Coordinated Multicell Beamforming with Local and Global Data Rate Constraints

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***Abstract*—This paper outlines how Singular Value Decomposition (SVD) technique can be used to simplify a multicell network with heterogenous users. The heterogenous users considered in this work comprise of single antenna local users and one multiantenna global user. The local users are attached to their respective base stations (BSs) while the global user is jointly served by multiple BSs. We consider downlink beamforming design using power minimization approach. It is shown that the data rate to the global user should be split equally amongst the serving BSs.**

***Index Terms*—Beamforming, SVD, downlink, SINR target, multiantenna, quality of service.**

I. Introduction

With technology trends of today, where wireless networks have data hungry users, it is necessary to consider cell densification that enhances frequency reusage [1], [2]. Multiantenna deployment at both mobile users and BSs also enable the mobile network to take advantage of the spatial diversity in order to increase the overall performance of the network. Various coordinated beamformer techniques have been developed for downlink beamforming in multiantenna wireless systems [3]–[6]. The use of generalized singular value decomposition (GSVD) for coordinated beamforming in MIMO system was examined in [7]. In [8], a multiuser multi-input multi-output (MU-MIMO) network was considered. The work in [8] showed that by introducing a limited number of zero-forcing constraints, the SINRs of all stream are decoupled and this reduces the problem to a multiuser multi-input single-output (MU-MISO) problem. The setback of this approach is the reduced degrees-of-freedom and inefficiency. Coordinated beamforming design with weighted power minimization was considered in [9] using Lagrangian duality theory. The work in [10] considered power minimization problem in a network wherein users are served by joint non-coherent multiflow beamforming. The authors in [10] emphasized that even though the users can be served by multiple transmitters, the information symbols are coded and transmitted independently. Coordinated beamforming with user fairness based SINR balancing techniques were also considered in [11]–[16].

In this paper, we aim to study joint downlink beamforming using power minimization approach. The users have known specific data rate targets that need to be satisfied. We consider optimal solution that meets the target SINRs for all users.

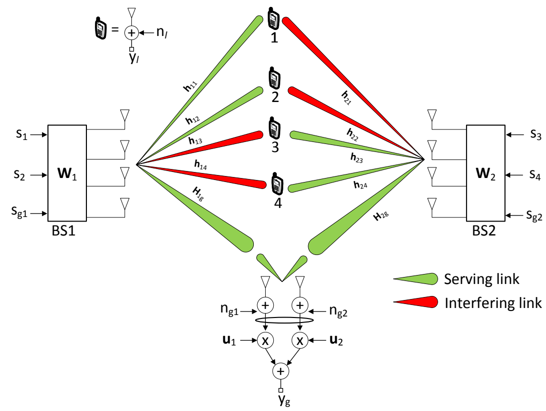


Fig. 1. Network topology. Both BS1 and BS2 serve two local user and one global user.

The set of users considered in this work consist of single antenna local users and a single multiantenna global user. The global user is served by multiple BSs. By using SVD, we decompose the multi-input multi-output (MIMO) channels between the BSs and the global user to form parallel and independent multi-input single-output (MISO) channels. The approach requires no phase synchronization between the BSs that are serving the global user. According to the proposed scheme, certain users are served by only a single base station (BS), however, one multiantenna terminal is served by two BSs. The latter user is known as global user which receives data from both BSs simultaneously. Hence, optimum split of data rate from different BS is also considered in this paper. Even though one global user is considered in this paper, it is possible to extend our work to multiple global users who can benefit from different channel conditions of both the BSs, especially when the users are at the cell edge.

The work is organized as follows: Section II presents the system model and assumptions; also it shows the mathematical framework and highlights the problem formulation. SVD based beamforming design is demonstrated in section III. Numerical Performance Analysis is conducted in section IV. Finally, the discussions are concluded in Section V.

II. System Model and Assumptions

This paper considers a network comprising of two base-stations denoted $n = [1, 2]$ as depicted in Figure 1. Each BS is equipped with M antennas and it serves L_n single antenna local users in the cell n. We denote a set of all local users and all BSs as $\mathcal{L}$ and $\mathcal{N}$ respectively. There is a global user denoted g being served by both BSs. The global user is equipped with two antennas. We assume that all the BSs operate in the same frequency and that all users experience considerable intercell interference.

A. Problem Formulation

In the downlink, the transmitted signal for l-th local user from n-th BS can be written as

$$\mathbf{x}_{nl}(t) = \mathbf{w}_{nl} s_l(t), \tag{1}$$

where $s_l(t) \in \mathbb{C}$ represents the information symbol at time t and $\mathbf{w}_{nl} \in \mathbb{C}^M$ is the unnormalised transmit beamforming vector for user l at n-th BS. Without loss of generality we assume that $s_l(t)$ is normalised such that $\mathbb{E}\{|s_l(t)|^2\} = 1$ and that all data streams are independent such that $\mathbb{E}\{s_l(t)s_j(t)^*\} = 0$ if $l \neq i$. In this paper, we assume perfect channel state information (CSI) knowledge at both the transmitter and the receiver. We denote the MIMO channel between the n-th BS and the global user g as $\mathbf{H}_{ng} \in \mathbb{C}^{N_r \times M}$, where N_r is the number of receive antennas at the global user. The intended signal at the global user is given by

$$\mathbf{r} = \mathbf{H}_{1g}\mathbf{w}_{1g_1} sg_1(n) + \mathbf{H}_{2g}\mathbf{w}_{2g_2} sg_2(n) \tag{2}$$

The global user deploys the receive beamformers $\mathbf{u}_1$ and $\mathbf{u}_2$. By using SVD, the channel matrices between the BSs can be written as

$$\mathbf{H}_{1g} = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{V}_1^H, \tag{3}$$
$$\mathbf{H}_{2g} = \mathbf{U}_2 \mathbf{\Lambda}_2 \mathbf{V}_2^H, \tag{4}$$

where $\mathbf{U}_1 \in \mathbb{C}^{N_r \times N_r}$ (respectively $\mathbf{U}_2 \in \mathbb{C}^{N_r \times N_r}$) and $\mathbf{V}_1 \in \mathbb{C}^{M \times M}$ (respectively $\mathbf{V}_2 \in \mathbb{C}^{M \times M}$) are the unitary matrices and $\mathbf{\Lambda}_1 \in \mathbb{C}^{N_r \times M}$ (respectively $\mathbf{\Lambda}_2 \in \mathbb{C}^{N_r \times M}$) is the diagonal matrix of the singular values of $\mathbf{H}_{1g}$ (respectively $\mathbf{H}_{2g}$) sorted in descending order. The SVD of the MIMO channels allows us to represent the global user as two virtual users denoted as g_1 and g_2. Denote $\mathbf{u}_1$ and $\mathbf{u}_2$ as the singular vectors corresponding to the largest singular values of $\mathbf{H}_{1g}$ and $\mathbf{H}_{2g}$ respectively. The decomposed received signal at the virtual users g_1 and g_2 can be written as

$$y_{g_1} = \mathbf{u}_1^H \Big[\mathbf{H}_{1g}\mathbf{w}_{1g} sg_1 + \mathbf{H}_{2g}\mathbf{w}_{2g} sg_2 + \mathbf{H}_{1g}\left(\mathbf{w}_{11}s_1 + \mathbf{w}_{12}s_2\right) + \mathbf{H}_{2g}\left(\mathbf{w}_{23}s_3 + \mathbf{w}_{24}s_4\right) + n_{g_1} \Big], \tag{5}$$

$$y_{g_2} = \mathbf{u}_2^H \Big[\mathbf{H}_{2g}\mathbf{w}_{2g} sg_2 + \mathbf{H}_{1g}\mathbf{w}_{1g} sg_1 + \mathbf{H}_{2g}\left(\mathbf{w}_{23}s_3 + \mathbf{w}_{24}s_4\right) + \mathbf{H}_{1g}\left(\mathbf{w}_{11}s_1 + \mathbf{w}_{12}s_2\right) + n_{g_2} \Big], \tag{6}$$

respectively. Lets us denote the effective channel vector between the n-th BS and the virtual user g_v as $\mathbf{q}_{n,g_v}$. The effective channels between the BSs and the virtual users can be written as

$$\mathbf{q}_{11} = \mathbf{u}_1^H \mathbf{H}_{1g}, \tag{7}$$
$$\mathbf{q}_{12} = \mathbf{u}_2^H \mathbf{H}_{1g}, \tag{8}$$
$$\mathbf{q}_{21} = \mathbf{u}_1^H \mathbf{H}_{2g}, \tag{9}$$
$$\mathbf{q}_{22} = \mathbf{u}_2^H \mathbf{H}_{2g}. \tag{10}$$

B. System Metric Design

All the users have specific data rate requirements in order to establish successful connections. Let us denote a set of local users belonging to the n-th BS as $\mathcal{L}_n \subset \mathcal{L}$. Let us define the correlation matrix of the channel from the n-th BS to l-th local user as $\mathbf{R}_{nl} = [\mathbf{h}_{nl}\mathbf{h}_{nl}^H]$. The correlation matrix of the channel from the n-th BS to the virtual user g_v is denoted as $\mathbf{G}_{ng_v} = [\mathbf{q}_{ng_v}^H \mathbf{q}_{ng_v}]$. The intracell and intercell interference powers experienced by the l-th local user are given as

$$I_n = \sum_{\substack{i=1 \\ i \neq l}}^{L_n} \mathbf{w}_{ni}^H \mathbf{R}_{nl} \mathbf{w}_{ni} + \mathbf{w}_{1g_1}^H \mathbf{R}_{nl} \mathbf{w}_{1g_1}, \tag{11}$$

$$I_p = \sum_{\substack{j=1 \\ p \neq n}}^{L_p} \mathbf{w}_{pj}^H \mathbf{R}_{pl} \mathbf{w}_{pj} + \mathbf{w}_{2g_2}^H \mathbf{R}_{pl} \mathbf{w}_{2g_2}, \tag{12}$$

respectively. The downlink SINR of the l-th local user at n-th BS is given by

$$\text{SINR}_l^n = \frac{\mathbf{w}_{nl}^H \mathbf{R}_{nl} \mathbf{w}_{nl}}{I_n + I_p + \sigma_l^2}. \tag{13}$$

where $\mathbf{h}_{nl} \in \mathbb{C}^{M \times 1}$ be channel vector between the n-th BS and the l-th local user, and σ^2 is the noise variance at the l-th local user. Respectively, the SINR of the virtual users g_{v1} and g_{v2} are given by

$$\text{SINR}_{g_1} = \frac{\mathbf{w}_{1g_1}^H \mathbf{G}_{11} \mathbf{w}_{1g_1}}{\sum_{i=1}^{L_1} \mathbf{w}_{1i}^H \mathbf{G}_{11} \mathbf{w}_{1i} + \sum_{j=1}^{L_2} \mathbf{w}_{2j}^H \mathbf{G}_{21} \mathbf{w}_{2j} + \sigma_{g_1}^2}. \tag{14}$$

$$\text{SINR}_{g_2} = \frac{\mathbf{w}_{2g_2}^H \mathbf{G}_{22} \mathbf{w}_{2g_2}}{\sum_{i=1}^{L_1} \mathbf{w}_{1i}^H \mathbf{G}_{12} \mathbf{w}_{1i} + \sum_{j=1}^{L_2} \mathbf{w}_{2j}^H \mathbf{G}_{22} \mathbf{w}_{2j} + \sigma_{g_2}^2}. \tag{15}$$

Therefore, the total data rate of the global user is given by

$$R_g = R_{g1} + R_{g2} = \log_2(1 + \text{SINR}_{g_1}) + \log_2(1 + \text{SINR}_{g_2}). \tag{16}$$

III. SVD Based Beamforming Design

We aim to operate with the minimum total transmission power that will guarantee all the users their specific data rate target. We denote the specific data rate for the l-th local user and the global user as r_l and r_g respectively. Our optimization problem is formulated as

$$\begin{aligned} &\min \sum_{n \in \mathcal{N}} \sum_{l \in \mathcal{L}} \|\mathbf{w}_{nl}\|_2^2 + \|\mathbf{w}_{1g_1}\|_2^2 + \|\mathbf{w}_{2g_2}\|_2^2, \\ &\text{s.t} \quad \log_2(1 + \text{SINR}_l) \geq r_l, \quad \forall l, \\ &\quad R_g \geq r_g. \end{aligned} \tag{17}$$

In [4], it was proved that at optimality, the constraints in (17) will be satisfied with equality. For analysis purpose, we convert the data rates in (17) to SINRs. The local user SINR is determined as $\text{SINR}_l = 2^{r_l} - 1$. By setting the data rate at virtual user $g1$ as a variable θ, where $0 \le \theta \le r_g$, the SINRs of the virtual users g_1 and g_2 can be written as $\text{SINR}_{g_1} = 2^{\theta} - 1$ and $\text{SINR}_{g_2} = 2^{(r_g - \theta)} - 1$, respectively. Respectively, let us define the total interference experienced by the l-th local user and virtual users g_1 and g_2 as

$$\begin{aligned} I_l = & \sum_{n\in\mathcal{N}} \sum_{k\neq l} \mathbf{w}_{pk}^H \mathbf{R}_{nk} \mathbf{w}_{pk} + \mathbf{w}_{1g_1}^H \mathbf{R}_{pg_1} \mathbf{w}_{1g_1} \\ & + \mathbf{w}_{2g_2}^H \mathbf{R}_{2g_2} \mathbf{w}_{2g_2}, \end{aligned} \quad (18)$$

$$I_{g_1} = \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} \mathbf{w}_{nl}^H \mathbf{R}_{nl} \mathbf{w}_{nl} + \mathbf{w}_{2g_2}^H \mathbf{G}_{1g_2} \mathbf{w}_{2g_2}, \quad (19)$$

$$I_{g_2} = \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} \mathbf{w}_{nl}^H \mathbf{R}_{nl} \mathbf{w}_{nl} + \mathbf{w}_{1g_1}^H \mathbf{G}_{2g_1} \mathbf{w}_{1g_1}. \quad (20)$$

Given the SINR thresholds of the l-th local user and the virtual user g_v as γ_l and γ_{g_v}, we rewrite (17) as

$$\begin{aligned} \min & \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} \mathbf{w}_{nl}^H \mathbf{w}_{nl} + \mathbf{w}_{1g_1}^H \mathbf{w}_{1g_1} + \mathbf{w}_{2g_2}^H \mathbf{w}_{2g_2} \\ \text{s.t} \ & \mathbf{w}_{nl}^H \mathbf{R}_{nl} \mathbf{w}_{nl} - \gamma_l I_l \ge \gamma_{nl} \sigma_l^2, \quad \forall l, \\ & \mathbf{w}_{1g_1}^H \mathbf{G}_{1g_1} \mathbf{w}_{1g_1} - \gamma_{g_1} I_{g_1} \ge \gamma_{g_1} \sigma_{g_1}^2, \quad \forall l, \\ & \mathbf{w}_{2g_2}^H \mathbf{G}_{2g_2} \mathbf{w}_{2g_2} - \gamma_{g_2} I_{g_2} \ge \gamma_{g_2} \sigma_{g_2}^2, \quad \forall l. \end{aligned} \quad (21)$$

The constraints set in (17) makes the whole problem non-convex but after necessary manipulation, the problem can be convexified. Let us denote $\mathbf{W}_{nl} = \mathbf{w}_{nl}\mathbf{w}_{nl}^H$, $\mathbf{W}_{1g_1} = \mathbf{w}_{1g_1}\mathbf{w}_{1g}^H$ and $\mathbf{W}_{2g_2} = \mathbf{w}_{2g_2}\mathbf{w}_{2g}^H$. We then use the rule $\mathbf{w}^H\mathbf{R}\mathbf{w} = \text{Tr}[\mathbf{R}\mathbf{w}^H\mathbf{w}] = \text{Tr}[\mathbf{R}\mathbf{W}]$ to rewrite the (18)-(21) as

$$\begin{aligned} I_l = & \sum_{n\in\mathcal{N}} \sum_{k\neq l} \text{Tr}[\mathbf{R}_{nk}\mathbf{W}_{pk}] + \text{Tr}[\mathbf{G}_{1g_1}\mathbf{W}_{1g_1}] \\ & + \text{Tr}[\mathbf{G}_{2g_2}\mathbf{W}_{2g_2}], \end{aligned} \quad (22)$$

$$I_{g_1} = \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} \text{Tr}[\mathbf{R}_{nl}\mathbf{W}_{nl}] + \text{Tr}[\mathbf{G}_{2g_2}\mathbf{W}_{2g_2}], \quad (23)$$

$$I_{g_2} = \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} \text{Tr}[\mathbf{R}_{nl}\mathbf{W}_{nl}] + \text{Tr}[\mathbf{G}_{1g_1}\mathbf{W}_{1g_1}], \quad (24)$$

$$\begin{aligned} \min & \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} \text{Tr}[\mathbf{W}_{nl}] + \text{Tr}[\mathbf{W}_{1g_1}] + \text{Tr}[\mathbf{W}_{2g_2}] \\ \text{s.t} \ & \text{Tr}[\mathbf{R}_{nl}\mathbf{W}_{nl}] - \gamma_l I_l \ge \gamma_{nl}\sigma_l^2, \quad \forall l, \\ & \text{Tr}[\mathbf{G}_{1g_1}\mathbf{W}_{1g_1}] - \gamma_{g_1} I_{g_1} \ge \gamma_{g_1}\sigma_{g_1}^2, \quad \forall l, \\ & \text{Tr}[\mathbf{G}_{2g_2}\mathbf{W}_{2g_2}] - \gamma_{g_2} I_{g_2} \ge \gamma_{g_2}\sigma_{g_2}^2, \quad \forall l, \\ & \mathbf{W}_{nl} \succeq 0, \quad \mathbf{W}_{nl} = \mathbf{W}_{nl}^H, \quad \text{rank}[\mathbf{W}_{nl}] = 1, \quad \forall n, \forall l, \\ & \mathbf{W}_{1g_1} \succeq 0, \quad \mathbf{W}_{1g_1} = \mathbf{W}_{1g_1}^H, \quad \text{rank}[\mathbf{W}_{1g_1}] = 1, \\ & \mathbf{W}_{2g_2} \succeq 0, \quad \mathbf{W}_{2g_2} = \mathbf{W}_{2g_2}^H, \quad \text{rank}[\mathbf{W}_{2g_2}] = 1, \end{aligned} \quad (25)$$

where, $\mathbf{W} \succeq 0$ means $\mathbf{W}$ positive semidefinite. The ranks of $\{\mathbf{W}_{nl}\}_{\forall n, \forall l}$, $\mathbf{W}_{1g_1}$, and $\mathbf{W}_{2g_2}$ are nonconvex. Nevertheless, relaxing all the rank constraints gives the following relaxed semidefinite optimization problem [5]

$$\begin{aligned} \min & \sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}} \text{Tr}[\mathbf{W}_{nl}] + \text{Tr}[\mathbf{W}_{1g_1}] + \text{Tr}[\mathbf{W}_{2g_2}] \\ \text{s.t} \ & \text{Tr}[\mathbf{R}_{nl}\mathbf{W}_{nl}] - \gamma_l I_l \ge \gamma_{nl}\sigma_l^2, \quad \forall l, \\ & \text{Tr}[\mathbf{G}_{1g_1}\mathbf{W}_{1g_1}] - \gamma_{g_1} I_{g_1} \ge \gamma_{g_1}\sigma_{g_1}^2, \quad \forall l, \\ & \text{Tr}[\mathbf{G}_{2g_2}\mathbf{W}_{2g_2}] - \gamma_{g_2} I_{g_2} \ge \gamma_{g_2}\sigma_{g_2}^2, \quad \forall l, \\ & \mathbf{W}_{nl} \succeq 0, \quad \mathbf{W}_{nl} = \mathbf{W}_{nl}^H, \forall n, \forall l, \\ & \mathbf{W}_{1g_1} \succeq 0, \quad \mathbf{W}_{1g_1} = \mathbf{W}_{1g_1}^H, \\ & \mathbf{W}_{2g_2} \succeq 0, \quad \mathbf{W}_{2g_2} = \mathbf{W}_{2g_2}^H, \end{aligned} \quad (26)$$

which can be solved to an arbitrary accuracy using SDP solvers like YALMIP [17]. We note that if the (26) is feasible, it will provide rank-1 matrices $\{\mathbf{W}_{nl}\}_{\forall n, \forall l}$, $\mathbf{W}_{1g_1}$, and $\mathbf{W}_{2g_2}$ [5], [18]. However, if the rank of $\{\mathbf{W}_{nl}^\star\}_{\forall n, \forall l}$, $\mathbf{W}_{1g_1}^\star$, and $\mathbf{W}_{2g_2}^\star$ are greater than one, we can use the randomization techniques to heuristically find the $\mathbf{w}_{nl}, \forall n, \forall l$, $\mathbf{w}_{1g_1}$, and $\mathbf{w}_{2g_2}$ [18]. Note that if rank of $\{\mathbf{W}_{nl}^\star\}_{\forall n, \forall l}$, $\mathbf{W}_{1g_1}^\star$, and $\mathbf{W}_{2g_2}^\star$ are greater than one, then the heuristic $\{\mathbf{w}_{nl}\}_{\forall n, \forall l}$, $\mathbf{w}_{1g_1}$, and $\mathbf{w}_{2g_2}$ will provide a lower bound for the minimum required transmission power. Apparently, (26) is a dual of a dual program (i.e., bidual) of (25) [19], [20].

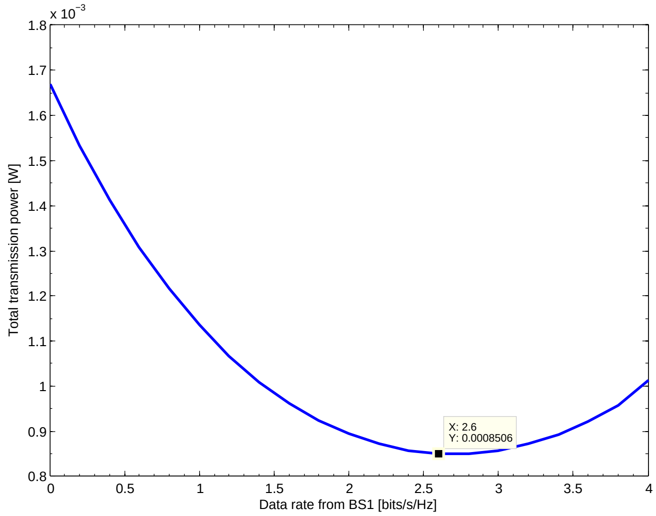
IV. Numerical Example

We consider a multicell multiuser network with two BSs and five users. Each BS is equipped with $M = 5$ antennae and it serves two single antenna local users. A global user is equipped with two receive antennae and it is served by both BSs. All BSs operate on the same frequency henceforth we assume all users experience significant intra-cell and inter-cell interference. Each user has a specific data rate target which needs to be satisfied for a successful connection. The channel vectors $\mathbf{h}_{nl}$ and $\mathbf{H}_{ng}$ were generated as i.i.d Gaussian random variables and the noise variance was set to $\sigma^2 = 1$ for all users. The random channels are generated between users and all BSs with zero mean and unity variance. The data rate targets for a pair of local users at each BS were set to 1.5 bits/s/Hz and 2 bits/s/Hz respectively. The data rate target for the global user was set to 4 bits/s/Hz.

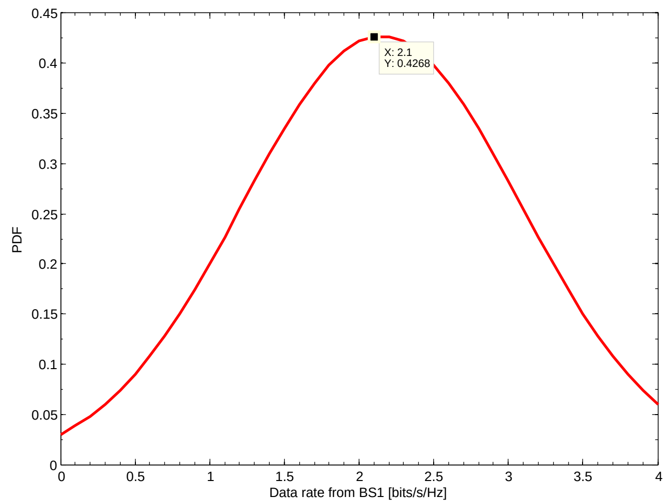
Subfigure 2a shows the total transmission power, for a single channel realization, when the data rate from BS1 to the global user is varied from 0 to 4 bits/s/Hz with step size $\delta = 0.1$ bits/s/Hz. We observe that the minimum total transmission power is achieved when BS1 contribute 2.6 bits/s/Hz of the 4 bits/s/Hz. It is possible that, for a given channel realization, all the data rate to the global user comes from only one BS. In subfigure 2b, we study the average data rate contributed by BS1 to the global user over 250 random channel realizations. As anticipated, we note that on average, BS1 will contribute 2 bits/s/Hz, whereas the remaining data rate will be contributed by BS2.

V. Conclusion

We studied a multicell multiuser network which simultaneously considers coordinated beamforming and joint transmission. The network consists of single antenna local users and



(a) Total transmission power for one channel realization.



(b) Average achievable data rate at the global user contributed by BS1.

Fig. 2. Performance analysis of the proposed SVD based beamformer.

one multi-antenna global user. The global user is served by more than one BS, whereas the local users are assigned to only one BS at a time. We considered beamforming design using power minimization criterion. The simulation results concluded that on average, the minimum total transmission power will be achieved if the data rate to the global user is equally shared between the BSs.

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