



Slots of Power-Law Profile as Acoustic Black Holes for Flexural Waves in Metallic and Composite Plates



E.P. Bowyer, V.V. Krylov *

Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK

ARTICLE INFO

Article history:

Received 30 October 2014

Received in revised form 30 January 2016

Accepted 1 February 2016

Available online 10 February 2016

Keywords:

Vibration damping

Acoustic black hole effect

Slots of power-law profile

ABSTRACT

A new method of damping flexural vibrations in plate-like structures based on the 'acoustic black hole effect' has been recently developed and investigated. As 'acoustic black holes', one-dimensional elastic wedges of power-law profile covered by narrow strips of absorbing layers near sharp edges have been used initially. The addition of such power-law profiled wedges to edges of rectangular plates or strips results in substantial increase in damping of resonant flexural vibrations in such plates or strips due to the more efficient absorption of flexural waves at the tips of power-law wedges. One of the problems faced by this method of damping is having the wedge tips exposed on the outer edges of the plate or strip. One of the solutions to this problem is to move the wedges inside a plate, so that they form edges of power-law slots within the plate. The present paper reports the results of the experimental investigations into the effects of such slots on damping flexural vibrations. The obtained experimental results show that introducing power-law profiled slots within plates represents an effective method of damping flexural vibrations, which is comparable with the method using power-law wedges at plate edges.

© 2016 The Authors. The Institution of Structural Engineers. Published by Elsevier Ltd. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Reduction of unwanted vibrations in different engineering structures remains a major problem that requires effective solution in each specific case under consideration. Passive damping of structural vibrations is usually achieved by adding layers of highly absorbing materials to the structure in order to increase energy dissipation of propagating (mostly flexural) waves [1–3]. Another well-known approach to suppression of resonant vibrations of different engineering structures is to reduce reflections of structural waves from their free edges [2,4].

To implement the latter approach in a more efficient way, a new physical phenomenon, the so-called 'acoustic black hole effect', has been recently investigated and adopted for vibration damping purposes [5–7]. The acoustic black hole effect is a relatively new physical phenomenon that helps to achieve a very efficient absorption (almost 100%) of flexural waves in plate areas of variable thickness gradually reducing to zero, where thickness variation with distance should follow a power-law profile. Such areas of variable thickness are often called 'acoustic black holes'. As 'acoustic black holes' for flexural waves, one-dimensional elastic wedges of power-law profile covered by narrow strips of absorbing layers near sharp edges have been used initially. Ideally, if the power-law exponent is equal or larger than two, the flexural wave never reaches the sharp edge of the wedge and therefore never

reflects back [5–8]. Thus, such an ideal power-law wedge can be considered as a one-dimensional acoustic black hole for flexural waves. In real situations though, zero reflection never happens because of ever present wedge truncations and other imperfections. However, the addition of even small amounts of absorbing materials at the sharp edges of real power-law wedges reduces the reflection coefficients drastically, and this constitutes the acoustic black hole effect [5,6]. It has been established theoretically [5,6] and confirmed experimentally [7,9] that the method of damping structural vibrations based on the acoustic black hole effect is very efficient even in the presence of edge truncations and other imperfections. In addition to the above-mentioned wedges of power-law profile, circular indentations of power-law profile made within plates can be also used [10–12]. Such circular indentations or their combinations materialise two-dimensional acoustic black holes that can be used as efficient dampers of structural vibrations.

The main advantage of using acoustic black holes over traditional methods of vibration damping utilising layers of highly absorbing materials attached to entire surfaces of structures is that it requires very small amounts of absorbing materials placed at the sharp edges of power-law wedges or at the centres of circular power-law profiled indentations to achieve comparable values of damping [9–13]. This is very attractive for vibration damping in light-weight structures used in aeronautical and automotive engineering. Note that the unique properties of acoustic black holes for flexural waves stimulated a number of recent investigations considering some of their new potential applications, not necessarily limited to vibration damping [14–18].

* Corresponding author: Tel.: +44 1509 227216.

E-mail address: V.V.Krylov@lboro.ac.uk (V.V. Krylov).

Table 1
Geometrical and material properties of plates and damping layers.

	Steel plate	Carbon composite plate	Damping layer
Thickness	5.04 mm	2.5 mm (single), 5 mm (combined)	0.08 mm
Young's modulus	190 GPa	85 GPa	–
Mass density	7000 kg/m ³	1600 kg/m ³	300 kg/m ³
Poisson's ratio	0.3	–	–
Loss factor	0.006	0.1–0.2	0.06

One should mention in this connection that controlling wave propagation over the given area does not necessarily mean damping and suppression of such waves. A very elegant way of controlling waves of different nature, including flexural waves in plates and Rayleigh surface waves in elastic half spaces, is based on the concept of 'transformation elastodynamics' that employs elements of metamaterials to change wave propagation path in a desirable way (see e.g. [19–23]). In particular, waves can be forced to propagate around the chosen area without penetrating into it, thus making the area (object) basically 'invisible' (cloaking) to the waves considered. This can be used in practise for isolation of sensitive objects, e.g. equipment or houses, not by suppressing or screening incident waves, but by redirecting their propagation path around the area.

Returning to controlling the propagation of flexural waves via their damping, it should be noted that one of the main practical problems faced by acoustic black holes formed by wedges of power-law profile is having the wedge tip exposed on the outer edges of the plate or strip. The tip of a wedge is not only delicate, but sharp, presenting an exposed structurally weak edge with a health and safety risk. A wedge on the edge of a strip or plate also presents a difficulty in integrating the structural components containing acoustic black holes into panels/plates that need securing at the edges. One possible solution to this problem is to use the above-mentioned circular indentations of power-law profile within plates [10–12], instead of wedges of power-law profile attached to their edges. Another promising solution to this problem is to move the power-law wedges inside plates, so that they form slots within the plates, materialising quasi-one-dimensional acoustic black holes for flexural waves. This solution will be the focus of this experimental work that explores the effects of the introduction of tapered slots into steel and composite plates on damping flexural vibrations in such plates. Note in this connection that one- and two-dimensional acoustic black holes in composite plates and panels have been investigated in the recently published work [24]. However, the application of slots of power-law profile as acoustic black holes to composite structures is rather new.

2. Experimental samples and their manufacturing

Eleven experimental samples have been manufactured for this investigation; six of which were made from 5 mm thick hot-drawn mild steel sheets; which are more resistant to mechanical stresses incurred in the manufacturing process than cold-drawn steel sheets, resulting in fewer internal defects. The dimensions of the steel rectangular plates were 320 × 240 mm, with a slot size of 100 × 75 mm. The other five samples have been made from 2.5 mm thick carbon composite. These samples were made using pre-preg carbon composite sheets that were layed up, cured, and then machined. The dimensions of the carbon composite plates were 280 × 175 mm, with a slot size of 140 × 80 mm. The estimated material properties of plates and viscoelastic damping layers are listed in Table 1.

A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm was used to produce the slots in both the steel and composite plates. There are two main problems encountered when utilising this method of manufacturing. The first being that at the centre of the slot, where the machining stresses and resulting heat are high and the material thickness is less than 0.2 mm, blistering in the steel can occur. This can lead to inaccurate results during a test due to irregularly varying thickness. The second is wedge tip tearing. This is particularly prominent in the composite samples where the tip thickness is less than 0.2 mm. This effect can occur during the profiling of the wedge itself or during the insertion of the central gully.

The six steel samples consisted of a plain reference plate, a punched slot reference plate and samples A–D containing slots of power-law profile (samples A and C are shown in Fig. 1). The wedges in slots are of power-law profile with the value of the power-law exponent $m = 2.2$. The five carbon composite samples consisted of a plain reference plate, a combined reference plate and samples E–G containing slots of power-law profile (samples E and F are shown in Fig. 2). The wedges in this case are of power-law profile with the value of the exponent $m = 4$. The longitudinal cross-sections of all profiled samples are shown in Fig. 3.

3. Experimental setup

The experimental setup has been designed to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate edges and introduce minimal damping to the system, see Fig. 4(a).

The excitation force was applied to three locations on the plate via an electromagnetic shaker attached to the plate using 'glue' and fed via a broadband signal amplifier. Position 1 – top dead centre, position 2 – to the side centrally located, and position 3 – bottom dead centre

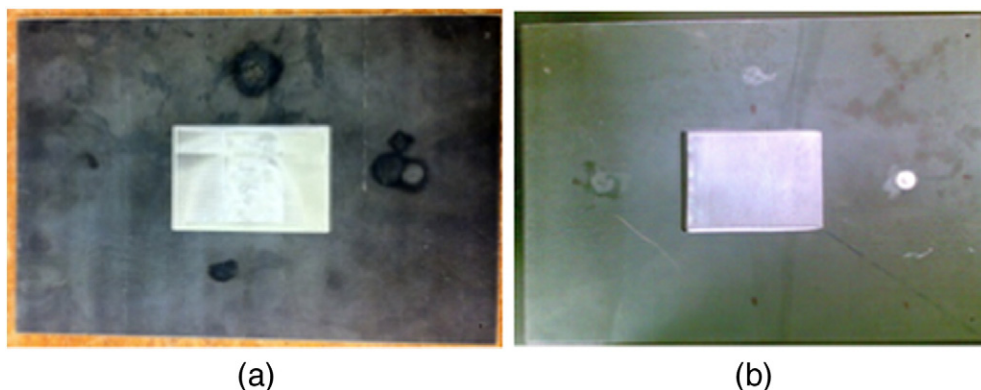


Fig. 1. Photograph of (a) sample A – steel, and (b) sample C – steel.

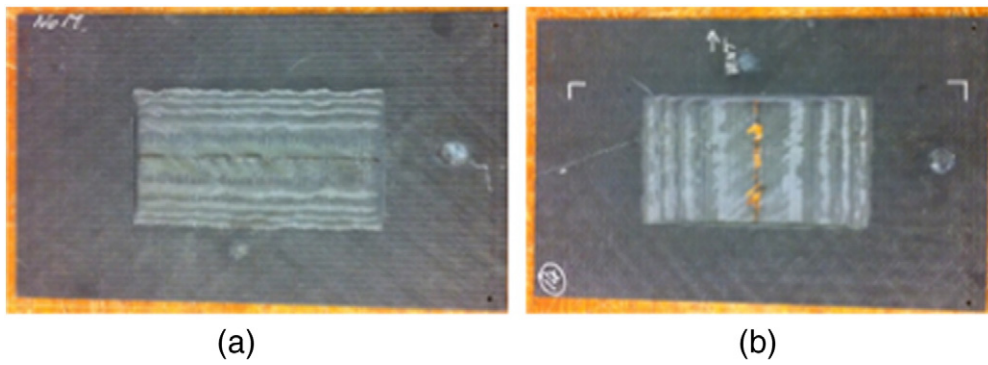


Fig. 2. (a) Sample E – Composite, and (b) sample F – Composite.

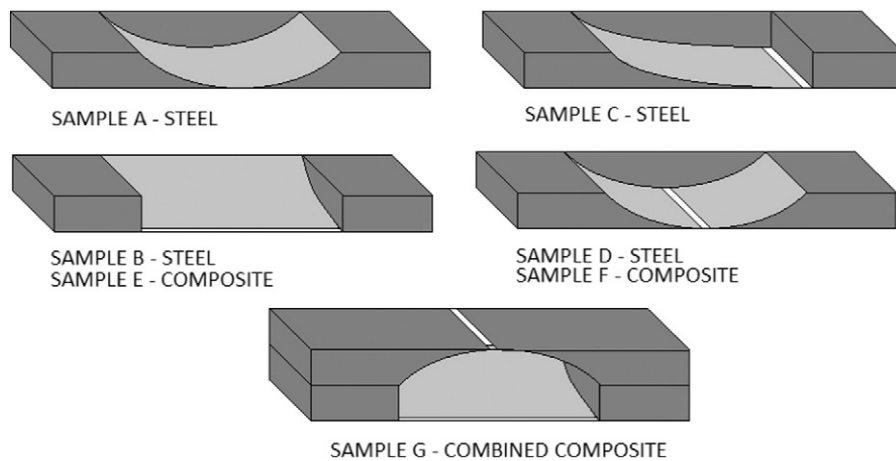


Fig. 3. Longitudinal cross-section through the slot area of samples A–G.

(Fig. 4(b)). The response was recorded by an accelerometer (B&K Type 4371) that was attached to the one surface, directly in line with the force transducer (B&K Type 8200), also attached using ‘glue’.

The acquisition of the point acceleration utilised a Bruel & Kjaer 2035 analyser and amplifier, a schematic is shown in Fig. 5. The frequency ranges of 0–9 kHz for the steel samples and 0–4.5 kHz for the carbon composite samples were used. These choices of the frequency ranges for the measurements correspond to typical frequencies of structural vibrations associated with audible structure-borne sound. In the case of composite samples, the frequency range has been reduced because of the higher intrinsic attenuation in composite materials.

4. Results and discussion

4.1. Tapered slots in steel plates

Initially, a long narrow wedge, of dimensions 95×70 mm similar in style to that seen on the external edge of a strip or plate [5–7] was machined into the slot in the steel plate (sample C). The tip of the wedge was free in order to allow for a more accurate physical comparison to an external wedge. This wedge is almost double the length of the wedges machined on the external edge of the plates that have previously been tested. The sides of the wedge in the slot however are fixed from wedge root to tip (Fig. 6). A damping layer was attached to the thinner region of the profile. All the tests in this section were performed at position 1, as shown in Fig. 4(b).

Fig. 7 shows the measured acceleration for sample C when compared to a plain reference plate, i.e. a plate without thickness indentations. The effect of adding an internal wedge into the slot is immediately obvious, with considerable damping of resonant peaks easily observed. Below

900 Hz, little to no damping is seen. The peak at 1.4 kHz appears to have been split into two smaller resonances, one at either side of the original peak. The specific contribution of the acoustic black hole effect appears in the reduction in resonant peaks, and therefore in damping increase, with the increase of frequency. Some other differences in the accelerances between sample C and the reference plate, including the splitting of resonance peaks, may partly be due to the breaking of geometrical symmetry from the reference plate to the tested plate. Furthermore, the plates have different total masses, which may alter the frequency response. A matching of peak amplitudes between the two samples occurs until about 6 kHz, after which the response of sample C flattens out the distinct resonant peaks seen in the reference sample. A maximum reduction from the reference plate of 11 dB can be observed at 5 frequencies: 1.4, 2.2, 5.0, 7.5 and 8.3 kHz.

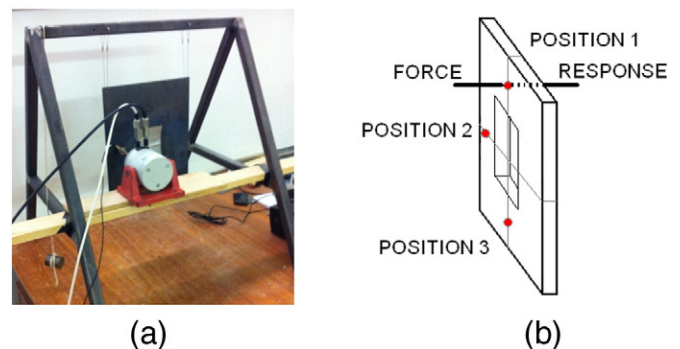


Fig. 4. (a) Experimental setup, (b) locations of the shaker (force) and of the accelerometer (response) on an experimental sample.

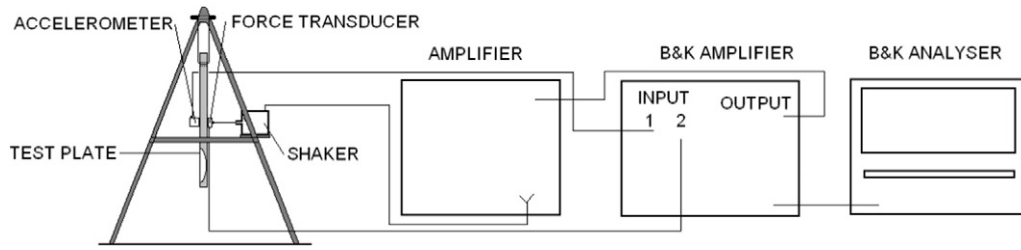


Fig. 5. Schematic of the experimental setup utilising a Bruel & Kjaer analyser.

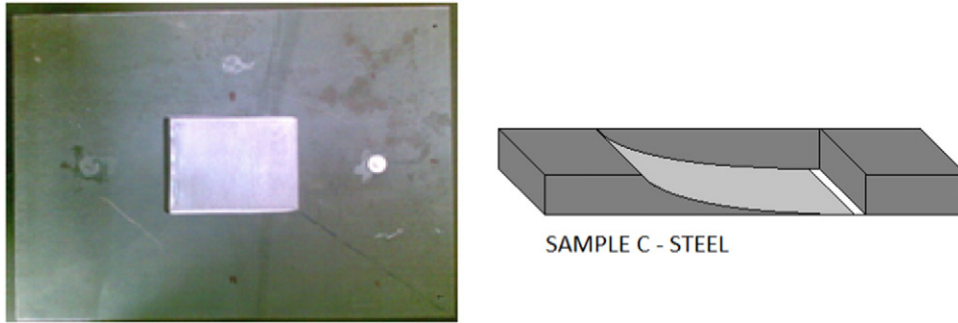


Fig. 6. Photograph and longitudinal cross-section of sample C – steel.

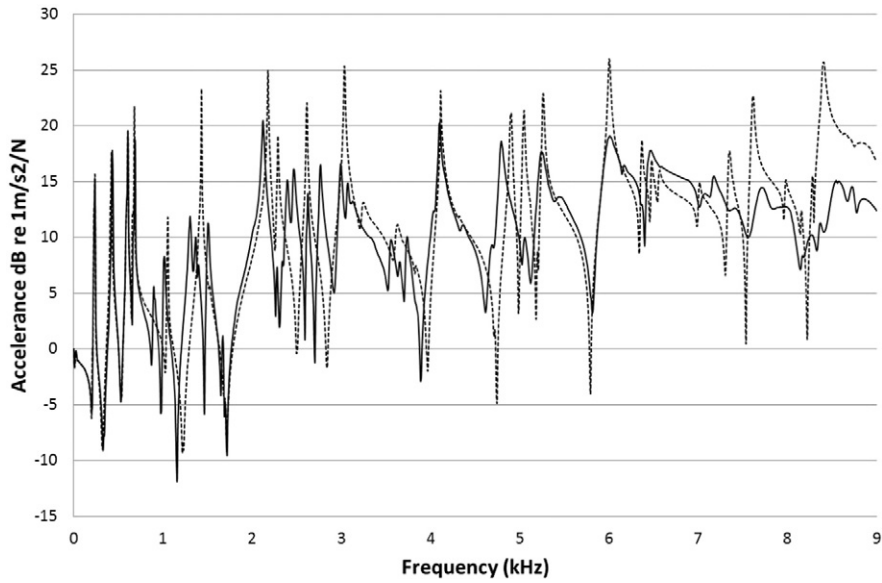


Fig. 7. Accelerance for sample C (solid line) compared to a reference plate (dashed line).

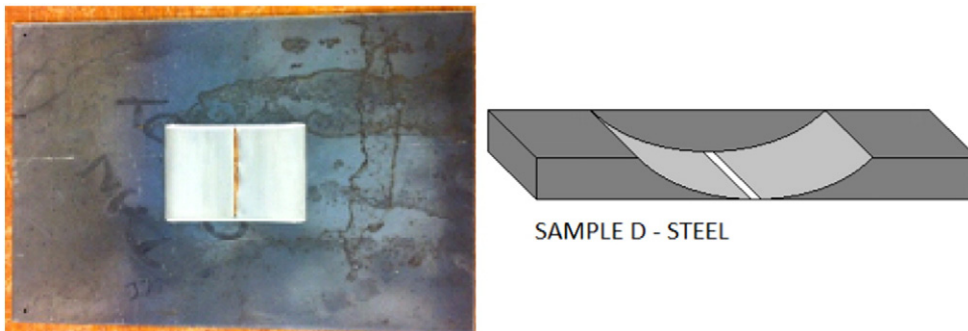


Fig. 8. Photograph and longitudinal cross-section of sample D – steel.

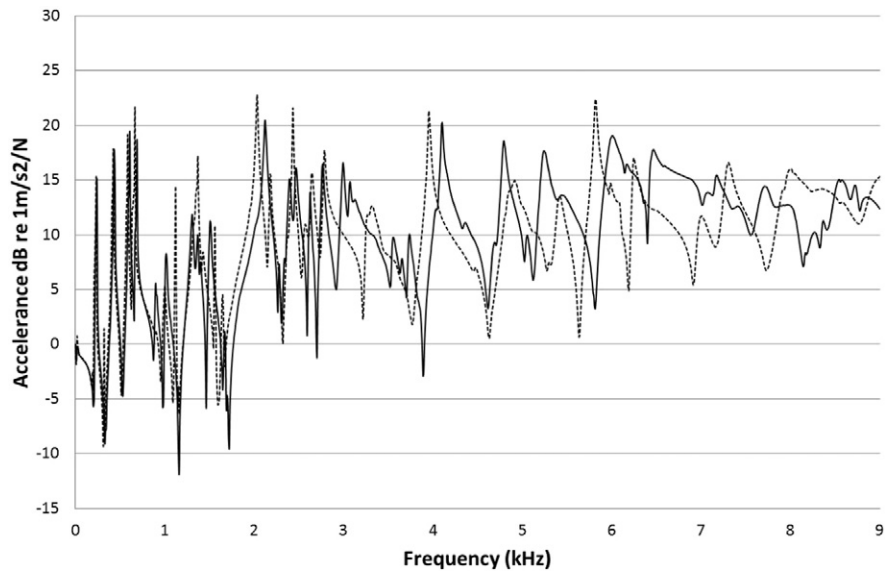


Fig. 9. Accelerance for sample C (solid line) compared to sample D (dashed line).

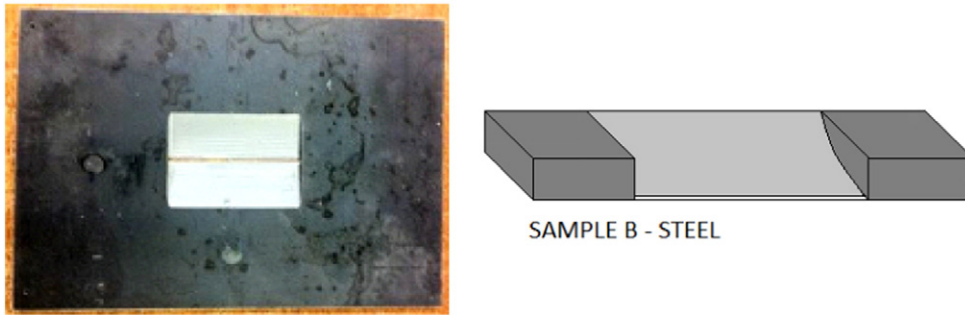


Fig. 10. Photograph and longitudinal cross-section of sample B – steel.

The next step was to investigate the effect of doubling the number of wedges (from 1 to 2) within the slot, Fig. 8. Although this would half the length of the existing wedge, it was expected that by doubling the number of internal wedges the damping capabilities of the slot would

increase. The abrupt edge of the slot present at 5 mm from the wedge tip would be removed, and the waves instead of being reflected would propagate into the wedge. Thus, there are now two wedges within the slot, both 47×70 mm.

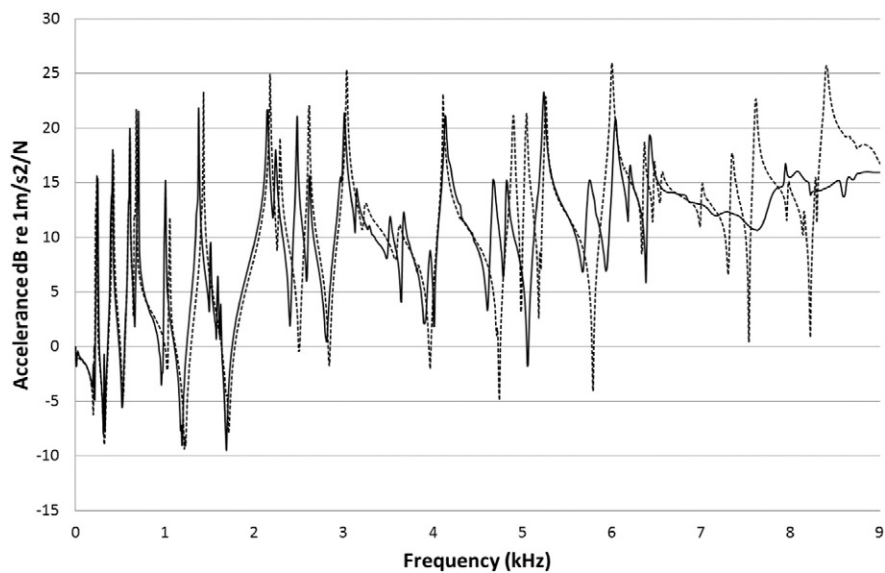


Fig. 11. Accelerance for sample B (solid line) compared to a reference plate (dashed line).

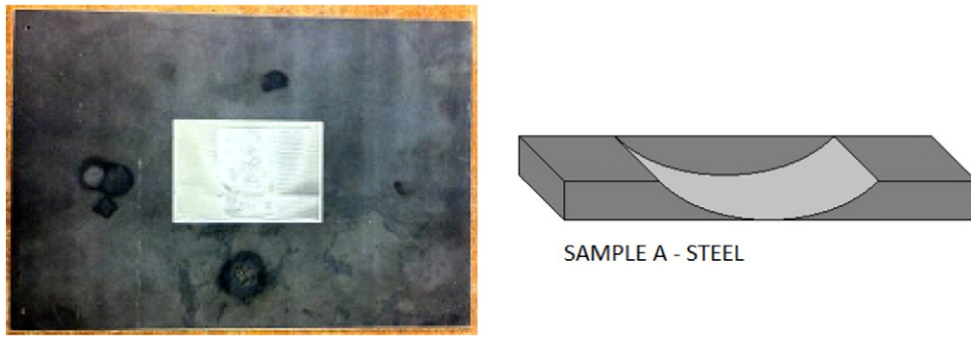


Fig. 12. Photograph and longitudinal cross-section of sample A – steel.

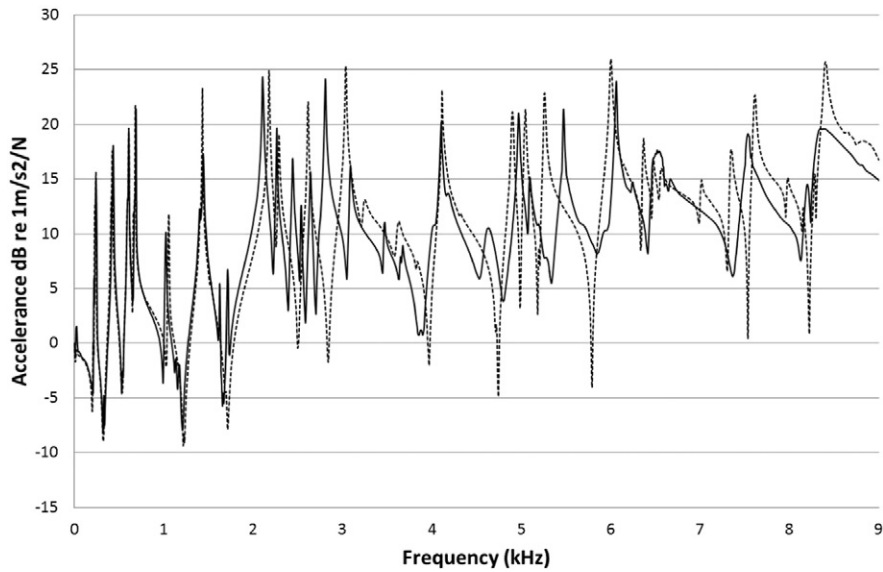


Fig. 13. Accelerance for sample A (solid line) compared to a reference plate (dashed line).

A comparison of measured accelerances for samples C and D is shown in Fig. 9. There is almost no difference between the two samples at low frequencies. However, with the increase of frequency, the accelerance for sample C oscillates in most efficient damping

performance with that for sample D, with the latter outperforming sample C at higher frequencies (above 5 kHz).

The next step is to consider slots of a greater tip width and to find out if they perform more effectively at damping flexural vibrations

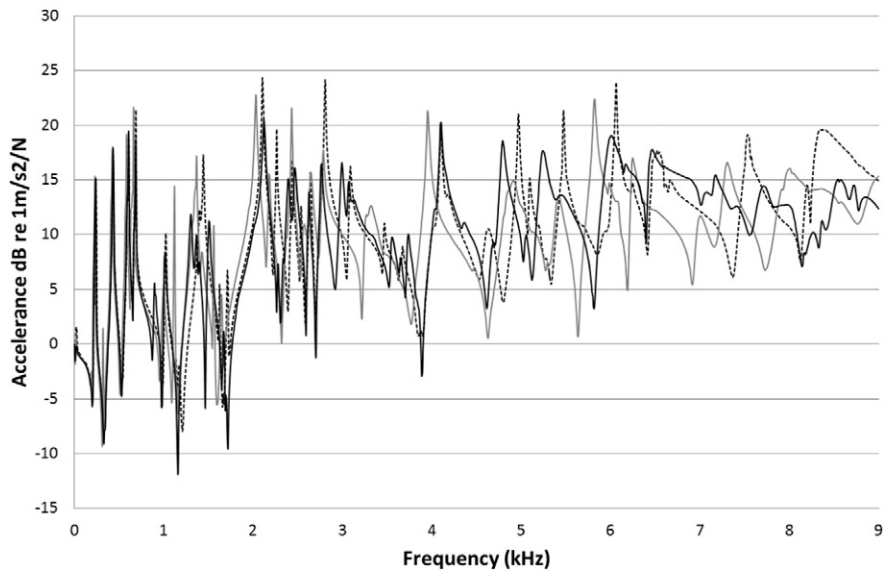


Fig. 14. Accelerance for sample C (solid black line) compared to sample D (solid grey line) and sample A (dashed line).

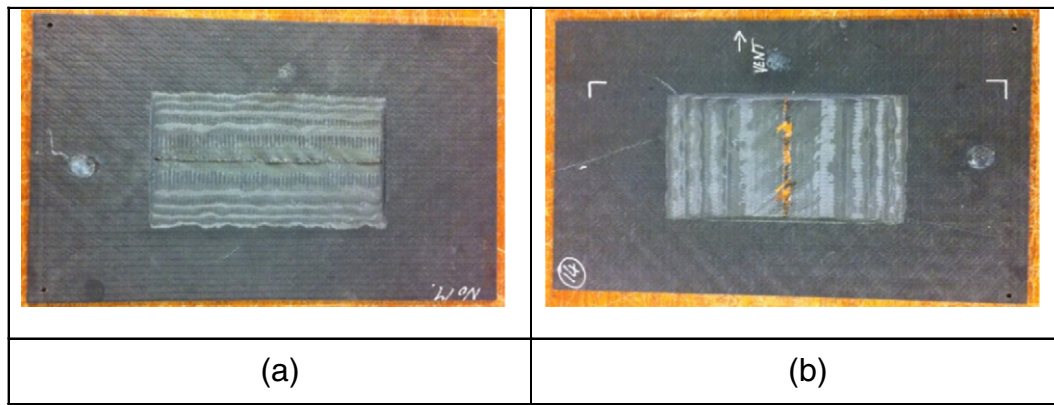


Fig. 15. Carbon fibre composites sample E (a) and sample F (b).

than shorter slots. In order to accommodate such an increase in width the wedges would have to be rotated in the slot, aligning the wedge tip line in parallel to the excitation to produce sample B with two short but wide wedges (see Fig. 10). The wedge dimensions are 95×37 mm. A two-wedge configuration was chosen due to the increased damping attained in certain frequency ranges and also the flexibility in excitation position gained with the addition of a second wedge.

From Fig. 11, it can be seen that when sample B is compared to a plain reference plate and excited from position 1, the damping achieved is less than that achieved when sample C or sample D configurations are used. As expected, at low frequencies there is little to no damping, and at 1 kHz an increase in 4 dB can be seen. At higher frequencies the damping increases. A maximum reduction from the reference plate of 8.5 dB can be seen at 7.6 kHz.

One of the drivers towards creating the slot plates was the protection of the wedge tip. However, they are still fragile and prone to tearing during and post manufacture. A possible solution to this problem could be to remove the central gully at all, creating a 'slot pit', sample A (see Fig. 12).

A comparison of sample A to the plain reference plate is shown in Fig. 13. A maximum reduction from the reference plate of 9 dB can be seen at 3 kHz, with a 6 dB reduction at 1.4 and 8.3 kHz. Despite these relatively large reductions in peak amplitude, the majority of peaks are reduced by only 0–3 dB. When the damping performance of sample

A is compared to those of samples C and D (see Fig. 14), it can be seen that sample A is clearly the least effective at damping flexural vibrations in the steel plate. Thus, the gap at the centre of a power-law slot is essential for increased damping performance.

4.2. Tapered slots in carbon composite plates

Composites have always been considered as the material of the future, and with their increasing popularity in the industry it would be a miss not to consider the effects of new damping methods on such materials. Effects of one- and two-dimensional acoustic black holes on vibration damping in composite plates and panels have been considered in the recent paper [24]. In this section, the effects of tapered power-law slots on damping flexural vibrations in composite plates are investigated.

A plain reference composite plate was used for comparison in this section, and, following [24], no additional visco-elastic damping layer was added to any of the composite samples. All the tests in this section were performed at position 1, as shown in Fig. 4(b). When used in a practical application, it is unlikely that the excitation of the plate will be from a single point source, and it will more than likely be excited from multiple sources. Therefore, the two slot configurations chosen to be tested in carbon composites plates were sample E and sample F (see Fig. 15), as they offer the greatest flexibility in excitation position.

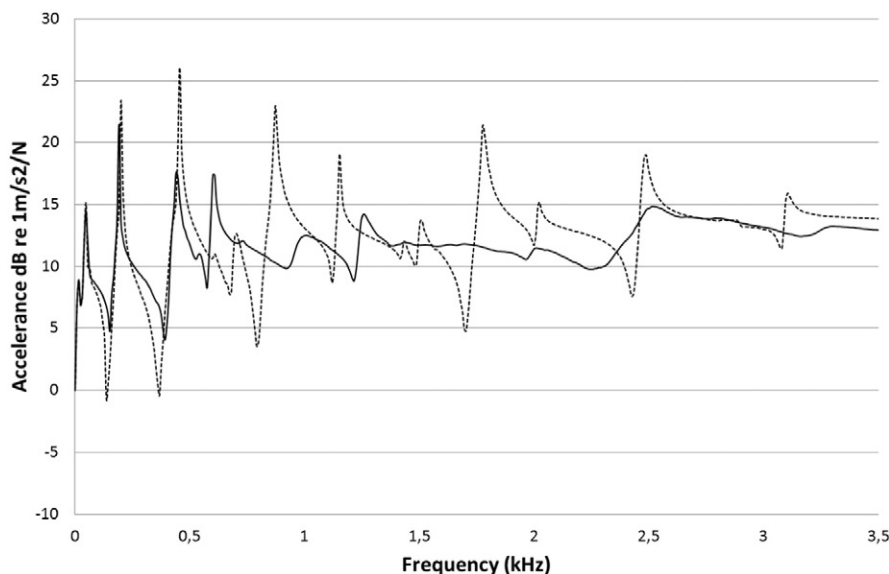


Fig. 16. Accelerance for sample F (solid line) compared to a reference plate (dashed line).

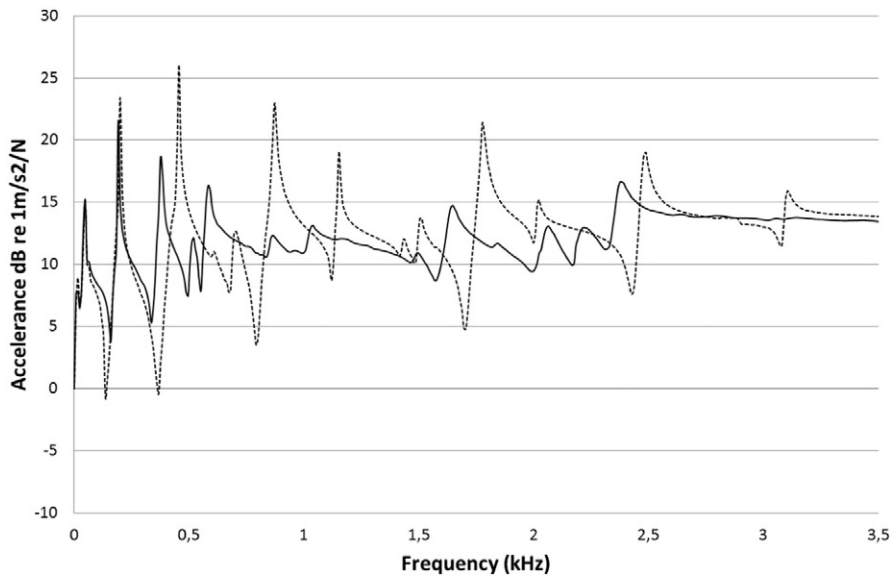


Fig. 17. Accelerance for sample E (solid line) compared to a reference plate (dashed line).

Fig. 16 shows the results for sample F as compared to a plain reference plate. At low frequencies (below 250 Hz) little to no damping is seen, and at higher frequencies the response is smoothed, with resonant peaks heavily damped if not completely removed. A maximum reduction from the reference plate is about 11 dB. The results for sample E compared to a plain reference plate are shown in Fig. 17. Again, at low frequencies there is little to no damping, whereas at higher frequencies peak amplitudes are reduced substantially. A maximum reduction from the reference plate is about 12 dB.

When compared to each other, the two samples show the same trends as their steel plate counterparts. The only differences in the observed trends are down to the changes in material properties of composite plates. One should note that, due to large material loss factors of composites, 0.1–0.2 (see Table 1), no additional damping layer was needed to obtain substantial reductions in the amplitude of the resonance peaks. A similar behaviour was observed in the case of one- and two-dimensional acoustic black holes in composite plates and panels [19].

4.3. The effect of vibration source position on achievable levels of damping in slotted plates

This section deals with the levels of vibration damping that can be obtained for different positions of the excitation point. When different excitation positions are considered, different modes are excited within the plates. Three steel samples were used: sample C, sample D and sample B, along with the two composite samples. First, sample C was excited in three different positions in relation to the slot (Fig. 4(b)).

From Fig. 18, it can be seen that, when sample C was excited from positions 1, 2 and 3, the position of the wedge root relative to the excitation position determined the level of damping achieved. Considering the general trend of these three excitation positions on sample C, the excitation position 1 has the closest proximity to the wedge root and it has the greatest damping capability. When excited at position 2, the damping capability of the sample is less than that at position 1. The increase in the damping performance over the frequency range is small; approximately only 1–2 dB per rotation of the excitation position. The

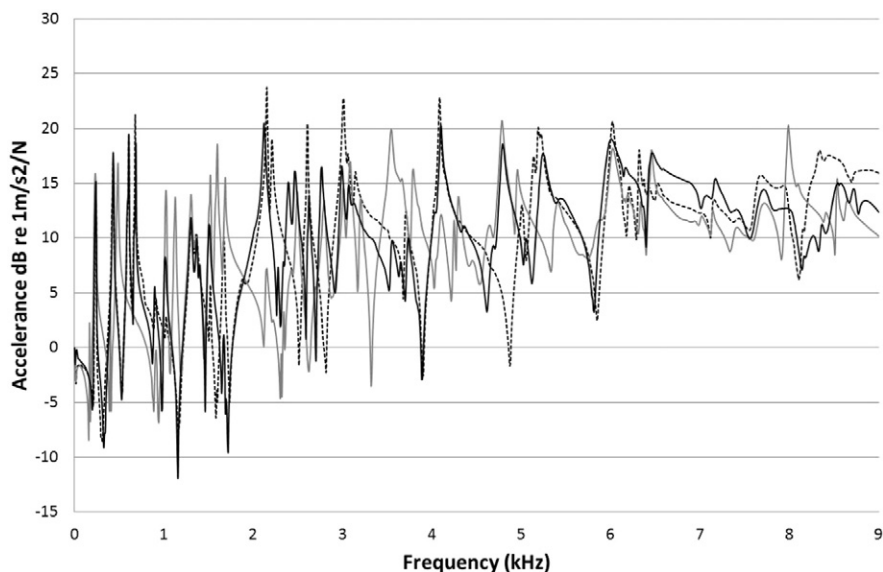


Fig. 18. Accelerance comparison for sample C excitation positions 1 (solid black line), 2 (grey line) and 3 (dashed line).

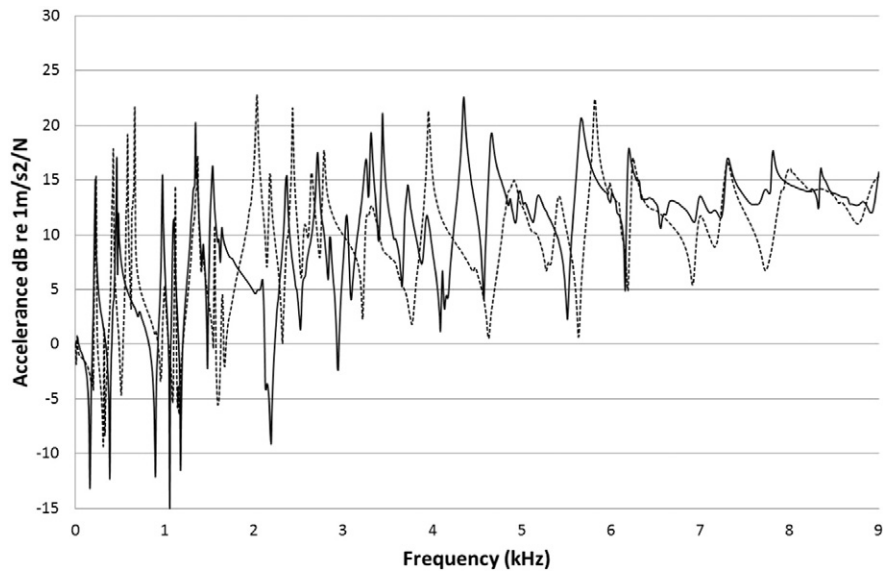


Fig. 19. Accelerance for sample D excitation positions 1 (dashed line) and 2 (solid line).

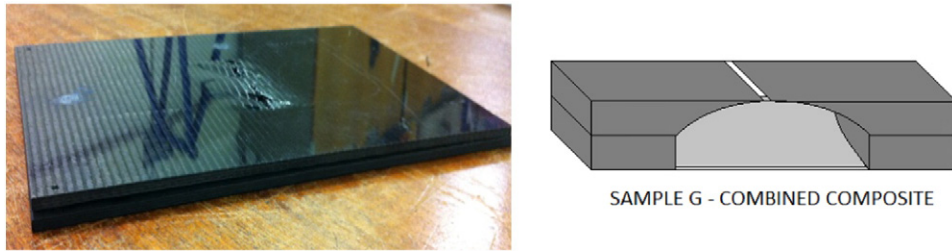


Fig. 20. Photograph and longitudinal cross-section for sample G.

results for sample D for excitation positions 1 and 2 are shown in Fig. 19. The results in this case are less clear than in sample C. Below 4.5 kHz the sample is more effective at reducing peak amplitudes when excited from position 1.

Note that excitation at any of the three excitation positions causes flexural waves to propagate out in all directions from the excitation

point. The excitation position directly adjacent to the wedge root is expected to result in the greatest reduction in resonant peaks due to the waves propagating directly into the wedge [7,9]. For sample C this situation occurs at excitation position 1. Samples B and E perform more effectively when excited from position 2, which is close to the wedge root in these cases. Thus, the sample configurations that had

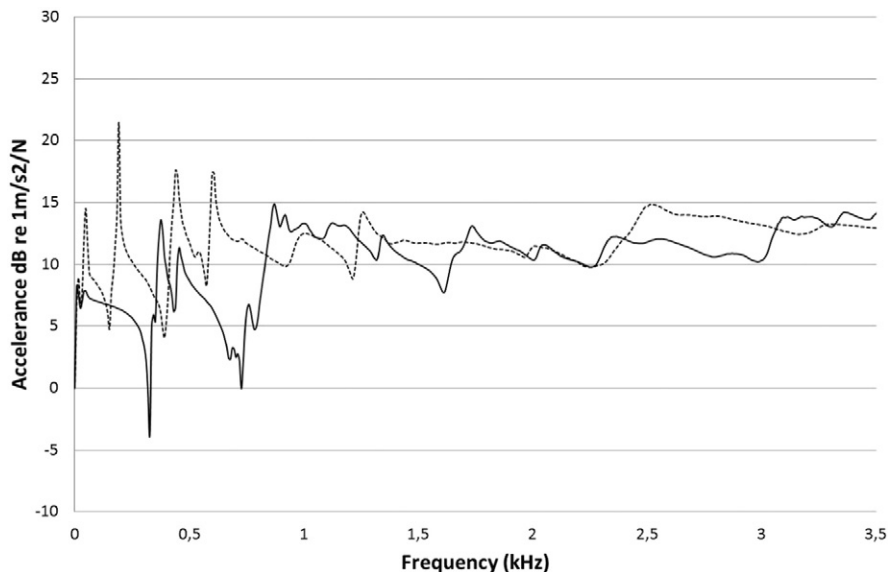


Fig. 21. Accelerance for sample G (solid line) compared to sample F (dashed line), excitation position 1.

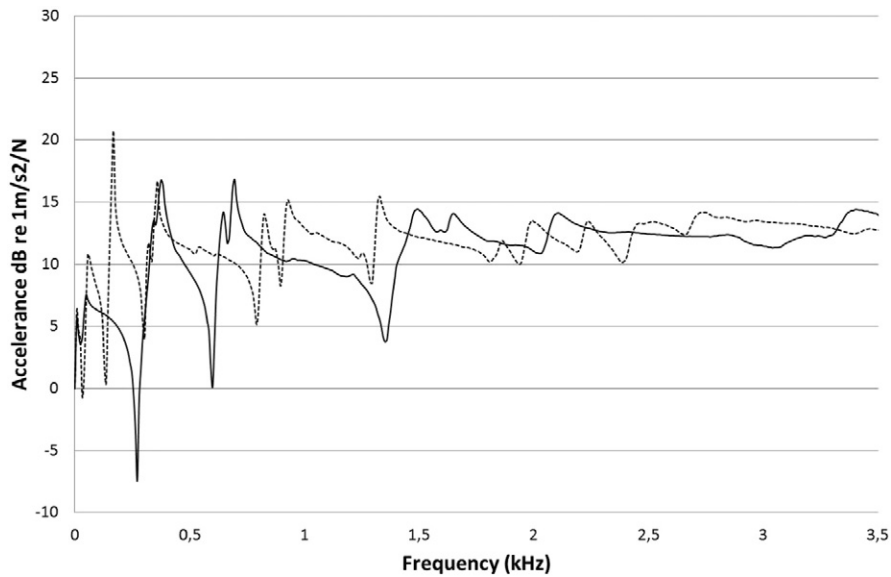


Fig. 22. Accelerance for sample G (solid line) compared to sample E (dashed line), excitation position 2.

the wedge root positioned adjacent to the excitation point yielded the greatest reduction in resonant peaks. The composite sample results (not shown here for brevity) concurred with the conclusions obtained above.

4.4. Combined composite plates with slots of power-law profile

Considering the effect of the source position in relation to the wedge root on the damping performance found in all samples, the possibility of combining two styles of slotted plates was considered in an attempt to obtain a more efficient damping performance. The composite samples that would best suit amalgamation, as they could be easily bonded together using a long cure epoxy resin, were samples E and F. These configurations when combined were expected to provide the greatest damping performance over the greatest number of excitation positions. Sample G (see Fig. 20) was excited in both positions 1 and 2 and was compared to the sample that produced the most effective damping for each respective excitation point. Both excitation positions for the combination plate were then compared. It should be noted that the photo in Fig. 20 shows the external view of the panel with the wedge being hidden inside. The full details of the structure of the panel are shown in the drawing on the right from the photo in Fig. 20.

Fig. 21 shows the results for sample G in comparison with sample F for excitation position 1. One can see that the samples respond in much the same way. A similar picture can be seen in Fig. 22 that shows the results for sample G when compared to sample E for excitation position 2. The above results imply that a combined carbon composite plate (sample G) is characterised by efficient damping of all the modes that are damped in the individual slot plates before amalgamation, irrespective of the position of the excitation point.

5. Conclusions

It has been demonstrated in this paper that slots of power-law profile located within plates materialise a specific type of quasi-two-dimensional acoustic black holes for flexural waves that occupy an intermediate position between one-dimensional black holes (outer wedges of power-law profile) and two-dimensional black holes (circular indentations of power-law profile). It has been shown that slots of power-law profile as acoustic black holes represent an effective method of damping structural vibrations, with the efficiency of damping approaching that for one-dimensional acoustic black holes [7,9].

Carbon composite plates with slots of power-law profile follow the same trends as the steel slot plates. It is remarkable that no damping layer is required for the composite plates to achieve comparable damping performance to the steel slot plates with added pieces of absorbing layers. This is due to large values of material loss factor for composites. As a result of this, changes in geometry of composite plates alone (by introducing slots of power-law profile) bring substantial increases in vibration damping.

It has been demonstrated that a combined carbon composite plate (sample G) is characterised by efficient damping of all the modes that are damped in the individual slot plates before amalgamation, irrespective of the position of the excitation point.

Further investigations of slots of power-law profile in metallic and composite plates would be required to explore their full potential as structural vibration dampers. In particular, it would be practically important to investigate the effects of combinations of slots (two or more) on damping structural vibrations, as it was done for combinations of circular indentations of power-law profile [12].

Acknowledgements

The research reported here has been partly supported by EPSRC grant EP/F009232/1.

References

- [1] Ross D, Kerwin E, Ungar E. Damping of plate flexural vibrations by means of viscoelastic laminae. In: Ruzicka JE, editor. Structural damping, 3; 1959. p. 44–87.
- [2] Heckl M, Cremer L, Ungar E. Structure Borne Sound. 2nd ed. Berlin: Springer-Verlag; 1988.
- [3] Mead DJ. Passive Vibration Control. Chichester: Wiley; 1998.
- [4] Vemula C, Norris AN, Cody GD. Attenuation of waves in plates and bars using a graded impedance interface at edges. J Sound Vib 1996;196:107–27.
- [5] Krylov VV. New type of vibration dampers utilising the effect of acoustic 'black holes'. Acta Acustica united with Acustica, 90; 2004. p. 830–7.
- [6] Krylov VV, Tilman FJBS. Acoustic black holes for flexural waves as effective vibration dampers. J Sound Vib 2004;274:605–19.
- [7] Krylov VV, Winward RETB. Experimental investigation of the acoustic black hole effect for flexural waves in tapered plates. J Sound Vib 2007;300:43–9.
- [8] Mironov MA. Propagation of a flexural wave in a plate whose thickness decreases smoothly to zero in a finite interval. Sov Phys Acoust 1988;34:318–9.
- [9] Bowyer EP, O'Boy DJ, Krylov VV, Horner JL. Effect of geometrical and material imperfections on damping flexural vibrations in plates with attached wedges of power law profile. Appl Acoust 2012;73:514–23.
- [10] Krylov VV. Propagation of Plate Bending Waves in the Vicinity of One- and Two-Dimensional Acoustic 'Black Holes'. Proceedings of the International Conference on

- Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2007), Rethymno, Crete, Greece, 13–16 June 2007 (on CD); 2007.
- [11] Georgiev VB, Cuenca J, Gautier F, Simon L, Krylov VV. Damping of structural vibrations in beams and elliptical plates using the acoustic black hole effect. *J Sound Vib* 2011;330:2497–508.
- [12] Bowyer EP, O'Boy DJ, Krylov VV, Gautier F. Experimental investigation of damping flexural vibrations in plates containing tapered indentations of power-law profile. *Appl Acoust* 2013;74:553–60.
- [13] Bayod JJ. Experimental study of vibration damping in a modified elastic wedge of power-law profile. *J Vib Acoust* 2011;133: (061003–1-7).
- [14] Clemente A, Torrent D, Sanchez-Dehesa J. Omnidirectional broadband insulating device for flexural waves in thin plates. *J Appl Phys* 2013;114 (214903–1-9).
- [15] Beck BS, Cunefare KA. Improved negative capacitance shunt damping with the use of acoustic black holes. *Proceedings of SPIE*, 9057. ; 2014 (90571Z-1-10).
- [16] Zhao L, Semperlotti F, Conlon SC. Enhanced vibration based energy harvesting using embedded acoustic black holes. *Proceedings of SPIE*, 9061. ; 2014 (90610L-1-7).
- [17] Denis V, Pelat A, Gautier F, Elie B. Modal overlap factor of a beam with an acoustic black hole termination. *J Sound Vib* 2014;333:2475–88.
- [18] Remillieux MC, Anderson BE, Le Bas PY, Ulrich TJ. Improving the air coupling of bulk piezoelectric transducers with wedges of power-law profiles: a numerical study. *Ultrasonics* 2014;54(5):1409–16.
- [19] Farhat M, Guenneau S, Enoch S. Broadband cloaking of bending waves via homogenization of multiply perforated radially symmetric and isotropic thin elastic plates. *Phys Rev B* 2012;85(2):020301.
- [20] Stenger N, Wilhelm M, Wegener M. Experiments on elastic cloaking in thin plates. *Phys Rev Lett* 2012;108(1):014301.
- [21] Brûlé S, Javelaud EH, Enoch S, Guenneau S. Experiments on seismic metamaterials: molding surface waves. *Phys Rev Lett* 2014;112(13):133901.
- [22] Colquitt DJ, Brun M, Gei M, Movchan AB, Movchan NV, Jones IS. Transformation elastodynamics and cloaking for flexural waves. *J Mech Phys Solids* 2014;72: 131–43.
- [23] Farhat M, Chen P-Y, Bağcı H, Enoch S, Guenneau S, Alù A. Platonic scattering cancellation for bending waves in a thin plate. *Sci Rep* 2014;4:4644.
- [24] Bowyer EP, Krylov VV. Experimental investigation of damping flexural vibrations in glass fibre composite plates containing one- and two-dimensional acoustic black holes. *Compos Struct* 2014;107:406–15.