

# Energy-Efficient Robust Resource Provisioning in Virtualized Wireless Networks

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**Abstract**—This paper proposes a robust resource allocation approach in virtualized wireless networks (VWNs) to address the uncertainty in channel state information (CSI) at the base station (BS) due to estimation error and mobility of users. In this set-up, the resources of an OFDMA-based wireless network are shared among different slices where the minimum reserved rate is considered as the quality-of-service (QoS) requirement of each slice. We formulate the robust resource allocation problem against the worst-case CSI uncertainty, aiming to maximize the overall energy efficiency (EE) of VWN in terms of a newly defined slice utility function. Uncertain CSI is modeled as the sum of its true estimated value and an error assumed to be bounded in a specific uncertainty region. The formulated problem suffers from two major issues: computational complexity and energy-efficiency degradation due to the considered error in the maximum extent. To deal with these issues, we consider a specific form of uncertainty region to solve the robust resource allocation problem via an iterative algorithm. The simulation results demonstrate the effectiveness of the proposed algorithms.

**Index Terms**—Energy-efficient resource provisioning, worst-case robust optimization, virtualized wireless networks.

## I. INTRODUCTION

Wireless network virtualization is a promising paradigm to improve the spectrum efficiency and enable service customization among slices belonging to different service providers via introducing abstraction and modularity in wireless networks [1]–[3]. In a single-cell VWN, different slices can share physical network resources (e.g., base station (BS)) and wireless resources (e.g., sub-carriers and power) where each slice comprises a set of users, and has its own QoS requirements. Due to the diverse QoS requirements of slices and wireless resource limitations, resource provisioning among slices is challenging and essential, which has drawn a lot of attentions recently, e.g., [4]–[10].

Generally, the resource provisioning problems considered in [4]–[10] are based on a common assumption that the accurate channel state information (CSI) of all users of different slices to BS is available. Also, these works consider the total throughput of VWN as an objective function, e.g., [7]–[10]. However, due to users' mobility, stochastic nature of wireless channels and delay in feedback channels, perfect CSI knowledge may not be available in practice. Besides, considering a utility function to investigate the energy efficiency is of high importance for wireless networks [11]. In this paper, we aim to focus on these two issues as follows. We first introduce a utility function for each slice based on its total rate and its

cost of transmit power. We show that how this per-slice utility function can increase the energy efficiency of VWN.

To immunize the performance of VWN against the uncertainty in the CSI values, we apply the worst-case optimization theory, which has been widely applied in the resource allocation in wireless networks, e.g., [12]–[15]. In this context, the uncertain parameter is modeled as an estimated value plus an error that is modeled as a bounded value in the specific region and the performance of network is maximized under the worst condition of error. It is well-known that the worst-case approach is capable to preserve the instantaneous VWN performance against the uncertain parameters, while it suffers from high computational complexity and total throughput reduction due to its conservative view of worst-case error [12], [13].

To deal with the above mentioned issues, we resort to the moderate version of robust optimization theory in which the error is assumed to have joint bounded and stochastic nature [12]. Also, instead of maximizing the throughput under the worst-case error condition, the throughput outage probability is preserved below the predefined threshold. By selecting the appropriate uncertainty region as well as variable-relaxation and transformation techniques, we convexify the resource allocation problem and propose an efficient two-level iterative solution algorithm.

Simulation results verify the energy efficiency of the proposed robust resource allocation algorithm in VWNs, based on the slice utility function. Specifically, they show how important the cost factor of utility function is in controlling the EE factor of VWN.

The rest of this paper is organized as follows. Section II introduces the system model and problem formulations, followed by Section III, where a solution to the robust problem and an iterative algorithm are proposed. In Section IV, the simulation results are presented. Finally, Section V concludes the paper.

## II. NETWORK MODEL AND PROBLEM FORMULATION

We consider the down-link transmission of OFDMA-based VWN with one central base station (BS) serving a set of slices, i.e.,  $\mathcal{G} = \{1, \dots, G\}$ , in which each slice  $g \in \mathcal{G}$  requires a minimum reserved rate  $R_g^{\text{rsv}}$ . Furthermore, each slice  $g \in \mathcal{G}$  has a set of users, i.e.,  $\mathcal{N}_g = \{1, \dots, N_g\}$ , where  $N_g$  is the total number of users in slice  $g$  and  $N = \sum_{g \in \mathcal{G}} N_g$  represents the total number of users in VWN. Considering the OFDMA

scheme, the total bandwidth  $B$  is equally divided into a set of sub-carriers, i.e.,  $\mathcal{K} = \{1, \dots, K\}$ , where each sub-carrier bandwidth  $B_c = B/K$  is assumed to be small compared to the coherent bandwidth of wireless channel. Thereby, the channel gain  $h_{n_g,k}$  of user  $n_g$  on sub-carrier  $k$  exhibits flat fading.

Let  $w_{n_g,k} \in \{0, 1\}$  be the sub-carrier allocation indicator for user  $n_g$  on sub-carrier  $k$ , where  $w_{n_g,k} = 1$  indicates that sub-carrier  $k$  is assigned to user  $n_g$ , and otherwise  $w_{n_g,k} = 0$ . Via exclusive orthogonal sub-carrier assignment imposed by OFDMA implementation issue, we have

$$C1: \sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} w_{n_g,k} \leq 1, \quad \forall k \in \mathcal{K},$$

which means that each sub-carrier is allocated to maximum one user. Due to the transmit power limitation of BS, we have

$$C2: \sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} \sum_{k \in \mathcal{K}} w_{n_g,k} P_{n_g,k} \leq P_{\max},$$

where  $P_{n_g,k}$  and  $P_{\max}$  are the allocated power to user  $n_g$  over sub-carrier  $k$  and maximum transmit power of BS, respectively. Therefore, the rate of user  $n_g \in \mathcal{N}_g$  is

$$R_{n_g}(\mathbf{P}, \mathbf{w}) = \sum_{k \in \mathcal{K}} w_{n_g,k} \log_2 \left( 1 + \frac{P_{n_g,k} h_{n_g,k}}{\sigma} \right),$$

where  $\mathbf{P} = [P_{n_g,k}]_{\forall n_g, g, k}$  and  $\mathbf{w} = [w_{n_g,k}]_{\forall n_g, g, k}$  are the allocated power vector and the sub-carrier assignment vector of all users, respectively. The constraint on the minimum rate reserved for each slice  $g \in \mathcal{G}$  is represented as

$$C3: \sum_{n_g \in \mathcal{N}_g} R_{n_g}(\mathbf{P}, \mathbf{w}) \geq R_g^{\text{rsv}}, \quad \forall g \in \mathcal{G}.$$

For energy-efficient design of VWN, we consider the following slice utility function,  $\forall g \in \mathcal{G}$ ,

$$U_g(\mathbf{P}, \mathbf{w}) = \sum_{n_g \in \mathcal{N}_g} R_{n_g}(\mathbf{P}, \mathbf{w}) - C_g^{\text{E}} \sum_{n_g \in \mathcal{N}_g} \sum_{k \in \mathcal{K}} w_{n_g,k} P_{n_g,k},$$

where the energy-cost coefficient of slice  $g \in \mathcal{G}$ ,  $C_g^{\text{E}}$  provides the trade-off between its achieved throughput and its power consumption. Aiming to maximize the sum utility of all slices, while satisfying the minimum required slice rates, the nominal VWN optimization problem is

$$\max_{\mathbf{P}, \mathbf{w}} \sum_{g \in \mathcal{G}} U_g(\mathbf{P}, \mathbf{w}), \quad (1)$$

subject to : C1 – C3.

In (1), perfect CSI knowledge is assumed. However, in practice, due to delay in feedback channel, user mobility, and error in the estimation, such CSI knowledge can be imperfect. To deal with this issue, we consider the uncertainty in CSI at the BS and introduce a robust counterpart of the above resource allocation problem.

The imperfect CSI is modeled as the sum of its estimated value and an additive error i.e.,

$$\mathbf{h}_{n_g} = \bar{\mathbf{h}}_{n_g} + \hat{\mathbf{h}}_{n_g}, \quad \forall n_g \in \mathcal{N}_g, \quad g \in \mathcal{G},$$

where  $\mathbf{h}_{n_g} = [h_{n_g,k}]_{\forall k}$  is the  $1 \times K$  uncertain CSI vector, and,  $\bar{\mathbf{h}}_{n_g} = [\bar{h}_{n_g,k}]_{\forall k}$  and  $\hat{\mathbf{h}}_{n_g} = [\hat{h}_{n_g,k}]_{\forall k}$  are, respectively, the  $1 \times K$  estimated CSI and error vectors of user  $n_g$ . In the context of worst-case robust optimization, the errors on the estimated values are trapped in the bounded region, called uncertainty region, defined as

$$\mathcal{E}_{n_g} = \{\mathbf{h}_{n_g} \mid \|\mathbf{h}_{n_g} - \bar{\mathbf{h}}_{n_g}\| \leq \epsilon_{n_g}\}, \quad \forall n_g \in \mathcal{N}_g, \quad \forall g \in \mathcal{G},$$

where  $\epsilon_{n_g} \geq 0$  is the uncertainty bound, assumed to be small, and  $\|\mathbf{x}\|$  denotes the norm function of vector  $\mathbf{x}$  [16].

The effect of uncertainty on  $\mathbf{h}_{n_g}$  can be represented by a new vector of variables in the throughput of each user. Let  $\hat{R}_{n_g}$  denotes the throughput of user  $n_g$  in the robust resource allocation, which depends on  $\mathbf{h} = [\mathbf{h}_{n_g}]_{\forall g, n_g}$ . When the uncertainty region shrinks to zero (i.e.,  $\epsilon_{n_g} = 0$ ), the total throughput of the nominal and robust optimization problems are identical i.e.,

$$R_{n_g}(\mathbf{P}, \mathbf{w}) = \hat{R}_{n_g}(\mathbf{P}, \mathbf{w}, \mathbf{h})|_{\epsilon_{n_g}=0}, \quad \forall n_g \in \mathcal{N}_g.$$

The objective of the worst-case approach is to find the optimal transmit power and sub-carrier allocation for each user that optimize their total throughput under the worst condition of error in the uncertainty region. In this approach, the robust VWN resource allocation problem based on (1) becomes [12]

$$\max_{\mathbf{P}, \mathbf{w}} \sum_{g \in \mathcal{G}} \hat{U}_g(\mathbf{P}, \mathbf{w}, \mathbf{h}), \quad (2)$$

subject to : C1 – C3,

where  $\hat{U}_g(\mathbf{P}, \mathbf{w}, \mathbf{h})$  is the robust counterpart of the utility  $U_g$ , mathematically expressed as

$$\hat{U}_g(\mathbf{P}, \mathbf{w}, \mathbf{h}) = \sum_{n_g \in \mathcal{N}_g} \min_{\mathbf{h}_{n_g} \in \mathcal{E}_{n_g}} \hat{R}_{n_g}(\mathbf{P}, \mathbf{w}, \mathbf{h}) - C_g^{\text{E}} \sum_{k \in \mathcal{K}} w_{n_g,k} P_{n_g,k}$$

In general, solving the robust counterpart (2) involves high computational complexity, because, in addition to the inherent computational complexity from (1), it has a new set of optimization variables with uncertain parameters, i.e.,  $\mathbf{h}_{n_g}$ . To reduce the computational complexity of (2), we treat each  $h_{n_g,k}$  as a bounded random variable. Then, we demonstrate how the inner minimization over  $\mathbf{h}_{n_g}$  is solved. Interestingly, we will also show that the proposed reformulation provides a trade-off between performance and robustness.

### III. ROBUST EE RESOURCE PROVISIONING ALGORITHM

The direct way to solve (2) is to obtain the inner minimization analytically, and then, solve the outer maximization, either numerically or analytically [16]. In the following, we will show how the inner and outer optimization problems can be solved.

#### A. Inner Optimization Problem

The inner optimization problem of (2) is

$$\min_{\mathbf{h}_{n_g} \in \mathcal{E}_{n_g}} \hat{R}_{n_g}(\mathbf{P}, \mathbf{w}, \mathbf{h}), \quad \forall n_g \in \mathcal{N}_g, \quad g \in \mathcal{G}. \quad (3)$$

For general definition of norm function of  $\mathcal{E}_{n_g}$ , a closed-form expression of  $\mathbf{h}_{n_g}$  cannot be obtained for a given values of  $\mathbf{P}$  and  $\mathbf{w}$  for (3). To simplify (3), following the same argument as in [16]–[18], we assume that  $h_{n_g,k}$  for all  $n_g \in \mathcal{N}_g$  and  $k \in \mathcal{K}$  are i.i.d. random variables with the probability distribution function (pdf) of  $f(\hat{h}_{n_g,k})$ . In this case, the uncertainty region is transformed into  $\hat{h}_{n_g,k} \in [-\varepsilon_{n_g,k}, \varepsilon_{n_g,k}]$ , where  $\varepsilon_{n_g,k}$  is the bound of uncertainty region for user  $n_g$  on sub-carrier  $k$ . Now, by utilizing the pdf of  $\hat{h}_{n_g,k}$ , the inner optimization problem of (3) is transformed into [17]

$$\min_{\mathbf{t}} \sum_{k \in \mathcal{K}} w_{n_g,k} t_{n_g,k}, \quad (4)$$

subject to:

$$\text{C4: } \Pr \left\{ \log_2 \left( 1 + \frac{P_{n_g,k} h_{n_g,k}}{\sigma} \right) < t_{n_g,k} \right\} > \eta_{n_g,k},$$

$$\text{C5: } \hat{h}_{n_g,k} \in [-\varepsilon_{n_g,k}, \varepsilon_{n_g,k}], \quad \forall k \in \mathcal{K}, \quad \forall n_g \in \mathcal{N}_g,$$

where  $\mathbf{t} = [t_{n_g,k}]_{\forall n_g, k}$  and  $t_{n_g,k} \geq 0$  is an auxiliary variable for this transformation. Also,  $0 < \eta_{n_g,k} < 1$  is the probability factor against the uncertain parameters. C4 can be simplified to  $t_{n_g,k} > \log_2 \left( 1 + \frac{P_{n_g,k} F^{-1}(\eta_{n_g,k})}{\sigma} \right)$  for all  $k \in \mathcal{K}$  and  $n_g \in \mathcal{N}_g$ . If  $f(\hat{h}_{n_g,k})$  has a uniform distribution over the interval  $[-\varepsilon_{n_g,k}, \varepsilon_{n_g,k}]$ , we have  $F^{-1}(\eta_{n_g,k}) = 2\varepsilon_{n_g,k} \eta_{n_g,k} + \bar{h}_{n_g,k} - \varepsilon_{n_g,k}$ . Therefore, the solution of (4) for all  $n_g$  and  $k$  is

$$\tilde{h}_{n_g,k} = 2\varepsilon_{n_g,k} \eta_{n_g,k} + \bar{h}_{n_g,k} - \varepsilon_{n_g,k}, \quad (5)$$

for all  $k \in \mathcal{K}$  and  $n_g \in \mathcal{N}_g$ . From (5), for  $0.5 \leq \eta_{n_g,k} < 1$ ,  $\bar{h}_{n_g,k} \leq \tilde{h}_{n_g,k}$ , and is in-line with the concept of worst-case robust optimization, in which the error is at its own maximum extent. For this, we will focus on this case in the rest of this paper.

The throughput of each user with uncertainty can be rewritten as  $\hat{R}_{n_g}(\mathbf{P}, \mathbf{w}) = \sum_{k \in \mathcal{K}} w_{n_g,k} \log_2 \left( 1 + \frac{P_{n_g,k} \hat{h}_{n_g,k}}{\sigma} \right)$ , for all  $n_g \in \mathcal{N}_g$  and  $g \in \mathcal{G}$ . Therefore, (2) is simplified to

$$\max_{\mathbf{P}, \mathbf{w}} \sum_{g \in \mathcal{G}} \hat{U}_g(\mathbf{P}, \mathbf{w}), \quad (6)$$

subject to : C1 – C2 and  $\widehat{\text{C3}}$ ,

where

$$\hat{U}_g(\mathbf{P}, \mathbf{w}) = \sum_{n_g \in \mathcal{N}_g} \hat{R}_{n_g}(\mathbf{P}, \mathbf{w}) - C_g^E \sum_{n_g \in \mathcal{N}_g} \sum_{k \in \mathcal{K}} w_{n_g,k} P_{n_g,k}$$

and

$$\widehat{\text{C3}}: \sum_{n_g \in \mathcal{N}_g} \hat{R}_{n_g}(\mathbf{P}, \mathbf{w}) \geq R_g^{\text{rsv}}, \quad \forall g \in \mathcal{G}.$$

By the new definition of  $\hat{U}_g(\mathbf{P}, \mathbf{w})$  and  $\tilde{h}_{n_g,k}$ , (2) is transformed into the optimization problem with two variable vectors  $\mathbf{P}$  and  $\mathbf{w}$ , similar to the nominal optimization problem in (1). Therefore, computational complexity of robust optimization problem (6) is downgraded to that of (1).

## B. Proposed Algorithm to Solve (6)

Due to the existence of both continuous and discrete variables  $\hat{R}_{n_g}(\mathbf{P}, \mathbf{w})$ , (6) is non-convex. To transform (6) to a convex optimization problem, following by [19], we apply techniques of variable transformation and relaxations. First, we relax the  $w_{n_g,k}$  as a continuous variable in interval  $[0, 1]$ . In the new definition,  $w_{n_g,k}$  indicates the portion of time that sub-carrier  $k$  is assigned to user  $n_g$  for a specific transmission frame. Consequently, C1 is changed to

$$\widetilde{\text{C1}}: w_{n_g,k} \in [0, 1] \text{ and } \sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} w_{n_g,k} \leq 1, \quad \forall k \in \mathcal{K}.$$

Furthermore, we consider a new variable  $x_{n_g,k} = w_{n_g,k} P_{n_g,k}$ , which transforms  $\hat{R}_{n_g}(\mathbf{P}, \mathbf{w})$  to

$$\tilde{R}_{n_g}(\mathbf{x}, \mathbf{w}) = \sum_{k \in \mathcal{K}} w_{n_g,k} \log_2 \left( 1 + \frac{x_{n_g,k} \tilde{h}_{n_g,k}}{\sigma w_{n_g,k}} \right), \quad \forall n_g \in \mathcal{N}_g.$$

Therefore, the utility function is simplified to

$$\tilde{U}_g(\mathbf{x}, \mathbf{w}) = \sum_{n_g \in \mathcal{N}_g} \tilde{R}_{n_g}(\mathbf{x}, \mathbf{w}) - C_g^E \sum_{n_g \in \mathcal{N}_g} \sum_{k \in \mathcal{K}} x_{n_g,k}.$$

In this context, C2 and  $\widehat{\text{C3}}$  are transformed into

$$\widetilde{\text{C2}}: \sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} \sum_{k \in \mathcal{K}} x_{n_g,k} \leq P_{\max} \text{ and}$$

$$\widetilde{\text{C3}}: \sum_{n_g \in \mathcal{N}_g} \tilde{R}_{n_g}(\mathbf{x}, \mathbf{w}) \geq R_g^{\text{rsv}}, \quad \forall g \in \mathcal{G},$$

respectively. Since  $\tilde{R}_{n_g}(\mathbf{x}, \mathbf{w})$  belongs to the class of convex functions represented as  $f(x, y) = x \log_2 \left( 1 + \frac{y}{x} \right) \quad \forall x, y \geq 0$  [20]. Therefore, the convexified robust counterpart of (1) is

$$\max_{\mathbf{x}, \mathbf{w}} \sum_{g \in \mathcal{G}} \tilde{U}_g(\mathbf{x}, \mathbf{w}), \quad (7)$$

subject to :  $\widetilde{\text{C1}}, \widetilde{\text{C2}}$  and  $\widetilde{\text{C3}}$ .

Now, we can solve (7) by solving the dual optimization problem and applying KKT conditions. Let  $\rho_k$ ,  $\lambda$  and  $\phi_g$  represent the Lagrange multipliers for constraints  $\widetilde{\text{C1}}$ ,  $\widetilde{\text{C2}}$  and  $\widetilde{\text{C3}}$ , respectively. Therefore, the Lagrange function for (7) is

$$\begin{aligned} \mathcal{L}(\mathbf{w}, \mathbf{x}, \lambda, \phi, \rho) = & \\ & - \sum_{g \in \mathcal{G}} \tilde{U}_g + \lambda \left( \sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} \sum_{k \in \mathcal{K}} x_{n_g,k} - P_{\max} \right) + \\ & \sum_{g \in \mathcal{G}} \phi_g \left( R_g^{\text{rsv}} - \sum_{n_g \in \mathcal{N}_g} \tilde{R}_{n_g} \right) + \sum_{k \in \mathcal{K}} \rho_k \left( \sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} w_{n_g,k} - 1 \right). \end{aligned} \quad (8)$$

Applying KKT conditions to (8), we obtain the optimal power solution of (7)  $\forall k \in \mathcal{K}$ ,  $n_g \in \mathcal{N}_g$  and  $g \in \mathcal{G}$  as

$$P_{n_g,k} = \left[ \frac{1 + \phi_g}{\ln(2)(\lambda + C_g^E)} - \frac{\sigma}{\tilde{h}_{n_g,k}} \right]_0^{P_{\max}}. \quad (9)$$

In order to obtain the solution for sub-carrier allocation, we obtain the following necessary condition for  $w_{n_g,k}$  for all  $k \in \mathcal{K}$  and  $n_g \in \mathcal{N}_g$

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**Algorithm 1** :Robust Slice Provisioning

**Initialization:** Set  $\mathbf{w}^*(l = 0) = 1$ ,  $\mathbf{P}_{n_g}^*(l = 0) = P_{\max}/K$ ,  $\forall n_g \in \mathcal{N}_g, g \in \mathcal{G}$ ,  $l = 0, l_{\max}, i_{\max}$  and  $0 < \zeta_m \ll 1$  for  $m = \{1, 2, 3\}$ .

**OL: Repeat**  $l = l + 1$ :

$$\lambda(l) = [\lambda(l-1) + \delta_\lambda \frac{\partial \mathcal{L}}{\partial \lambda}]^+,$$

$$\phi_g(l) = [\phi_g(l-1) + \delta_{\phi_g} \frac{\partial \mathcal{L}}{\partial \phi_g}]^+, \quad \forall g \in \mathcal{G}.$$

**IL: Repeat**  $i = i + 1$ :

Update  $\mathbf{P}^*(i)$  according to (9).

Update  $\mathbf{w}^*(i)$  according to (10).

**Until** ( $\|\mathbf{P}(i) - \mathbf{P}(i-1)\| \leq \zeta_1$ ) or  $i > i_{\max}$ .

**Until** ( $\|\phi_g(l) - \phi_g(l-1)\| \leq \zeta_2$  and  $\|\lambda(l) - \lambda(l-1)\| \leq \zeta_3$ ) or  $l > l_{\max}$ .

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$$w_{n_g, k}^* \begin{cases} = 0, & \frac{\partial \mathcal{L}(\mathbf{w}, \mathbf{x}, \lambda, \phi, \rho)}{\partial w_{n_g, k}^*} < 0, \\ \in [0, 1], & \frac{\partial \mathcal{L}(\mathbf{w}, \mathbf{x}, \lambda, \phi, \rho)}{\partial w_{n_g, k}^*} = 0, \\ = 1, & \frac{\partial \mathcal{L}(\mathbf{w}, \mathbf{x}, \lambda, \phi, \rho)}{\partial w_{n_g, k}^*} > 0, \end{cases}$$

where [21]

$$\frac{\partial \mathcal{L}(\mathbf{w}, \mathbf{x}, \lambda, \phi, \rho)}{\partial w_{n_g, k}^*} = (1 + \phi_g) \left( \log_2(1 + \gamma_{n_g, k}) - \frac{\gamma_{n_g, k}}{(1 + \gamma_{n_g, k}) \ln(2)} \right).$$

For holding the exclusive sub-carrier allocation of OFDMA, the sub-carrier  $k$  is allocated to user which satisfy the followings

$$w_{n'_g, k}^* = \begin{cases} 1, & n'_g = \max_{\forall n_g, \forall g} \frac{\partial \mathcal{L}(\mathbf{w}, \mathbf{x}, \lambda, \phi, \rho)}{\partial w_{n_g, k}^*}, \\ 0, & n_g \neq n'_g. \end{cases} \quad (10)$$

The iterative algorithm to allocate the optimal power and sub-carrier with uncertain CSI is presented in Algorithm 1. It starts with initialization of variables followed by an outer loop where Lagrange variables  $\lambda$  and  $\phi_g$  are updated for all  $g \in \mathcal{G}$  via gradient method, where  $0 < \delta_x \ll 1$  is the step size for Lagrange variable  $x$ . In the inner loop, the power and sub-carriers are computed from the updated values of Lagrange variables. The iterative processes are stopped when power and sub-carrier converge to the constant values.

#### IV. SIMULATION RESULTS

In this section, we investigate the proposed solution of the resource provisioning problem (2) via simulation results. For simulation settings, we consider two slices  $g_1$  and  $g_2 \in \mathcal{G}$  where BS has  $K = 64$  sub-carriers and  $\sigma = 1$ . The CSI is derived from Rayleigh fading distribution, modeled as  $h_{n_g, k} = \mathcal{X}d_{n_g}^{-\beta}$ , where  $\beta = 4$  is the path loss exponent,  $\mathcal{X}$  is exponential random variable with mean one, and  $d_{n_g}$  is the distance of user  $n_g$  from BS. For all the simulations, we set the minimum reserved rate  $R_g^{\text{rsv}} = 1.0$  bps/Hz for each slice,  $\eta_{n_g, k} = 0.9$ ,  $C_g^{\text{E}} = 3.0$ ,  $P_{\max} = 15$

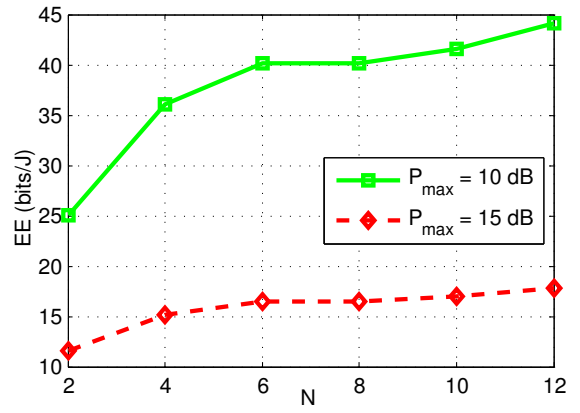


Fig. 1: EE factor versus number of users  $N$

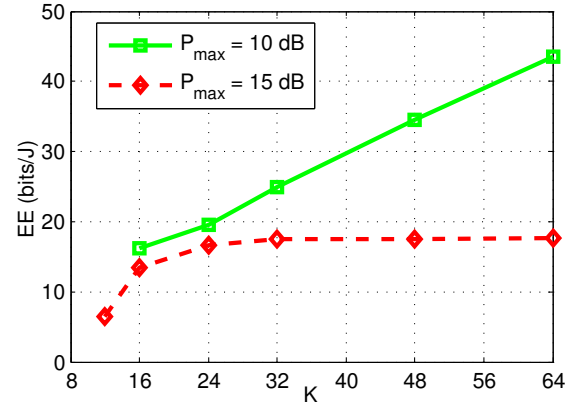


Fig. 2: EE versus number of subcarriers  $K$

dB, and  $\varepsilon_{n_g, k} = \varepsilon = 0.3, \forall n_g \in \mathcal{N}_g$  and  $\forall k \in \mathcal{K}$  unless otherwise stated. For the simulations in Figs. 1 and 2, all the users of slices are randomly located in the range of distance  $d_{n_g} \in \{0.2, 0.5\}$  Km. All the plotted results are obtained from the average of over 100 CSI realizations. To demonstrate the results, we define the energy efficiency (EE) factor as  $EE = \sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} R_{n_g}(\mathbf{P}, \mathbf{w}) / (\sum_{g \in \mathcal{G}} \sum_{n_g \in \mathcal{N}_g} \sum_{k \in \mathcal{K}} P_{n_g, k} + P_c)$  where  $P_c = -10$  dB is the constant signal processing power required at the BS [22].

Fig. 1 illustrates the total EE factor versus number of users  $N$  for different values of  $P_{\max}$ . The EE increases with increasing  $N$  for all considered  $P_{\max}$  due to the multi-user diversity gain, which increases the total rate leading to higher energy efficiency. From Fig. 2, increasing  $K$  also increases the EE factor. This is because that VWN has more options to assign sub-carriers with better channel gains to the users with increasing  $K$ . Thus, the total rate of VWN and hence EE would be increased. Figs. 1 and 2 indicate that the total EE reduces with increasing  $P_{\max}$ . This happens because higher power cannot help to increase throughput (due to the tradeoff between throughput and power cost in the defined utility) as much as required to compensate the power increase in the denominator of EE factor.

To analyze the behavior of EE with respect to  $\varepsilon_{n_g, k}$ , we con-

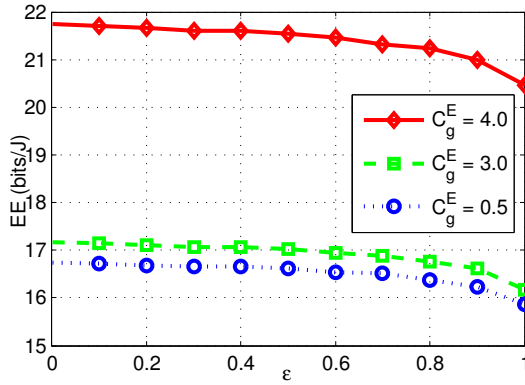


Fig. 3: EE factor versus  $\varepsilon$  and  $C_g^E$  for cell-center users scenario

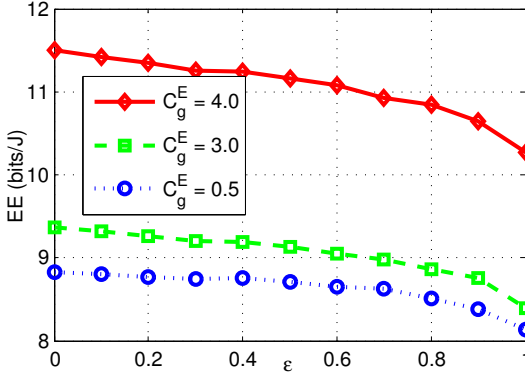


Fig. 4: EE factor versus  $\varepsilon$  and  $C_g^E$  for cell-edge users scenario

consider two scenarios based on the locations of users, 1) **Users at the cell-edge** or low-SNR scenario where  $d_{n_g} \in [0.4, 0.6]$  Km for all  $n_g$ , and 2) **Users at the cell-center** or high-SNR scenario where  $d_{n_g} \in [0.2, 0.3]$  Km for all  $n_g$ . It can be observed that increasing  $\varepsilon$  decreases the EE factor due to the conservative feature of the worst-case approach, where error is considered to the maximum extent. From both Figs. 3 and 4, increasing  $C_g^E$  increases the EE factor since for higher value of price, the VWN consumes less power. Consequently, EE is increased. Moreover, EE factor in Fig. 3 is higher than that in Fig. 4. This is because when users are located at the boarder of the cell, the rate of VWN decreases because of limited transmit power and large scale fading. Consequently, EE factor decreases.

## V. CONCLUSION

In this paper, we propose the robust resource provisioning policy for the OFDMA-based VWNs, aiming to maximize the total energy efficiency of network while satisfying the minimum rate requirements of all the slices. The non-convex problem is transformed into the convex one by applying the appropriate selection of uncertainty region, variable transformations and relaxations. Based on the solution of the convexified problem, an iterative algorithm is proposed. Via simulation results, the effects of system parameters including error in CSI on total EE factor of VWN is investigated.

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