Journal of Numerical Cognition
jnc.psychopen.eu | 2363-8761

PsychOpen publishing psychology

Research Reports

The Interaction of Procedural Skill, Conceptual Understanding and Working Memory in Early Mathematics Achievement

Camilla Gilmore*[a], Sarah Keeble[a], Sophie Richardson[b], Lucy Cragg[b]

[a] Mathematics Education Centre, Loughborough University, Loughborough, United Kingdom. **[b]** School of Psychology, University of Nottingham, Nottingham, United Kingdom.

Abstract

Large individual differences in children's mathematics achievement are observed from the start of schooling. Previous research has identified three cognitive skills that are independent predictors of mathematics achievement: procedural skill, conceptual understanding and working memory. However, most studies have only tested independent effects of these factors and failed to consider moderating effects. We explored the procedural skill, conceptual understanding and working memory capacity of 75 children aged 5 to 6 years as well as their overall mathematical achievement. We found that, not only were all three skills independently associated with mathematics achievement, but there was also a significant interaction between them. We found that levels of conceptual understanding and working memory moderated the relationship between procedural skill and mathematics achievement such that there was a greater benefit of good procedural skill when associated with good conceptual understanding and working memory. Cluster analysis also revealed that children with equivalent levels of overall mathematical achievement had differing strengths and weaknesses across these skills. This highlights the importance of considering children's skill profile, rather than simply their overall achievement.

Keywords: mathematics education, mathematical cognition, working memory, conceptual understanding, procedural skill

Journal of Numerical Cognition, 2017, Vol. 3(2), 400–416, doi:10.5964/jnc.v3i2.51

Received: 2016-05-26. Accepted: 2016-11-11. Published (VoR): 2017-12-22.

Handling Editors: Hans-Christoph Nuerk, Humboldt-Universität Berlin, Berlin, Germany; André Knops, Universität Tübingen, Tübingen, Germany; Silke Goebel, University of York, York, United Kingdom

*Corresponding author at: Mathematics Education Centre, Loughborough University, Epinal Way, Loughborough, LE11 3TU, UK. E-mail: c.gilmore@lboro.ac.uk

Individuals' success in dealing with numbers and quantities is related to their job prospects, income and quality of life (Gross, Hudson, & Price, 2009; OECD Skills Outlook 2013, 2013; Parsons & Bynner, 2005). However, many adults do not have the numeracy skills required for everyday activities such as shopping and budgeting (Department for Business Innovation & Skills, 2011). Importantly, these difficulties emerge early, for example in the UK 21% of children leave primary school without the expected level of mathematics skills, and 5% of 11-year-olds fail to achieve the mathematics skills expected of 7 year-olds (Gross, 2007). Moreover, individual differences in the early years of schooling are seemingly resistant to change, such that the strongest predictor of later mathematical achievement is early mathematics skills (Duncan et al., 2007), although the mechanisms underlying this relationship may be complex (Bailey, Watts, Littlefield, & Geary, 2014)

As a result, researchers have increased their efforts to understand the cognitive bases for mathematical achievement with a view to developing more effective teaching strategies. There has been a particular focus on the early years of education, based on an assumption that early interventions may be more effective and efficient (Heckman, 2007). However, in order to intervene we need to understand the ways in which cognitive factors impact on children's success with learning mathematics.

Research to date has made progress in identifying a set of skills that are related to success in mathematics achievement. Many researchers have adopted a multi-component framework which recognizes that both domain-specific and domain-general processes contribute to mathematical skills (Fuchs et al., 2010; Geary, 2004, 2011; LeFevre et al., 2010), and that there are multiple pathways to numerical competence (Vukovic et al., 2014). This work has identified at least three groups of cognitive skills that are associated with better mathematics outcomes. Two of these skills are specific to mathematics: the ability to accurately and efficiently perform arithmetical procedures and understanding of the concepts that underlie arithmetic; while the third, working memory, is domain-general. The role of each of these skills will be considered below.

Procedural Skill and Conceptual Understanding

Procedural skill is the ability to carry out a sequence of operations accurately and efficiently (Hiebert & Lefevre, 1986) or *knowing how-to* (Baroody, 2003). Research has established that procedural skill is related to both concurrent and future mathematics achievement (e.g. LeFevre et al., 2010; Mazzocco & Thompson, 2005). For example, Jordan, Glutting, and Ramineni (2010) found that a comprehensive assessment of number recognition, counting, number comparison and basic calculation skills was a strong predictor of concurrent mathematics achievement and achievement two years later. Similarly, Geary (2011) identified counting knowledge, the use of advanced strategies to solve addition problems and fluency with symbolic representations of number as important predictors of mathematics achievement five years later. However, as highlighted by multi-component models of mathematical cognition (e.g. Geary, 2004), procedural skills do not operate in isolation, but are situated within a broader framework encompassing both conceptual understanding and domain-general skills.

Conceptual knowledge encompasses understanding of the principles and relationships that underlie a domain (Hiebert & Lefevre, 1986) or *knowing why* (Baroody, 2003). Studies have identified that good conceptual understanding is important for success in mathematics. For example, young children's understanding of arithmetic concepts (e.g. commutativity and part-whole relations) is associated with their problem solving accuracy and strategy use (Canobi, 2004). Children's conceptual understanding of counting is associated with both age and mathematics achievement, although the relationship may be more complex than the relationship between procedural skill and mathematics achievement (LeFevre et al., 2006). Children with mathematics learning difficulties also have poorer conceptual understanding than their typically-developing peers (Geary, Hamson, & Hoard, 2000). Good conceptual understanding allows children to make adaptive strategy choices when solving problems, for example by using conceptually-based shortcuts, and therefore children with better conceptual understanding are often found to also have better procedural skill (see review by Rittle-Johnson & Schneider, 2015).

Much research has explored how conceptual understanding and procedural skill develop, with evidence supporting a model of iterative development, such that advances in conceptual understanding lead to developments in procedural skill and vice-versa (Rittle-Johnson, Siegler, & Alibali, 2001). However, there may be individual differences in the relationship between conceptual understanding and procedural skill, such that, for example, children with the same level of procedural skill may have differing levels of conceptual understanding (Canobi, 2004; Gilmore & Papadatou-Pastou, 2009). Aside from research into the development of conceptual understanding itself, large longitudinal studies of the development of mathematics achievement have tended to place more attention on procedural skills or basic numerical processing, compared with conceptual understanding (e.g. Geary, Hoard, Nugent, & Bailey, 2013; Jordan et al., 2010; LeFevre et al., 2010; Sasanguie, Van den Bussche, & Reynvoet, 2012). Consequently, we know less about the importance of conceptual understanding for long-term mathematics outcomes and how this relates to domain-general influences on mathematics achievement.

Importantly, while previous research has demonstrated that procedural skill and conceptual understanding are both predictors of mathematics achievement, we do not know if they have independent effects. In fact, the iterative model of development would predict that these effects are not independent, but rather levels of conceptual understanding moderate the relationship between procedural skill and mathematics achievement. We return to this point below.

Working Memory

Alongside recognition of the importance of domain-specific skills has come an increased understanding of the importance of domain-general skills for mathematics learning. In particular, an extensive body of research has identified executive function skills, including working memory, inhibition and shifting, as being critical for success with mathematics. The majority of research has focused on working memory, the ability to monitor and manipulate information held in mind, and revealed that this is related to both concurrent and future mathematics achievement across development (see reviews by Bull & Lee, 2014; Cragg & Gilmore, 2014; Raghubar, Barnes, & Hecht, 2010). Associations with mathematics achievement have been found for both verbal and visuo-spatial working memory, although there is mixed evidence regarding which is more important (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013), which could be explained by developmental changes (Li & Geary, 2013). Working memory has been found to be associated both with general measures of overall mathematics achievement and individual components of mathematics (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017; Friso-van den Bos et al., 2013). In particular dual-task studies have identified that working memory is required for performing arithmetical procedures (Hubber, Gilmore, & Cragg, 2014; Imbo & Vandierendonck, 2007). Working memory may play multiple roles in mathematics performance, including problem representation, the storage of interim solutions and to access information stored in long-term memory. Although it is well established that working memory is important for mathematics achievement, the nature of this relationship is unclear. Specifically, we do not know if levels of working memory may change the nature of the relationship between basic numerical skills and overall mathematics achievement. Working memory plays a stronger role in procedural skills, in comparison to conceptual understanding (Cragg et al., 2017), and consequently we would predict that levels of working memory moderate the relationship between procedural skills and mathematics achievement.

Interactions Among Predictors

The studies described above have demonstrated the importance of procedural skills, conceptual understanding and working memory for mathematics achievement. One limitation of this body of work is that little attention has been paid to the moderating effects of one skill on the relationship between other skills and mathematics achievement. Although studies have established that procedural skill, conceptual understanding and working memory each make independent contributions to mathematics achievement, they have tended to focus on main effects and not interactions (Fuchs et al., 2010; Geary, 2011; LeFevre et al., 2010). Consequently, we do not know whether the relationship between, for example, procedural skill and mathematics achievement is the same regardless of levels of conceptual understanding or working memory.

In fact, we predict that differences in mathematics achievement arise not only from children's differing levels of skill in each of these areas, but also from the interactions amongst them. These interactions might be crucial to understanding why some children succeed with mathematics and others struggle. The importance of one skill may depend on the levels of proficiency with other skills. This could come about because children may be able to compensate for weakness in one skill with strengths in another. For example, good conceptual understanding may help children overcome working memory difficulties by allowing them to identify and use shortcut strategies which have lower working memory demands than simple computation. In contrast, it may be that weaknesses in one skill exacerbate difficulties with another, e.g. poor conceptual understanding combined with poor procedural skill may be particularly problematic. It is therefore important to consider different profiles of skills and how these relate to differences in mathematics achievement.

There is some previous evidence to suggest that it is important to consider combinations of skills, and not just individual skills in isolation. LeFevre et al. (2006) found different patterns in the development of conceptual understanding for 5 to 7 year old children who had high, average or low levels of procedural skill. Less skilled children showed delayed development of conceptual understanding. Geary (2011) also reported similar findings. Canobi (2004) highlighted that 6 to 8 year old children with good levels of conceptual understanding may have widely varying levels of procedural skill. Consequently, mathematics achievement measurements may not be good indicators of children's combination of conceptual and procedural skills (Gilmore & Bryant, 2006).

A small number of studies have begun to explore the interactions among procedural skill, conceptual understanding and domain-general skills. Watchorn et al. (2014) found an interaction amongst conceptual understanding (of the addition-subtraction inversion principle), attention skills and calculation ability in a study with children aged 7 to 9 years old. Good attention was associated with better conceptual understanding, but only for children who also had good procedural skills. However, this study did not explore how the interaction among these skills was associated with mathematics achievement. Cowan et al. (2011) explored procedural skill, conceptual understanding and a range of domain-general skills, including working memory, in children aged 7 to 9 years old. They identified that conceptual understanding and working memory both partially mediate the relationship between procedural skill and mathematics achievement. This mediation analysis indicates that one way in which procedural skill is associated with mathematics achievement is through its association with conceptual understanding and working memory. However, it does not identify how conceptual understanding or working memory might change the nature of the relationship between procedural skill and mathematics achievement. For this, moderation analysis is required, or in other words an examination of the association between the interaction of these skills and mathematics achievement.

Here we explore the association between mathematics achievement and procedural skill, conceptual understanding and working memory in children who are at the early stages of learning mathematics. We predict that mathematics achievement will not only be associated with the main effects of procedural skills, conceptual understanding and working memory, but also with the interaction between them. Specifically, we predict that conceptual understanding and working memory will moderate the relationship between procedural skill and mathematics achievement. We focus on children who are at the earliest stages of formal mathematics learning in an effort to understand how early differences in mathematics achievement arise.

Method

Participants

Participants in the study were 75 children (41 male) in Year 1 of primary school (mean age = 6.2 years *SD* = .36). For the majority of children in the UK, Year 1 is the second year of schooling, however it is the first year in which children follow the National Curriculum and in which teaching becomes more formal and structured. The children attended two suburban primary schools where the majority of students were white British. The proportion of children at the schools eligible for free school meals, with special educational needs or who spoke English as an additional language was in line with or below the national average. Parents of all children in Year 1 at the schools were sent letters about the study and given the option to opt out. Ethical approval for the study was obtained from the Ethics Approvals (Human Participants) Sub-Committee at Loughborough University.

Tasks

The children completed tasks to assess their mathematics achievement, procedural skills (counting and arithmetic), conceptual understanding and working memory (verbal and visuo-spatial). The working memory tasks were presented on a laptop computer, the other tasks were presented verbally, or with cards and props as described below.

Mathematics Achievement

Mathematics achievement was measured using the Wechsler Individual Achievement Test–II UK (Wechsler, 2005) mathematics composite. This consists of two subtests: numerical operations and mathematics reasoning, which were administered according to the standard procedure. The numerical operations subtest is a pencil and paper measure of children’s knowledge of the number system (i.e. digit recognition, counting) as well as simple abstract calculations. The mathematics reasoning subtest comprises a series of word problems that assess broader mathematical reasoning, including questions about number, shape and arithmetic. Problems are presented verbally with visual support. Raw composite scores across the two subtests were calculated and used in the analysis.

Procedural Skill

Children's procedural arithmetic skills were assessed using two tasks. A total procedural skill score was calculated by adding accuracy scores across the two tasks. Cronbach's alpha for this combined score was 0.73.

Counting — Children were asked to complete five counting sequences. They were asked to count up to 10, on from 28, on from 45, backwards from 12, and backwards from 33. With the exception of the first trial children were stopped after they had given seven numbers, which ensured that they had crossed a decade boundary in each case. Each trial was scored as correct if children completed the whole series correctly and proportion correct scores were recorded.

Arithmetic — Children were shown a series of addition and subtraction problems (e.g. 3 + 2) printed onto cards and were asked to solve them using any strategy of their choice. There were four practice trials and eight experimental trials. All problems involved single digit addends. The items were presented in one of two orders, counterbalanced across participants. Number lines marked from 1 – 10 and 1 – 20 and a set of counters were provided for children to use if they wished. Proportion correct scores were recorded.

Conceptual Understanding

To assess conceptual understanding children played a game involving a puppet, adapted from Canobi (2004). Children watched the puppet solve an arithmetic problem using counters and were shown the example problem (including the answer) written in a booklet (e.g. 3 + 6 = 9). They were then shown four probe problems which were presented without answers and for each probe problem children were asked whether the puppet could use the example (completed) problem to solve each probe problem, or if he would need to use the counters to solve it. Of the four probe problems, three were related to the example problem and one was unrelated (e.g. 6 + 9 =). One of the related problems was identical (e.g. 3 + 6 =), one was related by commutativity (e.g. 6 + 3 =) and one was related by inversion (e.g. 9 – 3 =). The children were first asked to decide whether or not the example problem could help the puppet solve each probe problem ("Do you think that Rolo needs to use the cubes to work this one out, or can he work it out by looking back at this one?", and then asked to explain why ("Why does he need to use cubes?" or "How can this one help him?" as appropriate). Children were credited with a correct explanation if they mentioned the relevant relationship (e.g. for identical problems the numbers were "just the same"; for commutativity the addends were "swapped round"; for inversion the numbers were "added and then taken away").

Children completed two practice example problems, each with one identical, one unrelated and one related (addend + 1 rule) probe problem, followed by 24 experimental trials (six example problems each with 4 probe problems). Feedback was provided during the practice trials to help children understand the task. All trials were audio recorded. For each item children were given a score (0 or 1) for answering whether the items were related and a score (0 or 1) for identifying the relationship. If children correctly answered that the items were unrelated then they were also given credit for a correct explanation. Therefore, the total score for accuracy was out of 24 and for explanations was out of 24. Proportion correct scores were calculated for each measure and a total conceptual understanding score was calculated by adding the proportion correct scores for answers and explanations. Cronbach's alpha for this combined score was 0.77.

Working Memory Tasks

Children completed separate verbal and visuo-spatial working memory tasks. A total working memory score was calculated by adding the scores across the two tasks.

Verbal Working Memory — Verbal working memory was assessed via a sentence span task. Children heard a sentence with the final word missing and had to provide the appropriate word. After a set of sentences children were asked to recall the final word of each sentence in the set, in the correct order. There were three sets at each span length, beginning with sets of two sentences, and children continued to the next list length if they responded correctly to at least one of the sets at each list length. The total number of correctly recalled words was recorded.

Visuo-Spatial Working Memory — Visuo-spatial working memory was assessed via a complex span task. Children saw a series of 3 x 3 grids each containing three symbols and they had to point to the symbol that differed from the other two. After a set of grids children were asked to recall the position of the odd-one-out on each grid, in the correct order. There were three sets at each span length, beginning with sets of two grids, and children continued to the next span length if they responded correctly to at least one of the sets at each span length. The total number of correctly recalled locations was recorded.

Results

In the following sections we first report children's performance on the set of arithmetic tasks and working memory measures and the associations between them. The relationship between procedural skill, conceptual understanding and working memory and mathematics achievement is then investigated using linear regression including moderation (interaction terms). Finally, cluster analysis is used to examine the performance of subgroups of children.

Task Performance

Descriptive statistics for performance on the arithmetic and working memory tasks are presented in Table 1. There was a good range of performance on all of the tasks, with no evidence of floor or ceiling effects.

Table 1

Descriptive Statistics for all Tasks

Task	*M*	*SD*	Min	Max
WIAT Numerical Operations (raw score)	8.76	3.02	3	24
WIAT Mathematics Reasoning (raw score)	18.00	5.20	8	36
Counting (proportion correct)	0.69	0.27	0.20	1
Arithmetic (proportion correct)	0.74	0.27	0	1
Conceptual understanding accuracy (proportion correct)	0.60	0.12	0.25	0.83
Conceptual understanding explanations (proportion correct)	0.52	0.14	0.08	0.75
Verbal working memory (total item score)	5.39	3.34	0	15
Visuo-spatial working memory (total item score)	14.59	9.07	0	42

Zero-order correlations among the measures of conceptual understanding, procedural skill, working memory and mathematics achievement are reported in Table 2. In line with theoretical models and previous research, procedural skills, conceptual understanding and working memory all had a strong positive correlation with mathematics achievement. This does not, however, reveal the independent contribution of these skills, nor whether they interacted in their association with mathematics achievement.

Table 2

Correlations Among Measures of Mathematics Achievement, Procedural Skills, Conceptual Understanding and Working Memory

Measure	Mathematics Achievement	Procedural skills	Conceptual understanding
Mathematics Achievement			
Procedural skills	.624**		
Conceptual understanding	.620**	.426**	
Working Memory	.639**	.410**	.354**

Note. $n = 75$.
**$p < .01$.

The Relationship Between Working Memory, Conceptual and Procedural Skills and Mathematics Achievement

We then sought to understand the relationship between children's working memory, conceptual and procedural scores with overall mathematics achievement. We conducted a multiple linear regression including the main effects of procedural skill, conceptual understanding and working memory as well as all two-way interactions and the three-way interaction. Specifically, we tested whether levels of conceptual understanding and working memory moderated the relationship between procedural skill and mathematics achievement. This was conducted using the PROCESS macro for SPSS (Hayes, 2013). To aid interpretability of any interaction effects, all variables were *z*-transformed prior to entering into the analysis.

Table 3

Multiple Linear Regression Predicting Mathematical Achievement by Procedural Skill, Conceptual Understanding, Working Memory and all Interaction Terms

Predictor	b	*t*	*p*
Procedural skill	0.39	4.35	<.001
Conceptual understanding	0.23	2.63	.010
Working memory	0.27	3.12	.003
Procedural * Working memory	0.09	0.84	.407
Conceptual * Working memory	-0.00	-0.05	.964
Procedural * Conceptual	0.25	2.39	.020
Procedural * Conceptual * Working memory	0.22	2.65	.010

Note. DV = WIAT mathematics composite raw scores. $R^2 = .71$.

The model was significant, $F(7,67) = 23.3$, $p < .001$, and explained 71% of the variance in mathematics achievement scores. As shown in Table 3, procedural skill ($p < .001$), conceptual understanding ($p = .010$) and working memory ($p = .003$) were all significant independent predictors of mathematics achievement. Furthermore, the interaction between procedural and conceptual scores ($p = .020$) and the three-way interaction among procedural scores, conceptual understanding and working memory (p = .010) were also significant predictors of mathematics achievement.

To explore these interaction effects we plotted predicted mathematics achievement scores (z scores) for high (1 *SD* above the mean) and low (1 *SD* below the mean) values of each skill. The two-way interaction is depicted in Figure 1, which shows the effect of high or low conceptual understanding for high or low levels of procedural skill. This clearly shows that good procedural skill is more beneficial, in terms of overall mathematics achievement, for children who have high, compared to low levels of conceptual understanding.

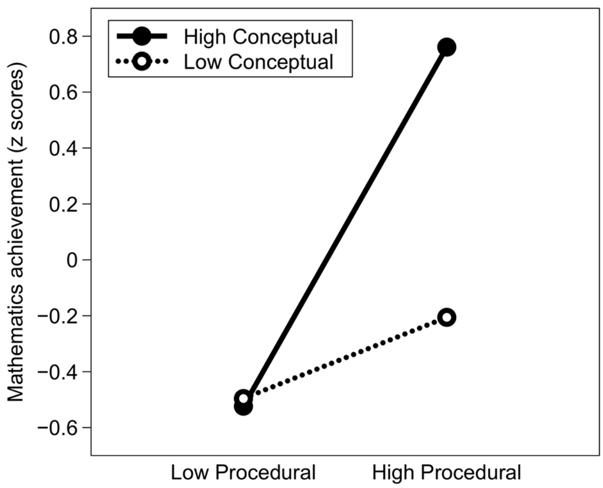

Figure 1. Predicted mathematics achievement (WIAT mathematics composite *z* scores) for high (1 *SD* above mean) and low (1 *SD* below mean) levels of procedural skill and conceptual understanding.

The three-way interaction between procedural skill, conceptual understanding and working memory on mathematics achievement is depicted in Figure 2. This suggests that the effect of high, compared to low, procedural skill differs according to levels of both conceptual understanding and working memory. There was a significant effect of procedural skill on mathematics achievement when associated with low conceptual understanding and low working memory, $t(67) = 2.45$, $p = .017$, high conceptual understanding and low working memory, $t(67) = 2.18$, $p = .033$, high conceptual understanding and high working memory, $t(67) = 3.63$, $p < .001$, but not with low conceptual understanding and high working memory, $t(67) = 0.11$, $p = .910$. To further explore the nature of this interaction we used the Johnson-Neyman technique (Hayes, 2013), which identifies values of a moderator for which an effect transitions between non-significant and significant. This identified that the interaction between procedural skill and conceptual understanding was significant for working memory scores above the 42% percentile (z-score of -.335) but non-significant for values below this. In other words, conceptual understanding moderated the relationship between procedural skill and mathematics achievement for working memory scores above this value. In contrast, for working memory scores below this value, procedural skill and conceptual understanding had independent effects on mathematics achievement.

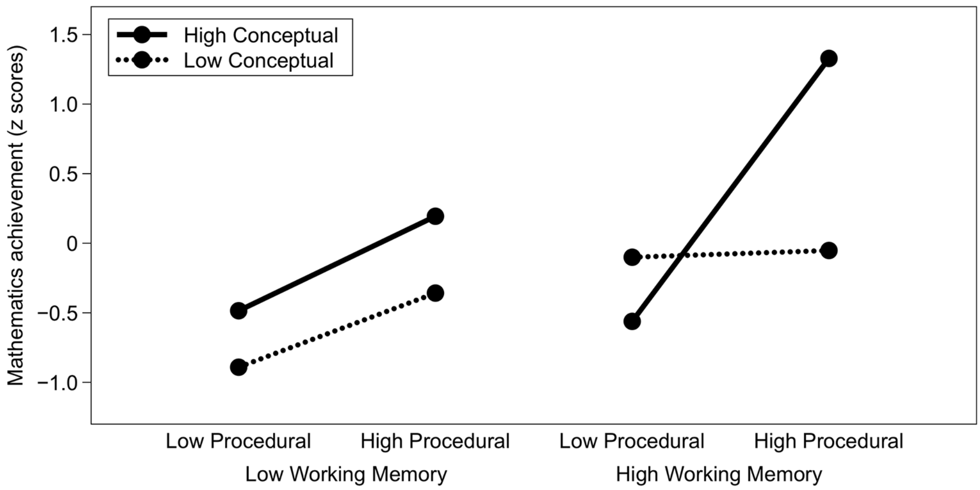

Figure 2. Predicted mathematics achievement (WIAT mathematics composite *z* scores) for high (1 *SD* above mean) and low (1 *SD* below mean) levels of procedural skill, conceptual understanding and working memory.

Cluster Analysis

Finally, we further explored the influence of these three skills on children's mathematics achievement by examining subgroups of children within the whole sample. We conducted a hierarchical cluster analysis (using Ward's method) on children's procedural skill, conceptual understanding and working memory. Examination of the dendogram indicated that 5 distinct clusters could be identified, and this solution accounted for 67.0% of the variance in scores.

Mean scores for each cluster on the measures of procedural skill, conceptual understanding and working memory are depicted in Figure 3. This suggests that children in Cluster 4 (n = 26) had overall strong performance across all measures, and children in Cluster 5 (n = 6) had overall weak performance across all measures, particularly procedural skill. However, children in Clusters 1 (n = 16), 2 (n = 12) and 3 (n = 15) show different profiles across the three measures. We compared performance across groups on each of the measures using one-way between-groups ANOVA. For procedural skill, $F(4,74) = 84.8$, $p < .001$, Bonferroni corrected t-tests indicated that children in Cluster 3 had significantly lower scores than children in either Cluster 1 ($p < .001$) or Cluster 2 ($p < .001$). For conceptual understanding, $F(4,74) = 21.9$, $p < .001$, children in Cluster 2 had significantly lower scores than children in Cluster 1 ($p < .001$). For working memory, $F(2,42) = 20.7$, $p < .001$, children in Cluster 1 ($p = .003$) and Cluster 3 ($p = .004$) had significantly lower scores than children in Cluster 2. Finally, we compared overall mathematics achievement (WIAT mathematics composite) scores across the clusters, $F(4,74) = 17.04$, $p < .001$. Children in Cluster 4 had the highest scores and children in Cluster 5 had the lowest scores, however mathematics achievement scores did not differ for children in Clusters 1, 2 or 3 (all p's > .2)

The differences among children in each of these subgroups can therefore be summarised as follows. Cluster 5 is a small group of children with generally low performance and Cluster 4 is a large group of children with generally high performance. Children in Cluster 1 had average levels of procedural skill and conceptual understanding, but poorer working memory. Children in Cluster 2 had average levels of procedural skill and working memory, but poorer conceptual understanding. Children in Cluster 3 had average levels of conceptual

PsychOpen
publishing psychology

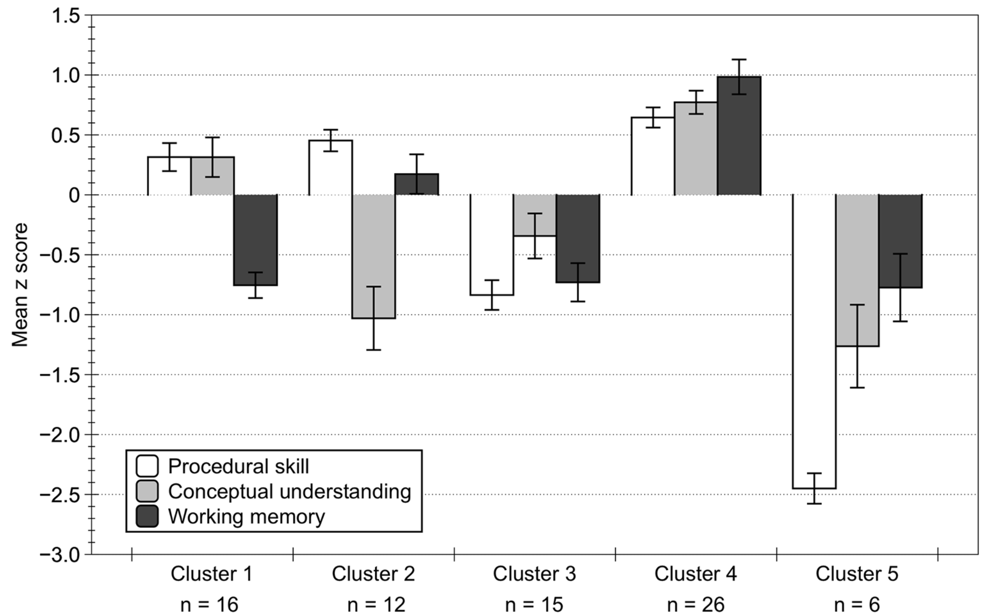

Figure 3. Procedural skill, conceptual understanding and working memory (mean *z*-scores) for each cluster.

understanding, but poorer procedural skill and working memory. Despite these differences, the performance of children in these three clusters did not differ on overall mathematics achievement.

Discussion

In line with multi-component frameworks of mathematics achievement (Fuchs et al., 2010; Geary, 2004; LeFevre et al., 2010) we found that procedural skill, conceptual understanding and working memory are all independently associated with overall mathematics achievement in children aged 5 to 6 years old. We have extended these frameworks, however, to show for the first time that the interactions between these factors are also important for understanding differences in mathematics achievement. The influence of each of these factors can only be fully understood by taking into account children's level of skill with the other factors. Specifically, levels of working memory and conceptual understanding moderated the relationship between procedural skill and mathematical achievement. Below we consider the implications of these findings for theories of mathematical cognition, and identify important questions for future research.

The significant 3-way interaction indicated that the relationship between procedural skill and mathematics achievement was moderated by levels of conceptual understanding and working memory. Good conceptual understanding is important to be able to identify and select computationally less demanding strategies, which can in turn reduce the reliance on procedural skills. However, this may only be possible if an individual has sufficient working memory resources. In line with this we found that for children with lower levels of working memory, procedural skill and conceptual understanding had independent effects on mathematics achievement and there was no additional benefit of having both good conceptual understanding and procedural skills. However, for children with higher levels of working memory, additional procedural skill conferred an extra benefit for mathematical achievement when associated with good conceptual understanding, compared to the

doi:10.5964/jnc.v3i2.51

effect on children with lower levels of conceptual understanding. This suggests that children are only able to apply good conceptual understanding in their problem solving, and thereby reduce computational demands, if they have the domain-general resources to allow them to identify how conceptual understanding is relevant. These results are consistent with those of Watchorn et al. (2014) and reinforce the important role of domain-general skills.

Models of mathematical cognition (e.g. Geary, 2004) propose that mathematics achievement depends on levels of procedural skill and conceptual understanding, and that these competencies in turn depend on domain-general skills including, but not limited to, working memory. We have found that, as well as having impact on levels of procedural skill and conceptual understanding themselves, differences in domain-general skills change the relationship between conceptual or procedural knowledge and mathematics achievement. One consequence of this is that it is harder to predict levels of achievement from measures of domain-specific skills alone, and that domain-general skills must also be taken into account.

Our cluster analysis highlighted subgroups of children with different profiles of performance. Interestingly, we found that weaknesses in different skills appeared to have equivalent impact on overall mathematics achievement. Specifically, there was no difference in mathematics achievement, measured by mathematics composite WIAT scores, for children in Clusters 1, 2 and 3, despite the fact that they had different profiles of procedural skill, conceptual understanding and working memory. This adds to previous evidence suggesting that there are multiple pathways to mathematical competence (Vukovic et al., 2014). This also suggests that some compensation is possible, which may occur through the use of different types of problem-solving strategies. For example, children with good conceptual and procedural skills may be able to compensate for poorer working memory through the use of efficient strategies with lower working memory demands (Cluster 1). Similarly, children with good procedural skill and working memory may be able to compensate for poorer conceptual understanding by being able to carry out less conceptually sophisticated, but more computationally demanding strategies accurately (Cluster 2). Finally, children with good conceptual understanding may compensate for poorer procedural skill and working memory by being able to identify conceptually-based shortcut strategies that are less computationally demanding. Compensating for weaknesses in one area through the use of alternative strategies is likely to require good attentional control, in order to inhibit prepotent strategies (e.g. Robinson & Dubé, 2013). This adds to the evidence of moderation effects, further demonstrating that the relationships between basic numerical skills and overall achievement are not independent of domain-general resources.

The results of our cluster analysis also have important implications for education. In a classroom setting we would expect groups of children such as those identified by our cluster analysis, to require different types of support for their mathematics learning. However, simply observing their scores on a mathematics achievement measure would not indicate this. This highlights the importance of examining more detailed profiles of children's performance when determining the types of support that they require. Our findings also have implications for the development of interventions. The moderating effect of working memory indicates that children's ability to benefit from interventions targeting either procedural skill or conceptual understanding is likely to depend on levels of working memory.

Here we focused on working memory as a measure of domain-general skills. Research which measures a broader range of executive functions, including inhibition and cognitive flexibility may reveal a more complex pattern of interactions between different executive function skills and arithmetic skills such that working memory, inhibition and cognitive flexibility interact in different ways with procedural skill and conceptual understanding. Research exploring these patterns may help to identify the roles played by inhibition and cognitive flexibility in mathematics learning, which have received less attention in comparison to the role of working memory (Bull & Lee, 2014; Cragg & Gilmore, 2014). In our analysis we used a composite measure of verbal and visuo-spatial working memory rather than separate scores. To explore whether this masked any differences in the pattern of relationships across the two forms of working memory, we repeated our analysis twice using the separate measures rather than composite scores. This revealed that there were no significant differences in the regression estimates for the separate measures, which suggests that verbal and visuo-spatial working memory play a similar role, in young children at least. There is some evidence that younger and older children may rely on different forms of working memory (Li & Geary, 2013, although see Cragg et al., 2017) and so this pattern should be tested further with older children.

One limitation of the current study is the cross-sectional and correlational nature of the data. It is therefore not possible to identify the extent to which these patterns represent causal relationships, or how they might change over time. For example, while children in Clusters 1, 2 and 3 had equivalent mathematics achievement at a single time point despite different patterns of skills, it is possible that the future learning trajectories for these groups may be very different. Recent use of latent growth modelling has started to identify the patterns of skills associated with faster growth in mathematics learning (e.g. Geary et al., 2009) and it would be interesting to explore how the interactions identified here are associated with differences in learning rates. Given that Geary et al. (2009) found that working memory and procedural skills were particularly important for high achievers, it is possible that children in Cluster 2 may make more rapid progress than children in Clusters 1 or 3.

Here we found that procedural skill was an important component of mathematics, with both independent and interactive effects. Inevitably because procedural skill is important for mathematics achievement there is some overlap in the items used to measure both procedural skill and overall achievement. This problem may be particularly the case for young children who have a limited range of mathematics knowledge. Therefore, it would be informative to replicate these findings in older children, for whom it may be possible to more clearly separate measures of procedural skill and overall mathematics achievement.

In conclusion, we have extended multi-component frameworks of mathematics by demonstrating that not only do procedural skill, conceptual understanding and working memory independently account for differences in mathematics achievement, but the interaction amongst these skills is also important. We have identified how the profile of children’s skills has impact on their mathematics performance, thereby elucidating multiple pathways to mathematics achievement. In particular, children who had strengths in each of these three areas were particularly advantaged in terms of their overall achievement. This emphasizes the complex nature of mathematics and the wide range of skills that are needed to succeed. This may provide one explanation for why so many children struggle with learning mathematics, because lower levels of proficiency in just one area can impact on overall mathematics performance. Consequently, there is a multitude of different ways in which children can have difficulties with mathematics. Researchers and educators should take account of the wide range of skills involved when attempting to understand differences in mathematics achievement and provide appropriate support to learners.

PsychOpen
publishing psychology

Funding

This work was funded by grant RES-062-23-3280 from the Economic and Social Research Council, UK. CG is funded by a Royal Society Dorothy Hodgkin Research Fellowship. The funders had no role in the research design, execution, analysis, interpretation or reporting.

Competing Interests

The authors have declared that no competing interests exist.

Acknowledgments

The authors have no support to report.

References

Bailey, D. H., Watts, T. W., Littlefield, A. K., & Geary, D. C. (2014). State and trait effects on individual differences in children's mathematical development. *Psychological Science, 25*, 2017-2026. doi:10.1177/0956797614547539

Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.),*The development of arithmetic concepts and skills: Constructive adaptive expertise*. New York, NY, USA: Routledge.

Bull, R., & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives, 8*(1), 36-41. doi:10.1111/cdep.12059

Canobi, K. H. (2004). Individual differences in children's addition and subtraction knowledge. *Cognitive Development, 19*(1), 81-93. doi:10.1016/j.cogdev.2003.10.001

Cowan, R., Donlan, C., Shepherd, D.-L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology, 103*(4), 786-803. doi:10.1037/a0024556

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education, 3*(2), 63-68. doi:10.1016/j.tine.2013.12.001

Cragg, L., Keeble, S., Richardson, S., Roome, H. E., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition, 162*, 12-26. doi:10.1016/j.cognition.2017.01.014

Department for Business Innovation & Skills. (2011). 2011 Skills for Life Survey: Headline findings (BIS Research Paper No. 57). Retrieved from https://www.gov.uk/government/publications/skills-for-life-survey-2011-important-findings

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428-1446. doi:10.1037/0012-1649.43.6.1428

Friso-van den Bos, I., van der Ven, S. H. G., Kroesbergen, E. H., & van Luit, J. E. H. (2013). Working memory and mathematics in primary school children: A meta-analysis. *Educational Research Review, 10*, 29-44. doi:10.1016/j.edurev.2013.05.003

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M., . . . Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology, 46*, 1731-1746. doi:10.1037/a0020662

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4-15. doi:10.1177/00222194040370010201

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology, 47*(6), 1539-1552. doi:10.1037/a0025510

Geary, D. C., Bailey, D. H., Littlefield, A., Wood, P., Hoard, M. K., & Nugent, L. (2009). First-grade predictors of mathematical learning disability: A latent class trajectory analysis. *Cognitive Development, 24*(4), 411-429. doi:10.1016/j.cogdev.2009.10.001

Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology, 77*(3), 236-263. doi:10.1006/jecp.2000.2561

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PLOS ONE, 8*(1), Article e54651. doi:10.1371/journal.pone.0054651

Gilmore, C. K., & Bryant, P. (2006). Individual differences in children's understanding of inversion and arithmetical skill. *The British Journal of Educational Psychology, 76*(2), 309-331. doi:10.1348/000709905X39125

Gilmore, C. K., & Papadatou-Pastou, M. (2009). Patterns of individual differences in conceptual understanding and arithmetical skill: A meta-analysis. *Mathematical Thinking and Learning, 11*(1-2), 25-40. doi:10.1080/10986060802583923

Gross, J. (2007). Supporting children with gaps in their mathematical understanding: The impact of the National Numeracy Strategy (NNS) on children who find mathematics difficult. *Educational and Child Psychology, 24*, 146-156.

Gross, J., Hudson, C., & Price, D. (2009). *The long term costs of numeracy difficulties.* London, United Kingdom: Every Child a Chance Trust & KPMG.

Hayes, A. F. (2013). *Introduction to mediation, moderation, and conditional process analysis: A regression-based approach*. New York, NY, USA: The Guilford Press.

Heckman, J. J. (2007). The economics, technology, and neuroscience of human capability formation. *Proceedings of the National Academy of Sciences of the United States of America, 104*(33), 13250-13255. doi:10.1073/pnas.0701362104

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ, USA: Lawrence Erlbaum.

Hubber, P. J., Gilmore, C., & Cragg, L. (2014). The roles of the central executive and visuospatial storage in mental arithmetic: A comparison across strategies. *Quarterly Journal of Experimental Psychology, 67*(5), 936-954. doi:10.1080/17470218.2013.838590

Imbo, I., & Vandierendonck, A. (2007). The development of strategy use in elementary school children: Working memory and individual differences. *Journal of Experimental Child Psychology, 96*(4), 284-309. doi:10.1016/j.jecp.2006.09.001

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences, 20*(2), 82-88. doi:10.1016/j.lindif.2009.07.004

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance: Pathways to mathematics. *Child Development, 81*(6), 1753-1767. doi:10.1111/j.1467-8624.2010.01508.x

LeFevre, J.-A., Smith-Chant, B. L., Fast, L., Skwarchuk, S.-L., Sargla, E., Arnup, J. S., . . . Kamawar, D. (2006). What counts as knowing? The development of conceptual and procedural knowledge of counting from kindergarten through Grade 2. *Journal of Experimental Child Psychology, 93*(4), 285-303. doi:10.1016/j.jecp.2005.11.002

Li, Y., & Geary, D. C. (2013). Developmental gains in visuospatial memory predict gains in mathematics achievement. *PLOS ONE, 8*(7), Article e70160. doi:10.1371/journal.pone.0070160

Mazzocco, M. M. M., & Thompson, R. E. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities Research & Practice, 20*(3), 142-155. doi:10.1111/j.1540-5826.2005.00129.x

OECD Skills Outlook 2013. (2013). *OECD Publishing.* Retrieved from http://www.oecd-ilibrary.org/education/oecd-skills-outlook-2013_9789264204256-en

Parsons, S., & Bynner, J. (2005). *Does numeracy matter more?* (National Research and Development Centre for Adult Literacy and Numeracy Research Report). London, United Kingdom: Institute of Education.

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences, 20*(2), 110-122. doi:10.1016/j.lindif.2009.10.005

Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge of mathematics. In R. Kadosh & A. Dowker (Eds.), *Oxford handbook of numerical cognition* (pp. 1118-1134). Oxford, United Kingdom: Oxford University Press.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346-362. doi:10.1037/0022-0663.93.2.346

Robinson, K. M., & Dubé, A. K. (2013). Children's additive concepts: Promoting understanding and the role of inhibition. *Learning and Individual Differences, 23*, 101-107. doi:10.1016/j.lindif.2012.07.016

Sasanguie, D., Van den Bussche, E., & Reynvoet, B. (2012). Predictors for mathematics achievement? Evidence from a longitudinal study. *Mind, Brain and Education, 6*(3), 119-128. doi:10.1111/j.1751-228X.2012.01147.x

Szucs, D., Devine, A., Soltesz, F., Nobes, A., & Gabriel, F. (2013). Developmental dyscalculia is related to visuo-spatial memory and inhibition impairment. *Cortex, 49*(10), 2674-2688. doi:10.1016/j.cortex.2013.06.007

Vukovic, R. K., Fuchs, L. S., Geary, D. C., Jordan, N. C., Gersten, R., & Siegler, R. S. (2014). Sources of individual differences in children's understanding of fractions. *Child Development, 85*(4), 1461-1476. doi:10.1111/cdev.12218

Watchorn, R. P. D., Bisanz, J., Fast, L., LeFevre, J.-A., Skwarchuk, S.-L., & Smith-Chant, B. L. (2014). Development of mathematical knowledge in young children: Attentional skill and the use of inversion. *Journal of Cognition and Development, 15*(1), 161-180. doi:10.1080/15248372.2012.742899

Wechsler, D. (2005). *Wechsler Individual Achievement Test 2nd Edition (WIAT II)*. London, United Kingdom: The Psychological Corporation.

PsychOpen is a publishing service by Leibniz Institute
for Psychology Information (ZPID), Trier, Germany.
www.zpid.de/en