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Spontaneous focusing on numerosity and the arithmetic advantage

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ABSTRACT

Children show individual differences in their tendency to focus on the numerical aspects of their environment. These individual differences in 'Spontaneous Focusing on Numerosity' (SFON) have been shown to predict both current numerical skills and later mathematics success. Here we investigated possible factors which may explain the positive relationship between SFON and symbolic number development. Children aged 4–5 years ($N = 130$) completed a battery of tasks designed to assess SFON and a range of mathematical skills. Results showed that SFON was positively associated with children's symbolic numerical processing skills and their performance on a standardised test of arithmetic. Hierarchical regression analyses demonstrated that the relationship between SFON and symbolic mathematics achievement can be explained, in part, by individual differences in children's nonsymbolic numerical processing skills and their ability to map between nonsymbolic and symbolic representations of number.

1. Introduction

For many children, the development of symbolic number knowledge (i.e. knowledge of number words and Arabic numerals) is a long and arduous process. Learning the number sequence by rote may happen very early on – children typically begin counting around the age of two – but it can take years to grasp the meanings of the words in the count list. While some children start school with a range of numerical skills (from counting, matching and ordering sets, to adding and subtracting small numbers), others have yet to understand that the last word in their count list represents the numerosity of the set as a whole (e.g. Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). In other words, they have yet to acquire the cardinal principle of counting (Gelman & Gallistel, 1978).

Recent research has highlighted the role of informal numerical experiences in the acquisition of formal symbolic number knowledge. In particular, Hannula and colleagues have demonstrated that preschoolers show individual differences in their tendency to focus on numerical information in informal everyday contexts. These individual differences in 'Spontaneous Focusing on Numerosity' (SFON) are related to children's counting skills (Hannula & Lehtinen, 2005; Hannula, Räsänen, & Lehtinen, 2007) and they predict later arithmetical success (Hannula, Lepola, & Lehtinen, 2010; Hannula-Sormunen, Lehtinen, & Räsänen, 2015; McMullen, Hannula-Sormunen, & Lehtinen, 2015).

1.1. Spontaneous focusing on numerosity (SFON)

SFON is a recently-developed construct which captures an individual's spontaneous focusing on the numerical aspects of their environment (e.g. Hannula & Lehtinen, 2005). The term "spontaneous" is used to refer to the fact that the process of "focusing attention on numerosity" is self-initiated or non-guided. That is, attention is not explicitly guided towards the aspect of number or the process of enumeration. The idea is that "SFON tendency indicates the amount of a child's spontaneous practice in using exact enumeration in her or his natural surroundings" (Hannula et al., 2010, p.395).

The measures used to assess children's SFON differ from typical enumeration measures. Firstly, children are not guided towards the numerical aspects of the tasks; researchers are careful to ensure that the numerical nature of the tasks is not disclosed. Secondly, the tasks always involve small numerosities so that all children have sufficient enumeration skills to recognise the numbers in the activities. This is important for ensuring that the tasks capture individual differences in focusing on numerosity rather than individual differences in enumeration skills. To demonstrate that SFON tasks are not measures of individual differences in accuracy of number recognition skills per se, previous studies have included guided

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focusing on numerosity (GFON) versions of the tasks. Hannula and Lehtinen (2005) and Hannula et al. (2010) showed that low-SFON children could perform the tasks when guided towards numerosity, thus their low-SFON scores can be interpreted as not focusing on numerosity rather than not having sufficient skills needed to perform the tasks.

1.2. The relationship between SFON and numerical skills

In a three-year longitudinal study, Hannula and Lehtinen (2005) tracked preschool children's counting skills together with their SFON. Results showed that children's SFON, measured at 4, 5, and 6 years, was significantly associated with the development of number word sequence production, object counting and cardinality understanding. Path analyses revealed a reciprocal relationship suggesting that SFON both precedes and follows the development of early counting skills.

Follow-up work demonstrated the domain specificity of SFON as a predictor of children's numerical skills. In another longitudinal study, Hannula et al. (2010) measured children's SFON together with their spontaneous focusing on a non-numerical aspect of the environment, namely, ‘Spontaneous Focusing on Spatial Locations’ (SFOL). Findings showed that SFON in preschool predicted arithmetic skills, but not reading skills, two years later in school. This relationship could not be explained by individual differences in nonverbal IQ, verbal comprehension or SFOL.

Further results from more recent studies have demonstrated an even longer-term role of SFON in predicting school mathematics achievement. Hannula-Sormunen et al. (2015) found that SFON in preschool is still a significant predictor of mathematics achievement at the age of 12, even after controlling for nonverbal IQ. This longer-term relationship was found not only for natural number and arithmetic skills, but for rational number conceptual knowledge as well (McMullen et al., 2015).

1.3. Why is SFON associated with a numerical advantage?

SFON is emerging as a key factor for explaining variations in children's numerical development. However, the mechanisms behind this relationship are not yet clear. In particular, we do not know why SFON provides a numerical advantage. Hannula et al. (2007) proposed that the more children focus on the numerical aspects of their environment, the more practice they acquire with enumeration and thus, the better their counting skills become. To explore this possibility, they looked at the relations between children's subitizing-based enumeration (i.e. the rapid perception of the numerosity of small sets, without counting), object counting and SFON. Regression analyses revealed a direct relationship between children's SFON and their number sequence production skills. In contrast, there was an indirect relationship between SFON and object counting that was explained by individual differences in subitizing-based enumeration skills. This provides some evidence to suggest that SFON promotes perceptual subitizing skills which in turn supports the development of children's counting skills.

Other research has investigated motivational factors in the development of children's SFON and early numerical skills. In one of the first SFON studies to be conducted outside of Finland, Edens and Potter (2013) explored the relationship between SFON and counting skills in 4-year-old children in US preschools. They obtained teacher reports of children's motivation, attentional self-regulation, persistence and interest in mathematics. They also measured children's self-selected activity choices during free-play in the classroom. In line with the results from Hannula and colleagues (e.g. Hannula & Lehtinen, 2005), Edens and Potter found a positive correlation between preschoolers' SFON and their object counting and number sequence production skills. In terms of the motivational factors, they found that teachers' reports of children's motivation and interest in mathematics were significantly correlated with children's counting skills, but not with children's SFON. Moreover, there was no relationship between children's SFON and their self-selected activity choices during free-play: High-SFON children did not choose overtly number-related activities in their classrooms. These findings suggest that SFON does not reflect children's interest in mathematics, or at least not their “overt” interest in mathematics.

Together these studies indicate that the factors underpinning the relationship between SFON and children's numerical development are more likely to be cognitive than affective. However, the precise mechanisms involved need further investigation. The current literature is sparse and somewhat limited in scope. Thus far, studies exploring the mechanisms of SFON have focused solely on its relationship with early counting skills. We do not know why SFON is related to children's later arithmetical development. We also do not know how SFON relates to more basic numerical competencies such as nonsymbolic processing skills or ‘number sense’ (Dehaene, 2001).

One possibility is that SFON works by increasing children's fluency with number symbols. High-SFON children may get more practice mapping between their newly-acquired symbolic representations of number (Arabic numerals and number words) and pre-existing nonsymbolic (approximate) representations. As children get practice with, and improve the precision of these mappings, their counting and arithmetic skills may develop. This is theoretically likely because we know from previous research that mapping ability is related to mathematics achievement (Booth & Siegler, 2008; Holloway & Ansari, 2009; Mundy & Gilmore, 2009; see De Smedt, Noël, Gilmore, & Ansari, 2013 for a review). For example, Mundy and Gilmore (2009) found that children aged 6–8 years showed individual differences in their ability to map between nonsymbolic representations (dot arrays) and symbolic representations (Arabic digits). These individual differences explained a significant amount of variation in children's school mathematics achievement.

Some initial support for this possibility comes from two recent studies. Firstly, Sella, Berteletti, Lucangeli, and Zorzi (2015) found that pre-counting children who spontaneously focused on numerosity did so in an approximate manner. Sella et al. suggest that high-SFON children might be more prone to comparing and estimating numerical sets from an early age thus improving the precision of their numerical representations. Secondly, Bull (2013) found that high-SFON children (aged 5–7 years) performed better than their low-SFON peers on a numerical estimation task, in which they had to assign a symbolic number word to a nonsymbolic array of dots. In other words, children who consistently focused on numerosity were better able to map between nonsymbolic and symbolic representations of number.

In addition to these studies, research exploring the transition from informal to formal mathematics knowledge has highlighted the role of mapping ability. In a one-year longitudinal study Purpura, Baroody, and Lonigan (2013) demonstrated that the link between children's informal and formal mathematics knowledge was fully explained by individual differences in symbolic number identification and the understanding of symbol to quantity relations. Here, informal mathematics knowledge was defined as “those competencies generally learned before or outside of school, often in spontaneous but meaningful everyday situations including play” (Purpura et al., 2013, p. 454). It is important to note that this informal mathematics is a separate construct to SFON (recall that SFON is a distinct attentional process rather than the spontaneous acquisition of mathematical skills). Therefore further research is needed to examine the nature of the relationships between SFON, mapping ability and early arithmetic skills.

1.4. The present study

The aim of the present study was to investigate possible factors which may explain the positive relationship between SFON and symbolic number development. Specifically, we sought to investigate whether the relationship between children's SFON and mathematical skills can be accounted for by individual differences in fluency with nonsymbolic and symbolic representations of number. We gave children aged 4–5 years a battery of tasks designed to assess SFON, nonsymbolic magnitude comparison, symbolic (Arabic digit) comparison, nonsymbolic-to-symbolic mapping and arithmetic skills. We also gave them a digit recognition task to determine their knowledge of number symbols. Furthermore, we obtained measures of visuospatial working memory (to control for the working memory component of the SFON Posting Task) and verbal skills (to control for the verbal component of the SFON Picture Task). The inclusion of these control measures is necessary to show that we are capturing individual differences in SFON, and not just individual differences in working memory or verbal skills.

Our predictions for the study were as follows: First, we predicted that SFON would show a significant positive correlation with children's mathematical skills, thus confirming the results of previous studies (Prediction 1). We tested this prediction using partial correlation analyses to control for age, working memory skills, verbal skills and Arabic digit recognition. Second, we predicted that the relationship between SFON and mathematical skills would be largely explained by individual differences in children's ability to map between nonsymbolic and symbolic representations of number (Prediction 2). We tested this prediction using hierarchical regression analyses with two mathematical outcome measures, symbolic number comparison and standardised arithmetic performance. These outcome measures have been shown to be closely related in previous studies (e.g. Fazio, Bailey, Thompson, & Siegler, 2014). The inclusion of the symbolic comparison measure allowed us to directly examine the nature of the relationship between children's SFON and their fluency with number symbols.

2. Method

2.1. Participants

Participants were 130 children (64 girls and 66 boys) aged 4.5–5.6 years ($M = 5.0$ years, $SD = .3$ years). Children were recruited from three primary schools in Nottinghamshire and Leicestershire, UK, which were of varying socio-economic status (SES)[1]: one low ($N = 32$), one medium ($N = 58$) and one high ($N = 40$). All children were in the second term of their first year of school. At this stage classes are very informal; learning is play-based and child-led, following the ‘Early Years Foundation Stage’ framework. Participation was voluntary and the children received stickers to thank them for taking part. Study procedures were approved by the Loughborough University Ethics Approvals (Human Participants) Sub-Committee.

Nine children were excluded from all the analyses for the following reasons: English was not their native language ($N = 2$), speech and language difficulties and/or selective mutism ($N = 2$), other special educational needs ($N = 3$), failure to identify numerical digits beyond 1 ($N = 2$). A further two children did not complete all of the measures at Time 2, leaving a total of 119 complete datasets.

2.2. Design

Children took part in two testing sessions scheduled one-week apart. During Session 1 they completed two SFON tasks and a visuospatial working memory task. During Session 2 they completed a series of computer-based numerical processing tasks (nonsymbolic comparison, symbolic comparison, digit recognition and nonsymbolic-to-symbolic mapping) followed by a standardised measure of arithmetic. Testing took place on a one-to-one basis with the researcher who was present at all times throughout each of the tasks. The tasks were presented in the same order for every child. Each task is described in turn below, in the order in which it was presented.

2.3. Tasks & procedure

Children were tested individually in a quiet room or corridor outside their classroom. The researcher ensured that the testing area was free from any numerical displays that might have prompted the children to focus on number (during Session 1) or helped them to solve a numerical problem (during Session 2). During testing Session 1, children were not told that the tasks were in anyway numerical or quantitative. Likewise, the children's parents and teachers were not informed of the numerical aspects of the study; rather, they were told that the study was focusing on children's general thinking skills.

Throughout all tasks children received general praise (e.g. “You're watching really nicely”) but no specific feedback was given. At the end of each task children were allowed to choose themselves a sticker.

2.3.1. SFON

Children completed two SFON measures, an imitation ‘Posting Task’ developed by Hannula and Lehtinen (2005) and a novel ‘Picture Task’ adapted from Hannula et al. (2009). The order of these tasks was counterbalanced.

2.3.1.1. Posting Task. The materials used in this task were a toy postbox (diameter = 28.7 cm, height = 42.0 cm), a pile of 20 blue letters (9.5 cm × 6.0 cm) and a pile of 20 yellow letters (9.5 cm × 6.0 cm).

The researcher introduced the materials by saying: “Here is Pete the Postman's postbox, and here are some letters. We have some blue letters and some yellow letters. Now, watch carefully what I do, and then you do just the same”. The researcher posted two yellow letters, one at a time, into the postbox followed by one blue letter. They then prompted the child: “Now you do just the same”. On the second trial the researcher posted one blue letter and one yellow letter and on the third and final trial they posted two blue letters and three yellow letters. The researcher progressed from one trial to the next by saying: “Okay, let's go again”.

All of the trials involved small numerosities (ranging 1 to 3 of each colour with a total of up to 5) to ensure that they were within the children's counting range. As outlined in the introduction, it is important that SFON tasks include small numerosities so that all children have sufficient enumeration skills to recognise the small numbers in the activities. This ensures that the tasks capture individual differences in focusing on numerosity and not individual differences in enumeration skills.

In line with Hannula and Lehtinen (2005) the researcher

[1] Based on the proportion of children eligible for free school meals compared to the national average.

recorded all verbal and nonverbal quantitative acts. These included (a) utterances including number words (e.g. "I did yellow three times … did you do yellow three times?"), (b) counting acts (e.g. "one, two, three, four … I want four"), (c) use of fingers (e.g. pointing) to denote numbers, (d) utterances referring to quantities or counting (e.g. "I think I did too many"), and (e) interpretation of the goal of the task as quantitative (e.g. "I don't know how many you did"). For each of the three trials children received a score of 0 or 1 depending on whether or not they spontaneously focused on numerosity. Children were scored as spontaneously focusing on numerosity if they posted the same total number of letters as the researcher[2] and/or if they presented any of the quantifying acts listed above (a-e). Note that because SFON scores for each trial were binary, a child who posted the correct number and a child who posted the correct number plus presented a quantifying act both received the same score of 1. Each child received a total SFON score out of three. Responses were coded by a single observer (the researcher). A second independent observer coded a random subset (20%) of the observation forms to establish inter-rater reliability. The inter-rater reliability was 1.00.

2.3.1.2. Picture Task. The materials used in this task were three cartoon pictures (16.0 cm × 12.0 cm) each laminated on A4 card. The pictures are shown in Fig. 1.

The researcher introduced the task by saying: "This game is all about pictures. I'm going to show you a picture, but I'm not going to see the picture. Only you get to see the picture. This means I need your help to tell me what's in the picture." On each of three trials, the researcher held up a picture in front of the child and said: "What can you see in this picture?" The researcher wrote down everything the child said. If the child was reluctant to speak, the researcher repeated their request: "Can you tell me what you can see?" If the child spoke too quietly, the researcher prompted them to speak a little louder. There was no time limit for children to respond. When the child finished the researcher asked: "Is that everything?" When the child was ready to move on the researcher introduced the next trial: "Let's look at another picture. Ready, steady …"

The pictures were presented in the same order for each child. Picture 1 showed a girl standing in the rain with a leaf umbrella and baby chicks (see Fig. 1a). Picture 2 showed a boy and a girl in a hot air balloon with houses and trees below (see Fig. 1b). Picture 3 showed a girl with a hat on holding a basket of flowers near the sea (see Fig. 1c). Importantly, all pictures contained several small arrays (of objects, people or animals) that could be enumerated, for example, "three chicks" (Picture 1), "two children" (Picture 2), "four flowers" (Picture 3). The set sizes of these arrays ranged from 1 to 9. As with the Posting Task, small numerosities were included so that all children would have sufficient enumeration skills to recognize the numbers in the activities.

For each of the three trials children received a score of 0 or 1 depending on whether or not they spontaneously focused on numerosity. Children were scored as spontaneously focusing on numerosity if their description contained any symbolic number word/s, regardless of whether they had enumerated the objects correctly. For example, if a child accurately described "three chicks" in Picture 1 they received a SFON score of 1. Likewise, if a child miscounted and described "four chicks" they too received a SFON score of 1. However, if a child described "some chicks" and made no other reference to number in their description then they received a SFON score of 0. Note that because SFON scores for each trial were binary, a child who mentioned number several times and a child who mentioned number only once both received the same score of 1. As with the Posting Task, each child received a total SFON score out of 3. The inter-rater reliability of two independent observers (who coded 20% of the observation forms) was 1.00.

2.3.2. Verbal skills

Children's verbal skills were indexed by the average number of words they uttered on the SFON Picture Task.

2.3.2.1. Average word count on the Picture Task. The SFON Picture Task required children to produce verbal descriptions of the pictures they were presented with. Given these verbal requirements, it is important to show that individual differences on this task were capturing individual differences in SFON, not just individual differences in verbal skills. Verbal skills were thus measured by adding up the number of words children uttered on each Picture Task trial and computing the average across all three trials.

The verbal descriptions of the pictures were recorded (written down) by the researcher during the testing phase of the Picture Task. The researcher wrote down the descriptions using shorthand (symbols and abbreviations for common words) allowing her to write as quickly as the children spoke. Overall, children showed large individual differences in the length of their picture descriptions. The word count ranged from 6.67 words to 68.67 words ($M = 26.98$, $SD = 9.74$). Verbal skills (average word count) are controlled for in the analyses presented in the Results section.

2.3.3. Working memory

Visuospatial working memory skills were measured using a visual search ('Spin-the-Pots') task adapted from Hughes and Ensor (2005).

2.3.3.1. Spin-the-Pots task. The materials were a circular silver tray (diameter = 39.5 cm), 11 different coloured paper cups (diameter = 7.0 cm, height = 9.5 cm), 9 stickers (2.0 cm × 2.0 cm) and an A3 piece of card.

The researcher randomly positioned each cup upside down around the rim of the circular tray. They then introduced the task to the child by saying: "Now we're going to play a finding game. Here are some cups. They are all different colours. Can you tell me what colours they are?" This question was intended to check whether the child could distinguish between all of the different colours. The researcher then placed each sticker on top of a cup, pointing out to the child that there were not enough stickers for all of the cups and that two cups would not have stickers. Next, they instructed the child: "Watch carefully whilst I hide the stickers under the cups. Later, you can have a go at finding them." The researcher hid all of the stickers and then covered the cups with a piece of card. They told the child: "Now, I'm going to spin the cups. Then you can choose one cup and see if there's a sticker inside." The researcher spun the cups and then removed the card for the child to choose a cup. If they found a sticker then they took it out and kept it beside them. The researcher continued by covering up the cups again and spinning them round before allowing the child to choose another. This continued until the child found all 9 stickers, or, until the maximum number of spins ($N = 18$) was reached. Each child received a score out of 18 depending on the number of errors they made.

2.3.4. Mathematical skills

Children completed four computer-based numerical processing tasks (all programmed using E-Prime software 2.0 and all presented on a 15 inch LCD laptop screen). Task instructions were presented on the laptop screen and they were read aloud by the

[2] This could be the correct number of colour 1 and the correct number of colour 2, or, the correct number in total regardless of colour.

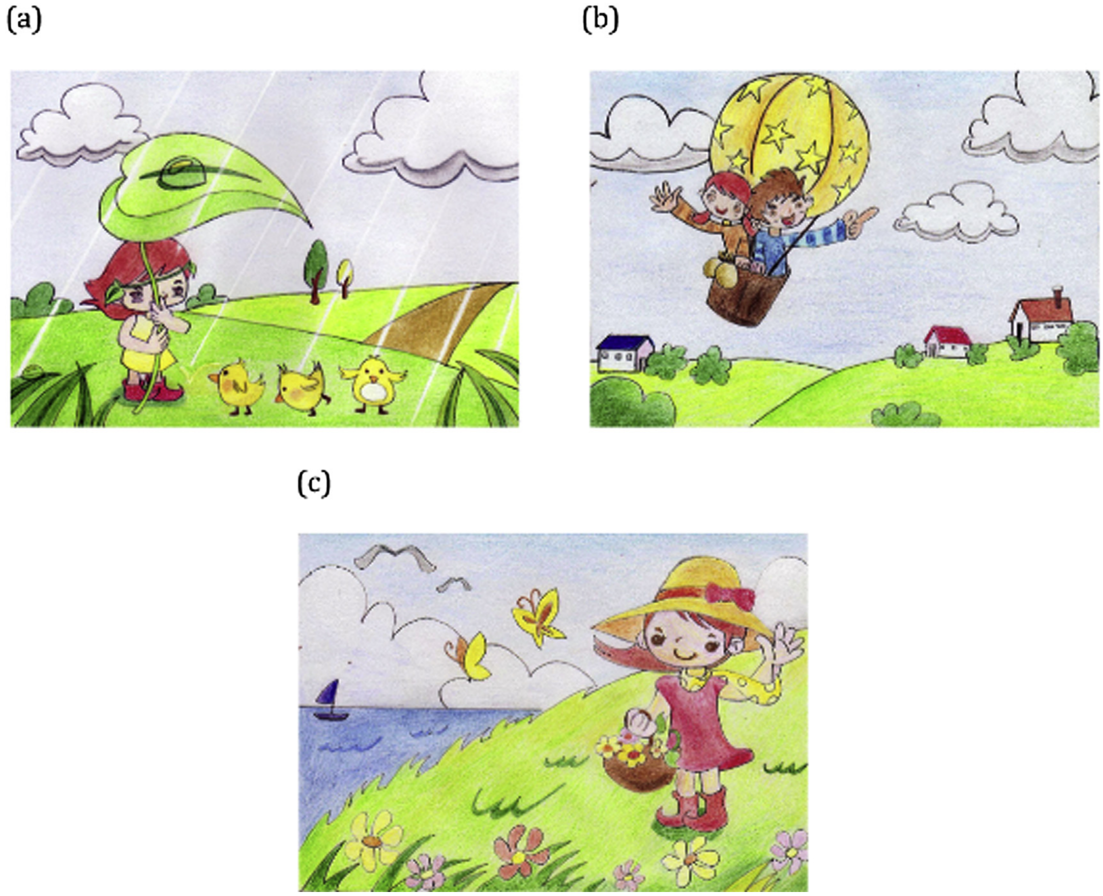

Fig. 1. The stimuli used in (a) the first, (b) the second and (c) the third trials of the SFON Picture Task.

researcher. Following the computer-based tasks children completed a standardised measure of arithmetic. The researcher was present at all times throughout each of the tasks.

2.3.4.1. Nonsymbolic comparison task. This task measured children's ability to compare nonsymbolic numerical stimuli. Children were presented with two arrays of dots and they were asked to select the more numerous of the two arrays. The task was incorporated into a game in which the children saw two fictional characters and were asked to quickly decide (without counting) who had the most marbles.

Numerosities ranged from 4 to 9 and the numerical distance between the two numbers being compared was either small (a distance of 1 or 2) or large (a distance of 3 or 4). Numerosities 1 to 3 were excluded because they are in the subitizing range. Dot arrays were generated randomly in accordance with previous numerosity experiments, such that no two dot arrays for the same quantity were the same. Stimuli were created using the method by Dehaene, Izard, and Piazza (2005) to control for continuous quantity variables such as dot size and envelope area. All dot arrays were black dots on a white circular background as shown in Fig. 2a. The side of the correct array was counterbalanced.

Each of 40 experimental trials began with a fixation cross for 1000 ms, followed by the two dots arrays (side-by-side) for 1250 ms, followed by a question mark until response. Stimuli presentation times were chosen based on pilot testing with children of the same age. Children responded by pointing to the character with the most marbles. The researcher recorded these responses via the ‘c’ (left bigger) and ‘m’ (right bigger) keys on a standard keyboard. The order of the trials was randomised and children were prompted to take a break after 20 trials.

The experimental trials were preceded by two blocks of four practice trials. In the first practice block children received no time limit; they were presented with a fixation cross followed by the two dot arrays until response. In the second practice block, the researcher introduced the experimental time limit of 1250 ms to prevent the children from counting. The researcher emphasised that it was a speeded game, and children were encouraged to have a guess if they were not sure. Each child received an accuracy score based on the proportion of items they answered correctly.

2.3.4.2. Symbolic comparison task. This task measured children's ability to compare symbolic numerical stimuli (visual number symbols). Children were presented with two Arabic digits and they were asked to select the numerically larger of the two. Numerosities ranged from 4 to 9. The problems were identical to the nonsymbolic problems, except the numerosities were presented as Arabic digits instead of dot arrays. Symbolic stimuli were black digits on a white circular background as shown in Fig. 2b.

Each of 40 experimental trials began with a fixation cross for 1000 ms, followed by the two Arabic digits (side-by-side) for 750 ms, followed by a question mark until response. Stimuli presentation times were chosen based on pilot testing with children of the same age. They varied across tasks to avoid floor and/or ceiling effects. In line with the nonsymbolic version of the task, children responded by pointing to the character with the larger number of marbles and the researcher recorded these responses on the computer.

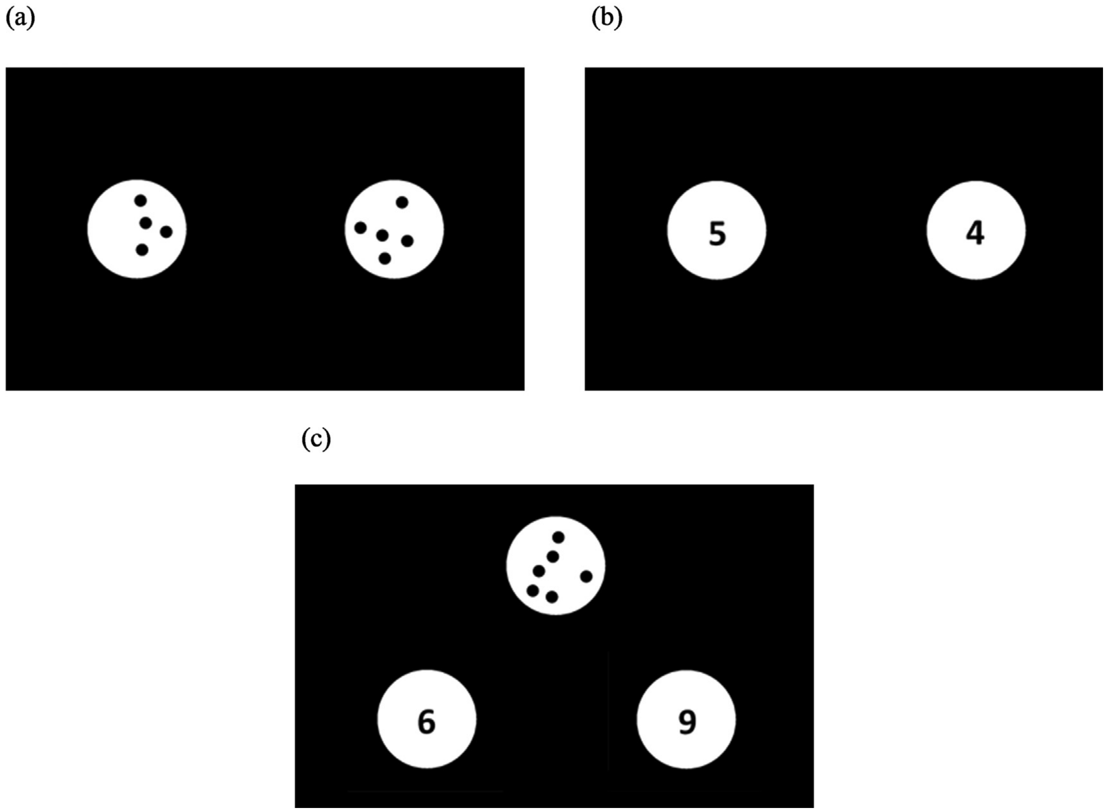

Fig. 2. The stimuli used in (a) the nonsymbolic comparison task, (b) the symbolic comparison task and (c) the numerical mapping task.

The experimental trials were preceded by two blocks of four practice trials. The first practice block had no time limit and the second practice block introduced the experimental time limit of 750 ms. All trials were presented in a random order and children were prompted to take a break half-way through. Each child received an accuracy score based on the proportion of items they answered correctly.

2.3.4.3. Digit recognition task. This task measured children's knowledge of Arabic digit stimuli. Children were asked to read aloud a series of Arabic digits (ranging from 1 to 9) presented one by one in a random order on the laptop screen. Children scored one point for each correct identification giving a total score out of 9.

2.3.4.4. Numerical mapping task. This task measured children's ability to map nonsymbolic numerical stimuli onto symbolic numerical stimuli. Children were presented with an array of dots and they were asked to quickly (without counting) decide which of two Arabic digits matched the numerosity of the dots. The task was adapted from Mundy and Gilmore (2009).

Numerosities ranged from 2 to 9 and the numerical distance between the two symbolic choices was either small (a distance of 1 or 2) or large (a distance of 3 or 4). The number range included small numerosities within the subitizing range because pilot testing revealed some children to be performing at chance with the larger numerosities. Stimuli were presented simultaneously with the dot array centred at the top and the symbolic (Arabic digit) stimuli at the bottom left and right hand sides of the screen (as shown in Fig. 2c).

Each of 40 experimental trials began with a fixation cross for 1000 ms, followed by the numerical stimuli for 2000 ms, followed by a question mark until response. Stimuli presentation times were chosen based on pilot testing with children of the same age. The dot array disappeared when the question mark appeared to prevent children from counting. Children responded by pointing to the digit that matched the numerosity of the dots. The researcher recorded these responses via the ‘c’ (left matching) and ‘m’ (right matching) keys on a standard keyboard.

As with the comparison tasks, the experimental trials were preceded by two blocks of four practice trials (first with no-time limit, then with the experimental time limit of 2000 ms). Again the researcher emphasised that this was a speeded game and children were encouraged to have a guess if they were not sure. Each child received an accuracy score based on the proportion of items they answered correctly.

2.3.4.5. Arithmetic task. The arithmetic subtest of the Wechsler Preschool and Primary Scale of Intelligence (WPPSI) was administered in accordance with the standard procedure. There were 20 questions in total. Questions 1 to 4 required children to make nonsymbolic judgements about size or quantity (e.g. “Here are some balls. Point to the one that is the biggest”), questions 5 to 8 required children to perform counting tasks with blocks (e.g. “Can you give me all of the blocks except four”) and questions 9 to 20 required children to mentally solve arithmetic word problems (e.g. “John had two pennies and his dad gave him one more. How many pennies did John have altogether?”). Children continued until they had answered four consecutive questions incorrectly. They received a raw score out of 20.

3. Results

First, we present the descriptive statistics for children's performance on each of the experimental tasks. Next, we explore the correlations among children's performance on the SFON and mathematical tasks (Prediction 1). Finally, we run a series of hierarchical regression models to test whether the relationships between SFON and mathematical skills can be accounted for by individual differences in children's ability to map between nonsymbolic and symbolic representations of number (Prediction 2).

3.1. Descriptive statistics

Fig. 3 shows the number of children focusing on numerosity from zero to three times on each SFON task. Children's performance on all other tasks is presented in Table 1. Together these demonstrate that children showed individual differences in SFON and a range of performance on the working memory and mathematical tasks. The two SFON tasks varied in terms of difficulty. Scores on the Posting Task were negatively skewed (55.5% of children obtained a score below the maximum of 3) and scores on the Picture Task were positively skewed (64.7% of children obtained a score above the maximum of 0). As a possible result of this, performance on these two tasks was not significantly correlated ($r_s = .06$, $p = .533$).

The order of the SFON tasks was counterbalanced therefore children's scores were checked for order effects. The results demonstrated no order effects: There was no significant difference in SFON on the Posting Task between children who completed the Posting Task first versus children who completed the Picture Task first ($t(117) = -.50$, $p = .618$); likewise, there was no significant difference in SFON on the Picture Task between children who completed the Posting Task first versus children who completed the Picture Task first ($t(117) = -1.08$, $p = .283$).

3.2. Correlations

Correlations between all variables are reported in Table 2. These show that the SFON tasks (while not significantly correlated with each other) were both positively related to performance on the mathematical tasks, thus lending support for Prediction 1. The correlation between SFON and arithmetic was .30 for the Posting Task and .47 for the Picture Task. These correlations are similar in magnitude to those found in previous SFON studies (e.g. Hannula et al., 2007). Importantly, they remain significant even after controlling for age, working memory skills, verbal skills (average word count on the SFON Picture Task) and Arabic digit recognition: Children's arithmetic scores were positively correlated with SFON scores on the Posting Task (*partial* $r = .29$, $p = .002$) and the Picture Task (*partial* $r = .42$, $p < .001$).

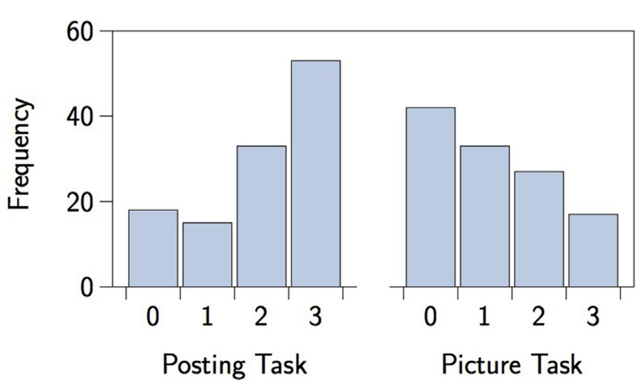

Fig. 3. Frequencies of spontaneous focusing on numerosity on each of the SFON tasks ($N = 119$).

Table 1
Descriptive statistics for performance on each of the nine measures ($N = 119$).

	M	SD	Range
SFON Posting	2.02	1.09	0–3
SFON Picture	1.16	1.07	0–3
Average word count	26.98	9.74	6.67–68.67
Working memory (errors)	6.30	3.53	0–11
Nonsymbolic comparison	.74	.12	.43–.95
Symbolic comparison	.75	.17	.38–1.00
Digit recognition	8.22	1.31	4–9
Numerical mapping	.73	.16	.38–1.00
Arithmetic (WPPSI raw score)	10.95	2.63	5–17

3.3. Hierarchical regression models

To explore the nature of these relationships we ran a series of hierarchical regression models. Specifically, we tested whether nonsymbolic skills and the mapping between nonsymbolic and symbolic representations could account for the relationship between SFON and mathematics achievement (Prediction 2). We used two mathematical outcome measures, symbolic comparison performance and arithmetic performance, both of which were highly correlated ($r = .65$, $p < .001$). For each of these dependent variables, we conducted a set of two models. In the first model baseline variables were entered in Step 1, followed by SFON in Step 2, and nonsymbolic comparison and mapping performance in Step 3. In the second model, the order of steps 2 and 3 were reversed.

As shown in Table 3, SFON was a significant predictor of symbolic comparison performance when entered in Step 2, before the nonsymbolic comparison and mapping tasks, but not when it was entered after these variables in Step 3. In other words, SFON did not explain significant variance in symbolic comparison performance once nonsymbolic comparison and mapping performance had been taken into account. This demonstrates that the relationship between SFON and symbolic processing skills can be accounted for by individual differences in nonsymbolic skills and mapping skills. With arithmetic performance as the dependent variable, we see a different pattern of results. SFON was a significant predictor of arithmetic when entered into the model at Step 2 and also at Step 3. This shows that SFON explains additional variance in arithmetic performance over that explained by nonsymbolic skills and mapping skills. Therefore, the relationship between SFON and arithmetic skills is only partly accounted for by individual differences in nonsymbolic skills and mapping skills.

4. Discussion

These results add to our limited understanding of how children's informal (spontaneous) interactions with number relate to their early mathematical skills. First, they replicate previous studies showing that SFON is associated with an arithmetic advantage (e.g. Hannula et al., 2010). Second, they extend previous findings by providing evidence that this association persists even after controlling for individual differences in Arabic digit recognition, verbal skills and working memory. Third, and most importantly, they advance our theoretical understanding of how SFON may exert its positive influence on arithmetic skills. Specifically, the findings suggest that SFON may lead to increased practice mapping between nonsymbolic and symbolic representations of number which improves symbolic fluency and, in part, leads to better counting and arithmetic skills.

Since mapping ability only partly accounted for the relationship between SFON and arithmetic skills, further research needs to explore the additional factors at play. We highlight two possibilities. One possibility is that SFON improves the precision with

Table 2
Zero-order correlations between all variables.

Variable	1	2	3	4	5	6	7	8	9
1. SFON Posting	_								
2. SFON Picture	.06	_							
3. Average word count	.03	.34***	_						
4. Working memory	−.04	−.08	.08	_					
5. Nonsymbolic comparison	.22*	.25**	.09	−.17	_				
6. Symbolic comparison	.22*	.31**	.10	−.25**	.55***	_			
7. Digit recognition	.11	.15	.05	−.25**	.23*	.50***	_		
8. Numerical mapping	.25**	.37***	.11	−.25**	.51***	.72***	.47***	_	
9. Arithmetic (WPPSI)	.30**	.47***	.17	−.15	.43***	.65***	.42***	.57***	_

Note. Spearman's r_s coefficients are reported for the correlation between the two SFON tasks. All other coefficients are Pearson's r. $N = 119$, two-tailed hypotheses, *$p < .05$, **$p < .01$, ***$p < .001$.

Table 3
Linear regression models predicting symbolic comparison and arithmetic performance.

Model	Step	Predictor	Symbolic comparison		Arithmetic (WPPSI)	
			β	ΔR^2	β	ΔR^2
1	1	Age	.00	.27***	.02	.20***
		Digit recognition	.46***		.40***	
		Working memory	−.14		−.06	
		Average word count	.09		.15	
	2	SFON Posting	.15	.07**	.23**	.19***
		SFON Picture	.22		.39***	
	3	Nonsymbolic comparison	.25**	.26***	.16	.09***
		Numerical mapping	.47***		.25**	
2	1	Age	.00	.27***	.02	.20***
		Digit recognition	.46***		.40***	
		Working memory	−.14		−.06	
		Average word count	.09		.15	
	2	Nonsymbolic comparison	.26***	.33***	.19*	.19***
		Numerical mapping	.49***		.37***	
	3	SFON Posting	.02	.002	.15*	.08***
		SFON Picture	.05		.29***	

Note. $N = 119$, *$p < .05$, **$p < .01$, ***$p < .001$.

which children execute arithmetic procedures. High-SFON children may get more practice counting and as a result they may develop more mature counting strategies, which lead to more accurate arithmetic calculations. We know that as children become more proficient at counting they become less reliant on finger counting and they start to use more mature counting strategies, e.g. ‘counting on’ as opposed to ‘counting all’. Numerous studies have related these advanced counting strategies to improved performance on arithmetic tasks (e.g. Geary, Hoard, Byrd-Craven, & DeSoto, 2004). Therefore, if SFON supports the acquisition of more mature counting strategies then it may also advance children's arithmetic skills, over and above the advantage provided by high-SFON children's mapping ability.

A second possibility is that SFON provides an arithmetic advantage because it makes children better at extracting and modelling numerical information from real-world contexts. We know that being able to construct a mental representation of an arithmetic problem (i.e. understanding the quantitative relations between, and the actions upon different numerical sets in a problem) is an important process in numerical problem solving (Kintsch & Greeno, 1985; Thevenot, 2010; Verschaffel & De Corte, 1993). Children with high-SFON tendency may not necessarily have more advanced computational skills; rather, they may be better at working out when (and which of) these computational skills need to be used. Note that the standardised arithmetic task used in the present study comprised several word-based problems in which children needed to extract and model numerical information from a real-world story context, e.g. buying apples, sharing sweets and losing toys.

As well as testing these possibilities, it would be valuable for future studies to examine issues surrounding causality. Data presented here is cross-sectional thus we can only tentatively specify the causal nature of SFON based on prior longitudinal research. Hannula and Lehtinen (2005) showed that children's SFON was reciprocally related to counting skills. This suggests that SFON and arithmetic skills are likely to develop together in a cumulative cycle. Further longitudinal work (and training studies) will allow us to determine whether SFON increases symbolic fluency, and therefore arithmetic skills, and/or vice versa.

4.1. Methodological considerations

In addition to these theoretical issues, the findings from the present study generate methodological discussion. Here we introduced a new picture-based task for measuring SFON. Children were shown a cartoon picture and they were asked to describe what was in the picture. The potential advantages of this task are threefold. Firstly, there are several competing dimensions on which one can choose to focus. Children may focus on the number of items in the picture (e.g. “three houses”) or they may focus on the colours of the items (“bright blue sky”) or the emotional content (“they look happy”). This contrasts with the Posting Task on which children can focus on little information other than the number of letters posted. Secondly, unlike the pretend play activities of the Posting Task, the Picture Task is suitable for participants of all ages. It may be administered with simple cartoon pictures for preschoolers and primary school-aged children or with more complex visual scenes for older children and adults (see Hannula et al., 2009 for an example picture-based task with adults). Importantly, this means that we can study SFON throughout development in a simple and consistent manner. Thirdly, the Picture Task is quick and easy to run. While the Posting Task needs to be administered on a one-to-one basis, the Picture Task may be flexibly administered in small or large whole group settings. Here participants would be required to write down their descriptions rather than orally responding. Thus, we would first need to consider whether written SFON responses differ from oral SFON responses.

Despite these potential advantages, the Picture Task is not without its limitations. In view of the verbal requirements of the task it is only appropriate for children who have developed verbal communication skills. It would not be suitable for measuring SFON in infants or children with speech and language difficulties, and it may need to be used cautiously with bilinguals. Given the verbal demands, it is necessary to control for children's verbal skills when using this Picture Task. Recall that in our current study verbal skills were measured by calculating the average number of words in children's picture descriptions. This average word count was positively correlated with children's SFON thus it was entered as a control variable in all our regression models. Future studies should employ similar controls for the length of children's picture descriptions, ideally by audio recording and transcribing children's verbal responses. In the current study children's picture descriptions were written down by the researcher at the time of testing which may not be an entirely reliable way of recording the responses.

A further methodological issue is that the Picture Task cannot be used interchangeably with the Posting Task because children's scores on these tasks were not significantly correlated. This lack of correlation may stem from the oppositely skewed distributions of children's performance on the two tasks. Scores on the Posting Task showed a tendency towards ceiling effects while scores on the Picture Task showed a tendency towards floor effects. Alternatively, the lack of correlation between the tasks may be due to the different task demands and response modes. We speculate on four key differences below.

First, one noticeable difference between the two SFON tasks is that the Posting Task requires a nonverbal response whereas the Picture Task requires a verbal response. Second, the tasks can be seen to differ in terms of their time frame for focusing on numerosity: In the Posting Task the child needs to focus on number immediately (right from the beginning of the researcher's model performance), whilst in the Picture Task the child has as much time as s/he wants to start focusing on numerosity. Third, the ambiguity of the number aspect differs in the two tasks: In the Posting Task the ambiguity is at the level of the general aim of the task (the child does not know the exact aim of the task), whereas in the Picture Task the ambiguity is at the level of how to proceed in the task (the child knows that the aim is to describe the picture but they do not know which aspects to focus on). Fourth, the tasks differ in that the Posting Task is an action-based task whereas the Picture Task is a perception-based task. Research suggests that action and perception are functionally dissociable streams of the visual system which may not be linked in early development (see Bertenthal, 1996, for a review of the development of perception, action and representations); thus it is perhaps not surprising that performance on the tasks is not correlated in four- and five-year-old children.

In view of these speculations, further research is needed to untangle the subtle differences between the tasks and the underlying SFON constructs that they are measuring. Both tasks show predictive validity of arithmetic skills therefore they both warrant further investigation.

4.2. Educational implications

Finally we turn to the educational implications of this research. Our findings show that SFON is an important factor in the development of children's early numerical skills. This raises interesting questions as to whether SFON is something that can be trained. Can we increase children's tendency to recognise and use numbers in informal everyday contexts? If so, do increases in SFON lead to better mathematical outcomes? Researchers have started to explore these issues. A preliminary small-scale intervention study found that preschool children's SFON was enhanced through social interaction in day care settings. This enhancement was associated with improved cardinality skills suggesting that SFON-based interventions may help to support children's early counting development (Hannula, Mattinen & Lehtinen, 2005).

4.3. Conclusions

The present study reveals one way in which SFON may exert its positive influence on arithmetic skills. Specifically, it shows that the relationship between SFON and arithmetic can be explained, in part, by individual differences in nonsymbolic number skills and the mapping between nonsymbolic and symbolic representations of number. In light of this, adult guidance in helping low-SFON children to recognise and use more everyday numerosities (i.e. by directing their attention to the exact number of objects in their surroundings) may help children to make the links between symbols (number words and numerals) and quantities. With more practice making these links, the precision of low-SFON children's mappings between nonsymbolic and symbolic representations of number may increase, and this may support the development of early counting and arithmetic skills.

We know that preschoolers show individual differences in numerical knowledge and these individual differences predict mathematics achievement throughout the primary and secondary school years (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Baroody, 2000; Duncan et al., 2007; Klibanoff et al., 2006; Reeve, Reynolds, Humberstone, & Butterworth, 2012; Stevenson & Newman, 1986). Moreover, we know that the more children engage with and enjoy informal numerical activities before school, the more they engage with formal mathematics throughout school and higher education (Linder, Powers-Costello, & Stegelin, 2011; Seefeldt & Galper, 2008; Van de Walle & Lovin, 2006). Therefore, the present research exploring children's early engagement with numbers may play a key role in identifying important factors that influence children's later success with mathematics.

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