

Received March 22, 2017, accepted April 21, 2017, date of publication May 8, 2017, date of current version June 28, 2017.

Digital Object Identifier 10.1109/ACCESS.2017.2702058

# Coalitional Games for Downlink Multicell Beamforming

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This work was supported by the Engineering and Physical Sciences Research Council of the U.K. under Grant EP/M015475/1.

**ABSTRACT** A coalitional game is proposed for multicell multi-user downlink beamforming. Each base station intends to minimize its transmission power while aiming to attain a set of target signal-to-interference-plus-noise-ratio (SINR) for its users. To reduce power consumption, base stations have incentive to cooperate with other base stations to mitigate intercell interference. The coalitional game is introduced where base stations are allowed to forge partial cooperation rather than full cooperation. The partition form coalitional game is formulated with the consideration that beamformer design of a coalition depends on the coalition structure outside the considered coalition. We first formulate the beamformer design for a given coalition structure in which base stations in a coalition greedily minimize the total weighted transmit power without considering interference leakage to users in other coalitions. This can be considered as a non-cooperative game with each player as a distinct coalition. By introducing cost for cooperation, the coalition formation game is considered for the power minimization-based beamforming. A merge-regret-based sequential coalition formation algorithm has been developed that is capable of reaching a unique stable coalition structure. Finally, an  $\alpha$ -Modification algorithm has been proposed to improve the performance of the coalition formation algorithm.

**INDEX TERMS** Downlink beamforming, power minimization, Nash equilibrium, coalitional game.

## I. INTRODUCTION

The ever increasing demand for wireless services and applications has made the radio spectrum a scarce resource. The efficiency of frequency use is becoming a key issue in designing wireless networks. In traditional cellular networks, the spectrum utilization is improved by employing single cell based frequency reuse techniques. The limitation of such arrangement is that a specific base station (BS) will treat signals from all other BSs as noise, which could severely degrade the system performance. A possible way to mitigate inter-cell interference is to jointly manage transmitters of all BSs and perform multiple antenna beamforming. For example, in [1], [2], and [3], signal level coordination has been considered in which each BS is equipped with a single antenna but coordinates with all other BSs to form a virtual large antenna array. A coordinated multicell downlink beamforming was considered in [4] where all BSs cooperate with each other only at the beamforming level. Though such coordination can effectively achieve the aim of interference mitigation,

in a coordinated multicell network, messages have to be exchanged between BSs through backhaul channels which will cause considerable cost and burden to the network.

## A. MULTICELL DOWNLINK BEAMFORMING

Coordinated downlink beamforming is an efficient method to suppress inter-cell interference and to improve overall performance of the network. Several downlink beamforming techniques for multicell networks have been proposed in recent years with different performance requirements. In [4], beamformers for all users in all cells were coordinately designed through minimizing the weighted total transmission power of all BSs while a set of target SINRs is satisfied. This method can be traced back to [5] in which a beamformer-power based iterative algorithm was proposed that allowed users to achieve a set of target SINRs. In [5], the uplink-downlink duality has been applied as the key technique that transforms the downlink beamforming problem to determine the optimal virtual uplink beamformers which is easier to

solve [6], while in [4], the Lagrangian duality was introduced to transform the original problem to an uplink problem where optimal uplink beamformers were designed using a distributed approach.

Another commonly applied beamforming technique is to fairly balance the SINRs of all users to the same level by maximizing the worst case SINR under a maximum transmission power constraint, which is named as SINR balancing. This problem has been studied in [7] for a single cell case, in which SINRs of all users are maximized and balanced to the same level. In [8], [9], and [10], the multicell SINR balancing problem has been considered, where SINRs of all users in all cells are balanced and maximized to the same level. To further improve the performance, multicell beamforming algorithms based on per BS SINR balancing criterion have been proposed in [11] and [12]. In these works, SINRs of users in different cells were maximized and balanced to different levels.

Though the coordination at the beamforming level can effectively reduce the exchange of information between BSs as compared to the signal level coordination, perfect channel reciprocal and strict synchronization are required. For example, in [4], the dual uplink powers need to be iteratively updated based on stable synchronization schemes. To avoid the disadvantages arising from full coordination, game theoretic beamforming techniques have been proposed in recent years. Different from the coordinated beamforming, in a noncooperative game, each BS is considered as a player that greedily maximizes its own utility. In [13], a game theoretic method for multiple-input-single-output (MISO) interference channel (IFC) was considered, in which each BS equipped with multiple antennas served only one user equipped with a single antenna. A two BSs based MISO IFC beamforming has been studied in [14] for both the competitive and the cooperative scenarios. In [14], it has been found that the Nash equilibrium point of the MISO IFC game is equivalent to the solution obtained through the maximum ratio transmission (MRT). The competition in a multicell multiuser network was studied using game theory in [15] where each BS employs downlink beamforming to greedily minimize its own transmission power. A sufficient and necessary condition for the existence and uniqueness of the Nash equilibrium has been discussed and similar to [14], the Nash equilibrium is not Pareto optimal and always inefficient. To improve the efficiency of the Nash equilibrium, pricing strategies have been applied in [15] and [16]. It has been proved that the pricing scheme can effectively improve the efficiency of the Nash equilibrium by compromising the independence of each BS to some extent.

The inefficiency of the Nash equilibrium obtained in game theoretic based multicell beamforming urges BSs to coordinate with other BSs. However, if certain requirements for the coordinated beamforming such as the availability of channel state information and strict synchronization are included as extra cost, not all BSs are guaranteed to benefit from full coordination. For this reason, the mechanism for cooperation

should allow each BS to selectively cooperate with a subset of BSs to maximize its own utility. The cooperative game that was applied to the MISO IFC channels considered strategic bargaining [17]. In [18], a MISO IFC beamforming based coalition formation was studied. Coalition structures obtained through the algorithm proposed in [18] have been proved always in a coalition structure stable set. However, in both [17] and [18], only the scenario that each BS serves only one user has been considered.

## B. COALITIONAL GAME THEORY

Coalitional game aims to balance the competitiveness and coordination by allowing players to partially cooperate with each other. A coalition is defined as a group of players jointly improving their benefits through cooperation within the group [19]. The most widely studied coalitional game is the game in characteristic form. In this form, the utility of a coalition depends only on the members of the coalition rather than any players outside the coalition [20]. Both characteristic form coalitional games with transferable utility (TU) and nontransferable utility (NTU) have been studied in [21] and [22], respectively. Based on the coalitional game in characteristic form, the canonical coalitional game has been considered in which players are assumed to always benefit from the formation of a larger coalition. Hence, the grand coalition (the coalition of all players) is the optimal structure. The aim of the canonical coalitional game is to find the optimal solution that can maintain the stability of the grand coalition [20]. The solution of a canonical coalitional game has been investigated in [23]–[26] and [27].

Another class of coalitional games is the coalitional game in partition form which was first introduced in [28]. Different from the games in characteristic form, utility of players in a coalition in partition form is highly dependent on the structure outside of the coalition. The aim of a partition form coalitional game is to study how a coalition structure is formed and whether such coalition structure is stable. For this purpose, several coalition formation mechanisms namely dynamic coalition formation have been proposed. A dynamic coalition formation game is studied in [29] in which a player is allowed to merge with existing coalitions at each stage. In [30], a split-merge based coalition formation mechanism has been considered. A sequential based coalition formation process where a coalition structure can be achieved through sequentially merging or splitting has been investigated in [31]. The stability of the coalition structures has been discussed in a variety of works. In [32], the stable coalition structures achieved through the equilibrium binding agreement method for both the positive and the negative externalities have been studied and compared. The work in [30] provides the concepts of both strong and weak stability as  $\mathbb{D}_c$ -stable and  $\mathbb{D}_{hp}$ -stable for the proposed split-merge coalition formation game with transferable utility. Instead of finding a stable structure, the stable coalition structure set has been considered in [31] as a collection of all possible stable coalitions. Other concepts such as the recursive core have also

been studied as the solution of partition form coalition game in [33].

### C. CONTRIBUTION

We consider a coalitional game based multicell multiuser downlink beamforming. The aim for each BS is to minimize its transmission power while satisfying a set of target SINRs for its users. In [4], this problem has been solved by coordinately designing beamformers over all BSs in the network. The work in [4] did not consider the cost arising from the cooperation and every BS had the incentive to cooperate with all other BSs to reduce its transmission power. However, in practice, BSs might not benefit from such full cooperation since the transmission power reduction for each BS through the coordination may not be worthy as compared to the cost incurred for forging cooperation. In this paper, we assume there is a cost for cooperation, which is linearly proportional to the number of BSs involved in the cooperation. Instead of cooperating with all BSs, a set of BSs are allowed to locally cooperate with each other by forming a coalition to maximize their benefits. We consider the coalitional game in partition form due to the fact that the transmission power of a BS depends not only on the coalition that it is attached to, but also on the structure of external coalitions. In [18], a merge-based coalition formation algorithm was proposed for the multicell MISO IFC problem. Based on the algorithm proposed in [18], a merge-regret based coalition formation algorithm is developed where BSs are allowed to split from the newly formed coalition if they do not benefit from the formation of the coalition. Instead of considering all potential combinations at each formation stage as stated in [18], in the proposed algorithm, by numbering all coalitions at each stage with a given numbering strategy, only a limited number of combinations is required. This is because once a combination is found to be valid, the corresponding coalition will be formed and the rest of the potential combinations will not be considered further. Hence, compared to [18], the proposed algorithm can reduce the computational complexity and coalition formation time. In addition, we investigated two coalition formation decision rules with strong independence and weak independence. To improve the performance of the proposed coalition formation algorithm, an  $\alpha$ -Modification algorithm has been developed for different decision rules. Finally, it is proved that by applying the proposed coalition formation algorithm, a unique and stable output coalition structure is obtained.

### D. ORGANIZATION

The remainder of this paper is organized as follows. In section II, the system model and problem formulation are given. Section III discusses the coalitional multicell beamforming with different coalition structures. In section IV, we formulate the coalition formation algorithm for the proposed multicell downlink beamforming game. The simulation results are provided in section V followed by conclusions in section VI.

**Notations:** Superscripts  $(\cdot)^T$  and  $(\cdot)^H$  represent the transpose and Hermitian transpose respectively;  $\mathbb{R}$  and  $\mathbb{C}$  stand for the real and complex spaces;  $\mathbf{I}$  denotes the identity matrix;  $E\{\cdot\}$  denotes the expectation operation;  $\mathbf{A} \geq \mathbf{0}$  means each element of matrix  $\mathbf{A}$  is nonnegative;  $\mathbf{A} > \mathbf{0}$  means that  $\mathbf{A} \geq \mathbf{0}$  and at least one element of matrix  $\mathbf{A}$  is positive.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. SYSTEM MODEL

A multicell multi-user wireless network consisting of  $J$  cells is considered. Let  $\Omega = \{1, \dots, J\}$  be the set of all cells. It is assumed there are  $K$  users in each cell. A MISO technique is employed, i.e. each BS is equipped with  $M$  antennas, while each user terminal has a single antenna.  $\mathbf{h}_{i,j,k} \in \mathbb{C}^{M \times 1}$  represents the channel vector from the  $i$ th BS to the  $k$ th user in the  $j$ th cell while  $s_{j,k}$  denotes the information symbol to the  $k$ th user in the  $j$ th cell, where  $E\{|s_{j,k}|^2\} = 1$ . To perform downlink beamforming, we denote  $\mathbf{u}_{j,k} \in \mathbb{C}^{M \times 1}$  as the downlink transmit beamformer vector for the  $k$ th user in the  $j$ th cell. Then, the received signal at the  $k$ th user in the  $j$ th cell can be written as

$$y_{j,k} = \sum_{l=1}^K \mathbf{h}_{j,j,k}^H \mathbf{u}_{j,l} s_{j,l} + \sum_{i=1, i \neq j}^J \sum_{m=1}^K \mathbf{h}_{i,j,k}^H \mathbf{u}_{i,m} s_{i,m} + z_{j,k} \quad (1)$$

where  $z_{j,k}$  in (1) is assumed to be complex additive white Gaussian noise with zero mean and variance  $\sigma_{j,k}^2$ . The SINR of the  $k$ th user in the  $j$ th cell is given by

$$\Gamma_{j,k}^{DL} = \frac{|\mathbf{u}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{l \neq k} |\mathbf{u}_{j,l}^H \mathbf{h}_{j,j,k}|^2 + \sum_{i \neq j} \sum_{m=1}^K |\mathbf{u}_{i,m}^H \mathbf{h}_{i,j,k}|^2 + \sigma_{j,k}^2}. \quad (2)$$

### B. MOTIVATION OF COALITIONAL GAME

We consider a problem where each BS optimizes its beamformers and transmission power subject to achieving a target SINR for its users. In traditional single cell based processing, the problem is solved by letting each BS to individually design beamformers for its own users and greedily minimize its own transmission power. The advantage of this method is that BSs do not need to communicate with each other; however, since BSs are competing with each other, the transmit power for each BS is always considerably high. Another way of solving the transmit power minimization problem is to coordinately design beamformers for all users in all cells, which can effectively mitigate the inter-cell interference to all users [4]. However, in [4], the cost arising from cooperation has not been taken into consideration; hence, BSs always have incentive to cooperate with each other to improve their performance. In practice, there is a cost for cooperation which may discourage certain BSs to cooperate with other BSs. Hence, in this work, by introducing cooperation cost, we consider a scenario that certain sets of BSs are allowed to form coalitions locally to maximize their overall gain. For a given coalition structure, the power minimization is used to

determine the optimal beamformers, while by employing the coalition formation algorithm, an optimal coalition structure is obtained.

### III. COALITIONAL BEAMFORMING PROBLEM

We begin by focusing on downlink transmit beamforming in the multicell multiuser scenario. The transmit beamformers of users in the  $j$ th cell are denoted in a matrix form as  $\mathbf{U}_j = [\mathbf{u}_{j,1}, \dots, \mathbf{u}_{j,K}]$ , where  $\mathbf{U}_j \in \mathcal{B}_j$  and  $\mathcal{B}_j$  is the strategy space of BS  $j$ , defined as

$$\mathcal{B}_j := \{\mathbf{U}_j \in \mathbb{C}^{M \times K}\} \quad (3)$$

Then, the strategy profile of all BSs is the joint set of all possible strategies, defined as

$$(\mathbf{U}_1, \dots, \mathbf{U}_J) \in \mathcal{X} := \mathcal{B}_1 \times \dots \times \mathcal{B}_J \quad (4)$$

For each BS, the aim is to minimize the transmission power while ensuring that the downlink SINRs of its users are greater than a set of target values, i.e.,  $\Gamma_{j,k}^{DL} \geq \gamma_{j,k}$ . Hence, the utility function of the  $j$ th BS is defined as the transmission power at BS  $j$ ,

$$p_j = \sum_{k=1}^K \|\mathbf{u}_{j,k}\|_2^2 = \|\mathbf{U}_j\|_F^2, \quad \forall j \in \Omega. \quad (5)$$

For a given set of players  $\Omega$ , a coalition structure  $\mathcal{S} = \{C_1, \dots, C_{N_s}\}$  is defined as a partition of  $\Omega$  with the following characteristics:  $\bigcup_{q=1}^{N_s} C_q = \Omega$  and  $C_x \cap C_y = \emptyset$  for any  $C_x, C_y \in \mathcal{S}$ . Based on the above definitions, the game in partition form can be expressed as [18]

$$\langle \Omega, \mathcal{X}, \mathcal{F}, (p_j)_{j \in \Omega} \rangle, \quad (6)$$

where  $\mathcal{F}$  is the partition function that assigns all possible partitions to the game. It is assumed that the utility of a player in a coalition cannot be transferred to other players in the same coalition, and this is known as nontransferable utility. In the following, the coalitional beamforming problem with various coalition structures is discussed.

#### A. NON-COOPERATIVE MULTICELL BEAMFORMING

In coalitional game theory, a special coalition structure is that all coalitions are singletons. In this case, each player is competing with all other players, which falls into the traditional strategic non-cooperative game (SNG). For the transmission power minimization problem, players are the BSs. Each BS will greedily minimize its own transmission power without constraining interference to users in other cells. The transmission power is the utility function of each BS. Hence, for the non-cooperative game, the beamforming strategy set of the  $j$ th BS is defined as

$$\mathcal{B}'_j := \{\mathbf{U}_j \in \mathbb{C}^{M \times K} : \Gamma_{j,k}^{DL}(\mathbf{U}_j, \mathbf{U}_{-j}) \geq \gamma_{j,k}, \forall k\} \quad (7)$$

where  $\mathbf{U}_{-j}$  is the strategy of all BSs except BS  $j$ . Then, the coalitional beamforming game with non-cooperative coal-

ition structure can be expressed as

$$\langle \Omega, \{\mathcal{B}'_j\}_{j \in \Omega}, (p_j)_{j \in \Omega} \rangle \quad (8)$$

This game has been considered in [15] and [34], for which the best response strategy of the  $j$ th BS is the solution of the following optimization problem:

$$\underset{\mathbf{U}_j}{\text{minimize}} \quad \|\mathbf{U}_j\|_F^2 \quad (9a)$$

$$\text{subject to} \quad \frac{|\mathbf{u}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{l \neq k} |\mathbf{u}_{j,l}^H \mathbf{h}_{j,j,k}|^2 + \eta_{j,k}} \geq \gamma_{j,k}, \quad \forall k \quad (9b)$$

where  $\eta_{j,k} = \sum_{i \neq j} \sum_{m=1}^K |\mathbf{u}_{i,m}^H \mathbf{h}_{i,j,k}|^2 + \sigma_{j,k}^2$  is the noise power plus the inter-cell interference from all of the users in all other BSs except BS  $j$ . It has been proven in [15] that if both the necessary and the sufficient conditions are satisfied, a unique Nash equilibrium point exists for the game and a set of best beamforming strategies  $\{\mathbf{U}_1^*, \dots, \mathbf{U}_J^*\}$  can be found as satisfying

$$p_j(\mathbf{U}_j^*) \leq p_j(\mathbf{U}_j), \quad \forall \mathbf{U}_j \in \mathcal{B}'_j, \quad \forall j \in \Omega, \quad (10)$$

where  $p_j(\mathbf{U}_j^*)$  is the transmission power of the  $j$ th BS at the equilibrium. It has also been found in [15] that for each BS, beam patterns of its users are independent of the value of the inter-cell interference. This means that for a given set of target SINRs, each BS can design a set of fixed beam patterns for its users regardless of the interference value  $\eta_{j,k}$ . Therefore, the strategies of the non-cooperative beamforming game can be reduced to a set of power allocation as follows:

$$\mathcal{B}'_j(\mathbf{p}) := \{\mathbf{p}_j \in \mathbb{R}_+^K : \Gamma_{j,k}^{DL}(\mathbf{p}_j, \mathbf{p}_{-j}) \geq \gamma_{j,k}, \forall k\} \quad (11)$$

where  $\mathbf{p}_j$  and  $\mathbf{p}_{-j}$  are power allocation of the  $j$ th BS and all other BSs except the  $j$ th BS respectively. By substituting the obtained beam patterns into (9b) and setting equality in (9b), the best response strategy of the  $j$ th BS can be obtained as

$$\mathbf{p}_j^* = \mathbf{G}_j^{-1} \boldsymbol{\eta}_j \quad (12)$$

where  $\boldsymbol{\eta}_j = [\eta_{j,1}, \dots, \eta_{j,K}]^T$ .  $\mathbf{G}_j$  is a matrix obtained from  $\mathbf{h}_{j,j,k}$  and beam patterns  $\bar{\mathbf{U}}_j$  of all users in the  $j$ th cell as defined in Claim (4) of [15]. Hence, once the intersection point of (12) is obtained, the Nash equilibrium is achieved. It has been discussed in [15] that the Nash equilibrium exists if and only if the following matrix is an  $\mathbf{M}$ -matrix:

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & -\mathbf{G}_{21} & \dots & -\mathbf{G}_{J1} \\ -\mathbf{G}_{12} & \mathbf{G}_2 & \dots & -\mathbf{G}_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{G}_{1J} & -\mathbf{G}_{2J} & \dots & \mathbf{G}_J \end{bmatrix} \quad (13)$$

where  $\mathbf{G}_{ij}$  is the inter-cell interference matrix from  $i$ th cell to the  $j$ th cell. By summarizing all of the above arguments, the best response strategies and utilities at the Nash equilibrium can be determined through algorithm I.



**Algorithm 1** Beamforming With Non-Cooperative Game

- 1) Determine the downlink beamformer  $\mathbf{u}_{j,k}$  for all users using the method proposed in [7] with a given set of interference.
- 2) Find the beam pattern  $\bar{\mathbf{u}}_{j,k}$  by  $\bar{\mathbf{u}}_{j,k} = \mathbf{u}_{j,k} / \|\mathbf{u}_{j,k}\|_2$ .
- 3) Substitute  $\bar{\mathbf{u}}_{j,k}$  into (9b) and set (9b) as equality to obtain (12).
- 4) For a given  $\eta_j$ , determine  $\mathbf{p}_j$  using (12).
- 5) Update  $\eta_j$  with  $\mathbf{p}_{-j}$  and repeat step 4 until the optimal  $\mathbf{p}_j^*$  is obtained.

**B. COORDINATED MULTICELL BEAMFORMING**

Another special coalition structure is that all of the BSs form a grand coalition and coordinately design beamformers for their users, which is known as fully coordinated multicell beamforming. By jointly designing beamformers for all users in all cells, inter-cell interference can be effectively mitigated and the transmission power of each BS can be reduced. For this case, the multicell power minimization problem has been formulated in [4] as follows:

$$\underset{\mathbf{U}_1, \dots, \mathbf{U}_J}{\text{minimize}} \sum_{j=1}^J \hat{\alpha}_j \|\mathbf{U}_j\|_F^2 \quad (14a)$$

$$\text{subject to} \frac{|\mathbf{u}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{l \neq k} |\mathbf{u}_{j,l}^H \mathbf{h}_{j,j,k}|^2 + \sum_{i \neq j} \sum_{m=1}^K |\mathbf{u}_{i,m}^H \mathbf{h}_{i,j,k}|^2 + \sigma_{j,k}^2} \geq \gamma_{j,k}, \quad \forall j, k, \quad (14b)$$

where  $\hat{\alpha}_j$  is the weighting factor assigned to the  $j$ th BS in the grand coalition  $\Omega$ . As stated in [4], the optimal solution of (14) can be obtained by solving a dual uplink problem for the same set of SINRs. By introducing the Lagrangian technique, (14) can be transformed to the following uplink problem:

$$\underset{\lambda_{1,1}, \dots, \lambda_{J,K}}{\text{minimize}} \sum_{j=1}^J \sum_{k=1}^K \hat{\lambda}_{j,k} \sigma^2 \quad (15a)$$

$$\text{subject to} \frac{\hat{\lambda}_{j,k} |\hat{\mathbf{u}}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{(i,m) \neq (j,k)} \hat{\lambda}_{i,m} |\hat{\mathbf{u}}_{j,k}^H \mathbf{h}_{j,i,m}|^2 + \hat{\alpha}_j \hat{\mathbf{u}}_{j,k}^H \hat{\mathbf{u}}_{j,k}} \geq \gamma_{j,k}, \quad \forall j, k, \quad (15b)$$

where  $\hat{\lambda}_{j,k}$  and  $\hat{\mathbf{u}}_{j,k}$  are the uplink power and receiver beamformer of the  $k$ th user in the  $j$ th cell in the grand coalition  $\Omega$ , respectively. The uplink power  $\hat{\lambda}_{j,k}$  can be iteratively obtained through the method proposed in [35], and the uplink beamformer  $\hat{\mathbf{u}}_{j,k}$  can be calculated through the following equation

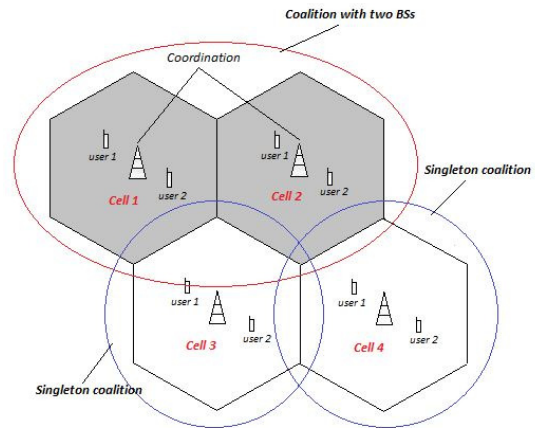
$$\hat{\mathbf{u}}_{j,k} = \left( \sum_{i=1}^J \sum_{m=1}^K \hat{\lambda}_{i,m} \mathbf{h}_{j,i,m}^H \mathbf{h}_{j,i,m} + \hat{\alpha}_j \mathbf{I} \right)^{-1} \mathbf{h}_{j,j,k} \quad (16)$$

According to [4], a downlink beamformer should be a scaled version of the corresponding uplink beamformer

as  $\mathbf{u}_{j,k} = \sqrt{\hat{\delta}_{j,k}} \hat{\mathbf{u}}_{j,k}$ , where the scaling factor  $\hat{\delta}_{j,k}$  can be obtained using (18) in [4]. Based on this, the fully coordinated beamforming proposed in [4] is summarized in algorithm 2.

**Algorithm 2** Fully Coordinated Beamforming

- 1) Iteratively find the uplink power  $\hat{\lambda}_{j,k}$ .
- 2) Determine the receiver beamformers using (16) for a given set of uplink power allocation.
- 3) Obtain the scaling factor  $\hat{\delta}_{j,k}$  using equation (18) in [4].
- 4) Calculate the downlink beamformers using  $\mathbf{u}_{j,k} = \sqrt{\hat{\delta}_{j,k}} \hat{\mathbf{u}}_{j,k}$ .



**FIGURE 1.** Multicell beamforming for a given coalition structure.

**C. BEAMFORMERS DESIGN FOR A GIVEN COALITION STRUCTURE**

After considering the fully non-cooperative and fully coordinated cases, we are interested in formulating the multicell downlink beamforming for a given coalition structure [36]. In the coalitional beamforming, disjoint cells merge to several coalitions and BSs in each coalition jointly design beamformers for their users. A typical coalitional form beamforming is shown in Figure 1, in which cell 1 and cell 2 has formed a coalition while cell 3 and cell 4 are singleton coalitions. The coordination exists only within each coalition, which means that the beamformers are designed with partial coordination and each coalition is still competing with other coalitions. Hence, to find the optimal beamformers for all users in all coalitions, the coalitional game can be transferred to a strategic non-cooperative sub game (SNSG), in which players are the coalitions. For a singleton coalition, the utility function is still the transmission power while for coalitions consisting of multiple cells, utility function is the weighted total transmission power of BSs within each coalition. Hence, in SNSG, each coalition competes with other coalitions by greedily maximizing its utility.

We consider a coalition structure  $S$  with  $N_s$  coalitions  $\{C_1, \dots, C_{N_s}\}$ , where  $\bigcup_{q=1}^{N_s} C_q = \Omega$ . Let  $\Omega_s = \{1, \dots, N_s\}$  be the set of players for the SNSG game with coalition structure  $S$  and coalition  $C_q$  is the  $q$ th player of the SNSG game,  $\forall q \in \Omega_s$ . For the SNSG game with coalition structure  $S$ , the utility function of coalition  $C_q$  is defined as

$$T_q = \sum_{j \in C_q} \sum_{k=1}^K \check{\alpha}_j \| \mathbf{u}_{j,k} \|_2^2 = \sum_{j \in C_q} \check{\alpha}_j \| \mathbf{U}_j \|_F^2 \quad (17)$$

where  $\check{\alpha}_j$  is the weighting factor of the  $j$ th BS with the coalition structure  $S$  and  $\sum_{j \in C_q} \check{\alpha}_j = 1$ . It should be noticed that for the single coalition,  $\check{\alpha}_j = 1$  and (17) reduces to (5). Define the beamformer matrix of coalition  $C_q$  in the coalition structure  $S$  as  $\mathbf{U}_q(S)$ . This is the strategy for coalition  $C_q$ .  $\mathbf{U}_{-q}(S)$  is defined as the beamforming strategy of all coalitions except coalition  $C_q$ . Then, by introducing downlink SINRs, the admissible strategy set for coalition  $C_q$  is defined as

$$\mathcal{B}_q(S) = \{ \mathbf{U}_q \in \mathbb{C}^{M \times K|C_q|} : \Gamma_{j,k}(\mathbf{U}_q(S), \mathbf{U}_{-q}(S)) \geq \gamma_{j,k}, \forall j \in C_q, \forall k \}, \quad (18)$$

where  $\gamma_{j,k}$  is the SINR target at the  $k$ th user in the  $j$ th cell for all  $j \in C_q$ . The interference induced by all BSs outside coalition  $C_q$  to the  $k$ th user in the  $j$ th cell can be written as  $\sum_{i \notin C_q} \sum_{m=1}^K | \mathbf{u}_{i,m}^H \mathbf{h}_{i,j,k} |^2$ . Then, the SNSG game for a given coalition structure  $S$  can be written as

$$\langle \Omega_s, \{ \mathcal{B}_q(S) \}_{q \in \Omega_s}, \{ T_q \}_{q \in \Omega_s} \rangle \quad (19)$$

The optimal strategy of the  $q$ th coalition for this game is obtained by solving the following optimization problem

$$\text{minimize}_{\mathbf{U}_j, \forall j \in C_q} \sum_{j \in C_q} \check{\alpha}_j \| \mathbf{U}_j \|_F^2 \quad (20a)$$

$$\text{subject to} \quad \frac{| \mathbf{u}_{j,k}^H \mathbf{h}_{j,j,k} |^2}{\sum_{l \neq k} | \mathbf{u}_{j,l}^H \mathbf{h}_{j,j,k} |^2 + \sum_{i \in C_q, m=1}^K | \mathbf{u}_{i,m}^H \mathbf{h}_{i,j,k} |^2 + \check{\eta}_{j,k}} \geq \gamma_{j,k}, \forall j \in C_q, \forall k, \quad (20b)$$

where  $\check{\eta}_{j,k}$  is the inter-coalition interference from BSs outside coalition  $C_q$  to the  $k$ th user in the  $j$ th cell plus noise power for all BSs  $j \in C_q$ . By rewriting the inter-coalition interference in a coalitional game form,  $\check{\eta}_{j,k}$  is given by

$$\check{\eta}_{j,k} = \sum_{x \in \Omega_s} \sum_{i \in C_x} \sum_{m=1}^K | \mathbf{u}_{i,m}^H \mathbf{h}_{i,j,k} |^2 + \sigma_{j,k}^2 \quad (21)$$

For a given coalition  $C_q$  in coalition structure  $S$  with multiple BSs and a given set of  $\check{\eta}_{j,k}$ , problem (20) can be solved using the method proposed in [4]. The optimal transmit beamformers can be obtained via the corresponding dual uplink problem. Then, similar to the fully coordinated case, by

introducing the Lagrangian duality, (20) can be transformed to the following optimization problem

$$\text{maximize}_{\check{\lambda}_{j,1}, \dots, \check{\lambda}_{j,K}, \forall j \in C_q} \sum_{j \in C_q} \sum_{k=1}^K \check{\lambda}_{j,k} \check{\eta}_{j,k} \quad (22a)$$

$$\text{subject to} \quad \check{\Sigma}_j \succeq (1 + \frac{1}{\gamma_{j,k}}) \check{\lambda}_{j,k} \mathbf{h}_{j,j,k} \mathbf{h}_{j,j,k}^H \quad (22b)$$

where

$$\check{\Sigma}_j = \sum_{i \in C_q} \sum_{m=1}^K \check{\lambda}_{i,m} \mathbf{h}_{i,j,k} \mathbf{h}_{i,j,k}^H + \check{\alpha}_j \mathbf{I} \quad (23)$$

and  $\check{\lambda}_{j,k}$  is the uplink power of the  $k$ th user in the  $j$ th cell with the coalition structure  $S$ . According to [4], problem (22) is equivalent to the following optimization problem

$$\text{minimize}_{\check{\lambda}_{j,1}, \dots, \check{\lambda}_{j,K}, \forall j \in C_q} \sum_{j \in C_q} \sum_{k=1}^K \check{\lambda}_{j,k} \check{\eta}_{j,k} \quad (24a)$$

$$\text{subject to} \quad \frac{\check{\lambda}_{j,k} | \check{\mathbf{u}}_{j,k}^H \mathbf{h}_{j,j,k} |^2}{\sum_{i \in C_q} \sum_{m=1}^K \check{\lambda}_{i,m} | \check{\mathbf{u}}_{i,k}^H \mathbf{h}_{i,j,k} |^2 + \check{\alpha}_j \check{\mathbf{u}}_{j,k}^H \check{\mathbf{u}}_{j,k}} \geq \frac{\gamma_{j,k}}{1 + \gamma_{j,k}} \quad (24b)$$

where  $\check{\mathbf{u}}_{j,k}$  is the uplink beamformer for the  $k$ th user in the  $j$ th cell with the coalition structure  $S$ . According to [4] and [15], the optimal uplink power  $\check{\lambda}_{j,k}$ ,  $\forall j \in C_q$ ,  $\forall k$ , can be determined through the following iterative fixed point method

$$\check{\lambda}_{j,k} = \frac{\gamma_{j,k}}{1 + \gamma_{j,k}} \cdot \frac{1}{\mathbf{h}_{j,j,k}^H \check{\Sigma}_j^{-1} \mathbf{h}_{j,j,k}}. \quad (25)$$

Once the optimal uplink power  $\check{\lambda}_{j,k}$  is obtained, the optimal receiver beamformer  $\check{\mathbf{u}}_{j,k}$  is the MMSE receiver, expressed as

$$\check{\mathbf{u}}_{j,k} = (\sum_{i \in C_q} \sum_{m=1}^K \check{\lambda}_{i,m} \mathbf{h}_{i,j,k} \mathbf{h}_{i,j,k}^H + \check{\alpha}_j \mathbf{I})^{-1} \mathbf{h}_{j,j,k}. \quad (26)$$

The solution of uplink power  $\check{\lambda}_{j,k}$  and receive beamformer  $\check{\mathbf{u}}_{j,k}$  depends only on the intra-coalition channels and weighting factors in coalition  $C_q$ , and it is independent of the interference induced by BSs outside  $C_q$ . It has been proved in [4] that the transmit beamformer  $\mathbf{u}_{j,k}$  is the scaled version of the receiver beamformer  $\check{\mathbf{u}}_{j,k}$ . Hence, the transmit beamformer  $\mathbf{u}_{j,k}$  should also be a scaled version of  $\check{\mathbf{u}}_{j,k}$ , where  $\check{\mathbf{u}}_{j,k}$  is the beam pattern of  $\check{\mathbf{u}}_{j,k}$  for the  $k$ th user in the  $j$ th cell with  $\| \check{\mathbf{u}}_{j,k} \|_2^2 = 1$  for the coalition structure  $S$ . This can be obtained as

$$\tilde{\mathbf{u}}_{j,k} = \frac{\check{\mathbf{u}}_{j,k}}{\| \check{\mathbf{u}}_{j,k} \|_2}. \quad (27)$$

Hence, for coalition  $C_q$  in  $S$ , we can design a fixed set of beam patterns for users inside  $C_q$  without considering any inter-coalition interference [36]. By writing the transmit

beamformer as  $\mathbf{u}_{j,k} = \sqrt{p_{j,k}} \tilde{\mathbf{u}}_{j,k}$ , where  $p_{j,k}$  is the downlink power allocated to the  $k$ th user in the  $j$ th cell, the weighted total power minimization (20) can be restated as

$$\underset{p_{j,1}, \dots, p_{j,K}, \forall j \in C_q}{\text{minimize}} \quad \sum_{j \in C_q} \sum_{k=1}^K \check{\alpha}_j p_{j,k} \quad (28a)$$

$$\text{s.t.} \quad \frac{p_{j,k} |\tilde{\mathbf{u}}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\sum_{l=1, l \neq k}^K p_{j,l} |\tilde{\mathbf{u}}_{j,l}^H \mathbf{h}_{j,j,k}|^2 + \sum_{i \in C_q, i \neq j} \sum_{m=1}^K p_{i,m} |\tilde{\mathbf{u}}_{i,m}^H \mathbf{h}_{i,j,k}|^2 + \check{\eta}_{j,k}} \geq \gamma_{j,k}, \quad \forall j \in C_q, \forall k. \quad (28b)$$

Since all SINR constraints should achieve equality when optimal beamformers  $\mathbf{u}_{j,k}^* = \sqrt{p_{j,k}^*} \tilde{\mathbf{u}}_{j,k}$  is obtained  $\forall j \in C_q, \forall k$ , constraints (28b) can be rewritten as

$$p_{j,k} \frac{|\tilde{\mathbf{u}}_{j,k}^H \mathbf{h}_{j,j,k}|^2}{\gamma_{j,k}} - \sum_{l=1, l \neq k}^K p_{j,l} |\tilde{\mathbf{u}}_{j,l}^H \mathbf{h}_{j,j,k}|^2 - \sum_{i \in C_q, i \neq j} \sum_{m=1}^K p_{i,m} |\tilde{\mathbf{u}}_{i,m}^H \mathbf{h}_{i,j,k}|^2 = \check{\eta}_{j,k}, \quad \forall j \in C_q, \forall k \quad (29)$$

To simplify our expression, it is assumed all the BSs in coalition  $C_q$  with ascending indexes are renumbered from 1 to  $|C_q|$ . Then, parameters can be re-denoted in the following way: denote  $\mathbf{h}_{(i)_x, (j,k)_q} \in \mathbb{C}^{M \times 1}$  as the channel vector from the  $i$ th BS in coalition  $C_x$  to the  $k$ th user of the  $j$ th cell in coalition  $C_q$  and  $\mathbf{u}_{(j,k)_q}$  as the downlink transmit beamformer vector for the  $k$ th user in the  $j$ th cell in coalition  $C_q$ .  $p_{(j,k)_q}$  represents the allocated power to the  $k$ th user in the  $j$ th cell in coalition  $C_q$ . The power allocation vector of coalition  $C_q$  in coalition structure  $S$  can be denoted as  $\check{\mathbf{p}}_q = [p_{(1)_q}^T, \dots, p_{(|C_q|)_q}^T]^T$ ,  $\forall C_q \in S$ , where  $\mathbf{p}_{(j)_q}$  is the power allocation vector of the  $j$ th cell in coalition  $C_q$ . By setting all SINR constraints in coalition  $C_q$  to equality, the following equation is obtained

$$\mathbf{F}_q \check{\mathbf{p}}_q = \check{\eta}_q \quad (30)$$

where  $\check{\eta}_q = [\check{\eta}_{(1)_q}^T, \dots, \check{\eta}_{(|C_q|)_q}^T]^T$  and  $\check{\eta}_{(v)_q}$  is the inter-coalition interference vector of the  $v$ th BS in coalition  $C_q$ .  $\mathbf{F}_q$  is a  $K|C_q| \times K|C_q|$  matrix with the following structure

$$\mathbf{F}_q = \begin{bmatrix} \mathbf{F}_q^{(1,1)} & \mathbf{F}_q^{(1,2)} & \dots & \mathbf{F}_q^{(1,|C_q|)} \\ \mathbf{F}_q^{(2,1)} & \mathbf{F}_q^{(2,2)} & \dots & \mathbf{F}_q^{(2,|C_q|)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_q^{(|C_q|,1)} & \mathbf{F}_q^{(|C_q|,2)} & \dots & \mathbf{F}_q^{(|C_q|,|C_q|)} \end{bmatrix} \quad (31)$$

where  $\mathbf{F}_q^{(j,i)}$  is a  $K \times K$  sub-matrix with the following entries

$$[\mathbf{F}_q^{(j,i)}]_{k,m} = \begin{cases} \frac{|\tilde{\mathbf{u}}_{(j,k)_q}^H \mathbf{h}_{(j,k)_q}|^2}{\gamma_{(j,k)_q}} & i = j, m = k, \\ -|\tilde{\mathbf{u}}_{(j,m)_q}^H \mathbf{h}_{(j,k)_q}|^2 & i = j, m \neq k, \\ -|\tilde{\mathbf{u}}_{(i,m)_q}^H \mathbf{h}_{(i,k)_q}|^2 & i \neq j \end{cases} \quad (32)$$

for  $j, i = 1, \dots, |C_q|$  and  $k, m = 1, \dots, K$ . The best response strategy of the sub-game is expressed as

$$\check{\mathbf{p}}_q = \mathbf{R}_q(\check{\mathbf{p}}_{-q}) = \mathbf{F}_q^{-1} \check{\eta}_q \quad (33)$$

It should be noticed that the best response strategy exists only if  $\mathbf{F}_q$  is invertible. By rewriting the inter-coalition interference in a matrix form, the best response strategy can be expressed as

$$\check{\mathbf{p}}_q = \mathbf{F}_q^{-1} \left( \sum_{\substack{x \in \Omega_s \\ x \neq q}} \mathbf{F}_{xq} \check{\mathbf{p}}_x + \mathbf{1} \sigma^2 \right) \quad (34)$$

where  $\mathbf{F}_{xq}$  is the inter-coalition interference matrix of size  $K|C_q| \times K|C_x|$  from the  $x$ th coalition to the  $q$ th coalition with the following structure

$$\mathbf{F}_{xq} = \begin{bmatrix} \mathbf{F}_{xq}^{(1,1)} & \mathbf{F}_{xq}^{(1,2)} & \dots & \mathbf{F}_{xq}^{(1,|C_x|)} \\ \mathbf{F}_{xq}^{(2,1)} & \mathbf{F}_{xq}^{(2,2)} & \dots & \mathbf{F}_{xq}^{(2,|C_x|)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{F}_{xq}^{(|C_q|,1)} & \mathbf{F}_{xq}^{(|C_q|,2)} & \dots & \mathbf{F}_{xq}^{(|C_q|,|C_x|)} \end{bmatrix} \quad (35)$$

in which  $\mathbf{F}_{xq}^{(j,i)}$  is a  $K \times K$  sub-matrix with the following entries

$$[\mathbf{F}_{xq}^{(j,i)}]_{k,m} = |\tilde{\mathbf{u}}_{(i,m)_x}^H \mathbf{h}_{(i)_x, (j,k)_q}|^2, \quad \begin{aligned} j &= 1, \dots, |C_q|, \\ i &= 1, \dots, |C_x|, \\ k &= 1, \dots, K, \\ m &= 1, \dots, K. \end{aligned} \quad (36)$$

It should be noticed that (34) is the best response obtained based on a coalition  $C_q$  with multiple BSs in which the downlink beam pattern  $\tilde{\mathbf{u}}_{j,k}$  is obtained using (26) and (27). However, for a singleton coalition, the downlink beam pattern should be determined using the method presented in algorithm 1. Based on the discussion above, the downlink beamforming for a given coalition structure is summarized in algorithm 3.

**Lemma 1:** For a given coalition structure  $S$ , the best response function (34) of the  $q$ th coalition is standard.

**Proof:** The proof is similar to the one presented in [15], hence not repeated here. ■

According to the fixed point theorem in [35], a standard function means that if the Nash Equilibrium (NE) of the SNSG for a given coalition structure exists, then the NE point is unique. The sufficient and necessary conditions for the existence and uniqueness of the NE for the non-cooperative game (9) has been given in [15]. In this paper, we only consider the case that the NE for the non-cooperative game exists. For a given coalition structure  $S$ , by rewriting the downlink SINRs for all users in all cells into the matrix form, the following equation can be obtained

$$\mathbf{G}' \check{\mathbf{p}}^* = \mathbf{1} \sigma^2 \quad (37)$$

where  $\mathbf{G}'$  is a matrix with the same structure of  $\mathbf{G}$  as defined in (13). The difference between  $\mathbf{G}'$  and  $\mathbf{G}$  is that they are obtained through different sets of beam patterns.

**Definition 1 (M-Matrix)** [37], [38]: A square matrix  $\mathbf{A}$  is said to be an M matrix if  $\mathbf{A}\mathbf{y} \geq \mathbf{0}$  implies  $\mathbf{y} \geq \mathbf{0}$  for all  $\mathbf{y}$ . A square matrix  $\mathbf{A}$  is a Z-matrix if all its off-diagonal elements are nonpositive.

To distinguish with the normal matrix inequality, we write

$$\begin{aligned} \mathbf{A} &\geq \mathbf{B} && \text{if } a_{i,j} \geq b_{i,j}, \forall i, j, \\ \mathbf{A} &> \mathbf{B} && \text{if } \mathbf{A} \geq \mathbf{B} \text{ and } \mathbf{A} \neq \mathbf{B}, \end{aligned}$$

where  $\mathbf{A}$  and  $\mathbf{B}$  are matrices with the same dimension;  $a_{i,j}$  and  $b_{i,j}$  are the  $(i, j)$ th elements of matrices  $\mathbf{A}$  and  $\mathbf{B}$ , respectively. Then, the following proposition holds:

**Proposition 1:** If the NE of the non-cooperative game (9) exists, the NE of the SNSG game for a given coalition structure  $S$ ,  $S \neq \Omega$  exists if there exists an inverse-positive matrix  $\mathbf{A}_1$ , satisfying  $\mathbf{A}_1 \leq \mathbf{G}'$ .

*Proof:* First, since matrix  $\mathbf{G}'$  is a square matrix with the same structure as  $\mathbf{G}$ , the matrix  $\mathbf{G}'$  is a Z-matrix. As proved in [15], if the NE of the non-cooperative game (9) exists,  $\mathbf{G}$  is an M-matrix.

Let  $\mathbf{A}_2$  be a Z-matrix satisfying  $\mathbf{A}_2 \geq \mathbf{G}$  and  $\mathbf{A}_2 \geq \mathbf{G}'$ . Since  $\mathbf{G}$  is an M-matrix,  $\mathbf{G}^{-1}$  exists and  $\mathbf{G}^{-1} > \mathbf{0}$ . In addition, since  $\mathbf{A}_2$  is a Z-matrix satisfying  $\mathbf{A}_2 \geq \mathbf{G}$ , according to [37], the inverse matrix  $\mathbf{A}_2^{-1}$  exists and  $\mathbf{G}^{-1} \geq \mathbf{A}_2^{-1} \geq \mathbf{0}$ . Hence,  $\mathbf{A}_2$  is an inverse-positive matrix.

If there exists an inverse-positive matrix  $\mathbf{A}_1$  satisfying  $\mathbf{A}_1 \leq \mathbf{G}'$ , then both  $\mathbf{A}_1$  and  $\mathbf{A}_2$  are inverse-positive and according to [38], they are also monotone. Since  $\mathbf{A}_1 \leq \mathbf{G}' \leq \mathbf{A}_2$ , then  $\mathbf{G}'$  is also monotone [36, Corollary 3.5], which means  $\mathbf{G}'^{-1}$  exists and  $\mathbf{G}'^{-1} > \mathbf{0}$ . Hence, equation (37) can be written as  $\check{\mathbf{p}}^* = \mathbf{G}'^{-1} \mathbf{1} \sigma^2 > \mathbf{0}$ , which means that there exists positive solutions  $\check{\mathbf{p}}^*$  for (37). Thus, the NE of the SNSG game exists. ■

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#### Algorithm 3 Coalitional Beamforming Algorithm

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- 1) Find the downlink beam pattern  $\tilde{\mathbf{u}}_{j,k}$  via Algorithm 1 and Algorithm 2 for singleton coalitions and coalitions with multiple BSs respectively.
  - 2) Substitute  $\tilde{\mathbf{u}}_{j,k}$  into (28b) to obtain equation (33).
  - 3) For a given  $\check{\eta}_q$ , determine  $\check{\mathbf{p}}_q$  using (33).
  - 4) Update  $\check{\eta}_q$  and repeat step 3 until the optimal  $\check{\mathbf{p}}_q^*$  is obtained.
  - 5) Calculate the optimal downlink beamformer using  $\mathbf{u}_{j,k}^* = \sqrt{p_{j,k}^*} \tilde{\mathbf{u}}_{j,k}$ .
- 

If coalition structure  $S$  is a partition with all singleton coalitions, algorithm 3 will reduce to algorithm 1; while if  $S$  is the grand coalition  $\Omega$ , algorithm 3 is equivalent to algorithm 2. For a given coalition structure, the best response strategy of the above SNSG game is obtained based on fixed sets of weighting factors for all coalitions in  $S$  with multiple BSs. This means that the change of weighting factors in a coalition will change the NE, which will be discussed in section IV.

## IV. COALITION FORMATION PROCESS

In this section, we focus on the coalition formation process. First, some definitions are given. The concept of *q-Deviation* has been proposed in [18] as a deviation rule that describes how a coalition structure transits to another coalition structure in the coalitional formation process. Here, we apply such concept to our problem and introduce the concept of  *$\alpha$ -Deviation*.

**Definition 2 ( $\alpha$ -Deviation):**  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$  represents the process of transiting the coalition structure  $S_n$  to the coalition structure  $S_{n+1}$  by merging coalitions in  $\Theta$  to a new coalition  $C_M = \bigcup \Theta$  with a given  $\alpha$ , where  $\Theta \subset S_n$ ;  $C_M \in S_{n+1}$  and  $S_{n+1} = S_n \setminus \Theta \cup C_M$ ;  $\alpha$  are the weighting factors in vector form for all BSs in  $C_M$  with  $\alpha \in \mathbb{R}_+^{|C_M|}$  and  $\|\alpha\|_1 = 1$ .

With the above definition, a coalition structure  $S_n$  can transit to coalition structure  $S_{n+1}$  by merging all coalitions in the set  $\Theta$  into one coalition. However, the new coalition  $C_M$  can be successfully formed only if BSs  $j \in \bigcup \Theta$  agree to such transition on the basis of a given decision rule. Several decision rules have been studied in [30] for both individual utility comparison and collective utility comparison. In our coalition formation problem, the individual utility based decision rule is applied by comparing utilities achieved in  $S_{n+1}$  and  $S_n$  for all BS  $j \in \bigcup \Theta$ . Both *Pareto Order* and *Majority Order* comparisons have been proposed in [30]. We apply both of these comparison rules to the coalition formation process.

As discussed in previous sections, in the existing works of multicell beamforming, beamformers for users of different BSs are coordinately designed without considering the cost of cooperation. However, in practice, such cooperation cost cannot be ignored. Hence, in our coalitional beamforming problem, we include the cost of cooperation for the formation of coalition. The cooperation cost of a BS is assumed to be linearly proportional to the size of the coalition it is attached to, i.e. the cooperation cost for the BS  $j$  in the coalition structure  $S$  is

$$\epsilon_j(S) = (|C| - 1)\epsilon, \quad j \in C \quad (39)$$

where  $\epsilon$  is the cost factor. For BSs that do not cooperate with other BSs,  $|C| = 1$ , hence zero cost.

In the coalitional beamforming, each BS intends to reduce its transmission power by cooperating with other BSs; hence, once a new coalition  $C_M$  is formed, the benefit for the BSs in  $C_M$  is the reduced transmission power. However, in practice, it cannot be guaranteed that all BSs will benefit from the deviation, especially when the cooperation cost is taken into consideration. Hence, the concept of *deviation gain* is introduced as the total benefit obtained through the deviation  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$  by each BS. We first define the *resource consumption* of the  $j$ th BS in the coalition structure  $S_n$  as

$$r_j(S_n) = p_j(S_n) + \epsilon_j(S_n), \quad \forall j. \quad (40)$$



Then, the *deviation gain* of BS  $j$  obtained by  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$  can be defined as

$$v_j(S_n \xrightarrow{\alpha, \Theta} S_{n+1}) = r_j(S_n) - r_j(S_{n+1}), \quad \forall j. \quad (41)$$

In addition, for deviation  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$ , it is assumed only BSs in  $\bigcup \Theta$  can decide whether to form coalition  $C_M$  and the rest of the BSs are not allowed to make decisions. Based on these definitions and rules, both strongly independent comparison and weakly independent comparison can be stated as follows:

*Strong Independence Comparison: Pareto Order*

$$\begin{aligned} S_{n+1} &\succ_P S_n \quad \text{iif} \\ v_j(S_n \xrightarrow{\alpha, \Theta} S_{n+1}) &\geq 0, \quad \forall j \in \bigcup \Theta, \text{ and} \\ \exists j \in \bigcup \Theta &\text{ satisfy } v_j(S_n \xrightarrow{\alpha, \Theta} S_{n+1}) > 0. \end{aligned} \quad (42)$$

In Pareto order comparison,  $S_n$  can transit to  $S_{n+1}$  only if the deviation gains of all BSs in  $\bigcup \Theta$  are nonnegative and at least one BS  $j \in \bigcup \Theta$  has positive deviation gain. Those BSs in  $\bigcup \Theta$  that have obtained negative deviation gains will decline to stay in the coalition  $C_M$  and the coalition structure  $S_{n+1}$  can not be achieved.

*Weak Independence Comparison: Majority Order*

$$\begin{aligned} S_{n+1} &\succ_M S_n \quad \text{iif} \\ | \{j | v_j(S_n \xrightarrow{\alpha, \Theta} S_{n+1}) > 0, \quad \forall j \in \bigcup \Theta \} | &> \\ | \{j | v_j(S_n \xrightarrow{\alpha, \Theta} S_{n+1}) < 0, \quad \forall j \in \bigcup \Theta \} |. \end{aligned} \quad (43)$$

Different to the Pareto order comparison, in majority order, if the majority of BSs in  $\bigcup \Theta$  have positive deviation gains of  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$ ,  $S_n$  is allowed to transit to  $S_{n+1}$ . Hence, in this way of comparison, no BS can unilaterally reject a coalition formation. In the following, we use  $\triangleright = \{\succ_P, \succ_M\}$  as the comparison strategy set that includes both of these comparison rules. Hence, it can be concluded that  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$  is reachable if and only if  $S_{n+1} \triangleright S_n$  holds.

#### A. COALITION FORMATION ALGORITHM

Based on the discussion above, a coalition structure can be reached through a coalition formation process on the basis of  $\alpha$ -Deviation. In this section, an algorithm for the coalition formation game is developed. As discussed in section III, for a given coalition structure, the beamformer design is an SNSG game, which means that for the deviation  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$ , BSs in  $\bigcup \Theta$  can only decide whether to stay in the newly formed coalition  $C_M$  after the coalition structure  $S_{n+1}$  is formed and the Nash equilibrium of the SNSG game with  $S_{n+1}$  is achieved. Hence, in our algorithm, a merge-regret formation strategy is adopted. The coalition formation is shown in Algorithm 4.

We assume the coalition formation process always starts from the non-cooperative game in which all coalitions are singletons. Each BS has a preset index and knows the indexes

#### Algorithm 4 Merge-Regret Based Coalition Formation Algorithm

- 1) **Input:**  $\Omega, b, \epsilon$
- 2) **Initialize:**  $n = 0, S_n, l = \min\{b, |S_n|\}$
- 3) **while**  $l \geq 2$
- 4) Each coalition generates  $b^*$   $l$ -combinations of  $S_n$  in lexicographical order;
- 5) **Initialize:**  $k = 1$
- 6)  $S_n$  temporarily transits to  $S_t$  by merging coalitions in the  $k$ th combination  $\Theta_k$  into a single temporary coalition  $C_t$ ;
- 7) Compute utility  $t_j(S_t)$  for all BSs  $j \in C_t$  with  $\alpha = \frac{1}{|C_t|} \mathbf{1}$  using Algorithm 3;
- 8) Each BS  $j \in C_t$  computes  $v_j(S_n \xrightarrow{\alpha, \Theta_k} S_t)$  using (40) and (41);
- 9) **if**  $S_t \triangleright S_n$  holds
- 10)  $n = n + 1$ ;
- 11) Update  $S_n = S_t$  and  $l = \min\{b, |S_n|\}$ ;
- 12) Number all coalitions in  $S_n$ ;
- 13) Go to step 4;
- 14) **elseif**  $k < b^*$
- 15) BSs in  $C_t$  split from  $C_t$  and re-form  $S_n$ ;
- 16)  $k = k + 1$ ;
- 17) Go to step 6;
- 18) **else**
- 19) BSs in  $C_t$  split from  $C_t$  and re-form  $S_n$ ;
- 20)  $l = l - 1$ ;
- 21) Go to step 4;
- 22) **Output:**  $S_n, \alpha$

of other BSs. It is assumed that BSs can communicate with each other and share coalition information without additional cost. For a coalition structure  $S_n$ , all coalitions are numbered in a specific way. Each coalition in  $S_n$  first generates a set of  $l$ -combinations with lexicographical order, where  $l = \min\{b, |S_n|\}$  and  $b$  is the maximum allowable size for merging [18]. Then, by using the same sequence of  $l$ -combinations, coalitions in the first  $l$ -combination merge into a temporary coalition  $C_t$  with a given  $\alpha = \frac{1}{|C_t|} \mathbf{1}$ , and the coalition structure  $S_n$  is temporarily transited to  $S_t$ . Then, BSs in  $C_t$  will decide whether  $C_t$  is valid. Based on algorithm 3, each BS in  $C_t$  will determine the best response beamforming strategy at the NE; then its transition gain can be calculated and sent to all other BSs in  $C_t$ . A decision can be made by all BSs in  $C_t$ . If  $S_t \triangleright S_n$  holds,  $C_t$  is valid and  $S_{n+1} = S_t$ ; else, BSs in  $C_t$  will split from the temporary coalition  $C_t$  and the original coalitions  $S_n$  will be reformed. Then, another temporary coalition with the next  $l$ -combination will be formed. The above process is repeated until a valid coalition structure is found. If for all  $l$ -combinations, no valid coalition structure  $S_{n+1}$  is found; this means  $l$  is too large for coalitions in  $S_n$  to transit to a new coalition structure via  $\alpha$ -Deviation. Then each coalition will generate a new set of  $l$ -combinations with lexicographical order by reducing  $l$

to  $l - 1$  and repeating the process of temporary coalition formation until  $l < 2$ . If no valid coalition structure  $S_l$  is found, algorithm stops and coalition structure  $S_n$  will be sustained; else,  $S_n$  is successfully transited to  $S_{n+1}$  and the coalition formation is continued.

In practice, the coalition formation process highly depends on the message exchange between BSs. In a coalition with multiple BSs, BSs cooperate with each other to design beamformers. In this process, the channel state information of users in the coalition needs to be exchanged through the backhaul network of all BSs in the coalition. In addition, once a coalition structure is reached, the newly formed coalitions need to be numbered for further merging process. Such numbering process requires BSs in different coalitions to exchange the information of members in each coalition. It should be noticed that for the beamformer design in a coalition, the channel state information of users needs to be constantly updated by each BS; hence, this process will cause an evident increase of power consumption. However, for the numbering process, the number of data bits passing through the backhaul network is very small, hence the overhead caused by such message exchange can be ignored.

### B. $\alpha$ -MODIFICATION ALGORITHM

As discussed in section III, in coalitional beamforming, beam patterns for users in a coalition depend only on the intra-coalition channels of the coalition and the weighting factors assigned to all BSs within the coalition. This means that by modifying weighting factors, beam patterns of all users in the coalition will be reshaped and the transmission power of each BS will be changed. In Algorithm 4, once a temporary coalition  $C_t$  is formed with a given  $\alpha = \frac{1}{|C_t|} \mathbf{1}$ , the coalition structure  $S_n$  is temporarily transited to  $S_t$ . If  $S_t \triangleright S_n$  holds, the formation of  $C_t$  is valid, then  $S_n$  will formally transit to  $S_{n+1}$ , where  $S_{n+1} = S_t$ ; else  $C_t$  will be split and the next temporary coalition will be formed. However, once a temporary coalition  $C_t$  is found invalid, by modifying the weighting factor vector  $\alpha$ , both the Nash equilibrium point of the SNSG game and the transmission power of BSs in  $C_t$  can be changed, which might lead  $S_t \triangleright S_n$  to hold and  $C_t$  to be valid.

The effect of this modification process can be seen in Figure 2, in which a network with two cells and two users in each cell is considered. As seen in Figure 2, if both the BSs individually design beamformers for their users in a competitive way, the Nash equilibrium can be obtained with transmission power of 0.45W and 0.8W for BS1 and BS2 respectively. When the two BSs coordinately design beamformers with  $\alpha = [0.4 \ 0.6]$ , the consumed power of BS 2 is reduced to 0.57W; however, the power consumed by BS1 is increased to 0.46W. This means that if the Pareto comparison is applied, even if the coordination cost is not taken into consideration, the two BSs will still decide not to join in a coalition. However, once  $\alpha$  is modified to  $[0.9 \ 0.1]$ , the power consumption of both the BSs has been reduced as compared to the competitive design. This will lead to a positive

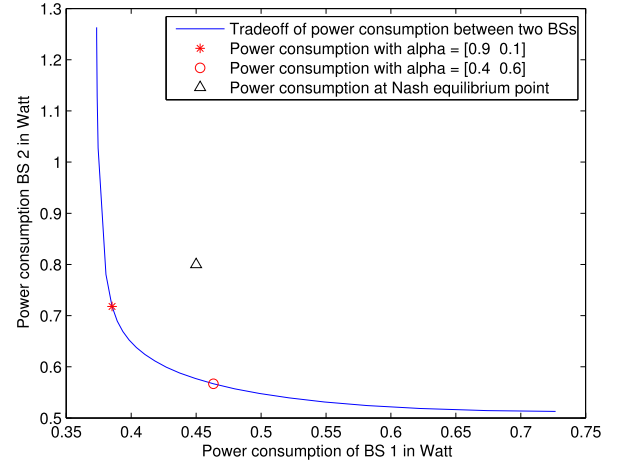


FIGURE 2. Power consumption of individual BSs for different beamforming design methods.

decision for the coalition formation. Hence, for the proposed Algorithm 4, by employing the  $\alpha$ -Modification scheme, there is a better chance to successfully deviate to a new coalition structure. To formulate the  $\alpha$ -Modification algorithm, we first define the *deviation gain ratio* of  $\alpha$ -Deviation as

$$\beta_j = \frac{v_j(S_n \xrightarrow{\alpha, \Theta} S_{n+1})}{r_j(S_n)}, \quad \forall j. \quad (44)$$

Then, if  $S_t \triangleright S_n$  does not hold,  $\alpha$  can be modified through the following equations:

$$\alpha_j = \alpha_j + \zeta, \quad \forall j \in \Omega_1, \text{ for Pareto order}; \\ \alpha_j = \alpha_j + \zeta, \quad \forall j \in \Omega_3, \text{ for Majority order}, \quad (45)$$

$$\alpha_j = \alpha_j - \zeta, \quad \forall j \in \Omega_2, \text{ for Pareto order}; \\ \alpha_j = \alpha_j - \zeta, \quad \forall j \in \Omega_4, \text{ for Majority order}, \quad (46)$$

where  $\zeta$  is the step size for updating.

For Pareto comparison,  $\Omega_1 = \{j | v_j(S_n \xrightarrow{\alpha, \Theta} S_t) < 0, \forall j \in C_t\}$  is the set of BSs with  $\beta_j < 0$  while  $\Omega_2 \subset C_t$  is a set of  $|\Omega_1|$  BSs with  $|\Omega_1|$  largest  $\beta$  values. It is obvious that  $\alpha$  can be modified only if  $\Omega_1$  satisfies  $|\Omega_1| < |\{j | v_j(S_n \xrightarrow{\alpha, \Theta} S_t) > 0, \forall j \in C_t\}|$ . However, in practice, with a very large  $|\Omega_1|$ , even if the above condition is satisfied, it will be still difficult to achieve  $S_t \triangleright_P S_n$  with the modified  $\alpha$  due to the limitation of flexibility for those BSs in  $\Omega_2$ . Hence, an upper bound  $\mu$  is introduced so that the Pareto comparison based  $\alpha$ -Modification is applicable only if  $|\Omega_1| \leq \mu < |\{j | v_j(S_n \xrightarrow{\alpha, \Theta} S_t) > 0, \forall j \in C_t\}|$ .

Different to the Pareto case, for majority comparison,  $\alpha_j$  of BSs in  $\Omega_3$  and  $\Omega_4$  will be modified, where  $\Omega_3$  is the set of  $N_m$  BSs with smallest  $|\beta_j|$  values and  $\beta_j < 0$ ; while  $\Omega_4 \subset C_t$  is a set of  $|\Omega_2|$  BSs with largest  $\beta$  values.  $N_m$  is the minimum number of BSs that need to improve  $\beta_j$  values and is obtained as

$$N_m = \lceil \frac{N_d + 1}{2} \rceil,$$

where  $\lceil \cdot \rceil$  is the ceiling function defined as  $\lceil x \rceil = \min\{y \in \mathbb{Z} \mid x \leq y\}$  and

$$N_d = |\{j \mid v_j(S_n \xrightarrow{\alpha, \Theta} S_t) < 0, \forall j \in C_t\}| \\ - |\{j \mid v_j(S_n \xrightarrow{\alpha, \Theta} S_t) > 0, \forall j \in C_t\}|. \quad (47)$$

Similar to the Pareto case, the upper bound  $\mu$  has been introduced so that the majority comparison based  $\alpha$ -Modification is applicable only if  $N_m \leq \mu < |\{j \mid v_j(S_n \xrightarrow{\alpha, \Theta} S_t) > 0, \forall j \in C_t\}|$ . Then, by introducing the maximum number of modifications  $N_M$  and integrating the Pareto case and the majority case, the  $\alpha$ -Modification algorithm is summarized in Algorithm 5.

#### Algorithm 5 $\alpha$ -Modification Algorithm

- 1) **Input:**  $S_t, C_t, \zeta, \mu, N_M, \{r_j(S_n), \forall j \in C_t\}$
- 2) **Initialize**  $\alpha^{(m)} = \frac{1}{|C_t|} \mathbf{1}, m = 0$
- 3) Compute utility  $t_j^{(m)}(S_t)$  for all  $j \in C_t$  with  $\alpha^{(m)}$  using Algorithm 3;
- 4) Each BS  $j \in C_t$  computes  $v_j^{(m)}(S_n \xrightarrow{\alpha, \Theta} S_t)$  and  $\beta_j^{(m)}$  using (40), (41), (44) and  $r_j(S_n)$ ;
- 5) **if**  $S_t \triangleright S_n$  holds
- 6)  $n = n + 1$ ;
- 7)  $S_n = S_t$ ;
- 8) Go to step 17;
- 9) **elseif**  $|\Omega_1| > \mu$  (or  $|\Omega_3| > \mu$ )
- 10) Go to step 17;
- 11) **elseif**  $m < N_M$
- 12) Update  $\alpha^{(m)}$  using (45) and (46);
- 13)  $m = m + 1$ ;
- 14) Go to Step 3;
- 15) **else**
- 16) Go to step 17;
- 17) **Output:**  $S_n, \alpha^{(m)}$

### C. STABLE COALITION STRUCTURES

In the coalition formation game, a main concern is that whether the output coalition structure is stable. To analyze the stability of the coalition structures obtained by the proposed algorithm, the following definition is given.

**Definition 3:**  $S_{n+1}$   $\alpha$ - $b$  dominates  $S_n$ , if there exists a combination  $\Theta \subset S_n, |\Theta| \leq b$ , with a given  $\alpha \in \mathbb{R}_+^{|\cup \Theta|}$  such that  $S_n \xrightarrow{\alpha, \Theta} S_{n+1}$ , and  $S_{n+1} \triangleright S_n$ . The  $\alpha$ - $b$  dominance can be written as  $S_{n+1} \gg^{\alpha-b} S_n$ .

On the basis of the above definitions, the solution of the coalition formation game can be obtained by introducing the concept of *Coalition Structure Stable Set* proposed in [31]. By defining the coalition formation game proposed in Algorithm 4 as  $(\mathcal{P}, \gg)$ , the Coalition Structure Stable Set can be defined as follows:

**Definition 4 (Coalition Structure Stable Set):** The set of coalition structures  $\mathcal{R} \subset \mathcal{P}$  is a coalition structure stable set of  $(\mathcal{P}, \gg)$  only if the following conditions are satisfied [31]:

**TABLE 1.** Some possible coalition structures for a multicell network with seven cells.

	Coalition structure
$S_1$	$\{\{1, 2\}, \{3, 5\}, \{4, 6\}, \{7\}\}$
$S_2$	$\{\{1, 2, 7\}, \{3, 5\}, \{4, 6\}\}$
$S_3$	$\{\{1, 2, 5\}, \{3, 7\}, \{4, 6\}\}$
$S_4$	$\{\{1, 4, 5\}, \{2, 6\}, \{3\}, \{7\}\}$
$S_5$	$\{\{1, 4, 5\}, \{2, 6, 7\}, \{3\}\}$

- \*  $\mathcal{R}$  is internally stable for  $(\mathcal{P}, \gg)$  if there does not exist  $S, S' \in \mathcal{R}$  such that  $S \gg^{\alpha-b} S'$ ;
- \*  $\mathcal{R}$  is externally stable for  $(\mathcal{P}, \gg)$  if for all  $S \in \mathcal{P}/\mathcal{R}$ , there exists  $S' \in \mathcal{R}$  such that  $S' \gg^{\alpha-b} S$ ;
- \*  $\mathcal{R}$  is a coalition structure stable set for  $(\mathcal{P}, \gg)$  if it is both internally and externally stable.

For the proposed coalition formation algorithm with a certain comparison rule (Pareto or majority), it is a sequential process in which coalitions can only merge into a larger size coalition. Such characteristic could guarantee the formation process always reach some stable points. This can be directly proved by considering an output coalition structure  $S_o$  of Algorithm 4. We first assume  $S_o$  is in the coalition structure stable set  $\mathcal{R}$  and a coalition structure  $S'_o$  can be found in  $\mathcal{R}$  that has  $S'_o \gg^{\alpha-b} S_o$ ; this means that  $S_o$  can further transit to  $S'_o$  via Algorithm 4 and  $S_o$  is not the output of  $(\mathcal{P}, \gg)$ , which contradicts to the assumption. In addition, we can assume that  $S_o$  is outside the coalition structure stable set  $\mathcal{R}$  that  $S_o \in \mathcal{P}/\mathcal{R}$ . Then, according to external stability, there should be a coalition structure  $S'_o$  in  $\mathcal{R}$  that satisfies  $S'_o \gg^{\alpha-b} S_o$ , which also contradicts to the assumption. Hence, the output coalition structures from Algorithm 4 must be in the coalition structure stable set.

**Proposition 2:** The coalition formation game  $(\mathcal{P}, \gg)$  can reach a unique coalition structure, if and only if the following conditions are satisfied:

- A numbering strategy is adopted to all coalition formations;
- If the  $\alpha$  is allowed to be modified, a given  $\alpha$ -Modification algorithm must be applied to all of the coalition formations;

**Proof:** This is a direct consequence from Definition 1 and Algorithm 4. If the  $\alpha$ -Modification algorithm is not employed, if there is a fixed numbering strategy only, the sequence of the coalition formation process is unique and the unique output is guaranteed.

Once the  $\alpha$ -Modification algorithm is employed, if for different coalition formation processes, different modification schemes are allowed, the sequence of the coalition formation process may be altered. Hence, to guarantee the uniqueness of the output, it is necessary to ensure that the same  $\alpha$ -Modification algorithm is employed to all of the coalition formations. ■

**TABLE 2.** Probability of performance improvement for different coalition transit process and target SINRs.

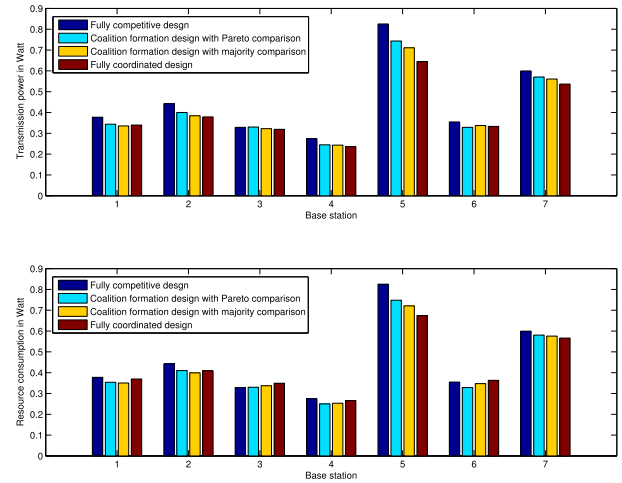
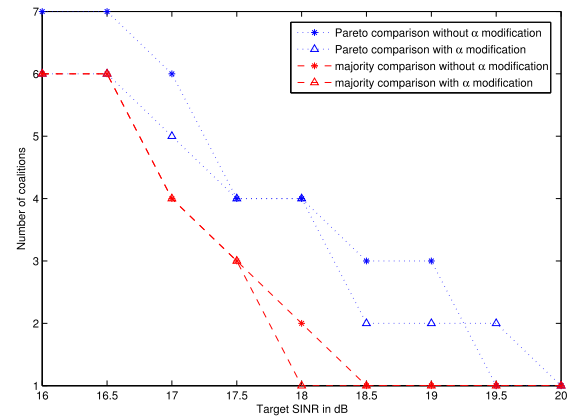
Coalition deviation	Probability of power reduction	
	Target SINR = 17dB	Target SINR = 18dB
$S_1 \xrightarrow{\alpha, \Theta} S_2$	98.21%	97.02%
$S_4 \xrightarrow{\alpha, \Theta} S_5$	98.27%	97.22%

## V. NUMERICAL RESULTS

Some numerical results for the proposed coalition formation algorithms are presented. We consider a multicell network with seven cells each serving two users. It is assumed that each BS employs six antennas while each user is equipped with a single antenna. To simplify the simulation setting, the target SINRs for all users in all of the cells are set to an identical value. The noise variance  $\sigma^2$  at the receiver of all users is set to 0.01W. The channel coefficients have been generated using zero mean complex Gaussian random variables. A distance dependent path loss model with path loss exponent of 3 is introduced to calculate channel gains for both inter-cell and intra-cell channels. The distance between a BS and its users is set to 0.9 kilometers for all BSs while the distance between any two neighboring BSs is set to six kilometers. It is assumed that the coalition formation game always starts from a non-cooperative game setting. Once a new coalition structure is reached, a postpositional numbering strategy is used so that the newly formed coalition is always numbered as the last coalition while all other coalitions are numbered in the ordinary way. We assume that the cooperation cost is characterized by the same dimension as power hence the cooperation cost factor  $\epsilon$  has a unit of Watt [36].

Before demonstrating the benefits of the proposed formation algorithm, we wish to consider the cases as in Table 1 to show merger of coalitions can potentially reduce the transmission power most of the times. As an example, as seen in Table 1, for the coalition structure transit  $S_1 \rightarrow S_2$ , the coalition  $\{7\}$  is forced to merge with the coalition  $\{1, 2\}$ . Out of 10000 various channel realizations, we observed 98.21% of the time, this random merge has reduced the total power consumption of all seven BSs as shown in Table 2. The table also indicated the percentage of power reduction for a different transit  $S_4 \rightarrow S_5$ . This shows the advantage of merging coalitions even in an arbitrary manner. However, the deterministic approach as proposed in this paper provides even better performance.

Figure 3 compares the performance of the proposed coalition formation algorithm with that of the fully competitive beamforming discussed in [15] and the fully coordinated beamformer design algorithm developed in [4]. In this case, target SINRs of all users are set to 18.5dB while the cooperation cost factor  $\epsilon$  is set to 0.005W. We observed that the fully coordinated design has the advantage of reducing the power consumption for individual BSs most of the time. For both the Pareto comparison and the majority comparison, the proposed algorithm resulted into lower power consumption

**FIGURE 3.** Transmission power and resource consumption of individual BSs for various beamforming design methods.**FIGURE 4.** Effect of  $\alpha$ -Modification algorithm on the number of coalitions.

for the individual BSs as compared to a fully competitive design. However, when the cooperation cost is included, BSs may not always benefit from cooperation. Hence, as shown in Figure 3, the proposed algorithm has a better performance in terms of resource consumption than the fully competitive design and fully coordinated design most of the time. Hence, when the cooperation cost is included, the proposed algorithm improves the performance of the network.

We then investigated the effect of  $\alpha$ -Modification algorithm on the proposed coalition formation algorithm. As seen in Figure 4,  $\alpha$ -Modification is very sensitive to the Pareto comparison, however for most of the cases,  $\alpha$ -Modification does not change the number coalitions significantly for



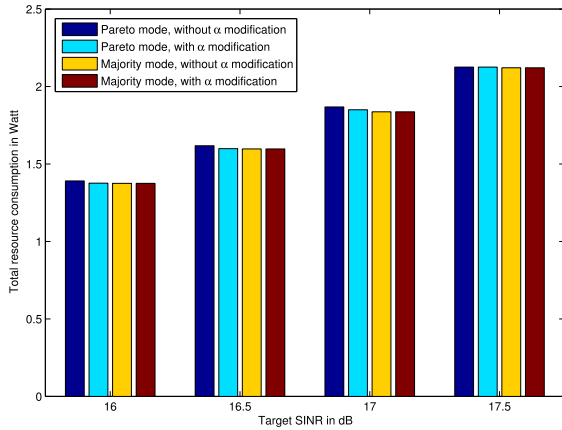


FIGURE 5. Effect of  $\alpha$ -Modification algorithm on the total transmission power.

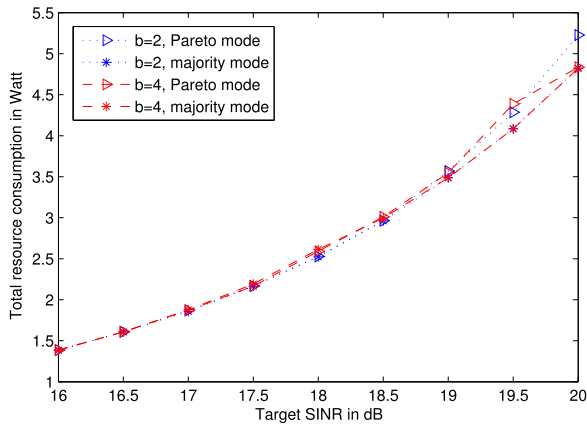


FIGURE 6. Total resource consumption versus various SINR targets.

majority comparison mode. Lower number of coalitions is likely to provide higher saving in the transmission power. Therefore, we notice that the majority comparison performs better than the Pareto comparison for most of the SINR targets. The effect of these four schemes on the total resource consumption is shown in Figure 5. As seen, all four schemes consume almost the same amount of resources, however, a closer look reveals that the majority comparison based algorithm provides more saving in terms of resource consumption.

Figure 6 compares the total resource consumption of the output obtained with different  $b$  values. As seen, both cases of  $b = 2$  and  $b = 4$  have resulted almost identical total resource consumption. Hence, the value of  $b$  has limited effect on the performance of the coalition formation process in terms of resource consumption. However, in practice, in addition to resource consumption, other factors such as the formation speed should be considered. Figure 7 shows the performance in terms of the number of temporary coalitions formed, for different  $b$  values. Larger the number of temporary coalitions more the time it takes for final coalition formation. As the SINR target increases, the number of temporary coalitions decreases and converges to almost the same value for both  $b = 2$  and  $b = 4$ . However, when the target SINR is small,

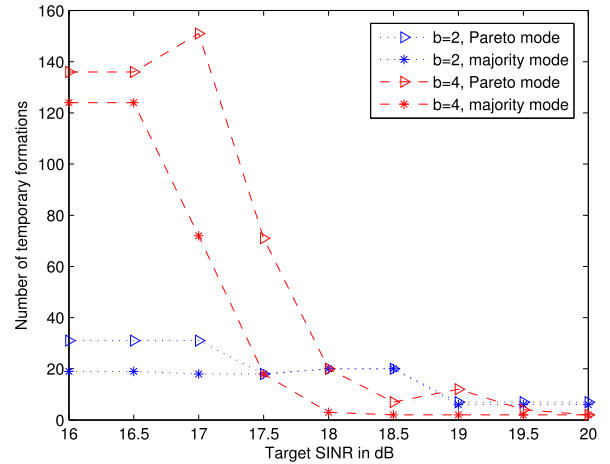


FIGURE 7. Number of temporary formations versus various SINR targets.

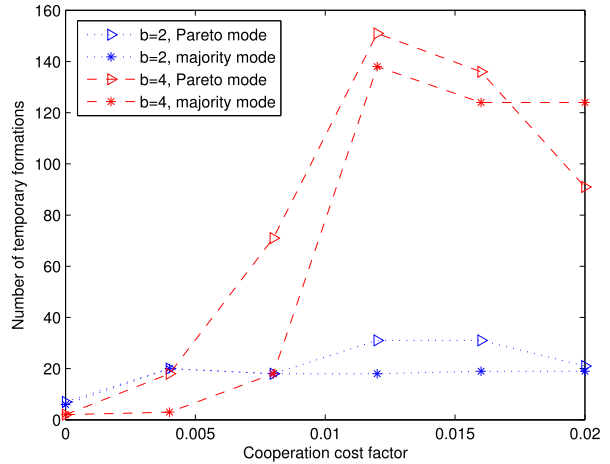


FIGURE 8. Number of temporary formations versus various cooperation cost factors.

the number of temporary coalitions is considerably low for  $b = 2$ . Hence, it is a good practice to choose a lower  $b$  value.

So far, we have analyzed the key parameters of the proposed algorithm and their effect on the coalition formation process. However, all of these simulations are based on the assumption that the cooperation cost factor is set to 0.005W. Now, we investigate the effect of the cooperation cost on the coalition formation process. As seen in Figure 8, once again a lower  $b$  value provides a better performance almost for all values of the cooperation cost.

## VI. CONCLUSION

We have proposed a multicell multiuser downlink beamforming technique using coalitional games. Due to the benefits of coordination, every BS has incentive to cooperate with other BSs via forming coalitions. However, when the associated cost is included, cooperation may not be preferred by all the BSs since any benefits in terms of reduction of transmission power may be overwhelmed by the cost of cooperation. We first considered the beamformer design for a given coalition structure and illustrated the process of finding downlink beamformers for all users. Then, we studied the

coalition formation game and proposed a merge-regret based sequential coalition formation algorithm. We have shown that the output of the proposed algorithm is in a coalition structure stable set. With certain constraints, the proposed algorithm can produce a unique stable coalition structure. The simulation results have shown that the majority mode can always provide a better performance. Generally, it is better to choose a smaller  $b$  value to accelerate the coalition formation process. As part of the proposed coalition formation algorithm, the  $\alpha$ -Modification algorithm has been developed for a range of comparison rules. The simulation results demonstrated that when the Pareto mode is used, the employment of  $\alpha$ -Modification algorithm can help to reduce the number of coalitions at the output, and decrease the total power consumption.

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