation through the Second Year project. In the early life of the course the Second Year Project was a much scaled-down version of the Final Year exercise; there was no oral presentation in the first term and the oral examination was replaced by a talk of 15 minutes duration to an audience of contemporaries, academic staff and First Year students. The argument was made for a minor project at this stage of the course to help students to mature in their approach to project work and hhas been gratified to see his theories vindicated.

In Appendix 16A are listed some Second Year project titles which were undertaken in the years 1978-79 to 1985-86. It will be seen that there has been an effort to set projects which seek to model engineering systems via mathematics; most of them require a computer-based solution to the model.

In the session 1986-87 the individual project at the Second Year level was replaced by a project carried out in groups of three or four. The aim of the project was to write a structured program in Fortran 77 to analyse a general two-dimensional pin-jointed structure and calculate the deformations, reaction forces and member forces under a specified loading. The decision to change to this style of project was taken partly because of the advantages of learning how to work in a small team and partly because of the need to train the students to write large-scale but structured programs. The groups had to assign tasks to their members, hold regular team meetings, report regularly to the member of staff who was the project co-ordinator and present their project reports both orally and in written form. The mark each student received was the sum of a group mark and an individual mark. Experience from the two occasions on which this exercise has been conducted has indicated that the new format is a definite benefit to the students. They are much more confident when entering their Industrial Year that they will be able to cope with the writing of software and are able to adapt more
easily to working in a team environment.

Also in Appendix 16 are some Final Year project titles; these cover the sessions 1979-80 to 1986-87. Again, the emphasis has been on the modelling of engineering systems, with the use of computer-based techniques for solution being predominant. In some cases the project has been jointly supervised with a member of staff from another engineering department. This has provided a useful external input to our marking and has always been an encouragement that we were marking a a level commensurate with other departments. $_{\text {at }}$.

The Re-Accreditation Party welcomed the inclusion of group projects and were of the opinion that the Second Year was the appropriate time. The Final Year project was felt to have the correct relative weighting but there was some concern that a student could be awarded an Honours Degree even though he had not gained a pass mark in the project; this has now been rectified.

### 7.3.3 Industrial Placement

Table 7.4 shows the growth of the demand for taking the Industrial Year which has taken place over the lifetime of the course. It should be borne in mind that some students obtain industrial sponsorship prior to entry on the course and this usually means that they will spend a year in industry before embarking on the academic programme; there were 5 such students entering the course in 1986.

The contacts which had been made in visits to industrial firms before the course started bore fruit when it came to placing the students in suitable organisations in the years during which the course found its feet.

|  | Table 7.4 |  |
| :---: | :---: | :---: |
| INDUSTRIAL PLACEMENT |  |  |
| Academic Year | Number in Second Year | Number going to Industrial <br> Training in following Session |
| $1978-79$ | 5 | 1 |
| $1979-80$ | 18 | 3 |
| $1980-81$ | 16 | 2 |
| $1981-82$ | 20 | 3 |
| $1982-83$ | 16 | 9 |
| $1983-84$ | 17 | 8 |
| $1984-85$ | 19 | 13 |
| $1985-86$ | 17 | 16 |
| $1986-87$ | 17 | 16 |
| $1987-88$ | 20 | 14 |

In their Industrial Year the students are required to keep a log book of the work they carry out and to write a dissertation on some aspect of that work. In Appendix 16C the titles of some of the dissertations from the years 1979-81 and from 1985-86 are shown; they indicate the variety of work that has been undertaken. Satisfactory completion of the dissertation and the log book are prerequisites for the award of the Diploma in Industrial Studies.

Each student is assigned an Academic Tutor from the Department and an Industrial Tutor from the host industrial organisation. They are responsible for monitoring the student's progress and assessing his performance during the year.

On their return to the Final Year of the course, the students are required to give a short talk on their work experiences to Second Year colleagues. Almost everyone
has indicated that they found the industrial training valuable and had no regrets about taking the year out. There is a general surprise that, whereas the level of mathematics which they were required to use was much lower than they had expected, the level of responsibility which they were given was higher than anticipated. Many of them were awarded sponsorships for the Final Year by their employers and several took up a graduate position with the firm in the following year.

As a result of the work undertaken in the Industrial Year, some students have suggested a Final Year project based on their dissertation. This has been welcomed by the Department.

Present indications are that the students have been well received in industry, judging by the comments made by the relevant Industrial Tutor on their assessment forms and by discussions between the Industrial Tutor and the Academic Tutor who visits the student twice during the training period. Even students whom the Academic Tutors considered of only modest ability have been rated very highly by their industrial counterparts; how pleasing it is to record this fact.

The assessment for the training year is completed by a joint report by the two tutors on the student's work, and successful students are awarded the Diploma in Industrial Studies of the University once they have gained the degree award of Bachelor of Engineering (BEng).

There is a general feeling within the University that on average those students who have spent a year in industry do better in their Final Year examinations than those who have not. However, this has not been shown conclusively to be largely attributable to the experience gained in the Industrial Year; part of the difficulty in assessing its role is the fact that it has been the more academically able students who have tended to opt for the Industrial Year. However, the author's considered view is that the extra maturity gained in that Year is a key factor in the student's performance; success in the Final Year relies heavily on the ability to plan one's
time, and this is notably more evident in the fourth-year student.

### 7.3.4 Engineering Applications

The Finniston Report (69) advocated strongly that all programmes leading to the award of Chartered Engineer should include four modules in Engineering Applications, dubbed EA1 to EA4. The modules were described briefly as:

EA1 An introduction to the fabrication and use of materials.

EA2 Application of engineering principles to the solution of practical problems based upon engineering systems and processes.

EA3 A structured introduction to industry under supervision, and involving a range of practical assignments.

EA4 Specific preparation for a first responsible post and a period carrying responsibility in that post with decreasingly close supervision.

The actual character of these modules is a matter for the constituent Institutions of the Engineering Council. Each Institution has expanded on these brief descriptions in its information to those who seek Membership. In its submission to the Re-Accreditation Party (204) prior to their visit in 1987, the Department of Engineering Mathematics put its case for its coverage of EA1 and EA2. It prefaced the case thus:
"The Mathematical Engineering degree course has always given high priority to engineering applications. These have evolved steadily since its inception to take account of the course philosophy and the nature of the industries most likely to employ Mathematical Engineering graduates.

A significant part of the EA1 material is taught by the University's Centre for

Industrial Studies (CIS) which for many years has provided on-campus training for the University's courses in engineering. From 1990 the University plans to commit all the resources of the Centre to engineering departments to enhance the facilities required to support the EA1 requirements of engineering degree courses. The provision of support for the EA activity in the post-1990 period, as in the past, is thus guaranteed."

The module EA1 was deemed to be catered for in the First Year by the courses in Introduction to Properties, Fabrication and Use of Materials, Introduction to Production Processes, and Engineering Measurement. The second of these courses is a compulsory full-time two week course in the CIS which takes place in the Easter vacation. It is a practical training course which is concerned with the basic production processes for metals. Consideration is also given to electrical and electronic control methods including numerically controlled machining. The two other courses are dove-tailed together to provide a study of the characteristics and behaviour of engineering materials together with an examination of the methods and equipment used in engineering measurements.

The EA2 activity occurs throughout the degree course. Especially singled out for mention were the Second and Third Year courses in Comprehensive Design, the Project work and the Modelling and Simulation course in Years I and II. The Second Year Design module requires students to work together in small groups, but each student presents an individual report. At the Final year level, work is carried out on both an individual basis and a group basis: this necessitates elements of planning and organisation. Assessment is based on written and verbal presentation. In both years, design is approached as a total activity and the core phases of market research, specification, conceptual and detailed design, manufacture and testing, and sales are studied and implemented.

The Re-Accreditation Party were of the opinion that the package which we had 'bought' from within the University was not ideally suited to our students. However, bearing in mind the relatively small numbers on the course and the
shortage of resources for the EA activity across the campus, it seemed unlikely that we would be able to effect much change.

### 7.3.5 Destinations of Graduates

One test of the success of a degree course is the ease with which its graduates find employment. At the time of writing nine cohorts of graduates have been produced and the overwhelming majority were in jobs within three months of graduation. Initially, most graduates took up posts in the field of engineering. However, the current crop of 17 new graduates contained six who have entered the world of finance and two who have continued their studies. The phenomenon of engineering students opting for a career in finance has been experienced by other engineering departments and is a reflection of our times, at least in the United Kingdom.

The predominant activity of our graduates over the years is the writing of software, either inside an industrial concern or commercially for a software house. It was this feature, which started to manifest itself early on in the lifetime of the course, which reinforced the author's belief that the computing element of the course needed to be increased. The contact which the author has maintained with many of the graduates has provided a continuing feed-back that has helped to justify the changes that have taken place in the course.

The graduates have commented that a firm base in computer programming is of prime importance. It has to be acknowledged that not many of them are using much high-powered mathematics and that very few really draw upon their engineering training to an appreciable extent. However, very few regret their choice of degree course and most were emphatic that the balance of subjects in the course was suited to their talents and aspirations at the time of entry. All valued the variety of options which were available in their Final Year and welcomed the broad spectrum of subjects studied which allowed them to keep a wide perspective for the future careers.

Appendix 16D contains a cross-section of posts held by our graduates.

### 7.4 Nottingham, Bristol and Eindhoven - Alternative Approaches

In this Section the courses in the area of mathematics combined with engineering which are offered at the Universities of Nottingham, Bristol and Eindhoven are summarised individually. Then a comparison is made between these courses and the one at Loughborough.

### 7.4.1 Mathematics with Engineering at Nottingham

This course is run by the Department of Theoretical Mechanics which belongs to the Engineering Faculty. The Nottingham course brochure (205) opens by asking 'Is this the course for you?' It suggests three categories of reader who would find the course worthy of serious consideration:
(i) those who are interested in studying mathematics further and who would like to apply it to real life problems,
(ii) those who wish to study engineering without specialising in any particular branch,
(iii) those who contemplate a career involving computers, engineering and mathematics.
The brochure continues:
'Industry needs mathematicians (both men and women) who understand engineering principles and can apply mathematics to the solution of engineering and industrial problems. The course in Mathematics with Engineering is designed to meet this need and, at the same time, to provide a mathematical education to honours degree level which gives the same career opportunities as are open to the graduate with a traditional mathematics degree. If your interest is in teaching mathematics, you will find that the engineering content gives you an increased appreciation of the power and importance of your subject. If you are contemplating doing research, a good honours degree in Mathematics with Engineering opens the way to study for a higher degree either in Mathematics or Engineering. Perhaps you
are thinking of a university course as a completion of your education rather than a training for a career, in which case the rigour of mathematical methods provides an excellent intellectual discipline while the engineering studies serve as a point of contact with industrial problems. Whichever of these reasons may attract you, this course will consolidate your existing knowledge, open out new areas of learning and develop habits of independent thought and inquiry.'

The course is designed to train mathematicians to apply their mathematics to the solution of practical problems. The three-year degree course leads to an Honours degree in mathematics with subsidiary engineering studies.

The mathematical studies comprise the following:
(a) Topics in pure mathematics, which are studied for the essential understanding of mathematical ideas and for the development of general methods of solution of mathematical problems.
(b) Branches of applied mathematics, which provide the theoretical basis for the major disciplines of civil, mechanical and electrical engineering.
(c) Numerical methods and the use of computers in the solution of engineering and other problems.
(d) Statistics and operational research techniques, which are applied in the managerial aspects of industry and commerce.

The Engineering courses are taught by the Engineering Departments and are part of their undergraduate programmes. The integration of the mathematics and the engineering studies may be observed in the course structure."

Table 7.5 shows the course structure; in all cases the lecture hours shown are supplemented by example classes. In addition to the examinations shown the students sit a paper in Engineering Mathematics which comprises multiple - choice questions and is sat by all freshmen engineers. The examinations in the Summer Term are a qualifying examination for entry to the Second Year of the course. After

Table 7.5

## FIRST YEAR

| Mathematical Analysis | (43) | Electrical Theory | (20) |
| :--- | :--- | :--- | ---: |
| Mathematical Methods I | (43) | Digital Electronics I* | (20) |
| Linear Algebra | $(10)$ | Mechanics \& Thermodynamics of Fluids*(40) |  |
| Numerical Analysis/Computing* | (14+20) | Properties of Materials | $(20)$ |
| Applied Mathematics* | $(46)$ | Engineering Dra. wing \& CAD* $\dagger$ | $(6+28)$ |

*Threse subjects contain a coursework element.
$\dagger$ Examination in December.
The hours shown in parentheses are lecture hours, except for Numerical Analysis/
Computing, and Engineering Drawing and CAD where the number before the + sign represents lecture hours and the second number relates to laboratory class hours.

## SECOND YEAR

Mechanics of Rigid Bodies
Linear Algebra
Differential Equations
Numerical Analysis
Computing Assignments $\dagger$
ONE SUBJECT FROM
Materials and Geotechnics II
(40) Manufacturing Systems II
(40) Applied Thermodynamics Fluid Mechanics
(40) Structure-Property Relationships
(30) Mechanics of Deformable Materials
(25) Vector Analysis (25)
(25) Statistics

Electronics and Communication
Network \& Signal Analysis/
Feedback Systems
$\dagger$ Assessed by coursework.

## THIRD YEAR

## Required subjects

Mathematical Methods III
Project and Dissertation
Optional subjects - The student selects a total of FIVE subjects as follows:

| (a) TWO, THREE or FOUR subjects from: | (b) ONE or TWO engineering subjects <br> from a wide range offered by the <br> Engineering Departments, for <br> example: |
| :--- | :--- |
|  |  |
|  | Environmental Engineering |
| Mathematical Methods IV | Advanced Communication Systems |
| Linear Elasticity and Viscoelasticity | Heat Transfer, Thermal Power |
| Finite Elasticity and Plasticity | Nuclear Engineering |
| Mechanics of Viscous Fluids | Manufacturing Systems III |
| Waves in Fluids | Materials \& Geotechnics III |
| Electromagnetic Waves | Highway and Traffic Engineering |
| Special Relativity for Engineering | Hydraulic and Public Health |
| Optimisation \& Applied Probability | Engineering |
|  | Software Engineering |
|  | Control Theory |

(c) Up to TWO subjects from a wide range offered by the Department of Mathematics, for example

| Mathematical Education | Regression Analysis |
| :--- | :--- |
| Time Series Analysis | Applied Statistics |
| Formal Computation | Graph Theory |
| Case Studies | Game Theory |

The lecture courses are 25 hours in duration except for those optional subjects listed under (b) which occupy 40 hours.

All subjects are equally weighted.
The Project disseratation has to be submitted before the end of the Spring Term.
these summer examinations a compulsory short course on Laplace transforms is given and a reading course is set on this topic for the following summer vacation. The work is examined in the Second Year as part of the Differential Equations module. A course in computer programming is also given after the summer examinations have taken place and before the students leave for their vacation.

In addition to the compulsory subjects in the Second Year the student is required to select one engineering option from a wide range of second year lecture courses offered by engineering departments. Over the first two years of the course the ratio of time spent on mathematical techniques to that spent on applied mathematics is approximately two to one.

Following the summer examinations in the Second Year an introductory course is given to prepare students for the Final Year; it covers Elasticity, Electromagnetism and Mathematical Methods. Coursework in these subjects is taken into account in the Final Year assessment and work in these subjects is examined in the corresponding Final Year examinations.

In the third year the student is allowed a wide choice to permit a selection of topics to best fit his interests and abilities. The engineering option can be a follow-up to the second-year option or an entirely new second or third year course.

The Final Year project involves the student working on a problem of practical interest which requires substantial analytical and computational techniques for its solution. The student writes an account of his work in the form of a dissertation which is assessed and counts towards the degree classification. Examples of recent project titles are listed in Appendix 17.

The final assessment is made up from $20 \%$ of the Second Year mark, $60 \%$ of the Final Year examination mark and $20 \%$ of the Project mark.

The course is run as a three year academic programme with no provision for an Industrial Year.

As with the Loughborough course, the graduates have found employment relatively straightforward to obtain. The posts that recent graduates have held immediately after completing the course are shown in Appendix 17. It will be seen that they bear a marked resemblance to those held by the Loughborough graduates.

### 7.4.2 Engineering Mathematics at Bristol

This course is run by the Department of Engineering Mathematics in the Faculty of Engineering. The leaflet issued to applicants to the course (206) sets out the overall objectives:
"Students are given a specialist training in applicable mathematics which is both broader and more fundamental in content than the mathematics courses usually given to students of engineering. The students are also introduced to the principles of Engineering Science. This education allows them to be highly versatile and is good preparation for professional or public service careers. It also enables them to fit more readily into an industrial environment than students who have attended more traditional mathematics courses."

As with Nottingham, the course is run primarily as a three year academic programme, although it welcomes applications from those candidates who wish to pursue a 1-3-1 sandwich training scheme. The outline course structure is shown in Table 7.6.

There is a clear statement that the First Year course is essentially Engineering Science. The modules in Applied Electricity, Strength of Materials, Mechanics of Fluids, Applied Mechanics, and Thermodynamics are provided by Engineering Departments and are shared with engineering students. The module in Mathematical Systems and Modelling introduces the student to the construction of

Table 7.6

## FIRST STAGE

| Applied Electricity | $(38,40)$ | Applied Mechanics I | $(28,14)$ |
| :--- | :--- | :--- | :--- |
| Automata Theory | $(8,0)$ | Computing | $(10,0)$ |
| Logic | $(8,0)$ | Probability Theory I | $(20,4)$ |
| Prolog | $(10,0)$ | Mathematical Modelling | $(24)$ |
| Microcomputer Engineering | $(10,0)$ | Mathematics I | $(41,44)$ |
| Professional Engineering Studies | $(12,0)$ | Mechanics of Fluids | $(24,24)$ |
| Strength of Materials | $(24,24)$ | Thermodynamics | $(16,24)$ |

Laboratories (in Heat Transfer, Aeronautics, Engines, Computing, Prolog, Electrical Engineering, Microcomputers, Civil Engineering) 37 afternoons.

The hours shown in parentheses are in the form (lecture hours, problem class hours).

## SECOND STAGE

| Applied Mechanics | $(10)$ | Complex Variables | (20) |
| :--- | :--- | :--- | :--- |
| Software Engineering | $(10+$ labs $)$ | Design Support Systems | $(15)$ |
| Finite Difference Methods | $(15)$ | Logic | $(14)$ |
| Mathematical Methods | $(30)$ | Prolog | $(16$, all labs) |
| Vector Calculus | $(10)$ | Vector Space Theory | $(20)$ |
| Control | $(34+$ labs) | Continuum Mechanics |  |
| Professional Engineering Studies (30) | Case Studies |  |  |

Hours shown in parentheses are lecture hours.

## THIRD STAGE

Compulsory Subject Professional Engineering Studies
Optional Subjects (Choose 4 whole subjects + one half subject)

| Non-linear Continuum Mechanics $\dagger$ | (15) | Continuous System Modelling $\dagger$ | (15) |
| :---: | :---: | :---: | :---: |
| Information Theory* | (30) | Control \& Systems Theory | (30) |
| Variational Methods* | (30) | Heat Transfer* | (30) |
| Artificial Intelligence | (30) | Analysis of Algorithms | (30) |
| Operations Research** | (30) |  |  |
| $\dagger$ Half Subject |  |  |  |
| *First 15 hours may be taken as a h **Either the first 15 hours or the hour examination. Whole subjects | If sub | a two-hour examination <br> ay be taken as a half subject wi e hour examination. |  |

mathematical models and includes a mini- project. Most lecture courses are supplemented by appropriate problem and laboratory classes. The total lecture time is of the order of eleven hours per week, three of these hours being taken up by mathematics. Problem classes occupy seven hours each week, whilst time in the laboratories is about four hours weekly.

A course in Professional Engineering Studies runs throughout all three years. It introduces engineering students to basic aspects of accountancy, economics, industrial law, contracting, and the place of the engineer in society.

The Second Year comprises specialist mathematics courses. Lectures total about ten per week. The courses are of two kinds: those relating to fundamental mathematical theory (viz. Complex Variable Theory, Vector Space Theory, Linear Systems, Mathematical Methods, Numerical Analysis, Probability Theory) and those which consider in detail some areas of the application of mathematics (viz. Applied Mechanics, Control Theory, Continuum Mechanics, Decision Theory). There are also lectures in structured programming and microprocessors.

One special feature of the Second Year are the courses in Vector Calculus and Vector Space Theory which are classified as guided reading courses. These comprise a set of notes which guide students through a recommended text, discuss certain difficulties and important issues, and suggest suitable examples to be tried. This exercise is part of an attempt to encourage students to read more widely and the general reduction in timetabled hours from those in the First Year is designed to allow more private study. Example classes are normally of a "general surgery" type, but some lecture courses organise individual classes.

A second special feature is the Case Study activity. The students are subdivided into subgroups of two or three and are presented with a problem derived from an actual industrial situation. The problem is given in the form of correspondence, data sheets, technical reports and memoranda. Each subgroup has to 'interpret the problem, create a mathematical model, obtain solutions and make
recommendations' (206). The subgroup holds weekly discussions with a member of the academic staff who acts as an industrial manager. Each Case Study lasts typically two or three weeks, which gives an element of time limitation. The students are required to submit a written report. Clements (142) has reported on the popularity of this activity with his students.

In the Final Year the students select a number of lecture courses from a wide choice of options. In addition a Mathematics project, supervised by a member of the Engineering Mathematics Department must be undertaken. The author's feeling is that these projects tend to be more of a general applied mathematical nature than they are specifically engineering-orientated.

As regards the career prospects of graduates, the most popular area of employment has been in software production. However, the list of employers shown in Appendix 17 indicates a range of companies willing to take on graduates, including large industrial concerns, Government agencies, and career choices outside engineering.

### 7.4.3 Mathematical Engineering at Eindhoven

When the OECD Report (14) was being written the course in Mathematical Engineering at the Technische Universiteit Eindhoven was only four years old. As with the course at Delft which had started five years earlier, the first two years of the course were identical to the first and second year curriculum of one or other of the engineering departments. The remaining three years of the course were mainly devoted to mathematics, with engineering modules being chosen to emphasise topics of a general and fundamental importance - for example, Control Engineering, Fluid Dynamics, Engineering Mechanics. There were three slants in the curriculum: the mathematical treatment of physical processes, the statistical aspects of industrial problems and the numerical solution of mathematical problems.

Plans were already being drawn up to reduce that period of the course which
was coincident with another engineering discipline to one year, instead of two. It was felt that the current students were spending time acquiring knowledge and skills in areas in which they would not be active.

When the author visited Eindhoven in May 1988 he found the academic staff in the Faculty of Mathematics and Computer Science in an apprehensive mood. The nominal five year programme (which could take upwards of six years to complete because of the requirement to pass all subjects) had been reduced to a nominal four year programme (with an upper limit of five years). This was Government policy for all degree courses in the Netherlands and had caused the staff real anxieties that the quality of their graduates would decrease.

The course aims to provide training in the solution of scientific, engineering or management problems by mathematical methods. The first two years give an introduction to mathematics and computer science followed by a two year specialisation in one of the areas of discrete mathematics, analysis, applied statistics, and decision theory.

After the four year programme of courses the student is required to work for approximately six months on an individual research project in his chosen specialisation; the results of his research are presented in a thesis. The project topic can be chosen from the Faculty's research programme or the research can be carried out in industry.

In the First Year the students take courses in economics, engineers in society, analysis, algebra, numerical mathematics and programming, electricity and magnetism, and introductory mechanics. The first topic occupies about $20 \%$ of the lecture hours whilst the last two topics account for about $16 \%$. In addition to the lectures there are problem classes and practical sessions.

In the Second Year, compulsory courses are given in Analysis, Numerical Methods, Probability and Statistics, Fourier Analysis, Differential Equations,

Theoretical Mechanics, Function Theory and Matrix Theory. Optional courses are offered in Mathematical Economics, Abstract Algebra, Discrete Mathematics, Further Analysis, Further Differential Equations, Continuum Mechanics, Regression Analysis and Stochastic Processes. There is a requirement to choose a package of options totalling at least a specified minimum number of hours. There is a seminar style course in mathematical model-building which is also compulsory.

The discrete mathematics specialisation includes courses in Abstract Algebra, Stochastic Processes, Function Theory, Numerical Mathematics, Analysis, Group Theory, Optimisation, Combinatories, Cryptology, and Complex Variable Theory. These are supplemented by a variety of optional courses.

Successful candidates from the course are awarded the ir - degree (equivalent to a master's degree).

### 7.4.4 Comparison of the Courses

The titles of the courses at Loughborough, Nottingham and Bristol reflect the fact that these courses have placed different emphases on the engineering element. The author believes that each title is opposite to the role of engineering in the course concerned. The same cannot really be said in the case of Eindhoven; the reduction in the engineering content since the course was initiated was seen as regrettable but not unduly harmful to the character of the academic programme.

As has already been stated, the Loughborough course had been established as an engineering course; its accreditation and re-accreditation by the Engineering Council and two member institutions would indicate that this is how it is seen by the engineering profession. Nottingham has endeavoured to keep pukka engineering courses throughout all three years of the academic programme whilst Bristol's second and third years contain modules which are more of an applied mathematics/theoretical physics nature than engineering. In the opinion of the author it is matter for regret that Bristol and Nottingham have not integrated the
engineering component more into the overall course structure and have therefore not gained accreditation by member institutions. Being unique is sometimes not as advantageous as being nearly unique.

Whereas the Loughborough course stated its aim as being to produce an effective mathematical model builder and solver it was intended to be a course which fitted into the Loughborough tradition. Nottingham wanted its graduates to understand engineering principles and to be able to apply mathematics to the solution of engineering problems; it saw itself as training mathematicians. Bristol are clear that their students are given a training in applicable mathematics together with an introduction to engineering science. It seems reasonable to conclude that the courses are listed above in decreasing order as regards the importance of the engineering aspect. Loughborough is the only course to include the element on total Engineering Design and this is a clear distinguishing feature. Loughborough includes a group project it its Second Year and Bristol has its Case Studies module; all three courses have an individual project in their final years. Loughborough has the lowest proportion of degree marks for this project but when the Second Year project is taken into account, the three universities weight project work approximately equally.

Both Loughborough and Bristol give time to the areas of economics, industrial law and the role of the engineer in society. However, Bristol includes the subjects in all three years, increasing the proportion with the years whereas Loughborough restricts its coverage to the Final Year.

There are differences to be found in the content of the courses and the impression is a marked one that Bristol, and to a lesser extent Nottingham approach the mathematics components in a more formal manner than Loughborough. However, there are many similarities between the three courses and they all have a distinctive position in the spectrum of the academic programms of their respective universities.

Judging by the destinations of graduates, the three courses fill similar needs for employers. The range of posts taken on graduation is similar for each course and the work carried out is comparable.

The course at Eindhoven is nearer to that at Bristol than to the ones at Nottingham and Loughborough but it is more of a mathematics course than that at Bristol. The general approach to mathematics is a formal one and the Final Year Project is effectively a mathematics project. It does, however, include a module in mathematical model-building. Given that engineering courses in the Netherlands, in common with most continental European countries, are more theoretical than those in the United Kingdom, then perhaps the course at Eindhoven is in the same relation to mainstream engineering courses as are the Loughborough, Nottingham and Bristol programmes in this country.

### 7.5 The Contribution of 'Mathematical Engineering'

Thus far, no mention has been made of recruitment to courses in the United Kingdom, other than to refer to the demise of the degree programmes at Aston and Warwick.

Conversations with colleagues at Bristol and Nottingham Universities have revealed a less than rosy picture. It is now more than ten years since all the three courses were in operation and it might have been expected that they would have become firmly established in the spectrum of degree courses. With the intake quotas for Nottingham and Bristol ranging usually between 12 and 15, and that for Loughborough between 15 and 20, then the annual "demand" for recruits, even allowing for the transient presence of other courses, has never been more than about 65. Yet at no time have all the courses currently in operation been over-subscribed.

There has not been a lack of effort on the part of the Universities concerned. Secondary schools with a large sixth form have all been sent a copy of at least one course brochure, together with a poster advertising the courses at Bristol, Queen

Mary College, Loughborough, Nottingham and Swansea.

In the late 1970's the majority of UCCA applicants for these courses fell into two categories: those who had mainly opted for mathematics, and those who had mainly opted for a mechanical engineering type of course; in each case the 'mathematical engineering' courses were effectively stocking-fillers. More recently, however, there has been a tendency for the majority of applicants to plump for 'mathematical engineering', with mathematics or mechanical engineering as the back-up.

It is perhaps too early yet to assess the impact of the graduates from these courses on industry. The time-scale is too short and the numbers so far have been too few to make any definite statements. However, the general view of employers is that those students who have entered industry have been a positive asset. Their range of background skills has allowed them to work more effectively in design terms than the traditional mathematics graduate.

The advantage to a department which has a large service commitment to engineering departments in having its own students has been considerable. It has allowed the author, for example, to carry out experiments in new teaching strategies before undertaking investigations with the mainstream engineers. Although the author has always enjoyed a good rapport with the latter groups of students it is more satisfactory to use one's own undergraduates for the first run through of a teaching innovation. From a practical point of view it is easier to plan, execute and assess a teaching experiment with a group of 15 students than with one of 80 or more. Certainly, many of the experiments in modelling and the use of case studies were first tried with the author's mathematical engineering undergraduates, and the informality of the teacher/student relationship that was possible with a small group led to more comprehensive feedback than was likely to be the case with a large group of students from another department, no matter how good the rapport.

It is noticeable how the morale of the staff in the author's Department
improved once the Mathematical Engineering course was initiated. However satisfying the service teaching of engineers may be, there is a special pride in teaching one's own undergraduates on a course which one has planned. In addition, there has been a small yet regular supply of students staying on to carry out research, which is a bonus in present circumstances.

However, the main reason for establishing an undergraduate course is because one believes that there is a real need to produce graduates with a particular blend of skills and knowledge. It is clear that a section of industry - the author would say the more enlightened section of industry - perceives a real need for graduates from hybrid courses of the nature this chapter has been describing. The problem lies in attracting sufficient good quality applicants from schools and colleges. These days 'small' is not beautiful and the viability of courses with only a few students entering each year is likely to be questioned. The hope must be that a sympathetic view prevails.

How to obtain more applicants remains the burning question. There seems to have been a move away from engineering courses towards the financial, accounting and management area. With the decrease in the number of 18 year olds due to continue for a number of years there will be increased competition to attract students.

Despite the competition the quality of entrants to the Loughborough course has shown a marked improvement. At the time of the first accreditation exercise in 1982, the average point score of ' $A$ ' Level entrants (based on $A=5, B=4$ etc) had just risen from 10 in previous sessions to 11.2 in the current session; the entry of October 1988 had an average point score of 14.9.

With 1992 only four years away, the gap between UK engineering graduates and their mainland European counterparts as regards depth of mathematics knowledge will prove an embarrassment. The mathematical engineer will be in the best position of United Kingdom engineers to meet the challenge of a united Europe.

Perhaps as 1992 draws nearer, the message may penetrate into the schools and the recruitment picture may improve.

The author is in no doubt that courses such as those at Loughborough, Nottingham and Bristol have an important role to play in the future of the engineering industry in the United Kingdom. That role will increase in importance as we move into the twenty-first century. They are the engineering courses of the future and it would be a tragedy if that future were threatened by a lack of vision today.

## Chapter 8

## Related Matters

### 8.1 Partial Differential Equations: Resolving Difficulties

The topic of partial differential equations is commonly introduced in second-year engineering courses. Most students find it the hardest part of their mathematical studies and there is a strong feeling amongst engineering staff that it is probably too difficult for their students. The author, in common with many of his colleagues, has constantly agonised over the presentation of this topic and has carried out a number of investigations with individual students in an attempt to improve that presentation.

Searl (149) compared the relatively small amount of theory that is usually provided in lectures to underpin the solution of partial differential equations with that accompanying the solution of ordinary differential equations. He argued that one of the reasons for this disparity was the difficulty associated with the former. It was important for a student after his first encounter with partial differential equations to be able to recognise the types of equation and the nature of the accompanying boundary conditions. The student should also be able to interpret these conditions in physical terms and have a working knowledge of the special properties of the solution of each type of equation.

The author had independently developed an approach similar to that recommended by Searl. The package of lectures which is described below has been distilled from the experience gained over several years lecturing on the topic and from the ideas of colleagues both at Loughborough and elsewhere. The presentation to second year Civil Engineers follows a treatment of Fourier series and is timed to coincide with that part of their Geotechnics course which deals with seepage in soils and, by prior arrangement, also with that part of the Building

Services stream which covers steady state heat flow. The students are shown that the governing equation in each case is Laplace's equation. Then follows a description of the Geotechnics laboratory experiment which uses conducting paper cut into a shape similar to that of the soil under a given dam; the distribution of electrostatic charge on the surface of the paper is also governed by Laplace's equation. The top edge of the paper is connected to one terminal of a voltage supply and the three other edges are earthed. With the aid of a galvanometer the lines of equal potential can be traced out; from these lines a flow net for the seepage problem can be constructed.

Attention is then turned to the steady-state temperature distribution in a thin rectangular plate. The plate is insulated on its two large faces so that heat can enter or leave the plate only along its edges. Following the treatment given in Bajpai, Mustoe and Walker, Advanced Engineering Mathematics (42) Laplace's equation is developed for the steady state distribution of temperature $\theta(x, y)$, viz

$$
\begin{equation*}
\frac{\partial^{2} \theta}{\partial x^{2}}+\frac{\partial^{2} \theta}{\partial y^{2}}=0 \tag{8.1}
\end{equation*}
$$

which applies at all points on the plate $0 \leq x \leq a, 0 \leq y \leq b$. The specific problem which is discussed is that where three edges are maintained at $0^{\circ} \mathrm{C}$ and the fourth at $100^{\circ} \mathrm{C}$. Heat will clearly flow from the hot edge, across the plate and out through the three other edges. At any point in the interior of the plate the temperature will change, rapidly at first, then more slowly, until steady-state is reached when no appreciable changes occur.

The boundary conditions are stated as

$$
\left.\begin{array}{lllll}
\theta=0 & \text { when } & x=0 & \text { for } & 0 \leq y \leq b  \tag{8.2}\\
\theta=0 & \text { when } & y=0 & \text { for } & 0 \leq x \leq a \\
\theta=0 & \text { when } & x=a & \text { for } & 0 \leq y \leq b \\
\theta=100 & \text { when } & y=b & \text { for } & 0<x<a
\end{array}\right\}
$$

It is argued that to solve (8.1) we effectively need two integrations with respect to $x$ and two with respect to $y$; each of these will introduce an arbitrary
element into the formula for $\theta$, which implies a need for four conditions to obtain a unique solution - two on $x$ and two on $y$.

The students are asked what other set of boundary conditions would specify a unique solution and it is suggested that they should consider the effect of insulating the edge $y=0$. After a discussion the condition

$$
\frac{\partial \theta}{\partial y}=0 \text { at } y=0,0 \leq x \leq a
$$

emerges.

The author believes that this carefully laid background is important, not just in its own right, but also because it provides motivation for his students. The solution of the equations and the application of the boundary conditions is a very long haul for both parties and the motivation is a vital sustaining force.

Having been shown the class the analytical solution for this problem the students are introduced to a simple numerical solution using a relaxation technique and the output, in graphical form, from a program using the same technique on a much finer mesh. The details are provided in Bajpai et al (42).

The usefulness of the analytical solution is discussed in view of its complicated nature, its inability to yield readily any graphical information about the nature of the solution and its unsuitability as a means of calculating temperatures.

The Building Services stream have been set an exercise by one of their lecturers which requires them to find numerically the steady-state temperature distribution in the cross-section of a chimney and this acts as a useful conclusion to the discussion of this part of the lecture package on partial differential equations.

Next, the one-dimensional diffusion equation is considered using as examples unsteady heat flow in a bar and the consolidation of a layer of wet soil which is allowed to drain excess pore water ; for the latter case see, for example, Craig (207). These problems introduce the idea of a steady-state solution and a transient.

With a long bar $0 \leq x \leq \ell$, the temperature $\theta(x, t)$ is governed by the equation

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=k \frac{\partial^{2} \theta}{\partial x^{2}} \tag{8.3}
\end{equation*}
$$

where $k$ is a diffusivity constant for the material of the bar.

It is agreed that suitable boundary conditions would be to specify the temperature at both ends of the bar and the initial temperature distribution along the length of the bar.

The first set of conditions examined is

$$
\left.\begin{array}{llll}
\theta=100 & \text { at } & x=0 & \text { for } t>0  \tag{8.4}\\
\theta=0 & \text { at } & x=\ell & \text { for } t>0 \\
\theta=100 & \text { at } & t=0 & \text { for } 0 \leq x \leq \ell
\end{array}\right\}
$$

This set is discussed in physical terms and then the mathematical statement provided. What is the steady-state distribution? Equation (8.3) reduces to

$$
\frac{\partial^{2} \theta}{\partial x^{2}}=0
$$

suggesting a linear profile; the nature of the boundary conditions leads to

$$
\begin{equation*}
\theta_{\mathrm{s}}=100\left(1-\frac{\mathrm{x}}{\ell}\right) \tag{8.5}
\end{equation*}
$$

The second set of conditions is

$$
\left.\begin{array}{llll}
\theta=100 & \text { at } & x=0 & \text { for } t>0  \tag{8.6}\\
\theta=50 & \text { at } & x=\ell & \text { for } t>0 \\
\theta=100 & \text { at } & t=0 & \text { for } 0 \leq x \leq \ell
\end{array}\right\}
$$

In this instance, the steady-state distribution is

$$
\begin{equation*}
\theta_{\mathrm{s}}=50+50\left(1-\frac{\mathrm{x}}{\ell}\right) \tag{8.7}
\end{equation*}
$$

The third set is

$$
\left.\begin{array}{llll}
\theta=100 & \text { at } & x=0 & \text { for } t>0  \tag{8.8}\\
\theta=0 & \text { at } & x=\ell & \text { for } t>0 \\
\theta=0 & \text { at } & t=0 & \text { for } 0 \leq x \leq \ell
\end{array}\right\}
$$

in which case the steady-state distribution is again given by

$$
\begin{equation*}
\theta_{\mathrm{s}}=100\left(1-\frac{\mathrm{x}}{\ell}\right) \tag{8.5}
\end{equation*}
$$

The three results are compared in terms of the physical situations which they represent. Now it is argued that

$$
\theta(x, t)=\theta_{S}(x)+\theta_{T}(x, t)
$$

where $\theta_{T}(x, t)$ is the transient part of the solution.
Then

$$
\frac{\partial \theta}{\partial t}=0+\frac{\partial \theta_{T}}{\partial t}
$$

and

$$
\frac{\partial^{2} \theta}{\partial x^{2}}=0+\frac{\partial^{2} \theta_{T}}{\partial x^{2}}
$$

since $\theta_{S}$ is a linear function of $x$. Hence (8.3) reduces to the equation

$$
\begin{equation*}
\frac{\partial \theta_{T}}{\partial t}=k \frac{\partial^{2} \theta_{T}}{\partial x^{2}} \tag{8.9}
\end{equation*}
$$

Furthermore, given that the steady state solution (8.5) satisfies the conditions
$\theta_{S}=100$ at $X=0$ and $\theta_{S}=0$ at $x=\ell$ then the conditions (8.4) reduce to

$$
\left.\begin{array}{lllcc}
\theta_{T}=0 & \text { at } & x=0 & \text { for } & t>0  \tag{8.10}\\
\theta_{T}=0 & \text { at } & x=\ell & \text { for } & t>0 \\
\theta_{T}=100 \frac{x}{\ell} & \text { at } & t=0 & \text { for } & 0 \leq x \leq \ell
\end{array}\right\}
$$

It is readily seen that for the second and third examples also

$$
\theta_{\mathrm{T}}=0 \text { at } \mathrm{x}=0 \text { and at } \mathrm{x}=\ell
$$

The solution for the first example is derived, having first suggested a time component which decays exponentially. The class is invited to solve the equation (8.3) for the two other sets of conditions; hopefully they will see the connection between the three solutions and save themselves much tedious calculation.

The class is now asked what would be the effect of insulating one end of the bar, say in the first example, and to obtain the solution in such a case to verify the conjecture. For the example of the consolidation problem the insulation condition corresponds to an impermeable boundary.

The mainstream Civil Engineers will be shown the solution of the consolidation equation for several sets of boundary conditions and have the topic reinforced in their Geotechnics lectures a few weeks later.

Finally, the wave equation is discussed briefly, the emphasis being on the nature of the boundary conditions and the resulting solution.

The solution of the Laplace equation is now effected by the method of separation of variables. The hope is that the motivation that has been engendered will keep spirits up whilst the tedium of the method is endured.

Experience has shown that it helps the students to offer up as potential solutions functions such as

$$
\theta=2 \sin 3 x \sinh 3 y
$$

and

$$
\theta=5 \cosh 2 x \cos 2 y
$$

The students can see that these are indeed solutions since they satisfy Laplace's equation. It is then readily accepted that it is reasonable to look for a solution which is based on building blocks of the form

$$
\theta(x, y)=X(x) \quad Y(y)
$$

When the equation

$$
X^{\prime \prime}(x) Y(y)+X(x) Y^{\prime \prime}(y)=0
$$

is derived, it is a relatively straightforward matter to obtain the rearrangement

$$
\begin{equation*}
\frac{X^{\prime \prime}(x)}{X(x)}=\frac{-Y^{\prime \prime}(y)}{Y(y)} . \tag{8.11}
\end{equation*}
$$

The rearranged equation becomes for the first suggested solution

$$
\frac{-18 \sin 3 x}{2 \sin 3 x}=-\left(\frac{9 \sinh 3 y}{\sinh 3 y}\right) \text { ie }-9=-(9)
$$

and in the second case

$$
\frac{20 \cosh 2 x}{5 \cosh 2 x}=-\left(\frac{-4 \cos 2 y}{\cos 2 y}\right) \text { ie } 4=-(-4)
$$

This makes more plausible the argument that each side of (8.11) must be a constant. Of course, it is well known that most people prefer to see specific examples of a method in action before attempting a formal generalisation.

It is important to emphasise the role of the boundary conditions in restricting the number of possible candidates for solution. The first task is to decide the sign of the constant referred to above and the consequences of choosing
it to be positive, negative or zero are shown. Then the boundary conditions are examined for clues to the sign required in the problem under consideration. It is agreed that a sine variation in $X$ is required and the constant is chosen as negative; this leads to the variation in $y$ being described by a sinh/cosh formulation. Hence a suitable solution is of the form

$$
\theta=\sin k x(A \sinh k y+B \cosh k y)
$$

The condition

$$
\theta=0 \text { when } y=0
$$

leads immediately to the choice of $B=0$; this has never presented a problem to the students to whom the author has lectured.

Since $\theta=0$ again when $x=a$, suitable values of $k$ are

$$
\frac{\pi}{a}, \frac{2 \pi}{a}, \frac{3 \pi}{a}, \ldots \text { etc. }
$$

It is easy to rule out $k=0$ and it is straightforward to point out that

$$
k=-\frac{\pi}{a}
$$

gives a solution of the same form as

$$
k=+\frac{\pi}{a}
$$

so that no new information is gained by considering negative values of $\mathbf{k}$.
Therefore, possible candidates for the required solution are

$$
\theta=A_{1} \sin \frac{\pi x}{a} \sinh \frac{\pi y}{a}, \theta=A_{2} \sin \frac{2 \pi x}{a} \sinh \frac{2 \pi y}{a} \ldots \text { etc }
$$

where the multiplying constant has been labelled differently in each case to allow maximum generality. It is then pointed out that Laplace's equation is linear and by reference back to the solution of linear ordinary differential equations covered in the previous session it is made more palatable when the general solution so far is written as

$$
\begin{equation*}
\theta=\sum_{n=1}^{\infty} A_{n} \sin \frac{n \pi x}{a} \sinh \frac{n \pi y}{a} \tag{8.12}
\end{equation*}
$$

The one remaining boundary condition is applied with the comment that it is only relevant for $0 \leq x \leq a$. Equation (8.12) reduces to the condition

$$
\begin{equation*}
\sum_{n=1}^{\infty} C_{n} \sin \frac{n \pi x}{a}=100,0 \leq x \leq a \tag{8.13}
\end{equation*}
$$

where

$$
C_{n}=A_{n} \sinh \frac{n \pi b}{a}
$$

This is now recognised as a Fourier Series problem and, using the techniques learned when covering that topic immediately prior to the module on partial differential equations, it is a simple matter to obtain

$$
\begin{aligned}
C_{n} & =\frac{400}{n \pi}, n \text { odd } \\
C_{n} & =0, n \text { even }
\end{aligned}
$$

so that the solution is finally obtained as

$$
\begin{equation*}
\theta(x, y)=\frac{400}{\pi} \sum_{n=1,3,5}^{\infty}\left[\frac{\sin \left(\frac{n \pi x}{a}\right) \sinh \left(\frac{n \pi y}{a}\right)}{n \sinh \left(\frac{n \pi b}{a}\right)}\right] \tag{8.14}
\end{equation*}
$$

It is then again pointed out how cumbersome this formula is and how difficult it is to obtain information about the nature of the solution without expending considerable effort.

Searl (149) criticised the method of separation of variables because the class of problems for which it will provide a solution is small. Furthermore, he cited the equation

$$
\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial x^{2}}, 0 \leq x \leq 1
$$

which, when accompanied by the conditions
$u_{x}(0, t)=0, u(1, t)=1$ and $u(x, 0)=0$
can be solved by the method but when accompanied by the conditions
$u_{x}(0, t)=0, u(1, t)=1$ and $u(x, 0)=0$
cannot be so solved; there was no clear reason why success in the first case should be countered by failure in the second. The author of this thesis agrees with these criticisms but he believes that there are a number of practical examples where the method does produce an answer and whilst engineering text books continue to quote the solution thus obtained, it is important to show the students how it had been derived. The consolidation of wet soil is a case in point; see Craig (207).

In summary, the topic of partial differential equations is a difficult one to teach and the ground needs to be laid carefully. By showing the relevance of such equations and demonstrating the relationship between the boundary conditions and the physical problem the lecturer can sustain the students' motivation through the tedium of the analytical solution. Of course, with other groups of engineers, relevant examples can be taken from their discipline.

### 8.2 Fourier Series: Using Tutorials to Advantage

There is a tendency amongst lecturers who teach second year students to make too many assumptions about the knowledge which they have carried forward from their first year studies. When he first taught the topic of Fourier series to second year engineers the author had assumed that most of his audience would be familiar with the properties of the functions $\sin \mathrm{mx}$ and $\cos \mathrm{mx}$. Tutorial classes had shown an inability to calculate the Fourier coefficients for a given function, but it was the testing of individual students, selected randomly, over a number of years that demonstrated the widespread ignorance of facts which had been taken for granted when preparing the lectures.

Before the package of lectures on Fourier series the students are given a tutorial class on the properties of $\sin \mathrm{mx}$ and $\cos \mathrm{mx}$. They are first asked to make a sketch of the graph of $\sin x$ for the domain $-6 \pi \leq x \leq 6 \pi$ and are asked to
point out its main features. The particular answers sought are its oddness and its periodic nature. The students are reminded of the definitions of an odd function and of a periodic function and it is seen that the function $\sin x$ has period $2 \pi$. They are then asked to repeat the exercise for $\sin 2 x$ and $\sin 3 x$. In the case of the first of these functions many students will produce graphs of $\sin x / 2$ or $2 \sin x$. These misconceptions must be ironed out firmly. Next the class is presented with the function

$$
f(x)=2 \sin x+3 \sin 2 x
$$

Is it periodic? If so, what is the period? Again there is confusion; some students will aver that the period is $\pi$ and some gentle but firm persuasion is required. Now the students have to tackle the function

$$
f(x)=a \sin x+b \sin 2 x+c \sin 3 x
$$

where $a, b$ and $c$ are constants.

Having correctly identified it as being periodic and of period $2 \pi$ the students are expected to comment that the function is odd. At this stage they are asked to repeat the complete exercise with the cosine function replacing the sine function. After they have obtained the required results they are asked to consider the function

$$
f(x)=d+a \cos x+b \cos 2 x+c \cos 3 x
$$

where $d$ is a constant.

There is a certain reluctance to accept that this function is periodic; even the replacement of $x$ by $x+2 \pi$ does not fully convince the doubters. To complete this part of the proceedings the class is confronted with the function

$$
\begin{gathered}
f(x)=a_{0}+a_{1} \cos x+b_{1} \sin x+a_{2} \cos 2 x+b_{2} \sin 2 x \\
+a_{3} \cos 3 x+b_{3} \sin 3 x
\end{gathered}
$$

where the $a_{i}$ and $b_{i}$ are constants. Is it odd? Is it even? Clearly not. But is it
periodic? Yes, and the period is $2 \pi$.

Next, the students are shown the graph reproduced in Figure 8.1 (a). It is easily identified as $f(x)=x$. However, the function $g(x)$ whose graph is presented in Figure 8.1 (b) proves somewhat harder to describe. Well then, concentrate on the interval $-\pi \leq x \leq \pi$. What is the function in that range? Yes, it is $\quad g(x)=x$. What do we notice about the function in the range $\pi<x<3 \pi$; it seems to be the same as that part between $-\pi$ and $\pi$ shifted along. Then we accept that the function is periodic. After some prompting the function is described thus:

$$
\left\{\begin{array}{l}
g(x)=x \\
g(x+2 \pi)=g(x)
\end{array} \quad-\pi<x<\pi\right.
$$

It takes careful argument to explain how the second part of the definition covers periodicity to the left in addition to the right. But what about the value of the function at $x=\pi$ and $-\pi$ and $-3 \pi$, etc? It is agreed to complete the definition by extending the first part of the definition to be

$$
g(x)=x \quad-\pi<x \leq \pi
$$

although the alternative

$$
g(x)=x \quad-\pi \leq x<\pi
$$

is recognised as equally plausible. Occasionally, a student will suggest that the definition $g(\pi)=0$ is in some way a more reasonable compromise. Then the function $h(x)$ whose graph is reproduced as Figure 8.1 (c) is displayed. The class is able to voice readily the observation that $h(x)$ is periodic and of period $2 \pi$ and then its definition is sought. First it is decided to tackle the portion between $x=-\pi$ and $x=\pi$. It is clear that a simple description would be

$$
\begin{aligned}
& h(x)=|x| \\
& h(x)=\left\{\begin{array}{rrr}
x & , \quad 0 \leq x \leq \pi \\
-x & -\pi<x<0
\end{array}\right.
\end{aligned}
$$



Figure $8.1(a)$


Figure $8.1(b)$


Figure 8.1(c)

Because of the periodicity, the definition is completed by the statement

$$
h(x+2 \pi)=h(x)
$$

What is the most obvious difference between $g(x)$ and $h(x)$ ? The former is discontinuous at $x= \pm \pi, \pm 3 \pi$, etc. Also, $h(x)$ is an even function, whereas $g(x)$ is odd.

Finally, the students are asked to compare $f(x), g(x)$ and $h(x)$ for the sub-domain $0 \leq x<\pi$. There is no distinction to be drawn. They are asked whether a good approximation to $h(x)$ will be a good approximation to $f(x)$ in the interval $0 \leq x<\pi$. It will. But will it be a good approximation to $g(x)$ ? Some doubt is expressed about the situation near $x=\pi$. However, it is agreed that a good approximation to $h(x)$ will also be a good approximation to $f(x)$ in the interval $0 \leq x<\pi$.

The third part of the tutorial returns to the functions $\cos x$ and $\sin x$. The students are asked to consider the integrals

$$
\begin{array}{ll}
I_{1}=\int_{-\pi}^{\pi} \sin m x \sin n x d x, & m \neq n \\
I_{2}=\int_{-\pi}^{\pi} \cos m x \cos n x d x, & m \neq n \\
I_{3}=\int_{-\pi}^{\pi} \sin ^{2} m x d x & I_{5}=\int_{-\pi}^{\pi} \frac{1}{2} d x
\end{array}
$$

The class is divided into four subgroups, each having a different integral from the
set $I_{1}$ to $l_{4}$. When it is reported that $I_{1}=0=l_{2}$ and that $I_{3}=\pi=I_{4}$ the result $I_{5}=\pi$ is contributed and it is explained that that the somewhat peculiar choice of $1 / 2$ as the integrand was to ensure that $I_{5}=I_{3}=I_{4}$. It is also pointed out that since $\sin ^{2} x$ and $\cos ^{2} x$ take the same set of values in the interval $-\pi \leq x \leq \pi$, then it is reasonable to expect that $I_{3}=I_{4}$ and that

$$
I_{3}+I_{4}=\int_{-\pi}^{\pi}\left(\sin ^{2} n x+\cos ^{2} n x\right) d x=2 I_{5}=2 \pi
$$

so that

$$
I_{3}=I_{4}=\pi
$$

Are there any other integrals which we should consider? It is suggested that we might examine

$$
\begin{aligned}
& I_{6}=\int_{-\pi}^{\pi} \sin m x \cos n x d x, \quad m \neq n \\
& I_{7}=\int_{-\pi}^{\pi} \sin n x \cos n x d x \\
& I_{8}=\int_{-\pi}^{\pi} \frac{1}{2} \sin n x d x
\end{aligned}
$$

and

$$
I_{9}=\int_{-\pi}^{\pi} \frac{1}{2} \cos n x d x
$$

Collecting the results that $I_{6}, I_{7}, I_{8}$ and $I_{9}$ are all zero, the observation is made that from the building blocks $1 / 2, \cos x, \sin x, \cos 2 x, \sin 2 x, \ldots$ the only products which do not vanish when integrated between $-\pi$ and $\pi$ are
$\sin ^{2} \mathrm{mx}$ and $\cos ^{2} \mathrm{nx}$, and perhaps we might add (1/2) ${ }^{2}$.

A comparison is drawn with a set of vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$ which are such that $\mathbf{a} . \mathbf{b}=\mathbf{b} . \mathbf{c}=\mathbf{c} . \mathbf{a}=0$. They would be said to be orthogonal; could we say that the functional building blocks are also orthogonal is some sense?

The way has now been paved for the development of the Fourier series approximation to a function which will take place in the next mathematics lecture. In the author's opinion this is the correct use for a tutorial - to supplement the work of the lecture by covering background material in an interactive way.

More recently, use has been made of a computer enhanced learning unit on Fourier Series. This shows how the various harmonics can be superimposed to create a Fourier Series approximation to a given function. The opportunity is also taken to compare odd and even extensions of a half-range function.

### 8.3 Examining the Examinations

A worry facing any examiner, however experienced, is that the examination paper he sets for his students will be fair: fair to his students and fair to his standards. He must stretch the good student but he must not leave a weaker student, who has worked hard all year, floundering in despair. One set of public examinations which has been looked to over many years as exemplifying the appropriate standard are those of the Engineering Council (previously the Council of Engineering Institutions); however, the author has gained the impression that the questions set today are more fragmentary than was the case some years ago.

Sometimes it is not clear what the purpose of a question is; sometimes several threads appear tangled. Consider the two questions below: Question 1 is taken from the CEI Part I Examination in Mathematics in 1978, whereas Question 2 appeared on the EC Part I Examination in Mathematics ten years later.

1 Interpret the complex number $e^{i \theta}$ on an Argand diagram. From the exponential definitions of the hyperbolic functions, show that
$\sinh i z=i \sin z$ and $\cosh i z=\cos z$
and hence obtain expressions for the real and imaginary parts of the complex function $\cosh (\pi z / a)$. Construct in the $z$ plane the semi-infinite rectangle defined by the points $(0,0),(0, a),(\infty, a)$, $(\infty, 0)$. Determine the area in the $w$ plane into which this rectangle is mapped by the transformation

$$
w=\cosh (\pi z / a)
$$

In particular, indicate the corresponding points in the $w$ plane and find the curves into which the sides of the rectangle are transformed.
(a) The complex number $z_{1}$ is shown in Figure 8.2 (a). Find $z_{1}^{20}$ and write it in Cartesian form giving the numerical values to the nearest integer.

If $z_{2}=-5+i 2$, use de Moivre's theorem to find in polar form the cube roots of $z_{2}$ and display them on an Argand diagram.
(b) In a certain engineering situation $Y=[1 /(R+i \omega L)]+i \omega C$. Derive the value of $\omega$ in terms of $R, L$ and $C$ so that $\operatorname{Im}(Y)=0$, and write down Y in this case.
(c) In Figure 8.2(b) OPQR is a parallelogram. Write down the vectors RQ and RP in terms of $\mathbf{z}$.
(i) If $z$ varies such that $|z-4|=3$ describe geometrically the locus of $\mathbf{Z}$ and also the corresponding locus of $\mathbf{Q}$.
(ii) If $z$ varies such that $\arg (z-4)=3 \pi / 4$ derive algebraically the equation of the path of $P$ and find the equation for $Q$.


Fig 8.2(a)


Fig 8.2(b)

In the first example there is an identifiable thread running through the question; each part leads on to the next in a coherent fashion. From the interpretation of $e^{i \theta}$ the candidate is asked to use the exponential function $e^{i z}$ and then $\cosh (\pi z / a)$ to tackle a complex transformation based on the latter. In contrast, Question 2 appears somewhat of rag-bag of techniques on the theme of complex variables. Once the candidate has recovered from the shock of the length of the question he has first to use the result that

$$
\{\mathrm{r} \theta\}^{20}=\mathrm{r}^{20} \underline{20 \theta}
$$

then he has to apply de Moivre's theorem to find cube roots. Next he is expected to use the idea of a complex conjugate to find the imaginary part of a complex fraction. He then has to focus his attention on the geometrical interpretation of the modulus and argument of a complex number.

The first observation to be made is that a candidate whose knowledge of complex variables was restricted to the basics could cope with most of the 1988 question but would be hard pressed to perform as credibly on its 1978
predecessor. Further, the more recent question guides the student much more carefully through the steps that he is expected to carry out; there is no real chance for an outstandingly good candidate to perform significantly better than one who is much less able.

Whereas there is something to be said for weak candidates being able to gain a reasonable mark on an examination paper, this should not be at the expense of the more able candidates being able to demonstrate their supremacy. The "solution" which the author has adopted is to set some questions on which the weaker candidates can score most of the marks necessary to pass the examination but to have other questions which are more demanding: an outstanding candidate could score $100 \%$ but a modest one should have to struggle hard to achieve a distinction. When an examination paper contains sufficient questions of the style and content of Question 2 for a high mark to be achieved on basics, it becomes dubious as to what it is intending to test. In fairness, it must be pointed out that the more recent style was adopted by a new Chief Examiner who believed (rightly) that the cause of so many candidates obtaining poor marks was their inability to carry out the most fundamental of mathematical tasks; he argued that it was preferable to test these fundamentals as opposed to taking them for granted when setting questions.

The Bristol University examination IACEM $\Omega$ Papers 1 and 2 are each of the style; answer six questions out of eight in three hours. Whereas the 1988 papers do contain some questions of the type in which the parts are related there are some curious bedfellows, for example Question 3 below. On the same paper are also to be found questions combining limits of functions of the form $f(x) / g(x)$ with work on fixed-point iteration and on constrained partial differentiation combined with a part on Laplace transforms of functions of the form $t . f(t)$.

3 (a) Find the values of $z$ for which

$$
z^{4}=(-2 \sqrt{3}-2 j)
$$

writing them in the form $x+j y$.
Sketch their positions on an Argand diagram.
(b) Explain why the following integrals are improper and show whether or not they exist
(i) $\int_{0}^{2} \frac{d x}{x^{2}-2 x+1}$
(ii) $\int_{0}^{\infty} \frac{\sinh (x)}{x \cosh (x)} d x$
(You are not required to evaluate them.)

It is of course a long-standing dilemma as to whether or not to try to cover all the main topics on the syllabus. This can lead to artificial hybrid questions of this type. An argument in favour of this approach is that to omit some topics would introduce an element of luck into the process, favouring those candidates who had been fortunate in their selection of topics to revise. It could be argued, equally convincingly, that a hybrid question does not allow a candidate to demonstrate his knowledge in depth on a topic. In public examinations such as those of the Engineering Council, the candidates have no direct contact with their examiners and are therfore at a disadvantage compared with their contemporaries who sit internal examinations. An internal examiner has the opportunity to guide his students in their revision.

4 (a) If $\mathbf{e}$ is an eigenvector of the matrix $A$, with eigenvalue $p$, show that $\quad A^{2} e=p^{2} e$ and hence that $\mathbf{e}$ is also an eigenvector of the matrix $\mathbf{A}^{2}$, with eigenvalue $p^{2}$.
(b) The matrix
$A=\left(\begin{array}{lll}3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3\end{array}\right)$
has 6 as one of its eigenvalues. Find all the eigenvalues and associated normalised eigenvectors of the matrix $\mathbf{A}$.

Question 4 is from Paper 2 of the same year. Although at first sight the question seems to have a coherence, the author believes that the first part is actually misleading. Granted part (b) does not include words like "hence" or "therefore", but a candidate might be forgiven for thinking that part (a) has some relevance to its successor; if not, what purpose does it serve? Why use the notation $\mathbf{A}$ for the matrix in both cases? Before the author is accused of cavilling, let the reader remember that candidates under examination stress do not always think clearly and a question which could unwittingly lead them up a blind alley is to be avoided.

If in answering Question 4 the candidate did calculate

$$
A^{2}=\left(\begin{array}{rrr}
12 & 8 & 8 \\
16 & 20 & 16 \\
8 & 8 & 12
\end{array}\right)
$$

he would wonder what use he could make of the fact that one eigenvalue of $A^{2}$ is 36 ; what is the advantage over the direct calculation of the eigenvalues of $\mathbf{A}$ ?

5 Use Gaussian elimination to determine the inverse of the matrix

$$
\mathbf{A}=\left(\begin{array}{lll}
0 & 2 & 3 \\
2 & 0 & 1 \\
3 & 1 & 2
\end{array}\right)
$$

if it exists.
Hence find values of $x, y$, and $z$ that satisfy the equations

$$
\begin{array}{r}
3 x+y+2 z=1 \\
2 y+3 z=1 \\
2 x+z=2
\end{array}
$$

Question 5 is taken from the same examination paper as Question 4; this can be compared with a question from the 1988 paper Engineering Mathematics IB set by the University of Nottingham - Question 6.

6 Use the Gauss-Jordan method (row operations) to invert the matrix A given by
$\mathbf{A}=\left[\begin{array}{rrr}2 & -4 & 0 \\ 3 & 0 & 5 \\ 0 & 3 & 2\end{array}\right]$
and hence solve the equations

$$
A x=b
$$

for each of the right-hand sides
(a) $\mathbf{b}=\left[\begin{array}{r}10 \\ 0 \\ -6\end{array}\right]$,
(b) $\mathbf{b}=\left[\begin{array}{r}12 \\ -3 \\ 9\end{array}\right]$,
(c) $\mathbf{b}=\left[\begin{array}{r}-4 \\ -3 \\ 3\end{array}\right]$.

For the matrix $A$ above, find the three values of $\lambda$ for which the equation $A x=\lambda x$ has a non-trivial solution.

In both cases one is tempted to ask why the candidate is being asked to solve the equations

$$
A \mathbf{x}=\mathbf{b}
$$

by first finding the inverse matrix $A^{-1}$. This is not the basis of a practical method of solution. The question of the relevance of the mathematics is appropriate in such instances. In the case of Question 5 it does seem a strange way of presenting the mathematics, especially since the candidate has to 'spot' that the equations need to be re-ordered before a direct comparison could be
made with the matrix $A$. If one were to solve the equations by inverting the coefficient matrix then the matrix to be inverted would be

$$
\left(\begin{array}{lll}
3 & 0 & 1 \\
0 & 2 & 3 \\
2 & 0 & 1
\end{array}\right)
$$

What is gained by giving the matrix in the form of $\mathbf{A}$ ? Question 6 suffers from a similar air of artificiality. If one wished to solve the system $\mathbf{A x}=\mathbf{b}$ for three right-hand sides then it would make sense to create a matrix $\mathbf{B}$ whose columns were the three given vectors. Then one could carry out an extended Gauss Jordan elimination on the augmented matrix

$$
\left(\begin{array}{rrr:rrr}
2 & -4 & 0 & 10 & 12 & -4 \\
3 & 0 & 5 & : & 0 & -3 \\
0 & 3 & 2 & : & -6 & 9 \\
0
\end{array}\right) .
$$

By way of contrast, most of the questions on the Nottingham paper were not open to these criticisms; they did hang together and were blending theory and calculation well. Question 7 is an example of such an approach.

7 The fixed-point iterative method for solving the equation $f(x)=0$ consists of rearranging the equation into the form $x=\phi(x)$ and then generating a sequence of approximations $x_{n+1}=\phi\left(x_{n}\right)$ which may or may not converge to the root of $f(x)=0$.
(a) Prove that if a root is known to lie in the interval (a, b) and that $\phi:(a, b) \xrightarrow{\text { into }}(a, b)$ with $\phi$ continuous on $[a, b]$ and differentiable on ( $a, b$ ) with $\left|\phi^{\prime}(x)\right|<1,(a<x<b)$, then the fixed-point iterative method $x_{n+1}=\phi\left(x_{n}\right)$ will converge to a root of the equation $x=\phi(x)$.
(b) Find a rearrangement of the equation $x^{2}-2 x-3=0$ which, when an initial approximation $x_{0}=4$ is taken, will converge to a root of the equation. This root should be calculated to an accuracy of THREE decimal places.
(c) State what is meant by the order of convergence of an iterative method and show that the fixed-point method has linear convergence when the above conditions on $\phi$ hold.

The author is aware that any examination paper could be dissected and crticised in this fashion. He is also aware that he has been guilty of some of the shortcomings that have been highlighted in the questions considered. For example, in 1973 question 8 was set to a first year class comprising both Chemical Engineers and Civil Engineers. No excuse can be offered for including part (ii): it is there merely to pad out the first part of the question on interpolation. If the argument were advanced that the first part is concerned with the approximation of a function which is specified by a set of tabulated values whereas the second part is concerned with an approximation of a function specified by a formula, then it would be a feeble one.
8. (i) The angle turned through by a shaft was measured and the results were tabulated as follows:

| $t$ (seconds) | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\theta$ (radius) | -0.002 | 0.058 | 0.149 | 0.283 | 0.471 | 0.707 |

Form a difference table for $\theta$ and use Newton-Gregory formulae to estimate $\theta$ when $t=0.3,0.15,0.7$ and 0.9 .
(ii) Obtain by any means the Maclaurin expansion of $\tan ^{-1} \mathrm{x}$ as far as the term in $\mathrm{x}^{5}$.
9. Evaluate $\int \sqrt{x} \ln x d x$
10. Solve the equation $z^{3 / 2}-4 z^{2}=0$

Questions 9 and 10 were set two years earalier to a comparable class. The questions could be charitably be described as "academic"; it is hard to see what purpose they serve. Would such integrals or such equations ever confront the students in real life - or even in their engineering lectures? It is highly unlikely. If the argument is that it is important for the students to acquire manipulative skills then surely it is possible to test such skills on more relevant examples.

For many students the written examination is the major element of assessment, if not the only element and in a large group of students there may be no personal knowledge which can be brought to light in an examination panel. It is therefore of the utmost importance that the examination papers, even at this level, are carefully scrutinised not merely for correctness but also to see whether each question achieves its objectives.

Some years ago the following question was typical of those to be found on examination papers for second year engineering undergraduates.
"Find the eigenvalues and eigenvectors of the matrix

$$
\mathbf{A}=\left(\begin{array}{rrr}
4 & 2 & -2 \\
1 & 3 & 1 \\
-1 & -1 & 5
\end{array}\right)
$$

For its complete solution the following steps must be effected:
(i) Form the determinant
$|A-\lambda I|=\left|\begin{array}{ccc}4-\lambda & 2 & -2 \\ 1 & 3-\lambda & 1 \\ -1 & -1 & 5-\lambda\end{array}\right|$
(ii) Expand the equation $|A-\lambda I|=0$
ie $\lambda^{3}-12 \lambda^{2}+44 \lambda-48=0$.
(iii) Solve this equation to obtain

$$
\lambda=2,4 \text { or } 6
$$

(iv) Solve the system of equations

$$
(A-21) x=0
$$

ie $\left(\begin{array}{rrr}2 & 2 & -2 \\ 1 & 1 & 1 \\ -1 & -1 & 3\end{array}\right)\left(\begin{array}{l}x_{1} \\ x_{2} \\ x_{3}\end{array}\right)=\left(\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$
to obtain $X=(1,-1,0)^{\top}$
by noting that, for example, the first row of $\mathbf{A}-21$ is equal to the third row subtracted from the second.
(v) Solve the system of equations
$(A-4 I) x=0$
ie $\left(\begin{array}{rrr}0 & 2 & -2 \\ 1 & -1 & 1 \\ -1 & -1 & 1\end{array}\right)\left(\begin{array}{l}x_{1} \\ x_{2} \\ x_{3}\end{array}\right)=\left(\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$,
by ignoring the third equation as being a repeat of the result of combining the first two, to obtain

$$
\mathbf{x}=(0,1,1)^{\top}
$$

(vi) Solve the system of equations

$$
(A-61) x=0
$$

ie $\left(\begin{array}{rrr}-2 & 2 & -2 \\ 1 & -3 & 1 \\ -1 & -1 & -1\end{array}\right)\left(\begin{array}{l}x_{1} \\ x_{2} \\ x_{3}\end{array}\right)=\left(\begin{array}{l}0 \\ 0 \\ 0\end{array}\right)$,
by working with the first and third equations, to obtain $\mathbf{x}=(1,0,-1)^{\top}$

There are additional skills required:
(a) Recognition of the need to solve the equation
$|A-\lambda| \mid=0$
to find the eigenvalues.
(b) To find the eigenvector associated with a particular eigenvalue, it is necessary to solve the equations

$$
(A-\lambda I) x=0
$$

(c) Of these individual equations, one will be redundant; the choice of the one to be ignored may effect the ease of solution.
(d) When the remaining equations are solved, there will be an undetermined parameter and it suffices to select a suitable member of the family of solutions in each case.

Furthermore, if the question had continued
"Hence find a matrix $\mathbf{P}$ such that $\mathbf{D}=\mathbf{P}^{-1} \mathbf{A} \mathbf{P}$ is diagonal and write down the matrix $D^{\prime \prime}$ then the extra skills required are
(e) Recognise that the required matrix $\mathbf{P}$ - the modal matrix - is formed by writing the eigenvectors of $\mathbf{A}$ as its columns.
(f) Know that the diagonal elements of $D$ are the eigenvalues of $\mathbf{A}$ in the same order as the eigenvectors have been entered into $\mathbf{P}$.

For the candidate who performs every step correctly then the question is a fair test of his understanding of the mechanism of calculating the eigenvalues and associated eigenvectors of a $3 \times 3$ matrix in addition to his ability to perform the calculations correctly. But what of the candidate who makes an arithmetical error early on? An omitted minus sign could well lead to the equation $\quad \lambda^{3}-12 \lambda^{2}+44 \lambda-56=0$ which does not factorise readily. And where does the candidate go from here? There is little hope of him guessing the correct values of $\lambda$, so that he would be unlikely to proceed further; without a correct value for $\lambda$ it would not be possible to obtain a set of equations which contained a redundant one, except by pure fluke. How much credit would, or should, an examiner give to a candidate who was able merely to describe the steps he would carry out?

How to set a question which allows a candidate to display his knowledge and ability having committed an early arithmetic error is a problem which has been tackled by colleagues at different institution in different ways. In 1987, Queen Mary College (208) gave the characteristic polynomial in factorial form as a hint accompanying the question. In 1988, Nottingham (209) gave the eigenvalues as part of the text of the question; in the same year, Bristol (210) gave one eigenvalue in the text.

The examiner must ask what he is trying to achieve by his question. There is always a worry in providing partial results as part of the text of an examination question; the author's experience has been that some candidates spend a very long time trying to detect an error so that they can obtain this result, whilst some employ dishonest means to dupe the examiner into believing that they have achieved the result. There is also the matter of how to apportion the total marks for the question amongst its constituent parts. Two different examiners could apportion the marks quite differently and a candidate could badly misjudge how many marks he is forgoing should he omit or give up on a particular component. There is a school of thought which argues that such uncertainty is part of the examination process; certainly, the author is not convinced that it is wise to show on the question paper the allocation of marks to each section of every question.

The author is of the opinion that some level of algebraic manipulation is important and should be tested; equally, the candidate should be required to demonstrate a knowledge of the stages in the process of solution. The style of question the author currently employs is epitomised by that which follows
"(a) Find the characteristic equation of the matrix

$$
\mathbf{A}=\left(\begin{array}{rrr}
4 & 2 & -2 \\
1 & 3 & 1 \\
-1 & -1 & 5
\end{array}\right)
$$

(b) A matrix $B$ has characteristic equation

$$
\lambda^{3}-2 \lambda^{2}-5 \lambda+6=0 ;
$$

find the eigenvalues of $B$.
(c) The matrix
$\mathbf{C}=\left(\begin{array}{rrr}1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1\end{array}\right)$
has eigenvalues $\lambda=0,1,3$. Find the associated eigenvectors."

This style of question allows, for example, a student who cannot factorise a cubic polynomial to demonstrate his knowledge and ability on parts (a) and (c); it also allows a student who is able to factorise the characteristic polynomial to demonstrate that skill. (In practice, the author would relate the matrix $\mathbf{C}$ to, say, the modelling of a compressor for a jet engine by three disks on a rotating shaft. The candidates would be expected to relate the eigenvalues and eigenvectors to the model and make suitable comments.)

### 8.4 Visual Images and the Role of Television

There is no doubt that the undergraduate of today relies more on visual stimuli than his counterpart of twenty years ago. The modern engineering student has grown up in a world where television is the dominant mass medium; he seldom reads a serious newspaper and hardly ever listens to the spoken word on radio for any significant length of time. Since the earliest days of his childhood, television has filled his mind with striking visual images; not for him the active participation of creating mental images to complement the radio broadcast of an adventure serial which was the joy of the author's early years.

To test these theories some informal experiments were conducted with groups of students using a twenty-minute excerpt from a television documentary. In the first experiment the students were asked to pay attention to both the commentary and to the pictures on the screen. Via a short questionnaire accompanied by a post-viewing discussion, it was quite obvious that the spoken word had made very little impact whereas many of the visual images had been retained. A second experiment was carried out with one half of the group listening to a tape recording of the commentary whilst the other half viewed the pictures with the sound turned off. The questionnaires and the discussion revealed that the latter half had maintained their interest and
retained more information. Even when note-taking was allowed the latter half showed a greater awareness of the content of the programme.

Of course, most mathematics lecturers make use of overhead projectors and/or the blackboard to present their material. It has been known for a mathematics lecturer to read for the full 50 minutes from the text-book which he had written, but that was an exception. However, even the more common form of presentation will result in loss of concentration in the learner after a while. Research has shown that attention from an audience fluctuates during the course of the lecture. After some fifteen to twenty minutes there is a decline in attention followed by a peak towards the end of the lecture: Brown \& Atkins (211). The decline in attention can be lessened by varying the activities during the lecture. Experience has shown that switching from overhead projector to blackboard to present supplementary results and then returning to the overhead projector to continue the main thrust of the lecture leads to an increase in the concentration of the students.

It has been known for some time that visual reinforcement of the spoken word results in the learner enhancing his retention the lecture material, and that the incorporation of some activity on the part of the learner leads to even greater retention. On the occasions when students have been asked to perform calculations on their pocket calculators during lectures on numerical techniques there is some indication that this activity has helped maintain their interest.

Stice (212) discusses four stages of learning
(i) concrete experience (CE), which emphasises personal involvement
(ii) reflective observation ( RO ), which involves watching and listening
(iii) abstract conceptualisation (AC), where learners actively experiment with situations.
(iv) active experimentation (AE), where learners actively experiment with situations.

Learners are categorised in four main groupings.
(i) Divergers, who prefer to learn by CE and RO. They are creative and understand people's behaviour patterns.
(ii) Assimilators, who learn primarily by RO and AC. They are more interested in abstract ideas than their practical value.
(iii) Convergers, who are strong on AC and AE . They like the practical application of ideas and are good at solving problems and making decisions.
(iv) Accommodators, whose learning preferences are AE and CE . They learn primarily from "hands-on" experiences and tend to act on feelings rather than logical analysis.

Significantly, engineers are to be found mostly amongst convergers, whereas mathematicians predominantly lie in the assimilator category. Stice argues that mathematics lecturers should enhance the presentation of their subject when teaching engineers not only by introducing applications but also by using visual demonstrations.

Experimental demonstrations have been used successfully on occasions to help illustrate certain mathematical ideas. For example, when introducing the topic of eigenvalues and eigenvectors, the author uses a model of a two-storey frame; the model consists of two heavy steel bars connected by steel bands to each other and a heavy steel base. By setting the two bars into horizontal vibration the idea of eigenvalues as normal frequencies and the eigenvectors as associated normal modes can be explained. Students have said that when they come to tackle abstract problems on finding the eigenvalues and eigenvectors of a given matrix it has helped to retain the image of the vibrating modes. The "live" nature of the experiment was a key feature for them.

To illustrate mathematical theories the use of animated pictures is helpful. The role of computer-animated movies in elucidating mathematical concepts was the subject of a study carried out by the author (213). He had found that the understanding of the theory of viscous flow between parallel plates was enhanced by first showing an extract of a computer-produced film which demonstrated the velocity profile and the idea of vorticity and by then showing a commercially - produced film of a laboratory experiment of the flow. The case was argued for the production of films which incorporated computer-animated sequences alongside sequences of real physical phenomena.

For many years the Open University has used three-dimensional models in its mathematics television programmes. These models are of a high quality and many lecturers have looked wistfully at these programmes, wishing that they could have such quality products at their disposal. Even with the use of video recorders, the sheer effort and time involved in arranging for perhaps two minutes of play-back in a lecture renders the idea impracticable.

The suggestion of computer-animated sequences lying alongside conventional film sequences was re-echoed by Berry (214) and (215). Berry had lectured at the Open University and was a strong advocate of the use of video in teaching mathematics; he believed that video was a valuable resource which was widely available but greatly under-used. He saw it being used to introduce real world applications, to assist in the mathematical modelling of real problems and to provide an insight into the physical interpretation of mathematical theory. As an example of the first use he suggested that the flow of liquid past a cylinder could be investigated by showing a video sequence of such a flow. For the second use he suggested that the validation of a simple model of laminar flow in a pipe could be effected by showing a video sequence of a real flow. In the third use he cited the example of illustrating the concept of the curl of a vector via a sequence which showed
(i) the motion of water in a bowl which rotated with constant angular velocity
(ii) the motion of water near to a bath plughole
(iii) the flow of water along a long rectangular tank.

The advantage of a video recording was that it could be stopped and started and replayed at will thus introducing an element of interaction.

The case for interactive video was argued by Gayeski and Williams (216). They described the system that they have developed in which sequences on the videotape are accessed by means of a microprocessor which controls the branching of a teaching program.

Berry is also an advocate of bringing simple experiments into the classroom (217). He gave an outline of how he introduces the topic of the mechanics of flight. First, he demonstrates how blowing down through a cotton reel onto a post card does not cause the card to be blown away: an illustration of Bernoulli's equation. Then he shows a sequence of flow past an aerofoil which includes the phenomenon of stalling. A study of vibrations begins with the showing of a video sequence of the Tacoma Narrows bridge in oscillation followed by a discussion on forced oscillations. Next, natural oscillations are discussed in terms of car suspension systems. A second video clip shows sea waves, vibrating chimneys and, finally, the collapse of the Tacoma Narrows bridge. Simple Harmonic Motion is investigated using a specially designed piece of apparatus in which masses of 50 g and 100 g can be induced into vertical oscillations with the aid of a variablespeed electric motor. It is possible to show the free oscillations before switching on the motor. With the motor switched on forced oscillations and resonance are demonstrated, then the experiment is repeated with a dashpot in place to demonstrate the effects of damping. At this stage, the conjectures generated by the experiment can be verified or refuted by formulating and solving a mathematical model for the system. Whilst carrying out the mathematical analysis, the students will be able to relate back to the experiment and to the video sequences, the visual images of which are vividly retained.

With the advent of video disc as a viable teaching medium there is obviously a whole area of activity waiting to be exploited. The facility of storing vast amounts of numerical and visual data on one disk and having almost immediate access to any part of it could have, and should have, an enormous effect on the presentation of lecture material. Once again, resources will be needed to make full use of the opportunity being offered.
"Traditional" universities and polytechnics have generally been slow to take up some of the challenges offered by the use of television in their teaching. Whatever one's opinions on the desirability of television as a medium for entertainment, it is a medium for instruction which opens up exciting possibilities; these possibilities must be converted into reality. The Open University has shown its sister institutions the way forward; they must tread this path for themselves.

### 8.5 Distance Learning

With the need for engineers to update their knowledge and skills during their professional careers there is an incoming role for distance learning to fulfill.

Rumble (217) quotes seven characteristics which he regards as essential for a comprehensive definition of distance education.
(i) Separation of teacher and student. The overall design of a system of distance education is based on the premise of there being a separation in distance and time of the teaching and the learning.
(ii) Influence of an educational organisation. Distance learning needs to be differentiated from private study at home; the key factor is that there is a parent educational institution that is consciously teaching its students.
(iii) Use of technical media. The recent growth of distance learning has been facilitated by the use of communications media to provide the basic elements of teaching.
(iv) Two-way communication between individual students and tutors. It is strongly emphasised that the student must have the opportunity for dialogue with his tutor and that the student can initiate the dialogue.
(v) Absence of group learning. Whilst occasional seminars are a possibility, especially now that new communications technology allows group interactions at a distance, distance education is characterised by the almost permanent absence of the learning group.
(vi) Industrialising education. The mass production and distribution of learning materials, together with the logistical and administrative problems of coping with dispersed populations of teachers and students, requires an industrial approach to management.
(vii) Privatisation of learning. This characteristic is really closely connected with ( $v$ ) and reflects the growing preference of adults in particular to study in private at home rather than participating in evening classes. Television has been a major factor in the relative growth of home-centred activities.

The best-known system of distance education in the United Kingdom has been the Open University. Its work has been described in many articles; for example, Berry (218). The Open University enrolled its first students in 1971 and operates on a 32 study week year, with courses beginning in February and finishing in September. Examinations are held in the two following months. In July and August week-long summer schools are held in conjunction with each study course. Because of the "Open" nature of its entry policy - students do not need to possess any formal qualifications - the first level of courses are Foundation courses which mainly teach skills and ideas usually taught in sixth
forms and their equivalent. The Foundation courses are built on by second and third level courses (with a few fourth level courses). Hence in many instances, a second level course will correspond to what is taught in the first year of a conventional university course.

A BA degree can be obtained by a student who holds six credits, with a further two being necessary to obtain a degree with honours. A full credit may be gained by passing, successfully a 32 week course, whereas 'half-credit courses' are 16 weeks in duration. Students are required to take two Foundation courses (almost without exception, this means two different faculties).

A teaching package for a typical mathematics course will comprise correspondence texts, audio-cassettes, television broadcasts, contact with a tutor, assessments and, perhaps, experimental work.

The correspondence texts are booklets written by the 'course team' and may refer to a textbook for supplementary reading. The booklets are designed to be self-learning material. Some basic skills in mathematics are best accomplished by practice and the use of an audio-cassette with accompanying written material can help in this activity. The student will play the tape, puti:t on pause while carrying out some task and play the next section to check on what he has done and set a new task.

Most people who are not Open University students have contact with its courses through the lecturers on broadcasts and it is assumed, wrongly, that these are the prime means of teaching. In fact, the programmes are designed to act as a reinforcement of the teaching and provide in many cases a 'visual enrichment' of a topic. The 25 minute programmes, broadcast on a weekly or fortnightly basis, also help a student working in isolation to pace his study. There has been a move in recent years to make video an option to the televised broadcast to allow the student the opportunity of a more active role.

Each student is assigned a part-time tutor who can provide extra teaching to the student by holding face-to-face tutorials or telephone conversations to iron out weaknesses. The main role of a tutor is to mark part of the student's assessment. Tutor-marked assignments consist of essays, problems and project work, although for mathematics it is the second form which predominates. Tutors have carefully structured marking notes to help ensure reasonably consistent marking around the United Kingdom and the tutor is expected to write helpful comments on the student's answers before returning the marked scripts to him.

A second form of continual assessment is the computer-marked assignment which consists mainly of short questions with a list of optional answers from 'which the 'correct' one and enters it on a specially-designed form. these assignments are posted to the appropriate marking centre and the results accumulated on the student's computer record.

Sparkes (219) quoted approximate figures for the in man-hours of teacher time required to produce one student-hour of study: Audio tapes 6 hours, tutor-text 50 hours, TV broadcasts 50 hours. Note that these figures do not include the number of man-hours of support staff required, nor, indeed, the cost of the technology involved.

Smith (220) drew attention to the growth of distance education students in universities and colleges of advanced education in Australia in the ten years since 1975. The number of such students in the universities had doubled whilst that in the colleges had trebled. The growth was attributed to five causes: the need to upgrade qualifications in response to technological change, the convenience of distance study as opposed to part-time study based on campus, the changing status of women, the growing respectability of the distance mede of education, and the trends towards privacy in Australian society.

Smith argued that distance education had had a significant effect on methods of teaching and learning in all sectors of Australian higher education. As an example he cited the case of alternative teaching methods. Distance educators had had to optimise on the use of print, graphics and audio-cassettes. They had been more ready than their campus-based colleagues to experiment with alternative methods of instruction because they needed to be. As a result the latter have been forced to consider new methods of teaching and have often used some of the materials produced. There was a feeling that there would be a convergence of teaching methods between the distance and mainstream modes although it was stressed that the face-to-face method of instruction is one of the most effective methods available and would ensure that the two modes of education did not converge totally.

The author's own teaching has benefitted from watching the Open University's programmes. He can only envy them the luxury of a course team approach and of the supporting skills of the BBC technicians and graphic artists. He sees an important role for distance education in providing post-experience courses and believes that campus-based universities will have to be prepared to operate in a distance education mode to cater for the needs of engineers, for example, to update their knowledge and skills. With a decreasing population of eighteen-year-olds, the universities must look to those already in the engineering profession to provide the necessary consumers of their teaching.

## Chapter 9

## A Proposed Teaching Model

### 9.1 Introduction

In this chapter the author sets out his stall by proposing a model for teaching mathematics to engineering undergraduates in the United Kingdom which will serve their needs in the 1990's and beyond. The basic premise on which the model is built is that the mathematics should be taught as one of the engineering subjects. This premise will undoubtedly offend some mathematical colleagues who decry those who have regard for the needs of the customer. However, if these needs are not satisfied then the customer will be forced to look elsewhere and, at the end of the day, the mathematics lecturer must ask himself what the purpose of his course of lectures really is. It would indeed be gratifying to think that the students appreciated the power and the beauty of mathematics, that they understood its innermost workings and even had an intrinsic interest in mathematics for its own sake. It would also be flying in the face of reality.

An engineering undergraduate has selected his particular discipline because he has a practical outlook on life; he wishes to be a manufacturer or a designer of things that work or that serve a pupose - aeroplanes, cars, pumps, bridges and the like. Were he interested in mathematics primarily for its own sake he would have elected to read for a mathematics degree; he did not so choose. Mathematics is a tool of his trade, and that fact must be accepted by the mathematics lecturer, however unpalatable that may be for him. Mathematics is, however, the most important of the tools that the engineer will possess and special care must be taken to ensure that he values it above all the other tools in his tool-box. In order to instil in the student an appreciation for the role of mathematics in his studies it is necessary to convince him of its value. That can only be done satisfactorily by demonstrating its worth through the medium of applications.

A commonly-held view is that mathematicians should preserve at all costs rigour in their teaching to engineers. The remark has been made on several occasions that one would not drive a car if one did not know how it worked. This is clearly not the case. Many, if not most, drivers in the United Kingdom have at best a very limited knowledge of what lurks underneath the bonnet of their car. A few are able to maintain their own cars quite satisfactorily themselves; on the other hand, there are some who will insist on carrying out repairs and maintenance beyond their competence. The overwhelming majority, however, take their cars to a garage for any major work that needs to be done. The crucial factor is to know when to take the car to the garage rather than attempt the work oneself.

Mathematics is the language of modern engineering and provides a thread which runs through all the constituent subject areas. It has the power to unify and to clarify ; it is the means of analysis and the means of synthesis ; it is the agent of generalisation. The mathematical modelling of engineering systems is a process which the modern engineer must master thoroughly if he is to survive in the complex world of tomorrow.

The reality facing the mathematics lecturer is one in which there is pressure to reduce contact hours, there is an increasing lack of mathematical knowledge among his freshman intake, and there is a need to incorporate fresh topics into his syllabus as new areas of study become more firmly established in engineering practice.

The choice facing the mathematics lecturer is clear: either he continues to succumb to these pressures and gives courses which he finds ever more unsatisfactory, or he can seize the initiative and make mathematics the keystone of undergraduate engineering. By a judicious mixture of techniques and applications, together with a programme of case studies, modelling exercises and laboratory classes, the students will be led towards an appreciation of the role of mathematics in their chosen discipline and will be sufficiently motivated to study
the subject with interest. Once the students have shown this interest, there is every hope that the engineering staff will follow suit.

In designing the teaching model which forms the subject matter of this chapter the following goals have been set.
(i) To give the student an understanding of basic mathematical concepts and an appreciation of the language of mathematics,together with an awareness of the need for rigour.
(ii) To provide a tool-kit of mathematical techniques which will last the student throughout his professional career.
(iii) To train the student in the application of those techniques and to give him an awareness of their limitations.
(iv) To train the student in the skills involved in the mathematical modelling of engineering systems.
(v) To provide a solid foundation in mathematics so that in his later career the engineer can acquire new skills and master those new techniques and concepts which enter his discipline.

In the following sections we examine what mathematics should form the core curriculum of the courses taught to engineers; what are the additional needs of the various engineering specialisms; how, when and by whom the mathematics should be taught; and what teaching aids and strategies should be employed. Finally, the problems associated with the teaching and implementation of the proposals are discussed.

### 9.2 What Mathematics Should Form the Core?

In 1970 the paper by Bajpai et al on the teaching of differential equations (21) was responsible for a radical re-appraisal of the teaching of mathematics to engineers. One outcome of that re-think was the design and implementation of a
course which integrated the numerical and analytical techniques. Since that revolution there has been a continuous evolution taking place in the teaching of engineering mathematics. Now it is time to be equally revolutionary.

Most professional engineers must expect to make frequent use of standard computer packages, and many during their careers will be more actively involved in the specification, design, writing and amending of programs. It is important that an appropriate form of computer education be provided to prepare the engineering student for the demands that are likely to be made upon him. In the past, education in computing has been treated as a 'craft-skill' rather than an integral part of his training. It is currently being argued that the software engineering industry itself is moving from a craft-based industry to a mathematics-based technology. There is a feeling that part of the explanation of why many programs today are of dubious accuracy and reliability is a lack of appreciation by the programmers of formal methods and the relevant mathematics. One level removed from the intelligent user of software is the designer of software, and one level removed from the latter is the designer of computers: these engineers will need to have a far greater knowledge of mathematics than the user, of course.

There can be no doubt that the new revolution in the teaching of mathematics to engineers must be the introduction of a substantial amount of discrete mathematics into the core syllabus. This may mean the relegation of many topics of long standing to a more minor role or even to an optional mathematics course which supplements the core material. So be it.

Another consequence of the increasing part being played by computer-based methods in engineering is the rise in the status of linear algebra. Continuous problems are made discrete, non-linear problems are linearised at each step of an iterative process and dynamic problems are broken up into finite time steps. The digital computer has pushed the analogue computer into a poor second place and the signals analysed are now discrete rather than continuous. Differential
equation systems lead to matrix equations where the coefficient matrix is commonly factorised into two triangular matrices. Hence the need to solve efficiently banks of linear equations.

For those who favour a rigorous approach to the teaching of mathematics to engineers, linear algebra offers the ideal medium. Because of its applicability to the solution of engineering systems it has a high motivational content. It is also an elegant subject and there is every expectation that, having been given an application-orientated approach, the student will accept a rigorous treatment of certain topics in the field of linear algebra.

The Finite Element method has now established itself as a widely-used technique for solving problems in many branches of engineering. Any core curriculum in engineering mathematics worthy of the name must prepare the ground for the student to be able to work with the method with success and assurance.

The growth in importance of Computer-Aided Design is making it essential for a foundation to be laid in certain areas of geometry. The geometric description and properties of curves and surfaces and the use of spline functions as approximators, for example, can lay claim for serious consideration as part of the core curriculum.

How can all these new areas of mathematics enter a crowded syllabus without a radical weeding out of the established topics that are already there? They cannot. It is always painful to say farewell to friends, especially those of many years standing, but the mathematics taught to engineers cannot stand still (and one might say become ossified) if it is to survive into the 1990's as a subject worthy of repect. No doubt, many hands will be raised in horror at the absences from the core, but the decisions as to what material remains in the core and what has to depart can be made on the grounds of practicality and realism.

The core curriculum syllabus is presented as Table 9.1. Topics have been grouped under ten sub-headings but it is recognised that some of the topicsecould sit equally happily under other sub-headings. When designing a lecture course based upon this core the lecturer should have the freedom to organise his coverage of the topics in a way which suits both his requirements and the needs of the students concerned.

The hours shown in Table 9.1 are lecture hours and a suggested partitioning between the three years of a United Kingdom engineering degree course are: 65 hours in the first year, 60 hours in the second year and 40 hours in the final year.

## Table 9.1

The core mathematics syllabus (165 hours)

## 1. Modelling with mathematics <br> (30 hours)

Introduction to mathematical modelling, modelling flowchart. Case studies chosen to illustrate techniques covered in the syllabus.
Programming in a high level language.
Flow diagrams, loops, arrays and subscripts, subprograms and files.
Use of software packages.
Model representation, behaviour, evaluation.
Sensitivity, validity, refinement of models.
Lumped parameter and distributed parameter modelling.
2. Discrete Mathematics (25 hours)
Mathematical Logic. Propositions, connectives, truth tables. Rules of inference, quantifiers, concepts of proof and program correctness. Boolean algebra.

Sets and functions. Sets, subsets, introduction to cardinality.

Operations on sets. Boolean algebra.
Definition of a function, domain, co-domain and range.
Composition of functions, one-to-one, onto, inverse functions.
Recursive definition of functions.
Number systems. Binary and hexadecimal systems, computer arithmetic, rational and real numbers.

The concept of computability.
Mathematical induction. Recursion.
Difference equations and their solution.
The Z- transform and its relation to digital systems. Algorithms. Concepts of proof of an algorithm and the estimate of its efficiency.

Sorting and searching algorithms.
3. Linear Algebra
(20 hours)
Vectors. Geometry and algebra of vectors.
Scalar and vector product.
Vector spaces. Linear independence, bases, dimensions.
Matrices. Matrix algebra, special kinds of matrix, rank.
Systems of linear equations. Gauss elimination, pivotting, residuals. LU decomposition. Comparison with iterative methods of solution. Ill-conditioned systems. Determinants. Simple examples of linear programming.
Linear transformations. Orthogonality.
Eigenvalues and eigenvectors. Computational aspects.
Quadratic forms.
4. Calculus

Evaluation of polynomials.
Differentiation, maximum and minimum values, points of inflection.

Sequences and limits. Iterative solution of non-linear equations. Limits of functions.

Interpolation. Series approximation of functions, Maclaurin and Taylor series.
Fourier series, orthogonal functions, harmonic analysis.
Integration, including Trapezoidal rule and Simpson's rule Applications of integration.

Functions of several variables, maximum and minimum values. Constrained problems. Simple numerical optimisation.

Least squares curve fitting.

## 5. Geometry

Plane curves, tangents and normals, curvature.
Simple coordinate systems, transformations in two and three dimensions including rotation of axes.

Line and plane.
Conics.
Curve sketching, including asymptotes.
Splines. Natural splines, B-splines, Bézier curves.
Mesh surfaces.
Geometric elements: line, triangle, tetrahedron.
Natural coordinates, area and volume coordinates.

## 6. Differential Equations

(20 hours)
Ordinary differential equations. Classification of differential equations, arbitrary constants, initial and boundary conditions. Introduction to existence and uniqueness of solutions. First order equations. Isoclines and the sketching of solutions. Complementary function and particular integral.

Variables separable. Integrating factors.
Euler's method, Runge-Kutta fourth order method. Simple ideas on predictor-corrector methods.
Second order equations, linear with constant coefficients. Complementary function and particular integral.

Laplace transforms.
Boundary value problems.
Systems of first order equations and their solution by Laplace transforms.

Eigenvalues.
Non-linear systems, phase plane, critical points and stability. Stiff systems.
Partial differential equations.
Derivation of Laplace's equation, boundary conditions. Simple relaxation method of solution.

Derivation of the diffusion equation, boundary and initial conditions. Introduction to explicit and implicit finite difference methods of solution.

The wave equation. Nature of boundary and initial conditions, and solution.

## 7. Probability and Statistics

(16 hours)
Dealing with data. Graphical representation of data,frequency distributions, measures of location and dispersion.

Probability. Events, conditional probability, random variables, probability distributions, expectation.
Probability models. Binomial, Poisson, Exponential and Normal distributions.

Inference. Large samples. Estimation, confidence limits and hypothesis testing. Small samples. $F$ and $t$ distributions. Statistical modelling. Simple linear regression.
Introduction to the design of experiments.
Use of statistical computer software.
8. Introduction to the Finite Element method (12 hours) Principles of minimum potential energy and virtual displacements.

Element stiffness matrices.
Idealisation of simple structures.
Principle of virtual work.
Introduction to and use of a Finite Element package.
9. Mathematics of Fields
(6 hours)
Gradient, divergence and curl. Physical interpretations. Conservative fields, scalar potential.
Examples of the application of Gauss', Stokes' and Green's theorems.
10. Complex Variables (8 hours)

Complex numbers, de Moivre's theorem and roots.
Complex mappings.
Poles, zeros.
Residues applied to Laplace transforms.

Commentary on the core curriculum

## 1. Modelling with mathematics

This component of the core is the one which is expected to require the greatest active participation by the students. Many of the models will be computer-based and it is important that the students be trained to use the computer from the earliest moment possible. There is no evidence yet that even the majority of engineering freshmen are conversant with the use of computers and therefore for the foreseeable future it must fall to the tertiary institutions to provide the initial training. Where there are large numbers of students involved there will clearly be a logistical problem. Careful timetabling arrangements will be required.

It is necessary that newcomers to using the computer should be eased in gently. Many students have a fear of inadequacy when making their first encounter with a terminal or a micro and it is wise to let them use a standard package to begin with until they gain their confidence. This aspect is examined further in Section 9.4, but it is mentioned here because of the importance of the computer in the modelling of engineering systems. The engineering student must come to regard the computer as a natural ally.

Initially, the models chosen will be able to be tackled using school level mathematics only. The emphasis should be on the modelling process itself. Later in the course it will be possible to study models that arise in the solution of real engineering problems and the level of mathematics involved can, of course, be higher.

It is recommended that some group activity be included, with each group being required to present a verbal and written report on their mathematical model.

## 2. Discrete Mathematics

It is a matter for growing concern for computer users and for those who rely on the results of computer programs that these programs are becoming more complex. It is now more difficult to verify that these programs are correct. Logic-programming is a increasingly important tool in the production of software and it is being predicted that it will be central to the development of the next generation of computer languages. Mathematical logic is crucial in the design of digital circuitry.

An understanding of sets, subsets and set operations is necessary to the provision of a precise means of discussion of the concepts relating to computing. The language of set theory is used to describe arrays, procedures and relations. Boolean algebra is a key element in the construction of digital logic.

It is important to have a clear concept of number and to understand the forms of arithmetic which are germane to computing. Mathematical induction is an effective technique for the proof of programs.

The computer modelling of systems relies heavily on difference equations and for those equations which are linear with constant coefficients the use of Z-transforms is recommended, since there is a connection with the processing of digital signals. For other equations a more direct recursive approach is more suitable.

In order to provide an insight into the working of compiler systems and simple editors a study of relations and finite state machines is suggested.

The section on algorithms acts as a link for several aspects of this module. The key concepts are those of the proof of an algorithm and of an estimate of its accuracy. Sorting and searching algorithms are suitable examples for this purpose.

## 3. Linear Algebra

It is to be hoped that the opportunity will be taken to present this part of the core material in a way which emphasises its fundamental importance in engineering mathematics. A clear understanding of the concepts involved is essential for the successful application of its techniques. So much of the modelling of engineering systems relies on linear algebra that it is arguable that it, rather than calculus and analysis, should be the foundation on which the engineering course is built. It should be possible to emphasise the links with geometry in addition to considering computer-based techniques, whilst presenting some of the material in a rigorous way to give the student some feel for the theoretical underpinning which is necessary for the confident application of standard methods of solution.

The topic of eigenvalues offers a case in point. Geometric transformations
in the plane make for a simple introduction to the subject. It can be shown how the mathematical models of certain simple vibrating systems can lead to an algebraic eigenvalue problem and how the algebraic solution helps to understand the problem qualitatively. Then, more realistic problems can be tackled using numerical methods which can be implemented by a computer package. It helps when presenting the power method to highlight the geometrical aspect: students are able to see that the method effectively rotates the initial vector of approximate solutions until it is virtually coincident with an eigenvector.

Having introduced the idea of linear programming in two variables via a graphical approach the opprtunity should be taken to explain how the simplex method hinges on Gaussian elimination.

It is important that students should understand clearly the concept of linearity. They should, for example, know the implications of classifying a differential equation as linear, and why linear programming is so-called.

## 4. Calculus

Although a reduced role is suggested for this subject, it would be foolish to suggest that it is unimportant. Rather, the opportunity should be taken to omit the more exotic examples of differentiation and integration which abound in many textbooks. Too often time is spent on discussing technical "tricks of the trade" such as the use of the substitution of $t=\tan x$ in order to evaluate certain definite integrals. Instead, the emphasis should be on the use of calculus to provide criteria for the safe and successful use of numerical methods to solve practical problems. Whilst it is useful to know how to derive the Maclaurin's series for $\sec \mathrm{x}$, and the range over which the series expansion is valid, it is probably more useful to know how to interpolate between tabulated values and what are the dangers of extrapolation.

Although the module is entitled 'calculus', it does contain a number of
topics usually classified as numerical methods. This is deliberate and is designed to emphasise the close connection between the latter and analytical methods.

## 5. Geometry

Although there is some geometry taught in schools it is considerably less than was the case a few years ago. It is surprising how many students seem to have little feel for geometrical ideas such as asymptotic behaviour and curvature. Certainly, many students have not encountered much in the way of coordinate geometry and very few have met the conic sections prior to tertiary level. As has already been mentioned, the spread of Computer-Aided Design in industry has led to an increased role for geometry. In addition, a geometrical approach to a problem can often cast sufficient light to lead to its solution. However, with the reduced exposure to geometrical thinking given in schools it will be necessary to cover some very basic material. Certainly it is expected that full use will be made of the computer in the presentation of some of the topics and the students should be given the opportunity to work interactively with some prepared software.

## 6. Differential Equations

Little justification need be given for the inclusion of this module. Differential equations form the basis of so many models of engineering systems that they have a leading role in any realistic core curriculum. What is not so easily justified is the time spent in courses on ordinary differential equations considering special methods of solution for equations whose appearance in such models is at best very rare. The emphasis should be on such matters as existence and uniqueness of solutions, complementary functions and partưlar integrals, and the nature of initial and boundary conditions. The widespread use of variables separable and integrating factor equations justifies their treatment, given the relative ease of obtaining the analytical solution. Once again, the analytical approach can help elucidate the underlying structure of the solution of a differential equation whilst a numerical approach can be used to obtain the
solution in more awkward cases.

Linear differential equations with constant coefficients are another class of equations which have widespread application. The opportunity should be taken to emphasise the linearity in this context.

Non-linear systems offer a good example of the value of linearisation of problems. Geometrical ideas are involved in the study of critical points in the phase plane and this provides a useful link with the previous module.

It is anticipated that the treatment of stiff systems would be confined to a simple example "on the board" using two equations, supplemented by an illustration of the phenomenon in a larger system with the aid of a computer program.

A recommended approach to the treatment of partial differential equations was given in Section 8.1 .

## 7. Probability and Statistics

The single most important aim of this module is to make the student aware that random variation is an ever-present perturbation of his experimental observations and the phenomena he seeks to analyse. However, it can be modelled and to a certain degree controlled.

It is assumed that every opportunity will be taken to use practical examples from engineering as illustrations throughout the course. An engineering undergraduate is unlikely to be motivated to study the Poisson distribution if it is introduced by the well-worn example of the deaths of Prussian officers due to horse-kicks, however amusing that might appear to be to the lecturer.

The lecturer should have available sets of suitable data for analysis during
the course. Care should be taken to emphasise the misleading ways in which data have been represented graphically. The shape of a data set can affect the accuracy of the various measures of location and dispersion commonly in use and this should be highlighted via the data sets available.

The assumptions associated with each of the probability models in the syllabus should be stressed and examples provided of the problems in choosing the "appropriate" model for a given set of data.

The estimation of population parameters from the corresponding sample statistic is a safer process with a large sample and this fact can be illustrated with suitably chosen data sets.

The fitting of a straight line to a set of data points by least squares can be extended to include a discussion of how regression analysis takes into account random variations in the data values. It is suggested that the initial discussion is qualitative and is followed by a study of the output from a computer-based analysis of a real data set using a standard software package.

Most final year projects in an undergraduate engineering course involve the setting up of an experiment, the collection of results and their susequent analysis. All too often, a student will come armed with his experimental results and ask what can be deduced from them. When it is suggested that he should have thought at the outset what he was endeavouring to show as a result of his experiments and then planned those experiments accordingly, the best one can hope for is a rueful smile. After a considerable amount of work on the part of the student, it is disappointing to have to tell him that there is, in fact, very little that can be deduced from his results. Hence the need in this module to spend some time pointing out the dangers inherent in carrying experiments that have not been carefully designed.

## 8. Introduction to the Finite Element Method

Many engineering departments teach finite elements as part of a structures course. The aim of this module is to lay the necessary mathematical foundations, linking with the work on matrix algebra.

## 9. Mathematics of Fields

The emphasis here is on physical interpretation to help explain the concepts. In the core curriculum all that is expected is the defintion of gradient, divergence and curl, their physical interpretation and examples of them in practical contexts, and explanation of the meaning of the three integral theorems and examples of them in action. Particular attention is to be paid to the idea of a conservative field and the implications.

## 10. Complex Variables

Elegant though the theory of functions of a complex variable may be, there is little room in a crowded syllabus for more than a cursory and utilitarian treatment of this subject. Apart from a grounding in complex arithmetic, the only topics to be covered are some simple complex mappings which lead to streamline flow past an aerofoil (which can be demonstrated on a micro without going into a detailed analysis) and sufficient work on residues to allow the inversion of Laplace transforms to be effected in standard cases.

### 9.3 Needs of the Engineering Specialisms

The core curriculum which was described in the last section cannot be expected to cater for the needs of all the individual specialisms. Traditionally, Civil Engineering students have covered less mathematics than, say, Electrical Engineers; the latter group have tended to have a mathematics course akin to that
given to Physics students in its content. Production and Manufacturing students have required the greatest coverage of statistics and some final year options in Electrical Engineering have needed a more extensive grounding in probability theory than is given to other engineering disciplines.

It is proposed that the core curriculum is augmented by a number of modules which are designed to provide the coverage of additional material required by individual departments. A selection of such modules is given below. One increasingly important group which will require a considerable amount of additional material in discrete mathematics are the information technologists and software engineers. The first four modules which follow extend the core material in discrete mathematics.

## Algebraic Structures

Further operations on sets.
Monoids, semi-groups and groups.
Homomorphisms and isomorphisms.
Quotient structures. Modular arithmetics.
The applications of the mathematics in this module include : fast addition, languages and grammars, dynamic memories, and state reduction for finite-state machines.

## Finite Rings and Fields

Rings.
Polynomial rings and irreducible polynomials.
Construction of fields. Discrete Fourier transforms.
The applications include : fast multiplication, FFT algorithms, cryptography, error- correcting codes, random number generators.

## Combinatorics and Graph Theory

Techniques for counting. Product rule, inclusion-exclusion, Polya enumeration. Directed and undirected graphs. Connectivity. Trees.

Existence problems.
Optimisation problems, including flows in networks.

## Lattices and Boolean Algebra

Partial ordering and lattices.
Boolean algebras and Boolean functions.
Minimisation of Boolean functions.
Applications include: switching theory, logic design of finite-state machines.

Other modules
Other modules offered would include the following:

Operations Research
Optimisation
Mathematics of Control
Finite Difference Methods for Partial Differential Equations
Advanced Finite Element Methods
Random Processes
Functional Analysis Computer-Aided Design
Microcomputer Simulation
Further Probability
Further Statistical Methods
Advanced Linear Algebra
Mathematical Methods in Field Theory
Solid Mechanics
Boundary Elements
Analysis of Systems

As examples, Civil Engineers might like to take the modules in Advanced Finite Element Methods and/or Boundary Elements as final year options, whereas Electrical Engineers might prefer to study Further Probability and Mathematical Methods in Field Theory.

### 9.4 Who Should Teach the Mathematics?

The teaching of mathematics to engineering undergraduates is surely too important to be left to engineers. There are a number of occupations in life where everyone could do the job as well as, if not better than, the holder of that occupation. Being a mathematics lecturer who specialises in teaching engineering undergraduates is such an occupation - he is frequently told by his engineering colleagues that they cannot understand why he needs so much time to teach his syllabus. They were not given so many lectures when they were undergraduates, and they are sure that they could teach the subject-matter in less time.

Of course they could. But their knowledge of mathematics is limited to small pockets of techniques and ideas and they do not possess the breadth and depth of experience necessary to appreciate the power and unifying nature of the subject. They can see the brush strokes on the canvas but they cannot stand back and see the composition of the painting. Their teaching would be unsatisfactory.

On the other hand, the teaching of mathematics to engineering undergraduates is too important to be left to mathematicians - at least to those mathematicians who have no real knowledge of the engineering discipline of the students whom they are teaching. The latter group will often accuse some of their colleagues of prostituting their art by attempting to make their teaching relevant to the needs of the students rather than giving a "pukka" mathematics course. However, it does seem reasonable to suggest that, on the contrary, a mathematician who has never been involved in the use of mathematics to solve an engineering problem should be encouraged to gain that experience in order to enhance his teaching.

Such a lecturer could be seconded for a year, say, to study courses in engineering to a level sufficient to give him a good working knowledge of engineering principles and of applications of mathematics, in addition to understanding the engineering approach. Ideally, some of his time could be spent
in industry working with engineers on the modelling of engineering systems. Such a scheme would require a sympathetic Head of Department and recognition from his university's hierarchy. Perhaps the Engineering Council could also play a role here by granting the participants in the scheme the status of Chartered Engineer after they have taught engineering undergraduates successfully for some years.

In many ways the ideal lecturer for teaching mathematics to engineers would have a first degree from a course akin to Mathematical Engineering. Such a person has the requisite mathematical knowledge, combined with a background training in the general principles of engineering. He will be able to appreciate the needs and aspirations of his engineering students and temper those needs and aspirations with the demands of providing a coherent and professional mathematics course.

## Combining efforts

There are occasions where the mathematics is taught most effectively by a combination of mathematician and engineer through the medium of a shared lecture. In a small number of instances a topic could be covered by a suitably qualified engineer; in a crowded syllabus every little helps. The topic of Finite Elements has been successfully taught to Second Year Civil Engineers at Loughborough by a colleague as part of his Structures course; he is an expert in the method and its application and he is mathematically the most able of his departmental colleagues. Collaboration has ensured that the students are given at the appropriate time a suitable grounding in matrix methods and theory.

If the mathematics lecturer has some engineering specialism, say fluid dynamics, he should be prepared to do some small amount of teaching in that area. In the author's case he was able to contribute to the teaching of streamlines and potential flows to second year students, and in a final year option in Offshore Engineering, he taught the techniques of wave prediction.

Another example where collaboration can be of benefit to all concerned is the mathematics lecturer being part of a joint research project with his engineering colleagues. If such a formal large-scale activity is not possible, then the mathematics lecturer can be on hand to give advice on small mathematical problems as they arise. This can be a challenging and enjoyable experience and it will help him to earn the respect of his engineering colleagues; even if he cannot solve the problem directly, he can probably put the engineer in contact with another mathematician who is able to provide the necessary assistance.

### 9.5 How and When Should the Mathematics be Taught?

Mathematics is too important to be relegated to the first two years of a three-year degree programme as is often the case in the United Kingdom. In many engineering courses in the U.K. mathematics is not even counted in the degree assessment; this is a disgrace in the context of engineering today. Is it any wonder that many engineering students do not take their mathematical studies as seriously as they should when their academic staff do not apparently value mathematics sufficiently highly to include it among those subjects on which the degree is assessed?

Part of the problem is that mathematics is too often seen by the engineering staff and students either as a cook-book of techniques, only some of which are directly relevant to their requirements, or as an esoteric subject which occasionally overlaps with their discipline. In many cases this is a fair criticism and some of the blame at least must be taken by the mathematics lecturer.

It should hardly need emphasising at this stage that the style of teaching expected in this model is that of the integrated approach. In addition, relevance and illustrative applications will be at the forefront.

If the mathematics course includes a modelling element then it can
justifiably lay claim to a place in all years of the engineering degree programme. In the first two years the emphasis will be on concepts and techniques, though these will be accompanied by case studies and a small amount of modelling, the latter being placed mainly in the second year. In the third year of the course the emphasis should be on modelling, with extra techniques and concepts being introduced as and when they are needed.

The main form of instruction would be the lecture, but with extensive use being made of audio-visual aids; refer to the next section for details. The lectures should preferably take place in the early morning and on separate days of the week. There should be a tutorial held for each student on a fortnightly basis with a maximum of ten students per tutorial group. The tutorial would usually be devoted principally to discussions of the type described in Sections 5.8, 6.4 and 8.2; the aim would be to encourage student participation and would be especially useful when considering modelling aspects of the course. Any spare time in the tutorial could be set aside to deal with individual difficulties which the students had encountered in recent lectures or on problems set in conjunction with the lecture material. The tutorial would normally be taken by the mathematics lecturer, but might be taken by a mathematical colleague if the lecture group were particularly large.

To back up the lectures and tutorials, weekly surgeries would be held for the whole group, staffed mainly by postgraduates, during which individual students could have particular problems ironed out. The attendance at these surgeries would be voluntary and appointments would be made by noon of the preceding day. Bookings could be made individually or jointly by students with a common problem or difficulty of understanding. Standard solutions to the problems set weekly by the lecturer would be available for inspection in the week following their setting, perhaps on microfiche which could be viewed in the university library on a signing-out arrangement, or perhaps by having them accessible on the mainframe computer via terminals. The attendance at surgeries would be monitored and if a student was having persistent difficulty in
understanding the lecture material he could receive specialist advice. This aspect is discussed further in Section 9.7.

In addition to the teaching arrangements described above there should be a regular mathematics laboratory component timetabled as part of the engineering laboratory programme. Part of this activity would involve learning to program on the computers owned by the engineering department or in a centrally-owned terminal/microcomputer laboratory. The coursework for the first year mathematics would consist of two or more programming exercises of the kind described in Section 3.5. In the second and final years the coursework would be in the form of modelling exercises which would need computer-based techniques for their solution. The students would be expected to make use of standard packages in their programs. In each year the appropriate work would be written up and submitted as a laboratory report.

The teaching of programming would be conducted by the mathematics lecturer in conjunction with his engineering colleagues as outlined in Section 3.5. It is not expecting too much to ask that the mathematics lecturer and the relevant engineering staff should be computer users; at the present time this is the norm, and in the future it should be the rule without exception.

## Examinations and assessment

In the fist two years the assessment would be based partly on a written examination, held at the end of the academic year, and partly on coursework which would comprise submitted reports on experiments carried out in the computer laboratory. The emphasis in the examination would not be on technical skills alone, but would include questions designed to test the students' understanding of concepts.

### 9.6 What Teaching Aids Should be Employed?

## (i) Textbooks and monographs

Although there is evidence that, even allowing for the second-hand market, students of engineering are buying fewer text-books than was the case a few years ago, they should be encouraged to purchase more text-books in order to acquire the habit of reading technical material. If the students are led to believe that the notes which they take during lectures form the totality of reading-matter that they need then they are being done a disservice. However, given that different engineering specialisms require different additional material on top of the core syllabus it might be argued that it would be wise to provide a text-book, or perhaps two, on the core syllabus only. A series of short monographs could be written on the additional topics, so that a particular student could select only those which were of direct relevance to him.

Many lecturers are in the habit of handing out comprehensive notes as an accompanient to their lectures. Whereas there is merit in producing occasional supplementary notes to expand the exposition of particular topics, it does seem somewhat excessive for a lecturer to hand out what is, in effect, a text-book in serial form.

The publishing world is currently facing a dilemma as regards the sales of academic books and they might be reluctant to consider the idea of a series of booklets as outlined above. The advent of desk-top publishing should make it possible to produce such a series economically whilst maintaining a high standard of presentation.

If the idea of updating the mathematical knowledge of practising engineers can be brought to fruition on a large scale then the publication of the monograph series would become a more viable proposition.

## (ii) Overhead transparencies and slides

Today's undergraduate has been brought up in a world of sophisticated graphics, glossy brochures and slick advertising. He will not warm to a lecturer who contents himself with a poorly-drawn diagram on the blackboard or on the overhead projector. If the lecturer's presentation is slipshod, why should he expect his students to take more care over their work?

With the widespread availability of word processors it is becoming noticeable how many speakers at conferences illustrate their talks with very well-prepared transparencies. The graphics work is of a high quality and even the format of text is attractive and striking. Why then, should the undergraduate be satisfied with a lower level of service?

No doubt, it will be argued that staff time and production costs militate against the wider use of such quality material. Perhaps it would be an effective use of resources were a library of graphical displays were produced and stored on disc so that the disc could be copied commercially; perhaps the material could be made available in book form so that it could be photocopied onto transparencies.

The use of slides is a preferred alternative to transparencies for the overhead projector for a number of lecturers. Not all lecture rooms have a slide projector, however, and the switching on and off of the slide projector and the overhead projector could become a distraction to both staff and students.

## (iii) Films

Film is a medium rarely used in lectures, especially in mathematics. The difficulties logistically and organisationally of arranging for a film to be shown have not helped make it more popular. Nowadays, with the wider availability of video recorders and the relative ease of purchasing or even making a tape it may well prove to be the case that, with few exceptions, film is no longer a viable
proposition for enhancing mathematics

## (iv) The Micro in the classroom

It is now possible to purchase a liquid crystal display unit which will fit on top of an overhead projector screen; the unit can be connected to an output port of a microcomputer. This allows a lecturer to show his audience the same screen display which he can see on his monitor so that he can control the display more easily whilst at the lecture bench. It is then possible to demonstrate programs interactively during the lecture in a room holding up to about thirty. For a larger audience, a large screen is more suitable. There is the problem of having a microcomputer installed in the room as a permanent fixture; the alternative of having to transport the equipment into the room and out again is not attractive, even when a compromise has been reached of leaving a monitor in the room and transporting the microcomputer and a disc drive in a boxed unit.

## (v) Television

It is unlikely that there will be a great need to make use of television to broadcast 'live' performances. Some institutions do.employ closed circuit television to relay lectures live into overflow theatres but it is not a practice which has much to commend it, given the relative ease of using a video-recording.
(vi) Video recordings

The facility for replaying video-recordings should be present in most, if not all, larger lecture theatres. Whilst there may be occasions when a complete Open University programme or some similar programme would be shown in its entirety, it is more likely that excerpts only would be needed and the equipment necessary to carry out the required editing should be provided.

Staff should be encouraged and assisted to produce 'home-made' tapes
which could show experimental demonstrations or perhaps a discussion on the formation of a mathematical model.

## (vii) Computer laboratory

The computer laboratory is an essential part of the mathematics course. Provision must be made for the students to have access to the laboratory for at least one half-day per week. Part of the time would be spent carrying out assignments which would be submitted as coursework. In addition to writing their own programs, the students would be encouraged to use prepared software and make extensive use of commercial packages.

In their final year mathematics course, where the emphasis is on modelling, it is to be expected that the use of the laboratory would be at its greatest.

## (viii) Three-dimensional models

Many students have difficulty thinking in three dimensions. The use of three-dimensional models of things like the surface representing a function of two variables, or a surface with normal and gradient vectors being shown would be of benefit to these students. Although a television display of such models is useful, there is added impact if the student can walk round the model or even handle it.

The live demonstration of a simple experiment such as the vibration of a two-storey frame has been known to make a long-lasting impact on students. When they are able to carry out the experiments for themselves interest is heightened and the impact is yet greater. Although the effort required to set up such experiments is sometimes considerable the benefit to the students justifies the extra expenditure. There is merit in arranging a mathematical laboratory of this nature as part of the students' experimental programme.

In the final year, where the emphasis is on modelling, the assessment
would take the form of a written report on the modelling exercises which had been carried out. Since it is assumed that the students would be required to gave a verbal presentation of their major engineering project, it seems unnecessary to ask them to do so in this part of their course.

### 9.7 Mathematics Learning Resource Centre

In these days of increasing pressure on university lecturers to devote more of their time to research there is almost a disincentive to pay more than scant attention to the needs of their students. If it is a situation which is unwelcome, it is one which must be faced. How can the students of tomorrow, who are likely to require more support in mathematics than the present cohort, be catered for adequately when that precious resource of staff availability is almost certain to diminish?

The solution proposed here is the establishment within each institution of a Mathematics Learning Resource Centre. This would be directed by a member of the academic staff who was a mathematician sympathetic to the needs of engineers and other students who are studying mathematics as a non-specialism. The Centre would, as its name implies, contain teaching and learning resource material, and would provide a focus for the support needed by students who are experiencing difficulty with their mathematics.

## Helping with difficulties

The first contact that a student made with the Centre would be via a diagnostic test which would be conducted in the first few days of the new session. The test would attempt to discover any particular weaknesses which the student possessed on entry to the institution. It might be possible to arrange for the test to be 'rough marked' by computer, with the student entering his answers on a terminal. The results would be available to the student, to his lecturer and to the Centre very quickly and this procedure would hopefully isolate any major
mathematical weaknesses of the student as early as possible. The individual answer scripts would be handed in to the Centre and would be studied in conjunction with the computer-produced results to provide more details about the nature of the student's particular difficulties in understanding concepts or in manipulation.

At this juncture it is important to point out that it is not envisaged that the Centre would take away from the individual lecturers their rights or their responsibilities. It is there to provide a back-up service to both staff and students and its relationship with both is of crucial importance. The former group must feel confidence in the Centre's ability to perform the supporting role that is needed and it is to be hoped that they would feel able to contribute to the resource material which was held by the Centre. In this respect one might draw a parallel with a medical general practitioner referring a patient to a specialist for advice and/or treatment which were beyond the knowledge and resources to provide. The patient remains under the general care of the doctor but has received specialist treatment for a particular condition.

On the other side of the coin, the resources of the Centre would be available if a student wished to make a visit on his own initiative. It could well happen that a student felt that he was having difficulty with a particular topic, or indeed with his mathematics course in general. He could call in to the Centre, as he might to the general student counselling service, or the Medical Centre, and if it were not possible to see a 'mathematics counsellor' immediately he could make an appointment to see him in the very near future. He could then discuss his problem with the counsellor. It might be that the student would be asked to take a short diagnostic test to help pinpoint the root cause of his problem.

Then, once the real nature of his problem had been diagnosed, remedial action could be recommended which would almost certainly mean utilising the resources of the Centre. Of course, it would be possible for a student to visit the Centre at any stage in his undergraduate (or even postgraduate) career, either
following a referral by his lecturer or personal tutor, or on his own initiative.

## BTEC entrants

One group of students that have especial difficulty with mathematics are the BTEC entrants. These students enter tertiary education with a very limited knowledge of mathematics and, whilst it is true that they have an advantage over their A-Level contemporaries in Drawing, Structures, Fluid Mechanics and other engineering subjects which they have studied at college, their difficulties in coping with the mathematics must not be underestimated. The time taken to assimilate concepts and master techniques cannot easily be reduced and yet the student with the weaker background is expected to make up his deficiencies at the same time as absorbing new concepts and acquring new skills. It is a tall order and one which is not often given much sympathetic treatment.

## Individualised learning

It is envisaged that the Centre will have under its control the resources to provide the facility for individualised learning. Students and others on short courses or those who want to top up some area of mathematics would be able to assemble the necessary material from the Centre and organise their own learning programme.

## Staffing the Centre

One resource of the Centre which has already been alluded to is that of staffing. It is envisaged that there would be a small number of full-time staff who are able to act as mathematical counsellors and who would be responsible for preparing resource materials and for building up a collection of these materials from other sources. Such staff might well be recruited initially from among the staff currently teaching undergraduate engineering mathematics in the institution concerned; in any event, it is to be expected that they would continue to teach their
undergraduate courses after being appointed to the Centre. Other academic staff could become associate members of the Centre and contribute to the development of its activities.

## Considerations of space

There is an impressive provision of audio-visual resources available in the library building at Plymouth Polytechnic, which the author was able to visit recently. These resources were being heavily used even at $9 \mathrm{p} . \mathrm{m}$. and the students to whom he spoke appreciated the accessibility of the resource material for the total time that the library building was open. Space provision is a continuing worry for all institutions and it would be quite unrealistic to suppose that it could be found easily for the Centre, however important its advocates might consider it to be. It has to be accepted that some of the resource material might have to be housed away from the main area of the Centre.

## Resource material

One simple resource which should be available in the Centre itself are the specimen solutions to problems set by the lecturers. The Centre would undertake to collate these solutions and put them into a standard format, perhaps on microfiche or onto a database held on the university mainframe computer. Experience with worked examples has shown them to be a valuable means of helping the student learn with a reduced direct contact from the lecturer.

In addition to the initial diagnostic test there would be a collection of such tests on individual topics which could be given to students in an attempt to help them to identify the roots of their weaknesses. There would also be information packs on these topics which would be written in a house style and would be designed to provide a fuller explanation of the topic concerned, say Maclaurin's series. When the student had worked through the material in the pack he could take a short test on the topic and if he was still having problems he could see a
mathematical counsellor to seek further remedial action. There is scope for incorporating audio cassettes and slides as part of the package. The slides would fit into a carousel and would be released into the slide viewer under the control of the cassette which would provide a commentary on the information shown on the slides. The cassette would also ask the user to pause from time to time to read further written material. It should be possible for the student to buy the information packs or some of the constituent material.

Among other audio-visual material to be brought into the collection would be videotapes which have been produced commercially or produced in house. The tape collection would clearly need to include Open University programmes and the hope would be that particular sequences could be isolated easily to save the student having to view most or all of a 25 minute programme for the sake of a two or three minute excerpt. There is much merit in having available videotaped experimental demonstrations or even videotaped lectures on specialised topics, particularly if the lecture required the assembly of equipment and presentation of such demonstrations. Lecturers would be encouraged to make use of these materials in their own lectures. Experience has shown that many staff would be happy to use such material if someone else had already taken the trouble to organise it.

One would like to believe that students could borrow audio cassettes and other material for overnight use but, regrettably, in today's society that idea would prove impracticable. One must be resigned to the fact that they would have to be restricted to using the material in the Centre or in the library. How could one check that a tape had not been tampered with? It is no use pretending that such a situation would not arise; there is widespread theft and vandalism taking place in institutional libraries. The 'disappearance' of audio-visual material from lecture rooms and seminar rooms also occurs at an alarming rate.

## Wider usage

Engineering staff who wished to update their mathematical knowledge would be encouraged to use the resources of the Centre, particularly in the vacations when student usage is expected to be slight. There is a clear need for engineers in industry to update their mathematical awareness and perhaps renew their skills in particular areas. If short courses were organised, perhaps of a few days duration or on a day release basis, the Centre could act as the focus of the activity. This would provide an income which would help to support its other activities.

## National network

There should be a national network of such centres which would cooperate in the production and distribution of materials. This cooperation would ease the problem of financial provision. There is already a network of libraries and computer centres which would facilitate the linking of the mathematics learning resource centres.

## Closing comments

Some readers may dismiss these ideas as fanciful. However, we are facing a crisis in the mathematical education of engineers and it does not seem likely that the present set-up in universities will be able to handle the situation. The network of Mathematical Learning Resource Centres offers a positive and practical means of coping with the impending crisis. Of course, there is the over-riding problem of cost. Each centre could only grow at the rate that its income would allow. In the early stages the provision of materials would be limited in quantity and range, but the author is confident that the usage of the centres by the students would justify further expenditure. If the country wants to improve the mathematical knowledge and skills of its engineers, as it must if it is to survive in the world of tomorrow, then it must invest in the future.

### 9.8 Implementation of the Teaching Model

In order to implement the model proposed in the previous sections a number of major changes will have to be made which will necessitate considerable efforts on the part of the lecturing staff.

## (i) Retraining of mathematics staff

As regards the core syllabus, the introduction of a sizeable amount of new material in the area of discrete mathematics means that most of the mathematics lecturers will be faced with the need to come to terms with yet another learning task. The previous generation had to cope with computer programming and numerical methods, when most of them had not covered such material in their own degree courses. The learning tended to be on a piecemeal basis, and many even today would regret their lack of formal training in this area. It is imperative that the situation is not repeated with the area of discrete mathematics. Accordingly, there should be a coherent programme of courses organised on a regional basis for mathematics lecturers to acquaint themselves with the concepts, techniques and applications of this topic.

It is also important that the mathematics staff have direct and regular contact with the engineering industry and it is recommended that arrangements be made for them to spend an initial period working in an industrial concern.

It has to be made clear that such efforts on the part of the staff should be recognised and, indeed, rewarded. If financial incentives are not present it is not likely that lecturers will take on board their retraining, however educationally desirable that might be.

## (ii) Retraining of engineering staff

Many engineering lecturers have suffered from an unsympathetic mathematics course as part of their undergraduate programme and feel indifferent or even antipathetic to mathematics as a result. They have not improved and updated their mathematics skills and perhaps feel embarrassed at their lack of knowledge. They, too, should be encouraged to redress this deficiency and rewarded for their efforts.

## (iii) Cooperation between mathematics and engineering staff

One way in which closer cooperation between the mathematics and the engineering staff could be achieved would be for the mathematics lecturer to be an integral member of the engineering course management team. This should be reciprocated by the engineering staff being invited to discuss regularly with the mathematics lecturer the content of his mathematics course to their students.

## (iv) Modelling workshops

The emphasis on modelling in the mathematics course given to the final year students demands that both the mathematics and the engineering lecturers be skilled in conducting workshop sessions. For this to be the case the staff must themselves have attended such sessions and a programme of workshops, organised on a regional basis needs to be established.

## (v) More resources for computing

In many institutions the provision for computing falls far short of that which is required to teach effectively a modern engineering course. If we are to take seriously the task of educating engineers for the future then institutions must be given the resources to build up their computing facilities to the level required.

## (vi) Establishment of Learning Resource Centres

There must be a concerted effort to set up Learning Resource Centres along the lines of the suggestions in the previous section. This would involve the appointment of a Director of the Centre who would be supported by a small full-time academic and secretarial staff. Space would have to be found for the Centre and sufficient funds provided to allow the Centre to find its feet. Only by establishing a national network of such centres would they be able to realise their full potential. However, it would be wise first to establish a few centres, one per region; if they prove to be successful, further centres would then be established.

## Final remarks

The teaching model proposed in this chapter is an attempt to come to terms with a worsening situation in the mathematical education of engineering undergraduates in the United Kingdom. Unless drastic action is taken soon, the situation will worsen dramatically. As has been seen all too often, educational changes are slow to take effect. Given the climate in universities and polytechnics, which seems likely to persist for some time, the idea of a Learning Resource Centre is crucial to the success of the teaching model. The other major change suggested is the radical overhaul of the syllabuses. The most important of the revisions are the inclusion of a substantial amount of discrete mathematics in the core material and the explicit emphasis on modelling .

The implementation of the model requires considerable effort on the part of all concerned, and that effort will not be forthcoming unless suitable rewards are provided. It took hard work, and not a little courage to champion the integrated approach in the early 1970's. Now that approach is widely accepted. What is being asked in order to implement the proposed model requires even harder work and an equal amount of courage. What sustained the advocates of the integrated approach was a belief in their vision of the way forward.

The author has the same belief in the model proposed in this chapter.

## Chapter 10

## Conclusions

10.1 Summary of the Research

An account of the more significant research into the mathematical education of engineering undergraduates which has been carried out in the last forty years forms the basis of Chapter 2. At the British Association meeting in 1948 there was a clear division of opinion between those who believed that mathematics should be taught as a subject in its own right, divorced from any applications, and those who argued that it should be taught in a way which emphasised its relevance to engineering. That division of opinion has continued to influence the teaching of mathematics to engineers, and the debate is still as heated today as it was two decades previously.

It is fair to say that those who believed in the latter approach have continually striven to make their teaching match the changing needs of the engineer by updating syllabuses and by bringing in new styles of instruction, even if the pace of change has not been as fast as might be desired. In 1966 the OECD Report gave a comprehensive review of the then current state of the mathematics taught to engineers in its member countries and set down somewhat ambitious target syllabuses; in additi:on, suggestions were made as to styles of teaching, and special attention was given to the need to increase the role of computer-based methods.

A major step forward was heralded by the paper by Bajpai et al (21) which showed how the topic of ordinary differential equations could be presented by an integrated approach which combined analytical and numerical methods of solution. As a result of that paper, a course was developed at Loughborough based on the integrated approach; the development was described in Chapter 3.

It is, of course, essential for the integrated approach to succeed that both the students and the teacher become familiar with computers and computer programming at the outset of their studies. Coursework has been set in co-operation with engineering staff which requires the writing of computer programs. In this way the mathematics is integrated with the engineering, in addition to the mathematics being integrated within itself. An example was provided in Section 3.4 of the integrated approach in action: to establish a model it was necessary to appeal to calculus, whereas the solution of the resulting equation was effected by employing a numerical technique.

The material that was produced in note-form as hand-outs to accompany the course was subsequently published in the form of three text-books.

Feed-back from other institutions has supported the local experience that students who are taught via the integrated approach do have a better motivation to learn mathematics and are more aware mathematically than those who followed the previously accepted approach.

One important aspect of any teaching system is the quality of the input. A diagnostic test conducted on freshman entrants revealed a worrying level of mathematical ignorance and incompetence. These deficiencies had to be taken into account when devising the syllabus and suitable remedial measures taken.

The OECD Report has frequently been used as a yardstick by which to judge subsequent progress. Ten years after its publication, the author and his colleagues (40) and (41) expressed concern that little progress had been made. They pointed out current shortcomings and suggested a way forward. A particular plea was for the inclusion of project work in the mathematics course.

The effect of the increased computing power available to students with regard to both pocket calculators and computers was the subject of Chapter 4. In the case of the fomer, it was necessary to train students in the correct use of their
calculators and to instil in them a proper regard for the limitations imposed on the result of a calculation by imprecise data. In the case of the latter, a terminal laboratory programme had been developed which comprised both the use of prepared software to compare different methods of solving a particular problem and the writing of short programs to tackle specific mathematical models.

There was concern that although the computing facilities available to students had increased considerably they had failed to keep pace with the demand. Drastic measures would have to be carried out in order to remedy the situation.

A natural development of the research was in the area of computer enhanced learning. The MIME project was established in 1983 to write and test software for microcomputers which would be used as an enhancement to the teaching and learning of mathematics. The project was described in Chapter 5. The first units to be produced were on topics in mechanics at the level of school sixth form and first year university.

The intention was that the software should be flexible in its use, capable of either serving as an aid to the teacher in the classroom by providing interactive demonstrations, or permitting individualised learning by students in a self-paced mode. In either case the software allowed the user to simulate experimental situations and behaved, in effect, like a sophisticated piece of laboratory equipment.

The problem of presenting topics in statics in a lively way provided a challenge which was overcome by using practical illustrations of the theory and by allowing the user to vary the parameters of the model. This latter aspect, in particular, provided the student with a deeper understanding of a topic. The opportunity was also taken to give the student a qualitative insight into certain problems which were beyond his mathematical competence to analyse quantitatively.

Comment received from teachers was generally favourable and indicated that the majority of students had benefitted from using the software. However, it was clear that the teachers would have preferred more guidance in the use of the software in their teaching and were rather reluctant to write their own supporting material. It appeared that the flexibilty which the project team had regarded as a strong point of their software was, in this aspect, not appreciated. Notwithstanding this point, the software had succeeded in its aim of enhancing the learning of mathematics.

The project was extended into the area of university mathematics, although on a reduced level of activity, due to a general lack of funding available for such work. The software units were designed to be used either in the lecture as a demonstration tool, or in the tutorial in a fully interactive fashion, or by the student in self-paced study. Examples of these three modes of use are given in Sections 5.7 to 5.9. Staff reaction to the software was generally encouraging, though again there was an indication that some who would have preferred more supporting documentation were unwilling to experiment with the programs; they were only interested in having a package which was effectively self-running.

The use of case studies in the curriculum formed the subject-matter of Chapter 6. There was a discussion on the role of modelling in the course which indicated the need for a shift in emphasis away from the teaching of techniques, however useful they may be, towards the formulation of mathematical models of engineering systems and the interpretation and validation of the solution of these models.

The value of a shared lecture in showing the interaction between mathematics and engineering was demonstrated in Section 6.3. The dialogue between the mathematics lecturer and his engineering colleague mirrored that interaction. Sections 6.4 and 6.5 demonstrated respectively the development of two simple models of the same engineering situation in a tutorial session and the development of a more complex model over a longer time-scale.

Modelling has a crucial role to play in the teaching of mathematics to engineers. It provides an additional motivation for the students and it allows them to participate in an active way to complement the presentation of case studies in the syllabus.

Chapter 7 was devoted to the comparative study of courses which combine a general grounding in engineering with a substantial input of mathematics. Such courses are few in number but they do fill a gap in the spectrum of engineering graduates and the evidence from industry is that the output from these courses is highly valued. Whilst the courses will never produce graduates in large numbers they do have an important role to play in attracting able entrants who might otherwise be lost to engineering.

Chapter 8 gathered together a number of topics which merit consideration in the build-up to the teaching model which formed the outcome of the research described in this thesis. In Section 8.1 the difficult subject of partial differential equations was treated; a method of presenting the material so that the students' motivation is retained whilst the essentials of the subject are covered was discussed. The bottom line was that the students should recognise the main types of equation, understand how they were derived, appreciate the nature of the accompanying boundary and/or initial conditions and understand the processes of solution, both numerical and analytical.

The use of the tutorial to provide a back-up to the lecture by involving the students in an interactive exercise was explained in Section 8.2, using the topic of Fourier series as a vehicle for discussion. Section 8.3 took a critical hat some examination questions and concluded that care was needed when setting questions to ensure that the skills which were supposedly being tested were not masked by the students' lack of other skills.

Two other aspects which were considered in Sections 8.4 and 8.5 respectively were the influence of visual images and the growth of distance learning. It was
felt that not enough attention was being paid to the dominance of television during the formative years of modern children and the consequent impact of visual stimuli relative to that made by the spoken word. The use of experimental demonstrations in the lecture was commended. The success of distance learning had not been fully recognised by campus-based institutions. They needed to make more use of distance learning materials in their teaching programmes; the particular case of videotape and videodisc was highlighted.

Chapter 9 faced up to the crisis in the teaching of mathematics to engineers which is upon us. It proposed a teaching model to take us towards the year 2000. It defined a core curriculum for all engineers which was designed to take 165 hours of lectures and proposed that mathematics should be taught in all years of the degree programme. There should be a greater emphasis on modelling and this would form the bulk of the final year work.

A substantial amount of discrete mathematics was included in the core curriculum and the time needed for this topic and the modelling work was found by removing much material of long standing which was relatively dead wood. Individual engineering specialisms could augment the core material by choosing from a portfolio of additional modules according to their needs.

It was argued in Section 9.4 that the mathematics lecturer needed to have some experience of applying mathematics to the solution of engineering problems and that he should be encouraged to spend some time in industry to this end. This would help him to be accepted by the engineering staff as an integral member of the teaching team.

The greater use of a variety of teaching materials was recommended in Section 9.6.

Crucial to the successful implementation of the teaching model, given the pressures on the mathematics lecturer and the general worsening and variable
nature of the quality of intake was the establishment in each institution of a Mathematics Learning Resource Centre. The Centre would provide a necessary support to the teaching programmes. It would be responsible for conducting diagnostic tests to discover particular weaknesses in individual students and would provide the relevant remedial material to help overcome those weaknesses. It would offer a mathematical counselling service to assist those students who had more serious mathematical problems and who needed closer guidance. It would stock information packs on individual topics and would contain a library of audio-visual material which could be used by staff to enhance their teaching or by students in self-paced mode.

The resources of the Centre would be available to staff, particularly from engineering departments, who wished to update their knowledge or to iron out their own mathematical shortcomings. It was envisaged that the Centre would be able to assist in the updating of engineers in industry. A call was made for a national network of such centres which would operate in a way similar to the audio-visual centres in universities and polytechnics.

The implementation of the teaching model required the retraining of both mathematics and engineering staff, the former in the application of mathematics in the engineering industry and the latter in the mathematical needs of modern engineering. In addition it was recommended that a programme of modelling workshops be established to provide staff with a training in the methodology and practice of modelling. More resources were needed to build up the computing facilities in universities and polytechnics to the required level. Finally, a concerted effort was needed to establish the network of Mathematics Learning Resource Centres.

### 10.2 Recommendations for Action

In this section a number of recommendations are made which are considered necessary for the improvement of the teaching of mathematics to engineering undergraduates.

## (i) Mathematics Departments

Unless mathematics departments desist from standing on their academic dignity they will find that the teaching of engineers is taken from them, with a consequent threat upon jobs. Customers will eventually have the last word, and it is surely preferable for the mathematicians to teach the mathematics, albeit in a way which they will find at first a culture shock, rather than lose it altogether. They must cooperate with their engineering colleagues.

## (ii) Engineering Departments

The mathematics course must be regarded as integral to the engineering curriculum. Staff should make an effort to update their mathematical skills and acquaint themselves with modern engineering practice. They should encourage their students to take their mathematical studies seriously.

## (iii) Universities

University policy-making bodies must awaken to the need for better mathematical provision if they are to make a serious contribution to the future needs of the country as regards the output of engineers. They should establish Mathematics Learning Resource Centres as a matter of urgency.

## (iv) Professional Institutions

The Engineering Council and The Institute of Mathematics and its

Applications should make a joint initiative to alert their members to the impending crisis in the provision of engineering graduates with a suitable mathematical training. They should take the lead in urging universities and the Government to take the problem seriously.

## (v) Government

The Government must face up to the urgent need to redress the current situation. At the end of the day, if funding is not available then the action that needs to be taken will be severely curtailed. We hear much about the need for a strong Britain, but a strong Britain needs a steady output of modern engineers who are trained to face tomorrow's world. Such training of necessity implies a grounding in the mathematics of at least the core curriculum proposed in Chapter 9. The Government must take its responsibility professionally and provide the required funding earmarked for the proper mathematical education of engineers.

### 10.3 Suggestions for Further Work

High on the list of priorities for further work is the development of material suitable for the Mathematics Learning Resource Centre. Particular attention needs to be paid to the preparation of remedial information packs and to the compilation of suitable diagnostic tests.

There is much to be done in the area of videotape and videodisc production. However, these items are expensive to manufacture and to use, and more research needs to be carried out into the effectiveness of the medium as a means of imparting information and of providing motivation. A teaching aid which is more easily and more economically manufactured is that of standard diagrams and graphs, probably produced by a microcomputer, which could be made available in the form of overhead transparencies or slides.

These slides could be combined with audio cassette and written material to form an individualised learning package. Such packages are relatively inexpensive to produce and to operate, and offer a simple means of self-learning. Research is needed into the viability of such packages and their effectiveness.

More effort is needed in the area of the use of micro-based software in teaching. Attractive though the MIME mechanics units were, the emphasis must now be on smaller units, targetted at more specific topics.

The most fundamental research that needs to be conducted is in the acquisition of mathematical concepts by undergraduate engineers. Much is being asked of them in the core curriculum and with a shifting base it is important that not too much is asked. In particular, students who enter with BTEC qualifications or their equivalent face difficulties which have not been fully explored. There is likely to be an increasing number of students who wish to convert from arts-based A levels or who have returned to the educational scene later in life, and these students will have a different kind of handicap to overcome. All these categories of student are important and their needs must be catered for. It is this area of research which is especially urgent.

### 10.4 Envoi

The work described in this thesis has been carried out because the author cares about mathematics. He also cares about the mathematical education of his students. The task which they face in coming to terms with a more demanding syllabus, having been given a less thorough grounding in school mathematics than hitherto, is an increasingly daunting one.

The research has been undertaken in an attempt to help the students tackle their task more successfully. It does matter that they are motivated by the mathematics being made relvant to their needs and interests; it does matter that
they come to regard mathematics as an important foundation for their studies both at and after tertiary education.

It is hoped that the reader of this thesis also cares about mathematics sufficiently to make an effort to improve his teaching to engineering undergraduates. Whilst the author has made some small contribution to the mathematical education of engineers, he is only too aware that he has at best lit a trail which others may wish to join him in treading. So far, the temptation to include a quotation has been resisted, but it would perhaps be apposite at the end of this thesis to echo the words of Cecil Rhodes, which encapsulate the author's feelings at this juncture.

## "So little done, so much to do"

## REFERENCES

1. ALLEN, D.N. de G. (1948)

Applicable mathematics from the teacher's point of view.
Engineering, 165, 5 Nov., 439.
2. POPE, J.A. (1948)

Applicable mathematics and the engineer.
Engineering, 165, 12 Nov., 462.
3. BICKLEY, W.G. (1948)

The need for an Institute of Applicable Mathematics.
Engineering, 165, 3 Dec., 534.
4. ANON. (1948)

Applicable mathematics.
Engineering, 165, 3 Dec., 541-542.
5. ANON. (1948)

Engineering, 165, 15 Oct., 376-377.
6. ANON. (1948)

Mathematics as a tool.
Engineering, 165, 24 Dec., 613-614
7. BANKS, J.T. (1988)

Private communication.
8. CHRISTOPHERSON, D.G. (1967)

The engineer in the university. E.U.P., London.
9. HART, R.W. \& WOOD, W.H. (1959)

Ratings of college mathematics courses by applied mathematicians. Am. Math. Mnthly., 66, June/July, 510-512.
10. JONES, C.W. (1963)

Applied mathematics in the context of engineering.
Inaugural lecture, Imperial College, London.
11. C.U.P.M. (1962)

Report No.5. Recommendations on the undergraduate mathematics program for engineers and physicists.
Am. J. Phys., 30, 569-582.
12. O.E.C.D. (1961)

Mathematics for physicists and engineers. Project STP17.
O.E.C.D., Paris.
13. O.E.C.D. (1964)

Engineering education in the computer age.
O.E.C.D., Rome.
14. O.E.C.D. (1966)

Mathematical education of engineers.
O.E.C.D., Paris.
15. SCOTT, M.R.; BROOKS, A.; LEE, A.W. \& RAMSAY, H.B. (1966)

The use of mathematics in the electrical industry. Pitman, London.
16. NOBLE, B. (1967)

Applications of undergraduate mathematics in engineering. Mathematical Association of America/MacMillan, New York.
17. JAMES, D.J.G. (1988)

A core mathematics curriculum for engineering undergraduate courses.
Paper presented to S.E.F.I. Seminar on Mathematics in Engineering Education, Plymouth, U.K.
18. KERR, E. (1970)

Certain thoughts following the O.E.C.D. Report (1965) on the mathematical education of engineers.
Int. J. Math. Educ. Sci. Technol., 1(4), 389-391.
19. DAVIES, M.W.H. (1970)

The requirements of the engineering department.
Int. J. Math. Educ. Sci. Technol., 1(4), 395-397.
20. BAJPAI, A.C. (1970)

The teaching task as seen from a Department of Mathematics. Int. J. Math. Educ. Sci. Technol., 1(4), 398-401.
21. BAJPAI, A.C.; CALUS, I.M. \& SIMPSON, G.B. (1970)

An approach to the teaching of ordinary differential equations.
Int. J. Math. Educ. Sci. Technol., 1(1), 39-54.
22. BAJPAI, A.C. (1971)

Service teaching of mathematics in universities and polytechnics.
Bull. I.M.A., 7(5), 146-151.
23. ELTON, L.R.B. (1971)

Aims and objectives in the teaching of mathematics to non-mathematicians.
Int. J. Math. Educ. Sci. Technol., 2(1), 75-81.
24. BAJPAI, A.C. (1972)

Rôle of integrated packages for teaching mathematics to non-specialists.
Bull. I.M.A., 8(7), 214-216.
25. MUSTOE, L.R. (1972)

Mathematics for Civil Engineers : interim report on a teaching experiment in engineering mathematics.
Bull. I.M.A., 8(8), 245-247.
26. SCOTT, M.R. (1972)

Objectives in the teaching of mathematics to engineers.
Bull. I.M.A., 8(8), 239-241.
27. LIGHTHILL, M.J. (1972)

Teaching problems at various interfaces.
Bull. I.M.A., 8(8), 238.
28. BAJPAI, A.C. \& FRANCIS, D.C. (1970)

A survey of mathematics in engineering degree courses in the United Kingdom.
Int. J. Math. Educ. Sci. Technol., 1(4), 297-308.
29. FRANCIS, D.C. (1972)

Differential equations in engineering courses.
Int. J. Math. Educ. Sci. Technol., 3(3), 263-268.
30. TAGG, E.D. (1970)

Some difficulties experienced by first year undergraduates - an attempt to remedy their lack of sixth form mathematics.
Int. J. Math. Educ. Sci. Technol., 1(2), 153-157.
31. KNOWLES, F. (1975)

An approach to applicable mathematics.
Math. Teaching, 56, 50-55.
32. CORNELIUS, M.L. (1972)

The transition from school to university mathematics. Math. Gaz., LVI, 397, 207-218.
33. CORNELIUS, M.L. (1974)

Is there a 'gap' between school and university mathematics? Int. J. Math. Educ. Sci. Technol., 5(2), 209-212.
34. FLEGG, H.G. (1974)

The mathematical education of scientists and technologists - a personal view.

Int. J. Math. Educ. Sci. Technol., 5(1), 65-74.
35. DAVIES, M.J. (1972)

A first year course in applied mathematics.
Int. J. Math. Educ. Sci. Technol., 3(1), 71-75.
36. SIDA, D.W. (1975)

Old and new challenges to applied mathematics.
Int. J. Math. Educ. Sci. Technol., 6(2), 205-210.
37. BAKER, J.E.; CRAMPIN, M. \& NUTTALL, J. (1973)

A crash course in calculus.
Int. J. Math. Educ. Sci. Technol., 4(4), 335-339.
38. McLONE, R.R. (1971)

On the relationship between curriculum, teaching and assessment of mathematics.
Int. J. Math. Educ. Sci. Technol., 2(4), 341-344.
39. POLLAK, H.O. (1976)

What industry wants a mathematician to know and how we want them to know it.
Educ. Stud. Math., 7, 109-112.
40. BAJPAI, A.C.; MUSTOE, L.R. \& WALKER, D. (1975)

Mathematical education of engineers: part I-a critical appraisal.
Int. J. Math. Educ. Sci. Technol., 6(3), 361-380.
41. BAJPAI, A.C.; MUSTOE, L.R. \& WALKER, D. (1976)

Mathematical education of engineers: part II - towards possible
solutions.
Int. J. Math. Educ. Sci. Technol., 7(3), 349-364.
42. BAJPAI, A.C.; MUSTOE, L.R. \& WALKER, D.
(1974) Engineering mathematics.
(1977) Advanced engineering mathematics.
(1980) Specialist techniques in engineering mathematics.

Wiley, Chichester.
43. BAJPAI, A.C.; CALUS, I.M. \& FAIRLEY, J.A.
(1973) Mathematics for engineers and scientists. Vol.1.
(1975) Numerical methods for engineers and scientists.
(1978) Statistical methods for engineers and scientists.

Wiley, Chichester.
44. BAJPAI, A.C.; CALUS, I.M.; FAIRLEY, J.A. \& WALKER, D. (1973)-

Mathematics for engineers and scientists. Vol.2.
Wiley, Chichester.
45. BAJPAI, A.C.; CLARKE, R.J.; DOUBLEDAY, J.M.; PAKES, H.W. \& STEVENS, T.J. (1972)

Fortran and Algol. Wiley, Chichester.
46. CALUS, I.M. (1979)

The rôle of programmed texts in the mathematical education of engineers and scientists.
Bull. I.M.A., 15(8/9), 212-215.
47. GODFREY, K.R. (1985)

Presenting mathematics in a first-year engineering course at university.
Int. J. Math. Educ. Sci. Technol., 16(2), 311-319.
48. STROUD, K.
(1970) Engineering mathematics.
(1986) Further engineering mathematics.

Macmillan, Basingstoke.
49. MUSTOE, L.R. (1977)

Mathematical education of engineers.
Proceedings of S.E.F.I. Conference on Essential Elements in Engineering Education. Copenhagen.
50. MUSTOE, L.R. (1978)

Where there's a will, there's a way. Bull. I.M.A., 14(11/12), 302-306.
51. MUSTOE, L.R. (1982)

Are you being served? - Mathematics for Civil Engineers. Civil Eng. Educ., IV(2), Fall, 15-17.
52. POPE, J.A. (1978)

The mathematical education of engineers - where next?
Bull. I.M.A., 14(11/12), 274.
53. SHERCLIFF, J.A. (1978)

Can mathematics form the heart of the engineering curriculum? Bull. I.M.A., 14(11/12), 279-281.
54. WAKELY, P.G. (1978)

The needs of industry.
Bull. I.M.A., 14(11/12), 291-294.
55. HUNTER, J. (1978)

Student self-learning in mathematics.
Bull. I.M.A., 14(11/12), 282-284.
56. BLACKBURN, D.A. (1978)

Teaching mathematics by modelling with mathematics.
Bull. I.M.A., 14(11/12), 285-288.
57. CLEMENTS, R.R. (1978)

The rôle of simulation/case studies in teaching the practical application of mathematics.
Bull. I.M.A., 14(11/12), 295-297.
58. CRAGGS, J.W. (1978)

Rigorous engineering mathematics.
Bull. I.M.A., 14(11/12), 307-308.
59. BRIGHOUSE, G.A. (1978)

The formative years.
Bull. I.M.A., 14(11/12), 298-300.
60. GIBBONS, R.F. (1978)

Varieties of mathematical experience.
Bull. I.M.A., 14(11/12), 301.
61. LIGHTHILL, M.J. (1979)

The mathematical education of engineers.
Bull. I.M.A., 15(4), 89-94.
62. GNEDENKO, B.V. \& KHALIL, Z. (1979)

The mathematical education of engineers.
Educ. Stud. Math., 10(1), 71-83.
63. LUND, M. \& CHRISTIANSEN, T. (1980)

The rôle of mathematics and physics in the first year of study at D.T.H. (a necessary evil?).

Eur. J. Eng. Educ., 4, 147-152.
64. HEARD, T.J. (1978)

The mathematical education of engineers at school and university. Report, Eng. Science Dept., Durham University.
65. BARRETT, K.E.; JAMES, D.J.G. \& STEELE, N.C. (1980)

The mathematical content of first year engineering courses in British universities and polytechnics.
Eur. J. Eng. Educ., 4, 153-163.
76. BAJPAI, A.C. (1985)

The rôle of mathematics in engineering education: a mathematician's view.
Int. J. Math. Educ. Sci. Technol., 16(3), 417-430.
77. SCANLAN, J.O. (1985)

The rôle of mathematics in engineering education: an engineer's view.
Int. J. Math. Educ. Sci. Technol., 16(3), 445-451.
78. CONFERENCE OF PROFESSORS OF APPLIED MATHEMATICS (1987)

Survey on science teaching.
79. CLEMENTS, R.R. (1985)

The curriculum into the 1990s - a personal view. Int. J. Math. Educ. Sci. Technol., 16(2), 233-238.
80. VARIOUS. (1985)

Proceedings of the First European Seminar of Mathematics in Engineering Education, Kassel.
Int. J. Math. Educ. Sci. Technol., 16(2), 152-242.
81. VARIOUS. (1985)

Proceedings of Anglo-Swedish Conference on the Teaching of Mathematics to Science and Engineering Students.
Int. J. Math. Educ. Sci. Technol., 16(2), 243-327.
82. VARIOUS. (1986)

Proceedings of the Second European Seminar on Mathematics in Engineering Education, Lyngby.
Int. J. Math. Educ. Sci. Technol., 17(5), 527-575.
83. VARIOUS. (1987)

Proceedings of the Third European Seminar on Mathematics in Engineering Education, Turin.
Int. J. Math. Educ. Sci. Technol., 18(5), 647-698.
84. RÅDE, L. (Ed.) (1988)

Teaching modern engineering mathematics. Chartwell-Bratt, Bromley.
85. FOWLER, W.T., JONG, I.C. \& ROGERS, B. (1976) Is the typical engineering freshman ready for engineering? Eng. Educ., 66, April, 754-756.
86. SNYDER, V.W. \& MERIAM, J.L. (1976)

A study of mathematical preparedness of students: the mechanics readiness test.
Eng. Educ., 69, Dec., 261-268.
87. MUSTOE, L.R. (1979)

Mathematical ability of Civil Engineering entrants.
Unpublished report.
88. FYFE, D.J. (1981)

Assessment of the mathematical ability of students entering advanced courses in science and engineering.
Bull. I.M.A., 17(10), 209-211.
89. ELTON, L.R.B. (1980)

Mathematical deficiencies in university entrants.
Eur. J. Sci. Educ., 2(1), 25-44.
90. GONZALEZ-LEON, E. (1980)

Remedial work in mathematics for students entering engineering courses at universities.
Int. J. Math. Educ. Sci. Technol., 11(1), 81-89.
91. GONZALEZ-LEON, E. (1981)

A survey of pre-requisite mathematical knowledge for first year engineering students.
Bull. I.M.A., 17(10), 212-214.
92. HOWARTH, M.J. \& SMITH, B.J. (1980)

Attempts to identify and remedy the mathematical deficiencies of engineering undergraduates at Plymouth Polytechnic.
Int. J. Math. Educ. Sci. Technol., 11(3), 377-383.
93. HUBBARD, R. (1986)

A comprehensive scheme to assist mathematically deficient tertiary entrants.
Int. J. Math. Educ. Sci. Technol., 17(2), 247-251.
94. KURZ, G. (1985)

Remedial courses in mathematics - scopes andproblems - a survey for the Federal Republic of Germany.
Int. J. Math. Educ. Sci. Technol., 16(2), 211-225.
95. TURNER, S. \& MUSTOE, L.R. (1977)

The usefulness of topics in modern mathematics - a quantitative assessment for degree level studies.
Bull. I.M.A., 13(2), 51-54.
96. WALKDEN, F. \& SCOTT, M.R. (1980)

Aspects of mathematical education. Int. J. Math. Educ. Sci. Technol., 11(1), 45-53.
97. POWER, T. (1985)

Mathematics for technician engineers: concepts in place of confusion. Int. J. Math. Educ. Sci. Technol., 16(3), 453-461.
98. SCHOOLS COUNCIL (1973)

16-19: growth and response, 2. Examination structure.
Evan/Methuen Educational, London.
(Schools Council working paper, no 46)
99. DEPARTMENT OF EDUCATION AND SCIENCE (1987)

Broadening A level studies. A guide for schools and colleges.
H.M.S.O., London.
100. SECONDARY EXAMINATIONS COUNCIL (1986)

Mathematics G.C.S.E. A guide for teachers.
Open University Press, Milton Keynes.
101. COCKCROFT, W. (1982)

Mathematics counts. Report of the Committee of Inquiry into the Teaching of Mathematics in Schools.
H.M.S.O., London.
102. SCHEY, H.M.; SCHWARTZ, J.L.; WALTON, W.V. \& ZACHARIAS, J.R. (1970)

A laboratory computer and calculus based course in mathematics. Int. J. Math. Educ. Sci. Technol., 1(2), 115-130.
103. BAJPAI, A.C. \& MUSTOE, L.R. (1974)

An educational philosophy for scientists and technologists using computers.
Int. J. Math. Educ. Sci. Technol., 5(4), 601-606.
104. MUSTOE, L.R. (1976)

Numerical methods on computer terminals. Part I-a philosophy. Internal report, Loughborough University of Technology.
Department of Engineering Mathematics.
105. ENGLISH, H.W.F.E. (1976)

Numerical methods on computer terminals. part II implementation and assessment.
Internal report, Loughborough University of Technology.
Department of Engineering Mathematics.
106. LEACH, D. \& HAMPTON, J. (1978)

A computer-based laboratory for mathematics teaching. Bull. I.M.A., 14(8/9), 223-226.
107. HUNDHAUSEN, J.R. (1980)

Advanced O.D.E. in an engineering environment: a computer-assisted approach.
Am. Math. Mnthly., 87, Oct., 662-669.
108. ERIKSSON, H. (1985)

The impact of computers on the teaching of university mathematics. Int. J. Math. Educ. Sci. Technol., 16(2), 247-251.
109. WINKELMANN, B. (1984)

The impact of the computer on the teaching of analysis. Int. J. Math. Educ. Sci. Technol., 15(6), 675-689.
110. ROWE, G.W. (1985)

The impact of computers on the teaching of mathematics to engineers.
Int. J. Math. Educ. Sci. Technol., 16(2), 181-185.
111. CANELOS, J. \& CARNEY, B.W. (1986)

How computer-based instruction affects learning. Eng. Educ., 76, Feb., 298-301.
112. MURAKAMI, H. \& HATA, M. (1986)

Mathematical education in the computer age.
In The Influence of Computers and Informatics on Mathematics and its Teaching.
C.U.P. (I.C.M.I. Study Series)
113. HARDING, R.D. (1983)

Applied mathematics and the computer revolution.
Bull. I.M.A., 19, Mar./Apr., 67-70.
114. HARDING, R.D. (1986)

A software tool-kit for computer-aided mathematics teaching. Bull. I.M.A., 22, May/June, 76-80.
115. JACQUES, I.B. \& JUDD, C.J. (1985)

Use of microcomputers in teaching numerical mathematics. Int. J. Math. Educ. Sci. Technol., 16(3), 347-354.
116. JAMES, D.J.G. \& WILSON, M.A. (1985)

The rôle of a 'micro' in a mathematical modelling course. Int. J. Math. Educ. Sci. Technol., 16(3), 391-405.
117. BAJPAI, A.C.; FAIRLEY, J.A.; HARRISON, M.C.; MUSTOE, L.R.; WALKER, D. \& WHITFIELD, A.H. (1984)

The MIME Project at Loughborough - a first report.
Int. J. Math. Educ. Sci. Technol., 15(6), 781-810.
118. BAJPAI, A.C.; FAIRLEY, J.A.; HARRISON, M.C.; MUSTOE, L.R.; WALKER, D. \& WHITFIELD, A.H. (1985)

Mathematics and the micro-some hints on software development.
Int. J. Math. Educ. Sci. Technol., 16(3), 407-412.
119. BAJPAI, A.C.; FAIRLEY, J.A.; HARRISON, M.C.; MUSTOE, L.R.; WALKER, D. \& WHITFIELD, A.H. (1987)

Mathematics and the micro - the MIME Project at Loughborough.
Collegiate Microcomputer, V(2), 128-134.
120. BAJPAI, A.C.; FAIRLEY, J.A.; HARRISON, M.C.; MUSTOE, L.R.; WALKER, D. \& WHITFIELD, A.H. (1987)

The MIME Project at Loughborough - a second report.
Int. J. Math. Educ. Sci. Technol., 18(2), 301-313.
121. BAJPAI, A.C. \& MUSTOE, L.R. (1986)

Computer enhanced learning units - the Loughborough experience. Paper presented to Conference on Computers in the Teaching of the Mathematical Sciences in Higher Education. Canterbury.
122. MUSTOE, L.R. \& DOWNEND, I.J. (1987)

Computer enhanced learning of mathematics - a laboratory-based approach.
Paper presented to Conference on Mathematics and Statistics Curricula in Higher Education for the 1990s. Edinburgh.
123. BAJPAI, A.C. \& DOWNEND, I.J. (1987)

Development of computer enhanced learning units under the MIME Project.
Bull. I.M.A., 23, Sep., 122-127.
124. BAJPAI, A.C. \& DOWNEND, I.J. (1988)

Computer enhanced learning in engineering mathematics.
Proceedings of a Conference on Computers in Engineering Education. Imperial College.
C.T.I.S.S., Bath.
125. JAMES, D.J.G. \& McDONALD, J.J. (Eds.) (1981)

Case studies in mathematical modelling.
Stanley Thomas, Cheltenham.
126. BURGHES, D.N. \& BORRIE, M.S. (1981) Modelling with differential equations. Ellis Horwood, Chichester.
127. BURGHES, D.N.; HUNTLEY, I.D. \& McDONALD, J.J. (1982)

Applying mathematics.
Ellis Horwood, Chichester.
128. CROSS, M. \& MOSCARDINI, A.O. (1985)

Learning the art of mathematical modelling. Ellis Horwood, Chichester.
129. HEATON, A.G. (1970)

Some aspects of the mathematical formulation and solution of real problems.
Int. J. Math. Educ. Sci. Technol., 1(2), 207-211.
130. FORD, B. \& HALL, G.G. (1970)

Model-building - an educational philosophy for applied mathematics. Int. J. Math. Educ. Sci. Technol., 1(1), 77-83.
131. McDONALD, J.J. (1977)

Introducing mathematical modelling to undergraduates. Int. J. Math. Educ. Sci. Technol., 8(4), 453-461.
132. McLONE, R.R. (1984)

Can mathematical modelling be taught?
In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester.
133. SEKHON, J.G.; SHANNON, A.G.; CHIARELLA, C. \& HORADAM, A.F. (1984)

Mathematical modelling and the preparation of industrial mathematicians.
In Teaching and Applying Mathematical Modelling; edited by Berry J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester.
134. BURLEY, D.M. \& TROWBRIDGE, E.A. (1984)

Experiences of mathematical modelling at Sheffield University. In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester.
135. BECKETT, P.M. (1982)

How mathematical modelling can be used as a source of inspiration. Int. J. Math. Educ. Sci. Technol., 13(2), 125-132.

## 136. HALL, G.G. (1984)

The assessment of modelling projects.
In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester.
137. GADIAN, A.M.; HUDSON, P.C.; O'CARROLL, M.J. \& WILLERS, W.P. (1984)

Experience with team projects in mathematical modelling.
In Teaching and Applying Mathematical Modelling; edited by Berry,
J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O.

Ellis Horwood, Chichester.
138. ROBSON, E.H. (1985)

The rôle of computer simulation.
Int. J. Math. Educ. Sci. Technol., 16(2), 255-258.
139. CLEMENTS, L.S. \& CLEMENTS, R.R. (1978)

The objectives and creation of a course of simulations/case studies for the teaching of engineering mathematics.
Int. J. Math. Educ. Sci. Technol., 9(1), 97-117.
140. CLEMENTS, R.R. (1982)

On the rôle of notation in the formulation of mathematical models. Int. J. Math. Educ. Sci. Technol., 13(5), 543-549.
141. CLEMENTS, R.R. (1982)

Initial experience of the use of simulations/case studies in the teaching of engineering mathematics.
Int. J. Math. Educ. Sci. Technol., 13(1), 111-116.
142. CLEMENTS, R.R. (1984)

The use of simulation and case study techniques in the teaching of mathematical modelling.
In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester.
143. ANDRIÉ, M. (1985)

Application-orientated mathematics in the education of engineers. Int. J. Math. Educ. Sci. Technol., 16(2), 157-162.
144. CRILLY, A.J.; KROPHOLLER, H.W. \& MUSTOE, L.R. (1976)

Some structural aspects of distillation.
Int. J. Math. Educ. Sci. Technol., 7(2), 133-150.
145. SHARP, J.J. \& MOORE, E. (1977)

Teaching mathematics to Civil Engineers.
Int. J. Math. Educ. Sci. Technol., 8(2), 127-131.
146. MACNAB, D.; MICKASCH, H.D. \& GEORGI, W. (1977)

Description and assessment of different methods of teaching engineering students mathematics.
Int. J. Math. Educ. Sci. Technol., 8(2), 219-228.
147. CLARK, M. (1980)

Mathematics as responsive tertiary education concerned with the needs of the students of the subject.
Int. J. Math. Educ. Sci. Technol., 10(4), 581-585.
148. CUMMING, B.L. \& McINTOSH, C. (1983)

Innovation despite austerity: P.S.I. in engineering mathematics.
Int. J. Math. Educ. Sci. Technol., 14(2), 201-204.
149. SEARL, J.W. (1984)

The trouble with partial differential equations.
Int. J. Math. Educ. Sci. Technol., 15(3), 275-284.
150. SEARL, J.W. (1985)

Mathematics for engineers and scientists: some teaching approaches.
Int. J. Math. Educ. Sci. Technol., 16(2), 275-283.
151. LAWES, P. (1985)

Manufacturing industry - the consumer of mathematics.
Int. J. Math. Educ. Sci. Technol., 16(3), 463-468.
152. EINARSSON, O. (1985)

How much mathematics is needed for an engineer working in industry?
Int. J. Math. Educ. Sci. Technol., 16(2), 289-293.
153. HENNIG, T. (1985)

Industry's demands on graduate engineers' knowledge of mathematics.
Int. J. Math. Educ. Sci. Technol., 16(2), 205-209.
154. SUNDSTRÖM, D. (1985)

Mathematics in industry.
Int. J. Math. Educ. Sci. Technol., 16(2), 295-296.
155. WILKINSON, T.S. (1985)

Mathematics in engineering : an industrial view. Int. J. Math. Educ. Sci. Technol., 16(2), 297-300.
156. GREEN, D.R. (1982)

Probability concepts in school pupils aged 11-16 years.
Ph.D. Thesis. Loughborough University of Technology.
157. MUSTOE, L.R. (1986)

Worked examples in engineering mathematics.
Wiley, Chichester.
158. MUSTOE, L.R. (1988)

Worked examples in advanced engineering mathematics.
Wiley, Chichester.
159. G.C.E. EXAMINING BOARDS. (1983)

Common cores at Advanced Level.
160. MUSTOE, L.R. (1976)
$2+2=3.99$.
Mathematics in Schools, 5(5), 31-34.
161. METCALF, M. (1982)

Fortran is alive and well.
CERN, Data Handling Division report, no. DD/82-19.
162. HARDING, R.D. (1974)

Computer aided teaching of applied mathematics.
Int. J. Math. Educ. Sci. Technol., 5(4), 447-455.
163. HAGGETT, A.J. \& LE MASURIER, D.W. (1985)

A computer appreciation course for first year mechanical engineering students.
Europ. J. Eng. Educ., 10(3/4), 345-351.
164. BEILBY, M.H. (1987)

Computer exercises in mathematics teaching programs. Proceedings of the Conference on Mathematics and Statistics
Curricula in Higher Education for the 1990s. The Polytechnic of Edinburgh. 239-247.
165. MACKIE, D. (1987)

Computer-based investigations in mathematics.
Proceedings of the Conference on Mathematics and Statistics Curricula in Higher Education for the 1990s. The Polytechnic of Edinburgh. 257-264.
166. TALL, D. (1987)

The complementary rôles of prepared software and programming in the learning of mathematics.
Bull. I.M.A., 23, Sep, 128-133.
167. JAEN GALLEGO, J.A., JUAN RUIZ, S., PRIETO FERNANDEZ, F.J. \& SALAMANCA, FERNANDEZ. (1985)

Computer-assisted instruction in linear programming. Europ. J. Eng. Educ., 10(3/4), 329-338.
168. MORRIS, J.M. (1985)

The impact of computer science on mathematics in the engineering curriculum.
Int. J. Math. Educ. Sci. Technol., 16(3), 437-443.
169. ROBSON, E.H. (1985)

Some implications of computing developments for engineering. Europ. J. Eng. Educ., 10(3/4), 285-289.
170. COMPUTER BOARD FOR UNIVERSITIIES AND RESEARCH COUNCILS. (1983)

Computer facilities for teaching in universities.
Report of a Working Party.
D.E.S., London.
171. BEEVERS, C.E. (1987)

The CALM Project - evaluation and development.
Proceedings of the Conference on Mathematics and Statistics Curricula in Higher Education for the 1990s. The Polytechnic of Edinburgh. 287-298.
172. HODGKINSON, D. (1987)

Algebraic computing in education.
Proceedings of the Conference on Mathematics and Statistics
Curricula in Higher Education for the 1990s. The Polytechnic of Edinburgh. 197-217.
173. FUGARD, T.B. (1987)

Experience in the use of MACSYMA in a problem-solving environment.
Proceedings of the Conference on Mathematics and Statistics Curricula in Higher Education for the 1990s. The Polytechnic of Edinburgh. 301-313.

LAURILLARD, D. (1988)
Evaluating computer-based learning.
Proceedings of a Conference on Computers in Engineering Education.
Imperial College.
C.T.I.S.S., Bath. 70-76.
175. ORTON, A. (Ed.) (1985)

Studies in mechanics learning. Centre for Studies in Science and Mathematics Education, University of Leeds.
176. MORGAN, W. (1964)

Elements of structure.
Pitman, London.
177. BAJPAI, A.C. \& MUSTOE, L.R. (1987)

Computer enhanced learning of mathematics.
Project Report.
C.T.I.S.S., Bath.
178. MUSTOE, L.R. (1988)

Computer enhanced learning of mathematics: a laboratory-based approach.
Proceedings of the Fifth European Seminar on Mathematics in
Engineering Education.
SEFI, Plymouth.
179. MUSTOE, L.R. (1988)

Computer enhanced learning of mathematics.
Proceedings of a Conference on Engineering Education in Europe.
SEFI, Leuven, Belgium.
180. POLLAK, H.O. (1968)

On some of the problems of teaching applications of mathematics. Ed. Stud. Maths., 1, 24-30.
181. MEDLEY, D.G. (1984)

Modelling, 'successful' and 'unsuccessful'.
In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester. 26-38.
182. MOSCARDINI, A.O., CURRAN, D.A..S., SAUNDERS, R., LEWIS, D.A. \& PRIOR, D.E. (1984)

Issues involved in the design of a modelling course.
In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester. 96-106.

MASON, J.H. (1984)
Modelling: what do we really want students to learn?
In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes, D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O. Ellis Horwood, Chichester. 215-234.
184. JAMES, D.J.G. \& STEELE, N.C. (1984)

Control theory ideas as a component of a modelling course.
In Teaching and Applying Mathematical Modelling; edited by Berry, J.S., Burghes; D.N., Huntley, I.D., James, D.J.G \& Moscardini, A.O.

Ellis Horwood, Chichester. 390-411.
185. EVANS, J.R. (1980)

Solving word problems and elementary mathematical modelling. Int. J. Math. Educ. Sci. Technol., 11(4), 517-522.
186. FYFE, D.J. (1982)

A computation of the dimensions of an underground cylindrical tank - an undergraduate modelling exercise.

Int. J. Math. Educ. Sci. Technol., 13(6), 685-692.
187. BURGHES, D.N. \& HUNTLEY, I.D. (1982)

Teaching mathematical modelling - reflections and advice. Int. J. Math. Educ. Sci. Technol., 13(6), 735-754.
188. GIORDIANO, F.R. \& POLLIN, J.M. (1982)

Developing an undergraduate mathematical modelling course.
Int. J. Math. Educ. Sci. Technol., 13(6), 755-762.
189. CROSS, M., MOSCARDINI, A.O. \& THORP, M. (1982)

Interactive computer simulation tools for use in teaching mathematical modelling.
Int. J. Math. Educ. Sci. Technol., 13(6), 763-778.
190. EVANS, J.R. \& McKINNEY, J.M. (1987)

The modelling process and creative thinking. Int. J. Math. Educ. Sci. Technol., 18(1), 1-8.
191. ANDREWS, J.G. \& McLONE, R.R. (1976)

Mathematical modelling. Butterworths, London.
192. OKE, K.H. (1984)

Mathematical modelling processes: implications for teaching and learning.
Ph.D. Thesis. Loughborough University of Technology.
193. BERRY, J.S. \& O'SHEA, T. (1982)

Assessing mathematical modelling.
Int. J. Math. Educ. Sci. Technol., 13(6), 715-724.
194. MURTHY, D.N.P. \& PAGE, N.W. (1981)

Teaching mathematical modelling to undergraduate engineering students.
Int. J. Math. Educ. Sci. Technol., 12(2), 235-243.
195. JAMES, D.J.G. \& WILSON, M.A. (1982)

Student problems with a modelling exercise.
Int. J. Math. Educ. Sci. Technol., 13(6), 789-796.
196. OKE, K.H. \& BAJPAI, A.C. (1982)

Teaching the formulation stage of mathematical modelling to students in the mathematical and physical sciences.
Int. J. Math. Educ. Sci. Technol., 13(6), 797-814.
197. RUBIN, R.L. (1982)

Mathematical model formulation.
Int. J. Math. Educ. Sci. Technol., 13(6), 725-734.
198. MORRIS, W.T. (1967)

On the art of modelling.
Management Sci., 13(12), B707-B717.
199 HANSEN, L.A. \& SPIES, K. (1988)
Private communication.
RICHARDS, E.J. (1973)
Some thoughts on mathematical education.
Int. J. Math. Educ. Sci. Technol., 4(4), 377-395.
LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY.
Report on the Working Party on Mathematical Modelling and Engineering Systems.
Loughborough University of Technology report, no ENG 74: 73.
FULLER, M. (1978)
Private communication.
LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY. DEPARTMENT OF ENGINEERING MATHEMATICS. (1982)

Application for exemption from the CEI Parts 1 and 2 examinations.
LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY. DEPARTMENT OF ENGINEERING MATHEMATICS. (1987)

Application for exemption from the Engineering Council Parts 1 and 2 examinations.

205 UNIVERSITY OF NOTTINGHAM. DEPARTMENT OF THEORETICAL MECHANICS. (1988)

Mathematics with engineering.
206 UNIVERSITY OF BRISTOL. DEPARTMENT OF ENGINEERING MATHEMATICS. (1988)

Study engineering mathematics at Bristol.
207 CRAIG, R.E. (1987)
Soil mechanics.
Van Nostrand Reinhold (UK), Wokingham.
208 UNIVERSITY OF LONDON. QUEEN MARY COLLEGE. (1987)
BSc. Examination, Engineering Mathematics III (QMC 622)
209 UNIVERSITY OF NOTTINGHAM. FACULTY OF ENGINEERING. (1988)
Part I Examination. Engineering Mathematics II, Paper A. (2367)
210 UNIVERSITY OF BRISTOL. (1988)
First stage examination for the degree of Bachelor of Engineering. Mathematics IS paper 2. (703)

211 BROWN, G. \& ATKINS, M. (1988)
Effective teaching in higher education.
Methuen, London. 10.

MUSTOE, L.R. (1970)
Computer-produced movies in mathematical education.
Int. J. Math. Educ. Sci. Technol., 1(4), 353-358.
BERRY, J.S. (1985)
Using video in the teaching of engineering science and applied mathematics.
Int. J. Math. Educ. Sci. Technol., 16(2), 197-203.
BERRY, J.S. \& HUNTLEY, I.D. (1986)
Video - a new resource in the teaching of mathematics.
Int. J. Math. Educ. Sci. Technol., 17(4), 403-405.

Interactive video in higher education.
In Video in Higher Education; edited by Zuber-Skeritt, O.
Kogan Paul, London.

BERRY, J.S. (1988)
Visualising mathematics.
Lecture given to the Annual Meeting of the British Association for the Advancement of Science. Oxford.

RUMBLE, G. (1988)
The planning and management of distance education. Croom Helm, London. 10-15.

BERRY, J.S. (1985)
Teaching at a distance - the Open University.
Int. J. Math. Educ. Sci. Technol., 16(2), 267-274.
SPARKES, J. (1983)
On choosing teaching methods to match educational aims.
In Distance Education: International Perspectives; edited by Stewart,
D., Keegan, D. \& Holmberg, B.

Croom Helm, London.
SMITH, P. (1987)
Distance education and educational change.
In Distance Education and the Mainstream; edited by Smith, P. \& Kelly, M.
Croom Helm, London.

## Appendix 1

## Short Course Syllabus

## Algebra and Analysis

## A Algebra (40 hours)

Real numbers. Complex numbers. Binomial theorem, Polynomials and rational functions. Vector spaces, linear dependence. Vector algebra. Linear transformations, Matrices. Systems of linear equations. Eigenvalues and Eigenvectors. Reduction to diagonal form for distinct Eigenvalues. Orthogonalizing. Norm. Quadratic forms. Classification. Geometry of the line, conic, plane, quadric (by vector and matrix methods).
Different coordinate systems.

## B Analysis (180 hours)

Functions of a real variable: Limits, continuity, continuous functions. Monotonic functions. Concept of inverse function. Differentiation, mean value theorem, maxima, minima and variation of functions.

Application to plane curves; tangent, normal, curvature.
Indeterminate forms.
Elementary functions.
Integration: the concept of integral as limit of a sum.
Relation between integration and differentiation. Methods of integration
Application of integration (eg areas, volumes, first and second moments). Improper integrals. Tables of integrals.

Series - Series with positive terms. Elementary convergence tests. Power series including Taylor series.

Ordinary differential equations - Equations of first order. Linear equation with constant coefficients. Use of Laplace transform.

Exact solutions. Singular points. Isoclines. Solutions in series. Application to special functions. Existence and uniqueness of the solution (no proof required).

Functions of several variables - Continuity. Differentiation. Taylor theorem. Maxima and Minima. Conditional extrema and Lagrange multipliers. Representaion of a function as a surface. Parametric integrals. Differentiation and integration of integrals. Multiple integrals, including the rule of change of variables. Line, surface and volume integrals. Vector fields: gradient, divergence, and theorems of Gauss, Green and Stokes.

Partial differential equations - Classification of 2nd order partial differential equations with constant coefficients. Solution by separation of boundary value problems. Solution by Laplace transform.

Functions of complex variable - differentiation. Analytic functions. Cauchy-Riemann conditions. Introduction to conformal mapping. Elementary functions.

## C Digital Computation (21 hours)

Functional organisation of a digital computer. Hierarchy of computing languages; machine, symbolic assembly, procedure-oriented, problem-oriented.

Instructions and procedures; flow diagrams; concept of stored programme and instruction modification; iterative procedures; automatic programming languages, sufficient details of one language, such as Algol 60 , to enable simple examples to be programmed; numerical and non numerical applications, data-structure and list processing.

## D Analogue Computation (4 hours)

An introduction to analogue computers with demonstration of solution of differential equations describing some typical engineering problems.

E Numerical Analysis ( 40 hours)
Basic ideas. Formulation, truncation and rounding errors. Simple error analysis. Chebyshev and least squares approximation. Economisation of series. Orthogonal polynomials. Introduction to finite and divided differences. Interpolation, differentiation, integration. Lagrange formulae.

Non linear equations.
Linear simultaneous equations by direct and iterative methods.
Matrix inversion. Systems of non-linear equations by the Newton-Raphson method.
Matrix eigen-value and eigen-vector determination.
Ordinary differential equations including boundary-value problems.
Runge-Kutta method; selected predictor-corrector method; deferred-correction method.
Partial differential equations. Finite-difference methods.
Ideas of stability.

## F Statistics and Probability (60 hours)

## Syllabus A (USA)

Probability theory (discrete case, 17 lectures; continuous case, 9 lectures). Sample space, event, random variable, function of a random variable. Probability, expectation, variance, moments, Chebyshev's inequality. Joint distribution, transformations of joint densities, conditional probabilities, Bayes' theorem, independence. Bernoulli trials, combinatorics, binomial distribution. Normal law, introduction to the law of large numbers and statement of the central limit theorem, Poisson distribution, elementary Markov chains.

Introduction to statistical inference ( 13 lectures). The formulation of statistical problems and the rationale behind the choice of statistical procedures. An introduction to estimation and sampling, with point and interval estimation. Elementary hypothesis testing, power of a test. Regression, a few examples of nonparametric methods.

## Syllabus B (UK)

Introduction to probability. Theorems including Bayes' theorem. Inherent variations in observed data. The normal distribution, its properties and estimation of its parameters. Simple tests of significance and confidence limits for samples from the normal distribution.

Basic principles of experimentation; statistical inference and testing. Initial steps in planning experiments; randomisation, replication and other means of improving quality of estimates. Simple Analysis of Variance. Sources of variation in data; the use of models to describe data. The regression model; estimation of parameters, etc. General curve fitting. The regression model; estimation of parameters, etc. General curve fitting. Statistical distributions; uniform, binomial, Poisson. Goodness of fit test.

## Appendix 2

## EVALUATION OF MATHEMATICS ABILITY

1 Estimate the values of
(i) $\frac{4.1 \times 16.9}{2.3 \times 0.95}$
(ii) $\frac{\pi^{3} \times 41.3}{11.2 \times 7.63}$

2 You measure a diameter as 3.618 cm but you know your measurement may be inaccurate by as much as 0.01 cm . How would you quote your answer?

3 You are given the formula

$$
\mathrm{f}=\frac{1}{2 \pi \sqrt{\mathrm{LC}^{3}}}
$$

Rearrange this formula to make
(i) L the subject
(ii) C the subject

4 What are the roots of the equation

$$
x^{3}-4 x^{2}=5 x ?
$$

5 You are given the formula

$$
y=\frac{x}{\left[1+(z / 3)^{2}\right]}
$$

If $x=20, y=2$, find $z$.
6 Find the height of the tower $\mathrm{TT}^{1}$ in terms of the measured distance d and angles $\alpha, \beta$.


7 Determine the coordinates of the centroid of the area enclosed by the parabola and the axes shown below.


8 The bending of a simple beam subjected to a uniformly distributed load w/unit length can be represented by the equation below where $y$ is the vertical displacement downwards and k is a constant.

$$
\mathrm{k} \frac{\mathrm{~d}^{2} \mathrm{y}}{\mathrm{dx}} \mathrm{x}^{2}=\frac{1}{2} \mathrm{wx} \mathrm{x}^{2}-\frac{1}{2} \mathrm{wLx}
$$

Find an expression for $y$.


9 By resolving forces to maintain equilibrium at the joints, find the forces in all members of the truss shown below:


10 Three tugs are each pushing with a force of 20 kN on a freighter as shown. What is the resultant force along the centre line of the ship? Will it move forwards or backwards?


11 A rectangular tank 3 m long, 1 m wide and 2.5 m deep weighs 4.5 tonnes. It floats on an even keel in fresh water of density $9871 \mathrm{~N} / \mathrm{m}^{3}$.
(i) A square hole width sides of 300 mm is made in the bottom of the tank. What force in Newtons would be required to hold a weightless, watertight plate in place on the hole?
(ii) A square vertical tube is welded to the hole in the tank. How high in the tube will the water rise above the bottom of the tank?
(iii) What volume of paraffin with a specific gravity of 0.88 must be poured into the tube to drive out the water?
(iv) Could you use glycerine (S. G. $=1.36$ ) instead of paraffin?

12 (i) A rectangular tank of the same dimensions and weight as in Q 11 is to carry an even load of sand. What volume of sand, weighing 1.5 tonnes $/ \mathrm{m}^{3}$ could be carried, before the tank sinks?
(ii) What would be the density of a solid block of material of the same size as the tank that would float with one of the 3 mx 1 m sides 0.5 m above water level? Is it likely to float like this?

## Appendix 3

## ENGINEERING MATHEMATICS SYLLABUSES

## Part A Courses

## 1 Mathematical Methods I

Differentiation including maxima and minima, Taylor and Maclaurin series.
Partial differentiation, total derivative and applications to errors.
Integration and applications.
Complex numbers.
Vector algebra.
First order separable and linear differential equations.
Second order differential equations with constant coefficients including the use of the Laplace Transform.

2 Numerical Methods I and Computer Programming
Introduction to basic ideas, errors etc.
Solution of nonlinear equations.
Solution of simultaneous linear equations by Gauss elimination.
Finite difference notation, interpolation, numerical differentiation and integration.
Curve fitting by method of least squares.
Flow diagrams and simple computer programs including formation of loops.
Subscripted variables.
Subprograms.

## 3 Statistics I

Descriptive statistics, basic probability theory, binomial, Poisson, and normal distributions.

Distribution of sums and differences, sample means and convergence to normality, elementary significance tests.

## Part B Courses

## 4 Mathematical Methods II

Functions of several variables, maxima and minima.
Matrix solution of linear equations, transformations and eigenvalues.
Fourier series and solution of partial differential equations by separation of variables.
5 Mathematical Methods III
Functions of several variables, change of variables.
Line, surface and volume integrals.
6 Vector Field Theory
Vector function, differentiation and applications.
Vector integrals and orthogonal curvilinear coordinates.
Differential operators, grad, div and curl.
Gauss' and Stokes' Theorems.
7 Functions of a Complex Variable
Analytic functions, Cauchy-Riemann conditions.
Conformal transformations.
Taylor and Laurent series, singularities, residues and contour integrals.

## 8 Numerical Methods II

Numerical linear algebra.
Determination of eigenvalues and eigenvectors.
Solution of ordinary differential equations, initial value and boundary value problems including computer methods.

9 Statistics II
Linear regression and correlation.
$F, t$ and $x^{2}$ tests.
Introduction to multiple regression.

## 10 Statistics III

Multiple regression.
Analysis of variance.
Design of experiments.

## Part C Courses (Term 1 only)

The following optional course, each comprising ten lecture hours, could be arranged for final year students from all Engineering Departments. No examinations would be held. Other courses could be arranged on request. Selected topics from the following syllabuses would be discussed.

## 11 Transform Calculus

Further work on Laplace Transforms, convolution theorem.
Introduction to Z transforms.
Fourier Transforms.

## 12 Mathematics for Telecommunications

Bessel functions.
Positive real functions, Hurwitz polynomials, stability criteria.

## 13 Introduction to Tensor Calculus

Tensor formulation and extension of vector field theorems.
General equations of motion and continuity of fluids.

## 14 Introduction to Operational Research

Linear programming and allocation problems.
Dynamic programming. Queueing.
Basic ideas on theory of games and Moneo Carlo methods.

## 15 Introduction to Stochastic Processes

Probability theory; laws, random variables and probability distributions.
Expectation, variance, generating functions.
Markov chains, Markov processes, renewal processes.
Introduction to queueing theory.

## 16 Introduction to Optimization Methods

Survey of numerical hill-climbing techniques.
Dynamic programming, Pontryagin's maximum principle, gradient methods in function space.

## 17 Numerical Methods III

Solution of partial differential equations.
Harmonic analysis.
Orthogonal polynomials.
18 Further Statistics
Similar to Course 9, Statistics II.

## Appendix 4

## Civil Engineering Examination Paper in Mathematics

 1971MATEEMATICS
Friday, 11 June, 1971.

$$
2.00 \text { p.m. to } 5.00 \mathrm{z} .=
$$

Attempt ONE question from SECTION A; and FIVE questions from SECTIONS B ani : including at least ONE from SECTION C.
(Note: SECTION A questions have twice the weight of the others).

## SECTION A

1. We seek the solution of the differential equation

$$
\frac{d y}{d x}=1+2 x y \text { for which } y(0)=1
$$

(a) Find the integrating factor for the equation and use it to obtain an expression for $y(1)$. Evaluate the resulting integral using Simpson's rule with 4 strips. What steps could you take to improve the accuracy and what are the drawbacks ?
(b) Now obtain an estimate of $y(1)$ using Euler's method with a step size of 0.25 . Comment on the result.
(c) Use the differential equation and the initial condition to obtain the Maclaurin expansion for $y$ as far as the term in $x^{4}$ and recelculate $y(1)$. Is the series useful ?
(d) Finally, use the standard series for $e^{x}$ to expand $\int e^{-x^{2}} d x$ in ascending powers of $x$ as far as the term in $x^{7}$. Use this expension to evaluate $y(\lambda)$ from (a). Comment.
2. (i) Show that $f(x)=2 x^{3}+4 x^{2}-2 x-5$ has a local maximum at $x \simeq-1.5$ and a local mininum at $x \simeq 0.2$.
By determining the sign of $f(x)$ at these points show that the equation $f(x)=0$ has 3 real roots.

What is meant by the statement that the sequence $\left\{u_{n}\right\}$ converges to a limit u ?

State the sufficient condition for the iterative formula $x_{n+1}=F\left(x_{n}\right)$ to converge to a root of the equation $x=F(x)$. Since the root is no: known precisely in advance, can you state an alternative definition of convergence which will be more useful in this context?

The equation $f(x)=O$ (above) can be rearranged to give

> (a)

$$
x=\sqrt{\frac{2 x+5}{2 x+4}}
$$

(b)

$$
x=\frac{1}{2} \sqrt{5+2 x-2 x^{3}}
$$

(c)

$$
x=\left(\frac{5+2 x-2 x^{2}}{2}\right)
$$

Show that each of the above will yield an iterative formula convergi=s to the root near $x=1$. Which formula do you think will converge ecs: rapidiy and which most slowly ? Test jour theory by applying these two formulae three times each to an initial approximation $x_{0}=1$.
(ii) The secant method for iterative solution of the eauation $f(x)=0$ does not use derivatives and hence is suitable for digital computers:

the formula is $x_{2}=\frac{x_{0} f\left(x_{1}\right)-x_{1} f\left(x_{0}\right)}{f\left(x_{1}\right)-f\left(x_{0}\right)}$ and, in general,

$$
x_{n+1}=\frac{x_{0} f\left(x_{n}\right)-x_{n} f\left(x_{0}\right)}{f\left(x_{n}\right)-f\left(x_{0}\right)}
$$

How is the formula affected if $f\left(x_{0}\right), f\left(x_{1}\right)$ are of the same sign ? Write a flow chart for the method applisd to a general function $f(x)$ to obtain any given accuracy. Incorpcrete a check for non-convergenes

## SECTION B

3. Find an expression for the total squared error in $y, S$, in fitting the curve $y=a e^{b x}$ to $n$ pairs of points $\left(x_{i}, y_{i}\right)$. Using partial differentiation find $:=s$ equations to determine $a, b$ for least $S$. Comrent.

Start again by taking logarithms of $y$ and use standard equations to determine $a$ and $b$. What is minimised here ?

Use the following data and compare your results with the given $y$ values.

| $x$ | 0 | 2 | 3 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 2 | 4 | 5 | 8 |

-. A uniform flexible cable is suspended from two pyions at the same neight as z=ow


The equation for the profile of the cable (referred to axes shown) is

$$
y=c\left(\cosh \frac{x}{c}-1\right)
$$

where $c$ is a constant.
Verify that this equation gives a minimum value of $y$ at the origin.
Find $s$, the half-length, in terms of the half-span $\ell$, and show that $s^{2}=h^{2}+2 c$ : where $h$ is the sag.
The radius of curvature, $R$, of a curve $y=f(x)$ is defined to be $\frac{d s}{d \psi}$ where $\frac{d y}{d x}=$ ter and $\frac{d x}{d s}=\cos \psi$.

Show that $\frac{d^{2} y}{d x^{2}}=\sec ^{3} \psi \frac{d \psi}{d s}$ and hence obtain

$$
R=\frac{\left[1+\left(\frac{d y}{d x}\right)^{2}\right]^{3 / 2}}{\frac{a^{2} y}{d x^{2}}}
$$

Find the minimum value of $R$ on the cable.
5. A sphere of density $\rho$ and radius $r$ weighs $\frac{4}{3} \pi r^{3} \rho$. It floats on water of density 1 submerged to a depth $h$. Show that the volume of water displaced by the sphere is $\frac{\pi}{3}\left(3 \mathrm{rh}^{2}-\mathrm{h}^{3}\right)$. If the density of the sphere is 0.4 find $a=\frac{h}{r}$ to 2 decimal places using the Newton-Raphson method with an initial approximatio $\alpha_{0}=1$. Why is $\alpha_{0}=1$ suggested ?
6. (i) Write a Fortran program to read in 2 complex numbers, celculate and punch out their moduli, arguments, sum and product. Your output showid contain suitable explanatory text.
(Hint: treat each complex number as a pair of real numbers). Simplify the expression $\frac{(1-i)^{2}}{-\sqrt{3}+i}$ end find its square roots, expressing them in the cartesian form.
 $(1,3,-1),(3,-2,1)$ respectively. ヨy scrmine a s:ivable vector groduct, Eine $\varepsilon$ unit vector $\hat{n}$ normal to the piane ABC. Yerae, by sonsidering ine component in direction $\hat{n}$ of a suitable vercor, detemene $\overrightarrow{\text { Win }}$ winere $N$ is the foot of the perven dicular from $D$ onto the plane $A B C$, and thus ojtain the coordinates of $N$.
8. (i) Solve, by any suitable method, tine differential equation

$$
\frac{d^{2} y}{d x^{2}}+4 \frac{d y}{d x}+3 y=e^{-3 x}
$$

subject to the conditions $y=1, \frac{d y}{d x}=0$ when $x=0$.
(ii) The plane motion of an electron normal to a uniform magnetic field is given by

$$
\frac{d^{2} x}{d t^{2}}=\omega \frac{d y}{d t} \text { and } \frac{d^{2} y}{d t^{2}}=-w \frac{d x}{d t}
$$

(where $\omega$ is a constant).
If $x=0, \frac{d x}{d t}=u, y=0$ and $\frac{d y}{d t}=0$ at time $t=0$, find, by the method of Laplace transforms or otherwise, the coordinates $x$ and $y$ at general time :

SECTION $C$
9. (i) A sampling inspection plan operetes as follows. Take a random sample of size 10 from a large batch. If none of the sample is defective, then accept the batch. If more than one are defective, then reject the batch. If exactly one is defective, take another sample of size 10 , and accept the batch only if this second sample contains no defectives. If a batch which is $5 \%$ defective is tested by this plan, what is the probability that (a) it is accepted after the first sample, (b) it is accepted ?
(ii) A machine-shop storekeeper finds that, over a long period, the average demand per week for a certain machine tool is 3. His stocking policy is to make up stock to 4 at the beginning of each week. Estimate the probability that he will fail to satisfy demand in a given week, and determine his stocking policy if the chance of running out is not to exceed $5 \%$.
10. (i) The average life of a 250 watt electria movor is 8 years, with a standara deviation of 2 years. The manufacturer replaces, free of cinarge, all motors that fail whilst under guarantee. Assuming that the motor lives are normally distributed, how long a gaarantee should he provide if he is willing to replace no more than $2 \%$ of all the motors he sells ? What proportion of motors will still be serviceable after 11 years ?

20/Continued ......
(ii) An automaric machine fills bags with cement; the machine has been se: to produce bag weights normally distributed about a mean of $60 \mathrm{lb}:=:=$ standard deviation of 2 lb. After a period, 4 bags are selected $\because=\Omega=$ the output and their weignts are found to be $55.7,55.4,57.4,56.7 \mathrm{i}$. Do you feel any action to be necessary ?

## L.R. Mustoe.

Appendix 5
Civil Engineering Computation Coursework Year I

LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY Dept. of Civil Engineering

## COMPUTATION COURSEWORK

## ARTIFICIAL LIGHTING INSTALLATION : POINT BY POINT

## INTRODUCTION

Consider a $4 \times 4$ array of light fittings arranged on a square grid such that the spacing is $S$ and the mounting height above the working plane is $H$


The direct illuminance ' $R$ ' on the working $p l a n e$ is a function of the distance ' $d$ ' of the receiving station from the light source, the Intensity $I_{\alpha}$ and the angle of incidence $\theta$ :

$$
B .=\frac{I \alpha \operatorname{Cos} \theta}{d^{2}} \quad I u x
$$

When more than one source is involved then the effect of each source must be considered in turn and the total illuminance obtained by a process of summation.

## OBJECTIVE:

Write a program to compute the direct illuminance at representative stations at any point within the central area $A B C D$.

## TECHNICAL INFORMATION

The light fittings may be assumed to have a BZ5 classification which has an intensity distribution of

$$
I \alpha=I_{0} \cos \alpha
$$

Where $I_{0}$ is the intensity vertically downwards, and the angle $a$ is measured from the downward vertical.

A suitable value for $I_{0}$ would be $1,000 / \pi$ candela per fitting.

LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY Dept. of Civil Engineering

## COMPUTATION COURSEWORK

## T1/6

## MAXIMUM SAFE LOADS IN COMPOSITE COLUMNS

## INTRODUCTION:

Short, hollow, metal-alloy pipes are available to be filled with concrete and use as support colums.

## OBJECTIVE:

Given a range of available column sizes you are to write a computer program which will tabulate the maximum load which may be safely applied to each size of colum for both filled and unfilled pipes.

TECHNICAL INFORMATION:


$$
\frac{D_{i}}{D_{0}}=0.8
$$

$E_{\text {Alloy }}=3 \times E_{\text {Concrete }}$
a Alloy $\ngtr 10 \mathrm{MN} / \mathrm{m}^{2}$
Available pipe sizes ( $D_{0}$ m $m$ ):
$100,125,150,175,200,225,250,300,400,500$.

TUTOR: Dr. Robins

## LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY

Dept. of Civil Engineering
COMPUTATION COURSEWORK

PROGRAMME OF WORK

## INTRODUCTION:

The many interdependent activities which occur in any construction process need to be scheduled such that a sensible and organised programme of work can be established. A very useful procedure to the Engineer is critical path analysis.

OBJECTIVE:
Write a computer program which lists the order of activities in a constructioa programme. Produce a table which states both the earliest and latest starttimes and the earliest and latest finish-times. Clearly identify the critical path and the amount of slack times (floats) for each activity.

TECHNICAL INFORMATION:

You will be given an example by your tutor.

In order to convert working days into calender dates and vice-versa you will need to account for:
a) The number of working days in a typical week and the resultant week-ends. (You may assume a 6 day working-week).
b) The actual day of the week on which work starts.
c) The date on which the work starts.
d) Statutory Holidays. (You may ignore these for the purpose of this exercise).
e) Year Ends and Leap Years. (This exercise will run from March-September 1986).

TUTOR:
Mr A Thorpe

# LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY Dept. of Civil Engineering 

## COMPUTATION COURSEWORK

FLOW BALANCE AT A PIPE JUNCTION

INTRODUCTION:
Accurate methods of predicting flow in pipe networks are of vital importance in the provision of an adequate water supply to all consumers in the area served.

## OBJECTIVE:

Write a computer program to determine the pressure head at any one junction in a pipe network and the resulting flows in each of the connecting pipes, in order to maintain a specified draw-off. The program should allow the specified draw off to be changed by the user after any calculation.

## TECHNICAL INFORMATION

Consider a number of pipes serving one draw-off point ' $X$ ' within a network syster; thus:-


If the pressure head ' H ' at the end of each pipe is known we may calculate the heȧ at the junction in order to maintain the specified draw-off flow ' $X$ ' and the resul= flows in each of the pipes using the following procedure.
i) Start with a reasonable assumption for the pressure ' $H$ ' at the node.
ii) Compute the head loss for each pipe, $H_{f i}=H_{i}-H$. (Note that the sign of $\mathrm{H}_{\mathrm{f}}$ defines the direction of flow).
iii) Compute the corresponding discharges $Q_{i}$ for each pipe from the general formula.
$H_{f}=R Q^{m}$
where $m$ is a constant
$R$ is a constant dependant upon the pipe length and diameter. $Q$ is the flow rate.
iv) Unless the head estimate is correct the algebraic sum of the flows at the nodes will not be zero and the excess or deficiency of inflow $L Q$ may be determined.
v) Calculate $\Sigma\left(Q_{i} / H_{f i}\right)$ for all pipes without regard to sign.
$v i)$ Determine the correction $\Delta H=m \sum\left(Q_{i}\right) / \Sigma\left(Q_{i} / H_{f i}\right)$.
vii) The corrected head at the node is $H^{*}=H+\Delta H$.
viii) Repeat steps i) - vii) until $\Delta H$ is within an acceptable tolerance.

## Appendix 6

## PROGRAMMING PROBLEMS

## 5 Specific Permeability of a Porous Medium

The specific permeability k of a porous medium consisting of particles of various sizes and shapes is an important parameter in the study of the movement of groundwater and in the design of wells. The following formula is used to calculate specific permeability:

$$
\begin{equation*}
k=\frac{1}{m\left[\left[(1-p)^{2} / p^{3}\right]\left[\sum\left(\theta_{j} / 100\right)\left(P_{j} / d_{j}\right)\right]^{2}\right.} \tag{1}
\end{equation*}
$$

where $\mathrm{k}=$ specific permeability of mixture, $\mathrm{mm}^{2}$
$\mathrm{p}=$ porosity (fraction of the mixture occupied by interstices)
$\mathrm{m}=$ packing factor
$\theta_{j}=$ particle shape factor
$P_{j}=$ percent by weight of material $j$ in the medium
$\mathrm{d}_{\mathrm{j}}=$ diameter of the particles of material j in the medium, mm
If all the particles of the mixture have the same shape factor, then equation (1) simplifies to:

$$
\begin{equation*}
k=\frac{1}{\left.m\left\{\left[(1-p)^{2} / p^{3}\right](\theta / 100)^{2}\left(\sum P_{j} / d_{j}\right)\right]\right)^{2}} \tag{2}
\end{equation*}
$$

Consider the following problem. We have a mixture of two types of sand, type A and type B. How does the specific permeability of the mixture vary as the proportions of $A$ and $B$ in the mixture are varied?
Because the particles of each type of sand are assumed to have the same shape factor, equation (2) is used. Write a program for carrying out the necessary calculations.

Take as sample data:
$\mathrm{d}_{\mathrm{a}}=0.08 \mathrm{~mm}$
$q=6.5$
$m=5$
$\mathrm{d}_{\mathrm{b}}=0.40 \mathrm{~mm}$
$\mathrm{p}=0.34$

## 6 Oxygen Deficit in a Polluted Stream

The calculation of the variation with time of the dissolved oxygen in a polluted stream is important in water-resources engineering. In this section, we will briefly discuss this topic and summarize a method for calculating $D_{t}$, the oxygen deficit in a polluted stream.

Organic matter in sewage decomposes through chemical and bacterial action. In this process, free oxygen is consumed, and the sewage is deoxygenated. A standard procedure for determining the rate of deoxygenation of sewage involves diluting a sewage sample with water containing a known amount of dissolved oxygen and determining the loss in oxygen after the mixture has been maintained at a temperature of a $20^{\circ} \mathrm{C}$ for a period of five days. The BOD for a period of twenty days at a temperature of $20^{\circ} \mathrm{C}$ is called the first-stage demand and denoted $\lambda_{20}$ (the subscripts indicate the temperature: $20^{\circ} \mathrm{C}$ ). For any temperature, T , the first stage demand $\lambda$ can be calculated by the formula

$$
\begin{equation*}
\lambda=\lambda_{20}(0.02 \mathrm{~T}+0.6) \tag{1}
\end{equation*}
$$

As previously mentioned, when sewage is discharged into a stream, oxygen is consumed in the decompositon of organic matter. At the same time, oxygen is absorbed from the air. However, deoxygenation and reoxygenation take place, in general, at different rates. Usually, reoxygenation lags behind deoxygenation and the dissolved oxygen decreases with time, reaches a minimum, and then increases. As the dissolved oxygen decreases, an oxygen deficit is said to occur. When the dissolved oxygen is at a minimum, the oxygen deficit is at a maximum.

The oxygen deficit of the polluted stream can be calculated from

$$
\begin{equation*}
D_{t}=\frac{K_{d} \lambda_{m}}{K_{r}-K_{d}}\left(10^{K_{d} t}-10^{-K_{t}}\right)+D_{0} \times 10^{-K_{t} t} \tag{2}
\end{equation*}
$$

where $D_{t}=$ oxygen deficit of the stream at time $t, \mathrm{mg} / \mathrm{l}$
$K_{d}=$ coefficient of deoxygenation
$\mathrm{K}_{\mathrm{r}}=$ coefficient of reoxygenation
$\mathrm{D}_{\mathrm{o}}=$ initial oxygen deficit, $\mathrm{mg} / \mathrm{l}$
$\lambda_{m}=$ first stage $B O D$ of polluted stream, $\mathrm{mg} / \mathrm{l}$
$t=$ elapsed time (when $t=0, D_{t}=D_{0}$ ), days

Note: Before we can calculate $D_{t}$ we have to perform the following preliminary calculations:

1 Calculate $K_{d}$ at temperature $T_{m}$ by use of the following equation:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{d}}=\mathrm{K}_{\mathrm{d}_{20}}(1.047)^{\left(\mathrm{T}_{\mathrm{m}}-20\right)} \tag{3}
\end{equation*}
$$

2 Calculate the BOD of the mixture of sewage and stream, (BOD) $)_{m}$ by use of following equation:

$$
\begin{equation*}
(\mathrm{BOD})_{m}=\frac{(\mathrm{BOD})_{S} Q_{S}+(\mathrm{BOD})_{R} Q_{R}}{Q_{S}+Q_{R}} \tag{4}
\end{equation*}
$$

3 Calculate $\lambda$, the first-stage BOD of the mixture at $20^{\circ} \mathrm{C}$ :

$$
\begin{equation*}
\lambda_{20}=\frac{(\mathrm{BOD})_{\mathrm{m}}}{0.68} \tag{5}
\end{equation*}
$$

if $\lambda_{20}$, as given by equation (5), is substituted into equation (1), we obtain the firststage BOD at temperature $T_{m}$

$$
\begin{equation*}
\lambda_{m}=\lambda_{20}\left(0.02 T_{m}+0.6\right)=\frac{(B O D)_{m}}{0.68}\left(0.02 T_{m}+0.6\right) \tag{6}
\end{equation*}
$$

With the data assumed above, and the use of equations (2) and (6), we can predict the effect of the sewage on the oxygen content of the stream by evaluating $D_{t}$ for several increments of time after the addition of the sewage.

Write a program for carrying out these calculations above.
Take as sample data
1 The temperature of the mixture of sewage and stream $=17.6^{\circ} \mathrm{C}$. This temperature will be denoted by $T_{m}$,
$2 \mathrm{~K}_{\mathrm{r}}=0.2$ (at temperature $\mathrm{T}_{\mathrm{m}}$ )
$3 \mathrm{~K}_{\mathrm{d}_{20}}=0.1\left(\mathrm{~K}_{\mathrm{d}}\right.$ at $\left.20^{\circ} \mathrm{C}\right)$
4 The biochemical oxygen demand of the stream ( BOD$)_{R}$, above the point at which the sewage discharges into it , is zero

5 The biochemical oxygen demand of the sewage, (BOD) $)_{S}$ is $145 \mathrm{mg} / 1$
$6 \mathrm{D}_{\mathrm{o}}=1.3 \mathrm{mg} / \mathrm{l}$
7 The rate of flow of the stream, $\mathrm{Q}_{\mathrm{R}}=23.9$ million ga:/day
8 The rate of flow of the sewage, $\mathrm{Q}_{\mathrm{S}}=3.5$ million gal'day

## 19 Deflection of a plate

Given a rectangular plate that is simply supported at all of its edges and loaded with a single concentrated force $P$ at the specified locatio: $(\xi, \eta)$


The deflection $\omega$ at any point on the plate is given by:

$$
\begin{equation*}
\omega=\frac{4 P}{\pi^{4} a b D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\left(m^{2} / a^{2}+n^{2} / b^{2}\right)} \sin \frac{m \pi \xi}{2} \sin \frac{n \pi \eta}{a} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \tag{1}
\end{equation*}
$$

where

$$
D=\frac{\mathrm{Eh}^{3}}{12\left(1-v^{2}\right)}
$$

and
$\mathrm{D}=$ flexural rigidity of the plate, kVimm
$\mathrm{E}=$ modulus of elasticity, $\mathrm{kN} / \mathrm{mm}^{2}$
$v=$ Poisson's ratio
$\mathrm{h}=$ plate thickness, mm
$\omega \quad=$ deflection, mm
$\mathrm{P} \quad=$ load, kN
$\mathrm{a}, \mathrm{b}=$ dimensions of the plate, mm
$\xi, \eta=$ location of the applied force, rm
$\mathbf{x , y}=$ location of the deflected point, nm
Divide the plate into a rectangular grid having 25 interior nodal points and write a FORTRAN program that will calculate the location of the interior nodal points and also calculate and print the value of the deflection at ez:h of these points.

The input to the program should be the dimensic-s of the plate $(a, b, h)$, the magnitude and location of the applied load ( $\mathrm{P}, \xi, \eta$ ), and the physical properties of the plate material ( $\mathrm{E}, \mathrm{v}$ ).

Take as sample data:
$a=30$,
65. $\mathrm{E}=$
$=210$,
$b=60, \quad \xi=10, \quad v=0.3$
$h=0.5, \quad \eta=20$.

## Appendix 7

Pre-University Knowledge
Questions and Test Paper

## RESEARCH PROJECT ON ENGINEERING MATHEMATICS/SCHOOL MATHEMATICS

This project investigates the strength of the links between mathematics at secondary level and the mathematics required by degree courses in engineering. Your answers will be helpful also to your own progress in the mathematics course here at LUT but will be treated in strict confidence.

Please enter your answers in the spaces provided, unless altemative answers are provided when you should ring the correct one.

## NAME:

## COURSE:

TYPE OF SCHOOL last attended:

| 1 | Private | 2 | Comprehensive | 3 | Sixth Form College |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | Technical College | 5 | Other (please specify) |  |  |

## PRE 'A' LEVEL QUALIFICATIONS

Please give details of the mathematics syllabus (eg JMB Syllabus A, SMP, MEI, etc)

|  | Grade | Year Taken | Board and Syllabus |
| :--- | :--- | :--- | :--- |
| CSE <br> Mathematics |  |  |  |
| GCE <br> 'O' Level <br> Mathematics |  |  |  |
| 'O' Level <br> Additional <br> Mathematics |  |  |  |

If none of the above please specify what course you followed.

## POST 'O' LEVEL QUALIFICATIONS

'A' LEVEL (If you repeated a mathematics 'A' Level, include both attempts. Show a Nuffield syllabus by a prefix $N$ )


OTHER If you do not have ' $A$ ' levels, please indicate the other post ' $O$ ' Level qualifications

| 1 | TEC | 2 ONC/OND | 3 | HNC/HND 4 Scotish Higher |
| :--- | :--- | :--- | :--- | :--- |
| 5 | Cambridge overseas HSC | 6 | Other (please specify) |  |

Please give details of mathematics marks at level 3 TEC or above and final specialism (eg mechanical, electrical)

## PRE-UNIVERSITY MATHEMATICS KNOWLEDGE

Attached are three lists of school-level mathematics. The first is a list for a proposed Common Core syllabus for all ' $A$ ' level boards; the second is a list of topics that the Physics panel expected to be covered in a mathematics syllabus; the third is a list prepared for a BP research project. Please indicate for each topic one of the following for the teaching:
A Covered thoroughly
B Covered fairly well
C Briefly mentioned
D Not covered
and one of the following for your understanding of the topic:
W Understand thoroughly
X Understand reasonably
Y Don't really understand
Z No idea at all
(Hence typical responses might be AW; AX; BY; DZ)
There may be some over lap between lists.

## LIST I

1 ..... 10 ..... 19
2 ..... 11 ..... 20
3 ..... 12 ..... 21
4 13 ..... 22
5 ..... 14 ..... 23
6 ..... 15 ..... 24
7 16 ..... 25
8 ..... 17 ..... 26
9 ..... 18 ..... 27

## LIST II

| 1 | 10 | 19 |
| :--- | :--- | :--- |
| 2 | 11 | 20 |
| 3 | 12 | 21 |
| 4 | 13 | 22 |
| 5 | 14 | 23 |
| 6 | 15 | 24 |
| 7 | 16 | 25 |
| 8 | 17 | 26 |
| 9 | 18 | 27 |

## LIST III

1427

2

15

28
$316 \quad 29$
4 . 17 30
$518 \quad 31$
$6 \quad 1932$
720 , 33
$8 \quad 21 \quad 34$
$922 \quad 35$
$1023 \quad 36$
1124
$12 \quad 25 \quad 38$
$13 \quad 26 \quad 39$

## LIST I

## TOPICS

1 Algebraic operations on polynomials and rational functions.

Factors of polynomials. The factor theorem.

2 Partial fractions

3 Positive and negative rational indices.

4 The general quadratic function in one variable, including solution of quadratic equations, completing the square, sketching graphs and finding maxima and minima.

5 Arithmetic and geometric progressions and their sums to $n$ terms. Sum to infinity of geometric series.

6 The use of the binomial expansion of $(1+x)^{n}$ when
(a) $\quad n$ is a positive integer, and
(b) $\quad n$ is rational and $|x|<1$.

7 The manipulation of simple algebraic inequalities. The function $|x|$.

## NOTES

Addition, subtraction, multiplication and division, and the confident use of brackets and surds.

> To include denominators such as
> $\quad(a x+b)(c x+d)(e x+f)$
> and $(a x+b)(c x+d)^{2}$
> and $(a x+b)\left(x^{2}+c^{2}\right)$.

8 Plane cartesian coordinates

9 Curves and equations in cartesian form.

10 Expression of the coordinates (or position vector) of a point on a curve in terms of a parameter.

11 Definition of the six trigonometric functions for any angle, knowledge of their periodic properties and symmetries.

12 Use of the sine and cosine formulae.
13 The angle between a line and a plane, between two planes, and between two skew lines in simple cases.
14 Circular measure, $s=r \theta, A=\frac{1}{2} r^{2} \theta$.
15 Knowledge and use of the fomulae for $\sin (A \pm B), \cos (A \pm B) \tan (A \pm B)$, $\sin A \pm \sin B$ etc. Knowledge of identities such as
$\sin ^{2} \mathrm{~A}+\cos ^{2} \mathrm{~A}=1$
$1+\tan ^{2} \mathrm{~A}=\sec ^{2} \mathrm{~A}$.
Expression of $a \cos \theta+b \sin \theta$ in the form $r \cos (\theta \pm \alpha)$.

Understanding of the relationship between a graph and the associated algebraic relation. In particular, ability to sketch curves such as $\mathbf{y}=\mathbf{k x}^{\mathrm{n}}$ for integral and simple rational n , $a x+b y=c$,
$\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$.
Geometrical properties of the parabola, ellipse and hyperbola are not included in the common core.

Knowledge of the effect of simple transformations as represented by $y=a f(x)$. $y=f(x)+a, y=f(x-a), y=f(a x)$. The relation of the equation of a graph to its symmetries.

The graphs of sine, cosine and tangent.

Confidence in the application of these formulae in simple cases is expected. In particular the confident use of double angle formulae is expected.

16 General solution of simple trigonometric equations, including graphical interpretation.

17 The approximations $\sin x \approx x, \tan x \approx x$, $\cos \mathrm{x}=1-\frac{1}{2} \mathrm{x}^{2}$.

18 Vectors in two and three dimensions; algebraic operations of addition and multiplication by scalars, and their geometrical significance; the scalar product and its use for calculating the angle between two lines; position vectors; vector equation of a line in the form $r=a+t b$.

19 Functions. The inverse of a one-one function. composition of functions. Graphical illustration of the relationship between a function and its inverse.

20 The exponential and logarithmic functions and their simple properties.

21 The idea of a limit and the derivative defined as a limit. The gradient of a tangent as the limit of the gradient of a chord.
Differentiation of standard functions.

22 Differentiation of sum, product and quotient of functions, and of composite functions. Differentiation of simple functions defined implicity or parametrically.

23 Applications of differentiation to gradients, tangents and normals, maxima and minima, curve sketching, connected rates of change, small increments and approximations.

The definition $a^{x}=e^{x \ln } a^{2}$

The derivatives of $x^{n}, \sin x, \cos x, \tan x$, $\sin ^{-1} x, \cos ^{-1} x, \tan ^{-1} x, e^{x}, a^{x}, \ln x$.

Skill should be expected in the differentiation of functions generated from standard functions by these operations.

24 Integration as the inverse of differentiation. Integration of standard functions.

25 Simple techniques of integration, including integration by substitution and by parts.

26 The evaluation of definite integrals with fixed limits.

27 The idea of area under a curve as the limit of a sum of area of rectangles. Simple applications of integration to plane areas and volumes of revolution.

The integrals of $x^{n}, 1 / x, e^{x}, \sin x, \cos x$, $1 /\left(1+x^{2}\right), 1 / N\left(1-x^{2}\right)$

The relationship with correspondin techniques of differentiation should te understood.

## LIST II

Competence in mathematics is as important for the correct handling of physical concepts and models as the physics itself. A core of mathematical ability is therefore an essential part of A-level physics. It is not intended that any questions should be set in which the main interest is the mathematics.

Students need to be able to do the following:

## Arithmetic

1 Recognise and use expressions in decimal and standard form (scientific) notation.
2 Use appropriate calculating aids (electronic calculator, tables or slide-rule) for addition, subtraction, multiplication, and division. Find arithmetic means, reciprocals, square roots, sines, cosines, tangents, exponentials and logarithms.

3 Take account of accuracy in numerical work and handle calculations so that significant figures are neither lost unnecessarily nor carried beyond what is justified.

4 Make approximate evaluations of numerical expressions (eg $\pi^{2} \approx 10$ ) and use such approximations to check the magnitude of machine calculations.

## Algebra

5 Change the subject of an equation. Most relevant equations involve only the simpler operations but may include positive and negative indices and square roots.

6 Solve simple algebraic equations. Most relevant equations are linear but some may involve inverse and inverse square relationships. Linear simultaneous equations and the use of the formula to obtain the solutions of quadratic equations are included.

7 Substitute physical quantities into physical equations using consistent units and check the dimensional consistency of such equations.

8 Formulate simple algebraic equations as mathematical models of physical situations.
9 Recognise and use the logarithmic forms of expressions like $a b, a / b, x^{n}, e^{k x}$.
10 Express small changes or errors as percentages and vice versa.

11 Comprehend and use the symbols, $<,>, \ll, \gg, \approx, /, \alpha,\langle x\rangle=\bar{x}, \Sigma, \Delta x, \delta x$.

12 Calculate areas of right-angled and isosceles triangles, circumferences and areas of circles, areas and volumes of rectangular blocks, cylinders and spheres.

13 Use Pythagoras' theorem, similarity of triangles, the angle sum of a triangle.
14 Use sines, cosines and tangents in physical problems; recall or calculate quickly values at $0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}$.

15 Recall $\sin \theta \approx \tan \theta \approx \theta$ and $\cos \theta \approx 1$ for small $\theta$, and recall $\sin ^{2} \theta+\cos ^{2} \theta=1$.
16 Understand the relationship between degrees and radians (defined as arc/radius), translate from one to the other and ensure that the appropriate system is used.

## Vectors

17 Find the resultant of two coplanar vectors, recognising situations where vector addition is appropriate.

18 Obtain expressions for components of a vector in perpendicular directions, recognising situations where vector resolution is appropriate.

## Graphs

19 Translate information between graphical, numerical, algebraic and verbal forms.
20 Select appropriate variables and scales for graph plotting.
21 Determine the slope and intercept of a linear graph in appropriate physical units.
22 Choose by inspection a straight line which will serve as the best straight line through a set of data points presented graphically.

23 Recall standard linear form $y=m x+c$ and rearrange relationships into linear form where appropriate.

24 Sketch and recognise the forms of plots of common simple expressions like $1 / x, x^{2}$, $1 / x^{2}, \sin x, \cos x, e^{-x}$.

25 Use logarithmic plots to test exponential and power law variations.
26 Understand and use the slope of a tangent to a curve as a means to obtain the gradient. Understand and use the notation $\mathrm{d} / \mathrm{dt}$ for a rate of change.

27 Understand and use the area below a curve where the area has physical significance.

## LIST III

| 1 | Coordinate Geometry of Circle | 21 Partial differentiation |
| :---: | :---: | :---: |
| 2 | Coordinate Geometry of Ellipse | 22 Integration as the limit of a sum |
| 3 | Coordinate Geomerry of Hyperbola | 23 Integration via substitution |
| 4 | Sketching Curves in polar coordinates | 24 Integration by parts |
| 5 | Adding/Subtracting Vectors | 25 Simpson's rule |
| 6 | Scalar Product of Vectors | 26 Equilibrium of coplanar forces |
| 7 | Equation of line in 3-D | 27 Couples |
| 8 | Equation of plane in 3-D | 28 Relative motion |
| 9 | Solving 3 simultaneous linear equations | 29 Moment of inertia |
| 10 | Inverse of $3 \times 3$ matrix | 30 Motion of a rigid body |
| 11 | Evaluating $3 \times 3$ determinants | 31 Bending moments |
| 12 | Argand diagram | 32 Mathematical Induction |
| 13 | $r \mathrm{e}^{\mathrm{i} \theta}$ form of complex numbers | 33 Calculation of standard deviation |
| 14 | sinh and cosh functions | 34 Simple probability |
| 15 | Maclaurin's expansion | 35 Binomial model |
| 16 | Newton-Raphson formula | 36 Poisson model |
| 17 | Formulation of differential equations | 37 Normal model |
| 18 | Solution of 1st order o.d.e. | 38 Correlation |
| 19 | Solution of 2nd order o.d.e. | 39 Sample means and confidence hypothesis |
| 20 | Simple harmonic motion | 40 Tests of statistical hypothesis |

## Comments on responses

The topics most conspicuously absent from the students' repertoires were

Lines \& Planes<br>Conics<br>The approximation $\sin x \simeq x$<br>Small change as \%<br>Vectors<br>Log plots of $x^{n}$<br>Inverse of a matrix.

In addition, many respondents reported uncertainty on the following
Polar coordinates
Matrices
Hyperbolic functions
Parameters
Maclaurin Series
Partial Differentiation

Most notably, many candidates who showed themselves unable to cope with the notation $f(x)$ had declared themselves competent in that area.

TEST 1: PRE-UNIVERSITY KNOWLEDGE

1 Simplify the expression $\frac{\left(x^{3}+a^{3}\right)}{\sqrt{x^{2}-a^{2}}} \frac{(x-a)^{5 / 2}}{\left(x^{2}+a^{2}\right)}$

2 Decompose the expression $\frac{8 x^{4}-2 x^{2}+7 x+6}{(3 x+2)\left(x^{2}+4\right)(x-5)^{2}}$ into partial functions.

Do not evaluate the numerators of these partial function; ie you may leave a fraction as
$\frac{A}{3 x+2}$ without evaluating $A$.
3 What is the value of $(64)^{1 / 3} .(25)^{-1 / 4}$ ?
4 Sketch the graph of $y=4+4 x-x^{2}$. Where does $y$ have a maximum or minimum value and what is this value?

5 Write out the first four terms and the nth term of $S_{n}=\sum_{i=1}^{n} a(1+r)^{i}$.
If $\mathrm{r}=\frac{1}{2}$, find the value of $\mathrm{S}_{\mathrm{n}}$ as $\mathrm{n} \rightarrow \infty$.
6 Write down the first four terms of $(1-2 x)^{-1 / 3}$. You need not simplify the coefficients for each term. Is the expansion always valid?
7 Sketch the curves $y=x^{2}, x^{4}, x^{\frac{1}{2}}$ for $x \geq 0$.
8 The graph of $y=f(x)$ is shown Sketch graphs of $y=2 f(x)$, $y=f(x)+4, y=f(x-4)$, $y=f(2 x)$.


9 Simplify the inequality $(y+7)(y-2)>0$ and sketch the graph of $z=(y+7)(y-2)$.
10 If $x=3 t+1$ and $y=t^{2}-4$, find the relationship between $y$ and $x$.
11 Sketch a graph of $y=\tan x$, pointing out special features.
12 What is the angle between the line $z=2 x, y=2$ and the plane $x+y+z=4$ ?
13 The minor arc of a circle subtends an angle $30^{\circ}$ at the centre of the circle. Find the length of this arc and the area of the sector so formed, given tht the radius of the circle is 10 cm .

14 What is $\sin (A+2 B)$, given that $\sin A=\frac{1}{12}, \sin B=\frac{1}{2}$ ?
15 Find $x$ in the range $0^{\circ}$ to $180^{\circ}$ for which $4 \sin x+3 \cos x=2.4$.
16 Estimate $\cos 2 x$ when $x=4.5^{\circ}$ without using a calculator (Take $\pi^{2} \cong 10$ ).
17 If $\mathbf{a}=(1,2,1), b=(-2,0,3), \mathbf{c}=(5,-2,4)$ find the angle between $(\mathbf{a}+\mathbf{b})$ and $\mathbf{c}$; leave your answer as an inverse cosine.

18 Sketch graphs of the function $f(x)=2 x^{3}+5$ and the inverse function $f^{-1}(x)$.
19 Simplify the expression $e^{x \ell n 2}$ and sketch its graph.
20 Find the derived functions of $\tan 2 x$ and $\mathrm{e}^{\frac{1}{2} x}$.
21 Find where $f(x)=x \sin x$ has a stationary point.
22 If $x=a t^{2}$ and $y=2 a t$, find $\frac{d y}{d x}$ in terms of $t$.
23 Find an indefinite integral of $\frac{1}{x}+\sin 2 x+\frac{1}{1+x^{2}}$.
24 Find $I=\int_{0}^{\pi / 2} x \cos x d x$.
25 Write down an integral which is the volume obtained by rotating the parabolic arc $y=x^{2}$ from $x=1$ to $x=4$ about the $x$-axis.

## Table A. 1

## Question Number

| 1 | 160 | 53 |
| :--- | ---: | ---: |
| 2 | 241 | 171 |
| 3 | 284 | 48 ( +77 as a decimal) |
| 4 | 229 | 57 ( +39 plots $)$ |
| 5 | 159 | 24 |
| 6 | 248 | 183 |
| 7 | 285 | 24 |
| 8 | 199 | 43 |
| 9 | 228 | 3 |
| 10 | 255 | 196 |
| 11 | 256 | 147 |
| 12 | 101 | 2 |
| 13 | 248 | 195 |
| 14 | 223 | 132 |
| 15 | 188 | 95 |
| 16 | 83 | 47 |
| 17 | 136 | 72 |
| 18 | 192 | 7 |
| 19 | 141 | 49 |
| 20 | 180 | 108 |
| 21 | 212 | 3 |
| 22 | 243 | 175 |
| 23 | 204 | 108 |
| 24 | 196 | 109 |
| 25 | 213 | 61 (+177 evaluations) |

## Appendix 8

## PROJECTILES STORY-LINE (First Draft)

## Sequence

1 Projection from cliff


2 Ground to ground


3 Hitting a fixed target


4 Projection from a cliff


5 Range on inclined plane


6 Impacts I


## Outline of sequence $\boldsymbol{\&}$ objectives

Particle dropped \& 3 particles projected with different horizontal velocities. Compare horizontal ranges and observe that vertical motions are the same.

Idea of parabolic flight path.
Possibility of choosing cliff height, display of velocities \& displacements. Energy conservation.

Application to dropping package from plane.

Demonstration trajectory.
Fixed angle, varying speeds to study range \& time of flight etc, maximum height.

Fixed speed, varying angle to discover maximum range and complementary angles. Possibility of choosing velocities, displacements, angle to horizontal, energy.

Given a velocity, choose the angle of projection to hit a fixed target.

Include a point outside parabola of safety.
Applications Hitting a golf green without and with a tree to limit possible angles. Perhaps include cricket and tennis.

Angle of projection $>0$. To complete the study of sequence 1.

Application Hitting a target area at the level of the base of the cliff.

Demonstration.
Choose up or down a plane. Vary angle of plane.
Find angle for maximum range.
Choice of items to display.

Ground to ground: study various quantities (height, range etc) for 1st three phases of motion.

Sequence
7 Impacts II


8 Impacts III


9 Air resistance


Outline of sequence \& objectives
On vertical wall. Possibility of rebounding to point of impact.

On inclined plane.
UP Will projectile continue up or start back down? DOWN Nature of each phase.
$\mathrm{R}=\mathrm{kv} ; \mathrm{R}=\mathrm{kv}{ }^{2} ; \mathrm{R}=\mathrm{kv}{ }^{\mathrm{n}}$
Study nature of trajectory as $\mathrm{k} \rightarrow \infty$.
Idea of terminal velocity extended from unit on vertical motion.

Also, allow the case of air density varying with height.

## PROJECTILE MOTION SCRIPT (First Draft)

## Section 1.1

## Horizontal Launch from Cliff

\left.| HEADING |  |
| :--- | :--- |
|  | PARAM |
| DIAGRAM |  |
| (OR TEXT) |  |$\right]$

## SCREEN DISPLAY



TEXT
A projectile is fired from the top of a cliff.

Let the height of the cliff be H , the initial velocity of projectile be $u$, the horizontal range be $R$ and the time of flight be $T$.

Input H in Metres
Consider $\mathrm{u}=0$.

| TIME | DISTANCE VERTICAL |  |
| :---: | :---: | :---: |
| ELAPSED | FALLEN | VELOCITY |
| $(\mathrm{s})$ | $(\mathrm{m})$ | $(\mathrm{m} / \mathrm{s})$ |
| - | - | - |
| - | - | - |
| Input u in $\mathrm{m} / \mathrm{s}$ |  |  |

TIME DISTANCE VERTICAL ELAPSED FALLEN VELOCITY
(s)
(m)
( $\mathrm{m} / \mathrm{s}$ )
-
-
-

NOTES
About 12 s in duration. Noise on impact.

Repeat first diagram with $\mathrm{H}, \mathrm{u}$ and R appeariang sequentially.

Noise on impact.

Show flight in "real time" then stop at intermediate stages, holding on last value.

Hold background diagram. Display new trajectory \& then repeat for both simultaneously.

Repeat the sequence for a second choice of $u$.
And for a third choice of $u$.


Replace diagram by the table.

Allow choice of parameters to be displayed (together with time elapsed) from the following:
Horizontal distance travelled, distance fallen, horizontal velocity, vertical velocity, angle of trojectory to horizontal.

Display to be halted at several stages of the motion. Table displayed for current instantaneous values. At end of motion replace diagram with full table.

## Micros in Mathematics Education (MIME) <br> MECHANICS

## MIME TEAM

(at Loughborough University of Technology)
Protessor A C Bajpai (Director)

## Authors:

Dr M C Harrison
MrLRMustoe
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Units avamable in the series:
Momentum and lmpacts
Projectile Motion
Linear Motion
Friction
Relative Motion
Angular Motion
Equilibrium
Vectors
Newton's Laws of Motion
Work, Energy. Power
Centus of Grivity
Circular Motion
Simple Harmonic Motion

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## Projectile Motion

| Author: | LR Mustoe |
| :--- | :--- |
| Work Cards: | J A Fairley |
| Software Adviser: | A H Whitfield |
| Senior Programmer: | I A Sutton |

## SUMMARY

This unit is concerned with the motion of objects projected into the air from a stationary starting-point. Horizonlal projection from a chiff top: the motion when projection is from a horizontal plane, including attempting to hit targets both above and at ground level; motion up on down an inclined plane; motion after impact: are all considered in the: absence of air resistance. Finally, some cases of resistance to the motion are examined.

## NOTE

It is assumed that the projectile can lye treated as a point mass, i.e. rotational eflects can be ignored. Further, the acceleration due to gravity is assumed constiant so that the motion takes place close to the earti's suilace.

The mathematical derivations of resulls and relintionshins suggested by the use of programs in this unil cam be fonmo in most standard text-books which include the lopic of projectile motion.

## CUNIENTS

## PART 0 Inexperienced User Guide

PART 1 Horizontal Launch from a Cliff
PART 2 Lanuclifrom Level Ground
2.1 Fixed Initial Angle, Varying Inilial Speed

27 Varving Initial Angle. Fixed Intiall Suened
23 Virying Initial Angle or Varying linitial Speed

## PART 3 Hita Targe

3.1 Elevaled Targe
3.2 Ground Targe
3.3 Ball Ganes

PART 4 Launch from a Cliff
4.1 General Angle of Projection
4.2 Hit a Target betow Cliff

PART 5 Landing on Inclined Planes
5.1 Launch up the Plane
S.2 Lanch down the Plane
5.3 Find Maximum Range

## PART 6 Motion after Impact

G. 1 Impati on Level Ground
6.2 Impact with Vertical Wall
6.3 Impact on linclined Plane

## PART 7 Resisted Motion

71 Examples
7.2 Variable Coefficient and budex
7.3 Variation of Air Density with Height

PART 8 Exil from Unit

## OPERATING INSIRUCTIONS

This unit is contained on iwo program dises: Dise 1 egrtants Parts 0-4 inclusive and Disc 2 contains Parts $5-8$ inelusive. Io use it on a BBC model 8 computer with a double disc dive. load Disc 1 in Drive 0 (the top one) and Disc 2 in Drive: 1; with a single disc drive. Ioad Disc 1 (even if one ol Parls 5.8 is required). Then while holding down the St IIFT key. press thi: BREAK key to start the unit. (Alternalively, iype

```
CHAIN "START"
```

and press the RETURN key.) Further instuactions on usayge ane given in PART 0 but note that:
(i) Pressing the ESCAPE key at any time restat is the current part or section
(ii) Pressing the BREAK key at any time relurns you to lhe contents to select another part or to exil from the unit.

## ACCURACY

1. The accuracy of all numerical calculations with na:al numbers is linited by the number of digits the computer uses to represent them
2. Computed results oulpul on the screen have usilatly tre:t rounded to the number of decimal places shown. Consequently the mumber may not be shown preaisily

## NOTES FOR IHE USER

## PART 0 Inexperienced User Guide

Use of this unit is explained including
current status page
subdivision of the unit
alphabetic input
numerical input
mu:nus
PSB
BREAK
ESCAPE

PART 1 Horizontal Launch off a Clif
The sequence begins by showing a iypica
raicectory of a projectite fired from a gun at a vilucity u horizontally to land on level ground a distance $H$ below the firing point and -thorizontal distance $R$.
You are invited to choose a value for H (252 is stitable). First the projectile is allowed to tall freely from this height; note the final vertical velocity.
Now you choose three horizontal velocities in turn (suitable choices are 26, 52, 78 or 40, 80, 100) Nolice the final vertical velocity in each case. See whelher you can discover a relationship belween vertical velocity and horizontal range. What is the shape of the trajectory?

For the last velocity chosen the potential and kinetic onergies are displayed at various stages in hee motion. What do you notice (casiest if the last velocily was 100 ms I? What does vour result sungerst as a possibte means of solvin! such pollens of projectile motion?

## PART 2 Launch from Level Ground

2.1 Fixed initial angle, varying initial speted The sequence begins by showing a typical frijectory of a projectile fired at speed wand imble $a$ to the horizontal; its horimontal tange is R and the greatest height achieved is H . First you choose values for " and 11 ( 60 and 100 are suitable). Then you choose two other values for $u$ ( 50 and 75 ).
What shape do you think the tajectories are?

What is the relationshp between mithial :p:at and:
(a) greatest height (b) horizontal rimgo
c) time of flight
(d) time to maximum height?
2.2 Varying initial angle, fixed initial speed An initial speed is selected (100) and theee angles: suitable values are $30,40,50$ or 30,60 . 0. Note the range each time. Tiy to fimt the angle which gives the maximum range. What do you notice about the other angles \{lo instance $40^{\circ}, 50^{\prime}$ or $30^{\prime}, 60^{\prime \prime}$ ?
2.3 Varying initial angle or varying initial spe:erl You can find out other features of this motion by keeping the angle of projection lixat and varying the speed or vice-versa
For the last example selerted potemlial, kinetir: and total energies are displayed at sevenal stages of the motion. Whiat do you notice?

## PART 3 Hit a Target

3.1 Elevated target

Four targets are presented and you have (1) choose the angle of projection you think wilt be suitable. If you miss three times in succession you are told the correct angle:(s), if you hit with the first attempt and miss ther: limes more you are told the missing ample. In cases of lailure the correct trajectories are displayed. Note the What and fouth cans:s. After the tourth example, liv to explain thi: third.
A selection of ramdon lang!as ate pre:athe:d for you to obtain further pratice.
3.2 Ground target

Next cnmes the problem of landing a golf taill on a green. You have to choose the anyle of pojection which will allow you to hit the target. Remember Part 2.2 results
3.3 Ball games

Three simple problems are considered; hitling a cricket ball over a lielder and perhaps: clearing the boundary, throwing a bill into the wicket keeper, hitting a lennis ball over a
demnis net and fanding in the count. You call bace the obstacles and choose position of boundary, speed and angle of projection.

## PART 4 Launch from a Cliff

4.1 General angle of projection

The example in Part 1 is extended.
A height for the cliff is chosen, an initial speed
of projection and an initial angle to the
horizontal the angle is positive above the
horizontal and negative belowl. Horizontal
and vertical distances form the point of projection
or horizontal and vertical velocities can be
displayed at half second intervals of projectile
lime.
Yout have three angles to select (70, 20, -40).
What shapes are the trajectories?
4.2 Hit a target

The last sequence gives a target area at the
level of the bottom of the cliff which has to be hit.

## PART 5 Landing on inclined Planes

### 5.1 Launch up the plane

An example is shown of a projectile landing on a plane inclined upwards to the horizonta at angle $\beta$. You are invited to choose a speed of projection, the angle $f$ and the angle of projection with the plane.

See what relationships you can deduce about ange, maximum height achieved and time o light.
5.2 Launch down the plane

Then the sequence is repeated with the plane inclined downwards to the horizontal.
5.3 Find the maximum range

We have the choice of a plane inclined ulwards or inclined downwards
We select a speed of projection and an anyle finclination for the plase. Nole the range along the plane. Select other angles to try to establish the one which gives maximum ange.

## PART 6 Motion aiter Impac

6.1 Impact on level yround

The first sequence shows a projectile: rebounding and follows it until its second impact.
After choosing an initial speed and an initia angle of projection wo select a value for the coefficient of restitution 0.5 is recommenteal) The initial trajectory and two rebounds ate followed and you are invited to decide the effect of rebounds on tinne of flighli, ramge and greatest height reached.
6.2 Impact with vertical wall

The projectile now meets a vertical watl in its trajectory and rebounds. Ayain an mitial speed and angle of projection are selrected Notice the velocities betore impact. Choose a value for the coeflicient of restitution 10.5 is suggested) and note the velocities altet impact.

The sequence is repoated showing distance travelled in the horizontal and vertical directions.
6.3 Impact on inclined plane

Three examples are given of motion up a plane following an impact, showing dillerem possibilities which arise.

Then an example is shown lur motion duwn : plane. Does this represent the only possibility?

## Part 7 Resisted Motion

7.1 Examples

Three cases are cunsidered: resistance proportional to velocity, velocity sefuared atu velocily to the power 1.5 . In each instance, the trajectory wilh no resistance present is displayed tor relerence.
7.2 Variable coellicient and intex

Then, resistance ellects are shown will, the: constant of pronortionality equal to 1 : you can choose a value for the constant as a thied
eximple. What changes do you notice as the constant increases?
What are the effects of increasing the power of the velocity from 1 throught 1.5 to 2 ?
7.3 Variation of air density with height

First the variation of air density with heieght is shown. The effects of this on the trajector are displayed. Then you are invited to find the angle of projection from level ground which gives maximum range in the cases:
(i) air density assumed constant (ii) air density varies.

## PART 8 Exit from Unit

The unit is terminated and the computer is left in its 'normal' state.

## PROJECTILE MOTION - TEACHER'S GUIDE

This unit is intended to enthance the stamlard material on projectile motion found in text tooks. To this end it anplites the principles of mechanics to some simplified but ;)aclicat problems. However, the power of the micio compult!! $i: ;$ used to animate and elucidale the motion for the benetit ol bott teacher and pupil. The wit is memu-driven and suitalla: for use either by the teacher with his class to demonstiale theoretical results obtained by him or by the pupil for spirded discovery - the guidance, of couse, being sumpied by the: teacher. In addition, boll teacher and pupil can use the wit experinentally as they would a piece of taboratory
apparatus. The menu-driven nature of the thit allows th: parts to be used in any order according to need or inulecd ter some parts to be omitted aecording to syllitus requitements. With each patt a possible course of action is
(i) the teacher introduces the concepts involved
(ii) under the guidance of the teacher. the pupils investigiate these concepts using the computer simulations as an atternative to traditional laboratory experiments
(iii) the teacher collects the ideas and suggested funclional relationships and uses mathomatical analysis" to derive results. The programs can be used to check out cerlall results and solve problems posed by the teachet.)

* Some parts may involve results which are beyond the levi: of mathematics apposite to the students. This is dotibe:rale? and if is honed that the opportunity will the taken to intmentas: students to ideas beyond the current syllitus.


## PART 1 Horizontal Launcli from a Clifl

The aims of this part ate to show that:
(i) the motion of a propectide fited horimontilly fom a point abuve level ghomed is the wes:allamt of two separate motions: ventical lall umber gravily and uniform horizontal vetocity.
(ii) hurizontal range is proportional to horizumit velocity,
(iii) the sum of the kimetic and potentiat anergines 1:s constanl during the motion.

## PART 2 Launch from Level Ground

The main ains of this patt ate to demonstiate: the: general properties of the parabolic trajectory and o ohtall the angle of projection which gives maximum horizontal range.

PART 3 Hit a Target
This part aims to introduce the concept of parabola of salety, and to show that a point within the paralsola of safety can be hit with a lixed initia parabola of satety can be hit with a lixed initial
speed using either of two angles of projection. The secund section is concerned with obstacles blocking the path of the projectile on its way to a lixed target.
Finally, some elementary aspects of cricket and tennis are considered.

## PART 4 Launch from a Cliff: General Angle

This part considers the continuation of the parabolic trajectory past the horizontal level of the point of projection.
PART 5 Landing on Inclined Planes
In this part the problem encountered with the projectile landing on a plane inclined upwards or downwards is examined. The question of angle of projection to achieve maximum lange is considered.

## PART 6 Motion after Impact

All previous parts dealt with motion top to the poin
of first impact. Rebounds are considered off level ground, a vertical wall and a plane inclined upwallds or downwards the last example is dealt with qualitatively).

## PART 7 Resisted Motion

This part considers qualitalively the effects of resistance to motion of the form
$k v^{\prime \prime}$ where $n=1,2$ and 1.5
The effect of variation of air density wilh height is also considered. in particular upon the angle of projection to achieve maximum horizontal range.

Appendix 10
Simultaneous Linear Equations
User Guide

## SOLUTION OF LINEAR EQUATIONS

## SUMMARY

This unit is intended for use as an aid in the teazning and learing oi solution meinods ior systems of linear equations. Various siyles of piopram are moluaed, some pernaps more suited to demonstration in lectures, whilst oiner more interactive programs may find better service in the iutorial situation. However the sotiware is used, every efiort has been mage to ensure comorete ease of use. Four meinods of solution are considered; Gauss eliminawon, marix inversion, decomposition and iteration.

## DISC CONTENTS

```
Gauss slimination
    ExEmple without pivoring
    Example with pivoting
    Fixed exampie of Gauss-jorcian
    Variadle Sysiems
Matrix inversion
    Fixed example using elimination
    Variable systems
Decomposition
    Fixed example - Crout LU
    Fixed example - Doolittle LU
Iteration
    Fixed example - Jacobi
    Fixed example - Gauss-Seidel
```


## OPERATING INSTRUCTIONS

The software is written for the BBC model B microcomputer and is contained on one 40 track disc. Place the dise in the drive (drive 0 it using a double dise drive); the sotware is then staned by holding down the SHIFT key and simultaneousty pressing the BREAK KEY. slight adjustment to the monitor display is allowed if reaured.
i) Pressing the EREAK key at any time returns the user to the contents with the section pointer at the first entry reasy for reselection.
ii) Pressing the ESCAPE key at any time returns the user to the contents with the section pointer at the current seation. effectively restanting the current part.

Apart from the above two keys this unit operates with only the four CURSOR keys and the RETURN key.

Note: It is not essential to use a colour monitor with this unit but it available, colour will ennance the presentation.

Gauss Elimination


Figure 1
Three fixed examples of $3 \times 3$ systems are used as demonstrations of the elimination procedure; the first without pivoting, the second using pivoting and the third extending the pivoting example to use the Gauss-Jordan method to further simplity the system. All three examples have exact solutions (no errors from round-off etc) and consequentiy the method here is all-important. The fourth program in this section allows solutions of a four equation system to be developed. This example is very flexibie the user has control of various aspects of the solution. The pivot row can be found for each of the foliowing cases:

1) No pivoting
2) Correct pivoting
3) 'Anti' pivoting

Having reduced the system to tejer riangular iorm. the ostion to 'scroll' the disptay back and review the soiution is avaiiable: at this siage aiso the 'restart' oztion allows the soiution to de cisoiayed apain trom the beginning with a furtior
 assumed or wnetner errors (if any) are to be printed at inose positions where $z$ zros are being generated. The solution may aiso be extenaed via the Gauss-Jorean method to enable the solution to be 'read off directly.

## MATRIX INVERSION



## Figure 2

The first program here solves a 3 equation system by matrix inversion. The inverse is found by elementary row operations. This program is intended as a fixed demonstration illustrating the method of inversion by row operations.

The second program solves a 4 variable system. A record of the determinant of both the coefficient matrix and the identity matrix is kept throughout the row operations.

## DECOMPOSITION



Figure 3
Two fixed examples are used to demonstrate the metiods of Crout and Doolittle. The working required to formulate the two triangular matrices from the original matrix is optional. The third program allows a 4 equation system to be reduced to upper- and lower- triangular form by either Crout or Doolittle's method.

## ITERATION

Two fixed examples are avaiiable, one on Jacobi's method the other Gauss-Seidet. The examples are on two equation systems such that the solutions can be monitored graphically. Two iteration schemes are used for each method, one of which converges whilst the other does not.

## Appendix 11

## NUMERICAL INTEGRATION

## SUPAMARY

This unit is intended for use as an aid in the teacning and learning of numerical integration. Various styles of program are included and every effort has been made to ensure that the sotware can be easily adapted to a wide range of teaching styles. Four methocs of integration may be examined in a higily interactive manner.

## DISC CONTENTS

1 Trapezium Rule
2 Simpson's Rule
3 Gauss-Legendre Quadrature (2-point)
4 Gauss-Legendre Quadrature (n-point)
5 Romberg Integration
6 Comparison of Methods

## COMHON ELEMENTS IN THE UNIT

With the exception of Romoerg integration, every oart in the unit altows a enoice o: whith function is to be studied. There are seven functions avaitade
$1 f(x)=x^{2}$
$2 f(x)=x^{3}-2 x^{2}+x+1$
$3 f(x)=\sin x$
$4 \quad f(x)=e^{x} \sin x$
$5 f(x)=1 /\left(x^{2}+1\right)$
$6 f(x)=\ln x$
$7 f(x)=e^{-\frac{1}{2} x^{2}}$

It is hoped that this selection will provide enough variation to bring out all the relevant features. Function 7 is provided to demonstrate the usefulness of numeri:= integration on functions that cannot be integrated analytically and can be demonstrated with reference to the normal probability integral. Functions 1 to $6 \varepsilon$. be examined in terms of their analytical and numerical solutions, showing the reia:: : accuracy of each method.

The interval $[\mathrm{a}, \mathrm{b}]$ over which integration is carried out can also be varied within centain limits. These timits depend on the function under consideration, but there should be enough flexibility to show how the integral and its accuracy depend on the interval.

In eash program the option to cnange function or interval is given in the main mene. io choose a function:
 with the aesired function.

AEETURNB Take the current function and return to the main menu
To change interval endpoints:

| $\leftarrow \rightarrow$ | Move a left \& right respectively between its lower limit (shown on the $x$-axis) and b |
| :---: | :---: |
| il | Move $b$ left $\&$ right respectively belween $a$ and the upper limit (also shown on the $x$-axis) |

<Relurn> Take the current values of $a$ and $b$ and return to the main menu.

## TRAPEZIUM RULE



When the trapezium rule is chosen from the main contents, the menu page will appear (as above). From this menu six options can be chosen. Throughout the running of the program there is a readout (on the right-hand side of the screen) of the current information. This shows $f(x)$ (the function under consideration), $a$ and $b$ (the limits of the interval under consideration), $n$ (the number of strips being used for the approximation), and the true and estimated values of this integral. Above this appears a graphical representation of the difference between true and estimated values of the integral for values of $n$ from 1 to 10. All of this iniormation refers to the function which is shown above the menu. At the start of the program the true integral is orawn in lignt blue, and the trapezoidal approximation is nighlighted over this in black.

There are six copions io the menu. To choose one of tiose couns move the wine ber
 then press <Return>. Tie options are as foliows;

## 1 Change function

This gives a submenu of seven funstions.
2 Change numiver of surips
This aliows the user to select a value of $n$ veiween 1 and 10 and displays the corresponding picture.

3 Change endpoints
This allows the user to choose the interval over which integration takes place.

4 Toggle colours
This option interchanges the highlighting of the pieture
between the true integral and the trapezoical approximation, so that when this option is selected successive presses of the <Return> key results in the alternate highlighting of the two integrals.

5 Error for each strip
The menu is removed and in its place there appears a graphical representation of how the total error on ( $\mathrm{a}, \mathrm{b}$ ) is divided up between the n strips. It should be noted that the height of the bars are not scaled to the picture above.

6 Quit
Leaves the program and returns to the main contents page.

A visual interpretation of the trapezium rule is generally quite easily grasped. Here it can be made more obvious by 'toggling' between pictures of the integral and it's approximation, on the same axes. This shows where the corners of trapeziums come from, and how near the approximation is to the real thing. By taking estimates for difierent values of $n$ it soon becomes clear that accuracy increases as $n$ increases. This fact is encapsulated in the error graph in the top right-hand corner of the display, and also shows how quickly the error drops. For larger values of $n$ the trapeziums and the curve itself become difficult to distinguish. This is when the 'error for each strip' option becomes more interesting. The relationship between tise error and the second derivative of the function is often stated but rarely demonstrated, leading to a disinterest in error calculation, or in working out how many strips are required to get to within a centain accuracy. When a larger va!ue of $n$ is used, the distribution of error along the interval can be seen to be the same 'shape' as $f^{\prime}(x)$. This is particularly obvious when $f(x)=x^{2}$ and the bars line up io form a constant, or when $f(x)=x^{3}+2 x \cdot 1$ and the bars stant to look like 2 line of constant slope.

These demonstrations can help to give a gese intuitive ijea of what is coing on. בנt iney need to be backed up by more solid facts. When the trapezium appreximat:on cormula is supptied, the student can work on particular examples, and compare his result with the true integral. without having to solve it analytically (particularly io: values of $n$ not catered for by the software $)$. Of course, for $f(x)=\exp \left(-x^{2} / 2\right)$ an analytical result cannot be supplied, and any result obtained can only be compared azainst another numerical result, iaken by a more accurate method. This exampie inereiore demonstrates the impsaance of numerical integration.

## SIMPSON'S RULE

The lavout of the dispiay and the options available from the menu are exactly the same as for the previous program. The number of strips is increased and decreased in steps of two. so that only even values of $n$ are permissible, and only these vaiues are snown in the error graph. As beiore the $\uparrow$ and $\downarrow$ cursor keys move the highlignt bar up and down the menu, and <Return> will seiect the highlighted cption.

A good basic uncersianoing of Simpson's rule can be gained by tospiing detween the picture of an integral and the picture of its Simpson approximation. The increased accurasy of Simpson's rule over the trapezoidal method means tha: the difference between the two integrals is only discernable for very low vatues of $n$, and the reea can be expianed just using $n=2$. The difierence is pantizuiarly ojvious for 'wizely' functions such as $f(x)=x^{3}+2 x-1$ or sin $x$ over a large interval. and the three poinis used tor the curve fitting can easily be shown.

For larger values of $n$, this diagram becomes less useful, but the 'error for each strip' option gets more interesting. In particular it shows that for each two-strip sestion, while there is an error for both, the two errors are of opposite sign and hence go some way towards cancelling each other out. In the case of the cubic function, of course. these two errors cancel each other out completely, and the approximation is exact.

The heights of the bars in the error graph are much lower than those of the previous program, and this comparison can show very quickly how much more accurate Simpson's rule is than the trapezium rule for similar amounts of calculating. Again, this intuitive introduction needs to be backed up by giving the appropriate formulae and working on fixed examples, and perhaps snowing how the formula is derived in $i:=$ first place.

## GAUSS-LEGENDRE QUADRATURE



Two sections on Gauss-Legendre integration are provided in the unit: 2-point and $n$ point. These serve to show two ways in which the sum of (ordinates $x$ weightings) can be represented geometrically; firstly as a tiapezium, and secondly as a series of rectangles. In both cases the function is shown on the lett-hand side of the display, with the scaled function (to the interval $[-1,1]$ on the right-rand side. The GaussLegenare approximation is then superimposed over the scaled function. The number of ordinates can be selected, from the menu, in the rage 2 to 6 . but the cnoice of functions and intervals is the same as before.

Once it has been pointed out why Gaussian iniegration takes diace over [-1, 1] it can sasily be shown that any funcuon over any interval $\{\mathrm{a}, \mathrm{b}]$ can de scaled down to $!-1,1]$ by $g(u)=1 / 2(b-a)\{(1,2[b-a] u+1 / 2[b+a])$ so that the
:niegiais $\int_{a}^{b} f(x) d x$ and $\int_{-1}^{1} g(u)$ cu are ine same. The two iunci:ans, snown sias by s:=e
on the display, are seen to be roughly the same shape, with the scaled version squashed or elongated, and the two areas look the same size. A student can be told that these areas are the same size. but here is visual evidence of it. Once this reiztioncaip has been esizblisnea ons can ac asout the tesk of iniegrathy this new function. Of course, once the ordinales and veigntings are siated it is simple to work on particular examples, but it is up to the teacher to snow where these values come from, and why they should produce such an ascurate result: in particular, the cases vihere this method produces exact results. The examples need to be pointed out, but further than this the reason for this exactness needs $t 0$ be explained. This information is quite separate from the software and only its implications can be demonstrated on the screen.

## COMPARISON OF METHODS



The controls and available choices in this section are much the same as for the other parts of the unit, except for changing the accuracy of the methods. Here the keys $\leftarrow \rightarrow$ are used to select the parameter to de changed (highlighted by a red box) and the keys it then increment or decrement the highlighted parameter. The limits for these as follows:

Trapezium $\quad 1$ to 30 strips
Simpson 2 to 30 strips (in steps of 2)
Gauss-Legenore 2 to 6 ordirates
Fomoerg $\quad$ نjses Tiapezum rule with $1: 20$ sirios. win 1 c: 2 i:=:ations

It shouid be pointed out that this program is intended pureiy as a demonsiration of tife comparative accuracies of the different methods. The time iaken for each calculation is shown (in hundredths of a second), but obviously this time is depengent on the machine and the algorititm used by the progiammer, and consecuently should be used as an indianan of the soeed meined rainer than a hare tast. These t:mes can also be
 iaken too slictly.

It can be shown that for the quadratic and cubic examples toth Simpson's (2 strip) and Gauss (2 ordinate) are exact, and can be arrived at quilkly, but for $e^{x} \sin x$, for example, Gauss (2 ordinate) suffers a severe crop in accuracy. in most cases it is clear that the trapezoidal rule is slow and inaccurate, but can be vastly improved by the simple application of the Romberg formula. Ol course it is for the class and teacher to introduce more complicated methods of numertal intepration, their

## Appendix 12

## CUBIC SPLINES

## SUMMARY

This unit is intended for use as an aid in the teacing and learning of the use of cubic splines. The progiam is ideally suited for use either by the student with the associated tutorial sneet or by the lectureritutor to ennance the normal teacning methods.

## OPERATING INSTRUCTIONS

The software is written ior the $\Xi 30$ Model B microcomputer and is coniained on one 40 track disc. Piace the dise in tie drive (divive 0 if using a ciouble dise crive); the software is then started by holding down the SHIFT key and simultaneously pressing the BREAK key. Slight edjusiment to the monitor display is aliowed it required.

Two further points on the unit operation:
i) Pressing the EREAK key at any time returns the micro to its default (switched on) state.
ii) Pressing the ミSCAP $\equiv$ key at any time restarts the current part.

Apart from the above two keys this unit operates with only the four CURSOR keys and the RETURN key.

Note: It is not essential to use a colour monitor with this unit but if available, coiour will ennance the presentation.

## USING THE PROGRAM SPLINES

The program displays both numerically and graphically eight data points in the ranges $x(0-55)$ and $y(0-34)$. These points (or subsets of them) may de connected by a set ot cubic splines as specified by the user. The up and cown cursor keys are used to move the hiohiight bar ud and down the menu: <Return> then seiects the current option. The options on the main menu periorm the ioliowing firetions:

## 1 PLOTSPUNES

This option connects the data points nominated in 3 below by a set of sblines using the curvature end conditions d:splayed in the dotiom right of the screen. The piot is superimeased on any winich may already be present.

## 2 MOVE POINTS

On seiecting this cption a marker below the x-axis points to the first caia point. This point may be moved in either direction in integer steps, using the cursor keys. As the point moves its entry in the $x, y$ table is updated. In the $x$-direction movement is bounded by the range restrictions ( 0,56 ) or in the case of internal points by the $x$-value of adjacent points. Pressing <Return> confirms the current point and moves the indicator to the next point which may then be repositioned in the same vay (if required). Subsequent pressing of <Return> will move the indicator along the ciata points. Foliowing confirmation of the last point a QUIT option appears. At this siage pressing the cursor key will change the state $Y(e s), N(0)$ and <Return> will then confirm the selection. 'No' here restants the movement cycle with the indicator at the first coordinate. 'Yes' will return to the main menu.

## 3 SEEETPONTS

This obtion allows sumsets of the eight points to de selected. A cox is cositioned around the numerisal values of the first point in the $x, y$ table; pressing any autsor key will 10 gyle the cotour ot the numbers in the dex between biack and white. fill

 coordinaies. Conifming the last coordinates will bring up a QUIT option as desc:ibed in 2 above.

## 4 VARY $S_{1}$ AND $s_{n}$.

The user via this copion may vary the value of $S$ at the firs: and las: ponts inciucec in the (white) set selected at 3 . The box surrounds the value of $S$ unoer observation; leturight cursor keys move the box to the other value, up.down cursor keys increase/decrease the currently highlighted value in sleps of 0.01 in the range -0.5 10 0.5 , <Return> confirms both values and returns to the main menu.

## 5 NEWFUNLOTION




Fig 1
This allows one of three functions (or none) to be selected from a list. The selected function is drawn in blue on the graph in order that approximations to it may be maje using the splines. Functions are selected in the same way as selections are made at the main menu. The selected function is printed in the appropriate box on the screen.

## 6 CLEARSCFEEN

As implied here, this option will clear the screen (only the graph window). The axes and data points are replaced as is the approximating function chosen in 5.

7 TASIE


Fig 2

With up to eight daia points in play there will be up to seven solines. This iable whish overlays the graph gives the cooefficients of each of the splanes in use. The vaiues in the table $\alpha_{i} \beta_{i} \gamma_{i} \delta_{i}$ are the coefficients of the $i$ th spline
$y=\alpha_{i}\left(x-x_{i}\right)^{3}+\beta_{i}\left(x-x_{i}\right)^{2}+\gamma_{i}\left(x-x_{i}\right)+\delta_{i}$
Notice that the top left hand box of the table containing the - symbol has invered colours; this invened highlight may be moved across the iable (ieftright) using the appropriate cursor keys. Whiie on the - symbol, if <Return> is pressed then the iajle disappears and the main menu comes back. If however <Return> is pressed whilst eitner the $\alpha \beta$ or $\gamma$ box is highlighted then the currently nominated column is prinied to 5 dp . for greater accurazy. Once expanded the leftright cursor keys will reurn to the iull table and move to the next coiumn.

## 8 PRESSBBAEAK TO QUIT PROGRAM

## TUTORIAL SHEET • SPLINES

1 Using the 'Select points' and 'Move points' options from the menu, select two points at

$$
x=13, y=15 \text { and } x=50, y=15
$$

With $S_{1}=S_{n}=0$. plot the connecting spline and write down the general form of the curve.

ת̄edeat this process with the points

$$
x=i 3 . y=25 \quad x=50 . y=15 .
$$

2 Fourn to the originat points

$$
x=13, y=15 \text { and } x=50, y=15
$$

but this time set $\mathrm{S}_{\boldsymbol{1}}=\mathrm{S}_{\mathrm{n}}=0.05$.
Using the tajle, write down the ecuation of the connesting apline.
Repeat this process with

$$
x=13 . y=25 \text { and } x=50, y=15
$$

3 Return to the original points and plot the connecting spline, using

$$
\text { i) } S_{1}=0.12 \quad S_{n}=-0.10
$$

ii) $S_{1}=-0.12 \quad S_{n}=-0.10$

Are the two splines of the same form?

4 Seiect tnree points at

$$
x=3, y=15 \quad x=30, y=20 \quad x=50, y=10
$$

and pu: $S_{1}=-0.3 S_{n}=0.20$.
Plot the conneating spline for each one. Now plo: the first of these cujics between $x=0$ and $x=40$.

Hint: The equation $y=\alpha_{i}\left(x-x_{i}\right)^{3}+\beta_{i}\left(x-x_{i}\right)^{2}+\gamma_{i}\left(x-x_{j}\right)+\bar{o}_{i}$

$$
\text { gives } \quad y^{\prime \prime}=6 \alpha_{i}\left(x-x_{i}\right)+2 \beta_{i}
$$

Hence $S_{i}\left(=y^{\text {n }}\right.$ evaluated at $\left.x\right)$ can be found at $x=40$.

5 Select the following points
$x=0, y=10 \quad x=10, y=14 \quad x=20, y=18$
$x=30, y=22 \quad x=40, y=26 \quad x=50, y=30$
with $S_{1}=0.5$ and $S_{n}=0$ and plot the splines. Look at the iable and graph and comment on the suitability of the curve p:otred to the set of cata points. Suggest an aigoritnm for guessing a value for $S$ for a general set of cata points.

i) $S_{1}=-0.50$
ii) $S_{1}=0.50$
iii) $S_{1}=0.00$

Comment on the effect of a cinange in bounciary conditions.

7 Experiment, and try to asproximale the sine funation watn
i) one spline (use $x_{1}=10$ and $x_{2}=50$ )
ii) two splines (use $x_{1}=10, x_{2}=23$ and $x_{3}=50$ )

Write down the equations you obtain and calculate the error in your approximation at $x=30$.

## CUBIC SPLINES

A cubic curve which passes through the points ( $\mathrm{x}_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}}$ ) and ( $\mathrm{x}_{\mathrm{i}+1}, \mathrm{y}_{\mathrm{i}+1}$ ) can be described by the equation

$$
\begin{equation*}
y=\alpha_{i}\left(x-x_{i}\right)^{3}+\beta_{i}\left(x-x_{i}\right)^{2}+\gamma_{i}\left(x-x_{i}\right)+\delta_{i} \tag{1}
\end{equation*}
$$

Note that at $x=x_{i}, y=y_{i}=\delta_{i}$
and at $x=x_{i+1}, y=y_{i+1}=\alpha_{i} h^{3}+\beta_{i} h^{2}+\gamma_{i} h+\delta_{i}$
where $h=x_{i+1}-x_{i}$ is assumed to be constant.
Now, from (1),
$y^{\prime}=3 \alpha_{i}\left(x-x_{i}\right)^{2}+2 \beta_{i}\left(x-x_{i}\right)+\gamma_{i}$
$y^{\prime \prime}=6 \alpha_{i}\left(x-x_{i}\right)+2 \beta_{i}$
If we let $S_{i}$ be the value of $y$ " at $x=x_{i}$
then $S_{i}=2 \beta_{i}$
We shall use the variables $S_{i}$ as the basic variables in our subsequent working. $\dagger$ First of all, we have $\beta_{i}=\frac{1}{2} S_{i}$
and since $S_{i+1}$ is the value of $y$ " at $x=x_{i+1}$
then

$$
S_{i+1}=6 \alpha_{i} h+2 \beta_{i}=6 \alpha_{i} h+S_{i}
$$

so that $\quad \alpha_{i}=\frac{S_{i+1}-S_{i}}{6 h}$

Further, $\quad y_{i+1}=\alpha_{i} h^{3}+\beta h^{2}+\gamma_{i} h+\delta_{i}$,
and via (2), (3), and (4)

$$
y_{i+1}=\left(\frac{S_{i+1}-S_{i}}{6}\right) h^{2}+\frac{S_{i}}{2} h^{2}+\gamma_{i} h+y_{i}
$$

from which we obtain

$$
\begin{equation*}
\gamma_{i}=\left(\frac{y_{i+1}-y_{i}}{h}\right)-\frac{h}{6}\left(S_{i+1}+2 S_{i}\right) \tag{5}
\end{equation*}
$$

$\dagger$ This effectively ensures that $y$ " is continuous at the points $x_{i}$.

We have assumed that $y$ ' is continuous at $x=x_{i}$.
For the interval $\left[\mathrm{x}_{\mathrm{i}-1}, \mathrm{x}_{\mathrm{i}}\right]$

$$
y^{\prime}=3 \alpha_{i-1}\left(x-x_{i-1}\right)^{2}+2 \beta_{i-1}\left(x-x_{i-1}\right)+\gamma_{i}
$$

so that at $\mathbf{x}=\mathrm{x}_{\mathrm{i}}$

$$
y_{i}^{\prime}=3 \alpha_{i-1} h^{2}+2 \beta_{i-1} h+\gamma_{i-1}
$$

For the interval $\left[\mathrm{x}_{\mathrm{i}}, \mathbf{x}_{\mathrm{i}+1}\right.$ ]

$$
y_{i}^{\prime}=3 \alpha_{i}(0)^{2}+2 \beta_{i}(0)+\gamma_{i}
$$

Hence

$$
3 \alpha_{i-1} h^{2}+2 \beta_{i-1} h+\gamma_{i-1}=\gamma_{i}
$$

$$
3\left(\frac{S_{i}-S_{i-1}}{6}\right) h+S_{i-1} h+\left(\frac{y_{i}-y_{i-1}}{h}\right)-\frac{h}{6}\left(S_{i}+2 S_{i-1}\right)
$$

ie

$$
=\left(\frac{y_{i+1}-y_{i}}{h}\right)-\frac{h}{6}\left(s_{i+1}+2 s_{i}\right)
$$

so that $\frac{h}{6}\left\{3 S_{i}-3 S_{i-1}+6 S_{i-1}-S_{i}-2 S_{i-1}+S_{i-1}+2 S_{i}\right\}=\frac{\Delta y_{i}}{h}-\frac{\Delta y_{i-1}}{h}$

$$
=\frac{\Delta^{2} y_{i-1}}{h}
$$

Therefore $\mathrm{S}_{\mathrm{i}-1}+4 \mathrm{~S}_{\mathrm{i}}+\mathrm{S}_{\mathrm{i}+1}=\frac{6}{\mathrm{~h}^{2}} \Delta^{2} \mathrm{y}_{\mathrm{i}-1}$.

To provide the necessary extra conditons we can choose either
(a) $\mathrm{S}_{1}=\mathrm{S}_{\mathrm{n}}=0$ (natural spline)
or
(b) $\mathrm{S}_{\mathrm{i}}$ is a linear extrapolation from $\mathrm{S}_{3}$ and $\mathrm{S}_{2}$, $S_{n}$ is a linear extrapolation from $S_{n-2}$ and $S_{n-1}$.
Hence $\frac{S_{2}-S_{1}}{h}=\frac{S_{3}-S_{2}}{h}$, ie $S_{1}-2 S_{2}+S_{3}=0$;
similarly $\mathrm{S}_{\mathrm{n}-2}-2 \mathrm{~S}_{\mathrm{n}-1}+\mathrm{S}_{\mathrm{n}}=0$.

In case (a) the equations for $\mathrm{S}_{\mathrm{i}}$ become, in matrix form,

$$
\left[\begin{array}{cccccccc}
4 & 1 & 0 & 0 & 0 & \ldots 0 & 0 & 0  \tag{7a}\\
1 & 4 & 1 & 0 & 0 & \ldots & 0 & 0 \\
0 & 1 & 4 & 1 & 0 & \ldots 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & . & 4 \\
0 & 1 \\
0 & 0 & 0 & 0 & 0 & \ldots & 1 & 4
\end{array}\right]\left[\begin{array}{c}
S_{2} \\
S_{3} \\
S_{4} \\
\cdots \\
S_{n-2} \\
S_{n-1}
\end{array}\right]=\frac{6}{h^{2}}\left[\begin{array}{c}
\Delta^{2} y_{1} \\
\Delta^{2} y_{2} \\
\Delta^{2} y_{3} \\
\cdots \\
\Delta^{2} y_{n-3} \\
\Delta^{2} y_{n-2}
\end{array}\right]
$$

In case (b) the equations are

$$
\left[\begin{array}{rrrrrr}
1 & -2 & 1 & 0 & 0 &  \tag{7b}\\
1 & 1 & 1 & 0 & 0 & \\
0 & 4 & 4 & 1 & 1 & \\
& & & & & 1 \\
1 & 4 & 1 \\
& & & & & 1
\end{array}\right]\left[\begin{array}{c}
S_{1} \\
S_{2} \\
S_{3} \\
\cdots \\
S_{n-1} \\
S_{n}
\end{array}\right]=\left[\begin{array}{c} 
\\
0 \\
\Delta^{2} y_{1} \\
\Delta^{2} y_{2} \\
\cdots \\
\Delta^{2} y_{n-2} \\
0
\end{array}\right]
$$

Having solved (7a) or (7b) for $\left\{S_{1}, S_{2}, \ldots, S_{n}\right\}$, we determine $\left\{\alpha_{i}, \beta_{i}, \gamma_{i}, \delta_{i} ; i=1, \ldots . n\right\}$ using equations (2) to (5).
Hence we can write the equations for each of the cubic spline curves; it is not unknown for two adjacent cubics to share the same equation.

## Appendix 13

UGC CTI (Computers in Teaching Initiative) SOFTWARE

## Lecturer's Questionnaire

Please fill in one of these questionnaires for each Unit that you have used.

1 Which Unit did you use? Which programs on this unit did you use? Give details.

2 In which teaching mode did you use the Unit?
(L) As part of a normal Lecture, using the programs (or parts of them) as additional aids
(T) In a Tutorial with a small group of students, either demonstrating or letting them run the programs for themselves
(D) As a Demonstration of the Unit, but not in the context of a normal lecture
(O) In an entirely different way. .

Answer Yes/No for each mode
(L)
(T)
(D)
(O)

If your answer to ( O ) was "Yes" then please give further details.

3 At what stage(s) in the teaching of this topic did you use the Software?
(i) In your introductory lecture?
(ii) In later lectures as the topic was developed?
(iii) At the end to consolidate the learning?
(iv) Later as an aid to revision?

Answer Yes/No for each question
(i)
(ii)
(iii)
(iv)

4 Did you have any difficulties in demonstrating and using the Unit? Give details.

5 As far as you were able to judge, was your students' response
(A) enthusiastic
(B) interested
(C) patient, but not particularly positive
(D) unresponsive
(E) extremely negative

## 6 Did you think that the Unit

(i) enabled you more easily to introduce/explain certain aspects of the topic?
(ii) enhanced the work that you normally do by more traditional methods?
(iii) helped consolidate the concepts for your students?
(iv) neither added nor detracted from yor usual methods of presenting this topic?
(v) made little or no contribution to the students' understanding of this work?

Answer Yes/No to each of the questions (i) to (v)
(i)
(ii)
(iii)
(iv)
(v)

7 Are there any other comments on this Unit that you would like to make?

8 Have you ever used any other Software on this topic? Give details.

9 Are there other areas of your syllabus where you think that traditional teaching might be enhanced by the production of a similar package?

UGC CTI (Computers in Teaching Initiative) SOFTWARE

## Student's Questionnaire

Please fill in one of these questionnaires for each Unit that you have used.
1 Which Unit did you use? Which programs of this Unit did you use? Give details.

2 Did you view the programs in the Unit
(L) during part of a normal Lecture on the topic in which the program(s) was just one of the aids used by the Lecturer
( T ) in a Tutorial either with the Lecturer/Demonstrator discussing the programs with a small group of students, or else running the program(s) yourself
(D) in a Demonstration of the Unit, but not in the context of a normal lecture
$(0)$ in some other way
Answer Yes/NO for each mode
(L)
(T)
(D)
(O)

If your answer to (O) was "Yes" then please give further details.

3 If you used the Unit in a Tutorial, for approximately how long were you running the programs?

4 If you ran the program(s) yourself, did you have any problems with the Unit? Give details.

5 Did the Unit help to increase your interest / motivation?
Answer on the scale (A) to (E)
(A) I found it highly motivating
(B) It made the topic more interesting for me
(C) I was interested, but no more than I would have been in the normal way
(D) It did not interest me very much
(E) I was frustrated / bored by the programs

If you want to enlarge upon your answer, please do so.

6 To what extent did the Unit aid your understanding?

Answer on the scale (A) to (E)
(A) I found it vey helpful and understood certain points much better after viewing the program(s)
(B) It helped to consolidate my ideas on this topic
(C) It was helpful, but I learnt no more than I would have done by more traditional methods of teaching
(D) It did not increase my understanding greatly
(E) It made the topic even more confusing

If you would like to enlarge upon your answer, please do so.

7 Are there any further Comments that you would like to make concerning this Unit?

## Appendix 14

## OUTLINE SYLLABUS FOR MATHEMATICAL ENGINEERING (after Richards (200))

## Year I

A Algebra and Matrices: linear equations, determinants, vectors, matrices.
B Calculus: differentiation and integration, differential equations, Fourier series, multiple integrals, partial differentiation.
C Complex Variables: complex algebra, Cauchy-Riemann equations, conformal transformations, complex integration.
D Mechanics: particles, systems of particles, rigid bodies, machines.
E Statistics: moments, etc, distributions, sampling, nomality, significance, regression.
F Linear Systems Theory: transforms, Laplace/Fourier, solution of DE's, transfer functions.
G Electrical Science: atomic structure, solid state devices, circuits, electromagnetism, power generation.
H Computing Project: basic numerical analysis and programming techniques applied to individual projects.

## Year II

A Field Theory: vector and tensor analysis, Gauss-Stokes-Green theorems, applications in elasticity, electrodynamics and fluid mechanics.
B Operational Research: linear programming, game theory, decision theory, queueing.
C Control Engineering: feedback, performance and stability, noise and random processes.
D Structural Analysis: simple structures, energy methods, continuous structures, instabilities.
E Materials Science: crystals, real materials, properties, metals and alloys, polymers, ceramics.
F Thermodynamics: laws, entropy, enthalpy, cycles, system applications.
G Advanced Statistics: estimation, variance and co-variance, multiple regression, stochastic processes.
H Design Project: fundamental design methods, applied to group design projects.

## Year III

A Fluid Mechanics: Navier-Stokes, real and ideal fluids, boundary layers and MAE, vortex methods, unsteady flows.
B Vibrations: beam and plate vibrations, model analysis, pole-zero, single and multiple response.
C Optimisation: calculus of variations, constraints, dynamic programming, gradient methods.
D Acoustic and Electromagnetic Radiation: wave equations, 1, 2 and 3 dimensions, radiation and propagation.
E Information Theory: communication and signal processing, sampling theorems and applications.
F Computing Science: advanced numerical and digital techniques, integral equations, approximations.
G Systems Engineering: the over-all design problem, analysis and synthesis, networks, applications.
H Research Project: individual research project.

## Appendix 15

## PROGRAMMING PROBLEMS

1 Number of turns required for a specified inductance
2 Water temperature for concrete mix
3 Current in a zero-resistance circuit
4 Effect of friction on velocity
5 Specific permeability of a porous medium
6 Oxygen deficit in a polluted stream
7 Van der Waal's equation
8 Power-law curve fitting to polymer flow data
9 Friction factor for turbulent flow in a smooth pipe
10 Shape factor for flow in a rectangular channel
11 Maximum oxygen deficit in a stream
12 Merging delay in traffic flow
13 Mean and variance
14 Reversing an array of numbers
15 Printing a list of numbers in descending order
16 Finding the mean and standard deviation of grouped data
17 The Bode plot
18 Thermal equilibrium in a plate
19 Deflection of a plate
20 Deflection of a Belleville spring
21 Boundary layer analysis
22 High-frequency transmission line
23 Accurate buckling load for a cantilevered shaft
24 Approximate buckling load for a cantillevered shaft
25 Currents in an electrical network
26 Stresses in concentric thick-walled cylinders
27 Currents in an electrical circuit near resonance
28 Natural frequencies of a mechanical system with three degrees of freedom
29 Temperature distribution in fluid flow

## 3 Current in a zero-resistance circuit

The current in an ideal (zero resistance) circuit is given by

$$
I=\frac{V}{|\omega L-1 / \omega C|}
$$

where I is the current in $\mathrm{A}, \mathrm{V}$ is the voltage in $\mathrm{V}, \omega$ is the angular frequency in $\mathrm{rad} / \mathrm{s}, \mathrm{L}$ is the inductance in $\mathrm{H}, \mathrm{C}$ is the capacitance in F .
(For certain values of $\mathrm{V}, \mathrm{L}$ and C there is a value of $\omega$ at which the current becomes infinitely large.)

It is required to increment $\omega$ over a specified range of values and output the values of $\omega$ and $I$ at each step. When $\mid \omega \mathrm{L}-1 / \omega \mathrm{C}$ is less than a specified small number $\varepsilon$ the output should consist of the value of $\omega$ and a suitable message.

Take as sample data
$V=10.0, \mathrm{~L}=0.001, \mathrm{C}=1.0 \times 10^{-9}, \varepsilon=0.0001$, lower limit on $\omega=10^{4} \mathrm{rad} / \mathrm{s}$, upper limit on $\omega=10^{8} \mathrm{rad} / \mathrm{s}$.

## 4 Effect of friction on velocity

If a body of a constant mass is acted on by a friction force, whose magnitude is a function of the velocity of the body, the velocity will tend to a limiting value in certain cases. One such case is where the coefficient of friction and the velocity are related by

$$
\mu=0.1758(v)^{0.09}
$$

where $\mu$ is the coefficient of kinetic friction and $v$ is the velocity of the body in $\mathrm{cm} / \mathrm{s}$. Consider the case of sliding friction, where the equation of motion is

$$
a=\frac{F}{m}-\mu g
$$

where F is the applied force in $\mathrm{N}, \mathrm{m}$ is the mass of the body in kg and g is the gravitational acceleration in $\mathrm{m} / \mathrm{s}^{2}$.

Since the velocity changes more rapidly in the early stages of motion, the time intervals at which the velocity is output should not be equal. (Try a geometric progression.)

Calculate the velocity of the body at a suitable set of times. Include a criterion for stopping the calculations.

Take as sample data

$$
\mathrm{F}=0.23, \mathrm{~m}=0.1, \mathrm{v}=0.20, \text { initial time step }=0.125 \mathrm{~s}, \mathrm{~g}=9.81
$$

## 26 Stresses in concentric thick-walled cylinders

Figure 1 shows a cross-sectional view of two long concentric thick-walled cylinders. At ambient temperature and in the unstressed state, these cylinders have inner and outer radii of $r_{1}$ and $r_{2}$, and $r_{2}$ and $r_{3}$. An internal pressure of $P$ above ambient is applied in such a way that no axial stresses are generated in the cylinders.

Using the Lame equations, the distributions of hoop and radial stresses may be expressed in dimensionless form for either cylinder as:

$$
\begin{equation*}
\frac{\sigma_{\theta \theta}}{\mathrm{P}}=\mathrm{Y}+\frac{\mathrm{Z}}{\mathrm{r}^{2}} \frac{\sigma_{\mathrm{r}}}{\mathrm{P}}=\mathrm{Y}-\frac{\mathrm{Z}}{\mathrm{r}^{2}} \tag{1}
\end{equation*}
$$



Fig 1 Concentric thick-walled cylinders
where r and $\theta$ are the polar coordinates shown in Fig 1. The parameters Y and Z are constants, whose values are determined by the boundary conditions for the particular cylinder. For the present problem, these conditions include:

$$
\sigma_{\mathrm{rr}}=-\mathrm{P} \text { at } \mathrm{r}=\mathrm{r}_{1}, \sigma_{\pi}=0 \text { at } \mathrm{r}=\mathrm{r}_{3},
$$

and the radial stresses are the same in both cylinders at $\mathrm{r}=\mathrm{r}_{2}$. Assuming the cylinders fit perfectly in the unstressed state, the hoop strains are also the same at $r=r_{2}$. Hoop strain is given by:

$$
\begin{equation*}
e_{\theta \theta}=\frac{1}{E}\left(\sigma_{\theta \theta}-v \sigma_{\pi}\right)=\frac{P}{E}\left(Y(1-v)+\frac{Z}{r^{2}}(1+v)\right) \tag{2}
\end{equation*}
$$

where E is Young's modulus and v is Poisson's ratio. The following four equations are obtained.

$$
\begin{align*}
& Y_{1} \frac{Z_{1}}{r_{1}^{2}}=-1 \\
& Y_{1}-\frac{Z_{1}}{r_{2}^{2}}=Y_{2}-\frac{Z_{2}}{r_{2}^{2}}  \tag{3}\\
& Y_{2}-\frac{Z_{2}}{r_{3}^{2}}=0 \\
& \frac{1}{E_{1}}\left(Y_{1}\left(1-v_{1}\right)+\frac{Z_{1}}{r_{2}^{2}}\left(1-v_{1}\right)\right)=\frac{1}{E_{2}}\left(Y_{2}\left(1-v_{2}\right)+\frac{Z_{2}}{r_{2}^{2}}\left(1+v_{2}\right)\right)
\end{align*}
$$

The subscripts 1 and 2 refer to the inner and outer cylinders respectively.
Problem specification. The ratios of the hoop and radial stresses to the internal pressure are to be determined as functions of radius for the concentric cylinders shown in Fig 1.

Take as sample data
$v_{1}=0.35, v_{2}=0.30, E_{1} / E_{2}=1.5, r_{1}=0.1, r_{2}=0.2$ and $r_{3}=0.4$, where the unit of radius is arbitrary.

## 27 Currents in an electrical circuit near resonance

Figure 1 shows an electrical circuit having inductance and capacitance, but no resistance. The potential $V \sin \omega t$ alternates with respect to time $t$ with an angular frequency $\omega$ and amplitude $\mathrm{V} . \mathrm{L}_{1}$ and $\mathrm{L}_{2}$ are inductances, $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ are capacitances, and $\mathrm{i}_{1}, \mathrm{i}_{2}$ are the currents in the circuit.


Fig 1 An electrical circuit

Equations may be written down for the conservation of currents at nodes of a network, and the summation of potential differences round closed loops. Hence, the relationship between the currents is

$$
\begin{equation*}
i_{1}-i_{2}-i_{3}=0 \tag{1}
\end{equation*}
$$

and, for the route followed by $i_{L}$ and $i_{2}$

$$
\begin{equation*}
\mathrm{L}_{1} \frac{\mathrm{di}_{1}}{\mathrm{dt}}+\frac{\mathrm{q}_{1}}{\mathrm{C}_{1}}+\mathrm{L}_{2} \frac{\mathrm{di}_{2}}{\mathrm{dt}}=\mathrm{V} \sin \omega t \tag{2}
\end{equation*}
$$

where $q_{1}$ is the electrical charge on the capacitance $C_{1}$. Since $i_{1}=d q_{1} / d t$, it is convenient to differentiate this equation to eliminate $q_{1}$ as follows:

$$
\begin{equation*}
L_{1} \frac{d^{2} i_{1}}{d t^{2}}+\frac{i_{1}}{C_{1}}+L_{2} \frac{d^{2} i_{2}}{d t^{2}}=\omega V \cos \omega t \tag{3}
\end{equation*}
$$

Similarly, for the route followed by $i_{1}$ and $i_{3}$

$$
\begin{equation*}
\frac{\mathrm{d}^{2} \mathrm{i}_{1}}{\mathrm{dt}^{2}}+\frac{\mathrm{i}_{1}}{\mathrm{C}_{1}}+\frac{\mathrm{i}_{3}}{\mathrm{C}_{2}}=\omega V \cos \omega t \tag{4}
\end{equation*}
$$

In the steady state, the three currents alternate with the same frequency as the applied potential. Since in circuits containing only inductance and capacitance the currents are $90^{\circ}$ out of phase with the applied potential, they may be represented by $i_{1}=I_{1} \cos \omega t$, $\mathrm{i}_{2}=\mathrm{I}_{2} \cos \omega \mathrm{t}$ and $\mathrm{i}_{3}=\mathrm{I}_{3} \cos \omega \mathrm{t}$. Substituting these expressions into equations (1), (3), (4) the following set of linear equations is obtained for the amplitudes of the currents

$$
\left[\begin{array}{ccc}
\frac{1}{\omega C_{1}}-\omega L_{1} & -\omega L_{2} & 0  \tag{5}\\
\frac{1}{\omega C_{1}}-\omega L_{1} & 0 & \frac{1}{\omega C_{2}} \\
1 & -1 & -1
\end{array}\right]\left[\begin{array}{c}
\mathrm{I}_{1} \\
\mathrm{I}_{2} \\
\mathrm{I}_{3}
\end{array}\right] \quad\left[\begin{array}{c}
\mathrm{V} \\
\mathrm{~V} \\
0
\end{array}\right]
$$

A condition known as resonance occurs in an alternating current circuit when its impedance approaches zero. The currents tend to infinity, although in real circuits the presence of even a small amount of resistance restricts them to finite, but large, values. Resonance occurs in the present circuit when the coefficient matrix in equations (5) is singular.

In the circuit shown in Fig 1, the amplitude of the applied potential is 300 V , the inductances are $\mathrm{L}_{1}=0.60 \mathrm{H}$ and $\mathrm{L}_{2}=0.20 \mathrm{H}$, while the capacitances are $\mathrm{C}_{1}=1 \mu \mathrm{~F}$ and $C_{2}=2.5 \mu \mathrm{~F}$. The effect on the current amplitudes is to be examined as the angular frequency approaches $1000 \mathrm{rad} / \mathrm{s}$, one of the resonant values. Also, the effect of a small inherent error in the specific value of $L_{1}$ on the currents near resonance is to be determined.

## Appendix 16

## A Second-Year Project Titles - a selection

Metal quenching
Pigment extrusion
Turbulent flow in a pipe network
Buckling of a centrally - loaded column
Computer calculations in magnetostatics
The stiffness method in structures
Stability investigation of a surge tank
The frequency response of linear systems
Microcomputer plotting of 2-D Laplacian fields
Road vehicle performance characteristics
Shop floor machine location
Lateral vibrations of beams
Fractal graphics
Errors in exhaust noise measurements
Recursive subdivision

## B Final Year Project Titles - a selection

Control of vehicles in an automated transportation system
Aerodynamics of car body profiles
Modelling hydraulic control systems
Effect of lateral loads on track movement
Flow patterns during compression modelling
Numerically controlled wiring of logic boards
Numerical representation of surfaces
Coordinate geometry in acoustic telemetry
Heat transmission through doubly glazed windows
Glass flow along canals
Car-following models
Modelling an electron lens
Flow in pipe networks
Computer-aided design of exhaust silencers
Noise radiation from an engine
Dispersion of effluent in a river
Bezier curve and surface manipulation
Shock waves in traffic
Control of a continuous furnace
Transfer function identification
C Selected Dissertation Titles from the Industrial Year
STL: Thermal Modelling Techniques in Electric Motor DesignBabcock Power: Acoustic Excitation of Manipulators
Rolls-Royce Ltd: Design and Analysis of Prop-fans
British Steel: A Mathematical Model of the Cooling System of a Hot-strip MillDunlop: Two-and Four-Wheeled Vehicle Stability
British Rail: Active and Passive Suspensions for Railway Vehicles
GEC Industrial Controls: Cold Mill Rolling Automation
CEGB: Corrosion of Nuclear Fuel Magnox Cladding during Transport
British Gas: The Development of a Program to Analyse Stress in Pipes.
Bass PLC: A Mathematical Model of a Maltkin
British Aerospace Dynamics: Feedforward Calculations in a Modern Guided WeaponSystem
1CI Engineering: Pipeline Specification Generation Systems
D Destination of Graduates - a sample
Space Systems Engineer British Aerospace Aircraft Group
Stress Engineer : British Aerospace Aircraft Group
Systems Engineer : British Aerospace Dynamics Group
Computer Systems Engineer : Ford Motor Company
Technologist : Dunlop
Computer Programmer- Dunlop
Engineer GEC Power EngineeringSoftware Engineer
: Self-Changing Gears
Consultant ..... LogicaEngineerEngineer: Marconi Avionics
Marconi Electronics Devices
Software EngineerScientific OfficerSystems Analyst- Nomalair-Garret
Design Engineer : Racal Electronics
Engineer Trainee : Rolls-Royce
Mathematical Programmer : Scicon
Control Technologist : Pilkington Brothers PLC
Software Engineer

## Appendix 17

## A Final Year Projects at Nottingham

Calculations of Water Inflow to Underground Mine Workings
Sound Transmission through Double Glazing
Flow of Granular Materials in Wedge-shaped Channels
the Pressure Swing Absorption Technique for Separating Gases
Surf Run-up and Backwash on a Beach
Optimisation of Methods for the Cutting of Panels
Percolation of Water through an Earth Dam
Stability of Liquid Films Flowing down Inclined Planes
Stress in the Vicinity of Elliptical Holes in Laminated Elastic Plates
Generation of Optimum Triangular Element Grids for Aerodynamic Calculations
Supercritical Free Surface Flow of a Fluid over a Ramp
A Time-marching Method for the Convection-Diffusion Equation
The Squashing and Buckling of Elastic Tubes
The Statistical Modelling of a Production Line
On Modelling the Internal Deflection of Mine Roadways
The Dynamics of a Four-wheeled Steered and Sprung Vehicle
A Model for Turbidity Currents in the Ocean
Artificial Density Techniques for Transonic Aerodynamic Flow
The Chattering of Railway Trucks
Phase Changes around a Buried Gas Pipeline

## B Employment of Nottingham Graduates

Some of the posts held by our recent graduates immediately after completing the course are:

Accountancy Trainees
Commercial Computing
Computer Analysts
Computer Programmers
Design Engineer
Engineer
Graduate Trainees
Graduate Trainee
(Quality Assurance)
Management Trainees
Market Research
Mathematicians
Networks Planner
Programmers - British Airways (London), Logica (London)
Research Students (MSc and PhD) and Research Assistants
Research Officer - Marconi (Chelmsford)

| Systems Analysts | - ICL (Kidsgrove), GEC Avionics (Kent) |
| :--- | :--- |
| Software Engineer/Analysts | - Central Electricity Generating Board (London), <br>  <br> Logica, (London), Scicon (Milton Keynes <br> and London) |
| Scientist | - Atomic Energy Authority (Risley) |

## C Final Year Projects at Bristol

Why political Opinion Polls Fail
The Election Database
An Electronic Cricket Scoreboard
Language Modelling for Speech Recognition
Time-dependent Motions in Electrothermal Systems
CAM Gears
Dynamics of the Golf Swing
Determination of CAM Profiles
Mathematical Modelling of an Aerial for Space Communication
Helicopter System Identification \& Control
An Orthodontic Expert System
Computer Simulation of Epidemics
A Computer Program to Play Mah-Jongg
Prolog in Forth
Computer Assisted Timetabling
Optimal Sail Design
Modelling of Non-linear Car Suspension Components
Linear Instability in Electrothermal Convection
Level-index Computer Arithmetic
Software for Speech Recognition

## D Employers of Bristol Graduates

British Aerospace
British Petroleum
British Oxygen Company (BOC)
British Telecom
Central Electricity Generating Board, Berkeley Nuclear Labs
Ferranti Computer Systems
Ford Motor Company, Research Labs
GEC Avionics
GEC Turbine Generators, Rugby
Hunting Engineering, Bedfordshire
ICI
Logica
Metal Box Co

## Motor Industries Research Associates

RACAL
Rolls Royce Aero Engines
Royal Airforce
Royal Navy
REME
Shell



[^0]:    $\dagger$ Coursework marks accounting for $20 \%$ of the total arise from laboratory work.
    $\dagger \dagger$ A practical design project is the assessed item.

    * Includes a component on Finite Elements.

