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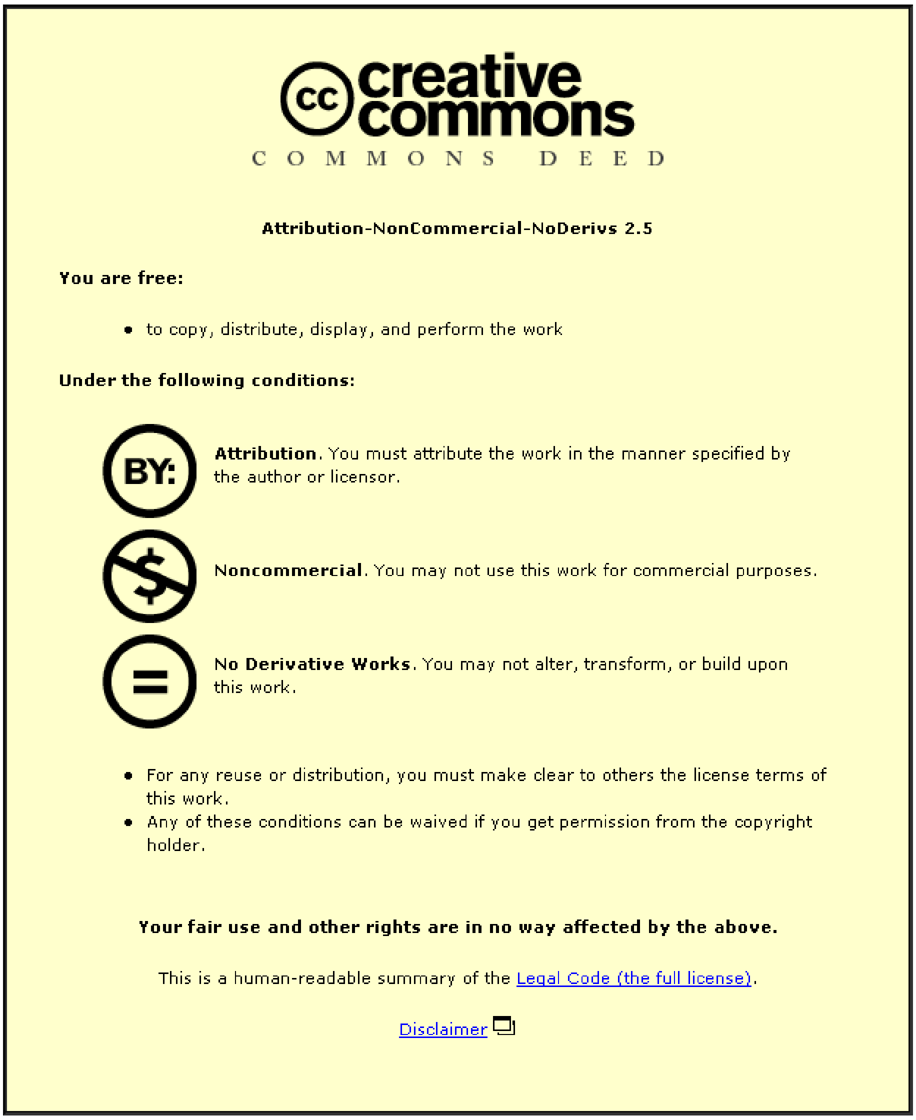

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THE APPLICATION OF FINITE ELEMENT TECHNIQUE TO LUBRICATION PROBLEMS

A THESIS
SUBMITTED TO LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY
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Abstract

The work presented in this thesis is concerned with the application of the finite element technique to solve general lubrication problems. Incompressible isothermal condition has been considered as a first step towards the lubricant film investigation.

Fluid finite elements of triangular and rectangular planform have then been developed and incorporated in a computer program especially developed for a finite element analysis. Although the elements are primarily two dimensional in the film region, it is possible to allow for a variation in the thickness of an oil film within an element. Other parameters affecting the lubricant such as shear forces, body forces, inertia forces, squeezing velocities and diffusion velocities can also be varied at each node. These elements have been extensively tested by considering standard lubrication problems such as squeesing pad, slider bearing and step bearing and the results obtained are shown to be an excellent agreement with those derived from theoretical solutions.

The analysis has then been extended to study the behaviour of the oil film between rotating annular discs where it is known that grooving on the disc affects the pressure distribution. The results indicate that grooving reduces the disc engaging time and that the engaging speed determined by surface velocity in the z direction is shown to be higher in radially grooved discs than in spirally grooved discs at a given squeezing pressure.

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Nomenclature

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A	area	5
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B	body force	8
B_x, B_y	body forces in x, y direction	19
B_{mx}, B_{my}	averaged body forces in x, y direction	23
C_f	centrifugal force per unit volume	25
C_{fm}	centrifugal force averaged along z direction	26
f_i	interpolation function at node i	6
F_x, F_y	friction forces in x, y direction	29
F_T	total friction force	56
h	film thickness	5
$\dot{H}$	thickness of porous region	29
I	functional	5
$\hat{i}$, $\hat{j}$	unit vectors in x, y direction	22
$[J]$	the Jacobian matrix	42
$\|J\|$	determinant of [J]	42
K	fluidity matrix	35
L	load carrying capacity	29
M	interpolation function	49
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n	unit outward normal	5
P_o, P_1	pressures on boundary	5
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r	polar coordinate	25
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		Page number
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x, y, z	coordinates	5
α, β, γ	multipliers	40
ε	arbitrary parameter	32
$\eta(x)$	continuous function	32
θ	angle	26
μ	fluid viscosity	5
ρ	density	8
τ_{xy}	shear stress on the surface normal to y axis in x direction	19
Φ	permeability	11
ω	angular velocity	25
∇	$\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}$	5

1. Introduction

The analysis of lubrication problems with a view to reduction of friction losses occurring between relatively moving surfaces have occupied researcher's attention for a long time. These problems can be recognized in almost all mechanisms which have moving parts and as such their investigation and study is important. These investigations have assumed added significance lately due to the energy crisis since the reduction of friction losses contributes greatly to energy savings.

Another aspect of lubrication analysis is the determination of ways of obtaining higher friction forces and friction torques between lubricated surfaces in wet clutches and brake discs. In this case friction has a positive influence in the engaging actions of the surfaces during operation of wet friction devices.

Behaviour of a lubricant film can be explained by the basic theory of Reynolds equation with ideal assumptions such as the lubricant is incompressible, isothermal and behaves as a Newtonian fluid and that the film thickness is known. Some simple standard problems can be solved theoretically, however, the solution of Reynolds equation for the analysis of general lubrication problems requires the use of numerical procedures such as the finite difference method and the finite element method. Furthermore, in practice, most applications encounter irregular configurations in geometry, arbitrary boundary conditions and varying film properties. Most of these difficulties can be overcome by the use of the finite element technique.

The work presented in this thesis is concerned with the applicaion of the finite element technique to lubrication problems including friction discs. Incompressible isothermal condition has been considered as a first step towards the lubricant film investigation. The starting point is the solution of generalized Reynolds equation as stated by Huebner (11) which includes various effects such as shear force, body force, squeeze action and diffusion effect. The inertia effect and detailed consideration of the diffusion effect have been additionally included in order that rotating disc problems can be investigated.

Fluid finite elements of both triangular and rectangular planform have been developed and have been incorporated in a computer program especially formulated for the presented analysis. Although the elements are primarily two dimensional in the film region analysis, thickness of the oil film can be varied within an element. Other parameters affecting the lubricant such as shear, body and inertia forces, squeeze and diffusion velocities can also be varied at each node.

These elements have been extensively tested by their application to classical lubrication problems such as squeezing square pad, slider and step bearings and the results derived have been compared with theoretical solutions. The computed results are shown to give good agreement with the theoretical solutions in all the cases that have been studied.

The analysis has been extended to the study of the behaviour of the oil film between rotating circular discs with flat surfaces as such results will be applicable to oil immersed brakes and clutches.

The effect of inertia forces has also been considered in this analysis.

Another aspect of wet type clutches which is of interest is the effect of oil grooves on the disc surface on the behaviour of the clutches. Experimental investigations indicate that grooves greatly affect the dynamic coefficient of friction. However, only a few analytical studies have been carried out in this field so far. In the present work radial and spiral groove arrangements have been investigated to determine the pressure distributions of wet clutches.

2. Literature Survey

2.1 General Lubrication Analysis Using Finite Element Technique

Owing to the development of computer technology, numerical analyses of fluid film lubrication problems have rapidly progressed. Recently the finite element technique has been successfully applied for the solution of Reynolds equation based on classical variational principles. This has come about partly by virtue of ease of the application to cater for arbitrary boundary conditions and partly, by the ease of handling complex geometric configurations. Variations in field properties such as changes of film thickness that occur in bearing pockets and oil grooves, and also the effects of external pressures can all be investigated by this type of the analysis.

Approximate solutions of incompressible isothermal lubrication problems using the variational principle was obtained by Hays(1). His analysis is based on the following assumptions:

1. Film thickness is small compared to other system dimensions.
2. Viscosity is assumed to be constant.
3. Reynolds number is assumed to be small consequently the flow is assumed to be laminar.

Based on these assumptions, the following theorem was derived "Of all the possible fluid motions within a region which are compatible with the equation of continuity and the prescribed boundary conditions, that motion which minimized the excess of the energy dissipation over twice the rate at which the work is being done by the specified surface tractions on the boundary, will be the true steady-state motion". Using this theorem, Hays presented the following function to be minimized:

$$I = \int_A \left\{ \frac{h^3}{12\mu} \nabla p \nabla p + \frac{\mu U U}{h} \right\} dA + 2 \int_S p\theta n \, ds$$

Subject to the following boundary conditions:

1. A specified constant pressure along the boundary or
2. If the pressure is not constant then the volume flow normal to the boundary must be zero and the normal pressure gradient across the boundary must satisfy the equation,

$$\nabla p = \frac{6\mu}{h^2} U \qquad \text{on } S$$

Subsequently the pressure distribution was assumed to be expressed in the form of an infinite series given by the equation

$$p = p_o + \frac{p_1 - p_o}{\pi} y + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (\sin(ny))(a_{nm} \cdot \sin(mx) + b_{nm} \cdot \cos(nx))$$

By determining the coefficients a_{nm}, b_{nm} which minimize the function I, Reynolds equation was solved. This approach was used to analyze the hydrodynamic behaviour of the finite width journal bearing and many valuable results were presented to determine such quantities as load capacity, moment, coefficient of friction, side flow, minimum film height and power loss. Hays further studied the characteristics of a finite width journal bearing under a cyclic sinusoidal load (2). Rectangular pad problems with flat and curved surfaces were also investigated by the same author (3), and results showed that the effects of curvature only become important as the film thickness decreases,and this may reduce the squeeze film capacity of the plate by several orders of magnitude.

Moore (4) experimentally verified Hays theoretical results for a flat rectangular pad and has developed an approximate analysis of the pressure distribution on a pad bearing consisting of two inclined plates. However, both studies assume that the boundary flow is zero.

An approximate hydrodynamic analysis based on variational principles has been presented by Tipei (5), though, again the limitation of the flow boundary condition is also evident in this work.

This restriction on boundary conditions has since been overcome by the application of finite element techniques. Reddi (6) applied the finite element technique to solve incompressible lubrication problems, and has investigated the effects of squeeze and shear action forces within the fluid. He stated the variational principle for an incompressible fluid to include the non-zero flow boundary condition. Advantage was then taken of the important assumption made in the application of the finite element technique, that the state of the field variable within an element may be described by values of the unknown variable at a finite number of nodes located on the boundary of an element. This implies that in lubrication problems, the pressure distribution in an element may be expressed in terms of unknown pressures at the nodes in the element. Reddi has used the triangular element for this analysis and assumed a pressure distribution of the form given below

$$p(x,y) = \langle f_i(x,y) \rangle \{a_j\}$$

where $f_i(x,y)$ are interpolation functions and a_j are related to the unknown pressures at the element nodes. The interpolation function was chosen such that p(x,y) satisfies the following conditions:

1. f_i is continuous within an element;
2. pressure along any inter-element boundary should be specified completely by nodal pressures on that boundary;
3. constant pressure state is included;
4. uniform pressure gradient is included;
5. a linear transformation of the coordinate system must not change the pressure representation within the element.

This method was used to investigate the squeeze pad, slider bearing and step bearing problems and the results were compared with other theoretical solutions. Simple triangular elements and composite quadrilateral elements were used, and no special treatment was required to account for the sudden change in film thickness. These predicted results were found to be in good agreement with those obtained from classical analyses. Reddi then extended the finite element technique to analyze the compressible fluid lubrication problems using quadrilateral elements (7). Calculations of the fluid matrices were achieved by means of Gaussian integration.

Wada, Hayashi and Migita (8),(9), have derived the solutions of infinite and finite width bearing problems using an assumed pressure distribution which is expressed by a high order algebraic equation. They have examined the effects of taking a number of coefficients in the pressure function for rotating journal bearing problems and have cuncluded that accurate results can be obtained by using only the first few elements of the high order algebraic equation.

Allan (10) has examined the characteristics of journal bearing with externally pressured oil pockets, using an iterative method to to determine the oil pressures and the flows through oil pockets. In this work it is shown that the finite element technique is a powerful and flexible method capable of handling any bearing surface and that the use of simple triangular elements is adequate to illustrate the approach and provide useful results. Allan has also presented full details of the computer program required to solve journal bearing problems.

Recent works have been concentrated mainly on the generalization of the finite element technique for the analysis of lubrication problems. A detailed explanation of solution procedure of a triangular squeezing pad problem has been developed by Booker and Huebner (11), to handle effects such as shear stress, squeeze action, body force, lubricant expansion due to heating and diffusion through the pad. The functional which must be minimized for the incompressible isothermal lubrication problems is given by the equation

$$I = \int_A \left[\left(\frac{\rho h^3}{24\mu} \nabla p - \rho h \bar{U} - \frac{\rho^2 h^3}{12\mu} \bar{B} \right) \nabla p + \left(\rho \frac{\partial h}{\partial t} + h \frac{\partial p}{\partial t} + \rho v_d \right) \right] dA + \int_{S_q} (\rho h \bar{u} n)\, p \, ds$$

Despite various flow action effects being taken into account in this analysis the consideration of the diffusion effect is incomplete and also the inertia forces are assumed to be negligible when compared with shear and pressure forces.

Huebner (12) has further extended this method to analyze thermo-hydrodynamic lubrication problems. The weighted residuals associated with Galerkin's criterion is used to solve the thermal energy equation which describes the temperature distribution in the lubricant film. An iterative procedure is applied to obtain self-consistent pressure and temperature distribution results. The importance of including thermal effects in the hydrodynamic analysis has been discussed by Huebner who has concluded that the use of an isothermal analysis may lead to overestimates of calculated values of the bearing load capacity and coefficient of friction.

Stafford (13) has carried out a modification of the method and has contributed to the existing suit of finite element sub-routines known as PAFEC (14). Isoparametric elements are used and the integrations of the system matrices are achieved by the Gaussian method. The effects of body force, inertia and diffusion are neglected in his analysis which was also extended to include the effect of structual distortion on the film by using an iterative approach.

The effect of pad deformation on bearing performance has been studied by Jain, Sinhasan and Singh (15) using a three dimensional finite element technique. The pressure field in the fluid film region is determined by the simultaneous solution of Reynolds equation and the relevant elasticity equations using an iterative approach.

Allaire, Nicholas and Gunter (16) have developed a systematic matrix approach using finite elements to minimize the bandwidth of the resulting algebraic equations. Also they have established an optimum method for dividing the bearing area into elements. Their error analysis indicates that the division of the bearing surface into elements is of great importance and that the alignment of the diagonal sides of triangular elements to the expected direction of the pressure gradient provides more accurate results for the same number of elements used. The significance of variable grid spacing was also pointed out by the authors in order to reduce errors in the results.

Das and Dancer (17) have presented an analysis of the oil flow and its frictional behaviour in diesel engine bearings. Various factors influencing the bearing performance have been investigated by the finite element method based on steady state lubrication theory. It is shown that the coefficient of friction and oil flow are dependent upon the basic geometric proportions such as the length, diameter, clearance etc. of the bearing. Results also indicate that engine load and manufacturing variations such as taper and misalignment have little influence on bearing performance.

An analysis of the non-Newtonian fluid effects in a finite width journal bearing using the finite element technique has been developed by Tayal, Shinhasan and Singh (18). The non-linear behaviour of the fluid was investigated by modifying the viscosity term at each stage of an iteration process.

2.2 Porous Region Analysis

Porous materials are widely used as bearing surfaces and clutch disc facings, and many analyses have been made to predict the characteristics of the porous region. It is assumed that the oil film region satisfies the Reynolds equation and the flow in the porous region satifies the Laplace or Poisson equation in which are substituted Darcy's law of porous media flow. The problem is solved by coupling these equations with the associated boundary conditions. Darcy's law gives the following expressions for the flow in the porous region

$$u = - \frac{\Phi}{\mu} \frac{\partial p}{\partial x}$$

$$v = - \frac{\Phi}{\mu} \frac{\partial p}{\partial y}$$

$$w = - \frac{\Phi}{\mu} \frac{\partial p}{\partial z}$$

and if these equations are substituted in the continuity equation in the porous region, the governing Laplace or Poisson equation is obtained. Various assumptions and simplifications have been made by many investigators to solve these equations.

Wu (19) has investigated analytically the squeeze film behaviour of a porous annular disc approaching a plain disc of the same dimensions by the use of Fourier-Bessel expansions. The assumptions made in the analysis were as follows:

1. The porous facing has uniform thickness and permeability.
2. The fluid is incompressible and has constant properties.
3. The no-slip condition is applicable to all liquid-solid interfaces. Results were presented giving the pressure distribution, load-carrying capacity and film thickness as the plates come into contact.

In this problem only part of the fluid will be squeezed out and the remaining part will flow out through the porous medium.

The combined effect of these two actions will reduce the pressure in the fluid film compared with that reached in a non-porous medium. Wu also concludes that porous effects are influenced by not only the permeability of material but also by the film thickness. Later Wu also studied the effect of including rotational inertia in the analysis (20). The effects of rotational inertia are shown to further reduce the film pressure and load carrying capacity and also to shorten the time required for reducing the film thickness. Wu has further applied the same approach as used for discs to study rectangular squeeze pad problems (21).

Disc problems have also been analyzed by Ting (22) who used an expression for the average pressure through the thickness of porous region and assumed that the mean pressures in the film and porous regions are equal at any radius for small values of thickness. Good agreement between the results evaluated by the simplified method and the Fourier-Bessel solutions are reported.

Another simplification applied to the integration of Laplace's equation has been made by Prakash and Vij (23) to examine squeeze film effects in circular, annular, elliptic and rectangular porous plates.

The squeeze film behaviour in an inclined porous slider bearing has been investigated by Bhat and Patel (24) and the cause of a porous composite slider bearing by Puri and Patel (25). The analysis adopted by Prakash et al. has been extended to these studies. Results show that the response time for a composite slider bearing is greater than that for an inclined slider bearing.

In most disc problems the surfaces have been assumed to be flat, but in practice, owing to elastic, thermal and uneven wear effects, modifications to the analysis are required to allow for plate distortion. Vora and Bhat (26), Gupta and Vora (27) have considered the effects of curvature of surfaces in their analysis of a squeeze film action between porous rotating circular plates. Expressions for the pressure and load carrying capacity of the disc are given in the form of exponential series. Again results obtained by the authors show that the effect of rotating fluid inertia is to reduce the load capacity.

Prakash and Tiwari (28) have analyzed the effects of surface roughness on the squeeze film action between rotating porous discs. They assume that the film thickness can be expressed by a combination of nominal film thickness and deviation of height from the nominal level. Their results indicate that the circumferential roughness increases while the radial roughness decreases the load carrying capacity at constant roughness values.

Application of the finite element technique to the analysis of porous regions in a variety of lubrication problems has been demonstrated by several workers. Rohde and Oh (29) have applied the technique to journal bearing problems with a compressible lubricant. They assumed the flow in the porous region only to be across its thickness, an assumption which is valid for a very thin porous region case. Eidelberg and Booker (30) have presented the technique for the analysis of squeeze films to take into account three dimensional flow in the film and porous regions. Their analysis is based on the following coupling conditions and boundary conditions:

1. The diffusion velocity in the film region interface is equal to the velocity normal to the surface in porous region, thus the model flow at the interface is zero.
2. Pressure in porous region is the same as the fluid film pressure at the interface.
3. The surrounding pressure is zero.
4. The flow at all the internal nodes is zero.

The simplest elements such as triangular elements and tetrahedra elements are used in the idealization of the film region and the porous region respectively. Applications have been made to solve problems involving irregular geometrical configurations and different material properties.

Malik, Sinhasan and Chandra (31) have recently reported the analysis of porous step bearings using rectangular and hexahedral quadratic elements. The effects of tangential velocity slip ignored previously (29), (30) have been taken into account in their results predicting load carrying capacity and coefficient of friction.

2.3 Disc Problem Analysis

The behaviour of the fluid film between annular discs has been examined both theoretically and experimentally by many investigators.

Archibald (32) has analyzed squeezing flow problems such as spherical bearings and circular plates. Jackson (33) has used an iterative procedure to solve the continuity and radial momentum equations to provide a better approximation of inertia effects that occur in the fluid film. Rotating conditions were also considered by Allen and McKillop (34) in their analysis of the problem of normal approach of two annular surfaces, one of which is rotating with respect to the other. Consideration of centrifugal forces acting on the fluid was based upon the assumption of Couette flow in the tangential direction.

As a conclusion they have stated that the theoretical results showed that the only effect of rotation on an ideal squeeze film between parallel surface, is due to the centrifugal forces, which tends to increase the rate of approach of the two surfaces. The authors have also presented some empirical results using various kinds of fluid and good agreements with theoretical results have been obtained in some cases.

Ludwig (35) has analyzed the engagement characteristics of wet type clutches mathematically and experimentally. He found that the grooving pattern on the plates had a pronounced effect on the dynamic coefficient of friction and that the spiral grooves produced a higher friction than radial grooves. Wu (36) has developed a model to simulate the engagement characteristics of a single pair of wet type clutch plates used in automatic transmissions. The equations are based on those derived from his previous studies (19), (20). Expressions that enable calculation of such quantities as film thickness, transmitted torque, interface temperature, heat generation rate and engine speed have been presented. Utilizing such calculation Wu has shown that viscous shear forces can produce significant amount of the clutch torque transmitted during the squeezing motion and that most of the energy is dissipated during the squeeze film region. Results also indicate that the permeability parameter and porous facing thickness ratio are of great importance in engagement. The effect of surface irregularities and grooving effects were neglected in Wu's analysis. El-Serbiny and Newcomb (37) have described a general model to simulate the engagement characteristics of a wet type friction clutch using the finite element technique. Oil groove effects and the heat generated by viscous shear are taken into account in their analysis of single and repeated engagements. The predicted effects of frictional behaviour agreed with trends observed from practical investigation. Porosity effects in the clutch materials were ignored in this analysis.

In the present work inertia effects have been included in a finite element analysis of the rotating disc problems when radial and spiral grooves are incorporated in one disc surface.

Before consideration of this and other problems a theoretical development of the generalized Reynolds equation is presented in Chapter 3. A solution to this equation using a finite element analysis is then incorporated into a computer program as outlined in Chapter 4.

3. Theoretical Development

3.1. Reynolds Equation

The derivation of the generalized Reynolds equation for incompressible isothermal steady state lubrication problems is based on the following assumptions:

1. The pressure throughout the film thickness remains constant.
2. The curvature of the bearing surface is large compared to the oil film thickness.
3. No slip between the bearing surface and adjacent layers of fluid film.
4. The lubricant is considered to be a Newtonian fluid.
5. The fluid flow is laminar.
6. The viscosity is temperature dependent.

The geometry and coordinate system for a fluid film and corresponding surfaces is shown in FIG.1.

Reynolds equation is derived from the consideration of the fluid continuity of flow and the equilibrium of a fluid element.

An infinitely small element of fluid of sides dx, dy and dz is shown in FIG.2. The fluid velocities on all the faces of the element and in the orthogonal direction x, y and z are assumed to be constant.

The incoming volume flow rate is given by the equation

$$u\,dy\,dz + v\,dx\,dz + w\,dx\,dy \tag{3.1}$$

and the out flow rate by

$$\left(u + \frac{\partial u}{\partial x} dx\right)dydz + \left(v + \frac{\partial v}{\partial y} dy\right)dxdz + \left(w + \frac{\partial w}{\partial z} dz\right)dxdy \tag{3.2}$$

From the continuity of flow, the net flow rate must be zero, which leads to the following relationship,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (3.3)$$

Next consider a column of fluid of sides dx, dy and height h as shown in FIG.3. The fluid influx and efflux rates per unit width are shown in the figure.

Volume flow in x direction : Influx $q_x dy$

Efflux $(q_x + \frac{\partial q_x}{\partial x} dx)dy$

Volume flow in y direction : Influx $q_y dx$

Efflux $(q_y + \frac{\partial q_y}{\partial y} dy)dx$

In the z direction, if the velocities of the lower and upper surfaces of the column are w_0 and w_1 respectively, the increase in volume can be expressed as

$$(w_o - w_1) \, dx \, dy$$

Using the condition of continuity of flow, the following relationship is obtained,

$$(q_x dy + q_y dx) + (w_o - w_1) \, dxdy = (q_x + \frac{\partial q_x}{\partial x} dx)dy + (q_y + \frac{\partial q_y}{\partial y} dy)dx$$

Cancelling (dxdy) which is arbitrary and non-zero gives

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + (w_1 - w_0) = 0 \qquad (3.4)$$

If the upper surface is permeable, the last term of eqn.(3.4) $(w_1 - w_0)$ can be explained by the squeeze and diffusion actions on the fluid then

$$w_1 - w_0 = \frac{\partial h}{\partial t} + v_d \tag{3.5}$$

where

$\frac{\partial h}{\partial t}$: the rate of change of height of the column, namely squeeze velocity

v_d : diffusion velocity

Substituting eqn.(3.5) in eqn.(3.4), the following expression is obtained

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial h}{\partial t} + v_d = 0 \tag{3.6}$$

Finally consider the equilibrium of a fluid element as shown in FIG.4. In this case, the forces consist of viscous shear stresses, body forces and fluid pressure, and resolving in the direction of the x axis gives the equation

$$p\,dydz + (\tau_{xz}+\frac{\partial \tau_{xz}}{\partial z}\,dz)dxdy + B_x dxdydz=(p + \frac{\partial p}{\partial x}\,dx)dydz + \tau_{xz}dxdy$$

which reduces to

$$\frac{\partial \tau_{xz}}{\partial z} + B_x = \frac{\partial p}{\partial x} \tag{3.7}$$

Similarly in the direction of the y axis

$$\frac{\partial \tau_{yz}}{\partial z} + B_y = \frac{\partial p}{\partial y} \tag{3.8}$$

In the z direction the pressure gradient is assumed to be zero.

$$\frac{\partial p}{\partial z} = 0 \tag{3.9}$$

According to the Newton's law of viscisity

$$\tau_{xy} = \mu \frac{\partial u}{\partial z} \tag{3.10}$$

$$\tau_{yz} = \mu \frac{\partial v}{\partial z} \tag{3.11}$$

Sustituting eqns (3.10) and (3.11) into (3.7) and (3.8) the following relationships are given

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z} (\mu \frac{\partial u}{\partial z}) + B_x \tag{3.12}$$

$$\frac{\partial p}{\partial y} = \frac{\partial}{\partial z} (\mu \frac{\partial v}{\partial z}) + B_y \tag{3.13}$$

The velocity gradient is obtained by integrating eqn.(3.12) with respect to z.

$$\frac{\partial u}{\partial z} = \frac{1}{\mu}\Big((\frac{\partial p}{\partial x}) z - \int_0^z B_x dz\Big) + \frac{C_1}{\mu} \tag{3.14}$$

Integrating again gives

$$u = \frac{\partial p}{\partial x}\int_0^z \frac{z}{\mu} dz - \int_0^z \frac{1}{\mu} \int_0^z B_x dzdz + \int_0^z \frac{C_1}{\mu} dz + C_2 \tag{3.15}$$

The constants of integration C_1 and C_2 are determined by the application of the following boundary conditions

$$u = U_1,\ v = V_1,\ w = W_1 \quad \text{at} \quad z = 0 \tag{3.16}$$

$$u = U_2,\ v = V_2,\ w = W_2 \quad \text{at} \quad z = h \tag{3.17}$$

$$p = P(x,y) \tag{3.18}$$

where P(x,y) is a specified function on a non-vanishing segment of the boundary.

Application of the boundary condition (3.16) to eqn.(3.15) yields

$$C_2 = U_1 \tag{3.19}$$

Using boundary condition (3.17) and the value of C_2

$$C_1 = \frac{1}{A_0} \left(U_2 - U_1 \right) \frac{\partial p}{\partial x} A_1 + \int_0^h \frac{1}{\mu} \int_0^z B_x dzdz \qquad (3.20)$$

where

$$A_0 = \int_0^h \frac{1}{\mu} dz \, , \qquad A_1 = \int_0^h \frac{z}{\mu} dz$$

Substituting values of C_1 and C_2 into eqn. (3.15), the velocity component is obtained as follows

$$u = \frac{\partial p}{\partial x} \left(\int_0^z \frac{z}{\mu} dz - \frac{A_1}{A_0} \int_0^z \frac{1}{\mu} dz \right) + U_1 + \frac{U_2 - U_1}{A_0} \int_0^z \frac{1}{\mu} dz + \bar{B}_x \qquad (3.21)$$

where

$$\bar{B}_x = \frac{1}{A_0} \int_o^z \frac{1}{\mu} dz \left(\int_o^h \frac{1}{\mu} \int_o^z B_x dzdz \right) - \int_o^z \frac{1}{\mu} \int_o^z B_x \, dzdz$$

Similarly in the y direction

$$v = \frac{\partial p}{\partial y} \int_o^z \frac{z}{\mu} dz - \frac{A_1}{A_0} \int_0^z \frac{1}{\mu} dz + V_1 + \frac{V_2 - V_1}{A_0} \int_o^z \frac{1}{\mu} dz + \bar{B}_y \qquad (3.22)$$

where

$$\bar{B}_y = \frac{1}{A_0} \int_o^z \frac{1}{\mu} dz \left(\int_o^h \frac{1}{\mu} \, {}_o^z \, B_y \, dzdz \right) - \int_o^z \frac{1}{\mu} \int_o^z B_y \, dzdz$$

The velocity in the z direction, w can be obtained by substituting eqns.(3.21) and (3.22) into the continuity eqn.(3.3) and is as follows

$$w = - \int_o^z \frac{\partial u}{\partial x} dz - \int_o^z \frac{\partial v}{\partial y} dz + W_1 \qquad (3.23)$$

where W_1 is the magnitude of the velocity w at z = 0.

The average velocities in the x and y directions can be expressed as

$$\bar{u} = \frac{1}{h}\int_{o}^{h} u\,dz \qquad (3.24)$$

$$\bar{v} = \frac{1}{h}\int_{o}^{h} v\,dz \qquad (3.25)$$

and the volume flows as

$$q_x = h \cdot \bar{u} = \int_{o}^{h} u\,dz \qquad (3.26)$$

$$q_y = h \cdot \bar{v} = \int_{o}^{h} v\,dz \qquad (3.27)$$

Substituting these values in the continuity eqn. (3.6), the following expression is obtained

$$\frac{\partial}{\partial x}\left(\int_{o}^{h} u\,dz\right) + \frac{\partial}{\partial y}\left(\int_{o}^{h} v\,dz\right) + \frac{\partial h}{\partial t} + v_d = 0 \qquad (3.28)$$

Substituting the values of velocities from eqn.(3.21) and (3.22) in the above expression, the generalized Reynolds equation in vector form is obtained

$$- \nabla G\, \nabla \;\; = \nabla(h\,\mathbf{U}) + \nabla(\Delta \mathbf{U} A_2) + \nabla B + \frac{\partial h}{\partial t} + v_d \qquad (3.29)$$

where

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y}$$

$$G = \int_{o}^{h}\int_{o}^{z} \frac{z}{\mu}\,dzdz - \frac{A_1}{A_0}\int_{o}^{h}\int_{o}^{z} \frac{1}{\mu}\,dzdz$$

$$\mathbf{U} = U_1\,\hat{i} + V_1\,\hat{j}$$

$$\Delta\mathbf{U} = (\,U_2 - U_1\,)\,\hat{i} + (\,V_2 - V_1\,)\,\hat{j}$$

$$A_2 = \frac{1}{A_0}\int_{o}^{h}\int_{o}^{z} \frac{1}{\mu}\,dzdz$$

$$\mathbb{B} = \hat{i}\int_o^h \overline{B}_x \; dz + \hat{j}\int_o^h \overline{B}_y \; dz$$

3.2 Incompressible Isothermal Lubrication

In this section development of the incompressible form of the Reynolds equation is presented. The viscosity of the lubricant is assumed to be constant . Then eqn.(3.15) can be expressed as

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\int_o^z z \; dz - \frac{1}{\mu}\int_o^z\int_o^z B_x \; dzdz + \frac{1}{\mu}\int_o^z C_1 \; dz + C_2 \qquad (3.30)$$

The body forces are expressed by averaged values in z direction to simplify the analysis

$$\begin{aligned} B_{mx}(x,y) &= \frac{1}{h}\int_o^h B_x \; dz \\ B_{my}(x,y) &= \frac{1}{h}\int_o^h B_y \; dz \end{aligned} \qquad (3.31)$$

Using these averaged values for the body forces in eqn.(3.30), the following form can be obtained

$$u = \frac{z^2}{2\mu}\frac{\partial p}{\partial x} - \frac{z^2}{2\,\mu} B_{mx} + \frac{C_1}{\mu} z + C_2 \qquad (3.32)$$

Applying the boundary conditions (3.16) and (3.17), the velocity component of the fluid in incompressible condition is given in the equation

$$u = \frac{z(z-h)}{2\mu}\left(\frac{\partial p}{\partial x} - B_{mx}\right) + \frac{z}{h}(U_2 - U_1) + U_1 \qquad (3.33)$$

Similarly

$$v = \frac{z(z-h)}{2\,\mu}\left(\frac{\partial p}{\partial y} - B_{my}\right) + \frac{z}{h}(V_2 - V_1) + V_1 \qquad (3.34)$$

The corresponding volume flows are

$$q_x = \int_o^h u \, dz = - \frac{h^3}{12\mu} (\frac{\partial p}{\partial x} - B_{mx}) + \frac{h}{2} (U_1 + U_2) \qquad (3.35)$$

$$q_y = \int_o^h v \, dz = - \frac{h^3}{12\mu} (\frac{\partial p}{\partial y} - B_{my}) + \frac{h}{2} (V_1 + V_2) \qquad (3.36)$$

The volume flow gradients are obtained by differentiating eqns.(3.35) and (3.36) with regard to x and y

$$\frac{\partial q_x}{\partial x} = - \frac{1}{12\mu} \frac{\partial}{\partial x} (h^3 \frac{\partial p}{\partial x} - B_{mx}) + \frac{\partial}{\partial x} \left[\frac{h(U_1 + U_2)}{2} \right] \qquad (3.37)$$

$$\frac{\partial q_y}{\partial y} = - \frac{1}{12\mu} \frac{\partial}{\partial y} (h^3 \frac{\partial p}{\partial y} - B_{my}) + \frac{\partial}{\partial y} \left[\frac{h(V_1 + V_2)}{2} \right] \qquad (3.38)$$

Finally substituting these gradients in eqn.(3.28), the generalized Reynolds equation for incompressible isothermal steady state is obtained in the form

$$\frac{1}{12\mu} \left[\frac{\partial}{\partial x} (h^3 \frac{\partial p}{\partial x}) + \frac{\partial}{\partial y} (h^3 \frac{\partial p}{\partial y}) \right] = \frac{\partial}{\partial x} (h\bar{u}) + \frac{\partial}{\partial y} (h\bar{v})$$

$$+ \frac{1}{12\mu} \left[\frac{\partial}{\partial x} (h^3 B_{mx}) + \frac{\partial}{\partial y} (h^3 B_{my}) \right] + \frac{\partial h}{\partial t} + v_d \qquad (3.39)$$

where

$$\bar{u} = \frac{U_1 + U_2}{2} , \qquad \bar{v} = \frac{V_1 + V_2}{2}$$

This Reynolds equation is solved with boundary conditions such that along part of the boudary S_p shown in FIG.8 , the pressure is specified by

$$p = P(x,y) \qquad \text{on} \qquad S_p \qquad (3.40)$$

and along the remainder of the boundary S_q, the volume flow per unit boundary length is specified by

$$q = q_x\hat{i} + q_y\hat{j} \qquad \text{on} \qquad S_q \tag{3.41}$$

3.3 Inclusion of the Centrifugal Force in the Generalized Reynolds Equation

Normally inertia effects in fluids are of little significance and as such are neglected in Reynolds equation. Since in this project, discs are to be analyzed and Reynolds equation is applied to disc problems, the inertia effects are of interest and the theory has been extended to inclide these.

A typical arrangement is shown in FIG.5, where the lower surface has an angular velocity of ω_1 and the upper surface has an angular velocity of ω_2. The centrifugal force per unit volume is

$$C_f = \rho r \omega^2 \tag{3.42}$$

Assuming that the angular velocity varies linearly with height

$$\omega = \omega_o z + \omega_1 \tag{3.43}$$

where

$$\omega_o = \frac{\omega_2 - \omega_1}{h}$$

The centrifugal force can now be expressed as

$$C_f = \rho r(\omega_o z + \omega_1)^2 \tag{3.44}$$

The average centrifugal forces are given by the equations

$$C_{fm} = \frac{1}{h}\int_o^h C_f \, dz$$

$$= \frac{\rho r}{3h\omega_o}\left[(\omega_o h + \omega_1)^3 - \omega_1{}^3\right] \qquad (3.45)$$

And the average centrifugal forces in x and y direction are as follows

$$C_{fmx} = \frac{\rho r \cos\theta}{3h\omega_o}\left[(\omega_o h + \omega_1)^3 - \omega_1{}^3\right] \qquad (3.46)$$

$$C_{fmy} = \frac{\rho r \sin\theta}{3h\omega_o}\left[(\omega_o h + \omega_1)^3 - \omega_1{}^3\right] \qquad (3.47)$$

Considering the equilibrium of a fluid element

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xz}}{\partial z} + B_{mx} + C_{fmx} \qquad (3.48)$$

$$\frac{\partial p}{\partial y} = \frac{\partial \tau_{yz}}{\partial z} + B_{my} + C_{fmy} \qquad (3.49)$$

Proceeding as before and using continuity equation, Reynolds equation including the inertia terms is obtained as follows

$$\frac{1}{12\mu}\left[\frac{\partial}{\partial x}(h^3 \frac{\partial p}{\partial x}) + \frac{\partial}{\partial y}(h^3 \frac{\partial p}{\partial y})\right]$$
$$= \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) + \frac{1}{12\mu}\left[\frac{\partial}{\partial x}(h^3 B_{mx}) + \frac{\partial}{\partial y}(h^3 B_{my})\right]$$
$$+ \frac{1}{12\mu}\left[\frac{\partial}{\partial x}(h^3 C_{fmx}) + \frac{\partial}{\partial y}(h^3 C_{fmy})\right] + \frac{\partial h}{\partial t} + v_d \qquad (3.50)$$

3.4 Application to Porous Annular Discs

The flow field in the porous region of a disc is governed by the Laplace equation which is coupled to the Reynolds equation governing the film region. In the film region, the pressure distribution can be expressed in a two dimensional form as before. However, in the porous region of the disc, the pressure changes across the thickness and accordingly a three dimensional approach has to be made. A three dimensional finite element technique would be suitable for the solution of the three dimensional Laplace equation and would overcome the complicated surface configuration.

As a primary analysis, a simplified investigation of the porous region is presented here. The investigation is confined to squeezing pads, one of which has a porous facing. This model has been adopted, because it is nearest to automotive applications such as clutch plates and oil immersed disc brakes. The model under consideration is shown in FIG.6.

The analysis is based on the following assumptions:

1. The porous facing has constant permeability.
2. The pressure in surrounding field is zero.
3. Squeezing action on the fluid is the most dominant effect and other effects are neglected.

These assumptions are in addition to those applicable to the film region.

The fluid velocities for a porous region can be derived from Darcy's law (38).

$$\dot{u} = -\frac{\Phi}{\mu}\frac{\partial \dot{p}}{\partial x}$$

$$\dot{v} = -\frac{\Phi}{\mu}\frac{\partial \dot{p}}{\partial y} \qquad (3.51)$$

$$\dot{w} = -\frac{\Phi}{\mu}\frac{\partial \dot{p}}{\partial z}$$

The three dimensional continuity equation can be expressed as follows (39)

$$\nabla (\rho\dot{U}) + \dot{A}\,\frac{\partial p}{\partial t} = 0 \qquad (3.52)$$

where

$$\dot{U} = \dot{u}\hat{i} + \dot{v}\hat{j} + \dot{w}\hat{k}$$

$\dot{A}$ = porosity

Using Darcy's equation (3.51), the continuity equation becomes

$$\nabla(\frac{\rho\Phi}{\mu}\,\nabla\dot{p})= \dot{A}\,\frac{\partial p}{\partial t} \qquad (3.53)$$

The associated boundary conditions between the film region and porous region are

$$\left.\begin{array}{l} v_d(x,y) = \dot{U}(x,y,h)\;\hat{n} \qquad (3.54) \\ p(x,y) = \dot{p}(x,y,h) \qquad (3.55) \end{array}\right\} \text{at} \quad h = \dot{h}$$

where

$\hat{n}$ = unit normal vector to a boundary surface

Also the boundary conditions between the porous region and the surrounding field are given by

$$\left.\begin{array}{lll} p = 0 & \text{on boundary} & S \\ \dot{p} = 0 & \text{on boundary} & \dot{S} \end{array}\right\} \qquad (3.56)$$

$$\dot{U} = (x,y,h)\ \hat{n} = 0 \qquad \text{on} \qquad h = H \qquad (3.57)$$

3.5 Friction Force, Friction Torque and Load Carrying Capacity of Bearings and Discs.

The theory developed so far can be extended to assess the performance of bearings and discs in so far as their load and torque carrying capacity is concerned. Friction forces and torques indicate the power loss for bearings. However for discs these values are more significant because they govern their engagement capacity.

The load carrying capacity L can be expressed as

$$L = \int_A p(x,y)\ dA \qquad (3.58)$$

and the friction forces by

$$F_x = \int_A \tau_x\Big|_{z=0,h}\ dA \qquad (3.59)$$

$$F_y = \int_A \tau_y\Big|_{z=0,h}\ dA \qquad (3.60)$$

where A is the area concerned.

From eqns. (3.10) and (3.11) the shear stresses are

$$\tau_x = \mu \frac{\partial u}{\partial z}$$

$$\tau_y = \mu \frac{\partial v}{\partial z}$$

and the velocity gradients are derived from eqns. (3.33) and (3.34)

$$\frac{\partial u}{\partial z} = \frac{(2z-h)}{2\mu} \left(\frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{(U_2 - U_1)}{h} \tag{3.61}$$

$$\frac{\partial v}{\partial z} = \frac{(2z-h)}{2\mu} \left(\frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{(V_2 - V_1)}{h} \tag{3.62}$$

Substituting these values into eqns.(3.10) and (3.11), the components shear stresses are now expressed as

$$\tau_x = \frac{(2z-h)}{2} \left(\frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{\mu}{h} U \tag{3.63}$$

$$\tau_y = \frac{(2z-h)}{2} \left(\frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{\mu}{h} V \tag{3.64}$$

where

$$U = U_2 - U_1 \quad , \quad V = V_2 - V_1$$

The shear stresses for the lower surface , z = 0 are given by the equations

$$- \tau_{x1} = - \frac{h}{2} \left(\frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{\mu}{h} U \tag{3.65}$$

$$- \tau_{y1} = - \frac{h}{2} \left(\frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{\mu}{h} V \tag{3.66}$$

and the shear stresses for the upper surface, $z = h$ by

$$\tau_{x2} = \frac{h}{2} \left(\frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{\mu}{h} U \tag{3.67}$$

$$\tau_{y2} = \frac{h}{2} \left(\frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{\mu}{h} V \tag{3.68}$$

The friction forces are obtained by substituting eqns.(3.67) and(3.68) into eqns.(3.59) and (3.60)

$$F_{x1} = \int_A \frac{h}{2} \left(\frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) - \frac{\mu}{h} U \quad dA \tag{3.69}$$

$$F_{x2} = \int_A \frac{h}{2} \left(\frac{\partial p}{\partial x} - B_{mx} - C_{fmx} \right) + \frac{\mu}{h} U \quad dA \tag{3.70}$$

$$F_{y1} = \int_A \frac{h}{2} \left(\frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) - \frac{\mu}{h} V \quad dA \tag{3.71}$$

$$F_{y2} = \int_A \frac{h}{2} \left(\frac{\partial p}{\partial y} - B_{my} - C_{fmy} \right) + \frac{\mu}{h} V \quad dA \tag{3.72}$$

For disc problems, determination of friction torque is required as a measure of their performance. The torque field for a disc is shown in FIG.7. The friction torque of the area ΔA is expressed as

$$\Delta T_e = \quad r \ (\ \tau_x \ \sin\theta + \tau_y \ \cos\theta \ ) \ \Delta A \tag{3.74}$$

The total friction torques of the lower and upper surfaces can be obtained by integrating eqn. (3.73) over the appropriate area and take the form

$$T_{e_1} = \int_A r \ (\ \tau_{x_1} \sin\theta + \tau_{y_1} \cos\theta \ ) \ dA \tag{3.74}$$

$$T_{e_2} = \int_A r \ (\ \tau_{x_2} \sin\theta + \tau_{y_2} \cos\theta \ ) \ dA \tag{3.75}$$

4. Application of The Finite Element Technique

4.1 Variational Principles

The generalized Reynolds equation (3.50) describes the behaviour of film lubrication when the film thickness and other variables such as body forces,surface velocities, centrifugal forces, squeezing velocities and the diffusion velocities together with appropriate boundary pressure and flow conditions (3.40), (3.41) are known. Variational principles can be applied for the solution of this equation.

The integral I is a functional which has independent variables x, y and unknown function p(x,y).

$$I(p) = \iint_A F\left(x,y,p, \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial^2 p}{\partial x^2}, \frac{\partial^2 p}{\partial y^2}, \frac{\partial^2 p}{\partial x \; \partial y}\right) dxdy = \text{constant} \qquad (4.1)$$

Variational calculus is used for the determination of function p which minimizes I(p).

Let the function

$$p = p^* + \varepsilon\eta(x)$$

where ε is an arbitrary parameter and $\eta(x)$ is a continuous function having zero values on the boundaries. In order that function I(p) be a minimum at $p = p^*$, the following conditions must be satisfied

$$\left.\frac{dI}{d\varepsilon}\right|_{\varepsilon=0} = 0$$

$$\left.\frac{d^2I}{d\varepsilon^2}\right|_{\varepsilon=0} \geq 0 \qquad (4.2)$$

Substituting eqn.(4.2) into (4.1) a Euler - Lagrange form of equation is obtained,

$$\frac{\partial^2}{\partial x^2}\left(\frac{\partial F}{\partial p_{xx}}\right)+\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial F}{\partial p_{xy}}\right)+\frac{\partial^2}{\partial y^2}\left(\frac{\partial F}{\partial p_{yy}}\right)$$

$$-\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial p_x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial p_y}\right)+\frac{\partial F}{\partial p} = 0 \qquad (4.3)$$

The function p that minimizes this functional satisfies the Reynolds equation (3.50) and the boundary conditions (3.40) and (3.41) is given by the equation

$$I(p) = \int_A\left[\left\{\frac{h^3}{24\mu}\nabla p - hU - \frac{h^3}{12\mu}(B_m+C_{fm})\right\}\nabla p+\left(\frac{\partial h}{\partial t}+v_d\right)p\right]dA+\int_{S_q} Q\,p\,ds \quad (4.4)$$

4.2 Development of Fluidity Matrices

The finite element technique has been applied for the determiation of an approximate pressure distribution in a two dimensional field. Initially the field under consideration is subdivided into smaller elements having a finite number of nodes. Approximate interpolation functions are chosen to express pressure and other variable variations within these elements. These functions satisfy the boundary continuity criteria.

The approximate variation of various variables can be expressed as

$$p = N\,\{p\} = \sum_{i=1}^{r} N_i(x,y)\;p_i \qquad (4.5)$$

$$U_x = N \{U_x\} = \sum_{i=1}^{r} N_i(x,y)U_{x_i}$$

$$U_y = N \{U_y\} = \sum_{i=1}^{r} N_i(x,y)U_{y_i}$$

$$B_{mx} = N \{B_{mx}\} = \sum_{i=1}^{r} N_i(x,y)B_{mx_i} \qquad (4.6)$$

$$B_{my} = N \{B_{my}\} = \sum_{i=1}^{r} N_i(x,y)B_{my_i}$$

$$C_{fmx} = N \{C_{fmx}\} = \sum_{i=1}^{r} N_i(x,y)C_{fmx_i}$$

$$C_{fmy} = N \{C_{fmy}\} = \sum_{i=1}^{r} N_i(x,y)C_{fmy_i}$$

$$\frac{\partial h}{\partial t} = N \{\frac{\partial h}{\partial t}\} = \sum_{i=1}^{r} N_i(x,y) \frac{\partial h}{\partial t}_i$$

$$v_d = N \{v_d\} = \sum_{i=1}^{r} N_i(x,y) v_{d_i}$$

The field values (4.5) and (4.6) are substituted into the functional of eqn.(4.4) which is then minimized with respect to the nodal pressure p_i of the element

$$\frac{\partial I}{\partial p_i} = 0 \qquad i = 1, 2, \cdots\cdots r \qquad (4.7)$$

and considering the whole domain

$$\sum_{i=1}^{N} \frac{\partial I}{\partial p_i} = 0 \tag{4.8}$$

Substituting (4.5) and (4.6) into (4.4) each derivatives of p becomes

$$\nabla p = \frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}$$

$$= \left[\sum_{i=1}^{r} \frac{\partial N_i}{\partial x} p_i\right] + \left[\sum_{i=1}^{r} \frac{\partial N_i}{\partial y} p_i\right]$$

$$\nabla p \nabla p = \left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2$$

$$= \left(\sum_{i=1}^{r} \frac{\partial N_i}{\partial x} p_i\right) \times \left(\sum_{j=1}^{r} \frac{\partial N_j}{\partial x} p_j\right) + \left(\sum_{i=1}^{r} \frac{\partial N_i}{\partial y} p_i\right) \times \left(\sum_{j=1}^{r} \frac{\partial N_j}{\partial y} p_j\right)$$

and on differentiation with respect to p_i,

$$\frac{\partial}{\partial p_i}(\nabla p) = \frac{\partial N_i}{\partial x} + \frac{\partial N_i}{\partial y}$$

$$\frac{\partial}{\partial p_i} \nabla p \nabla p = \left(\frac{\partial N_i}{\partial x}\right)\left[\sum_{j=1}^{r} \frac{\partial N_j}{\partial x} p_j\right] + \left(\frac{\partial N_i}{\partial y}\right)\left[\sum_{j=1}^{r} \frac{\partial N_j}{\partial y} p_j\right]$$

$$\frac{\partial}{\partial p_i} (p) = N_i$$

Thus equation (4.8) can be expressed, using the fluidity matrices and nodal values as follows

$$\left[K_p\right]\{p\} = -\left[K_{U_x}\right]\{U_x\} - \left[K_{U_y}\right]\{U_y\} - \left[K_{B_{mx}}\right]\{B_{mx}\} - \left[K_{B_{my}}\right]\{B_{my}\}$$

$$-\left[K_{C_{fmx}}\right]\{C_{fmx}\} - \left[K_{C_{fmy}}\right]\{C_{fmy}\} - \left[K_{\dot{h}}\right]\left\{\frac{\partial h}{\partial t}\right\} - \left[K_{v_d}\right]\{v_d\} + \{q\} \tag{4.9}$$

where matrices $[K_P]$, $[K_{U_x}]$, $[K_{U_y}]$,.... are of size r×r and matrices $\{p\}$, $\{U_x\}$, $\{U_y\}$,.... are of size r×1

$$\text{Pressure : } K_{P_{ij}} = -\int_A \left[\frac{h^3}{12\mu}\left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right)\right] dA \tag{4.10}$$

$$\text{Shear : } K_{U_{x_{ij}}} = \int_A h \frac{\partial N_i}{\partial x} N_j \, dA$$

$$K_{U_{y_{ij}}} = \int_A h \frac{\partial N_i}{\partial y} N_j \, dA$$

$$\text{Body force: } K_{B_{mx_{ij}}} = \int_A \frac{h^3}{12\mu}\frac{\partial N_i}{\partial x} N_j \, dA$$

$$K_{B_{my_{ij}}} = \int_A \frac{h^3}{12\mu}\frac{\partial N_i}{\partial y} N_j \, dA \tag{4.11}$$

Centrifugal

$$\text{force : } K_{C_{fmx_{ij}}} = K_{B_{mx_{ij}}}$$

$$K_{C_{fmy_{ij}}} = K_{B_{my_{ij}}}$$

$$\text{Squeeze : } K_{\dot{h}_{ij}} = -\int_A N_i N_j \, dA$$

$$\text{Diffusion: } K_{v_{d_{ij}}} = K_{\dot{h}_{ij}}$$

$$\text{Flow: } q_i = \int_{S_q} Q N_i \, dA$$

where q_i is the outward flow across the boundary S_i associated with the node i . The half-boundary S_i is either side of the node as shown in FIG.8.

Equation (4.9) can be solved provided n_i nodal pressures and flows of the rest of the nodes ($N - n_i$) are both known. All other nodal forcing values such as body forces, must also be known in order to solve this equation.

Equation (4.9) can be expressed in matrix form as

$$\begin{bmatrix} K_p \end{bmatrix} \{p\} = \{q\} - \begin{bmatrix} K_a \end{bmatrix} \{a\} \qquad (4.12)$$

These matrices can be partitioned and rearranged as follows into known and unknown value matrices

$$\begin{bmatrix} K_{p_{11}} & K_{p_{12}} \\ K_{p_{21}} & K_{p_{22}} \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} - \begin{bmatrix} K_{a_{11}} & K_{a_{12}} \\ K_{a_{21}} & K_{a_{22}} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} \qquad (4.13)$$

Equation (4.13) can then be subdivided into two separate matrix equations

$$\{q_1\} = \begin{bmatrix} K_{p_{11}} \end{bmatrix} \{p_1\} + \begin{bmatrix} K_{p_{12}} \end{bmatrix} \{p_2\} - \begin{bmatrix} K_{a_{11}} \end{bmatrix} \{a_1\} - \begin{bmatrix} K_{a_{12}} \end{bmatrix} \{a_2\} \qquad (4.14)$$

$$\{q_2\} = \begin{bmatrix} K_{p_{21}} \end{bmatrix} \{p_1\} + \begin{bmatrix} K_{p_{22}} \end{bmatrix} \{p_2\} - \begin{bmatrix} K_{a_{21}} \end{bmatrix} \{a_1\} - \begin{bmatrix} K_{a_{22}} \end{bmatrix} \{a_2\} \qquad (4.15)$$

Rearranging eqn.(4.14) gives the following equations

$$\{Q_1\} = \begin{bmatrix} K_{p_{11}} \end{bmatrix} \{p_1\}$$

$$\{p_1\} = \begin{bmatrix} K_{p_{11}} \end{bmatrix}^{-1} \{Q_1\} \qquad (4.16)$$

where

$$\{Q_1\} = \{q_1\} - K_{p_{12}} \{p_2\} + K_{a_{11}} \{a_1\} + K_{a_{12}} \{a_2\} .$$

Since all nodal pressures become known from eqn.(4.16), these can be substituted into eqn.(4.15) to obtain the corresponding flows.

Once the nodal pressure at each node has been determined, the component flows (flows in an element) can be obtained by applying the pressures to the original equation(4.7).

4.2.1 Development of Fluidity Matrices for Triangular Elements

In this project the triangular element system is presented as a primary development of F.E.Technique for lubrication problems.

The elements are connected at the nodes which are located on the corners of triangles as shown in FIG.8, and are assumed to have linear variation of states. This variation is represented by a linear interpolation polynominal of the form

$$N_i(x,y) = a_i + b_i x + c_i y \tag{4.17}$$

The constants a_i, b_i and c_i are chosen so that $N_i = 1$ at node i and $N_i = 0$ at the other two nodes: that is

$$\begin{aligned} a_i &= (x_i y_k - x_k y_i) / 2A \\ b_i &= (y_i - y_k) / 2A \\ c_i &= (x_k - x_j) / 2A \end{aligned} \tag{4.18}$$

where

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = \text{(area of a triangle)} \quad .$$

The pressure distribution p can be expressed as

$$p = \sum_{i=1}^{3} N_i(x,y).p_i$$

$$= \sum_{i=1}^{3} (a_i+b_ix+c_iy).p_i \tag{4.19}$$

The other field values are defined in a similar manner. The film thickness variation can also be expressed by using an interpolation function such as

$$h = N\ \{h\} = \sum_{i=1}^{3} N_i(x,y)h_i \tag{4.20}$$

The fluidity matrices are then described as follows. Since the derivations of N in eqn.(4.10) can be expressed as

$$\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} = (b_ib_j + c_ic_j) = \text{const}$$

and

$$h^3 = \left\{\sum_{i=1}^{3} N_ih_i\right\}^3 = (N_1h_1 + N_2h_3 + N_3h_3)^3$$

Hence the pressure matrix assumes the following form

$$K_{P_{ij}} = -\frac{\rho}{12\mu}(b_ib_j + c_ic_j)\int_A (N_1h_1 + N_2h_2 + N_3h_3)^3\, dA$$

The integrated result of interpolation functions over the area of a triangular element is presented by Zienkiewicz (4) and is as follows

$$\int_A N_1^{\alpha} N_2^{\beta} N_3^{\gamma} \, dA = \frac{\alpha! \; \beta! \; \gamma!}{(\alpha + \beta + \gamma+2)!} \, 2A \qquad (4.21)$$

Accordingly the fluidity matrix is given by the following expression

$$K_{P_{ij}} = - \frac{1}{480A\mu} (b_i b_j + c_i c_j) \, B \qquad (4.22)$$

where $B = \left(\sum_{k=1}^{3} h_k^2\right)\left(\sum_{\ell=1}^{3} h_\ell\right) + h_1 h_2 h_3$

Other fluidity matrices can be expressed in a similar manner by the equations

$$K_{U_{x_{ij}}} = \frac{b_i}{24} \sum_{k=1}^{3} h_k (1+\delta_{kj}) \qquad (4.23)$$

$$K_{U_{y_{ij}}} = \frac{c_i}{24} \sum_{k=1}^{3} h_k (1+\delta_{kj}) \qquad (4.24)$$

$$K_{B_{mx_{ij}}} = \frac{b_i}{1440\,\mu} \, G \qquad (4.25)$$

$$K_{B_{my_{ij}}} = \frac{c_i}{1440\,\mu} \, G \qquad (4.26)$$

$$K_{C_{fmx_{ij}}} = K_{B_{mx_{ij}}} \qquad (4.27)$$

$$K_{C_{fmy_{ij}}} = K_{B_{my_{ij}}} \qquad (4.28)$$

$$K_{\dot{h}_{ij}} = - \frac{1}{12} A \, (1 + \delta_{ij}) \qquad (4.29)$$

$$K_{v_{d_{ij}}} = K_{\dot{h}_{ij}} \qquad (4.30)$$

where

$$\delta_{ij} = 1 \qquad \text{when} \quad i = j$$

$$\delta_{ij} = 0 \qquad \text{when} \quad i \neq j$$

$$G = \left(\sum_{k=1}^{3} h_k^2\right)\left(\sum_{k=1}^{3} h_k\right) + 2h_j^2\left(\sum_{k=1}^{3} h_k\right) + h_j\left(\sum_{k=1}^{3} h_k^2\right) + 2h_1h_2h_3$$

4.2.2 Development of Fluidity Matrices of Rectangular Elements

In this derivation of fluidity matrix a different type of natural coordinate system has been used. FIG.9 shows the two coordinate systems. The Cartesian coordinates are expressed in terms of the natural coordinate as follows

$$\begin{aligned} x &= \frac{1}{4}\Big[(1-s)(1-t)x_1 + (1+s)(1-t)x_2 \\ &\qquad + (1+s)(1+t)x_3 + (1-s)(1+t)x_4\Big] \\ y &= \frac{1}{4}\Big[(1-s)(1-t)y_1 + (1+s)(1-t)y_2 \\ &\qquad + (1+s)(1+t)y_3 + (1-s)(1+t)y_4\Big] \end{aligned} \qquad (4.31)$$

and the interpolation function for a linear rectangular element is

$$N_i(x,y) = \frac{1}{4}(1+ss_i)(1+tt_i) \qquad (4.32)$$

Hence the pressure distribution can be described as

$$p = \sum_{i=1}^{4} \frac{1}{4}(1+ss_i)(1+tt_i)p_i \qquad (4.33)$$

Transformation of the coordinates from Cartesian to natural is carried out as follows

$$\frac{\partial N}{\partial x} = \frac{\partial N}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial N}{\partial t}\frac{\partial t}{\partial x} \tag{4.34}$$

$$\frac{\partial N}{\partial y} = \frac{\partial N}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial N}{\partial t}\frac{\partial t}{\partial y} \tag{4.35}$$

$$\frac{\partial s}{\partial x} = \frac{\frac{\partial y}{\partial t}}{|J|}$$

$$\frac{\partial s}{\partial y} = \frac{-\frac{\partial x}{\partial t}}{|J|}$$

$$\frac{\partial t}{\partial x} = \frac{-\frac{\partial y}{\partial s}}{|J|}$$

$$\frac{\partial t}{\partial y} = \frac{\frac{\partial x}{\partial s}}{|J|}$$

Also

$$dxdy = |J|\,dsdt$$

where J is the determinant of the Jacobian matrix [J] given by

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} \end{bmatrix}$$

Eqns. (4.34) and (4.35) can be expressed as

$$\frac{\partial N}{\partial x} = \frac{1}{|J|}\left(\frac{\partial N}{\partial s}\frac{\partial y}{\partial t} - \frac{\partial N}{\partial t}\frac{\partial y}{\partial s} \right) \tag{4.36}$$

$$\frac{\partial N}{\partial y} = \frac{-1}{|J|}\left(\frac{\partial N}{\partial s}\frac{\partial x}{\partial t} - \frac{\partial N}{\partial t}\frac{\partial y}{\partial s} \right) \tag{4.37}$$

The derivatives of N in eqn.(4.10) can now be expressed as

$$\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} = \frac{1}{|J|\ |J|}\left(\frac{\partial N_i}{\partial s}\frac{\partial y}{\partial t} - \frac{\partial N_i}{\partial t}\frac{\partial y}{\partial s}\right)\left(\frac{\partial N_j}{\partial s}\frac{\partial y}{\partial t} - \frac{\partial N_j}{\partial t}\frac{\partial y}{\partial s}\right) \qquad (4.38)$$

$$\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} = \frac{1}{|J|\ |J|}\left(\frac{\partial N_i}{\partial s}\frac{\partial x}{\partial t} - \frac{\partial N_i}{\partial t}\frac{\partial x}{\partial s}\right)\left(\frac{\partial N_j}{\partial s}\frac{\partial x}{\partial t} - \frac{\partial N_j}{\partial t}\frac{\partial x}{\partial s}\right) \qquad (4.39)$$

From eqn.(4.31) and eqn.(4.32)

$$\frac{\partial N_i}{\partial s} = \frac{s_i}{4}(1+tt_i)$$

$$\frac{\partial N_i}{\partial t} = \frac{t_i}{4}(1+ss_i)$$

also

$$\frac{\partial x}{\partial s} = \frac{1}{4}\{(x_2-x_1) + (x_3-x_4)\} = a_1$$

$$\frac{\partial x}{\partial t} = \frac{1}{4}\{(x_4-x_1) + (x_3-x_2)\} = a_2$$

$$\frac{\partial y}{\partial s} = \frac{1}{4}\{(y_2-y_1) + (y_3-y_4)\} = a_3$$

$$\frac{\partial y}{\partial t} = \frac{1}{4}\{(y_4-y_1) + (y_3-y_2)\} = a_4$$

Hence the determinant $|J|$ becomes

$$|J| = \frac{\partial x}{\partial s}\frac{\partial y}{\partial t} - \frac{\partial x}{\partial t}\frac{\partial y}{\partial s}$$

$$= a_1 a_4 - a_2 a_4 \qquad (4.40)$$

Rearranging eqns.(4.38) and (4.39)

$$\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} = \frac{1}{16\,|J|\,|J|}\{a_4 s_i(1+tt_i)-\ a_3 t_i(1+ss_i)\}\{a_4 s_j(1+tt_j)-a_3 t_j(1+ss_j)\}$$

$$\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} = \frac{1}{16\,|J|\,|J|}\{a_2 s_i(1+tt_i)-\ a_1 t_i(1+ss_i)\}\{a_2 s_j(1+tt_j)-a_1 t_j(1+ss_j)\} \qquad (4.41)$$

If the film thickness and viscosity are constant, the integration of eqn.(4.10) takes the following form after substituting eqns.(4.40) and (4.41),

$$K_{P_{ij}} = -\int_A \frac{h^3}{12\mu}\left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right)\ dA$$

$$= \frac{-h^3}{192\mu\ |J|\,|J|}\int_{-1}^{1}\int_{-1}^{1}\Big[\{a_4 s_i(1+tt_i)-a_3 t_i(1+ss_i)\}$$

$$\{a_4 s_j(1+tt_j)-a_3 t_j(1+ss_j)\}+\{a_2 s_i(1+tt_i)-a_1 t_i(1+ss_i)\}$$

$$\{a_2 s_j(1+tt_j)-a_1 t_j(1+ss_j)\}\Big]|J|\ dsdt$$

This expression can be simplified by expressing it as a sum of the following terms

$$A = s_i s_j(a_2^2+a_4^2)\int_{-1}^{1}\int_{-1}^{1}(1+tt_i)(1+tt_j)\ dsdt$$

$$B = t_i t_j(a_1^2+\ a_3^2)\int_{-1}^{1}\int_{-1}^{1}(1+ss_i)(1+ss_j)\ dsdt$$

$$C = s_i t_j(a_1 a_2+\ a_3 a_4)\int_{-1}^{1}\int_{-1}^{1}(1+ss_j)(1+tt_i)\ dsdt$$

$$D = -\ s_j t_i(a_1 a_2+\ a_3 a_4)\int_{-1}^{1}\int_{-1}^{1}(1+ss_i)(1+tt_j)\ dsdt \quad .$$

Integrating these terms, the following results are obtained

$$\begin{aligned} A &= 4s_i s_j (a_2^2 + a_4^2)(\frac{1}{3}t_i t_j + 1) \\ B &= 4t_i t_j (a_1^2 + a_3^2)(\frac{1}{3}s_i s_j + 1) \\ C &= -4s_i t_j (a_1 a_2 + a_3 a_4) \\ D &= -4s_j t_i (a_1 a_2 + a_3 a_4) \end{aligned} \tag{4.42}$$

The fluidity matrix $K_{P_{ij}}$ therefore becomes

$$K_{P_{ij}} = \frac{-h^3}{192\mu \, |J|} (A+B+C+D) \tag{4.43}$$

Similarly, other fluidity matrices for the rectangular element can be expressed as

$$\begin{aligned} K_{U_{x_{ij}}} &= \frac{h}{4} \{s_i a_4 (\frac{1}{3}t_i t_j + 1) - t_i a_3 (\frac{1}{3}s_i s_j + 1)\} \\ K_{U_{y_{ij}}} &= \frac{h}{4} \{s_i a_2 (\frac{1}{3}t_i t_j + 1) - t_i a_1 (\frac{1}{3}s_i s_j + 1)\} \\ K_{B_{mx_{ij}}} &= \frac{h^3}{48\mu} \{s_i a_4 (\frac{1}{3}t_i t_j + 1) - t_i a_3 (\frac{1}{3}s_i s_j + 1)\} \\ K_{B_{my_{ij}}} &= \frac{h^3}{48\mu} \{s_i a_2 (\frac{1}{3}t_i t_j + 1) + t_i a_1 (\frac{1}{3}s_i s_j + 1)\} \\ K_{C_{fmx_{ij}}} &= K_{B_{mx_{ij}}} \\ K_{C_{fm_{ij}}} &= K_{B_{my_{ij}}} \end{aligned} \tag{4.44}$$

$$K_{\dot{h}_{ij}} = \frac{|J|}{4}(\frac{1}{3}s_i s_j+1)(\frac{1}{3}t_i t_j+1)$$

$$K_{v_{d_{ij}}} = K_{\dot{h}_{ij}}$$

If the film thickness is variable in an element, it is assumed that the thickness can also be expressed using the interpolation function N_i, as follows

$$h = \sum_{i=1}^{4} N_i(x,y)\ h_i = \frac{1}{4}\sum_{i=1}^{4} (1+ss_i)(1+tt_i)\ h_i \qquad (4.45)$$

The matrix $K_{P_{ij}}$ in this case is given by the equation

$$K_{P_{ij}} = -\frac{1}{192\mu|J|}(A'+B'+C'+D') \qquad (4.46)$$

where

$$A' = s_i s_j(a_2^2+ a_4^2)\int_{-1}^{1}\int_{-1}^{1} h^3(1+tt_i)(1+tt_j)\ dsdt$$

$$B' = t_i t_j(a_1^2+ a_3^2)\int_{-1}^{1}\int_{-1}^{1} h^3(1+ss_i)(1+ss_j)\ dsdt$$

$$C' = -s_i t_j(a_1 a_2-a_3 a_4)\int_{-1}^{1}\int_{-1}^{1} h^3(1+ss_j)(1+tt_j)\ dsdt$$

$$D' = -s_j t_i(a_1 a_2-a_3 a_4)\int_{-1}^{1}\int_{-1}^{1} h^3(1+ss_i)(1+tt_j)\ dsdt$$

Substituting eqn.(4.45) into eqn.(4.46) and rearranging terms, the quantities A' to D' can be expressed in the form

$$A' = 4s_i s_j(a_2^2+ a_4^2)\sum_{\ell=1}^{4}\sum_{m=1}^{4}\sum_{n=1}^{4}\{h_\ell h_m h_n(\frac{1}{3}s_{12}+1)(\frac{1}{5}t_{21}+\frac{1}{3}t_{22}+1)\}$$

$$B' = 4t_i t_j(a_1^2 + a_3^2)\sum_{\ell=1}^{4}\sum_{m=1}^{4}\sum_{n=1}^{4}\{h_\ell h_m h_n(\frac{1}{3}t_{12}+1)(\frac{1}{5}s_{21}+\frac{1}{3}s_{22}+1)\}$$

$$C' = -4s_i t_j(a_1 a_2 - a_3 a_4)\sum_{\ell=1}^{4}\sum_{m=1}^{4}\sum_{n=1}^{4}\Big[h_\ell h_m h_n\{\frac{1}{5}s_{11}s_j+\frac{1}{3}(s_{12}+s_{13}s_j)+1\}$$

$$+\{\frac{1}{5}t_{11}t_i+\frac{1}{3}(t_{12}+t_{13}t_i)+ 1\}\Big]$$

$$D' = -4s_j t_i(a_1 a_2 - a_3 a_4)\sum_{\ell=1}^{4}\sum_{m=1}^{4}\sum_{n=1}^{4}\Big[h_\ell h_m h_n\{\frac{1}{5}s_{11}s_i+\frac{1}{3}(s_{12}+s_{13}s_i)+1\}$$

$$+\{\frac{1}{5}t_{11}t_j+\frac{1}{3}(t_{12}+t_{13}t_j)+ 1\}\Big] \cdot \quad (4.47)$$

where $t_{11} = t_\ell t_m t_n$

$t_{12} = t_\ell t_m + t_m t_n + t_n t_\ell$

$t_{13} = t_\ell + t_m + t_n$

$s_{11} = s_\ell s_m s_n$

$s_{12} = s_\ell s_m + s_m s_n + s_n s_\ell$

$s_{13} = s_\ell + s_m + s_n$

$t_{21} = t_{11}t_j + (t_{11} + t_{12}t_j)t_i$

$t_{22} = (t_{12} + t_{13}t_j) + (t_{13} + t_j)t_i$

$s_{21} = s_{11}s_j + (s_{11} + s_{12}s_j)s_i$

$s_{22} = (s_{12} + s_{13}s_j) + (s_{13} + s_j)s_i$

Similarly other matrices are derived as

$$K_{U_{x_{ij}}} = \frac{s_i a_4}{16}\sum_{\ell=1}^{4}\Big[(\frac{1}{3}s_\ell s_j+1)\{\frac{1}{3}(t_\ell t_i+t_i t_j+t_j t_\ell)+1\}\Big]$$

$$-\frac{t_i a_3}{16}\sum_{\ell=1}^{4}\Big[(\frac{1}{3}t_\ell t_j+1)\{\frac{1}{3}(s_\ell s_i+s_i s_j+s_j s_\ell)+1\}\Big] \cdot \quad (4.48)$$

$$K_{U_{y_{ij}}} = \frac{s_i a_2}{16} \sum_{\ell=1}^{4} \left[\left(\frac{1}{3} s_\ell s_j + 1\right) \left\{ \frac{1}{3} (t_\ell t_i + t_i t_j + t_j t_\ell) + 1 \right\} \right]$$

$$- \frac{t_i a_1}{16} \sum_{\ell=1}^{4} \left[\left(\frac{1}{3} t_\ell t_j + 1\right) \left\{ \frac{1}{3} (s_\ell s_i + s_i s_j + s_j s_\ell) + 1 \right\} \right] \tag{4.49}$$

$$K_{B_{mx_{ij}}} = \frac{1}{768\mu} \sum_{\ell=1}^{4} \sum_{m=1}^{4} \sum_{n=1}^{4} (s_i a_4 s_{23} t_{24} - t_i a_3 s_{24} t_{23}) \tag{4.50}$$

$$K_{B_{my_{ij}}} = \frac{1}{768\mu} \sum_{\ell=1}^{4} \sum_{m=1}^{4} \sum_{n=1}^{4} (s_i a_2 s_{23} t_{24} - t_i a_1 s_{24} t_{23}) \tag{4.51}$$

$$K_{\dot{h}_{ij}} = \frac{-|J|}{4} \left(\frac{1}{3} s_i s_j + 1\right) \left(\frac{1}{3} t_i t_j + 1\right) \tag{4.52}$$

$$K_{v_{d_{ij}}} = K_{\dot{h}_{ij}} \tag{4.53}$$

where $s_{23} = \frac{1}{5} s_\ell s_m s_n s_j + \frac{1}{3} (s_\ell s_m + s_m s_n + s_n s_j + s_j s_\ell) + 1$

$$s_{24} = \frac{1}{5} s_{25} + \frac{1}{3} s_{26} + 1$$

$$s_{25} = s_\ell s_m s_n + s_\ell s_m s_i + s_\ell s_m s_j + s_\ell s_n s_i + \cdots$$

$$s_{26} = s_\ell s_m + s_\ell s_n + s_\ell s_i + s_\ell s_j + s_m s_n + \ldots$$

$$t_{23} = \frac{1}{5} t_\ell t_m t_n t_j + \frac{1}{3} (t_\ell t_m + t_m t_n + t_n t_j + t_j t_\ell) + 1$$

$$t_{24} = \frac{1}{5} t_{25} + \frac{1}{3} t_{26} + 1$$

$$t_{25} = t_\ell t_m t_n + t_\ell t_m t_i + t_\ell t_m t_j + t_\ell t_n t_i + \ldots$$

$$t_{26} = t_\ell t_m + t_\ell t_n + t_\ell t_i + t_\ell t_j + t_m t_n + \ldots$$

4.3 Analysis of The Porous Region

The pressure distribution for a surface with a porous region can be found by solving equation (4.9) together with the Laplace equation (3.53) and the associated boundary conditions (3.54)-(3.57). Fig.10 shows the finite element idealization of a porous region.

By expressing the diffusion term $[K_{v_d}].\{v_d\}$ as the nodal diffusion flow $\{q'\}$ in equation (4.9) and by expressing the fluidity matrix terms $[K_a]$ $\{a\}$, except the pressure term, the rearranged form of the equation is obtained

$$\{q\} = [K_p]\ \{p\} + [K_a]\ \{a\} + \{q'\} \tag{4.54}$$

The Laplace equation (3.53) is solved in a similar manner to the Reynold's equation using variational principles. The functional to be minimized is given by

$$\bar{I}(\bar{p}) = \int_\Omega \left(\frac{\rho\Phi}{\mu}\nabla\bar{p}\nabla\bar{p}\right)d\Omega + \int_\Omega \vec{U}.p\,d\Omega + \int_S \dot{Q}\,p\,dS \tag{4.55}$$

Hence

$$\frac{\partial\bar{I}}{\partial\bar{p}_i} = 0 \qquad\qquad i = 1,2,\ldots,\bar{N} \tag{4.56}$$

The pressure distribution as before is assumed to be linear within an element. For the analysis of the three dimensional porous region tetrahedral elements are used. For these elements the pressure distribution is expressed as

$$\bar{p} = \lfloor M \rfloor\ \{\bar{p}\} = \sum_{i=1}^{4} M_i(x,y,z).\bar{p}_i \tag{4.57}$$

$$M_i = a_i + b_i x + c_i y + d_i z \tag{4.58}$$

where

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} = \text{adj} \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix}$$

$$6V = \begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix} = 6 \quad \text{(Volume of the tetrahedron defined by nodes 1,2,3,4.)}$$

Substituting eqn.(4.57) into eqn.(4.56)

$$\frac{\partial \bar{I}}{\partial \bar{p}_i} = \int_\Omega \frac{\rho\phi}{\mu} \left[\frac{\partial M_i}{\partial x} \sum_{j=1}^{4} \left(\left[\frac{\partial M_j}{\partial x} \right] \{\bar{p}\} \right) + \frac{\partial M_i}{\partial y} \sum_{j=1}^{4} \left(\left[\frac{\partial M_j}{\partial y} \right] \{\bar{p}\} \right) + \frac{\partial M_i}{\partial z} \sum_{j=1}^{4} \left(\left[\frac{\partial M_j}{\partial z} \right] \{\bar{p}\} \right) \right] d\Omega$$

$$+ \int_\Omega M_i \sum_{j=1}^{4} (M_j)\{\bar{U}\} d\Omega + \int_S Q N_i \, dS = 0 \tag{4.59}$$

Using the interpolation function (4.58) with eqn.(4.59) and the integration formula (40)

$$\int_\Omega M_i^\alpha M_j^\beta M_k^\gamma d\Omega = \frac{\alpha!\beta!\gamma!}{(\alpha+\beta+\gamma+3)!} \, 6V \tag{4.60}$$

The flow equation for a porous region is obtained as follows

$$\{\bar{q}\} = [\bar{K}_p] \{\bar{p}\} + \{\bar{q}'\} \tag{4.61}$$

where

$$K_{P_{ij}} = -\frac{\rho\Phi V}{\mu}(b_i b_j + c_i c_j + d_i d_j)$$

$$\overline{q}_i = \int_S Q\, N_i ds \qquad \text{(on surfaces)}$$

$$q_1' = \int_\Omega M_i \sum_{j=1}^{4} [M_j]\{\overline{U}\} d\Omega$$

To couple the analysis pertaining to the film and porous region conditions, equations (4.54) and (4.61) are reordered and partitioned into submatrices as follows

$$\begin{Bmatrix} \overline{q}_1 \\ \overline{q}_2 \end{Bmatrix} = \begin{vmatrix} \overline{K}_{11} & \overline{K}_{12} \\ \overline{K}_{21} & \overline{K}_{22} \end{vmatrix} \begin{Bmatrix} \overline{p}_1 \\ \overline{p}_2 \end{Bmatrix} - \begin{Bmatrix} \overline{q}_1' \\ 0 \end{Bmatrix} \tag{4.62}$$

where $\overline{q}_1$, $\overline{p}_1$, $\overline{q}_1'$ share nodes with the film region. The associated boundary condition (3.54) gives the following relationship for the flow

$$\{q'\} + \{\overline{q}_1'\} = \{0\} \tag{4.63}$$

The pressures at the common boundary of the film and the porous regions (3.55) is given by

$$\{p\} = \{\overline{p}_1\} \tag{4.64}$$

The matrix equation (4.62) can be rewritten as

$$\{\overline{q}_1\} = [\overline{K}_{11}]\{\overline{p}_1\} + [\overline{K}_{12}]\{\overline{p}_2\} - \{\overline{q}'_1\} \tag{4.65}$$

$$\{\overline{q}_2\} = [\overline{K}_{21}]\{\overline{p}_1\} + [\overline{K}_{22}]\{\overline{p}_2\} \tag{4.66}$$

Equations (4.54), (4.63), (4.64) and (4.65) yield

$$\{\bar{q}_1+q\} = [\bar{K}_{11}+K_p]\{\bar{p}_1\} + [\bar{K}_{12}]\{\bar{p}_2\} + [K_a]\{a\} \tag{4.67}$$

and combining eqn.(4.67) and eqn.(4.66) the following result is obtained

$$\begin{Bmatrix} \bar{q}_1+q \\ \bar{q}_2 \end{Bmatrix} = \begin{bmatrix} \bar{K}_{11}+K_p & \bar{K}_{12} \\ \bar{K}_{21} & \bar{K}_{22} \end{bmatrix} \begin{Bmatrix} \bar{p}_1 \\ \bar{p}_2 \end{Bmatrix} + \begin{bmatrix} K_a & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} a \\ 0 \end{Bmatrix} \tag{4.68}$$

Equation (4.68) can be expressed in a simplified form as

$$\{\bar{\bar{q}}\} = [\bar{\bar{K}}]\{\bar{\bar{p}}\} + [\bar{\bar{K}}_a]\{\bar{\bar{a}}\} \tag{4.69}$$

This equation can be solved in the same way as that described in Section 4.2.

4.4 Determination of the Load Carrying Capacity of Bearings, Friction Force and Friction Torque

The load carrying capacity of bearings L_e was written in the form of an integral (3. 58) in Section 3.5. The calculation can by finalized by substituting the pressure values (4.5) into equation (3.58)

$$L_e = \int_A \sum_{i=1}^{r} N_i(x,y)p_i \; dA$$

For the triangular element, the interpolation function is

$$N_i(x,y) = a_i + b_i x + c_i y$$

and using the integration formulae

$$\int_A N_i dA = \frac{1}{3} A$$

the load carrying capacity for the triangular element can be expressed as

$$L_e = \frac{1}{3} A \sum_{i=1}^{3} p_i \tag{4.70}$$

Similarly the load carrying capacity L_e for a rectangular element is given by

$$L_e = \frac{1}{4} A \sum_{i=1}^{4} p_i \tag{4.71}$$

Total load carrying capacity of the domain is

$$L = \sum_{e=1}^{E} L_e \qquad (\text{ E : number of elements}) \tag{4.72}$$

The friction forces in an element are given by eqns.(3.69) - (3.72). Substituting the pressure and other field values from eqns.(4.5), (4.6) into eqns.(3.69) - (3.72), the friction forces in the form of an interpolation function and nodal values can be obtained as follows:

$$F_{x1} = -\frac{1}{2} \int_A \Big(\sum_{i=1}^{r} N_i h_r\Big)\Big(\sum_{j=1}^{r} \frac{\partial N_j}{\partial x} p_j\Big) dA - \frac{1}{2} \int_A \Big(\sum_{i=1}^{r} N_i h_i\Big)\Big(\sum_{j=1}^{r} N_j B_{mx_j}\Big) dA$$
$$- \frac{1}{2} \int_A \Big(\sum_{i=1}^{r} N_i h_i\Big)\Big(\sum_{j=1}^{r} N_j C_{fmx_j}\Big) dA + \mu \int_A \frac{\sum_{j=1}^{r} N_j U_j}{\sum_{i=1}^{r} N_i h_i} dA \tag{4.73}$$

$$F_{x2} = \frac{1}{2} \int_A \Big(\sum_{i=1}^{r} N_i h_i\Big)\Big(\sum_{j=1}^{r} \frac{\partial N_j}{\partial x} p_j\Big) dA + \frac{1}{2} \int_A \Big(\sum_{i=1}^{r} N_i h_i\Big)\Big(\sum_{j=1}^{r} N_j B_{mx_j}\Big) dA$$
$$+ \frac{1}{2} \int_A \Big(\sum_{i=1}^{r} N_i h_i\Big)\Big(\sum_{j=1}^{r} N_j C_{fmx_j}\Big) dA + \mu \int_A \frac{\sum_{j=1}^{r} N_j U_j}{\sum_{i=1}^{r} N_i h_i} dA \tag{4.74}$$

$$F_{y1} = -\frac{1}{2}\int_A (\sum_{i=1}^{r} N_i h_i)(\sum_{j=1}^{r} \frac{\partial N_j}{\partial y} p_j)dA - \frac{1}{2}\int_A (\sum_{i=1}^{r} N_i h_i)(\sum_{j=1}^{r} N_j B_{myj})dA$$

$$- \frac{1}{2}\int_A (\sum_{i=1}^{r} N_i h_i)(\sum_{j=1}^{r} N_j\, c_{fmyj})dA + \mu \int_A \frac{\sum_{j=1}^{r} N_j V_j}{\sum_{j=1}^{r} N_i h_i} dA \qquad (4.75)$$

$$F_{y2} = \frac{1}{2}\int_A (\sum_{i=1}^{r} N_i h_i)(\sum_{j=1}^{r} \frac{\partial N_j}{\partial y} p_j)dA + \frac{1}{2}\int_A (\sum_{i=1}^{r} N_i h_i)(\sum_{j=1}^{r} N_j\, B_{myj})dA$$

$$+ \frac{1}{2}\int_A (\sum_{i=1}^{r} N_i h_i)(\sum_{j=1}^{r} N_j\, c_{fmyj})dA + \mu \int_A \frac{\sum_{j=1}^{r} N_j V_j}{\sum_{i=1}^{r} N_i h_i} dA \qquad (4.76)$$

To simplify the integration of the last term an average thickness $\overline{h}$ is used in place of $\sum N_i h_i$.

For the triangular element, substituting the interpolation function (4.17) into eqns.(4.73) - (4.76), the friction forces derived are

$$F_{x_1} = -\frac{1}{12}\sum_{i=1}^{3} b_i p_i - \frac{A}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}\{h_i(1+\delta_{ij})(B_{mxj}+C_{fmxj})\} + \frac{\mu A}{3\overline{h}}\sum_{i=1}^{3} U_i$$

$$F_{x_2} = \frac{1}{12}\sum_{i=1}^{3} b_i p_i + \frac{A}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}\{h_i(1+\delta_{ij})(B_{myj}+C_{fmxj})\} + \frac{\mu A}{3\overline{h}}\sum_{i=1}^{3} U_i \qquad (4.77)$$

$$F_{y_1} = -\frac{1}{12}\sum_{i=1}^{3} c_i p_i - \frac{A}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}\{h(1+\delta_{ij})(B_{myj}+C_{fmyj})\} + \frac{\mu A}{3\overline{h}}\sum_{i=1}^{3} V_i$$

$$F_{y_2} = \frac{1}{12}\sum_{i=1}^{3} c_i p_i + \frac{A}{6}\sum_{i=1}^{3}\sum_{j=1}^{3}\{h_i(1+\delta_{ij})(B_{myi}+C_{fmyj})\} + \frac{\mu A}{3\overline{h}}\sum_{i=1}^{3} V_i$$

For the rectangular element, substituting the interpolation function (4.32) into eqns.(4.73) - (4.76), and using transformations (4.31) - (4.37) the friction forces for a rectangular element can be expressed as

$$\begin{aligned} F_{x1} &= - C_{100} - C_{101} - C_{102} + C_{103} \\ F_{x_2} &= \ \ C_{100} + C_{101} + C_{102} + C_{103} \\ F_{y1} &= - C_{100} - C_{104} - C_{105} + C_{106} \\ F_{y2} &= \ \ C_{100} + C_{104} + C_{105} + C_{106} \end{aligned} \tag{4.78}$$

where

$$C_{100} = 2|J|\{\frac{1}{3} C_3C_6 + (C_1C_5 + \frac{1}{3} C_4C_7)\}$$

$$C_{101} = 2|J|\{(C_1C_8 + \frac{1}{3} C_4C_{11}) + \frac{1}{3} (C_3C_{10} + \frac{1}{3} C_2C_9)\}$$

$$C_{102} = 2|J|\{(C_1C_{12} + \frac{1}{3} C_4C_{15}) + \frac{1}{3} (C_3C_{14} + \frac{1}{3} C_2C_{13})\}$$

$$C_{103} = \frac{\mu|J|}{\bar{h}} \sum_{i=1}^{4} U_i$$

$$C_{104} = 2|J|\{(C_1B_8 + \frac{1}{3} C_4B_{11}) + \frac{1}{3} (C_3B_{10} + \frac{1}{3} C_2B_9)\}$$

$$C_{105} = 2|J|\{(C_1B_{12} + \frac{1}{3} C_4B_{15}) + \frac{1}{3} (C_3B_{14} + \frac{1}{3} C_2B_{13})\}$$

$$C_{106} = \frac{\mu|J|}{\bar{h}} \sum_{i=1}^{4} V_i$$

$$C_1 = \frac{1}{4} \sum_{i=1}^{4} h_i$$

$$C_2 = \frac{1}{4} \sum_{i=1}^{4} s_it_ih_i$$

$$C_3 = \frac{1}{4} \sum_{i=1}^{4} s_ih_i$$

$$C_4 = \frac{1}{4} \sum_{i=1}^{4} t_ih_i$$

$$C_5 = \frac{1}{4} \sum_{i=1}^{4} (s_ia_4 - t_ia_3)p_i$$

$$C_6 = - \frac{1}{4} \sum_{i=1}^{4} a_3s_it_ip_i$$

$$C_7 = \frac{1}{4} \sum_{i=1}^{4} a_4s_it_ip_i$$

$$C_8 = \frac{1}{4} \sum_{i=1}^{4} B_{mx_i} \qquad C_{12} = \frac{1}{4} \sum_{i=1}^{4} C_{fmx_i}$$

$$C_9 = \frac{1}{4} \sum_{i=1}^{4} s_i t_i \; B_{mx_i} \qquad C_{13} = \frac{1}{4} \sum_{i=1}^{4} s_i t_i \; C_{fmx_i}$$

$$C_{10} = \frac{1}{4} \sum_{i=1}^{4} s_i \; B_{mx_i} \qquad C_{14} = \frac{1}{4} \sum_{i=1}^{4} s_i \; C_{fmx_i}$$

$$C_{11} = \frac{1}{4} \sum_{i=1}^{4} t_i \; B_{mx_i} \qquad C_{15} = \frac{1}{4} \sum_{i=1}^{4} t_i \; C_{fmx_i}$$

$$B_8 = \frac{1}{4} \sum_{i=1}^{4} B_{my_i} \qquad B_{12} = \frac{1}{4} \sum_{i=1}^{4} C_{fmy_i}$$

$$B_9 = \frac{1}{4} \sum_{i=1}^{4} s_i t_i \; B_{my_i} \qquad B_{13} = \frac{1}{4} \sum_{i=1}^{4} s_i t_i \; C_{fmy_i}$$

$$B_{10} = \frac{1}{4} \sum_{i=1}^{4} s_i \; B_{my_i} \qquad B_{14} = \frac{1}{4} \sum_{i=1}^{4} s_i \; C_{fmy_i}$$

$$B_{11} = \frac{1}{4} \sum_{i=1}^{4} t_i \; B_{my_i} \qquad B_{15} = \frac{1}{4} \sum_{i=1}^{4} t_i \; C_{fmy_i}$$

This leads to the determination of friction forces in an element. Further more the total friction forces are obtained by summing these values over the domain area, as follows

$$\begin{aligned} F_{T_{x_1}} &= \sum_{i=1}^{E} F_{x_{1_i}} \\ F_{T_{x_2}} &= \sum_{i=1}^{E} F_{x_{2_i}} \\ F_{T_{y_1}} &= \sum_{i=1}^{E} F_{y_{1_i}} \\ F_{T_{y_2}} &= \sum_{i=1}^{E} F_{y_{2_i}} \end{aligned} \qquad (4.79)$$

For the rotational disc problems the evaluation of friction torque becomes necessary, since it is a measure of the disc performance.

The friction torque T_e for an element as shown in FIG.7 is given by

$$T_e = (F_x \sin\theta + F_y \cos\theta) \cdot r \cdot A \tag{4.80}$$

the coordinates of the point C are

$$x_c = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$y_c = \frac{1}{n} \sum_{i=1}^{n} y_i$$

where (x_i, y_i) : the coordinates of each node in the element

n : number of nodes in the element

and the radius r can be expressed as

$$r = (x_c^2 + y_c^2)^{\frac{1}{2}} \tag{4.81}$$

By substituting eqn.(4.81) into eqn.(4.80) the elemental torque is given by the following expression

$$T_e = (F_x \sin\theta + F_y \cos\theta)(x_c^2 + y_c^2)^{\frac{1}{2}} \cdot A \tag{4.82}$$

The total friction torque T_T is then calculated by summing the elemental torques over the domain area and is found to be

$$T_T = \sum_{e=1}^{E} T_e \tag{4.83}$$

where E : number of elements in the domain area

4.5 Development of the Computer Program

In this section listings of the computer program developed for the lubrication analysis is presented. The program was written in standard FORTRAN for the PRIME 400 Computer of Loughborough University of Technology.

4.5.1 Flow Charts

The computer program developed in this work consists of several subroutines. FIG.11 contains the main flow chart required in the calculations and the subroutine system is listed in FIG.12. The purpose of each subroutine is also presented in TABLE.1, and details of the important subroutines are shown in FIG.13 to 17. The complete program is presented from page 136 onwards.

4.5.2 Data input

As a guide for using the program, details of input data are presented here.

1. RESULT ; Are details of calculations needed ? YES or NO
2. COOR ; Coordinate system Cartesian or polar coordinate? XY or PO
3. UNIT1; Only when polar coordinate is used, input the unit of angle.

   DEG or RAD
4. IUNIT2 ; Which velocity unit is used, XY or angular vel. ?

   Input 1 for XY, or 2 for angular vel.. ☐
5. TEST NAME ; Restricted to 80 characters ☐
6. ELEMENT DETAILS ; Input NCOE, NODES, NELE, NNEL. ☐ ☐ ☐ ☐

   where NCOE : Number of dimension (two-dimension:2)

   NODES: Total number of nodes

   NELE : Total number of elements

   NNEL : Number of nodes per element
7. DENVIS ; Is density and viscosity constant throughout the system?

   YES or NO
8. INPUT DENSITY AND VISCOSITY OF EACH ELEMENT

ELEMENT No.	ELEMENT TYPE	NNEL	DENSITY	VISCOSITY
☐	☐	☐	☐	☐

(example) 1 3 3 1.0E-10 1.0

2 3 3 1.3E-9 1.7

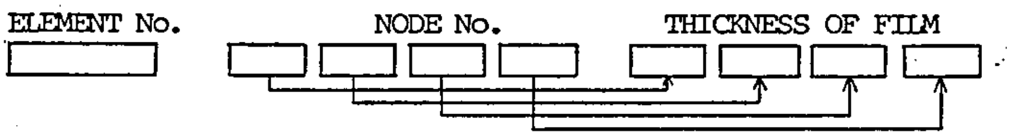
3 3 3 1.1E-10 1.0

(ELEMENT TYPE)

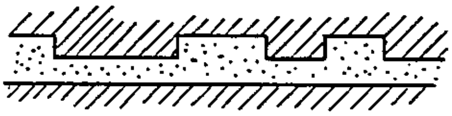
3 : triangular

4 : rectangular

9. INPUT NODE No. AND THICKNESS OF FILM AT EACH NODE

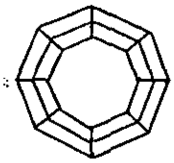


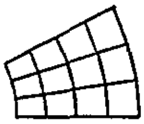
(note) Change in thickness is discribed in the change of upper surface, therefore lower surface is treated flat.



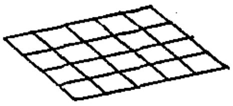
10. LTYPE ; Choose the type of mesh generation. $\boxed{1}$ or $\boxed{2}$ or $\boxed{3}$
where

LTYPE 1 Circular or sector shape system meshed uniformly

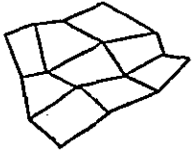




LTYPE 2 Parallelogram shape system meshed equally

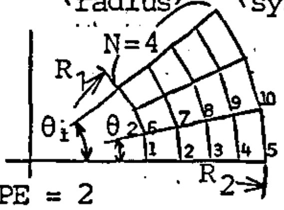


LTYPE 3 Arbitrary shape system



11. Only when LTYPE = 1

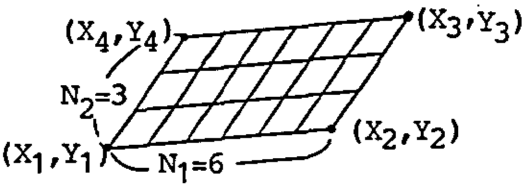
☐	☐	☐	☐	☐
R_1	R_2	θ_1	θ_2	N
(inner radius)	(outer radius)	(angle of system)	(angle of division)	(number of division through radius)



when circular shape is used, input $(360-\theta_2)$ for θ_1

12. Only when LTYPE = 2

☐	☐	☐	☐	☐	☐
X_1,Y_1	X_2,Y_2	X_3,Y_3	X_4,Y_4	N_1	N_2



13. Only when LTYPE = 3

☐	☐
ELEMENT No.	X , Y

(ex.) 1 2.0 1.0
2 1.5 1.0
3 1.0 1.0
⋮
(NELE)

14. BOUNDARY CONDITION NQ and NP ☐ ☐

where

NQ : Number of nodes where flow values are known as boundary conditions

NP : Number of nodes where pressure is known as a boundary condition

(note. NQ + NP = NODES)

15. BC ; Are values of boundary conditions same throughout the whole system ? $\boxed{\text{YES}}$ or $\boxed{\text{NO}}$

16. Only when BC = YES , input the numbers of node where pressures are known as boundary condition

☐ (ex.) 1 2 4 6 7

Then input the values of boundary conditions

☐	☐
value of flow	value of pressure

17. Only when BC = NO , input BC type and value at each node

	☐	☐	☐
	NODE No.	BC TYPE	VALUE OF BC
(ex.)	1	2	0.0
	2	1	0.0
	3	2	1.0

(BC TYPE)
1 : flow is known
2 : pressure is known

18. BCA ; Are flow action values (shear action, body force etc) same throughout the system ? $\boxed{\text{YES}}$ or $\boxed{\text{NO}}$

19. BCAC ; Input the values of flow actions at each node

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
NODE No.	UX1	UX2	UY1	UY2	BX1	BX2	BY1	BY2	$\frac{\partial H}{\partial t}$	Vd

where
UX1 : velocity of lower surface in X-direction
UX2 : velocity of upper surface in X-direction
UY1 : velocity of lower surface in Y-direction
UY2 : velocity of upper surface in Y-direction
BX1 : body force of lower surface in X-direction
$\frac{\partial H}{\partial t}$: squeeze velocity in Z-direction
Vd : diffusion velocity in porous surface

when angular velocity is used, input format is as follows

☐	☐	☐	$\boxed{0.0}$	$\boxed{0.0}$	☐	☐	☐	☐	☐	☐
NODE No.	ω_1	ω_2	0.0	0.0	BX1	BX2	BY1	BY2	$\frac{\partial H}{\partial t}$	Vd

where
ω_1 : angular velocity of lower surface
ω_2 : angular velocity of upper surface

5. Application to Standard Lubrication Problems

The validation of the finite element analysis outlined in Chapter 4 was established by its application to standard lubrication problems and making a comparison with the results obtained from analytical solutions of Reynolds equation. These finite element idealizations and the associated software were used to solve such problems as the rectangular squeezing pad, slider bearing, step bearing etc. The squeezing rectanguler pad problem was first investigated to determine the accuracy of including the squeezing effect in the analysis and afterwards the slider bearing and step bearing problems were considered to deal with lubricated surfaces of both infinite and finite width.

For all the analyses discussed in this chapter, the film viscosity is taken as

$$\mu = 0.102 \ \mathrm{kg \cdot sec} / \mathrm{m}^2$$

5.1 Rectangular Squeezing Pad

5.1.1 Theoretical Analysis

The geometry of a rectangular pad is shown in FIG.18 and the pressure distribution in such pads has been determined by the solution of Reynolds equation using variational techniques (3). The pressure on the pad surface can be expressed by the equation

$$p = \frac{\mu A^2}{h^3} \frac{\partial h}{\partial t} \sum_{k,}^{\infty} \sum_{\ell=1,3,5,\cdots}^{\infty} B_{k\ell} \sin k\phi \cos \theta \qquad (5.1)$$

where

$$\phi = \frac{\pi x}{A}$$

$$\theta = \frac{\pi y}{B}$$

$$B_{k\ell} = \frac{192R^2}{\pi^4 k\ell(R^2\ell^2 + k^2)} \qquad k,\ell = 1,3,5,\ldots\infty$$

$$R = \frac{B}{A}$$

and the corresponding load carrying capacity is obtained by integrating the pressure over the area to give the infinite series

$$L = \frac{4\mu A^3 B}{\pi^2 h^3}\frac{\partial h}{\partial t} \sum_{k,\ell=1,3,5,\cdots}^{\infty}\sum^{\infty} \frac{1}{k\ell} B_{k\ell} \qquad (5.2)$$

5.1.2 Finite Element Model and Results

A finite element model was developed for the determination of pressure distribution in the pad. Owing to the symmetry of the bearing, only a quarter of the pad was idealized. Triangular fluid finite elements were used for this idealization which are shown as (a) to (e) in FIG.19. The properties of the pad and fluid used in the calculation were as follows

$A = 1.0$ m

$B = 1.0$ m

$R^2 = 1.0$

$\mu = 0.102 \times 10^{-5}$ kg·sec / mm^2 (viscosity)

$h = 0.01$ mm (film thickness)

$\frac{\partial h}{\partial t} = 10$ mm / sec (squeezing velocity)

The circumferential pressure was assumed to be zero in this case.

The pressure distribution in the pad at $y = 0.0$ along the direction and along the diagonal as predicted by the theoretical approach and the finite element method are shown in FIG's 20,21 and 22. The effect of the finite element mesh size on the convergence of results is shown in TABLE 2 for the maximum pressure and TABLE 3 for the load carrying capacity of the pad. In spite of the assumption of linear pressure within the elements, very satisfactory agreement between the two approaches was obtained as is obvious from the figures. The maximum error obtained was of the order of 6.20 % when using 8 elements. It appears that the maximum pressure at the midpoint is overestimated for the coarser mesh elements. This is to be expected because of the nature of the formulation. However these results can be considerably improved to give less than 1 % error by utilizing a finer finite element mesh.

5.2 Infinite Width Slider Bearing

5.2.1 Theoretical Analysis

As one of the various analyses of standard lubrication problems, the case of an infinite width slider bearing shown in FIG.23 is studied in this section. The properties of the bearing and the fluid used in this analysis are as follows

$B = 0.5$ m

$h_o = 1.0 \times 10^{-5}$ m

$U_x = -15.0$ m / sec (velocity)

$\mu = 0.102$ kg sec / m² (viscosity)

The pressure distribution has been given in many references (38), (39), and can be expressed in the form

$$p = \frac{6}{h_o^2}\mu\, U_x B\, K_p \tag{5.3}$$

where

$$K_p = \frac{1}{m}\left\{\frac{2\,m + 2}{-2(2+m)(1+m\frac{x}{B})^2}\right\} + \frac{1}{(1 + m\frac{x}{B})} - \frac{1}{2 + m}$$

$$m = \frac{h - h_o}{h_o}$$

Again the corresponding load carrying capacity is obtained by integrating the pressure over the plate area of unit width and is given by the equation

$$L = \frac{6\,\mu U B^2}{h^2} K_p \tag{5.4}$$

and the friction force at the two bearing surfaces is expressed by the equation

$$F_h = -\frac{\mu U}{h_o} B \frac{\ell og_e(\,1 + m\,)}{m} + \frac{W\,m\,h_o}{2\,B} \qquad \text{(at } h{=}h\text{)}$$

$$F_o = -\frac{\mu U}{h_o} B \frac{\ell og_e(1 + m)}{m} - \frac{W\,m\,h_o}{2\,B} \qquad \text{(at } h{=}0\text{)} \tag{5.5}$$

5.2.2 Finite Element Model and Results

Finite element layouts of a slider bearing are shown as (a) to (c) in FIG.24. Number of elements was chosen to be 20, 60 and 240 respectively in order to examine the effects of the mesh size on the convergence of results. The calculation is made by specifying zero oil flow in the y direction as the width in y direction is assumed to be infinite and the circumferential pressure is assumed to be zero. The effect of graded mesh is also studied in this section using 20 triangular elements. The graded mesh patterns are shown in TABLE 8. Five patterns of grading are studied.

The results of pressure distribution are shown in TABLE 4 and these values are also compared with those derived from the theoretical solution and another finite element solution (13). The percentage errors in the pressures at different values of x are also given in TABLE 5 and it can be seen that if the number of elements is around 20 the percentage error is less than 1 %. The magnitude of errors of load carrying values and friction forces are presented in TABLEs 6 and 7 and again accurate results are obtained when the number of elements is approximately 20. The effect of the grading of meshes is also examined and the results are shown in TABLE 8. Five grading patterns were decided as follows

(a) : regular mesh

(b) : converged mesh around the area where maximum pressure is assumed to be obtained and roughly meshed in other area (c): converged more intensely around the area of maximum pressure from the result of (b)

(d): converged more intensely from the result of (c)

(e): converged more intensely from the result of (d)

It can be seen from the results in TABLE 8 that the grading of meshes is extremely effective method of getting more accurate results using a certain elements. The percentage error of pressure in grading pattern (c) is less than 3 % while that of regular meshing pattern (a) is 12 %. However, extreme convergence of grading mesh gives poorer accuracy which can be seen in the results of patterns (d) and (e).

5.3 Step Bearing

The significance of the finite element analysis of a step bearing lies in its ability to study the effects of abrupt changes in the film thickness, which leads to the investigation of the effect of oil grooves of discs on the moving surfaces. The configuration of groove surfaces can be assumed to be the combination of two step bearings whose geometry is illustrated in FIG.25.

5.3.1 Infinite Width Step Bearing

The theoretical solutions of an infinite width step bearing (40) to predict the pressure distribution in the two flow regions are given by the equations

$$p = \frac{p^*}{c_1} x \qquad (\text{ region (I): } 0 \leqq x \leqq c_1)$$

$$p = \frac{p^*}{c_2 - c_1} (c_2 - x) \qquad (\text{ region (II): } c_1 \leqq x \leqq c_2) \qquad (5.6)$$

where

$$p^* = \frac{6 \mu U}{h_1^2} (h^* - 1) \Big/ \left(\frac{h^{*3}}{c_2 - c_1} + \frac{1}{c_1} \right) \qquad (= \text{maximum pressure})$$

$$h^* = \frac{h_2}{h_1}$$

and the corresponding load carrying capacity per unit width can be determined from the relationship

$$L = \frac{1}{2} P_o \, c_2 \tag{5.7}$$

5.3.2 Finite Width Step Bearing

Theoretical solutions of the pressure distributions are obtained by using a Fourier sine series expansion (40) and is described in series form as

$$p = \sum_{n=1,3,5\cdots}^{\infty} \frac{P_n}{\sinh \frac{n\pi c_1}{b}} \sin \frac{n\pi z}{b} \sinh \frac{n\pi x}{b} \qquad : \text{region (I)} \tag{5.8}$$

$$p = \sum_{n=1,3,5\cdots}^{\infty} \frac{P_n}{\sinh \frac{n\pi(c_2-c_1)}{b}} \sin \frac{n\pi z}{b} \sinh \frac{n\pi(c_2-x)}{b} \qquad : \text{region (II)} \tag{5.9}$$

where

$$P_n = \frac{24\,\mu U\, b\,(h_2 - h_1)}{n^2\,\pi^2 [h_1^3 \coth \frac{n\pi c_1}{b} + h_2^3 \coth \frac{n\pi(c_2 - c_1)}{b}]}$$

The corresponding load carrying capacity of the bearing is given by the equation

$$L = \sum_{n=1,3,5\cdots}^{\infty} \frac{2\,b^2\,P_n}{n^2\,\pi^2} \left[\frac{\cosh \frac{n\pi c_1}{b} - 1}{\sinh \frac{n\pi c_1}{b}} + \frac{\cosh \frac{n\pi(c_2 - c_1)}{b} - 1}{\sinh \frac{n\pi(c_2 - c_1)}{b}} \right] \tag{5.10}$$

5.3.3 Finite Element Models and Results

Bearing size and film properties used in this analysis were taken as follows

$c_1 = 0.5$ m

$c_2 = 0.6$ m

$b = 1.1$ m (for a finite width bearing)

$h_1 = 1.7 \times 10^{-5}$ m

$h_2 = 1.0 \times 10^{-5}$ m

$U = 15.0$ m/sec

and the element layouts used in this bearing analysis are shown as (a) to (c) in FIG.26.

Results of the pressure distribution of an infinite width bearing are shown graphically in FIG.27 and since the pressure of an infinite width bearing linearly distributed in the x direction, the computed results show good agreement with these obtained theoretically.

Results of a finite width step bearing are also presented in FIG.28 and these results show that as finite element mesh size approaches that taken in FIG.26 (c) the pressure distribution is within two or three percent of that calculated from the theoretical solution.

In summarizing the above the finite element technique is particularly amenable for the solution of lubrication problems and is a very versatile tool as it is capable of dealing with complex geometrical shapes that cannot be readily solved by conventional analytical methods.

6. Application to Annular Disc Problems

The finite element technique developed in this work was applied to the annular disc problems as a first step to studying the performance of oil-immersed brakes.

Most investigations (32),(33),(34),(35),(36) and (37) of the behaviour of an oil film between discs consider rotating and squeezing effects between flat surfaces. However, most brake discs used in practice have grooves cut in on the surfaces in order to supply cooling oil efficiently over the surfaces especially during long brake applications. Various kinds of grooving patterns have been experimented with but most popular are radial and spiral grooves. The effects of the grooves on the hydrodynamic behaviour have been studied by only a few investigators (35),(37), and little details of the pressure distribution on the grooved surfaces have been presented.

Extending our knowledge of the flow behaviour between rotating discs is the purpose of this chapter which is divided into two sections. In the first section characteristics of the film between flat discs have been examined and results have been compared with the theoretical solutions, and in the second section the effects of grooves on the pressure distribution have been studied. Typical radial and spiral groove patterns were chosen for this investigation.

6.1 Behaviour of the Film between Flat Discs

6.1.1 Theoretical Analysis

FIG.29 shows the geometry of a disc. The pressure distribution of the fluid film between such two flat discs with rotating and squeezing motions has been investigated both theoretically and experimentally (34), and the derivative of the pressure has been expressed in the form

$$\frac{\partial p}{\partial r} = \frac{6\mu \frac{\partial h}{\partial t}}{h^3}(r - \frac{r_o^2}{r}) + 0.3 \rho\omega^2 r \qquad (6.1)$$

with boundary conditions

$$p = 0 \qquad \text{at} \quad r = r_1, r_2 \qquad (6.2)$$

where

r_o denotes a radius of flow separation.

Integrating eqn (6.1) with regard to r enables the pressure distribution to be determined from the equation

$$p = (3\mu \frac{\frac{\partial h}{\partial t}}{h^3} + 0.15\rho\omega^2) r^2 - \frac{6\mu \frac{\partial h}{\partial t}}{h^3} r_o^2 \, \ell og|r| + c \qquad (6.3)$$

The radius of flow separation r_o can be obtained by substituting the boundary conditions (6.2) into eqn(6.3) and

$$r_o^2 = (0.5 + 0.025 \frac{\rho\omega^2 h^3}{\mu \frac{\partial h}{\partial t}}) \frac{(r_1^2 - r_2^2)}{\ell og|r_1| - \ell og|r_2|} \qquad (6.4)$$

Also the integration constant c is obtained by substituting eqns. (6.2) and (6.4) into eqn.(6.3) so that

$$c = (\frac{3\mu \frac{\partial h}{\partial t}}{h^3} + 0.15\rho\omega^2) \left[(r_1^2 - r_2^2) \frac{\ell og|r|}{\ell og\frac{|r_1|}{|r_2|}} - r_1^2 \right] \qquad (6.5)$$

A maximum pressure p_{max} is calculated at $\frac{\partial p}{\partial r} = 0$ and

$$p_{max} = \left(3\mu \frac{\frac{\partial h}{\partial t}}{h^3} + 0.15\rho\omega^2 \right) r^{*2} - \frac{6\mu \frac{\partial h}{\partial t}}{h^3} r_o^2 \, \ell og \left| r^* \right| + c \quad (6.6)$$

where r^* is the radius where $\frac{\partial p}{\partial r} = 0$ and r^{*2} is expressed as

$$r^{*2} = \frac{6\mu \frac{\partial h}{\partial t}}{h^3} r_o^2 \Big/ \left(\frac{6\mu \frac{\partial h}{\partial t}}{h^3} + 0.3\rho\omega^2 \right) \quad (6.7)$$

Calculations have been made using the above equation where the properties of the discs and the fluid are taken as follows

$r_1 = 1.0$ m

$r_2 = 2.0$ m

$h = 1.0, \ 0.5$ mm

$\mu = 1.0$ kg sec / m^2

$\frac{\partial h}{\partial t} = -1.0$ m / sec

$\omega = 0.0 , 1.0 , 2.0 , 3.0 , 4.0$ rad / sec

6.1.2 Finite Element Model and Results

Two types of finite element layout shown in FIG.30 and FIG.31 have been used for the analyses of disc problems.

A 60 degree sector shown in FIG.30 can be applied to the non-rotating disc problems, however, when the discs rotate, neither oil pressure nor oil flow can be determined at the boundaries denoted by the edge nodes (numbered 1,2,3,4,5,31,32,33,34,35). The pressures have to be calculated at those nodes. And at the node 2 in FIG.30 for example, the flow q_2 is expressed as follows

$$q_2 = (\ q_2 \text{ in the element } E_1) + (\ q_2 \text{ in the element } E_2) + (q_2 \text{in the element } E_3)$$

as each flow in each element at node 2 can not be specified, total flow at node 2 q_2 remains unknown. However, at node 7, the total flow q_7 can be expressed as follows

$$\begin{aligned} q_7 &= (q_7 \text{ in the element } E_2) + (q_7 \text{ in } E_3)+(q_7 \text{ in } E_4) \\ &\quad +(q_7 \text{ in } E_9) + (q_7 \text{ in } E_{10}) + (q_7 \text{ in } E_{11}) \\ &= 0 \end{aligned}$$

so the boundary condition can be specified as $q_7 = 0$. When applying symmetrical abbreviation to the element layout this kind of consideration has to be included .

Results of both non-rotating and rotating discs with flat surfaces are presented in FIG.32.

For the squeeze motion analysis both the theoretical result and that given by the finite element method show a very good agreement at low speeds of rotation, the effect of inertia appears to be rather larger at high rotational speeds.

6.2 Behaviour of the Film between Grooved Discs

Many groove patterns, some of which are presented in FIG.33, have been practically used for brake and clutch discs. Radial, spiral and waffle patterns are largely adopted both for papercomposited discs and for sintered alloy discs. However, in many cases the depth and size of the groove configurations have been chosen on the basis of experimental investigations.

The finite element technique developed in this work can readily be applied to investigate the pressure distribution of the film between grooved discs. Two typical groove patterns, namely, radial and spiral patterns have been chosen for this investigation. Finite element idealizations for these patterns using 320 triangular elements are shown in FIG.34, and FIG.35. General data for the calculation are given below.

inside radius of a disc = 1.0 m
outside radius of a disc = 2.0 m
depth of groove = 0.5 mm
film thickness = 1.0 mm
viscosity = 1.0 kg sec / m^2
squeezing speed = -1.0 m / sec
angular velocities = 0.0 , 1.0 rad / sec

These values are chosen to enable a comparison to be made with results of the flat surface disc problem analyzed in the previous section.

Calculated results of pressure distributions are presented in FIGS.36 to 43. FIGS.36 and 37 show the effects of groove pattern to the pressure distribution of discs with simple squeezing motion and complex squeezing and rotating motions respectively. FIGS.38 and 39 show the effects of rotating motion on radially and spirally grooved discs respectively. The contour diagrams of the pressure distribution of radially grooved discs are shown in FIGS.40 and 41, and those of spirally grooved discs are shown in FIGS.42 and 43. These four figures are presented in order that the distortion of the pressure distribution throughout the disc surface should be understood.

Results of pressure distributions, that is, variation of pressure with θ, of radially and spirally grooved discs during single squeezing motion compared with the results of the grooveless disc are shown in FIG.36. Maximum pressures are seen at the center of disc facings and minimum pressures are seen at the center of grooves. It is found that grooves on the disc surface reduce the pressure greatly and that radial grooves cause a higher maximum pressure, a lower minimum pressure and more drastic change of the pressure over the surface than spiral grooves which give a more even pressure over the surface.

Pressure distributions during squeezing and rotating motions are presented in FIG.37. These curves indicate that the pressure decreases because of the centrifugal force and grooves affect the drop of pressures more than flat surface. The maximum pressure decreases in the radially grooved discs with increase of rotating speed. Differences between maximum and minimum pressures become greater in both radially and spirally grooved discs when discs rotate.

In the radially grooved disc the pressure at the groove is lower since the length of the groove is shorter and the width of the groove in the oil flow direction is wider, both of which reduce the resistance to flow pass more in radial groove than in spiral groove.

Rearrangement of results in accordance with the rotating motion effect on radially grooved disc is shown in FIG.38 and that on spirally grooved disc is shown in FIG.39. It is more clearly seen that the effect of the rotating motion reduces both maximum and minimum pressures in radially grooved discs, however, in spirally grooved discs the maximum pressure keeps the same level but only minimum pressure reduces.

FIGS.38 and 39 also indicate that the positions of peak pressures are moved in the circumferential direction in accordance with the rotating motion. The peak points move about three degrees in radially grooved discs, while ten degrees in spirally grooved discs.

Each value of calculated maximum and minimum pressures is presented in TABLE.9. It is found that grooves on the disc surface reduce the pressure greatly to about 80 % of that of the flat surface and the effect of rotating motion on pressure distributions is greater when using grooved discs.

The distortion of the pressure distribution due to the pattern of grooves and the rotating motion throughout the disc surface can be understood more clearly by using contour plots which are shown in FIGS.40 to 43. By comparing FIG.40 and FIG.41, it is found that the pressure distributes very simply and with less distortion on radially grooved discs, and that the points of maximum pressure move only in circumferential direction and not in radial direction when discs rotate.

Contours of the pressure distribution of the spirally grooved discs shown in FIGS.42 and 43 indicate that spiral grooves cause the greater distortion of pressure distribution than radial grooves. The points of maximum pressure are also found to move only in the circumferential direction when discs rotate.

It is of great interest that the pressure gradient at the area A_p shown in FIG.43 is very large while this phenomenon is not found in the result of radially grooved discs in FIG.41. This indicates that in spiral grooves pressure goes down to the atmospheric pressure even inside the grooves. Contour diagrams are very useful figures for researchers to understand the overall pressure distribution of complicated configurations.

7. Conclusions

A finite element application to lubrication problems which includes rotating annular discs has been successfully developed. Although limited to incompressible isothermal conditions, the solution of the generalized Reynolds equation developed in this work includes various effects such as shear force, body force, squeeze and diffusion effects. Furthermore, inertia effects have also been considered as these are essential when investigating disc problems. Thickness of the oil film can be varied within an element, which overcomes irregular configurations of the film thickness such as grooving.

The finite element technique has been validated by solving standard lubrication problems such as squeezing pad, slider bearing and step bearing. The comparison with the theoretical results presented in chapter 5 shows very satisfactory agreements between two methods. The increasingly finer grading of meshes generally provides a better accuracy and this has also been established in this work.

The results of flat disc problems show that the finite element technique developed here can be a powerful tool for the investigation of clutch disc problems, however, irregular configuration of surfaces such as groovings requires the finite element idealization of the whole disc instead of considering symmetrical sections of the disc. The inertia effect is found to be greater than theoretical results which are based on the assumption of the Couette flow in the tangential direction.

The investigation of the grooved disc problems by using the finite element technique presented in chapter 6 leads to the following conclusions.

1. Radial grooves cause higher pressure and greater pressure gradients than spiral grooves.
2. Rotating motion affects the oil film pressure more in radially grooved surfaces than in spirally grooved surfaces. The pressure decreases more rapidly in radially grooved discs than in spirally grooved discs as the discs increase in speed. This indicates that the engaging speed, which is represented by the surface velocity in z direction, is higher in radially grooved discs than in spirally grooved discs for a given squeezing pressure.

As for future research, wider and more intensive investigations of discs are worthwhile for the analyses of brake discs. Also the iterative calculation will enable the thermal analyses of dynamic engaging characteristics of wet type clutch discs to be made.

REFERENCES

(1) HAYS, D.F., "A Variational Approach to Lubrication Problems and the Solution of the Finite Journal Bearing", Journal of Basic Engineering A.S.M.E. Series D, 81 (1959) pp.13-23.

(2) HAYS, D.F., "Squeeze Film: A Finite Journal Bearing with a Fluctuating Load", Journal of Basic Engineering, A.S.M.E. Series D, 83 (1961), pp.579-588.

(3) HAYS, D.F., "Squeeze Film for Rectangular Plates", Journal of Basic Engineering, A.S.M.E. Series D, 58 (1963), pp.243-246.

(4) MOORE, D.F., "On the Inclined Non-Inertial Sinkage of a Flat Plate", Journal of Fluid Mechanics, 20 (1964), pp.321-330.

(5) TIPEI, N., Theory of Lubrication, Stanford University Press, Stanford, Calif. (1962) pp.168.

(6) REDDI, M.M., "Finite Element Solution of the Incompressible Lubrication Problem", Journal of Lubrication Technology, A.S.M.E., Series F, 91 (1969), pp.524-533.

(7) REDDI, M.N., and CHU, T.Y., "Finite Element Solution of the Steady State Compressible Lubrication Problem", Journal of Lubrication Technology, A.S.M.E., Series F, 22 (1970), pp.495-503.

(8) WADA, S., HAYASHI, H., and MIGITA, M., "Application of Finite Element Method to Hydrodynamic Lubrication Problems (Part 1, Infinite-Width Bearings)", Bulletin of Japan Society of Mechanical Engineers, 14 (1971), pp.1222-1233.

(9) WADA, S. and HAYASHI, H., "Application of the Finite Element Method to Hydrodynamic Lubrication Problems (Part 2, Finite Width Bearing)", Bulletin of Japan Society of Mechanical Engineers, 14 (1971), pp.1234-1244.

(10) ALLAN, T., "The Application of Finite Element Analysis of Hydrodynamic and externally Pressurized Pocket Bearings", Wear 19 (1972), pp.169-206.

(11) BOOKER, J.F. and HUEBNER, K.H., "Application of Finite Elements to Lubrication: An Engineering Approach", Journal of Lubrication Technology, A.S.M.E., Series F, 24 (1972), pp.313-323.

(12) HUEBNER, K.H., "Application of Finite Element Methods to Thermodydrodynamic Lubrication", Int. Journal for Numerical Methods in Engineering, 8 (1974), pp.139-165.

(13) STAFFORD, A.C., "The Application of Finite Element Methods in the Field of Lubrication", Ph.D. Thesis, Nottingham University, (1979).

(14) Pafec Ltd., 'PAFEC 75', Pafec Ltd., Nottingham (1978).

(15) JAIN, S.C., SINHASAN, R. and SINGH, D.V., "Effect of Bearing Pad Deformation on the performance of Finite Fixed-Pad Slider Bearing", Wear, 76 (1982), pp.189-198.

(16) ALLAIRE, P.E., NICHOLAS, J.C. and GUNTER, E.J., "Systems of Finite Elements for Finite Bearings", Journal of Lubrication Technology, A.S.M.E., Series F, (1977) pp.187-197.

(17) DAS, P.K. and DANCER, S.B., "An Analysis of Flow and Friction in Diesel Engine Bearings", 14th Int. Congress on Combustion Engines, Helsinki,(1981) pp.D80-1 -D80-22.

(18) TAYAL, S.P., SINHASAN, S.P. and SINGH, D.V., "Analysis of Hydrodynamic Journal Bearing with Non-Newtonian Power Law Lubrication by the Finite Element Method", Wear, 71 (1981), pp.15-27.

(19) WU, H., "Squeeze-Film Behaviour for Porous Annular Disks", Journal of Lubrication Technology, A.S.M.E., Series F, 92 (1970), pp.593-596.

(20) WU, H., "The Squeeze Film Between Rotating Porous Annular Disks", Wear, 18 (1971), pp.461-470.

(21) WU, H., "An Analysis of the Squeeze Film Between Porous Rectangular Plates", Journal of Lubrication Technology, A.S.M.E., Series F, (1972), pp.64-68.

(22) TING, L.L., "A Mathematical Analog for Determination of Porous Annular Disk Squeeze Film Behaviour including the Fluid Inertia Effect", Journal of Basic Engineering, Series (1972), pp.417-421.

(23) PRAKASH, J. and VIJ, S.K., "Load Capacity and Time-Height Relations for Squeeze Films Between Porous Plates", Wear, 24 (1973), pp.309-322.

(24) BHAT, M.V. and PATEL, C.M., "The Squeeze Film in An Inclined Porous Slider Bearing", Wear, 66 (1981), pp.189-193.

(25) PURI, V.K. and PATEL, C.M., "The Squeeze Film in A Porous Composite Slider Bearing", Wear, 70 (1981), pp.197-206.

(26) VORA, K.H. and BHAT, M.V., "The Load Capacity of A Squeeze Film Between Curved Porous Rotating Annular Plates", Wear, 65 (1980), pp.36-46.

(27) GUPTA, J.L. and VORA, K.H., "The Effect of Rotational Inertia on the Squeeze Film Load Between Porous Annular Curved Plates", Wear, 79 (1982), pp.235-240.

(28) PRAKASH, J. and TIWARI, K., "Effect of Surface Roughness on the Squeeze Film Between Rotating Porous Annular Disks", Journal of Mechanical Engineering Science, 24 (1982), pp.155-161.

(29) ROHDE, S.M. and OH, K.P., "Higher Order Finite Element Methods for the Solution of Compressible Porous Bearing Problems", Int. Journal for Numerical Methods in Engineering, 9 (1975), pp.903-911.

(30) EIDELBERG, B.E. and BOOKER, J.F., "Application of Finite Element Methods to Lubrication: Squeeze Films Between Porous Surfaces", Journal of Lubrication Technology, A.S.M.E., Series F, 98 (1976), pp.175-180.

(31) MALIK, M., SINHASAN, R. and CHANDRA, M., "Finite Element Analysis fo A Porous Thrust Slider", Wear, 76 (1982), pp.1-13.

(32) ARCHIBALD, F.R., "Load Capacity and Time Relations for Squeeze Films", A.S.M.E., 78 (1956), pp.29-35.

(33) JACKSON, J.D., "A Study of Squeezing Flow", Applied Scientific Research, Section A 11(1963), pp.148-152.

(34) ALLEN, C.W. and McKILLOP, A.A., "An Investigation of the Squeeze Film Between Rotating Annuli", Journal of Lubrication Technology, A.S.M.E., Series F, 92 (1970), pp.435-441.

(35) LUDWIG, L.P., "Engagement Characteristics of Wet Type Clutches", S.A.E. Paper 58-76 B, (1958).

(36) WU, H., "An Analysis of the Engagement of Wet-Clutch Plates", Wear, 24 (1973), pp.23-33.

(37) EL-SHERBINY, M.G. and NEWCOMB, T.P., "Numerical Simulation of the Engagement Characteristics of A Wet Clutch", Conference on Oil-Immersed Brakes and Clutches, Institution of Mechanical Engineers (1977), pp.81-92.

(38) CAMERON,A., "Basic Lubrication Theory", Ellis Horwood LTD, (1981), pp67 - 75.

(39) FULLER, "Theory and Practice of Lubrication for Engineers",(1970), pp166

(40) ARCHIBALD,F.R., "A Simple Hydrodynamic Thrust Bearing", Trans. of the ASME, May (1950), pp393 - 400.

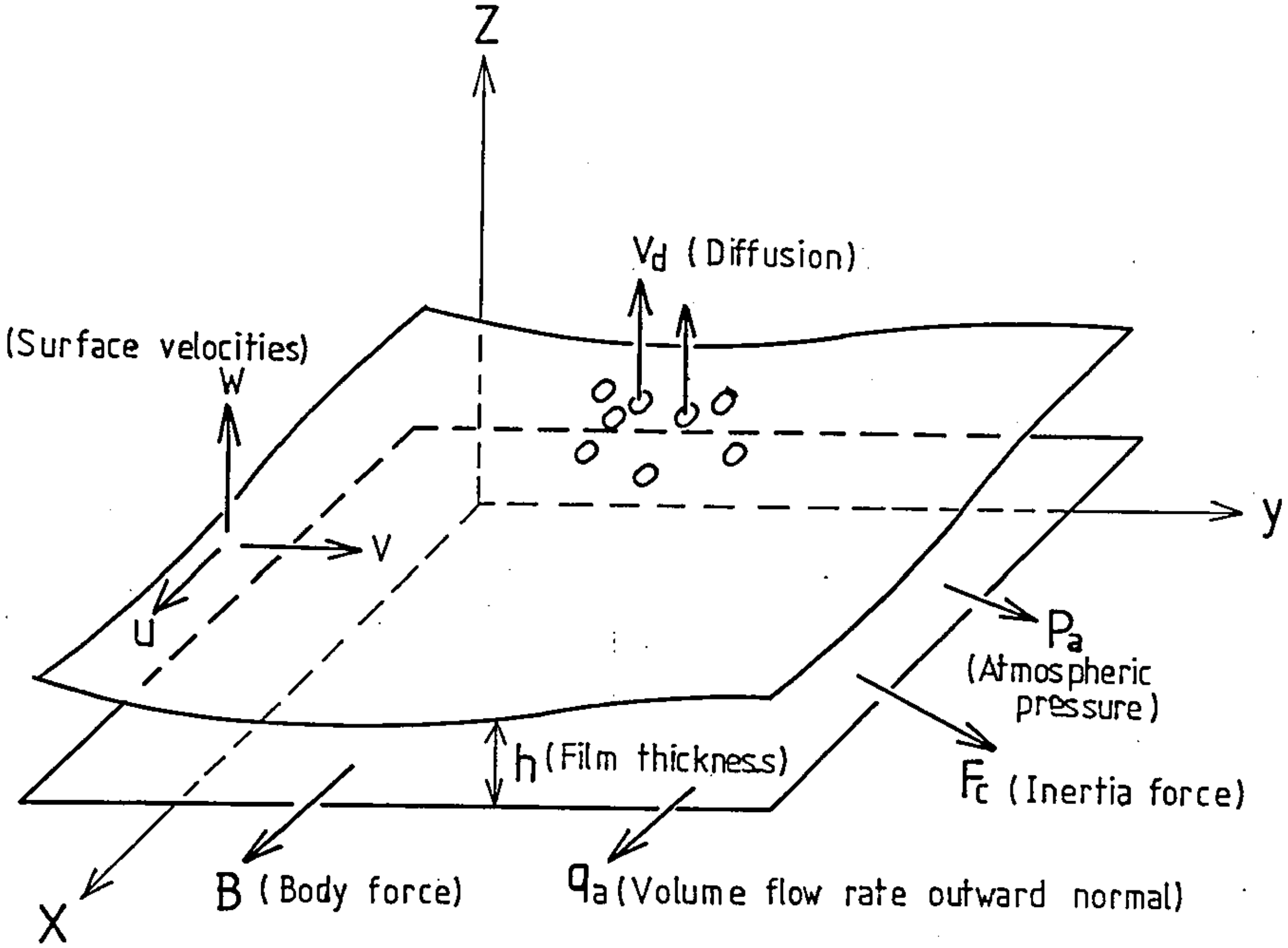


FIG. 1 Geometry and coordinate system for a fluid film and corresponding surfaces.

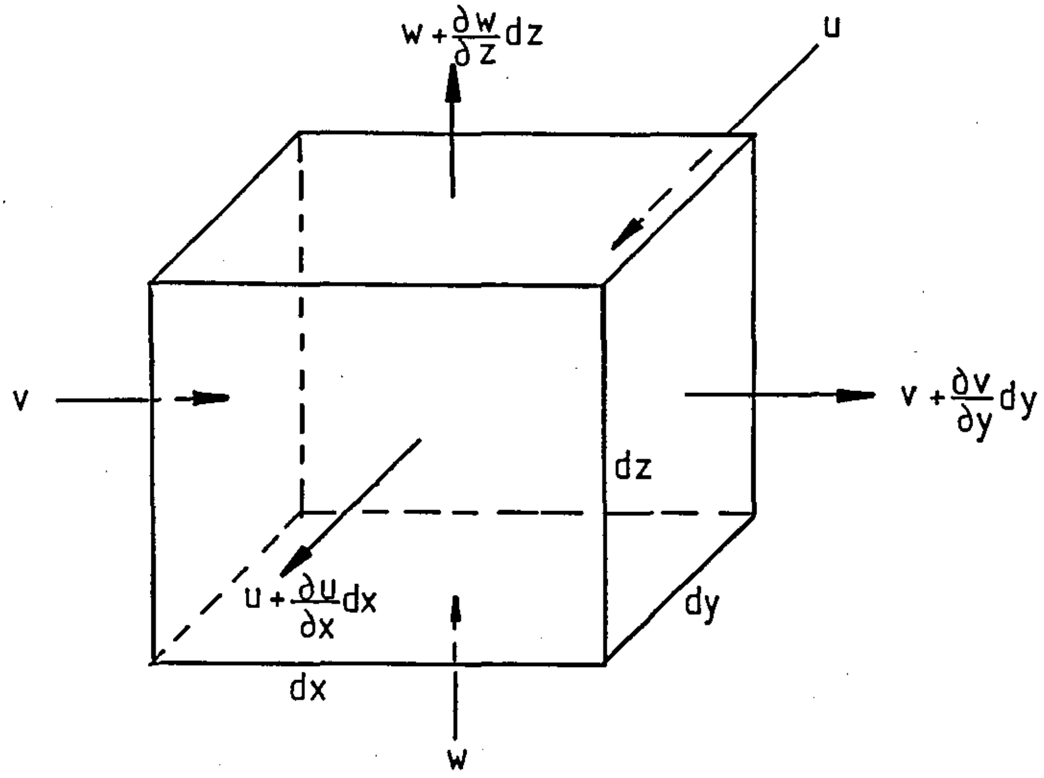


FIG.2 Continuity of flow of a fluid element.

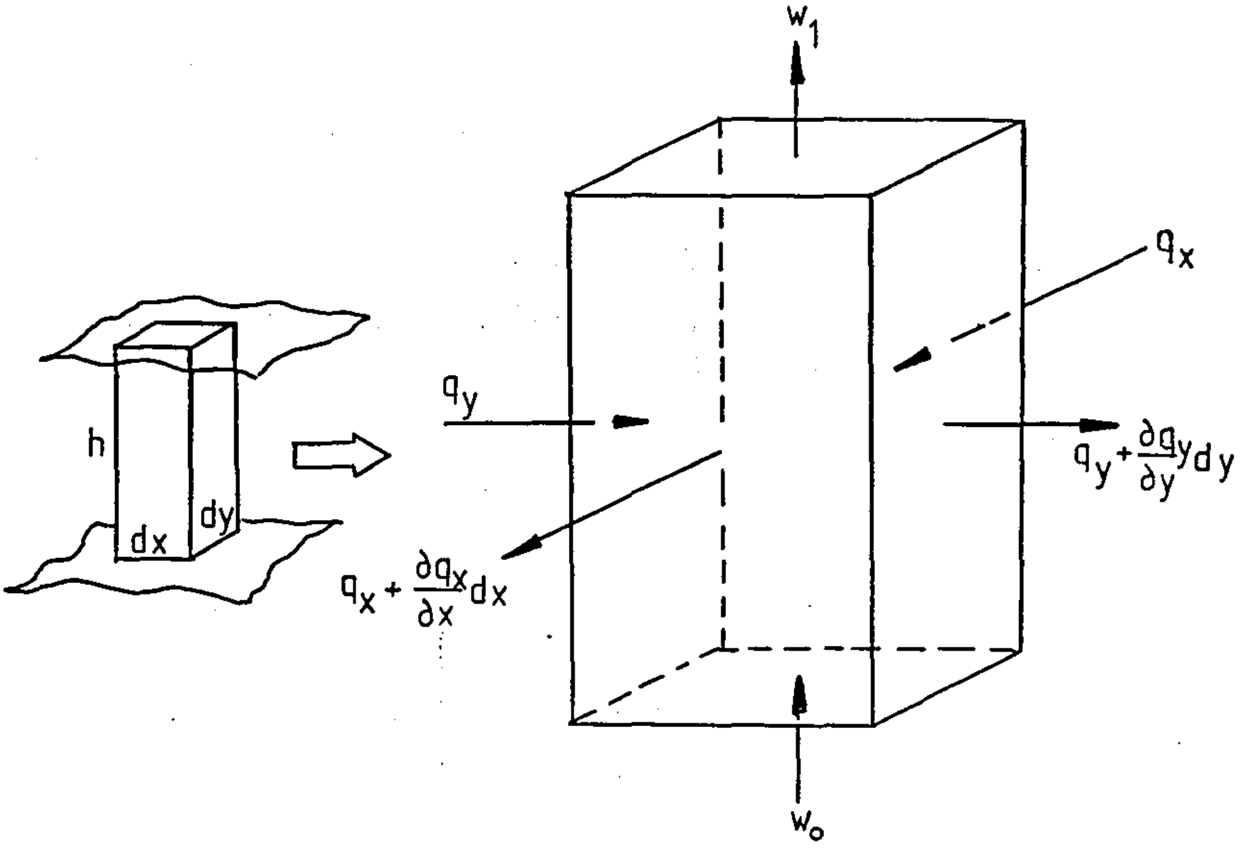


FIG.3 Continuity of flow of a column of fluid.

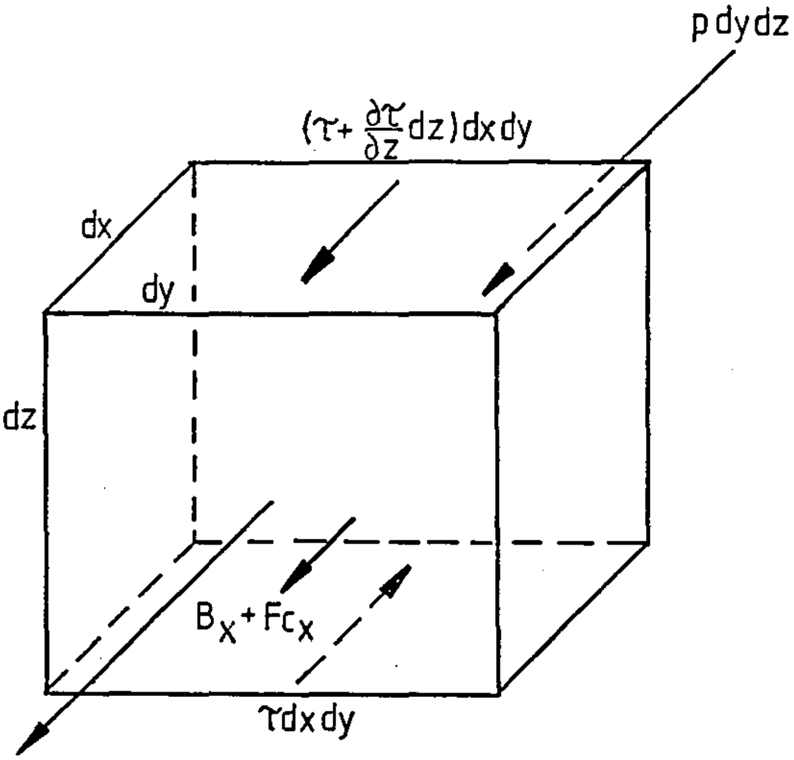


FIG.4 Equilibrium of a fluid element.

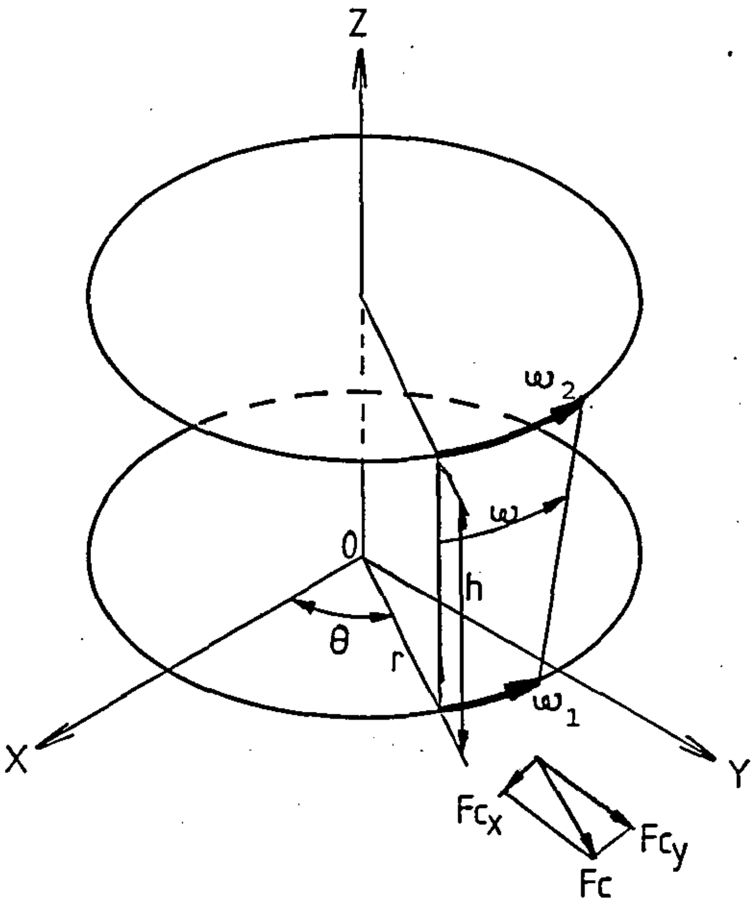


FIG.5 Description of Centrifugal force action.

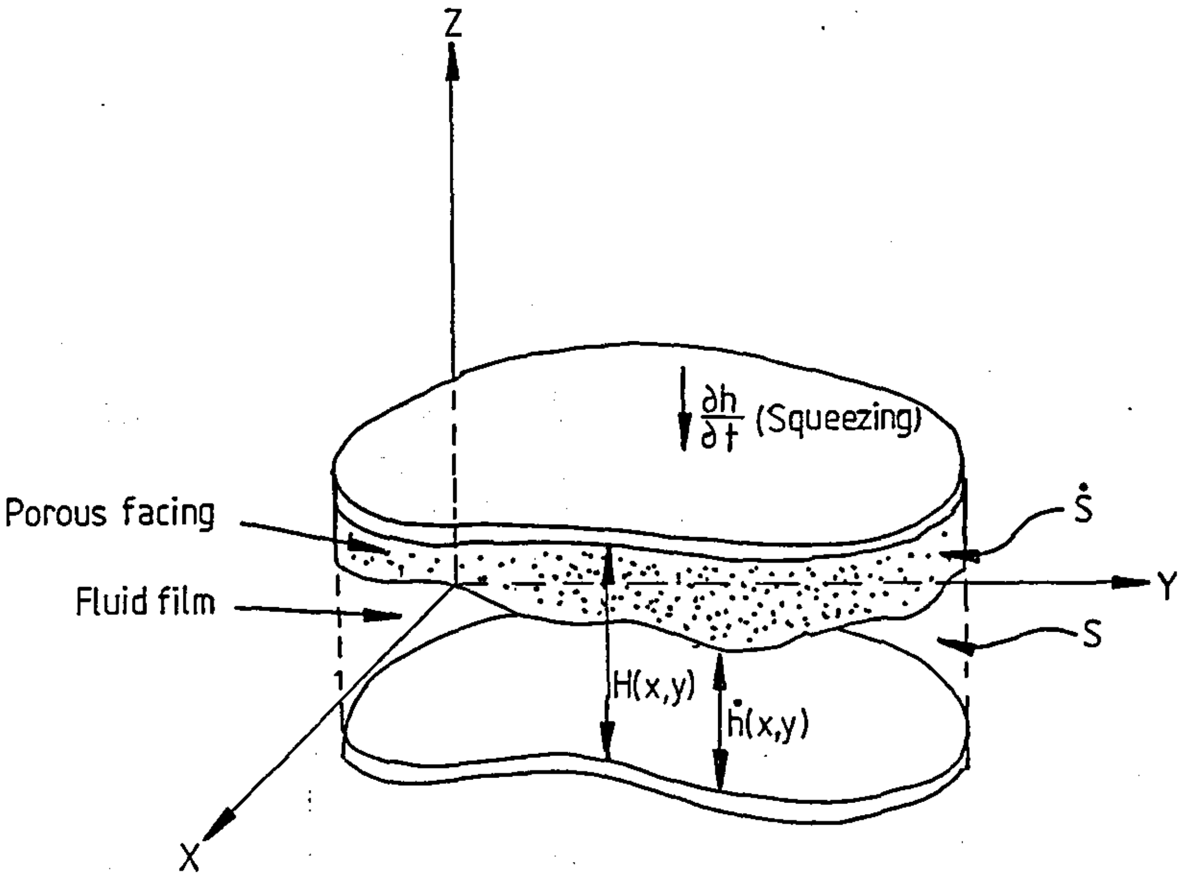


FIG.6 Geometry of a porous system.

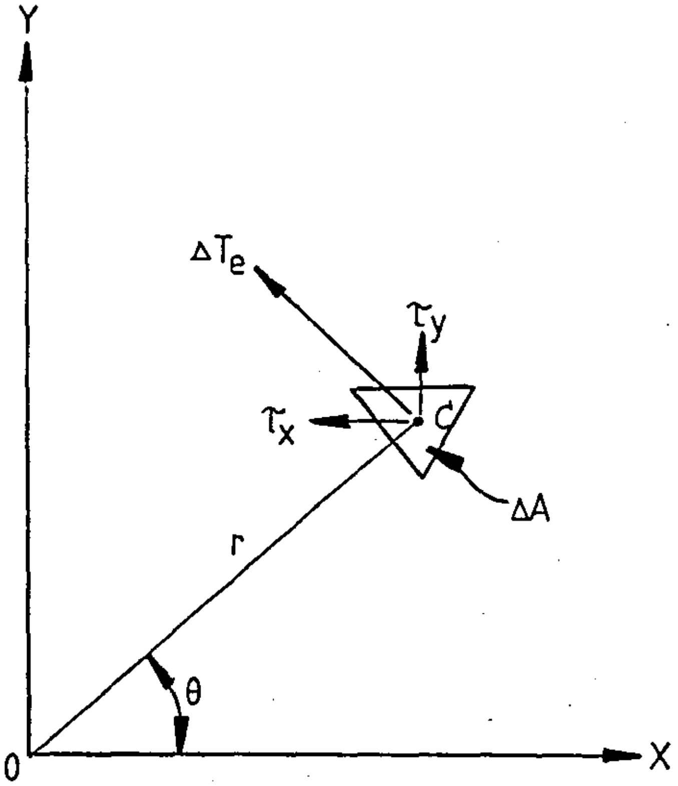


FIG.7 Representation of a torque field.

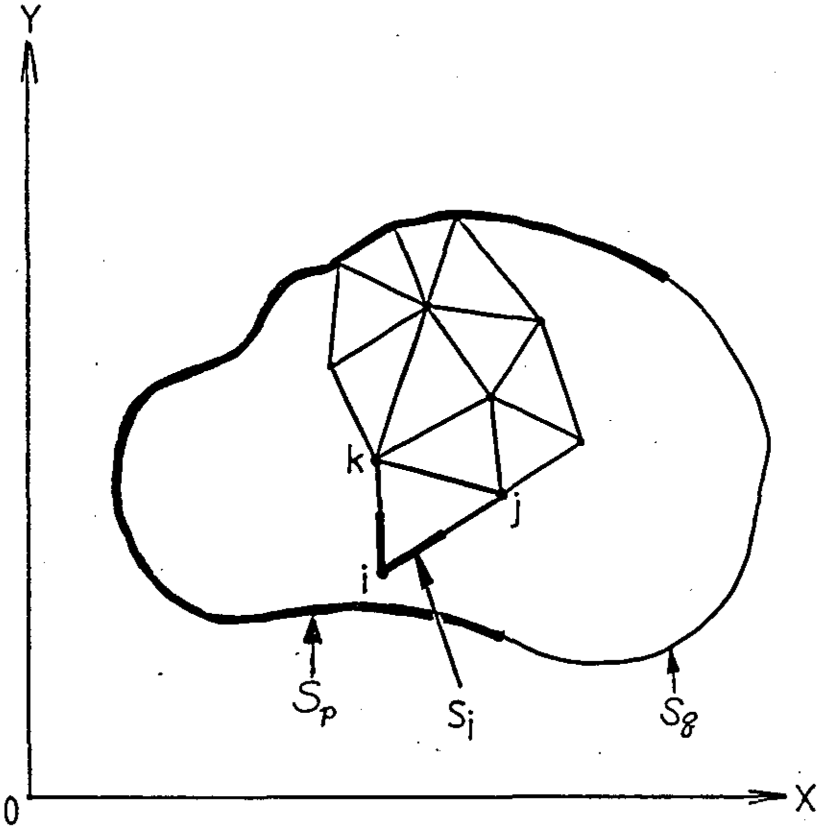


FIG.8 Lubricant domain and F.E. idealization.

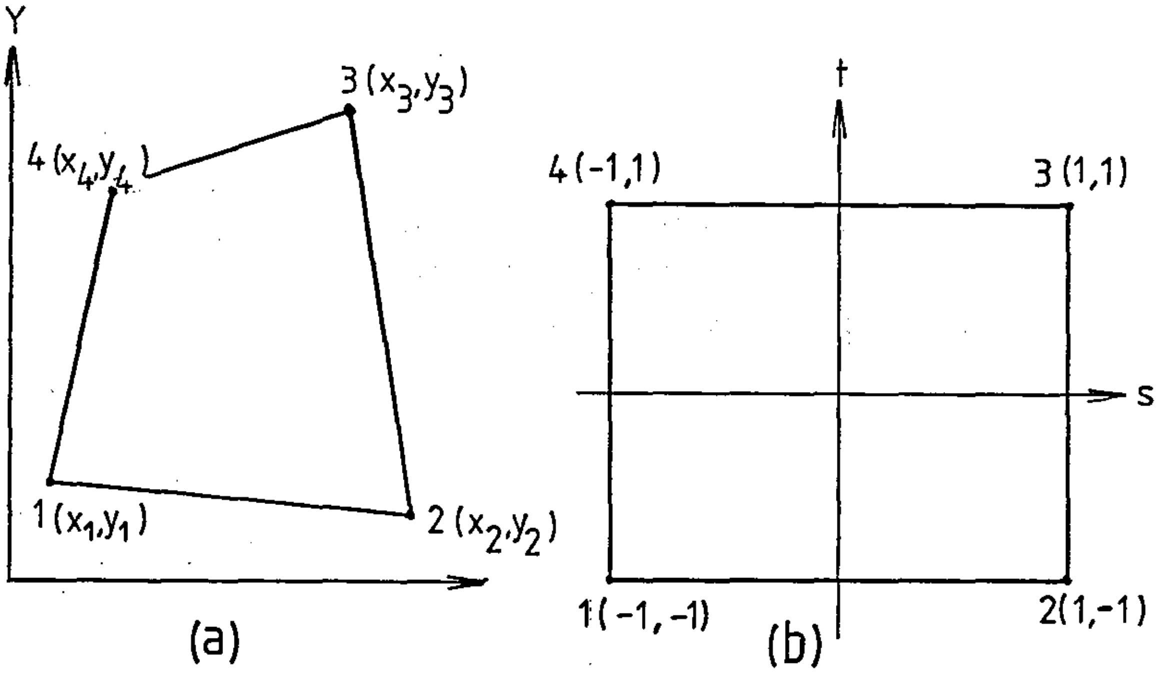


FIG.9 Natural coordinates for a quadrilateral element.

(a) Cartesian Coordinates (b) Natural Coordinates.

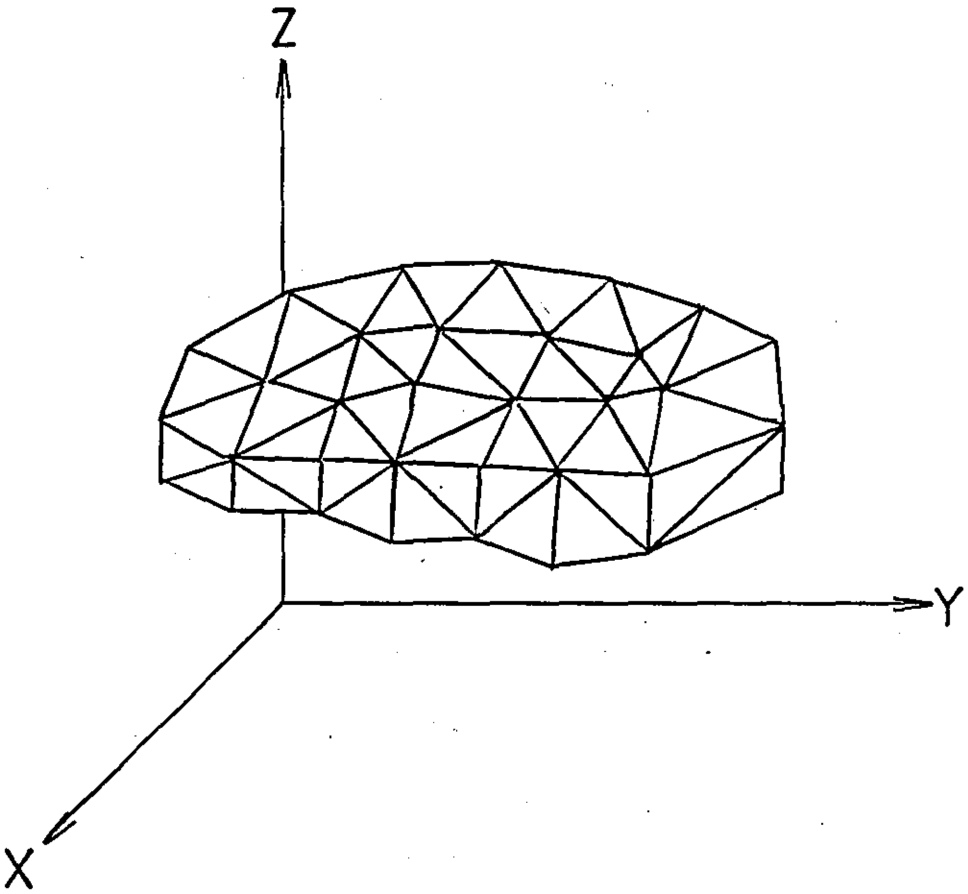


FIG.10 F.E. idealization of a porous region.

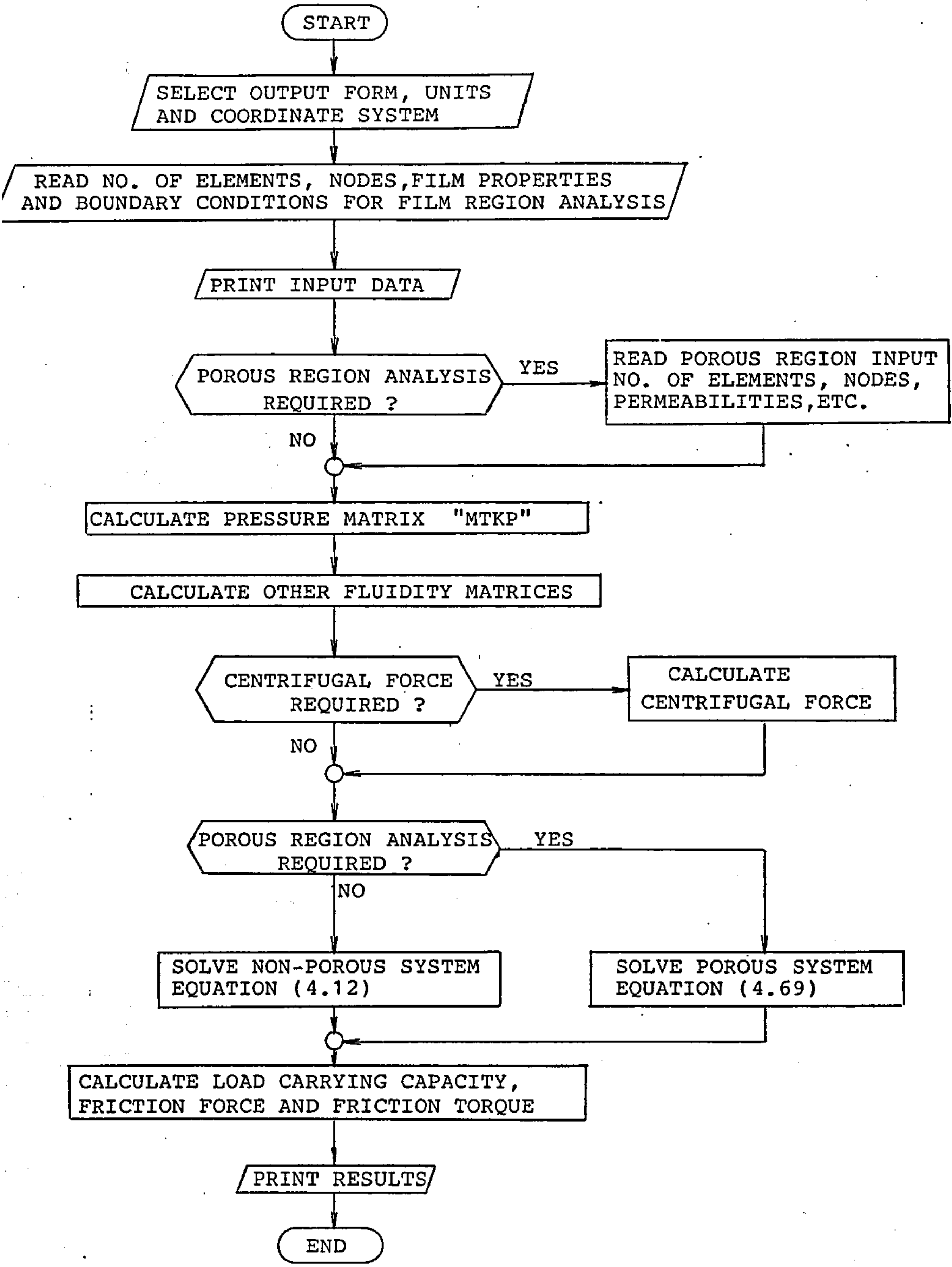


FIG.11 MAIN FLOWCHART

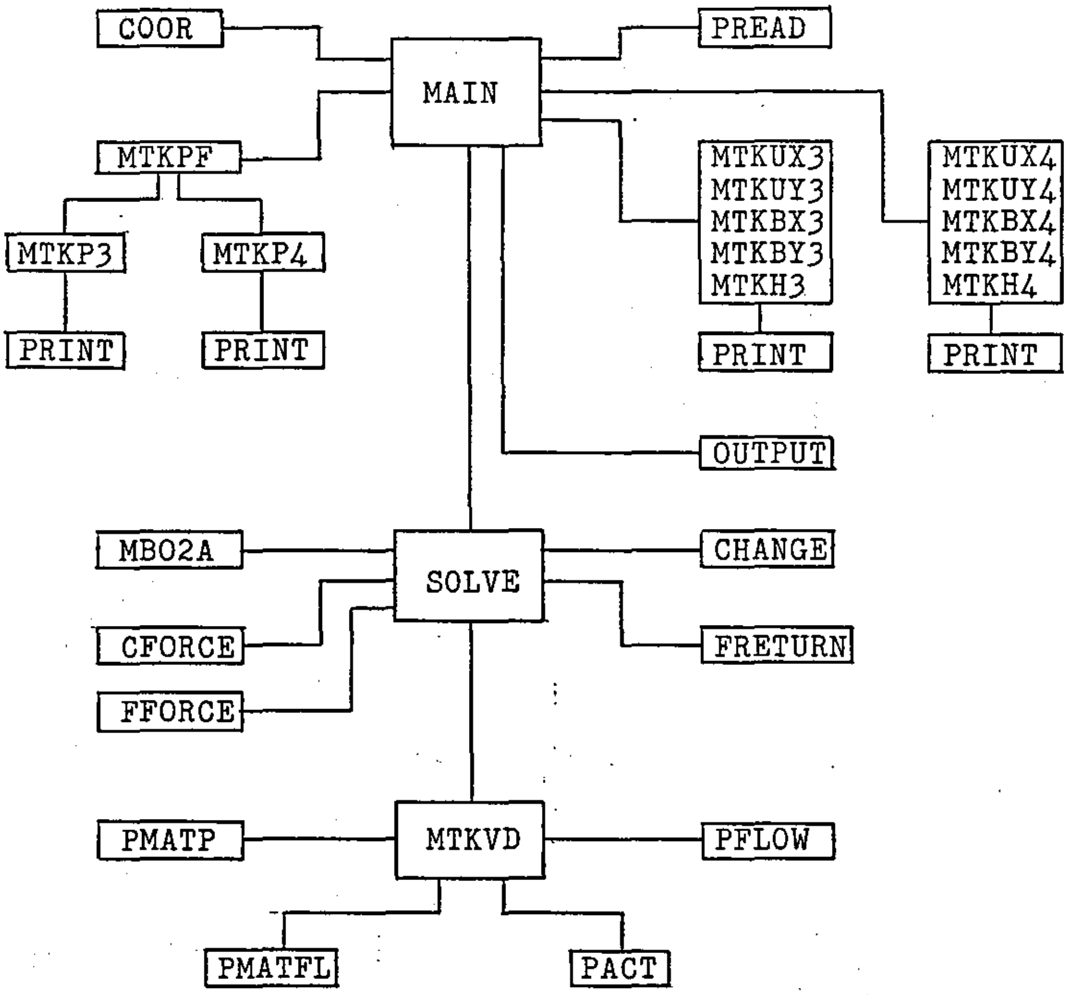


FIG. 12 SUBROUTINE SYSTEM

TABLE 1 List of routines

ROUTINE	PORPOSE
MTKPF	General pressure matrix routine
MTKP	Pressure matrix routine
MTKP3	for triangular element
MTKP4	for rectangular element
MTKUX	X-direction shear action matrix routine
MTKUX3	for triangular element
MTKUX4	for rectangular element
MTKUY	Y-direction shear action matrix routine
MTKUY3	for triangular element
MTKUY4	for rectangular element
MTKBX	X-direction body force action matrix routine
MTKBX3	for triangular element
MTKBX4	for rectangular element
MTKBY	Y-direction body force action matrix routine
MTKBY3	for triangular element
MTKBY4	for rectangular element
MTKH	Squeeze action matrix routine
MTKH3	for triangular element
MTKH4	for rectangular element
SOLVE	Routine for solving the system equation(4.12),(4.69) and for the calculation of friction forces,torques and load capacity
PRINT	Routine for listing of the global matrix
COOR	Routine for arranging the coordinate system from various type of input
MB02A	Routine for calculating the inverse of an matrix
CFORCE	Routine for the calculation of centrifugal forces
FFORCE	Routine for the calculation of friction forces and torques
output	Routine for printing pressures,flows and other flow variables
MTKVD	General routine for porous region analysis
PREAD	Routine for reading input for porous analysis
PMATP	Pressure matrix routine for porous region
PMATFL	Other fluidity matrix routine for porous region
PFLOW	Routine for arranging boundary conditions for porous region
PACT	Routine for arranging fluidity action values for porous region
CHANGE	Routine for changing the system from film to porous
FRTURN	Routine for returning the system from porous to film

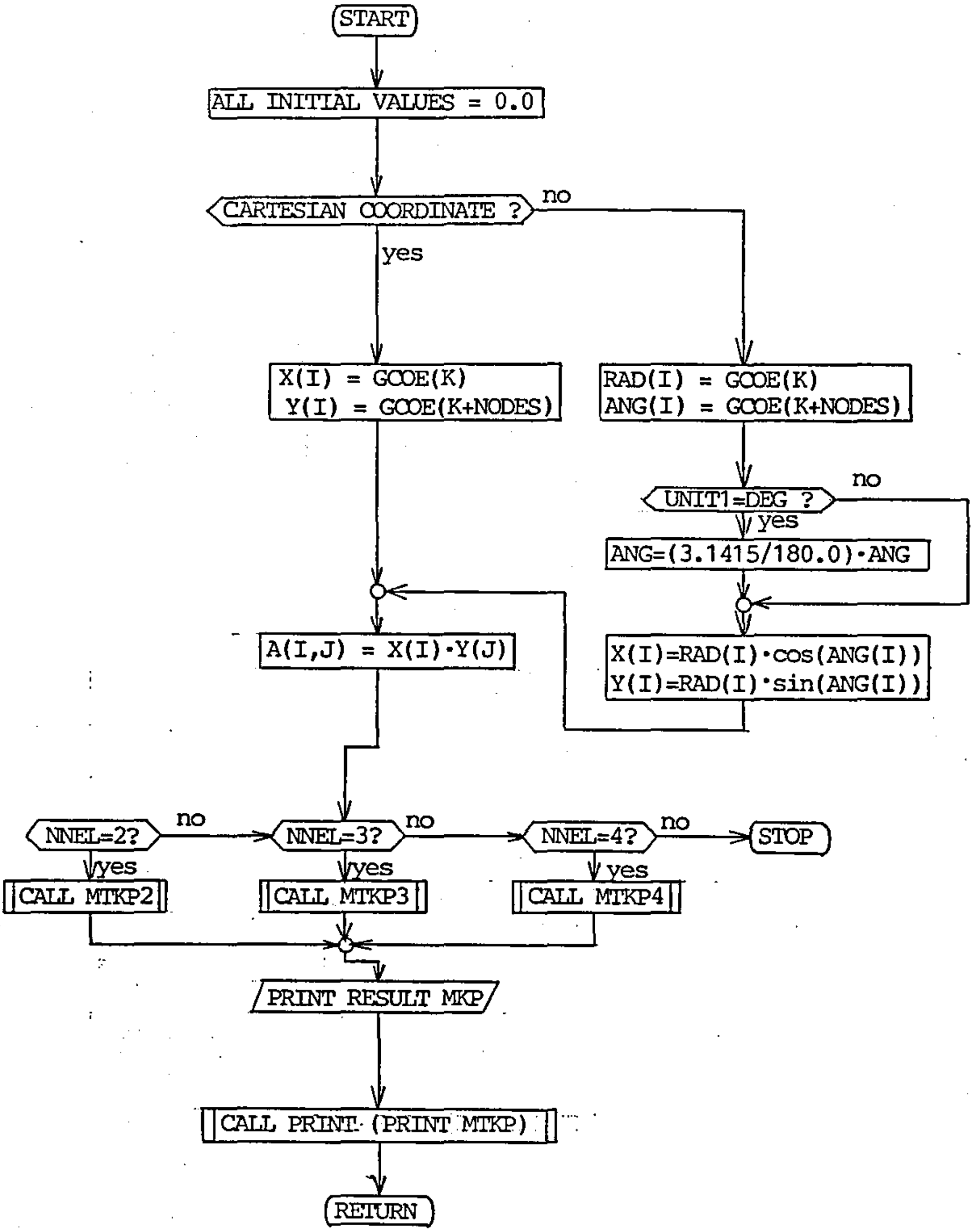


FIG. 13 SUBROUTINE MTKPF

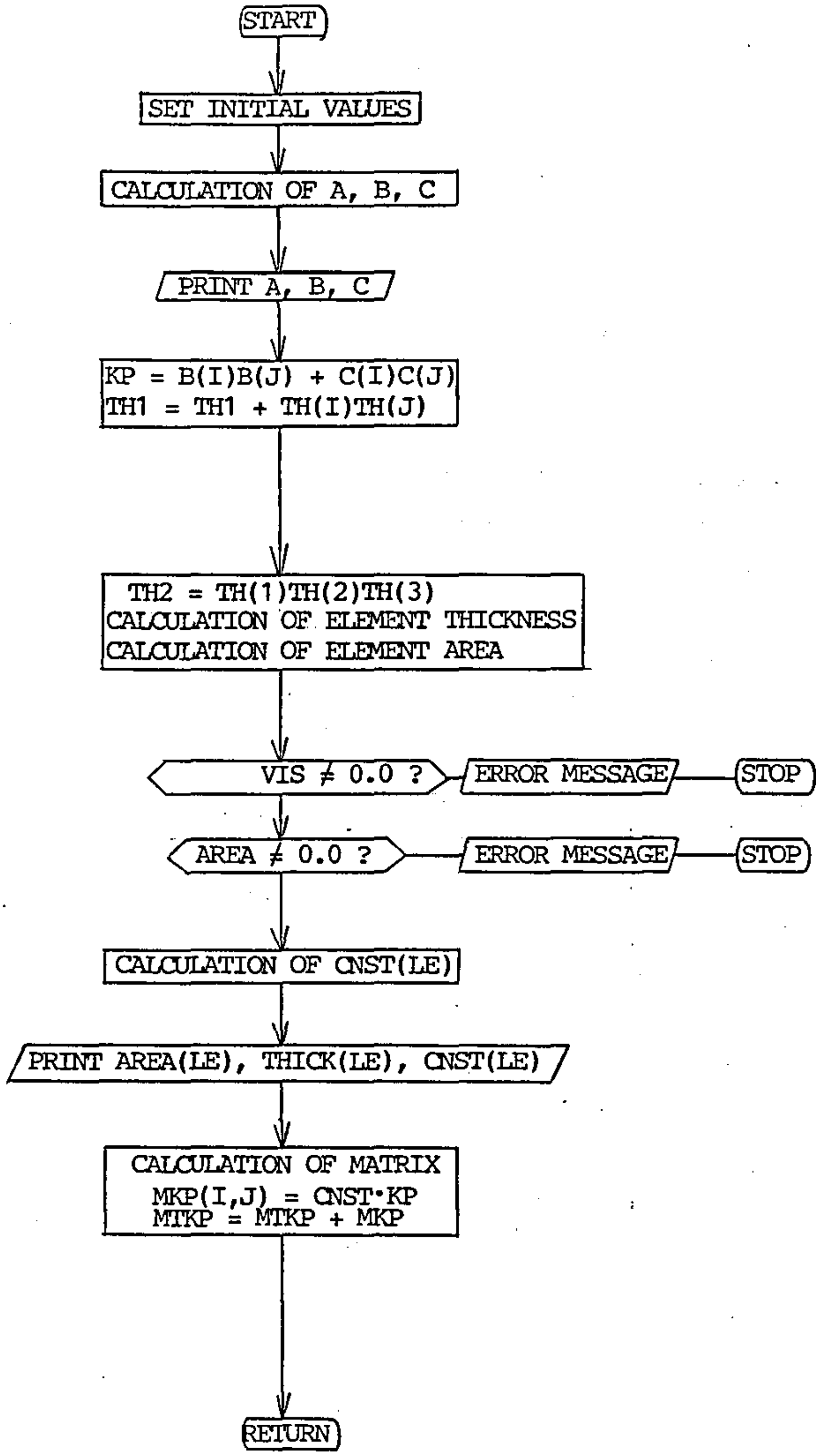


FIG.14 SUBROUTINE MTKP3

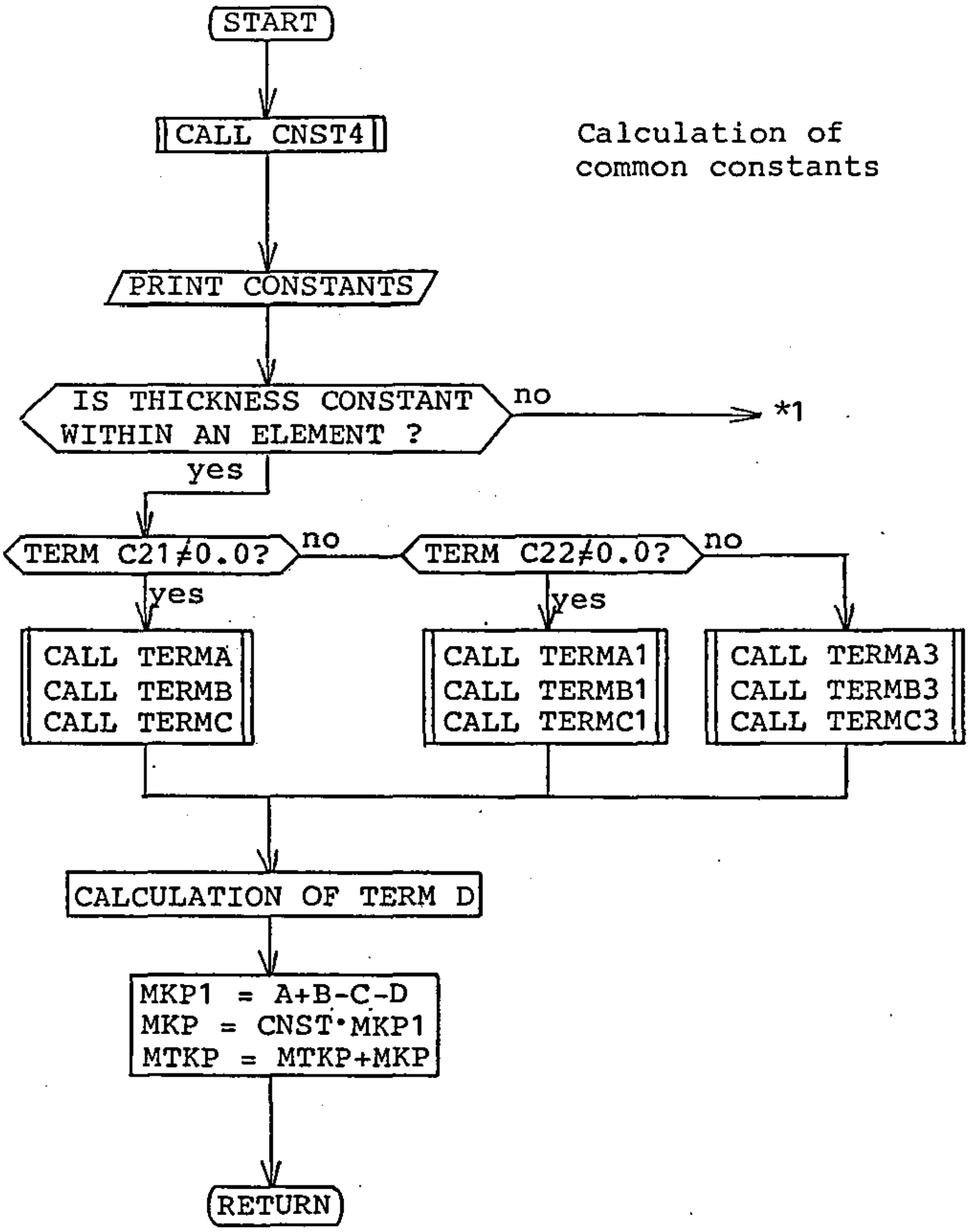


FIG. 15 SUBROUTINE MTKP4

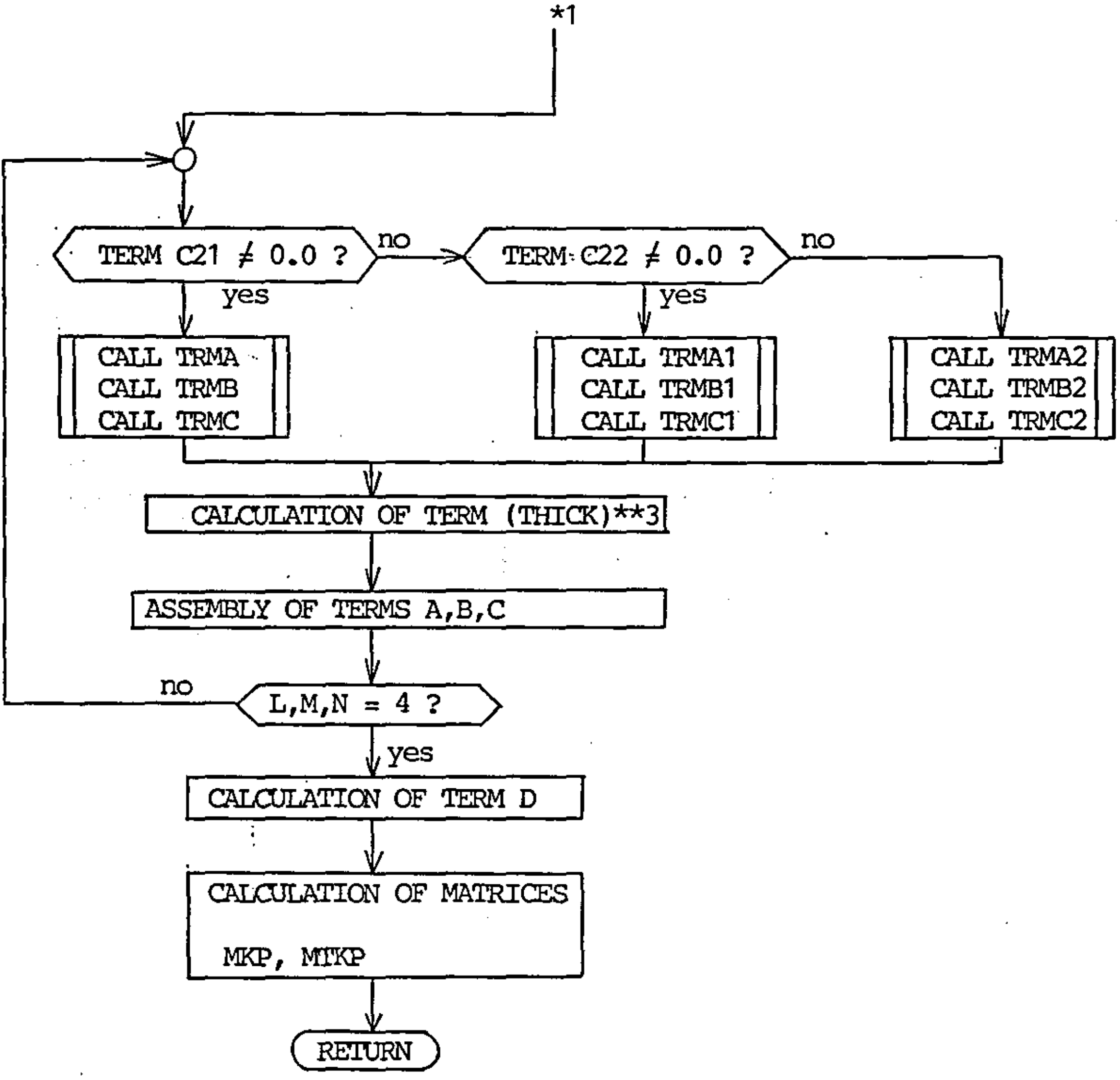


FIG.15 SUBROUTINE MTKP4 (continued)

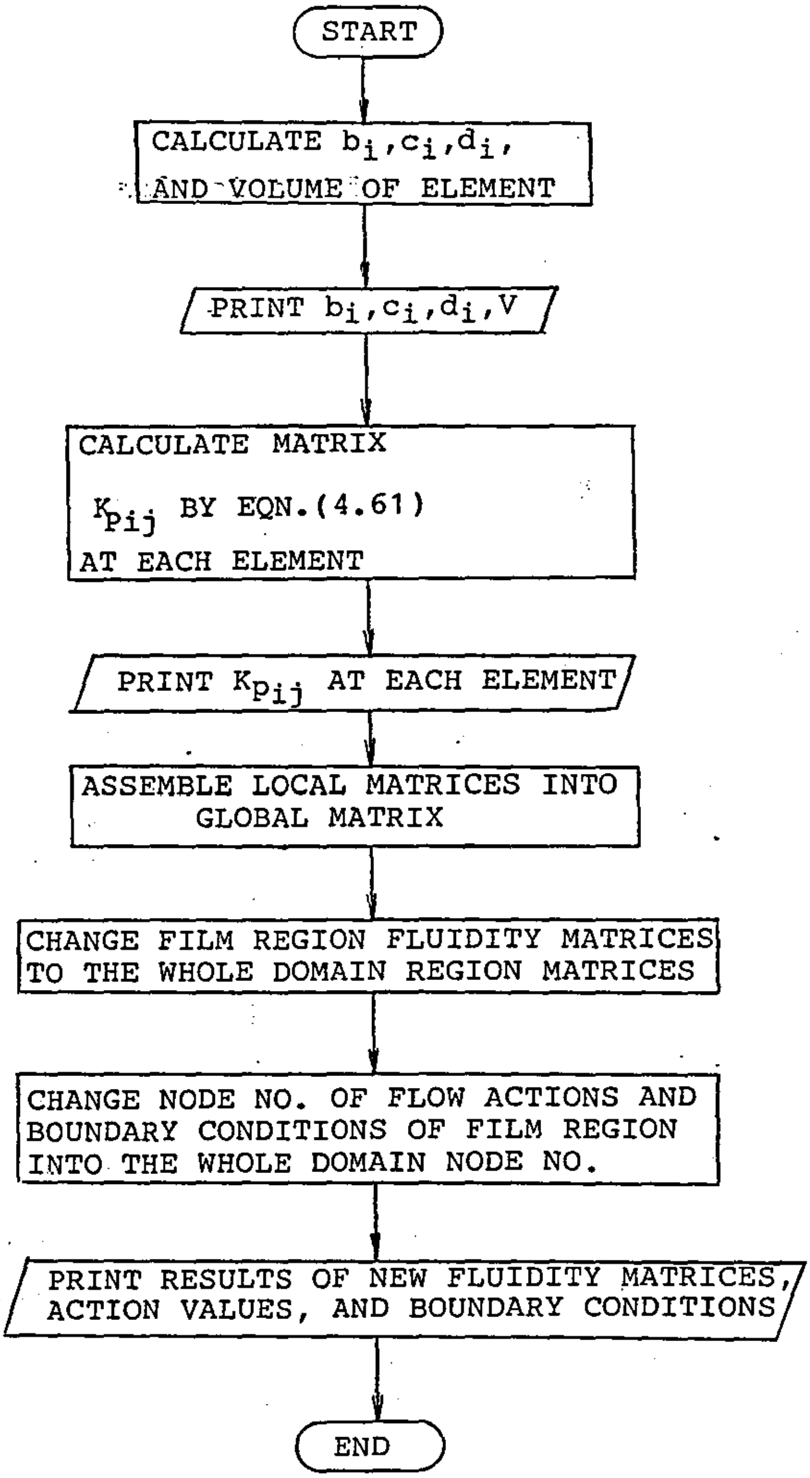


FIG. 16 SUBROUTINE MTKVD

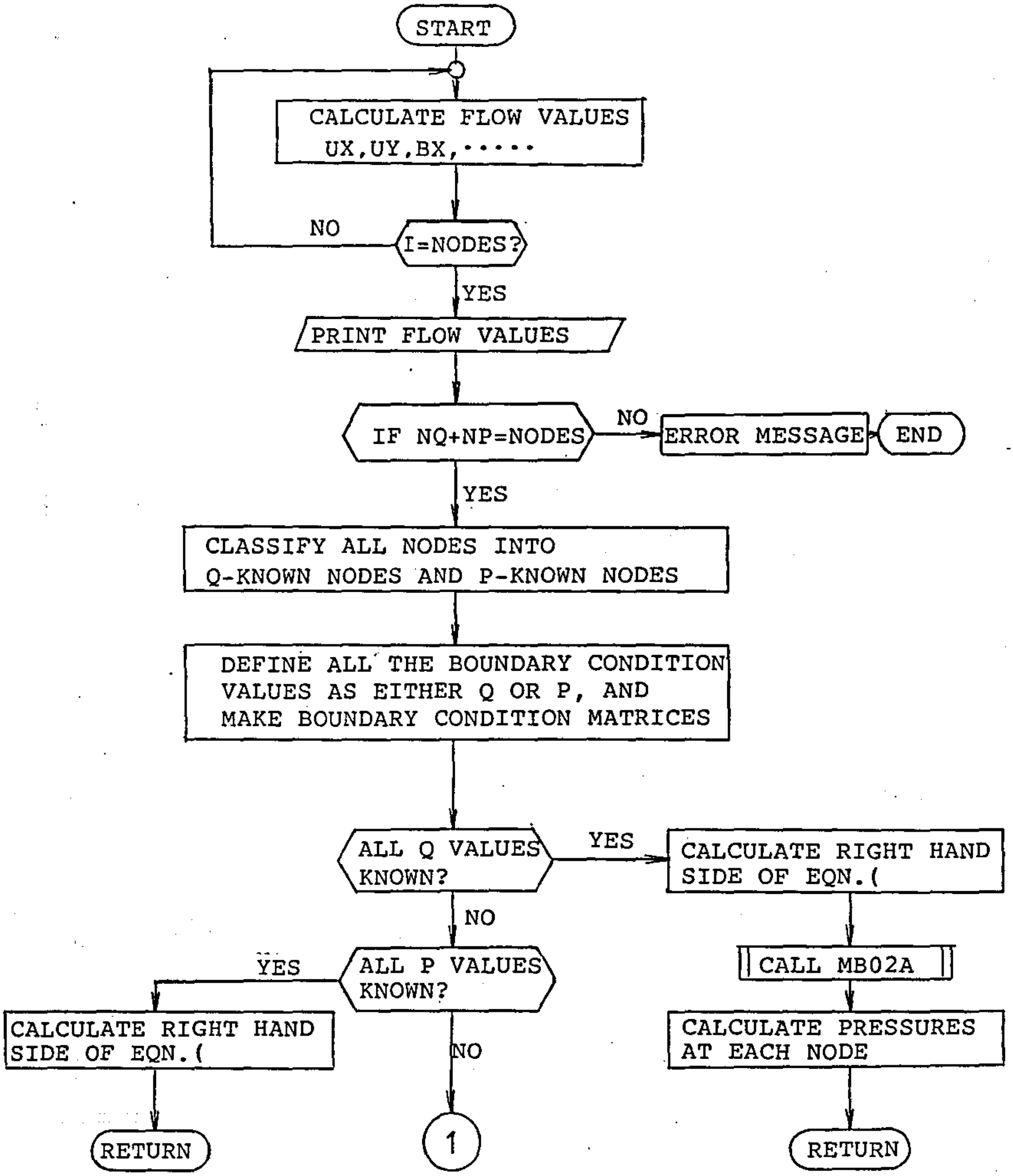


FIG.17 SUBROUTINE SOLVE

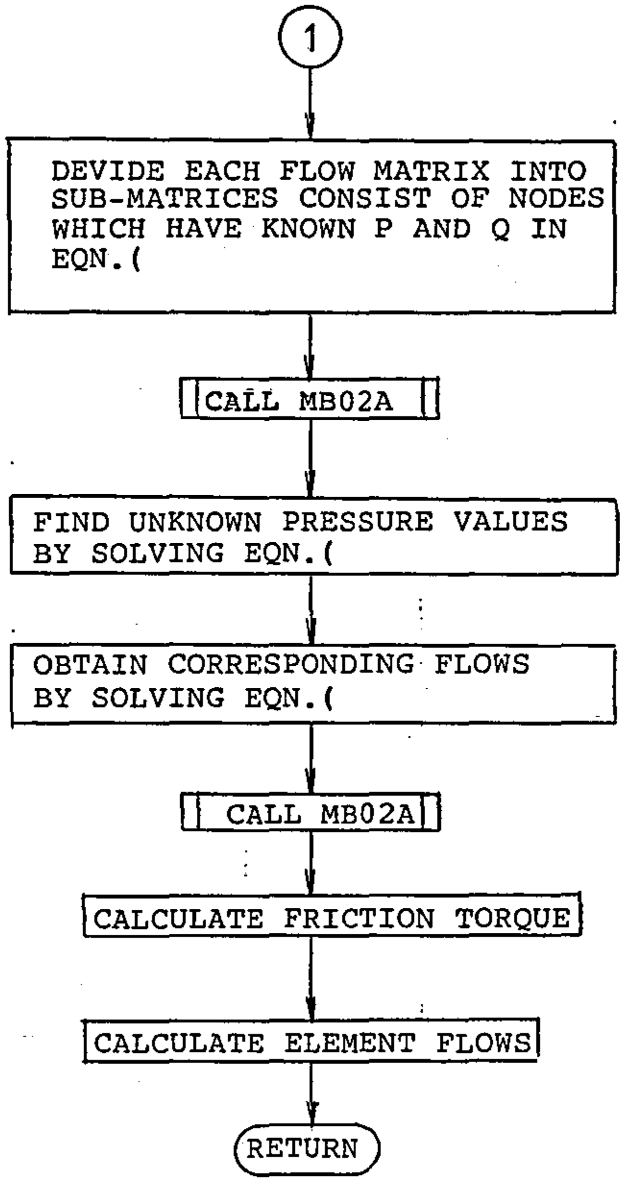


FIG.17 SUBROUTINE SOLVE (continued)

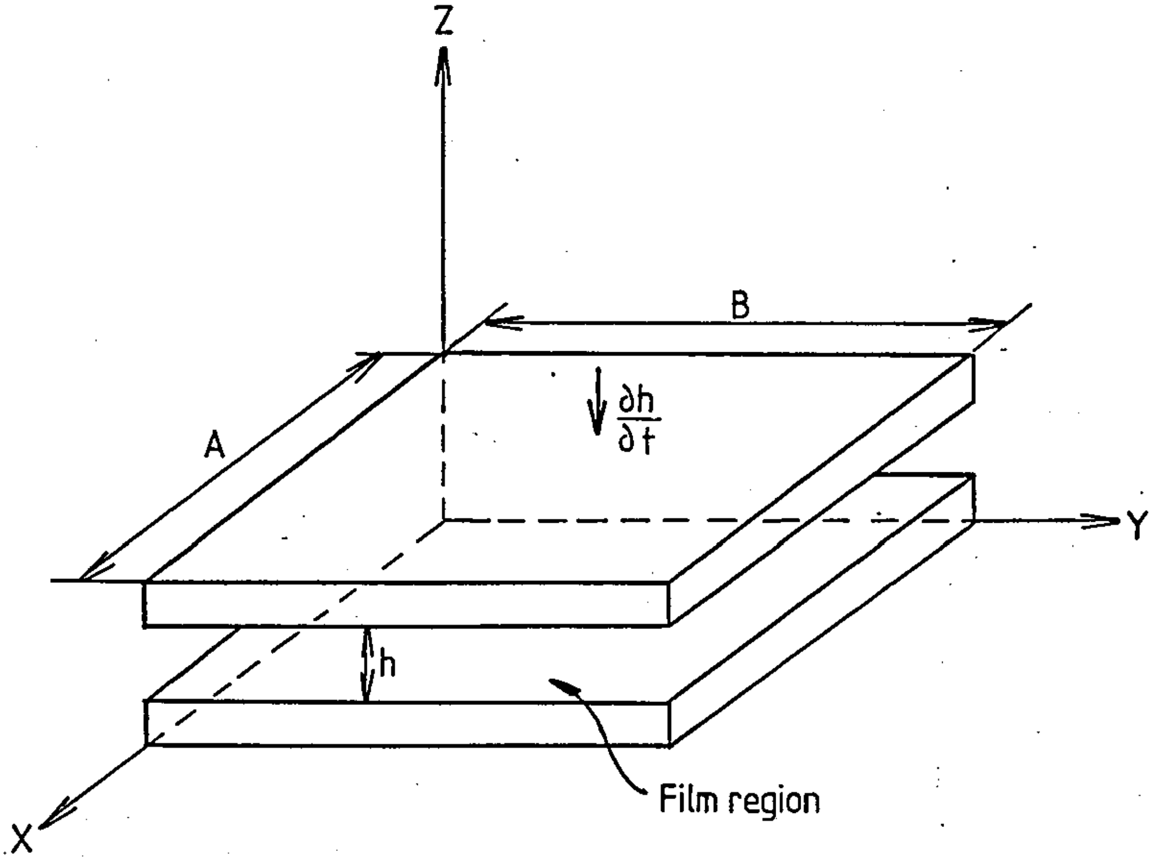


FIG. 18 Geometry of a rectangular squeezing pad.

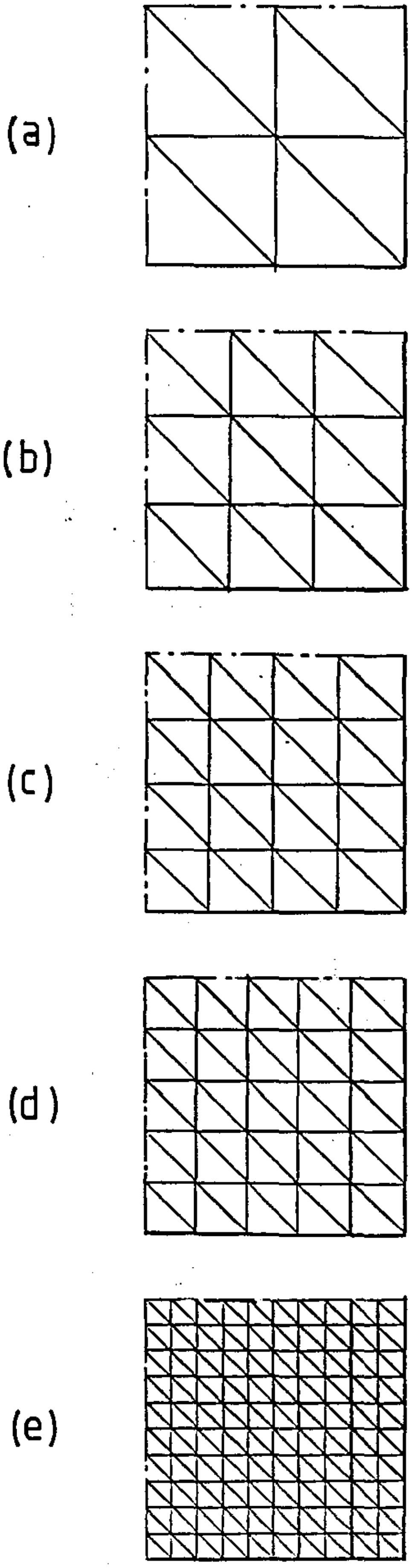


FIG. 19 Finite element layouts of a rectangular pad.

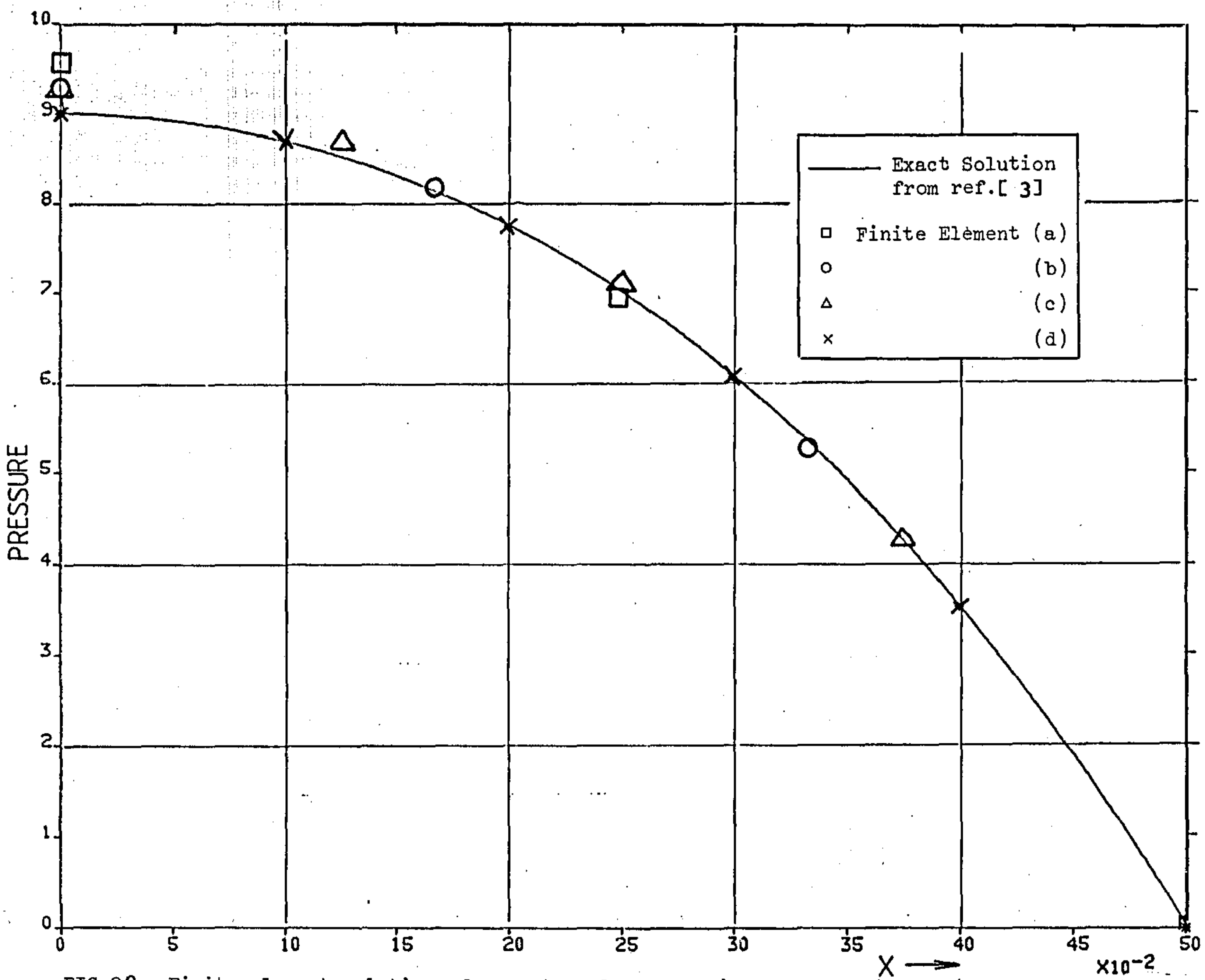


FIG.20 Finite element solution of a rectangular squeezing pad.

(Section Y = 0)

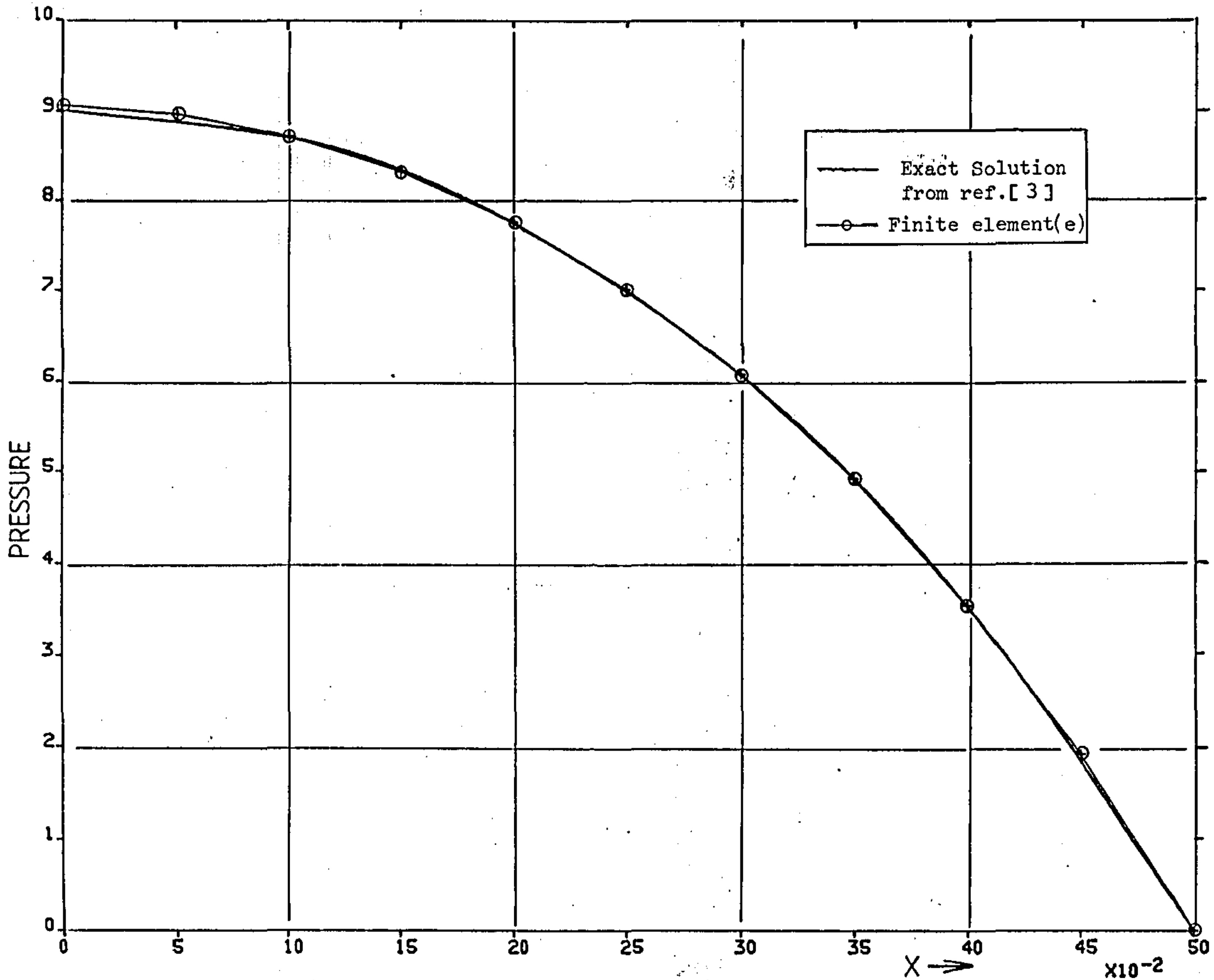


FIG.21 Finite element solution of a rectangular squeezing pad. (Section Y = 0)

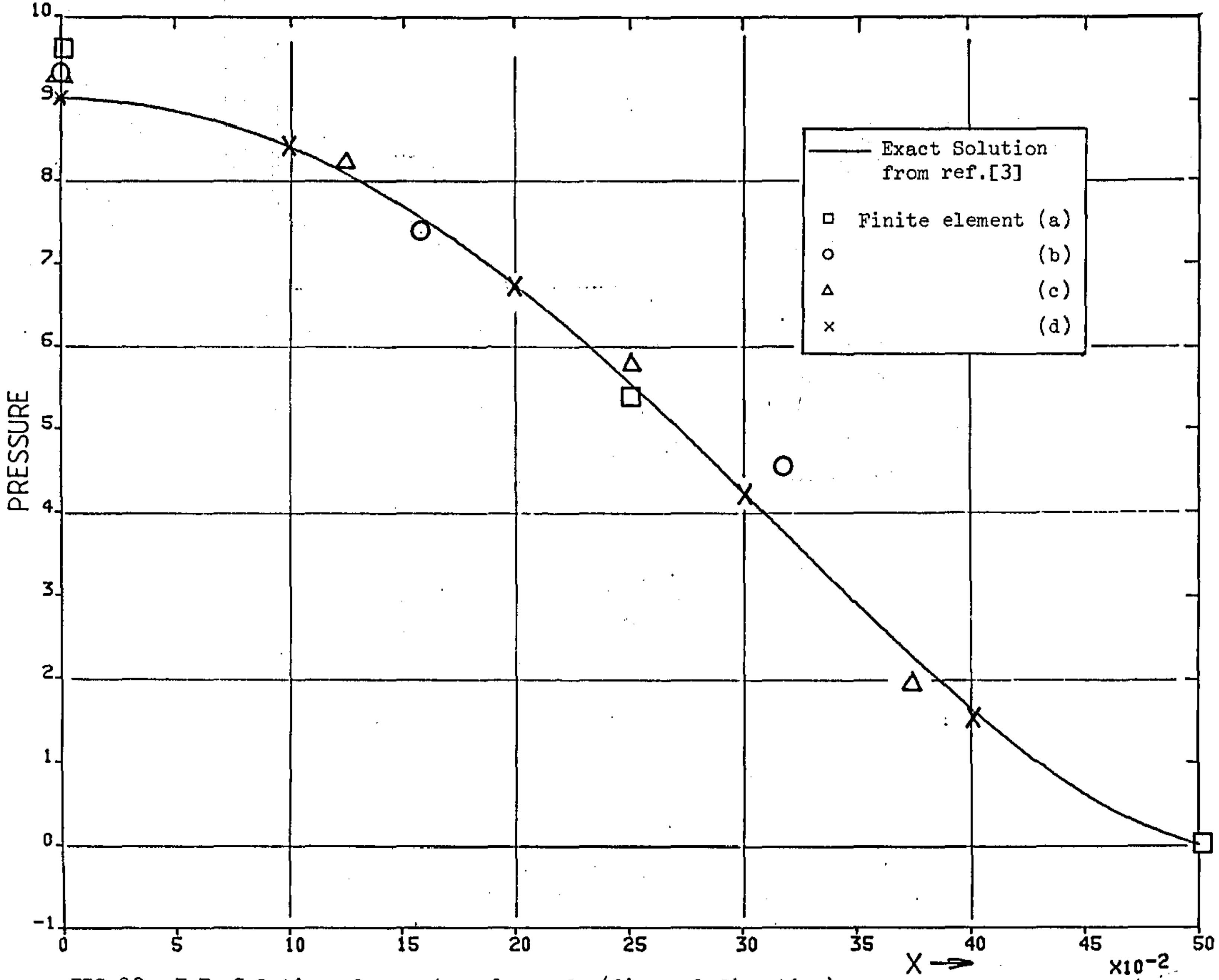


FIG.22 F.E. Solution of a rectangular pad (diagonal direction)

TABLE.2 Maximum pressure values for rectangular squeezing pad at (x,y) = (0.0 , 0.0)

Analysis Methods No. of elements	Finite Element Technique					Exact Solution (ref. 3)
	8	18	32	50	200	
Pressure	9.5625	9.3282	9.3178	9.1614	9.0626	9.0041
% Errors	+6.20	+3.60	+3.48	+1.75	+0.65	

TABLE.3 Percentage errors of load carrying capacities of rectangular squeezing pad

Analysis Methods No. of elements	Finite Element Technique					Exact Solution
	8	18	32	50	200	
Load carrying capacity	4.6990	4.4996	4.4346	4.3778	4.3262	4.3017
% Errors	9.23	4.60	3.09	1.77	0.57	———

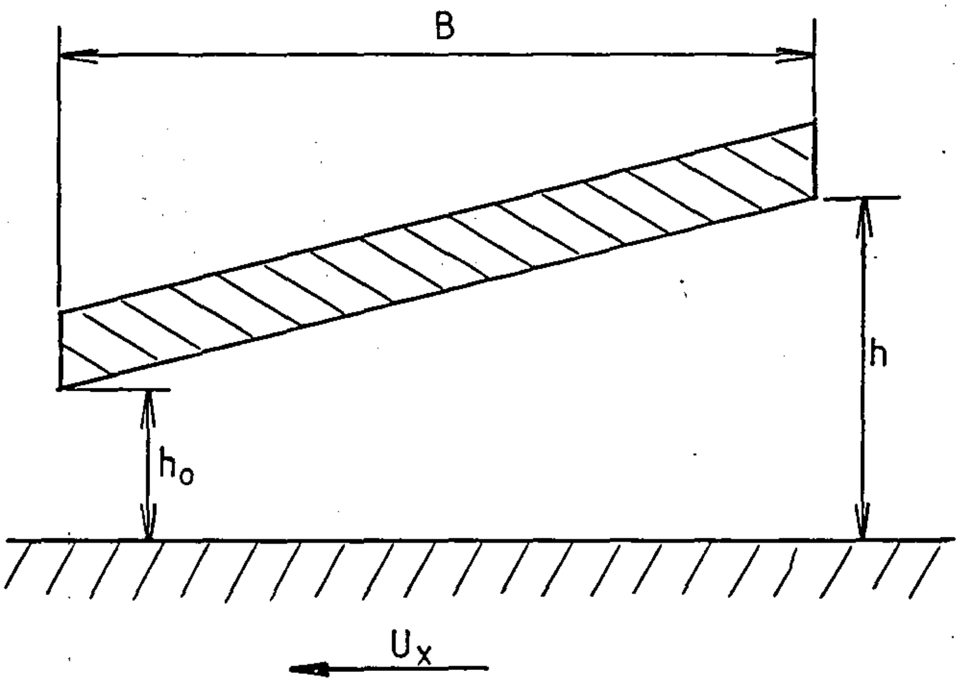


FIG. 23 Geometry of a slider bearing

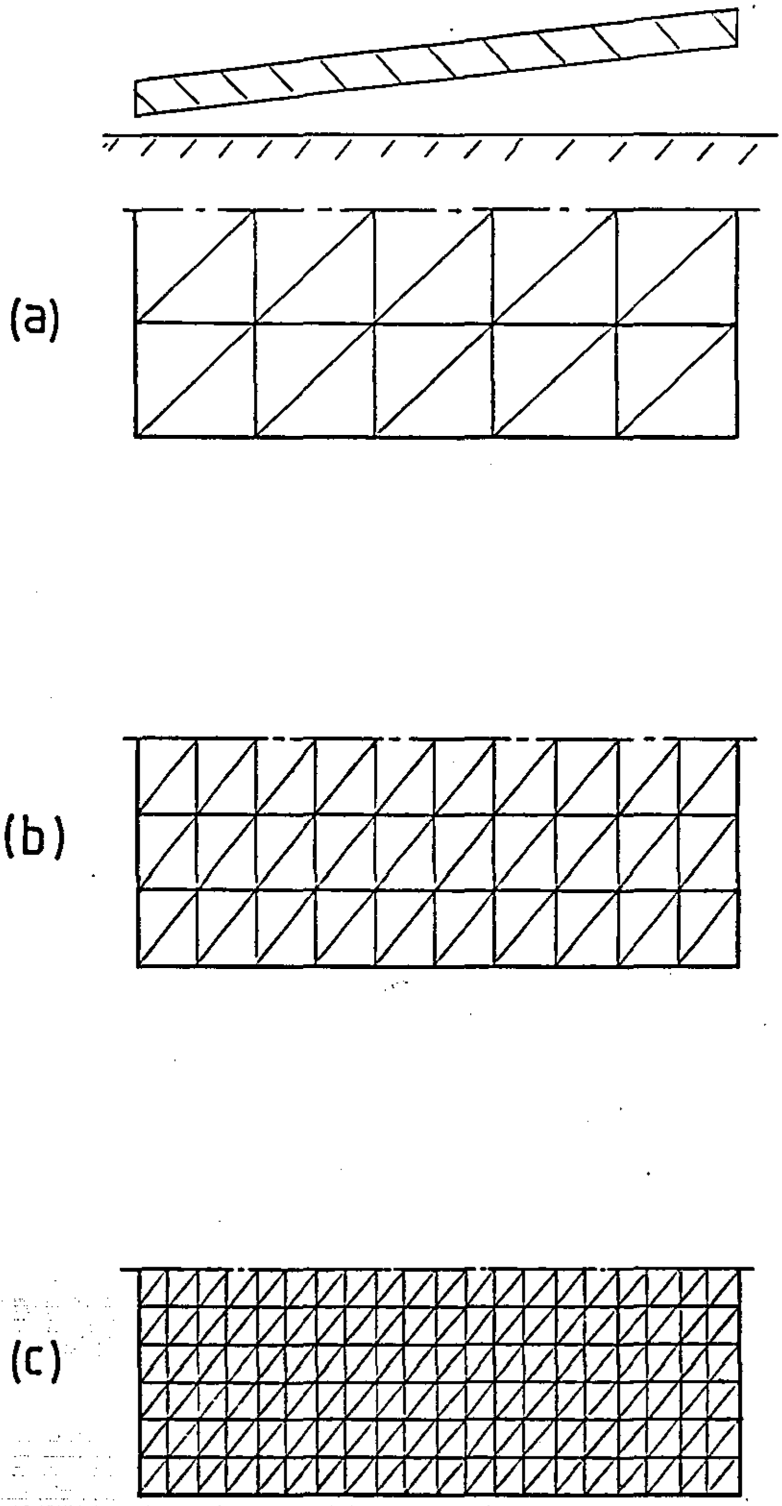


FIG.24 Finite element layouts of a slider bearing.

TABLE.4 Pressure profiles for infinitely wide slider bearing (regular meshes)

x (m)	Exact Solution	Number of Element in Flow Direction							
		LUB6				PAFEC (ref.[13])			
		5	10	20	50	5	10	20	30
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05	14552.2	------	13868.0	14384.0	14527.0	13901.8	14530.4	14548.0	14549.2
0.10	16557.1	14614.0	15952.0	16417.0	16534.0	16395.8	16540.1	16553.5	16554.3
0.15	15091.2	——	14631.0	14994.0	15074.0	14952.6	15079.1	15088.4	15088.9
0.20	12671.3	11628.0	12331.0	12607.0	12660.0	12594.7	12663.0	12669.2	12669.6
0.25	10105.7	——	9854.6	10066.0	10098.0	10051.8	10100.0	10104.2	10104.5
0.30	7665.3	7169.8	7480.7	7627.4	7659.6	7630.6	7661.5	7664.3	7664.5
0.35	5432.8	——	5344.5	5408.6	5429.1	5411.0	5430.4	5432.2	5432.3
0.40	3420.9	3235.3	3396.5	3406.9	3418.8	3408.1	3419.5	3420.5	3420.6
0.45	1616.9	——	1594.3	1610.7	1616.0	1611.7	1616.3	1616.7	1616.8
0.50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE.5 Percentage error of pressure values from the exact solution (regular meshes)

x (m)	Exact Solution	Number of Element in Flow Direction							
		LUB6				PAFEC (ref.[13])			
		5	10	20	50	5	10	20	30
0.0	0.0	——	——	——	——	——	——	——	——
0.05	14552.2	——	4.70	1.16	0.17	4.47	0.15	0.03	0.02
0.10	16557.1	11.74	3.65	0.85	0.14	0.97	0.10	0.02	0.02
0.15	15091.2	——	3.05	0.64	0.11	0.92	0.08	0.02	0.02
0.20	12671.3	8.23	2.69	0.51	0.09	0.60	0.07	0.02	0.01
0.25	10105.7	——	2.48	0.39	0.08	0.53	0.06	0.01	0.01
0.30	7665.3	6.46	2.41	0.49	0.07	0.45	0.05	0.01	0.01
0.35	5432.8	——	1.63	0.45	0.07	0.40	0.04	0.01	0.00
0.40	3420.9	5.43	0.71	0.41	0.06	0.37	0.04	0.01	0.00
0.45	1616.9	——	1.39	0.38	0.06	0.32	0.04	0.01	0.00
0.0	0.0	——	——	——	——	——	——	——	——

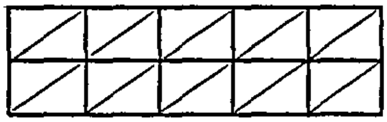
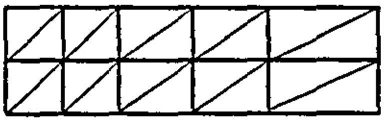
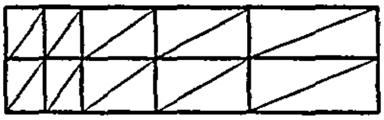
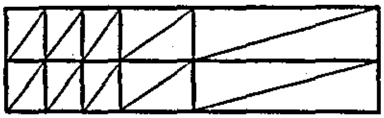
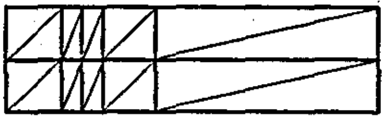
TABLE. 6 Percentage errors of load carring capacities from the exact solution

Analysis methods No. of elements	Finite Element Technique				Exact Solution (ref.[38])
	5	10	20	50	
Load carrying capacity(x10^7)	0.3667	0.4221	o.4415	0.4467	0.4468 x 10^7
% Error	17.93	5.54	1.20	0.03	———

TABLE. 7 Percentage errors of friction force values from the exact solution

Analysis methods No. of elements		Finite Element Technique				Exact Solution
		5	10	20	50	
Friction force	upper surface	3340.18	3340.18	3369.96	3369.62	3383.49
	lower surface	3340.32	3340.32	3337.63	3338.97	3353.73
% Error	upper surface	1.28	1.28	0.40	0.41	———
	lower surface	0.40	0.40	0.48	0.44	———

TABLE.8 Comparison of errors of various graded-mesh elements

	Graded Meshes	Values and Percentage Errors from the Exact Solution Values		
		Maximum Pressure	Load Carrying Capacity	Friction Force
(a)		14614.0 (11.74%)	$0.3667\mathrm{x}10^{7}$ (17.93)	3340.18 (1.28)
(b)		15239.2 (7.96)	0.3962 (11.33)	3349.66 (1.00)
(c)		16081.9 (2.87)	0.4199 (6.02)	3351.69 (0.94)
(d)		15802.1 (4.56)	0.4181 (6.41)	3345.93 (1.11)
(e)		15058.7 (9.05)	0.3944 (11.72)	3301.27 (2.43)
	Exact Solution	16557.1	$0.4468\mathrm{x}10^{7}$	3383.49

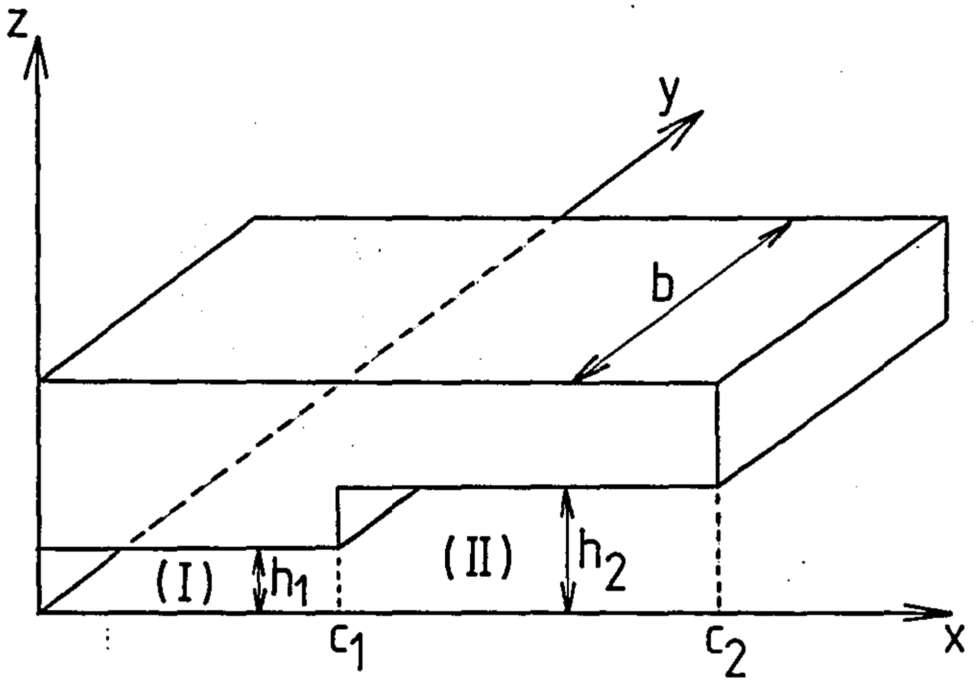


FIG.25 Geometry of a step bearing

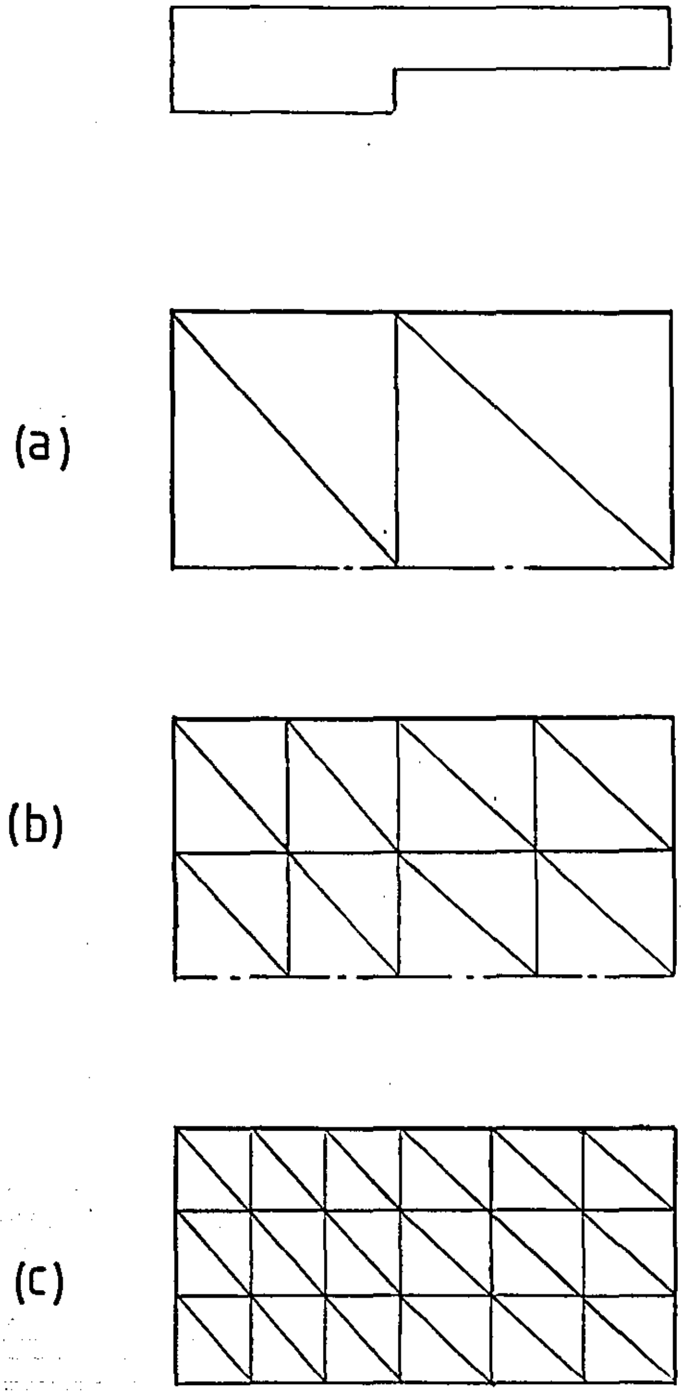


FIG.26 Finite element layouts of a step bearing.

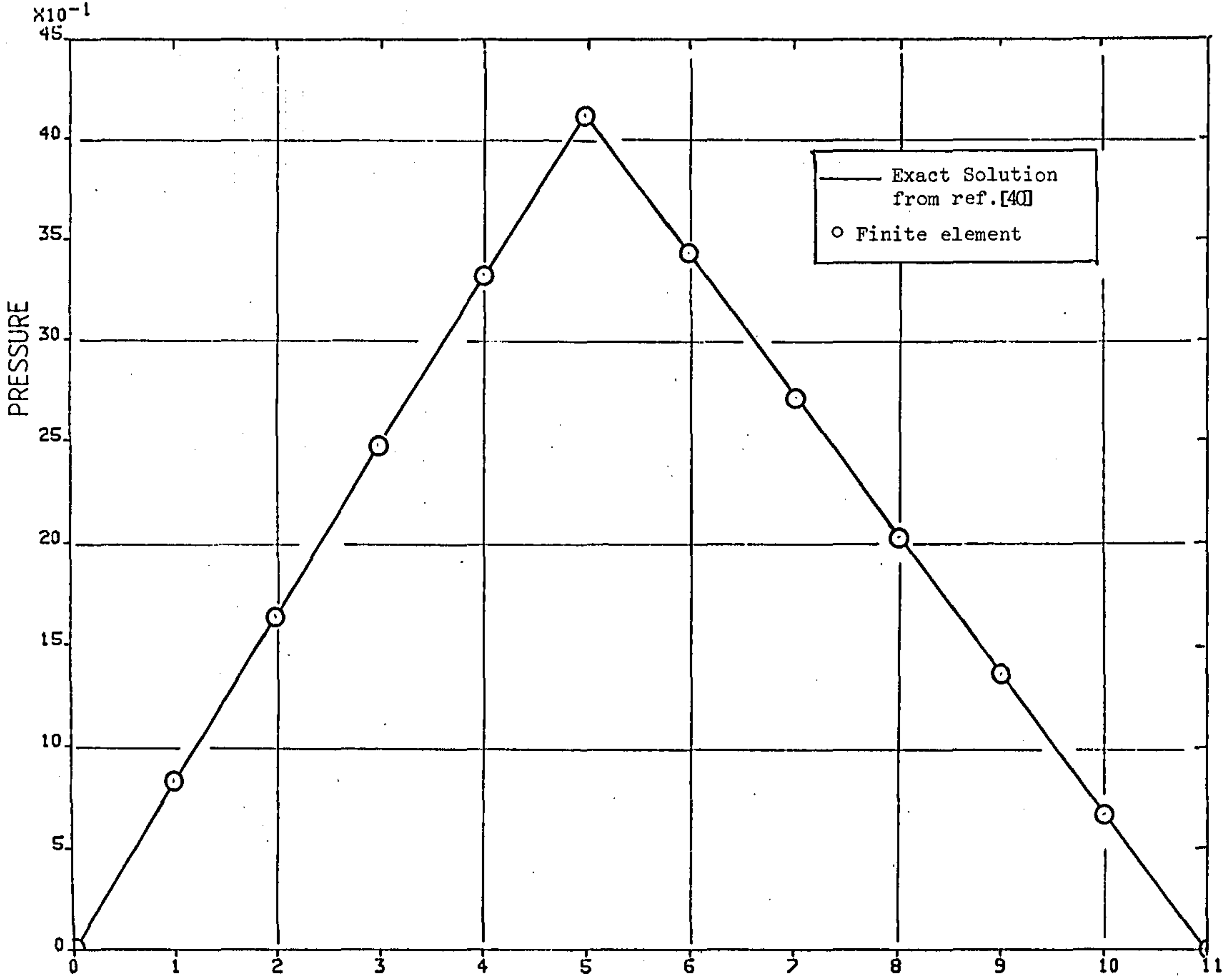


FIG.27 F.E. Solution of an infinite width step bearing.

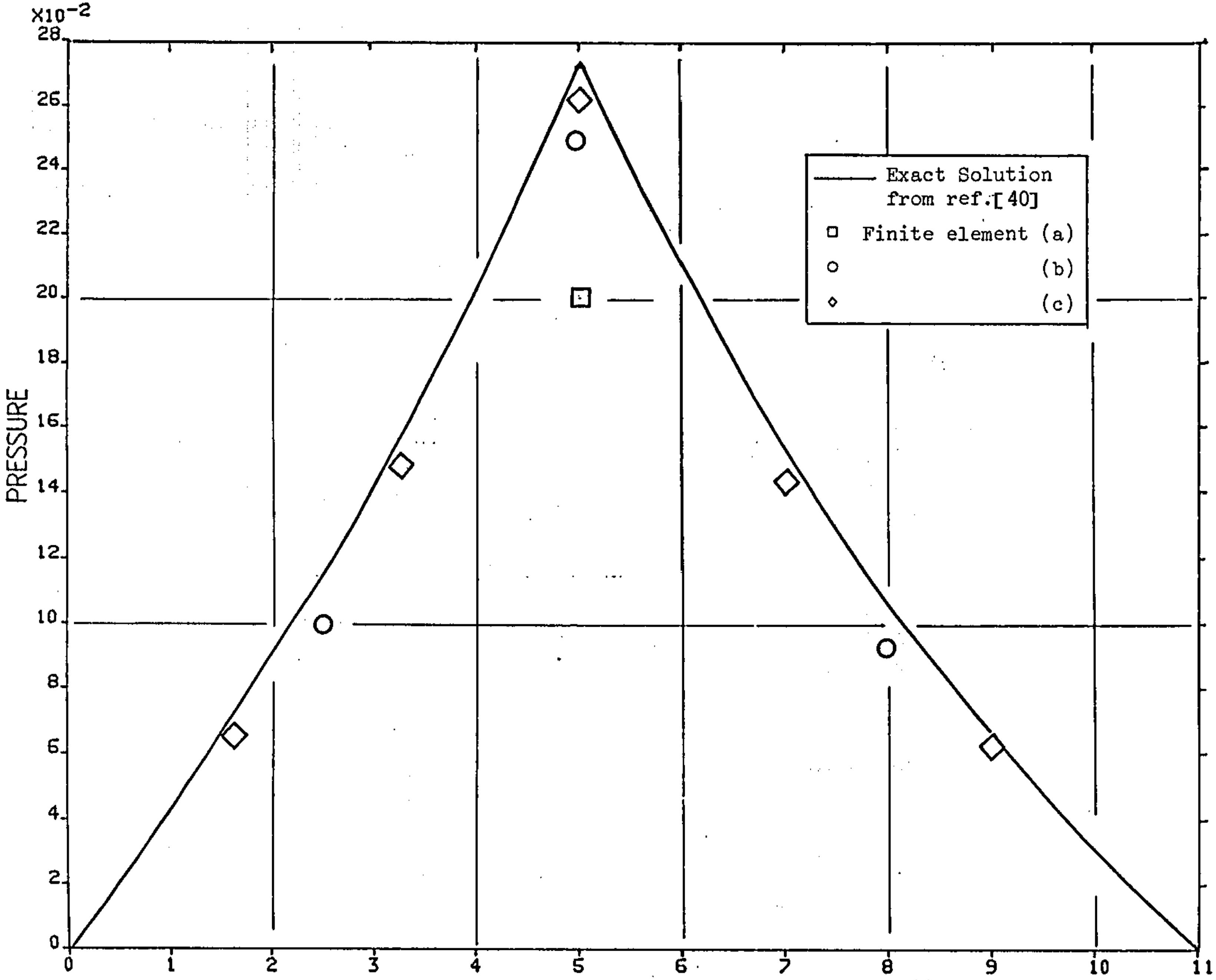


FIG.28 F.E. Solution of a finite width step bearing.

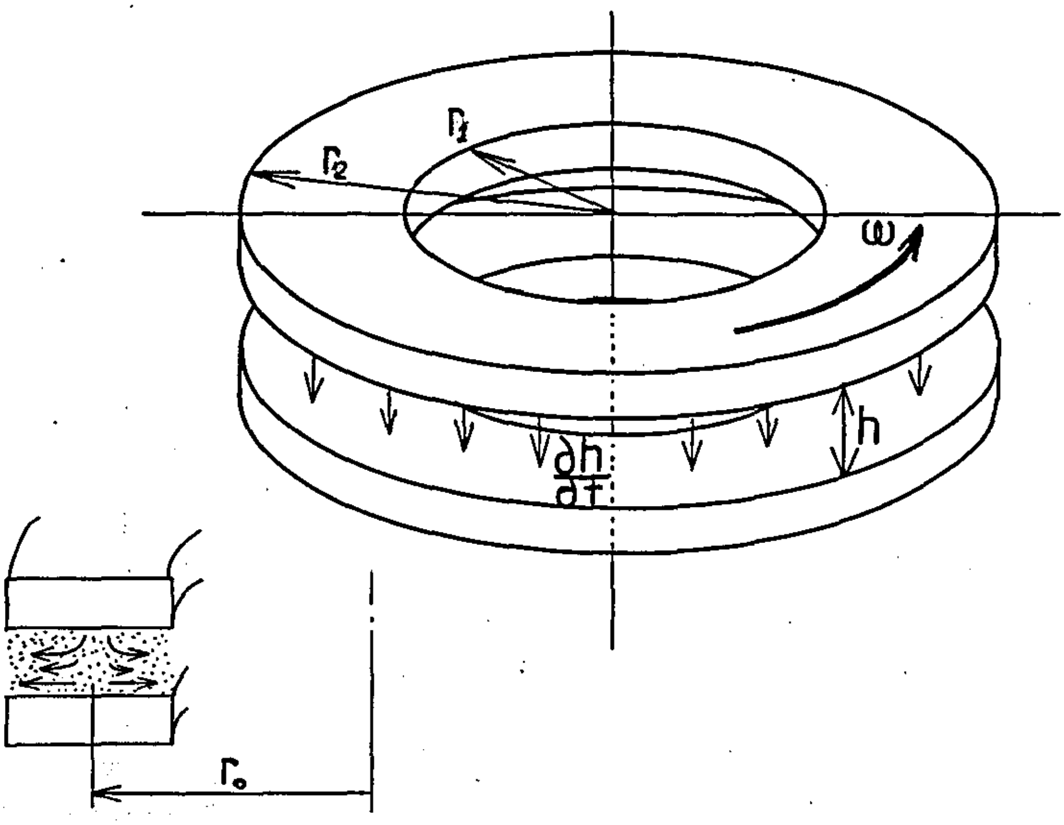


FIG.29 Geometry of discs.

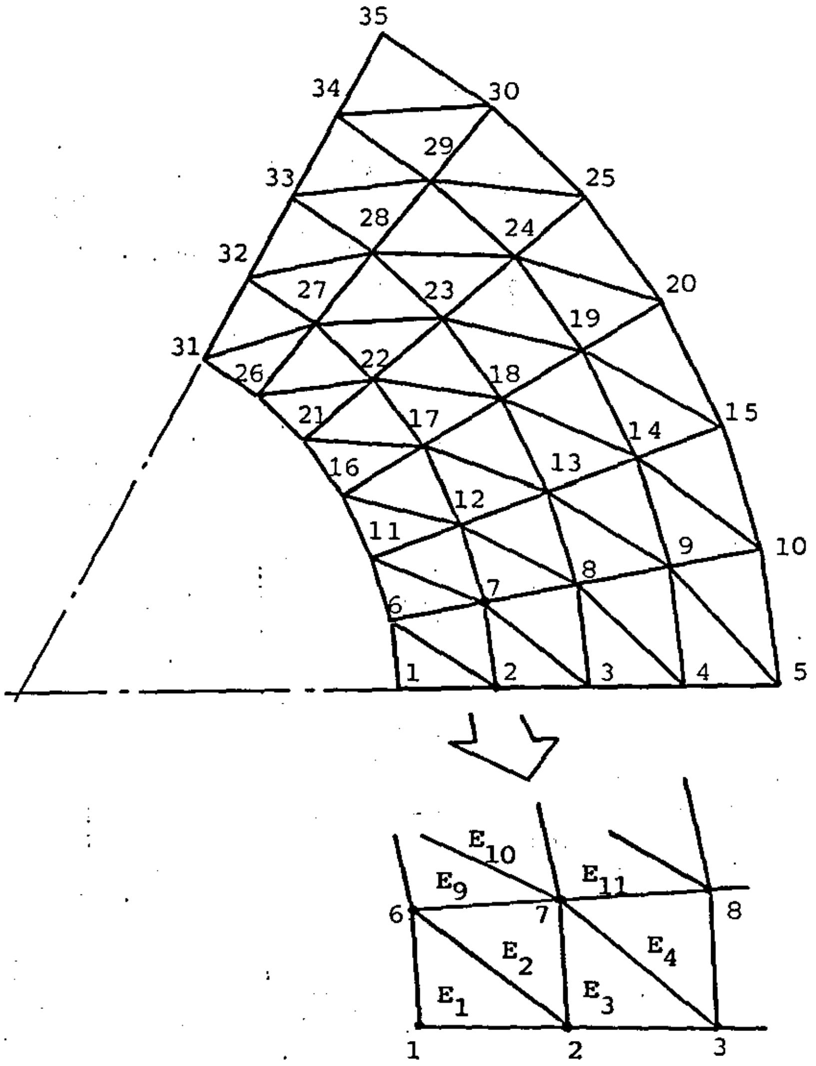


FIG. 30 Finite Element Layout of a 60 degree sector

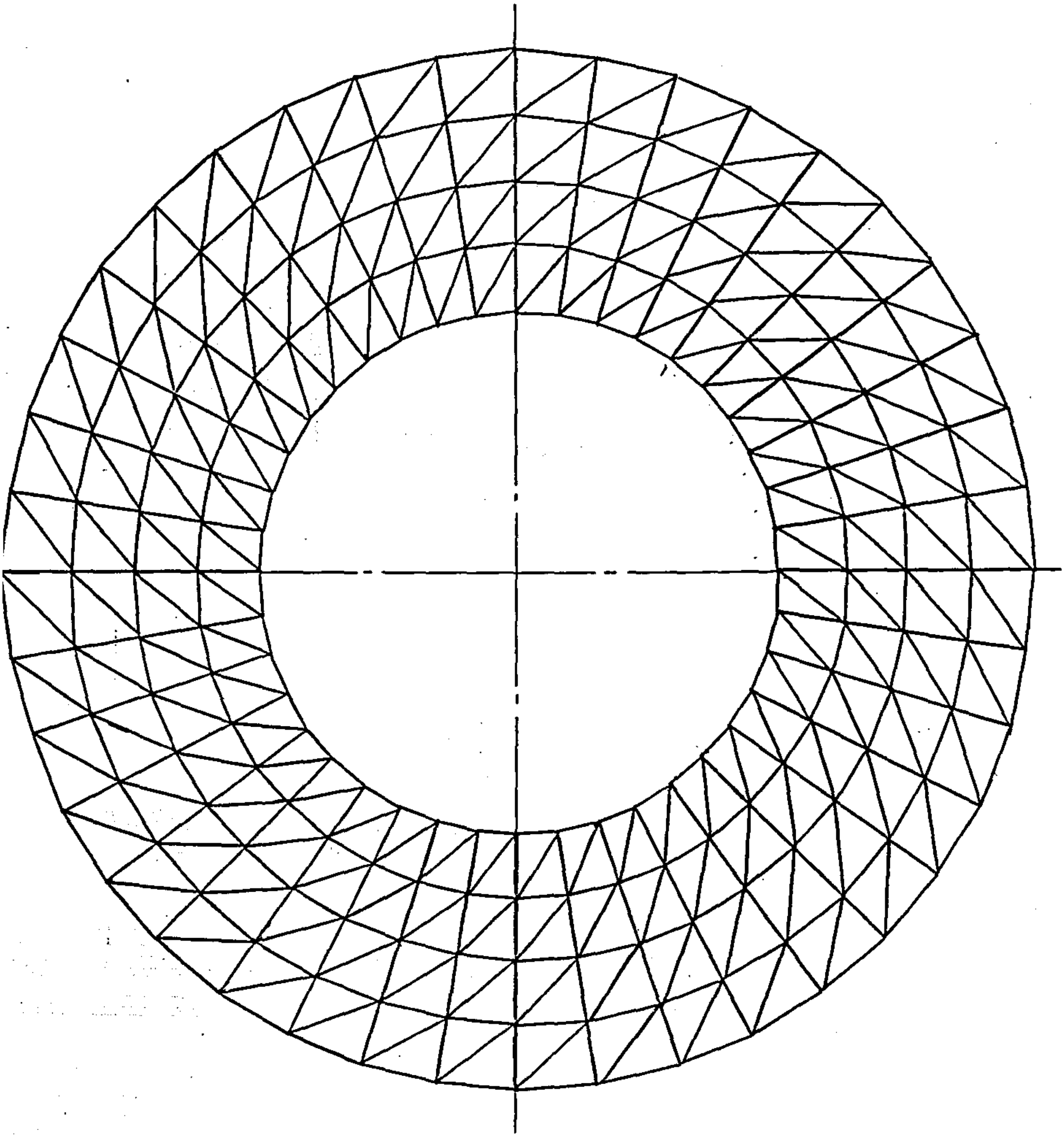
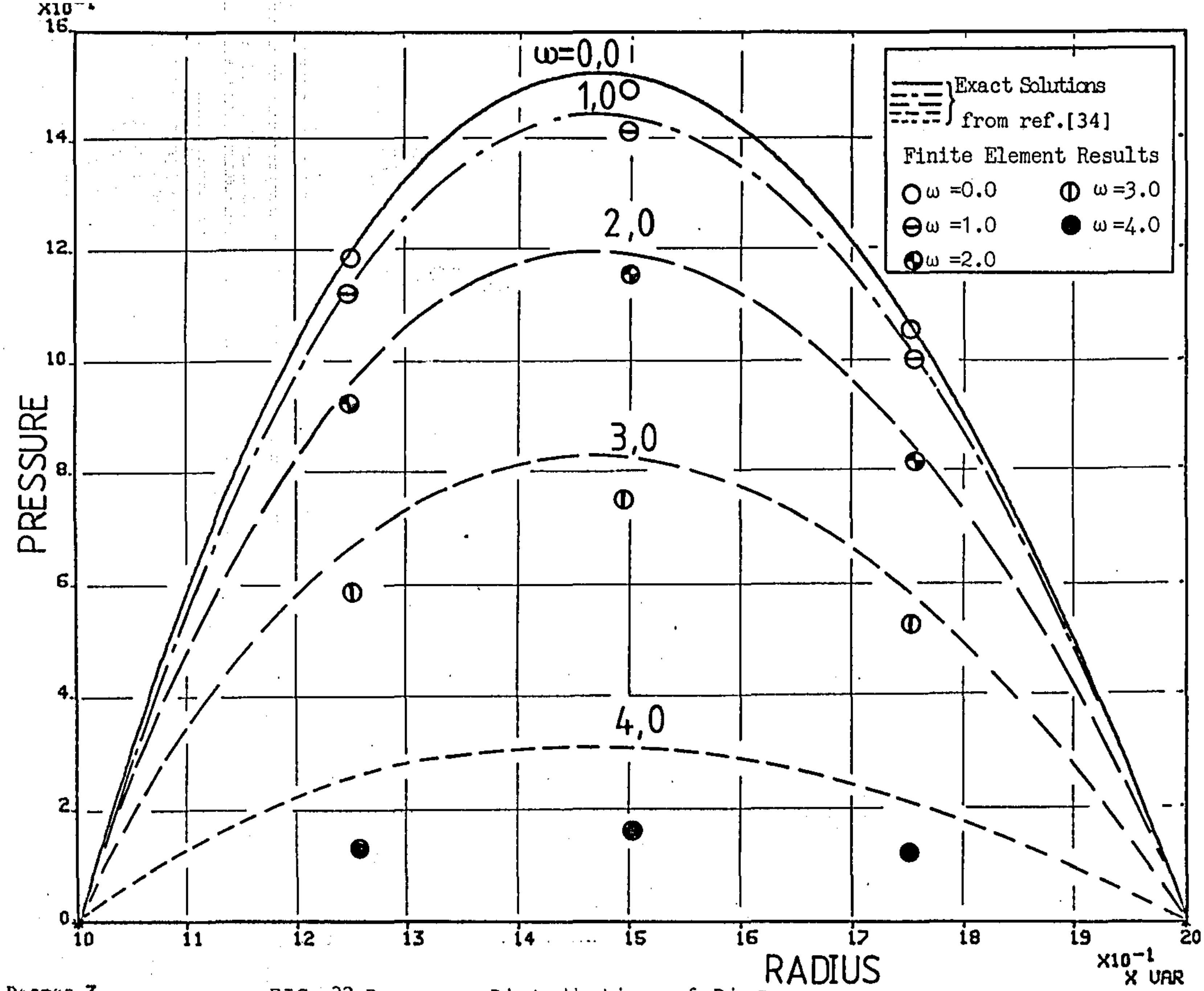
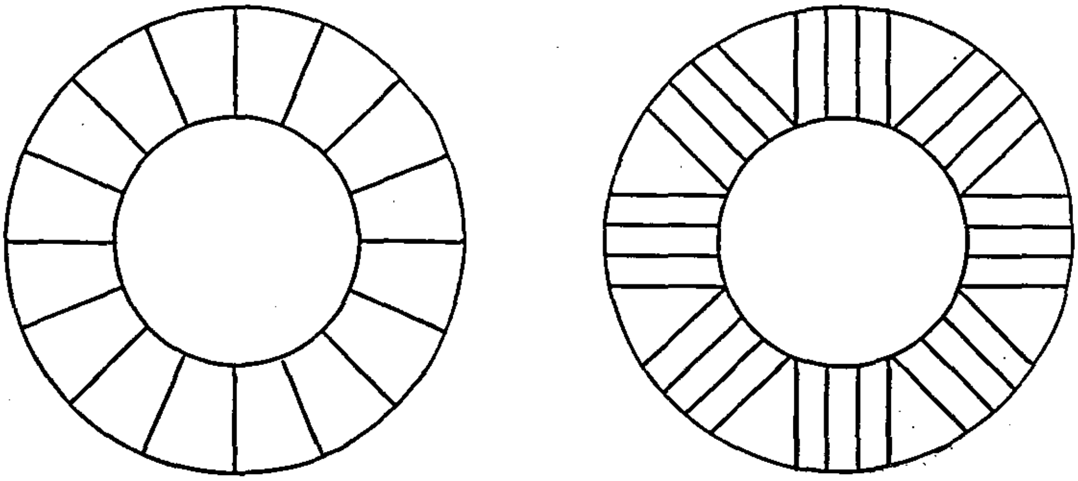


FIG.31 Finite element layout of a rotating disk



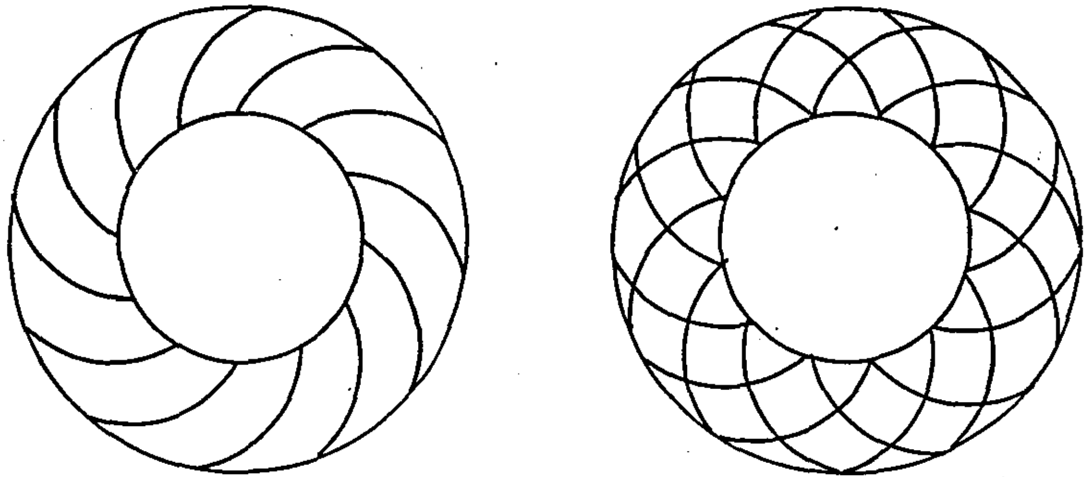
Degree 3

FIG. 32 Pressure Distribution of Disc



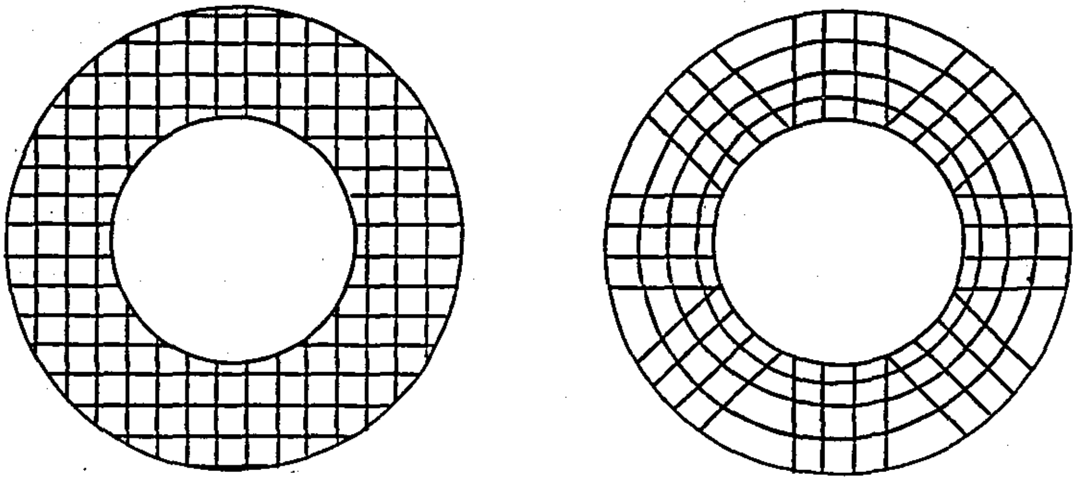
(a) radial (I)

(b) radial (II)



(c) spiral

(d) double spiral



(e) waffle

(f) radial (I) + circumferential

FIG. 33 Examples of groove patterns

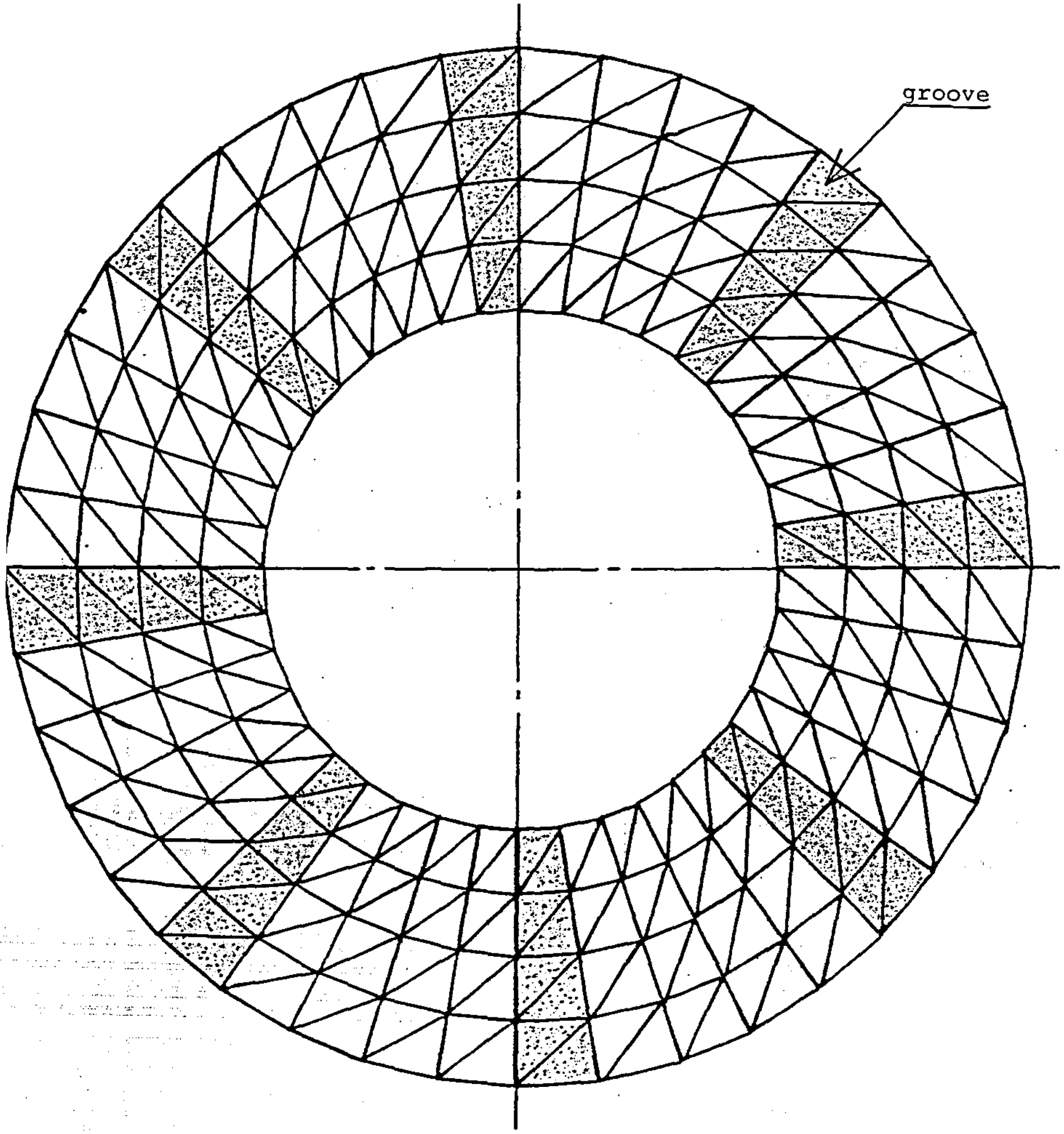


FIG.34 Finite element layout of a radial grooved disk

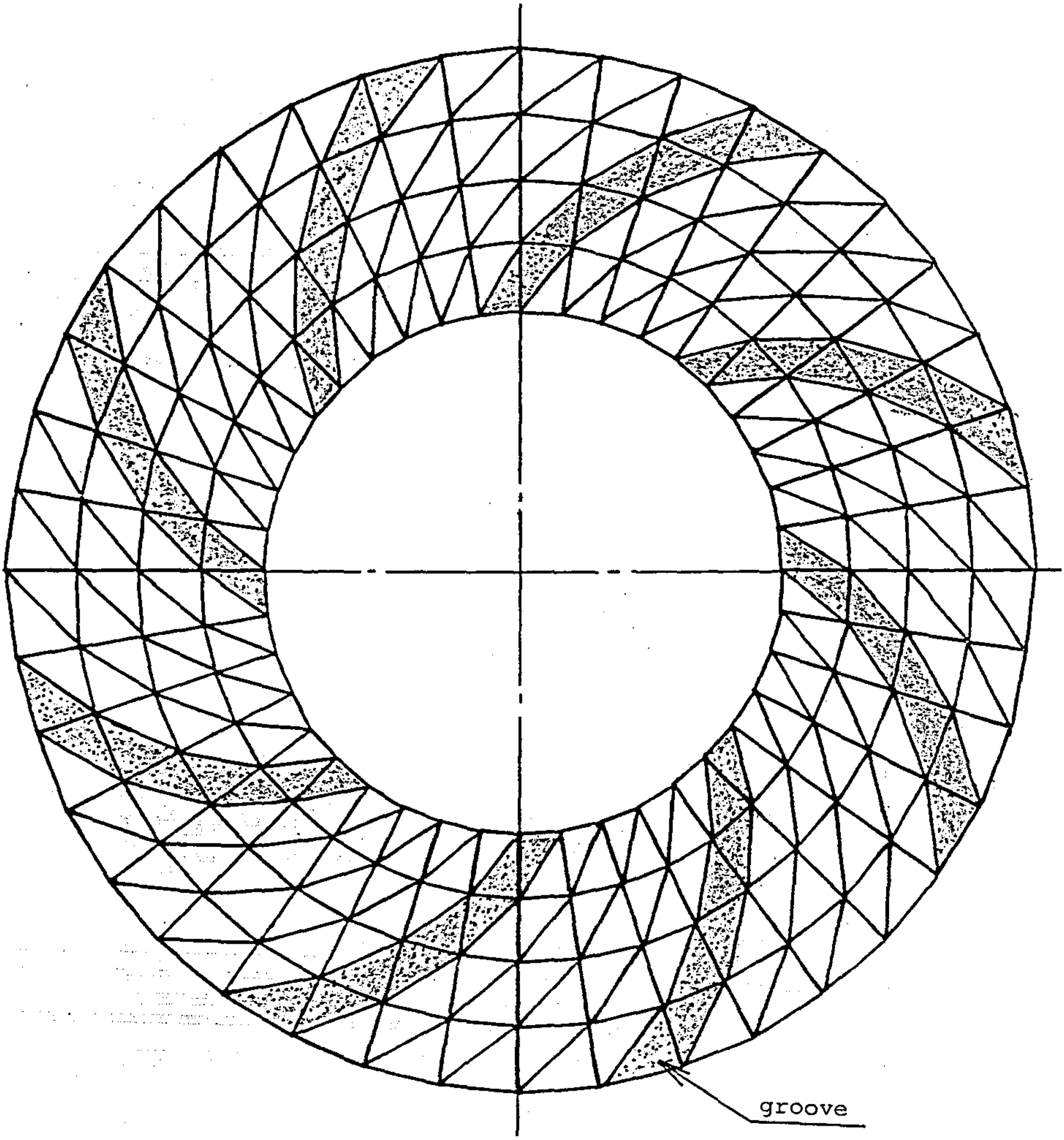


FIG.35 Finite element layout of a spiral grooved disk

r_1 = 1.0 m
r_2 = 2.0 m
depth of groove = 0.5 mm
film thickness = 1.0 mm
squeezing speed = -1.0 m/sec
angle velocity = 0.0 rad/sec

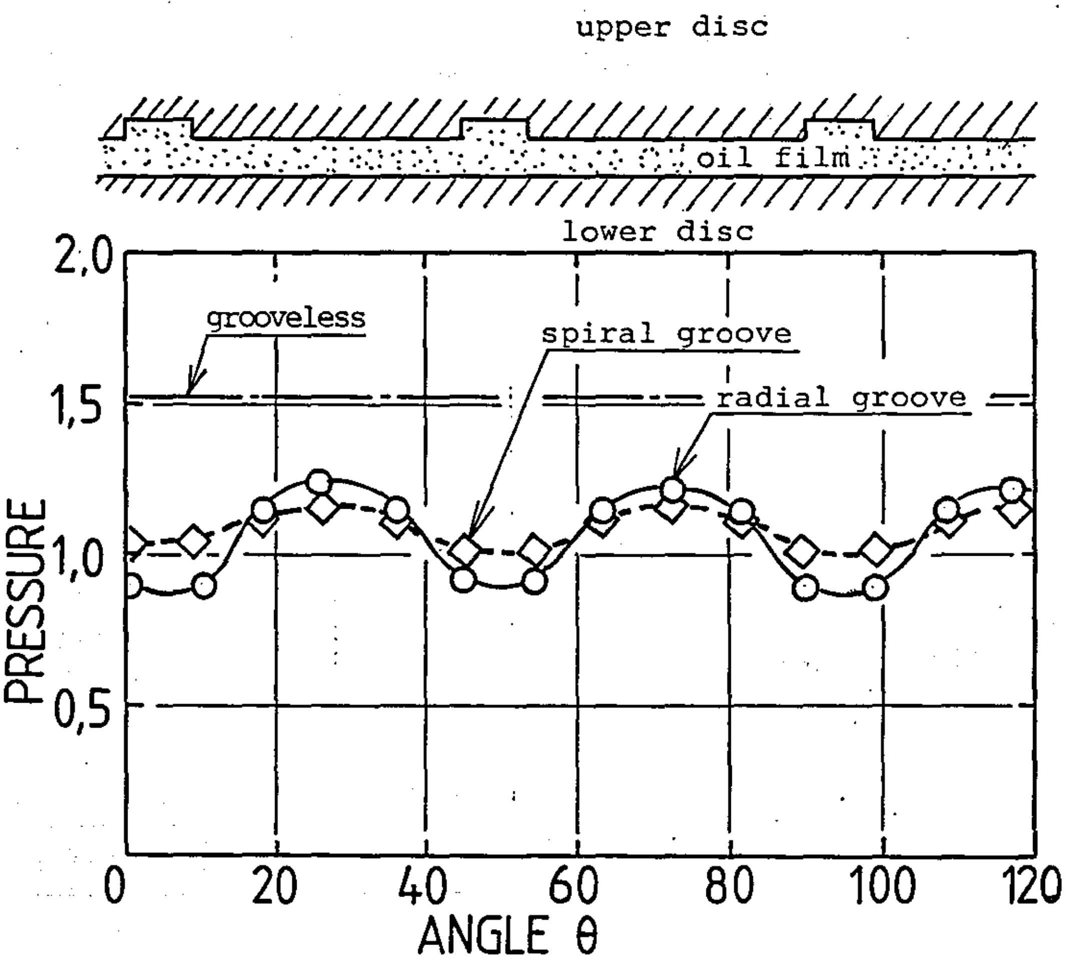


FIG.36 Pressure distribution between grooved discs (at r=1.5m)

r_1 = 1.0 m
r_2 = 2.0 m
depth of groove = 0.5 mm
film thickness = 1.0 mm
squeezing speed =-1.0 m/sec
angle velocity = 1.0 rad/sec

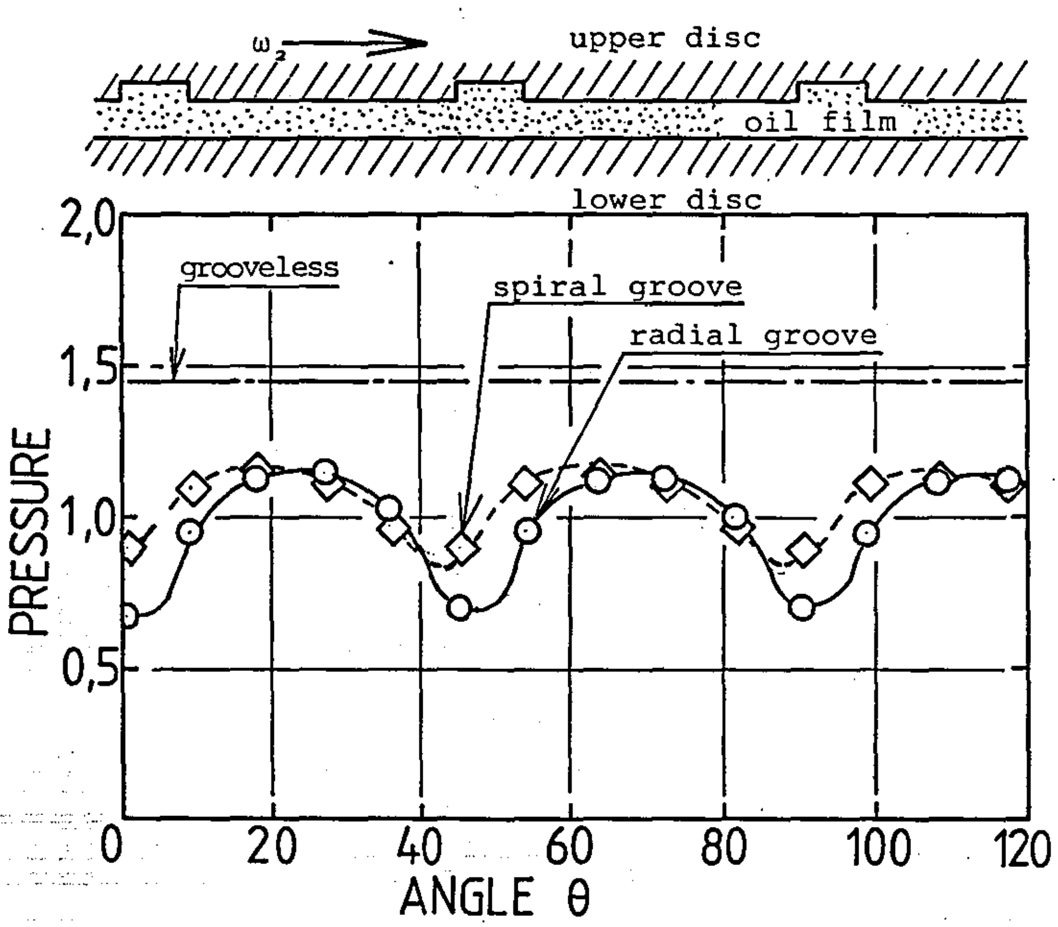


FIG.37 Pressure distribution between grooved disc with rotation (at r = 1.5m)

groove pattern = radial
r_1 = 1.0 m
r_2 = 2.0 m
depth of groove = 0.5 mm
film thickness = 1.0 mm
squeezing speed =-1.0 m/sec
angle velocity = 0 , 1.0 rad/sec

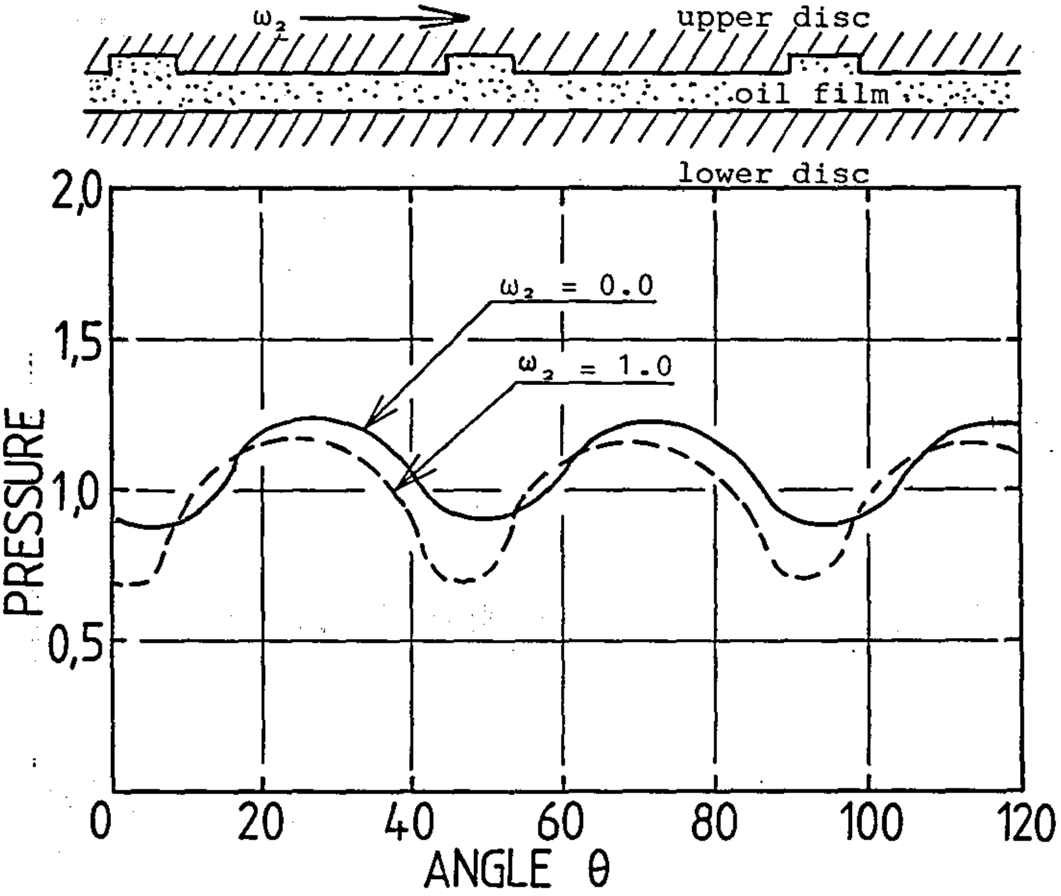


FIG. 38 Pressure distribution of radial grooved disc (at r=1.5m)

groove pattern = spiral
r_1 = 1.0 m
r_2 = 2.0 m
depth of groove = 0.5 mm
film thickness = 1.0 mm
squeezing speed =-1.0 m/sec
angle velocity = 0 , 1.0 rad/sec

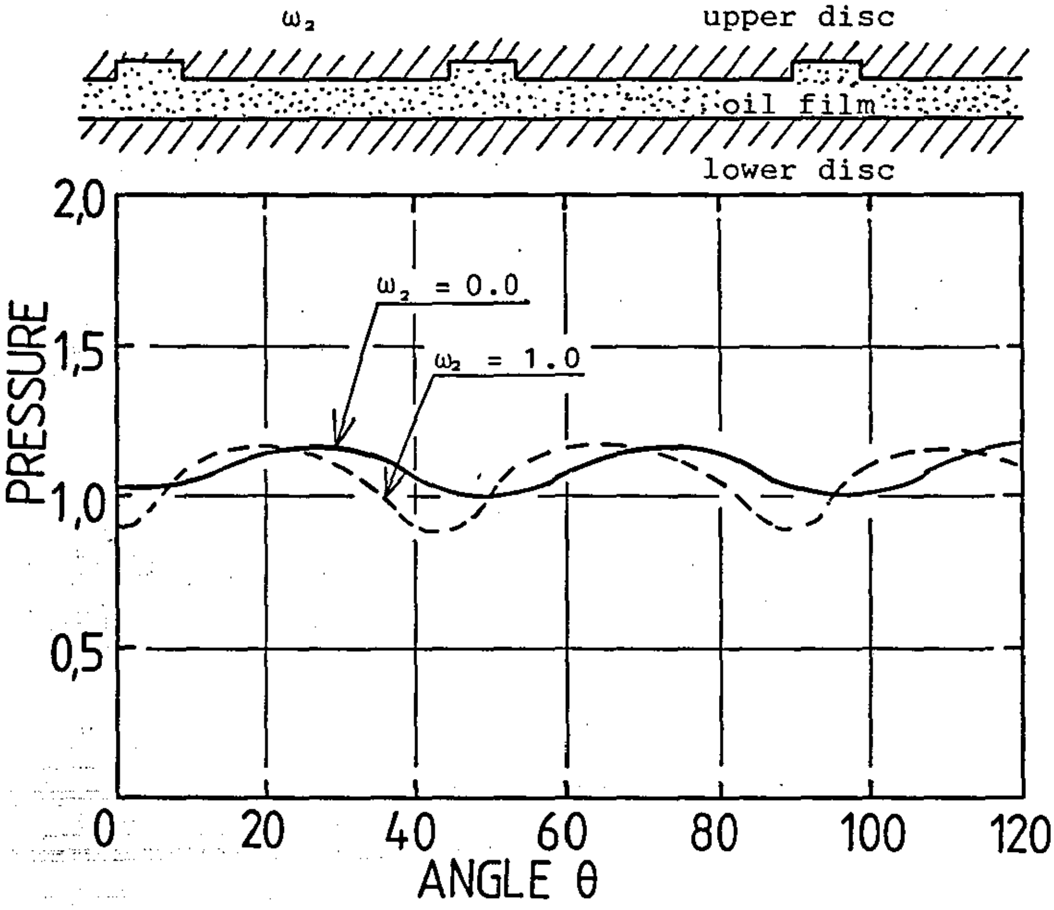


FIG. 39 Pressure distribution of spiral grooved disc (at r=1.5m)

TABLE. 9 Maximum, minimum pressures of grooved discs and flat disc at the radius r = 1.5 m

Motions	Groove Patterns Pressures	Radial	Spiral	Flat
Squeezing	Maximum Pressure	1.25	1.17	1.52
	Minimum Pressure	0.91	1.02	
	Max.-Min.	0.34	0.15	0.0
Squeezing + Rotating	Maximum	1.15	1.17	1.45
	Minimum	0.70	0.85	
	Max.-Min.	0.45	0.37	0.0

Effect of Grooves (percentage change)	
Radial	Spiral
82 %	77 %
60 %	67 %
——	——
79 %	81 %
48 %	59 %
——	——

Effect of Rotating Motion (change of pressure)	Maximum	-0.1	0.0	-0.07
	Minimum	-0.21	-0.17	
	Max.-Min.	0.11	0.22	——

FIG.40 PRESSURE DISTRIBUTION OF SQUEEZE DISC (RADIAL GROOVES) (non-rotating)

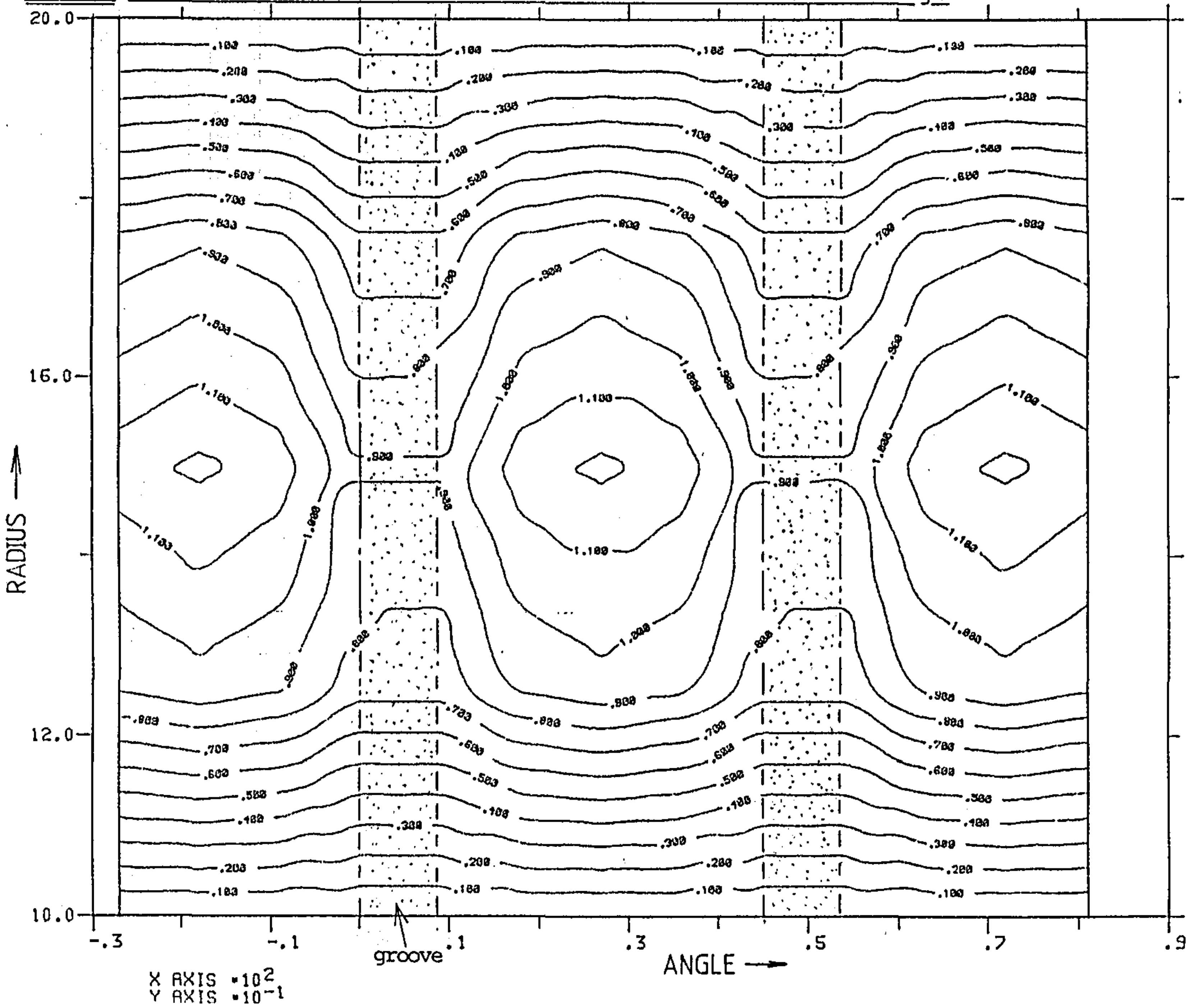


FIG.41 PRESSURE DISTRIBUTION OF SQUEEZE DISC (RADIAL GROOVES) (rotating)

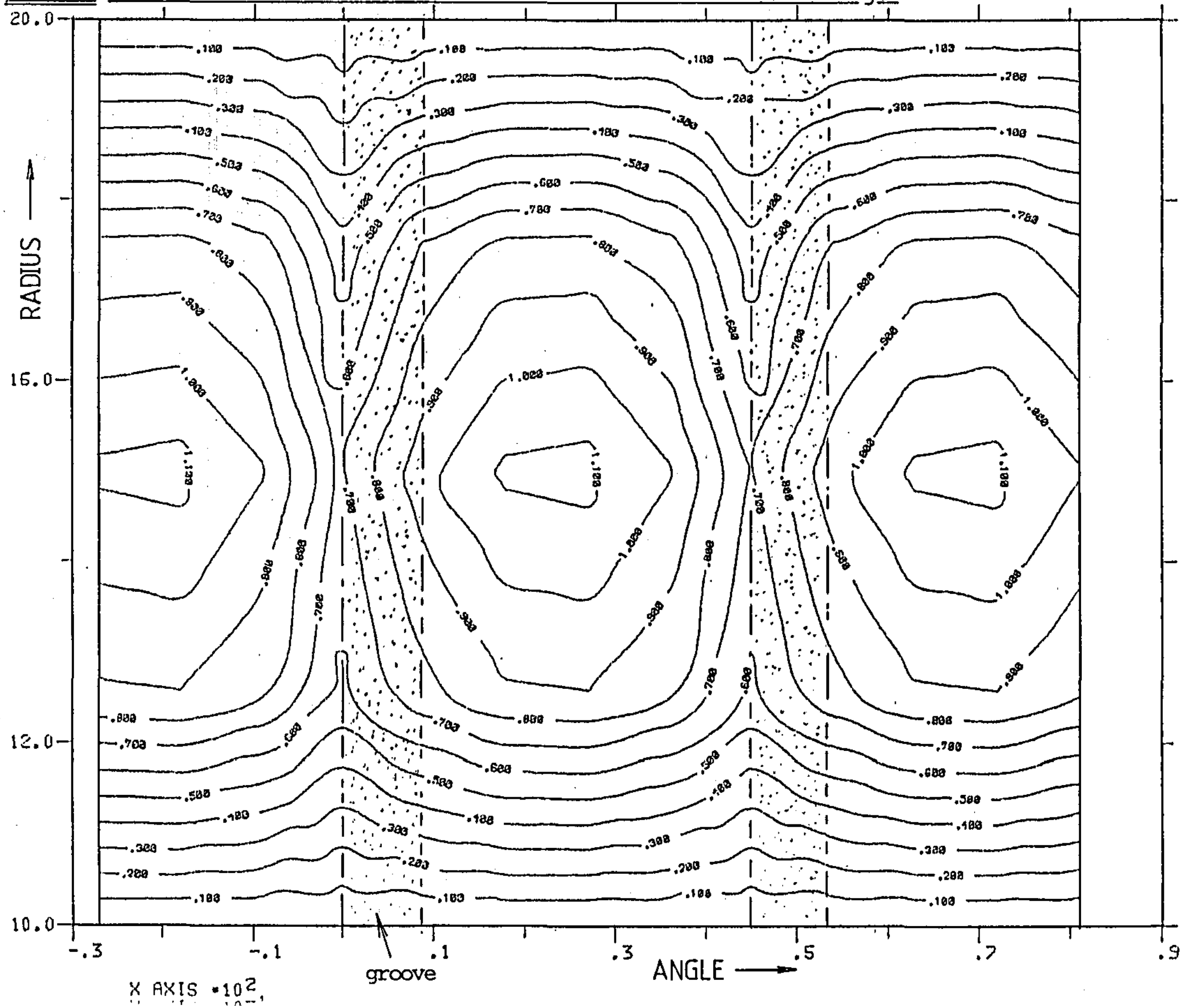


FIG.42 PRESSURE DISTRIBUTION OF SQUEEZE DISC (SPIRAL GROOVES) (non-rotating)

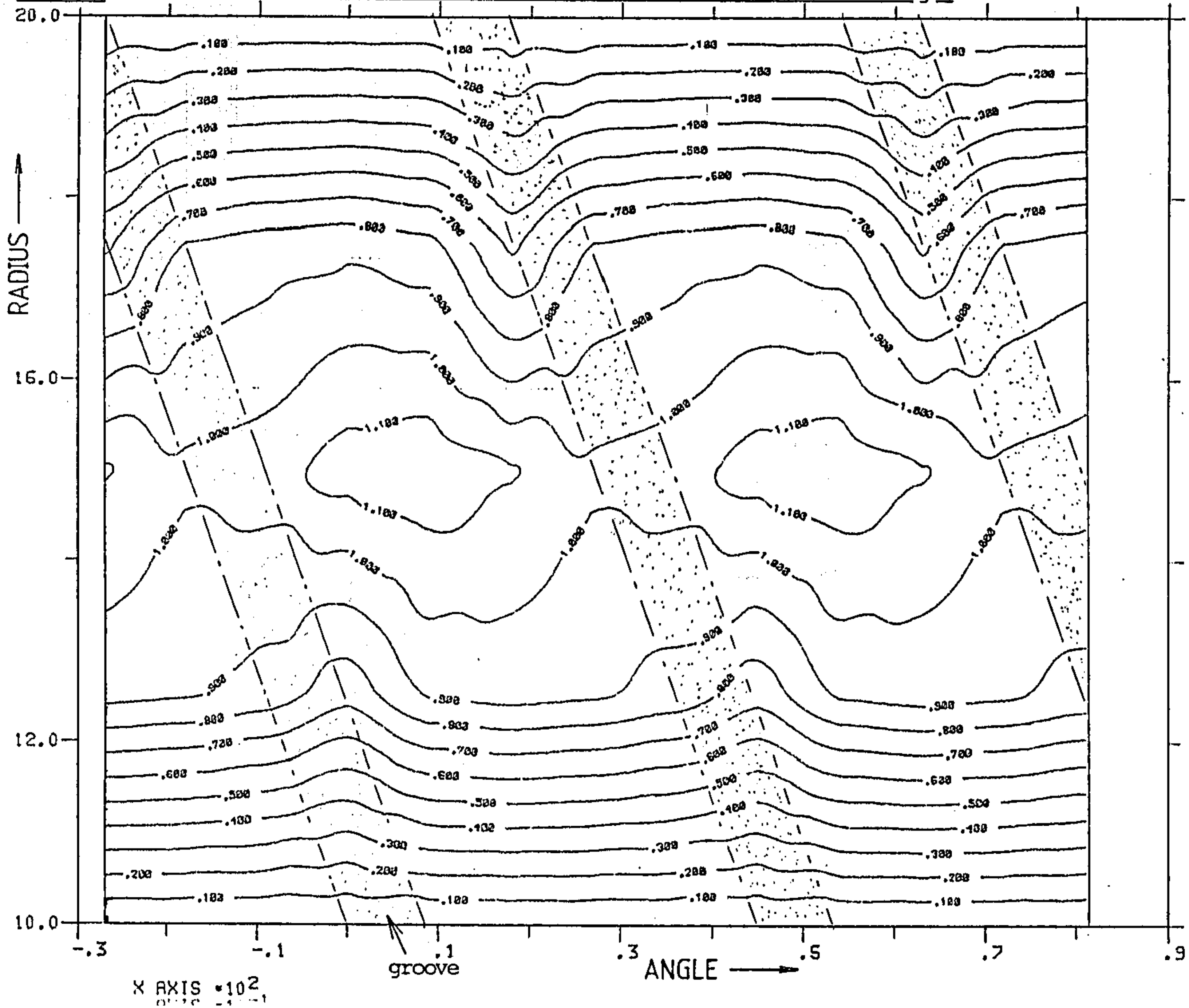
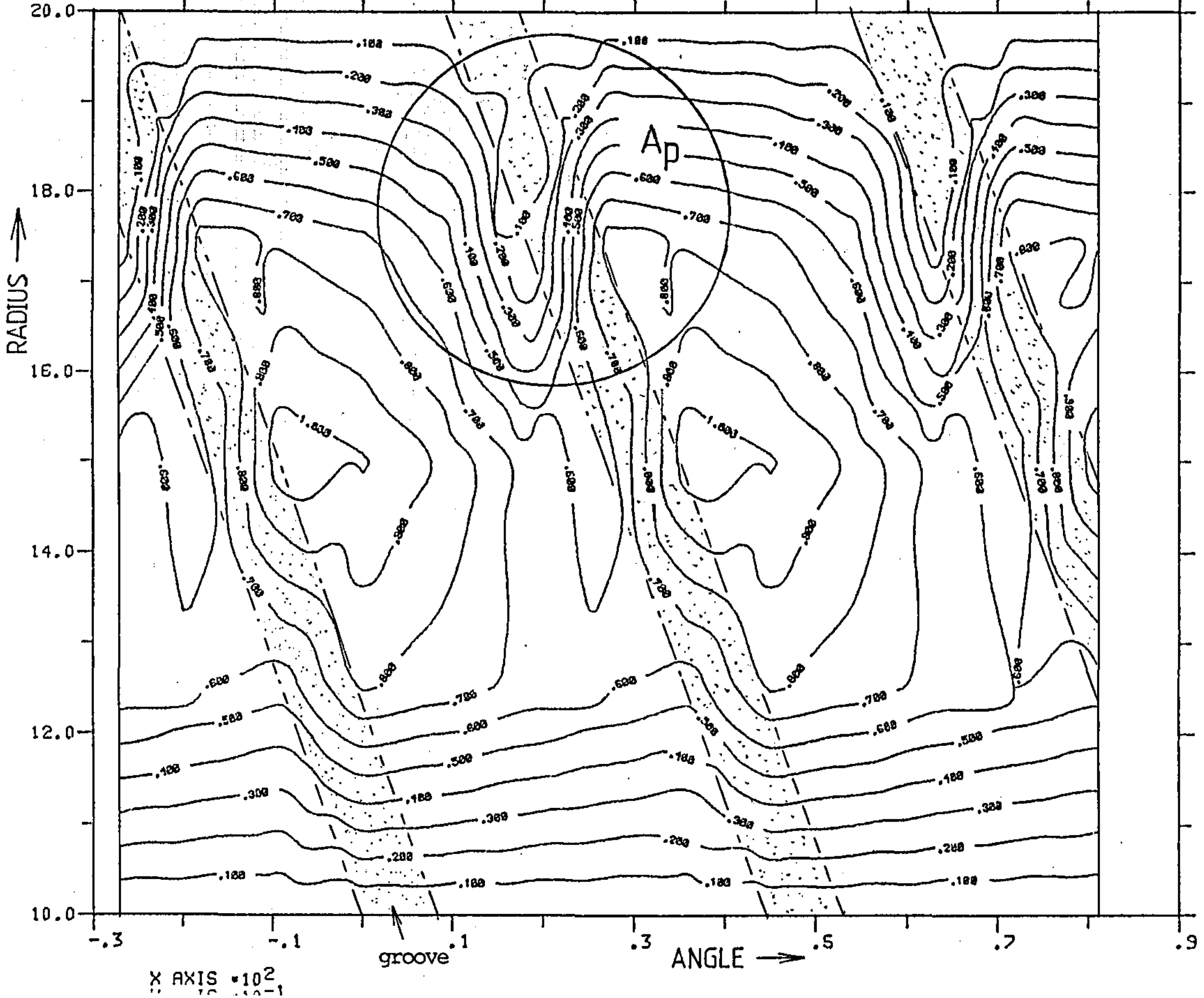


FIG.43 PRESSURE DISTRIBUTION OF SQUEEZE DISC (SPIRAL GROOVES) (rotating)



```
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C-----------------------------------------------------------------------------C
C                                                                             C
C          LL        UU      UU      BBBBBBBB              66                 C
C          LL        UU      UU      BB      BB          66                   C
C          LL        UU      UU      BB      BB        66                     C
C          LL        UU      UU      BBBBBBBBB        666666                  C
C          LL        UU      UU      BB       BB    66      66                C
C          LL        UUU    UUU      BB       BB    66      66                C
C          LLLLLLLL    UUUUUU        BBBBBBBBBB       666666                  C
C                                                                             C
C                                                                             C
C   ******************************************************************        C
C   **                                                              **        C
C   **    THE APPLICATION OF FINITE ELEMENT METHOD TO LUBRICATION   **        C
C   **                                                              **        C
C   ******************************************************************        C
C                                                                             C
C     TWO DIMENSION F.E.M. FOR LUBRICATION PROBLEMS    PROGRAM NAME...LUB6    C
C-----------------------------------------------------------------------------C
C-(NOMENCLATURE)-----------------------------------------C
C                                                        C
C                                                        C
C     NCOE              : NO. OF COORDINATE              C
C     NNEL       : NO. OF NODE PER ELEMENT               C
C     NELE       : TOTAL NO. OF ELEMENT                  C
C     IBPS(I,1)  : ELEMENT SEQUENCE COUNTER  (LE)        C
C     IBPS(I,2)  : ELEMENT TYPE CODE                     C
C                   { 3= TRIANGULAR ELEMENT }            C
C                   { 4=QUADRILATERAL ELEMENT }          C
C                   { 5=PARALLELOGRAM,RECTANGLE }        C
C     IBPS(I,3)  : NO. OF NODES PER ELEMENT (NNEL)       C
C     IBPS(I,4)  : NO. OF DATA ITEM                      C
C     SECPR(I,1) : FLUID FILM DENSITY (DEN)              C
C     SECPR(I,2) : FLUID FILM VISCOSITY (VIS)            C
C     TH(LE,NNEL): FLUID FILM THICKNESS                  C
C     NNOD       : NODAL NO.                             C
C     NDF        : TYPE OF KNOWN VALUE                   C
C                 { 1 : FLOW VALUE     }                 C
C                 { 2 : PRESSURE VALUE}                  C
C                                                        C
C     BCVL       : KNOWN VALUE OF FLOW OR PRESSURE       C
C     BCAC(I,1)  : SHEAR ACTION IN X-DIRECTION : (UX1)   C
C                  {LOWER SURFACE VELOCITY (H=0.0) }     C
C                  /ANGLAR VELOCITY OF THE LOWER SURFACE\ C
C                  \WHEN USING IUNIT2=2                / C
C                                                        C
C     BCAC(I,2)  : SHEAR ACTION IN X-DIRECTION : (UX2)   C
C                  {UPPER SURFACE VELOCITY  (H=H  ) }    C
C                  /ANGLAR VEL. OF THE UPPER SURFACE  \  C
C                  \WHEN USING IUNIT2=2               /  C
C                                                        C
C     BCAC(I,3)  : SHEAR ACTION IN Y-DIRECTION : (UY1)   C
C                  (0.0  IN POLAR COORDINATE)            C
C     BCAC(I,4)  : SHEAR ACTION IN Y-DIRECTION : (UY2)   C
C                  (0.0  IN POLAR COORDINATE)            C
C     BCAC(I,5)  : BODY FORCE ACTION IN X-DIRECTION (BX1) C
C     BCAC(I,6)  : BODY FORCE ACTION IN X-DIRECTION (BX2) C
C     BCAC(I,7)  : BODY FORCE ACTION IN Y-DIRECTION (BY1) C
C     BCAC(I,8)  : BODY FORCE ACTION IN Y-DIRECTION (BY2) C
C     BCAC(I,9)  : SQUEEZE ACTION (H)                    C
C     BCAC(I,10) : DIFFUSION ACTION (VD)                 C
C     NODES      : TOTAL NO. OF NODES                    C
C     CFX(I)     : CENTRIFUGAL FORCE IN X-DIRECTION      C
C     CFY(I)     : CENTRIFUGAL FORCE IN Y-DIRECTION      C
C                                                        C
C                                                        C
C--------------------------------------------------------C
C
```

```
C   C------------------------C
C   C                        C
C   C     MAIN PROGRAM       C
C   C                        C
C   C------------------------C
C
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER R,W

      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1           NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2           GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3           A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK2/MTKP(200,200),MKP(200,200),MTKP1(200,200),MP(400,4,4)
      COMMON/BLK3/MTKUX(200,200),MKUX(400,4,4)
      COMMON/BLK4/MTKUY(200,200),MKUY(400,4,4)
      COMMON/BLK5/MTKBX(200,200),MKBX(400,4,4)
      COMMON/BLK6/MTKBY(200,200),MKBY(400,4,4)
      COMMON/BLK7/MTKH(200,200),MKH(400,4,4)
      COMMON/BLK8/MTKVD(200,200),MKVD(400,4,4)
      COMMON/BLK9/Q(400),P(400),QIN,QOUT,WW,QEL(400,4)
      COMMON/BLK10/TEST
      COMMON/BLK13/DELTA(4,4)
      COMMON/BLK14/ CFX(400),CFY(400)
      COMMON/BLK23/NDQ(400),NDP(400)
C
C
C-----READ INPUT DATA
      R=5
      W=6
      WRITE(6,1000)
 1000 FORMAT('DO YOU NEED TO PRINT THE DETAILS OF CALCULATIONS,',
     1     'YES...OR...NO...OR...XX (XX : ONLY THE FINAL RESULT)')
      READ(5,1100) RESULT
      WRITE(6,1011) RESULT
 1011 FORMAT(1H+,T100,'____',A4)
 1001 WRITE(6,1002)
 1002 FORMAT('CHOOSE THE COORDINATE SYSTEM.    ',
     1/'...NOTICE...Only when the polar coordinate is used,',
     2/'              the centrifugal force will be considered..',
     3    /'IF YOU USE X-Y COORDINATE.........PLEASE KEY IN :XY',
     4   /'IF YOU USE POLAR COORDINATE........PLEASE KEY IN :PO')
      READ(5,1100) COOR
      WRITE(6,1012) COOR
 1012 FORMAT(1H+,T55,'____',A3)
      IF(COOR.NE.'XY') GO TO 1004
      IUNIT2=1
      GO TO 1009
 1004 IF(COOR.NE.'PO') GO TO 1001
      WRITE(6,1005)
 1005 FORMAT('CHOOSE THE UNIT OF ANGLE.',
     1     /,'IF YOU USE UNIT DEGREE.....PLEASE KEY IN`DEG`',
     2     /,'IF YOU USE UNIT RADIAN.....PLEASE KEY IN `RAD`')
      READ(5,1100) UNIT1
      WRITE(6,1013) UNIT1
 1013 FORMAT(1H+,T48,'____',A3)
      WRITE(6,1007)
 1007 FORMAT('WHICH VELOCITY DO YOU USE, X-Y VEL.OR ANGULAR.',
     1  /'IF X-Y VEL......PLEASE KEY-IN    1',
     2  /'IF ANGULAR VEL..PLEASE KEY-IN    2')
      READ(5,1008) IUNIT2
 1008 FORMAT(I1)
      WRITE(6,1014) IUNIT2
 1014 FORMAT(1H+,T35,'    ',I2)
```

```
C
C-----READ ELEMENT PROPERTIES
 1009 WRITE(6,6)
    6 FORMAT(///,'Calculation Start.',
     1//,'READING  TEST NAME ')
      READ(R,1010) TEST
      WRITE(6,10)
   10 FORMAT('READING NCOE,NODES,NELE,NNEL')
      READ(R,*) NCOE,NODES,NELE,NNEL
      WRITE(6,30)
   30 FORMAT('READING IBPS(I,J),SECPR(I,K)')
      READ(R,1100) DENVIS
      IF(DENVIS.EQ.'YES') GO TO 21
      DO 20 I=1,NELE
   20 READ(R,*) (IBPS(I,J),J=1,4),(SECPR(I,K),K=1,2)
      GO TO 51
   21 READ(R,*) (IBPS(1,J),J=1,4),(SECPR(1,K),K=1,2)
      DO 22 I=2,NELE
      SECPR(I,1)=SECPR(1,1)
      SECPR(I,2)=SECPR(1,2)
      DO 22 J=1,4
      IBPS(I,J)=IBPS(1,J)
   22 CONTINUE
   51 WRITE(6,50)
   50 FORMAT('READING IBPS(I,1),NME(I,J),TH(I,K)')
      DO 40 I=1,NELE
   40 READ(R,*) IBPS(I,1),(NME(I,J),J=1,NNEL),(TH(I,K),K=1,NNEL)
C-----READ COODINATES OF NODES
      IF(COOR.NE.'PO') GO TO 73
   71 WRITE(6,72)
   72 FORMAT('READING IGELCO(I,J)',
     1       ' IN POLER COORDINATE')
      GO TO 74
   73 WRITE(6,70)
   70 FORMAT('READING I,GELCO(I,J)')
   74 READ(5,*) LTYPE
      IF(LTYPE.EQ.1) GO TO 61
      DO 60 I=1,NODES
   60 READ(R,*) I,(GELCO(I,J),J=1,NCOE)
      GO TO 65
   61 READ(5,*) RA1,RA2,THITA1,THITA2,ND
      DO 62 I=1,NODES
      NA=1+ND
      NB=(I-1)/NA
      GELCO(I,1)=RA1+(I-1-NA*NB)*(RA2-RA1)/ND
      GELCO(I,2)=THITA2*NB
   62 CONTINUE
   65 NOD=2*NODES
      DO 75 L=1,NOD
      GCOE(L)=0.0
   75 CONTINUE
         DO 80 I=1,NELE
      DO 80 J=1,NNEL
       DO 80 K=1,NODES
      IF(NME(I,J).EQ.K) GO TO 90
      GO TO 80
   90 GCOE(K)=GELCO(K,1)
      GCOE(K+NODES)=GELCO(K,2)
   80 CONTINUE
C
C
```

```
C-----READ BOUNDARY CONDITIONS
C
      WRITE(6,901)
  901 FORMAT('READING NQ,NP')
      READ(R,*) NQ,NP
      WRITE(6,902)
  902 FORMAT('READ BC   YES  OR  NO ')
      READ(5,1100) BC
      WRITE(6,91)
   91 FORMAT('READING NNOD(I),NDF(I),BCVL(I)')
      IF(BC.EQ.'YES') GO TO 906
      DO 92 I=1,NODES
   92 READ(R,*) NNOD(I),NDF(I),BCVL(I)
      J=0
      K=0
      DO 905 I=1,NODES
      IF(NDF(I).NE.1) GO TO 904
      K=K+1
      NDQ(K)=NNOD(I)
      GO TO 905
  904 J=J+1
      NDP(J)=NNOD(I)
  905 CONTINUE
      GO TO 911
  906 READ(5,*) (NDP(I),I=1,NP)
      K=0
      DO 907 I=1,NODES
      NNOD(I)=I
      NDF(I)=2
      DO 908 J=1,NP
      IF(I.EQ.NDP(J)) GO TO 907
  908 CONTINUE
      K=K+1
      NDQ(K)=I
      NDF(I)=1
  907 CONTINUE
      READ(5,*) BCVLP,BCVLQ
      DO 909 I=1,NP
      BCVL(NDP(I))=BCVLP
  909 CONTINUE
      DO 910 I=1,NQ
      BCVL(NDQ(I))=BCVLQ
  910 CONTINUE
  911 WRITE(6,93)
   93 FORMAT('READING NNOD(I),BCAC(I,J)')
      READ(R,1100) BCA
      IF(BCA.EQ.'YES') GO TO 98
      DO 94 I=1,NODES
   94 READ(R,*) NNOD(I),(BCAC(I,J),J=1,10)
      GO TO 96
   98 READ(R,*) NNOD(1),(BCAC(1,J),J=1,10)
      DO 99 I=2,NODES
      DO 99 J=1,10
      BCAC(I,J)=BCAC(1,J)
   99 CONTINUE
 1010 FORMAT(A80)
 1100 FORMAT(A3)
C
C-----WRITE THE TITLE AND INPUT DATA TO CHECK
C
```

```
C
C-----WRITE THE TITLE AND INPUT DATA TO CHECK
C
   96 WRITE(W,100)
  100 FORMAT(///////////////////////,
     1/,2X,'CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC',
     2/,2X,'C',48X,'C',
     3/,2X,'C  THE APPLICATION OF F.E.M. TO THE LUBRICATION  C',
     4/,2X,'C',48X,'C',
     5/,2X,'CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC')
      WRITE(W,101) TEST
  101 FORMAT(//,'TEST NAME...',3X,A80)
      IF(RESULT.NE.'YES') GO TO 106
      WRITE(W,105)
  105 FORMAT(//,2X,'CCCCCCCCCCCCCCCC',/,2X,'C',14X,'C',
     1/,2X,'C  INPUT DATA  C',
     2/,2X,'C',14X,'C',/,2X,'CCCCCCCCCCCCCCCC')
      WRITE(W,2001) TEST
 2001 FORMAT(//,A80)
      WRITE(W,2002) NCOE,NODES,NELE,NNEL
 2002 FORMAT(4I5)
      NN=NELE
      IF(DENVIS.EQ.'YES') NN=1
      DO 2003 I=1,NN
 2003 WRITE(W,2004) (IBPS(I,J),J=1,4),(SECPR(I,K),K=1,2)
 2004 FORMAT(4I5,2E10.3)
      IF(NNEL.EQ.4) GO TO 3010
      DO 2005 I=1,NELE
 2005 WRITE(W,2006) IBPS(I,1),(NME(I,J),J=1,NNEL),(TH(I,K),K=1,NNEL)
 2006 FORMAT(I4,2X,3I4,3F10.5)
      GO TO 3013
 3010 DO 3011 I=1,NELE
 3011 WRITE(W,3012) IBPS(I,1),(NME(I,J),J=1,NNEL),(TH(I,K),K=1,NNEL)
 3012 FORMAT(I4,2X,4I4,4F10.5)
 3013 DO 2007 I=1,NODES
 2007 WRITE(W,2008) I,(GELCO(I,J),J=1,NCOE)
 2008 FORMAT(I4,2X,2F10.5)
      DO 2009 I=1,NODES
 2009 WRITE(W,2010) NNOD(I),NDF(I),BCVL(I)
 2010 FORMAT(2I4,2X,F10.5)
      NOD=NODES
      IF(BCA.EQ.'YES') NOD=1
      DO 2011 I=1,NOD
 2011 WRITE(W,2012) NNOD(I),(BCAC(I,J),J=1,10)
 2012 FORMAT(I4,10(2X,F6.3))
      WRITE(W,2013) NQ,NP
 2013 FORMAT(2I4)
  106 WRITE(W,110)
  110 FORMAT(/,/,'ELEMENT NO.',3X,'EL.TYPE',3X,'NODE NO.',
     1       14X,'FILM THICKNESS')
      IF(NNEL.EQ.4) GO TO 121
      DO 120 I=1,NELE
  120 WRITE(W,130) IBPS(I,1),IBPS(I,2),(NME(I,J),J=1,NNEL),
     1(TH(I,K),K=1,NNEL)
  130 FORMAT(1H ,5X,I3,7X,I2,4X,3I4,3(3X,E10.3))
      GO TO 123
  121 WRITE(W,122) IBPS(I,1),IBPS(I,2),(NME(I,J),J=1,NNEL),
     1(TH(I,K),K=1,NNEL)
  122 FORMAT(1H ,5X,I3,7X,I2,4X,4I4,4(3X,E10.3))
  123 IF(COOR.EQ.'PO') GO TO 141
```

```
  123 IF(COOR.EQ.'PO') GO TO 141
      WRITE(W,140)
  140 FORMAT(/,/,'COORDINATE ARRAY OF NODAL POINT',
     1//,9X,'NODE NO.',3X,'X-COORDINATE',2X,'Y-COORDINATE')
      GO TO 143
  141 WRITE(W,142)
  142 FORMAT(//,'COORDINATE ARRAY OF NODAL POINT',
     1        //,9X,'NODE NO.',9X,'RADIAL',8X,'ANGLE')
  143 DO 150 I=1,NODES
  150 WRITE(W,160) I,GCOE(I),GCOE(I+NODES)
  160 FORMAT(1H ,12X,I3,6X,E10.3,5X,E10.3)
      WRITE(W,161)
  161 FORMAT(///,'THE FILM PROPERTIES',
     1//,6X,'ELEMENT NO.',10X,'DEN',12X,'VIS')
      IF(DENVIS.EQ.'YES') GO TO 164
      DO 162 I=1,NELE
  162 WRITE(W,163) I,(SECPR(I,K),K=1,2)
  163 FORMAT(1H ,12X,I3,1X,2(5X,E10.3))
      GO TO 169
  164 WRITE(W,165) NELE,(SECPR(1,K),K=1,2)
  165 FORMAT(/8X,'1 ~',I4,1X,2(5X,E10.3))
  169 WRITE(W,170) NQ,NP
  170 FORMAT(////,'I-------------------------------I',
     1/,'I   BOUNDARY CONDITION VALUE   I',
     2/,'I-----------------------------I',
     3//,'NQ=',I3,5X,'NP=',I3,
     4//,'BCVL  TYPE 1 : FLOW VALUE (Q)',
     5/,6X,'TYPE 2 :   PRESSURE (P)',
     6//,1X,'NODE',4X,'TYPE',7X,'BCVL')
      DO 175 I=1,NODES
  175 WRITE(W,180) NNOD(I),NDF(I),BCVL(I)
  180 FORMAT(1H ,1X,I3,4X,I3,3X,E10.3)
      IF(IUNIT2.EQ.2) GO TO 1181
      WRITE(W,181)
  181 FORMAT(//,'C',21('-'),'C',/'C  VALUES OF ACTIONS  C',
     1 /,'C',21('-'),'C',/,9X,'UX1'9X,'UX2',9X,'UY1',9X,'UY2',
     2 9X,'BX1',9X,'BX2',9X,'BY1',9X,'BY2')
      GO TO 1183
 1181 WRITE(W,1182)
 1182 FORMAT(//,'VALUES OF ACTIONS',
     1 /,10X,'W1',10X,'W2',9X,'---',9X,'---',
     2  9X,'BX1',9X,'BX2',9X,'BY1',9X,'BY2')
 1183 NOD=NODES
      IF(BCA.EQ.'YES') NOD=1
      DO 182 I=1,NOD
  182 WRITE(W,183) NNOD(I),(BCAC(I,J),J=1,8)
  183 FORMAT(I3,8(2X,E10.3))
      WRITE(W,187)
  187 FORMAT(/,11X,'H',11X,'VD')
      DO 188 I=1,NOD
  188 WRITE(W,189) NNOD(I),(BCAC(I,J),J=9,10)
  189 FORMAT(I3,2(3X,E10.3))
      WRITE(6,190)
  190 FORMAT(/,'Finished Reading and Writing Input Data')
C
      DO 300 I=1,NNEL
      DO 300 J=1,NNEL
      DELTA(I,J)=0.0
      DELTA(I,I)=1.0
  300 CONTINUE
C
```

```
C
      CALL MTKPF
      WRITE(1,191)
C
      IF(NNEL-3.0) 410,420,430
  410 GO TO 500
  420 CALL MTKUX3
      WRITE(1,192)
      CALL MTKUY3
      WRITE(1,193)
      CALL MTKBX3
      WRITE(1,194)
      CALL MTKBY3
      WRITE(1,195)
      CALL MTKH3
      WRITE(1,196)
      CALL MTKVD3
      WRITE(1,197)
      GO TO 450
C
  430 CALL MTKUX4
      WRITE(1,192)
      CALL MTKUY4
      WRITE(1,193)
      CALL MTKBX4
      WRITE(1,194)
      CALL MTKBY4
      WRITE(1,195)
      CALL MTKH4
      WRITE(1,196)
      CALL MTKVD4
      WRITE(1,197)
  450 CALL SOLVE
      WRITE(1,198)
      CALL OUTPUT
      WRITE(1,199)
      GO TO 500
C
  191 FORMAT('[ Finished Calculation of MTKPF  ]')
  192 FORMAT('[ Finished Calculation of MTKUXF ]')
  193 FORMAT('[ Finished Calculation of MTKUYF ]')
  194 FORMAT('[ Finished Calculation of MTKBXF ]')
  195 FORMAT('[ Finished Calculation of MTKBYF ]')
  196 FORMAT('[ Finished Calculation of MTKHF  ]')
  197 FORMAT('[ Finished Calculation of MTKVDF ]')
  198 FORMAT('[ Finished Calculation of SOLVE  ]')
  199 FORMAT('[ Finished Calculation of OUTPUT ]')
C
  500 WRITE(1,200)
  200 FORMAT('Your Calculation has been finished.')
C
C
      CALL EXIT
      END
```

```
      SUBROUTINE MTKPF
C
C-----*** THIS SUBROUTINE MAKES MATRIX OF PRESSURE ****
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
C
CC
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1           NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2             GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3             A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK2/MTKP(200,200),MKP(200,200),MTKP1(200,200),MP(400,4,4)
      COMMON/BLK15/X(4),Y(4)
      DIMENSION RAD(400),ANG(400)
C
      WRITE(W,50)
   50 FORMAT(//////,2X,'CCCCCCCCCCCCCCCCC',/,2X,'C',15X,'C',
     1/,2X,'C  CALCULATION  C',
     2/,2X,'C',15X,'C',/,2X,'CCCCCCCCCCCCCCCCC')
      WRITE(W,100)
  100 FORMAT(///'I',45('-'),'I',
     1/,'I  1. CALCULATION OF ELEMENT PRESSURE MATRIX  I',
     2/,'I',45('-'),'I'//)
C-----LET ALL INITIAL VALUES ZERO.
      DO 140 I=1,NNEL
      RAD(I)=0.0
      ANG(I)=0.0
      X(I)=0.0
      Y(I)=0.0
      DO 140 LE=1,NELE
      B(LE,I)=0.0
      C(LE,I)=0.0
      AA(LE,I)=0.0
      DO 140 J=1,NNEL
      A(I,J)=0.0
  140 CONTINUE
      DO 150 I=1,NODES
      DO 150 J=1,NODES
      MKP(I,J)=0.0
      MTKP(I,J)=0.0
  150 CONTINUE
      DO 200 LE=1,NELE
      IF(RESULT.NE.'YES') GO TO 201
      WRITE(W,160) LE
  160 FORMAT(//2X,'ELEMENT  NO.',I3)
C-----ARRANGEMENT OF COORDINATE
  201 IF (COOR.EQ.'PO') GO TO 221
      DO 220 I=1,NNEL
      X(I)=GCOE(NME(LE,I))
      Y(I)=GCOE(NME(LE,I)+NODES)
      IF(RESULT.NE.'YES') GO TO 220
      WRITE(W,210) NME(LE,I),X(I),Y(I)
  210 FORMAT(1H ,6X,'NODE NO.',I3,3X,'(X,Y)= (',2(3X,E10.3),' )')
  220 CONTINUE
      GO TO 227
  221 DO 226 I=1,NNEL
      RAD(I)=GCOE(NME(LE,I))
      ANG(I)=GCOE(NME(LE,I)+NODES)
      AN=ANG(I)
      IF(UNIT1.EQ.'DEG') GO TO 222
```

```
      IF(UNIT1.EQ.'DEG') GO TO 222
      IF(UNIT1.EQ.'RAD') GO TO 223
  222 ANG(I)=3.141592654/180.0*ANG(I)
  223 X(I)=RAD(I)*DCOS(ANG(I))
      Y(I)=RAD(I)*DSIN(ANG(I))
      IF(RESULT.NE.'YES') GO TO 226
      WRITE(W,225) NME(LE,I),UNIT1,RAD(I),AN,X(I),Y(I)
  225 FORMAT(6X,'NODE NO.',I3,3X,'(RAD,ANG(',A3,'))= (',E10.3,
     1 1X,E10.3,')',3X,'(X,Y)= (',E10.3,1X,E10.3,')')
  226 CONTINUE
C-----CALCULATION OF A(I,J)
  227 DO 230 I=1,NNEL
      DO 230 J=1,NNEL
      A(I,J)=X(I)*Y(J)
  230 CONTINUE
C
C-----CALCULATION OF PRESSURE MATRIX MTKP OF ALL TYPES OF ELEMENT
      IF(NNEL.EQ.2) GO TO 1
      IF(NNEL.EQ.3) GO TO 2
      IF(NNEL.EQ.4) GO TO 3
      WRITE(1,1200)
 1200 FORMAT(/'ERROR...PLEASE USE NNEL=2,3 OR 4 OTHERWISE ',
     1        'CALCULATION CANNOT BE DONE.')
      CALL EXIT
    1 GO TO 240
    2 CALL MTKP3(LE)
      GO TO 240
    3 CALL MTKP4(LE)
      GO TO 240
C
C-----PRINT THE RESULT OF MTKP
  240 IF(RESULT.NE.'YES') GO TO 200
      WRITE(W,1000)
 1000 FORMAT(4X,'ELEMENT PRESSURE MATRIX')
      DO 250 I=1,NNEL
  250 WRITE(W,1100) ((NME(LE,I),NME(LE,J),MKP(NME(LE,I),
     1                 NME(LE,J))),J=1,NNEL)
 1100 FORMAT(6X,4('MKP(',I2,I3,1X,')=',E10.3,2X))
C
  200 CONTINUE
C
      IF(RESULT.EQ.'XX') GO TO 270
      WRITE(W,260)
  260 FORMAT(///'I',38('-'),'I',
     1/,'I   2. RESULT OF GLOBAL MATRIX (MTKP)  I',
     2/,'I',38('-'),'I',//)
C
      CALL PRINT(NODES,MTKP,W)
C
  270 RETURN
      END
```

```
      SUBROUTINE MTKP3(LE)
C
C     THIS ROUTINE IS TO CALCULATE PRESSURE MATRIX MTKP
C     OF TRIANGLE ELEMENT
C
      IMPLICIT REAL*8(A-H,K,M,O-Z)
      INTEGER W
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1        NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2        GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3        A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4        RESULT,UNIT1,IUNIT2
      COMMON/BLK2/MTKP(200,200),MKP(200,200),MTKP1(200,200),MP(400,4,4)
      COMMON/BLK15/X(4),Y(4)
      DIMENSION KP(4,4),CNST(400),THICK1(400)
C
C
      AA(LE,1)=A(2,3)-A(3,2)
      AA(LE,2)=A(3,1)-A(1,3)
      AA(LE,3)=A(1,2)-A(2,1)
      B(LE,1)=Y(2)-Y(3)
      B(LE,2)=Y(3)-Y(1)
      B(LE,3)=Y(1)-Y(2)
      C(LE,1)=X(3)-X(2)
      C(LE,2)=X(1)-X(3)
      C(LE,3)=X(2)-X(1)
      IF(RESULT.NE.'YES') GO TO 35
      WRITE(W,10)
   10 FORMAT(4X,'CONSTANT A,B,C')
      DO 20 I=1,NNEL
   20 WRITE(W,30) LE,I,AA(LE,I),LE,I,B(LE,I),LE,I,C(LE,I)
   30 FORMAT(1H ,5X,'A(',I2,I2,')=',E10.3,5X,'B(',I2,I2,')=',
     1        E10.3,5X,'C(',I2,I2,')=',E10.3)
C
C------ ELEMENT PRESSURE MATRIX KP CALCULATION
C
C------ CALCULATION OF MKP
   35 AREA(LE)=0.0
      CNST(LE)=0.0
      DEN=SECPR(LE,1)
      VIS=SECPR(LE,2)
      THICK1(LE)=0.0
      TH1=0.0
      TH2=0.0
      DO 50 I=1,NNEL
      DO 50 J=1,NNEL
      KP(I,J)=(B(LE,I)*B(LE,J))+(C(LE,I)*C(LE,J))
      TH1=TH1+TH(LE,I)**2.0*TH(LE,J)
   50 CONTINUE
      TH2=TH(LE,1)*TH(LE,2)*TH(LE,3)
      THICK1(LE)=TH1+TH2
      AR=((A(1,2)+A(2,3)+A(3,1))-(A(2,1)+A(3,2)+A(1,3)))/2.0
      AREA(LE)=DABS(AR)
C
      IF(VIS.NE.0.0) GO TO 55
      WRITE(1,51)
```

```
      WRITE(1,51)
      GO TO 100
   55 IF(AREA(LE).NE.0.0) GO TO 56
      WRITE(1,52)
      GO TO 100
   56 CNST(LE)=(-1.0)*DEN*THICK1(LE)/(480.0*VIS*AREA(LE))
      IF(RESULT.NE.'YES') GO TO 65
      WRITE(W,60) AREA(LE),THICK1(LE),CNST(LE)
   60 FORMAT(/,4X,'AREA(LE)=',E10.3,3X,'THICK1(LE)=',E10.3,
     1        4X,'CNST(LE)=',E10.3)
   65 DO 70 I=1,NODES
      DO 70 J=1,NODES
      MKP(I,J)=0.0
   70 CONTINUE
C
C-----CALCULATION OF ELEMENT MATRIX MKP, AND ASSEMBLY  MATRIX MTKP
      DO 80 I=1,NNEL
      DO 80 J=1,NNEL
      IN=NME(LE,I)
      JN=NME(LE,J)
      MKP(IN,JN)=CNST(LE)*KP(I,J)
      MP(LE,I,J)=MKP(IN,JN)
      MTKP(IN,JN)=MTKP(IN,JN)+MKP(IN,JN)
   80 CONTINUE
C
   51 FORMAT('ERROR: As VIS=0.0, CNST(LE) will be infinite.',
     1'Please check the value of VIS.')
   52 FORMAT('ERROR: As AREA(LE)=0.0 CNST(LE) becomes infinite.',
     1'Please check the value of AREA(LE).')
  100 RETURN
      END
```

```
      SUBROUTINE MTKP4(LE)
C
C-----THIS ROUTINE IS TO CALCULATE THE PRESSURE MATRIX MTKP OF QUADRILATERAL ELEMEN
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1          NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2          GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3          A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4          RESULT,UNIT1,IUNIT2
      COMMON/BLK2/MTKP(200,200),MKP(200,200),MTKP1(200,200),MP(400,4,4)
      COMMON/BLK15/X(4),Y(4)
      COMMON/BLK16/T(4),S(4)
      COMMON/BLK17/THICK4(400)
      COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C6(400),C8(400)
      DIMENSION TA(4,4),TB(4,4),TC(4,4),TD(4,4),MKP1(4,4)
C
C-----CALCULATION OF EACH CONSTANT VALUE
C
      CALL CNST4(X,Y,A,LE,CC1,CC2,CC3,CC4,CC6,CC8,
     1          C10,C11,C12,C13,C14,C15,C16,C17,C18,C19,
     2          C21,C22,C23,C40,C41,AREA,T,S)
C
      C1(LE)=CC1
      C2(LE)=CC2
      C3(LE)=CC3
      C4(LE)=CC4
      C6(LE)=CC6
      C8(LE)=CC8
      DEN=SECPR(LE,1)
      VIS=SECPR(LE,2)
      WRITE(W,100) LE
  100 FORMAT(///,'LE=',I2,/)
C
C
C-----CALCULATION OF EACH TERM A,B,C,D
C
C----- 1 WHEN THICKNESS IS CONSTANT WITHIN AN ELEMENT
C
C
      CALL INTEG(2,2,2,ITGA)
      CALL INTEG(2,2,2,ITGB)
      CALL INTEG(2,2,2,ITGC)
      CALL INTEG(2,2,2,ITGD)
      CALL INTEG(2,2,2,ITGE)
C
      WRITE(W,101) C21,C40,C21,C41
  101 FORMAT(//,'C21,C40,C21,C41',/,4(E10.3,3X))
      WRITE(W,102) ITGA,ITGB,ITGC,ITGD,ITGE
  102 FORMAT(//,'ITGA   ITGB   ITGC   ITGD   ITGE',/5(E10.3,3X))
      CONST=(-1.0)*DEN*TH(LE,1)/(12.0*VIS)
      DO 11 I=1,4
      DO 11 J=1,4
      IF(IBPS(LE,2).EQ.5) GO TO 15
      IF(C21.EQ.0.0) GO TO 14
C-----ROUTINES FOR GENERAL QUADRILATERAL ELEMENT
      CALL TERMA(C10,C11,C12,C21,C22,C23,C40,C41,T,S,TA,W)
      CALL TERMB(C13,C14,C15,C21,C22,C23,C40,C41,T,S,TB,W)
      CALL TERMC(C10,C17,C18,C19,C21,C22,C23,C40,C41,T,S,TC,W)
      GO TO 16
C
   14 IF(C22.EQ.0.0) GO TO 15
```

```
   14 IF(C22.EQ.0.0) GO TO 15
C-----ROUTINES FOR TROTEZIUM ELEMENT
      CALL TERMA1(C10,C11,C12,C22,C23,T,S,TA)
      CALL TERMB1(C13,C15,C22,C23,T,S,TB)
      CALL TERMC1(LE,TH,C10,C17,C18,C19,C22,C23,T,S,TC)
      GO TO 16
C
C-----ROUTINES FOR RECTANGLE AND PARALLELOGRAM ELEMENTS
   15 CALL TERMA3(C10,C12,C23,T,S,TA)
      CALL TERMA3(C13,C15,C23,S,T,TB)
      CALL TERMC3(C10,C17,C18,C19,C23,T,S,TC)
C----- CALCULATION OF TERM D
   16 TD(J,I)=TC(I,J)
C----- CALCULATION OF MKP AND GLOBALMATRIX MTKP
      IN=NME(LE,I)
      JN=NME(LE,J)
      MKP1(I,J)=TA(I,J)+TB(I,J)-TC(I,J)-TD(I,J)
      MKP(IN,JN)=CONST*MKP1(I,J)
      MP(LE,I,J)=MKP(IN,JN)
      MTKP(IN,JN)=MTKP(IN,JN)+MKP(IN,JN)
   11 CONTINUE
C
      IF(RESULT.NE.'YES') GO TO 12
C
C-----PRINT THE RESULT OF MKP,MTKP
      WRITE(W,7) LE,TH(LE,1),DEN,VIS,CONST
    7 FORMAT(//'ELEMENT NO.=',I5,/,5X,'THICK4,DEN,VIS,CONST',
     1        /,5X,4(E10.3,2X))
      DO 6 I=1,4
      WRITE(W,9) TA(I,J),TB(I,J),TC(I,J),TD(I,J),MKP1(I,J)
    9 FORMAT('A    B    C    D    MKP1',/,5(E10.3,3X))
    6 CONTINUE
   12 RETURN
      END
```

```
      SUBROUTINE MTKUX3
C
C-----THIS SUBROUTINE MAKES MATRIX OF SHEAR ACTION IN X-DIRECTION
C
      IMPLICIT REAL*3(A-H,M,O-Z)
      INTEGER W
C
C
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1            NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2              GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3              A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),N,COOR,
     4             RESULT,UNIT1,IUNIT2
      COMMON/BLK3/MTKUX(200,200),MKUX(400,4,4)
      COMMON/BLK11/THICK2(400,4)
      COMMON/BLK13/DELTA(4,4)
C
C-----LET ALL INITIAL VALUES ZERO.
      DO 100 I=1,NODES
      DO 100 J=1,NODES
      MTKUX(I,J)=0.0
  100 CONTINUE
      DO 200 I=1,NODES
      IF(BCAC(I,1).NE.0.0) GOTO 300
  200 IF(BCAC(I,2).NE.0.0) GOTO 300
      RETURN
  300 WRITE(W,800)
C-----CALCULATION OF ELEMENT MATRIX MKUX AND GLOBAL MATRIX MTKUX
      DO 700 LE=1,NELE
      DO 500 I=1,NNEL
      DO 500 J=1,NNEL
      THCK=0.0
      DO 400 K=1,NNEL
      THCK=THCK+TH(LE,K)*(1.0+DELTA(K,J))
  400 CONTINUE
      THICK2(LE,J)=THCK
      IN=NME(LE,I)
      JN=NME(LE,J)
      MKUX(LE,I,J)=DEN*THICK2(LE,J)*B(LE,I)/24.0
      MTKUX(IN,JN)=MTKUX(IN,JN)+MKUX(LE,I,J)
  500 CONTINUE
C
C-----PRINT THE RESULT
      IF(RESULT.NE.'YES') GO TO 700
      WRITE(W,600) LE
  600 FORMAT('MKUX MATRIX    ELEMENT NO.',I2)
      DO 610 I=1,NNEL
  610 WRITE(W,620) ((LE,NME(LE,I),NME(LE,J),MKUX(LE,I,J)),J=1,NNEL)
  620 FORMAT(3X,3('MKUX(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
  700 CONTINUE
      IF(RESULT.EQ.'XX') GO TO 1000
  800 FORMAT(////5X,43('-'),
     1/,5X,'I  RESULT OF SHEAR ACTION MATRIX (MTKUX)  I',
     2/,5X,43('-'))
      WRITE(W,900)
  900 FORMAT(//,31('-'),
     1       /,'I  TOTAL GLOBAL MATRIX MTKUX  I',
     2       /,31('-'))
C
      CALL PRINT(NODES,MTKUX,W)
C
 1000 RETURN
      END
```

```
      SUBROUTINE MTKUY3
C
C-----THIS SUBROUTINE MAKES MATRIX OF SHEAR ACTION IN Y-DIRECTION
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1            NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2              GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3              A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4             RESULT,UNIT1,IUNIT2
      COMMON/BLK4/MTKUY(200,200),MKUY(400,4,4)
      COMMON/BLK11/THICK2(400,4)
      COMMON/BLK13/DELTA(4,4)
C
C-----LET ALL INITIAL VALUES ZERO.
      DO 100 I=1,NODES
      DO 100 J=1,NODES
      MTKUY(I,J)=0.0
  100 CONTINUE
      DO 200 I=1,NODES
      IF(IUNIT2.EQ.2) GO TO 101
      IF(BCAC(I,3).NE.0.0) GOTO 300
      IF(BCAC(I,4).NE.0.0) GOTO 300
  101 IF(BCAC(I,1).NE.0.0) GO TO 300
      IF(BCAC(I,2).NE.0.0) GO TO 300
  200 CONTINUE
      RETURN
  300 WRITE(W,800)
C-----CALCULATION OF ELEMENT MATRIX MKUY AND GLOBAL MATRIX MTKUY
      DO 700 LE=1,NELE
      DO 500 I=1,NNEL
      DO 500 J=1,NNEL
      THCK=0.0
      DO 400 K=1,NNEL
      THCK=THCK+TH(LE,K)*(1.0+DELTA(K,J))
  400 CONTINUE
      THICK2(LE,J)=THCK
      IN=NME(LE,I)
      JN=NME(LE,J)
      MKUY(LE,I,J)=DEN*THICK2(LE,J)*C(LE,I)/24.0
      MTKUY(IN,JN)=MTKUY(IN,JN)+MKUY(LE,I,J)
  500 CONTINUE
C
C-----PRINT THE RESULT
      IF(RESULT.NE.'YES') GO TO 700
      WRITE(W,600) LE
  600 FORMAT('MKUY MATRIX   ELEMENT NO.',I2)
      DO 610 I=1,NNEL
  610 WRITE(W,620) ((LE,NME(LE,I),NME(LE,J),MKUY(LE,I,J)),J=1,NNEL)
  620 FORMAT(3X,3('MKUY(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
  700 CONTINUE
      IF(RESULT.EQ.'XX') GO TO 1000
  800 FORMAT(////5X,37('-'),
     1/,5X,'I   RESULT OF GLOBAL MATRIX (MTKUY)  I',
     2/,5X,37('-'))
      WRITE(W,900)
  900 FORMAT(//,25('-'),
     1/'{  TOTAL GLOBAL MATRIX  I',
     2/,25('-'))
C
      CALL PRINT(NODES,MTKUY,W)
 1000 RETURN
      END
```

```
      SUBROUTINE MTKBX3
C
C-----THIS SUBROUTINE MAKES MATRIX OF BODY FORCE ACTION
C----- IN X-DIRECTION
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
C
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1            NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2              GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3              A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4             RESULT,UNIT1,IUNIT2
      COMMON/BLK5/MTKBX(200,200),MKBX(400,4,4)
      COMMON/BLK12/THICK3(400,4)
C
C-----LET ALL INITIAL VALUES ZERO.
      DO 100 I=1,NODES
      DO 100 J=1,NODES
      MTKBX(I,J)=0.0
  100 CONTINUE
      DO 200 I=1,NODES
      IF(BCAC(I,5).NE.0.0) GOTO 300
      IF(BCAC(I,6).NE.0.0) GOTO 300
      IF(IUNIT2.NE.2) GO TO 200
      IF(BCAC(I,1).NE.0.0) GO TO 300
      IF(BCAC(I,2).NE.0.0) GO TO 300
  200 CONTINUE
      RETURN
  300 WRITE(W,700)
C
C-----CALCULATION OF ELEMENT MATRIX MKBX AND GLOBAL MATRIX MTKBX
      DO 600 LE=1,NELE
      G1=TH(LE,1)+TH(LE,2)+TH(LE,3)
      G2=TH(LE,1)**2+TH(LE,2)**2+TH(LE,3)**2
      G3=TH(LE,1)*TH(LE,2)*TH(LE,3)
      DO 400 I=1,NNEL
      DO 400 J=1,NNEL
      IN=NME(LE,I)
      JN=NME(LE,J)
      THICK3(LE,J)=G1*G2+2.0*G1*(TH(LE,J)**2)+G2*TH(LE,J)+2.0*G3
      VISCOS=VIS*1440.0
      MKBX(LE,I,J)=DEN*THICK3(LE,J)*B(LE,I)/VISCOS
      MTKBX(IN,JN)=MTKBX(IN,JN)+MKBX(LE,I,J)
  400 CONTINUE
C
C-----PRINT THE RESULT
      IF(RESULT.NE.'YES') GO TO 600
      WRITE(W,500) LE
  500 FORMAT('MKBX MATRIX  ELEMENT NO.',I2)
      DO 510 I=1,NNEL
  510 WRITE(W,520) ((LE,NME(LE,I),NME(LE,J),MKBX(LE,I,J)),J=1,NNEL)
  520 FORMAT(3X,3('MKBX(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
  600 CONTINUE
      IF(RESULT.EQ.'XX') GO TO 1000
  700 FORMAT(////5X,'I',44('-'),'I',
     1/,5X,'I  RESULT OF BODY FORCE MATRIX (BX)  I',
     2/,5X,'I',44('-'),'I')
      WRITE(W,800)
  800 FORMAT(//,31('-'),
     1    /,'I  TOTAL GLOAL MATRIX MTKBX  I',
     2     /,31('-'))
C
      CALL PRINT(NODES,MTKBX,W)
 1000 RETURN
      END
```

```
      SUBROUTINE MTKBY3
C
C-----THIS SUBROUTINE MAKES MATRIX OF BODY FORCE ACTION
C----- IN Y-DIRECTION
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
C
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1            NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2              GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3              A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4             RESULT,UNIT1,IUNIT2
      COMMON/BLK6/MTKBY(200,200),MKBY(400,4,4)
      COMMON/BLK12/THICK3(400,4)
C
C-----LET ALL INITIAL VALUES ZERO.
      DO 100 I=1,NODES
      DO 100 J=1,NODES
      MTKBY(I,J)=0.0
  100 CONTINUE
      DO 200 I=1,NODES
      IF(BCAC(I,7).NE.0.0) GOTO 300
      IF(BCAC(I,8).NE.0.0) GOTO 300
      IF(IUNIT2.NE.2) GO TO 200
      IF(BCAC(I,1).NE.0.0) GO TO 300
      IF(BCAC(I,2).NE.0.0) GO TO 300
  200 CONTINUE
      RETURN
  300 WRITE(W,700)
      DO 600 LE=1,NELE
      G1=TH(LE,1)+TH(LE,2)+TH(LE,3)
      G2=TH(LE,1)**2+TH(LE,2)**2+TH(LE,3)**2
      G3=TH(LE,1)*TH(LE,2)*TH(LE,3)
      DO 400 I=1,NNEL
      DO 400 J=1,NNEL
      IN=NME(LE,I)
      JN=NME(LE,J)
      THICK3(LE,J)=G1*G2+2.0*G1*(TH(LE,J)**2)+G2*TH(LE,J)+2.0*G3
      VISCOS=VIS*1440.0
      MKBY(LE,I,J)=DEN*THICK3(LE,J)*C(LE,I)/VISCOS
      MTKBY(IN,JN)=MTKBY(IN,JN)+MKBY(LE,I,J)
  400 CONTINUE
C-----PRINT THE RESULT
      IF(RESULT.NE.'YES') GO TO 600
      WRITE(W,500) LE
  500 FORMAT('MKBY MATRIX   ELEMENT NO.',I2)
      DO 510 I=1,NNEL
  510 WRITE(W,520) ((LE,NME(LE,I),NME(LE,J),MKBY(LE,I,J)),J=1,NNEL)
  520 FORMAT(3X,3('MKBY(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
  600 CONTINUE
      IF(RESULT.EQ.'XX') GO TO 1000
  700 FORMAT(////5X,'I-----------------------------------I',
     1          /,5X,'I  RESULT OF BODY FORCE MATRIX (BY)  I',
     2          /,5X,'I-----------------------------------I')
      WRITE(W,800)
  800 FORMAT('TOTAL GLOBAL MATRIX MTKBY')
      CALL PRINT(NODES,MTKBY,W)
 1000 RETURN
      END
```

```
      SUBROUTINE MTKH3
C
C-----THIS SUBROUTINE MAKES MATRIX OF SQUEEZE ACTION
C-----*** MKH=(-1)*AREA*DEN*(1.0+DELTA)/12.0 ***
C
      IMPLICIT REAL*3(A-H,M,O-Z)
      INTEGER W
C
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1           NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2             GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3             A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK7/MTKH(200,200),MKH(400,4,4)
      COMMON/BLK13/DELTA(4,4)
C
C-----LET ALL INITIAL VALUES ZERO.
      DO 100 I=1,NODES
      DO 100 J=1,NODES
      MTKH(I,J)=0.0
  100 CONTINUE
      DO 200 I=1,NODES
  200 IF(BCAC(I,9).NE.0.0) GOTO 300
      RETURN
  300 WRITE(W,700)
C-----CALCULATION OF ELEMENT MATRIX MKH AND GLOBAL MATRIX MTKH
      DO 600 LE=1,NELE
      DO 400 I=1,NNEL
      DO 400 J=1,NNEL
      IN=NME(LE,I)
      JN=NME(LE,J)
      MKH(LE,I,J)=-1.0*AREA(LE)*DEN*(1.0+DELTA(I,J))/12.0
      MTKH(IN,JN)=MTKH(IN,JN)+MKH(LE,I,J)
  400 CONTINUE
C
C-----PRINT THE RESULT
      IF(RESULT.NE.'YES') GO TO 600
      WRITE(W,500) LE
  500 FORMAT('MKH MATRIX   ELEMENT NO.',I2)
      DO 510 I=1,NNEL
  510 WRITE(W,520) ((LE,NME(LE,I),NME(LE,J),MKH(LE,I,J)),J=1,NNEL)
  520 FORMAT(3X,3('MKH(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
  600 CONTINUE
      IF(RESULT.EQ.'XX') GO TO 1000
  700 FORMAT(///'I', 33('-'),'I',
     1/,'I  RESULT OF SQUEEZE MATRIX (MTKH)  I',
     2/,'I',33('-'),'I')
      WRITE(W,800)
  800 FORMAT(//,30('-'),
     1     /,'I  TOTAL GLOBAL MATRIX MTKH  I',
     2      /,30('-'))
C
      CALL PRINT(NODES,MTKH,W)
 1000 RETURN
      END
```

```
      SUBROUTINE MTKVD3
C
C-----THIIS SUBROUTINE MAKES MATRIX OF DIFFUSION ACTION
C-----MKVD=(-1.0)*AREA*DEN*(1.0+DELTA)/12.0
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1          NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(100,3),VIS,
     2            GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3            A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4           RESULT,UNIT1,IUNIT2
      COMMON/BLK8/MTKVD(200,200),MKVD(400,4,4)
      COMMON/BLK13/DELTA(4,4)
C
C-----LET ALL INITIAL VALUES ZERO.
      DO 100 I=1,NODES
      DO 100 J=1,NODES
      MTKVD(I,J)=0.0
  100 CONTINUE
      DO 200 I=1,NODES
  200 IF(BCAC(I,10).NE.0.0) GOTO 300
      RETURN
  300 WRITE(W,700)
C-----CALCULATION OF ELEMENT MATRIX MKVD AND GLOBAL MATRIX MTKVD
      DO 600 LE=1,NELE
      DO 400 I=1,NNEL
      DO 400 J=1,NNEL
      IN=NME(LE,I)
      JN=NME(LE,J)
      DELTA(I,J)=0.0
      DELTA(I,I)=1.0
      MKVD(LE,I,J)=-1.0*AREA(LE)*DEN*(1.0+DELTA(I,J))/12.0
      MTKVD(IN,JN)=MTKVD(IN,JN)+MKVD(LE,I,J)
  400 CONTINUE
C
C-----PRINT THE RESULT
      IF(RESULT.NE.'YES') GO TO 600
      WRITE(W,500) LE
  500 FORMAT('MKVD MATRIX   ELEMENT NO.',I2)
      DO 510 I=1,NNEL
  510 WRITE(W,520) ((LE,NME(LE,I),NME(LE,J),MKVD(LE,I,J)),J=1,NNEL)
  520 FORMAT(3X,3('MKVD(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
  600 CONTINUE
      IF(RESULT.EQ.'XX') GO TO 1000
  700 FORMAT(////5X,'I',43('-'),'I',
     1/,5X,'I  RESULT OF GLOBAL MATRIX (MTKVD)  I',
     2/,5X,'I',43('-'),'I')
      WRITE(W,800)
  800 FORMAT(//,31('-'),
     1       /,'I  TOTAL GLOBAL MATRIX MTKVD  I',
     2       /,31('-'))
C
      CALL PRINT(NODES,MTKVD,W)
 1000 RETURN

      END
```

```
      SUBROUTINE MTKUX4
C
C-----THIS ROUTINE IS TO CALCULATE THE MATRIX  MTKUX
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1          NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2          GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3          A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK3/MTKUX(200,200),MKUX(400,4,4)
      COMMON/BLK16/T(4),S(4)
      COMMON/BLK17/THICK4(400)
      COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C6(400),C8(400)
C
      DO 10 I=1,NODES
      DO 10 J=1,NODES
      MTKUX(I,J)=0.0
   10 CONTINUE
      DO 1 I=1,NODES
      IF(BCAC(I,1).NE.0.0) GO TO 5
      IF(BCAC(I,2).NE.0.0) GO TO 5
    1 CONTINUE
      RETURN
    5 WRITE(W,100)
  100 FORMAT(////5X,'********************************************',
     1     /,5X,'** RESULT OF SHEAR ACTION MATRIX (MTKUX) **',
     2     /,5X,'********************************************')
C
C-----CALCULATION OF MKUX AND MTKUX
      DO 30 LE=1,NELE
      CONST=DEN*THICK4(LE)/36.0
      DO 20 I=1,4
      DO 20 J=1,4
      IN=NME(LE,I)
      JN=NME(LE,J)
      B1=S(I)*(T(I)*T(J)+3.0)*(C3(LE)*S(J)+3.0*C8(LE))
      B2=T(I)*(S(I)*S(J)+3.0)*(C3(LE)*T(J)+3.0*C4(LE))
      MKUX(LE,I,J)=CONST*(B1-B2)
      MTKUX(IN,JN)=MTKUX(IN,JN)+MKUX(LE,I,J)
CCCCC----THIS PART IS NOT YET COMPLETED -----------CCCCCCCCCCCCCCCCCCCCCC
C     CALL TERMU3(T,S,TH,LE,DEN,C3,C4,C8,MKUX)
C     DO 20 I=1,4
C     DO 20 J=1,4
C     IN=NME(LE,I)
C     JN=NME(LE,J)
C     MTKUX(IN,JN)=MTKUX(IN,JN)+MKUX(LE,I,J)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
   20 CONTINUE
      IF(RESULT.NE.'YES') GO TO 30
      WRITE(W,110) LE
  110 FORMAT(///'ELEMENT MATRIX MKUX,/,2X,ELEMENT NO.',I2)
      DO 120 I=1,NNEL
  120 WRITE(W,130) ((LE,NME(LE,I),NME(LE,J),MKUX(LE,I,J)),J=1,NNEL)
  130 FORMAT(3X,4('MKUX(',I3,',',I2,',',I3,1X,')=',E10.3,2X))


   30 CONTINUE
      IF(RESULT.EQ.'XX') GO TO 150
      WRITE(W,140)
  140 FORMAT(///'TOTAL GLOBAL MATRIX MTKUX')
C
      CALL PRINT(NODES,MTKUX,W)
C
  150 RETURN
      END
```

```
      SUBROUTINE MTKUY4
C
C-----THIS ROUTINE IS TO CALCULATE THE MATRICES OF MKUY AND MTKUY
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),MNOD(400),
     1          NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2          GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3          A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK4/MTKUY(200,200),MKUY(400,4,4)
      COMMON/BLK16/T(4),S(4)
      COMMON/BLK17/THICK4(400)
      COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C6(400),C8(400)
C
      DO 10 I=1,NODES
      DO 10 J=1,NODES
      MTKUY(I,J)=0.0
   10 CONTINUE
      DO 1 I=1,NODES
      IF(IUNIT2.EQ.2) GO TO 11
      IF(BCAC(I,3).NE.0.0) GO TO 5
      IF(BCAC(I,4).NE.0.0) GO TO 5
   11 IF(BCAC(I,1).NE.0.0) GO TO 5
      IF(BCAC(I,2).NE.0.0) GO TO 5
    1 CONTINUE
      RETURN
    5 WRITE(W,100)
  100 FORMAT(///,5X,'******************************************',
     1         /,5X,'**RESULT OF SHEAR ACTION MATRIX (MTKUY) **',
     2         /,5X,'******************************************')
C
C-----CALCULATION OF MKUY AND MTKUY
      DO 30 LE=1,NELE
      CALL TERMU3(T,S,TH,LE,DEN,C1,C2,C6,MKUY)
      DO 20 I=1,4
      DO 20 J=1,4
      IN=NME(LE,I)
      JN=NME(LE,J)
      MTKUY(IN,JN)=MTKUY(IN,JN)+MKUY(LE,I,J)
   20 CONTINUE
      IF(RESULT.NE.'YES') GO TO 30
      WRITE(W,110) LE
  110 FORMAT(///'ELEMENT MATRIX MKUY,/,2X,ELEMENT NO.',I2)
      DO 120 I=1,NNEL
  120 WRITE(W,130) ((LE,NME(LE,I),NME(LE,J),MKUY(LE,I,J)),J=1,NNEL)
  130 FORMAT(3X,4('MKUY(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
   30 CONTINUE
      WRITE(W,140)
  140 FORMAT(///'TOTAL GLOBAL MATRIX MTKUY')
C
      CALL PRINT(NODES,MTKUY,W)
C
      RETURN
      END
```

```
      SUBROUTINE MTKBX4
C
C-----THIS ROUTINE IS TO CALCULATE THE MATRIX O MTKBX
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1          NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2          GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3          A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK5/MTKBX(200,200),MKBX(400,4,4)
      COMMON/BLK16/T(4),S(4)
      COMMON/BLK17/THICK4(400)
      COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C6(400),C8(400)
C
      DO 10 I=1,NODES
      DO 10 J=1,NODES
      MTKBX(I,J)=0.0
   10 CONTINUE
      DO 1 I=1,NODES
      IF(BCAC(I,5).NE.0.0) GO TO 5
      IF(BCAC(I,6).NE.0.0) GO TO 5
      IF(IUNIT2.NE.2) GO TO 1
      IF(BCAC(I,1).NE.0.0) GO TO 5
      IF(BCAC(I,2).NE.0.0) GO TO 5
    1 CONTINUE
      RETURN
    5 WRITE(W,100) LE
  100 FORMAT(///,5X,'*****************************************',
     1         /,5X,'** RESULT OF BODY FORCE MATRIX (MTKBX)**',
     2         /,5X,'*****************************************')
C
C-----CALCULATION OF MKBX AND MTKBX
      DO 30 LE=1,NELE
      CALL TERMB3(T,S,TH,LE,C3,C4,C8,DEN,VIS,MKBX)
      DO 20 I=1,4
      DO 20 J=1,4
      IN=NME(LE,I)
      JN=NME(LE,J)
      MTKBX(IN,JN)=MTKBX(IN,JN)+MKBX(LE,I,J)
   20 CONTINUE
      IF(RESULT.NE.'YES') GO TO 30
      WRITE(W,110)
  110 FORMAT('ELEMENT MATRIX MKBX/2X,ELEMENT NO.',I2)
      DO 120 I=1,NNEL
  120 WRITE(W,130) ((LE,NME(LE,I),NME(LE,J),MKBX(LE,I,J)),J=1,NNEL)
  130 FORMAT(3X,4('MKBX(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
   30 CONTINUE
      WRITE(W,140)
  140 FORMAT(///'TOTAL GLOBAL MATRIX MTKBX')
C
      CALL PRINT(NODES,MTKBX,W)
C
      RETURN
      END
```

```
      SUBROUTINE MTKBY4
C
C-----THIS ROUTINE IS TO CALCULATE  MKBY AND MTKBY
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1            NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2            GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3            A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK6/MTKBY(200,200),MKBY(400,4,4)
      COMMON/BLK16/T(4),S(4)
      COMMON/BLK17/THICK4(400)
      COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C6(400),C8(400)
C
      DO 10 I=1,NODES
      DO 10 J=1,NODES
      MTKBY(I,J)=0.0
   10 CONTINUE
      DO 1 I=1,NODES
      IF(BCAC(I,7).NE.0.0) GO TO 5
      IF(BCAC(I,8).NE.0.0) GO TO 5
      IF(IUNIT2.NE.2) GO TO 1
      IF(BCAC(I,1).NE.0.0) GO TO 5
      IF(BCAC(I,2).NE.0.0) GO TO 5
    1 CONTINUE
      RETURN
    5 WRITE(W,100)
  100 FORMAT(///,5X,'****************************************',
     1         /,5X,'** RESULT OF BODY FORCE MATRIX MTKBY **',
     2         /,5X,'****************************************')
      DO 30 LE=1,NELE
      CALL TERMB3(S,T,TH,LE,C1,C6,C2,DEN,VIS,MKBY)
      DO 20 I=1,4
      DO 20 J=1,4
      IN=NME(LE,I)
      JN=NME(LE,J)
      MTKBY(IN,JN)=MTKBY(IN,JN)+MKBY(LE,I,J)
   20 CONTINUE
      IF(RESULT.NE.'YES') GO TO 30
      WRITE(W,110) LE
  110 FORMAT('ELEMENT MATRIX MKBY   ELEMENT NO. ',I2)
      DO 120 I=1,NNEL
  120 WRITE(W,130) ((LE,NME(LE,I),NME(LE,J),MKBY(LE,I,J)),J=1,NNEL)
  130 FORMAT(3X,4('MKBY(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
   30 CONTINUE
      WRITE(W,140)
  140 FORMAT('TOTAL GLOBAL MATRIX MTKBY')
C
      RETURN
      END
```

```
      SUBROUTINE MTKH4
C
C-----THIS ROUTINE IS TO CALCULATE MKH AND MTKH OF QUADRILATERAL
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1            NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2            GELCO(400,2),CCOE(800),BCVL(400),BCAC(400,10),
     3            A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK7/MTKH(200,200),MKH(400,4,4)
      COMMON/BLK16/T(4),S(4)
      COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C6(400),C8(400)
C
      DO 10 I=1,NODES
      DO 10 J=1,NODES
      MTKH(I,J)=0.0
   10 CONTINUE
      DO 1 I=1,NODES
      IF(BCAC(I,9).NE.0.0) GO TO 5
    1 CONTINUE
      RETURN
    5 WRITE(W,100)
  100 FORMAT(///,5X,44('*'),
     1          /,5X,'** RESULT OF SQUEEZE ACTION MATRIX MTKH**',
     2          /,5X,44('*'))
C-----CALCULATION OF MKH AND MTKH
      DO 30 LE=1,NELE
      DO 20 I=1,NNEL
      DO 20 J=1,NNEL
      IN=NME(LE,I)
      JN=NME(LE,J)
      T1=T(I)+T(J)
      T2=T(I)*T(J)
      S1=S(I)+S(J)
      S2=S(I)*S(J)
      C21=C1(LE)*C8(LE)-C3(LE)*C6(LE)
      C22=C2(LE)*C3(LE)-C1(LE)*C4(LE)
      C23=C2(LE)*C8(LE)-C4(LE)*C6(LE)
      C24=2.0*(T2*C22/3.0+C22)
      C25=2.0*(T1*C21+T2*C23)/3.0+2.0*C23
      H1=(-1.0)*DEN/8.0
      H2=(C24*S1+C25*S2)/3.0+C25
      MKH(LE,I,J)=H1*H2
      MTKH(IN,JN)=MTKH(IN,JN)+MKH(LE,I,J)
      WRITE(W,14) LE,I,J,T1,T2,S1,S2
   14 FORMAT(//,'LE,I,J,T1,T2,S1,S2',/,I3,4X,2I4,5X,4(F5.2,2X))
      WRITE(W,11) C21,C22,C23,C24,C25
   11 FORMAT('C21,C22,C23,C24,C25',/,5(E10.3,3X))
      WRITE(W,12) H1,H2
   12 FORMAT('H1,H2',/,2(E10.3,5X))
   20 CONTINUE
      IF(RESULT.NE.'YES') GO TO 30
      WRITE(W,110) LE
  110 FORMAT('ELEMENT MATRIX  MKH  ELEMENT NO.',I2)
      DO 120 I=1,NNEL
  120 WRITE(W,130) ((LE,NME(LE,I),NME(LE,J),MKH(LE,I,J)),J=1,NNEL)
  130 FORMAT(3X,4('MKH(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
   30 CONTINUE
      WRITE(W,140)
  140 FORMAT(///'TOTAL GLOBAL MATRIX MTKH')
C
C
      CALL PRINT(NODES,MTKH,W)
C
      RETURN
      END
```

```
      SUBROUTINE MTKVD4
C
C-----THIS ROUTINE IS TO CALCULATE MKVD AND MTKVD
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1             NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2             CELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3             A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,C00R,
     4             RESULT,UNIT1,IUNIT2
      COMMON/BLK8/MTKVD(200,200),MKVD(400,4,4)
      COMMON/BLK16/T(4),S(4)
      COMMON/BLK18/C1(400),C2(400),C3(400),C4(400),C6(400),C8(400)
C
      DO 10 I=1,NODES
      DO 10 J=1,NODES
      MTKVD(I,J)=0.0
   10 CONTINUE
      DO 1 I=1,NODES
      IF(BCAC(I,10).NE.0.0) GO TO 5
    1 CONTINUE
      RETURN
    5 WRITE(W,100)
  100 FORMAT(///,5X,'*************************************',
     1          /,5X,'* RESULT OF DIFUSION ACTION MATRIX  *',
     2           /5X,'*************************************')
C
C-----CALCULATION OF MKVD AND MTKVD
      DO 30 LE=1,NELE
      DO 20 I=1,NNEL
      DO 20 J=1,NNEL
      IN=NME(LE,I)
      JN=NME(LE,J)
      T1=T(I)+T(J)
      T2=T(I)*T(J)
      S1=S(I)+S(J)
      S2=S(I)*S(J)
      C21=C1(LE)*C8(LE)-C3(LE)*C6(LE)
      C22=C2(LE)*C3(LE)-C1(LE)*C4(LE)
      C23=C2(LE)*C8(LE)-C4(LE)*C6(LE)
      C24=2.0*C21*(S2/3.0+1.0)
      C25=2.0/3.0*(S1*C22+S2*C23)+2.0*C23
      VD1=(-1.0)*DEN/8.0
      VD2=(T1*C24+T2*C25)/3.0+C25
      MKVD(LE,I,J)=VD1*VD2
      MTKVD(IN,JN)=MTKVD(IN,JN)+MKVD(LE,I,J)
   20 CONTINUE
      IF(RESULT.NE.'YES') GO TO 30
      WRITE(W,110) LE
  110 FORMAT('MTKVD MATRIX  ELEMENT NO.' I2)
      DO 120 I=1,NNEL
  120 WRITE(W,130) ((LE,NME(LE,I),NME(LE,J),MKVD(LE,I,J)),J=1,NNEL)
  130 FORMAT(3X,4('MKVD(',I3,',',I2,',',I3,1X,')=',E10.3,2X))
   30 CONTINUE
      WRITE(W,140)
  140 FORMAT(///'TOTAL GLOBAL MATRIX MTKVD')
C
      CALL PRINT(NODES,MTKVD,W)
C
      RETURN
      END
```

```
      SUBROUTINE PRINT(NODES,MTK,W)
C
C----- THIS SUBROUTINE IS TO PRINT OVERALL MATRIX
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
C
      DIMENSION JJ(200,5),MTK(200,200)
C-----DEFINITION OF WRITING FORM.
      ICASE=0
      DO 200 IN=1,NODES
      IF(5*IN.GE.NODES) GO TO 250
  200 CONTINUE
  250 ICASE=IN
      IF(ICASE.EQ.1) GO TO 550
      INN=ICASE-1
      DO 300 J=1,INN
      DO 300 K=1,5
      JJ(J,K)=5*(J-1)+K
  300 CONTINUE
      DO 500 J=1,INN
      WRITE(W,350) (JJ(J,K),K=1,5)
  350 FORMAT(1H ,14X,5(I3,12X))
      DO 400 I=1,NODES
  400 WRITE(W,450) I,(MTK(I,JJ(J,K)),K=1,5)
  450 FORMAT(1H ,3X,I3,3X,5(E10.3,4X))
  500 CONTINUE
  550 IK=NODES-5*(ICASE-1)
      DO 560 K=1,5
      JJ(ICASE,K)=5*(ICASE-1)+K
  560 CONTINUE
      WRITE(W,600) (JJ(ICASE,K),K=1,IK)
  600 FORMAT(1H ,14X,5(I2,12X))
      DO 650 I=1,NODES
  650 WRITE(W,700) I,(MTK(I,JJ(ICASE,K)),K=1,IK)
  700 FORMAT(1H ,3X,I3,3X,5(E10.3,4X))
      RETURN
      END
```

```
      SUBROUTINE SOLVE
C
C     THIS SUBROUTINE FIND THE UNKNOWN VALUES OF FLOW AND PRESSURE.
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
C
C  C----NOMENCLATURE------------------------------C
C  C                                              C
C  C           NO. OF KNOWN P VALUE---NP          C
C  C           NO. OF KNOWN Q VALUE---NQ          C
C  C           NODE NO. OF KNOWN P --NDP          C
C  C           NODE NO. OF KNOWN Q --NDQ          C
C  C           MEAN SURFACE VELOCITY --UX,UY      C
C  C           MEAN BODY FORCE    ----BX,BY       C
C  C                                              C
C  C                                              C
C  C----------------------------------------------C
C
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1           NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2             GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3             A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK2/MTKP(200,200),MKP(200,200),MTKP1(200,200),MP(400,4,4)
      COMMON/BLK3/MTKUX(200,200),MKUX(400,4,4)
      COMMON/BLK4/MTKUY(200,200),MKUY(400,4,4)
      COMMON/BLK5/MTKBX(200,200),MKBX(400,4,4)
      COMMON/BLK6/MTKBY(200,200),MKBY(400,4,4)
      COMMON/BLK7/MTKH(200,200),MKH(400,4,4)
      COMMON/BLK8/MTKVD(200,200),MKVD(400,4,4)
      COMMON/BLK9/Q(400),P(400),QIN,QOUT,WW,QEL(400,4)
      COMMON/BLK14/ CFX(400),CFY(400)
      COMMON/BLK20/UX(400),UY(400),BX(400),BY(400),H(400),VD(400)
      COMMON/BLK21/FX1,FX2,FY1,FY2,RF1,RF2,TQ1,TQ2
      COMMON/BLK22/TFX1,TFX2,TFY1,TFY2,TRF1,TRF2,TORQ1,TORQ2
      COMMON/BLK23/NDQ(400),NDP(400)
      DIMENSION QQ1(400),QQ2(400),QQ3(400),QQ4(400),
     1          QQ5(400),QQ6(400),QQ7(400)
      DIMENSION QQ12(400),QQ13(400),QQ14(400),QQ15(400),
     1          QQ16(400),QQ17(400)
      DIMENSION QQ21(400),QQ22(400),QQ23(400),QQ24(400),
     1          QQ25(400),QQ26(400)
      DIMENSION QQ27(400),Q1(400),Q2(400),P1(400)
C
      IP=3
C-----------------------------------------------C
C     CALCULATION  OF FLOW ACTION VALUES        C
C-----------------------------------------------C
      IF(IUNIT2.EQ.2) GO TO 20
      DO 10 I=1,NODES
      CFX(I)=0.0
      CFY(I)=0.0
      UX(I)=-1.0*(BCAC(I,1)-BCAC(I,2))/2.0
      UY(I)=-1.0*(BCAC(I,3)-BCAC(I,4))/2.0
   10 CONTINUE
      GO TO 35
C
   20 DO 30 I=1,NODES
      RAD=GCOE(I)
      ANG=GCOE(I+NODES)
      IF(COOR.EQ.'RAD') GO TO 21
      ANG=3.141592654/180.0*ANG
   21 SN=DSIN(ANG)
      CS=DCOS(ANG)
      UX1=RAD*BCAC(I,1)*SN
      UX2=RAD*BCAC(I,2)*SN
```

```
      UX2=RAD*BCAC(I,2)*SN
      UY1=RAD*BCAC(I,1)*CS
      UY2=RAD*BCAC(I,2)*CS
      UX(I)=(UX1-UX2)/2.0
      UY(I)=(-1.0)*(UY1-UY2)/2.0
   30 CONTINUE
   35 DO 40 I=1,NODES
      H(I)=BCAC(I,9)
      VD(I)=BCAC(I,10)
   40 CONTINUE
C----------------------------------C
C     CALCULATION OF BODY FORCES   C
C----------------------------------C
      DO 41 LE=1,NELE
      DO 41 I=1,NNEL
      IN=NME(LE,I)
      BX(IN)=(BCAC(IN,5)+BCAC(IN,6))/2.0
      BY(IN)=(BCAC(IN,7)+BCAC(IN,8))/2.0
   41 CONTINUE
C
C-----------------------------------------C
C    CALCULATION OF CENTRIFUGAL FORCE     C
C-----------------------------------------C
      CALL CFORCE
      DO 50 I=1,NODES
      BX(I)=BX(I)+CFX(I)
      BY(I)=BY(I)+CFY(I)
   50 CONTINUE
C
C------------------------------------------------------C
C     PRINT ALL THE FLOW ACTION VALUES CALCULATED.     C
C------------------------------------------------------C
      WRITE(W,51)
   51 FORMAT(//,'I',40('-'),'I',
     1   /'I  FLOW ACTION VALUES (MEAN VALUES)      I',
     2   /'I  INCLUDING CENTRIFUGAL FORCES.         I',
     3   /'I',40('-'),'I',
     4   //5X,'SHEAR ACTION (X-DIRECTION)..........UX',
     5    /5X,'SHEAR ACTION (Y-DIRECTION)..........UY',
     6    /5X,'CENTRIFUGAL FORCE (X-DIRECTION).....CFX',
     7    /5X,'CENTRIFUGAL FORCE (Y-DIRECTION).....CFY',
     8    /5X,'TOTAL BODY FORCE (X-DIRECTION)......BX (=BX+CFX)',
     9    /5X,'TOTAL BODY FORCE (Y-DIRECTION)......BY (=BY+CFY)',
     1    /5X,'SQUEEZE ACTION (Z-DIRECTION).........H',
     2    /5X,'DIFFUSION ACTION (Z-DIRECTION)......VD',
     1   //,1X,'NODE',6X,'UX',10X,'UY',9X,'CFX',9X,'CFY',
     2      10X,'BX',10X,'BY',11X,'H',10X,'VD')
      DO 52 I=1,NODES
   52 WRITE(W,53) I,UX(I),UY(I),CFX(I),CFY(I),BX(I),BY(I),H(I),VD(I)
   53 FORMAT(2X,I3,8(2X,E10.3))
C-----Q(I)=MTKP(I,J)*P(J)  I,J=1,NODES
      IF(NODES.LE.(NP+NQ)) GO TO 70
      WRITE(W,60) NODES,NP,NQ
   60 FORMAT(1H1,////10X,'** SOLVE THE EQUATIONS **',
     1//,10X,'NODES=',I4,10X,'NP=',I4,10X,'NQ=',I4,
     2//,10X,'*** INCONSISTENT  ***STOP***')
      STOP
   70 CONTINUE
```

```
C
C--------------------------------------------------------C
C           CLASSIFICATION OF CONSTRAINED NODES          C
C--------------------------------------------------------C
      J=0
      K=0
      DO 100 I=1,NODES
      Q(I)=0.0
      P(I)=0.0
      IF(NDF(I).EQ.1) GO TO 110
      IF(NDF(I).EQ.2) GO TO 120
  110 J=J+1
      Q(I)=BCVL(I)
      GO TO 100
  120 K=K+1
      P(I)=BCVL(I)
  100 CONTINUE
C
      DO 250 I=1,NODES
C-----LET ALL INITIAL VALUES ZERO.
      QQ1(I)=0.0
      QQ2(I)=0.0
      QQ3(I)=0.0
      QQ4(I)=0.0
      QQ5(I)=0.0
      QQ6(I)=0.0
      QQ7(I)=0.0
  250 CONTINUE
      IF(NP.NE.0) GO TO 500
C
C--------------------------------------------C
C  CASE-1   IF ALL FLOW VALUES ARE KNOWN     C
C--------------------------------------------C
      DO 350 I=1,NODES
      DO 300 J=1,NODES
      QQ1(I)=QQ1(I)+MTKP(I,J)*P(J)
      QQ2(I)=QQ2(I)+MTKUX(I,J)*UX(J)
      QQ3(I)=QQ3(I)+MTKUY(I,J)*UY(J)
      QQ4(I)=QQ4(I)+MTKBX(I,J)*BX(J)
      QQ5(I)=QQ5(I)+MTKBY(I,J)*BY(J)
      QQ6(I)=QQ6(I)+MTKH(I,J)*H(J)
      QQ7(I)=QQ7(I)+MTKVD(I,J)*VD(J)
  300 CONTINUE
      Q(I)=Q(I)+QQ1(I)-(QQ2(I)+QQ3(I)+QQ4(I)+QQ5(I)+QQ6(I)+QQ7(I))
  350 CONTINUE
C
C-----CALCULATE THE INVERSED MATRIX OF MTKP.
      CALL MB02A(MTKP,MKP,NODES,IP)
      DO 400 I=1,NODES
      DO 400 J=1,NODES
C-----THE MATRIX MTKP HAS BEEN INVERTED.
      P(I)=P(I)+MTKP(I,J)*Q(J)
  400 CONTINUE
      GO TO 955
C
  500 IF(NQ.NE.0) GO TO 700
```

```
C----------------------------------------------C
C   CASE-2  IF ALL PRESSURE VALUES ARE KNOWN.  C
C----------------------------------------------C
      DO 650 I=1,NODES
      DO 600 J=1,NODES
      QQ1(I)=QQ1(I)+MTKP(I,J)*P(J)
      QQ2(I)=QQ2(I)+MTKUX(I,J)*UX(J)
      QQ3(I)=QQ3(I)+MTKUY(I,J)*UY(J)
      QQ4(I)=QQ4(I)+MTKBX(I,J)*BX(J)
      QQ5(I)=QQ5(I)+MTKBY(I,J)*BY(J)
      QQ6(I)=QQ6(I)+MTKH(I,J)*H(J)
      QQ7(I)=QQ7(I)+MTKVD(I,J)*VD(J)
  600 CONTINUE
      Q(I)=Q(I)+QQ1(I)+(QQ2(I)+QQ3(I)+QQ4(I)+QQ5(I)+QQ6(I)+QQ7(I))
  650 CONTINUE
      GO TO 955
C
C-------------------------------------C
C   CASE-3   FORM STANDARD PROBLEM     C
C-------------------------------------C
C
C1    DIVIDE ALL Q AND P INTO KNOWN Q1,P2 AND UNKNOWN Q2,P1
C     1    FIND UNKNOWN VALUE P1(I)
C     Q1(KNOWN)=MTKP11*P1(UNKNOWN)+MTKP12*P2(KNOWN)
C
  700 DO 830 I=1,NQ
C-----LET ALL INITIAL VALUES ZERO.
      QQ1(I)=0.0
      QQ2(I)=0.0
      QQ12(I)=0.0
      QQ13(I)=0.0
      QQ14(I)=0.0
      QQ15(I)=0.0
      QQ16(I)=0.0
      QQ17(I)=0.0
      QQ21(I)=0.0
      QQ22(I)=0.0
      QQ23(I)=0.0
      QQ24(I)=0.0
      QQ25(I)=0.0
      QQ26(I)=0.0
      QQ27(I)=0.0
      DO 800 J=1,NQ
      MTKP1(I,J)=MTKP(NDQ(I),NDQ(J))
      QQ12(I)=QQ12(I)+MTKUX(NDQ(I),NDQ(J))*UX(NDQ(J))
      QQ13(I)=QQ13(I)+MTKUY(NDQ(I),NDQ(J))*UY(NDQ(J))
      QQ14(I)=QQ14(I)+MTKBX(NDQ(I),NDQ(J))*BX(NDQ(J))
      QQ15(I)=QQ15(I)+MTKBY(NDQ(I),NDQ(J))*BY(NDQ(J))
      QQ16(I)=QQ16(I)+MTKH(NDQ(I),NDQ(J))*H(NDQ(J))
      QQ17(I)=QQ17(I)+MTKVD(NDQ(I),NDQ(J))*VD(NDQ(J))
  800 CONTINUE
      QQ1(I)=QQ12(I)+QQ13(I)+QQ14(I)+QQ15(I)+QQ16(I)+QQ17(I)
      DO 810 J=1,NP
      QQ21(I)=QQ21(I)+MTKP(NDQ(I),NDP(J))*P(NDP(J))
      QQ22(I)=QQ22(I)+MTKUX(NDQ(I),NDP(J))*UX(NDP(J))
      QQ23(I)=QQ23(I)+MTKUY(NDQ(I),NDP(J))*UY(NDP(J))
      QQ24(I)=QQ24(I)+MTKBX(NDQ(I),NDP(J))*BX(NDP(J))
      QQ25(I)=QQ25(I)+MTKBY(NDQ(I),NDP(J))*BY(NDP(J))
      QQ26(I)=QQ26(I)+MTKH(NDQ(I),NDP(J))*H(NDP(J))
      QQ27(I)=QQ27(I)+MTKVD(NDQ(I),NDP(J))*VD(NDP(J))
  810 CONTINUE
      QQ2(I)=QQ21(I)+QQ22(I)+QQ23(I)+QQ24(I)+QQ25(I)+QQ26(I)+QQ27(I)
      Q1(I)=Q(NDQ(I))-(QQ1(I)+QQ2(I))
  830 CONTINUE
```

```
C2----ALL FLOW VALUES Q1 HAVE BEEN CALCULATED.
C-----FIND UNKNOWN PRESSURE P1.
C-----MAKE INV. MATRIX OF MTKP1(NDQ(I),NDQ(I))
      IF(NQ-1)950,840,850
  840 MTKP1(1,1)=1.0/MTKP1(1,1)
      GO TO 860
C
  850 CALL MB02A(MTKP1,MKP,NQ,IP)
      IF(RESULT.NE.'YES') GO TO 860
      WRITE(W,851)
  851 FORMAT(//,2X,37('C'),
     1          /,2X,'C',35X,'C',
     2      /,2X,'C  RESULT OF INVERTED MATRIX MTKP1  C',
     3      /2X,'C',35X,'C',
     4      /,2X,37('C'),
     5      //,5X,'NDQ(I)')
      WRITE(W,852) (NDQ(I),I=1,NQ)
  852 FORMAT(5X,I5)
C
      CALL PRINT(NQ,MTKP1,W)
C
C-----THE MATRIX MTKP1 HAS BEEN INVERTED.
  860 CONTINUE
      DO 900 I=1,NQ
      P1(I)=0.0
      DO 900 J=1,NQ
      P1(I)=P1(I)+MTKP1(I,J)*Q1(J)
  900 CONTINUE
      DO 901 I=1,NQ
      P(NDQ(I))=P1(I)
  901 CONTINUE
C
C3----ALL PRESSURE VALUES ARE KNOWN NOW.
C-----THEN FIND UNKNOWN FLOW VALUES Q2.
C
      DO 950 I=1,NP
C3-1  LET ALL INITIAL VALUES ZERO.
      Q2(I)=0.0
      QQ21(I)=0.0
      QQ22(I)=0.0
      QQ23(I)=0.0
      QQ24(I)=0.0
      QQ25(I)=0.0
      QQ26(I)=0.0
      QQ27(I)=0.0
C
C3-2  CALCULATION OF Q2
C-----Q2(I)=KP21*P1+KP22*P2
      DO 910 J=1,NODES
      QQ21(I)=QQ21(I)+MTKP(NDP(I),J)*P(J)
      QQ22(I)=QQ22(I)+MTKUX(NDP(I),J)*UX(J)
      QQ23(I)=QQ23(I)+MTKUY(NDP(I),J)*UY(J)
      QQ24(I)=QQ24(I)+MTKBX(NDP(I),J)*BX(J)
      QQ25(I)=QQ25(I)+MTKBY(NDP(I),J)*BY(J)
      QQ26(I)=QQ26(I)+MTKH(NDP(I),J)*H(J)
      QQ27(I)=QQ27(I)+MTKVD(NDP(I),J)*VD(J)
  910 CONTINUE
      Q2(I)=QQ21(I)+QQ22(I)+QQ23(I)+QQ24(I)+QQ25(I)+QQ26(I)+QQ27(I)
      Q(NDP(I))=Q2(I)
  950 CONTINUE
C-----ALL FLOW VALUES HAVE BEEN CALCULATED
```

```
C     CALCULATION OF INWARD and OUTWARD FLOW (Qin,Qout)     C
C-----------------------------------------------------------C
  955 QIN=0.0
      QOUT=0.0
      DO 970 I=1,NODES
      IF(Q(I).LT.0.0) GOTO 960
      QOUT=QOUT+Q(I)
      GO TO 970
  960 QIN=QIN+Q(I)
  970 CONTINUE
      QIN=DABS(QIN)
C5----CALCULATION OF LOAD CAPACITY (WW)
      WW=0.0
      DO 990 LE=1,NELE
      PP=0.0
      DO 980 I=1,NNEL
      PP=PP+P(NME(LE,I))/NNEL
  980 CONTINUE
      WW=WW+PP*AREA(LE)
  990 CONTINUE
C---------------------------------------------------C
C      CALACULATION OF FRICTION FORCE AND TORQUE.   C
C---------------------------------------------------C
C
      DO 1000 I=1,NODES
      DO 1000 J=1,4
      IF(BCAC(I,J).NE.0.0) GO TO 1001
 1000 CONTINUE
      GO TO 1100
 1001 WRITE(W,1002)
 1002 FORMAT(///,'*',60('*'),'*',
     1/'*  CALCULATION OF FRICTION FORCE AND TORQUE OF EACH ELEMENT  *',
     2  /'*',60('*'),'*')
      WRITE(W,1003)
 1003 FORMAT(//,16X,'LOWER SURFACE',35X,'UPPER SURFACE',
     1  //,'ELEMENT',2X,'X-Direction',2X,'Y-Direction',3X,
     2'RESULTANT',5X,'F.TORQUE',4X,'X-Direction',2X,'Y-Direction',
     3  3X,'RESULTANT',4X,'F.TORQUE',/,)
      TFX1=0.0
      TFY1=0.0
      TRF1=0.0
      TFX2=0.0
      TFY2=0.0
      TRF2=0.0
      TORQ1=0.0
      TORQ2=0.0
      DO 1010 LE=1,NELE
C
      CALL FFORCE(LE)
C
C     TOTAL FRICTION FORCE CALCULATION
      TFX1=TFX1+FX1
      TFY1=TFY1+FY1
      TRF1=TRF1+RF1
      TFX2=TFX2+FX2
      TFY2=TFY2+FY2
      TRF2=TRF2+RF2
      IF(COOR.NE.'PO') GO TO 1004
C
C     TOTAL TORQUE CALCULATION
      TORQ1=TORQ1+TQ1
      TORQ2=TORQ2+TQ2
 1004 WRITE(W,1005) LE,FX1,FY1,RF1,TQ1,FX2,FY2,RF2,TQ2
 1005 FORMAT(3X,I3,3X,4(E10.3,3X),1X,4(E10.3,3X))
 1010 CONTINUE
```

```
C-----------------------------------------C
C     CALCULATION OF ELEMENT FLOWS        C
C-----------------------------------------C
C
      IF(RESULT.EQ.'XX') GO TO 1006
 1100 DO 2000 LE=1,NELE
C
      DO 2000 I=1,NNEL
C     INITIAL VALUES ARE ZERO
      EP=0.0
      EUX=0.0
      EUY=0.0
      BXEL=0.0
      BYEL=0.0
      EH=0.0
      EVD=0.0
      DO 2001 J=1,NNEL
      JN=NME(LE,J)
      EP=EP+MP(LE,I,J)*P(JN)
      EUX=EUX+MKUX(LE,I,J)*UX(JN)
      EUY=EUY+MKUY(LE,I,J)*UY(JN)
      BXEL=BXEL+MKBX(LE,I,J)*BX(JN)
      BYEL=BYEL+MKBY(LE,I,J)*BY(JN)
      EH=EH+MKH(LE,I,J)*H(JN)
      EVD=EVD+MKVD(LE,I,J)*VD(JN)
 2001 CONTINUE
      QEL(LE,I)=EP+EUX+EUY+BXEL+BYEL+EH+EVD
 2000 CONTINUE
      IF(RESULT.EQ.'XY') GO TO 1006
      WRITE(W,2010)
 2010 FORMAT(///,19('*'),/'*',17X,'*',
     1        /,'*  ELEMENT FLOWS  *',/,'*',17X,'*',/,19('*'))
      DO 2015 LE=1,NELE
      IF(NNEL.EQ.4) GO TO 2012
      WRITE(W,2011) LE,(NME(LE,I),I=1,3),(QEL(LE,I),I=1,3)
 2011 FORMAT(/,2X,'ELEMENT NO.',I3,3X,'NODE NO.',3(9X,'NO.',I3),
     1        /19X,'ELEMENT FLOW',3(3X,E12.5))
      GO TO 2015
 2012 WRITE(W,2013) LE,(NME(LE,I),I=1,4),(QEL(LE,I),I=1,4)
 2013 FORMAT(//,2X,'ELEMENT NO.',I3,/5X,'NODE NO.',4(9X,'NO.',I3),
     1        /5X,'ELEMENT FLOW',4(3X,E12.5))
 2015 CONTINUE
 1006 RETURN
      END
```

```
C                             CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                             C    (MB02A)                  C
C                             C  THIS ROUTINE IS            C
C                             C  TO MAKE INVERSED MATRIX    C
C                             C                             C
C                             CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C
      SUBROUTINE MB02A(A,C,M,IP)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(200,200),C(200,200)
      DIMENSION D(400),IND(400),JND(400)
C
      AMAX=0.0
      DO 2 I=1,M
      JND(I)=I
      IND(I)=I
      DO 2 J=1,M
      IF(DABS(A(I,J))-AMAX)2,2,3
    3 AMAX=DABS(A(I,J))
      I4=I
      J4=J
    2 CONTINUE
      D(1)=1.0
      MM=M-1
      DO 11 J=1,MM
      IF(I4-J)6,6,4
    4 D(1)=-D(1)
      ISTO=IND(J)
      IND(J)=IND(I4)
      IND(I4)=ISTO
      DO 5 K=1,M
      STO=A(I4,K)
      A(I4,K)=A(J,K)
      A(J,K)=STO
    5 CONTINUE
    6 IF(J4-J)8,8,9
    9 D(1)=-D(1)
      ISTO=JND(J)
      JND(J)=JND(J4)
      JND(J4)=ISTO
      DO 12 K=1,M
      STO=A(K,J4)
      A(K,J4)=A(K,J)
      A(K,J)=STO
   12 CONTINUE
    8 AMAX=0.0
      J1=J+1
      DO 11 I=J1,M
      STO=-A(I,J)/A(J,J)
      DO 10 K=1,M
      A(I,K)=A(I,K)+STO*A(J,K)
      IF(K-J)10,10,15
   15 IF(DABS(A(I,K))-AMAX)10,10,17
   17 AMAX=DABS(A(I,K))
      I4=I
      J4=K
   10 CONTINUE
```

```
10 CONTINUE
   A(I,J)=STO
11 CONTINUE
   DO 18 I=1,MM
   D(I+1)=D(I)*A(I,I)
18 CONTINUE
   DET=D(M)*A(M,M)
   PROD1=1.0
   IF(IP-2)99,19,16
16 PROD1=1.0/DET
19 DO 20 J=1,M
   DO 21 K=1,J
   C(K,J)=0.0
21 CONTINUE
   DO 22 K=J,M
   C(K,J)=A(K,J)
22 CONTINUE
   C(J,J)=1.0
   PROD=PROD1
   DO 30 I=1,MM
   I2=M-I
   I1=I2+1
   STO1=C(I1,J)
   C(I1,J)=D(I1)*STO1*PROD
   IF(DABS(STO1)-DABS(A(I1,I1)))25,25,26
25 STO=STO1/A(I1,I1)
   DO 27 K=1,I2
   C(K,J)=C(K,J)-STO*A(K,I1)
27 CONTINUE
   PROD=PROD*A(I1,I1)
   GO TO 30
26 STO=A(I1,I1)/STO1
   DO 28 K=1,I2
   C(K,J)=A(K,I1)-STO*C(K,J)
28 CONTINUE
   PROD=-PROD*STO1
30 CONTINUE
   C(1,J)=D(1)*C(1,J)*PROD
20 CONTINUE
   DO 40 I=1,M
   K=IND(I)
   DO 40 J=1,M
   L=JND(J)
   A(L,K)=C(J,I)
40 CONTINUE
99 CONTINUE
   RETURN
   END
```

```
C                                             CCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                             C  THIS SUBROUTINE IS        C
C                                             C  TO PRINT OUT RESULTS OF   C
C                                             C  VARIOUS PROPERTIES        C
C                                             CCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE OUTPUT
C
C
      IMPLICIT REAL*3(A-H,M,O-Z)
      INTEGER W
C
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1           NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2             GELCO(400,2),CCOE(800),BCVL(400),BCAC(400,10),
     3             A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK9/Q(400),P(400),QIN,QOUT,WW,QEL(400,4)
      COMMON/BLK10/TEST
      COMMON/BLK14/ CFX(400),CFY(400)
      COMMON/BLK22/TFX1,TFX2,TFY1,TFY2,TRF1,TRF2,TORQ1,TORQ2
C
      WRITE(W,10)
   10 FORMAT(/////,27('C'),/,'C',25X,'C',
     1    /,'C  RESULT OF CALCULATION  C',
     2   /,'C',25X,'C',/27('C'))
      WRITE(W,100) TEST
  100 FORMAT(//,2X,'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX',
     2        /,2X,'X   1. RESULT OF SYSTEM FLOWS AND PRESSURES   X',
     4        /,2X,'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX',
     5//,2X,'TEST NAME...',A80,
     6///,2X,'NODE',8X,'GLOBAL CO-ORDINATES',13X,'FLOW',10X,'PRESSURE')
      IF (COOR.EQ.'PO') GO TO 120
      WRITE(W,110)
  110 FORMAT(2X,'NUMBER',9X,'X',13X,'Y',14X,'Q(I)',9X,'P(I)')
      GO TO 140
  120 WRITE(W,130)
  130 FORMAT(2X,'NUMBER',6X,'RADIUS',8X,'ANGLE',14X,'Q(I)',9X,'P(I)')
  140 DO 200 I=1,NODES
  200 WRITE(W,300) I,(GELCO(I,J),J=1,NCOE),Q(I),P(I)
  300 FORMAT(3X,I3,5X,E10.3,4X,E10.3,6X,E12.5,4X,E12.5)
      WRITE(W,400) QIN,QOUT,WW
  400 FORMAT(///,2X,'XXXXXXXXXXXXXXXXXXX',
     1         /2X,'X  2. TOTAL FLOW  X',
     2         /2X,'XXXXXXXXXXXXXXXXXXX',
     3         //,5X,'Inward Flow.....Qin=',E12.5,
     4         /,5X,'Outward Flow...Qout=',E12.5,
     5      //,2X,22('X'),/,2X,'X  3. LOAD CAPACITY  X',/2X,22('X'),
     6         //,5X,'Load capacity...WW=',E12.5)
      IF(COOR.EQ.'PO') GO TO 700
      WRITE(W,600)
  600 FORMAT(//'END OF PRINT')
  700 DO 750  I=1,NODES
      IF(CFX(I).NE.0.0) GO TO 800
      IF(CFY(I).NE.0.0) GO TO 800
  750 CONTINUE
      PRNO=1.0
      GO TO 905
  800 WRITE(W,900)
  900 FORMAT(/,7X,'CENTRIFUGAL FORCE HAS BEEN CONSIDERED.')
  905 DO 910 I=1,NODES
```

```
  905 DO 910 I=1,NODES
      DO 910 J=1,4
      IF(BCAC(I,J).NE.0.0) GO TO 950
  910 CONTINUE
      GO TO 1002
C
C-----PRINT THE RESULT OF FRICTION FORCES AND FRICTION TORQUES
  950 WRITE(W,1000) TFX1,TFY1,TRF1,TFX2,TFY2,TRF2
 1000 FORMAT(///2X,29('X'),/,2X,'X  4. TOTAL FRICTION FORCE  X',
     1  /,2X,29('X'),//,16X,'LOWER SURFACE',29X,'UPPER SURFACE',
     2 //,4X,'X-Direction',3X,'Y-Direction',3X,'RESULTANT',7X,
     3 'X-Direction',2X,'Y-Direction',3X,'RESULTANT',
     4//,5X,3(E10.3,3X),3(3X,E10.3))
      IF(PRNO.EQ.1.0) GO TO 1002
      WRITE(W,1001) TORQ1,TORQ2

 1001 FORMAT(///,2X,30('X'),/2X,'X  5. TOTAL FRICTION TORQUE  X',
     1  /2X,30('X'),//,5X,'TOTAL TORQUE ON LOWER SURFACE..TQ1=',E12.5,
     2               //5X,'                 UPPER SURFACE..TQ2=',E12.5)
 1002 WRITE(W,600)
      RETURN
      END
```

```
C                               CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                               C  THIS SUBROUTINE IS TO              C
C                               C  CALCULATE THE CENTRIFUGAL FORCES   C
C                               C  IN ROTATING PROBLEM                C
C                               CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE CFORCE
C
C     THIS ROUTINE IS AVAILABLE FOR ONLY TRIANGULAR ELEMENT
C     BECAUSE OF THE TREATMENT OF MASS.
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
C
      COMMON/BLK1/ NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1             NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2             GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3             A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK14/ CFX(400),CFY(400)
      DIMENSION RAD(400),ANG(400),AG(400),S(400)
      DIMENSION CF(400),FC(400),FX(400),FY(400)
C
      IF(COOR.NE.'PO') GO TO 1000
      IF(IUNIT2.NE.2) GO TO 1000
C
      DO 10 I=1,NODES
      IF(BCAC(I,1).NE.0.0) GO TO 20
      IF(BCAC(I,2).NE.0.0) GO TO 20
   10 CONTINUE
      RETURN
C
   20 WRITE(W,103)
C-----ALL INITIAL VALUES ZERO
      FCX=0.0
      FCY=0.0
      DO 30 I=1,NODES
      CFX(I)=0.0
      CFY(I)=0.0
      RAD(I)=0.0
      ANG(I)=0.0
      S(I)=0.0
      FC(I)=0.0
      FX(I)=0.0
      FY(I)=0.0
   30 CONTINUE
C
C-----ARRANGEMENT OF COORDINATE
      DO 40 I=1,NODES
      RAD(I)=GCOE(I)
      ANG(I)=GCOE(I+NODES)
      IF(UNIT1.EQ.'RAD') GO TO 45
      ANG(I)=ANG(I)*3.141592654/180.0
   40 CONTINUE
   45 DO 50 LE=1,NELE
      DO 50 I=1,NNEL
C-----CALCULATION OF CENTRIFUGAL FORCE
      K=NME(LE,I)
      AG(K)=GCOE(K+NODES)
      AV1=BCAC(K,1)
      AV2=BCAC(K,2)
      IF(AV1-AV2.EQ.0.0) GO TO 60
      AV3=(AV2-AV1)/TH(LE,I)

      F1=(AV3*TH(LE,I)+AV1)**3-AV1**3
      F2=RAD(K)*DEN/(3.0*TH(LE,I)*AV3)
      F=F1*F2
      GO TO 61
   60 F=RAD(K)*DEN*AV1**2
```

```
   60 F=RAD(K)*DEN*AV1**2
C-----ASSEMBLY STEP
   61 FC(K)=FC(K)+F
      S(K)=S(K)+1.0
      IF(UNIT1.EQ.'DEG') GO TO 62
      IF(RESULT.NE.'YES') GO TO 50
      IF(LE.GT.100) GO TO 50
      WRITE(W,100) LE,K,RAD(K),AG(K)
  100 FORMAT(/,2X,'ELEMENT NO.=',I3,3X,'NODE NO.=',I3,
     1//6X,'RADIUS=',E10.3,5X,'ANGLE=',E10.3,'RADIAN')
      GO TO 63
   62 IF(RESULT.NE.'YES') GO TO 50
      WRITE(W,101) LE,K,RAD(K),AG(K)
  101 FORMAT(/,2X,'ELEMENT NO.=',I3,3X,'NODE NO.=',I3,
     1 //6X,'RADIUS=',E10.3,5X,'ANGLE=',E10.3,'DEG')
   63 WRITE(W,102) AV1,AV2,F1,F2,F
  102 FORMAT(6X,'LOWER SURFACE ANGULAR VELOCITY....',E10.3,
     1        /6X,'UPPER SURFACE ANGULAR VELOCITY....',E10.3,//
     2 6X,'F1=',E10.3,3X,'F2=',E10.3,6X,'CENTRIFUGAL FORCE=',E10.3,
     3 4X,'( = F1*F2 )')
   50 CONTINUE
      DO 70 I=1,NODES
      IF(S(I).EQ.0.0) GO TO 70
      FX(I)=FC(I)*DCOS(ANG(I))
      FY(I)=FC(I)*DSIN(ANG(I))
      CF(I)=FC(I)/S(I)
      CFX(I)=FX(I)/S(I)
      CFY(I)=FY(I)/S(I)
   70 CONTINUE
C
  103 FORMAT(//,5X,'C---------------------------------------------C',
     1        /,5X,'C',45X,'C',
     2        /5X,'C      CALCULATION OF CENTRIFUGAL FORCE        C',
     3        /,5X,'C',17X,'(MEAN FORCE VALUE)',10X,,'C',
     4        /5X,'C---------------------------------------------C')
      WRITE(W,104)
  104 FORMAT(//2X,'MEAN CENTRIFUGAL FORCE AT EACH NODE.'/)
      DO 80 I=1,NODES
   80 WRITE(W,105) I,S(I),FC(I),CFX(I),CFY(I)
  105 FORMAT(,6X,'NODE NO.=',I3,3X,'S(I)=',E10.3,5X,
     1        'FC(I)=',E10.3,3X,'CFX=',E10.3,3X,'CFY=',E10.3)
 1000 RETURN
      END
```

```
C                                          C  THIS ROUTINE IS TO CALCULATE     C
C                                          C  THE FRICTION FORCES AND TORQUES  C
C                                          CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      SUBROUTINE FFORCE(LE)
C
C-----THIS ROUTINE IS TO CALCULATE THE FRICTION FORCES AND FRICTION TORQUES
C     FOR TRIANGLE ELEMENT.
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      INTEGER W
C
      COMMON/BLK1/NCOE,NODES,NELE,NNEL,NP,NQ,NME(400,4),NNOD(400),
     1            NDF(400),IBPS(400,4),SECPR(400,4),DEN,TH(400,4),VIS,
     2            GELCO(400,2),GCOE(800),BCVL(400),BCAC(400,10),
     3            A(4,4),AA(400,4),B(400,4),C(400,4),AREA(400),W,COOR,
     4            RESULT,UNIT1,IUNIT2
      COMMON/BLK9/Q(400),P(400),QIN,QOUT,WW,QEL(400,4)
      COMMON/BLK13/DELTA(4,4)
      COMMON/BLK20/UX(400),UY(400),BX(400),BY(400),H(400),VD(400)
      COMMON/BLK21/FX1,FX2,FY1,FY2,RF1,RF2,TQ1,TQ2
C
      TX1=0.0
      TX2=0.0
      TY1=0.0
      TY2=0.0
      UX1=0.0
      UY1=0.0
      CX=0.0
      CY=0.0
      THICKM=0.0
      DO 1001 I=1,NNEL
      IN=NME(LE,I)
      TX1=TX1+B(LE,I)*P(IN)
      TY1=TY1+C(LE,I)*P(IN)
      UXI=2.0*UX(IN)
      UYI=2.0*UY(IN)
      UX1=UX1+UXI
      UY1=UY1+UYI
      THICKM=THICKM+TH(LE,I)
      DO 1001 J=1,NNEL
      JN=NME(LE,J)
      TX2=TX2+TH(LE,I)*(DELTA(I,J)+1.0)*BX(JN)
 1001 CONTINUE
      THICKM=THICKM/3.0
      TX11=TX1/12.0
      TX21=TX2*AREA(LE)/6.0
      TX31=VIS*AREA(LE)*UX1/(3.0*THICKM)
      TY11=TY1/12.0
      TY21=TY2*AREA(LE)/6.0
      TY31=VIS*AREA(LE)*UY1/(3.0*THICKM)
C
      IF(UX1.NE.0.0) GO TO 120
      FX1=0.0
      FX2=0.0
      GO TO 130
  120 FX1=(-1.0)*(TX11+TX21)+TX31
      FX2=TX11+TX21+TX31
  130 IF(UY1.NE.0.0) GO TO 140
      FY1=0.0
      FY2=0.0
      GO TO 150
  140 FY1=(-1.0)*(TY11+TY21)+TY31
      FY2=TY11+TY21+TY31
C
C-----CALCULATION OF FRICTION TORQUE
  150 RF1=DSQRT(FX1**2+FY1**2)
      RF2=DSQRT(FX2**2+FY2**2)
      DO 1002 I=1,NNEL

      CX=GELCO(NME(LE,I),1)+CX
```

```
      CX=GELCO(NME(LE,I),1)+CX
      CY=GELCO(NME(LE,I),2)+CY
 1002 CONTINUE
      CX=CX/3.0
      CY=CY/3.0
      TQ1=(-1.0)*CY*FX1+CX*FY1
      TQ2=(-1.0)*CY*FX2+CX*FY2
      RETURN
      END
```

```
      SUBROUTINE CNST4(X,Y,A,LE,C1,C2,C3,C4,C6,C8,
     1                 C10,C11,C12,C13,C14,C15,C16,C17,C18,C19,
     2                 C21,C22,C23,C40,C41,AREA,T,S)
C
C-----THIS ROUTINE IS TO CALCULATE THE COMMON CONSTANTS FOR MATRICES
C----- OF PRESSURE AND OTHER ACTIONS.
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION X(4),Y(4),A(4,4),XX(4,4),YY(4,4),AREA(400),T(4),S(4)
C
C
      DO 1 I=1,4
      DO 1 J=1,4
      XX(I,J)=0.0
      YY(I,J)=0.0
      XX(I,J)=X(I)-X(J)
      YY(I,J)=Y(I)-Y(J)
    1 CONTINUE
      AREA(LE)=0.0
      AREA1=DABS((A(1,2)+A(2,3)+A(3,1))-(A(2,1)+A(3,2)+A(1,3)))
      AREA2=DABS((A(1,3)+A(3,4)+A(4,1))-(A(3,1)+A(4,3)+A(1,4)))
      AREA(LE)=0.5*(AREA1+AREA2)
C
      C1=(XX(3,4)-XX(2,1))/4.0
      C2=(XX(3,4)+XX(2,1))/4.0
      C3=(YY(3,4)-YY(2,1))/4.0
      C4=(YY(3,4)+YY(2,1))/4.0
      C6=(XX(3,2)+XX(4,1))/4.0
      C8=(YY(3,2)+YY(4,1))/4.0
C
      C10=C1*C1+C3*C3
      C11=2.0*(C1*C6+C3*C8)
      C12=C6*C6+C8*C8
      C13=C10
      C14=2.0*(C1*C2+C3*C4)
      C15=C2*C2+C4*C4
      C16=C10
      C17=C1*C6+C3*C8
      C18=C14/2.0
      C19=C2*C6+C4*C8
      C21=C1*C8-C3*C6
      C22=C2*C3-C4*C1
      C23=C2*C8-C4*C6
      C40=C23+C21
      C41=C23-C21
      T(1)=-1.0
      T(2)=-1.0
      T(3)=1.0
      T(4)=1.0
      S(1)=-1.0
      S(2)=1.0
      S(3)=1.0
      S(4)=-1.0
      RETURN
      END
```

```
      SUBROUTINE TERMA(C10,C11,C12,C21,C22,C23,C40,C41,T,S,A,W,RESULT)
C
C------THIS ROUTINE IS TO CALCULATE THE TERM 'A' IN MATRIX OF MKP
C------FOR QUADRILATERAL ELEMENT (C21.NE.0.0)
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 ITGA,ITGB,ITGF,ITGG,ITGH
      DIMENSION A(4,4),T(4),S(4)
      INTEGER W
C
C------CALCULATION OF TERM A
C
      CALL INTEGA(C21,C40,C21,C41,ITGA)
      CALL INTEGB(C21,C40,C21,C41,ITGB)
      CALL INTEGF(C21,C40,C21,C41,ITGF)
      CALL INTEGG(C21,C40,C21,C41,ITGG)
      CALL INTEGH(C21,C40,C21,C41,ITGH)
C
      WRITE(W,2) C21,C40,C21,C41
    2 FORMAT(//'C21,C40,C21,C41',/,4(3X,E10.3))
      WRITE(W,3) ITGA,ITGB,ITGF,ITGG,ITGH
    3 FORMAT(//'ITGA,ITGB,ITGF,ITGG,ITGH'/,5(E10.3,3X))
      DO 1 I=1,4
      DO 1 J=1,4
      T1=T(I)+T(J)
      T2=T(I)*T(J)
      S1=S(I)+S(J)
      S2=S(I)*S(J)
      A1=T2/C21
      C24=(-1.0)*A1*C22/C21
      C25=(T1-A1*C23)/C21
      C26=(-1.0)*C22*C24
      C27=(-1.0)*(C22*C25+C23*C24)
      C28=1.0-C23*C25
C
      C31=C10*C25+C11*C24
      C33=C12*C25
      C34=C10*C26/C21
      C35=(C10*C27+C11*C26)/C21
      C36=(C10*C28+C11*C27+C12*C26)/C21
      C37=(C11*C28+C12*C27)/C21
      C38=C12*C28/C21
C
      A11=S2*(C31/3.0+C33)/4.0
      A21=S2/16.0
      A22=C34*ITGH+C35*ITGG+C36*ITGF+C37*ITGB+C38*ITGA
      A2=A21*A22
      A(I,J)=A11+A2
      IF(RESULT.NE.'YES') GO TO 20
      WRITE(W,9) I,J
    9 FORMAT(//,'I,J',/2I5)
      WRITE(W,10) C10,C11,C12,C21,C22,C23,C40,C41
   10 FORMAT(//'TERM-A C10,C11,C12,C21,C22,C23,C40,C41',/,6(E10.3,2X),/,
     1       2(E10.3,2X))
      WRITE(W,11) C24,C25,C26,C27,C28,C31,C33,C34,C35,C36,C37,C38
   11 FORMAT(//,'C24,C25,C26,C27,C28, C31,C33,C34,C35,C36,C37,C38
     1',/,5(E10.3,2X)/,7(E10.3,2X))
      WRITE(W,12) A11,A21,A22,A2,A(I,J)
   12 FORMAT(//,'A11,A21,A22,A2,A(I,J)',/,4(E10.3,2X),5X,E10.3)
    1 CONTINUE
C
   20 RETURN
      END
```

```
      SUBROUTINE TERMB(C13,C14,C15,C21,C22,C23,C40,C41,T,S,B,W,RESULT)
C
C-----THIS ROUTINE IS TO CALCULATE THE TERM 'B' IN MATRIX MKP FOR
C-----QUADRILATERAL ELEMENT (C21.NE.0.0)
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 ITGA,ITGB,ITGF,ITGG,ITGH
      DIMENSION B(4,4),T(4),S(4)
      INTEGER W
C
C-----CALCULATION OF TERM B
C
      CALL INTEGA(C21,C40,C21,C41,ITGA)
      CALL INTEGB(C21,C40,C21,C41,ITGB)
      CALL INTEGF(C21,C40,C21,C41,ITGF)
      CALL INTEGG(C21,C40,C21,C41,ITGG)
      CALL INTEGH(C21,C40,C21,C41,ITGH)
C
      DO 1 I=1,4
      DO 1 J=1,4
      T1=T(I)+T(J)
      T2=T(I)*T(J)
      S1=S(I)+S(J)
      S2=S(I)*S(J)
      B1=C13/C21
      C24=(-1.0)*B1*C22/C21
      C25=(C14-B1*C23)/C21
      C26=(-1.0)*C22*C24
      C27=(-1.0)*(C22*C25+C23*C24)
      C28=C15-C23*C25
      C31=C24*S1+C25*S2
      C33=C25
      C34=S2*C26/C21
      C35=(S1*C26+S2*C27)/C21
      C36=(S1*C27+S2*C28+C26)/C21
      C37=(S1*C28+C27)/C21
      C38=C28/C21
C
      B11=T2*(C31/3.0+C33)/4.0
      B21=C34*ITGH+C35*ITGG+C36*ITGF+C37*ITGB+C38*ITGA
      B22=T2/16.0
      B(I,J)=B11+B21*B22
C
      IF(RESULT.NE.'YES') GO TO 20
      WRITE(W,9) I,J
    9 FORMAT(//,'I,J',3X,2I5)
      WRITE(W,10) C13,C14,C15,C21,C22,C23,C40,C41
   10 FORMAT(//,'TERM-B  C13,C14,C15,C21,C22,C23,C40,C41',
     1          /,6(E10.3,2X),/,2(E10.3,2X))
      WRITE(W,11) C24,C27,C28,C31,C33,
     1           C34,C35,C36,C37,C38
   11 FORMAT(//,'C24,C27,C28,C31,C33',/,
     1           'C34,C35,C36,C37,C38',/,5(E10.3,3X),/,5(E10.3,3X))
      WRITE(W,12) B11,B21,B22,B(I,J)
   12 FORMAT(//,'B11,B21,B22,B(I,J)',/,3(E10.3,3X),5X,E10.3)
    1 CONTINUE
   20 RETURN
      END
```

```
      SUBROUTINE TERMC(C10,C17,C18,C19,C21,C22,C23,C40,C41,
     1                T,S,C,W,RESULT)
C
C-----THIS ROUTINE IS TO CALCULATE THE TERM 'C' IN MATRIX MKP FOR
C------QUADRILATERAL ELEMENT (C21.NE.0.0)
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 ITGA,ITGB,ITGF,ITGG,ITGH
      DIMENSION T(4),S(4),C(4,4)
      INTEGER W
C
      CALL INTEGA(C21,C40,C21,C41,ITGA)
      CALL INTEGB(C21,C40,C21,C41,ITGB)
      CALL INTEGF(C21,C40,C21,C41,ITGF)
      CALL INTEGG(C21,C40,C21,C41,ITGG)
      CALL INTEGH(C21,C40,C21,C41,ITGH)
C
      DO 1 I=1,4
      DO 1 J=1,4
      C60=C10*T(I)/C21
      C61=C17*T(I)/C21
      C62=(-1.0)*C22*C60/C21
      C63=(C10+C18*T(I)-C22*C61-C23*C60)/C21
      C64=(C17+C19*T(I)-C23*C61)/C21
      C65=(-1.0)*C22*C62
      C66=(-1.0)*(C22*C63+C23*C62)
      C67=C18-C22*C64-C23*C63
      C68=C19-C23*C64
      C70=C62+C63*S(J)
      C71=C64
      C72=C65*S(J)/C21
      C73=(C65+C66*S(J))/C21
      C74=(C66+C67*S(J))/C21
      C75=(C67+C68*S(J))/C21
      C76=C68/C21
C
      CA=S(I)*T(J)*(C70/3.0+C71)/4.0
      CB1=S(I)*T(J)/16.0
      CB2=C72*ITGH+C73*ITGG+C74*ITGF+C75*ITGB+C76*ITGA
      CB=CB1*CB2
      C(I,J)=CA+CB
      IF(RESULT.NE.'YES') GO TO 20
      WRITE(W,9) I,J
    9 FORMAT(//,'I,J',/,2I5)
      WRITE(W,10) C60,C61,C62,C63,C64,C65,C66,C67,C68,C70,C71,C72,
     1        C73,C74,C75,C76
   10 FORMAT(//,'TERM-C  C60,C61,C62,C63,C64,C65,C66,C67,C68',/,
     1 '      C70,C71,C72,C73,C74,C75,C76',
     2       5(E10.3,3X),/,4(E10.3,3X),/,2(E10.3,3X))
      WRITE(W,11) CA,CB1,CB2,CB,C(I,J)
   11 FORMAT(//,'CA,CB1,CB2,CB,C(I,J)',/,5(E10.3,3X))
    1 CONTINUE
   20 RETURN
      END
```

```
      SUBROUTINE INTEGA(A,B,C,D,INT)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2
      AA=A+B
      BB=A-B
      CC=C+D
      DD=C-D
      INT1=AA*DLOG(DABS(AA))/A+BB*DLOG(DABS(BB))/A
      INT2=CC*DLOG(DABS(CC))/C+DD*DLOG(DABS(DD))/C
      INT=INT1-INT2
      RETURN
      END
C
C
      SUBROUTINE INTEGB(A,B,C,D,INT)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2,INT3
      AA=A+B
      BB=A-B
      CC=C+D
      DD=C-D
      INT1=(B/A-D/C)
      INT2=AA*BB*DLOG(DABS(AA/BB))/(2.0*A**2)
      INT3=CC*DD*DLOG(DABS(CC/DD))/(2.0*C**2)
      INT=INT1+INT2-INT3
      RETURN
      END
C
C
      SUBROUTINE INTEGC(A,B,C,D,INT)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2
C
      T1=-1.0
      T2=1.0
      X1=A*T1+B
      X2=A*T2+B
      AA=A/C
      BB=B-A*D/C
      R=AA/BB
      AR=DABS(R)
      IF(DABS(X1)-DABS(X2)) 1,2,2
    1 XMAX=X2
      XMIN=X1
      GO TO 3
    2 XMAX=X1
      XMIN=X2
    3 IF(DABS(XMAX)-AR) 10,20,20
   10 CALL INTG01(AA,BB,X1,X2,INT)
      RETURN
   20 IF(DABS(XMIN)-AR) 30,40,40
   40 CALL INTG02(AA,BB,X1,X2,INT)
      RETURN
   30 IF((X1+X2).LT.0.0) GO TO 50
      IF(R-0.0) 60,70,70
   60 R=(-1.0)*R
   70 CALL INTG01(AA,BB,XMIN,R,INT1)
      CALL INTG02(AA,BB,R,XMAX,INT2)
      GO TO 100
   50 IF(R-0.0) 80,90,90
   90 R=(-1.0)*R
   80 CALL INTG01(AA,BB,XMAX,R,INT1)
      CALL INTG02(AA,BB,R,XMIN,INT2)
  100 INT=INT1+INT2
      RETURN
```

```
      SUBROUTINE INTG01(A,B,X1,X2,INT)
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2,INT21,INT22
      INT1=DLOG(DABS(A))*(DLOG(DABS(X2))-DLOG(DABS(X1)))
      DO 1 N=1,50
      INT21=(((-1.0)*B*X2/A)**N)/(N*N)
      INT22=(((-1.0)*B*X1/A)**N)/(N*N)
      INT2=INT21-INT22
      INT1=INT1+INT2
      IF(INT2/INT1-0.001) 2,2,1
    1 CONTINUE
    2 INT=INT1
      RETURN
      END
C
C
      SUBROUTINE INTG02(A,B,X1,X2,INT)
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2,INT21,INT22
      INT1=(DLOG(DABS(B*X2))**2-DLOG(DABS(B*X1))**2)/2.0
      DO 1 N=1,50
      INT21=(((-1.0*A)/(B*X2))**N)/(N*N)
      INT22=(((-1.0*A)/(B*X1))**N)/(N*N)
      INT2=INT21-INT22
      INT1=INT1+INT2
      IF(INT2/INT1-0.001) 2,2,1
    1 CONTINUE
    2 INT=INT1
      RETURN
      END
C
C
      SUBROUTINE INTEGD(A,B,C,D,INT)
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2,INT3,INT4
      AA=A+B
      BB=A-B
      CC=C+D
      DD=C-D
      INT1=DLOG(DABS(BB))/(C*DD)-DLOG(DABS(AA))/(C*CC)
      INT2=DLOG(DABS(DD))/(C*DD)-DLOG(DABS(CC))/(C*CC)
      A1=(1.0-A)/(1.0+A)
      B1=(1.0-B)/(1.0+B)
      INT3=DLOG(DABS(B1))-DLOG(DABS(A1))
      INT4=C*(C*B-D*A)
      INT=INT1-INT2+INT3/INT4
      RETURN
      END
C
C
      SUBROUTINE INTEGE(A,B,C,D,INT)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2,INT3,INT4
      AA=A+B
      BB=A-B
      CC=C+D
      DD=C-D
      INT1=(DLOG(DABS(BB))/(DD*DD)-DLOG(DABS(CC))/(CC*CC))/(2.0*C)
      INT2=A*A/(2.0*C*(A*D-B*C)**2)
      INT3=DLOG(DABS(BB*CC/(AA*DD)))
      INT4=2.0*C*(B*C-A*D)/(A*CC*DD)
      INT=INT1-INT2*(INT3+INT4)
      RETURN
      END
C
C
      SUBROUTINE INTEGF(A,B,C,D,INT)
C
```

```
      REAL*8 INT,INT1,INT2
C
      A1=((A**3+B**3)*DLOG(DABS(A+B)))/(3.0*A**3)
      A2=((A**3-B**3)*DLOG(DABS(A-B)))/(3.0*A**3)
      A3=2.0*(B/A)**2/3.0
      INT1=A1+A2-A3
C
      C1=((C**3+D**3)*DLOG(DABS(C+D)))/(3.0*C**3)
      C2=((C**3-D**3)*DLOG(DABS(C-D)))/(3.0*C**3)
      C3=2.0*(D/C)**2/3.0
      INT2=C1+C2-C3
C
      INT=INT1-INT2
      RETURN
      END
C
C
C
      SUBROUTINE INTEGG(A,B,C,D,INT)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2
C
      A11=(A**4-B**4)/(4.0*A**4)
      A12=DLOG(DABS((A+B)/(A-B)))
      A13=B/(2.0*A)
      A14=1.0/3.0+(B/A)**2
      INT1=A11*A12+A13*A14
C
      C11=(C**4-D**4)/(4.0*C**4)
      C12=DLOG(DABS((C+D)/(C-D)))
      C13=D/(2.0*C)
      C14=1.0/3.0+(D/C)**2
C
      INT2=C11*C12+C13*C14
      INT=INT1-INT2
C
      RETURN
      END
C
C
C
      SUBROUTINE INTEGH(A,B,C,D,INT)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT,INT1,INT2
C
      A11=(A**5+B**5)/(5.0*A**5)
      A12=DLOG(DABS(A+B))
      A13=(A**5-B**5)/(5.0*A**5)
      A14=DLOG(DABS(A-B))
      A15=2.0*((B/A)**2/3.0+(B/A)**4)/5.0
      INT1=A11*A12+A13*A14-A15
C
      C11=(C**5+D**5)/(5.0*C**5)
      C12=DLOG(DABS(C+D))
      C13=(C**5-D**5)/(5.0*C**5)
      C14=DLOG(DABS(C-D))
      C15=2.0*((D/C)**2/3.0+(D/C)**4)/5.0
      INT2=C11*C12+C13*C14-C15
      INT=INT1-INT2
C
      RETURN
      END
C
CCCCCCCCCCCTHESE ROUTINE HAVE NOT COMPLETED  YET.  CCCCCCCCCCCCCCCCCCCCCC
C
C

C
```

```
      SUBROUTINE TRMA1(LE,TH,C10,C11,C12,C22,C23,T,S,A)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4),T(4),S(4),TH(400,4)
C
C-----CALCULATION OF TERM-A  WHEN C21=0.0
C
      DO 1 I=1,4
      DO 1 J=1,4
      A(I,J)=0.0
      TA1=0.0
      TA2=0.0
      TA=0.0
      C1=T(I)*T(J)
      C2=T(I)+T(J)
      C3=1.0
      DO 2 L=1,4
      DO 2 M=1,4
      DO 2 N=1,4
C
      T1=T(L)*T(M)*T(N)
      T2=T(L)*T(M)+T(M)*T(N)+T(N)*T(L)
      T3=T(L)+T(M)+T(N)
      S1=S(L)*S(M)*S(N)
      S2=S(L)*S(M)+S(M)*S(N)+S(N)*S(L)
      S3=S(L)+S(M)+S(N)
      THICK=TH(LE,L)*TH(LE,M)*TH(LE,N)
C
      CALL INTEG1(C1,C2,C3,T1,T2,T3,TA1)
      CALL INTEG2(C22,C23,C10,C11,C12,S1,S2,S3,TA2)
      TA10=TA1*TA2
      TA=TA+THICK*TA10
    2 CONTINUE
      A(I,J)=S(I)*S(J)*TA/1024.0
    1 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE TRMB1(LE,TH,C13,C14,C15,C22,C23,T,S,B)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION B(4,4),T(4),S(4),TH(400,4)
C
C-----CALCULATION OF TERM-B  WHEN C21=0.0
C
      DO 1 I=1,4
      DO 1 J=1,4
      B(I,J)=0.0
      TB1=0.0
      TB2=0.0
      TB10=0.0
      TB=0.0
      C1=S(I)*S(J)
      C2=S(I)+S(J)
      C3=1.0
      DO 2 L=1,4
      DO 2 M=1,4
      DO 2 N=1,4
      T1=T(L)*T(M)*T(N)
      T2=T(L)*T(M)+T(M)*T(N)+T(N)*T(L)
      T3=T(L)+T(M)+T(N)
      S1=S(L)*S(M)*S(N)
      S2=S(L)*S(M)+S(M)*S(N)+S(N)*S(L)
      S3=S(L)+S(M)+S(N)
      THICK=TH(LE,L)*TH(LE,M)*TH(LE,N)
C
      CALL INTEG1(C13,C14,C15,T1,T2,T3,TB1)
      CALL INTEG2(C22,C23,C1,C2,C3,S1,S2,S3,TB2)
```

```
      TB10=TB1*TB2
      TB=TB+THICK*TB10
    2 CONTINUE
      B(I,J)=T(I)*T(J)*TB/1024.0
    1 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE TRMC1(LE,TH,C1,C2,C3,C4,C22,C23,T,S,C)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION C(4,4),T(4),S(4)
C
C------CALCULATION OF TERMC  WHEN C21=0.0
C
      DO 1 I=1,4
      DO 2 J=1,4
      C(I,J)=0.0
      TC1=0.0
      TC=0.0
      A1=C1*T(I)
      A2=C2*T(I)
      A3=C3*T(I)+C1
      A4=C4*T(I)+C2
      A5=C3
      A6=C4
      DO 2 L=1,4
      DO 2 M=1,4
      DO 2 N=1,4
      T1=T(L)*T(M)*T(N)
      T2=T(L)*T(M)+T(M)*T(N)+T(N)*T(L)
      T3=T(L)+T(M)+T(N)
      S1=S(L)*S(M)*S(N)
      S2=S(L)*S(M)+S(M)*S(N)+S(N)*S(L)
      S3=S(L)+S(M)+S(N)
      THICK=TH(LE,L)*TH(LE,M)*TH(LE,N)
      CALL INTG11(A1,A2,A3,A4,A5,A6,T1,T2,T3,C15,C16)
      C10=C15*S(J)
      C11=C15+C16*S(J)
      C12=C16
      CALL INTEG2(C22,C23,C10,C11,C12,S1,S2,S3,TC1)
      TC=TC+THICK*TC1
    2 CONTINUE
      C(I,J)=S(I)*T(J)*TC/1024.0
    1 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE TRMA3(T,S,TH,LE,C10,C11,C12,C13,C14,C15,
     1                 C17,C18,C19,C23,A,B,C,D)
C
C-----THIS SUBROUTINE IS FOR (C21=0.0,C22=0.0)CASE
C-----THICKNESS CHANGES WITHIN AN ELEMENT
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4),B(4,4),C(4,4),D(4,4),T(4),S(4),TH(400,4)
C
      DO 1 I=1,4
      DO 1 J=1,4
      A1=0.0
      B1=0.0
      C1=0.0
      A(I,J)=0.0
      B(I,J)=0.0
      C(I,J)=0.0
      D(J,I)=0.0
      DO 2 L=1,4
      DO 2 M=1,4
      DO 2 N=1,4
      T1=T(L)*T(M)*T(N)
```

```
      T3=T(L)+T(M)+T(N)
      S1=S(L)*S(M)*S(N)
      S2=S(L)*S(M)+S(M)*S(N)+S(N)*S(L)
      S3=S(L)+S(M)+S(N)
      THICK=TH(LE,L)*TH(LE,M)*TH(LE,N)
      CALL INTEG5(I,J,T,T1,T2,T3,TA1)
      CALL INTEG7(C10,C11,C12,S,S1,S2,S3,TA2)
      A1=A1+THICK*TA1*TA2
C
      CALL INTEG5(I,J,S,S1,S2,S3,TB1)
      CALL INTEG7(C13,C14,C15,T,T1,T2,T3,TB2)
      B1=B1+THICK*TB1*TB2
C
      C24=C10*(T2*T(I)+T1)+C18*T1*T(I)
      C25=C17*(T2*T(I)+T1)+C19*T1*T(I)
      C26=C10*(T(I)+T3)+C18*(T(I)*T3+T2)
      C27=C17*(T(I)+T3)+C19*(T(I)*T3+T2)
      C28=C24/5.0+C26/3.0+C18
      C29=C25/5.0+C27/3.0+C19
      CALL INTEG6(J,C28,C29,S,S1,S2,S3,TC1)
      C1=C1+THICK*TC1
    2 CONTINUE
      A(I,J)=S(I)*S(J)*A1/(256.0*C23)
      B(I,J)=T(I)*T(J)*B1/(256.0*C23)
      C(I,J)=S(I)*T(J)*C1/(256.0*C23)
      D(J,I)=C(I,J)
    1 CONTINUE
      RETURN
      END
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C THESE ROUTINES ARE FOR QUADRILATERAL ELEMENT WITH SAME THICKNESS
C
C
      SUBROUTINE TERMA1(C10,C11,C12,C22,C23,T,S,A)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION A(4,4),T(4),S(4)
C
C-----CALCULATION OF TERM-A
C
      C24=C10/C22
      C25=(C11-C23*C24)/C22
      C26=C12-C23*C25
      C42=C23+C22
      C43=C23-C22
      DO 1 I=1,4
      DO 1 J=1,4
      T2=T(I)*T(J)
      S2=S(I)*S(J)
      A1=S2/8.0*(T2/3.0+1.0)
      A2=2.0*C25+C26/C22*DLOG(DABS(C42/C43))
      A(I,J)=A1*A2
    1 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE TERMB1(C13,C15,C22,C23,T,S,B)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION B(4,4),T(4),S(4)
C
C-----CALCULATION OF TERM-B
C
      C42=C23+C22
      C43=C23-C22
      DO 1 I=1,4
      DO 1 J=1,4
```

```
      T2=T(I)*T(J)
      S1=S(I)+S(J)
      S2=S(I)*S(J)
      C24=S2/C22
      C25=(S1-C23*C24)/C22
      C26=1.0-C23*C26
C
      B1=T2/8.0
      B2=C13/3.0+C15
      B3=2.0*C25+C26/C22*DLOG(DABS(C42/C43))
      B(I,J)=B1*B2*B3
    1 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE TERMC1(C10,C17,C18,C19,C22,C23,T,S,C)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION C(4,4),T(4),S(4)
C
C-----CALCULATION OF TERM C
C
      C42=C23+C22
      C43=C23-C22
      DO 1 I=1,4
      DO 1 J=1,4
      C24=2.0*(C10*T(I)/3.0+C18)
      C25=2.0*(C17*T(I)/3.0+C19)
      CA1=C24*S(J)/C22
      CA2=(C24+C25*S(J)-CA1*C23)/C22
      CA3=C25-CA2*C23
      C1=S(I)*T(J)/16.0
      C2=2.0*CA2+CA3/C22*DLOG(DABS(C42/C43))
      C(I,J)=C1*C2
    1 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE TERMA3(C10,C12,C23,T,S,A)
C
C-----CALCULATION OF TERM A AND B FOR C21=0.0 C22=0.0 CASE)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION A(4,4),T(4),S(4)
C
      DO 1 I=1,4
      DO 1 J=1,4
      T2=T(I)*T(J)
      S2=S(I)*S(J)
      A1=S2/(4.0*C23)
      A2=T2/3.0+1.0
      A3=C10/3.0+C12
      A(I,J)=A1*A2*A3
    1 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE TERMC3(C10,C17,C18,C19,C23,T,S,C)
C
C-----CALCULATION OF TERM-C IN C21=0.0 C22=0.0 CASE
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION C(4,4),T(4),S(4)
C
      DO 1 I=1,4
```

```
      C24=2.0*(C16*T(I)/3.0+C18)
      C25=2.0*(C17*T(I)/3.0+C19)
      C1=S(I)*T(J)/(3.0*C23)
      C2=C24*S(J)/3.0+C25
      C(I,J)=C1*C2
    1 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE INTEG5(I,J,T,T1,T2,T3,ITG5)
C
C---INTEGRATION OF (1+T*TL)(1+T*TM)(1+T*TN)(1+T*TI)(1+T*TJ)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 ITG5
      DIMENSION T(4)
C
      T11=T1*T(J)+(T1+T2*T(J))*T(I)
      T12=(T2+T3*T(J))+(T3+T(J))*T(I)
C
      ITG5=T11/5.0+T12/3.0+1.0
C
      RETURN
      END
C
C
C
      SUBROUTINE INTEG6(J,C1,C2,T,T1,T2,T3,ITG6)
C
C-----INTEGRATION OF (1+T*TL)(1+T*TM)(1+T*TN)(1+T*TJ)(C1*T+C2)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 ITG6
      DIMENSION T(4)
C
C
      T11=C1*(T1+T2*T(J))+C2*T1*T(J)
      T12=C1*(T3+T(J))+C2*(T2+T3*T(J))
C
      ITG6=T11/5.0+T12/3.0+C2
      RETURN
      END
C
C
C
      SUBROUTINE INTEG7(C1,C2,C3,T,T1,T2,T3,ITG7)
C
C----INTEGRATION OF (1+T*TL)(1+T*TM)(1+T*TN)(C1*T**2+C2*T+C3)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 ITG7
      DIMENSION T(4)
C
C
      T11=C1*T2+C2*T1
      T12=C1+C2*T3+C3*T2
C
      ITG7=T11/5.0+T12/3.0+C3
C
      RETURN
      END
C
C
C
      SUBROUTINE TERMU3(T,S,TM,LE,DTM,C3,C4,CC,NP)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION T(4),S(4),TM(400,4),AP(400,4,1)
```

```
      UA=0.0
      UB=0.0
      DO 1 L=1,4
      UA1=(T(I)*T(J)+(T(I)+T(J))*T(L))/3.0+1.0
      UA2=(C3*S(J)*S(L)+(S(J)+S(L))*C3)/3.0+C8
      UA=UA+TH(LE,L)*UA1*UA2
      UB1=(S(I)*S(J)+(S(I)+S(J))*S(L))/3.0+1.0
      UB2=(C4*T(J)*T(L)+(T(J)+T(L))*C3)/3.0+C4
      UB=UB+TH(LE,L)*UB1*UB2
    1 CONTINUE
      A=0.0
      B=0.0
      CNST1=S(I)*DEN/16.0
      CNST2=T(I)*DEN/16.0
      A=CNST1*UA
      B=CNST2*UB
      AB(LE,I,J)=A-B
    2 CONTINUE
C
      RETURN
      END
C
      SUBROUTINE TERMY3(T,S,TH,LE,DEN,C1,C2,C6,AB)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      DIMENSION T(4),S(4),AB(400,4,4)
      DO 2 I=1,4
      DO 2 J=1,4
      UA=0.0
      UB=0.0
      DO 1 L=1,4
      UA1=(C1*(T(L)+T(J))+C2*T(L)*T(J))/3.0+C2
      UA2=(S(L)*(S(I)+S(J))+S(I)*S(J))/3.0+1.0
      UB1=(C1*(S(L)+S(J))+C6*S(L)*S(J))/3.0+C6
      UB2=(T(L)*(T(I)+T(J))+T(I)*T(J))/3.0+1.0
      UA=UA+TH(LE,L)*UA1*UA2
      UB=UB+TH(LE,L)*UB1*UB2
    1 CONTINUE
      CNST1=T(I)*DEN/16.0
      CNST2=S(I)*DEN/16.0
      A=CNST1*UA
      B=CNST2*UB
      AB(LE,I,J)=A-B
    2 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE TERMB3(T,S,TH,LE,C3,C4,C8,DEN,VIS,A)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION TH(400,4),T(4),S(4),A(400,4,4)
      DO 1 I=1,4
      DO 1 J=1,4
      A1=0.0
      B1=0.0
C
      DO 2 L=1,4
      DO 2 M=1,4
      DO 2 N=1,4
      THICK=TH(LE,L)*TH(LE,M)*TH(LE,N)
      T1=T(L)*T(M)*T(N)
      T2=T(L)*T(M)+T(M)*T(N)+T(N)*T(L)
      T3=T(L)+T(M)+T(N)
      S1=S(L)*S(M)*S(N)
      S2=S(L)*S(M)+S(M)*S(N)+S(N)*S(L)
      S3=S(L)+S(M)+S(N)
      CALL INTEG5(I,J,T,T1,T2,T3,TA1)
      CALL INTEG6(J,C3,C8,S,S1,S2,S3,TA2)
      [illegible]
```

```
      CALL INTEG6(J,C3,C4,T,T1,T2,T3,TB2)
      B1=B1+THICK*TB1*TB2
    2 CONTINUE
      CNST1=S(I)*(DEN)/(3072.0*VIS)
      CNST2=T(I)*(DEN)/(3072.0*VIS)
      A(LE,I,J)=A(LE,I,J)+CNST1*A1-CNST2*B1
    1 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE INTEG1(C10,C11,C12,T1,T2,T3,INT1)
C
C
C
C-----INTEGRATION OF (1+TTL)(1+TTM)(1+TTN)(C10*T**2+C11*T+C12)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT1
      T12=C10*T2+C11*T1
      T14=C10+C11*T3+C12*T2
      INT1=2.0*(T12/5.0+T14/3.0+C12)
      RETURN
      END
C
C
C
      SUBROUTINE INTG11(C1,C2,C3,C4,C5,C6,T1,T2,T3,C15,C16)
C
C-----INTEGRATION OF (1+TTL)(1+TTM)(1+TTN)(C10T**2+C11T+C12)
C-----WHEN C10=C1*S+C2   C11=C3*S+C4  AND C12=C5*S+C6
      IMPLICIT REAL*8(A-H,M,O-Z)
C
      C20=C1*T2+C3*T1
      C21=C2*T2+C4*T1
      C22=C1+C3*T3+C5*T2
      C23=C2+C4*T3+C6*T2
      C15=2.0*(C20/5.0+C22/3.0+C5)
      C16=2.0*(C21/5.0+C23/3.0+C6)
      RETURN
      END
C
C
      SUBROUTINE INTEG2(C1,C2,C10,C11,C12,T1,T2,T3,INT2)
C
C
C-----INTEGRATION OF (1+TTL)(1+TTM)(1+TTN)(C10T**2+C11T+C12)/(C1T+C2)
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT2
      T10=C10*T1/C1
      T11=(C10*T2+C11*T1-T10*C2)/C1
      T12=(C10*T3+C11*T2+C12*T1-T11*C2)/C1
      T13=(C10+C11*T3+C12*T2-T12*C2)/C1
      T14=(C11+C12*T3-T13*C2)/C1
      T15=C12-T14*C2
      TT1=2.0*(T10/5.0+T12/3.0+T14)
      TT2=T15/C1*DLOG(DABS((C1+C2)/(C1-C2)))
      INT2=TT1+TT2
      RETURN
      END
C
C
C
      SUBROUTINE INTG21(C1,C3,C4,C10,C11,C12,T1,T2,T3,
     1                   C35,C36,C37,C38,C39,C40,C41,C42,C43,C44,C45)
C
C-----INTEGRATION OF (1+TTL)(1+TTM)(1+TTN)(C10*T**2+C11*T+C12)/(C1*T+C2)
C-----WHEN C2=C3*S+C4
```

```
      T10=C10*T1/C1
      C21=(-1.0)*T10*C3/C1
      C22=(C10*T2+C11*T1-T10*C4)/C1
      C23=(-1.0)*C3*C21/C1
      C24=(-1.0)*(C3*C22+C4*C21)/C21
      C25=(C10*T3+C11*T2+C12*T1-C4*C22)/C1
      C26=(-1.0)*C3*C23/C1
      C27=(-1.0)*(C3*C24+C4*C23)/C1
      C28=(-1.0)*(C3*C25+C4*C24)/C1
      C29=(C10*T3+C11*T2+C12*T1-C4*C25)/C1
      C30=(-1.0)*C3*C26/C1
      C31=(-1.0)*(C3*C27+C4*C26)/C1
      C32=(-1.0)*(C3*C28*C4*C27)/C1
      C33=(-1.0)*(C3*C29+C4*C28)/C1
      C34=(C11+C12*T3-C4*C29)/C1
      C35=(-1.0)*C3*C30
      C36=(-1.0)*(C3*C31+C4*C30)
      C37=(-1.0)*(C3*C32+C4*C31)
      C38=(-1.0)*(C3*C33+C4*C32)
      C39=(-1.0)*(C3*C34+C4*C33)
      C40=C12-C4*C34
      C41=2.0*C30
      C42=2.0*C31
      C43=2.0*(C23/3.0+C32)
      C44=2.0*(C24/3.0+C33)
      C45=2.0*(T10/5.0+C25/3.0+C34)
      RETURN
      END
C
C
C
      SUBROUTINE INTG22(C1,A1,A2,A3,A4,A5,A6,A7,A8,T1,T2,T3,
     1      C40,C41,C42,C43,C44,C45,C46,C47,C48,C49,C50,C51,C52)
C
C
C-----INTEGRATION OF (1+TTL)(1+TTM)(1+TTN)(C10T**2+C11T+C12)/(C1T+C2)
C-----WHEN C2=A1*S+A2  C10=A3*S+A4  C11=A5*S+A6 AND C12=A7*S+A8
      IMPLICIT REAL*8(A-H,M,O-Z)
C
C
      C13=A3*T1/C1
      C14=A4*T1/C1
      C15=(-1.0)*C13*A1/C1
      C16=(A3*T2+A5*T1-C13*A2-C14*A1)/C1
      C17=(A4*T2+A6*T1-C14*A2)/C1
      C18=(-1.0)*C15*A1/C1
      C19=(-1.0)*(C15*A2+C16*A1)/C1
      C20=(A3*T3+A5*T2+A7*T1-C16*A2-C17*A1)/C1
      C21=(A4*T3+A6*T2+A8*T1-C17*A2)/C1
      C22=(-1.0)*C18*A1/C1
      C23=(-1.0)*(C18*A2+C19*A1)/C1
      C24=(-1.0)*(C19*A2+C20*A1)/C1
      C25=(A3+A5*T3+A7*T2-C20*A2-C21*A1)/C1
      C26=(A4+A6*T3+A8*T2-C21*A2)/C1
      C27=(-1.0)*C22*A1/C1
      C28=(-1.0)*(C22*A2+C23*A1)/C1
      C29=(-1.0)*(C23*A2*C24*A1)/C1
      C30=(-1.0)*(C24*A2+C25*A1)/C1
      C31=(A5+A7*T3-C25*A2-C26*A1)/C1
      C32=(A6+A8*T3-C26*A2)/C1
      C33=(-1.0)*C27*A1
      C34=(-1.0)*(C27*A2+C28*A1)
      C35=(-1.0)*(C28*A2+C29*A1)
      C36=(-1.0)*(C29*A2+C30*A1)
      C37=(-1.0)*(C30*A2+C31*A1)
      C38=A7-C31*A2-C32*A1
      C39=A8-C32*A2
      C40=2.0*C27
      C41=2.0*C28
      C42=2.0*(C18/3.0+C33)
```

```
      C45=2.0*(C14/5.0+C21/3.0+C32)
      C46=C33/C1
      C47=C34/C1
      C48=C35/C1
      C49=C36/C1
C
      C50=C37/C1
      C51=C38/C1
      C52=C39/C1
C
      RETURN
      END  .
C
C
C
      SUBROUTINE INTG31(A1,A2,A3,C1,C2,C3,C4,C5,S1,S2,S3,INT31)
C
C-----INTEGRATION OF (1+SSL)(1+SSM)(1+SSN)(A1S**2+A2S+A3)*(C1S**4+C2S**3+C3*S**2+C4*S+C5
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 INT31
      CC1=A1*S1
      CC2=A1*S2+A2*S2+A3*S1
      CC3=A1*S3+A2*S2+A3*S1
      CC4=A1+A2*S3+A3*S2
      CC5=A2+A3*S3
C
      C50=CC1*C2+CC2*C1
      C51=CC1*C4+CC2*C3+CC3*C2+CC4*C1
      C52=CC2*C5+CC3*C4+CC4*C3+CC5*C2+A3*C1
      C53=CC4*C5+CC5*C4+A3*C3
      C54=A3*C5
      INT31=2.0*(C50/9.0+C51/7.0+C52/5.0+C53/3.0+C54)
      RETURN
      END
C
C
C     SUBROUTINE INTEG32(A1,A2,C1,C2,C3,C4,C5,C6,S(L),S(M),S(N),
C    1                    S1,S2,S3,INT32)
C
C
C
C     SUBROUTINE INTG41(A1,A2,A3,C1,C2,C3,C4,C5,C6,D1,D2,D3,
C    1                    S(L)<S(M),S(N),S1,S2,S3,INT41)
C
C
C
C     SUBROUTINE INTEG42(A1,A2,C1,C2,C3,C4,C5,C6,C7,D1,D2,D3,
C    1                    S(L),S(M),S(N),INT42)
C
C
      SUBROUTINE INTEG (N,C1,C2,ITG)
C
C
      IMPLICIT REAL*8(A-H,M,O-Z)
      REAL*8 ITG,ITG1,ITG2,ITG3
C
C-----THIS ROUTINE IS TO CALCULATE THE INTEGRATION OF X**N*LOG(A)
C
      C10=C1+C2
      C11=C1-C2
      A1=FLOAT(N)
      A2=A1/2.0
      A3=AINT(A2)
      IF(A2.EQ.A3) GO TO 100
C-----WHEN N IS ODD NUMBER
      TG1=0.0
      TG2=0.0
      NN=(N+1)/2
      DO 1 I=1,NN
```

```
    1 CONTINUE
      ITG1=2.0*TG2/(A1+1.0)
      TG3=(C1**(N+1)-C2**(N+1))/((N+1)*C1**(N+1))
      ITG2=TG3*DLOG(DABS(C10/C11))
      ITG=ITG1+ITG2
      GO TO 10
C
C------WHEN N IS EVEN NUMBER
  100 TG1=0.0
      TG2=0.0
      NN=(N+2)/2
      DO 2 I=1,NN
      TG1=(C2/C1)**(2*I-2)/(A1+1.0-2.0**(I-1))
      TG2=TG1+TG2
    2 CONTINUE
      ITG1=2.0*TG2/(A1+1.0)
      TG3=(C1**(N+1)+C2**(N+1))/((A1+1.0)*C1**(N+1))
      TG4=(C1**(N+1)-C2**(N+1))/((A1+1.0)*C1**(N+1))
      ITG2=TG3*DLOG(DABS(C10))
      ITG3=TG4*DLOG(DABS(C11))
      ITG=ITG2+ITG3-ITG1
   10 RETURN
      END


BOTTOM
```