

# AN ACTIVITY THEORY ANALYSIS OF GROUP WORK IN MATHEMATICAL MODELLING

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*In this paper we analyse the activity of a group of engineering undergraduate students while working on a mathematical modelling task. Using Cultural-Historical Activity Theory as analytical framework, we focus our attention on their social interactions to understand how these mediate the collective sense making of the group and determine in great part the outcome of the activity. We conclude that a key factor to students' mathematical learning in collaborative tasks is the quality of peer interactions which stems from students' competences, such as communicative and inter-personal skills.*

## BACKGROUND

Many lecturers and researchers agree that the development of problem-solving and mathematical modelling skills is an important aspect of the education of undergraduates studying Science, Technology, Engineering and Mathematics (STEM). For example, in their seminal paper, Blum and Niss (1991) presented several arguments in favour of including aspects of modelling and problem-solving in mathematics instruction, amongst which are the development of skills and attitudes such as open-mindedness, self-reliance, confidence and critical thinking.

In many cases, pedagogical implementations of mathematical modelling and problem-solving are accompanied by collaborative group work. The potential benefits of team collaboration to student learning have been well documented (see for example Laal & Ghodsi 2012). In particular, several studies of collaborative work in mathematics suggest it can help the students' process of modelling and problem-solving, and hence contribute positively to their learning, but that this relation is complex and still not well understood. For example, Lowrie (2011) reports on the tensions between collaborative learning and the use of "genuine" artefacts in problem-solving mathematical tasks. And Clark et al. (2014) suggest that effective group work collaboration depends on the type of problem choice that can elicit certain behaviours and the establishment of a "group synergy" that can lead to increased group interaction and activity.

However, most research on collaborative group work in mathematical problem-solving and modelling has been done from a cognitive perspective, e.g. comparing the characteristics and behaviours of novice and expert modellers/problem-solvers and focusing mainly on the heuristics of the process (i.e. selection of key variables and appropriate assumptions to the problem, construction of relations between variables, etc.). For instance, Paterson and Watt (2014) describe how third year undergraduate students working in groups failed to solve a mathematical problem

because ‘they ignored explicit constraints, over-generalised earlier examples and left a number of erroneous assumptions unchallenged’ (p. 19). Further, Soon et al. (2011) describe first year undergraduate students’ difficulties with mathematical modelling as an inability to connect “real life contexts” and “mathematical representations”.

Of the few studies that take a socio-cultural perspective, Goos et al. (2002) is a significant example. Taking a Vygotskian perspective, these authors reconceptualise metacognition (or “learning to learn”) as a social practice, and they conclude that:

the interplay between transactive challenges and metacognitive decisions was significant in creating zones of proximal development that shaped problem solving outcomes, since challenges eliciting clarification and justification of strategies stimulated further monitoring that led to errors being noticed or fruitful strategies being endorsed (p. 218).

However, in this paper we will argue that there are other relevant issues that remain insufficiently explored in the literature and that may have a significant effect on the outcome of small group collaborative mathematical activity. The aim of this paper, therefore, is to investigate how the social interactions that occur in small group collaborative work affect the outcome of a mathematical modelling task, and particularly how these interactions contribute to mathematical sense making.

To this aim we observed students in a one semester second year undergraduate mathematics for engineering course at an English research-intensive university. A feature of this course is the use of mathematical modelling tasks as a complement to traditional style lectures; in order to solve these tasks, students work in small groups (4 -5 members). In this paper, we selected a one-hour episode in which the students failed to produce a mathematically correct solution to the task. The analysis of this particular episode allowed us to gain important insights into issues that we will argue are significant in shaping collaborative mathematical activity. The research question guiding our analysis was: How do social interactions in a small group collaborative work influence the students’ mathematical sense making and the outcome of the activity?

## **THEORETICAL FRAMEWORK AND METHODOLOGY**

In order to study how social interactions influence an activity such as small group collaborative work, we found Cultural-Historical Activity Theory (CHAT), as described by Engeström (1987), a helpful analytical framework. CHAT’s well-known “triangle” (Figure 1) draws attention to the complexity of (social) factors mediating human activity.

Human activity, in this case collaborative learning in a mathematical modelling task, is our unit of analysis; subjects engage in this object-oriented activity with the purpose of obtaining an outcome (e.g. learn something, solve a problem). The community, the rules and the division of labour represent the social/collective elements of the activity, which interact between them and, along with the tools, mediate the activity.

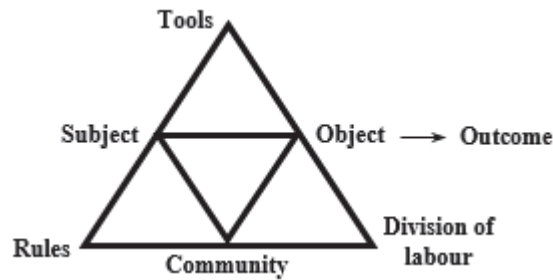


Figure 1. The structure of a human activity system (Engeström 1987).

The episode analysed in this paper was audio-recorded and transcribed. The second author, who observed the episode, also took notes that were included in the transcript. The transcript was then coded separately by the two authors according to the elements in the CHAT triangle and the results compared. Meanings were negotiated between the authors (there were no major discrepancies) and shared interpretations were achieved.

## CHAT ANALYSIS OF COLLABORATIVE GROUP WORK IN A MATHEMATICAL MODELLING TASK

### The activity

The activity described in this paper was the modelling task shown in Figure 2.

There is an object of mass  $m$  attached to the end of a spring as in the illustration. **Hooke's law** states that the force exerted over a spring that is stretched  $x$  units from its resting position (equilibrium) is proportional to the distance  $x$ .

Use Newton's second law of motion (Force is equal to mass times acceleration) to model this type of motion (known as simple harmonic motion) with a second order differential equation. Then, solve the equation.

Now suppose that a spring with a 3 kg mass is held stretched 0.6 m beyond its equilibrium position by a force of 20 N. If the spring begins at its equilibrium position (when  $t=0$  what is  $x$ ?) but a push gives it an initial velocity of 1.2 m/s (when  $t=0$  what is the velocity?), find the position of the mass after  $t$  seconds.

Figure 2. The modelling task

The topic of the task, on harmonic motion, was second order Ordinary Differential Equations (ODE) and students were asked to work in groups (of their own selection) to solve the problem and write a brief report of their solution to be handed in at the end of the tutorial session (1 hour). Students had already covered how to solve second order ODEs in previous lectures and also during their first year course. Students were reminded of the modelling cycle (Blum & Borromeo-Ferri, 2009), and the lecturer handed out a sheet on “effective group work” and explained it to the group. The task was not formally assessed; the lecturer introduced it as “preparation”

for their coursework, and offered to give constructive feedback to each group the following week.

### **The subjects of the activity**

The group that we followed started with 4 members (all pseudonyms): (1) Steve, who described himself as not confident with mathematics. He usually does not attend lectures; (2) Mike, who attends most lectures, said he was “catching up” with the topic and brought with him a textbook on engineering mathematics. He said he usually revises only before the exam; (3) Hank, who said he was confident with the material, attends all lectures and usually spends 30 minutes (“but no more”) revising problems seen during lectures. He always brings with him his lectures notes; and, (4) Tom, who attends lectures regularly but does not feel confident with mathematics.

After 10 and 15 minutes, respectively, two other students joined the group (Steve sent a mobile text to them asking to come and help him with the task): (5) George, who said he found the material difficult and that he is not confident with mathematics in general; and, (6) Alan, who described himself in similar terms as George but usually helps Steve with his mathematics. Both do not attend lectures regularly.

It can be noticed from the description of the members of this team that only Hank feels confident and indeed he takes the lead throughout most of the activity. However, contrary to common descriptions found in the literature where one team member (usually a “high achiever”) dominates and determines the group’s ideas, in this case important ideas came from various group members who took the lead at different points in time. In fact, it was observed that throughout the activity all members of the group went “in and out” of the mathematical problem, sometimes taking a “back seat” and then “coming back” to contribute to the discussion, sometimes forming sub-groups to discuss unrelated issues (e.g. sports activities).

### **The object of the activity**

It can be assumed that, for the subjects of the activity, their object was to solve the mathematical problem and, as a result, produce a group report. Members of the group ratified this objective at different times. For example, after George joined the group, Steve and Hank said:

Steve:            So what do we actually have to do?

Hank:            Make a report (and Mike proceeds to read the task).

### **The community and the tools**

In this case, the community was formed of the members of the group but it was not a “community of practice” in the sense of Wenger (1998). This “community” was formed spontaneously and for the purpose of solving this one task, and then dissolved. Members did not share any history in relation to the practice (although they have a history of knowing each other in other practices), or any shared discourse associated with mathematical collaborative work. There were no “master” and “apprentices”.

Due to the informal and spontaneous nature of this community, there was an attempt to build shared meanings and understandings and to collaborate in achieving the object of activity. This interactivity and willingness to build consensus made this activity genuinely collaborative (as opposed, for example, to a co-operative task). There was also a constant questioning and challenging of the work throughout the task, e.g. Mike: ‘Are you sure it’s supposed to be an  $H$  there?’ or Hank: ‘Think. Any ideas people?’ or George: ‘What am I doing here?’ Alan: ‘All right, then you divide it by  $x$ ’. George: ‘Oh, yeah, yeah’. Throughout the activity, peers took different roles as the group tried to make sense of the problem. For instance, around 15 minutes into the task the group had been struggling in defining variables and constants but had come to a shared agreement that Newton’s second law of motion ( $F = ma$ ) could be differentiated into  $\frac{dF}{dx} = m$  ( $F$  differentiated with respect to  $x$ ,  $m$  is a constant and  $a$  is a variable). However, differentiating again (to obtain a second order differential equation) would yield a zero! At that point Hank, unable to come up with a constructive idea, took a back seat to read from the workbook and Steve and George “took over” the task. George then came with an idea:

- George: I was thinking that, I was thinking I must put, like, because when  $F$  is the same in both, so we must put  $kx = ma$  and then, take it from there.
- Steve: Yeah. But you know that  $k$  is obviously a constant.
- George: Yeah,  $k$  is a constant.
- Steve: And mass is a constant, therefore you’ve got  $a$  and  $F$ .
- George: Sure. So wait, and isn’t force not the same in both of these things, so, would  $a$  not equal to  $k$ ?

While George’s first idea could have eventually resulted in a satisfactory solution, the interaction with Steve resulted in an equation which contained mathematical errors:  $m = kx$ . Taking then his peers’ idea as a resource, Hank continued to elaborate on the problem by trying to manipulate their expression, eventually getting stuck again.

Our interpretation of the type of interactions like the one described above is that peers’ ideas proposed at each moment in time become the “tools” that the group use in their process of sense making. Also, the relations in the group change and evolve (peers take different roles) and, as a consequence, the meaning making process can take unexpected directions according to the tools at hand. It would be simplistic to explain this group’s difficulties as *only* a matter of learning to distinguish between variables and constants or to establish sound relations between variables. How the members of the group (the community) interact between them and with the “tools” available at the time mediates the outcome of the activity in fundamental ways, shaping what is learnt individually and collectively.

### **The rules and the division of labour**

The rules of the activity (how members of the group interact in order to achieve the outcome of the task) can be explicit but often also implicit. This adds to the

complexity of the activity, as members interpret these rules in different ways. For example, while some group members were adamant that they should not use external help to solve the problem, others seem to be more willing to use external resources to advance their solution. This also impacts on the tools available to the group:

Steve: Literally, I'm just going to go on Google and google this to see if there is an equation for it.

Hank: Half the fun though.

Mike: No, you can't do that.

It is also important how different group members perceive their peers and are perceived by the group. This has an effect on whose ideas are considered valuable or worthy of taking into account. For instance Tom, who had not been participating in the discussions until then, suggested the course's workbook might help find a solution to the task. But this suggestion was followed by hesitation:

Tom: But, I don't know, from these two pages I don't know where to go from there.

As a result, the group dismissed his comment and went back to work on a previous idea. Indeed, in the division of labour some declare themselves "out" from the beginning (e.g. Steve: 'I'm pretty useless to be honest (...) I told him to come down because this isn't my thing'), even though they might change their "status" as the activity evolves and they feel they can contribute to the task. Newcomers (e.g. lecturer, new team members) disturb the division of labour by introducing new ideas, which are taken in differently by group members that react (or not) to this new information.

Furthermore, new ideas (externally sourced or internally proposed) are of little use if these are not specific, that is, if there is not a "connection" between the new idea and the current group's understanding of the problem (i.e. does it make sense in relation to the group's thinking of the problem at the time?). As seen above in the case of Tom, his idea of searching the workbook for help was discarded because it was not clear to the group how it could advance the solution. His hesitation ("I don't know where to go from there") meant that his suggestion was not worth considering. Also, his status in the group (as not having participated fully until then) meant that his voice was less heard. Similarly, suggestions of external help (e.g. Google) are rejected because the rules of the group are that this kind of help constitutes "cheating" or does not help someone "learn" and that some of these sources are not always reliable (a belief expressed in another occasion), hence having a lesser status.

In a few occasions, ideas that could have steered the group's thinking in the right direction did not materialise because they were either communicated without confidence or not clear enough to connect with the group's current thinking. For example, around 30 minutes into the task, Hank identified that a key variable in the problem is the distance ( $x$ ), but he was unable to identify the variable with respect to which it varies ( $\frac{dx}{?}$ ). This hesitation meant that the idea was discarded and the group

had gone back to a previous state where they tried to differentiate  $k$  (a constant!) with respect to the acceleration ( $\frac{dk}{da}$ ), getting a zero after differentiating twice.

Even if a new idea comes from an authority (e.g. lecturer) it still has to connect to the current thinking of the group in order to be useful, or in other words, to become a “tool” for sense making. For example, a couple of minutes after the previous episode, the lecturer approached the table and gave the group the following information:

Lecturer: The acceleration, that’s the derivative of the velocity, velocity is the derivative of the distance.

The current thinking of the group was that they should, somehow, differentiate an expression twice to get a second order differential equation. They did not realise that the first expression  $kx = ma$  was already a second order ODE and that they just had to “spell out” the acceleration as a second derivative of the distance w.r.t. time:  $\frac{d^2x}{dt^2}$ .

After this intervention by the lecturer, the group’s struggles continued without approaching a satisfactory solution. In jest, Hank said: ‘This is going to be a long hour, isn’t it?’

## Conclusions

Our research question was: How do social interactions in a small group collaborative work influence the students’ mathematical sense making and the outcome of the activity? We have tried to answer this question from a CHAT perspective:

The composition of the community (with their members’ individual histories of previous and present engagement with mathematics), the rules (explicit and implicit) and the division of labour (which influences whose ideas are valuable or not) shape in unique ways the social interactions that occur in a group activity. These interactions determine the tools that are available to the group, which in turn mediate the sense making process and influence the outcome of the activity.

In our particular community, there was no one who was mathematically confident enough to challenge the shared meanings constructed by the group, even though they all had studied second order ODEs before. While some research studies (Patterson & Watt, 2014; Goos et al., 2002) suggest the challenging of ideas as an important factor in the success of problem-solving collaborative group work (something like “playing the devil’s advocate”), our data suggests that this is not always enough. In our group there was challenging but students were unable to respond to the challenge or express their peers’ contributions in a way that could shift the collective meanings towards more productive thinking that could positively achieve the object of the activity. Even when potentially fruitful ideas were introduced into the activity, these were not presented or communicated in a way that could have been connected with the group’s sense making process at the time, that is, the group could not transform the ideas into effective tools for the achievement of the outcome, and therefore were lost, side-tracked or discarded. Hence, the meanings produced by the group were mathematically incorrect and the object of the activity was not achieved.

What could have helped this group achieve a better outcome, and so what can we learn from this analysis? We believe the key factors in achieving a positive outcome reside in the quality of interactions that are produced during collaborative work, and hence, in the shared construction of effective tools that can contribute to a sense making process that results in a mathematically correct outcome. However, the skills necessary for these interactions to occur are not normally an explicit or even implicit part of school/university mathematics curricula or assessment. Competences such as communicative skills (e.g. active listening, reflection, effective speaking) or interpersonal skills (e.g. negotiation, assertiveness) are not normally associated with mathematics, a subject that remains largely individualistic. We believe therefore that the explicit teaching of these skills within more socially-oriented mathematical pedagogies can benefit students' mathematical learning, engagement and achievement.

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