Arbitrary and Necessary: Part 2 Assisting Memory

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1 Curriculum divide: arbitrary and necessary

In part 1 (Hewitt, 1999), I put forward a way of viewing the mathematics curriculum in terms of those things which cannot be known by a student without that student being informed (arbitrary), and those things which *someone* can work out and know to be correct (necessary) without being informed by others

What is arbitrary is arbitrary for everyone, in that no one can know the arbitrary without being informed by others. The arbitrary concerns names and conventions which have been established within a culture and which need to be adopted by students if they are to participate and communicate successfully within this culture The arbitrary is in the realm of memory as students cannot work out these things through their own awareness. So the students' role is to memorise the arbitrary.

A teacher needs to inform students of the arbitrary and to assist with the task of memorisation If teachers do not do so, students are left to invent names and conventions, which they are perfectly able to do, and which may serve an educational purpose, but are unlikely to coincide with the names and conventions adopted within the mathematics culture.

If something is necessary, this does not imply that *everyone* can know it through their own awareness It does mean that with sufficient and appropriate awareness, it is possible for the necessary to be known for certain without the need to be informed by others. The necessary concerns properties and relationships and these lie in the realm of awareness A student's role is to educate their own awareness, so that the student comes to know for certain that a certain property or relationship must be so

A teacher's role is to provide suitable tasks to help students with their role of educating their own awareness. Should a teacher choose to inform a student of what is necessary, then I describe this as *received wisdom* which a student either has to memorise, or has to carry out additional work to try to turn this received wisdom into something which they can come to know through their own awareness

	ARBITRARY		NECESSARY	
TEACHER	teacher - informs	teacher - does not inform	teacher - informs	teacher - gives appropriate activity
STUDENT	students - have to memorise	students - have to invent	Received wisdom students - have to memorise unless they succeed in using their awareness to come to know	students - use awareness to come to know

Figure 2 A summary of teacher choices and consequential student ways of working

Figure 1 gives a summary of the above and although part 1 of the article discusses other aspects of what I call *givens* and *generates*, these will not be discussed further within this piece. Instead, I develop in detail the implications for teaching of this 'arbitrary' and 'necessary' divide, and discuss practical ways forward for teachers who plan lessons and work in the classroom with sensitivity to this divide This article concentrates on teaching and learning those aspects which are arbitrary and a further article (part 3 educating awareness) will relate to teaching and learning the necessary

2 Implications for teaching: the realm of memory

The arbitrary lies within the realm of memory, so a teacher's role is twofold. Firstly, the arbitrary needs to be provided, and secondly, students' need assistance with their task of successfully memorising the arbitrary The second role is just as important as the first, since without students successfully memorising it falls back on the teacher to inform students once again and the memorisation process starts all over again.

Memorising the arbitrary is an important issue, even though this is not where mathematics lies. The learning of names and conventions plays a vital role in engaging with mathematics and communicating with others about mathematics, but is not mathematics itself It can be a useful exercise for a mathematics department to look at their scheme of work and ask the question: *how much of this scheme of work is about memorising names and conventions*? Such work can sometimes be mistakenly viewed as time spent on mathematics when the real purpose of learning names and conventions is to facilitate work on where the mathematics really lies – with what is necessary

So although this section addresses the serious job which has to be done in informing students of the arbitrary and assisting them in their task of memorisation, time spent on this will only be of value if the learning of names and conventions is not seen as an end in itself, but as being purposeful in supporting work on what is necessary.

3 What is memory?

Sometimes, the term 'memory' is used in a relatively wide context, almost as if everything we retain is held in our 'memory'. This metaphor of storage space is not one which I find particularly useful when considering how people learn and the work which needs to be done in order for something newly encountered to be available at a later date, for certain actions to become more crafted or for certain meanings to be

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developed. I am particularly interested in looking at learning in terms of the nature of the work that someone needs or does not need to do in order for that learning to take place

Thus, I will not answer the question 'What is memory?', because I wish to attend to the human activity which is involved in a learning process rather than attend to the mental or physical phenomena that are collectively called 'memory'. I am more concerned with the question 'What is it to memorise?', and this is a question I will attempt to answer, firstly through differentiating between the activity of memorising and other learning activity.

In the mid-eighteenth century, Rousseau (1762/1986) was critical of the teaching of geometry:

> Instead of making us discover proofs, they are dictated to us; instead of teaching us to reason, our memory only is employed (p. 110)

Proofs can, in his words, be reasoned rather than memorised. In my phraseology, something which is necessary is presented as if it were arbitrary (received wisdom) and to be memorised. Quite different work is required from learners if they are to reason (rather than memorise), with abilities other than those required to memorise being evoked. Rousseau, for example, subsequently refers to imagination. Reasoning requires using awareness of relationships and properties relating to the theorem which is to be proved, whereas memorising does not require this. It is perfectly possible to memorise a proof as a collection of arbitrary symbols placed in an order and certain particular positions. I must confess to knowing this is the case since I passed some of my mathematics degree examinations on this basis. Reproducing the symbols involved in a proof does not imply that there is meaning given to the symbols' relationship to the mathematics involved in the theorem

Skemp (1987) observed that within his research:

> there seemed to be a qualitative difference between two kinds of learning which we may call habit learning, or rote-memorizing, and learning involving understanding, which is to say intelligent learning. (p. 15)

As one of my students pointed out in a reflection of his:

> Personally, I cannot learn things by memorising a vast amount of facts because I am not good at this. I learn things by trying to understand them. In this way, I avoid having to learn a lot of facts. This is why I prefer maths and science because my knowledge of these subjects is more dependent upon my understanding of them rather than my ability to memorise facts.

Such comments indicate noticing at a personal level the difference between memorising and other types of activity where what is produced is in some way personally generated through one's own awareness rather than reproducing someone else's awareness or knowledge. What, though, is involved in trying to memorise? Memorisation is still a complex activity which, contrary to Skemp's phraseology above, often requires 'intelligent' activity.

Wood (1988) talked of:

> the *multiplicity* of intellectual activities that we commonly (and mistakenly) lump together under headings like 'memory' (p. 60, *his emphasis*)

He discussed the contrast between pre-school children's difficulty with certain memory tasks in experiments he carried out and the fact that many children would, for example, recall where they had last seen a camera when told that it had been lost and asked to try to find it. Wood commented that:

> The abilities deployed by preschoolers in the situations just outlined [finding the camera] are different in kind from those demanded in the first experiments presented, which demanded 'deliberate' memorization. Basically, what young children learn and remember are things that arise as a 'natural' and often *incidental* consequence of their activities. No one needed to alert the preschoolers in the playgroup to the fact that they would be expected to remember the location of an object. Setting out deliberately to commit a body of information to memory is a quite different affair from such examples of natural or spontaneous remembering, where what is subsequently recalled is something one literally handled, attended to or in some way had to take cognizance of in the course of doing a practical activity (p. 60, *his emphasis*)

The difference between *setting out deliberately to commit a body of information to memory* and other ways in which something may be recalled, such as the last place a camera was seen, is very significant in terms of the nature of the personal work involved. To consider this difference, try to memorise the 'word' *pimolitel* and try to be able to say the 'word' at the end of reading this article, without having to look back to re-read it. I also invite you to recall some detail of a room which you have visited on very few occasions, perhaps even only once

These two examples differ in the following ways:

(a) the work which is carried out at the time of first encountering the word/room;

(b) the work carried out in the time between that first encounter and the request to remember the word or to recall detail of the room;

(c) the work required at the time when remembering/recalling is required.

Furthermore, I suggest the differences are usually as shown in Figure 2.

Time	Nature of Work Required	
	Remembering an arbitrary word	Recalling detail of a room rarely visited
(a) At time of encounter	Conscious activity to 'commit to memory'	No conscious activity to 'commit to memory'. However, engagement in some activity (perhaps nothing associated with the detail of the room itself) whilst in the room.
(b) Between encounter and request to remember/recall	Frequent practice	None.
(c) At time of request to remember/recall	Conscious activity to reproduce word	Conscious activity to 'enter back' into being in the room and articulating (perhaps for the first time) some detail

Figure 2 Differences between remembering and recalling

My anecdotal experience of carrying out a similar activity with a number of groups of people is that there was limited success at remembering the word successfully. Usually less than half were successful at remembering the word after approximately an hour. Those who were successful, and indeed some of those who were not, reported carrying out some work during the hour and all who engaged with the task (as some – perhaps you? – chose not to) reported conscious activity at the time of being presented with the word. I have found that there was very little success at recall after periods of weeks, despite the fact that some people had put in a considerable amount of conscious effort in the first few minutes, perhaps longer, of being told the word.

Regarding the request for recalling the detail of a room rarely visited, all were able to express some detail, which is not surprising since otherwise a different room may have been chosen. However, this does not take away from the fact that for many people a considerable amount of detail was expressed, none of which was ever consciously 'committed to memory'. In fact, no conscious work to hold this information had been carried out at all in advance of my request, because no one had known that such a request was ever going to be made. Furthermore, there may have been long periods of time – months, perhaps years – between the last, and perhaps only, visit to the room and this request being made.

I will restrict my use of the term *memorisation* to describe a conscious process where someone, *at the time of receiving information and thereafter*, activates techniques to hold that information so that it will be available at a future time. This contrasts with the activity of recalling details of a room, which is concerned with affectivity, the power of imagery and accessing images. (The detail of what is involved with recalling is not something I will explore further here.)

4 Assisting memory

The realm of memory concerns those things which are arbitrary – names and conventions – due to their nature as socially agreed entities. One role a teacher has is to inform students of what is arbitrary. However, if the teacher's role stops there, then the students are left with having to motivate themselves to do the work required at the time to begin the memorisation process, and to summon the discipline to carry on with the required practice in the weeks and months to come.

So what can a teacher do to assist students in their task of memorising? One common answer is for a teacher to provide regular testing. This provides an opportunity for students to practise those things which they have successfully remembered, and to become aware of those things which they have not. This can be a useful exercise, if time and attention is given for students to identify which things they had not remembered and to begin the process of memorisation over again. However, all too often time and attention is not devoted to such activities: instead, some teachers feel that their job is done by providing the test and moving on to other topics once the answers are given.

This means that only those things which had been memorised are practised, and the other half of the job is left unattended to. At the same time, new things are told to the students and more is required to be memorised. So gradually more things are forgotten and time is not devoted to renewing and maintaining the memorisation process. A half-life process of forgetting begins and continues: little wonder that students retain relatively little, that teachers feel obliged to teach the same things year after year and that students end up getting the questions wrong anyway in their formal examinations.

I will consider what is involved with memorising and some ways in which a teacher might act in an attempt to help students memorise and use names and conventions within an appropriate mathematical context. The process for memorising a name involves not only memorising the name but also linking it to appropriate properties and relationships (see Figure 3).

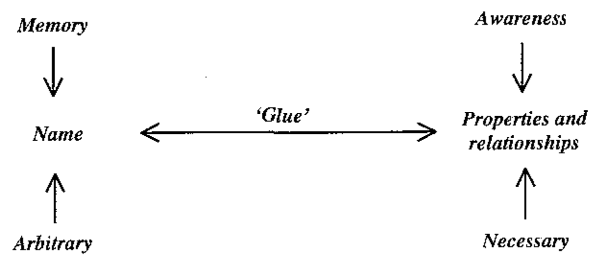

Figure 3 Linking a name with its associated properties and relationships

Students may be able to memorise a word, such as 'isosceles', but associating it with particular properties of a triangle is another matter. This 'glue' to help stick the name to certain properties or relationships also needs to be addressed, as well as the actual memorisation of the name itself. So the learning of a name (itself in the realm of memory) is linked to the realm of awareness through properties and relationships associated with that name. It is a pedagogic decision as to when attention is given to each of these realms. Is the learning of the name stressed initially? Or are the properties and relationships stressed initially? Or are both addressed simultaneously? I will consider each possibility in turn.

4.1 Initial stressing of names and conventions

Stressing names can be useful in that some words are unusual, such as *commutative*, and not that easy to say for some students. Emma Leaman has talked to me of spending a while with students asking them to chant a new word, so that they become practised in the physical movements of their mouths, tongues, etc., required to say such a word. This helps with the practice of saying the word, but there is still the job of helping students link the word with certain properties. A common approach to form this link is to state it as part of a definition for the word: for an example, see Figure 4.

> <u>***Definition***</u>
>
> ***An operation, $\oplus$, on a set S, is commutative if $x \oplus y = y \oplus x$ for all x and y in S.***

Figure 4 One way of introducing a word

This keeps the emphasis on memory Not only is the word something to be memorised, but the presentation of a definition can encourage a student to feel that the *property*, which forms part of the definition, has to be memorised as well. At this stage, the property is not necessarily something of which students are aware.

Word first – properties later can lead to an *extra* burden being placed on memory with both the word and the properties being placed within the realm of memory This will only change if the student manages to develop an awareness of the property from what has been stated in the definition The need for generality within a definition can mean that the language does not make the property particularly accessible for students Furthermore, time is not always given to help students turn a statement given by the teacher (received wisdom) into something which they know through their own awareness.

If time and attention are given to this, then the property can return to its rightful place of being within the realm of awareness. However, too often this is not the case and so mathematics can feel like something to be memorised rather than understood. The issue of helping the 'glue' to stick - the association between word and properties - still has to be addressed. A definition only introduces this link; it does nothing to help establish it

Imagery and association are commonly used by those who specialise in the ability to remember vast collections of arbitrary things (see, for example, Luria, 1969). This can be utilised by teachers by offering images to help link words and conventions with properties and relationships Indeed, it is used anyway by students themselves as it is a technique frequently deployed within daily life to help memorise certain things For example, memorising a telephone number through the visual and kinaesthetic image gained from a finger moving across the keys on the phone A friend of mine describes moving his finger over an imaginary set of keys and watching where it goes when he is asked by someone for a particular phone number.

In many classes, I have worked with a *Think of a number* task (see OU, 1991 for more detail) where I describe a number of operations I carry out on an unknown number and give the final result. The object of the lesson from my perspective is to help students articulate the inverse operations involved in getting back to my number and be able to write the expressions involved in 'correct' algebraic notation. During this task, I have used aural images to help draw attention to an aspect of the arbitrary notation. For example, in one lesson I was describing what I was doing to my number and had got the following already written on the board:

$$\frac{25\left(\frac{x-16}{4}+16.289331\right)-2}{43.92}$$

Then I was going to add 15, and students often write the addition sign continuing on from the top line of the expression, rather than writing it after the division sign.

$$\frac{25\left(\frac{x-16}{4}+16.289331\right)-2+15}{43.92}$$

So, I offered an image to help students retain this arbitrary convention within algebraic notation. As I said *add*, I pointed the chalk to the left-hand end of the division sign and whistled a single rising note as I moved along the division line to the right-hand end, and then stopped whistling. My hand continued moving the chalk in the same direction a little past the division line and wrote a plus sign

$$\frac{25\left(\frac{x-16}{4}+16.289331\right)-2}{43.92}+$$

The aural image is unusual and I have found that it sometimes stays with students as they begin to write their own expressions. Sometimes I hear students making a similar noise as they are working, and making the noise as they are moving along the division line and about to put an addition or subtraction sign after it Images can be retained without the need for anything to be memorised Along with the image comes the movement and the position of the addition or subtraction sign

Mnemonics such as SOHCAHTOA are frequently used to help students memorise which ratios are to be given the names of 'sine', 'cosine' and 'tangent' However, there is an issue about another image sometimes used in connection with trigonometry This is a triangle (see Figure 5) which is offered to help students construct an appropriate equation depending upon whether it is the angle, the length of the Opposite side, or the length of the Hypotenuse which is to be found

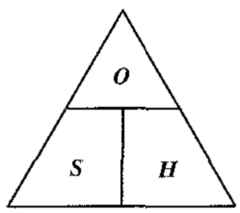

Figure 5 An image designed to help with trigonometry

If it is the angle θ which is to be found, then the image is read as $\sin \theta = \frac{O}{H}$ because visually the 'O' is on top of the 'H'. Likewise, if the Hypotenuse were required, then it would be read as $\frac{O}{\sin \theta} = H$ If it were the Opposite which is required, then it is read as $O = \sin \theta \times H$, because the 'S' and the 'H' are next to one another on the same line. This is clearly an aid to memory.

However, there is no attempt to educate someone's awareness of why these variations are like they are, only to help someone memorise them. Several teachers have expressed to me how successful this image is, and I believe them. It is a powerful image which assists *memory* However, the manipulation of an equation is in the realm of awareness, and this image completely avoids any sense of the necessary. This image only helps equations of the form $a = \frac{b}{c}$ and a student meeting a different form of equation, such as $v = u + at$, will be none the wiser as to how to manipulate this equation

The triangular image produces short-term and highly limited results It helps a student to ignore awareness and memorise instead and as such compares poorly with addressing directly the need for students to be able to manipulate linear equations through awareness, where students know what they are doing and why they are doing it, and that their awareness can be applied to an infinite variety of equations

4.2 Initial stressing of properties

Stressing the properties first can be done through devising a task where such properties are likely to become the focus of attention. At the same time, a teacher needs to make a conscious decision not to introduce the associated word until later. For example, consider two tasks: the first might involve using the numbers 1, 2 and 3 along with '-' to make as many different answers as possible where the rules are that the numbers cannot be 'combined' by juxtaposition to make 12, for example, and where all three numbers must be used once and only once This is followed by a second, similar, task where the same numbers are used but with '+'.

It is likely that students will comment (among other things) on the fact that although several different answers are possible with '-' (such as $3 - 2 - 1 = 0$; $2 - 3 - 1 = -2$; $1 - 2 - 3 = -4$), only one answer is possible using '+'. Discussion of why this may be the case can lead to a conscious awareness that $3 - 2$ and $2 - 3$, for example, are different, whereas $3 + 2$ and $2 + 3$ are the same The time when students are consciously aware of appropriate properties is a time for introducing the associated word. The word then has an awareness to which it can become glued - there is a hook on which the name can be hung.

The general principle here is only to introduce a word when the associated properties are already established and form the focus of students' attention. This helps the initial contact between the word and its related properties To help the 'glue' to stick will require this to be only the beginning of the process, with the activity continuing, such as looking at other operations or other sets of numbers, with the word being frequently used by the teacher and also with the teacher encouraging its general use within discussions.

Before leaving this example, I will point out that this is an example of an artificially created pedagogic task which is designed to 'force' an awareness. I use the word 'force' in this context following Gattegno's use of the word (see, for example, Gattegno, 1987), to mean that engagement in the task is likely to result in a student becoming aware of a property which was pre-determined by the teacher.

I mentioned that in order to help the glue to stick, it would be important for the word to be continually used within discussions Although students may be encouraged to use the word, in reality some may feel reluctant to do so. So what can a teacher do in order that even reluctant students do actually practise using the word? The idea of 'forcing' can be used here as well, by devising an artificial task which requires (indeed *demands* as part of the structure of the activity) that the word to be used again and again.

As an example, consider some words associated with triangles such as 'equilateral', 'isosceles', 'right-angled', 'acute' and 'obtuse'. The task involves taking copies of the set of triangles in Figure 6. Students work in pairs where only one person has drawings of the target shapes in Figure 7 This student has to get their partner to create each target shape in turn. However, there are strict rules of communication:

(a) no pointing, either directly (i.e. physically pointing) or indirectly (i e. "the one next to you") by either party is allowed;

(b) only the person with the sheet of target shapes is allowed to speak;

(c) all references to triangles must start with one or more of the following words: equilateral, isosceles, right-angled, acute-angled, obtuse-angled

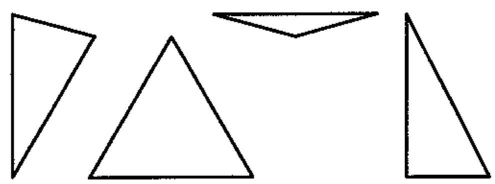

Figure 6 Starting triangles

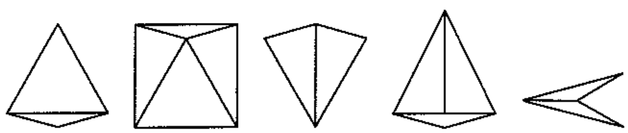

Figure 7 Target shapes

These artificially created rules are designed to 'force' the practice of naming whilst attention is with a different task, in this case creating the target shapes. Thus, rather than someone looking at the drawing of a target shape and physically moving the actual triangles into that shape, the rules of the task forces language to take a mediating role in achieving the desired goal (see Figure 8).

This is an explicit, pedagogic example of the role that all signs play as mediated activity As Vygotsky (1978) commented, his analysis provided:

> a sound basis for assigning the use of signs to the category of mediated activity, for the essence of sign use consists in man's [*sic*] affecting behavior through signs (p. 54)

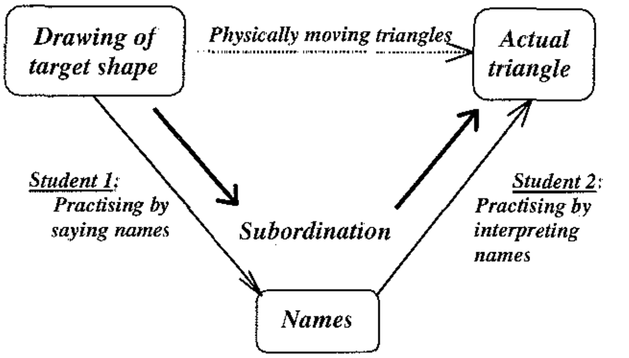

Figure 8 A pedagogically designed task to subordinate the practice of naming within an artifically created task

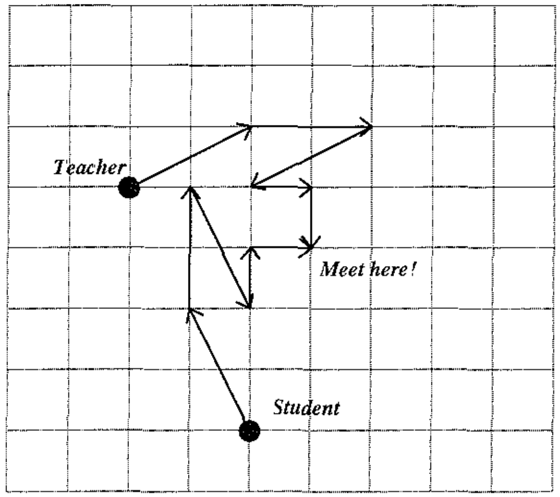

Figure 9 A game of Do we meet?

Such a task involves the features of subordination (Hewitt, 1996), where something is required to be practised in order to carry out a different task. Furthermore, the aspect being subordinated, in this case the naming of shapes, has a direct effect on the main task. For example, if student 1 uses an incorrect naming of the triangle they wanted student 2 to move, then they will notice student 2 picking up a different triangle from the one they intended. Thus, student 1 gets feedback on the accuracy or otherwise of what is said.

Likewise, student 2's interpretation of what student 1 says is given feedback by what student 1 says (e.g. "No, I would like the acute-angled isosceles triangle to be moved, not the obtuse-angled triangle"). In fact, several things are being practised through this task, not least properties of triangles, in order to direct and manipulate the triangles into their appropriate positions. Thus, attention is with the mathematics of properties of triangles whilst the arbitrary components, the names, are still being practised. This means that the practising of the arbitrary does not mean that mathematical activity need take a back seat. Subordination offers a way in which the two can be combined within a suitable pedagogically designed task.

Subordination is achieved through designing the rules of a task so that students are 'forced' into having to go through something (call it A) in order to work on the main task (call it B). In the above example, A was a set of names and B was the task of fitting triangles together in a particular way. However, the general principle can be applied to other situations. For example, in the task *Do we meet*? teacher and student have different starting positions on a grid. The student is asked to move somewhere else and draw a line to where they have moved. The teacher will move at the same time according to an unsaid rule connected to the student's movement. The aim is for the student (without knowing the rule in advance) to move to a place where they will make the teacher meet up with them at the same place. See Figure 9 for an example of a game with the teacher's (unsaid) rule being a 90° rotation clockwise.

Once students have watched a game or two, they can play this in pairs with the same or different rules for movement. The game offers many possibilities of learning about transformations and symmetry which I will not go into here. However, whilst this learning is taking place, it is possible to subordinate another part of the mathematics curriculum at the same time: vectors and vector notation. At present, the game is played by a student actually drawing each of the lines. Instead of this, an artificial (and pedagogically significant) rule can be created whereby the student is not allowed to draw or point, but is only allowed to write a vector, such as $\binom{-1}{2}$.

The teacher (or whomever is playing that role) then draws the movement according to the written vector, followed by their own movement according to their secret geometric rule (there is the option, of course, of also writing out the teacher's movement as a vector as well). This forces practice of the arbitrary conventions of vector notation whilst progress is being made with issues regarding transformations and symmetry. I describe this as *practice through progress* - practising one part of the curriculum whilst progressing in another part of the curriculum.

I argued in Hewitt (1996) that this type of practice is particularly effective in helping something become fluent, which classically is what is required with the adoption of the arbitrary. The nature of the arbitrary is such that it is required to be adopted and used with fluency rather than be the focus of attention and conscious consideration. Thus, practising the arbitrary whilst progressing with the necessary is particularly powerful, since effective practise of the arbitrary takes place whilst time is not 'wasted' doing this, since simultaneously attention is with the mathematics of the necessary.

The computer can be a powerful resource for the notion of subordinating certain names and conventions, since there always needs to be a way in which a student communicates with the computer. As Ainley (1997) comments on the role a computer plays with its 'teacher' (Ainley prefers to consider a student as 'teaching' the computer, as opposed to the computer 'teaching' the student):

> the computer is pedantic: it disciplines communication with its teacher by only accepting instructions which follow particular conventions (p 93)

Careful choice by software developers can 'force' communication to be through arbitrary names and conventions which then have to be practised whilst the software's main task involves opportunities and material to work on the necessary. [1]

4.3 Initial equal stressing of both properties and names

Names need to be provided by a teacher since they are arbitrary: properties are necessary and as such can be known through awareness. So initial equal stressing will require attention to be placed on becoming aware of new properties, whilst introducing a new name at the same time. Taking the name 'isosceles', this would mean an initial task whereby an objective is for students to become aware of the properties associated with isosceles triangles whilst being informed at the outset of the word 'isosceles'.

One such task involves a large collection of different triangles (or drawings of triangles). A teacher may point to one triangle and make the statement: "This is an isosceles triangle", then point to another triangle making the same statement, or perhaps saying: "This is not an isosceles triangle" Students are then invited to make similar statements about a different triangle as to whether they think it is or is not an isosceles triangle. After each statement, the teacher will say whether the statement is correct or not

Variations of this activity can involve students being invited to draw a triangle which is an isosceles triangle, or it may be that the teacher introduces more names than just one. I stress here that such a task is designed for a class of students who have not met the term 'isosceles' before This is *not* a practice task nor one designed to test whether someone has remembered the word (although, of course, it could be used in that way). Thus, it starts with using a name which students have not met before and whose associated properties the students are unaware of at this stage. In using this type of task with classes, I have found no need to explain what it is I want them to do. They are invariably intrigued and wish to work out what an isosceles triangle is Of course, some choose to be lazy and want me to tell them, but it does not take long for them to realise that I am not going to do this.

An equivalent example using technology comes with the use of one particular file from the *Active geometry* [2] collection of files and tasks for use with Geometer's Sketchpad (or Cabri-Géomètre) This file - *Quadinc* - addresses the naming of quadrilaterals and has a quadrilateral on the screen which can be manipulated. At any given shape the quadrilateral takes, the names associated with that shape appear on the screen (see Figure 10 for an example when the current shape of the quadrilateral brings the words 'kite' and 'cyclic' to the screen).

Some teachers feel that they need to teach students the properties of each of the names before using this file.

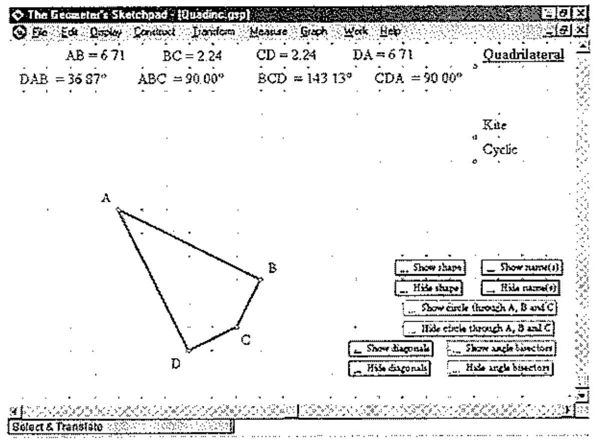

Figure 10 All the names associated with the shape appear on the screen and this changes dynamically as the shape is dynamically changed

However, the file offers the opportunity for students to meet these names (some of which will be met for the first time) and abstract related properties at the same time

Such tasks call upon students to work in a way similar to how they had to work as very young children When beginning to learn their first language, no one explained the first collection of words they learnt, since they did not have the language to understand any explanations offered. Words were used by adults, and young children were left with having to abstract meaning from the context within which the words were used The fact that these young children gained a first language and are now sitting in mathematics classrooms communicating through that language means that all students have this power to abstract properties. Sadly, this power is one which appears rarely to be called upon in mathematics classrooms with many teachers feeling as if they have to explain properties to students and give definitions for words they use

This approach of asking students to abstract meaning of words whilst engaging in a task which involves the use of those words has links to what Lave and Wenger (1991) term 'legitimate peripheral participation':

> It [legitimate peripheral participation] concerns the process by which newcomers become part of a community of practice. A person's intentions to learn are engaged and the meaning of learning is configured through the process of becoming a full participant in a sociocultural practice. This social process includes, indeed it subsumes, the learning of knowledgeable skills (p. 29)

Young children begin to participate within the practice of talking in a particular language, whilst carrying out practices associated with that language. Likewise, these same children, but now as older students in mathematics classrooms engaging with the above computer file, are being asked to participate within a practice which is initially unknown to them. Lave and Wenger go on to claim:

For newcomers then the purpose is not to learn *from* talk as a substitute for legitimate peripheral participation; it is to learn *to* talk as a key to legitimate peripheral participation (p. 109, *their emphasis*)

Perhaps too often teaching approaches expect students to learn *from* talk (e.g. receiving explanations and definitions from a teacher) rather than learning *to* talk in particular ways (e.g. being called upon to abstract properties associated with certain words whilst engaging in an activity which uses those words).

5 Summary

The division of the mathematics curriculum into arbitrary and necessary brings implications for the respective jobs that teachers and students have in the classroom. Within this article, I have considered implications for students and teachers of the issue of learning and teaching those things which are arbitrary - names and conventions. The arbitrary is in the realm of memory and students have the job of memorising. Teachers have the job of informing students of the arbitrary but also of assisting students with the task of memorisation. (By the way, what was that word I asked you to memorise?)

This might be done through use of *imagery and association*, or through *subordinating* the arbitrary to work which involves consideration of what is necessary - *practice through progress*. Alternatively, the realm of awareness can be brought in for those situations which concern the naming of properties. The name can be used whilst students are engaged in using their awareness to *abstract* the properties which are associated with the name.

This side of the divide - the arbitrary - is not where mathematics is to be found, and yet a considerable amount of lesson time and textbook space can be given over to introducing, practising and testing the arbitrary. I suggest that this is an indication that the job of assisting students to memorise the arbitrary is not as effective as it might be, and that considering this task further as a serious pedagogic challenge might bear considerable fruit with more effective memorisation resulting in time being freed for where mathematics really lies - with what is necessary.

The fact that mathematics does not lie with the arbitrary does not take away the vital role names and conventions play within the learning of mathematics. Both provide ways in which mathematical activity can progress. That they can be seen in this light is what is important for students. Too often, students are asked to memorise and practise a name or convention as if it were an end in itself rather than a vehicle through which to communicate mathematical awareness and progress with mathematical tasks.

Too often, a student is asked to memorise a name *per se* rather than to experience an awareness of a mathematical property which has sufficient significance to merit naming, since the property is one which links in with current and future mathematical activity. The learning of the arbitrary needs to be put in its rightful place of being inextricably linked with educating (mathematical) awareness. The third and final part of this trilogy of articles (to appear in the next issue of this journal) will be involved with the pedagogic challenge of educating (mathematical) awareness.

Notes

[1] Nathalie Sinclair has developed a web-based version of *Do we meet*? (which is called *Meeting Lulu*) at http://hydra.educ.queensu.ca/maths/ which has the feature of forcing communication through entering vectors and co-ordinates at one stage in carrying out the task

[2] Active Geometry for either Geometer's Sketchpad or Cabri-Géomètre is available from the Association of Teachers of Mathematics, 7 Shaftesbury Street, Derby DE23 8YB, U.K.

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