Adaptive Pilot-Duration and Resource Allocation in Virtualized Wireless Networks with Massive MIMO

Rajesh Dawadi*, Saeedeh Parsaeefard†, Mahsa Derakhshani‡, Tho Le-Ngoc*

*Department of Electrical & Computer Engineering, McGill University, Montreal, QC, Canada

† Iran Telecommunication Research Center, Tehran, Iran

‡Department of Electrical & Electronic Engineering, Imperial College, London, UK

Email: rajesh.dawadi@mail.mcgill.ca; s.parsaeifard@itrc.ac.ir; m.derakhshani@imperial.ac.uk; tho.le-ngoc@mcgill.ca

***Abstract*—This paper investigates the resource allocation problem for a virtualized wireless network (VWN) in which each base station (BS) is equipped with a large number of antennas and due to the pilot contamination error, the perfect estimation of channel state information (CSI) is not available. In this case, the duration of pilot sequence transmission plays a critical role on the achieved VWN throughput. Therefore, we consider this parameter as a new optimization variable and propose a novel utility function for the resource allocation problem. The proposed optimization problem is non-convex with high computational complexity. To address this issue, by applying relaxation and variable transformation techniques, we propose a two-step iterative algorithm in which the allocation of power, sub-carrier and number of antennas is first established and then used to optimize the pilot duration. Simulation results reveal that proper pilot duration design improves the VWN performance.**

***Index Terms*—Massive multiple-input multiple-output, pilot contamination error, virtualized wireless networks.**

I. Introduction

Wireless networks virtualization is a promising approach to improve network spectral efficiency wherein the wireless resources, e.g., power, sub-carriers, and antennas are shared between different wireless service providers (also called slices) [1], [2]. In a virtualized wireless network (VWN), each slice consists of a set of users and requires a minimum reserved rate. Therefore, similar to other conventional wireless network scenarios, efficient resource allocation algorithm is of high importance in order to achieve the maximum network performance while preserving the required rate of each slice.

Resource management in VWNs has received growing interest lately [3]–[8]. For instance, in [3], a novel admission control policy to provide the quality-of-service (QoS) requirement of each slice is introduced. In [4], the concept of wireless virtualization is extended to the LTE network by formulating a resource sharing algorithm. In [5], a resource allocation algorithm is proposed by considering both time and space division multiplexing so that effective isolation is ensured between slices, while the resource utilization is optimized. In [6], a game theoretic approach is applied to consider the possible interactions among the network operator, slices and the users such that the network tries to manage the spectrum among slices while the slices try to provide the required QoS for their users.

Application of massive multiple-input multiple-output (MIMO) in the base-station (BS) of wireless networks has been recently proposed to make extensive use of the degrees of freedom to increase the spectral and energy efficiencies. Channel state information (CSI) estimation in a massive MIMO environment poses a serious challenge and significantly complicates the resource allocation problem. Ideally, the pilot sequences transmitted by the users to assist the BS in estimating the CSI of the users should be mutually orthogonal. However, accommodating a large number of users in the neighboring BSs makes it necessary to reuse the orthogonal sequences among users, which creates an interference and causes imperfect CSI estimation.

There has been a growing interest in the research community to address the issue of mitigating the pilot contamination effects recently. For instance, in [9], various alternatives for precoding and cooperative methods have been presented to alleviate pilot contamination. In [10], the maximum number of admissible users in a down-link pilot-contaminated time division duplexing (TDD) massive MIMO system is derived and an algorithm is proposed to achieve the individual user capacity. In [11], an optimal power and pilot duration allocation algorithm is proposed in a conventional wireless network with massive MIMO by considering different power for data signal and training signal transmission.

In the context of VWNs, [12] has studied the benefits of applying massive MIMO on the achieved network throughput and shown that the feasibility region of optimization problem is considerably expanded and the overall system throughput is improved as a result. Both perfect and imperfect channel estimation have been considered with *fixed* pilot duration in [12]. In this paper, we aim to investigate whether and to what extent adaptive pilot duration allocation can improve the network performance. Based on our knowledge, there is no related work dealing with adaptive pilot duration allocation in a massive-MIMO based VWN.

The rest of this paper is organized as follows. Section II explains the system model along with the problem formulation. Section III discusses the proposed algorithm followed by illustrative results described in Section IV. Finally, Section V presents the concluding remarks.

II. System Model

We consider the up-link transmission of an orthogonal frequency division multiplexing (OFDMA) VWN with a single base-station (BS) equipped with an array of $\mathcal{N} = \{1, \ldots, N\}$ antennas. The BS serves a set of slices, $\mathcal{S} = \{1, \cdots, S\}$. Slice $s \in \mathcal{S}$ consists of a set of single-antenna users, $\mathcal{U}_s = \{1, \cdots, U_s\}$ and requires a minimum rate R_s^{rsv}. The total number of users in the system is $U = \sum_{s \in \mathcal{S}} U_s$, and we assume $U \ll N$.

The total available bandwidth is divided into a set of sub-carriers, $\mathcal{K} = \{1, \ldots, K\}$ and for OFDMA, each sub-carrier is exclusively assigned to at most a single user at a time. The sub-carrier assignment indicator is denoted as

$$\beta_{u_s,k} = \begin{cases} 1, & \text{if sub-carrier } k \text{ is allocated to user } u_s, \\ 0, & \text{otherwise.} \end{cases}$$

We denote the sub-carrier assignment vectors for the BS, each slice, and each user as $\boldsymbol{\beta} = [\boldsymbol{\beta}_1, \cdots, \boldsymbol{\beta}_S]$, $\boldsymbol{\beta}_s = [\boldsymbol{\beta}_{u_s}]_{u_s=1}^{U_s}$, and $\boldsymbol{\beta}_{u_s} = [\beta_{u_s,1}, \cdots, \beta_{u_s,K}]$, respectively. The up-link pilot duration for user u_s with sub-carrier k in slice s is denoted by $\tau_{u_s,k}$. Correspondingly, the pilot duration vectors of the system, slice s, and user u_s are $\boldsymbol{\tau} = [\boldsymbol{\tau}_1, \cdots, \boldsymbol{\tau}_S]$, $\boldsymbol{\tau}_s = [\boldsymbol{\tau}_{u_s}]_{u_s=1}^{U_s}$, and $\boldsymbol{\tau}_{u_s} = [\tau_{u_s,1}, \cdots, \tau_{u_s,K}]$, respectively.

Let $N_{u_s,k}$ be the number of antennas allocated to user u_s on sub-carrier k. The antenna allocation vector of the system, slice s, and user u_s can be denoted as $\mathbf{N} = [\mathbf{N}_1, \cdots, \mathbf{N}_S]$, $\mathbf{N}_s = [\mathbf{N}_{u_s}]_{u_s=1}^{U_s}$ and $\mathbf{N}_{u_s} = [N_{u_s,1}, \cdots, N_{u_s,\mathcal{K}}]$, respectively. Also, let $\mathbf{P} = [\mathbf{P}_1, \cdots, \mathbf{P}_S]$, $\mathbf{P}_s = [\mathbf{P}_{u_s}]_{u_s=1}^{U_s}$ and $\mathbf{P}_{u_s} = [P_{u_s,1}, \cdots, P_{u_s,\mathcal{K}}]$ be the allocated power vectors of the system, slice s, and user u_s. respectively, where $P_{u_s,k}$ is the power allocated to user u_s in sub-carrier k. The variables used in the system model are listed in Table I.

The channel state information vector of user u_s at sub-carrier k is denoted by $\mathbf{h}_{u_s,k} \in \mathbf{C}^{1 \times N_{u_s,k}}$, where the channel coefficients are given by, [13],

$$h_{u_s,k,N_{u_s,k}} = \chi_{u_s,k,N_{u_s,k}} \sqrt{d_{u_s}},$$

where $\chi_{u_s,k,N_{u_s,k}}$ represents the small-scale fading coefficient of the link from the user u_s on the sub-carrier k to the antenna $N_{u_s,k}$ and d_{u_s} represents the large-scale link power attenuation due to path-loss and shadowing.

In order to perform sub-carrier and power allocation aiming to maximize the transmission rate, the user-link fading coefficients need to be estimated by using the up-link pilot signals. To this end, all users simultaneously transmit orthogonal pilot sequences at a specific part of the coherence time interval of T. Ideally, the pilot sequences transmitted from the users to their BS and the neighboring BSs should be mutually orthogonal to allow CSI estimation for all the users. However, since the number of orthogonal pilot sequences that can be used within a fixed T and bandwidth is limited, reuse of the orthogonal pilot sequences is unavoidable in neighboring BSs. Therefore, a BS will get the same orthogonal sequences not only from users in its coverage area but also from users in the neighboring areas, causing pilot contamination. In this case, the BS would be unable to accurately estimate the CSI of all users due to interference from users in neighboring BSs.

TABLE I
LIST OF VARIABLES

Notations	Definitions
$\mathcal{S}$	Set of slices
$\mathcal{U}_S$	Set of users in slice s
$\mathcal{K}$	Set of sub-carriers
$\boldsymbol{\beta}$	Sub-carrier assignment vector
$\mathbf{h}_{u_s,k}$	CSI vector of user u_s in slice s in sub-carrier k
$\tau_{u_s,k}$	Pilot duration for user u_s in sub-carrier k
$N_{u_s,k}$	Antennas allocated to user u_s in sub-carrier k
$P_{u_s,k}$	Power allocated to user u_s in sub-carrier k
T	Coherence interval
σ^2	Noise power spectral density
$\gamma_{u_s,k}$	SINR of user u_s in sub-carrier k
$R_{u_s,k}$	Rate of user u_s in slice s and sub-carrier k
c_s^{N}	Pricing factor for antenna for slice s
c_s^{P}	Pricing factor for transmit power for slice s
c_s^{τ}	Pricing factor for pilot duration for slice s
$N_s^{\min}$	Min. antennas for slice s
$N_s^{\max}$	Max. antennas for slice s

If $\widetilde{\mathbf{h}}_{u_s,k}$ is the estimated channel vector taking into account the pilot contamination error and $\mathbf{f}_{u_s,k} \in \mathbf{C}^{N_{u_s,k} \times 1}$ is the precoding vector for user u_s and on sub-carrier k, the received signal at the BS after Maximal Ratio Combining (MRC) can be expressed as, [13]

$$y_{u_s,k} = \sqrt{P_{u_s,k}}\, \widetilde{\mathbf{h}}_{u_s,k} \mathbf{f}_{u_s,k} x_{u_s,k} + I_{u_s,k} + \mathbf{z}_{u_s,k} \mathbf{f}_{u_s,k}, \quad (1)$$

where $I_{u_s,k} = \sum_{s \in \mathcal{S}} \sum_{u'_s \neq u_s} \sqrt{P_{u'_s,k}} \widetilde{\mathbf{h}}_{u'_s,k} \mathbf{f}_{u_s,k} x_{u'_s,k} - \sqrt{P_{u_s,k}} \sum_{s \in \mathcal{S}} \sum_{u'_s \in \mathcal{U}_s} \mathbf{e}_{u'_s,k} \mathbf{f}_{u_s,k} x_{u'_s,k}$ is the interference received from other users due to contamination error. Also, $\mathbf{z}_{u_s,k}$ denotes the additive white Gaussian noise (AWGN) with zero mean and power spectral density σ assumed to be 1 for simplicity. Note that in the above expression for $I_{u_s,k}$, $\mathbf{e}_{u_s,k} = \widetilde{\mathbf{h}}_{u_s,k} - \mathbf{h}_{u_s,k}$ and the elements of $\mathbf{e}_{u_s,k}$ are random variables (RVs) with zero mean and variance $\frac{P^{\text{Pilot}}_{u_s,k} d_{u_s}}{P^{\text{Pilot}}_{u_s,k} d_{u_s} + 1}$ [13], and $P^{\text{Pilot}}_{u_s,k} = \tau_{u_s,k} P_{u_s,k}$ is the power used for the pilot signal.

For MMSE-based channel estimation, $\widetilde{\mathbf{h}}_{u_s,k}$ contains i.i.d RVs with zero mean and variance $\frac{P^{\text{Pilot}}_{u_s,k} d^2_{u_s}}{P^{\text{Pilot}}_{u_s,k} d_{u_s} + 1}$ [13], and for the MRC precoding vector, $\mathbf{f}_{u_s,k} = \widetilde{\mathbf{h}}^{\text{H}}_{u_s,k}$. Since $\mathbf{e}_{u_s,k}$ and $\widetilde{\mathbf{h}}_{u_s,k}$ are independent, $\forall u_s \in \mathcal{U}_s$ and $\forall k \in \mathcal{K}$, and $\mathbf{f}^{\text{Imperf}}_{u_s,k}$, $\mathbf{e}_{u_s,k}$ are independent, $\forall u_s \in \mathcal{U}_s$ and $\forall k \in \mathcal{K}$, the received SINR at the BS from the user u_s at sub-carrier k is given by

$$\gamma_{u_s,k} = \frac{P_{u_s,k} \|\widetilde{\mathbf{h}}_{u_s,k}\|^4}{P_{u_s,k} \|\mathbf{e}_{u_s,k} \widetilde{\mathbf{h}}^{\text{H}}_{u_s,k}\|^2 + \|\widetilde{\mathbf{h}}_{u_s,k}\|^2}. \quad (2)$$

Substituting the variance of $\widetilde{\mathbf{h}}_{u_s,k}$ in the above expression yields the received SINR, $\forall u_s \in \mathcal{U}_s$, $\forall k \in \mathcal{K}$,

$$\gamma_{u_s,k} = \frac{N^2_{u_s,k} P_{u_s,k} \zeta_{u_s,k}}{N_{u_s,k} P_{u_s,k} \zeta_{u_s,k} \left(\frac{P^{\text{Pilot}}_{u_s,k} d_{u_s}}{P^{\text{Pilot}}_{u_s,k} d_{u_s} + 1}\right)^2 + N_{u_s,k} \zeta_{u_s,k}}, \quad (3)$$

where $\zeta_{u_s,k} = (\frac{P^{\text{Pilot}}_{u_s,k} d^2_{u_s}}{P^{\text{Pilot}}_{u_s,k} d_{u_s}+1})^2$. By substituting $P^{\text{Pilot}}_{u_s,k} = \tau_{u_s,k} P_{u_s,k}$, we get, for all $u_s \in \mathcal{U}_s$ and $k \in \mathcal{K}$,

$$\gamma_{u_s,k} = \frac{\tau_{u_s,k} \rho^2_{u_s,k} d^2_{u_s}}{1 + (1+\tau_{u_s,k})\rho_{u_s,k} d_{u_s}/\sqrt{N_{u_s,k}}}, \tag{4}$$

where we substituted $P_{u_s,k} = \rho_{u_s,k}/\sqrt{N_{u_s,k}}$ [13]. As $N_{u_s,k} \to \infty$, we get $\gamma_{u_s,k} = \tau_{u_s,k}\rho^2_{u_s,k} d^2_{u_s} = \tau_{u_s,k} P^2_{u_s,k} d^2_{u_s} N_{u_s}$. Hence, the rate of user u_s on sub-carrier k for a fraction $\tau_{u_s,k}$ of the total up-link frame time T is

$$R_{u_s,k} = \frac{T - \tau_{u_s,k}}{T} \log_2\left(1 + \tau_{u_s,k} P^2_{u_s,k} d^2_{u_s} N_{u_s}\right). \tag{5}$$

Thus, the total rate of user u_s is

$$R_{u_s}(\mathbf{P}, \boldsymbol{\beta}, \mathbf{N}, \boldsymbol{\tau}) = \sum_{k\in\mathcal{K}} \beta_{u_s,k} R_{u_s,k}$$

which is a function of $\mathbf{P}, \boldsymbol{\beta},\ \mathbf{N},$ and $\boldsymbol{\tau}$. Now, we define the utility function of a slice s as

$$\begin{aligned} &F_s(\mathbf{P}, \boldsymbol{\beta}, \mathbf{N}, \boldsymbol{\tau}) = \sum\nolimits_{u_s\in\mathcal{U}_s} R_{u_s}(\mathbf{P}, \boldsymbol{\beta}, \mathbf{N}, \boldsymbol{\tau}) - \\ &- c^{\text{N}}_s \sum_{u_s\in\mathcal{U}_s}\sum_{k\in\mathcal{K}} \beta_{u_s,k} N_{u_s,k} - c^{\text{P}}_s \sum_{u_s\in\mathcal{U}_s}\sum_{k\in\mathcal{K}} \beta_{u_s,k} P_{u_s,k} \\ &- c^{\tau}_s \sum_{u_s\in\mathcal{U}_s}\sum_{k\in\mathcal{K}} \beta_{u_s,k} \tau_{u_s,k}, \end{aligned} \tag{6}$$

where c^{N}_s, c^{P}_s and c^{τ}_s are pricing factors for the number of allocated antennas, the transmit power and the up-link pilot duration for slice s, respectively. Considering these three pricing factors, $F_s(\mathbf{P}, \boldsymbol{\beta}, \mathbf{N}, \boldsymbol{\tau})$ is an increasing function of total rate of VWN while it is a decreasing function of the total consumed resource of the VWN for each slice, i.e., power, antenna and pilot duration, which is novel in this context and can be considered as a total revenue minus the costs for each slice. Since in an OFDMA system, a sub-carrier in a BS can be allocated to one user, we have the following constraint:

$$\text{C1: } \beta_{u_s,k} \in \{0,1\}, \text{and} \sum\nolimits_{s\in\mathcal{S}} \sum\nolimits_{u_s\in\mathcal{U}_s} \beta_{u_s,k} \le 1, \forall k \in \mathcal{K}.$$

The sum of the total transmit power for each user u_s over all sub-carriers k allocated to it poses another constraint, i.e.

$$\text{C2: } \sum\nolimits_{k\in\mathcal{K}} \beta_{u_s,k} P_{u_s,k} \le P^{\max}_{u_s}, \quad \forall u_s \in \mathcal{U}_s, \quad \forall s \in \mathcal{S},$$

where $P^{\max}_{u_s}$ is the maximum transmit power of user u_s. Since the minimum rate reservation per each slice s is R^{rsv}_s, we have

$$\text{C3: } \sum\nolimits_{u_s\in\mathcal{U}_s} R_{u_s} \ge R^{\text{rsv}}_s, \qquad \forall s \in \mathcal{S}.$$

If $N^{\min}_s$ and $N^{\max}_s$ are the minimum and maximum numbers of antennas that can be allocated for the slice s, then we have [14]

$$\text{C4: } \sum\nolimits_{u_s\in\mathcal{U}_s} \sum\nolimits_{k\in\mathcal{K}} \beta_{u_s,k} N_{u_s,k} \in \{N^{\min}_s, N^{\min+1}_s, \cdots, N^{\max}_s\},$$

for each slice $s \in \mathcal{S}$. Finally, for each user, the up-link pilot duration $\tau_{u_s,k}$ has a limitation, i.e.,

$$\text{C5: } 0 < \tau_{u_s,k} < T, \quad \text{if } \beta_{u_s,k} = 1.$$

Algorithm 1 : Iterative Algorithm

Initialization: Set each element of $\boldsymbol{\tau}(l=0)$, $\boldsymbol{\beta}(l=0)$, $\mathbf{P}_{u_s}(l=0)$, and $\mathbf{N}(l=0)$ to $0.3T$, 1, $P^{\max}_{u_s}/K$ and $N^{\max}_s$, respectively, for all $u_s \in \mathcal{U}_s$ and $s \in \mathcal{S}$. Initialize $l_{\max} \gg 1$, $0 < \varepsilon \ll 1$, $\boldsymbol{\lambda}(l=0)$, $\boldsymbol{\psi}(l=0)$ and $\boldsymbol{\theta}(l=0)$.

Step 1: Obtain the optimum values of $\mathbf{P}^*_{u_s}(l)$, $\boldsymbol{\beta}^*(l)$, $\mathbf{N}^*(l)$ using Alg. 1.A for fixed $\boldsymbol{\tau}(l)$.

Step 2: For fixed $\mathbf{P}^*_{u_s}(l)$, $\boldsymbol{\beta}^*(l)$, $\mathbf{N}^*(l)$, find the optimal pilot duration $\boldsymbol{\tau}^*(l)$ using Alg. 1.B.

Stop if $||\mathbf{P}^*_{u_s}(l+1) - \mathbf{P}^*_{u_s}(l)|| \le \varepsilon$, where $||\mathbf{x}||$ is the norm of vector $\mathbf{x}$, otherwise, set $l := l+1$ and go to **Step 1**.

Therefore, the resource allocation problem is

$$\max_{\mathbf{P},\boldsymbol{\beta},\mathbf{N},\boldsymbol{\tau}} \sum\nolimits_{s\in\mathcal{S}} F_s(\mathbf{P}, \boldsymbol{\beta}, \mathbf{N}, \boldsymbol{\tau}), \tag{7}$$
$$\text{subject to : C1} - \text{C5}.$$

(7) is an inherently non-convex optimization problem involving four sets of optimization variables. Thus, finding optimal solution of (7) leads to high computational complexity. To tackle this issue, in the next section, we propose a two-step iterative algorithm with a low computational complexity.

III. Proposed Algorithm

In the proposed **Algorithm 1** to solve problem (7), we first apply the framework proposed in [12] to derive the optimum values of $\mathbf{P}$, $\boldsymbol{\beta}$, and $\mathbf{N}$ for a fixed $\boldsymbol{\tau}$ in Step 1. Then, for the obtained values of $\mathbf{P}$, $\boldsymbol{\beta}$, and $\mathbf{N}$, we derive the optimal value of $\boldsymbol{\tau}$ in Step 2. The derived solution of iteration l is denoted as $\mathbf{P}^*(l), \boldsymbol{\beta}^*(l), \mathbf{N}^*(l),$ and $\boldsymbol{\tau}^*(l)$ and the overall solution process can be represented as

$$\underbrace{\boldsymbol{\tau}(0) \to \mathbf{P}(0), \boldsymbol{\beta}(0), \mathbf{N}(0)}_{\text{Initialization}} \to \ldots \underbrace{\boldsymbol{\tau}^*(l) \to \mathbf{P}^*(l), \boldsymbol{\beta}^*(l), \mathbf{N}^*(l)}_{\text{Iteration } l} \to \tag{8}$$
$$\underbrace{\boldsymbol{\tau}^* \to \mathbf{P}^*, \boldsymbol{\beta}^*, \mathbf{N}^*}_{\text{Optimal solution}}$$

Steps 1 and 2 are repeated until the convergence conditions are met. Also, to simplify the algorithm, we just focus on the case that $\tau_{u_s,k} P^2_{u_s,k} d^2_{u_s} N_{u_s} \gg 1$ which is a reasonable assumption in massive MIMO context due to large value of N_{u_s}.

A. Algorithm 1.A

For a fixed value of $\boldsymbol{\tau}$, (6) involves three sets of variables $\mathbf{P}, \boldsymbol{\beta},$ and $\mathbf{N}$, which is still a combinatorial function containing both discrete and continuous variables. To simplify this problem, we relax the sub-carrier assignment indicator to be continuous in the interval $[0,1]$ which will relax the constraints explained above. Considering the variable transformations $x_{u_s,k} = \beta_{u_s,k} P_{u_s,k}$ and $y_{u_s,k} = \beta_{u_s,k} N_{u_s,k}$, the optimization problem of this step can be written as

$$\max_{\boldsymbol{\beta},\mathbf{x},\mathbf{y}} \sum\nolimits_{s\in\mathcal{S}} \widetilde{F}_s(\boldsymbol{\beta}, \mathbf{x}, \mathbf{y}), \tag{9}$$
$$\text{subject to :}$$

$$
\begin{aligned}
&\widetilde{\mathrm{C1}}: \beta_{u_s,k} \in [0,1],\\
&\quad \sum\nolimits_{s\in\mathcal{S}} \sum\nolimits_{u_s\in\mathcal{U}_s} \beta_{u_s,k} \le 1, \quad \forall k \in \mathcal{K}\\
&\widetilde{\mathrm{C2}}: \sum\nolimits_{k\in\mathcal{K}} x_{u_s,k} \le P_{u_s}^{\max}, \quad \forall u_s \in \mathcal{U}_s, \quad \forall s \in \mathcal{S},\\
&\widetilde{\mathrm{C3}}: \sum\nolimits_{u_s\in\mathcal{U}_s} \widetilde{R}_{u_s} \ge R_s^{\mathrm{rsv}}, \quad \forall s \in \mathcal{S},\\
&\widetilde{\mathrm{C4}}: N_s^{\min} \le \sum\nolimits_{u_s\in\mathcal{U}_s} \sum\nolimits_{k\in\mathcal{K}} y_{u_s,k} \le N_s^{\max}, \forall s \in \mathcal{S},\\
&\widetilde{\mathrm{C5}}: \beta_{u_s,k}\tau_{u_s,k} < T, \forall k \in \mathcal{K}, \forall u_s \in \mathcal{U}_s, \forall s \in \mathcal{S}
\end{aligned}
$$

where $\widetilde{F}_s(\boldsymbol{\beta},\mathbf{x},\mathbf{y}) = \sum_{u_s\in\mathcal{U}_s} \widetilde{R}_{u_s}(\mathbf{x},\boldsymbol{\beta},\mathbf{y}) - c_s^{\mathrm{N}} \sum_{u_s\in\mathcal{U}_s} \sum_{k\in\mathcal{K}} y_{u_s,k} - c_s^{\mathrm{P}} \sum_{u_s\in\mathcal{U}_s} \sum_{k\in\mathcal{K}} x_{u_s,k} - c_s^{\tau} \sum_{u_s\in\mathcal{U}_s} \sum_{k\in\mathcal{K}} \beta_{u_s,k}\tau_{u_s,k}$.

By assuming $\tau_{u_s,k} P_{u_s,k}^2 d_{u_s}^2 N_{u_s} \gg 1$ and new sets of $x_{u_s,k}$, $y_{u_s,k}$ and $\beta_{u_s,k}$, the total rate of user u_s, R_{u_s}, is a convex function [12]. Consequently, (9) is a convex optimization problem which can be solved by Lagrange dual function, defined as

$$
\begin{aligned}
&\mathcal{L}(\boldsymbol{\beta},\mathbf{x},\mathbf{y},\lambda_{u_s},\phi_g,\theta_s,\psi_s) = \qquad (10)\\
&-\sum\nolimits_{s\in\mathcal{S}} \widetilde{F}_s + \sum\nolimits_{u_s\in\mathcal{U}_s} \lambda_{u_s}\Big(\sum_{k\in\mathcal{K}} x_{u_s,k} - P_{\max}\Big)\\
&+\sum\nolimits_{s\in\mathcal{S}} \phi_s\Big(R_s^{\mathrm{rsv}} - \sum\nolimits_{u_s\in\mathcal{U}_s} \widetilde{R}_{u_s}\Big)\\
&+\sum\nolimits_{s\in\mathcal{S}} \theta_s\Big(N_s^{\min} - \sum\nolimits_{u_s\in\mathcal{U}_s} \sum\nolimits_{k\in\mathcal{K}} y_{u_s,k}\Big)\\
&+\sum\nolimits_{s\in\mathcal{S}} \psi_s\Big(\sum\nolimits_{u_s\in\mathcal{U}_s} \sum\nolimits_{k\in\mathcal{K}} y_{u_s,k} - N_s^{\max}\Big),\\
&+\sum\nolimits_{u_s\in\mathcal{U}_s} \eta_{u_s}\Big(\sum_{k\in\mathcal{K}} \beta_{u_s,k}\tau_{u_s,k} - T\Big),
\end{aligned}
$$

where the Lagrange multipliers are $\lambda_{u_s}, \phi_s, \theta_s, , \psi_s$ and η_{u_s} for the relaxed constraints $\widetilde{\mathrm{C2}}$, $\widetilde{\mathrm{C3}}$, $\widetilde{\mathrm{C4}}$ and $\widetilde{\mathrm{C5}}$, respectively. To solve (10), we apply iterative gradient descent method introduced in Alg. 1.A where l_1 is the iteration number. In Alg. 1.A, the dual variables can be updated as

$$\lambda_{u_s}(l_1+1) = \left[\lambda_{u_s}(l_1) + \delta_{\lambda_{u_s}} \frac{\partial\mathcal{L}}{\partial\lambda_{u_s}}\right]^+, \forall u_s \in \mathcal{U}, \qquad (11)$$

$$\phi_s(l_1+1) = \left[\phi_s(l_1) + \delta_{\phi_s} \frac{\partial\mathcal{L}}{\partial\phi_s}\right]^+, \quad \forall s \in \mathcal{S}, \qquad (12)$$

$$\theta_s(l_1+1) = \left[\theta_s(l_1) + \delta_{\theta_s} \frac{\partial\mathcal{L}}{\partial\theta_s}\right]^+, \quad \forall s \in \mathcal{S}, \qquad (13)$$

$$\psi_s(l_1+1) = \left[\psi_s(l_1) + \delta_{\psi_s} \frac{\partial\mathcal{L}}{\partial\psi_s}\right]^+, \quad \forall s \in \mathcal{S}, \qquad (14)$$

$$\eta_{u_s}(l_1+1) = \left[\eta_{u_s}(l_1) + \delta_{\eta_{u_s}} \frac{\partial\mathcal{L}}{\partial\eta_{u_s}}\right]^+, \forall u_s \in \mathcal{U}_s. \qquad (15)$$

where $\delta_{\lambda_{u_s}}, \delta_{\phi_s}, \delta_{\theta_s}, \delta_{\psi_s}$ and $\delta_{\eta_{u_s}}$ are the small positive step sizes for their dual variables.

Now, to update the primal variables, $P_{u_s,k}$, $N_{u_s,k}$ and β_{u_s}, differentiating (10) with respect to each of the variables and setting them to zero, we get following expression for the updated value in the iteration $l+1$ as

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Algorithm 1.A: Resource Allocation

Initialization: Set $\boldsymbol{\tau}(l_1 = 0) = 0.3T$, $\boldsymbol{\beta}(l_1 = 0) = \boldsymbol{\beta}(l)$, $\mathbf{P}_{u_s}(l_1 = 0) = \mathbf{P}_{u_s}(l)$, and $\mathbf{N}(l_1 = 0) = \mathbf{N}(l)$ for all $u_s \in \mathcal{U}_s$ and $s \in \mathcal{S}$. Initialize $l_1^{\max} \gg 1$, $0 < \varepsilon \ll 1$, $\boldsymbol{\lambda}(l_1 = 0)$, $\boldsymbol{\psi}(l_1 = 0)$ and $\boldsymbol{\theta}(l_1 = 0)$.

1: Update dual variables, λ_{u_s}, ϕ_s, θ_s and ψ_s, by gradient descent method for all $s \in \mathcal{S}$ from (11)-(15).

2: Using the above updated parameters for iteration (l_1+1), compute $P_{u_s,k}^*(l_1+1)$ and $N_{u_s,k}^*(l_1+1)$ for all $u_s \in \mathcal{U}_s$, and $k \in \mathcal{K}$ using (16) and (17), respectively.

3: Perform sub-carrier allocation for all $u_s \in \mathcal{U}_s$ and for all $k \in \mathcal{K}$ from (19).

Stop if $||\mathbf{P}(l_1+1) - \mathbf{P}(l_1)|| \le \varepsilon$ or $l_1 \ge l_1^{\max}$, otherwise, set $l_1 := l_1 + 1$ and go to **1**.

$$P_{u_s,k}(l_1+1) = \qquad (16)$$
$$\frac{(T-\tau_{u_s,k}(l))}{T}\left[\frac{2(1+\phi_s)}{\ln(2)(\lambda_{u_s} + c_s^P)}\right]_0^{P_{u_s}^{\max}},$$

and,

$$N_{u_s,k}(l_1+1) = \qquad (17)$$
$$\frac{(T-\tau_{u_s,k}(l))}{T} \times \left[\frac{1+\phi_s}{\ln(2)(\psi_s - \theta_s + c_s^N)}\right]_0^{N_s^{\max}},$$

where $[x]_a^b = \min\{b, \max\{x, a\}\}$. Differentiating (10) with respect to $\beta_{u_s,k}$, we have

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\beta_{u_s,k}} =& c_s^{\tau} \sum\nolimits_{u_s\in\mathcal{U}_s} \tau_{u_s,k}(l) + \eta_{u_s}\tau_{u_s,k}(l) + \qquad (18)\\
&\frac{(1+\phi_s)(T-\tau_{u_s,k}(l))}{T} \times\\
&\left(\log_2(\tau_{u_s,k}(l)P_{u_s,k}(l_1)^2 d_{u_s}^2 N_{u_s,k}(l_1) - \frac{3}{\ln(2)}\right),
\end{aligned}
$$

and hence, as shown in [14],

$$\beta_{u_s,k}^*(l_1+1) = \begin{cases} 1, & \text{if } \frac{\partial\mathcal{L}}{\partial\beta_{u_s,k}} > 0,\\ 0, & \text{otherwise.}\end{cases} \qquad (19)$$

The iteration is repeated until $||\mathbf{P}(l_1+1) - \mathbf{P}(l_1)|| \le \varepsilon$ or $l_1 \ge l_1^{\max}$, where $l_1^{\max}$ is the maximum number of iterations for Algorithm 1.A.

B. Algorithm 1.B

For the derived values of $\mathbf{P}^*(l), \mathbf{N}^*(l)$and, $\boldsymbol{\beta}^*(l)$, the optimization problem for finding $\boldsymbol{\tau}^*(l)$ is

$$\max_{\boldsymbol{\tau}, 0<\tau_{u_s,k}<T} \sum\nolimits_{s\in\mathcal{S}} \widehat{F}_s(\boldsymbol{\tau}), \qquad (20)$$

where

$$
\begin{aligned}
\widehat{F}_s(\boldsymbol{\tau}) =& \sum\nolimits_{u_s\in\mathcal{U}_s} R_{u_s}(\boldsymbol{\tau}) - c_s^{\mathrm{N}} \sum_{u_s\in\mathcal{U}_s} \sum_{k\in\mathcal{K}} \beta_{u_s,k}(l) N_{u_s,k}(l)\\
&- c_s^{\mathrm{P}} \sum_{u_s\in\mathcal{U}_s} \sum_{k\in\mathcal{K}} \beta_{u_s,k}(l) P_{u_s,k}(l) - c_s^{\tau} \sum_{u_s\in\mathcal{U}_s} \sum_{k\in\mathcal{K}} \beta_{u_s,k}(l)\tau_{u_s,k}.
\end{aligned}
$$

The optimum value of $\boldsymbol{\tau}$ from (20) can be obtained by setting $\frac{\partial\widehat{F}_s(\boldsymbol{\tau})}{\partial\tau_{u_s,k}} = 0$ where $0 < \tau_{u_s,k} < T$. To derive this optimal value,

Algorithm 1.B: Pilot Duration Allocation

Initialization:

Set $\boldsymbol{\beta}(l_2 = 0) = \boldsymbol{\beta}^*(l_1)$, $\mathbf{P}_{u_s}(l_2 = 0) = \mathbf{P}^*_{u_s}(l_1)$, and $\mathbf{N}(l_2 = 0) = \mathbf{N}^*(l_1)$ for all $u_s \in \mathcal{U}_s$ and $s \in \mathcal{S}$, and $l_2^{\max} \gg 1$, $a_{u_s,k}(l_2 = 0) = 0$, $b_{u_s,k}(l_2 = 0) = T$, $0 < \varepsilon \ll 1$, and $c_{u_s,k}(l_2 = 0) = T/2$, and calculate $f_{u_s,k}(l_2 = 0) = f_{u_s,k}(c_{u_s,k}(l_2 = 0))$.

Iterative Bisection Method for all u_s and k:

1) For $c_{u_s,k}(l_2) = (a_{u_s,k}(l_2) + b_{u_s,k}(l_2))/2$, calculate $f_{u_s,k}(c_{u_s,k}(l_2))$ and $f_{u_s,k}(a_{u_s,k}(l_2))$

- If $f_{u_s,k}(a_{u_s,k}(l_2)) \times f_{u_s,k}(c_{u_s,k}(l_2)) < 0$, $b_{u_s,k}(l_2) = c_{u_s,k}(l_2)$
- Else, $a_{u_s,k}(l_2) = c_{u_s,k}(l_2)$

2) Consider $\mathbf{c}(l_2) = [c_{u_s,k}(l_2)]_{\forall u_s,k}$ and $\mathbf{f}(l_2) = [f_{u_s,k}(l_2)]_{\forall u_s,k}$,

Stop if:

- $\|\mathbf{c}(l_2) - \mathbf{c}(l_2 - 1)\| < \varepsilon$ or
- $\|\mathbf{f}(l_2) - \mathbf{f}(l_2 - 1)\| < \varepsilon$ or $l_2 > l_2^{\max}$
- Otherwise $l_2 = l_2 + 1$, go to **1**.

we apply the iterative bisection method where we consider

$$f_{u_s,k}(\tau_{u_s,k}) = \frac{T}{\tau_{u_s,k}} - \log_2(\tau_{u_s,k} P^{*2}_{u_s,k}(l) d^2_{u_s,k} N^*_{u_s,k}(l) - 1 - c^\tau_s T,$$

and the iteration number l_2 as summarized in Alg. 1.B.

IV. Simulation Results

To study the performance of Algorithm 1, we simulate a scenario of a VWN with a single BS serving two slices each with $U_s = 4$ users per slice where $P^{\max}_{u_s} = 0$ dB and $R^{\text{rsv}} = R^{\text{rsv}}_s = 2$ bps/Hz. The users are distributed uniformly in the coverage area of the BS. The total number of sub-carriers is $K = 4$ and the total transmission frame duration is set to $T = 1$ s. To study the effects of changing pricing factors, we consider 3 scenarios: (1) Set 1 where $c^\tau_s = c^N_s = c^P_s = 0$, (2) Set 2 where $c^\tau_s = 0.5$, $c^P_s = 1$, and $c^N_s = 0.07$ and (3) Set 3 where $c^\tau_s = 1$, $c^P_g = 2$ and $c^N_s = 0.09$. Obviously, Set 3 has more restricted price parameters than Set 2 and Set 1, while Set 2 has moderate pricing factors compared to other sets. We compare Alg. 1 with the approach using *fixed* $\boldsymbol{\tau}$ where $\tau_{u_s,k}/T = 0.3$ for all u_s and k. The simulation results are averaged over 100 trials and we set the total rate to zero when there is an infeasibility in the solution.

Fig. 1 plots the total rate versus R^{rsv} for the three different sets by applying Algorithm 1 as well as for a fixed pilot duration. The results indicate that the total rate achieved decreases with increasing R^{rsv} due to the fact that at higher R^{rsv}, C3 cannot be fulfilled all the time. Since we set the total rate to zero when there is an infeasibility, the average rate decreases with increasing R^{rsv}. The overall system throughput is improved by using adaptive pilot duration in Alg. 1. Moreover, the total rate decreases as the values of c^τ_s, c^P_s and c^N_s increase. This is obvious since the utility function (6) is defined as a non-increasing function of pricing factors.

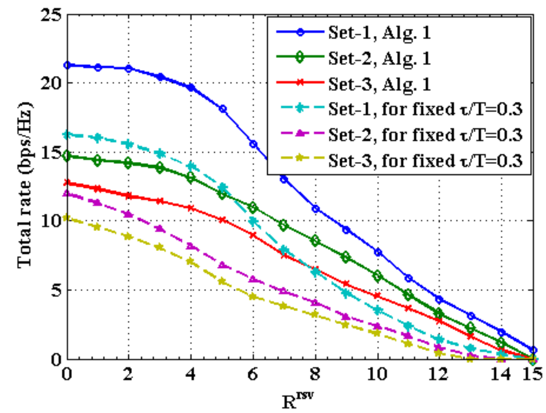

Fig. 1. Total rate versus R^{rsv}

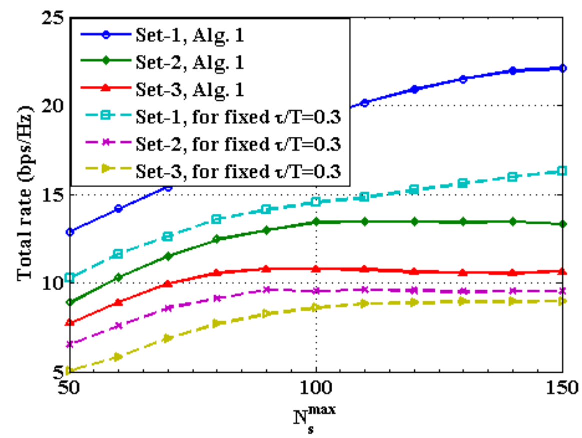

Fig. 2. Total rate versus $N^{\max}_s$

From Fig. 2, the total rate increases with increasing $N^{\max}_s$ as expected due to the multiplexing gain of massive MIMO. Again, the overall system performance is improved by adaptive pilot duration via Alg. 1 as compared to the approach using *fixed* pilot duration. Similarly, as the costs increase across Sets 1, 2, and 3, the total achieved rate decreases as expected by (6).

To get more insight about the effects of the pricing factors on the VWN performance, in Fig. 3 and 4, we plot the total rate versus c^τ_s and c^N_s with c^P_s fixed, and c^τ_s and c^P_s with c^N_s fixed, respectively. As seen in these two figures, the total rate is not a convex function with respect to the pricing factors. Specifically, the values of c^τ_s and c^N_s have significant effects on decreasing the total rate of the VWN.

V. Conclusion

In this paper, we examined the resource allocation in a massive MIMO-based VWN. In consideration of possible pilot contamination errors, we formulated an optimization problem to adaptively assign uplink pilot duration, power, antennas and sub-carriers to users in order to maximize the overall system throughput. We developed a low-complexity two-step iterative algorithm wherein the first step finds the optimal

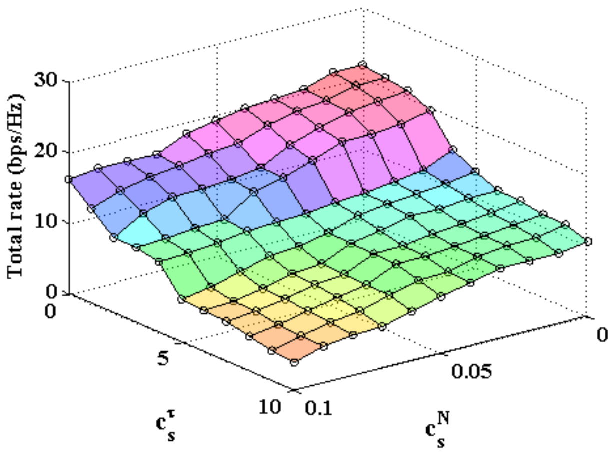

Fig. 3. Total rate versus c_s^τ and c_s^N for fixed c_s^P

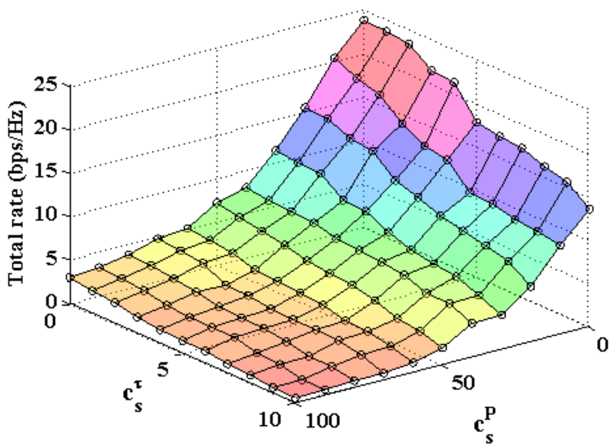

Fig. 4. Total rate versus c_s^τ and c_s^P for fixed c_s^N

power, antenna and sub-carrier allocation to be used in the second step to optimize the up-link pilot duration. Simulation results indicate a significant system performance improvement offered by the proposed scheme using adaptive pilot duration as compared to the scenario with fixed pilot duration.

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