13th International Congress on Mathematical Education
Hamburg, 24-31 July 2016

EMOTIONS IN UNDERGRADUATE MATHEMATICAL MODELLING GROUP WORK

Paul Hernandez-Martinez and Helen Harth

Loughborough University

Taking a socio-cultural perspective of affect in education, we use observations of two groups of undergraduate engineering students to explore the role of emotions on the students' mathematical thinking and learning while working collaboratively on a mathematical modelling coursework assignment outside the classroom. Our analysis revealed complex interrelations between patterns of emotions and aspects of mathematical learning. We conclude that 'negative' feelings might sometimes lead to positive consequences on the activity of individuals and conversely, that 'positive' feelings do not necessarily lead to positive outcomes. Hence, pedagogical practices should aim to foster a range of emotions that can open possibilities for students' success.

Research into the role of emotions (and affect in general) in mathematical thinking and learning has gained considerable interest in the past few decades. However, most of this literature has studied emotions from a psychological perspective, often seeing emotions as separate from an individual's cognition. More recently, there has been a shift in perspective, with an associated effort to explain affective issues from a socio-cultural view (Roth, 2007). In this paper we take the latter perspective.

Our aim is to study the role of emotions in the mathematical (social) activity of undergraduate engineering students working in small groups. Here, we analyse two separate groups of four students each, working on a group mathematical modelling coursework assignment (Figure 1) as part of a second year 12-week compulsory engineering mathematics course at an English research-intensive university. The course was designed according to ideas on mathematics collaborative learning, mathematical modelling and the development of employability skills (e.g. effective communication, presentation skills, et cetera). Our analysis focused on the students' emotions expressed within these collaborative situations, and the role that these affective factors had in the students' mathematical activity. Hence, our research questions were: (1) How and why emotions emerge within students' group work while working on a mathematical modelling assignment? and (2) What is the role of emotions on these students' mathematical thinking and learning?

THE WIND CHILL FACTOR
If you ever have waited in a bus stop or taken a walk during a cold winter day then you know that it feels colder when the wind blows. This cooling sensation is caused by the combined effect of temperature and wind, and is known as the wind chill. Your group's task is to come up with a mathematical formula (a model) to calculate the wind chill. You have to use the modelling cycle seen in lectures to explain your understanding of the problem and the assumptions that you made in order to arrive at your model. Once you have your model, you have to argue how you would test it and how you would refine it.

Figure 1: The mathematical modelling assignment

THEORETICAL PERSPECTIVE

According to Radford (2015) affective factors in general, and emotions in particular, are socio-cultural in nature; these might be expressed by a particular person but always relate to a socio-cultural and historical world in which the individual participates, shaping and organizing in turn the individual's motives and emotions. It is through emotions that we make meaning of ourselves, the world we inhabit and our place in it (Roth 2007; Solomon 1978). This view involves concepts of the self and entails moral and ethical dimensions (Radford 2015). This implies that in education (as in life in general), affective factors are inseparable from and constitutive of the learning that occurs, rather than just mere influences on the learning process (Gresalfi 2009). Furthermore, Vygotsky (1999, p.244) explains that emotions develop and "appear in new relations with other elements of mental life". For Leont'ev (cited in Radford 2015, p.35) emotions become related to the motives of the individual, for example, "in the form of interest, desire or passion". These perspectives emphasise the close relationship that exists between emotions and cognition, and the role that emotions have in potentially shifting our thoughts.

METHODOLOGY

The study involved video and audio observational data of students' group mathematical activities outside the classroom, observed over three weeks and fourteen project meetings (the first group met seven times over seven hours approximately while the second group met three times over four hours approximately). Episodes that were deemed to contain emotional reactions from the group members were transcribed and coded in an open manner, according to what the researchers interpreted as the effect that these emotions had in the activity of the groups (e.g. affecting engagement, helping produce a mathematical solution, relating to students' identities, et cetera). The groups were included in this comparative analysis because they produced similar mathematical solutions, yet they faced different challenges during their problem-solving processes.

FINDINGS

Our findings revealed the reciprocal relationship of students' emotional engagement with the task and their learning of mathematics. Successful and unsuccessful collaborative discussions over the time of the project within each group revealed different effects that emotions had on the students' social and mathematical thinking and learning.

For instance, in Group#1, feelings of frustration appeared during the first few meetings when the objectives set by the group were clearly not met. This frustration had important consequences for the group's dynamics, for example, when one of student's ideas were constantly dismissed. This student eventually ended up taking a back seat for the remaining of the task. This frustration turned sometimes into anger at comments that were made by group members that were interpreted as not helping to advance a solution to the problem. For example, during meeting 5 students expressed:

Student1: We spent four weeks but have a model that doesn't work. How is the presentation going (to Student3 and Student4)? Here is something that doesn't work.

Student2: It's difficult so no shame in failing.

Student1: (*In anger*) Yeah, but he (the lecturer) gave us the skills.

This hopelessness at not being able to use mathematics effectively to solve the task (in fact, the students felt they were not doing any mathematics at all) influenced their engagement and participation. For example, the lack of effective communication among the group members resulted in the erroneous use of resources, e.g. using software without a clear rationale for what the group was doing and therefore, not being able to link the use of these resources with the problem. Further, inefficient division of labour stemmed from strategic approaches to learning (e.g. "it's only 5% of the marks"). During the final meetings, it became more noticeable how their mathematical identities played a significant role in the development of their solution to the problem, and how these identities interacted with their emotions. Having been relatively successful mathematics students at school, the two students that were leading the group (Student1 and Student2) seemed to have unrealistic expectations of what they could achieve mathematically. Wanting to be the best group in the class but feeling frustrated at their failure to solve the problem posed an important emotional contradiction in them. Eventually, taking a strategic approach (looking for potential solutions in the lecture notes rather than trying to build a model themselves) resulted in a solution to the task (Student3: "We found it somewhere, I guess is in the word document"), and although this solution was not mathematically sound (it is uncertain if they realised this or not at the time), at that point the group's dynamics took a turn. They now concentrated on making a presentation that could 'stand out from the rest' and they became almost overconfident. Clearly, having found a solution to the problem, albeit a wrong one, provoked in them emotions that turned into a renewed engagement in the task.

For Group#2, the emotions expressed were of a very different nature. Throughout the meetings of this group, there were always laughter and humour, something that helped the group largely to cope effectively with challenging situations. When the group was unable to progress the solution to the problem during meetings, the students seemed happy to work individually and come back with fresh ideas. This 'relaxed' atmosphere meant that all students contributed and engaged in the task. There appeared to be feelings of real happiness when these students perceived they had achieved understanding of the problem or had overcome particular difficulties. An important factor that also contributed to these dynamics was the presence of a clear leader that encouraged all team members to participate, help and implement their ideas. In general, the process of solving the task was more efficient in this group: there were less meetings that took less time and the division of labour was much more equitable than in Group#1. In this group, their mathematical identities played an important role too. Students saw themselves as not very good at mathematics (Student6: "I'm terrible at maths") but they also wanted to 'stand out from the rest' not as the best in mathematics but as friendly and entertaining (e.g. wanting to "crack a couple of jokes" during their presentation).

In this group as well there might have been a contradiction between their apparently 'relaxed' attitude towards the task and the 'seriousness' of it, which might have been reflected on their final solution to the problem: this was not (mathematically) very different from that of the first group.

DISCUSSION AND CONCLUSIONS

Our analysis revealed how emotions interrelate with "other elements of mental life" (Vygostky 1999), shifting in fundamental ways students' motives for engagement and thoughts. Laughter, enjoyment, hopelessness, frustration, bewilderment, interest and other emotional processes expressed by these engineering students over the time of their collaborative modelling project evolved to reveal complex interrelations between patterns of emotions and aspects of mathematical learning.

For example, for Group#1, sometimes frustration closed the doors to effective communication and understanding but at other times it also pushed students to persevere and seek for more mathematically sophisticated strategies, particularly when these emotions interconnected with students' self-concepts with a history of previous achievements in school mathematics. Emotions also linked to the moral and ethical dimensions of students (Radford 2015) by, for instance, provoking anger at comments that were deemed unhelpful or removed from the 'rules of the game', i.e. the lecturer had provided them with the necessary skills to find a suitable solution.

Happiness and laughter in Group#2 combined also with students' identities (i.e. not the best in mathematics) by creating a more relaxed atmosphere in which engagement and participation was encouraged. Students felt that they could explore ideas easily without the fear of being judged too harshly (although they were keen to make jokes at each other). However, there is also the possibility that the environment created by these emotions inhibited at times the mathematical learning: students might have felt that because they were having a good time and because they did not expect too much of themselves mathematically speaking, their (erroneous) solution was adequate enough.

We feel that by contrasting these two cases we have a better understanding of the role that emotions had in these students' mathematical thinking and learning. Our analysis showed that we should step away from thinking that emotions that are typically considered as 'negative' (e.g. frustration, anxiety, anger) will always have negative consequences on the activity of individuals. Similarly, emotions that are typically considered as 'positive' (e.g. happiness, laughter) might not always have positive outcomes. So, for these engineering students engaged in the modelling task, their identities and histories, and the dynamics of their group created in particular contexts led to different patterns of activity and affective responses. And although their solutions to the task were similar their learning, as revealed by their interactions, was very different.

The implications of this study for practice are, in our view, that mathematical pedagogies should consider activities that foster a range of emotions; for example, the design of group assignments should consider that a socially relaxed, enjoyable atmosphere should also incite challenge and that this might enable students' potential for success.

REFERENCES

Gresalfi, M. S. (2009). Taking up opportunities to learn: Constructing dispositions in mathematics classrooms. *Journal of the Learning Sciences*, 18(3), 327-369.

Radford, L. (2015). Of Love, Frustration, and Mathematics: A Cultural-Historical Approach to Emotions in Mathematics Teaching and Learning. In B. Pepin & B. Roesken-Winter (Eds.). *From beliefs to dynamic affect systems in mathematics education* (pp. 25-49): Springer International Publishing.

Roth, M. W. (2007). Motive, emotion and identity at work: A contribution to third-generation cultural historical activity theory. *Mind, Culture and Activity*, 14(1-2), 40-63.

Solomon, R. (1978). Emotions and anthropology: The logic of emotional world views. *Inquiry*, 2, 189-199.

Vygotsky, L. S. (1999). *Collected works* (vol. 6). New York: Plenum.