

How do Undergraduates Read Mathematical Texts? An Eye-Movement Study

Lara Alcock, Tom Kilbey, Matthew Inglis

Mathematics Education Centre, Loughborough University, UK

This paper reports on an eye-movement study of undergraduate mathematical reading behaviours. The eye movements of 38 undergraduate students were recorded as they read a multi-page textbook section on graph theory; participants then took a short comprehension test. This abstract reports basic results showing that neither reading time nor processing effort – measured via mean fixation durations – predicted comprehension test performance: students who read for longer or tried harder did not necessarily learn more. The conference report will include more detailed analysis of participants' eye movements: it will explore their relative attention to different parts of the text and the extent to which they shift their attention back and forth during learning, and will analyse the extent to which these behaviours differ across more and less effective learners.

Introduction

Undergraduate mathematics students are expected to learn in part by reading lecture notes and textbooks. But do they read effectively? Research shows that perhaps they do not. Interview studies indicate that when reading textbook passages, students tend to respond unhelpfully when facing confusion (Shepherd & van de Sande, 2014); eye-movement studies indicate that when reading a single purported proof, students tend to make less effort than mathematicians to study logical relationships between its claims (Inglis & Alcock, 2012).

This report will extend work of both types by reporting a study in which 38 undergraduate mathematics students read an extended graph theory textbook passage while their eye movements were recorded, then took a short comprehension test. It will report descriptive statistics showing dramatic variation in students' reading times and comprehension test scores, and analyses of differences in reading behaviours of more and less successful students, including their attempts to link different parts of the text, their relative attention to different representation types, and their relative attention to definitions, theorems, proofs and examples.

Theoretical Background

There has been increasing interest in recent years in undergraduates' mathematical reading behaviours and their consequences for comprehension. This has arisen in part because many lecture-based learning situations demand that students learn from written mathematics (Weber & Mejía-Ramos, 2014), and in part because researchers recognised that earlier work on proof had tended to focus on proof construction rather than on other issues such as comprehension (Mejía-Ramos & Inglis, 2009). Mathematicians have argued that comprehension tests and other activities related to proof evaluation can and should be used as a way to support critical engagement with complex mathematical arguments (Conradie & Frith, 2000; Kasman, 2006), and mathematics educators have done theoretical and empirical work in developing proof comprehension tests (Mejía-Ramos, Fuller, Weber, Rhoads & Samkoff, 2012).

Empirical study of broader mathematical reading behaviours nevertheless remains in its infancy, although studies of two types are contributing in different ways. First, interview

studies indicate that when reading textbook passages, students tend to respond unhelpfully to confusion: compared with more mathematically experienced readers, they can be inattentive to details, insensitive to confusion or error, and less likely to seek resolution via careful re-reading (Shepherd & van de Sande, 2014). Such observations provide insight into suboptimal reading behaviours, but interview studies are necessarily subject to issues of reactivity (Russo, Johnson & Stephens, 1989): reporting aloud while learning can be expected to influence the behaviours under study.

Second, eye-movement studies indicate that compared with mathematicians, undergraduates attend less to the words of purported proofs and less to the logical relationships between their claims (Inglis & Alcock, 2012). Related work has demonstrated that self-explanation training can improve both attention to logical relationships and consequent comprehension (Hodds, Alcock & Inglis, 2014), but eye-movement work in this area has so far been restricted to the study of single proofs. It is thus limited in external validity: when studying lecture notes or textbooks, students need to understand extended passages of mathematical information; single proofs form part of such passages, but a student need not restrict their attention in this way.

The exploratory study reported here takes a step toward bringing together these approaches, studying eye movements of undergraduate mathematics students as they read an extended passage from graph theory text.

Method

The textbook section used was taken from the introductory chapter of the open-source textbook *Algorithmic Graph Theory* (Joyner, Nguyen & Phillips, 2011). Graph theory was considered appropriate because it requires few prerequisites and it commonly involves both verbal and algebraic arguments and diagrams. The first part of the chapter was formatted for eye tracking, with a standard font size but larger than usual spaces between lines; one definition and one diagram were repeated where this resulted in their being more separated from related content than they were in the book, and references to computer representations of graphs were removed. The resulting file took up 16 screens and included introductory material on vertices, edges, orders and sizes of graphs, adjacency and degree of vertices, regular graphs, subgraphs, walks, trails and paths, and connected, complete and cycle graphs. It contained several definitions, two sets of worked problems, two theorems with short proofs, one proposition with a lengthier proof, several diagrams, and passages of explanatory text.

A comprehension test was designed based on problems from the end of the chapter; because the number of questions on the included content was small, these were augmented with questions from a local graph theory course. Questions included multiple-choice items on basic definitions, drawing a graph and answering questions about its properties, and proving unseen results. The maximum score was 20.

Participants were mathematics students who had not taken a course in graph theory; each took part individually in exchange for a £6 inconvenience allowance. Participants were informed about the study's purpose and told that after the reading phase they would be asked to answer some questions without access to the textbook section. The eye-tracker was calibrated in each case, then participants read at their own pace. When they had finished, they were given 15 minutes to attempt the comprehension test and were asked to report their scores in earlier core

mathematics courses; from these we constructed a prior performance measure. Forty students participated, and eye-movement data from 38 was of sufficient quality for analysis.

Results

Basic descriptive results are reported here; more detailed analyses of differences in reading behaviours are summarised and will be reported in detail at the conference.

Participants' prior performance scores ranged from 38% to 91% with a mean of 64.5%, meaning that they were representative of the student body as a whole (UK universities typically require 40% to pass a course and 70% for a first-class degree). Comprehension test scores ranged from 1 to 19 out of 20 with a mean of 9.68, and showed a moderate correlation with prior performance ($r = .34$, $p = .036$). This is unsurprising: one would expect mathematically stronger students to do better in both, but short-term learning from a single text and longer-term learning from more materials obviously demand different skills.

Total reading durations varied widely, ranging from 13 to 35 minutes with a mean of 20.5 minutes; nevertheless they did not significantly correlation with comprehension test score ($r = -.08$, $p = .645$). This is striking: if longer study time does not reliably need to greater learning, then some students must use their reading time considerably more effectively. A similar result was found for mean fixation durations, where longer fixation durations are associated with greater processing effort (Rayner, 1998). Mean fixation durations were not significantly correlated with comprehension test scores ($r = -.08$, $p = .624$). Thus neither time nor effort predicted learning outcomes in the obvious way.

To investigate more localised differences in reading behaviour we divided the text into areas of interest (AOIs) (Tobii Technology, 2010), one for each title, quote, definition, example, theorem, proof, diagram, problem, and worked solution. To assess participants' attempts to link different parts of the text, we analysed participants' total *visit counts*, where a visit is a set of consecutive fixations in an AOI. When controlling for reading time there was no significant difference between higher- and lower-performing students on this measure. We note, however, that visit count is only a proxy for shifts of attention – studies of single proofs have considered between-line saccades (Inglis & Alcock, 2012) – and it is not obvious how best to study this aspect of reading behaviour for extended passages of text.

To assess participants' distribution of attention across different types of text we calculated the proportions of their reading times spent on these types. Students who performed better in the comprehension test paid less attention to examples and more to definitions and theorems, a result consistent with long-established arguments about the need for students to understand the importance of definitions in mathematics (Vinner, 1991). We note, however, that effect sizes were small, and that there remain numerous questions about how best to study and understand differences in mathematical reading behaviours. We will report on the details of these analyses and discuss the methodological issues further at the conference.

Discussion

This study was designed to explore undergraduates' mathematical reading behaviours during study of an extended textbook passage. Eye-tracking allows us to do this in an unobtrusive way because it provides behavioural measures without requiring participants to articulate their

thoughts aloud. Of course, it has limitations: one commonly-offered critique is that eye-movement analyses require students to read on a screen, and that this is different from reading mathematics on paper with a pen in hand. While this is indisputable, reading on a screen is a common activity in contemporary education: both students and mathematicians routinely access information in this way. More importantly, it cannot account for between-participant differences: all participants in the reported study were in the same position. Nevertheless, we agree that external validity remains an issue, and future research might well look to use mobile recording methods to study mathematical reading ‘in the wild’ (cf. Savic, 2015).

In the meantime, our early analyses indicate that obvious variations in reading duration and effort do not account for differences in learning effectiveness, and that explanations for this must therefore reside in other aspects of reading behaviour. At the conference we will report in detail on participants’ relative attention to different aspects of the text, analyse the extent to which this differs across more and less effective learners, and discuss follow-up research questions that would be open to investigation using a variety of methodological approaches.

References

- Conradie, J., & Frith, J. (2000). Comprehension tests in mathematics. *Educational Studies in Mathematics*, 42, 225–235.
- Hodds, M., Alcock, L., & Inglis, M. (2014). Self-explanation training improves proof comprehension. *Journal for Research in Mathematics Education*, 45, 62–101.
- Inglis, M., & Alcock, L. (2012). Expert and novice approaches to reading mathematical proofs. *Journal for Research in Mathematics Education*, 43, 358–390.
- Joyner, D., Nguyen, M.V., & Phillips, D. (2011). Algorithmic graph theory and Sage. Retrieved from <https://code.google.com/p/graphbook/>.
- Kasman, R. (2006). Critique that! Analytical writing assignments in advanced mathematics courses. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 16, 1–15.
- Mejía-Ramos, J. P., Fuller, E., Weber, K., Rhoads, K., & Samkoff, A. (2012). An assessment model for proof comprehension in undergraduate mathematics. *Educational Studies in Mathematics*, 79, 3-18.
- Mejía-Ramos, J. P., & Inglis, M. (2009). Argumentative and proving activities in mathematics education research. In F.-L. Lin, F.-J. Hsieh, G. Hanna & M. de Villiers (Eds.), *Proceedings of the ICMI Study 19 conference: Proof and Proving in Mathematics Education* (Vol. 2, pp. 88-93), Taipei, Taiwan.
- Rayner, K. (1998). Eye movements in reading and information processing: 20 years of research. *Psychological Bulletin*, 124, 372-422.
- Russo, J. E., Johnson, E. J., & Stephens, D. L. (1989). The validity of verbal protocols. *Memory and Cognition*, 17, 759-769.
- Savic, M. (2015). The incubation effect: How mathematicians recover from proving impasses. *Journal of Mathematical Behavior*, 39, 67–78.
- Shepherd, M. D., & van de Sande, C. C. (2014). Reading mathematics for understanding—from novice to expert. *Journal of Mathematical Behavior*, 35, 74–86.
- Tobii Technology (2010). Tobii eye tracking: An introduction to eye tracking and Tobii Eye Trackers. Stockholm, Sweden: Tobii Technology AP.
- Vinner, S. (1991). The role of definitions in teaching and learning. In Tall, D. O., editor, *Advanced Mathematical Thinking*, chapter 5, pages 65–81. Kluwer, Dordrecht.
- Weber, K., & Mejía-Ramos, J. P. (2014). Mathematics majors’ beliefs about proof reading. *International Journal of Mathematical Education in Science and Technology*, 45, 89–103.