# Decode-and-Forward Buffer-Aided Relay Selection 

## in Cognitive Relay Networks

Gaojie Chen, Member, IEEE, Zhao Tian, Student Member, IEEE, Yu Gong, Member, IEEE, and Jonathon Chambers, Fellow, IEEE


#### Abstract

This paper investigates decode-and-forward (DF) buffer-aided relay selection for underlay cognitive relay networks in the presence of both primary transmitter and receiver. We propose a novel buffer aided relay selection scheme for the cognitive relay network, where the best relay is selected with the highest signal-to-interferenceratio (SIR) among all available source-to-relay and relay-to-destination links while keeping the interference to the primary destination within a certain level. A closed-form expression for the outage probability of the proposed relay selection scheme is obtained. Both simulation and theoretical results are shown to confirm performance advantage over the conventional max-min relay selection scheme, making the proposed scheme attractive for cognitive relay networks.


## Index Terms

Cognitive relay networks, relay selection, buffer-aided decode-and-forward relay

## I. Introduction

Cognitive relay networks (CRNs) provide a promising way to exploit the advantages of both cognitive radio and cooperative relay networks [1]. While spectrum sharing in a CRN can be realized through various approaches including spectrum underlay, overlay and interweave [2], the underlay approach has most practical interest as the interference from the secondary users to the primary users is strictly limited. In a typical underlay CRN, beside the primary users, there are secondary users including secondary source, destination and a number of relay nodes. Relay selection provides an efficient way to achieve diversity gain in the CRN, because when only the best relay (rather than all relays) is selected for transmission, the
G. J. Chen, Z. Tian, Y. Gong and J. A. Chambers are with the Advanced Signal Processing Group, School of Electronic, Electrical and Systems Engineering, Loughborough University, Loughborough, Leicestershire, UK, E-mails: g.chen@1boro.ac.uk.
interference to the primary users is also limited. The system with relay selection generally works in two phases: in the first phase, the source transmits data to the selected relay; in the second phase, the selected relay forwards the data to the secondary destination. In a CRN, the best relay is selected to maximize the transmission rate between the secondary and destination nodes while keeping the interference to the primary users within a pre-required level. In the first phase, in particular, because the transmission power from source to every potential relay is limited according to the same interference constraint at the primary users, the received signal-to-noise (SNR) at the relays becomes correlated [3]. This may imply that the relay selection process among all candidates is mutually dependent, and so full diversity cannot always be achieved even when all relevant channel coefficients are identically and independently distributed (i.i.d.). This is very different from the conventional relay selection where the best relay is usually selected among independent candidates.

Nonetheless, relay selection in CRNs has attracted much attention recently. In [4], a max-min like selection decode-and-forward (SDF) relay scheme was proposed for a CRN, and outage analysis was based on the assumption that there are multiple independent links between the secondary source and primary user. This, however, contradicts the assumption that there is only one secondary source and primary user in the system. Moreover, although this simplified the outage analysis since the relay selection process can be assumed (wrongly) to be independent, the analytical result is not accurate. Some early works (e.g. [5]) on CRNs also failed to consider the dependence in the relay selection. The dependency in cognitive relay selection was identified in [3], and a "half" selection decode-and-forward (SDF) relay scheme was proposed to break the dependency in the relay selection. In the first phase of this approach the source broadcasts data to all relays and only in the second phase applies the relay selection. A similar relay selection technique was also considered in [6] so that the outage performance could be analyzed. The "half" relay selection (e.g. [3], [6]) is not the most efficient in making use of the relays, and generates security risk, because all relays (rather than only the selected relay) are involved in transmission in the first phase. Alternatively, the relay selection dependency problem can be avoided by assuming the link between the secondary source and primary user is constant, but this only applies to some specialized systems such as when the secondary source and primary user have little mobility (e.g. [7]).

Most current relay selection approaches (including the aforementioned) are for CRNs with no primary transmitters. In practice, both a primary transmitter and receiver may be present [8], [9], for which the interference from the primary transmitter to the secondary users cannot be ignored. This motivates us
to investigate relay selection in a more general CRN with both a primary transmitter and receiver being present. It has been recently recognized that the performance of conventional relay selection can be further improved by relaxing the constraint that the best source-to-relay and relay-to-destination links for a packet transmission must be determined altogether. This is achieved by introducing a data buffer at the relay nodes [10]-[15]. Such buffer-aided relay selection is even more useful in the CRN: because now the best source-to-relay and relay-to-destination links are selected separately, the dependency in the conventional max-min based relay selection can thereby be de-correlated.

Of particular interest is the max-link relay selection where the best link is always selected with the highest signal-to-noise (SNR) among all available source-to-relay and relay-to-destination links [10]. In this paper, considering the interference from/to the primary users, we propose a so-called max-SIR-link relay selection scheme for the CRN, where the best relay is selected with the highest signal to the primary interference ratio at the corresponding receiving nodes while satisfying the interference constraint at the primary receivers. The main contributions of this paper are listed as follows:

- Proposing DF buffer-aided max-SIR-link relay selection in the underlay CRN. As the proposed relay selection only lets the selected relay join the transmission at any one time, it is more efficient at de-correlating the relay selection process than the aforementioned "half" relay selection (e.g. [3], [6]), an important issue in cognitive relay selection. To the best of our knowledge, this is the first relay selection scheme for a CRN with both primary transmitter and receiver available.
- Deriving the closed-form expression of the outage probability for the proposed relay selection scheme. With the presence of both the primary transmitter and receiver, the analysis is much more involved than those for both the conventional and the existing cognitive relay selection schemes. The analysis not only provides deep insight in understanding the proposed scheme but also shows a potential approach to analyze similar systems in the future.

We next introduce the proposed relay selection scheme in the context of a CRN.

## II. Max-SiR-Link Relay Selection

## A. System model

The cognitive relay network with buffers at the relays is shown in Fig. 1, where there is one secondary source node $(S S)$, one secondary destination node $(S D)$, one primary source $(P S)$, one primary destination $(P D)$ and the number of DF relays $S R_{k}, k \in(1,2, \ldots, K)$. All nodes are half-duplex and do not transmit
and receive simultaneously. Each relay is equipped with a data buffer $Q_{k}(1 \leq k \leq K)$ of finite size $L$ (in the number of data packets), and the data packets in the buffer follow the "first-in first-out" rule. For simplicity of exposition, we assume no direct link between the secondary source $(S S)$ and the secondary destination $(S D)$ as path loss or shadowing is assumed to render it unusable [16].


Fig. 1. The system model of the CRN within buffered relay selection.

All channels in Fig. 1 related to secondary transmission can be divided into three groups: secondary transmission channels for $S S \rightarrow S R_{k}$ and $S R_{k} \rightarrow S D$ with channel coefficients as $h_{s r_{k}}(t)$ and $h_{r_{k} d}(t)$, secondary interfering channels for $P S \rightarrow S R_{k}$ and $P S \rightarrow S D$ with coefficients as $h_{p r_{k}}(t), h_{p d}(t)$, and primary interfering channels for $S S \rightarrow P D$ and $S R_{k} \rightarrow P D$ with coefficients as $h_{s p}(t)$ and $h_{r_{k} p}(t)$ respectively. The instantaneous and average channel gains are defined as $\gamma_{a b}(t)=\left|h_{a b}(t)\right|^{2}$ and $\lambda_{a b}=$ $\mathrm{E}\left|h_{a b}(t)\right|^{2}$ respectively, where $a b \in\left\{s r_{k}, s p, r_{k} d, r_{k} p, p r_{k}, p d, p p\right\}$. For convenience in development, the time index $t$ is ignored in the rest of the paper unless necessary.

We assume all channels are quasi-static Rayleigh fading so that the channel coefficients remain unchanged during one packet duration but independently vary from one packet time to another. We also assume that channels within every group are i.i.d. fading, but channels for different groups may have different average gains, or we have $\lambda_{s r_{k}}=\lambda_{r_{k} d}, \lambda_{p r_{k}}=\lambda_{p d}$ and $\lambda_{s p}=\lambda_{r_{k} p}$ for all $k$. This is a more practical assumption than those in many existing approaches where all channels are assumed to be i.i.d. fading (e.g. [?], [6], [10]). Exact knowledge of all instantaneous channels is assumed to be available at the secondary relay and destination nodes ${ }^{1}$. All channel noises are assumed to be zero mean

[^0]additive-white-Gaussian-noise (AWGN).

In the underlay cognitive system, the secondary transmission nodes including $S S$ and $S R_{k}$ are only allowed to share the spectrum with the primary user $P D$ if the corresponding interfering power to $P D$ is below a pre-defined level $I_{t h}$, so that we have

$$
\begin{equation*}
P_{s s} \gamma_{s p} \leq I_{t h} \quad \text { and } \quad P_{s r_{k}} \gamma_{r_{k} p} \leq I_{t h}, \quad k=1, \cdots, K \tag{1}
\end{equation*}
$$

where $P_{s s}$ and $P_{s r_{k}}$ are the transmission powers for $S S$ and $S R_{k}$ respectively.

If the relay $S R_{k}$ is selected to receive data from the secondary source $S S$, due to the interference from the primary source $P S$, the received signal at $S R_{k}$ is given by

$$
\begin{equation*}
\mathbf{y}_{s r_{k}}=\sqrt{P_{s s}} h_{s r_{k}} \mathbf{s}+h_{p r_{k}} \sqrt{P_{p s}} \mathbf{s}^{\prime}+\mathbf{n}_{r_{k}} \tag{2}
\end{equation*}
$$

where $\mathbf{s}$ and $\mathbf{s}^{\prime}$ are transmission vectors from $S S$ and $P S$ respectively, $P_{p s}$ is the transmission power of the primary source which is assumed to be unity without losing generality and $\mathbf{n}_{r_{k}}$ is the noise vector at $S R_{k}$. From (2), and with the power constraint as in (1), the received SIR at $S R_{k}$ is obtained as

$$
\begin{equation*}
\operatorname{SIR}_{s r_{k}}=\frac{P_{s s}\left|h_{s r_{k}}\right|^{2}}{P_{p s}\left|h_{p r_{k}}\right|^{2}}=\frac{I_{t h} \gamma_{s r_{k}}}{\gamma_{s p} \gamma_{p r_{k}}} . \tag{3}
\end{equation*}
$$

As in [9], we focus on the interference-limited scenario wherein the interference power from the primary source is dominant relative to the noise so that the noise effects can be ignored. Therefore the instantaneous capacity for $S S \rightarrow S R_{k}$ is approximated as $C_{s r_{k}} \approx(1 / 2) \log _{2}\left(1+\operatorname{SIR}_{s r_{k}}\right)$.

On the other hand, if the relay $S R_{k}$ is selected to forward data to the secondary destination $S D$, the received signal at $S D$ is given by

$$
\begin{equation*}
\mathbf{y}_{r_{k} d}=\sqrt{P_{s r_{k}}} h_{r_{k} d} \mathbf{s}+h_{p d} \sqrt{P_{p s}} \mathbf{s}^{\prime}+\mathbf{n}_{d}, \tag{4}
\end{equation*}
$$

where $\mathbf{n}_{d}$ is the noise vector at $S D$. From (4) and (1), the SIR at the secondary destination is obtained as

$$
\begin{equation*}
\operatorname{SIR}_{r_{k} d}=\frac{P_{s r_{k}}\left|h_{r_{k} d}\right|^{2}}{P_{p s}\left|h_{p d}\right|^{2}}=\frac{I_{t h} \gamma_{r_{k} d}}{\gamma_{r_{k} p} \gamma_{p d}} . \tag{5}
\end{equation*}
$$

Similarly, with the interference dominating the noise, the instantaneous capacity for $S R_{k} \rightarrow S D$ is approximated as $C_{r_{k} d} \approx(1 / 2) \log _{2}\left(1+\operatorname{SIR}_{r_{k} d}\right)$.

## B. Selection rule

In the max-SIR-link relay selection for the CRN, at any time, the best transmission link with the highest SIR is selected among all available source-to-relay and relay-to-destination links. A source-to-relay or a relay-to-destination link is considered available when the buffer of the corresponding relay node is not full nor empty respectively. To be specific, if a source-to-relay link is selected, the source node transmits one data packet to the corresponding relay node. If the selected relay can successfully decode the data, the decoded packet is stored in the buffer and the number of data packets in the buffer is increased by one. On the other hand, if a relay-to-source link is selected, the corresponding relay transmits the earliest stored packet in the buffer to the destination. If the destination can successfully decode the packet, the number of packets in the buffer is decreased by one.

The best selected relay (either for transmission or reception) in the max-SIR-link scheme can be obtained as $R_{b e s t}=\arg \max _{S R_{k}}\left\{\mathrm{SIR}_{s r_{k}}, \operatorname{SIR}_{r_{k} d}\right\}$. While the SIRs for the source-to-relay and relay-to-destination link are given by (3) and (5) respectively, we have

$$
\begin{equation*}
R_{b e s t}=\arg \max _{S R_{k}}\left\{\frac{\max _{S R_{k}: \Psi\left(Q_{k}\right) \neq L}\left\{\frac{I_{t h} \gamma_{s r_{k}}}{\gamma_{p r_{k}}}\right\}}{\gamma_{s p}}, \frac{\max _{S R_{k}: \Psi\left(Q_{k}\right) \neq L}\left\{\frac{I_{t h} \gamma_{r_{k} d}}{\gamma_{r_{k} p}}\right\}}{\gamma_{p d}}\right\} \tag{6}
\end{equation*}
$$

where $\Psi\left(Q_{k}\right)$ gives the number of data packets in the buffer $Q_{k}$.
The outage probability can be defined as the probability that the selected link is in outage as

$$
P_{\text {out }} \triangleq \begin{cases}\mathbb{P}\left\{(1 / 2) \log _{2}\left(1+\operatorname{SIR}_{s r_{k}}\right)<C_{t h}\right\} & \text { for relay reception, }  \tag{7}\\ \mathbb{P}\left\{(1 / 2) \log _{2}\left(1+\operatorname{SIR}_{r_{k} d}\right)<C_{t h}\right\} & \text { for destination reception }\end{cases}
$$

where $C_{t h}$ is the target rate, and the factor $1 / 2$ captures the fact that it takes two time slots to transmit any packet from the source to the destination. Next, we perform the outage probability analysis.

## III. Outage Probability Analysis

This section analyzes the outage probability of the max-SIR-link relay selection in the CRN. At any time, the numbers of data packets in every buffer form a "state". Because there are $K$ available relays and every relay is equipped with a buffer of size $L$, there are $(L+1)^{K}$ states in total. The $l$-th state vector is defined as

$$
\begin{equation*}
\mathbf{s}_{l}=\left[\Psi_{l}\left(Q_{1}\right), \cdots, \Psi_{l}\left(Q_{K}\right)\right]^{\mathrm{T}}, \quad l=1, \cdots,(L+1)^{K} \tag{8}
\end{equation*}
$$

where $\Psi_{l}\left(Q_{k}\right)$ gives the number of data packets in buffer $Q_{k}$ at state $s_{l}$. It is clear that $0 \leq \Psi_{l}\left(Q_{k}\right) \leq L$.
We assume that state $s_{l}$ corresponds to the pair of $\left(K_{1}, K_{2}\right)$, where $K_{1}$ and $K_{2}$ are the numbers of available links for source-to-relay and relay-to-destination transmission at state $s_{l}$ respectively. By considering all possible available links for $K_{1}$ and $K_{2}$, the outage probability of the overall system can be obtained as

$$
\begin{equation*}
P_{\text {out }}=\sum_{l=1}^{(L+1)^{K}} \pi_{l} \bar{p}_{s_{l}}^{\left(K_{1}, K_{2}\right)} \tag{9}
\end{equation*}
$$

where $\bar{p}_{s_{l}}^{\left(K_{1}, K_{2}\right)}$ is the outage probability when the state is at $s_{l}$, and $\pi_{l}$ is the stationary probability for the state $s_{l}$. The following two sub-sections show the calculation of $\bar{p}_{s_{l}}^{\left(K_{1}, K_{2}\right)}$ and $\pi_{l}$ respectively.
A. $\bar{p}_{s_{l}}^{\left(K_{1}, K_{2}\right)}$ : outage probability for state $s_{l}$

According to (6) and the theory of order statistics [19], if there are $K_{1}$ source-to-relay links available, the cumulative distribution function (CDF) of $X_{1}=\max _{S R_{k}: \Psi\left(Q_{k}\right) \neq L}\left\{\frac{\gamma_{s r_{k}}}{\gamma_{p r_{k}}}\right\}$ is given by

$$
\begin{equation*}
F_{X_{1}}(x)=\left(\frac{x}{L_{1}+x}\right)^{K_{1}} \tag{10}
\end{equation*}
$$

where $L_{1}=\frac{I_{t h} \lambda_{s r_{k}}}{\lambda_{p r_{k}}}$. Then the CDF of $X=X_{1} / \gamma_{s p}$ is given by

$$
F_{X}(x)= \begin{cases}1, & \text { if } \quad K_{1}=0,  \tag{11}\\ 1-\frac{L_{1}}{\lambda_{s p x}} x^{\frac{L_{1}}{\lambda_{s p x}} \operatorname{Ei}\left(1, \frac{L_{1}}{\lambda_{s p x}}\right),} & \text { if } K_{1}=1, \\ \left(\frac{\lambda_{s p} x}{L_{1}}\right)^{K_{1}-1} \frac{\left.\left.\mathcal{M G}([0],[]],\left[\left[K_{1}-1, K_{1}\right]\right],[]\right], \frac{L_{1}}{\lambda_{s p x}}\right)}{\Gamma\left(K_{1}\right)}, & \text { elsewhere },\end{cases}
$$

where $\operatorname{Ei}(1, a)=\int_{1}^{\infty} \frac{\exp (-t a)}{a} d t, a>0, \Gamma(\bullet)$ is the Gamma function, and $\mathcal{M G}([[],[]],[[\bullet, \bullet],[]], \bullet)$ is the Meijer G function [20].

Proof: See Appendix I.


$$
F_{Y}(y)= \begin{cases}1, & \text { if } K_{2}=0,  \tag{12}\\ 1-\frac{L_{2}}{\lambda_{p d} y} e^{\frac{L_{2}}{\lambda_{p d} y}} \operatorname{Ei}\left(1, \frac{L_{2}}{\lambda_{p d} y}\right), & \text { if } K_{2}=1, \\ \left(\frac{\lambda_{p d} y}{L_{2}}\right)^{K_{2}-1} \frac{\mathcal{M G}\left([[0],[]],\left[\left[K_{2}-1, K_{2}\right],[]\right], \frac{L_{2}}{\lambda_{p d} y}\right)}{\Gamma\left(K_{2}\right)}, & \text { elsewhere. }\end{cases}
$$

Because $X$ and $Y$ are independent, the CDF of $Z=\max (X, Y)$ is obtained as $F_{Z}(z)=F_{X}(z) F_{Y}(z)$. It is then from (6) and (7) that

$$
\begin{equation*}
\bar{p}_{s_{l}}^{\left(K_{1}, K_{2}\right)}=F_{Z}\left(\gamma_{t h}\right)=F_{X}\left(\gamma_{t h}\right) F_{Y}\left(\gamma_{t h}\right), \tag{13}
\end{equation*}
$$

where $\gamma_{t h}=2^{2 C_{t h}-1}$, and $C_{t h}$ is defined in (7) which is the target data rate.

## B. $\pi_{l}$ : stationary distribution probability for state $s_{l}$

The Markov chain can be used to model the transitions between the buffer states. Suppose at time $t$, the state is at $s_{l}$. At time $t+1$, if the received data can be successfully decoded, there must be one relay either receiving or transmitting a data packet, so that the number of packets in the corresponding buffer is increased or decreased by one respectively. Depending on which relay receives or transmits data, at time $t+1$, the buffers may move from state $s_{l}$ to several possible states. We assume the set $U_{l}$ contains all states which can be reached from $s_{l}$ in one step.

Because the channels within secondary transmission, secondary interfering and primary interfering groups are i.i.d. fading, it is clear from (3) and (5) that the SIRs for all channels are i.i.d. so that the probability to select any link is $1 /\left(K_{1}+K_{2}\right)$. Further noting that the state remains unchanged if outage occurs (or the decoding is not successful), the probabilities that the state $s_{l}$ moves to a state in $U_{l}$ is given by

$$
\begin{equation*}
p_{s_{l}}=\frac{1-\bar{p}_{s_{l}}^{\left(K_{1}, K_{2}\right)}}{K_{1}+K_{2}} . \tag{14}
\end{equation*}
$$

We denote $\mathbf{A}$ as the $(L+1)^{K} \times(L+1)^{K}$ state transition matrix, where the entry $\mathbf{A}_{n, l}=P\left(X_{t+1}=\right.$ $\left.s_{n} \mid X_{t}=s_{l}\right)$ which is the transition probability to move from state $s_{l}$ at time $t$ to state $s_{n}$ at time $(t+1)$. With the above analysis, we have

$$
\mathbf{A}_{n, l}= \begin{cases}\bar{p}_{s_{l}}^{\left(K_{1}, K_{2}\right)}, & \text { if } \quad s_{n} \notin U_{l},  \tag{15}\\ p_{s_{l}}, & \text { if } s_{n} \in U_{l}, \\ 0, & \text { elsewhere }\end{cases}
$$

Because the transition matrix $\mathbf{A}$ is column stochastic, irreducible and aperiodic ${ }^{2}$, the stationary state

[^1]probability vector is obtained as (see [22], [23])
\[

$$
\begin{equation*}
\boldsymbol{\pi}=(\mathbf{A}-\mathbf{I}+\mathbf{B})^{-1} \mathbf{b}, \tag{16}
\end{equation*}
$$

\]

where $\boldsymbol{\pi}=\left[\pi_{1}, \cdots, \pi_{(L+1)^{K}}\right]^{\mathrm{T}}, \mathbf{b}=(1,1, \ldots, 1)^{T}, \mathbf{I}$ is identity matrix and $\mathbf{B}_{n, l}=1, \forall n, l$.
Finally, from (9), the outage probability for the max-SIR-link scheme is given by

$$
\begin{equation*}
P_{\text {out }}=\operatorname{diag}(\mathbf{A}) \pi . \tag{17}
\end{equation*}
$$

Particularly, if the relay buffer size $L \rightarrow \infty$, similar to that in [10], it can be shown that probabilities for $K_{1}=K$ and $K_{2}=K$ are one. Thus we have

$$
\begin{equation*}
\lim _{L \rightarrow \infty} P_{o u t}=\left(\frac{\lambda_{s p} \lambda_{p d} z^{2}}{L_{1} L_{2}}\right)^{K-1} \frac{\mathcal{M G}\left([[0],[]],[[K-1, K],[]], \frac{L_{1}}{\lambda_{s p z} z}\right) \mathcal{M G}\left([[0],[]],[[K-1, K],[]], \frac{L_{2}}{\lambda_{p d} z}\right)}{\Gamma^{2}(K)} \tag{18}
\end{equation*}
$$

These results are next verified by Monte Carlo simulations.

## IV. Numerical Simulations

In the simulations below, the pre-defined level $I_{t h}=1$, and the average channel gains are set as $\lambda_{s r_{k}}=\lambda_{r_{k} d}=30 \mathrm{~dB}, \lambda_{s p}=\lambda_{r_{k} p}=10 \mathrm{~dB}$ and $\lambda_{p r_{k}}=\lambda_{p d}=10 \mathrm{~dB}$. The transmission powers of the primary transmitter and channel noise are normalized to unity.


Fig. 2. Theoretical and simulation outage probability $v s$ target rate for the proposed max-SIR-link relay selection.

Fig. 2 verifies the theoretical analysis for the proposed max-SIR-link scheme with simulations. We have
performed extensive simulations with different number of relays and buffer sizes. While all simulation results match the theoretical analysis, only a few are shown in Fig. 2 for better illustration. It is clearly shown that the outage probability decreases as the number of relays and buffer size increases. For example, for target rate $\gamma_{t h}=0.5$ bits per channel use (BPCU), when the number of relays and buffers ( $K, L$ ) increase from $(2,2)$ to $(5,5)$, the outage probability drops about 40 dB . It is not surprising that higher diversity is obtained with more relays and higher coding gain is obtained with larger buffer size. For better illustration, only theoretical results for the proposed scheme are shown in the following simulation.

Fig. 3 compares the outage probabilities of the proposed max-SIR-link, conventional max-min and no relay selection schemes, where the number of relays is set as $K=3$, different relay buffer sizes for the proposed approach are applied which are set as $L=1,5,50, \infty$, respectively. It is clearly shown that the proposed relay selection (even with $L=1$ ) has significantly better outage performance than the conventional max-min scheme, while both relay selection schemes are superior to the no-relay scheme in outage performance. Fig. 3 also shows that, for the proposed approach, the outage performance improves with larger buffer size, but the improvement becomes less significant when the buffer size is large enough. Particularly with $L=50$, the outage performance is almost the same as that for $L \rightarrow \infty$.


Fig. 3. Outage probability comparison for the proposed max-SIR-link, conventional max-min and no-relay selection schemes.

## V. Conclusions

This paper proposed DF buffer-aided max-SIR-link relay selection for an underlay CRN, in the presence of both primary source and destination. In the proposed scheme, the best relay corresponds to the highest

SIR among all available source-to-relay and relay-to-destination links while keeping the interference at the primary user within a pre-defined level. The closed-form expression of the outage probability of the proposed scheme was obtained, which matches exactly the simulation results. Both theoretical and simulation results showed that the proposed scheme has significantly better outage performance than the conventional max-min scheme, making it an attractive scheme in a CRN.

## Appendix I - Proof of (11)

 $X_{1}=\max _{S R_{k}: \Psi\left(Q_{k}\right) \neq L}\left\{\frac{\gamma_{s r_{k}}}{\gamma_{p r_{k}}}\right\}$ is obtained as in (10). The PDF of exponentially distributed $\gamma_{s p}$ is given by $f_{\gamma_{s p}}(\gamma)=\left(1 / \gamma_{s p}\right) e^{-\gamma / \gamma_{s p}}$.

Because $X_{1}$ and $\gamma_{s p}$ are independent, the CDF of $X=X_{1} / \gamma_{s p}$ is obtained as

$$
\begin{equation*}
F_{X}(x)=\int_{0}^{\infty}\left(\frac{x \gamma}{L_{1}+x \gamma}\right)^{K_{1}} \frac{1}{\gamma_{s p}} e^{-\frac{\gamma}{\gamma_{s p}}} d \gamma . \tag{19}
\end{equation*}
$$

For (19), if $K_{1}=0$, we have

$$
\begin{equation*}
F_{X}(x)=\int_{0}^{\infty} \frac{1}{\gamma_{s p}} e^{-\frac{\gamma}{\gamma_{s p}}} d \gamma=1, \tag{20}
\end{equation*}
$$

if $K_{1}=1$, we have

$$
\begin{equation*}
F_{X}(x)=\int_{0}^{\infty}\left(\frac{x \gamma}{L_{1}+x \gamma}\right) \frac{1}{\gamma_{s p}} e^{-\frac{\gamma}{\gamma_{s p}}} d \gamma=1-\frac{L_{1}}{\lambda_{s p} x} e^{\frac{L_{1}}{\lambda_{s p x}}} \operatorname{Ei}\left(1, \frac{L_{1}}{\lambda_{s p} x}\right), \tag{21}
\end{equation*}
$$

and if $K_{1}>1$, we have

$$
\begin{equation*}
F_{X}(x)=\int_{0}^{\infty}\left(\frac{x \gamma}{L_{1}+x \gamma}\right)^{K_{1}} \frac{1}{\gamma_{s p}} e^{-\frac{\gamma}{\gamma_{s p}}} d \gamma=\left(\frac{\lambda_{s p} x}{L_{1}}\right)^{K_{1}-1} \frac{\mathcal{M G}\left([[0],[]],\left[\left[K_{1}-1, K_{1}\right],[]\right], \frac{L_{1}}{\lambda_{s p} x}\right)}{\Gamma\left(K_{1}\right)} . \tag{22}
\end{equation*}
$$

## References

[1] Q. Zhang, J. Jia, and J. Zhang, "Cooperative relay to improve diversity in cognitive radio networks," IEEE Commun. Mag., vol. 47, no. 2, pp. 111-117, Feb. 2009.
[2] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognative radios: an information theoretic perspective," Proceedings of the IEEE, vol. 97, no. 5, pp. 894-914, May. 2009.
[3] Z. Yan, X. Zhang, and W. Wang, "Exact outage probability of cognitve relay networks with maximun transmit power limits," IEEE Commun. Lett., vol. 15, no. 12, pp. 1317-1319, Dec. 2011.
[4] J. Lee, H. Wang, J. G. Andrew, and D. Hong, "Outage probability of cognitve relay networks with interference constraints," IEEE Trans. Wireless Commun., vol. 10, no. 2, pp. 390-395, Feb. 2011.
[5] Y. Guo, G. Kang, W. Zhou, and P. Zhang, "Outage performance of relay-assisted cognitive-radio system under spectrum-sharing constraint," IET Electronic Letters, vol. 46, no. 2, pp. 182-184, Jan. 2010.
[6] J. Hong, B. Hong, T. W. ban, and W. Choi, "On the cooperative diveristy gain in underlay cognitive radio systems," IEEE Trans. Commun., vol. 60, no. 1, pp. 209-219, Jan. 2012.
[7] G. Chen, G. Yu, and J. A. Chambers, "Study of relay selection in a multi-cell cognitive network," Accepted for IEEE Wireless Commun. Lett., Apr. 2013.
[8] P. Yang, L. Luo, and J. Qin, "Outage performance of cognitive relay networks with interference from primary user," IEEE Commun. Lett., vol. 16, no. 10, pp. 1695-1908, Oct. 2012.
[9] T. Q. Duong, P. L. Yeoh, V. N. Q. Baoand, M. Elkashlan, and N. Yang, "Cognitive relay networks with multiple primary transceivers under spectrum-sharing," IEEE Signal Process. Lett., vol. 19, no. 11, pp. 741-744, Nov. 2012.
[10] I. Krikidis, T. Charalambous, and J. S. Thompson, "Buffer-aided relay selection for cooperative diversity systems without delay constraints," IEEE Trans. Wireless Commun., vol. 11, no. 5, pp. 1957-1967, May 2012.
[11] H. Liu, P. Popovski, E. Carvalho, and Y. Zhao, "Sum-rate optimization in a two-way relay network with buffering," IEEE Commun. Lett., vol. 17, pp. 95-98, Jan. 2013.
[12] A. Ikhlef, D. S. Michalopoulos, and R. Schober, "Max-max relay selection for relays with buffers," IEEE Trans. Wireless Commun., vol. 11, no. 3, pp. 1124-1135, May 2012.
[13] N. Zlatanov, R. Schober, and L. Lampe, "Buffer-aided relaying in a three node network," in Proc. 2012 IEEE International Symposium on Information Theory Proceedings, Cambridge, Massachusetts, USA, pp. 781-785, July 2012.
[14] N. Zlatanov, R. Schober, and P. Popovski, "Buffer-aided relaying with adaptive link selection," to appear in IEEE J. Sel. Areas Commun., vol. 31, Aug. 2012.
[15] B. Xia, Y. Fan, J. Thompson, and H. V. Poor, "Buffering in a three-node relay network," IEEE Trans. Wireless Commun., vol. 7, pp. 4492-4496, Nov. 2008.
[16] D. S. Michalopoulos and G. K. Karagianidis, "Performance analysis of single relay selection in Rayleigh fading," IEEE Trans. Wireless Commun., vol. 7, no. 11, pp. 3718-3724, Oct. 2008.
[17] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environment," IEEE Trans. Wireless Commun., vol. 6, no. 2, pp. 649-658, Feb. 2007.
[18] K. Hamdi, W. Zhang, and K. B. Letaief, "Power control in cognitive radio systems based on spectrum sensing side information," IEEE Intl. Conf. Commun., Glasgow, UK, June 2007.
[19] H. A. David, "Order statistics second edition," John Wiley Sons Ltd, 1981.
[20] A. Gilat and V. Subramaniam, "Numerical methods for engineers and scientists: An introduction with applications using Matlab," John Wiley Sons Ltd, 2011.
[21] C. M. Grinstead and J. L. Snell, "Introduction to probability: Second revised edition, Chapter 11 Markov Chains," American Mathematical Society, 1991.
[22] J. R. Norris, "Markov chains," Cambridge University Press, 1998.
[23] A. Berman and R. J. Plemmons, "Nonnegative matrices in the mathematical sciences," Society of industrial and applied mathematics, 1994.


[^0]:    ${ }^{1}$ The CSI is usually estimated through pilots and feedback (e.g. [17]), and the CSI estimation without feedback may also be applied (e.g [18]). Further detail of the CSI estimation is beyond the scope of this paper.

[^1]:    ${ }^{2}$ Column stochastic means all entries in any column sum up to one, irreducible means that it is possible to move from any state to any state, and aperiodic means that it is possible to return to the same state at any steps [21], [22].

