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Cultural contexts for European research and design practices in mathematics education

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DEDICATION. Our young friends Mustafa Alpaslan and Zişan Güner Alpaslan passed away in a terrible car accident on July 31, 2015. Both were in CERME9: Mustafa in TWG12 (History of Mathematics Education) and his wife Zişan in TWG13 (Early Years Mathematics). They both were brilliant researchers and active members of ERME community. In CERME9, Mustafa was elected as a representative of young researchers in the ERME Board. Mustafa took part in the preparation of this panel: we enjoyed his background in history of mathematics and his deep knowledge of the history of Turkish mathematics education, that made him very sensitive to the issue of cultural contexts. This text is dedicated to both of them.

Abstract. *The authors and guest-authors of this contribution worked together to prepare a plenary panel at CERME9. Starting from acknowledging the diversity of cultural contexts in which we work, the following questions are addressed:*

- *What do we mean by cultural contexts in European Research in Mathematics Education?*
- *How do cultural influences challenge the universality of research and design practices and their outcomes?*
- *Which (hidden) values in different cultural contexts influence research and design practices?*
- *How could cultural awareness among Mathematics Education researchers be raised?*

The authors give concrete examples of cultural differences and their impact on research and design practices in Mathematics Education. They address various cultural contexts, covering mathematics, classrooms, research community contexts and international comparisons. The centrality and nature of theories in addressing mathematics educational context are also discussed. Young researchers as guest-authors contribute further experiences and reflections. The joint reflection offers multiple suggestions for raising cultural awareness in Mathematics Education research and design practices and policy issues.

Keywords: *differences in cultural contexts, research practices, theoretical approaches as epistemic cultures, raising cultural awareness.*

1. INTRODUCTION

While approaching the complex topic of cultural contexts and their impacts on research and design practices, we addressed four main questions:

- What do we mean by cultural contexts in European Research in Mathematics Education?
- How do cultural influences challenge the universality of research and design practices and their outcomes?
- Which (hidden) values in different cultural contexts influence research and design practices?
- How could cultural awareness among Mathematics Education researchers be raised?

We have settled on three themes as a focus for our input and subsequent discussion, presented respectively by Maria G. Bartolini Bussi, Barbara Jaworski and Susanne Prediger. These are:

1. Mathematics and Mathematics Education; how we analyse mathematical concepts; how these ways of explaining and analysing mathematical concepts can be developed for the curriculum and influence the curriculum.
2. Classrooms, teachers and students – how the cultures which underpin interaction and communication, and the use of language, enable or restrict attention to classroom approaches to mathematics, ways of conducting research and the ethical and moral principles in Mathematics Education.
3. Research approaches and theoretical perspectives, and ways in which they underpin research interpretations, the ways in which research findings emerge in research communities and are presented in published works.

We also include reflections from a group of young researchers: Annica Andersson, Mustafa Alpaslan, Edyta Nowinska and Marta Pytlak; represented in the panel and in this writing by Edyta Nowinska.

Section 2 addresses the three themes. In Section 3 we have the reflections of the young researchers. Section 4 presents a synthesis of ideas, looking backwards and then forwards towards taking up cultural issues and in Section 5 we offer questions for our ongoing practices as mathematics educators.

2. MEETING CULTURAL DIFFERENCES ON DIFFERENT LEVELS - THREE THEMES

2.1 Mathematics in cultural contexts (Maria G. Bartolini Bussi)

Every thought, when coming towards the other,
questions itself about its own unthought.
(Jullien, 2006, p. vi)

2.1.1 Introduction

The mathematics developed in the West by professional mathematicians is, in some sense, *near-universal* (see Barton, 2009, who introduced the acronym NUC, that is Near-Universal Conventional mathematics, p. 10). This mathematics had become dominant all over the world, mainly for its century old effective applications to the development of science and technology. Nevertheless, the process of *mathematical enculturation* (Bishop, 1988) is, at least at the beginning, strongly dependent on the local context (often, although not always, identified with a country or a region where a language is spoken). Yet, sometimes people are so embedded in their own context as to ignore that in other contexts the "same" mathematical objects (yet, are these objects really the same?) might have had a different history and might convey even different meanings. It is only when, for some reasons, one is forced to exit her safe "niche" that she may become aware of that. There is a very positive feature in such a dialogue: "every thought, when coming towards the other, questions itself about its own unthought" as strongly claims Jullien (2006, p. vi) in his beautiful discussion of Chinese and European-Greek cultures. This awareness encourages the exploration of the geography and history of mathematical thinking (Bartolini Bussi, Baccaglini, & Ramploud, 2014).

2.1.2 First example: whole numbers

My first example focuses on some aspects of whole numbers. This choice seems provocative: are there things more universal than numbers, at least small whole numbers, when we move from one context to another, from one language to another?

There is an extended classical literature on the history of numbers (e.g., Menninger, 1969; Ifrah, 1985) and on the use of numbers in far contexts, where, for instance, the body parts are used to represent whole numbers (Saxe, 2014) or spatial arrangements substitute the lack of number words in complex arithmetical calculations (Butterworth & Reeve, 2008).

Moreover, Barton (2009) tells the story of the verbal roles of number words in Maori.

> In Maori, prior to European contact, numbers in everyday talk were like actions. […] Our awareness of this old Maori grammar of number suddenly sharpened when we tried to negate sentences that used numbers. […] To negate a verb in Maori the word *kaore* is used. […] Unlike English, where negating both verbs and adjectives requires the word 'not', in Maori, to negate an adjective a different word is used, *ehara.* (Barton, 2009, p. 5)

In Maori language to negate the sentences "there is a big house" and "there are four hills" two different wording of "not" are used. When this verbal feature of Maori numbers was ignored, the mathematics vocabulary process, translated from English, acted against the original ethos of the Maori language.

One might think that these examples are relevant only for historians or anthropologists or ethno mathematicians, but the pragmatic use of numbers in everyday communication offers some surprising evidence in familiar languages too.

There are examples (in many languages) where expressions with numbers are used to denote indefinite quantities. Bazzanella, Pugliese, and Strudsholm (2011) analyse translation problems between different languages. For instance, the Italian expression "Do you want two spaghetti?" that does not mean exactly "two" but "a few", might produce very funny episodes, in spoken communication between an Italian host and a not Italian guest, with the latter puzzled by the idea of eating exactly "two spaghetti" for dinner. In this case, the number is used in indeterminate or vague meaning. In other cases, what is focused is the starting point of the measuring process: for instance, an Italian speaker says "8 days" or "15 days" to mean one week or two weeks (e.g., "8 days from today" means "a week from today"). Actually in Italy a week is 7 days like everywhere, but, in similar expressions, it seems that today (the "zero" day) is counted.

Philosophers studied *vagueness* since antiquity. More recently, Black (1937) transferred the philosophical attention on vagueness to human language and Quine (1960) introduced his famous *principle of indeterminacy* of translation:

> There is no need to insist that the native word can be equated outright to any one English word or phrase. Certain contexts may be specified in which the word is to be translated one way and others in which the word is to be translated in another way (Quine, 1960, p. 69).

Humans do not really need to be precise in every situation and use vagueness and indeterminacy in thinking and communication, but are not always aware of that. It is worthwhile to reflect on the vague meanings of whole numbers that depend on the cultural context, for the importance they might have in the educational setting.

From the perspective of a mathematics teacher: is the everyday use of vague numbers consistent with the use of numbers in the mathematics classroom (e.g., just half a glass, please)? What about different utterances in the mathematics classroom, when the focus shifts from communication ("Be attentive for two minutes, please!") to an arithmetic statement ("two times ten is twenty")? What about multi-cultural classrooms where translation issues are interlaced with mathematical issues? Might vagueness foster or inhibit the construction of number meanings?

From the perspective of a researcher in Mathematics Education, is vagueness to be taken into account, as a tool, by researchers in studies on arithmetic teaching and learning in the mathematics classroom? Is the presence of either precise or vague meaning of numbers related to the development of the two core systems described by neuropsychologists for representing either small numbers of individual objects in a precise way or magnitudes in an approximate way (Feigenson et al., 2004)?

Some observation may be made also about curriculum development, when a cultural lens is used. We refer to two very recent "twin" papers prepared for the panel on "Traditions in Whole Numbers Arithmetic", to be held on the occasion of the "Primary School Study on Whole Numbers" (the 23rd ICMI Study, Macau, China, June 3-7, 2015). They are authored by Bartolini Bussi (2015) and Sun (2015) and address very popular approaches to whole numbers in the West and in China.

Sun (2015) reconstructs the ancient Chinese tradition of whole number arithmetic and its strong connection with today's curriculum. She emphasizes both linguistic and historic-epistemological perspectives. From the linguistic perspective, Sun's paper reads (p. 141):

> Unlike English and most Indo-European languages, written Chinese is logographic rather than alphabetic, and uses the radical ("section header") as the basic writing unit. Most (80-90%) of characters are phono-semantic compounds, combining a semantic radical with a phonetic radical. *Thus, the large majority of words have a compound, or part-part-whole structure. This differs from the phonetically based structure of writing in most Western languages, in which order is more important than the combination of parts* (my emphasis).

Then Sun mentions "classifiers" or measure units, which, as in many East Asian languages, are required when using numerals with nouns (spoken numbers). In Chinese, each type of object that is counted has a particular classifier associated with it. As a weakening of this rule, today it is often acceptable to use the generic classifier "ones" in place of a more specific classifier. As a character, "ones" (classifier) is pronounced *gè* and written 个 which represents a bamboo shoot. This feature highlights the focus on separate units since the ancient ages in China to present days.

Mathematics in ancient China was identified with arithmetic and numbers, and, in particular, with calculation[1]. To calculate with whole numbers in China, straight rods (also named counting sticks) were used 2500 years ago and influenced the early representation of digits (see Fig. 1). The rods had square cross sections to prevent them from rolling and were carried in hexagonal bundles (Fig. 2) consisting of 271 pieces with 9 rods on each edge (Lam & Ang, 2004, p. 44, see Fig. 3): whenever calculation was needed, they were brought out and computation was performed on a flat surface.

[1] Actually, in Chinese 数学 (*shùxué*, i.e., mathematics) literally means "number study" whilst in Greek μάθημα (*mathema*) means "knowledge, what has to be learnt".

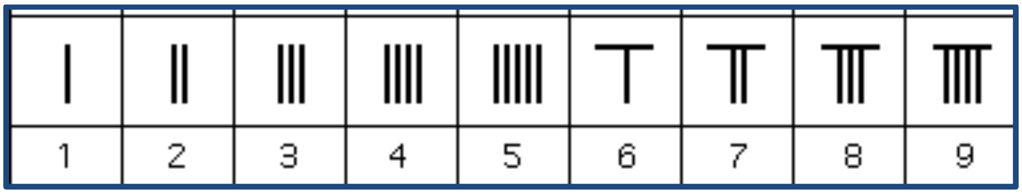

Figure 1: The first nine numerals from rods traditio

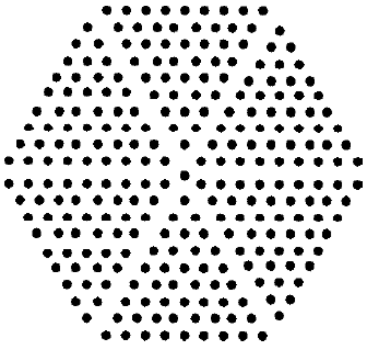

Figure 2: Cross-section of a hexagonal bundle of rods

Figure 3: A fragment in Banpo museum

Fig. 2 shows the structure of the bundles of rod numerals and appears in ancient pottery. The photograph in Fig. 3 was taken by the author in Banpo, a neolithic settlement close to Xi'an, dating back 5600-6700 years, and shows a fragment of a potsherd. The explanation reads: "This potsherd makes the concept of point and surface, number and shape together. It proves Banpo ancestors have certain knowledge of mathematics."

This focus on numbers as discrete quantities has remained in the tradition of teaching arithmetic. Even today, Chinese spoken numerals are the following:

Chinese spoken numerals (and English literal translation):

一个、二个、……;一十一个、一十二个、…………..; 二十个，二十一个….

One ones, two ones, …. one tens & one ones, one tens & two ones …two tens, two tens & one….

The tools for calculation in ancient China were rods or, later, *suànpán*, the Chinese abacus. Drawing on both tools, spoken numerals are transparent for place value. Actually, place value was (and still is) considered an overarching principle for whole number arithmetic. No specific chapter on place value appears in Chinese textbooks, as place value representation is the only way to approach numbers in language, in written representation and calculation practice. Finally, Sun concludes:

> By comparison, if the number concept is represented by the number line, used with calculations by counting up or down, or skip counting, the associativity of addition is developed less naturally than with the composition/decomposition model incorporated into the *suànpán* […] The *suànpán* makes place value explicit, and the calculation procedures of combining ones with ones, tens with tens, and so forth, are built into its structure. The model for numbers provided by the *suànpán* may be contrasted with the number line, which is a continuous, non-digital model for numbers, and is not naturally connected with place value. (Sun, 2015, pp. 151-154)

One might contend that discrete quantities and figured numbers such as the ones in Fig. 2 and Fig. 3 were known and used in ancient Greece since the age of Euclid and earlier. But these figural representations of numbers did not show any connection with verbal and written representation of numbers in calculation at that age. Hence, place value came later (through the Arabic mediation) and had to fight against the practical ways of calculating by means of abacus as a famous picture shows (Menninger, 1969, p. 350).

As far as the number line is concerned, Bartolini Bussi (2015) reconstructs the origin of this number representation, dating back to Euclid and reconsidered in the 17^{th} century when scholars such as Descartes and Wallis exploited the synergy between arithmetic and geometry. Euclid's use of straight line to represent numbers (in the Book 7 of the Elements, Heath, Vol. 2, p. 277 ff.) is interpreted by Netz (1999).

> Often the proof is about "any integer", a quantity floating freely through the entire space of integers, where it has no foothold, no barriers. […] A dot representation implies a specific number, and therefore immediately gives rise to the problem of the generalisation from that particular to a general conclusion, from the finite to the infinite. Greek mathematicians need, therefore, a representation of a number which would come close to the modern variable. This variable […] is the line itself. The line functions as a variable because nothing is known about the real size of the number it represents (Netz, 1999, p. 268).

This interpretation puts the number line into a Western cultural process, where the issue of variable, generality and proof are approached following Euclid's style.

We may conclude that rods and *suànpán* on the one hand and the number line on the other hand are cultural artefacts characterizing the Chinese and the Western curriculum. As cultural artefacts, they reveal valuable information about the society that made or used them and, when continuity between tradition and today's practices is maintained, foster the students' cultural awareness of the role mathematics played in their society. This was not always the case, in mathematics curricula, as the second example shows.

2.1.3 Second example: fractions

The second example concerns the idea of fraction that appears in the language of everyday life, but is perceived as advanced and difficult in Mathematics Education. I have recently published with two colleagues (Bartolini Bussi et al., 2014) a short piece of speculation about the geography and the history of the idea of fraction. We observed that in most Eastern languages (Chinese, Japanese, Korean, Burmese and similar) fractions were written and are still read bottom up, i.e., reading first the denominator and then the numerator. This idea mirrors the genesis of fraction as a part of a whole: to know first in how many parts the whole has been broken and to tell later how many pieces one takes. This way of writing and reading fractions was presented also in the Liber abaci, the text, authored in Latin by Leonardo Fibonacci drawing on Arabic sources, that introduced into Europe the so-called Hindu-Arabic notation and written algorithms (Cajori, 1928, p. 269). It is still extant in some European languages developed in countries which were for centuries under the influence of Arabic and Persian culture (e.g., Turkey). Then the story of fraction names in the European languages diverged, going farther from the genesis and adopting a top-down writing and reading, with the additional puzzle of using ordinals for the denominator. A similar process happened in some Eastern countries (e.g., Myanmar) under the effects of colonialism that cut the roots with local traditions in schools.

From the perspective of a mathematics teacher: the awareness of the gap between the genesis and representation of fractions may possibly be used to support low achievers. Actually, the inversion of numerator and denominator and the use of cardinal numbers, based on the Chinese reading and writing ("three parts, take one!" "five parts, take two!") had an immediate positive effect on the performances of dyscalculic students in the task of quick positioning of a given fraction between 0 and 1 on a number line (Bartolini Bussi et al., 2014).

From the perspective of a curriculum developer: how do we consider the present trend of "imitating" Western curricula to innovate early schooling in many developing countries, when it cuts the roots with local languages and everyday experience, especially in those cases when the local language is closer to the genesis of the mathematical idea (see also Boero, 2013, pp. 25-26)?

2.1.4 Third example: Infinity and limit

The third example shortly refers to a recent study carried out by Kim, Ferrini-Mundy, and Sfard (2012) about students' colloquial and mathematical discourses on infinity and limit. The study involved large samples of USA and Korean undergraduate students. The findings of the study were interesting and the interpretation even more interesting. In a part of the study, the students were asked to create sentences with the words infinite and infinity (a separate sentence for each of these words). The two ethnic groups showed very different productions. In the USA group infinite was used in the context of real-life phenomena, whilst in the Korean group the context of the sentences was predominantly abstract and mathematical with a disconnection between colloquial and mathematical discourses on infinity. USA speakers produced processual sentences, whilst Korean speakers were more likely to produce structural sentences that are closer to formal mathematical discourse.

> In English, there is an obvious lexical continuity throughout all levels of infinity discourse. The principal link that keeps all kinds of English infinity talk together as a cohesive whole is the use of the single word infinite throughout all relevant themes and levels, and in both informal and formal versions of infinity discourse. […] (Kim et al., 2012, p. 93).

In Korean, in contrast, there is a lexical rupture between levels. For instance, the Korean term for infinity (in everyday language) is taken from the Chinese *mu-su* (verbatim *numberlessness*) or *mu-gung* (verbatim *endlessness*), whilst the formal term (in mathematics) is taken from the Chinese *mu-han* (verbatim *boundlessness*). There is no emphasis on the shared part *mu* that means *none*. Hence, there is an evident break between everyday and mathematical terms (Kim et al., 2012).

It seems that English and, more generally, those European languages that were developed under the influence of the Greek thought, defined a path towards the mathematical idea of infinite and infinity (as it was constructed by mathematicians in the 19th century) that in some sense may model also the slow process of students from these cultures through the idea of potential and actual infinity. But this process is not likely to be the same for students coming from cultures where these ideas were developed only in schools (and not in everyday life), after and under the pressure of Western mathematics.

From the perspective of a researcher in Mathematics Education: have these findings the potential to question the faith in the "universal" validity of studies about the obstacles met by Western students in advanced mathematics?

2.1.5 Concluding remarks

In my contribution, I have considered only language to hint at the local culture and context. The examples contain arguments towards different and even opposite strategies for developing mathematics curricula: to exploit local languages and everyday experience with a slow transition from colloquial discourse to mathematical formal discourse versus to start from scratch ignoring the relationships between colloquial discourse and mathematical formal discourse. Is the difference related to the focus, i.e., elementary versus advanced mathematics? In all cases, however, it seems that considering the history and geography of mathematical thinking and the parallel development of language are essential for explaining and analysing the didactical phenomena to be considered in the design and implementation of mathematical curricula (Boero, 2013). Without this attention, it is likely that researchers from different contexts do not even understand each other and cannot exploit each other's findings (Bartolini Bussi & Martignone, 2013). This is one of the reasons why in the ongoing ICMI Study 23 (*Primary Study on Whole Numbers*, http://www.umac.mo/fed/ICMI23) a mandatory cover document about the context of each submitted paper was required in the Discussion Document. The papers which were submitted offered evidence not only of language differences (with strong effects on the arithmetic taught in primary school) but also of different societal norms, customs, institutional conditions, values and theoretical approaches, in one word, of different cultures.

In this section, I have offered mathematical examples, maybe strongly unexpected mathematical examples, as the audience's reaction showed in the panel and beyond. They question the idea of mathematics as a "universal language" or *lingua franca* of the modern world (Kim et al., 2012). In addition to general arguments about cultural relativism, mathematical examples offer a staggering evidence that cannot be ignored. They contribute to determine the contexts, the gap between them and the possibility/impossibility of easily transposing theories, findings and methodologies from one context to another, both in research and in classroom practice (Ramploud, 2015) against the naïve myth of "universality" of Mathematics Education research. In this case, it is the stunning difference that produces information.

The examples come from ethnomathematics, from the pragmatic use of numbers in everyday communication and from the comparison of mathematics curricula. In a more general way, examples may come from the curiosity about the history and geography of mathematical thinking. They concern both elementary and advanced mathematical thinking, with a potential conflict concerning continuity versus discontinuity between everyday and mathematical language. In particular, I challenge the presumed "universality" of number words and of mathematics language and, as a consequence, of theories, methodologies and findings in the studies on Mathematics Education.

2.2 Researchers, teachers, students in Mathematics Education and their values in cultural contexts (Barbara Jaworski)

The previous section (2.1) has pointed out that sometimes people are so embedded in their own context, the safe 'niche', as to ignore differences in other contexts. What seem like "the same" mathematical objects might have had a different history and might convey even different meanings. It is only when, for some reasons, someone is forced to exit her safe niche that she may become aware of that. The section focused on mathematics itself and the ways in which it is represented in writing and talking in differing cultures. Here, I take up this theme of the "safe niche" to look more broadly at context in mathematics learning, teaching and research in Mathematics Education to seek out cultural distinctions and anomalies. This will take us beyond mathematics itself into educational issues, particularly those that stem from the ways in which mathematics is regarded in education and society.

2.2.1 Introduction – who we are, and how this influences our work as Mathematics Education researchers

When we engage in research which involves human participants, we have *moral* and *ethical* responsibility towards our research participants (Pring, 2004). Pring writes:

> I shall argue that education itself is a moral practice … Ideally the 'practice' should be in the hands of moral educators (who themselves should manifest the signs of moral development). (2004, p. 12)

As researchers in Mathematics Education, we are required to attend to ethical issues in our research. Pring (2004) goes further to suggest that we are tasked with a *moral* agenda where research in education is concerned. A question for us all is what such morality involves. For example, we need to be aware of the *values* we bring to interactions, decisions, interpretations and judgments (Bishop, 2001; Chin, Leu, & Lin, 2001), how they relate to mathematics itself, and how they fit with the cultures in which our research takes place. These cultures are manifested in our lives and work, the societies and systems of which we are a part.

As a *mathematics* educator, I have ways of seeing and arguing rooted in the mathematics which has formed a central part of my studies and professional life; this is likely to distinguish me from educators in other subjects or from scholars in the natural sciences or humanities. Mathematics itself has cultural resonances, related to moral questions and values within society, as I shall discuss further below. Indeed, Section 2.1 has drawn attention to many aspects of mathematics and how these vary across parts of the world. However, other cultures are also central to our activity. As a *researcher*, I belong to a different culture from that of a teacher I work with although we are both interested in the learning and teaching of mathematics – I have university and research values; the teacher has school and teaching values (Jaworski, 2008). Here, culture is related to where we work and the values associated with the job we do. My own values are theory-related, since an expectation of a university role is to engage with theory and research as well as the university as an institution; a teacher is concerned with school values, students' characteristics and needs, and societal and political demands such as examination results and league tables of 'effective' schools.

Being a teacher involves different expectations and values in different settings, particularly across national boundaries. Such differences are highlighted by a Finnish colleague, Kirsti Hemmi, who wrote to me as follows about her experiences as a teacher of mathematics in Finland and in Sweden:

> I was recruited to Sweden to teach Finnish speaking children in the beginning of [19]80s. Since then, over thirty years, I have worked in the cross-section of these two educational cultures and experienced and witnessed other teachers' similar experiences about the very different attitudes towards what it means to be a primary school teacher and towards what kind of skills and understanding we expect children to develop in reading, writing and mathematics during the first school years. In this work the different cultural-educational traditions really collided in various ways, not only through the Finnish and Swedish teachers' different educational backgrounds but also through the character of the teaching materials, especially in mathematics produced in these two countries. Sometimes the differences were concrete, sometimes they were hard to articulate. (Kirsti Hemmi, personal communication)

Kirsti Hemmi's words provide insight into differences between cultural settings where, more superficially, there might be expected to be common understandings and ways of interpreting educational issues. When we work as researchers across national boundaries, how we understand each other becomes central to the ways in which we undertake research. In a personal communication, Heidi Krzywacki (from Finland) wrote to me about her experiences of conducting professional development research in Peru, to provide new ideas about Mathematics Education and teacher education not common in the current educational reality in Peru. She writes:

> I have reflected on some issues related to language in international cooperation and development work that we have had with Peruvian partners for developing their education system. For example, it took a while to understand that we had no common apprehension of action research: for the Finnish partners it was used for referring to a methodological approach, but Peruvian partners interpreted it (after translation) as personal reflections.
> (Heidi Krzywacki, Personal Communication)

These words alert us to differences in perception that may be ignored, perhaps dangerously for the ensuing research, because the language of communication hides important subtleties of meaning. Since differences in practices and cultural values underpin what is possible in educational environments, researchers working in these environments must be alert to such differences and must factor them into a research study; not easy to achieve, and requiring awareness and sensitivity. In the next section, I offer further examples to highlight issues which arise from cultural norms, sensitivities and differing values.

2.2.2 Cultural contexts and their influence on how we think and behave

I start with examples from my own experience. I have worked as a teacher and as a researcher in several countries including Pakistan and Norway, where I come from a different culture (call it, rather superficially, a *British* culture) from the people in whose country I am working. In Norway, we share western ways of thinking and a Christian tradition, but there are differences, some subtle, but nevertheless important. One example, which I met very early in relationships with Norwegian colleagues, is the *Law of Jante*, created by the Dano-Norwegian author Aksel Sandemose (Sandemose, 1933/2005) – the idea that there is a pattern of group behaviour towards individuals within Scandinavian communities that negatively portrays and criticizes individual success and achievement as unworthy and inappropriate (Wikipedia; (http://en.wikipedia.org/wiki/Law_of_Jante accessed 20-4-15).

> Most Danes seem to [be] much more reserved and humble in everyday life. These rules refrain people from "judging a book by its cover," as they encourage assuming that they are no better than the person they are meeting. (Gratale, 2014).

In discussions with colleagues in Norway about research approaches and the teaching of mathematics, it became an issue for me to take a more modest, or 'humble' stance on my own perspectives. Thus, awareness of culture impacted on how I as a researcher interacted with colleagues and approached my research role.

In Pakistan, the cultural differences are more obvious, and religion plays an important role – the Muslim religion and associated social values permeate how people think and what is possible in schools and classrooms (Farah & Jaworski, 2005). For example, when working with teachers in a master's programme in Pakistan, I emphasised the *value* (in mathematics) of *asking questions* about the mathematics in which we engaged. My argument was that inquiry approaches to mathematics, involving questioning of relationships and procedures, encourage students to go beyond the procedural towards more conceptual understandings of the mathematics in focus (Jaworski, 1994). One teacher chose to write an essay about 'questioning as a pedagogic approach'. She drew attention to the fact that in Pakistani society questioning is largely discouraged because it shows a lack of respect for parents, teachers or anyone senior in the community (Jaworski, 2001, p. 312). This observation led to our addressing questioning not only as a pedagogic approach in mathematics, but also as an issue of *values* in the Pakistani society impinging on what is possible in the mathematics classroom. We see here issues related to pedagogic practice designed to improve the learning of mathematics and specific societal norms, alongside the communicative difficulties across cultural boundaries.

Anjum Halai addresses such issues from within her own and another cultural context, Mathematics Education in Pakistan and in East Africa, raising wider social issues that have a compelling need to be addressed.

> Recognition of learners who are marginalized due to socio-economic status, gender, language or other factors would mean questioning deep seated assumptions that underpin the organising structure and process of classrooms, in this case mathematics classrooms. For example, in patriarchal societies with roles defined on the basis of gender, teachers often subscribe to the dominant social and cultural views that boys are inherently better in mathematics thereby marginalizing girls in terms of participation in mathematics. (Halai, 2014, p. 69)

For researchers, not questioning those deeply held cultural views, which limit participation of both, boys and girls, is to take a moral position. Halai positively recommends *questioning*, claiming that it is through questioning that we challenge entrenched discriminatory practices. This clearly raises issues for researchers who wish to conduct research without offending their respondents/participants, but who nevertheless see a moral dimension to their questioning of values, both in teaching and learning mathematics and in the wider society. Halai writes further:

> For skills development, processes of teaching and learning in the mathematics classrooms would move away from routine memorization of procedures and algorithmic knowledge towards participatory learning involving application of mathematics knowledge to problems. Mathematics knowledge embedded in the history and culture of the learners would be a significant element of the cultural capital being re-distributed. This would socio-culturally embed mathematics learning and reduce alienation of learners with school mathematics. (Halai, 2014, p. 69)

We see here serious challenges for researchers cross-culturally: while respecting the cultures in which we work as researchers, and without alienating those with whom we work, we need to address what we know to be good didactic and pedagogic practices in mathematics for the good of the students whose lives depend on it. These words suggest that research and educational development go hand in hand to promote practices which theory and research support as more likely to promote mathematical learning.

Diverse perspectives on what constitutes good learning of mathematics and how this relates to cultural perspectives have permeated Mathematics Education's recent history, in both developed and developing worlds. Mathematics is seen in many countries as an essential ingredient of a good education, having "exchange value" for entry to diverse disciplines and work opportunities (e.g., Williams, 2011). However, for many people mathematics appears to be outside their comprehension, creating serious sociocultural antipathies, as the next section reflects.

2.2.3 Perceptions of mathematics and mathematical achievement in diverse cultures and systems

In Mathematics Education, teachers and educators have the task of promoting mathematical learning and understanding among the students with whom they work, and researchers study the processes, practices and outcomes of this work. In 1990, writing from an ICMI study focusing on the *Popularisation of Mathematics*, Howson and Kahane (1990, p. 2/3) wrote that, in most developed countries, the public image of mathematics is bad. They quote from their respondents: "All problems are already formulated"; "Mathematics is not creative", "Mathematics is not part of human culture", "The only purpose of mathematics is for sorting out students". Moreover, "the image of mathematicians is still worse: arrogant, elitist, middle class, eccentric, male social misfits. They lack antennae, common sense, and a sense of humour". We might ask *why* mathematics elicits such negative responses from a wide range of people. In more recent years, in a study of students' views of mathematics in secondary classrooms in the UK, Nardi and Steward (2003) characterized students' views as expressing *tedium*, *isolation*, *rote learning*, *elitism* and *depersonalization* (pp. 355-360) – students were T.I.R.E.D, of/with mathematics. These findings are an indictment on the students' experiences of mathematics in their schooling. Such unwelcome messages challenge the educational *status quo* in the cultures to which they relate and impact on systems and practices. The challenges for researchers, and associated responsibilities, go beyond pointing out the failings towards a recognition of where persistent practices are failing learners.

For example, in the UK, long-standing practices which resist challenge involve the grouping students into 'sets' for mathematics based on achievement within the system. Such setting, based on achievement, is much less common in other disciplinary areas. Setting practices have discriminated against certain groups of students, leaving them to be defined as ‘low achieving’ or even ‘low ability’ (Boaler & Wiliam, 2001). In particular, setting regimes and associated forms of national assessment were found by Cooper and Dunne (2000) to discriminate again girls and students of lower social class. Why such practices are maintained, given the research evidence against them, is a cultural phenomenon, deeply embedded in the educational system and perceptions of policy-makers and teachers. In a study based on the ways in which committed teachers interpreted mathematics teaching in two schools in the UK, Boaler (1997) showed that differences in school organisation and teaching approach led to different ways in which students perceived and succeeded with mathematics, with differential effects for boys and girls. Such research findings at a national level beg questions about educational practices and learning outcomes in other countries which are addressed through international comparisons in mathematics.

In international comparisons of mathematics achievement, successive IEA studies have shown that several European countries perform relatively poorly in contrast with achievements in some eastern countries (Mullis, Martin, Gonzalez, & Chrostowski, 2004). We might ask whether ‘TIRED’ students are unlikely to achieve highly; or perhaps whether forms of ‘setting’ can be linked to national outcomes. It seems clear that the outcomes of testing students in international comparisons reflect cultural perspectives in mathematics and in education. As well as comparing learner outcomes in these countries, such studies beg many questions about the educational systems, classroom practices and education of teachers to which student learning outcomes relate. More recent studies, such as the TEDS-M study (Tatto et al., 2012), have taken up some of these questions.

For example, comparing national results in the TEDS-M study of teacher education, Kaiser and colleagues (2014) address the question: “What are the professional competencies of future mathematics teachers [in the countries to which the study relates]?” They write:

> In the secondary study, participants from Chinese Taipei outperformed all other participants, in relation to MCK [mathematical content knowledge] as well as MPCK [mathematical pedagogic content knowledge]. Participants from Russia, Singapore, Poland and Switzerland followed the Chinese Taipei prospective teachers with their achievements in MCK, German and US American prospective teachers achieved slightly above the average, … .
>
> These results point to interesting differences between prospective teachers for primary level and secondary level and confirm the superior performance of Eastern prospective teachers compared to their Western counterparts in most areas.
>
> In contrast, in Scandinavian countries, North and South America, and in countries shaped by US-American influence such as the Philippines or Singapore a so-called “progressive education” with child-centred approaches characterises school and teacher education. (Kaiser et al., 2014, pp. 42f)

By focusing on national achievements, these authors alert us to the differing systems and beliefs which underpin educational achievements. They suggest that links can be seen between these findings and those from surveys of student achievement in the same countries, and point towards the differing ‘orientations’ between countries as contributing to the findings of the study:

> The studies explore amongst others the extent to which a country’s culture can be characterised by an individualistic versus a collectivistic orientation using the cultural-sociological theory of Hofstede (1986). The collectivism-individualism antagonism describes the extent to which the individuals of a society are perceived as autonomous, the role and the responsibility of the individual for knowledge acquisition plays an important role. (Kaiser et al., 2014, p. 44)

Our attention is drawn here to the differing beliefs which shape an education system, and which are rooted in the collectivist-individualist debate which underpins the values on which educational practice is based. It is far from simple to address such established positions, especially when they are supported by politics and legislation; nevertheless as moral educator-researchers we cannot ignore such challenges.

2.2.4 Values as a determinant of difference

Questions related to *values* have been addressed explicitly in research looking at the importance of values in relation to classroom mathematics. The values that a teacher holds are influential on the ways in which curriculum content is addressed in the classroom. For example, Alan Bishop has written:

> If a teacher continually chooses to present opportunities for investigation, discussion and debate in her class, we might surmise she values the ability for logical argument. (Bishop, 2001, p. 238)

Maybe further, we might surmise she believes that logical argument is important to mathematical conceptualization. Bishop argues that beliefs and values are reflexively linked for teachers deciding how to bring mathematics to the students in their classroom. For the TIRED students in the study reported by Nardi and Steward, we might wonder about the beliefs and values underpinning the teaching of these students. Although such issues are not addressed explicitly in their paper, Nardi and Steward write nevertheless: “The students seemed to resent mathematical learning as a *rote learning* activity that involves the manipulation of unquestionable rules and yields unique methods and answers to problems.” (p. 362) Thus, *promoting rote learning* can be perceived by some as a legitimate way of teaching mathematics and is related to classroom values underpinning findings in this study.

In research in Taiwan, Chin, Leu, and Lin (2001) compared and contrasted the beliefs and values of two teachers and concluded that the process of making their values explicit had effects on their teaching related to the particular values they espoused. In one case, the teacher came to realize that using language that is more familiar to students, encouraging students to express mathematical ideas in their own language and only later moving to formal expressions, has positive outcomes for students’ mathematical learning.

Language also formed a central issue for Lee (2006) in a study of her own teaching approaches in a UK comprehensive school. She pointed to the importance of the transition from students’ own natural language to mathematical language, designing and exploring classroom approaches that promoted students’ development of mathematical language. Students had to get used to using mathematical terms and expressing in their own ways the meanings of these terms. Lee, acting as teacher-researcher, stood out against the prevailing ethos in her school in relation to the wider educational system. Her research demonstrates possibilities for promoting the development of mathematical language in the classroom and stands as a beacon for other teachers within the system.

These examples point to morality issues at the classroom level, involving teachers working with their students. However, teachers have to work within the prevailing system which imposes values beyond their own activity and that of their students. The educational system, with its ways of organising schools, curriculum and examinations, is formed within the nation’s societal and political forces which are culturally determined.

2.2.5 Ways of knowing and being

Cultural determination within any society might be seen in terms of ‘ways of knowing and being’. Research by Belenky and colleagues (1986) pointed to “Women’s Ways of Knowing”, drawing attention to the ways in which women perceive their various ‘worlds’ differently from male perceptions. Such worlds are the focus of Holland and colleagues (1998) who have suggested that humans make sense within "figured worlds", or figurative, narrativised or dramatized worlds:

> [A figured world is] a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others. (Holland et al., 1998, p. 52)

Ways of knowing or *figured worlds* can be seen to underpin the ways in which Mathematics Education is approached and practiced in culturally different educational settings. The examples provided above are all situated with respect to such worlds which may be local national or cross-national. Researchers working in cross-national frames may be unaware of the worlds of their partners, of ways of interpreting constructs and concepts, of overpowering societal or political forces. Issues arising may present tensions and dilemmas which need to be exposed, discussed and deconstructed in reports from the research.

I have drawn attention to cultural values permeating mathematics learning and teaching in and beyond national boundaries raising moral and ethical questions for teachers, educators and researchers. Research activity and practices cannot be divorced from the educational values that permeate societies and are promoted by systems, politics and legislation. The researcher is more than a recorder of practices and issues and cannot avoid involvement. We therefore need much more cognisance of ways in which research questions cut across ways of knowing and being in the worlds in which we do our research. This in itself is a research agenda.

2.2.6 Key concepts

The following is a list of the key concepts addressed in this section:

1. The moral and ethical nature of Mathematics Education practices and associated responsibilities of practitioners and researchers;
2. The centrality of *values* to educational research and practice in mathematics;
3. Perceptions of mathematics and their relation to didactic and pedagogic practices;
4. Differences of perception of mathematics educational practice rooted in cultural contexts and issues for researchers in challenging established ways of being;
5. International studies with cross-national comparisons and the challenges they raise for culture-bound practices;
6. Figured worlds which narrativise the human collective in Mathematics Education and require recognition and acknowledgement in their power to condition beliefs and values, and hence teaching and learning.

2.3 Research approaches and research communities as and in cultural contexts (Susanne Prediger)

The previous two sections have discussed differences in cultural contexts concerning *the mathematics* itself (in Section 2.1) and the contexts of *teaching and learning mathematics* (in Section 2.2). For both cultural contexts, it was shown how the hidden assumption about the universality of our own practices and values (of writing and doing mathematics, ways of teaching, of educating teachers) must be challenged. Instead, the mathematical and educational practices and values are deeply shaped by the culture we live in. Problems of intercultural misunderstandings can appear when we are not aware of this cultural boundedness and assume that approaches or knowledge can easily be transferred between cultural contexts.

In both sections, the "culture" in "cultural contexts" mainly referred to countries or regions, where intercultural reflections can be triggered by international comparisons. However, Maria G. Bartolini Bussi has already mentioned the differences between everyday language and mathematics language about whole numbers and Barbara Jaworski has already mentioned cultural differences within a country, for example, the researchers' culture versus the teachers' culture. This widened use of the construct "culture" is in line with modern conceptualizations, not only as national culture but as *a system of shared meanings, values and practices* shared by a group of people, also within a country (Geertz, 1973; Knorr Cetina, 1999). Specifically, Knorr Cetina (1999) has coined the term epistemic cultures and showed how implicit values and practices can shape the work of research communities and their research approaches.

Within the Mathematics Education community, especially the diversity of different theoretical approaches and the underlying research practices have been discussed as different cultural contexts. This section reports briefly on the raising awareness on these more subtle cultural contexts and discusses how strategies invented for dealing with diversity of research approaches can be transferred to other cultural differences. Thus, in this section, the relation between research approaches and cultural contexts is twofold: on the one hand, the research approaches are themselves considered *as* epistemic cultures with their own values and practices, on the other hand, research approaches have emerged *in* different international contexts and different research communities, and their specific characteristics have always shaped the epistemic cultures.

On this meta-level, the mathematics is more implicit: although the theories and research approaches deal with mathematics and its epistemological specificity, the reflection on how to combine different approaches is not so specific to mathematics anymore.

2.3.1 Different research approaches – a further cultural context influencing research and design practices

Since 2005, the ERME community has gained an increasing awareness of the existence of different theoretical approaches. At CERME congresses, the methodological discourse was installed by establishing a Working Group which is ongoing now for 10 years (Artigue, Bartolini Bussi, Dreyfus, Gray, & Prediger, 2005 at CERME5; Arzarello, Bosch, Lenfant, & Prediger, 2007 at CERME6; and successors). Successively, the awareness increases that theoretical approaches are always connected to research practices, this section hence talks about research approaches at large, including the theories framing the research as well as the underlying aims and values.

No empirical finding exists independent from the way it is generated within a theoretical approach, even if this theoretical approach is not made explicit. However, the complexity of mathematics teaching and learning can be conceptualized in very different ways, depending on the chosen theory and research approach. Fig. 4 sketches an example (presented and discussed in Prediger, 2010) of the empirical problem that immigrant students have difficulties with mathematical word problems. This problem can be conceptualized, described and explained by different lenses which are connected to different theoretical approaches, either in an individual cognitive perspective or a social perspective: Some focus (micro-sociologically) on the culture of mathematics classroom, and others also focus (macro-sociologically) on connections to students' social background and structures in society (cf., Sierpinska & Lerman, 1996, for the difference between cognitive and social perspectives at large).

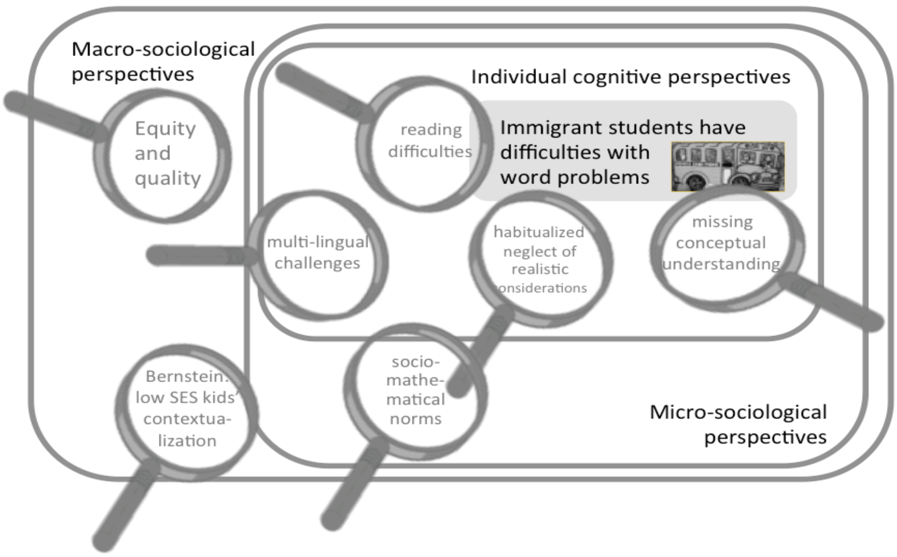

Figure 4: Different theoretical approaches to the same empirical problem (similar to Prediger, 2010, p. 183)

In each case, the activated approach influences (but not determines) the way the problem is researched. The original problem changes its character, since the theoretical approach shapes the problem into a so-called conceptualized phenomenon (Bikner-Ahsbahs, Prediger, & the Networking Theories Group, 2014, p. 238). Additionally, the specific conceptualization of the problem influences the design consequences drawn for overcoming it. If the problem is conceptualized as students' missing conceptual understanding, the design might focus on fostering understanding. In contrast, the conceptualization as a habitualized neglect of realistic consideration due to inadequate sociomathematical norms in the classroom might lead to changing the sociomathematical norms about how to deal with word problems. If the immigrant students are mainly considered as students with underprivileged social background, a more explicit teaching might be claimed as consequences drawn from Bernstein's theoretical approach. In contrast, the conceptualization as multilingual students might result in fostering students' academic language or raising more general issues of equity (Prediger, 2010). Especially, considering challenges relating to a specific group of students (such as here immigrant students) allows for macro-sociological perspectives on issues of equity, but can also be treated as a purely cognitive problem within a specific mathematical topic.

Even this very rough outline of alternative approaches and design consequences shows how each theoretical approach influences research and design practices. This sketched example can raise the cultural awareness that *no empirical finding exists independent from its conceptualization within a (more or less explicit) theoretical approach.*

It also shows that the diversity of approaches is not only another example of cultural differences (here difference of epistemic cultures), but that it is also *necessary* in order to grasp different aspects of the complexity of mathematics teaching and learning. Having acknowledged the necessity of different epistemic cultures is however not enough to make use of the diversity, because the diversity can mainly become fruitful if the research approaches are connected (Artigue et al., 2005). As classroom reality is always complex and multi-facetted, connecting different theoretical approaches and research practices is promising in order to grasp a higher complexity at the same time.

2.3.2 *Dealing with diverse research approaches and theories as an issue of methodological reflection*

How can the discipline of Mathematics Education make use, more systematically, of the diversity of different epistemic cultures? As dealing with the diversity was identified as an important challenge for the international community, a subgroup of the CERME working group started to work more intensively in order to elaborate the methodological reflections, first on theories alone, later more widely on research approaches and the underlying epistemic cultures (Bikner-Ahsbahs, Prediger, & the Networking Theories Group, 2014).

Given the high complexity of mathematics teaching and learning, *one big unified theory* is not a realistic and adequate goal. Instead, the group developed the idea of aiming at connecting two or three approaches each. For this purpose, the group developed a landscape of so-called networking strategies by which these connections can be realized (Prediger, Bikner-Ahsbahs, & Arzarello, 2008).

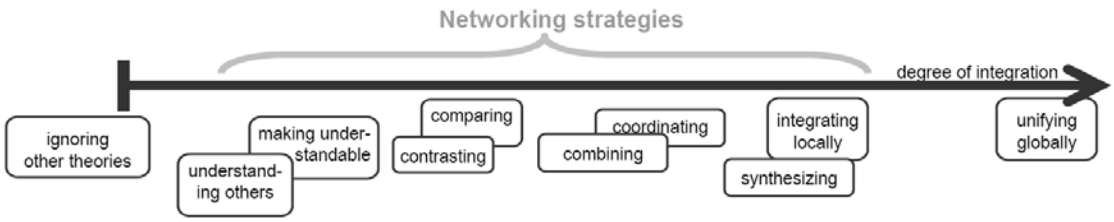

Figure 5: A landscape of strategies for connecting theoretical approaches (Prediger et al., 2008, p. 170)

Practical experiments with comparing and contrasting different theoretical approaches in different settings led to an increasing awareness that theoretical approaches influence each step in a research process, not only the analysis of data as the initial example in Fig. 4 might suggest:

- initial identification of a problem in classroom practice, loosely framed;
- conceptualisation of the classroom problem;
- transformation of the problem into more focused research questions;
- development of research design (including methodological choices on kinds of data, sample…);
- collecting data;
- choice and formulation of a research question;
- data analysis;
- consequences for a design of learning opportunities;
- evaluation of learning opportunities;
- …. .

It was therefore an interesting experience to convert research problems from one research approach into another, because this required changes in every step of the research practice as well. These experiences showed the strong intertwinement between theories and research practices and hence the meaning of Knorr Cetina's (1999) construct "epistemic cultures".

Although there is no shared unique definition of theory or theoretical approach among Mathematics Education researchers (see Assude, Boero, Herbst, Lerman, & Radford, 2008), many authors emphasize the double role of theory being the outcome of research and, at the same time, the background theory guiding the research practice as a framework (Assude et al., 2008). Radford (2008) takes this intertwinement into account by describing theories as "way[s] of producing understandings and ways of action based on […] basic principles, which include implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective; a methodology [.… and] a set of paradigmatic research questions" (Radford, 2008, p. 320). Radford's triplet includes the so-called background theories (Mason & Waywood, 1996) with many hidden assumptions and general philosophical stances which often remain implicit.

For example, adopting a macro-sociological perspective on the example problem in Fig. 4 immediately makes the researcher think about the students' background and how this is related to the mathematics learning. Relevant questions in this perspective must include societal questions. At the same time, the mathematics might be able to be treated in a more generic sense. In contrast, the cognitive perspective would focus on the specificity of the mathematical topic and its obstacles and might neglect the students' background as this question is not considered as important. Which approach is chosen might also depend on the national or regional contexts: where equity issues are prominently discussed, the macro-sociological perspective has entered Mathematics Education earlier than in other countries or regions.

Making crucial aspects of an approach explicit is therefore a major task when connecting theories and research approaches. That is why understanding another theory and making the own theory understandable was specified as the first networking strategy (cf. Fig. 5), since these two strategies already require big efforts of the researchers for an "intercultural communication".

Before applying networking strategies with higher degrees of integration like combining, coordinating or synthesizing, the compatibility of the approaches in view must carefully been checked. This is necessary to avoid inconsistencies in the built network (cf. Bikner-Ahsbahs et al., 2014).

2.3.3 Theoretical approaches and research and design practices as embedded in different research communities and institutional backgrounds

Conceptualizing theories as ways of producing understandings and ways of action corresponds to culturalistic conceptualizations of research as being conducted in *communities of practice"* (Wenger, 1998). The construct *practice* is explained by Wenger as socially bound to its community:

> The concept of practice connotes doing, but not just doing in and of itself. It is doing in a historical and social context that gives structure and meaning to what we do. In this sense, practice is always social practice. Such a concept of practice includes both the explicit and the tacit. It includes what is said and what is left unsaid; what is represented and what is assumed. It includes language, tools, documents, images, symbols, well-defined roles, specified criteria, codified procedures, regulations and contracts that various practices make explicit for a variety of purposes. But it also includes all the implicit relations, tacit conventions, subtle cues, untold rules of thumb, recognizable intuitions, specific perceptions, well-tuned sensitivities, embodied understanding, underlying assumptions, and shared world views. Most of these may never be articulated, yet they are unmistakable signs of membership in communities of practice and are crucial to the success of their enterprises. (Wenger, 1998, p. 47)

This conceptualization of research as always being bound in epistemic cultures, or here more precisely in research communities of practice, makes it clear that the choice of a theoretical approach is not a free individual choice but always limited and fired by the specific research community and their institutional background.

The fact that Maria G. Bartolini Bussi, in Section 2.1, focuses on the mathematics itself and Barbara Jaworski in Section 2.2 on teachers, classrooms and values of the society, might also be traced back to the different research communities and institutional backgrounds in which the researchers work: Being located in the mathematics or more attached to the general education department might affect the choices about focus, perspective and theoretical approach. In this sense, every research is strongly influenced by the community of practice and its institutional background.

This point is also made by Marta Pytlak in Section 3.2: Different institutional conditions influence the research practices we can choose. For the individual young researcher, it might hence be challenging to switch between the communities.

2.3.4 Learning for cultural contexts on other levels?

After ten years of methodological reflection on the diversity of research approaches and strategies and issues for dealing with it, we can ask whether we can learn something from this cultural context on the research level for other cultural contexts (e.g., on the mathematical level or the classroom level, see Section 2.1 and 2.2 as well as 3). Are there aspects of cultural awareness that can be transferred from the level of research cultures to other levels of cultural contexts raised in other sections?

Here are some preliminary aspects which should be further discussed:

- Being aware of differences is the first step to avoid too hasty generalizations. It is worth *not to take the own assumptions for granted universally*.
- Differences do not only pose problems, but also offer *chances* since they provide a larger variety of options and increase the repertoire of research and design practices.
- Beyond openly visible differences, there are always substantial differences in more subtle, implicit layers which are more difficult to communicate. That is why *understanding* other contexts and *making* the own context *understandable* is a challenge in itself which should be taken seriously.
- *Comparing and contrasting* cultural contexts is a strategy that allows making implicit thought and aspects explicit.
- *Converting* from one cultural context to another requires very careful adaptations or even transpositions in order to adjust to the cultural context soundly.
- *Combining* different contexts requires very careful considerations of *checking compatibility* in order to avoid inconsistencies.

3. YOUNG RESEARCHERS' EXPERIENCES AND REFLECTIONS

The previous sections give concrete examples in which the *invisible culture* of mathematical thoughts (Section 2.1), classrooms, their norms and values (Section 2.2 and 2.3) had been made visible throughout a deep analysis of particular cultural contexts. These examples indicate cultural determinants of the mathematics taught and learned in mathematics classes. In the given cultural context (determined by its history, tradition, institutions, systems of values, ideologies), the invisible culture is so natural and obvious that its participants might take it for granted also for other contexts.

From the point of view of researchers it is necessary to be aware of these determinants. This awareness requires a holistic view of the learning process as a part of a certain context and its culture. The reflections worked out in Section 2.4 explain from a meta-level how researchers' awareness of the cultural dimension of learning processes might be affected by the tradition of dealing with particular theoretical perspectives being characteristic for their community. These reflections point out another level of an invisible culture – the invisible culture of scientific thought in Mathematics Education – and explain well-established strategies for raising the cultural awareness on this level.

The reflections presented by three experienced researchers motivated a group of four young researchers (Annica Andersson, Mustafa Alpaslan, Edyta Nowinska, and Marta Pytlak) to analyse and discuss their own experience in conducting and communicating research in Mathematics Education. Some results of these processes are presented in this section. They document

- the essential role of an intercultural discourse (Section 3.1 and 3.3) in raising the cultural awareness of research practices,
- the influence of the institutional context affecting the effectiveness of such a discourse (Section 3.2), and
- the need for researchers' awareness of the invisible culture underpinning ways of thinking and acting in mathematics classes in order to design effective methods for improvement of teaching and learning practices (Section 3.4).

Section 3.5 presents a discussion of the critical points raised by the young researchers and puts them in relation to the reflections presented in the previous sections. The discussion emphasizes the need for the *cultural* and *theoretical sensitivities* for better understanding of the surrounding invisible cultures in research practices in Mathematics Education.

3.1 International perspectives on local phenomena – A personal experience (guest-author Annica Andersson)

In my reflections, I focus on my experiences from Mathematics Education research and teacher education in the diverse cultural settings I have had the opportunity to work in, for example, Sweden, Denmark, Colombia, Australia, Papua New Guinea and Greenland. These cultural experiences have shown that *our own languages, contexts and cultures may become visible when we see them from the outsider's perspective, or when others confront us with questions motivating us to reflect on our own use of the languages, contexts and cultures we participate in or are familiar with.* This has been one of my richest learning experiences when communicating my cultural research with others.

For my thesis research, focusing on students' narratives about their hating/ disliking/worrying about/ Mathematics and Mathematics Education (cf. Andersson & Valero, 2015), I collected my data in Swedish upper secondary schools. The fact that I analysed my data while being outside Sweden, in an English-speaking environment (Australia), and communicated my research within an international research group in Aalborg (Denmark), facilitated me to explain and express the data to people of other languages and cultures. Consequently, I recognised some aspects influencing the ways of thinking and acting in school contexts in Sweden, and realized that languages, cultures and contexts fluctuate and are not stable. The opportunity to communicate this research within an international community raised my awareness on how one's own cultural context may be different from other contexts and how it influences the ways of acting as a teacher or researcher and, consequently, also teaching methods, research questions and theoretical approaches.

For example, Elin, a mathematics teacher I collaborated with, talked about herself as being a "Curling teacher" (Andersson, 2011). Curling is a culturally-bounded winter ice sport where competitors sweep the ice in front of a stone to get it in the best position. The metaphor of a *curling* teacher is transferred from the term "curling parent", which, in Sweden, refers to parents who "sweep the way", hence serve their children to get the right, or best, positions, solving possible problems and tensions beforehand and thus make children's lives as smooth and easy as possible. The idea of a "curling teacher" was culturally-bounded and not obvious for an international research community.

The international (or rather inter-cultural) discourse on my research contributed to make my research problems, approaches and results clear and understandable for external research communities. My experiences allow me to argue that there is value in raising discussions about understanding Mathematics Education research as culturally developed and situated. Here the question arises on how to value the "universality" of research results within an international community. It seems that this question has to be discussed from the background of the cultural contexts in which the research questions appear to be relevant.

3.2 Divergent expectations in different research communities – A personal experience (guest-author Marta Pytlak)

ERME conferences (CERMEs) are a great opportunity to get access to a research community in Mathematics Education, and get insight into new research problems, methods and theoretical approaches. Here, problems related to school mathematics, teaching and learning situations in real settings in mathematics classes are discussed from different theoretical perspectives. Theories developed in Mathematics Education are considered as important scientific achievements and research tools. CERME papers document research and development work of their authors and their quality must satisfy scientific criteria.

However, the institutional context of my work as a researcher in Mathematics Education in Poland – Faculty of Mathematics – has other criteria to evaluate my work and publications. Since Mathematics Education is not recognized here as a scientific discipline, my work is evaluated on the basis of the same criteria as the work of mathematicians. Papers published in CERME proceedings or in Mathematics Education journals are not considered as results of scientific work, regardless of the content. I am expected to focus in my work on problems relevant for a scientific work in mathematics and to deal with mathematical theories. Methods, approaches and theoretical constructs developed in Mathematics Education seem to be irrelevant in this context. This hinders the development of research communities in Mathematics Education in Poland and makes the communication within an international community very difficult.

In my PhD thesis, I focused on the development of algebraic thinking in elementary school students. In the whole process of my PhD project I used theoretical constructs, approaches and methods developed and used in international communities in Mathematics Education. While discussing my research within such communities, I received constructive responses regarding the novelty and importance of my research questions and results. Because of references to problems and literature known in Mathematics Education my work was understandable for others.

Due to my institutional context in Poland, it was nevertheless important to adapt my final version of the PhD to the institutional expectations and reduce the part related to theories in Mathematics Education. Instead, I wrote one chapter with elaboration on advanced mathematical theories relevant for my work. Addressing some historical aspects of the development of algebra allowed me to make some links between this chapter and other chapters in my thesis. The changes made to satisfy the institutional criteria for a PhD thesis brought into my work new aspects. But they also shifted the focus from theories which I had used to conceptualize particular problems in Mathematics Education to the more "universal" mathematical theories. Consequently, this changed the way that this work is embedded in the discourse of the European Mathematics Education community.

My experience indicates one of many challenges for researchers in Mathematics Education in Poland which sometimes hinder the development of research communities and the access of the small group of Polish researchers to an international community.

3.3 Cultural biases in review procedures – A personal experience (guest-author Mustafa Alpaslan)

My reflections are related to my experiences as a PhD student and graduate assistant working in teacher education for pre-service middle school (ages 11 to 14) mathematics teachers at the Middle East Technical University, in Ankara, Turkey.

My research interests focus on the integration of history of mathematics in the education of pre-service mathematics teachers. One component of my master thesis was to investigate Turkish pre-service middle school mathematics teachers' knowledge of history of mathematics and to develop a valid test for this investigation. Since the history of mathematics is a large area, the scope of the test was restricted to the historical and institutional context of mathematics taught and learned in Turkish middle schools. Mathematics curricula, textbooks and guidelines for mathematics teachers' competencies were used as a reference frame for decisions concerning this restriction.

Besides items reflecting various cultures' contributions to the historical development of mathematics, one item addressed the history of Turkish mathematical language. In it, Mustafa Kemal Atatürk's contribution to create a new mathematical language according to the new Turkish language, with Latin alphabet rather than the old Ottoman Turkish with Arabic alphabet, was captured. In Turkey, this contribution is valued as an element of our cultural identity and the awareness of it is seen as a part of mathematics teachers' professional knowledge. Atatürk's reason for preferring the new Turkish was that it was actually the spoken language in the public, thus it would provide easier and more meaningful understanding of geometrical concepts.

The context-bound nature of my research seemed to be very natural in discussions with researchers from Turkey. However, a discussion with researchers from *the International Study Group on the Relations between the History and Pedagogy of Mathematics* raised my awareness of the fact, that some items of the test designed in my research may not be understandable for this intercultural community. For example, one item in form of a multiple-choice question asking to mark Atatürk's contributions to the development of mathematics in Turkey was interpreted by the reviewers coming from other countries as an inadequate conceptualization of the investigated construct (teachers' knowledge of history of mathematics). This challenged me to provide additional information about the specific characteristic of my research context. By giving more context information, the reviewers decreased their doubts about the missing universality and significance of my research results. By my context-bound argumentation justifying my research questions, approach and results in the cultural context of mathematics teacher education in Turkey, I succeeded to publish an article from my master's thesis (see Alpaslan, Işıksal, & Haser, 2014). The cultural context was accepted by the reviewers as an essential contribution making my research understandable for them and for the potential international readers of this paper. Increased cultural awareness may help to avoid biases in review procedures.

3.4 Implementation of design research in new contexts – A personal experience (Edyta Nowinska)

In 2011, a group of researchers in Mathematics Education from Germany was assigned by the German foreign aid organization MISEREOR to support teacher education and the development of the quality of mathematics classes on the Indonesian island Sumba in order to educate the learners better for their career opportunities. For this aim, a long-term design research project had been conducted. The focus was on teaching and learning mathematics at the beginning of a secondary school, in particular on learners' cognitive habitus in learning mathematics.

Our first analysis revealed that the Sumbanese learners have difficulties with critical thinking and mathematical reasoning: On each level in the school system there, learners are used to learn by memorizing, answer collectively and wait until the teacher tells them what is correct. They are not used to ask questions and practice monitoring (cf. Sembiring et al., 2008). Our further observations showed that there were some culture-bound variables influencing students' learning behaviour and hindering this kind of thinking and reasoning in mathematics classes. Critical thinking and rational reasoning are not essential characteristics of the Sumbanese culture, neither in the religion based on myths and legends, nor in everyday routines and system of values. This society exhibits a short-term point of view rather than a pragmatic future-oriented perspective based on critical thinking and precise planning.

It seems that the cultural context determines the ways of thinking and acting of teachers and learners. This determination results in culturally acquired epistemological obstacles, and beliefs that there are no alternative ways of acting and thinking. Thus, prior to the implementation of the teaching and learning concept developed on the basis of design principles worked out in the context of German secondary schools (cf. Cohors-Fresenborg & Kaune, 2005), a group of Indonesian pre-service mathematics teachers collaborated with the German educators to reflect on their (unconscious) ways of acting and change their own learning attitudes and teaching practice.

In this process of learning and reflecting, the pre-service teachers constructed mental models for mathematical reasoning, a system of metaphors enabling them to understand the core ideas of stepwise controlled mathematical argumentations and new beliefs about mathematics. They were astonished by the fact that the so-called mathematical "rules" can be derived and explained and that the meaning of symbols and signs can be negotiated in social interactions in class. The experience that Sumbanese learners are able to engage in such social interactions convinced the participating pre-service teachers that changes in the cognitive behaviour of the learners are possible to achieve, although the intended cognitive behaviour is not typical for the attitudes accumulated in the culture and everyday practices of the local society.

After two years of teacher professionalization courses and adapting the intervention developed in the context of German secondary schools to the Indonesian context, remarkable improvements in teaching and learning mathematics and in learners' competencies have been achieved (Nowinska, 2014). This was possible due to our holistic view of teaching and learning as a part of a certain cultural context and a culture itself. Neglecting this context and avoiding the interactive process with the participants of our project would change our intervention to a kind of indoctrination or anarchy and result in new forms of acting without understanding.

This cultural awareness and sensitivity motivated us to provide in our publications (cf. Nowinska, 2014) some explanations concerning the cultural context of our research, complementarily to the theoretical framework guiding our perception and conceptualization of "problems" in Mathematics Education. However, our experience suggests that it is a difficult task for reviewers to relate research problems and results from research and design practice in Mathematics Education to the context of this practice. It seems that in evaluating and reviewing design and research practices, the cultural aspect is often neglected and the major attention is paid to theoretical considerations and novelty of results. This may lead to trivialization of research problems and results in Mathematics Education.

3.5 Critical points raised by young researchers (Nowinska, Andersson, Pytlak, Alpaslan)

Internationalization of research in Mathematics Education (including international research conferences, publications, and collaborative and/or comparative cross-country research projects) challenges researchers to be aware of various contexts and their power to influence research and design practices in particular countries, cultures, societies, institutions and communities (cf. Atweh & Clarkson, 2001). The experiences described by four young researchers in this section indicate the essential role played by an international discourse in initiating reflections on one's own ways of thinking and acting as a researcher, educator or designer in a particular community, culture and society. Such a discourse challenges researchers to see their own practices from a broader perspective and may contribute to making them understandable for others.

The experience described by Annica Andersson indicates possible benefits that can result from participating and working in various contexts and from discourse within an international community of researchers for perception and better understanding of the unique characteristic of their own cultural context. Cultural differences and similarities become visible first as results of reflection and comparisons. Thus, an international discourse requires and facilitates cultural awareness.

Evidence of possible difficulties emerging in such a discourse is given in the reflections of Mustafa Alpaslan. His decisions while conducting his research project were affected by the historical context of the development of mathematical language in Turkey, yet this context was not made explicit when submitting a paper to an international community. Consequently, the novelty and importance of his research could not be understood by the reviewers coming from other cultural context until additional reflections on the cultural context of his research had been made explicit.

Similar challenges, yet related to the institutional context, are mentioned in the reflections provided by Marta Pytlak. The institutional criteria used to evaluate the work of many Polish researchers in mathematics education hinder the development of research communities and their work on problems related to teaching and learning of mathematics in schools.

Designing, justifying and evaluating indirect didactical actions is at the heart of research practices aiming at better understanding and improving teaching and learning processes (cf. Sierpinska, 1998). The experience described by Edyta Nowinska indicates that interventions and design principles resulting from design research cannot be seen as universal solutions for educational problems (cf. Plomp, 2013), even if some of these problems seem to be shared in various cultures.

From the perspective of researchers, seeing *similarities* in the nature of educational problems may contribute to a better understanding of these problems. However, complementarily, crucial *differences* between contexts where they appear should be considered. They provide some strategic guidelines for raising cultural awareness in our "daily" design and research practices: The challenge is to perceive the unique characteristic of the cultural context in which the design and research practices are conducted or have to be transferred into and to make them understandable for the participants of research as well as for readers and reviewers of research papers. Hofstede's dimensions of national culture (e.g., individualism versus collectivism, masculinity versus femininity, uncertainty avoidance, and long term orientation) (Hofstede, Hofstede, & Minkov, 2010) can be used to facilitate the *cultural sensitivity* needed to perceive such characteristics, complementarily to the *theoretical sensitivity* guiding researchers' perception and conceptualization of "problems" in mathematics education.

The reflections written by the four young researchers give insight into the complexity and variety of contexts that have to be taken into consideration to raise one's own cultural awareness. Some factors motivating and facilitating this kind of awareness can be identified. It seems that internalization of research and design in mathematics education, in particular the movement of young researchers within the international community, bring the importance of this theme again and again to light.

Critical points raised in reflections exposed by the young researchers:

- An international discourse initiates and facilitates reflections on one's own practices in a particular community of researchers, in a culture and society. It challenges researchers to raise cultural awareness and supports this awareness by providing new perspectives for contrasting one's own ways of acting and thinking with the ways of others.
- The challenge for raising cultural awareness of researchers is to make explicit the implicit decisions associated with their own design and research processes. It seems that a discursive approach within a community of researchers from different contexts may initiate, facilitate and raise cultural awareness of individuals.
- Writing about one's own research without justifying the choices of research questions and methods in the particular context of this research may not be understood by reviewers from another cultural context. Cultural awareness can help to avoid biases in review procedures.
- Internalization and globalization of design and research in mathematics education support transfer of knowledge and experience among researchers, in particular curricula and teaching interventions. However, teaching interventions cannot be implemented to a new context when the details of these interventions do not make sense in this context. Adaptation of design principles must take the local context into consideration.

Not only theoretical considerations but also various aspects of the particular social, historical, cultural and institutional context make design and research activities understandable.

4. LOOKING BACK AND LOOKING FORWARD

4.1 Looking back: Cultural differences in various contexts

The experiences and reflections presented in Sections 2 and 3 by experienced and young researchers all report on cultural differences which were only partly explicit and on how the differences or their implicitness affected the research or design practices.

However, these rich examples are located on different (of course overlapping) levels of cultural contexts (cf. Fig. 6), covering the mathematics itself (Section 2.1), but also contexts of mathematics classrooms and the societies (Sections 2.2 and 3.1, 3.3 and 3.4), or the institutional contexts of the research communities (Section 2.3 and 3.2).

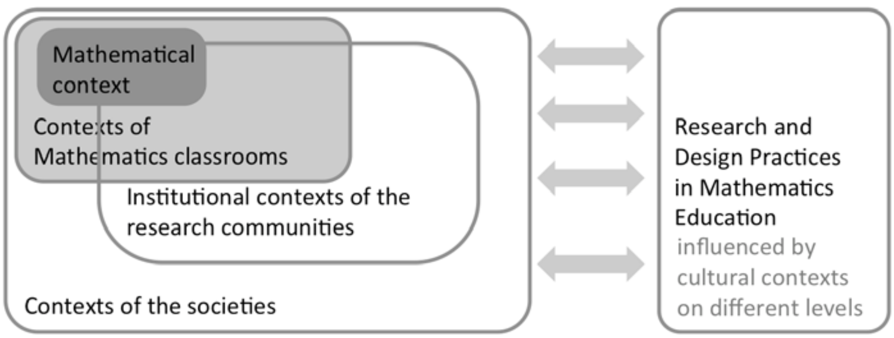

Figure 6: Nested cultural context on different levels which all influence the research and design practices in mathematics education

On the one hand, none of these differences are really surprising. In principle, we (should) know that these kinds of differences exist, and the international comparative studies on classroom cultures have shown such kind of differences systematically (cf. Stigler, Gonzales, Kawanka, Knoll, & Serrano, 1999; Clarke, Emanuelsson, Jablonka, & Mok, 2006). Additionally, the research on multicultural issues within one country has shown how differences in the societal context affect students in classrooms (e.g., Secada, 1992; and many others).

On the other hand, our principal awareness does often not reach our everyday research and design practices or it reaches them late, as a result of our practices instead as an input of them. The examples have shown that our daily research is often implicitly guided by the hidden assumption that contexts, problems, and outcomes are or should be universal and not shaped by the cultural relativity, briefly, the *hidden assumption of universality*. Instead, we plead for constantly questioning this hidden assumption of universality of research and design practices and outcomes and for raising cultural awareness. This seems necessary to establish the ERME community as an *epistemic community* using the multicultural diversity of its individual members and working groups to produce, widen and enrich our knowledge.

4.2 Looking forward: Raising cultural awareness

Being aware of differences and overcoming the hidden assumptions of the universality of research and development practices and outcomes, we and others can act in the directions outlined below:

- *when addressing mathematics,* we can try to be aware of cultural contingencies; we can challenge our own ways of perceiving and expressing mathematical constructs;
- *when reading other's papers,* we can avoid naïve transfers of constructs, approaches and outcomes from other cultural contexts; for example, what worked in Spain need not in Poland;
- *when reading other's papers,* we can systematically investigate the adequacy of transfers, not only for results, but also for theoretical constructs and approaches;
- *when writing papers*, we can describe explicitly our own cultural context, paying attention to the ways it affects what we write about research methodology and findings;
- *when conducting own research and development*, we can try to learn from other cultural contexts in order not to take for granted our own conditions.

Being aware of dominances and overcoming the hidden assumption of the universality of research and development practices and outcomes, we can attend to the following in our work with others:

- *when collaborating with colleagues from other cultural contexts,* we can take enough time to learn about other cultural contexts and consider differences; and we can exploit the gap between us, in order to become aware of our own unthoughts;
- *when importing research to other countries*, we can discuss and apply methodologies that allow us to be sensitive to the cultural contexts we join;
- *when acting as reviewers or editors for journals or conferences,* we can try to avoid the dominance of Western empirical research, where the sole adoption of the Western format of presentation (theoretical framework, research questions, methodology, findings) risk to hide new and fresh ideas, which have the potential to enrich our vision of the world;

- *when responsible for policy issues in international communities, we can follow good practices of* ICMI and ERME to strictly try to realize regional balances in all committees in order to alleviate cultural dominance;
- *when responsible for policy issues in international communities,* we can try to foster the standards for making explicit cultural contexts in writing and review guidelines (as, for example, in the practices seen in ERME guidelines, ICMI studies…);
- *when supervising PhD students from non-affluent countries,* we can reflect how far to impose own values on their research.

5. CONCLUSION: RAISING CULTURAL AWARENESS AS A COMMUNITY TASK

The focus here is both back onto mathematics and forward onto cultural awareness. We reflect on what we have written above and encourage questioning of our future practice. Our challenges in developing our own awarenesses place us in a position of responsibility to our academic and research communities in promoting more global values with respect to the cultural dimensions within which we work. Our intention here is to encourage discussion, and possibly debate, through which issues can be raised and addressed widely.

5.1 What makes mathematics central and different from other disciplinary areas?

In Section 2.1, we have raised issues relating to the ways in which mathematics is represented, symbolised and conceived in different parts of the world. Examples have shown how certain representational forms can foster or promote differing conceptions, some of which might be seen to limit how mathematics is understood. Perceptions of universality in mathematics can therefore be dangerous if they remain implicit and resist being challenged. Even the most experienced mathematics educators can learn from alternative cultural representations and build richer representational frames.

Section 2.2 has shown that teaching approaches and the ways in which mathematics is offered to learners carry a high responsibility with regard to learning outcomes. Narrow insistence on the internal consistency of mathematics as disseminated through representational forms and adherence to procedural rules, without corresponding attention to the underpinning concepts, have to bear a responsibility for public perceptions of mathematics and for mathematical achievement in diverse cultures. Richer cultural awarenesses can open up mathematical discourses that promote access and understanding and a broader willingness to engage and succeed with mathematics. However, these processes of innovation must themselves respect cultural differences.

5.2 How can a focus on mathematics take into account the moral and ethical issues of education for all?

The *moral* and the *ethical* are human constructs: being moral and ethical places responsibilities on mathematics educators as human beings. We have responsibilities to our disciplines of mathematics and education, and importantly to the people whose education in mathematics we promote. The intrinsic educational levels here present a complex weaving of responsibilities: educating students in mathematics; educating teachers in teaching mathematics; educating new researchers in theory and research; educating mathematics educators who educate at all of these levels. The awarenesses referred to in Section 5.1 with regard to mathematics underpin this edifice: we have to weigh the issues in deciding how best to interpret the role of educator. For example, the mathematics teacher who, with thoroughly good intentions, over-simplifies a mathematical concept to avert the struggles of the learner may not assist in the complexities in appreciating the concept; or a lecturer who emphasizes the fine details of a proof without attention to the sociohistorical origins of the proof may be true to mathematical rigor but leave a student mystified. Awareness of the choices we exercise as educators requires moral and ethical judgments in how we operate in our professional roles.

5.3 How can a focus on mathematics address the figured worlds of all who learn and teach?

The concept of *figured worlds* (Holland et al., 1998; cf. Section 2.2.5) recognizes that human beings are located within a complex synergy of cultures: societal, disciplinary, professional, familial, philosophical and personal, to name a few. In professional human relations, many of these 'worlds' are hidden; we have seen clear examples of this in the sections above. While any one or group cannot be expected to discern the extent of such complexity, we can be expected to be aware that it exists. This requires us to give overt attention to difference and meaning and their (potential) relationships to the concepts we address. It requires a willingness to be open, to encourage our learners to express their own conceptions and inform us of how things are done and seen in their contexts. As educators we have to do our best to make moral and ethical choices within our own knowledge and what we hear and learn by listening to others.

5.4 Which theories, research and design approaches can grasp the complexity and cultural differences?

As the reality of mathematics teaching and learning is very complex with all these different nuances and challenges, no single isolated theory, no single research or design approach can do justice to this complexity (cf. Section 2.3). Mathematics Education research that really contributes to relevant innovations in educational practices requires us to connect different approaches.

Raising cultural awareness also supports us to become aware of the strengths and limits of different epistemic cultures. This applies for Mathematics Education as well as for every other subject matter education and general education and requires not only the efforts of single researchers but the whole community.

5.5 What are the big issues that we should be addressing?

We wish to be true to mathematics as we know it. However, we can always learn more to enrich our own perceptions. As educators we are challenged continually to explore how best to bring mathematics and its learning and teaching to our learners. However, the biggest issue is not the question 'how can we bring mathematics and its learning and teaching to our learners?', but, 'how can we make *exploration* of such a question the basis of our professional activity'? We have to keep addressing this issue. For learners to see that their teachers, at any level of education, are also learners, questioning the very practices in which the teacher and learners are engaged, can be empowering and exhilarating, although it can also be frustrating for those who seek closure.

Closure is a philosophical position, related to seeking certainty and end points, and is something we have to address overtly. Rather than seeing closure in this concept, or this idea, or this issue, all participants can see themselves at their own stage of the educational journey, where the coming stages are open for participation. This very idea needs cultural reorientation in many contexts: for example, recognition that if we seek the 'right answer' to a mathematics problem, what is *right* may depend on a range of contextual factors; if we seek to *define* a mathematical entity, the very act of definition excludes other possibilities; if we present a Mathematics Education thesis in Poland, it will be judged differently from the same thesis in Spain. This is not to say there are no right answers, or to undermine the value of definitions, or to reject the judgments made in different communities; rather, while agreeing an answer or a definition, the limitations and exclusions of such acts need to be recognised and (insofar as we are able) addressed. Therefore, there are no end points, but many choices, challenges, and judgments. We have to embrace diversity and seek out alternative meanings and roots. Those more experienced are there to help all engage, to scaffold their growth of understanding, encourage progress and develop awareness, not to set limits or close off possibilities. This is the moral challenge for all of us!

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