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Corresponding Author: Mr. Trevor Davies,

Corresponding Author's Institution: Loughborough University

First Author: Trevor Davies

Order of Authors: Trevor Davies

Abstract: This is essentially a review paper describing progress made in treating mechanisms and machines as networks. Some of the terminology that is helpful to this approach is explained. Relevant elements of graph theory are mentioned. The original aim was to find a robust procedure for finding the instantaneous relative motion of all pairs of bodies within a kinematic chain. The manner in which this was achieved produced several other results that have found unanticipated applications. These are mentioned and publications are cited. Lessons have been learned and these are discussed in Section 11.

A network approach to mechanisms and machines: some lessons learned

Paper MECHMT-D-14-00258

T. H. Davies, (mcthd@lboro.ac.uk)

Highlights

- Graph theory is used to assemble matrices in adaptations of Kirchhoff's equations.
- Those matrices, when transposed, are used again in virtual power equations.
- 25 publications are cited that make use of these equations.
- 10 lessons learned are explained in a discussion section.
- One lesson introduces dual laws: the zeroth laws of mechanics.

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A network approach to mechanisms and machines: some lessons learned

(An abbreviated title of fewer than 40 characters, including spaces: A network approach to MMT)

T.H.Davies[1], Wolfson School of Manufacturing and Mechanical Engineering, Loughborough University, Loughborough, Leicestershire, UK, LE11 3TU

Abstract

This is essentially a review paper describing progress made in treating mechanisms and machines as networks. Some of the terminology that is helpful to this approach is explained. Relevant elements of graph theory are mentioned. The original aim was to find a robust procedure for finding the instantaneous relative motion of all pairs of bodies within a kinematic chain. The manner in which this was achieved produced several other results that have found unanticipated applications. These are mentioned and publications are cited. Lessons have been learned and these are discussed in Section 11.

Keywords:

circuit; constraint; cutset; freedom; graph; screw

1. Introduction

The author is glad of this opportunity to thank Erskine Crossley for his many acts of kindness and generosity and to join with others to pay tribute to the work he has done for IFToMM and as editor of the *Journal of Mechanisms*, the forerunner of this journal. In particular, the author can bear witness to the many contributions Erskine Crossley made to good international relations. But this is a technical paper and so it is appropriate to explain the stimulus Erskine Crossley provided that led to research interests of the author.

Erskine Crossley was the first to mention graph theory in the author's presence. Graph theory [1] [2] is a branch of topology concerned with the interconnections within a network of objects. Graph theory has found many applications; most relevant to this paper are applications in electrical network theory, more frequently called electrical circuit theory.

[1] mcthd@lboro.ac.uk (Trevor Davies),

Mechanism and machines can be thought of as coupling networks. Waldron [3] provides rules that apply to couplings arranged in series and in parallel. Like electrical networks, indirect couplings containing cross bracing pose special problems [4]. Baker [5] proposed a simple example that has subsequently proved well-suited as a demonstration for theories that have followed. One solution [6] required the adaptation of Kirchhoff’s voltage law. Subsequent work [7] [8][2] [9] [10] [11] [12] has led to the adaptation of Kirchhoff's current law as well, and two virtual power equations that use matrices that are identical to those needed for the adaptations of Kirchhoff's laws except for being transposed. All four equations are reproduced in this paper; the adaptations of Kirchhoff's laws equations (1,2) in section 7.2 and the virtual power equations (3,4) in section 8.2. Several applications have been found for the equations [13] [14] [15] [16] [17] [18] [19] [20] [21] [22] [23] [24] [25] [26] [27] [28] [29] [30] [31]; further details are provided in section 10.

Nomenclature

a	the rank of the network unit action matrix $\left\lfloor \hat{\mathbf{A}}_N \right\rfloor_{dk,C}$
b_{ij}	the element in row i, column j, of circuit matrix $[\mathbf{B}_M]_{l,F}$
c	degree of constraint of a direct coupling
c_{ij}	degree of constraint of bodies i and j of a coupling network
C	gross degree of constraint of a coupling network, Σc
C_N	nett degree of constraint of a coupling network
d	minimum order of the screw system, $1 \leq d \leq 6$
e	number of couplings in a coupling network and edges of coupling graph G_C
f	gross degree of freedom of a direct coupling
f_{ij}	degree of freedom of bodies i and j of a coupling network
F	gross degree of freedom of a coupling network, Σf
F_N	nett degree of freedom of a coupling network
k	number of independent cutsets of a graph
l	number of independent circuits (loops) of a graph
m	the rank of the network unit motion matrix $\left\lfloor \hat{\mathbf{M}}_N \right\rfloor_{dl,F}$
n	number of bodies in a coupling network and nodes of coupling graph G_C
q_{ij}	the element in row i, column j, of cutset matrix $[\mathbf{Q}_A]_{k,C}$
$\{r, s, t; u, v, w\}$	motion screw components in ray-coordinates
$\{R, S, T; U, V, W\}$	action screw components in axis-coordinates

Vectors

$[\mathbf{A}_l]_{dl}$	dl action components for all l circuits
$[\mathbf{M}_k]_{dk}$	dk motion components for all k cutsets
$[\varphi]_C$	magnitudes of C action screws
$[\psi]_F$	magnitudes of F motion screws

[2] Online versions of papers [8, 10-12, 17, 32] can be found from the website of Loughborough University, UK: search for Library, Institutional Repository, Author, Davies, T.H.

Matrices

$\lfloor \hat{\mathbf{A}}_D \rfloor_{d,C}$	unit action matrix of the direct couplings of a coupling network
$\lfloor \hat{\mathbf{A}}_N \rfloor_{dk,C}$	network unit action matrix of a coupling network N
$[\mathbf{B}_i]_{F,F}$	diagonal matrices with diagonal elements corresponding to row *i* of $[\mathbf{B}_M]_{l,F}$; in practice identification is by the circuit label, e. g. $[\mathbf{B}_b]_{F,F}$ for circuit b.
$[\mathbf{B}_M]_{l,F}$	circuit matrix of motion graph G_M
$\lfloor \hat{\mathbf{M}}_D \rfloor_{d,F}$	unit motion matrix of the direct couplings of a coupling network
$\lfloor \hat{\mathbf{M}}_N \rfloor_{dl,F}$	network unit motion matrix of a coupling network N
$[\mathbf{Q}_i]_{C,C}$	diagonal matrices with diagonal elements corresponding to row *i* of $[\mathbf{Q}_A]_{k,C}$; in practice, identification is by the cutset label, e. g. $[\mathbf{Q}_a]_{C,C}$ for cutset *a*.
$[\mathbf{Q}_A]_{k,C}$	cutset matrix of action graph G_A

2. Couplings

Central to the network approach described in this paper is the *coupling*. This term is applied to any means by which an *action* can be transmitted between two bodies that are sufficiently stiff to be regarded as rigid. Furthermore, a coupling must be capable of being disassembled without resort to cutting. This means that welded and riveted joints are not regarded as couplings, nor are joints formed by adhesion. Action is a term that is sometimes used [11] [12] [32] as shorthand for a wrench on a screw of any pitch, including a pitch that is zero, namely a force, and a pitch that is infinite, namely a torque. The coupling could be either direct, indirect or a hybrid comprising direct and indirect couplings in parallel. Except where it is necessary to make a distinction, all couplings mentioned are direct couplings. The term coupling is chosen as the name of a superset comprising passive and active couplings, the latter providing sinks or sources of power. Examples of couplings of both kinds have been listed [10]. Important subclasses of passive couplings mentioned in this paper are contact couplings, often referred to as kinematic pairs, and elastic couplings.

As well as the capability of transmitting an action, many couplings also permit relative motion of the bodies they couple. Motion is a term sometimes used [11] [12] [32] as shorthand for the first time derivative of displacement, geometrically described as a twist rate on the screw of any pitch, including a pitch that is zero, namely an angular velocity, and the pitch that is infinite namely translational velocity. A coupling is characterised by two screw systems [33], a *c*-system of actions that can be transmitted and an *f*-system of motions that can be allowed, and:

$$c + f = d,$$

where c and *f* are often referred to as the degrees of constraint and freedom of the coupling. The sum *d* could be said to be the dimension of the problem, having normally a maximum value of six. Simplification results from disregarding some of the actions couplings are capable of transmitting and then *d* will be less than six. Examples are to be found in section 10.

The action and motion screws systems of couplings are said to be reciprocal to one another because a screw of one system cannot expend power in conjunction with any of the screws of the other system. Note the use of the term power rather than work. The term work would be appropriate if motion is interpreted as infinitesimal displacements, as Ball [34] does. Here, and elsewhere [11] [12] [33] [35], the choice is made to divide all infinitesimal displacements by an infinitesimal time interval. Both approaches are equally valid.

3. Coupling networks

The following definition of the coupling network is expressed in terms that have similarities with the definition of a graph that appears later. A *coupling network* N consists of a non-empty finite set of bodies and a finite set of couplings linking pairs of those bodies. At least one path exists from each body of N to every other body of N, through couplings and other bodies of N. In other words, to borrow a term from graph theory, a coupling network is *connected*, that is to say, in one piece, rather than *disconnected*, in two or more parts.

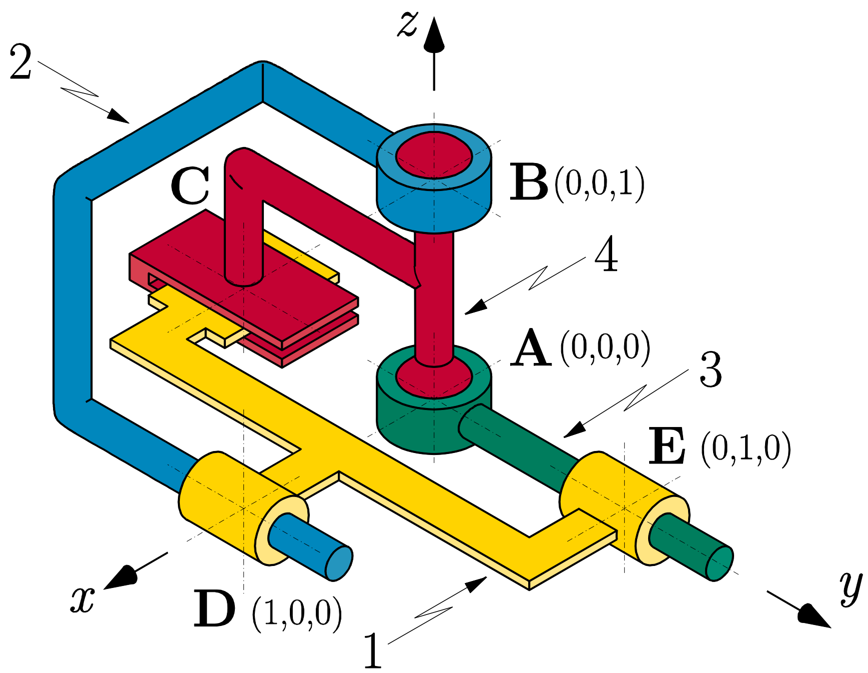


Figure. 1 A spatial kinematic chain

A coupling network has a characteristic gross degree of freedom $F = \Sigma f$ and a characteristic gross degree of constraint $C = \Sigma c$, where the summations are over all couplings. Coupling networks have another pair of characteristics of greater importance: these are the nett degree of freedom F_N and the nett degree of constraint C_N where, $0 \le F_N \le F$ and $0 \le C_N \le C$. The nett degree of freedom F_N has been called *M*, the degree of mobility, but mobility has another meaning [36]. It is also the "complex velocity response at a point in a linear system to a unit force excitation applied at the same point or another point in the system (inverse of mechanical impedance)". Coupling networks for which $F_N = 0$ are immobile structures that will not concern us here. Most structures are welded, riveted or made integral by adhesive so, owing to the restrictions placed on the meaning of a coupling, relatively few structures are coupling networks.

In the 1960s formulae were available for finding F_N, but they did not always work. One associated difficulty lead to a breakthrough. It had been identified [4] that finding the degree of freedom f_{ij} of two indirectly coupled bodies *i* and *j* is difficult if cross bracing exists. The task was to devise a general robust procedure that determines f_{ij} for any pair of bodies. Fig. 1 shows coupling network N that is a spatial kinematic chain, devised by Baker [5], and used since [6] [12] [32] as a test bed for some of the research cited in this paper.

Note for the publishers.

For the on-line version a supplementary video based on Figure 1 is submitted with this manuscript. The title is "Davies video". This is a suitable point in the manuscript to draw the reader's attention to it.

The kinematic chain is artificially contrived so that the elements of all matrices associated with it are 0, -1 or +1. Note that, for bodies two and three, the planar (ebene) coupling labelled C provides cross coupling. This is more evident in the coupling graph Fig. 2. One solution requires an adaptation of Kirchhoff's circulation law for mechanical problems. This approach resulted in a formula for F_N. Later, the problem of finding a formula for C_N was also achieved. Progress towards those two goals is explained in tandem wherever appropriate.

4. Kinematic chains, mechanisms and machines

The term *kinematic chain* is often applied to coupling networks for which $F_N > 0$. In introductory texts on Mechanisms and Machines it is frequently found that a mechanism is described as a kinematic chain for which a "fixed member" has been selected. Once a fixed member has been chosen, all other choices of fixed member are often referred to as inversions of that mechanism.

This approach places an unnecessary emphasis on the identification of a "fixed member", yet says nothing about connections that must be made from the kinematic chain to active couplings in order that useful power can flow. Arguments have been given [10] in favour of a definition of mechanism in terms of content, rather than usage. The approach involving content requires the identification of bodies of the kinematic chain as *terminal bodies* [37], pairs of which are called *ports*. The terminals of a port are a pair of bodies of the kinematic chain that are intended to be made integral with terminal bodies of another coupling or network. If only one port is identified the kinematic chain is an example of a 1-port device, in other words the kinematic chain creates an indirect coupling between the two terminal bodies of the port.

A *mechanism* is a kinematic chain with two or more *ports*. In this context a port could be defined as a pair of terminal bodies through which power can be transmitted to or from a port of another network. The following are two examples of definitions of a port. "*A pair of terminals at which a signal may enter or leave a network is called a port."* [38]; "*A terminal pair to which an input is applied or from which an output is extracted is called a port*." [39]. For a mechanism, the term "signal" is inappropriate and "an input … an output" is unnecessarily vague.

Many mechanisms have only one input port and only one output port; mechanisms with several input ports are likely to be classified as manipulators; mechanisms with more than one output port are rare, the crank-driven needle and awl mechanism of a shoe welt sewing machine is one example [40]. Two or more ports may have one terminal body in common. This is often so when the common body is the one that is called the fixed member or frame.

A *machine* is a mechanism with all ports connected to active couplings or to the ports of indirect couplings that contain active couplings. Such indirect couplings may also contain passive couplings; for example an electrical motor has its own bearings. If the active coupling is a source of power these indirect couplings are often called *actuators*.

In order to adapt Kirchhoff's laws to coupling networks it is necessary to involve graph theory, the subject of the next section.

5. Directed graphs

A simple description of a graph is that it is a set of nodes (points or vertices), some or all pairs of which are connected by lines called edges. We will be concerned only with directed graphs, also called digraphs, within which all edges have an arrowhead thereby making the two nodes incident with each edge an ordered pair. A formal definition now follows.

A directed *graph* G consists of a non-empty finite set V(G) of elements called *nodes* (or vertices) and a finite family E(G) of ordered pairs of elements of V(G) called directed *edges*. The term "family" is used here, as in [2], to accommodate graphs within which multiple edges terminate in the same pair of nodes. We will not be concerned with graphs containing edges that terminate in the same node; such an edge is called a *loop*. The definition of coupling networks provided earlier is modelled on this definition of graphs. This is made possible by incorporating jointed structures for which $F_N = 0$ within coupling networks.

There are several useful terms used in graph theory. Within a graph, a *walk* is a finite sequence of edges. If all edges are distinct the walk is called a *trail*. If, in addition, the vertices are distinct, except possibly for the first and last, then the trail is a *path*. A trail is said to be *closed* if the first and last vertices are the same. A closed path is a cycle or *circuit*.

A graph is *connected* if and only if there is a path between each pair of vertices. A disconnecting set in a connected graph G is a set of edges whose removal disconnects G. A *cutset* is a disconnecting set, no proper subset of which is a disconnecting set. The removal of the edges in a cutset always leaves a graph with exactly two components. A connected graph with no circuits is a *tree* each edge of which is called a *branch* the only member of a cutset. A *spanning tree* is a connected subgraph that contains all the nodes of a graph, but no circuit. The edges not included in the spanning tree are called *chords* and the addition of any chord creates a circuit. Associated with each chord is a fundamental circuit, associated with each branch is a fundamental cutset.

6. Coupling graphs, motion graphs and action graphs

A coupling graph G_C is a graph within which each of the *n* nodes represents a body of a coupling network N and each of the *e* edges represents a coupling of N. These couplings are direct couplings but some indirect couplings such as rolling contact bearings and Hooke’s coupling can be regarded as direct provided that the investigation does not concern their interior actions and motions.

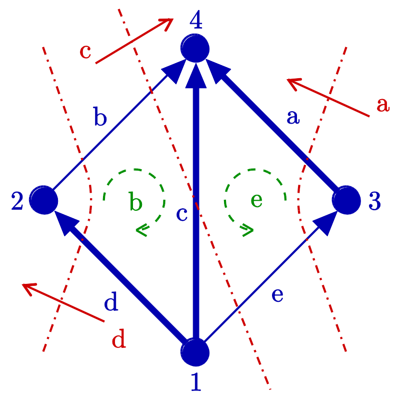


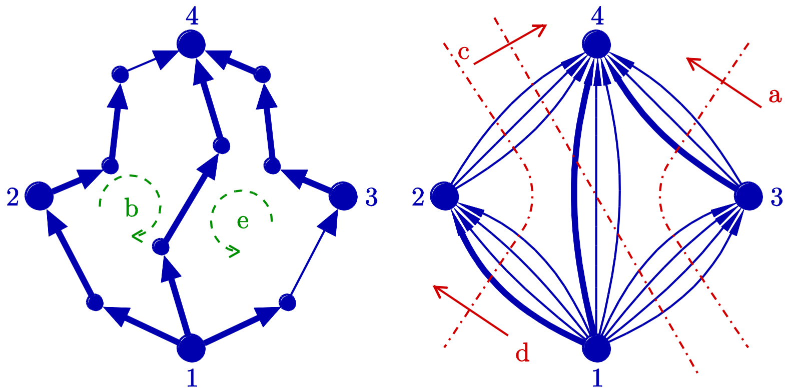
Figure. 2 The coupling graph G_C of the kinematic chain shown in Fig. 1

6.1 The coupling graph: its chords, branches, circuits and cutsets

A coupling graph will be said to have *l* chords and *l* fundamental circuits; it also has *k* branches and *k* fundamental cutsets. Fig. 2 shows the coupling graph G_C of the kinematic chain N shown in Fig. 1, with the arbitrarily selected spanning tree drawn with thick lines. Features of Fig.2 are now described. Here, and elsewhere in this paper, the presentation is provided in tandem where appropriate to emphasise the dual nature of the subject.

The edges **b** and **e** of G_C drawn with thin lines are the chords of the spanning tree. Each independent circuit contains one chord; all other edges are branches. Within these circuits there are arcs labelled **b** and **e** with arrowheads that assign a positive sense that can be arbitrarily chosen but, in this example, the choice corresponds with the positive sense assigned to the associated chords.

The edges **a**, **c** and **d** of G_C drawn with thick lines are the branches of the spanning tree. Each independent cutset contains one branch; all other edges are chords. Dashed lines are drawn through each cutset of edges. Arrows labelled **a**, **c** and **d** cutting these dashed lines assign a positive sense that can be arbitrarily chosen but, in this example, the choice corresponds with the positive sense assigned to the associated branches.



(a) (b)

Figure. 3 Graphs of the kinematic chain shown in Fig. 1:
a) motion graph G_M; b) action graph G_A

6.2 Motion and action graphs

From the coupling graph G_C it can be helpful to create a motion graph G_M and an action graph G_A. For the kinematic chain shown in Fig.1 these graphs are described below.

The motions allowed by a coupling having *f* degrees of freedom can be spanned by *f* independent motion screws. Each of these *f* screws can be represented in a motion graph G_M. The motion graph G_M is created by replacing each edge of G_C that represents an *f* degree of freedom coupling by *f* edges in series. Fig. 3a shows the motion graph for the kinematic chain of Fig. 1.

The actions transmitted by a coupling having *c* degrees of constraint can be spanned by *c* independent action screws. Each of these *c* screws can be represented in an action graph G_A. The action graph G_A is created by replacing every edge of G_C that represents a *c* degree of constraint coupling by *c* edges in parallel. Fig. 3b shows the action graph for the kinematic chain of Fig. 1.

The minimum number of parameters (independent motion magnitudes) necessary to provide the magnitudes of *all* motions within a coupling network is the nett degree of freedom F_N. Alternatively, F_N can be said to be the degree of overfreedom or excess freedom.

For a coupling network that is a tree,

$$F_N = F.$$

For coupling networks that contain one or more circuits comprised of two or more couplings,

$$0 \leq F_N \leq F.$$

Circuits can *reduce* freedoms.

The minimum number of parameters (independent action magnitudes) necessary to provide the magnitudes of *all* actions within a coupling network is the nett degree of constraint C_N. Alternatively, C_N can be said to be the degree of overconstraint or excess constraint.

For a coupling network that is a tree,

$$C_N = 0.$$

For coupling networks that contain one or more circuits comprised of two or more couplings,

$$C \geq C_N \geq 0.$$

Circuits can *increase* constraints.

7. Adaptations of Kirchhoff's laws

In this section matrices are needed that contain components of screws. Subscripts outside the square brackets around matrices signify the number of rows and columns respectively. A cap on a matrix signifies that the screws are normalised. The task of assembling equations is explained with the aid of the kinematic chain shown in Fig.1 and, in particular, the cylindrical coupling D having an axis through (1, 0, 0) parallel with the *y*-axis.

A notation is used that may be unfamiliar to the reader. This notation has been used before [11,12,17,32]; it is listed in the Introduction and explained in greater detail in section 11.3. The adaptations of the laws are now presented in tandem.

Kirchhoff's voltage law, when adapted for coupling networks, states that for each of the l independent circuits, the d components of screws spanning the motion screws of couplings of a circuit sum to zero when measured by reference to the same global frame. Thereby, dl equations can be written that impose conditions on the F unknowns. Some of these equations may prove to be redundant however. The circuit law equation can be written

Kirchhoff's current law, when adapted for coupling networks, states that for each of the k independent cutsets, the d components of screws spanning the action screws of couplings of a cutset sum to zero when measured by reference to the same global frame. Thereby, dk equations can be written that impose conditions on the C unknowns. Some of these equations may prove to be redundant however. The cutset law equation can be written as:

$$\left\lfloor \hat{\mathbf{M}}_N \right\rfloor_{dl,F} [\psi]_F = [\mathbf{0}]_{dl} . \qquad (1)$$

as:

$$\left\lfloor \hat{\mathbf{A}}_N \right\rfloor_{dk,C} [\varphi]_C = [\mathbf{0}]_{dk} . \qquad (2)$$

7.1 The vectors of unknown magnitudes

The vector $[\psi]_F$ = [r_a, s_a, t_a, r_b, s_b, t_b, t_c, u_c, v_c, s_d, v_d, s_e, v_e]^T contains F unknown magnitudes of motions spanning the motion screw systems of the couplings listed in the same order as they appear in the columns of $\hat{\mathbf{M}}_N$. For example, in the kinematic chain shown in Fig. 1, coupling D allows motions that belong to a fifth special 2-system of motion screws [33]. This system is spanned by any two screws of unequal pitch with ISA sharing the cylinder axis. Most conveniently the screws selected are those with zero and infinite pitch, namely angular velocity of magnitude s_d about the cylinder axis, the (local) y_d-axis, and translational velocity of magnitude v_d in the direction of the y-axis.

The vector $[\varphi]_C$ = [U_a, V_a, W_a, U_b, V_b, W_b, R_c, S_c, W_c, R_d, T_d, U_d, W_d, R_e, T_e, U_e, W_e]^T contains C unknown magnitudes of actions spanning the action screw systems of the couplings listed in the same order as they appear in the columns of $\hat{\mathbf{A}}_N$. For example, for the kinematic chain shown in Fig. 1, coupling D transmits actions that belong to a fifth special 4-system of action screws [33]. This system is spanned by any four screws reciprocal with the motion screws. A convenient set comprises torques (couples) parallel to the x- and z-axes of magnitudes R_d and T_d respectively, together with forces along the x- and (local) z_d-axes of magnitudes U_d and W_d respectively.

7.2 The network unit motion and unit action matrices

The network unit motion matrix

$$\left\lfloor \hat{\mathbf{M}}_N \right\rfloor_{dl,F} = \begin{bmatrix} \left\lfloor \hat{\mathbf{M}}_D \right\rfloor_{d,F} [\mathbf{B}_1]_{F,F} \\ \left\lfloor \hat{\mathbf{M}}_D \right\rfloor_{d,F} [\mathbf{B}_2]_{F,F} \\ \vdots \\ \left\lfloor \hat{\mathbf{M}}_D \right\rfloor_{d,F} [\mathbf{B}_l]_{F,F} \end{bmatrix},$$

where $\left\lfloor \hat{\mathbf{M}}_D \right\rfloor_{d,F}$, the direct coupling unit motion matrix, is determined by the geometry and $[\mathbf{B}_i]_{F,F}$, i = 1, 2, …, l by the topology as represented by the motion graph.

The network unit action matrix

$$\left\lfloor \hat{\mathbf{A}}_N \right\rfloor_{dk,C} = \begin{bmatrix} \left\lfloor \hat{\mathbf{A}}_D \right\rfloor_{d,C} [\mathbf{Q}_1]_{C,C} \\ \left\lfloor \hat{\mathbf{A}}_D \right\rfloor_{d,C} [\mathbf{Q}_2]_{C,C} \\ \vdots \\ \left\lfloor \hat{\mathbf{A}}_D \right\rfloor_{d,C} [\mathbf{Q}_k]_{C,C} \end{bmatrix},$$

where $\left\lfloor \hat{\mathbf{A}}_D \right\rfloor_{d,C}$, the direct coupling unit action matrix, is determined by the geometry and $[\mathbf{Q}_i]_{F,F}$, i = 1, 2, …, k by the topology as represented by the action graph.

7.3 Direct coupling unit motion and unit action matrices

The direct coupling unit motion matrix $\left[\hat{\mathbf{M}}_D\right]_{d,F}$ contains the d components of each of the F unit motion screws with respect to the global frame of reference with its origin at the centre of the spherical coupling A.

For example, for the kinematic chain shown in Fig.1, the 10th and 11th columns of $\left[\hat{\mathbf{M}}_D\right]_{6,13}$, shown as a submatrix below, are the motion components for the f = 2 cylindrical coupling located at D.

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

When these normalised screws are multiplied by the 10th and 11th elements of $[\psi]_{13}$, s_d and v_d respectively, the two motion screws are obtained of body two relative to body one. Note that the sixth element of the 10th column, when multiplied by s_d, is a velocity along the z-axis of a point on an imaginary extension of body two located at the global origin. This velocity results from the angular velocity s_d about the (local) y_d-axis recorded in the second element of the 10th column.

The direct coupling unit action matrix $\left[\hat{\mathbf{A}}_D\right]_{d,C}$ contains the d components of each of the C unit action screws with respect to the global frame of reference with its origin at the centre of the spherical coupling A.

For example, for the kinematic chain shown in Fig.1, the 10th to the 13th columns of $\left[\hat{\mathbf{A}}_D\right]_{6,17}$, shown as a submatrix below, are the action components for the c = 4 cylindrical coupling located at D.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

When these normalised screws are multiplied by the 10th to the 13th elements of $[\varphi]_{17}$, R_d, T_d, U_d and W_d respectively, the four action screws are obtained that are exerted by body one on body two. Note that the second element of the 13th column, when multiplied by W_d, is the (negative) moment about the y-axis. This moment results from the force W_d along the (local) z_d-axis recorded in the sixth element of the 13th column.

7.4 The circuit matrix of G_M, the cutset matrix of G_A, and diagonal matrices derived from them

The matrices $[\mathbf{B}_i]_{F,F}$, i = 1, 2, …, l are diagonal matrices in which the diagonal elements of the i^{th} matrix are those of the i^{th} row of the circuit matrix $[\mathbf{B}_M]_{l,F}$ of the motion graph G_M.
Each element b_{ij} of $[\mathbf{B}_M]_{l,F}$ is 0, +1, or -1: b_{ij} is zero if circuit i does not include edge j; +1 if the positive sense of circuit i is in the same direction as the positive sense of the edge j that it includes; and -1 if those positive senses are opposed. For example, for the kinematic chain shown in Fig.1, the columns 10 and 11 of $[\mathbf{B}_M]_{2,13}$ are:

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

The first row confirms that edge d is a member of circuit b and the positive direction assigned to the circuit corresponds with that of the edge. The second row confirms that edge d does not belong to circuit e. Subsequently, in the diagonal matrix $[\mathbf{B}_b]_{13,13}$, the 10th and 11th diagonal elements are both one whereas, in $[\mathbf{B}_e]_{13,13}$, these elements are zero.

A consequence is that, for the kinematic chain of Fig.1, in columns 10 and 11 of the network unit action matrix $\left[\hat{\mathbf{M}}_N\right]_{12,13}$ the first six rows are identical to those of $\left[\hat{\mathbf{M}}_D\right]_{6,13}$ and all elements of the last six rows are zero.

The matrices $[\mathbf{Q}_i]_{C,C}$, i = 1, 2, …, k are diagonal matrices in which the diagonal elements of the i^{th} matrix are those of the i^{th} row of the cutset matrix $[\mathbf{Q}_A]_{k,C}$ of the action graph G_A.
Each element q_{ij} of $[\mathbf{Q}_A]_{k,C}$ is 0, +1, or -1: q_{ij} is zero if cutset i does not include edge j; +1 if the positive sense of cutset i is in the same direction as the positive sense of the edge j that it includes; and -1 if those positive senses are opposed. For example, for the kinematic chain shown in Fig.1, the columns 10 - 13 of $[\mathbf{Q}_A]_{3,17}$ are:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

The last row confirms that edge d is a member of cutset d and the positive direction assigned to the cutset corresponds with that of the edge. The other two rows confirm that edge d does not belong to cutsets a and c. Subsequently, in the diagonal matrix $[\mathbf{Q}_d]_{17,17}$, the 10th - 13th diagonal elements are all one whereas, in $[\mathbf{Q}_a]_{17,17}$ and $[\mathbf{Q}_c]_{17,17}$, these elements are zero.

A consequence is that, for the kinematic chain of Fig.1, in columns 10 - 13 of the network unit action matrix $\left[\hat{\mathbf{A}}_N\right]_{18,17}$ the last six rows are identical to those of $\left[\hat{\mathbf{A}}_D\right]_{6,17}$ and all elements of the first 12 rows are zero.

7.5 Results

If there is overconstraint, the rank m of $\left[\hat{\mathbf{M}}_\mathbf{N}\right]_{dl,F}$ is less than dl, the number of rows, and so

$$C_N = dl - m$$

rows are redundant. The remaining m independent equations impose m constraints on the F unknown

If there is overfreedom, the rank a of $\left[\hat{\mathbf{A}}_\mathbf{N}\right]_{dk,C}$ is less than dk, the number of rows, and so

$$F_N = dk - a$$

rows are redundant. The remaining a independent equations impose a constraints on the C unknown magnitudes. Thereby, these *F* unknowns can be expressed in terms of F_N primary variables, where

$$F_N = F - m.$$

For the kinematic chain shown in Fig.1, *m* is 10, C_N is two and F_N is three.

For every pair of bodies {*i*, *j*} of a coupling network, equation (1) makes it possible to identify a set of f_{ij} independent motion screws that span the screw system of all motions of which bodies *i* and *j* are capable. Furthermore, equation (1) also expresses the magnitudes of each of these motion screws in terms of the magnitudes of F_N of them. Subject to some restrictions, there is freedom to choose which F_N motion screw magnitudes shall belong to this set.

magnitudes. Thereby, these *C* unknowns can be expressed in terms of C_N primary variables, where

$$C_N = C - a.$$

For the kinematic chain shown in Fig.1, *a* is 15, F_N is three and C_N is two.

For every pair of bodies {*i*, *j*} of a coupling network, equation (2) makes it possible to identify a set of c_{ij} independent action screws that span the screw system of all actions that can be transmitted between bodies *i* and *j*. Furthermore, equation (2) also expresses the magnitudes of each of these action screws in terms of the magnitudes of C_N of them. Subject to some restrictions, there is freedom to choose which C_N action screw magnitudes shall belong to this set.

Because the foregoing is a brief summary of the full investigation [12], tables 1 and 2 below give the results in detail.

Table 1: Results obtained from the solution of equation (1) for the kinematic chain shown in Figure 1.

Pairs of bodies	Label of direct coupling	Motion components			
		f	Direct couplings with *F* unknowns	f_{ij}	After assembly, using $\{s_a, t_a, v_c\}$ as primary variables
1, 2	d	2	$\{0, s_d, 0, 0, v_d, 0\}$	1	$\{0, 0, 0, 0, v_c, 0\}$
1, 3	e	2	$\{0, s_e, 0, 0, v_e, 0\}$	2	$\{0, -s_a, 0, 0, v_c, 0\}$
1, 4	c	3	$\{0, 0, t_c, u_c, v_c, 0\}$	2	$\{0, 0, t_a, 0, v_c, 0\}$
2, 3	Absent		N/A	1	$\{0, s_a, 0, 0, 0, 0\}$
2, 4	b	3	$\{r_b, s_b, t_b, 0, 0, 0\}$	2	$\{0, 0, t_a, 0, 0, 0\}$
3, 4	a	3	$\{r_a, s_a, t_a, 0, 0, 0\}$	2	$\{0, s_a, t_a, 0, 0, 0\}$

Table 2: Results obtained from the solution of equation (2) for the kinematic chain shown in Figure 1.

Pairs of bodies	Label of direct coupling	Action components			
		c	Direct couplings with C unknowns	c_{ij}	After assembly, using $\{U_b, W_e\}$ as primary variables
1, 2	d	4	$\{R_d, 0, T_d, U_d, 0, W_d\}$	1	$\{0, U_b, 0, U_b, 0, -U_b\}$
1, 3	e	4	$\{R_e, 0, T_e, U_e, 0, W_e\}$	2	$\{0, 0, 0, -U_b, 0, W_e\}$
1, 4	c	3	$\{R_c, S_c, 0, 0, 0, W_c\}$	2	$\{0, -U_b, 0, 0, 0, (U_b - W_e)\}$
2, 3	Absent		N/A		N/A
2, 4	b	3	$\{0, 0, 0, U_b, V_b, W_b\}$	2	$\{0, U_b, 0, U_b, 0, -U_b\}$
3, 4	a	3	$\{0, 0, 0, U_a, V_a, W_a\}$	2	$\{0, 0, 0, -U_b, 0, W_e\}$

One further matter is included here that is not mentioned in [12]. Suppose that the kinematic chain were to be used as a 1-port coupling network with bodies two and three, the pair of original interest, as the terminals of the port. Suppose also that those bodies are now grasped by someone, one body gripped in each hand. The person who is gripping the two bodies is behaving as another 1-port coupling network but one that is a six dof serial manipulator with built-in active couplings called muscles. The appearance of s_a in column six, row four, of table 1 indicates that bodies two and three are capable of relative rotation about the y-axis. Note that s_b, s_d or s_e could have been chosen as primary variables instead. The actions that can be transmitted from body two to body three are thereby restricted to the 5-system of action screws that are all reciprocal to that rotation. These actions are spanned by $\{R_f, T_f, U_f, V_f, W_f\}$, because $s_f S_f, = 0$. Whereas c_{23} was previously zero, now that the human coupling has been added thereby internalising these actions, it is now five.

8. Virtual power equations

There is an alternative way of finding the number of primary variables F_N and C_N and, in addition, an alternative way of expressing the magnitudes of all motions and actions in terms of those primary variables.

8.1 The cutset motion and circuit action vectors

Instead of starting with F unknown coupling motion components, dk unknown cutset motion components can be used instead. These dk motion

Instead of starting with C unknown coupling action components, dl unknown circuit action components can be used instead. These dl action components are subject to *C* conditions, some of which may prove to be redundant. The *C* action components cannot expend or generate power in conjunction with the *dk* motions and so the *C* actions must be regarded as virtual actions.

components are subject to *F* conditions, some of which may prove to be redundant. The *F* motion components cannot expend or generate power in conjunction with the *dl* actions and so the *F* motions must be regarded as virtual motions.

The *dk* unknowns must be assembled in a cutset motion vector $[\mathbf{M}_k]_{dk}$. Using Fig. 3b as an example wherein $d = 6$ and $k = 3$, the first six elements of $[\mathbf{M}_k]_{18}$ are the six unknown components for cutset *a*, namely:

$$[r_a, s_a, t_a; u_a, v_a, w_a]^\mathrm{T}.$$

There follows six components that are identical except that the subscript *a* is replaced by *c*, and six more subscripted by *d*.

The *dl* unknowns must be assembled in a circuit action vector $[\mathbf{A}_l]_{dl}$. Using Fig. 3a as an example wherein $d = 6$ and $l = 2$, the first six elements of $[\mathbf{A}_l]_{12}$ are the six unknown components for circuit *b* namely:

$$[R_b, S_b, T_b; U_b, V_b, W_b]^\mathrm{T}.$$

There follows six components that are identical except that the subscript *b* Is replaced by *e*.

8.2 The transposed network unit action and unit motion matrices

To apply the *C* conditions vector $[\mathbf{M}_k]_{dk}$ must be pre-multiplied by the transpose of the network unit action matrix $\lfloor \hat{\mathbf{A}}_\mathbf{N} \rfloor_{dk,C}$ used in equation (2). Thus:

$$\lfloor \hat{\mathbf{A}}_N^T \rfloor_{C,dk} [\mathbf{M}_k]_{dk} = [\mathbf{0}]_C . \qquad (3)$$

The *C* rows of $\lfloor \hat{\mathbf{A}}_N^T \rfloor_{C,dk}$ can be reduced to *a* rows by eliminating the C_N redundant ones.

To apply the *F* conditions vector $[\mathbf{A}_l]_{dl}$ must be pre-multiplied by the transpose of the network unit motion matrix $\lfloor \hat{\mathbf{M}}_\mathbf{N} \rfloor_{dl,F}$ used in equation (1). Thus:

$$\lfloor \hat{\mathbf{M}}_N^T \rfloor_{F,dl} [\mathbf{A}_l]_{dl} = [\mathbf{0}]_F . \qquad (4)$$

The *F* rows of $\lfloor \hat{\mathbf{M}}_N^T \rfloor_{F,dl}$ can be reduced to *m* rows by eliminating the F_N redundant ones.

For a coupling represented by a chord of G_C, the coupling motion components are those of the corresponding circuit of G_C. For a coupling represented by a branch of G_C, the motion components are the sum of the motion components of the circuits of G_C to which the branch belongs.

For a coupling represented by a branch of G_C, the coupling action components are those of the corresponding cutset of G_C. For a coupling represented by a chord of G_C, the action components are the sum of the action components of the cutsets of G_C to which the chord belongs.

The kinematic chain shown in Fig. 1 has no utility except as a geometrically and topologically simple example to demonstrate principles involved. Useful examples are described in the next two sections.

9. Dual coupling networks

The work described so far raises the question as to whether, for a coupling network N with network matrices $\hat{\mathbf{M}}_N$ and $\hat{\mathbf{A}}_N$ there exists a dual coupling network N* with network matrices $\hat{\mathbf{M}}_N^*$ and $\hat{\mathbf{A}}_N^*$ such that $\hat{\mathbf{M}}_N^*$ and $\hat{\mathbf{A}}_N^*$ are identical to $\hat{\mathbf{A}}_N$ and $\hat{\mathbf{M}}_N$ respectively? Dual coupling networks have been created and the procedure for creating them has been explained in detail [32], the chosen example is the coupling network N shown in Fig. 1 and its dual. The procedure requires the identification of dual couplings and dual coupling graphs. The duals of some simple planar kinematic chains have also been described [8] [17]; the latter is mentioned again in the next section.

Such studies are an aid to an understanding screw theory and graph theory. Furthermore, whereas actions are difficult to imagine in a coupling network N, it is relatively easy to imagine the geometrically identical screws that that describe the motions that can take place within the dual network N*.

10. Applications

The first two subsections involve coupling networks for which the geometry can be greatly simplified by ignoring some of the constraints. A consequence is that the dimension *d* can be less than six thereby making the matrices considerably smaller.

10.1 Planar kinematic chains

Studies [17] have been made of the duals of planar kinematic chains that are in critical configurations. By confining attention to motion screws belonging to the fifth special 3-system of screws, a dimension *d* of three can be used in assembling equation (1) with the consequence that matrix $\hat{\mathbf{M}}_N$ is much smaller than it would otherwise be. A complete kinematic analysis of a Stephenson kinematic chain is provided using equation (1) and this is shown to be identical to the results of a static analysis of the dual of the kinematic chain using equation (2).

10.2 Gear trains, friction and efficiency

Equations (2, 4) have limited utility when applied to a kinematic chain for reasons that are discussed later in section 11. These equations do have value however for studies of the statics of machines operating at a constant speed. The two-stage epicyclic gear train shown in Fig. 4 provides an example of the use of all four equations [11].

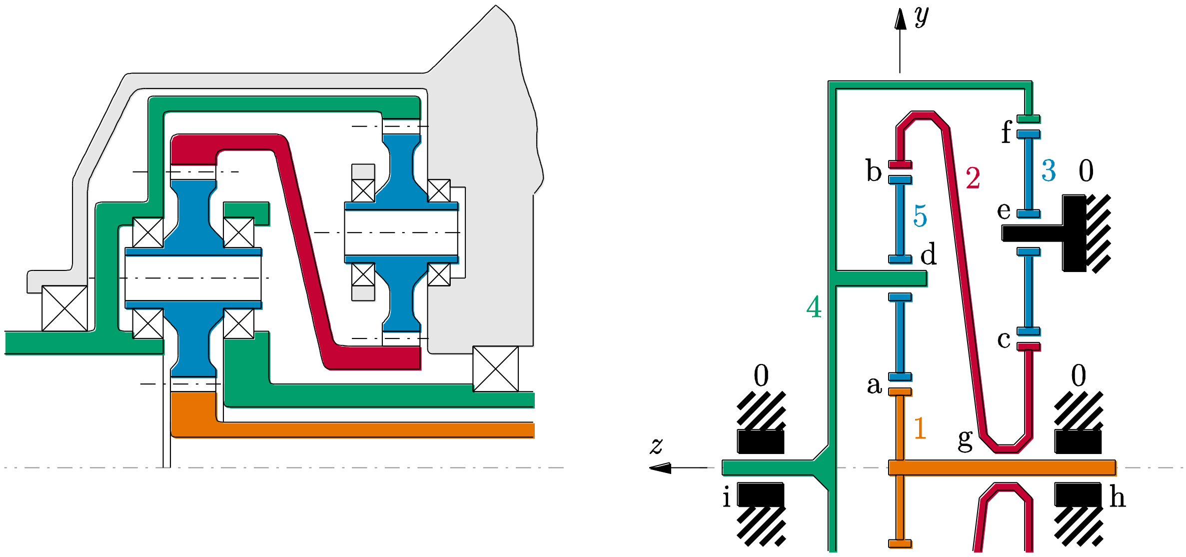


Figure. 4 A two-stage epicyclic gear train and a schematic diagram of it

In order to use equations 1 and 3 for kinematic analysis no modification is needed. In order to use equations 2 and 4 for the statics problem however, the gear train must be supplemented by two 1-port coupling networks that provide a source and sink for power, an electric motor and a fan for example. Both of these 1-port coupling networks contain an active coupling that transmits torque about the *z*-axis; they will also have bearings with the centre lines on the *z*-axis, but these duplicate the role played by bearings that exist within the gear train and can be ignored.

A major problem remains. The two extra actions supplement the many actions that could exist attributable to overconstraint. Because equations 2 and 4 can only analyse internal actions those actions attributable to overconstraint cannot be avoided. The problem is thereby far more complex than it needs to be. The extended coupling network can be greatly simplified however without impairment to the basic statics problem by taking the following steps.

- ❖ All but one planet in each stage is ignored.
- ❖ All moving parts are assumed to exist in the $z = 0$ plane.
- ❖ Both kinds of coupling, meshing gears and bearings, are assumed to be ($c = f = 1$) couplings by ignoring all other freedom and constraint.

Both the motion screws and the remaining action screws both belong to second special 2-systems of screws. These special screw systems differ geometrically however. Angular velocities have ISA parallel with the *z*-axis in the *x* = 0 plane, whereas forces have ISA parallel with the *x*-axis in the *z* = 0 plane. As Shai and Pennock [41] have observed of a similar gear train, the system is now identical to a sequence of levers.

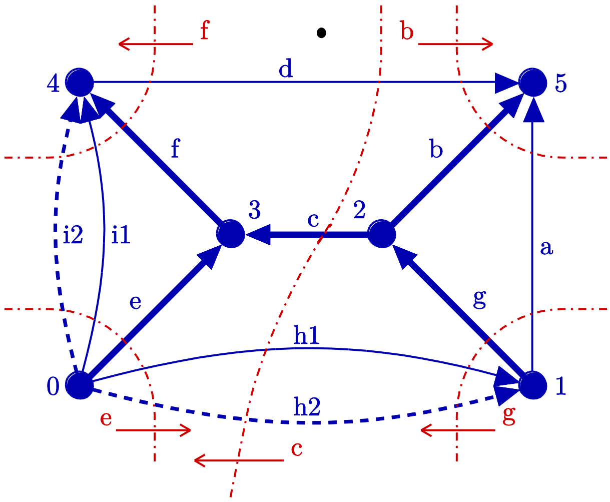


Figure. 5 The coupling graph G_C of the gear train shown in Fig. 4 when it is augmented by two active couplings represented by edges h2 and i2

For equation 2 two additional active couplings are needed and so, in Fig. 5, there are two edges from node 0 to node 1, and two edges from node 0 to node 4. The two additional edges h2 and i2 representing active couplings are shown as dashed lines. Fig. 5 is also the action graph G_A because *c* = 1 for all couplings. The five independent cutsets are identified in Fig. 5 by chain-dotted lines. Because *f* = 1 for all couplings, again Fig. 5 is also the motion graph except that edges h2 and i2 can be omitted. The four independent internal circuits are then obvious.

Cazangi and Martins [13] employ equation (1) for the analysis of two gear trains; one has two degrees of freedom, two forward ratios and one backward; the second has three degrees of freedom, three forward ratios and one backward.

Laus *et al* [14] employ equations 1 and 2 for studies of the efficiency of an epicyclic gear train and a Humpage gear train. For both, account is taken of friction, including gear tooth friction.

Tischler *et al* [15] uses equation (4) for a study of friction in multi-loop linkages. This may be the only occasion that equation (4) has been used for an application except for the epicyclic gear train described above.

10.3 Kinematic chains in critical configurations

Tischler [16] uses equation (1) in a study of critical configurations of a RCCC kinematic chain; Davies and Laus [17] do likewise for a planar 6-Link Stephenson kinematic chain.

10.4 The use of symbolic screw components

In a study to predict the slop that results from clearances in couplings of the Melbourne dextrous finger, Tischler *et al* [18] use symbolic screw components so that the analysis is valid throughout the cycle of configurations instead of only at one instantaneous configuration.

10.5 The use of virtual couplings (Assur groups)

An Assur group does not introduce additional constraints. For example, for a planar manipulator it can comprise PPR couplings in series; for a spatial manipulator PPPRRR or PPPS couplings in series. Equation (1) proves to be very useful; the primary variables can be either those of couplings of the manipulator or, for inverse kinematics, couplings of the Assur group.

Several workers have used Assur groups in combination with equation (1). Erthal *et al* [19] use them for a study of vehicle suspension; Campos *et al* [20] for the inverse kinematics of serial manipulators and [21] for the inverse kinematics of parallel manipulators. Inverse kinematics also gets attention from Simas *et al* [22].

There is work reported by Guenther *et al* [23] and Santos *et al* [24] [25] on the study of underwater manipulators. Simas *et al* [26] [27] and Rocha *et al* [28] report on work to avoid collisions and for carrying out tasks such as remote repair. Ribeiro *et al* [29] [30] describe the use of virtual chains in studies of cooperating robots. Recently, Ponce Saldias *et al* [31] [42] have extended the application of equation 1 and Assur groups to the modelling of the human knee to aid pre-operative planning.

11. Discussion

In this section some lessons learned from the foregoing are discussed.

11.1 If there is a “fixed” member in a mechanism, does it matter which it is?

In his lengthy notes that he includes in his English translation of Reuleaux [43], Kennedy [44] argues that a *machine* is defined by many in terms of what it does whereas, ideally, it should be defined in terms of what it comprises. In [10] this criticism is extended to some definitions provided by IFToMM [36]. In section 4 some extracts from [10] are repeated in order to draw attention to the fact that there is not necessity to identify an element (body/link/member) that is fixed. Of course, there are mechanisms, such as some handheld tools, wherein the term "fixed" is irrelevant.

For studies of kinematics and statics, the significance of a fixed member is unimportant. It is accepted of course that if acceleration, the second derivative of displacement, is a feature then it is essential to identify an inertial member, most frequently the earth.

11.2 A directed graph provides a concise and easily accessible record of a user-selected sign convention.

Anyone who has learned, or taught, elementary mechanics using free body diagrams may remember the tedium involved in using arrows twice, once on each of two directly coupled bodies. Likewise, for kinematics, it is necessary to distinguish the motion of body A relative to body B and body B relative to body A.

A directed graph has merits. A positive sense assigned to an edge by using an arrowhead indicates which, of two possibilities, will be regarded as the positive sense in any analysis. The choice of direction is an arbitrary decision. The coupling graph G_C in Fig. 5 of the gear train shown in Fig. 4 has nine edges so there are 512 possible different sets of directed edges. Fig. 3 provides evidence that it is the author’s practice to assign the positive direction away from the node labelled with the lower number. It is suggested here that the directed graph provides a concise store of a sign convention of the user’s choice that can be read at a glance.

11.3 In order to write the reciprocity condition it is sufficient to remember *rR*

In recent publications [11] [12] [17] [32] the author has chosen to represent the reciprocity condition for motion and action screws as follows:

$$rR + sS + tT + uU + vV + wW = 0.$$

Where {*r, s, t*} are the {*x, y, z*} components of angular velocity; {*u, v, w*} are components of the velocity of a point located at the origin; {*R, S, T*} are the components of moments measured at the origin; and {*U, V, W*} are the components of forces. The simple layout in the equation above is easily remembered and easily keyboarded. Others may prefer asterisks and exotic curly fonts. Note that *R* - *W* is sequential whereas $\mathscr{L} – \mathscr{R}$ is not; *T* is the moment about the *z*-axis, often the moment of Torque, and *u* and *v* are easily remembered velocity components of the origin along the *x*- and *y*-axes respectively. Furthermore, *p* is available for the pitch of a screw.

11.4 Mechanical network theory can be much more complex than electrical DC network theory.

Suppose that a coupling graph G_C, such as the one shown in Fig. 2, is also the graph of an electrical network. To keep matters simple suppose also that every one of the *e* edges corresponds either to a battery, or a resistor.

A coupling graph has *l* independent circuits and chords. For the equivalent electrical network there are therefore *le* elements in the voltage law equation matrix. For the equivalent mechanical matrix $\hat{\mathbf{M}}_N$, the number of elements is *Fdl.* The ratio is: *Fdl/le* = *Fd/e*.

A coupling graph has *k* independent cutsets and branches. For the equivalent electrical network there are therefore *ke* elements in the current law equation matrix. For the equivalent mechanical matrix $\hat{\mathbf{A}}_N$, the number of elements is *Cdk.* The ratio is: *Cdk/ke* = *Cd/e*.

Summary of results drawn from examples mentioned in this paper are provided in Table 3 below.

Table 3: The size of matrices relative to those of a topologically identical DC electrical network

Coupling network	*d*	*e*	Circuit law		Cutset law	
			F	***Fd/e***	*C*	***Cd/e***
Fig. 1	6	5	13	**78/5**	17	**102/5**
Stephenson III, a 6-link planar kinematic chain [17]	6	7	6	**36/7**	20	**180/7**
	3	7	6	**18/7**	N/A	
Simplified epicyclic gear train, Fig. 4	2	11	N/A		11	**2**
	2	9	9	**2**	N/A	

Judging by the ratio of the number of elements in matrices, *Fd/e* and *Cd/e*, the complexity of the coupling network problems are generally much greater than those of a simple DC network having the same topology.

11.5 Which equations are best?

For kinematic chains it has been observed that C, C_N, and matrix $\hat{\mathbf{A}}_N$ are larger, sometimes much larger, than F, F_N and matrix $\hat{\mathbf{M}}_N$ respectively. This suggests that, for statics of machines, equation 4 is superior to equation 2 and, for kinematics, equation 1 is superior to equation 3 which may explain why Jean Bernoulli never wrote about virtual actions.

11.6 Actions attributable to overconstraint cannot be measured by geometry and topology

Overconstraint is potentially dangerous, so awareness of its existence is important. This topic is also discussed in section 11.8. For kinematic chains equations 2 and 4 are incapable of providing the magnitudes of actions. These equations can enable all C actions that can exist within a kinematic chain that are attributable to overconstraint to be expressed in terms of a set of C_N actions that are chosen as primary variables. The magnitudes of these C_N actions remain unknown however; they are dependent on tolerances, shape, manufacturing errors, temperature and material properties.

11.7 The dual zeroth laws of mechanics

The zeroth law of thermodynamics is fundamental, very simple, and too obvious for much notice to be taken of it. The decision to number the law as the zeroth law is attributed to Fowler and Guggenheim [48].The law can be stated in several ways, Fowler and Guggenheim write:

If two thermal assemblies are each in thermal equilibrium with a third assembly, then all three are in thermal equilibrium with each other.

The following dual laws for actions and motions within coupling networks can be expressed in tandem.

The action law	The motion law
An action can be transmitted around a circuit comprising bodies and couplings provided that <u>all</u> those couplings are capable of transmitting that action.	*Two bodies separated by a cutset of couplings can have relative motion provided that <u>all</u> those couplings are capable of allowing that motion.*

Because the dual laws above, like the zeroth law of thermodynamics, are fundamental, very simple, and too obvious for much notice to be taken of them, maybe it is appropriate that they be called the dual zeroth laws of mechanics.

In this paper, with its focus on coupling networks, it is appropriate to write the law in its dual form; the symmetry of duality is also appealing. If duality is ignored the action law can be stated in a simpler way as:

An action cannot exist without a circuit capable of transmitting it.

This simple law becomes apparent when actions are internalised as they must be to employ equations (2, 4). It may have been overlooked because Isaac Newton was a free body diagram man: he never internalised actions.

Turning to the motion law, it is obvious that two bodies can be in relative motion without being members of a coupling network. In these circumstances it could be said that the only coupling is a *null coupling* that allows any motion.

11.8 Does elastic design get sufficient attention?

The existence of overconstraint can result in fatigue failure. Attempts to limit the dangerous consequences of overconstraint are of two kinds. One is kinematic design whereby additional freedom is introduced thereby increasing F_N and, by doing so, reducing C_N. This is certainly the preferred route for precision instruments. The second kind is to employ elastic design whereby, by changes in certain dimensions or a change of materials, some parts are made sufficiently compliant to allow limited elastic deformation.

Most writers concentrate attention on their speciality, either the kinematic approach or the elastic approach. Professor Michael French, an academic and a writer on the subject of engineering design, is an exception. He is an unrepentant generalist exemplified by his statement: "Never ask a specialist; they always give the wrong answer." Ouch! In his book [45], there is a chapter titled Kinematic and Elastic Design. It is a very good balanced account of the two approaches with several examples from gear trains that were in production at the time of publication.

11.9 Screw theory is addictive. All papers and books that mention screw theory should be required to print a warning: screw theory can damage your career.

The reader will understand the author's reluctance to provide evidence for this assertion but two addicts are mentioned if only because they are long since dead. In *A History of Mathematics*, Cajori [46] writes about Julius Plücker (1801-1868) [47], one of the founding fathers of screw theory; the following is an extract.

"In Germany J. Plücker's researches met with no favour. His method was declared to be unproductive as compared with the synthetic method of J. Steiner and J. V. Poncelet! His relations with C. G. J. Jacobi were not altogether friendly. Steiner once declared that he would stop writing for *Crelle's Journal* if Plücker continued to contribute to it. The result was that many of Plücker's researches were published in foreign journals, and that his work came to be better known in France and England than in his native country. The charge was also brought against Plücker that, although occupying the chair of physics, he was no physicist. This induced him to relinquish mathematics, and for nearly 20 years to devote his energy to physics. Important discoveries on Fresnel's wave-surface, magnetism and spectrum-analysis were made by him. But towards the close of his life he returned to his first love, mathematics, and enriched it with new discoveries. By considering space as made up of lines he created a "new geometry of space."

Another major contributor to screw theory was Sir Robert Stawell Ball (1840-1913) [34]. He also had a day job. In 1892 he was appointed as Lowndean Professor of Astronomy and Geometry at Cambridge University at the same time becoming director of the Cambridge Observatory. He was in great demand as a popular speaker on astronomy. His important contributions to screw theory however were ignored for around 70 years.

So, perhaps the best way of defeating drug traffickers is to ignore them.

11.10 Actions and motions rarely appear in the same textbook

Mention of Robert Ball brings back memories of something written [11] on the occasion of symposium held in 2000 to celebrate the hundredth anniversary of the publication of his book, *A Treatise on the Theory of screws* [34]. It is worth mentioning again.

Can you imagine a University's Department of Electrical Engineering advertising for two posts; one for a teacher of Electrical Circuit Theory (electrical currents) and another for a teacher of Electrical Circuit Theory (potential differences)? Electrical currents and potential differences are "through" and "across" variables respectively, as are actions and motions. Yet, despite being geometrically identical, actions and motions (first order time derivative of displacements) are often taught using separate textbooks and very often by different teachers. There is, of course, much more to kinematics than motion defined in this way.

12. Conclusions

Graph theory has an important role to play in assembling *dl* simultaneous equations for kinematic analysis and *dk* simultaneous equations for statics analysis. The matrices assembled for those equations can be used again, when transposed, in two virtual power equations that also provide kinematics and statics analysis. Graph theory also contributes concepts and terminology to these virtual power equations; notably the concepts of cutset motions and circuit actions. One further outcome is a pair of dual topological laws, called here the zeroth laws of mechanics.

It was Erskine Crossley who sowed the seed.

13. Acknowledgements

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Figure captions

Figure	Caption
1	A spatial kinematic chain
2	The coupling graph G_C of the kinematic chain shown in Fig. 1
3	Graphs of the kinematic chain shown in Fig. 1: a) motion graph G_M; b) action graph G_A
4	A two-stage epicyclic gear train and a schematic diagram of it
5	The coupling graph G_C of the gear train shown in Fig. 4 when it is augmented by two active couplings represented by edges h2 and i2

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A network approach to mechanisms and machines: some lessons learned

(An abbreviated title of fewer than 40 characters, including spaces: A network approach to MMT)

T.H.Davies[1], Wolfson School of Manufacturing and Mechanical Engineering, Loughborough University, Loughborough, Leicestershire, UK, LE11 3TU

Abstract

This is essentially a review paper describing progress made in treating mechanisms and machines as networks. Some of the terminology that is helpful to this approach is explained. Relevant elements of graph theory are mentioned. The original aim was to find a robust procedure for finding the instantaneous relative motion of all pairs of bodies within a kinematic chain. The manner in which this was achieved produced several other results that have found unanticipated applications. These are mentioned and publications are cited. Lessons have been learned and these are discussed in Section 11.

Keywords:

circuit; constraint; cutset; freedom; graph; screw

1. Introduction

The author is glad of this opportunity to thank Erskine Crossley for his many acts of kindness and generosity and to join with others to pay tribute to the work he has done for IFToMM and as editor of the *Journal of Mechanisms*, the forerunner of this journal. In particular, the author can bear witness to the many contributions Erskine Crossley made to good international relations. But this is a technical paper and so it is appropriate to explain the stimulus Erskine Crossley provided that led to research interests of the author.

Erskine Crossley was the first to mention graph theory in the author's presence. Graph theory [1] [2] is a branch of topology concerned with the interconnections within a network of objects. Graph theory has found many applications; most relevant to this paper are applications in electrical network theory, more frequently called electrical circuit theory.

[1] mcthd@lboro.ac.uk (Trevor Davies),

Mechanism and machines can be thought of as coupling networks. Waldron [3] provides rules that apply to couplings arranged in series and in parallel. Like electrical networks, indirect couplings containing cross bracing pose special problems [4]. Baker [5] proposed a simple example that has subsequently proved well-suited as a demonstration for theories that have followed. One solution [6] required the adaptation of Kirchhoff's voltage law. Subsequent work [7] [8][2] [9] [10] [11] [12] has led to the adaptation of Kirchhoff's current law as well, and two virtual power equations that use matrices that are identical to those needed for the adaptations of Kirchhoff's laws except for being transposed. All four equations are reproduced in this paper; the adaptations of Kirchhoff's laws equations (1,2) in section 7.2 and the virtual power equations (3,4) in section 8.2. Several applications have been found for the equations [13] [14] [15] [16] [17] [18] [19] [20] [21] [22] [23] [24] [25] [26] [27] [28] [29] [30] [31]; further details are provided in section 10.

Nomenclature

a	the rank of the network unit action matrix $\lfloor \hat{\mathbf{A}}_N \rfloor_{dk,C}$
b_{ij}	the element in row i, column j, of circuit matrix $[\mathbf{B}_M]_{l,F}$
c	degree of constraint of a direct coupling
c_{ij}	degree of constraint of bodies i and j of a coupling network
C	gross degree of constraint of a coupling network, Σc
C_N	nett degree of constraint of a coupling network
d	minimum order of the screw system, $1 \le d \le 6$
e	number of couplings in a coupling network and edges of coupling graph G_C
f	gross degree of freedom of a direct coupling
f_{ij}	degree of freedom of bodies i and j of a coupling network
F	gross degree of freedom of a coupling network, Σf
F_N	nett degree of freedom of a coupling network
k	number of independent cutsets of a graph
l	number of independent circuits (loops) of a graph
m	the rank of the network unit motion matrix $\lfloor \hat{\mathbf{M}}_N \rfloor_{dl,F}$
n	number of bodies in a coupling network and nodes of coupling graph G_C
q_{ij}	the element in row i, column j, of cutset matrix $[\mathbf{Q}_A]_{k,C}$
$\{r, s, t; u, v, w\}$	motion screw components in ray-coordinates
$\{R, S, T; U, V, W\}$	action screw components in axis-coordinates

Vectors

$[\mathbf{A}_l]_{dl}$	dl action components for all l circuits
$[\mathbf{M}_k]_{dk}$	dk motion components for all k cutsets
$[\varphi]^{C}$	magnitudes of C action screws
$[\psi]_F$	magnitudes of F motion screws

[2] Online versions of papers [8, 10-12, 17, 32] can be found from the website of Loughborough University, UK: search for Library, Institutional Repository, Author, Davies, T.H.

Matrices

$\lfloor \hat{\mathbf{A}}_D \rfloor_{d,C}$	unit action matrix of the direct couplings of a coupling network
$\lfloor \hat{\mathbf{A}}_N \rfloor_{dk,C}$	network unit action matrix of a coupling network N
$[\mathbf{B}_i]_{F,F}$	diagonal matrices with diagonal elements corresponding to row *i* of $[\mathbf{B}_M]_{l,F}$; in practice identification is by the circuit label, e. g. $[\mathbf{B}_b]_{F,F}$ for circuit b.
$[\mathbf{B}_M]_{l,F}$	circuit matrix of motion graph G_M
$\lfloor \hat{\mathbf{M}}_D \rfloor_{d,F}$	unit motion matrix of the direct couplings of a coupling network
$\lfloor \hat{\mathbf{M}}_N \rfloor_{dl,F}$	network unit motion matrix of a coupling network N
$[\mathbf{Q}_i]_{C,C}$	diagonal matrices with diagonal elements corresponding to row *i* of $[\mathbf{Q}_A]_{k,C}$; in practice, identification is by the cutset label, e. g. $[\mathbf{Q}_a]_{C,C}$ for cutset *a*.
$[\mathbf{Q}_A]_{k,C}$	cutset matrix of action graph G_A

2. Couplings

Central to the network approach described in this paper is the *coupling*. This term is applied to any means by which an *action* can be transmitted between two bodies that are sufficiently stiff to be regarded as rigid. Furthermore, a coupling must be capable of being disassembled without resort to cutting. This means that welded and riveted joints are not regarded as couplings, nor are joints formed by adhesion. Action is a term that is sometimes used [11] [12] [32] as shorthand for a wrench on a screw of any pitch, including a pitch that is zero, namely a force, and a pitch that is infinite, namely a torque. The coupling could be either direct, indirect or a hybrid comprising direct and indirect couplings in parallel. Except where it is necessary to make a distinction, all couplings mentioned are direct couplings. The term coupling is chosen as the name of a superset comprising passive and active couplings, the latter providing sinks or sources of power. Examples of couplings of both kinds have been listed [10]. Important subclasses of passive couplings mentioned in this paper are contact couplings, often referred to as kinematic pairs, and elastic couplings.

As well as the capability of transmitting an action, many couplings also permit relative motion of the bodies they couple. Motion is a term sometimes used [11] [12] [32] as shorthand for the first time derivative of displacement, geometrically described as a twist rate on the screw of any pitch, including a pitch that is zero, namely an angular velocity, and the pitch that is infinite namely translational velocity. A coupling is characterised by two screw systems [33], a *c*-system of actions that can be transmitted and an *f*-system of motions that can be allowed, and:

$$c + f = d,$$

where c and *f* are often referred to as the degrees of constraint and freedom of the coupling. The sum *d* could be said to be the dimension of the problem, having normally a maximum value of six. Simplification results from disregarding some of the actions couplings are capable of transmitting and then *d* will be less than six. Examples are to be found in section 10.

The action and motion screws systems of couplings are said to be reciprocal to one another because a screw of one system cannot expend power in conjunction with any of the screws of the other system. Note the use of the term power rather than work. The term work would be appropriate if motion is interpreted as infinitesimal displacements, as Ball [34] does. Here, and elsewhere [11] [12] [33] [35], the choice is made to divide all infinitesimal displacements by an infinitesimal time interval. Both approaches are equally valid.

3. Coupling networks

The following definition of the coupling network is expressed in terms that have similarities with the definition of a graph that appears later. A *coupling network* N consists of a non-empty finite set of bodies and a finite set of couplings linking pairs of those bodies. At least one path exists from each body of N to every other body of N, through couplings and other bodies of N. In other words, to borrow a term from graph theory, a coupling network is *connected*, that is to say, in one piece, rather than *disconnected*, in two or more parts.

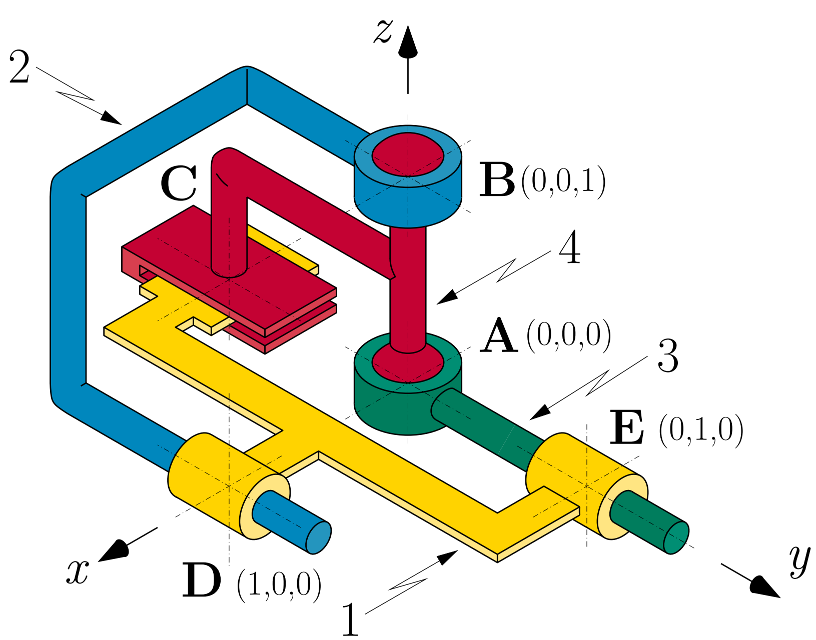


Figure. 1 A spatial kinematic chain

A coupling network has a characteristic gross degree of freedom $F = \Sigma f$ and a characteristic gross degree of constraint $C = \Sigma c$, where the summations are over all couplings. Coupling networks have another pair of characteristics of greater importance: these are the nett degree of freedom F_N and the nett degree of constraint C_N where, $0 \leq F_N \leq F$ and $0 \leq C_N \leq C$. The nett degree of freedom F_N has been called M, the degree of mobility, but mobility has another meaning [36]. It is also the "complex velocity response at a point in a linear system to a unit force excitation applied at the same point or another point in the system (inverse of mechanical impedance)". Coupling networks for which $F_N = 0$ are immobile structures that will not concern us here. Most structures are welded, riveted or made integral by adhesive so, owing to the restrictions placed on the meaning of a coupling, relatively few structures are coupling networks.

In the 1960s formulae were available for finding F_N, but they did not always work. One associated difficulty lead to a breakthrough. It had been identified [4] that finding the degree of freedom f_{ij} of two indirectly coupled bodies i and j is difficult if cross bracing exists. The task was to devise a general robust procedure that determines f_{ij} for any pair of bodies. Fig. 1 shows coupling network N that is a spatial kinematic chain, devised by Baker [5], and used since [6] [12] [32] as a test bed for some of the research cited in this paper.

Note for the publishers.

For the on-line version a supplementary video based on Figure 1 is submitted with this manuscript. The title is "Davies video". This is a suitable point in the manuscript to draw the reader's attention to it.

The kinematic chain is artificially contrived so that the elements of all matrices associated with it are 0, -1 or +1. Note that, for bodies two and three, the planar (ebene) coupling labelled C provides cross coupling. This is more evident in the coupling graph Fig. 2. One solution requires an adaptation of Kirchhoff's circulation law for mechanical problems. This approach resulted in a formula for F_N. Later, the problem of finding a formula for C_N was also achieved. Progress towards those two goals is explained in tandem wherever appropriate.

4. Kinematic chains, mechanisms and machines

The term *kinematic chain* is often applied to coupling networks for which F_N □ 0. In introductory texts on Mechanisms and Machines it is frequently found that a mechanism is described as a kinematic chain for which a "fixed member" has been selected. Once a fixed member has been chosen, all other choices of fixed member are often referred to as inversions of that mechanism.

This approach places an unnecessary emphasis on the identification of a "fixed member", yet says nothing about connections that must be made from the kinematic chain to active couplings in order that useful power can flow. Arguments have been given [10] in favour of a definition of mechanism in terms of content, rather than usage. The approach involving content requires the identification of bodies of the kinematic chain as *terminal bodies* [37], pairs of which are called *ports*. The terminals of a port are a pair of bodies of the kinematic chain that are intended to be made integral with terminal bodies of another coupling or network. If only one port is identified the kinematic chain is an example of a 1-port device, in other words the kinematic chain creates an indirect coupling between the two terminal bodies of the port.

A *mechanism* is a kinematic chain with two or more *ports*. In this context a port could be defined as a pair of terminal bodies through which power can be transmitted to or from a port of another network. The following are two examples of definitions of a port. "*A pair of terminals at which a signal may enter or leave a network is called a port."* [38]; "*A terminal pair to which an input is applied or from which an output is extracted is called a port*." [39]. For a mechanism, the term "signal" is inappropriate and "an input … an output" is unnecessarily vague.

Many mechanisms have only one input port and only one output port; mechanisms with several input ports are likely to be classified as manipulators; mechanisms with more than one output port are rare, the crank-driven needle and awl mechanism of a shoe welt sewing machine is one example [40]. Two or more ports may have one terminal body in common. This is often so when the common body is the one that is called the fixed member or frame.

A *machine* is a mechanism with all ports connected to active couplings or to the ports of indirect couplings that contain active couplings. Such indirect couplings may also contain passive couplings; for example an electrical motor has its own bearings. If the active coupling is a source of power these indirect couplings are often called *actuators*.

In order to adapt Kirchhoff's laws to coupling networks it is necessary to involve graph theory, the subject of the next section.

5. Directed graphs

A simple description of a graph is that it is a set of nodes (points or vertices), some or all pairs of which are connected by lines called edges. We will be concerned only with directed graphs, also called digraphs, within which all edges have an arrowhead thereby making the two nodes incident with each edge an ordered pair. A formal definition now follows.

A directed *graph* G consists of a non-empty finite set V(G) of elements called *nodes* (or vertices) and a finite family E(G) of ordered pairs of elements of V(G) called directed *edges*. The term "family" is used here, as in [2], to accommodate graphs within which multiple edges terminate in the same pair of nodes. We will not be concerned with graphs containing edges that terminate in the same node; such an edge is called a *loop*. The definition of coupling networks provided earlier is modelled on this definition of graphs. This is made possible by incorporating jointed structures for which $F_N = 0$ within coupling networks.

There are several useful terms used in graph theory. Within a graph, a *walk* is a finite sequence of edges. If all edges are distinct the walk is called a *trail*. If, in addition, the vertices are distinct, except possibly for the first and last, then the trail is a *path*. A trail is said to be *closed* if the first and last vertices are the same. A closed path is a cycle or *circuit*.

A graph is *connected* if and only if there is a path between each pair of vertices. A disconnecting set in a connected graph G is a set of edges whose removal disconnects G. A *cutset* is a disconnecting set, no proper subset of which is a disconnecting set. The removal of the edges in a cutset always leaves a graph with exactly two components. A connected graph with no circuits is a *tree* each edge of which is called a *branch* the only member of a cutset. A *spanning tree* is a connected subgraph that contains all the nodes of a graph, but no circuit. The edges not included in the spanning tree are called *chords* and the addition of any chord creates a circuit. Associated with each chord is a fundamental circuit, associated with each branch is a fundamental cutset.

6. Coupling graphs, motion graphs and action graphs

A coupling graph G_C is a graph within which each of the *n* nodes represents a body of a coupling network N and each of the *e* edges represents a coupling of N. These couplings are direct couplings but some indirect couplings such as rolling contact bearings and Hooke’s coupling can be regarded as direct provided that the investigation does not concern their interior actions and motions.

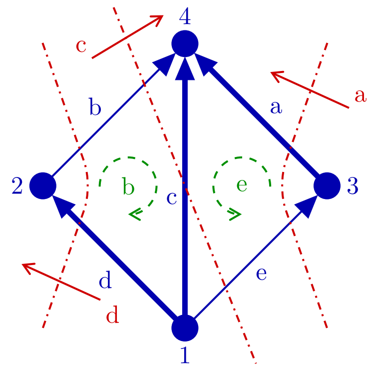


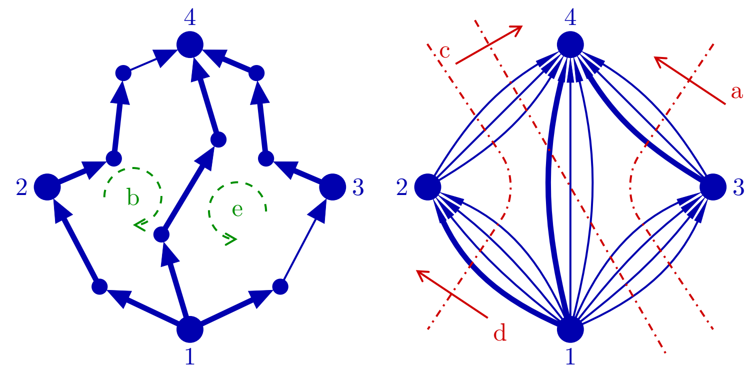
Figure. 2 The coupling graph G_C of the kinematic chain shown in Fig. 1

6.1 The coupling graph: its chords, branches, circuits and cutsets

A coupling graph will be said to have *l* chords and *l* fundamental circuits; it also has *k* branches and *k* fundamental cutsets. Fig. 2 shows the coupling graph G_C of the kinematic chain N shown in Fig. 1, with the arbitrarily selected spanning tree drawn with thick lines. Features of Fig.2 are now described. Here, and elsewhere in this paper, the presentation is provided in tandem where appropriate to emphasise the dual nature of the subject.

The edges **b** and **e** of G_C drawn with thin lines are the chords of the spanning tree. Each independent circuit contains one chord; all other edges are branches. Within these circuits there are arcs labelled **b** and **e** with arrowheads that assign a positive sense that can be arbitrarily chosen but, in this example, the choice corresponds with the positive sense assigned to the associated chords.

The edges **a**, **c** and **d** of G_C drawn with thick lines are the branches of the spanning tree. Each independent cutset contains one branch; all other edges are chords. Dashed lines are drawn through each cutset of edges. Arrows labelled **a**, **c** and **d** cutting these dashed lines assign a positive sense that can be arbitrarily chosen but, in this example, the choice corresponds with the positive sense assigned to the associated branches.



(a) (b)

Figure. 3 Graphs of the kinematic chain shown in Fig. 1:
a) motion graph G_M; b) action graph G_A

6.2 Motion and action graphs

From the coupling graph G_C it can be helpful to create a motion graph G_M and an action graph G_A. For the kinematic chain shown in Fig.1 these graphs are described below.

The motions allowed by a coupling having *f* degrees of freedom can be spanned by *f* independent motion screws. Each of these *f* screws can be represented in a motion graph G_M. The motion graph G_M is created by replacing each edge of G_C that represents an *f* degree of freedom coupling by *f* edges in series. Fig. 3a shows the motion graph for the kinematic chain of Fig. 1.

The actions transmitted by a coupling having *c* degrees of constraint can be spanned by *c* independent action screws. Each of these *c* screws can be represented in an action graph G_A. The action graph G_A is created by replacing every edge of G_C that represents a *c* degree of constraint coupling by *c* edges in parallel. Fig. 3b shows the action graph for the kinematic chain of Fig. 1.

The minimum number of parameters (independent motion magnitudes) necessary to provide the magnitudes of *all* motions within a coupling network is the nett degree of freedom F_N. Alternatively, F_N can be said to be the degree of overfreedom or excess freedom.

For a coupling network that is a tree,

$$F_N = F.$$

For coupling networks that contain one or more circuits comprised of two or more couplings,

$$0 \leq F_N \leq F.$$

Circuits can *reduce* freedoms.

The minimum number of parameters (independent action magnitudes) necessary to provide the magnitudes of *all* actions within a coupling network is the nett degree of constraint C_N. Alternatively, C_N can be said to be the degree of overconstraint or excess constraint.

For a coupling network that is a tree,

$$C_N = 0.$$

For coupling networks that contain one or more circuits comprised of two or more couplings,

$$C \geq C_N \geq 0.$$

Circuits can *increase* constraints.

7. Adaptations of Kirchhoff's laws

In this section matrices are needed that contain components of screws. Subscripts outside the square brackets around matrices signify the number of rows and columns respectively. A cap on a matrix signifies that the screws are normalised. The task of assembling equations is explained with the aid of the kinematic chain shown in Fig.1 and, in particular, the cylindrical coupling D having an axis through (1, 0, 0) parallel with the *y*-axis.

A notation is used that may be unfamiliar to the reader. This notation has been used before [11,12,17,32]; it is listed in the Introduction and explained in greater detail in section 11.3. The adaptations of the laws are now presented in tandem.

Kirchhoff's voltage law, when adapted for coupling networks, states that for each of the l independent circuits, the d components of screws spanning the motion screws of couplings of a circuit sum to zero when measured by reference to the same global frame. Thereby, dl equations can be written that impose conditions on the F unknowns. Some of these equations may prove to be redundant however. The circuit law equation can be written

Kirchhoff's current law, when adapted for coupling networks, states that for each of the k independent cutsets, the d components of screws spanning the action screws of couplings of a cutset sum to zero when measured by reference to the same global frame. Thereby, dk equations can be written that impose conditions on the C unknowns. Some of these equations may prove to be redundant however. The cutset law equation can be written as:

$$\left[\hat{\mathbf{M}}_N\right]_{dl,F}\left[\psi\right]_F = \left[\mathbf{0}\right]_{dl}. \qquad (1)$$

as:

$$\left[\hat{\mathbf{A}}_N\right]_{dk,C}\left[\varphi\right]_C = \left[\mathbf{0}\right]_{dk}. \qquad (2)$$

7.1 The vectors of unknown magnitudes

The vector $[\psi]_F$ = $[r_a, s_a, t_a, r_b, s_b, t_b, t_c, u_c, v_c, s_d, v_d, s_e, v_e]^T$ contains F unknown magnitudes of motions spanning the motion screw systems of the couplings listed in the same order as they appear in the columns of $\hat{\mathbf{M}}_N$. For example, in the kinematic chain shown in Fig. 1, coupling D allows motions that belong to a fifth special 2-system of motion screws [33]. This system is spanned by any two screws of unequal pitch with ISA sharing the cylinder axis. Most conveniently the screws selected are those with zero and infinite pitch, namely angular velocity of magnitude s_d about the cylinder axis, the (local) y_d-axis, and translational velocity of magnitude v_d in the direction of the y-axis.

The vector $[\varphi]^C$ = $[U_a, V_a, W_a, U_b, V_b, W_b, R_c, S_c, W_c, R_d, T_d, U_d, W_d, R_e, T_e, U_e, W_e]^T$ contains C unknown magnitudes of actions spanning the action screw systems of the couplings listed in the same order as they appear in the columns of $\hat{\mathbf{A}}_N$. For example, for the kinematic chain shown in Fig. 1, coupling D transmits actions that belong to a fifth special 4-system of action screws [33]. This system is spanned by any four screws reciprocal with the motion screws. A convenient set comprises torques (couples) parallel to the x- and z-axes of magnitudes R_d and T_d respectively, together with forces along the x- and (local) z_d-axes of magnitudes U_d and W_d respectively.

7.2 The network unit motion and unit action matrices

The network unit motion matrix

$$\left[\hat{\mathbf{M}}_N\right]_{dl,F} = \begin{bmatrix} \left[\hat{\mathbf{M}}_D\right]_{d,F}\left[\mathbf{B}_1\right]_{F,F} \\ \left[\hat{\mathbf{M}}_D\right]_{d,F}\left[\mathbf{B}_2\right]_{F,F} \\ \vdots \\ \left[\hat{\mathbf{M}}_D\right]_{d,F}\left[\mathbf{B}_l\right]_{F,F} \end{bmatrix},$$

where $\left[\hat{\mathbf{M}}_D\right]_{d,F}$, the direct coupling unit motion matrix, is determined by the geometry and $[\mathbf{B}_i]_{F,F}$, $i = 1, 2, \ldots, l$ by the topology as represented by the motion graph.

The network unit action matrix

$$\left[\hat{\mathbf{A}}_N\right]_{dk,C} = \begin{bmatrix} \left[\hat{\mathbf{A}}_\mathbf{D}\right]_{d,C}\left[\mathbf{Q}_1\right]_{C,C} \\ \left[\hat{\mathbf{A}}_\mathbf{D}\right]_{d,C}\left[\mathbf{Q}_2\right]_{C,C} \\ \vdots \\ \left[\hat{\mathbf{A}}_\mathbf{D}\right]_{d,C}\left[\mathbf{Q}_k\right]_{C,C} \end{bmatrix},$$

where $\left[\hat{\mathbf{A}}_D\right]_{d,C}$, the direct coupling unit action matrix, is determined by the geometry and $[\mathbf{Q}_i]_{F,F}$, $i = 1, 2, \ldots, k$ by the topology as represented by the action graph.

7.3 Direct coupling unit motion and unit action matrices

The direct coupling unit motion matrix $\left[\hat{\mathbf{M}}_D\right]_{d,F}$ contains the d components of each of the F unit motion screws with respect to the global frame of reference with its origin at the centre of the spherical coupling A.

For example, for the kinematic chain shown in Fig.1, the 10th and 11th columns of $\left[\hat{\mathbf{M}}_D\right]_{6,13}$, shown as a submatrix below, are the motion components for the f = 2 cylindrical coupling located at D.

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

When these normalised screws are multiplied by the 10th and 11th elements of $[\psi]^{13}$, s_d and v_d respectively, the two motion screws are obtained of body two relative to body one. Note that the sixth element of the 10th column, when multiplied by s_d, is a velocity along the z-axis of a point on an imaginary extension of body two located at the global origin. This velocity results from the angular velocity s_d about the (local) y_d-axis recorded in the second element of the 10th column.

The direct coupling unit action matrix $\left[\hat{\mathbf{A}}_D\right]_{d,C}$ contains the d components of each of the C unit action screws with respect to the global frame of reference with its origin at the centre of the spherical coupling A.

For example, for the kinematic chain shown in Fig.1, the 10th to the 13th columns of $\left[\hat{\mathbf{A}}_D\right]_{6,17}$, shown as a submatrix below, are the action components for the c = 4 cylindrical coupling located at D.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

When these normalised screws are multiplied by the 10th to the 13th elements of $[\varphi]^{17}$, R_d, T_d, U_d and W_d respectively, the four action screws are obtained that are exerted by body one on body two. Note that the second element of the 13th column, when multiplied by W_d, is the (negative) moment about the y-axis. This moment results from the force W_d along the (local) z_d-axis recorded in the sixth element of the 13th column.

7.4 The circuit matrix of G_M, the cutset matrix of G_A, and diagonal matrices derived from them

The matrices $[\mathbf{B}_i]_{F,F}$, i = 1, 2, …, l are diagonal matrices in which the diagonal elements of the i^{th} matrix are those of the i^{th} row of the circuit matrix $[\mathbf{B}_M]_{l,F}$ of the motion graph G_M.
Each element b_{ij} of $[\mathbf{B}_M]_{l,F}$ is 0, +1, or -1: b_{ij} is zero if circuit i does not include edge j; +1 if the positive sense of circuit i is in the same direction as the positive sense of the edge j that it includes; and -1 if those positive senses are opposed. For example, for the kinematic chain shown in Fig.1, the columns 10 and 11 of $[\mathbf{B}_M]_{2,13}$ are:

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

The first row confirms that edge d is a member of circuit b and the positive direction assigned to the circuit corresponds with that of the edge. The second row confirms that edge d does not belong to circuit e. Subsequently, in the diagonal matrix $[\mathbf{B}_b]_{13,13}$, the 10th and 11th diagonal elements are both one whereas, in $[\mathbf{B}_e]_{13,13}$, these elements are zero.

The matrices $[\mathbf{Q}_i]_{C,C}$, i = 1, 2, …, k are diagonal matrices in which the diagonal elements of the i^{th} matrix are those of the i^{th} row of the cutset matrix $[\mathbf{Q}_A]_{k,C}$ of the action graph G_A.
Each element q_{ij} of $[\mathbf{Q}_A]_{k,C}$ is 0, +1, or -1: q_{ij} is zero if cutset i does not include edge j; +1 if the positive sense of cutset i is in the same direction as the positive sense of the edge j that it includes; and -1 if those positive senses are opposed. For example, for the kinematic chain shown in Fig.1, the columns 10 - 13 of $[\mathbf{Q}_A]_{3,17}$ are:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

The last row confirms that edge d is a member of cutset d and the positive direction assigned to the cutset corresponds with that of the edge. The other two rows confirm that edge d does not belong to cutsets a and c. Subsequently, in the diagonal matrix $[\mathbf{Q}_d]_{17,17}$, the 10th - 13th diagonal elements are all one whereas, in $[\mathbf{Q}_a]_{17,17}$ and $[\mathbf{Q}_c]_{17,17}$, these elements are zero.

A consequence is that, for the kinematic chain of Fig.1, in columns 10 and 11 of the network unit action matrix $\left\lfloor \hat{\mathbf{M}}_N \right\rfloor_{12,13}$ the first six rows are identical to those of $\left\lfloor \hat{\mathbf{M}}_D \right\rfloor_{6,13}$ and all elements of the last six rows are zero.

A consequence is that, for the kinematic chain of Fig.1, in columns 10 - 13 of the network unit action matrix $\left\lfloor \hat{\mathbf{A}}_N \right\rfloor_{18,17}$ the last six rows are identical to those of $\left\lfloor \hat{\mathbf{A}}_D \right\rfloor_{6,17}$ and all elements of the first 12 rows are zero.

7.5 Results

If there is overconstraint, the rank m of $\left\lfloor \hat{\mathbf{M}}_\mathbf{N} \right\rfloor_{dl,F}$ is less than dl, the number of rows, and so

$$C_N = dl - m$$

rows are redundant. The remaining m independent equations impose m constraints on the F unknown

If there is overfreedom, the rank a of $\left\lfloor \hat{\mathbf{A}}_\mathbf{N} \right\rfloor_{dk,C}$ is less than dk, the number of rows, and so

$$F_N = dk - a$$

rows are redundant. The remaining a independent equations impose a constraints on the C unknown magnitudes. Thereby, these F unknowns can be expressed in terms of F_N primary variables, where

$$F_N = F - m.$$

For the kinematic chain shown in Fig.1, m is 10, C_N is two and F_N is three.

For every pair of bodies $\{i, j\}$ of a coupling network, equation (1) makes it possible to identify a set of f_{ij} independent motion screws that span the screw system of all motions of which bodies i and j are capable. Furthermore, equation (1) also expresses the magnitudes of each of these motion screws in terms of the magnitudes of F_N of them. Subject to some restrictions, there is freedom to choose which F_N motion screw magnitudes shall belong to this set.

magnitudes. Thereby, these C unknowns can be expressed in terms of C_N primary variables, where

$$C_N = C - a.$$

For the kinematic chain shown in Fig.1, a is 15, F_N is three and C_N is two.

For every pair of bodies $\{i, j\}$ of a coupling network, equation (2) makes it possible to identify a set of c_{ij} independent action screws that span the screw system of all actions that can be transmitted between bodies i and j. Furthermore, equation (2) also expresses the magnitudes of each of these action screws in terms of the magnitudes of C_N of them. Subject to some restrictions, there is freedom to choose which C_N action screw magnitudes shall belong to this set.

Because the foregoing is a brief summary of the full investigation [12], tables 1 and 2 below give the results in detail.

Table 1: Results obtained from the solution of equation (1) for the kinematic chain shown in Figure 1.

Pairs of bodies	Label of direct coupling	Motion components			
		f	Direct couplings with F unknowns	f_{ij}	After assembly, using $\{s_a, t_a, v_c\}$ as primary variables
1, 2	d	2	$\{0, s_d, 0, 0, v_d, 0\}$	1	$\{0, 0, 0, 0, v_c, 0\}$
1, 3	e	2	$\{0, s_e, 0, 0, v_e, 0\}$	2	$\{0, -s_a, 0, 0, v_c, 0\}$
1, 4	c	3	$\{0, 0, t_c, u_c, v_c, 0\}$	2	$\{0, 0, t_a, 0, v_c, 0\}$
2, 3	Absent		N/A	1	$\{0, s_a, 0, 0, 0, 0\}$
2, 4	b	3	$\{r_b, s_b, t_b, 0, 0, 0\}$	2	$\{0, 0, t_a, 0, 0, 0\}$
3, 4	a	3	$\{r_a, s_a, t_a, 0, 0, 0\}$	2	$\{0, s_a, t_a, 0, 0, 0\}$

Table 2: Results obtained from the solution of equation (2) for the kinematic chain shown in Figure 1.

Pairs of bodies	Label of direct coupling	Action components			
		c	Direct couplings with C unknowns	c_{ij}	After assembly, using $\{U_b, W_e\}$ as primary variables
1, 2	d	4	$\{R_d, 0, T_d, U_d, 0, W_d\}$	1	$\{0, U_b, 0, U_b, 0, -U_b\}$
1, 3	e	4	$\{R_e, 0, T_e, U_e, 0, W_e\}$	2	$\{0, 0, 0, -U_b, 0, W_e\}$
1, 4	c	3	$\{R_c, S_c, 0, 0, 0, W_c\}$	2	$\{0, -U_b, 0, 0, 0, (U_b - W_e)\}$
2, 3	Absent		N/A		N/A
2, 4	b	3	$\{0, 0, 0, U_b, V_b, W_b\}$	2	$\{0, U_b, 0, U_b, 0, -U_b\}$
3, 4	a	3	$\{0, 0, 0, U_a, V_a, W_a\}$	2	$\{0, 0, 0, -U_b, 0, W_e\}$

One further matter is included here that is not mentioned in [12]. Suppose that the kinematic chain were to be used as a 1-port coupling network with bodies two and three, the pair of original interest, as the terminals of the port. Suppose also that those bodies are now grasped by someone, one body gripped in each hand. The person who is gripping the two bodies is behaving as another 1-port coupling network but one that is a six dof serial manipulator with built-in active couplings called muscles. The appearance of s_a in column six, row four, of table 1 indicates that bodies two and three are capable of relative rotation about the y-axis. Note that s_b, s_d or s_e could have been chosen as primary variables instead. The actions that can be transmitted from body two to body three are thereby restricted to the 5-system of action screws that are all reciprocal to that rotation. These actions are spanned by $\{R_f, T_f, U_f, V_f, W_f\}$, because $s_f S_f, = 0$. Whereas c_{23} was previously zero, now that the human coupling has been added thereby internalising these actions, it is now five.

8. Virtual power equations

There is an alternative way of finding the number of primary variables F_N and C_N and, in addition, an alternative way of expressing the magnitudes of all motions and actions in terms of those primary variables.

8.1 The cutset motion and circuit action vectors

Instead of starting with F unknown coupling motion components, dk unknown cutset motion components can be used instead. These dk motion

Instead of starting with C unknown coupling action components, dl unknown circuit action components can be used instead. These dl action components are subject to C conditions, some of which may prove to be redundant. The C action components cannot expend or generate power in conjunction with the dk motions and so the C actions must be regarded as virtual actions.

The dk unknowns must be assembled in a cutset motion vector $[\mathbf{M}_k]_{dk}$. Using Fig. 3b as an example wherein d = 6 and k = 3, the first six elements of $[\mathbf{M}_k]_{18}$ are the six unknown components for cutset a, namely:

$$[r_a, s_a, t_a; u_a, v_a, w_a]^T.$$

There follows six components that are identical except that the subscript a is replaced by c, and six more subscripted by d.

components are subject to F conditions, some of which may prove to be redundant. The F motion components cannot expend or generate power in conjunction with the dl actions and so the F motions must be regarded as virtual motions.

The dl unknowns must be assembled in a circuit action vector $[\mathbf{A}_l]_{dl}$. Using Fig. 3a as an example wherein d = 6 and l = 2, the first six elements of $[\mathbf{A}_l]_{12}$ are the six unknown components for circuit b namely:

$$[R_b, S_b, T_b; U_b, V_b, W_b]^T.$$

There follows six components that are identical except that the subscript b Is replaced by e.

8.2 The transposed network unit action and unit motion matrices

To apply the C conditions vector $[\mathbf{M}_k]_{dk}$ must be pre-multiplied by the transpose of the network unit action matrix $\lfloor \hat{\mathbf{A}}_N \rfloor_{dk,C}$ used in equation (2). Thus:

$$\lfloor \hat{\mathbf{A}}_N^T \rfloor_{C,dk} [\mathbf{M}_k]_{dk} = [\mathbf{0}]_C . \qquad (3)$$

The C rows of $\lfloor \hat{\mathbf{A}}_N^T \rfloor_{C,dk}$ can be reduced to a rows by eliminating the C_N redundant ones.

For a coupling represented by a chord of G_C, the coupling motion components are those of the corresponding circuit of G_C. For a coupling represented by a branch of G_C, the motion components are the sum of the motion components of the circuits of G_C to which the branch belongs.

To apply the F conditions vector $[\mathbf{A}_l]_{dl}$ must be pre-multiplied by the transpose of the network unit motion matrix $\lfloor \hat{\mathbf{M}}_N \rfloor_{dl,F}$ used in equation (1). Thus:

$$\lfloor \hat{\mathbf{M}}_N^T \rfloor_{F,dl} [\mathbf{A}_l]_{dl} = [\mathbf{0}]_F . \qquad (4)$$

The F rows of $\lfloor \hat{\mathbf{M}}_N^T \rfloor_{F,dl}$ can be reduced to m rows by eliminating the F_N redundant ones.

For a coupling represented by a branch of G_C, the coupling action components are those of the corresponding cutset of G_C. For a coupling represented by a chord of G_C, the action components are the sum of the action components of the cutsets of G_C to which the chord belongs.

The kinematic chain shown in Fig. 1 has no utility except as a geometrically and topologically simple example to demonstrate principles involved. Useful examples are described in the next two sections.

9. Dual coupling networks

The work described so far raises the question as to whether, for a coupling network N with network matrices $\hat{\mathbf{M}}_N$ and $\hat{\mathbf{A}}_N$ there exists a dual coupling network N* with network matrices $\hat{\mathbf{M}}_N^*$ and $\hat{\mathbf{A}}_N^*$ such that $\hat{\mathbf{M}}_N^*$ and $\hat{\mathbf{A}}_N^*$ are identical to $\hat{\mathbf{A}}_N$ and $\hat{\mathbf{M}}_N$ respectively? Dual coupling networks have been created and the procedure for creating them has been explained in detail [32], the chosen example is the coupling network N shown in Fig. 1 and its dual. The procedure requires the identification of dual couplings and dual coupling graphs. The duals of some simple planar kinematic chains have also been described [8] [17]; the latter is mentioned again in the next section.

Such studies are an aid to an understanding screw theory and graph theory. Furthermore, whereas actions are difficult to imagine in a coupling network N, it is relatively easy to imagine the geometrically identical screws that that describe the motions that can take place within the dual network N*.

10. Applications

The first two subsections involve coupling networks for which the geometry can be greatly simplified by ignoring some of the constraints. A consequence is that the dimension *d* can be less than six thereby making the matrices considerably smaller.

10.1 Planar kinematic chains

Studies [17] have been made of the duals of planar kinematic chains that are in critical configurations. By confining attention to motion screws belonging to the fifth special 3-system of screws, a dimension *d* of three can be used in assembling equation (1) with the consequence that matrix $\hat{\mathbf{M}}_N$ is much smaller than it would otherwise be. A complete kinematic analysis of a Stephenson kinematic chain is provided using equation (1) and this is shown to be identical to the results of a static analysis of the dual of the kinematic chain using equation (2).

10.2 Gear trains, friction and efficiency

Equations (2, 4) have limited utility when applied to a kinematic chain for reasons that are discussed later in section 11. These equations do have value however for studies of the statics of machines operating at a constant speed. The two-stage epicyclic gear train shown in Fig. 4 provides an example of the use of all four equations [11].

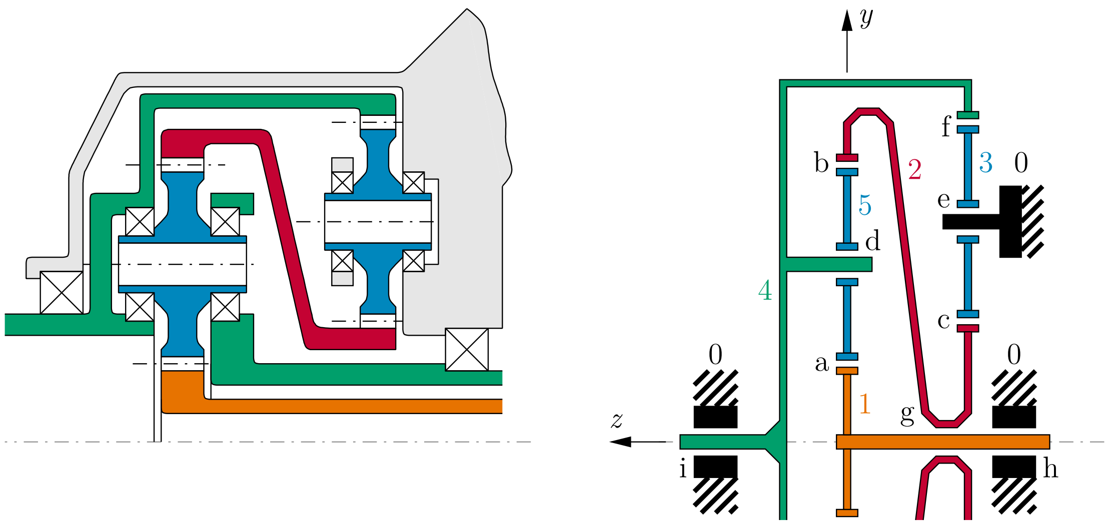


Figure. 4 A two-stage epicyclic gear train and a schematic diagram of it

In order to use equations 1 and 3 for kinematic analysis no modification is needed. In order to use equations 2 and 4 for the statics problem however, the gear train must be supplemented by two 1-port coupling networks that provide a source and sink for power, an electric motor and a fan for example. Both of these 1-port coupling networks contain an active coupling that transmits torque about the *z*-axis; they will also have bearings with the centre lines on the *z*-axis, but these duplicate the role played by bearings that exist within the gear train and can be ignored.

A major problem remains. The two extra actions supplement the many actions that could exist attributable to overconstraint. Because equations 2 and 4 can only analyse internal actions those actions attributable to overconstraint cannot be avoided. The problem is thereby far more complex than it needs to be. The extended coupling network can be greatly simplified however without impairment to the basic statics problem by taking the following steps.

- ❖ All but one planet in each stage is ignored.
- ❖ All moving parts are assumed to exist in the $z = 0$ plane.
- ❖ Both kinds of coupling, meshing gears and bearings, are assumed to be ($c = f = 1$) couplings by ignoring all other freedom and constraint.

Both the motion screws and the remaining action screws both belong to second special 2-systems of screws. These special screw systems differ geometrically however. Angular velocities have ISA parallel with the *z*-axis in the *x* = 0 plane, whereas forces have ISA parallel with the *x*-axis in the *z* = 0 plane. As Shai and Pennock [41] have observed of a similar gear train, the system is now identical to a sequence of levers.

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Figure. 5 The coupling graph G_C of the gear train shown in Fig. 4 when it is augmented by two active couplings represented by edges h2 and i2

For equation 2 two additional active couplings are needed and so, in Fig. 5, there are two edges from node 0 to node 1, and two edges from node 0 to node 4. The two additional edges h2 and i2 representing active couplings are shown as dashed lines. Fig. 5 is also the action graph G_A because c = 1 for all couplings. The five independent cutsets are identified in Fig. 5 by chain-dotted lines. Because f = 1 for all couplings, again Fig. 5 is also the motion graph except that edges h2 and i2 can be omitted. The four independent internal circuits are then obvious.

Cazangi and Martins [13] employ equation (1) for the analysis of two gear trains; one has two degrees of freedom, two forward ratios and one backward; the second has three degrees of freedom, three forward ratios and one backward.

Laus *et al* [14] employ equations 1 and 2 for studies of the efficiency of an epicyclic gear train and a Humpage gear train. For both, account is taken of friction, including gear tooth friction.

Tischler *et al* [15] uses equation (4) for a study of friction in multi-loop linkages. This may be the only occasion that equation (4) has been used for an application except for the epicyclic gear train described above.

10.3 Kinematic chains in critical configurations

Tischler [16] uses equation (1) in a study of critical configurations of a RCCC kinematic chain; Davies and Laus [17] do likewise for a planar 6-Link Stephenson kinematic chain.

10.4 The use of symbolic screw components

In a study to predict the slop that results from clearances in couplings of the Melbourne dextrous finger, Tischler *et al* [18] use symbolic screw components so that the analysis is valid throughout the cycle of configurations instead of only at one instantaneous configuration.

10.5 The use of virtual couplings (Assur groups)

An Assur group does not introduce additional constraints. For example, for a planar manipulator it can comprise PPR couplings in series; for a spatial manipulator PPPRRR or PPPS couplings in series. Equation (1) proves to be very useful; the primary variables can be either those of couplings of the manipulator or, for inverse kinematics, couplings of the Assur group.

Several workers have used Assur groups in combination with equation (1). Erthal *et al* [19] use them for a study of vehicle suspension; Campos *et al* [20] for the inverse kinematics of serial manipulators and [21] for the inverse kinematics of parallel manipulators. Inverse kinematics also gets attention from Simas *et al* [22].

There is work reported by Guenther *et al* [23] and Santos *et al* [24] [25] on the study of underwater manipulators. Simas *et al* [26] [27] and Rocha *et al* [28] report on work to avoid collisions and for carrying out tasks such as remote repair. Ribeiro *et al* [29] [30] describe the use of virtual chains in studies of cooperating robots. Recently, Ponce Saldias *et al* [31] [42] have extended the application of equation 1 and Assur groups to the modelling of the human knee to aid pre-operative planning.

11. Discussion

In this section some lessons learned from the foregoing are discussed.

11.1 If there is a “fixed” member in a mechanism, does it matter which it is?

In his lengthy notes that he includes in his English translation of Reuleaux [43], Kennedy [44] argues that a *machine* is defined by many in terms of what it does whereas, ideally, it should be defined in terms of what it comprises. In [10] this criticism is extended to some definitions provided by IFToMM [36]. In section 4 some extracts from [10] are repeated in order to draw attention to the fact that there is not necessity to identify an element (body/link/member) that is fixed. Of course, there are mechanisms, such as some handheld tools, wherein the term "fixed" is irrelevant.

For studies of kinematics and statics, the significance of a fixed member is unimportant. It is accepted of course that if acceleration, the second derivative of displacement, is a feature then it is essential to identify an inertial member, most frequently the earth.

11.2 A directed graph provides a concise and easily accessible record of a user-selected sign convention.

Anyone who has learned, or taught, elementary mechanics using free body diagrams may remember the tedium involved in using arrows twice, once on each of two directly coupled bodies. Likewise, for kinematics, it is necessary to distinguish the motion of body A relative to body B and body B relative to body A.

A directed graph has merits. A positive sense assigned to an edge by using an arrowhead indicates which, of two possibilities, will be regarded as the positive sense in any analysis. The choice of direction is an arbitrary decision. The coupling graph G_C in Fig. 5 of the gear train shown in Fig. 4 has nine edges so there are 512 possible different sets of directed edges. Fig. 3 provides evidence that it is the author’s practice to assign the positive direction away from the node labelled with the lower number. It is suggested here that the directed graph provides a concise store of a sign convention of the user’s choice that can be read at a glance.

11.3 In order to write the reciprocity condition it is sufficient to remember *rR*

In recent publications [11] [12] [17] [32] the author has chosen to represent the reciprocity condition for motion and action screws as follows:

$$rR + sS + tT + uU + vV + wW = 0.$$

Where {*r, s, t*} are the {*x, y, z*} components of angular velocity; {*u, v, w*} are components of the velocity of a point located at the origin; {*R, S, T*} are the components of moments measured at the origin; and {*U, V, W*} are the components of forces. The simple layout in the equation above is easily remembered and easily keyboarded. Others may prefer asterisks and exotic curly fonts. Note that *R* - *W* is sequential whereas $\mathscr{L} – \mathscr{R}$ is not; *T* is the moment about the *z*-axis, often the moment of Torque, and *u* and *v* are easily remembered velocity components of the origin along the *x*- and *y*-axes respectively. Furthermore, *p* is available for the pitch of a screw.

11.4 Mechanical network theory can be much more complex than electrical DC network theory.

Suppose that a coupling graph G_C, such as the one shown in Fig. 2, is also the graph of an electrical network. To keep matters simple suppose also that every one of the *e* edges corresponds either to a battery, or a resistor.

A coupling graph has *l* independent circuits and chords. For the equivalent electrical network there are therefore *le* elements in the voltage law equation matrix. For the equivalent mechanical matrix $\hat{\mathbf{M}}_N$, the number of elements is *Fdl.* The ratio is: *Fdl/le* = *Fd/e*.

A coupling graph has *k* independent cutsets and branches. For the equivalent electrical network there are therefore *ke* elements in the current law equation matrix. For the equivalent mechanical matrix $\hat{\mathbf{A}}_N$, the number of elements is *Cdk.* The ratio is: *Cdk/ke* = *Cd/e*.

Summary of results drawn from examples mentioned in this paper are provided in Table 3 below.

Table 3: The size of matrices relative to those of a topologically identical DC electrical network

Coupling network	*d*	*e*	Circuit law		Cutset law	
			F	***Fd/e***	*C*	***Cd/e***
Fig. 1	6	5	13	**78/5**	17	**102/5**
Stephenson III, a 6-link planar kinematic chain [17]	6	7	6	**36/7**	20	**180/7**
	3	7	6	**18/7**	N/A	
Simplified epicyclic gear train, Fig. 4	2	11	N/A		11	**2**
	2	9	9	**2**	N/A	

Judging by the ratio of the number of elements in matrices, *Fd/e* and *Cd/e*, the complexity of the coupling network problems are generally much greater than those of a simple DC network having the same topology.

11.5 Which equations are best?

For kinematic chains it has been observed that C, C_N, and matrix $\hat{\mathbf{A}}_N$ are larger, sometimes much larger, than F, F_N and matrix $\hat{\mathbf{M}}_N$ respectively. This suggests that, for statics of machines, equation 4 is superior to equation 2 and, for kinematics, equation 1 is superior to equation 3 which may explain why Jean Bernoulli never wrote about virtual actions.

11.6 Actions attributable to overconstraint cannot be measured by geometry and topology

Overconstraint is potentially dangerous, so awareness of its existence is important. This topic is also discussed in section 11.8. For kinematic chains equations 2 and 4 are incapable of providing the magnitudes of actions. These equations can enable all C actions that can exist within a kinematic chain that are attributable to overconstraint to be expressed in terms of a set of C_N actions that are chosen as primary variables. The magnitudes of these C_N actions remain unknown however; they are dependent on tolerances, shape, manufacturing errors, temperature and material properties.

11.7 The dual zeroth laws of mechanics

The zeroth law of thermodynamics is fundamental, very simple, and too obvious for much notice to be taken of it. The decision to number the law as the zeroth law is attributed to Fowler and Guggenheim [48].The law can be stated in several ways, Fowler and Guggenheim write:

If two thermal assemblies are each in thermal equilibrium with a third assembly, then all three are in thermal equilibrium with each other.

The following dual laws for actions and motions within coupling networks can be expressed in tandem.

The action law	The motion law
An action can be transmitted around a circuit comprising bodies and couplings provided that <u>all</u> those couplings are capable of transmitting that action.	*Two bodies separated by a cutset of couplings can have relative motion provided that <u>all</u> those couplings are capable of allowing that motion.*

Because the dual laws above, like the zeroth law of thermodynamics, are fundamental, very simple, and too obvious for much notice to be taken of them, maybe it is appropriate that they be called the dual zeroth laws of mechanics.

In this paper, with its focus on coupling networks, it is appropriate to write the law in its dual form; the symmetry of duality is also appealing. If duality is ignored the action law can be stated in a simpler way as:

An action cannot exist without a circuit capable of transmitting it.

This simple law becomes apparent when actions are internalised as they must be to employ equations (2, 4). It may have been overlooked because Isaac Newton was a free body diagram man: he never internalised actions.

Turning to the motion law, it is obvious that two bodies can be in relative motion without being members of a coupling network. In these circumstances it could be said that the only coupling is a *null coupling* that allows any motion.

11.8 Does elastic design get sufficient attention?

The existence of overconstraint can result in fatigue failure. Attempts to limit the dangerous consequences of overconstraint are of two kinds. One is kinematic design whereby additional freedom is introduced thereby increasing F_N and, by doing so, reducing C_N. This is certainly the preferred route for precision instruments. The second kind is to employ elastic design whereby, by changes in certain dimensions or a change of materials, some parts are made sufficiently compliant to allow limited elastic deformation.

Most writers concentrate attention on their speciality, either the kinematic approach or the elastic approach. Professor Michael French, an academic and a writer on the subject of engineering design, is an exception. He is an unrepentant generalist exemplified by his statement: "Never ask a specialist; they always give the wrong answer." Ouch! In his book [45], there is a chapter titled Kinematic and Elastic Design. It is a very good balanced account of the two approaches with several examples from gear trains that were in production at the time of publication.

11.9 Screw theory is addictive. All papers and books that mention screw theory should be required to print a warning: screw theory can damage your career.

The reader will understand the author's reluctance to provide evidence for this assertion but two addicts are mentioned if only because they are long since dead. In *A History of Mathematics*, Cajori [46] writes about Julius Plücker (1801-1868) [47], one of the founding fathers of screw theory; the following is an extract.

"In Germany J. Plücker's researches met with no favour. His method was declared to be unproductive as compared with the synthetic method of J. Steiner and J. V. Poncelet! His relations with C. G. J. Jacobi were not altogether friendly. Steiner once declared that he would stop writing for *Crelle's Journal* if Plücker continued to contribute to it. The result was that many of Plücker's researches were published in foreign journals, and that his work came to be better known in France and England than in his native country. The charge was also brought against Plücker that, although occupying the chair of physics, he was no physicist. This induced him to relinquish mathematics, and for nearly 20 years to devote his energy to physics. Important discoveries on Fresnel's wave-surface, magnetism and spectrum-analysis were made by him. But towards the close of his life he returned to his first love, mathematics, and enriched it with new discoveries. By considering space as made up of lines he created a "new geometry of space."

Another major contributor to screw theory was Sir Robert Stawell Ball (1840-1913) [34]. He also had a day job. In 1892 he was appointed as Lowndean Professor of Astronomy and Geometry at Cambridge University at the same time becoming director of the Cambridge Observatory. He was in great demand as a popular speaker on astronomy. His important contributions to screw theory however were ignored for around 70 years.

So, perhaps the best way of defeating drug traffickers is to ignore them.

11.10 Actions and motions rarely appear in the same textbook

Mention of Robert Ball brings back memories of something written [11] on the occasion of symposium held in 2000 to celebrate the hundredth anniversary of the publication of his book, *A Treatise on the Theory of screws* [34]. It is worth mentioning again.

Can you imagine a University's Department of Electrical Engineering advertising for two posts; one for a teacher of Electrical Circuit Theory (electrical currents) and another for a teacher of Electrical Circuit Theory (potential differences)? Electrical currents and potential differences are "through" and "across" variables respectively, as are actions and motions. Yet, despite being geometrically identical, actions and motions (first order time derivative of displacements) are often taught using separate textbooks and very often by different teachers. There is, of course, much more to kinematics than motion defined in this way.

12. Conclusions

Graph theory has an important role to play in assembling *dl* simultaneous equations for kinematic analysis and *dk* simultaneous equations for statics analysis. The matrices assembled for those equations can be used again, when transposed, in two virtual power equations that also provide kinematics and statics analysis. Graph theory also contributes concepts and terminology to these virtual power equations; notably the concepts of cutset motions and circuit actions. One further outcome is a pair of dual topological laws, called here the zeroth laws of mechanics.

It was Erskine Crossley who sowed the seed.

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Figure captions

Figure	Caption
1	A spatial kinematic chain
2	The coupling graph G_C of the kinematic chain shown in Fig. 1
3	Graphs of the kinematic chain shown in Fig. 1: a) motion graph G_M; b) action graph G_A
4	A two-stage epicyclic gear train and a schematic diagram of it
5	The coupling graph G_C of the gear train shown in Fig. 4 when it is augmented by two active couplings represented by edges h2 and i2

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Title: A network approach to mechanisms and machines: some lessons learned

Mechanism and Machine Theory

mcthd@lboro.ac.uk (Trevor Davies)

Table 1: Results obtained from the solution of equation (1) for the kinematic chain shown in Figure 1.

Pairs of bodies	Label of direct coupling	Motion components			
		f	Direct couplings with F unknowns	f_{ij}	After assembly, using $\{s_a, t_a, v_c\}$ as primary variables
1, 2	d	2	$\{0, s_d, 0, 0, v_d, 0\}$	1	$\{0, 0, 0, 0, v_c, 0\}$
1, 3	e	2	$\{0, s_e, 0, 0, v_e, 0\}$	2	$\{0, -s_a, 0, 0, v_c, 0\}$
1, 4	c	3	$\{0, 0, t_c, u_c, v_c, 0\}$	2	$\{0, 0, t_a, 0, v_c, 0\}$
2, 3	Absent		N/A	1	$\{0, s_a, 0, 0, 0, 0\}$
2, 4	b	3	$\{r_b, s_b, t_b, 0, 0, 0\}$	2	$\{0, 0, t_a, 0, 0, 0\}$
3, 4	a	3	$\{r_a, s_a, t_a, 0, 0, 0\}$	2	$\{0, s_a, t_a, 0, 0, 0\}$

Table 2: Results obtained from the solution of equation (2) for the kinematic chain shown in Figure 1.

Pairs of bodies	Label of direct coupling	Action components			
		c	Direct couplings with C unknowns	c_{ij}	After assembly, using $\{U_b, W_e\}$ as primary variables
1, 2	d	4	$\{R_d, 0, T_d, U_d, 0, W_d\}$	1	$\{0, U_b, 0, U_b, 0, -U_b\}$
1, 3	e	4	$\{R_e, 0, T_e, U_e, 0, W_e\}$	2	$\{0, 0, 0, -U_b, 0, W_e\}$
1, 4	c	3	$\{R_c, S_c, 0, 0, 0, W_c\}$	2	$\{0, -U_b, 0, 0, 0, (U_b - W_e)\}$
2, 3	Absent		N/A		N/A
2, 4	b	3	$\{0, 0, 0, U_b, V_b, W_b\}$	2	$\{0, U_b, 0, U_b, 0, -U_b\}$
3, 4	a	3	$\{0, 0, 0, U_a, V_a, W_a\}$	2	$\{0, 0, 0, -U_b, 0, W_e\}$

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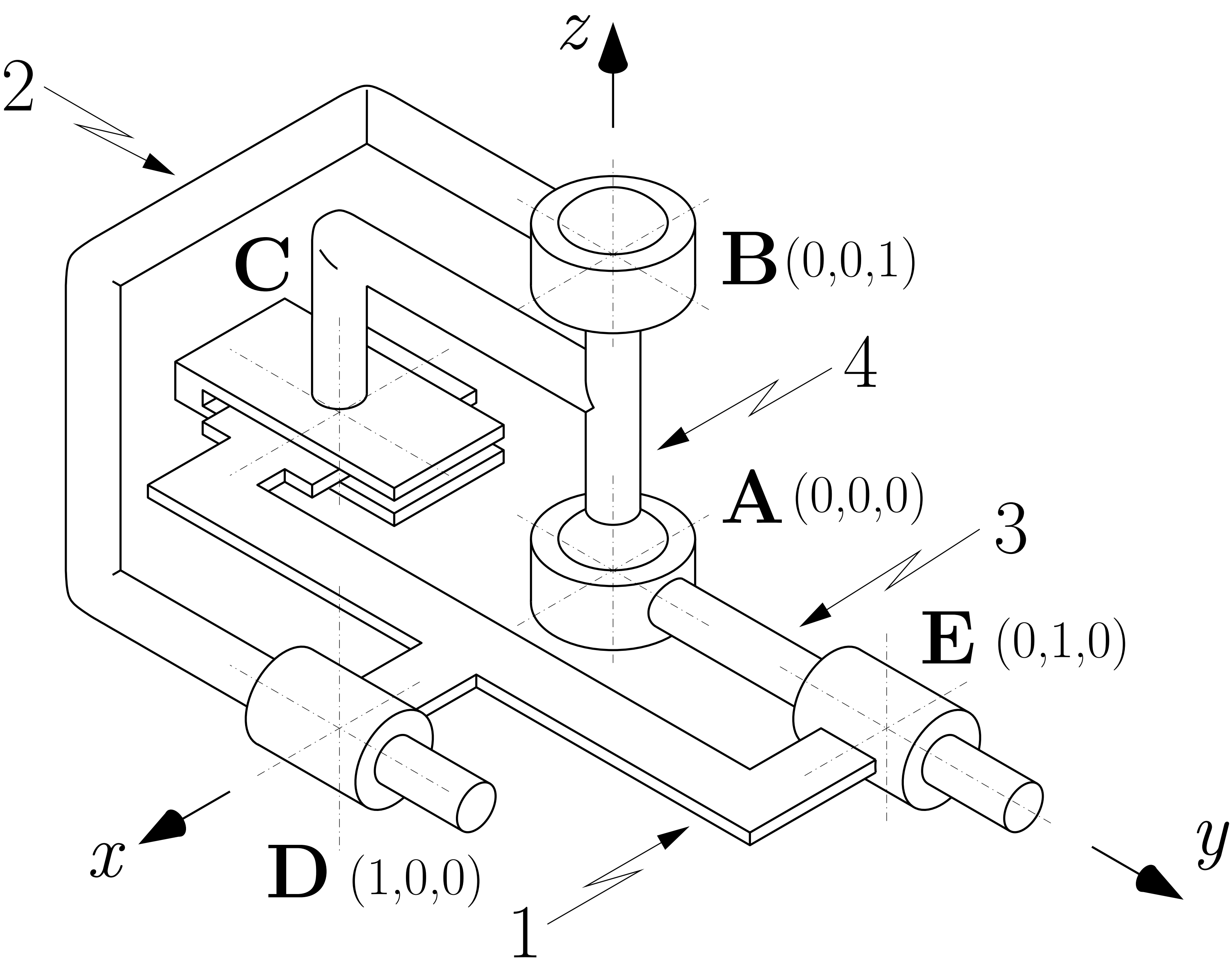

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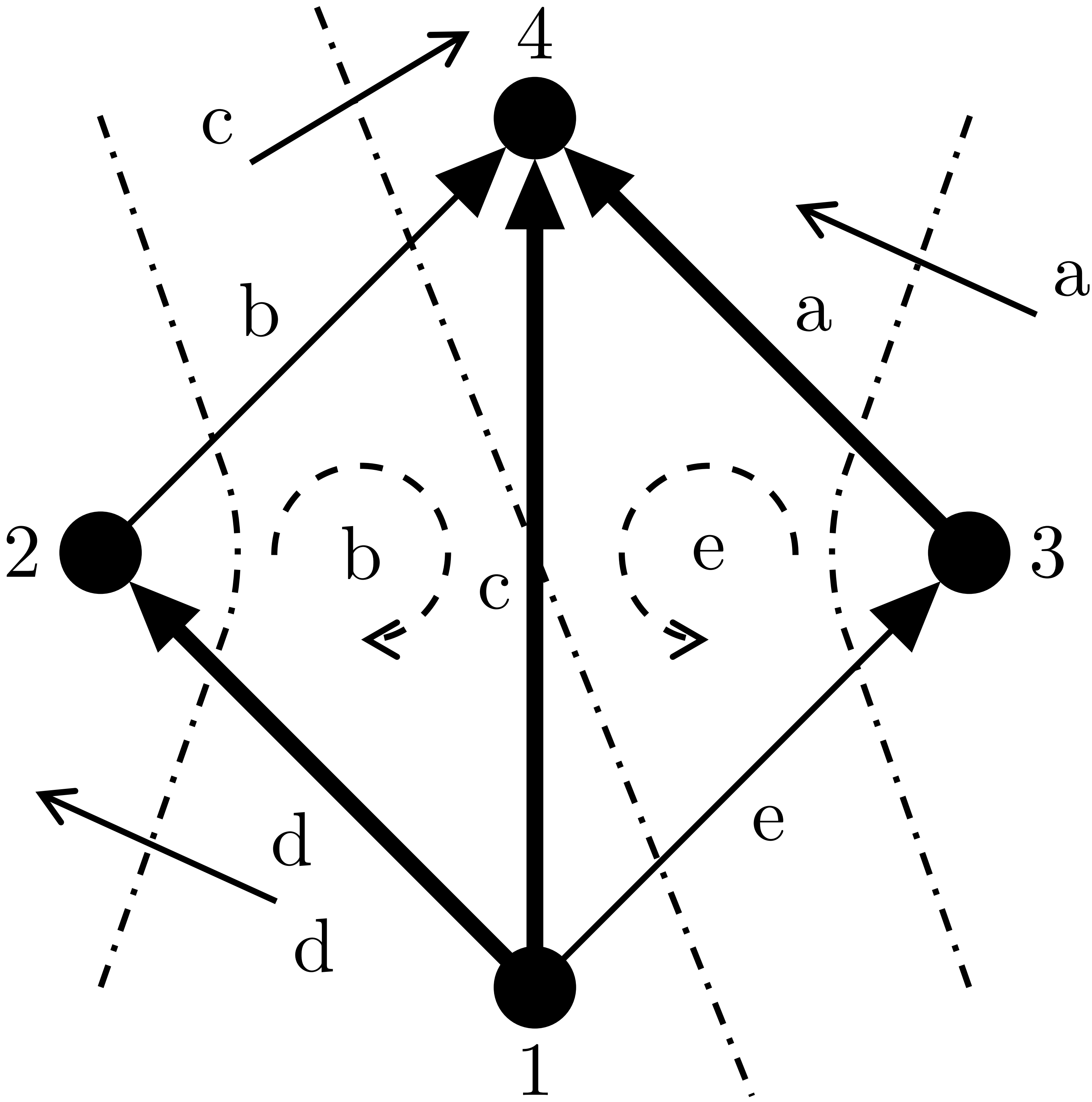

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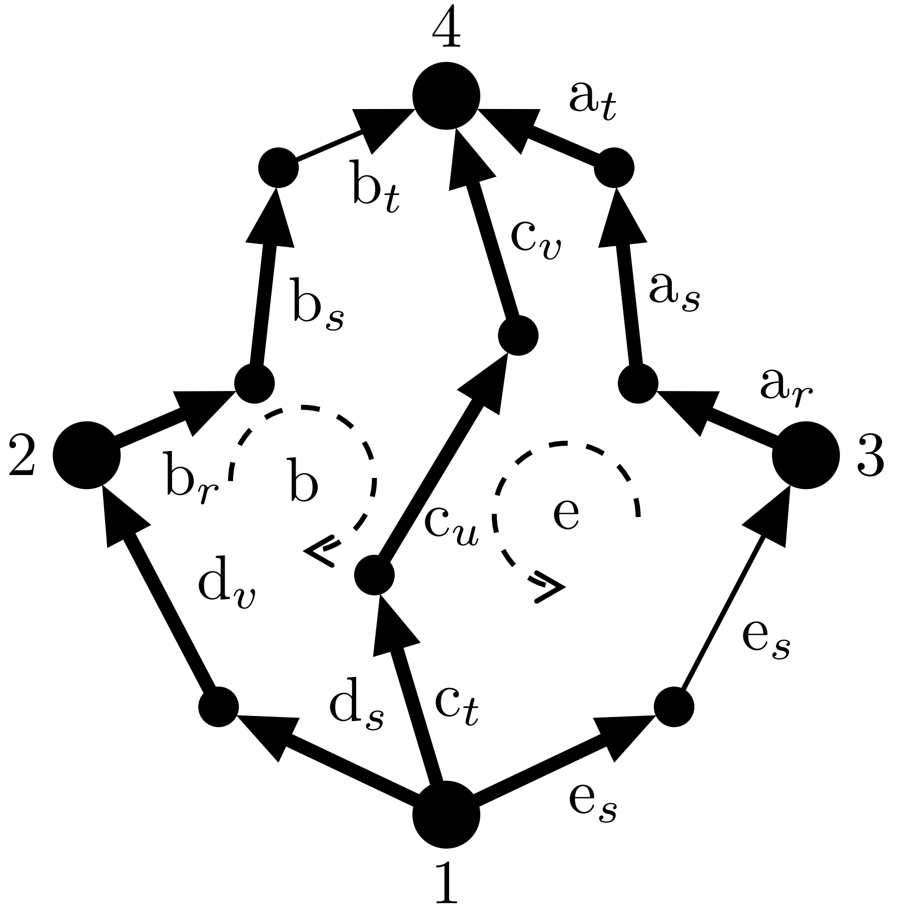

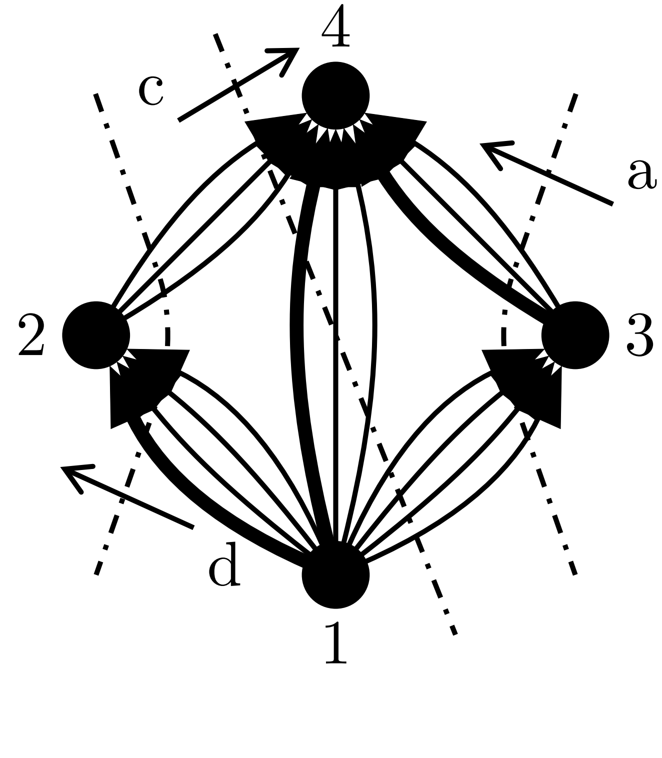

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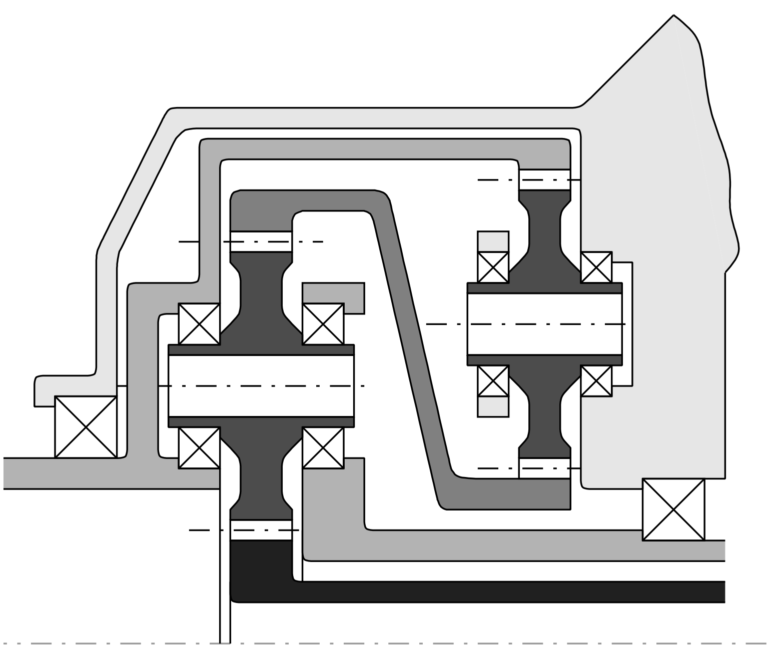

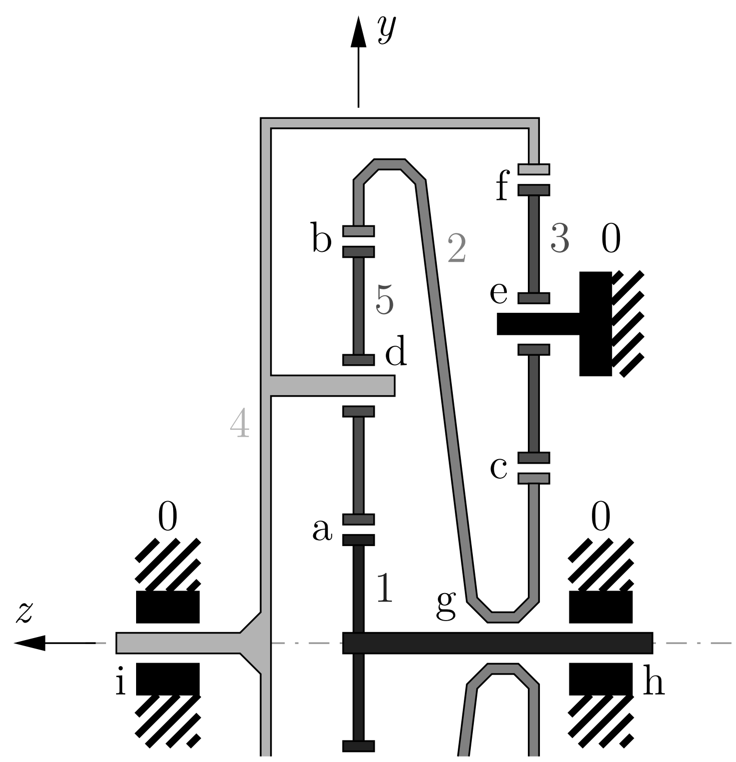

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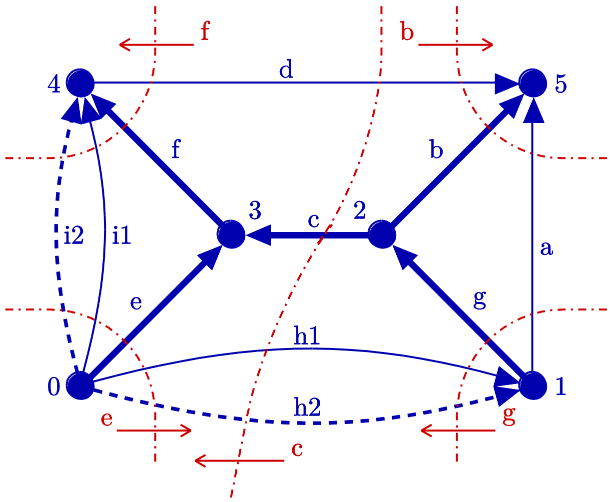

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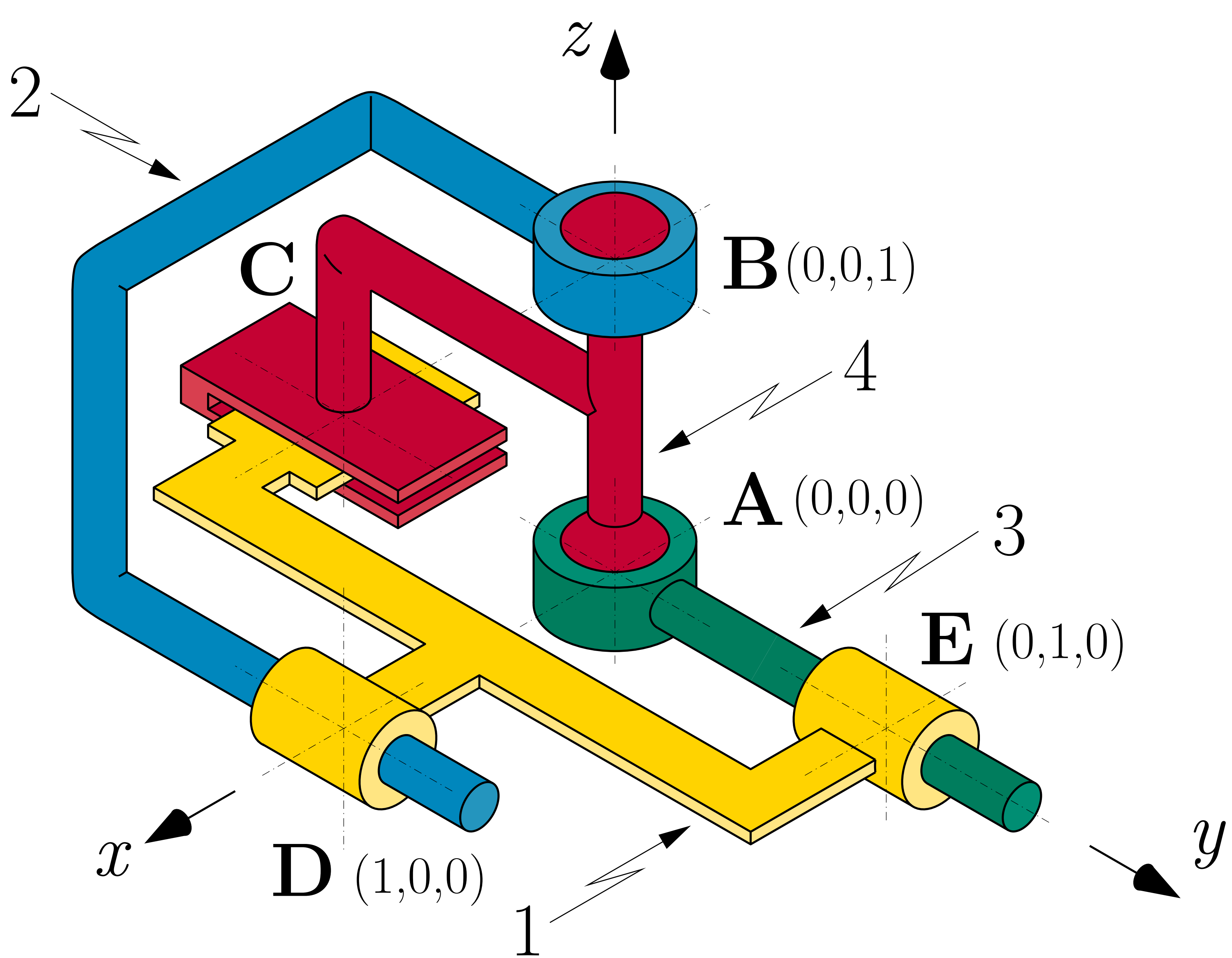

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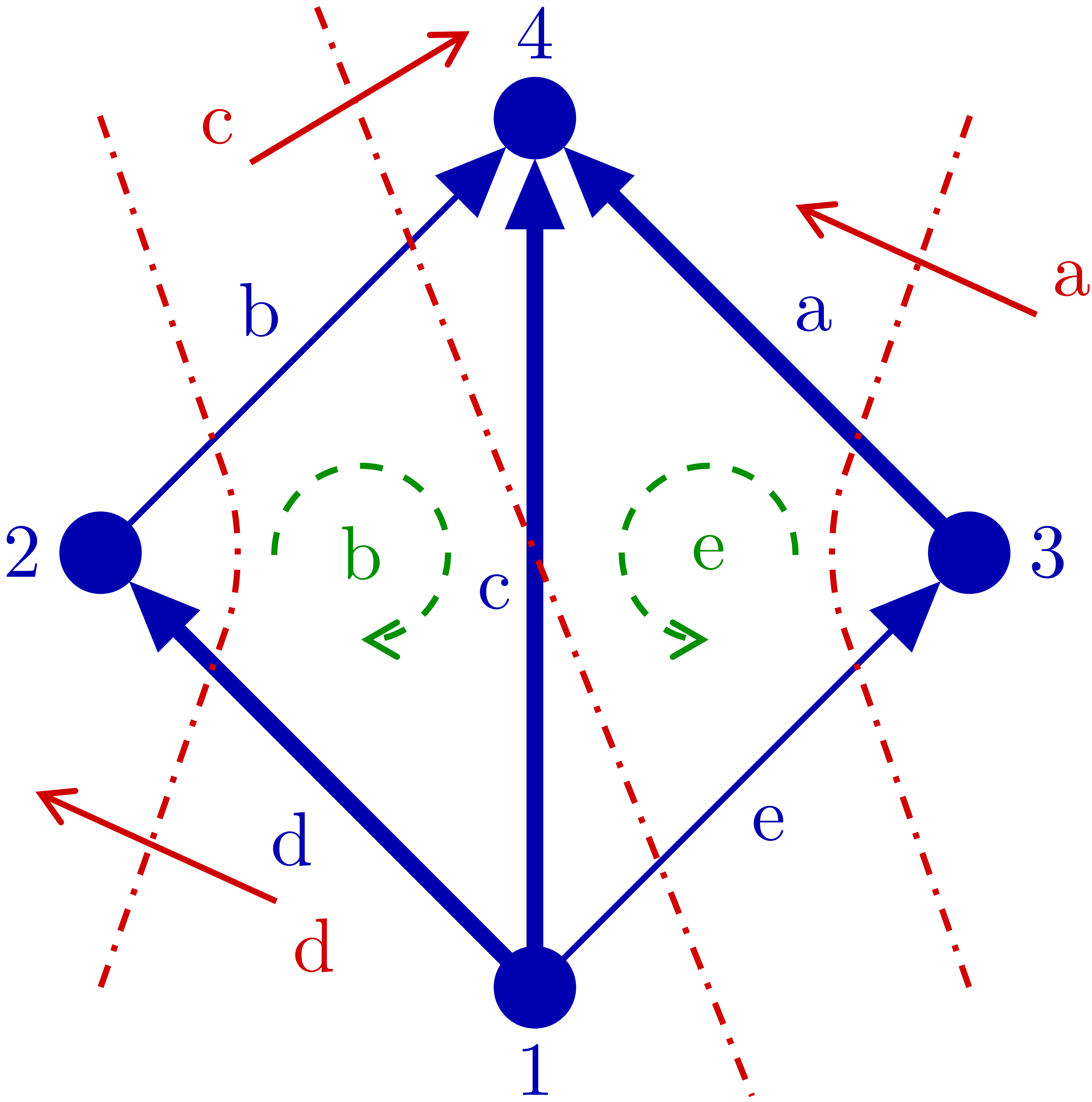

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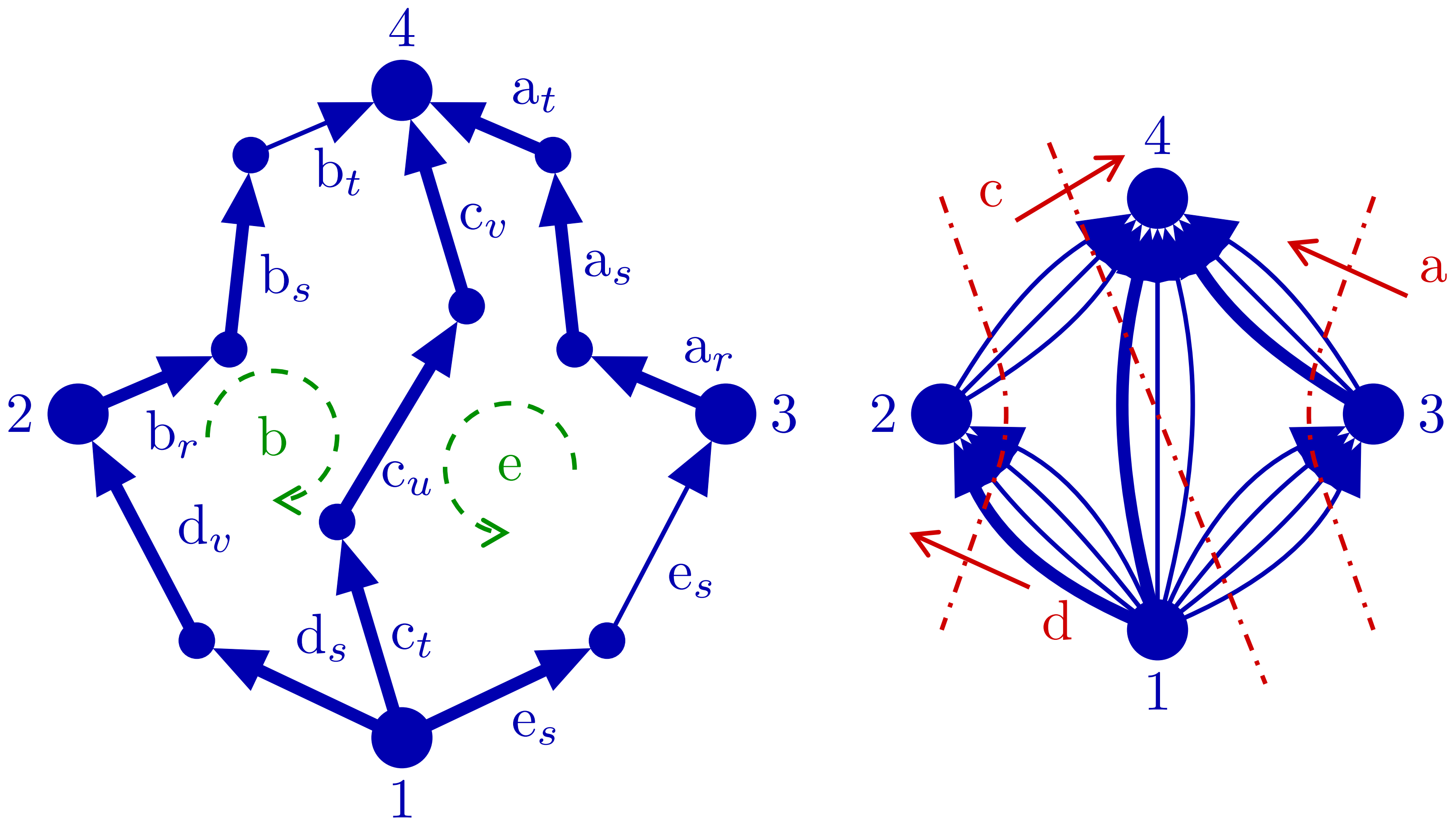

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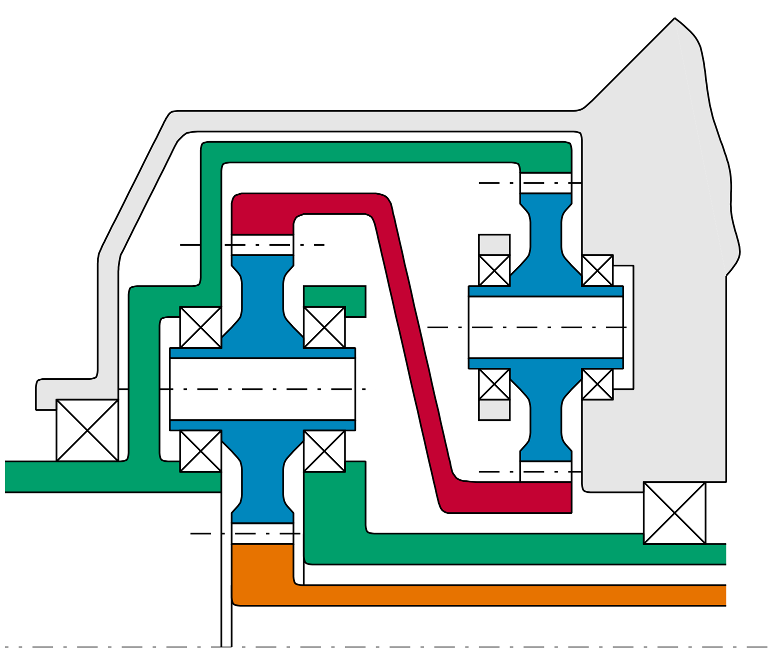

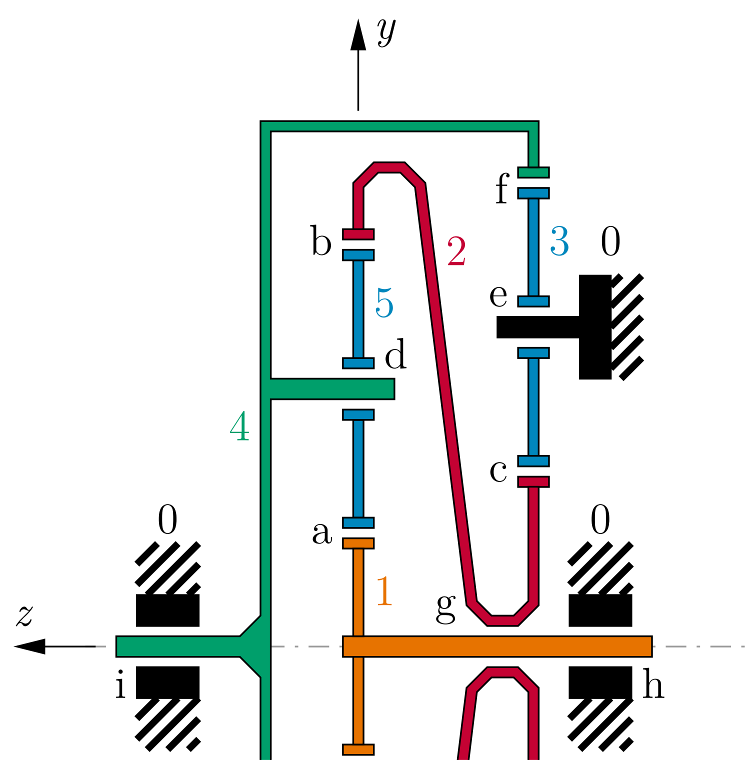

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