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Giant Conductance Oscillations in a Normal Mesoscopic Ring Induced by an SNS Josephson Current

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Abstract

A theoretical explanation of giant conductance oscillations observed in normal mesoscopic rings with superconducting "mirrors" is proposed. The effect is due to resonant tuning of Andreev levels to the Fermi level, which enhances the transparency of the system to the normal current. The mechanism is demonstrated for a one-dimensional model system.

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Recently, in a series of experiments by Petrashov et al. [1], new unusual properties of mesoscopic silver rings in contact with superconducting islands ("mirrors") were observed. These "mirrors" could be placed across the current leads (L-case) as in Fig. 1a, or as shown in Fig. 1b at the stubs connected to the ring perpendicular to the normal current flow (T-case). The three most striking features revealed in experiments are as follows:

(i) the amplitude of $hc/2e$-periodic Aharonov-Bohm (AB) oscillations was at least 100 times larger in the L-case than in the ring without "mirrors"; in the T-case the enhancement of $hc/2e$-oscillations was about 10 times, and $hc/4e$-oscillations were observed as well;

(ii) in the T-case the normal resistance of the system grows approximately twice, as the superconductivity of the "mirrors" is being suppressed by magnetic field (above 500 Gs);

(iii) the effects were totally absent in the case when one branch of the ring was completely crossed by a superconducting strip.

The confinement idea proposed in the original paper [1], i.e. that the effect is due to confinement of quasiparticles in the ring by the "mirrors", qualitatively explains the L-case behaviour, but meets difficulties both in obtaining a proper magnitude of the effect in the L-case (the maximum enhancement being of order 10), and in explaning the effect in the T-case, where the quasiparticles are not confined in a longitudinal direction.

An alternative explanation suggested by de Vegvar and Glazman [2] is based on a supposition that the "mirrors" induce superconductivity in a significant portion of the ring itself. These superconducting parts play the role of filters, through which only Cooper pairs can pass. However, this mechanism neither accounts for the significant difference between L- and T-configurations, nor for the complete absence of the effect when one branch of the ring was crossed by a superconductor (as the authors of Ref. [2] point out themselves).

In this paper we present some theoretical arguments which seem to explain the physical nature of the observed phenomena; they are supported by a model calculation. In our analysis we assume that the phase breaking length $L_\phi = (D\tau_\phi)^{1/2}$ and the normal metal coherence length $L_T = (\hbar D/k_B T)^{1/2}$ are larger than the characteristic sample size L. Here

τ_ϕ^{-1} is the inelastic scattering rate, D is the diffusion constant of quasiparticles, k_B is the Boltzmann constant, and T is temperature. We will also assume that the ring is weakly coupled to the normal reservoirs (source and sink of normal current).

We start from a qualitative discussion. If an electric current is made to pass through a mesoscopic normal ring, it is well known that its magnitude will oscillate as a function of the magnetic flux threading the ring. These normal AB oscillations persist if the ring is brought in contact with two superconducting mirrors. However, now a new group of quasiparticles starts to contribute to the oscillations: the electrons (or holes) that undergo Andreev reflections at the NS-boundary (between the ring and one of the mirrors). We will see below that their contribution may dominate the AB oscillations under certain conditions.

In a ring which is weakly connected to external reservoirs, the quantized energy levels of quasiparticles are well defined [6]. In our case these levels are formed by reflections both at NN-interfaces (between the ring and the normal reservoirs) and at NS-boundaries with the "mirrors". The energy levels and are therefore sensitive to the magnetic flux threading the ring, Φ, *and* to the superconducting phase difference between the mirrors, $\Delta\phi$. The corresponding quasiparticle states carry both the normal- and the Josephson [6,4] current.

Since the normal current in the linear response limit is carried only by the quasiparticles on the Fermi surface, it is resonantly enhanced each time an energy level — driven by the magnetic flux and/or the superconducting phase difference — passes the Fermi energy. Simultaneously, the Josephson current changes sign.

Therefore the relation can be guessed

$$I_q \sim \frac{dI_J}{d\Delta\phi}. \tag{1}$$

As a matter of fact, using a 1D model (to be described below) we are able to derive the relation valid in the ballistic case:

$$\lim_{\epsilon\to 0} \frac{I_q}{V\epsilon} = \left(\frac{eL}{4\hbar v_F} \frac{dI_J}{d\Delta\phi} + \frac{e^2}{\pi\hbar} \right). \tag{2}$$

Here ϵ is the probability for an electron to leave the ring for one of the normal reservoirs through the NN-interface (see below).

The second term in (2) is due to the fact that the energy levels in the ring are formed not only by Andreev reflections at NS-boundaries, but by the normal reflections at NN-interfaces as well. Therefore their shift in a magnetic field (and thus the conductance [3]) can not be completely accounted for by an expression like (1), where the magnetic flux, Φ, enters in a gauge invariant combination with the phase difference between the superconducting "mirrors", $\Delta\phi$. There must be a contribution to the level shift that depends solely on Φ.

The relation (2) shows that *if* there is a Josephson current in the system, it leads to a resonant contribution to the normal current for certain values of the magnetic flux. Indeed, the phase dependence of the Josephson current in an SNS contact has a sawtooth-like form, which would give rize to δ-function shaped peaks when its derivative is taken with respect to phase [4]. The question is, how can a stationary supercurrent flow between two finite and small superconducting "mirrors"?

Let us first consider a standard SNS-junction in an external magnetic field, $\mathbf{B}$, parallel to the boundary between the normal layer and superconducting half-spaces (Fig. 2a). The phase difference between the superconductors is [4]

$$\Delta\phi(y) = \phi_0 - \frac{2eBLy}{\hbar c} \equiv \phi_0 - 2\pi\frac{2\Phi(y)}{\Phi_0}, \tag{3}$$

where y is the coordinate on an axis that lies in the plane of the normal layer and is normal to the magnetic field, ϕ_0 is an arbitrary phase difference between two isolated superconductors, and L is the width of the normal layer. The quantity $\Phi(y_0)$ is evidently the magnetic flux through the part of the junction between the lines $y = 0$ and $y = y_0$, and $\Phi_0 = hc/e$. The distribution of Josephson current is schematically shown in Fig.2a; each current line encircles one half flux quantum (Josephson vertex).

In a real experimental situation the normal stubs connecting the metal ring with the "mirrors" have a finite width. This creates the situation when equal and opposite Josephson currents flow along the edges of the stubs (see Fig.2b). In order to establish such a current distribution, the phase difference between the superconducting "mirrors" tunes to each given value of the penetrating magnetic flux (via changing the constant ϕ_0 in (3)). Therefore the partial Josephson currents will oscillate, contributing to the AB oscillations in the normal conductance of the ring, though their sum (net Josephson current between the "mirrors") is always zero.

Now we can discuss when this contribution can play a major role. It is known that the amplitude of the Josephson current density is not sensitive to an increase in the area of an SNS junction [4]. On the other hand, the amplitude of the conductance oscillations in a normal metal ring decreases by a factor $\sqrt{N_\perp}$ as its cross-section grows [5] [the number of channels in a ring with the cross-section area A is $N_\perp \sim A/\lambda_F^2$]. The reason is that in the former case it is the effective momentum of the electron-hole excitations that carry the Josephson current, which is quantized:

$$p_\parallel^{(e)} - p_\parallel^{(h)} = \sqrt{2m(E_F + E) - p_\perp^2} - \sqrt{2m(E_F - E) - p_\perp^2} \simeq \frac{2mE}{\sqrt{E_F - p_\perp^2}}, \tag{4}$$

while in the latter case it is the electronic momentum itself:

$$p_\parallel \simeq \sqrt{2mE_F - p_\perp^2} + \frac{mE}{\sqrt{2mE_F - p_\perp^2}}. \tag{5}$$

The first term in (5), which is absent in (4), causes fast oscillations when the partial current is integrated over the transverse momentum $p_\perp$. This is the reason of drastical reduction of the "normal" AB oscillations' amplitude in a ring with large cross-section. In this case the "Josephson" contribution will be dominating.

The above considerations lead us to the conclusion that the ratio of the amplitude of AB oscillations in a ring with superconducting "mirrors" to the amplitude of AB oscillationsits in a "normal" ring is of the order of $\sqrt{N_\perp}$.

Let us make some numerical estimates. Taking for the cross-section area the value $A \approx 5 \cdot 10^{-11}\text{cm}^2$, qhich is consistent with the experiment [1], and $\lambda_F \sim 10^{-8}\text{cm}$, we find that the enhancement of the AB conductance oscillations due to the proposed mechanism is $\simeq 700$ times compared to the ring without superconducting "mirrors". (The actual value of Ref. [1] was up to 400 times).

The above qualitative speculations can be corroborated by a model calculation. The model system is shown in Fig.3. It contains all the physically significant features of the experimental setup (Fig.1); the normal part of the current from reservoir I to reservoir II through leads 3, 1, and 4 is controlled by the Josephson current between the “mirrors” through leads 1 and 2.

All the normal scattering in our model is confined to the nodes A and B. These are described by identical 6×6 S-matrices , which relate the incoming and outgoing wave amplitudes of the quasiparticles in the 1D wires. We use real matrices parametrized by a real number $0 \leq \epsilon \leq 1/2$ [6], which in the limit $\epsilon \ll 1$ (weak coupling to the reservoirs, i.e. small transition probability from lead 1 to leads 3 and 4) have the form

$$\mathbf{S} = \begin{pmatrix} -\epsilon/2 \cdot \hat{\mathbf{1}} & (1-\epsilon/2) \cdot \hat{\mathbf{1}} & \sqrt{\epsilon} \cdot \hat{\mathbf{1}} \\ (1-\epsilon/2) \cdot \hat{\mathbf{1}} & -\epsilon/2 \cdot \hat{\mathbf{1}} & \sqrt{\epsilon} \cdot \hat{\mathbf{1}} \\ \sqrt{\epsilon} \cdot \hat{\mathbf{1}} & \sqrt{\epsilon} \cdot \hat{\mathbf{1}} & (-1+\epsilon) \cdot \hat{\mathbf{1}} \end{pmatrix}. \tag{6}$$

Here $\hat{\mathbf{1}}$ is the unit 2×2-matrix, which reflects the fact that in the presense of an NS-boundary we have to use the two-component wave function of the quasiparticle even in the normal leads, in order to account for the electron-hole correlations created by Andreev reflections [4]. Eq.(6) reflects the fact that the electron- and holelike excitations are not mixed in the nodes A and B. On the other hand, we assume that at the interfaces of leads 1, 2 and the superconducting islands only Andreev reflection takes place, and the normal reflection is absent.

The calculations to be described are straightforward. We are interested in the linear response value of the normal conductance. Therefore, the problem is reduced to the calculation of the transition probability of a quasiparticle from, say, reservoir I to reservoir II. The initial scattering matrices can be replaced by effective matrices of dimensionality 4×4, $\mathcal{S}_A$ and $\mathcal{S}_B$. They only relate the quasiparticles in leads 3, 4, and the portion of lead 1 between A and B to each other and include the effects of Andreev reflections (see [7] for details). We have a standard Landauer configuration with two reservoirs connected by a 1D wire with scatterers A and B, but with two-component wave functions of the quasiparticles; the components are mixed by Andreev reflections at NS interfaces. The conductance is then obtained as [8]

$$G = \frac{2e^2}{h} \cdot 2 \int_0^\infty d\xi \, (T_0^> + R_a^>) \left(- \frac{\partial n_F(\xi)}{\partial \xi} \right) + \eta, \tag{7}$$

where $T_0^>$ ($R_a^>$) is the normal transition (Andreev reflection) probability for an electron incident from the left normal reservoir; $n_F(\xi)$ is the Fermi distribution, and ξ is energy measured from the Fermi level. The term η in (7) quickly oscillates as a function of the electron momentum (as $\sim \exp 2p_F L$, L being the length of lead 1) [7] and is exactly zero in the case of time reversal symmetry [8].

After averaging over fast spatial oscillations on the scale of $\lambda_F \ll L$, the coefficients in (7) take the following values [7]:

$$T_0^>(\xi) \approx R_a^>(\xi) \approx \epsilon^2 \left(|a_+(\xi)|^{-2} + |a_-(\xi)|^{-2} \right). \tag{8}$$

The resonant denominators $|a_\pm(\xi)|^2$ vanish close to the energies $\xi_n^\pm$ of the Andreev levels in lead 1:

$$|a_\pm(\xi)|^{-2} \approx \sum_n \left\{ \left(\frac{2L}{\hbar v_F} \right)^2 \cdot \left(\left(\xi - \xi_n^\pm \right)^2 + \epsilon^2 \left(\frac{\hbar v_F}{4L} \right)^2 \right) \right\}^{-1}; \tag{9}$$

$$\xi_n^\pm = \frac{\pi \hbar v_F n}{L} + \frac{\hbar v_F}{2L} (\pi \mp 2\pi \frac{2\Phi}{\Phi_0}). \tag{10}$$

Here v_F is the Fermi velocity and $\Phi_0 = hc/e$ is the magnetic flux quantum. In place of the superconducting phase difference, $\Delta\phi$, only the (dimensionless) magnetic flux Φ/Φ_0 through the loop formed by the normal wires 1, 2 and the superconducting mirrors enters expression (10) for the Andreev energies. This is due to the fact that the Josephson current in lead 1 must be exactly cancelled by the one in lead 2. This condition fixes the phase difference between the superconducting mirrors.

Provided that the Andreev level separation exceeds the level width $\Delta E = \epsilon \hbar v_F / 4L$, a condition which is satisfied for small enough ϵ, we can calculate the integral in (7) using the Poisson summation formula. At zero temperature one finds the expression

$$G = \epsilon\frac{e^2}{h}\left(1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-2|n|\epsilon}\cos\left(n\cdot 2\pi\frac{2\Phi}{\Phi_0}\right)\right). \tag{11}$$

If we compare this result to the well known expression for the Josephson current in a planar SNS-junction [4],

$$I_J(\Delta\phi) = \frac{8ev_F}{\pi L}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin n\Delta\phi}{n}, \tag{12}$$

we see that indeed, in the weak coupling limit ($\epsilon \to 0$) we have the relation between the normal conductance and the Josephson current given by equation (2).

The above calculations are directly generalized to the case of $N_\perp > 1$ transverse modes in the normal wire, provided that they are not mixed by scattering. Then the conductance (11) should be simply multiplied by $N_\perp$. The fact that the amplitude of its oscillations now can exceed the conductance quantum, $2e^2/h$, reflects the ballistic character of the system under consideration.

In conclusion, we have demonstrated that the normal conductivity of a mesoscopic ring with superconducting "mirrors" is sensitive to the Josephson current between them. The corresponding contribution to the conductance is resonant and in a many-channel case gives rize to greatly enhanced Aharonov-Bohm conductivity oscillations in the system. The results provide an explanation to recent experimental results by Petrashov et al. [1].

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FIGURES

FIG. 1. Sketch of the experimental setup of a mesoscopic ring with superconducting “mirrors” used in Ref. [1]. *I, II* are normal reservoirs, S - “mirrors” (small superconducting islands). The magnetic field is normal to the picture plane. (a) L-configuration. (b) T-configuration.

FIG. 2. (a) Distribution of Josephson current in an SNS-junction in the presence of a magnetic field; (b) Josephson current in a mesoscopic ring with superconducting “mirrors”

FIG. 3. Model system where 1, 2, 3, and 4 are ideal normal-conducting 1D leads. Nodes A, B are described by real scattering matrices (see text). The directions of normal (I_q) and Josephson (I_J) currents are schematically shown. The magnetic field $\mathbf{B}$ is normal to the plane of the picture, and the distance between the “mirrors” equals L.