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HEALING CAPILLARY FILMS

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Summary We investigate, by means of theoretical arguments, numerical simulation and numerics, the closing of a circular cavity (healing) in a thin liquid film. We assume that the process is dominated by capillary forces. The final stages of the evolution can be described by means of self-similar solutions to the problem. A comparison with experimental data is also presented.

INTRODUCTION AND MAIN RESULTS

We study the healing process of a viscous thin film driven by surface tension. Such phenomena occur in lots of physical and industrial processes [1], [2]. The effect of surface tension suggests a fourth-order nonlinear partial differential equation that has completely different behaviours from the nonlinear diffusion equation for the buoyancy-driven processes. The novel part of this study is to seek a self-similar solution for the fourth-order equation that describes the dynamics of the film thickness.

We consider a converging thin film driven by surface tension in axisymmetric geometry. We assume that the air-fluid interface is long and thin, so the flow is mainly one-dimensional, and the lubrication approximation holds. We assume that initially viscous fluid of height $\tilde{h}_0$ fills the gap between $\tilde{r}_0$, the location of a lock gate, and $\tilde{r}_{out}$, the location of an outer boundary; or equivalently, the thin film spreads toward the origin ($\tilde{r} = 0$) from an initial condition that takes the form of a step function. After defining dimesionless variables, one can obtain the following partial differential equation:

$$\frac{\partial h}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)\right)\right) = 0. \tag{1}$$

We now look for a self-similar solution for equation (1) of the form

$$h(r,t) = (t_0 - t)^{\alpha} f\left(\xi \equiv \frac{r}{(t_0 - t)^{\beta}}\right), \tag{2}$$

where t_0 denotes the dimensionless time for the (circular) front of the air-fluid interface to reach the origin ($r = 0$). Immediately, from dimensional analysis, the form of solution (2) suggests that $\alpha = \frac{(4\beta-1)}{3}$, and we can rewrite equation (1) as an ordinary differential equation (ODE):

$$-\frac{(4\beta-1)}{3}f + \beta\xi\frac{\mathrm{d}f}{\mathrm{d}\xi} + \frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi f^3\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi\frac{\mathrm{d}f}{\mathrm{d}\xi}\right)\right)\right) = 0. \tag{3}$$

We look now at a distance R from the center of the hole. Since fluid has to enter through $r = R$ in order to fill the hole, a certain flow rate $J = 2\pi rh^3\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)\right)$ has to be established and it is natural to assume that it is constant near t_0. This leads to an exponent $\beta = \frac{2}{5}$. On the other hand, the contact line must move with a nonzero velocity $v = h^2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)\right)$. Numerical evidence suggests the presence of a region near the contact line where such condition implies $\beta \approx 0.48\ldots$.

Numerical evidence

We solved (1) with no flux boundary condition at $r = 10$ and an initial step at $r = 1$. The evolution is sketched in figure 1(a). In figure 1(b) we represent the same profiles rescaled according to the similarity exponent $\beta = \frac{2}{5}$ and comparison with the similarity solution obtained from equation (3).

EXPERIMENTS

Laboratory experiments have been designed and conducted to verify the self-similar solutions we obtained from the theoretical model. We first prepare a clean oil-wetting slide glass as the flat substrate. We then place a Teflon container above the slide glass which forms a non-wetting circular outer boundary. A plastic cylinder is then placed at the center of the circular area created by the Teflon container, and used as a cylindrical lock gate in our experiments.

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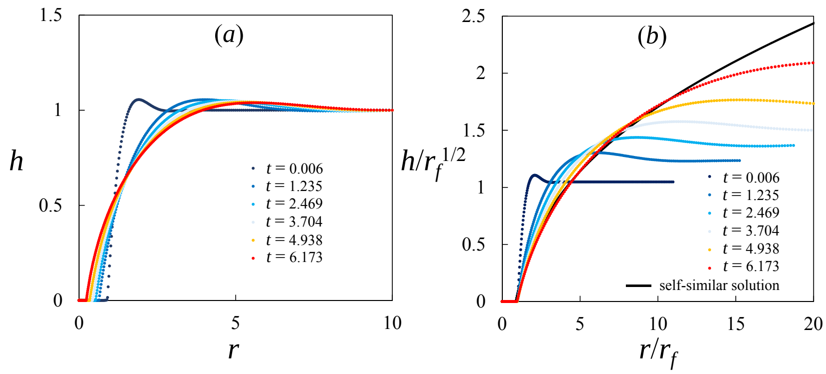

Figure 1: Time evolution of the profile shape from direct numerical simulation: (*a*) raw data; (*b*) rescaled data. The collapse of the profile shapes occurs near the location of the propagating front. The black curve represents the prediction from the self-similar solution with $\beta = 2/5$.

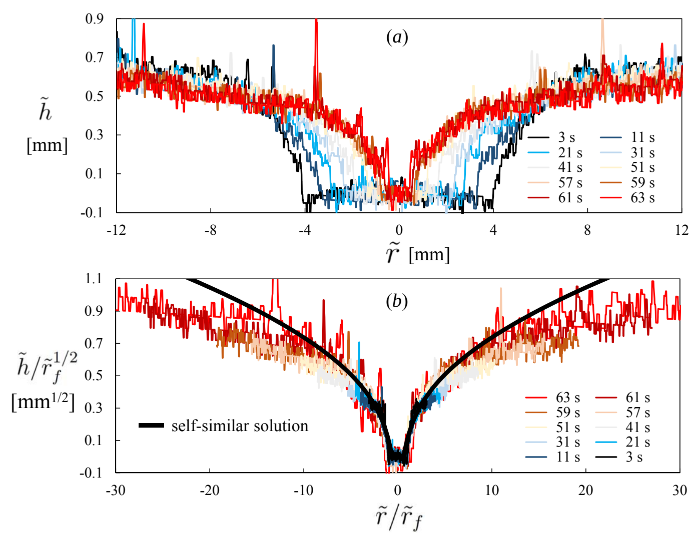

Figure 2: Time evolution of the shape of the fluid-fluid interface: (*a*) raw experimental data, and (*b*) rescaled experimental data.

In each experiment, we first fill the gap between the lock gate (plastic cylinder) and the outer boundary (Teflon container) with a viscous fluid (e.g., silicone oil). Upon the removal of the lock gate, the fluid film spreads toward the center, driven by surface tension. A digital camera (Nikon 7100) is placed right above the setup, and takes pictures of the healing thin films from the top. The propagating front appears to be circular, and we can measure the radius of the circular front $\tilde{r}_f(\tilde{t})$ as a function of time $\tilde{t}$.

We use different silicone oils with varying viscosities (e.g., 100 cst, or 500 cst) as the working fluid. In our post-experimental data analysis, based on the calibration, we are able to conduct the interface shapes $\tilde{h}(\tilde{r}, \tilde{t})$ at different times $\tilde{t}$. The profile shapes from a representative experiment are shown in figure 2*a*. Silicone oil of viscosity $\mu \approx 100$ cst and surface tension $\gamma \approx 20$ mN/m was used in this experiment. The time for the front to reach the origin is $\tilde{t} \approx 40$ s. The rescaled profile shape based on our theoretical arguments is also shown in figure 2*b*. The theoretical prediction from the self-similar solution is also plotted as the black curve in figure 2*b*.

References

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