Partition of Mixed-Mode Fractures in 2D Elastic Beams with Through-Thickness Shear Forces

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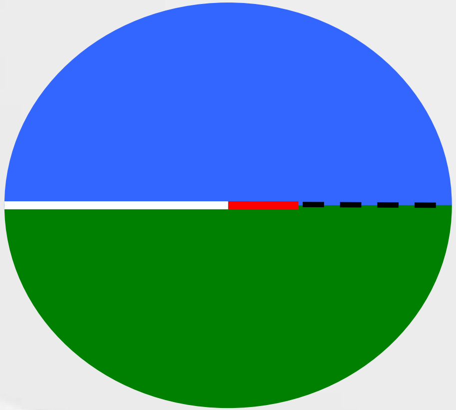
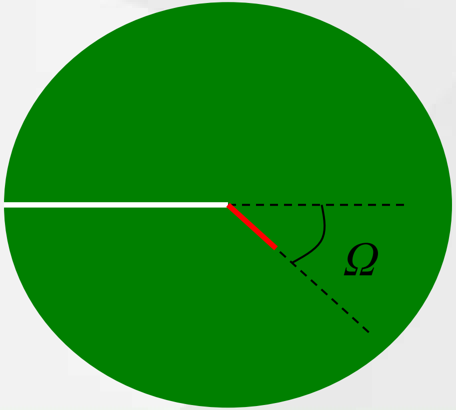
Interfacial cracks

- Cracks tend to propagate along interfaces in laminated materials because they **represent a plane of weakness**.

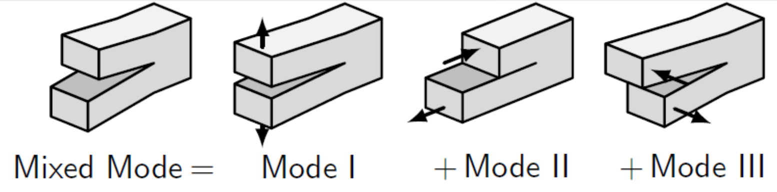


- They **do not kink** in order to propagate under pure mode I opening conditions, as they would tend to in an isotropic material.

- Interfacial cracks therefore propagate in a **mixed-mode** with a combination of mode I opening, mode II shearing, and/or mode III tearing.



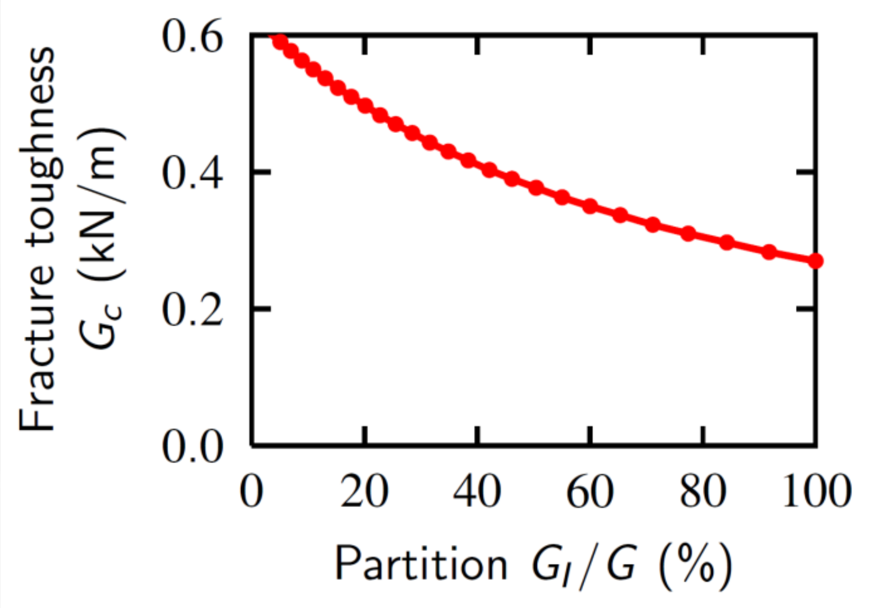
Fracture toughness



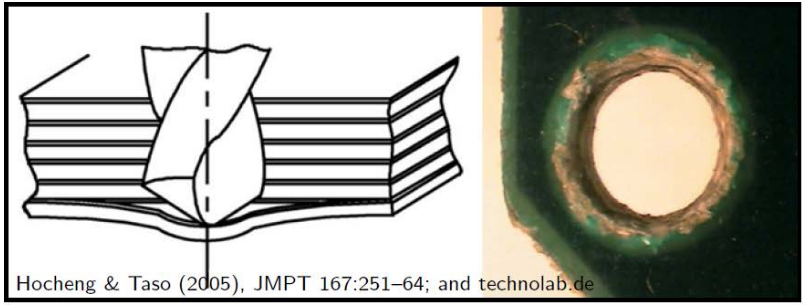
- Fracture toughness depends on the fracture mode partition.

- Predicting fracture toughness requires the knowledge of the partition of a mixed-mode fracture.

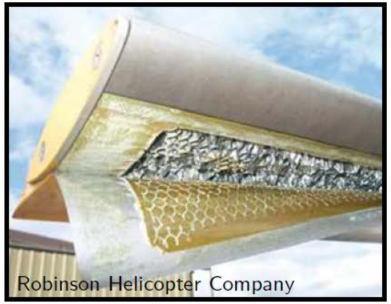
- Essential to have a correct analytical partition theory to predict the fracture toughness.



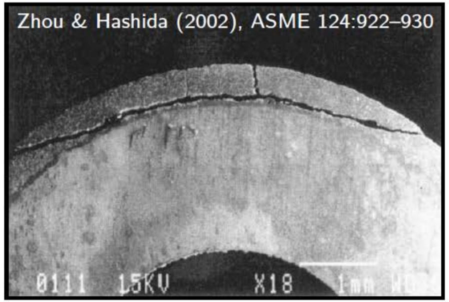
One-dimensional fractures



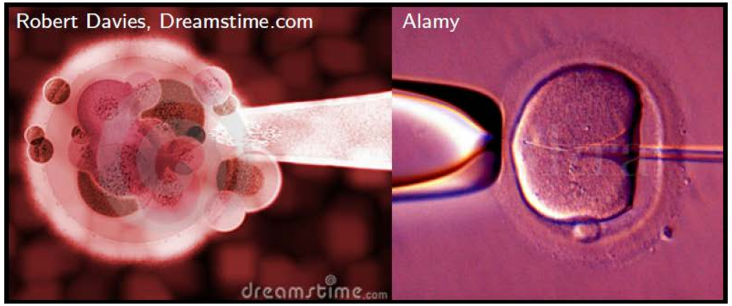
▲ Delamination during drilling



Helicopter blade delamination ▲



▲ Thermal barrier cracking

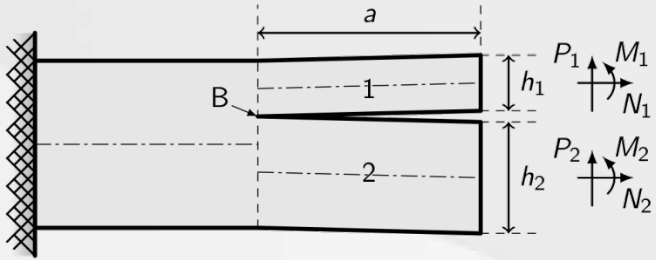


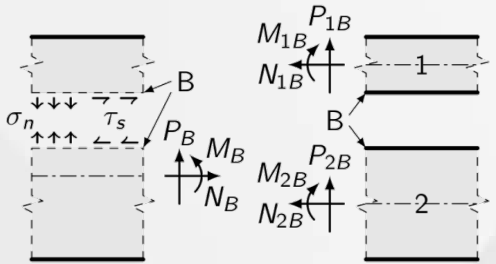
Needle puncture of red blood cell/IVF treatment ▲

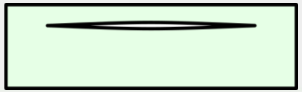
Mixed-mode interfacial fracture

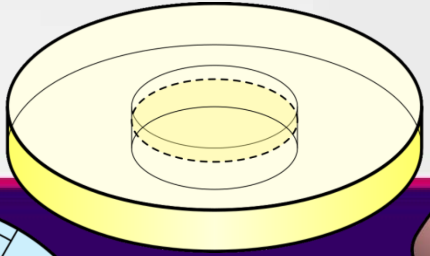
- 1D fracture of DCB is fundamental case for study
  - Bending moments M_1 and M_2
  - Axial forces N_1 and N_2
  - Shear forces P_1 and P_2
  - $\eta = E_2/E_1$, $\mathrm{N} = \nu_2/\nu_1$, $\gamma = h_2/h_1$

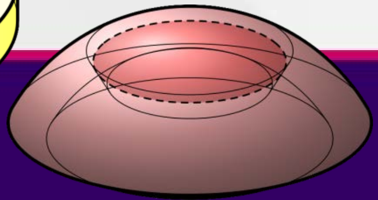
Double cantilever beam (DCB)









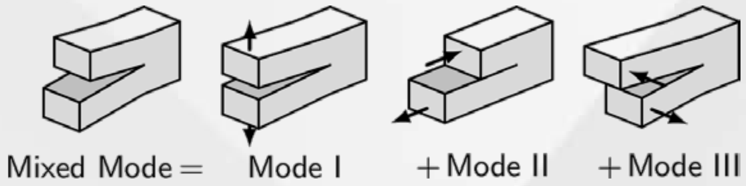


Total energy release rate (ERR)

- **Quadratic form** and **non-negative definite**
- Partition total ERR G into its **pure mode components**, G_I and G_{II}
- Use the **orthogonal pure fractures modes**

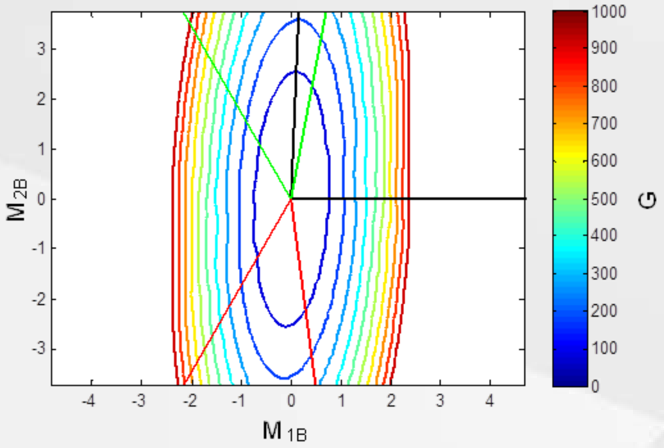
$$G = \begin{Bmatrix} M_{1B} \\ M_{2B} \\ N_{1B} \\ N_{2B} \end{Bmatrix}^T [C] \begin{Bmatrix} M_{1B} \\ M_{2B} \\ N_{1B} \\ N_{2B} \end{Bmatrix}$$

$$C_{ij} = f\left(E_1, E_2, \nu_1, \nu_2, h_1, h_2, b\right)$$



Pure fracture modes

Contours of ERR with $E = 1$, $b = 1$, $h = 1$, $\gamma = 1$, $\eta = 1$



- The inner product matrix transforms the $\{M_{1B} \quad M_{2B}\}$ vectors into ERR space
- In ERR space, orthogonality between two $\{M_{1B} \quad M_{2B}\}$ vectors means

$$\{M_{1B} \quad M_{2B}\}_1 [C] \{M_{1B} \quad M_{2B}\}_2{}^T = 0$$

- Orthogonal pairs of $\{M_{1B} \quad M_{2B}\}$ vectors exist that represent pure fracture modes
  - Denote pure mode I as $\{1 \quad M_{2B}/M_{1B}\} = \{1 \quad \theta_1\}$
  - Denote pure mode II as $\{1 \quad M_{2B}/M_{1B}\} = \{1 \quad \beta_1\}$, etc.
  - With θ_i and $\beta_i = f(E_1, E_2, \nu_1, \nu_2, h_1, h_2, b)$

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ERR partitions general theory

- Euler beam partitions: $G_{IE} = c_{IE}\left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3}\right)\left(M_{1B} - \frac{M_{2B}}{\beta_1'} - \frac{N_{1B}}{\beta_2'} - \frac{N_{2B}}{\beta_3'}\right)$

$$G_{IIE} = c_{IIE}\left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3}\right)\left(M_{1B} - \frac{M_{2B}}{\theta_{1'}} - \frac{N_{1B}}{\theta_{2'}} - \frac{N_{2B}}{\theta_{3'}}\right)$$

- Timoshenko beam partitions:

$$G_{IT} = c_{IT}\left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_{1B}}{\beta_2} - \frac{N_{2B}}{\beta_3} - \frac{P_{1B}}{\beta_4} - \frac{P_{2B}}{\beta_5}\right)^2 \qquad G_{IIT} = c_{IIT}\left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_{1B}}{\theta_2} - \frac{N_{2B}}{\theta_3} - \frac{P_{1B}}{\theta_4} - \frac{P_{2B}}{\theta_5}\right)^2$$

- 2D elasticity partitions:

$$G_{I\text{-2D}} = c_{I\text{-2D}}\left(M_{1B} - \frac{M_{2B}}{\beta_{1\text{-2D}}} - \frac{N_{1B}}{\beta_{2\text{-2D}}} - \frac{N_{2B}}{\beta_{3\text{-2D}}} - \frac{P_{1B}}{\beta_{4\text{-2D}}} - \frac{P_{2B}}{\beta_{5\text{-2D}}}\right)^2 \qquad G_{II\text{-2D}} = c_{II\text{-2D}}\left(M_{1B} - \frac{M_{2B}}{\theta_{1\text{-2D}}} - \frac{N_{1B}}{\theta_{2\text{-2D}}} - \frac{N_{2B}}{\theta_{3\text{-2D}}} - \frac{P_{1B}}{\theta_{4\text{-2D}}} - \frac{P_{2B}}{\theta_{5\text{-2D}}}\right)^2$$

General 2D elasticity partition theory

- Bending moments M_{1B} and M_{2B} and axial forces N_{1B} and N_{2B}
- Revisit the orthogonal pure fracture modes (θ_i, β_i)
  - Condition using beam theories does not produce the same stress distribution in 2D elasticity theory
  - Apply a correction factor for 2D elasticity to the part of the condition that represents the intact portion of the beam
  - Calibrate correction factor for θ_{1-2D} using $\theta_1 \leq \theta_{1-2D} \leq \theta_1'$
  - Obtain other pure modes (θ_{2-2D} , β_{1-2D} , β_{2-2D} , etc.) using orthogonality

Timoshenko beam partition theory

- Crack tip through-thickness shear forces P_{1B} and P_{2B} only
  - $M_{1B} = M_{2B} = N_{1B} = N_{2B} = 0$

$$G_{\theta_{P-T}} = \frac{1}{2b^2 h_1 \kappa\mu}\left(1 + \frac{\theta_{P-T}^2}{\gamma}\right) \qquad G_{\beta_{P-T}} = \frac{1}{2b^2 h_1 \kappa\mu}\left(1 + \frac{\beta_{P-T}^2}{\gamma} - \frac{(1+\beta_{P-T})^2}{1+\gamma}\right)$$

- $(\theta_{P-T}, \beta_{P-T}) = (-1, \gamma) \quad \therefore \quad G_{II} = 0$
- Shear correction factor $\kappa = 5/6$

2D elasticity partition theory

- Crack tip through-thickness shear forces P_{1B} and P_{2B} only
  - $M_{1B} = M_{2B} = N_{1B} = N_{2B} = 0$

$$G_{\theta_{P-2D}} = \frac{1}{2b^2 h_1 \kappa(\gamma)\mu}\left(1 + \frac{\theta_{P-2D}^2}{\gamma}\right), G_{\beta_{P-2D}} = \frac{1}{2b^2 h_1 \kappa(\gamma)\mu}\left(1 + \frac{\beta_{P-2D}^2}{\gamma} - \frac{(1+\beta_{P-2D})^2}{1+\gamma} c(\gamma)\right)$$

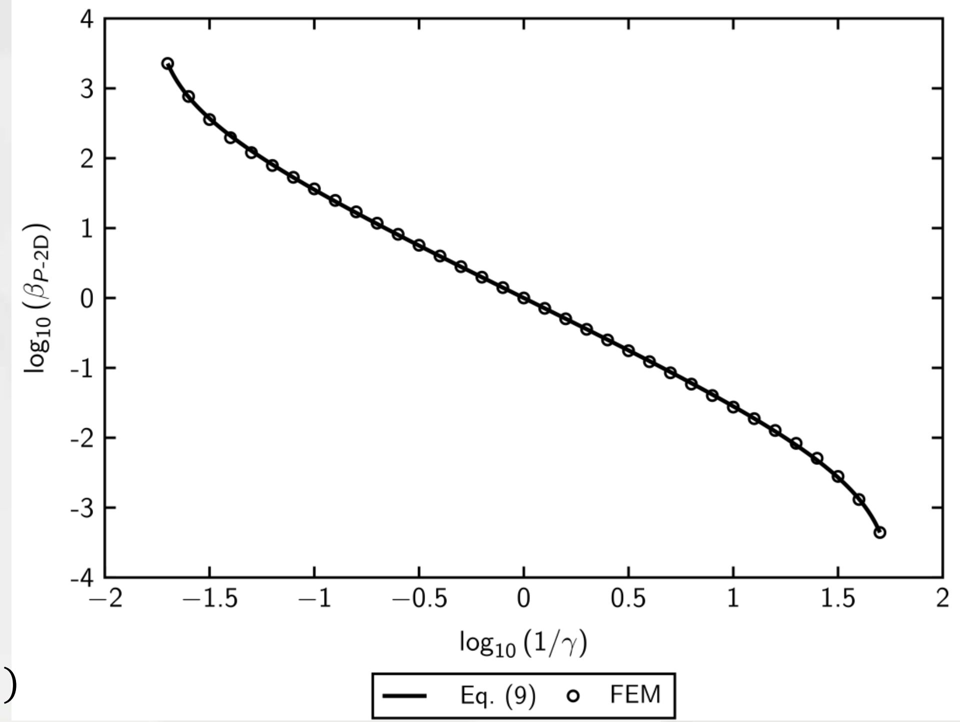
- $(\theta_{P-2D}, \beta_{P-2D}) = (??, \ ??)$
- Shear correction factor now γ dependent $\kappa(\gamma)$
- $G_{II} \neq 0$ and introduce pure-mode-II correction factor $c(\gamma)$

Shear Force Pure Modes

- $(\theta_{P-2D}, \beta_{P-2D})$
- FEM simulations
- $-1.7 \le \log_{10}(1/\gamma) \le 1.7$

- Pure mode I θ_{P-2D}
  - $G_{II} = 0, \;\; \theta_{P-2D} = -1$
  - $\therefore P_{2B} = -P_{1B}$

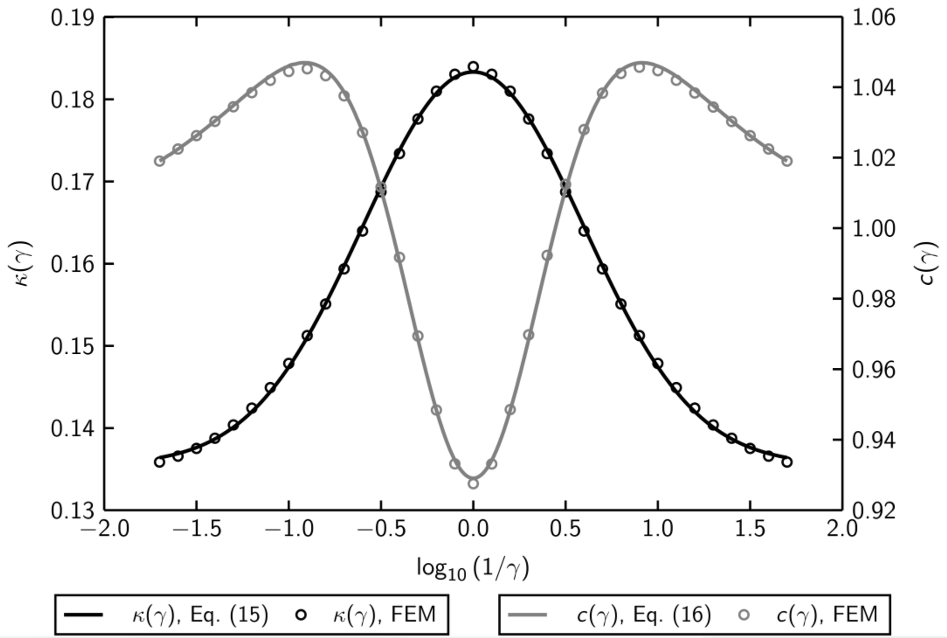
- Pure mode II β_{P-2D}
  - $G_I = 0$
  - $\beta_{P-2D} = \gamma \exp(-1.986060 \, \text{atanh}(0.563483\gamma_i))$



Shear & Pure Mode II Correction Factors

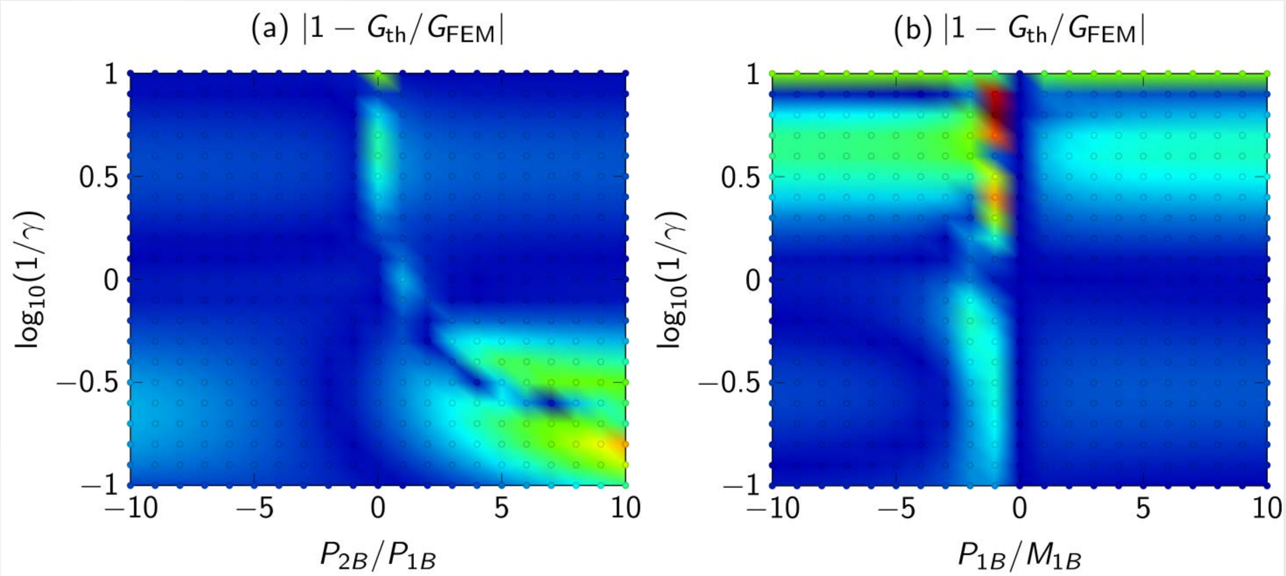
- FEM Simulations
- $M_{1B} = M_{2B} = N_{1B} = N_{2B} = 0$
- $-1.7 \le \log_{10}(1/\gamma) \le 1.7$

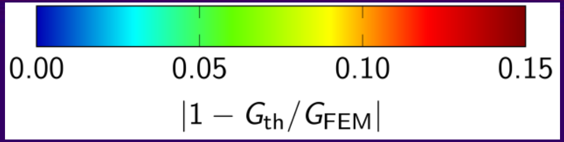
- Shear Correction Factor
  - $\kappa(\gamma)$
  - $P_{2B}/P_{1B} = \theta_{P-2D} = -1$
- Pure-mode-II ERR Correction Factor
  - $c(\gamma)$
  - $P_{2B}/P_{1B} = \beta_{P-2D}$



Numerical Verification

$$\frac{1}{10} \leq \gamma \leq 10$$





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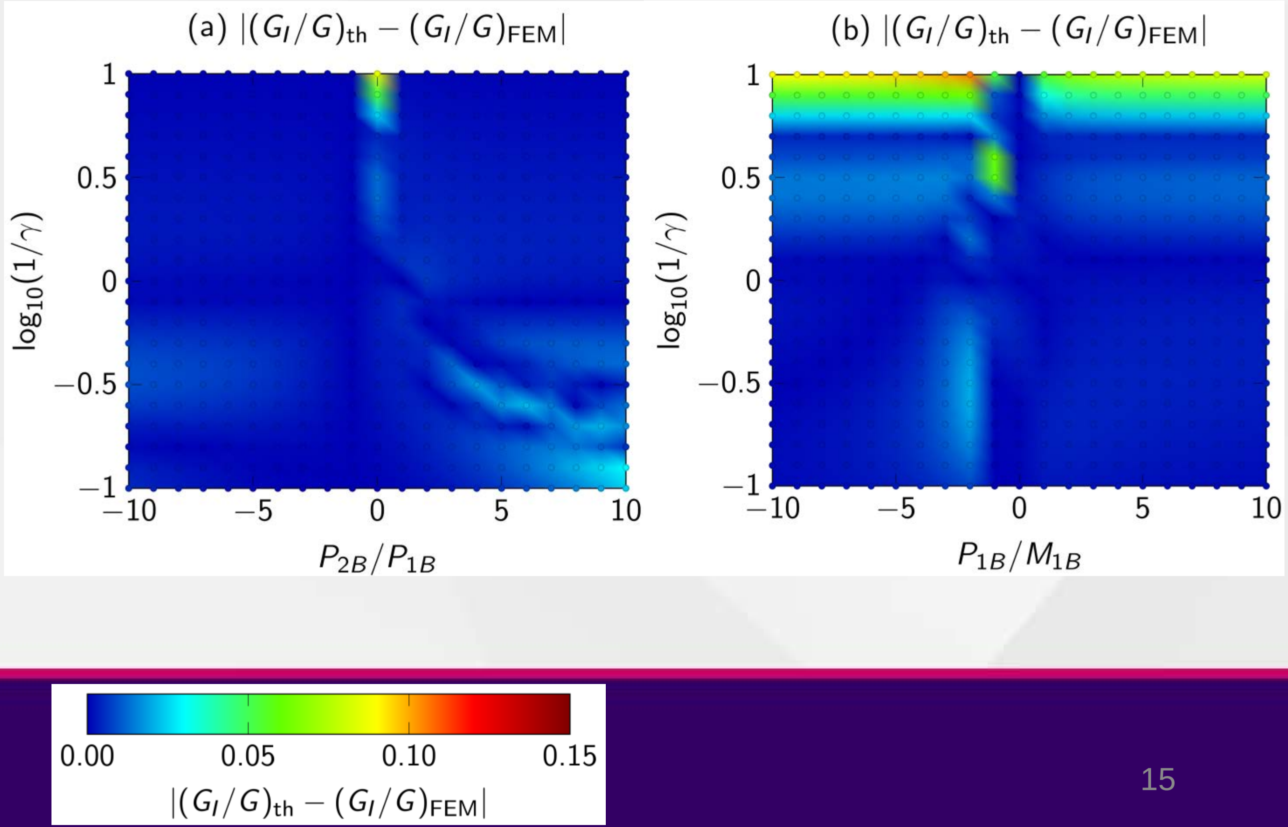
Numerical Verification

$$\frac{1}{10} \leq \gamma \leq 10$$



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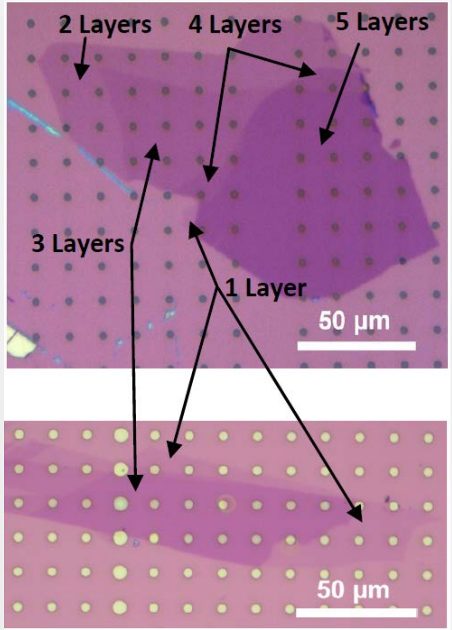
Blister Test

- Interface fracture toughness



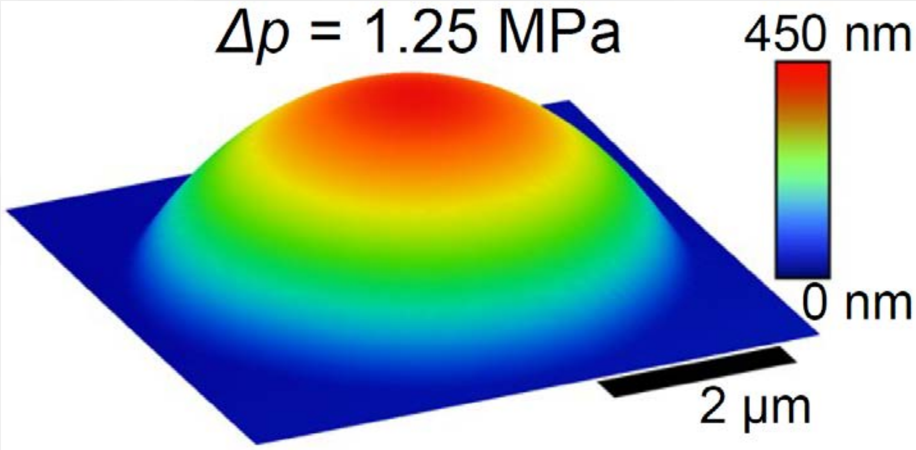
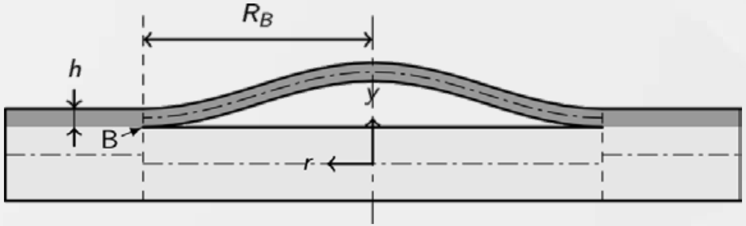
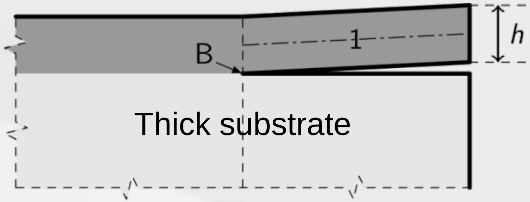


Image from Koenig (2011)

Adhesion of graphene membranes

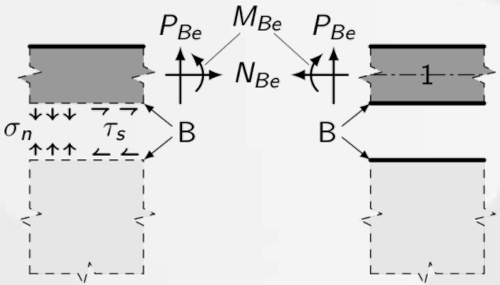
$$\gamma = h_2/h_1 \to \infty$$





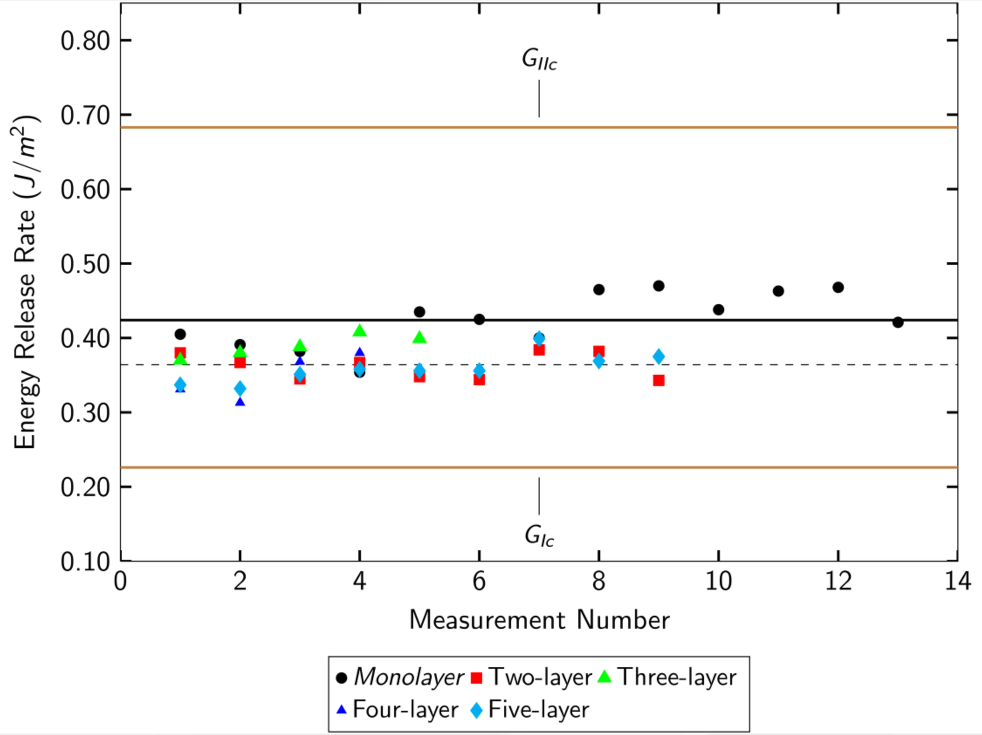
$$G_I = \frac{6M_{Be}^2}{Eh^3}(1-\nu^2)\left(1 - \frac{N_{Be}h}{4.450M_{Be}} - \lambda\right)^2 0.6227$$

$$G_{II} = \frac{6M_{Be}^2}{Eh^3}(1-\nu^2)\left(\frac{N_{Be}h}{2.697M_{Be}}\right)^2 0.3773$$



Adhesion of graphene membranes

- Pressure loaded blister test
  - Linear failure critierion
  - $G_{Ic} = 0.226 \ J/m^2$
  - $G_{IIc} = 0.683 \ J/m^2$



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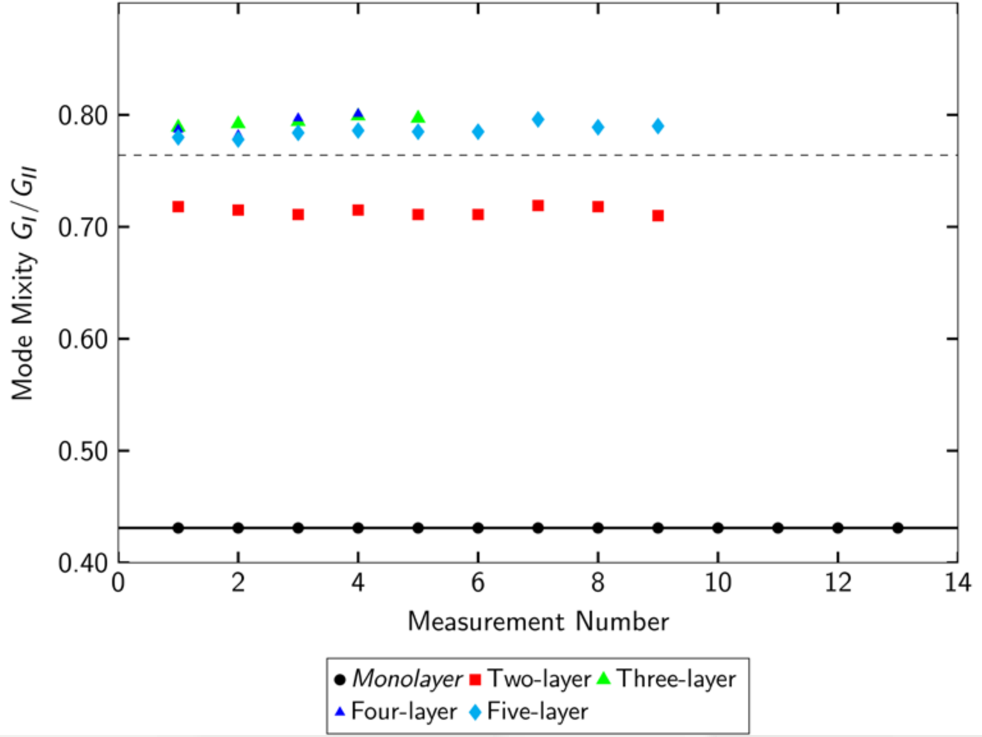
Adhesion of graphene membranes

- Pressure loaded blister test
  - Linear failure critierion
  - $G_{Ic} = 0.226\ J/m^2$
  - $G_{IIc} = 0.683\ J/m^2$

  - $\rho_{mono} = G_I/G_{II} = 0.431$
  - $\rho_{multi} = G_I/G_{II} = 0.764$



Experimental validation

- Pressure loaded blister test – Koenig et al. (2011)
  - Linear failure critierion
  - $G_{Ic} = 0.226\ J/m^2$ and $G_{IIc} = 0.683\ J/m^2$

- Point loaded blister – Zong et al. (2012)
  - Experimental Results
  - $\delta/R_B = 0.2309$, $E = 1\text{TPa}$, $nt = 1.7\text{nm}$ and $n = 5$.
  - $G_{exp} = 0.438\ J/m^2$
  - Mode mixity $\rho_{th} = G_I/G_{II} = 0.381$
  - Linear failure criterion $G_{th} = 0.438\ J/m^2$

Conclusion

- 2D elasticity partition theory
  - Developed for general loading conditions (bending moments, axial forces and shear forces).
  - Numerically verified for a number of loading conditions
- Application to:
  - Adhesion of graphene membranes
  - Adhesion energy has been explained and well-predicted

Thank you very much for your attention

Questions are now welcome

- Submitted for publication at Composite Structures
  - Partition of mixed-mode fractures in 2D elastic orthotropic laminated beams under general loading (2016).