

1     **An ice-sheet scale comparison of eskers with modelled subglacial**  
2                                    **drainage routes**

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13

### Abstract

14    Eskers record the signature of channelised meltwater drainage during deglaciation providing  
15    vital information on the nature and evolution of subglacial drainage. In this paper, we  
16    compare the spatial pattern of eskers beneath the former Laurentide Ice Sheet with  
17    subglacial drainage routes diagnosed at discrete time intervals from the results of a  
18    numerical ice-sheet model. Perhaps surprisingly, we show that eskers predominantly occur  
19    in regions where modelled subglacial water flow is low. Eskers and modelled subglacial  
20    drainage routes were found to typically match over distances of <10 km, and most eskers  
21    show a better agreement with the routes close to the ice margin just prior to deglaciation.  
22    This supports a time-transgressive esker pattern, with formation in short (<10 km) segments

1 of conduit close behind a retreating ice margin, and probably associated with thin, stagnant  
2 or sluggish ice. Esker forming conduits were probably dominated by supraglacially fed  
3 meltwater inputs. We also show that modelled subglacial drainage routes containing the  
4 largest concentrations of meltwater show a close correlation with palaeo-ice stream  
5 locations. The paucity of eskers along the terrestrial portion of these palaeo-ice streams and  
6 meltwater routes is probably due to the prevalence of distributed drainage and the high  
7 erosion potential of fast-flowing ice.

8 **Key words:** Esker; Ice Sheet; Numerical Modelling; Subglacial Drainage; Conduits

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## 11 **1. Introduction**

12 Eskers are slightly sinuous ridges composed of glaciofluvial sand and gravel that are  
13 deposited in subglacial, englacial or supraglacial drainage channels (e.g. Banerjee and  
14 McDonald, 1975; Brennand, 2000; Storrar et al., 2014a). They can extend for tens to  
15 hundreds of kilometres (taking into account small gaps), reach in excess of 50 m in height  
16 and are typically arranged roughly parallel to each other (e.g. Prest et al., 1968; Banerjee  
17 and McDonald, 1975; Shilts, 1984; Shreve, 1985a,b; Aylsworth and Shilts, 1989; Clark and  
18 Walder, 1994; Boulton et al., 2009; Storrar et al., 2014a,b). Eskers may therefore provide  
19 vital information about channelised drainage. This is significant because observations from  
20 modern ice sheets reveal the important role of water in lubricating the bed, and facilitating  
21 rapid ice-flow (Bartholomew et al., 2011; Sundal et al., 2011). In particular, the configuration  
22 of the subglacial drainage network and how it evolves to accommodate water inputs is  
23 critical (e.g. Budd et al., 1979; Alley et al., 1986; Iken and Bindshadler, 1986). Two end-

1 member drainage configurations are typically envisaged (e.g. Walder and Fowler, 1994): (i)  
2 an efficient channelised system commonly associated with lower water pressures, lower ice  
3 velocities, and higher water discharges; and (ii) an inefficient distributed system (e.g. linked  
4 cavities, braided canals and porous till layer) commonly associated with higher water  
5 pressures, higher ice velocities, and lower water discharges. However, because of the  
6 difficulties in directly observing the drainage of water at the bed of ice masses, we have a  
7 limited understanding of the distribution and geometry of the subglacial drainage network  
8 and a lack of data at the spatial and temporal scales necessary to constrain or test subglacial  
9 hydrological models (e.g. Hewitt, 2011; Werder et al., 2013).

10 It is presently physically difficult to investigate eskers located under contemporary ice  
11 sheets. The imprint of eskers recorded on the bed of former ice sheets has a clear  
12 advantage over data from contemporary ice sheets because we can directly observe the  
13 expression of meltwater drainage over large spatial scales. However, despite the use of  
14 eskers to reconstruct and constrain ice-retreat histories (e.g. Dyke and Prest, 1987; Margold  
15 et al., 2013), very few studies have investigated their pattern at the ice-sheet scale  
16 (Aylsworth and Shilts, 1989; Clark and Walder, 1994; Storrar et al., 2014a,b). This is because  
17 it is not known whether eskers form synchronously in long conduits (cf. Brennand, 1994), or  
18 if they represent a time-integrated signature of drainage deposition throughout deglaciation  
19 (e.g. Banerjee and McDonald, 1975; Shilts, 1984; Dyke and Dredge, 1989; Kleman et al.,  
20 1997; Hooke and Fastook, 2007). Consequently, it is difficult to extract and interpret  
21 information about where they form in relation to subglacial water generation and ice  
22 dynamics.

23 A new approach to understanding the pattern of eskers is to compare their distribution and  
24 orientation with numerical models of subglacial meltwater drainage at discrete intervals

1 throughout deglaciation. In this study our aim is to compare the expression of eskers on the  
2 bed of the former Laurentide Ice Sheet (LIS) with subglacial drainage routes predicted from  
3 the results of a numerical ice sheet model. This builds on previous work that has compared  
4 eskers and hydraulic gradient routes at glaciers (e.g. Syverson et al., 1994), but represents  
5 the first attempt to compare modelled drainage with eskers at the ice-sheet scale and at  
6 discrete time intervals.

7

## 8 **2. Methods**

### 9 *2.1 Mapping esker networks*

10 This paper uses 3,749 interpolated eskers (mostly >2 km long) mapped by Storrar et al.,  
11 (2013). The crestlines of esker ridges were mapped from Landsat 7 Enhanced Thematic  
12 Mapper (ETM)+ imagery of Canada, which has a resolution of ~30 m, and ~15 m in the  
13 panchromatic band. Eskers were typically mapped at a scale of 1:40,000 and were identified  
14 based on the criteria set out by Margold and Jansson (2012). Shorter eskers (< 2 km long)  
15 were more difficult to identify in Landsat imagery. Comparison with mapping from aerial  
16 photographs suggests that approximately 75% of eskers were identified, and that 81% of  
17 those missed are less than 2 km long (Storrar et al., 2013).

18 To enable the effective comparison of eskers with modelled subglacial drainage routes we  
19 used the interpolated esker dataset produced by Storrar et al. (2014a). This dataset was  
20 derived by interpolating a straight line (over short distances in the majority of cases)  
21 between aligned esker ridges that appear genetically related (i.e. formed in the same  
22 conduit), and merging with the mapped ridges to produce a single esker. It was produced to  
23 fill gaps that may have resulted from fragmentary deposition, post-depositional erosion or

1 submergence beneath lakes, and is therefore thought to give a better indication of where  
2 the esker forming conduits were located (Storrar et al., 2014a). We refer to this dataset  
3 throughout the paper simply as ‘eskera’.

4

## 5 2.2 *Modelling subglacial meltwater drainage*

6 Subglacial meltwater drainage was modelled using the method outlined in Livingstone et al.  
7 (2013a,b). Hydraulic potential surfaces ( $\phi$ ) of the LIS were calculated from the Shreve  
8 equation (Shreve, 1972):

$$9 \quad \phi = \rho_w g h + \rho_i g H, \quad (1)$$

10 where  $\rho_w$  is the density of water;  $\rho_i$  is the density of ice;  $g$  is the acceleration due to gravity;  
11  $h$  is the bed elevation; and  $H$  is the ice thickness. We calculated the subglacial drainage  
12 routes every 500 years for the period between 12 and 7 ka BP, which encompassed the  
13 largest retreat distance (hundreds of kilometres) during deglaciation and was over the  
14 predominantly hard crystalline bedrock on the Canadian Shield (see Dyke, 2004). The bed  
15 elevation data ( $h$ ) were constructed at 5 km resolution from Gebco\_08 digital elevation  
16 model (DEM), and the palaeo-ice surfaces and palaeo-bed topographies (corrected for  
17 isostasy) were derived from ice-sheet model output from one of the higher probability runs  
18 (LT9927) from the ensemble-based analyses of the LIS using 3D-MUN Glacial Systems  
19 Model (GSM) (Tarasov et al., 2012). The 3D-MUN GSM is calibrated against a large set of  
20 observational constraints, including geological and geomorphological evidence and is able to  
21 reproduce ice stream locations and ice-margin positions (Stokes and Tarasov, 2010; Tarasov  
22 et al., 2012). LT9927 is from the sub-ensemble of runs used by Livingstone et al., (2013).  
23 Their analysis shows that the modelled distribution of subglacial lakes and major drainage

1 routes is a robust result achieved irrespective of the model run used from this sub-ensemble  
2 (see Fig. 8 from Livingstone et al., 2013). Given this and the time required for analysing each  
3 run, we base our analysis just on LT9927 for this study. The 1° longitude by 0.5° latitude  
4 resolution model output was re-gridded at 5 km cell size. Subglacial drainage routes (i.e., the  
5 direction that water flows) were constructed from the hydraulic potential surfaces, using  
6 simple GIS routing tools as per Livingstone et al. (2013a,b). Basal meltwater production  
7 (cm/yr) generated from 3D-MUN GSM was used to weight flow accumulation down  
8 subglacial drainage routes. Each cell was given an accumulative basal meltwater value of all  
9 the cells that flow into, hereafter referred to as the 'modelled subglacial flow  
10 concentration'. Output cells with a high flow accumulation represent a drainage route along  
11 which subglacial meltwater is concentrated. To allow for basal meltwater production caused  
12 by likely sub-grid topographic variation, we set a minimum basal meltwater output (0.1  
13 cm/yr) in regions of the bed where the temperature was 0 to -2 °C below pressure melting  
14 point. Meltwater may also enter the subglacial system from supraglacial sources, although  
15 these are not reproduced here because of the difficulty of modelling this process. Thus, we  
16 use basal meltwater production simply to indicate where meltwater is likely to concentrate,  
17 rather than suggesting that all meltwater is necessarily *produced* at the bed.

18

### 19 2.3 *Comparison of eskers and modelled subglacial drainage routes*

20 To our knowledge, a comparison between the pattern of eskers and modelled subglacial  
21 drainage routes has not been previously undertaken at the ice sheet scale. Thus, in this  
22 analysis, we explore first-order relationships between the location and orientation of eskers

1 and modelled drainage routes (Fig. 1). Approaches for these two comparisons are described  
2 below.

### 3 *2.3.1 Spatial conformity of eskers and modelled subglacial flow concentration*

4 Output cells with a high subglacial flow concentration indicate regions where large volumes  
5 of meltwater are routed, and these represent potential meltwater conduit locations (Fig.  
6 1a). These should correspond to esker locations as this is where Röthlisberger channels are  
7 theorised to form (e.g. Shreve, 1972; Röthlisberger, 1972).

8 To investigate how the spatial pattern of eskers relates to the routing of subglacial  
9 meltwater beneath the LIS we compared modelled subglacial flow concentration with the  
10 esker pattern at 500-year time-slices from 12 to 7 ka BP (Fig. 1a). At each time slice we  
11 extracted flow concentration values of all cells that contain eskers and all cells covered by  
12 ice in the model domain. We also identified cells that match with the spatial extent of  
13 palaeo-ice stream locations recently compiled in Margold et al. (2015), and cells that match  
14 the terrestrial portion of the palaeo-ice sheet bed (i.e., where the mapping was carried out)  
15 (Storrar et al., 2013). The probability density function of modelled subglacial flow  
16 concentration was calculated for each of the variables extracted and results displayed as a  
17 ratio between each probability density function and the model as a whole (i.e., the  
18 population). Statistical significance was evaluated with a binomial test.

19 To further identify any spatial match between eskers and major subglacial drainage routes  
20 we used a flow concentration of  $>20$  cm/yr to identify potential meltwater conduit locations  
21 (Fig. 1a). A value of 1 was assigned to cells where the flow concentration exceeded 20 cm/yr  
22 and a value of 0 to those that did not (see also Livingstone et al., 2013b). This was done for

1 every time-slice and the values (0s and 1s) then summed together to produce a composite  
2 map of potential meltwater conduits and their persistence over time.

### 3 *2.3.2 Directional conformity of eskers and modelled subglacial drainage routes*

4 Eskers provide a composite history of the former drainage and direction of meltwater flow  
5 beneath an ice sheet throughout deglaciation. To try and disentangle what is assumed to be  
6 a time-integrated pattern, the match between the orientations of eskers and modelled  
7 subglacial drainage routes beneath the LIS were compared at 500-year intervals between 12  
8 and 7 ka BP (Fig. 1b) (Livingstone et al., 2013a). This comparison was used to investigate (i)  
9 how far up-ice from the margin eskers align with the modelled drainage directions; (ii)  
10 whether eskers show a better alignment closer to or further away from the ice margin; and  
11 (iii) whether any particular time-slice is matched to the entire esker pattern.

12 A tracking algorithm was developed to compare the directional conformity of eskers and  
13 modelled drainage routes (see supplementary information). The tracking proceeds up-ice,  
14 moving from the margin of the LIS towards its centre. Tracking was initiated for all eskers at  
15 each time slice, with esker-model agreement starting the first time an esker's orientation  
16 matched the modelled subglacial drainage direction of an overlying cell and continuing until  
17 the agreement first breaks down (Fig. 1b). The length ( $L$ ) that each esker was tracked and  
18 the percentage ( $f$ ) of overall length tracked ( $(\sum L_{\text{tracked}} / \sum L_{\text{total}}) \times 100$ ) was used to investigate  
19 the upstream conformity of eskers with modelled drainage directions. We also compared (i)  
20 the timing and (ii) the distance from the ice margin between the best and average fits of the  
21 modelled subglacial drainage and esker directions at each point along an esker that was  
22 tracked in  $\geq 2$  time periods. This was used as a measure of when (timing) and where  
23 (distance from the ice-margin) eskers best fit the modelled drainage directions.



1 There are three main parameters in the algorithm that could be varied: (i) the maximum  
2 angle of divergence ( $\theta$ ) allowed between the orientations of the esker and modelled  
3 drainage direction; (ii) the length scale at which the eskers are smoothed ( $E$ ); and (iii) the  
4 spatial scale ( $W$ ) used to smooth the modelled subglacial drainage direction grid. Preferred  
5 values for the parameters are  $45^\circ$  for  $\theta$  and a smoothing scale of 5 km for  $E$  and  $W$ , which is  
6 the same resolution at which the model output was re-gridded and drainage routes  
7 constructed. The supplementary section details a sensitivity analysis of  $\theta$ ,  $E$  and  $W$ .

8

### 9 2.3.3 *Limitations*

10 There are three major limitations to our preliminary approach. First, the results use the  
11 predictions of a numerical model, which is not reality. However, the 3D-MUN GSM includes  
12 basic glaciological physics and is calibrated against a large set of observational constraints  
13 (see Tarasov et al., 2012), including relative sea-level data, present-day rates of surface  
14 uplift, and an ice margin chronology ( $\pm 250$  to 1000 year uncertainty) derived from  
15 geological and geomorphological evidence (Dyke, 2004). LT9927 is therefore glaciologically  
16 self-consistent and is a sample from the most well-constrained distribution of possible  
17 deglaciation chronologies to date. Future extension of this study to a sub-ensemble from  
18 the calibration will enable uncertainty quantification of our initial results herein.

19 Secondly, there is a large difference between the initial resolution of the numerical ice  
20 model used to derive the ice-surface and bed topographies ( $\sim 50$  km resolution) and the  
21 (tens of metre) scale at which eskers were mapped. Even the smoothed (5 km resolution)  
22 ice-surfaces used to generate the hydraulic potential surfaces are relatively coarse, which  
23 might artificially create drainage routes that are further apart than they would be in reality.

1 However, away from the ice margin and ice streams/outlet glaciers, ice sheets are relatively  
2 smooth and a 50 km resolution is adequate.

3 Thirdly, this paper uses a simple representation of subglacial hydrology, which provides a  
4 first approximation of where water should seek to flow and pond relative to the ice-sheet  
5 model selected. Crucially, the subglacial drainage system and overlying ice are not  
6 dynamically coupled and it does not allow drainage configurations to evolve. We therefore  
7 assume that water pressure is equal to ice-overburden pressure uniformly across the bed.  
8 This is a reasonable assumption when averaged over coarse ice-sheet scales where  
9 geometry is the key control (Livingstone et al., 2013a,b), but unlikely to be valid at smaller  
10 scales dominated by local processes (e.g. Gulley et al., 2012).

11

## 12 **4. Results**

### 13 *4.1 Spatial conformity of eskers and modelled subglacial flow concentration*

14 Fig. 2 reveals that the probability of palaeo-ice streams occurring in regions with high  
15 subglacial flow concentrations is significantly higher than chance, even if the resolution is  
16 reduced 10-fold (binomial test for flow concentrations  $>20$  cm/yr,  $p < 0.001$ ). In contrast,  
17 cells that match esker locations show a lower probability for subglacial flow concentration  
18 values  $>100$  cm/yr. At values  $<100$  cm/yr eskers occur significantly more often than you  
19 would expect by chance (binomial test,  $p < 0.001$  and  $p < 0.01$  if the resolution is reduced  
20 10-fold) and, importantly, display a stronger correlation than the mapping domain. Eskers  
21 exhibit a higher probability than ice streams for subglacial flow concentration values  $<20$   
22 cm/yr.

1 The spatial pattern between eskers and potential subglacial meltwater conduits (subglacial  
2 flow concentration of  $>20$  cm/yr) is shown in Fig. 3. Whilst localised agreement can be  
3 identified, for example in the District of Keewatin, the general pattern is of eskers occurring  
4 in the gaps between potential meltwater conduits. This arrangement is particularly  
5 noticeable in the North West Territories and Quebec (Figs. 3b,c). Moreover, there is no clear  
6 correlation between mapped eskers and the persistence of potential meltwater conduits  
7 throughout deglaciation (Fig. 3). This is demonstrated in the Ungava Bay region (Fig. 3c),  
8 where conduits occur repeatedly and in similar locations throughout deglaciation, but  
9 eskers are rare.

10

#### 11 4.2 *Directional conformity of eskers and modelled subglacial drainage routes*

12 The length of directional conformity,  $L$ , between esker ridges and modelled subglacial  
13 drainage routes ranges from  $<1$  km to  $>100$  km (Fig. 4). The tracking algorithm matches with  
14 65% of the total length of all eskers. The median length of  $L$  is 5.4 km, the 5<sup>th</sup> and 95<sup>th</sup>  
15 percentiles are 0.5 km and 42 km respectively, and there is a long tail of outliers that  
16 includes matches of  $>100$  km in all except the 7.5 and 7 ka time-slices. Length distributions  
17 for  $L$  are similar at each modelled time-slice, but there is a shorter tail of outliers compared  
18 to the eskers (Fig. 4). Indeed, longer eskers conform less well to the direction of modelled  
19 subglacial drainage routes, with the fraction of  $>\sim 50$  km long eskers tracked  $<20\%$  (Fig. 5).

20 The difference in distance from the ice margin between the best and average fits of the  
21 modelled subglacial drainage and esker directions (Fig. 6a) reveals a positively skewed  
22 distribution (skew: +3.09), with the peak off-set towards negative values (i.e. the fit is better  
23 than average when closer to the ice margin). Fig. 6b illustrates the difference in time

1 between the best and average fits of the modelled subglacial drainage and esker directions.  
2 The result is a positively skewed distribution (skew: +2.2), with the peak off-set towards  
3 negative values, i.e. the fit is better than average when eskers are formed during later time-  
4 slices.

5 To evaluate whether or not the distribution of  $L$  may have been produced by chance we  
6 compared the results against the probability of tracking continuing along a randomly  
7 wandering esker (Fig. 5). As  $\theta$  is  $\pm 45^\circ$ ,  $90^\circ$  of a possible  $180^\circ$  forward arc, the probability  $p$   
8 of a 5 km step (i.e., matching  $E$ ) randomly orientated with respect to the previous one also  
9 being tracked is 0.5. The chance of proceeding to the  $n$ th step is  $p^n$ . We ran five stochastic  
10 simulations using the length distribution of the eskers. Fig. 5 shows the probability density  
11 of the eskers (blue line), the tracked eskers ( $L$ ) (red line), and an average of the five  
12 stochastic simulations (black line). In initial tracking acquisition, and progressing from the  
13 first to the second 5 km step,  $L$  does little better than random. This may be due to resolution  
14 issues, measurement 'noise', or local factors dominating ice flow at this scale or near the  
15 margin. Further tracking (>10 km) occurs much more frequently than we would expect by  
16 chance, which implies an association between eskers and the modelled subglacial drainage  
17 direction output.

18

## 19 **5. Discussion**

### 20 *5.1 Spatial association between eskers, palaeo-ice stream locations and modelled* 21 *subglacial flow concentration*

22 Our results show that eskers do not coincide with the dominant drainage routes where high  
23 concentrations of meltwater (>100 cm/yr) are modelled and routed subglacially (Figs. 2-3).

1 Indeed, there is a highly significant statistical tendency for eskers to occur in regions of low  
2 subglacial flow concentration ( $<100$  cm/yr), in the gaps between major drainage routes (i.e.,  
3 left of the vertical line in Fig. 2). The highest modelled subglacial flow concentrations are  
4 associated with palaeo-ice stream locations (Fig. 2), where eskers are rare (Fig. 7). In  
5 particular, major marine terminating palaeo-ice stream troughs with large drainage basins,  
6 such as Hudson and M'Clure straits, are predicted to have had significant volumes of  
7 subglacial meltwater routed down them (Fig. 3). This is unsurprising given ice streams  
8 require warm-based ice, produce large volumes of subglacial water by enhanced frictional  
9 melting (Engelhardt and Kamb, 1997; Kamb, 2001) and typically occur in topographic  
10 troughs (cf. Winsborrow et al., 2010) where meltwater is focussed. The pattern of  
11 meltwater drainage is therefore strongly controlled by the evolution of fast ice-flow in the  
12 3D-MUN GSM. Significantly, fast ice-flow in the model evolves freely as basal ice approaches  
13 the pressure melting point, and it is able to reproduce most major palaeo-ice streams  
14 (Stokes and Tarasov, 2010) though the shallow ice approximation employed and model grid  
15 resolution would caution against over interpretation of small-scale flow features.

16 There are a number of explanations for the relative absence of eskers on the terrestrial  
17 portions of Laurentide palaeo-ice stream beds where subglacial water was concentrated  
18 (Fig. 7). First, eskers tend to form on hard beds in Röthlisberger channels, while ice streams  
19 are more typically floored by soft sediments that are predicted to drain water through a  
20 distributed network (Clark and Walder, 1994). Secondly, increased ice creep associated with  
21 high basal velocities will promote the formation of high pressure distributed drainage  
22 networks (e.g. Kamb, 1987; Bell, 2008). Thirdly, any eskers that are deposited are unlikely to  
23 be preserved due to the enhanced erosional potential of fast-flowing ice (Boulton, 1996).

1 The formation of eskers in inter ice-stream regions, characterised by low subglacial flow  
2 concentration, may seem counter-intuitive because conduit location and growth are  
3 thought to be associated with high water discharges (e.g. Shreve, 1972; Schoof, 2010;  
4 Hewitt, 2011; Werder et al., 2013). Indeed, it implies that large volumes of subglacial  
5 meltwater were not significant in the formation of conduits and eskers beneath the LIS, a  
6 point further demonstrated by the absence of suitable palaeo-subglacial lake locations on  
7 the relatively flat Canadian Shield (Livingstone et al., 2013a). An alternative source of water  
8 is therefore needed, and the most likely option is the penetration of surface meltwater to  
9 the bed (e.g. Zwally et al., 2002). Critically, our model does not allow moulin formation or  
10 supraglacial inputs, instead assuming that all meltwater was generated subglacially.  
11 However, eskers on the Canadian Shield become more frequent later during deglaciation  
12 (e.g. Prest, 1968; Aylsworth and Shilts, 1989), and this coincides with increased rates of  
13 atmospheric warming and ice-margin recession (Storrar et al., 2014b). This suggests a  
14 genetic link between supraglacial meltwater inputs and esker formation (see also Shilts,  
15 1984; St-Onge, 1984; Aylsworth and Shilts, 1989; Hooke and Fastook, 2007; Burke et al.,  
16 2012).

17 Basal conditions in inter ice-stream regions may also have been conducive to subglacial  
18 conduit and esker formation. For instance, slower ice flow, where creep closure and glacial  
19 erosion are reduced, is more suitable for R othlisberger channel formation, and is consistent  
20 with observations of eskers in other deglaciated and modern landscapes (e.g. Price, 1966).  
21 Moreover, conduits could have been relatively small compared to the size of the esker  
22 produced, with growth occurring slowly by the melt out of dirty ice rather than by large  
23 discharges through a large channel (e.g. Clark and Walder, 1994; Warren and Ashley, 1994;  
24 Hooke, 2005; Hooke and Fastook, 2007). However, sedimentological investigations of eskers

1 comprising horizontally bedded sand and gravel deposits extending all the way across their  
2 width suggests conduits can also be the same size as the esker (e.g. Brennand, 1994).

3

#### 4 5.2 *Directional conformity of eskers and modelled subglacial drainage routes*

5 The length of agreement between modelled subglacial drainage directions and eskers is  
6 typically <10 km (Fig. 4) and the ability to track eskers is improved when the ice-margin is  
7 closer (Fig. 6a), just prior to deglaciation (Fig. 6b). In addition, no single modelled time-slice  
8 is able to reproduce the entire esker pattern. This suggests that the majority of eskers were  
9 deposited time-transgressively in short segments of conduit close to the retreating ice  
10 margin (as frequently inferred; e.g. Banerjee and McDonald, 1975; Bolduc et al., 1987;  
11 Shilts, 1984; St-Onge, 1984; Dyke and Dredge, 1989; Hooke and Fastook, 2007). This  
12 interpretation is consistent with a supraglacially fed drainage system, where more  
13 meltwater is produced and able to penetrate to the bed at lower elevations and in thin ice,  
14 near the margin (cf. Hooke and Fastook, 2007; Storrar et al., 2014b).

15 There are also rare cases where agreement between modelled subglacial drainage  
16 directions and eskers can be tracked for hundreds of kilometres up ice (Figs. 4-5). These  
17 might be explained in two ways: (i) synchronous formation in subglacial conduits that  
18 penetrated deep into the ice sheet interior (e.g. Brennand, 2000); or (ii) incremental  
19 formation in a stable conduit that migrated up-ice during uniform ice retreat. Where the  
20 pattern of retreat was uniform, we cannot distinguish between options (i) and (ii). However,  
21 we suggest that synchronous formation of very long (>100 km) eskers is unlikely given the  
22 poor correlation with subglacial meltwater concentration (Figs. 2-3), and the high elevation  
23 and great thickness of ice in the interior of the LIS (e.g. Peltier, 2004; Tarasov et al., 2012),

1 which would have suppressed the generation and penetration of surface melt to the bed, and  
2 increased conduit closure by ice creep, respectively.

3

#### 4 5.3 *Further work*

5 This paper uses a simple model to compare the spatial pattern of esker networks with  
6 modelled subglacial drainage routes and as such a certain degree of fallibility is unavoidable.  
7 The next step is to re-run the analysis using a higher model resolution, or to take numerical  
8 models that can account for dynamic changes in subglacial water pressure and drainage  
9 configuration (e.g. Schoof, 2010; Hewitt, 2011; Werder et al., 2013) and apply them to real  
10 world situations, where the results can be compared to geomorphological data. In  
11 particular, sediment erosion, transport and deposition need to be included in these models  
12 to produce the landform imprint that results from subglacial drainage routing and evolution.  
13 Our hope is that this preliminary effort offers a useful template for future work.

14

## 15 **6. Conclusions**

16 In this paper, we explore a new approach that compares the expression of eskers beneath  
17 the former LIS with subglacial drainage routes predicted from the results of a numerical ice  
18 sheet model at specific time-steps during deglaciation.

19 Eskers are more abundant in non-ice-stream regions where modelled subglacial flow  
20 concentration is low. Moreover, we find that the length of agreement between modelled  
21 subglacial drainage directions and eskers is typically <10 km, and the agreement is improved  
22 when eskers lie close to the ice-margin just prior to deglaciation. This suggests the majority  
23 of eskers were deposited time-transgressively in <10 km segments of conduits close to a



1 retreating ice margin, and in thin, sluggish or stagnant ice where conduit closure by ice  
2 creep is less significant. We suggest that supraglacial meltwater inputs were important for  
3 supporting Röthlisberger channel and esker forming processes.

4 We find that modelled subglacial drainage routes dominated by high flow concentration are  
5 associated with palaeo-ice stream locations. The paucity of eskers along the terrestrial  
6 portion of palaeo-ice streams is probably a result of the high velocities and soft-sediment  
7 beds that promote distributed rather than channelized drainage, while any eskers that do  
8 form are unlikely to be preserved beneath fast-flowing ice.

9

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15

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12

### 13 **Figures**

14 **Figure 1:** Cartoons illustrating the methods used to investigate the relationship between  
15 eskers and modelled subglacial drainage routes. A. Spatial conformity of eskers and  
16 modelled subglacial flow concentration. The drainage routes are the total concentrated flow  
17 being transferred into each downstream cell, with each cell weighted by the basal  
18 meltwater production (cm/yr) output from 3D-MUN GSM. In this example, the darker  
19 colours indicate greater meltwater flow concentration, which pick-out a dendritic drainage  
20 network. B. Directional conformity of eskers and modelled subglacial drainage routes.  
21 Eskers are traced from the ice-margin up-ice until the agreement with the modelled  
22 drainage directions breaks down. Directional conformity between modelled drainage routes

1 and eskers represents the length along which an esker forming meltwater conduit could  
2 have occurred.

3 **Figure 2:** Spatial relationship of eskers, the domain used to map the eskers (i.e. terrestrial  
4 portion of the former LIS bed) and ice streams with varying subglacial flow concentration  
5 (ranging from 1 to 100000 cm/yr). Results are displayed as a ratio between each probability  
6 density function and the model as a whole (i.e., the population). The 20 cm/yr drainage cut-  
7 off we use to produce Fig. 3 is shown by the vertical line. Values above 1 indicate that  
8 subglacial flow concentration happens more often than you would expect by a random  
9 distribution. Numbers in brackets are the number of cells used to produce each probability  
10 density function. Note how eskers are associated with lower subglacial flow concentrations  
11 and ice streams with higher subglacial flow concentrations.

12 **Figure 3:** Spatial association between eskers and potential meltwater conduit locations. A.  
13 Composite of all possible modelled subglacial drainage routes from 12-7 ka BP (Fig. 1a, see  
14 text for details). The purple lines represent potential meltwater conduits that were  
15 relatively persistent over time and the light blue lines are those that only formed during one  
16 modelled time-slice. Panels B and C are close-ups of Keewatin and Quebec, respectively.

17 **Figure 4:** Box and whisker plots showing the length distribution of the mapped eskers (ME)  
18 and the agreement between eskers and modelled subglacial drainage directions at each  
19 time-slice. The black indented line is the median, the edges of the box are the 25<sup>th</sup> and 75<sup>th</sup>  
20 percentiles, the whiskers are 1.5 times the interquartile range and the crosses are outliers.  
21 Note the median length tracked is <10 km, although with some matches of >100 km.

22 **Figure 5:** Probability density plot showing the distribution of eskers (blue line), tracked  
23 eskers ( $L$ ) (red line), and the average and standard deviation of five stochastic simulations



1 that look at the probability of tracking continuing along a randomly wandering esker (see  
2 text for details) (black line). The simulations use the length distribution of the eskers. As  $\theta$  is  
3  $\pm 45^\circ$ ,  $90^\circ$  of a possible  $180^\circ$  forward arc, the probability  $p$  of a 5 km step randomly  
4 orientated with respect to the previous one also being tracked is 0.5.

5 **Figure 6:** Histogram showing, for any point that is tracked in  $\geq 2$  time periods: A. the  
6 difference in distance from the ice margin between the best and average fits of the  
7 modelled subglacial drainage and esker directions. Negative values indicate that the ice  
8 margin is closer when the modelled drainage direction best fits the esker direction. Bins are  
9 in 50 km intervals. B. The difference in time between the best and average fits of the  
10 modelled subglacial drainage and esker directions. If the best fit between the modelled  
11 subglacial drainage and esker directions occurred later than the average fit (i.e. when the  
12 ice-sheet was smaller) the result is negative. Bins are in 500 year intervals.

13 **Figure 7:** The distribution of inferred palaeo-ice streams (from Margold et al., 2015) and  
14 mapped eskers (Storrar et al., 2013) beneath the former Laurentide Ice Sheet. In general,  
15 eskers are rarely found on the bed of palaeo-ice streams.

Figure 1

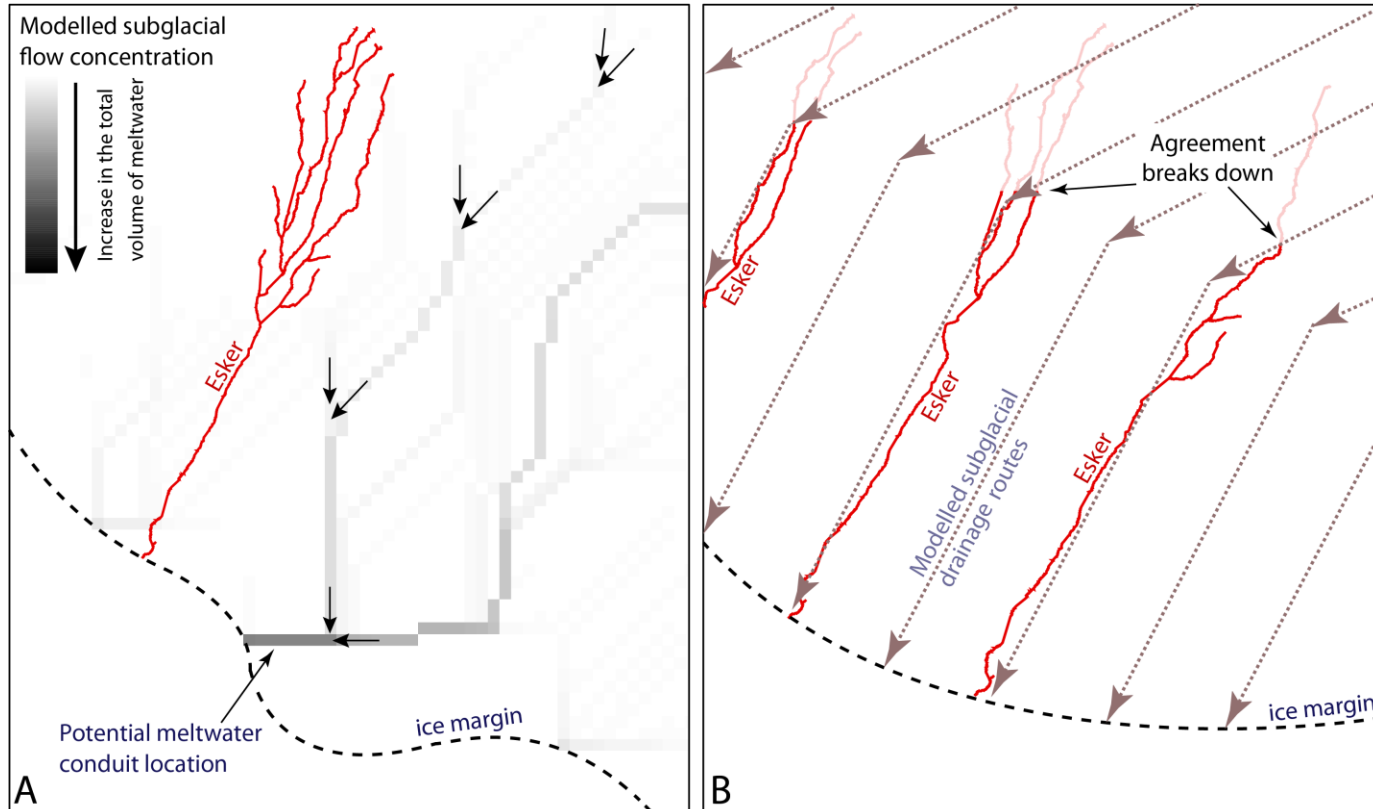


Figure 2

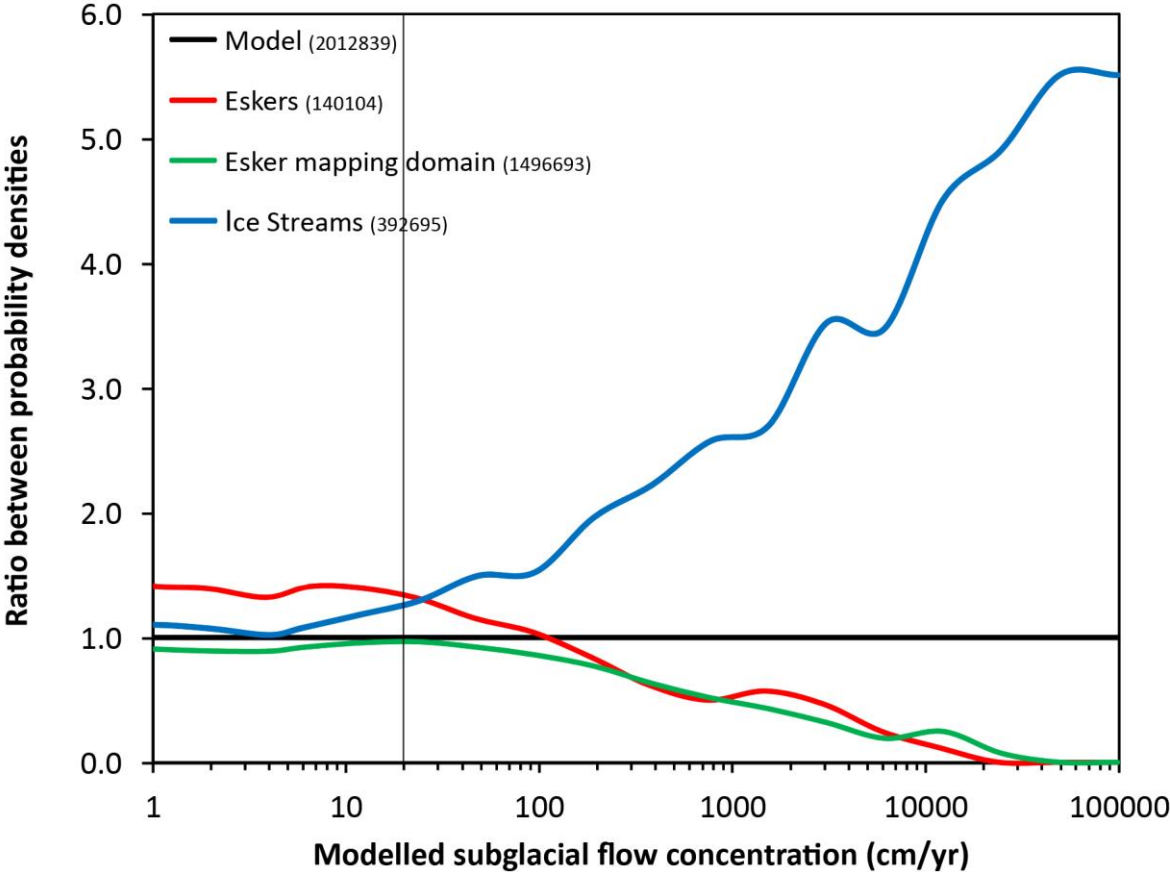


Figure 3a

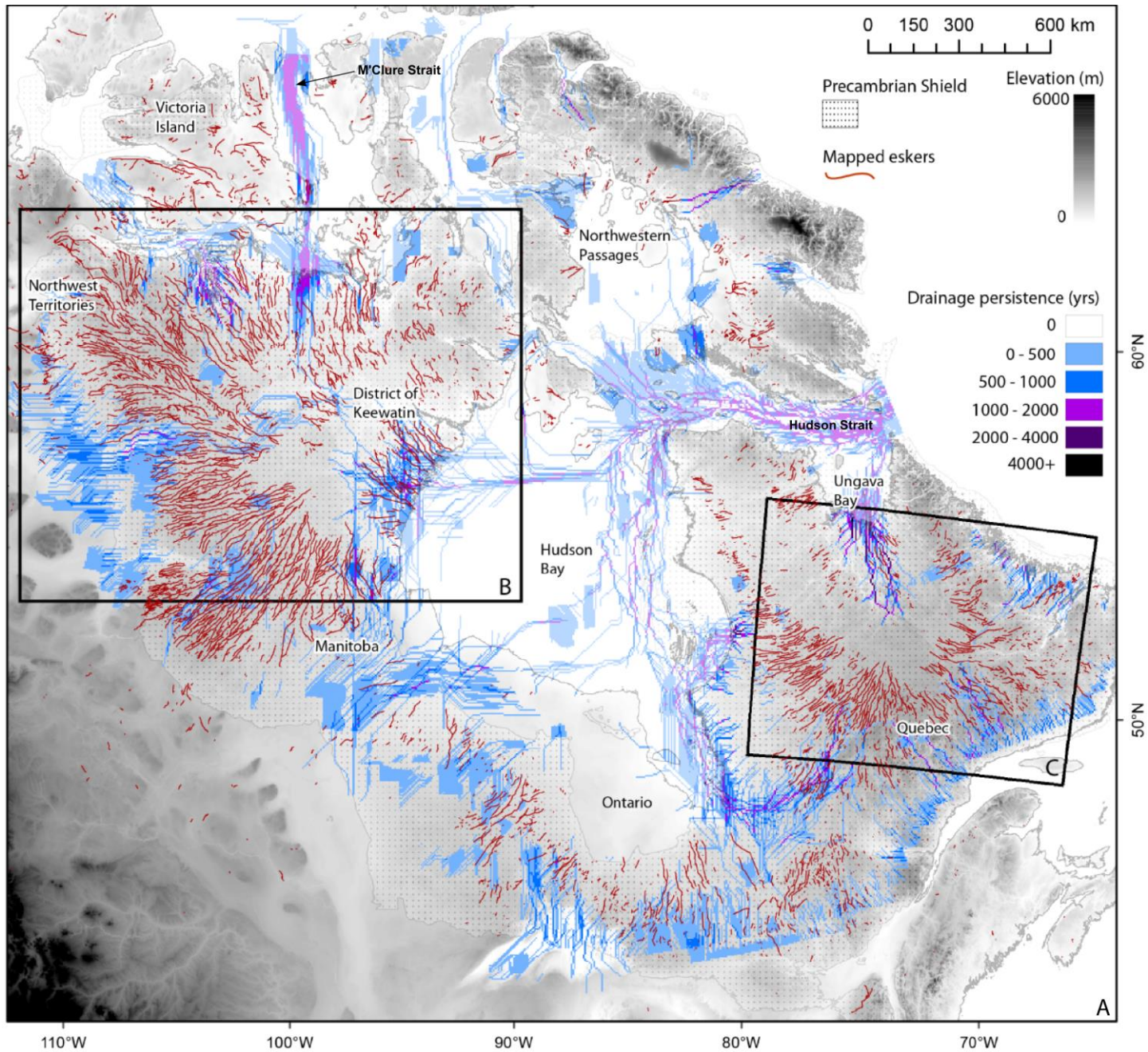




Figure 3b,c

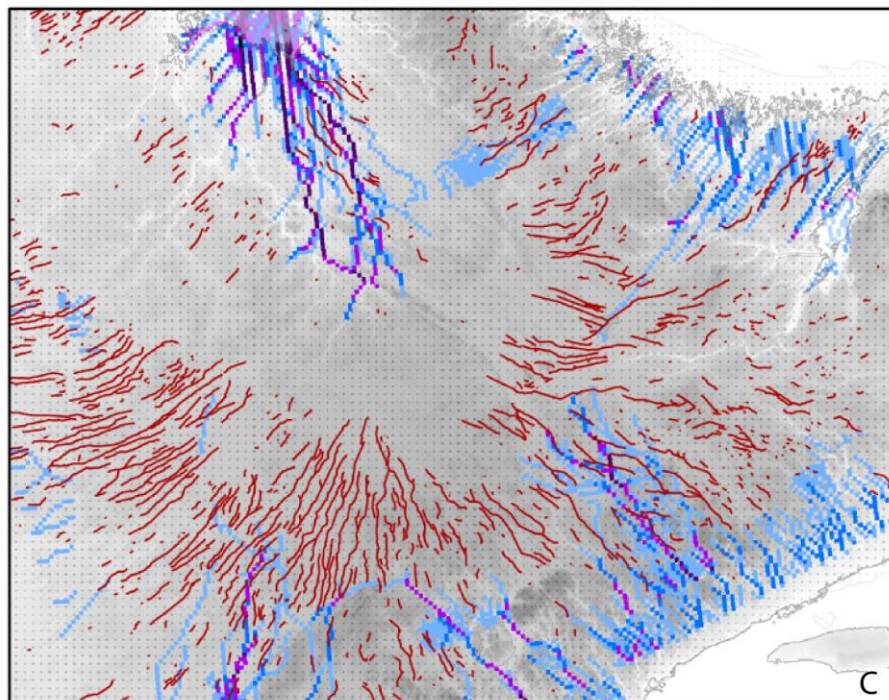
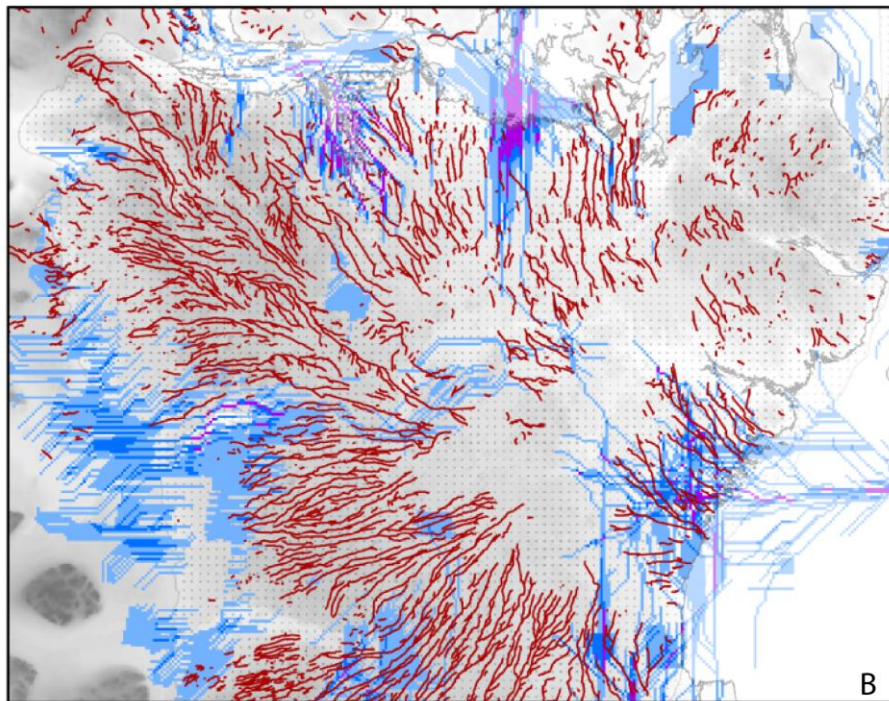


Figure 4

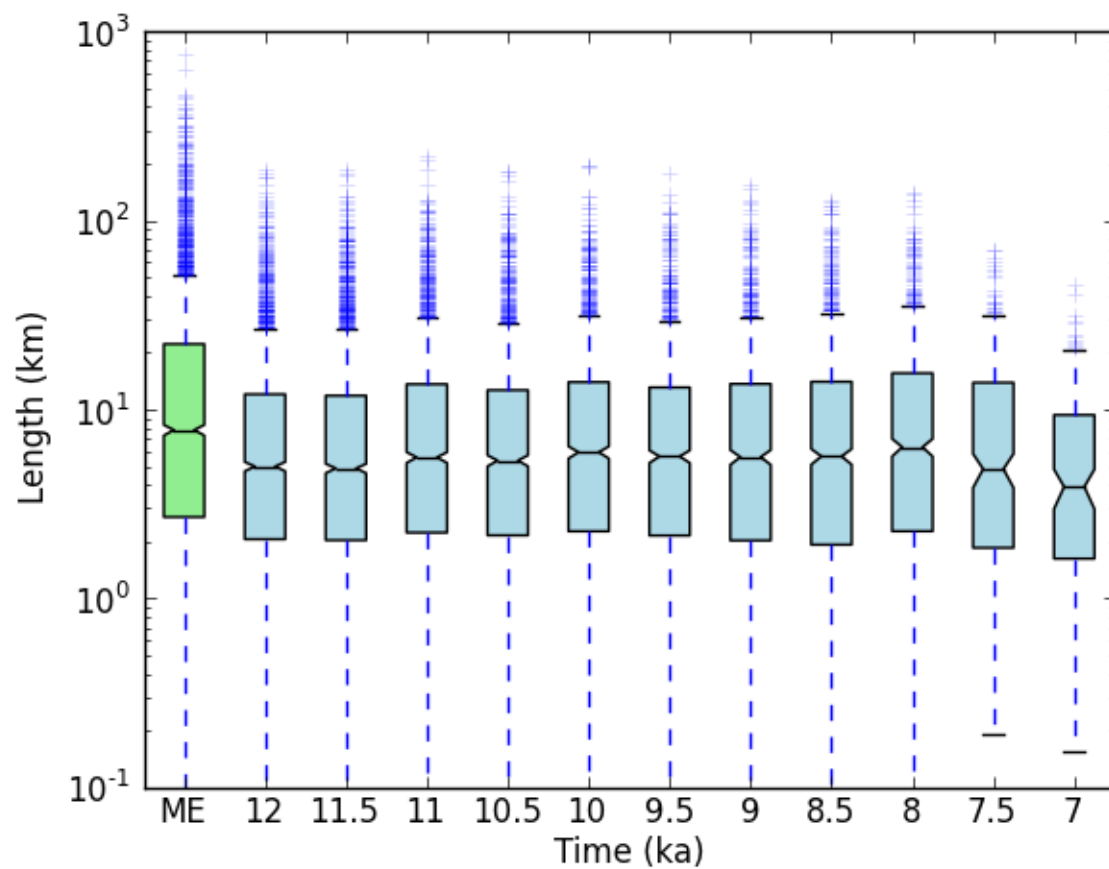


Figure 5

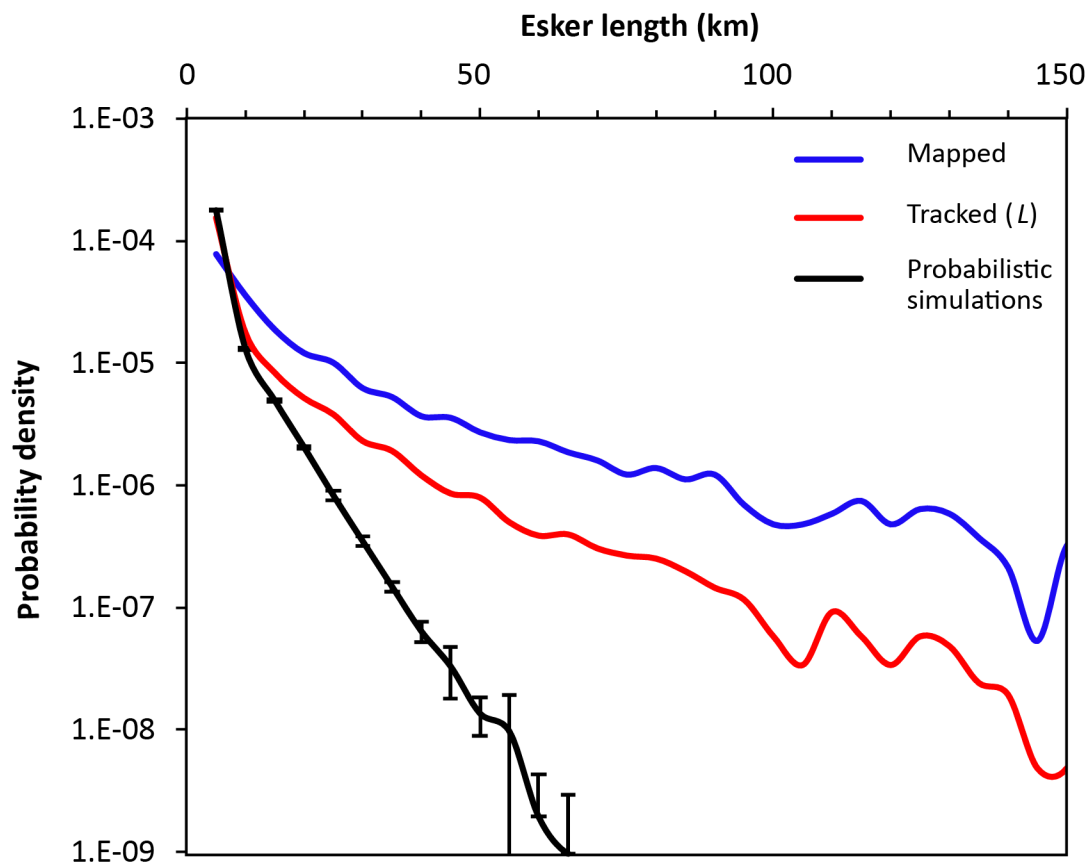


Figure 6

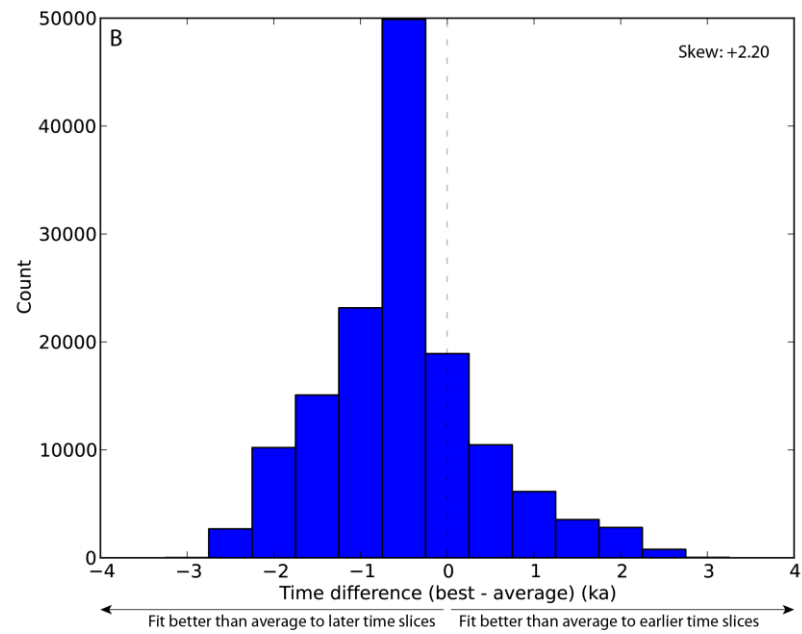
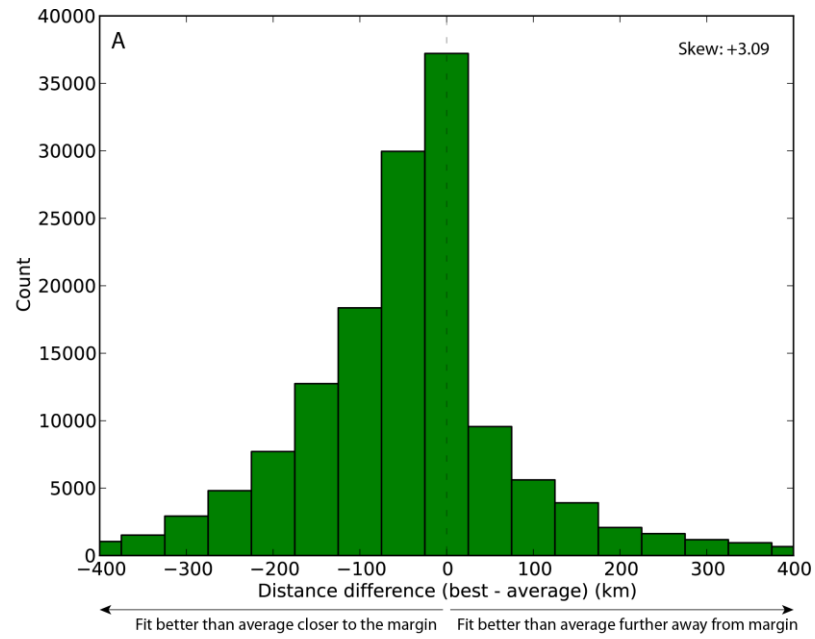




Figure 7

