



## A survey on the variability of dynamic stiffness data of identical vehicles

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### ABSTRACT

The variation in material quality, production tolerances and joint conditions in nominally 'identical' vehicles means that the level of noise and vibration perceived in the cabin will vary from car to car. To assess the level of variability in the low and medium frequency range a series of measurements have been conducted on a small set of nominally identical sedan vehicles. Measurements are presented in the form of dynamic stiffness data for twenty three vehicle mounting and response points and for three different translational directions. In this paper, the methodology used to analyse the large data set is shown. The level of variability in the corresponding frequency response functions (FRFs) from all the vehicles are then presented and compared for different locations and the different loading measurement directions. The mean value is calculated for each data set and compared and the variability is also presented as a function of frequency for a selected set of the dynamic stiffness data. The intention is to demonstrate how the level of FRF variability changes depending upon the excitation/response location chosen and how this can be related to the frequency band characteristics.

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### 1. INTRODUCTION

The variation in material quality, production tolerances and joint conditions in vehicles makes the level of noise and vibration in the passenger cabin variable from car to car. For the premium market, the assurance of a low level of noise and vibration in the passenger cabin is essential for every individual vehicle, to minimise customer returns and warranty claims. The difference between nominally identical vehicles has been studied before. Wood and Joachim (1,2) reported a variability of up to 15 dB in noise transfer functions measured in the interior cabin of six vehicles. They (3) later repeated the measurements on two body types of sedans and wagons and demonstrated that the variability in structural borne noise of two models which share the same front structure is the same.

A large study of structure-borne and air-borne noise levels inside passenger cabins of 99 nominally identical passenger cars and 57 nominally identical pick-up trucks have been measured in order to determine the scatter in the measured Frequency Response Functions (FRFs) (4,5), where it is shown that a 5 to 10 dB variation is found. Hill et al. (6,7) investigated the statistical distribution of noise levels of over 1130 vehicles. Their findings showed the limited effect of temperature and humidity on variability of noise transfer function at low frequencies. A normalised standard deviation of 0.2 at 20 Hz is measured which decreases to 0.05 at 100 Hz with a variability in sound pressure level of 5 dB.

These large studies illustrate the problem faced by automotive designers; by the time the vehicles have been produced in sufficient numbers, it is too late to alter the design. Of more interest is the ability to measure small sample sizes of incomplete vehicles (or components) and determine

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variability parameters from these. In this study, a small sample size of vehicles without engines and suspensions (including subframes) are used to illustrate a method for determining variability and assessing large amounts of information easily.

To assess the level of variability in this study, a series of measurements have been conducted on a small set of nominally identical vehicles provided by Jaguar Land Rover for analysis and data processing. The variability in dynamic stiffness of different measurement locations is compared in this paper. The study is unique as usually noise transfer functions are measured (providing an overall measure of the variability as different transfer paths contribute to it). For dynamic stiffness, the local modes play a more crucial role and its variability can be a measure of variability in different locations on a vehicle.

Measurements are conducted on five new executive sedans and are repeated on the fifth car which provided six ensembles, all of which have the powertrain, suspensions and sub-frames removed. The measurements were conducted on the attachments points in three perpendicular directions, providing the response where structural loading applies to the vehicle, induced by the road surface or engine. Twenty three measurements points in three directions were made available for frequencies up to 500 Hz corresponding to a range of interest for interior noise and vibration. Techniques to analyse data for small sample sizes and compare the variability in dynamic stiffness are presented here. To convey the importance of information, a colour coded table based on the location of the measurements are sorted from the highest to the lowest variability. The study is important as comparisons with low sample numbers of experimental measurements are often the only way to validate and gain confidence in numerical CAE models.

## 2. Measurement and analysis techniques

The test vehicles considered in this paper were nominally identical production examples where the suspension, engine, transmission and subframes were removed. The data made available derived from 23 connecting points where dynamic stiffness was measured. The abbreviations that are used in this paper for mounting position are given in Table 1, where the two sides of the vehicle are identified by “L” for the left and “R” for the right side of the vehicle (all were right hand drive). Three directions are identified by “X” along the vehicle, “Y” transverse direction and “Z” perpendicular to the ground. The vehicles were mounted on soft mounts to provide an approximate free-free boundary condition and to isolate them from environmental excitations.

Table 1: Abbreviations for excitation points

Abbreviation	Full name
FARB	Front Anti-Roll Bar
FLCA	Front Lower Control Arm
FSUS	Front SUSpension
FUCA	Front Upper Control Arm
RSUB	Rear SUB-frame
RSUS	Rear SUSpension
ENGN	Engine Mount (ENGiNe)
TRMNT_P	Transmission MouNT
CTRBR	Central Bearing

As removal of the suspension leaves a mounting hole, aluminium blocks were used in their place, which allows exciting the structure by an impact hammer and measuring the acceleration in the same direction as the excitation with an accelerometer, the block also allowing excitation in the other two translational directions, see Figure 1 for an example. A similar configuration is used for other measurement points.

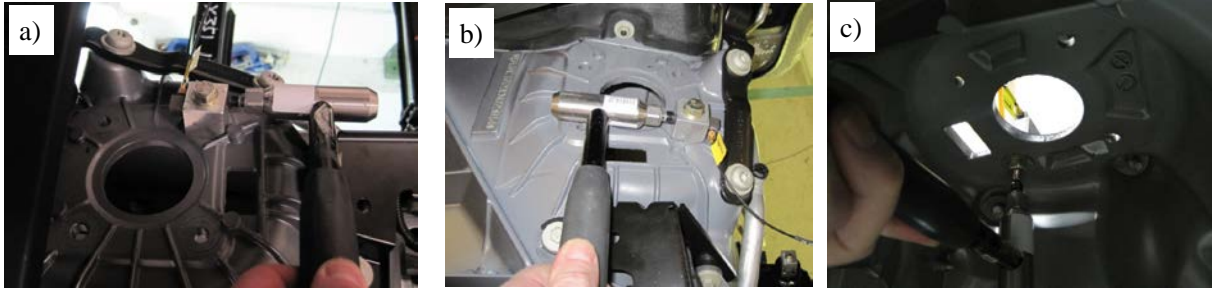


Figure 1: Measurement of dynamic stiffness at the front suspension (FSUS); a) measurement in X direction. b) measurement in Y direction. c) measurement in Z direction.

The measured dynamic stiffness for four different locations (and directions) are shown in Figure 2 where units of the y axis are removed due to the commercial confidentiality and a dB scale is used. A Dynamic stiffness measured at the front right anti-roll bar mount in the Y direction is shown in Figure 2(a) with and the front right lower control arm position 1 measured in the Z direction in Figure 2(b). These are the most variable amongst all other measurements. While the variability is higher at a certain frequency range in the position shown in Figure 2(a), the variability is less frequency dependant in the position shown in Figure 2(b). The least variable cases are shown in Figure 2(c) and (d) where there is a good match between six measurements.

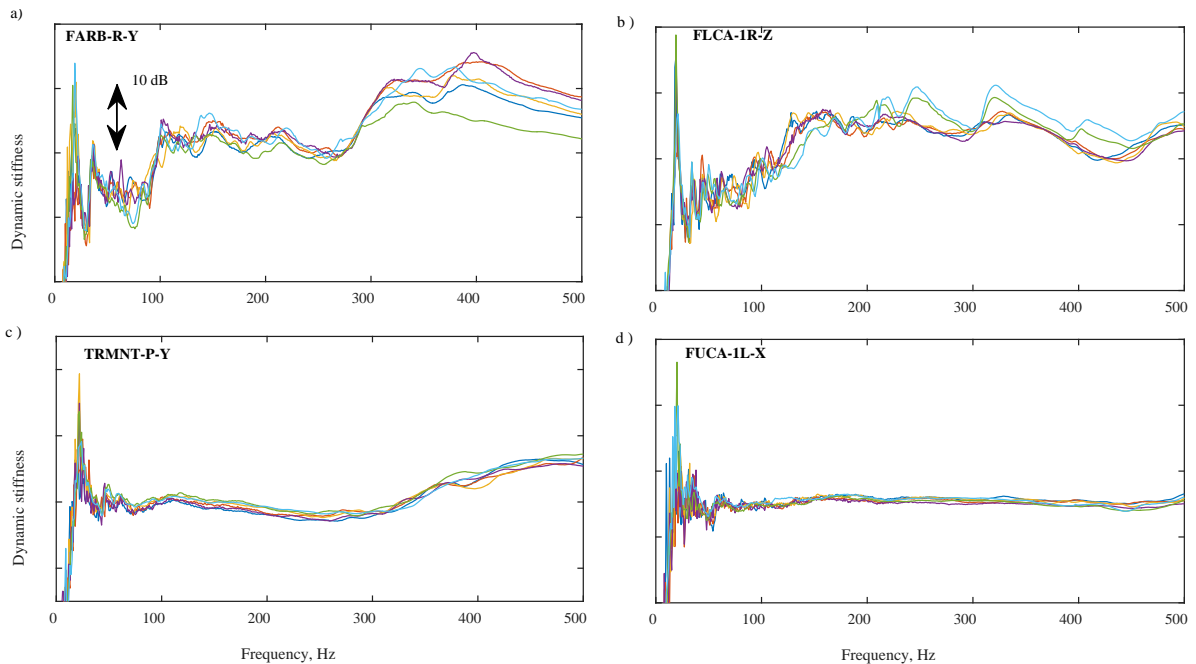


Figure 2: Dynamic stiffness measured at: a) Right front anti-roll bar mount measured in Y direction, b) Right front lower control arm position 1 measured in Z direction, c) Transmission mount measured in Y Direction and d) front left upper control arm position 1 measured in X direction.

Although it is possible to compare the measurements directly to observe the variability (as in Figure 2, which is subjective and prone to individual interpretation), this becomes impractical for large data sets (even with low sample numbers of vehicles), therefore a measure to rank the variability of different measurement points is required to make an analysis of data possible. The standard deviation is used as a means to quantify the variability here. Six different measurements exist for each dynamic stiffness. The standard deviation can be obtained by considering all the measurements at a specific frequency,

$$s(f) = \left( \frac{1}{n-1} \sum_{i=1}^n (FRF_i(f) - \overline{FRF}(f))^2 \right)^{\frac{1}{2}} \quad (1)$$

where  $s$  is the standard deviation,  $f$  is the frequency,  $n$  is the number of measurements,  $FRF_i$  refers to the dynamic stiffness of the  $i^{\text{th}}$  car, and  $\overline{FRF}(f)$  is the mean of the  $FRF$ s at a specific frequency and can be obtained from the following equation,

$$\overline{FRF}(f) = \frac{1}{n} \sum_{i=1}^n FRF_i(f) \quad (2)$$

The dimension of the standard deviation is that of the data, therefore, a normalised standard deviation is used here to make it possible to compare data from different sources. The mean value of the sample at each frequency is used to normalise the standard deviation,

$$\hat{s}(f) = \frac{s(f)}{\overline{FRF}(f)} \quad (3)$$

where  $\hat{s}$  is the normalised standard deviation as a function of frequency. The Root Mean Square (RMS) of the normalised standard deviation over frequency is used to obtain a single value for the variability for each measurement set. This allows comparing variabilities in dynamic stiffness that are represented by a single number. Equation (4) is used to obtain the RMS value of the standard deviation.

$$s_{RMS} = \sqrt{\frac{1}{m} \sum_{k=1}^m \hat{s}(f_k)^2} \quad (4)$$

where  $s_{RMS}$  is the root mean square of the standard deviation and  $m$  is the number of frequency points. If an average value over a frequency band is required then equation (4) can be used by limiting  $m$  to that specific band. This allows an assessment of the variability to be undertaken at different frequency bands for a single measurement or to compare with other measurements. In this paper, the data was analysed from 50 Hz to 500 Hz.

### 3. Variability in dynamic stiffness

The RMS values of the normalised standard deviation for dynamic stiffness are shown in Table 2. The following methodology is used in organising the table.

- i. The similar excitation points are colour coded together, for example four upper control arms are shown in blue.
- ii. The values of variability for each direction of excitation are sorted in ascending order in three columns of Table 2.
- iii. An identical colour map is used for values in three columns of RMS of standard deviation ( $s_{RMS}$ ) to make the comparison easier. The high values which appear at the top are marked with red background and the small values appear with blue background.
- iv. For each column, the mean, the standard deviation and the range of the values in that column are given at the bottom of the table.

Measurement points that are at close proximity have similar variability in most of the cases. The variability in dynamic stiffness is the highest in Y direction for the front anti roll bar (FARB), rear sub-frame (RSUB) and central bearing mounting positions (CTRBR). In X and Z direction, the maximum averaged normalised standard deviation is the same value of 0.175 implying the same maximum variability while by comparing standard deviations of values in each column it can be noticed that the distribution of variability values ( $s_{RMS}$ ) for Z direction is much lower than X and Y direction.

The lowest variability in dynamic stiffness belongs to the front upper control arm in X direction (FUCA). The variability in dynamic stiffness measured at the transmission mount (TRMNT\_P) is amongst the lowest in all three directions. For dynamic stiffness measured at the central bearing mounting position the level of variability is similar but it is higher than the front control arm (FUCA) in the X direction. In the Y direction, the variability in the front upper control arms (FUCA) is almost three times those value in the X direction but still they have lower variability compared to other components.

Table 2: RMS value of Standard Deviation for Dynamic stiffness of different measurement points sorted based on the variability value.

No.	Excitation Point in X.	$S_{RMS}$	No.	Excitation Point in Y.	$S_{RMS}$	No.	Excitation Point in Z.	$S_{RMS}$
1	ENGN_1L_X	0.175	1	FARB_R_Y	0.209	1	FLCA_1R_Z	0.175
2	FARB_R_X	0.17	2	RSUB_1R_Y	0.198	2	FSUS_1L_Z	0.136
3	RSUB_2R_X	0.161	3	RSUB_1L_Y	0.194	3	FLCA_1L_Z	0.133
4	FSUS_1L_X	0.161	4	CTRBR_L_Y	0.191	4	CTRBR_R_Z	0.126
5	CTRBR_R_X	0.158	5	FSUS_1R_Y	0.16	5	FARB_L_Z	0.125
6	RSUS_1L_X	0.155	6	FUCA_2R_Y	0.15	6	FUCA_1R_Z	0.123
7	RSUS_1R_X	0.152	7	CTRBR_R_Y	0.15	7	FARB_R_Z	0.119
8	RSUB_2L_X	0.147	8	FARB_L_Y	0.149	8	ENGN_1L_Z	0.117
9	FSUS_1R_X	0.14	9	FUCA_1R_Y	0.149	9	FSUS_1R_Z	0.116
10	RSUB_1R_X	0.125	10	RSUS_1R_Y	0.13	10	FUCA_2R_Z	0.116
11	FLCA_1L_X	0.12	11	FLCA_1L_Y	0.128	11	CTRBR_L_Z	0.116
12	FARB_L_X	0.116	12	FSUS_1L_Y	0.124	12	ENGN_1R_Z	0.114
13	FLCA_1R_X	0.112	13	FUCA_1L_Y	0.123	13	RSUB_1R_Z	0.114
14	RSUB_1L_X	0.111	14	ENGN_1L_Y	0.123	14	FUCA_1L_Z	0.113
15	FLCA_2R_X	0.081	15	FLCA_1R_Y	0.119	15	FLCA_2L_Z	0.113
16	FUCA_2R_X	0.075	16	FLCA_2R_Y	0.116	16	RSUB_2R_Z	0.112
17	ENGN_1R_X	0.074	17	FUCA_2L_Y	0.115	17	RSUB_2L_Z	0.112
18	CTRBR_L_X	0.067	18	ENGN_1R_Y	0.113	18	FLCA_2R_Z	0.109
19	FLCA_2L_X	0.058	19	FLCA_2L_Y	0.111	19	RSUB_1L_Z	0.105
20	TRMNT_P_X	0.056	20	RSUB_2L_Y	0.11	20	TRMNT_P_Z	0.104
21	FUCA_1R_X	0.056	21	RSUS_1L_Y	0.104	21	RSUS_1L_Z	0.099
22	FUCA_2L_X	0.049	22	RSUB_2R_Y	0.099	22	FUCA_2L_Z	0.093
23	FUCA_1L_X	0.045	23	TRMNT_P_Y	0.062	23	RSUS_1R_Z	0.084
Mean		0.111	Mean		0.136	Mean		0.116
St. dev.		0.044	St. dev.		0.036	St. dev.		0.017
Range		0.13	Range		0.146	Range		0.091

In general in the X direction, it seems that the variability in the rear of the vehicle is higher than the front. In the Y direction, the variability of the measurement points on the front of the vehicle occupy the middle of the table while the measurements on different points on the rear section of the vehicle

occupy the top and bottom of the table, noticeably, the variability in dynamic stiffness measured at rear sub-frame mounting points 1 left and right (RSUB\_1R\_Y and RSUB\_1L\_Y) have a high value of variability of 0.19 while those measured at point 2 have a variability of half the former ones of about 0.1. In the Z direction, the variability in the front is higher than the rear which is in contrast to the X direction.

#### **4. Conclusions**

The variability in the dynamic response of a vehicle is assessed by analysing a set of measurements that were provided by Jaguar Land Rover. Dynamic stiffness was measured at twenty-three different mounting points in three translational directions for five nominally identical cars. The frequency range of measurements was limited to 500 Hz at which interior structure-borne noise is the dominant source. The normalised standard deviation is used as a measure of variability, averaged over frequency. The analysis provided insight into the nature of the variability. Dynamic stiffnesses with measurement points that are at close proximity have similar variability. In general, the variability was higher in Y direction, which could indicate that the forcing direction was more difficult, thus increasing the experimental measurement error. The range of variability between different measurement points was higher in Y direction as well. The values for variability were closest together in Z direction with a mean value that was similar to measurements in X direction and smaller than the mean of the variability in Y direction. The rear of the car was more variable in X direction in contrast to Z direction which could possibly be due to the need for increased body stiffness near to the engine connection points.

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