Nonresonant Charged-Particle Acceleration by Electrostatic Waves Propagating across Fluctuating Magnetic Field

A. V. Artemyev,[1,*] A. I. Neishtadt,[1,2] A. A. Vasiliev,[1] and L. M. Zelenyi[1]
[1]Space Research Institute RAS, Moscow 117997, Russia
[2]Department of Mathematical Sciences, Loughborough University, Loughborough LE11 3TU, United Kingdom
(Received 25 May 2015; published 7 October 2015)

In this Letter, we demonstrate the effect of nonresonant charged-particle acceleration by an electrostatic wave propagating across the background magnetic field. We show that in the absence of resonance (i.e., when particle velocities are much smaller than the wave phase velocity) particles can be accelerated by electrostatic waves provided that the adiabaticity of particle motion is destroyed by magnetic field fluctuations. Thus, in a system with stochastic particle dynamics the electrostatic wave should be damped even in the absence of Landau resonance. The proposed mechanism is responsible for the acceleration of particles that cannot be accelerated via resonant wave-particle interactions. Simplicity of this straightforward acceleration scenario indicates a wide range of possible applications.

DOI: 10.1103/PhysRevLett.115.155001 PACS numbers: 52.20.Dq, 52.25.Gj, 94.05.Pt

Resonant interactions of charged particles with electromagnetic and electrostatic waves play the crucial role in the formation of high-energy particle populations in various laboratory and cosmic plasma systems: Earth ionosphere (e.g., [1]) and magnetosphere (e.g., [2,3]), shock waves (e.g., [4]), stellar atmospheres (e.g., [5]), laboratory plasma [6], etc. The Cherenkov condition $\mathbf{k}\cdot\mathbf{v}=\omega$ for the resonant interaction provides the relation between the wave vector $\mathbf{k}$, the wave frequency ω, and the particle velocity $\mathbf{v}$. For plasmas in a strong magnetic field this condition is expanded into a series of cyclotron resonances (e.g., [7]). In the case of waves with significant amplitudes, the resonant condition can be rewritten in a form of inequality. For example, in the case of electrostatic waves interacting with nonrelativistic particles the resonant condition takes the form $\mathbf{v}\cdot\mathbf{k}\in[\omega-k\sqrt{q\varphi_0/m},\omega+k\sqrt{q\varphi_0/m}]$, where $k=|\mathbf{k}|$, φ_0 is the wave scalar potential amplitude, and m and $q>0$ are the particle mass and charge [8,9]. Thus, the resonance acquires the finite width in the velocity space $\sim\sqrt{q\varphi_0/m}$. For wave ensemble (or inhomogeneous plasma) the resonance width is determined by the competition of the wave dispersion (or the spatial dispersion of wave properties) and the wave amplitude [10,11]. For the overwhelming majority of applications, the wave-particle interactions were considered in the vicinity of the resonance where the action of the wave electromagnetic field on particles becomes the most effective. Nonresonant particles (e.g., particles with velocities much smaller than $\omega/k-\sqrt{q\varphi_0/m}$) are practically not affected by the wave electromagnetic field because the action of this field can result only in reversible particle energy variations within a limited range [12]. Thus, the effect of the resonant interaction serves as a base for the consideration of wave excitation and charged-particle acceleration [6,13,14].

In this Letter, we show an additional potentially important mechanism responsible for the effective interaction of particles and electrostatic waves in the absence of a resonance. We demonstrate that the supplementary stochastization of the charged-particle motion far from the resonance in the velocity space results in effective wave-particle interaction induced by the destruction of correlations of wave actions on particles.

We consider one of the simplest systems including the wave-particle interaction: the Landau damping of the electrostatic wave in the presence of a weak background magnetic field [15–17]. Such a configuration assumes that particles gyrorotate in the (x,y) plane around the weak background magnetic field B_z and interact with the electrostatic wave $\varphi=\varphi_0\cos(ky-\omega t)$. This interaction becomes resonant if the initial velocity is larger than the phase wave velocity ω/k [see the scheme in Fig. 1(a)]. Very intense waves (with the amplitude $\varphi_0>cB_z/\sqrt{c^2k^2-\omega^2}$) can trap relativistic charged particles into the surfatron regime of acceleration [9,18–20], while for waves with a small amplitude the efficient resonant scattering is responsible for wave damping or amplification (or nonlinear saturation) [21].

In this Letter, we consider the system with a particle located in the velocity space far from the resonance [see the scheme in Fig. 1(b)]. In this case, the action of the wave electric field on the particle could be averaged and the particle energy is only slightly oscillating around the initial value. Such energy conservation results from the separation of time scales of the particle motion with velocity $\mathbf{v}$ and the variation of the electric field, e.g., $|\mathbf{v}\cdot\mathbf{k}-\omega|\gg\Omega_0$ where Ω_0 is the particle Larmor (gyro) frequency. This separation guarantees that the high-frequency perturbations cannot significantly destroy the initial trajectories in the phase space [12]. However, if the initial trajectory is more

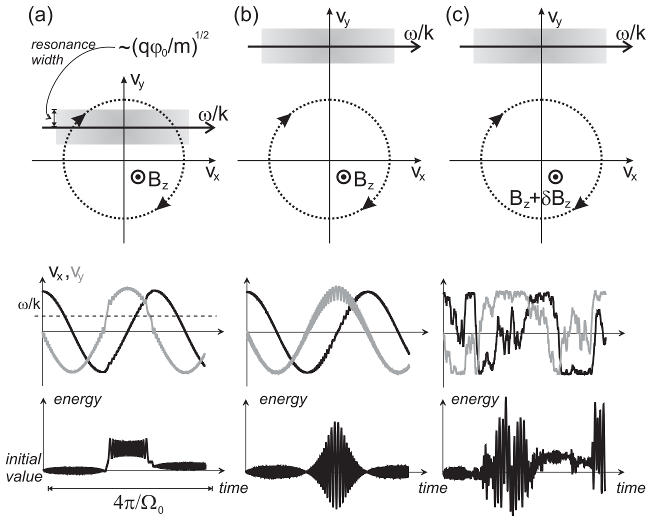

FIG. 1. Schematic view of the system: (a) with the resonant wave-particle interaction, (b) without a resonant interaction, and (c) without a resonant interaction but with magnetic field fluctuations. Particle velocity and energy are shown for two periods of gyrorotation around the background magnetic field $2\pi/\Omega_0$.

complicated [e.g., if the background magnetic field B_z is inhomogeneous in the (x, y) plane, see the scheme in Fig. 1(c)], then the average action of the wave electric field is not equal to zero. The additional stochastization of charged-particle motion due to small-scale inhomogeneities of B_z results in the destruction of correlations between wave phases in two subsequent moments of time when the particle reaches the closest position relative to the wave in the velocity space [i.e., when $|\mathbf{v}| = v_y$, see the scheme in Fig. 1(c)]. Thus, the wave action (even nonresonant) can destroy the initial particle trajectory in velocity space and result in a significant variation of particle energy. To demonstrate this effect we use the test particle approach.

The motion of the particle with the charge q and the rest mass m in the plane (x, y) transverse to the magnetic field $B_z\mathbf{n}_z$ is described by the following equation:

$$\dot{\mathbf{p}} = \Omega_0 m[1 + b_z(x, y, t)][\mathbf{v} \times \mathbf{n}_z] + \mathbf{n}_y qk\varphi_0 \sin(ky - \omega t) + \Omega_0 m\mathbf{e}(x, y, t), \quad (1)$$

where the wave is assumed to be propagating along the $\mathbf{n}_y$ direction, $\Omega_0 = qB_z/mc$, $\mathbf{p} = m\gamma\mathbf{v}$, $\gamma = \sqrt{1 + \mathbf{p}^2/m^2c^2}$, and the particle velocity is $\mathbf{v} = \dot{\mathbf{r}}$. Function $b_z(x, y, t)$ describes fluctuations of the magnetic field normalized on B_z. For simplicity these fluctuations are modeled as an ensemble of 10^3 plane waves [22] propagating with phases $\phi_{k,\theta} = k(x\cos\theta + y\sin\theta - v_\phi t)$. The corresponding induction electric field $\mathbf{e}(x, y, t)$ is calculated from Maxwell equations. The propagation velocity v_ϕ is the same for all waves in the ensemble (see distributions of k and θ in [22]). To characterize the amplitude of the main (electrostatic) wave we use a dimensional parameter, $\varepsilon = k\varphi_0/B_z$. The electromagnetic field fluctuations are characterized by dimensionless amplitude $\delta = \sqrt{\langle b_z^2\rangle}$ where the averaging is performed over the spatial area $L \times L$ and $2\pi/L$ is the minimum wave number of plane waves included into the ensemble of b_z fluctuations [22]. Note that in the system with $\varphi_0 = 0$ (i.e., when the wave is absent) and $v_\phi = 0$ (i.e., stationary fluctuations) the particle energy is conserved, i.e., $\gamma = \text{const}$.

We consider two systems: with stationary magnetic field fluctuations $v_\phi = 0$ and with the induction electric field $v_\phi \neq 0$. The second system is more realistic, while the simplified first system allows demonstrating the main effect of stochastization of charged-particle motion with conserved energy. Figure 2 shows five examples of particle trajectories calculated at different values of δ and v_ϕ. In

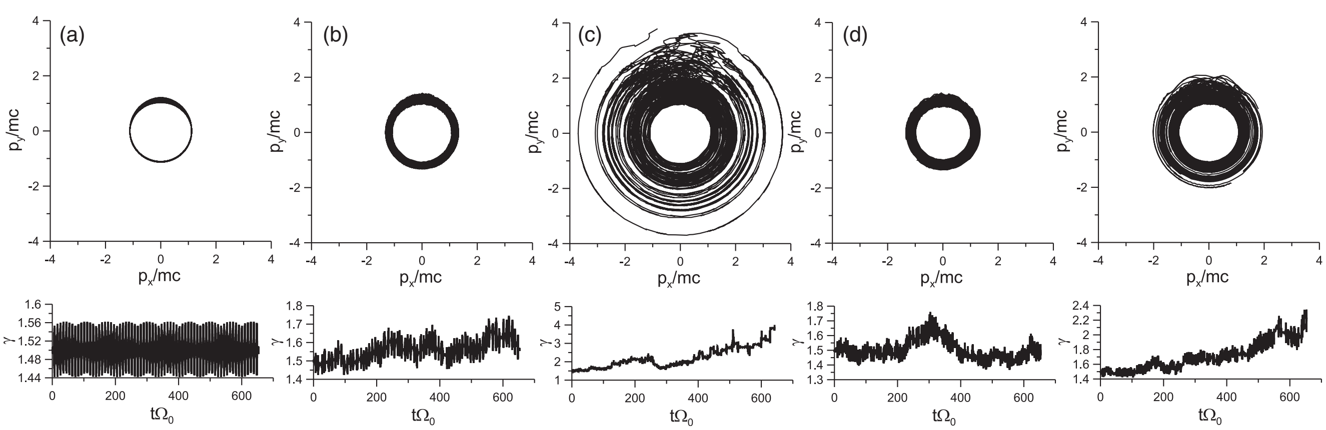

FIG. 2. Five examples of particle trajectories in the momentum space for different amplitudes of magnetic field fluctuations, electrostatic wave, and induction electric field: (a) $\varepsilon = 2$, $\delta = 0$, $v_\phi = 0$; (b) $\varepsilon = 2$, $\delta = 2.5$, $v_\phi = 0$; (c) $\varepsilon = 2$, $\delta = 5.0$, $v_\phi = 0$; (d) $\varepsilon = 0$, $\delta = 2.5$, $v_\phi = 0.005c$; (e) $\varepsilon = 2$, $\delta = 2.5$, $v_\phi = 0.005c$. The corresponding profiles of the particle energy are shown in the bottom panels. Wave frequency is $\omega/\Omega_0 = 100$, while the initial particle energy is $\gamma = 1.5$. The resonant value of momentum p_y/mc is $(\omega/kc)/\sqrt{1 - (\omega/kc)^2} \approx 7.5$ (i.e., all five examples correspond to the nonresonant interaction).

Fig. 2(a) we present an example of the trajectory obtained by the numerical integration of system (1) with $\delta = 0$, $v_\phi = 0$. The particle rotates around the background magnetic field and the particle energy oscillates around the initial value. Because of the low particle initial energy the resonant interaction with the wave is impossible and there is only reversible variation of the particle energy, i.e., the particle cannot escape from the close vicinity of its initial trajectory (this is the effect of the eternal adiabaticity for nonresonant low-dimensional systems, see [12]). Figures 2(b) and 2(c) show two examples of particle trajectories for systems with $\delta = 2.0$ and $\delta = 5.0$ (for both systems $v_\phi = 0$). Destruction of the initial adiabaticity of the charged-particle motion by magnetic field fluctuations [i.e., charged-particle diffusion in the coordinate space (x, y), see, e.g., [23]] results in the efficient particle diffusion in the momentum space. Thus, the particle energy in this case varies significantly in absence of wave-particle resonances. Figure 2(d) demonstrates the particle trajectory in the system with $v_\phi \neq 0$ and $\varepsilon = 0$. Electromagnetic field fluctuations induce a particle diffusion in the momentum space. This diffusion is significantly enhanced if we switch on the electrostatic wave [see Fig. 2(e) with $\varepsilon \neq 0$]. A comparison of Figs. 2(a), 2(d), and 2(e) shows that the nonresonant electrostatic wave by itself cannot produce the irreversible variations of particle energy, but can significantly amplify these variations produced by electromagnetic field fluctuations. Thus, in both cases ($v_\phi = 0$ and $v_\phi \neq 0$) we observe charged-particle acceleration by a nonresonant wave due to stochastization of the particle motion. We note that independently of the magnetic field amplitude (strong fluctuations can result in local magnetic field reversals), the main electrostatic wave is not in resonance with particles and, as a result, cannot accelerate particles in the absence of an additional stochastization.

To demonstrate that this effect is statistically important we integrate trajectories of the ensemble containing 10^4 particles. Figure 3 shows that the averaged energy $\langle\gamma\rangle$ of the particle ensemble is constant for $\delta = 0$ and increases for $\delta > 0$. Moreover, for a system with $v_\phi \neq 0$ the averaged energy increases significantly faster for $\varepsilon \neq 0$ than for $\varepsilon = 0$. Therefore, the particle energy grows diffusively, while all particles of the ensemble remain nonresonant (i.e., for all particles $|\mathbf{v}| < \omega/k - \sqrt{q\varphi_0/m}$). In the self-consistent system the observed acceleration of particles should result in wave damping. However, the mechanism of such wave damping differs from the Landau one [14] because there are no particles that satisfy the Cherenkov resonant conditions. We observe the same effect of the intensification of particle acceleration by nonresonant waves in both systems with stationary magnetic field fluctuations ($v_\phi = 0$) and with the induction electric field ($v_\phi \neq 0$).

Our results show that the inclusion of the background magnetic field spatial variations can result in particle

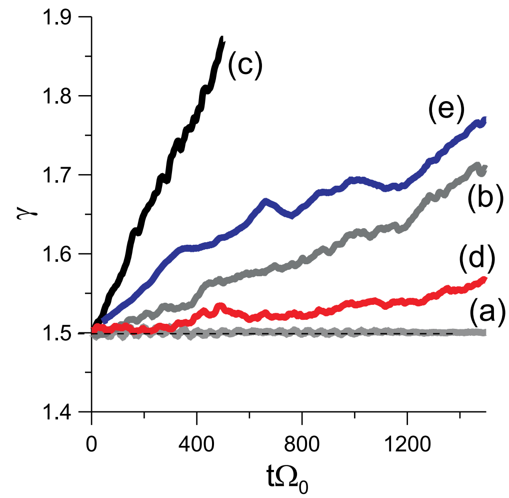

FIG. 3 (color online). Five examples of the averaged energy time profiles for particle ensembles (10^4 particles). System parameters are the same as in Fig. 2.

acceleration (wave damping) even in the system without resonant interactions. Such damping resembles wave scattering by small-scale density inhomogeneities when the wave energy is dissipated diffusively [24]. However, in contrast to resonant scattering in plasma with fluctuations (see, e.g., [25] and references therein), the proposed mechanism can be responsible for wave damping in cold plasma (without resonant particles). The realization of this mechanism is very straightforward and, thus, one can anticipate a wide range of possible applications. Although we consider charged-particle acceleration by an electrostatic wave propagating transversely relative to the background magnetic field, the same scheme of the nonresonant acceleration can be applied for both an oblique wave propagation and for electromagnetic waves. The further analytical consideration should provide a theoretical description of a nonresonant diffusion of charged particles in the velocity space. Such a description could supplement the quasilinear theory of resonant scattering of charged particles by waves [14].

An interesting effect of the nonresonant charged-particle acceleration can be found for a long term dynamics. If the particle has enough time, the diffusive nonresonant acceleration can result in energy gain large enough for the particle to approach the resonance (i.e., $|\mathbf{v}|$ becomes comparable to $\omega/k - \sqrt{q\varphi_0/m}$). In this case, the initially nonresonant particle can be trapped by the wave and accelerated in the resonant regime. For the electrostatic wave propagating transversely to the background magnetic field such trapped acceleration corresponds to the surfatron regime [18]. An example of the change of the particle acceleration regime from the nonresonant (diffusivelike) acceleration to the resonant surfatron acceleration is shown in Fig. 4. Thus, in the system with magnetic field fluctuations, initially nonresonant particles can become resonant

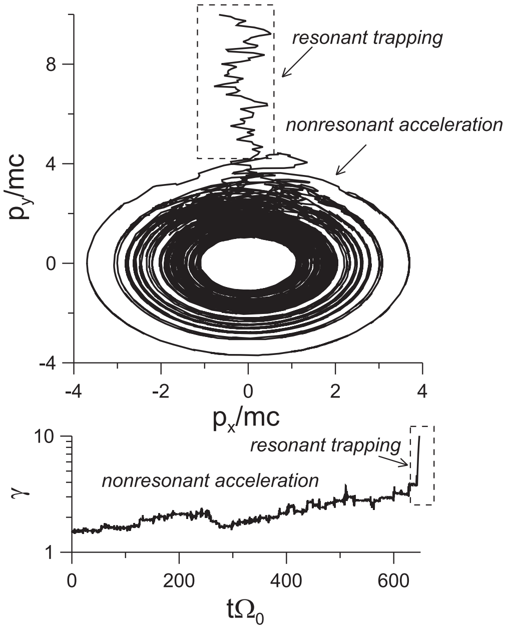

FIG. 4. An example of the particle trajectory containing two acceleration regimes: the nonresonant acceleration precedes the resonant surfatron trapping. System parameters are the same as for Fig. 2, but $\varepsilon = 10$, $\delta = 2.5$, $v_\phi = 0$.

and start efficiently interacting with the wave. This effect should result also in wave damping (or amplification) in the system where one could expect to observe the conservation of the wave energy due to the absence of resonant particles.

In conclusion, we demonstrated that the stochastization of charged-particle motion by magnetic field fluctuations (both stationary and dynamic) can result in the nonresonant acceleration by the electrostatic wave. The magnetic field fluctuations provide destruction of the adiabaticity of charged-particle motion, while the nonresonant interaction of stochastic particles with the wave electric field leads to a significant random variation of the charged-particle energy. This effect is observed both in systems with initially conserved particle energy (i.e., with stationary magnetic field fluctuations) and with a variation of energy (i.e., with a finite induction electric field). Because of nonresonant interaction with the electrostatic wave the energy of the particle ensemble grows with time diffusively and the rate of the corresponding acceleration depends on the intensity of the magnetic field fluctuations. Obtained results support the idea that waves can be damped in realistic systems even without resonant wave-particle interaction in the presence of sufficiently pronounced variations of the background magnetic field.

The work was supported by the RFBR No. 13-01-00251 (A. V. A. and A. A. V.), and by Leading Scientific Schools, project NSh-2964.2014.1 (A. I. N.).

*ante0226@gmail.com

[1] C. C. Chaston, C. Salem, J. W. Bonnell, C. W. Carlson, R. E. Ergun, R. J. Strangeway, and J. P. McFadden, Phys. Rev. Lett. **100**, 175003 (2008).
[2] R. M. Thorne, W. Li, B. Ni, Q. Ma, J. Bortnik, L. Chen, D. N. Baker, H. E. Spence, G. D. Reeves, M. G. Henderson, C. A. Kletzing, W. S. Kurth, G. B. Hospodarsky, J. B. Blake, J. F. Fennell, S. G. Claudepierre, and S. G. Kanekal, Nature (London) **504**, 411 (2013).
[3] F. S. Mozer, O. Agapitov, V. Krasnoselskikh, S. Lejosne, G. D. Reeves, and I. Roth, Phys. Rev. Lett. **113**, 035001 (2014).
[4] Y. Kuramitsu and V. Krasnoselskikh, Phys. Rev. Lett. **94**, 031102 (2005).
[5] V. Petrosian, Space Sci. Rev. **173**, 535 (2012).
[6] P. H. Diamond, S.-I. Itoh, and K. Itoh, *Modern Plasma Physics* (Cambridge University Press, Cambridge, 2010).
[7] A. B. Mikhailovskii, *Theory of Plasma Instabilities: Instabilities of a Homogeneous Plasma*, 1st ed. (Consultants Bureau, New York, 1974) pp. 1–308.
[8] C. F. F. Karney, Phys. Fluids **21**, 1584 (1978).
[9] H. Karimabadi, K. Akimoto, N. Omidi, and C. R. Menyuk, Phys. Fluids B **2**, 606 (1990).
[10] V. D. Shapiro and R. Z. Sagdeev, Phys. Rep. **283**, 49 (1997).
[11] D. Shklyar and H. Matsumoto, Surv. Geophys. **30**, 55 (2009).
[12] V. I. Arnold, V. V. Kozlov, and A. I. Neishtadt, *Mathematical Aspects of Classical and Celestial Mechanics*, 3rd ed., Encyclopedia of Mathematical Sciences (Springer-Verlag, New York, 2006).
[13] A. B. Mikhailovskii, *Electromagnetic Instabilities in an Inhomogeneous Plasma* (Institute of Physics Publishing, New York, 1992) pp. 1–189.
[14] D. A. Gurnett and A. Bhattacharjee, *Introduction to Plasma Physics* (Cambridge University Press, Cambridge, 2005).
[15] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).
[16] R. Z. Sagdeev and V. D. Shapiro, JETP Lett. **17**, 279 (1973).
[17] M. A. Malkov and G. M. Zaslavskii, Phys. Lett. A **106**, 257 (1984).
[18] T. Katsouleas and J. M. Dawson, Phys. Rev. Lett. **51**, 392 (1983).
[19] A. A. Chernikov, G. Schmidt, and A. I. Neishtadt, Phys. Rev. Lett. **68**, 1507 (1992).
[20] A. P. Itin, A. I. Neishtadt, and A. A. Vasiliev, Physica (Amsterdam) **141D**, 281 (2000).
[21] F. Valentini, P. Veltri, and A. Mangeney, Phys. Rev. E **71**, 016402 (2005).
[22] P. Veltri, G. Zimbardo, A. L. Taktakishvili, and L. M. Zelenyi, J. Geophys. Res. **103**, 14897 (1998).
[23] F. Chiaravalloti, A. V. Milovanov, and G. Zimbardo, Phys. Scr. **T122**, 79 (2006).
[24] A. I. Akhiezer, I. A. Akhiezer, and R. V. Polovin, *Non-Linear Theory and Fluctuations: Plasma Electrodynamics* (Pergamon Press, New York, 1975).
[25] C. Krafft, A. S. Volokitin, and V. V. Krasnoselskikh, Astrophys. J. **778**, 111 (2013).