The Economics of Information and Piecewise Linear Limited Liability Profit Sharing Contracts

by

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Abstract

This thesis makes a theoretical contribution to the design of profit sharing contracts which maximise the surplus a principal extracts from an agency relationship, whereby a pay floor limits the liability of an agent in low profit states, and information is either unilaterally or bi-laterally asymmetric.

In the first of three problems examined by the thesis, we explore the impact of imposing a floor to agent pay in a bi-lateral information asymmetry game, in which the agent is prior informed about the marginal productivity of capital. The principal (investor) privately observes the realisation of an ex ante uncertain opportunity cost of capital, and the ex post allocation of the agent is a fixed share of revenue net of some proportion of capital cost The principal separates agents through a contract menu parametising by agent type, a capital stock decision and associated proportional division of capital cost. We find that only the information private to the agent creates an inefficiency in the optimal level of investment, and that the welfare costs of this private information are mitigated by limiting the agent's liability.

In the second information problem with limited agent liability, we consider moral hazard due to unobservable effort. We advance the theory of incentives by analysing a contingent share ratio contract in which the ratio of agent to principal pay is a constant whose value depends on the attainment of a pre-specified profit target. We reveal an efficiency loss in comparison to lump-sum bonus schemes and show the significance to the principal of precise information concerning agent risk aversion. We then derive (sufficient) conditions for Pareto improvements from capital substitution by the agent and the availability of additional signals used to infer effort.

Finally, we examine optimal contract design by an investor for a combined problem of moral hazard and adverse selection, in which an agent whose pay cannot be less than zero, supplies unobservable effort, *and* is prior informed about the marginal productivity of effort and capital

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The following verses taken from the The Holy Qur'an (96:1-8) make clear to whom thanks is inevitably due:

> 'Read! In the name of thy Lord and Cherisher, Who created-
> Created man out of a leech-like clot:
> Read! And Thy Lord is Most Bountiful,-
> He Who taught (the use of) the Pen,-
> Taught man that which he knew not.
> Nay, but man doth transgress all bounds,
> In that he looketh upon himself as self-sufficient.
> Verily, to thy Lord is the return (of all)'.

Table of Contents

CHAPTER 3

INVESTMENT, HIDDEN PRODUCTIVITY AND LIMITED LIABILITY PROFIT SHARING CONTRACTS

CHAPTER 4

DICHOTOMOUS INCENTIVE CONTRACTS, UNOBSERVABLE EFFORT AND LIMITED LIABILITY

CHAPTER 5

DICHOTOMOUS INCENTIVE CONTRACTS, CAPITAL CONTRIBUTION AND ADDITIONAL SIGNALS

CHAPTER 6

DICHOTOMOUS LIMITED LIABILITY PROFIT SHARING CONTRACTS WITH UNOBSERVABLE EFFORT AND HIDDEN PRODUCTIVITY

CHAPTER 7

CONCLUSION

Table of Notation

B	Discontinuous increase in agent pay at $X = X_C$
e	Agent effort
e^*	First-best effort for pure moral hazard problem
$e_A(\theta)$	First-best effort for pure adverse selection problem
E	Agent capital contribution
$F(\theta)$	Cumulative probability distribution of θ
$F_1(\theta)$	Probability density function of θ
$G(\varepsilon)$	Cumulative probability distribution of ε
$G_1(\varepsilon)$	Probability density function of ε
H	Production function
I	Total capital partnership investment
J	Inverse function of H'
K	Capital investment
$K_A(\theta)$	First-best investment for pure adverse selection problem
-L	Agent pay floor
M	Contract menu
-Q(e)	Agent utility of effort e
$r \in [\underline{r}, \bar{r}]$	True opportunity cost of capital
$\hat{r} \in [\underline{r}, \bar{r}]$	Reported opportunity cost of capital
r_c	Critical realised opportunity cost of capital
R(r)	Cumulative probability distribution of r
$R_1(r)$	Probability distribution function of r
S	Share base
$\underline{U}$	Agent reservation utility
$U^a(\theta, \hat{\theta})$	Expected utility of a type θ agent reporting type $\hat{\theta}$
V^a	Agent's opportunity cost of capital (net of initial investment)
V^p	Principal's opportunity cost of capital (net of initial investment)

X	Profit[1]
X_C	Target profit
$\alpha \in [0,1]$	Proportional deductibility of capital costs from the share base
χ	Constant relative risk aversion coefficient
χ_a	Absolute risk aversion coefficient
χ_r	Relative risk aversion coefficient
$\varepsilon \in [\varepsilon_0, \varepsilon_1]$	Exogenous uncertainty variable
$\bar{\varepsilon}(e(\theta))$	Threshold realised uncertainty for target attainment at effort $e(\theta)$
ϕ	$\theta + \pi \frac{F(\theta)}{F_1(\theta)}$
Γ	First order moral hazard condition for maximal agent effort
φ	Actual agent pay ($= \pi S$)
$\lambda \in [0,1]$	Contingent proportional reduction in share ratio
$\lambda_C \in (0,1)$	Critical (contingent) proportional reduction in share ratio
$\mu \in [\mu_0, \mu_1]$	Monitoring signal
$\pi \in (0,1)$	Profit/revenue sharing ratio
$\theta \in [\underline{\theta}, \bar{\theta}]$	True productivity type
$\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$	Reported productivity type
ρ	Actual principal pay
$\Omega^a(\varphi)$	Agent utility of pay φ
$\Omega^p(\rho)$	Principal utility of pay ρ
$\Psi(\mu)$	Cumulative probability distribution of μ

[1] Depending on the context this may sometimes mean revenue net of all costs except capital costs

CHAPTER 1

INTRODUCTION

1.1 Information and agency

Since the pioneering work of Mirrlees (1971,1974) and Jensen and Meckling (1976), the study of the economics of information has evolved to understand how economic agents attempt to deal with their ignorance concerning information which affects them. The decisions that these agents take are intended to either acquire new information or to avoid some cost attributed to their ignorance. When this information is distributed asymmetrically among agents, then these decisions are to determine the design of contracts which attempt to mitigate the cost of ignorance, and/or induce the revelation of relevant private information.

One strand of the literature on contract theory (see review by Hart and Holmstrom (1985)) which has focused on the internal organisation of the firm, uses agency theory as the representative contract paradigm. An agency is essentially an economic arrangement in which two or more individuals share an outcome (say profit or revenue) which depends on the ex post realisation of an ex ante uncertain economic environmental variable (referred to as the state of nature) and a productive input (say effort) supplied by at least one of the agents

An agency may be viewed as a game. As a way of solving the game, the agency is embedded in a market that determines the expected utility of all the players except one, who extracts all the rent from the game. This latter player (referred to as the principal) offers a contract to the other players on a take-it or leave-it basis. Thus all bargaining power resides with the principal, in contrast to games in which bargaining power is distributed amongst players who determine the ex post allocation of the expected surplus according to their relative bargaining strengths (Binmore et al (1986)).

An agency may commonly arise because the principal requires an agent(s) to undertake a task which is either too complicated or too costly for the principal to do himself. Some examples of principal-agent relationships include (amongst many others) that of insurer and insured, lawyer and client, and government and regulated industry.

There are many ways to structure the distribution of information in an agency. This structure greatly affects the nature of the game. A *moral hazard* problem arises when an informational asymmetry arises after the contract has been signed An *adverse selection* problem exists when the agent has relevant private information before the contract is signed Lastly, a *signalling* game arises when the informed party is able to reveal private information via individual behaviour prior to formalising the contract.

1.2 Limited liability share contracts

In this thesis, we consider problems of moral hazard and adverse selection We examine a principal-agent relationship in which we impose a limited liability fee schedule that awards the agent a piecewise-linear share, that is one for which the allocation to the agent over each domain of possible outcomes is some fixed (domain dependent) proportion of the outcome, but for which his monetary reward cannot be less than some threshold amount (which can be zero). The motivation for examining limited liability contracts derives from the frequency with which these conditions are present in everyday life. This fact is illustrated by the relatively low criminal penalties applied in the case of bankruptcy In the case of employment contracts, many legal restrictions implicitly or explicitly limit the worker's liability. Such restrictions include laws which impose a minimum wage, and laws which exonerate workers from liability for damages caused during the execution of a contract[1]. Additionally, limited liability is studied in the context of (piecewise) linear (profit/revenue) sharing contracts The main reasons for this (apart from their simplicity and the abundance of linear schemes witnessed in practice) are two-fold.

Firstly, it has been claimed that linear (cash) sharing contracts give rise to advantages over their fixed wage counterparts at both a microeconomic and macroeconomic level. At a microeconomic level, profit sharing contracts have been cited as productivity enhancing. Jones and Svejnar (1985) point to the positive effects of profit sharing on improving the awareness and sense of responsibility that workers feel, which gives rise to greater labour tenure and a resultant increase in firm specific human capital that increases productivity. Cable and Fitzroy (1980) identify productivity affects that emanate from a greater identification between workers and management. The most notable criticism of productivity enhancement from profit sharing is due to a "free-rider" problem (Jensen and Meckling (1979)), in which shirking may arise in large organisations where group incentive schemes (viz-a-viz individual incentive schemes) give each worker only a small fraction of the incremental profit derived from their additional effort.

At a macroeconomic level, Weitzman (1983, 1984, 1985, 1986, 1987) has cited the potential of these contracts to cure stagflation (the coexistence of high inflation and unemployment) if adopted as the economy-wide method for worker remuneration. According to Weitzman, stagflation is caused by the relative inflexibility of money wages in the face of product-demand shocks, with a monopolistically competitive market structure underlying a wage-push inflationary spiral. According to Weitzman, by driving a wedge between the marginal and average cost of labour, economy-wide profit sharing creates a persistent excess demand for labour which ensures that the economy stays at full employment. Since unemployment is then cured, monetary policy can successfully be

[1] Limited liability piecewise linear profit sharing contract forms are also witnessed in Islamic venture capital financing, whereby a (zero capital participating) entrepreneur shares profits but not losses (Vogel and Hayes (1998))

used to target inflation alone. Weitzman's theory is certainly not without its critics (see amongst others, Eaton (1985), Levine (1987), Nuti (1987), Blanchflower and Oswald (1988), Wadhwani (1987), Estrin and Wadhwani (1990)). For example, Nuti (1987) and Estrin and Wadhwani (1990) cite the failure to link increased risk bearing without some measure of worker participation in the decisions of the firm as a major flaw in the argument, since incumbent workers are in reality likely to restrict employment expansion given its affect on their share ratio. Some critics (for example, Nuti (1987)) instead consider the marginal cost of a worker to be the sum of base wage and profit share and not just the base wage, thereby inferring no such comparative employment advantage of the type proffered by Weitzman. Others (for example, Wadhwani (1987)) suggest a failing in the theory since Weitzman does not supplement his theory with a description of how wage parameters are determined, instead being content to specify that they are fixed exogenously The employment creating potential offered by profit sharing implicitly assumes that profit sharing firms could lower total compensation to workers in the short run. However, since all firms (both share and wage) must offer the same total remuneration to workers in the long run, a weakness arises since no link between the short and long run is provided.

Notwithstanding fervent criticism, Weitzman's claims have been so influential that the British government decided to subsidise profit related pay schemes in the 1987 Finance Act. That fixed wages are so widely adopted as the standard remuneration paradigm may be more as a result of inertia than economic rationale. Throughout this thesis, the piecewise-linear share scheme is adopted as the basic contract form.

The second motivation for imposing a (piecewise) linear form of fee schedule, is that linear share contracts appear to be robust. As will become apparent in the following chapter, when effort cannot be verified, and cannot therefore be contracted upon, incentive contracts must be designed to give the agent a self-interested reason to supply a greater effort. These contracts ensure that the agent's remuneration is affected in a positive way by the effort that he supplies. A fixed wage contract cannot possibly realise this effect. Further, Holmstrom and Milgrom (1987) have shown that linear schemes as incentive contracts ensure that the agent's behaviour and the payoffs to both principal and agent change only very slightly when small changes are made in the specifications of an agency game. In contrast, "models that derive optimal rules in which small differences in outcomes lead to large differences in compensation are invariably based on the assumption that the agent finds it impossible, or very expensive, to cause small changes in individual outcomes. The optimal rule in such cases is usually inordinately sensitive to the distributional assumptions of the model" (Holmstrom and Milgrom (1987) pp 325-326) Thus, given this consideration, there is a case to bring for examining problems in which a linear form of share contract is imposed ex ante.

1.3 Outline of the thesis

In this section we will outline the scope and contribution of the thesis As a whole, the thesis makes a theoretical contribution to the understanding of the way in which limiting the liability of an agent in the context of informational asymmetry affects the design of contracts by a principal with whom the agent transacts a profit sharing agreement.

Chapter 2 qualitatively presents some of the more relevant theory from received literature concerning the economics of information. We explain the ideas of moral hazard and adverse selection, providing illustration with the use of frequently cited examples Game theoretic concepts used in the thesis are explained in detail. We then provide a formal literature review of limited liability in the context of information asymmetries, thereby identifying the contributions of this thesis in relation to existing literature.

Chapter 3 examines a problem in which an investor entirely funds the project of an agent who is privately informed as to the marginal productivity of capital. In contrast to the approach taken in a majority of the profit sharing literature, we emphasise the importance of the way in which the surplus generated for apportionment between principal (investor) and agent (entrepreneur) is defined with respect to the deductibility of capital costs. A bilateral information asymmetry model is developed, in which both the agent and the principal are separately endowed with private information which determines the utility of the other party to the contract. The principal privately observes the realisation of an ex ante uncertain unit capital cost and since some proportion of this cost is shared with the agent, the agent requires a floor to his ex post allocation in order to participate. It is found that only the information private to the agent creates an inefficiency in the optimal level of investment for the principal. The welfare costs of private agent information upon investment are mitigated when the liability of the agent is limited.

Chapters 4 and 5 examine a moral hazard problem in which an agent with limited liability supplies unobservable effort. In the absence of the restriction of non-negative pay (see 2 2.1.1), a risk averse agent can be induced to supply the socially optimal (first-best) effort by the use of the threat of unlimited punishment for low outcomes (profit/revenue) (Holmstrom (1979), Gjesdal (1976), Mirrlees (1974)). It is also well known (Harris and Raviv (1979), Holmstrom (1979), Shavell (1979)) that the first-best effort is supplied by a risk neutral agent who is made sole residual claimant through the use of a franchise contract However, in the presence of a liability constraint, both of these solutions are unenforceable. Dichotomous incentive contracts, in which the order of the realised outcome in relation to a pre-specified performance target precipitates one of two mutually exclusive fee schedules, may provide sufficient incentive pressure to elicit the first-best effort.

In Chapter 4 we examine contingent profit sharing ratio contracts, in which the ratio of agent to principal pay is a constant whose value depends on the achievement, or otherwise, of a performance target. In contrast to lump-sum bonus contracts, in which inducement to greater effort can be achieved by offering the agent a fixed performance bonus, analogous inducement in contingent share ratio contracts is achieved by the threat of a lower share ratio. However, the effect of this threat is in general ambiguous, since conflicting incentive pressure is generated though a lower share of all outcomes below the performance target, but a greater jump in the lump-sum element of pay otherwise

Under conditions in which risk neutrality precludes the use of contingent share ratio schemes in eliciting greater effort from the agent, we examine the impact of agent risk aversion. We find that the precise degree of risk aversion of the agent is crucial, and that there exists a range of risk aversion for which the threat of a lower share ratio must be sufficiently severe as to not render such schemes counterproductive to the objectives of the principal.

In Chapter 5 we then extend the analysis of Chapter 4 to consider ways in which the incentive pressure created by dichotomous contracts can be supplemented by marginal substitution of capital by the agent, and derive sufficient conditions for a Pareto improvement. We also explore the interaction of a monitoring technology on the use of dichotomous incentive schemes, whereby the benefit to dichotomous schemes of an informative but noisy signal is illustrated

Chapter 6 synthesises the information asymmetries examined in isolation by the preceding chapters to consider a problem of adverse selection with moral hazard in which an agent who supplies unobservable effort is prior informed as to the marginal productivity of a venture with respect to (hereafter w.r.t.) effort and capital. We derive conditions for which a contingent share ratio scheme parametised on a message variable signalled by the self-selection of a contract by the agent, can implement the optimal pure adverse selection outcome

Chapter 7 draws together the conclusions of the thesis and outlines some areas for future research

In the following chapter we provide a brief discussion of some of the key concepts in the study of the economics of information. This is intended to explain some of the background theory to the approaches taken in subsequent chapters, and to illustrate how problems of information asymmetry vary according to the information structure (specifically "who has what information and when ?"). We also formally provide a review of the literature on limited liability in the context of information asymmetries.

CHAPTER 2

THEORY AND REVIEW

2.1 Introduction

The aim of this chapter is two fold. The first objective is to collect in one place an explication of essential game theoretic concepts which are applied repeatedly in the thesis. In providing a review of received literature essential to solution methodologies applied to situations of moral hazard (see Essential concept: The trade-off between insurance and incentives 2.2.1.1) and adverse selection (see Essential concept: The revelation principle 2.2.2.1), we will intuitively explain the conflicts which exist between informed and uninformed economic agents for each of these informational problems. We also discuss the equilibrium concept applied to both the moral hazard and adverse selection games which we will examine (see Essential concept: subgame perfect equilibrium 2 2 1 2).

Further, in order to place the contributions made by this thesis beyond a statement of the relevance with respect only to the existing literature which discusses information asymmetries in the context of limited liability (see 2.3), we will also discuss alternative model formulations, thereby providing a game theoretic modelling backdrop in which to embed the particular features of problems tackled by the thesis. Since the information problems we consider specifically examine a monopolistic non-repeated relationship between a single agent and a single principal, the discussion of alternative informational settings for moral hazard extends to multi-period (2 2.1.3) and (separately) multi-agent (2 2.1.4) frameworks, and for adverse selection, the discussion includes competition between principals (2.2.2.2) and (separately) multi-period (2 2 2.3) relationships.

Figure 2 1 clarifies where the modelling features of the problems tackled by the thesis fit into alternative and sometimes wider informational settings.

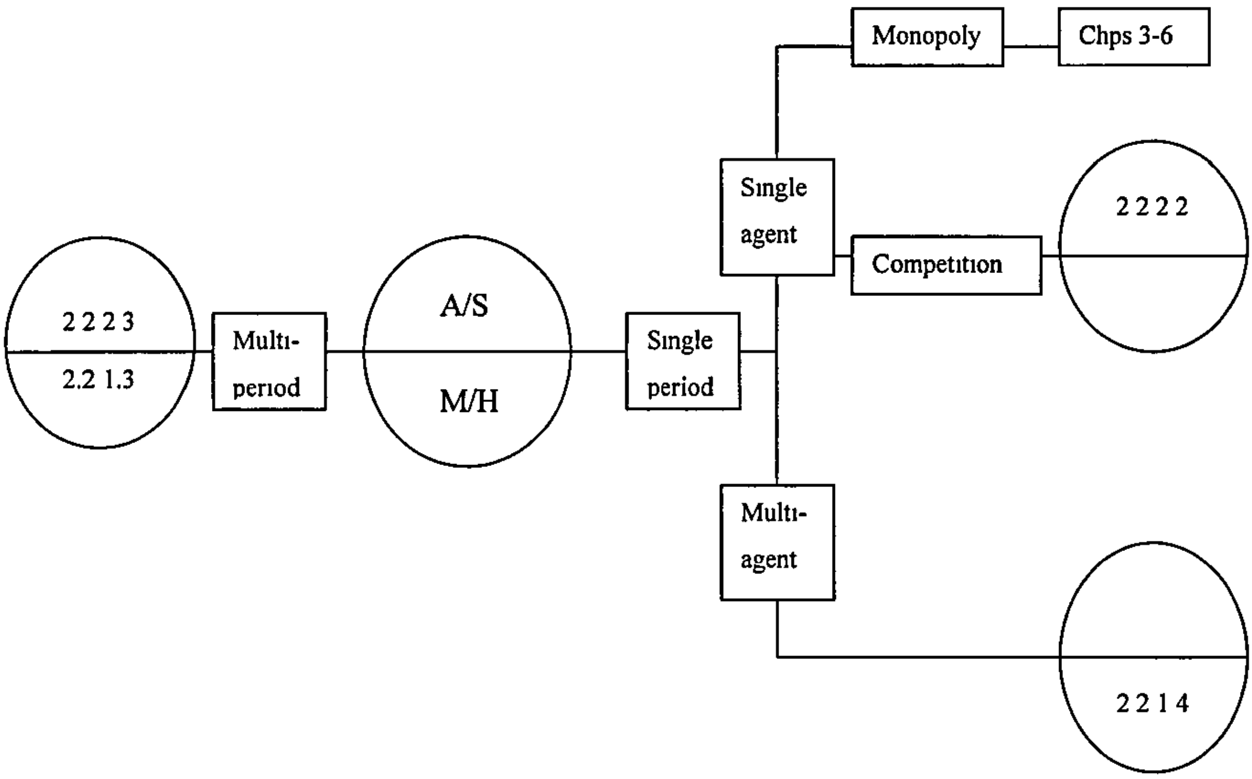

Figure 2.1

Shown in the uppermost part of the ovoids in Figure 2.1 are the subsection numbers discussing alternative models for problems of adverse selection (A/S). The corresponding lowermost part of the ovoids relate to subsections which discuss moral hazard (M/H).

The second objective of this chapter is to survey the literature which posits within the overlap of information asymmetry and limited liability studies (see 2.3). We provide an original classification for this literature which essentially divides the non-continuation[1] studies between incentives and non-incentives, wherein effort is a choice variable[2] for the

[1] By non-continuation we mean that once a mechanism triggering bankruptcy has occurred, such as the default on a loan note, the game ends, with no possibility of a bail out or subsequent firm activity.

[2] By this we mean that effort is not contracted because it is not observable It is important to note that as long as effort is chosen by the agent (to maximise his own utility), the problem is classified as one of

agent in the former case but not the latter. In the context of this classification we will further discuss the contributions made by the thesis.

2.2 The Economics of Information

The information asymmetries that we examine in this thesis are those that give rise to a problem of either moral hazard, adverse selection, or both moral hazard and adverse selection. A moral hazard problem arises when an asymmetry arises after the contract has been signed, whereas an adverse selection problem arises before the contract has been signed. It is important to note that we are only interested in information asymmetries that give rise to conflicts of interest between principal and agent. If this were not the case, then all relevant information would be automatically revealed, rendering any information asymmetry irrelevant.

2.2.1 Moral Hazard

A familiar situation in which a conflict of interest exists in a principal-agent framework is that of investor and entrepreneur. This conflict arises because an agent (entrepreneur) supplies costly effort in return for an allocation of the outcome (profit/revenue), whereas the principal (investor) enjoys the outcome net of this allocation without supplying effort. If the effort of the agent were verifiable[3] by the principal, then this effort could be contractually specified, and would be enforceable given the existence of suitable institutions (for example, a court of law). However, where effort is not verifiable (provable), the principal must achieve an internalising of incentives to supply effort by suitable contract design. Incentive contracts therefore exist in order to limit the cost to the

incentives even if it contains elements of adverse selection, i e precontractual information which is private to the agent

[3] A distinction is made between verifiable and observable The principal may be able to observe the effort of the agent However, without being able to verify (prove) his observation, the effort cannot be a contract variable since it is unenforceable.

principal of his ignorance concerning the effort supplied by an agent after contract acceptance.

2.2.1.1 Essential Concept: The trade-off between insurance and incentives

An agent is said to be risk averse with repect to (hereafter w.r.t.) money when the loss in utility that he attributes to a given decrease in his wealth, exceeds the gain in utility that he attributes to an equal increase in wealth. Such an agent is not indifferent to a fair bet. That is, if for example he stood an equal chance of gaining or losing a fixed amount of money, an outcome contingent on the realisation of an ex ante uncertain random event (say the flip of a coin), his expected utility (read welfare) would be less than zero. In order to accept the offer of a fair bet, he would in addition require a side payment equal to the premium that he attaches to the risk involved. The greater his degree of risk aversion, the greater would be the premium required to just leave him indifferent between accepting or rejecting the bet. In contrast, a risk neutral agent requires no such premium. His expected utility from the fair bet is zero, equal to the expected (mean) reward.

Consider a risk averse agent who is to supply an unverifiable effort prior to the realisation of an ex ante uncertain state of nature, where productive output is increasing in both the effort supplied and the realised state. The effort cannot be inferred by the principal from (perfectly) observed output since the principal is unable to observe the realised state of nature[4]. Faced with this situation, how is the principal best (for himself) able to design a fee schedule (contract) which the agent is willing to accept? The principal would like to give as much incentive to the agent as possible to supply greater effort. If higher outcomes are indicative of higher effort by the agent, then the way to achieve this would

[4] The agent should therefore not be given a fixed wage contract since he has no incentive to provide effort, given that his effort is not observable, and can always declare (possibly falsely) that the realised state was unfavourable upon observation of the outcome by the principal

be to allocate the agent a greater reward for higher outcomes[5]. However, when the agent is risk averse, there is a problem. By giving the agent greater incentives through greater risk sharing, the principal shares less of the risk. This is unfortunate, since the principal (being risk neutral) is willing to accept risk without requiring a premium to do so. The same, however, is not true of the agent. Therefore, if the agent is to be given greater incentives, he must also be given a greater risk premium for bearing risk. This risk compensation is costly to the principal since the outcome is allocated entirely between the principal-agent pair, and what the principal rewards the agent by way of risk premium, he must forego himself.

This trade-off between incentives and insurance for the agent is a key issue in the incentives literature. The need to give the risk averse agent an incentive to provide greater effort by bearing greater risk results in suboptimal risk sharing. The welfare loss of the principal due to the need to pay the agent a risk premium is a cost of asymmetric information[6]when the agent is risk averse.

It is worth noting that in contrast to the risk averse case, if the agent were risk neutral, he could be given maximum incentives to supply effort by becoming the sole residual claimant[7] (i.e. sole claimant to the ex ante uncertain outcome). The agent would require no risk premium for bearing this risk, and the principal could extract the entire rent that the agent expects by imposing a fee that leaves the agent with exactly his reservation utility (the utility that the agent could achieve if he had not accepted the incentive

[5] An unsatisfying feature of optimal fee schedules is that the allocation to the agent may not necessarily be increasing over the entire range of outcomes In fact, it is even possible that the fee schedule may be decreasing over some (non-empty) interval of outcomes A condition which is sufficient to guarantee monotonicity of the fee schedule is the *monotone likelihood ratio property* (P. Milgrom (1981)) This condition implies that, for any given outcome, increased effort leads to relatively greater probability weight on all higher profit levels (thereby implying first order stochastic dominance)

[6] The Lagrange multiplier, or shadow price, of the constraint which depicts the agent as choosing an effort level which is individually optimal (the moral hazard constraint) is greater than zero.

[7] A franchise contract

contract offered by the principal). Since risk aversion of the agent forces the principal to bear some of this risk by sharing the outcome, the agent's compensation becomes less sensitive to his performance compared to the franchise contract. The reduced effort of the agent when risk averse therefore results in losses for the principal relative to the franchise case.

2.2.1.2 Essential Concept: Subgame perfect equilibrium

The formalisation of the single-period moral hazard problem[8] is to derive the conditions that the optimal fee schedule must satisfy as a result of a constrained optimisation problem. Specifically, the problem is to maximise the expected utility of the principal subject to participation, and effort choice by the agent which is individually maximal[9].

[8] For details of a formal model along these lines, see for example Stiglitz (1974, 1975), Harris and Raviv (1979), Holmstrom (1979), or Shavell (1979). Also, see Grossman and Hart (1983) and Laffont and Tirole (1986).

[9] The optimisation is a double maximisation problem The principal must maximise his expected utility subject to the moral hazard constraint An approach which is commonly taken is to replace the moral hazard constraint with the first-order *necessary* condition for a maximum Since this condition is only a locally true necessary condition for a stationary point (local minima, saddle point, or local but not global maxima), and is not equivalent to the actual moral hazard constraint, we may find that we include too many efforts that locate other stationary points In order to isolate the effort which is globally maximal, it is sufficient to assume the following conditions in order to eliminate this problem (Rogerson (1985a))

(1) The distribution function of outcomes is convex in effort This indicates a stochastically diminishing marginal productivity of effort, and guarantees uniqueness of the agent's effort choice by ensuring that the expected utility of the agent is concave in effort

(2) The distribution function exhibits the monotone likelihood ratio property

Another approach (Grossman and Hart (1983)) to solve the double maximisation problem uses a two-stage method in which one analyses the characteristics of the optimal fee schedule independently of whether or not we identify the optimal effort This approach breaks the principal's problem up into a computation of the costs and benefits of the different actions that can be taken by the agent Under the assumption that the agent's preferences over income lotteries are independent of the action that he takes (utility is additively separable in money and effort), the cost minimisation problem becomes a fairly straightforward convex

There is an underlying sequence to the decisions of the players (the principal-agent pair) in this non-cooperative extensive form game. Given this sequentiality, the solution concept that is being applied is that of *subgame perfect equilibrium*[10] 'This solution concept requires that at each point in time, each player chooses an optimal strategy, given the situation that has been reached, and assuming that the other player will do likewise' (Macho-Stadler and Perez-Castrillo (1997) p8).

We will illustrate the concept of subgame perfect equilibrium for the (single period) moral hazard (hidden effort) game[11]. The first to act is the principal, who, knowing the future behaviour of the agent for each possible contract that he can offer (given that the principal is assumed to know the utility function of the agent), offers that contract which maximises his own expected utility. The future behaviour of the agent is depicted by two decisions Firstly the agent decides whether to accept or reject the contract proposed by the principal, whilst anticipating his future choice of effort were he to accept the contract. The agent accepts if he is able to extract from the relationship no less than the utility he can achieve in the best alternative relationship (the reservation utility). The second decision is a choice of effort. The agent is assumed to always choose an effort which maximises his own (expected) utility.

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programming problem A deficiency in this approach, however, is that it does not generalise to allow the principal to be risk averse

[10] For equilibrium concepts, see Eichberger, J (1993)

[11] The special features of the canonical moral hazard setting are (Sappington (1991)):

(1) Symmetry of precontractual beliefs If for example the principal and agent did not share the same beliefs about the distribution of ex post outcomes given effort, then such a neat separation between incentive and insurance issues might not be possible Implicit in the strong assumption of symmetric beliefs is that both parties are able to anticipate fully all possibilities that might arise during their relationship

(2) The agent can be costlessly bound to the terms of any contract he agrees to. Even though the agent may earn a return below his reservation utility when the state is realised, he is unable to abrogate or renegotiate the contract he has signed. His commitment, in this sense, is therefore perfect

(3) The outcome is perfectly, and publicly observable

The subgame perfect equilibrium solution concept is applied throughout the game theoretic modelling of the asymmetric information problems considered in this thesis. In the following section we briefly consider multi-period (2.2.1.3) and monitoring/multi-agent (2.2.1.4) extensions to the moral hazard setting so far considered

2.2.1.3 Alternatives: Multi-period contracts

If the principal-agent relationship is repeated, then the insurance-incentive trade-off, which prohibits a first-best outcome for the principal in a single period setting, may be avoided in a dynamic setting.

Suppose, for example, that both the principal and agent value future rewards[12] as much as they value current rewards (no discounting case), and that their relationship is indefinitely repeated[13]. If the agency is repeated a sufficiently large number of times, then by compensating the agent on the basis of an average of outcomes over time, the agent can be induced to supply the effort which is most preferred by the principal. The reason for this is that randomness in the agent reward becomes negligible, and the agent consequently faces very little risk (income uncertainty). Therefore, incentives can be provided for the risk averse agent without the welfare reducing need to pay a risk premium. The agent receives his reservation utility contingent on an average outcome sufficiently close to a pre-specified target[14]. The importance of not discounting is that the agent is dissuaded from supplying less than the requisite amount of effort by the threat of

[12] Which are ex post allocations in each period of the ex ante uncertain outcome (profits/revenue) for that period

[13] The first studies of dynamic interactions are those of Rubinstein (1979), Radner (1981,1985), and Rubinstein and Yaari (1983).

[14] Intuitively we would expected that frequent repetition allows us to converge to the efficient (first-best) solution In this framework, incentives are not determined by the fee schedule, but rather on average effort, and this information becomes very precise when the number of periods is large

punishment in future periods. Were the payoff from future periods to be discounted, the significance of this threat would become diluted in any current period[15].

When future payoffs are discounted and/or the duration of the agency relationship is more limited, the conflict between risk sharing and incentives re-emerges However, gains (Pareto-improvements[16]) generally arise when agent compensation in each period is based on past outcomes[17]as well as the current outcome This is the so-called "memory result", which suggests a rationale for long-term contracts. A criticism of this result is that it critically relies on assuming that the agent's reward is needed for immediate consumption[18]. When the agent has access to credit markets on the same terms as the principal, the advantage afforded by long-term contracts in improving upon a series of short-term contracts by providing consumption-smoothing opportunities, can be removed[19]. In this setting, a long-term contract will be no better than a series of repeated short-term contracts[20]. Access to credit markets is also important when considering whether or not the optimal long-term contract can be implemented[21]by a sequence of optimal short-term contracts. The conditions in which such implementation obtain are that the agent must be able to access credit markets (in order to smooth consumption in

[15] Not discounting future payoffs allows incentives to be better distributed over time

[16] The Pareto-efficiency concept is used to judge social welfare outcomes When there is moral hazard, Pareto-efficiency is used to imply the second-best outcome The first-best obtains when moral hazard is absent.

[17] See Lambert (1983), Rogerson (1985b), and Stiglitz and Weiss (1983).

[18] This point is raised by Fudenberg, Holmstrom and Milgrom (1990) and Malcomson and Spinnewyn (1988)

[19] This is because in long-term contracts, the principal serves as a "bank", lending to the agent in bad periods and drawing repayment from the agent in good periods This role of the principal is redundant when the agent has access to credit markets

[20] Fudenberg, Holmstrom and Milgrom (1990)

[21] Implementation in this sense means that the subgame perfect equilibrium of the short-term contracts leads to efforts and consumptions that coincide with those obtained under the long-term contract

the way that the internalisation of this process is achieved by a long-term contract), and that the long-term contract is re-negotiation proof[22].

2.2.1.4 Alternatives: Monitoring and many agents

Incentive contracts, which base a risk averse agent's reward on ex post realised outcome, compensate the agent according to observed productive output. An alternative approach is to base the agent's reward on an imperfect, and publicly observable signal of productive input (effort). For example, the signal may be a true observation of the effort with additive noise[23] (say a standard normal distribution). However, given that the signal is noisy, a question arises as to whether the principal would prefer to ignore the signal and compensate the agent according to an incentive contract depending only on observed output, or reward the agent according to both the observed output and the imperfect signal as to productive input The answer[24] is that whenever the signal together with observed output provides more information as to agent effort than output alone, then it is optimal for the principal to reward the agent according to the signal and the observed output. The extra incentives created by basing the agent's compensation on both pieces of information outweigh the increased risk exposure of the agent caused by use of the imperfect monitor. When a principal contracts with more than one agent, it may be possible to achieve monitoring of agents by exploiting the information available from their relative performances[25]. If there is a common environmental parameter, which is outside the control of agents whose productivity is equally affected, then basing rewards on the

[22] A contract is renegotiation-proof if at the beginning of any intermediate period, no new contract or renegotiation is possible that would be preferred by the principal and agent. See Dewatripont (1989) and Chiappori, Macho-Stadler, Rey and Salanie (1994)

[23] i e. observed effort is the true effort plus a random variable whose value is known only to the agent Observed effort cannot therefore be used by the principal to perfectly infer actual effort since any signal of the latter is 'garbled' by the additive random variable (noise)

[24] See Holmstrom (1979), Harris and Raviv (1979), and Shavell (1979)

[25] For details of this work see Green and Stokey (1983), Lazear and Rosen (1981), and Nalebuff and Stiglitz (1983a,b)

relative performance of identical tasks provides motivation without imposing excessive risks[26]. Rank order tournaments, which compensate agents according to the ordinal ranking of their output, provide such incentives. For example, the agent with the greatest output might be awarded a fixed compensation, which is greater than the equal reward which all other agents receive. Note that the ideal incentive scheme will generally be a combination of individualised and relative performance schemes[27]. These schemes supplement the ordinal nature of a tournament with a cardinal dimension.

2.2.2 Adverse Selection

The definitive feature of incentive contracts is that they exist to limit the cost of the principal's ignorance from an information asymmetry that arises in the process of contract execution. At the time of contract acceptance, both principal and agent share common beliefs about all aspects of the agency. In contrast, an adverse selection problem exists when, *prior* to contract acceptance, the agent is informed as to some relevant aspect of the agency about which the principal is ignorant. For example, an investor may be less informed than an entrepreneur about the marginal productivity of capital to be invested; a regulator may be less informed than a regulated firm about the market in which it operates; or a client may be less informed than his lawyer as to the chances of winning a case should he decide to employ the lawyer's services.

In the course of executing the contract, the information about which the principal was previously ignorant may be revealed. However, for a single repetition of the agency, this will be of no use to the principal. The principal must therefore attempt to limit the cost of his ignorance by making a strategic decision at the point in time at which a contract (or contracts) is offered. Critical to the analysis of adverse selection problems that we

[26] Incentive premiums due to risk aversion are reduced because the tournament provides insurance against random events, which are beyond the control of the agent's, and which affect them equally

[27] Nalebuff and Stiglitz (1983b)

consider in this thesis is the *revelation principle* (established by Gibbard (1973), Green and Laffont (1977), and Myerson (1979)).

2.2.2.1 Essential concept: The revelation principle

This principle asserts that any equilibrium outcome which is the result of a potential contract under any non-truth-telling mechanism can be replicated by the equilibrium outcome of some truth-telling mechanism.

In order to understand the intuition behind this principle, consider a lawyer/defendant relationship. Suppose that the defendant completely trusts the integrity of the lawyer, and that the lawyer needs certain information from the defendant in order to be able to best defend him in court. The defendant has nothing to gain from misrepresenting the truth to his lawyer, since then the lawyer may play the wrong strategy on his behalf, thus threatening his case. It is therefore possible to replicate the equilibrium of a non-truth telling mechanism where the defendant represents himself (assuming he is able to defend himself as well with representation as he is alone), with a truth-telling mechanism in which the defendant is represented by the lawyer. For the same reason, a firm will reveal its accounts truthfully to an accountant in order that the accountant can effect the best strategy to limit taxation liability.

More formally '...for any response-plan equilibrium of any choice mechanism, there is an equivalent incentive-compatible mechanism giving all types of all players the same expected payoffs' (Myerson (1979) p66). Thus, without any loss in generality, a principal may be restricted to policies/contracts/mechanisms which require the agent to truthfully reveal private information, and which give the agent no incentive to lie (incentive-compatible) In a regulator/firm setting[28], where production cost information is private to the firm and the regulator determines a price and transfer (tax or subsidy) policy, the

[28] Baron and Myerson (1982)

regulator could declare a policy which does not induce a truthful response from the firm as to its actual cost structure. The firm will report a cost structure that maximises its own expected utility against the declared policy. An alternative policy, which induces truthful cost reporting by the firm is the following: the regulator asks the firm to divulge its cost structure; the regulator then calculates the cost report that would maximise the firm's expected profits against the original policy given the true cost structure; the regulator then enforces the regulations which would have been enforced in the original policy in response to this (calculated) cost report[29].

Since the revelation principle permits a principal to restrict attention to those mechanisms (contracts) which induce truthful (type) revelation by an agent, determining which contract is optimal for the principal reduces to identifying from amongst the set of truth-telling contracts, which contract gives the principal the greatest expected utility. The reason is that any equilibrium of a non-truth-telling mechanism can be replicated by the equilibrium of a truth-telling mechanism. Therefore, no non-truth-telling mechanism can yield a better outcome for the principal than the best amongst the set of truth-telling mechanisms.

The analytical simplification that the revelation principle affords essentially derives from the consequent mitigation of the need for the principal to interpret the lies of an agent(s). A feature of adverse selection problems is that it is sometimes optimal for a principal to offer a set of contracts (contract menu) in which the selection of a contract by an agent uniquely identifies the agent's type. This is an example of a *direct revelation* mechanism, where agents truthfully reveal their type by disclosing their preference between contracts. For example, an insurance company can exploit the fact that low risk agents are more willing to buy partial coverage than their high risk counterparts. For higher risk types, the

[29] Another example is the use of taxation bands. Tax payers have an incentive to understate income If instead tax codes set the same tax across bands, then there is no incentive to misrepresent income, and the same amount of tax is collected in the truth revealing equilibrium

benefit of a lower insurance premium is outweighed by the anticipated cost of only partial coverage.

However, it is not necessarily the case that a principal's utility is maximised by separating out each type of agent. Instead, subsets of types may, in general, be pooled[30] (Hirschleifer and Riley (1992) pp319-325). Whether it is optimal for the principal to offer as many or fewer contracts than there are types will in general depend on the prior assessment of the principal as to the probability that the agent is of a particular type, and the indifference curves of the agent types, as expressed by their utility functions (Rasmusen (1989) chp 7).

In the next two subsections we briefly mention two adverse selection settings which highlight the issues which arise from an alternative modelling of the adverse selection problems which we consider in this thesis. The first concerns how competition between principals (2.2.2.2), which is the polar case w.r.t. the monopolistic framework assumed in Chapters 3 and 6, affects the outcome of a single period adverse selection problem. The second concerns the extension of a single period relationship with adverse selection to one which is multi-period (2.2.2.3)[31].

[30] When each type of agent chooses the same strategy, the equilibrium which results is said to be pooling, otherwise it is separating An equilibrium is fully revealing if the agent's choice of contract always conveys his private information to the principal Between pooling and fully revealing equilibria are imperfectly separating equilibria, also called semi-separating, partially separating, partially revealing, and partially pooling Note, however, that the distinction between pooling and separating has nothing to do with the equilibrium concept A model might have multiple Nash equilibria, some pooling and some separating Moreover, a single equilibrium, even a pooling one, can include several contracts, but if it is pooling the agent always chooses the same strategy, regardless of type If the agent's equilibrium strategy is mixed, the equilibrium is pooling if the agent always picks the same mixed strategy (Rasmusen (1989) chp 7).

[31] See schematic in Figure 2.1 for an overview of these alternatives in relation to the modelling context of problems tackled by this thesis.

2.2.2.2 Alternatives: Competition between principals

The adverse selection problems that we consider in this thesis depict a single principal contracting with a single agent whose type is privately known (see Chapters 3 and 6). The principal is a monopolist who attempts to extract the maximum rent from the relationship with the agent. In this subsection we briefly discuss how the conclusions reached for this type of adverse selection problem may be different if we instead consider a situation in which several principals compete in order to attract agents, whereupon each principal is constrained to earn no more than zero expected profits in equilibrium.

By way of example, consider the purchase of insurance by agents who wish to alter their pattern of income across states of nature (accident and no-accident), and who are either high or low risk types. In the absence of competition, insurance policies which maximise the utility of a principal offer full insurance to the high risk agent and extract the entire surplus of the low risk agent who receives partial coverage[32](Hirschleifer and Riley (1992) chp 11). With only two agent types, it is always optimal for the principal to separate the two risk types.

However, in the presence of the additional constraint that principals earn zero expected profits, the separating equilibrium may become dominated by a pooling equilibrium if the probability of the agent being low risk is sufficiently high. The reason is that offering a pooling contract, whose price is an average of the full information contract prices, is attractive to high risk types because it is cheap, due to the greater weight in averaging given to the lower insurance price. The pooling contract is also attractive to low risk types since it affords them greater coverage, albeit at a slightly higher price.

[32] The greater the probability that the agent is high risk, the lower is the coverage of the low risk agent When the probability of high risk is sufficiently large, there is no insurance coverage for the low risk agent.

However, since principals will earn positive expected profits from pooling agents who are mostly low risk, a pooling contract will not be robust to competition. It is possible that a competing principal can offer a contract which maintains the price per unit of coverage but offers lower coverage. The competing principal would thereby attract only low risk agents[33] willing to accept lower coverage, giving themselves a positive expected profit, and leaving the original pooling contract insuring only high risks with negative profits[34].

In summary, competition amongst principals precludes the existence of stable pooling (Nash) equilibria. Therefore, if the best separating equilibrium contracts are dominated by a pooling contract in competition, only an equilibrium which supports a mixed strategy is possible (Rothschild and Stiglitz (1976), Dasgupta and Maskin (1986)), in contrast to the case of a monopolist principal in which a pure strategy (Nash) equilibrium will always exist[35].

2.2.2.3 Alternatives: Dynamic adverse selection

Given an adverse selection problem, the principal's objective is to extract private information from the agent via the contract, introducing the least amount of inefficiency and cost possible. At issue when the relationship is repeated is whether repetition can help the principal in his search for information. In problems with hidden actions, repetition mitigates moral hazard since it allows incentives to be better distributed over

[33] This is referred to as "cream skimming"

[34] This conclusion obtains when the equilibrium concept is Nash. However, a Wilson equilibrium concept (Wilson (1980)), which requires that no new contract could be offered that makes positive profits even after all contracts that would make negative profits as a result of its entry are withdrawn, legitimises the pure strategy pooling equilibrium This is because if principals realise that the newly introduced competing contract is rendered unprofitable when the old pooling contract is withdrawn, it will not be introduced

[35] Riley (1985) derives sufficient conditions for the existence of a Nash equilibrium Crucially, the rate at which the marginal cost of signalling activity declines with improvement in "quality" (lower risk) must be sufficiently large.

time. However, mitigation of adverse selection problems through repetition does not necessarily follow, since if contracts in each period were to be conditioned on previously revealed information, the agent would have an even greater incentive to misrepresent his type compared to the static case (the "ratchet effect").

Critical to the design of optimal contracts in a repeated relationship is whether the principal commits[36] to not using information that is revealed through time in revising the contracts in each period Also crucial is the correlation of agent types in each period.

Baron and Besanko (1984) consider a two-period model in which the agent knows his type in each period just prior to the start of that period, and the principal commits to not using information which is revealed over time[37] With no correlation between types, the period one contract is the optimal static contract, and the second period contract is the optimal symmetric information contract because both agent and principal have the same information about period two type at the time the contract is signed (start of period one). If types across periods are perfectly correlated, the multi-period contract is the repetition of the optimal static contract. Since in this case the principal commits to not use his knowledge of the period two agent type in the second period contract, the multi-period contract is not sequentially rational and Pareto improvements are possible through renegotiation. With imperfect correlation, the period one contract is the optimal static contract and the period two contract is intermediate between the optimal static asymmetric and full information contracts.

When instead the principal makes no commitment to ignore information that is revealed over time, the ratchet effect becomes admissible. When the agent in each of two periods

[36] Note that the notion of commitment also exists in the static case, where the principal commits to not renegotiating the single-period contract between himself and the agent once the agent reveals information

[37] Note that commitment is (weakly) desirable since the principal can commit to the strategy that he would choose in the absence of commitment This induces the agent to take the same decisions as in the noncommitment case

is either low or high productivity, there is perfect correlation between types in each period, and where transfers to the agent are linear in output[38], Freixas, Guesnerie and Tirole (1985) apply backward induction to derive the optimal contract in each period. The period two contract is the optimal static contract given the (Bayesian) updated beliefs from the period one announcement made by the agent (which may or may not be truthful). Low productivity agents, who have no incentive to pass themselves off as high productivity types in either period, achieve no rents in each period. However, high productivity types trade off the discounted information rents possible in period two by pooling with low productivity types in period one, with the possible gains from separation in period one (whereupon the period two information rent is zero). Thus the optimal period one contract may induce separating[39], pooling or semi-separating equilibria, where in the latter equilibrium the high productivity agent plays a mixed strategy in which he sometimes reveals his true productivity. There is also a sense in which the search for information leads the principal to be more generous in the first period, in that the allocation to the agent is greater than the optimal static contract, recognising the greater tendency of the agent to misrepresent his type when the principal may revise future contracts to incorporate previously revealed information.

In the next section, we formally review the literature on information asymmetries in the context of limited liability.

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[38] With non-linear schemes, since high transfers required to induce high productivity firms to separate are appealing to low productivity firms, low productivity firms may pass themselves off as high productivity firms in the first period and then quit to achieve the reservation amount in period two ("take the money and run").

[39] With continuous types, no mechanism which induces type revelation is feasible, let alone desirable (Laffont and Tirole (1988))

2.3 Literature survey: Asymmetric information and limited liability

We now formally review the literature which posits within the overlap of limited liability and information asymmetry studies. An original[40] classification of this literature is provided below, after which each category is discussed in turn.

2.3.1 Classification

The literature on asymmetric information and limited liability broadly decomposes into continuation and non-continuation studies (see Figure 2.2). Continuation issues motivate the bankruptcy debate, which has focused on the design of bankruptcy laws intended to avoid the inefficient liquidation of distressed firms and the promotion of economically viable firms as ongoing concerns, when typically, debt holders cannot observe the efficiency of a firm.

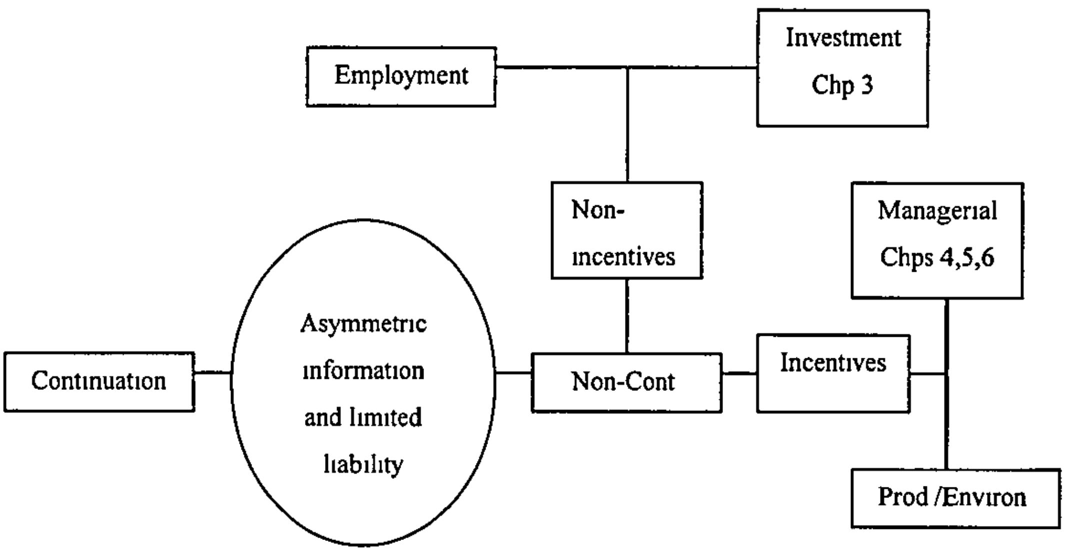

Figure 2.2

[40] The only aspect of this classification which is not original is the delineation of investment literature between ex ante and ex post information asymmetries (see 2 3 4), which is taken from Innes (1993b)

Non-continuation studies are static in the sense that there is no possibility of continuation following a trigger mechanism such as the default on a loan note.

Non-continuation studies further polarise into *incentives* and *non-incentives* literature.

The incentives literature examines how limiting the liability of an economic agent affects the motivation of the agent to supply effort which is only privately observable. This literature is divided between *managerial,* and *products & environmental* studies

Managerial studies consider a principal-agent setting in which a principal offers an agent a contract to supply an unobservable effort, and for which the principal provides a safety net limiting the share of ex post loss allocated to the agent. The principal-agent relationship is one of employer-employee.

However, products & environmental studies consider the incentives that contracts for the production of goods, or the provision of services, give to firms which can take costly preventative measures to reduce the risk of causing injury to third parties, be they consumers of a product or the general public, when the liability for loss that the injurer may cause is limited. In these studies, the principal-agent relationship is one of regulator and firm.

The non-incentives literature is divided between *investment* and *employment* studies, in which actions are publicly observable but some aspect(s) of information affecting the welfare of contracting parties is asymmetrically distributed between them

Investment studies[41] consider the allocation of capital to an entrepreneur whose project quality is not publicly observable, and who is either allocated no share of loss outcomes or may only share ex post outcomes above some threshold amount.

Employment studies consider the effects on employment of enabling firms not to commit to paying employee wages in low profit states, when only the firms can observe the realisation of an exogenous random productivity parameter.

We now review each subcategory, giving only a brief review of some of the aspects of the bankruptcy debate, since continuation issues form no part of the contribution made by this thesis.

2.3.2 Managerial

When a risk neutral agent supplies unobservable effort *prior* to observing an uncertain productivity state, a franchise contract is optimal[42] (Harris and Raviv (1979)), whereby the agent is sole residual claimant, and the principal extracts the entire surplus through a fixed fee (see 2 2.1.1). The agent participates given that his expected utility is no less than his reservation amount.

However, when the agent is constrained to earn no less than some minimum pay, the principal is no longer able to award the agent a franchise contract (Innes (1990, 1993a), Park (1995), Kim (1997)) This is because if the agent pays the principal a franchise fee which extracts the entire rent, then for some low states, the outcome net of this fee may

[41] This literature considers the debt contract to be the basic contract paradigm since the entrepreneur undertakes to pay the investor a fixed return from ex post profits, a guarantee which is compromised by liability limitations.

[42] For a risk neutral agent, the franchise contract is optimal independent of the order of state observation and the supply of effort What matters is that the contract is negotiated prior to the observation of the productivity state

be less than the minimum (floor) amount. Since a franchise contract clearly delivers too great a share of the realised surplus to the agent, the principal must resort to a sharing contract in which the agent supplies less effort because his reward is less sensitive to the effort that he supplies[43].

An important difference between franchise contracts[44] and share agreements in the context of liability limitations concerns the discretion exercised by agents when deciding between investment projects of differing risk When agents supply observable effort, Basu (1992) suggests that a pure rental agreement permits an agent to maximise the value of the insurance afforded by a liability constraint, by choosing (at cost to the investor) a project which does not maximise the expected (joint) surplus[45]. In contrast, a pure share agreement will always induce the agent to choose a project which maximises the expected (joint) surplus. However, in a richer framework in which effort is instead unobservable, Sengupta (1997)[46] has shown that the relative dominance of share contracts over pure rental agreements is sensitive to the actual value of the expected (joint) surplus, and that liability limitations alone are not sufficient to guarantee the dominance of share tenancy contracts over pure rental agreements.

[43] In the same way that risk aversion of the agent was costly to the principal, limited liability is similarly costly Note however that although bankruptcy constraints may function much as does risk aversion in models where the principal contracts with a single agent, notable distinctions concerning the (subgame) dominance of truth-telling by agents who also possess private information about the productivity state, may arise when the possibility of multiple agents working in correlated environments is admitted (Demski et al (1988))

[44] also referred to as pure rental agreements in the sharecropping literature.

[45] expected revenue net of effort and all other costs

[46] Sengupta (1997) generalised the earlier work of Basu (1992), in which effort is observable and the agent chooses from amongst projects which differ in risk

A qualification to the approach taken by Sengupta, in which he assumes a widely held view that agents whose liability is limited will take excessive risks, is illustrated by Suen (1995). Suen argues that even in the absence of direct bankruptcy costs, risk neutral agents will have an incentive to avoid bankruptcy in a multi-period setting, because bankruptcy means ending the game and foregoing a valuable option when the expected value of future income flows are positive[47].

Another variant of the sharecropping framework considered by Sengupta, is the link between the credibility of the commitment that tenant farmers (agents) may make to pay the fixed rent of a franchise contract, and their personal wealth (Shetty (1988)) When landlords (principals) can completely and costlessly appropriate the assets of tenant farmers, the efficient choice of effort is only made with fixed rent contracts for those tenants with sufficient wealth to guarantee the landlord full rental payment for all output realisations. For tenants with lower wealth levels, a disincentive to supply effort arises for the same reason that contracts which award an agent less than his marginal product may reduce effort. The effort of a tenant farmer will increase with his wealth when default is a possibility since the landlord may confiscate the tenant's assets if the rent is not paid in full.

However, the *optimal*[48] design of an incentive contract when the liability of the agent is limited centres on the possibility of motivating the agent to supply greater effort by the use of incentive targets. Holmstrom (1979) following on from the work of Gjesdal (1976) had noted that the first best was possible (for a risk averse or risk neutral agent) when

[47] An implication of limited liability in a multi-period setting is also that the resultant effort shirking from the use of a share contract viz-a-viz fixed rent contract is mitigated by the threat of the loss of a valuable continuation option

[48] An incentive contract is optimal when it minimises the incentive costs of contracting

sufficiently large penalties[49] for outcomes below some threshold can be imposed. But when limited agent liability precludes the principal motivating the agent by penalties, he must instead motivate the agent by discontinuously increasing agent pay for outcomes above some threshold[50].

That lump-sum bonuses or penalties of any size may improve incentive contracts was formally proved by Lewis (1980) for a strictly risk averse agent. Additionally, lump-sum bonuses or penalties may also motivate risk neutral agents, as shown by Innes (1990), whereupon the importance of the monotonicity of the pay of the principal over the range of profit outcomes was again[51] established to be a key feature in whether agents can be motivated to supply greater effort when penalties of unrestricted size cannot be enforced If the pay schedule of the principal is constrained to be monotone non-decreasing, as is the case for debt contracts in which the principal receives the lesser of the entire surplus or the promised amount, then the agent will strictly share the benefits of his marginal effort over some (non-empty) range of possible outcomes, thus ensuring that the conflict of interests caused by sharing profits but none of the effort costs of generating those profits will remain[52]. However, the first best (efficient) outcome was shown in Innes

[49] Mehta (1993) discusses the impact of imposing bounds on rewards or punishments upon attaining efficient (first-best) incentive contracts If efficiency is to be possible, then the inference about effort from the realised outcome must be stronger when rewards or punishments are constrained in magnitude.

[50] In essence, bonuses are a mirror image of penalties, except that when an agent is risk averse, the incentive pressure exerted by a punishment and a reward of equal measure are different Risk averse agents receive a greater fall in utility from a penalty than they do an increase in utility from a bonus of equal size When liability is limited, risk aversion therefore causes bonus schemes to become an "expensive" device for motivating greater effort from the agent

[51] When the pay of the agent discontinuously increases at some threshold outcome, the pay of the principal discontinuously decreases Therefore, even though agent pay may be monotone non-decreasing, the pay of the principal will be not be monotonic, as it will be both increasing and decreasing over the range of possible outcomes

[52] In Innes (1990), the debt contract is optimal (assuming a monotonic likelihood ratio property and a monotone contract constraint) when the output price is fixed and only the output is uncertain However, decomposing revenue uncertainty into price *and* output uncertainty, which permits a much wider set of

(1990) to be possible if the agent is instead awarded a contract in which he is allocated the entire profit, and therefore loses none of the marginal benefits of his effort, when high profit states (above some threshold) are realised[53]. For this latter contract, the payoff profile[54] of the principal is non-monotonic.

Thus, the emphasis in Innes (1990) was to show with a monotonic likelihood ratio property (hereafter MLRP) that the debt contract was not optimal, and that when contracts are allowed to be non-monotonic, that superior contract forms exist The fixed bonus element to incentive contracts discussed in Lewis (1980) and Innes (1990) has been further considered by Kim (1997) and Park (1995). These authors stress that the most efficient form of incentive contract when the liability of the agent is limited is a pure step contract, in which the agent receives nothing if the outcome is less than the target performance level, and receives a fixed amount if the target is achieved. Therefore, if it is not possible that this contract form implements the first best allocation, then under the same conditions, no incentive contract exists which can achieve this objective.

Kim and Park also consider a fixed ratio profit sharing contract in which they derive a necessary condition to ensure that a lump-sum bonus can motivate a risk neutral agent to supply the first best effort level whilst leaving the agent in expectation of no more than his reservation utility. In Chapters 4 to 6, we consider an alternative form of dichotomous incentive contract which has not been discussed by the incentives literature, in which the fee schedule of an agent with limited liability is a (piecewise-linear) profit sharing contract in which the share ratio, which divides the ex post profit between the agent and the principal, depends on the realised outcome. Specifically, we allow a proportional increase in profit sharing ratio upon the achievement of a performance target instead of a

possible contract forms, Innes (1993a) shows that the pure debt contract is almost never optimal Instead, the optimal contract form will be a combination of pure debt, commodity futures contracts, and a multiple of commodity call option contracts.

[53] Innes refers to this as a "live-or-die" contract.

[54] being a mapping of all possible outcomes to the ex post pay of the principal

lump-sum bonus of arbitrary measure. The crucial feature of this contract type that differentiates it from lump-sum bonus contracts in Kim (1997) and Park (1995), is that the lump-sum increase in pay derives from the threat of a lower share ratio for outcomes less than the performance target. As a result, conflicting incentive pressure is created since the agent's share of the value of his (costly) marginal effort is reduced for outcomes less than the target, whilst a greater jump in the lump-sum element of pay acts to increase incentive pressure.

In Chapter 4 we analyse this contract form for both a risk neutral and risk averse agent, showing that the effective use of contingent share ratio contracts as a means to elicit greater effort relies crucially on technology considerations. We also show the importance of the availability to the principal of precise information concerning the degree of risk aversion of the agent.

In Chapter 5, we then separately consider how two factors which affect incentive pressure can be incorporated into dichotomous contracts. In the first we examine capital contribution by the agent, and derive sufficient conditions for a (Pareto) improvement from the use of contingent share contracts through substitution of the principal's capital with that of the agent. In the second we examine additional signals and illustrate an improvement to dichotomous schemes which is provided by the use an imperfect monitoring technology.

In Chapter 6, we then extend the pure moral hazard setting of Chapters 4 and 5 by combining the information asymmetries examined in isolation by the preceding chapters (including Chapter 3, see 2.3.4) to consider a problem of adverse selection with moral hazard in which an agent who supplies unobservable effort is prior informed as to the marginal productivity of a venture w.r.t. effort and capital. We derive conditions for which a contingent share ratio scheme parametised on a message variable signalled by the self-selection of a contract by the agent, can implement the optimal pure adverse selection outcome.

Contracts which provide incentive effects, such as promotions and bonuses, are likely in practice to be only privately enforceable in the sense that they contain terms which may preclude the allocation of market traded instruments, such as financial derivatives, as a means to replicate the contract payoff profile of the agent. However, an important link between financial derivative contracts and incentive contracts has been established by Selender and Zou (1994), who showed the results of Innes (1990) to be a special case of their analysis, whereby the (strong) assumption of MLRP could be relaxed in favour of assuming first order stochastic dominance[55]. Selender and Zou showed that under limited liability, there exists a necessary and sufficient condition for standard share-derivative contracts to resolve the moral hazard problem In addition to having large enough expected profits, the manager must be able to hold combinations of call and put options in excess of the underlying assets[56]. This means that the slope of the contract (i.e. the relative changes of contractual value with changes in firm value) is greater than one over some domain of end-of-period firm value. This result therefore shows how incentive contracts that induce first best allocations may be achieved using real-world arrangements as opposed to abstract theoretical constructs[57].

In all of the above studies, the agent supplies effort prior to the realisation of uncertainty. However, limited liability constraints not only preclude the use of franchise contracts when a risk neutral agent supplies effort before observing the realised state of nature, but also render franchise contracts redundant when the agent sees the realised state of nature *prior* to supplying effort (Sappington (1983a)). If the liability of the agent is limited, then the agent is essentially free to dissociate himself from the principal once he has (privately) observed the realised state of nature, where by so doing he will be able to

[55] of the probability distribution of outcomes induced by effort given exogenous uncertainty.

[56] However, Selender and Zou also discuss the existence of institutional constraints which may not permit individuals to hold combinations of call and put options which allocate them more underlying shares, when the options expire, than are actually traded in the market place In this case, cash viz-a-viz physical settlement may circumvent such problems.

[57] such as the "live-or-die" contract of Innes (1990).

secure his reservation amount. The type of incentive contract that the principal must design is again significantly affected. In particular, the agent will never earn less than his reservation utility, and will receive rents in some states (Pitchford (1998)).

To illustrate this idea, consider an example[58] of a risk neutral agent who can supply one of two possible efforts. The agent is awarded some fixed amount depending on whether the outcome is a success or a failure. Suppose that in order to induce the agent to supply the greater effort, the principal must allocate him a reward for a success which is greater than his reward for a failure by some fixed amount. Absent some specified floor to agent pay for both outcomes, the principal can induce the greater effort and extract all of the rent by imposing a franchise fee (as discussed previously), thereby leaving the agent in expectation of his reservation utility. However, when there exists a floor which becomes binding in the event of a failure[59], differentiation of payments between the outcomes by the necessary amount intended to induce the greater effort can only be achieved at the expense of awarding the agent a rent. The first-best (franchise) outcome is therefore again precluded.

In Sappington (1983a), the agent supplies hidden effort after observing the realisation of a productivity state which is unobservable to the principal. As a consequence, the agent has an expanded strategy set in comparison to the case in which he supplies effort prior to seeing the productivity state. It is shown that in all states except the one in which the agent is most productive, and perhaps in certain very unproductive states, that the contract which is optimal for the principal will induce outcomes that are ex post Pareto inefficient. Output is ex post Pareto efficient when the agent's marginal disutility from generating an additional unit of output coincides with the principal's valuation of such output[60].

[58] This example was taken from Macho-Stadler and Perez-Castrillo (1997) pp64-66.

[59] That is, when the reward in the event of a failure for the unlimited case is below the specified floor

[60] This is in contrast to the result of Harris and Raviv (1979) (which has an equivalent information structure) whereby enforceability of contracts in all states facilitates ex post Pareto efficiency by the use of

The reason for preclusion of Pareto efficiency in Sappington (1983a), is that if the principal designs the contract such that the agent is compensated for producing an inefficiently small output in the lower states of nature, then he reduces the magnitude of the payment to the agent needed to induce a higher level of output in the more productive states The trade-off between inducing inefficiency in lower states and reducing the size of the payment to the agent in higher states is sensitive to the probability of occurrence of the lower states. If lower states are more probable then the trade off is less beneficial to the principal since an inefficient production level is a more likely realisation, whereas if higher states are more probable, the benefit of reducing the magnitude of the payment to the agent in higher states is likely to outweigh the detriment of an inefficient outcome for lower states. It is also the case that because the benefits associated with inducing an inefficient outcome in any state are realised only when higher states of nature occur, there are no incentives for the principal to induce an inefficient outcome in the highest state of nature. To see the importance of the liability constraint in this context, note that in contrast, absent any floor on the payoff that the agent can receive, the principal could restore efficiency and expand output for the lower states, extracting any surplus thereby allocated to the agent by use of a franchise contract.

Finally, a minority of the literature on the combination of limited liability with unobservable effort in a principal-agent model, examines issues whose scope extends beyond the optimal design of incentive contracts. Banerjee and Timothy (1990) ask whether it is optimal to increase taxes when introducing a limitation in the liability for losses of an owner-manager who finances investment using a debt contract with creditors who compete with one another By allocating a government subsidy to creditors for a reduction in the rate of interest, the owner-manager is given an incentive to increase effort whilst a rise in taxes maintains his wealth following lower borrowing costs. Since higher effort means a lower chance of bankruptcy, creditors are able to further reduce the risk

dichotomous contracts (dual and mutually exclusive contingent fee schedule rules) and the additional use of an imperfect monitoring technology for effort premium[61] and therefore the interest charge, thus effecting an overall welfare improvement when the subsidy is financed through the increase in taxation.

Brander and Spencer (1989) present a theory of the firm which examines the relationship between the financial structure of the firm and the effort and output decisions of the owner-manager. Substitution of borrowed funds for equity investment by the owner-manager induces lower levels of effort, since the number of states of nature in which the owner-manager receives a return from his effort is reduced. As a consequence, firm output is lower and the probability of bankruptcy is higher. Also, for any given financial structure, the effort supplied by a monopolist is less than that of a competitive owner.

Lastly, Lawaree and Audenrode (1992) establish the restrictions that limited liability imposes on the use of punishment threats following an audit of an agent employed to reduce costs, and who attempts to shirk by always passing off a privately observable firm specific cost parameter (which is either high or low) as being of a high cost type. If auditing is imperfect and liability is unlimited, then even though a high cost agent is erroneously punished with positive probability, the principal can compensate the high cost agent in expected value for possible mistakes. However, when liability is limited, compensation for punishment errors must be in full, and therefore since the principal cannot distinguish a truthful high cost agent from a cheating low cost agent, it is never profitable for the principal to perform the audit[62]. In Lawaree and Audenrode (1996), these authors also demonstrate that both agent types will receive a positive rent, and that even[63] the most efficient agent (with low cost parameter) will not produce at his first best effort level.

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[61] If liability is unlimited, then loans are always repaid in full and a competitive loan market guarantees that the owner-manager effort and the rate of interest are independent

[62] i e "if you cannot convict an innocent, do not audit".

[63] This is in contrast to "non-distortion at the top", in which the contract for the most efficient agent, i e the agent for whom no other type would pass themselves off as, is efficient

2.3.3 Products and Environmental

In the managerial model considered in section 2.3.2, the most important impact of limited liability on incentive contracts was to render a franchise contract, which is optimal when liability is unlimited, potentially unenforceable and therefore suboptimal.

An analogous first best rule exists in the law of tort to apportion the costs of remedial measures following an accident which causes harm to third parties, where the liability of the injurer is unlimited Shavell (1987) and Landes and Posner (1987) showed that a rule of strict liability with contributory negligence generates incentives for efficient accident prevention for both firms and consumers.

However, when firms can inflict harm which exceeds their net worth, then a rule of strict liability becomes unenforceable. Mandatory purchase of insurance by firms, specifying the care level, may mitigate this problem by inducing firms to supply socially efficient preventative care, whereupon the insurer creates additional incentives by conducting random and imperfect costly monitoring (Jost (1996)).

In the absence of the motivation to supply socially efficient preventative care, such as is the case with the mandatory purchase of insurance (Jost (1996)), a firm will only be motivated to take (privately observable) actions which reduce expected harm by the amount of damages for which they can be held liable. Endres and Ludeke (1998) identify three important ranges of liability for the allocation of the remedial costs following an accident. For low and high levels of liability, the firm will either choose no prevention care or the optimal care level chosen in an equilibrium characterised by the first best rule when liability is unlimited. However, for intermediate levels of liability only an equilibrium in mixed strategies, which can be motivated for example by lack of information about payoffs of firms and potential claimants, is possible, for which care levels are inefficient.

In the same way that limited agent liability motivates the use of performance bonuses in managerial models (see section 2.3.2) when punishments cannot be enforced, Strand (1994) illustrates that the first best care level of a firm can be implemented by a government subsidy which rewards the firm when accidents do not occur, and confiscates the entire assets of the firm in accident states. However, implementation of the first best is sensitive to the preference of the government to the distribution of income between itself and the firm[64]. When firm profits are less important to the government than its own revenues, only the second best is possible, with lower subsidies and less than the first best care level exercised by the firm

Government subsidies in Strand (1994) are mirrored in Leonard and Van Audenrode (1996) and Shavell (1997) by wage premiums paid to employees who cannot be held responsible for mistakes which cause harm to third parties, and who therefore receive wage premiums (wages above spot market rates) in order to motivate greater (unobservable) care Leonard and Van Audenrode empirically establish that limited (employee) liability results in fewer quits and firings through its affect on wage premiums, whilst Shavell explores the social welfare implications of a rule where the firm pays damages equal to harm in the event of an accident and employees receive supernormal wages because their liability is limited. Shavell challenges conventional wisdom that damages equal to harm induces the socially efficient outcome, a rule based on the rationale that as a result, product prices reflect the full social cost of production, thereby inducing customers to make socially correct purchase decisions. However, since supernormal wages create additional firm expense, whereas for society they are a costless transfer payment from firm to employee, damages greater than harm may be necessary to induce firms to pay high enough wage premiums. In contrast, since supernormal wages induce a social welfare loss by firms passing these costs onto consumers, thereby charging prices which exceed social cost, a rule of damages less than harm may be socially preferable in order to reduce prices by reducing wage premiums. The social

[64] i e the firm's profits and governments revenues have equal weight in the social welfare function

efficiency of a rule of damages equal to harm when employee liability is limited is therefore ambiguous.

Costly accident prevention care may in reality be one of several factors under firm control which affects the expected harm of an accident. The risk of an environmental accident is also determined by the activity level of the firm. Posey (1993) explains the affect that limited liability has on both preventative care and firm activity levels. The expected cost of care and the expected marginal benefit of activity are reduced when liability is limited. This is because costs and benefits are weighted by the probability that an accident does not occur, whereas for unlimited liability, expected costs and benefits equal their realised values. Consequently, because limited liability also reduces the marginal benefit of prevention costs and the marginal cost of risky activity, the affect of limited liability may be to either decrease or increase care and activity levels.

Finally, other interesting variants of accident prevention care and limited liability include the possibility that lenders can bear responsibility for the cost of accidents caused by firms which they finance. Pitchford (1995) establishes that partial lender liability, where the lender[65] compensates the victim for less than the total harm, together with a minimum equity requirement of the firm delivers the highest level of efficiency (compared to the social optimum when prevention care is contractable) Extensions of the economic analysis of tort to multi-party accidents with unobservable accidents are considered by Feess and Hege (1998). If punitive damages were possible, as is the case of unlimited liability, then the efficient liability rule is that each party pays the total damage in the event of an accident. However, when this is not possible, a fundamental dilemma arises because each party only pays a fraction of the total harm, ensuring inefficiently low levels of care. It is shown that asymmetry between parties in their impact on the stochastic damage function can be exploited to improve ex ante incentives to prevent an accident. The power of incentives can be increased and the liability rule may be efficient if each

[65] The loan contract specifies the repayment in an accident state and a no accident state

injurer has to pay a disproportionate share of those outcomes which are more likely their fault than the fault of others, in the sense that if the defendant in question was less careful than they should have been, then the probability of an accident outcome had risen more than it would have if another injurer had been less careful. This implies departures from the proportional rule (or constant splitting rule), based on the statistical information contained in the circumstances of the accident. By applying this idea in an optimal way, Feess and Hege find that efficient liability rules exist as long as the asymmetry across injurers is sufficiently large.

In the next section we discuss the non-incentives literature, for which effort is observable, whilst agent liability is limited and information is asymmetric.

2.3.4 Investment

In the absence of symmetric information about project quality, or a means by which project quality can be signalled prior to the commitment of capital, the financing of entrepreneurial projects by investors will take place subject to informational constraints. In this case, information asymmetry is *ex ante*, the entrepreneur being privately informed about the probability distribution of (costlessly observable) ex post profits. In contrast, an *ex post* information asymmetry occurs when the only asymmetry which arises concerns the observability of ex post profits. In this case, the entrepreneur is able to freely observe ex post profits, whereas observability by the investor is costly.

The literature which examines both ex ante and ex post information asymmetries when the entrepreneur and possibly the investor are constrained to earn no less than some minimum amount, studies a principal-agent model in which investment is governed by a debt contract. Even with identical risk preferences, the use of a debt contract leads to a conflict of goals between the entrepreneur and the investor. When the liability of the entrepreneur is limited, the entrepreneur has an incentive to commit investor capital to higher risk projects (Stiglitz and Weiss (1981)). Asymmetric information about project quality then creates a need for the investor to attempt to minimise the costs of this ignorance.

In contrast, if the liability of the entrepreneur is unlimited, then the ignorance of the investor has no associated cost since the return promised by the debt contract[66] is guaranteed. Additionally the impact of investor ignorance and the conflict of goals that arises with debt contracts and limited entrepreneurial liability, is not evident when an investor is instead allotted equity shares[67]. This is because equity shares make no promise of a fixed return, a promise which is compromised when the liability of the entrepreneur is limited. Also, the claims to the residual surplus per share are equal, whereas the claims of debt holders subjugate those of equity investors[68].

When information asymmetry is ex post, whereupon investors can only observe ex post profits at a cost, the debt contract is an optimal financial arrangement (Townsend (1979), Gale and Hellwig (1985), Williamson (1987)). This is because a debt contract is the least costly arrangement inducing the entrepreneur to truthful revelation of ex post profits, in which observation occurs only for a lower interval of (default) states, in which the promised amount exceeds the realised surplus. This type of information asymmetry also leads to suboptimal lending arrangements, in which entrepreneurs are (equally) unable to borrow the entire capital that they would like to given the quoted interest rate (Gale and

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[66] The fixed return being determined for example by the competitive market rate of interest for loans

[67] Notwithstanding the possibility of different risk preferences and insider-outsider conflicts (Meade (1986), Myers and Majluf (1984)) In Myers and Majluf (1984) for example, asymmetric information about real investment projects and assets-in-place causes a conflict of interest between existing security holders and new equity financiers, causing firms to forego valuable investment opportunities

[68] The priority of claimants in bankruptcy also provides a link between financial structure and consumer markets (Appelbaum (1992)) In bankruptcy, debt holders have a higher priority than consumers, whereas equity holders have a lower priority Obligations to debt holders therefore affects the risk facing consumers even when the firm provides product warranties. As a result, the firm will be fully equity financed if consumers are risk averse and security holders are risk neutral, thereby optimally shifting risk away from consumers

Hellwig (1985)), or in which some would-be borrowers receive loans whilst others do not (Williamson (1987)).

The availability of credit also motivates much of the discussion of ex ante information asymmetries[69], whereby the entrepreneur is endowed with private information about the probability distribution of ex post profits, prior to the commitment of capital by investors. Important to this literature is whether loan amounts are the same for each entrepreneur (Stiglitz and Weiss (1981), DeMeza and Webb (1987)), or whether the loan size itself is a contract variable (Innes (1993b), Milde and Riley (1988), Jaffee and Russell (1976)).

When loan sizes are fixed, it is not possible for investors to differentiate the risk of investment opportunities between entrepreneurs. As a result, since investors charge an interest premium in respect of the possibility of default, entrepreneurs with lower risk projects are driven from the market for loans (Stiglitz and Weiss (1981)). As a consequence, and in order to prevent the saturation of the loans market with poor quality (high risk) investment opportunities, investors must ration the supply of credit instead of reducing excess demand by increasing the rate of interest.

In contrast, DeMeza and Webb (1987) derive an over-investment result. This result is however consistent with that of Stiglitz and Weiss because DeMeza and Webb permit

[69] An important body of literature also attempts to provide an explanation other than tax shielding for the use of debt contracts when ex ante information asymmetry obtains Jensen and Meckling (1976) argue that there are agency costs with both equity and debt financing with an optimum mixture minimising the total agency cost Ross (1977) suggests that the manager of a firm whose wage depends on current and future firm value uses debt to signal firm value to the market The dependence of his wage on current firm value gives him the incentive to signal, while a penalty in the case of bankruptcy prevents him from overstating the value. Leland and Pyle (1977) suggest that an owner-manager uses the proportion of equity that he holds as a signal of firm quality Lastly, Narayanan (1988) identifies an advantage to risky debt not possible with equity finance, whereby its use by profitable firms keeps inferior firms out of the market even when the market is unable to perfectly distinguish between firms of different quality This reduces financing costs when firms are pooled by lenders

entrepreneurial projects to differ according to expected return. This means that the marginal project financed in DeMeza and Webb has the lowest probability of success, whereas the reverse is true of Stiglitz and Weiss.

Innes (1993b), in generalising the analysis of DeMeza and Webb, went on to illustrate the importance of assuming only monotone payoff profiles for the investor. For a non-monotone contract, in which investors receive the entire profit if it is below some threshold, and nothing otherwise, Innes was able to show that entrepreneurs of higher quality in the sense of MLRP can signal their type by preferring the non-monotonic contract over the debt contract, even when investment amounts are fixed. Further, if investment is not fixed, a separating equilibrium is possible, in which high quality entrepreneurs will signal their type by underinvesting in their projects. The cost of underinvesting is minimised, however, if entrepreneurs of high quality projects may also choose to finance their projects with investment contracts which are non-monotonic.

The intuition that the amount of loan risk 'purchased' by uninformed investors depends on the size of loans to entrepreneurs, was first highlighted by Jaffee and Russell (1976). Milde and Riley (1988), in extending the scope of the analysis of Jaffee and Russell, also demonstrated that the way in which exogenous uncertainty affects productivity determines how entrepreneurs signal their types to investors when the loan size is a contract variable. Since larger loans attract greater interest charges per unit of capital, entrepreneurs are able to signal their quality types by their willingness to accept greater lending costs for larger loans. For multiplicative uncertainty, higher quality entrepreneurs accept larger loans than they would otherwise, in order to distinguish themselves from poorer quality loan applicants. For additive uncertainty, higher quality entrepreneurs accept smaller loans. Therefore, technology considerations alone may determine whether over or under investment obtains when signalling phenomena are admissible.

It is evident from the literature reviewed above, which characterises optimal financial arrangements between asymmetrically informed investors and entrepreneurs, that as far as we are aware, no contributions exist which attempt to examine the ramifications of limited entrepreneurial liability when a profit sharing agreement governs the allocation of ex post profits between investor and entrepreneur. In Chapter 3 we examine the implications of imposing a floor to entrepreneurial pay when both the investor and the entrepreneur are privately endowed with information which determines the optimal capital investment The entrepreneur is privately informed about the marginal productivity of capital, whilst the investor becomes privately informed about the unit cost of capital which is shared with the entrepreneur.

We find for this bi-lateral information asymmetry problem, whereupon the investor must design a contract menu which induces him to truthfully reveal the cost of capital when realised, that the only inefficiency in the optimal investment schedule is due to the productivity information privately endowed to the entrepreneur prior to contract acceptance. The private observation of the realised cost of capital to the investor only serves to reduce welfare by restricting the space of feasible[70] mechanisms Additionally, imposing a pay floor for the entrepreneur is overall welfare enhancing since it reduces the extent of the information rents which the investor must award the entrepreneur in order to elicit truthful reporting of the entrepreneur's actual productivity type. This welfare enhancing feature of limited liability in profit sharing agreements has received no mention in the literature. However, whilst our contribution is a significant departure from the existing literature which examines investment arrangements between asymmetrically informed entrepreneurs and lenders in the context of a debt contract, a comparison is possible with some of the conclusions reached therein.

[70] By feasible mechanisms we mean contract menus which are incentive compatible and induce the borrower to participate

2.3.5 Employment

It is well known that incentive compatible contracts, in which risk averse firms privately observe an ex post realised productivity state, lead to underemployment (Hart (1983)). The reason is that risk averse firms prefer employee contracts in which wages are lower in relatively less productive states. However, since firms have an incentive to understate the realised productivity state, incentive compatible contracts can only be conditioned on total wages by way of employment. As a consequence, employment is inefficiently low in less productive states.

When risk neutral firms have limited liability, a claim of bankruptcy limits the payment a firm must make to workers. Because workers will therefore prefer to accept contracts which preclude firms from ever entering into bankruptcy, contracts appear to represent firms as having extreme disutility from negative returns (Kahn and Scheinkman (1985)). As a result, an artificial concavity is introduced into the utility function of limited liability risk neutral firms[71], ensuring that they behave as if they were risk averse[72].

Kahn and Scheinkman show that the consequence of this artificial concavity in the utility function of a risk neutral firm, when leisure is a normal good, is the existence of some productivity state, whereby higher state realisations yield overemployment, and lower state realisations yield underemployment[73]. If leisure is an inferior good, then ex post underemployment obtains for all states.

[71] This is in contrast to risk loving behaviour induced by limited liability for risk neutral firms when there is no incentive compatibility constraint

[72] Another way to view this is that the firm must be concerned about the allocation of profit across states, since the profit in low states must be adequate w r t the firm's liquidation value

[73] For the full information case, employment is efficient, but workers may not be fully insured, since the firm bankruptcy constraint ensures that workers cannot be paid more than the firm produces

The explanation of an underemployment result is also offered by Farmer (1984,1985), in which the return to the factors of production is extended to include the cost of debt finance. A contract in which a risk neutral firm with superior information is given the entire residual risk is rendered unfeasible in low states when the firm has limited liability. This means that since a constant utility level for factors of production cannot be guaranteed in every state, with a possible loss in low states, the factors of production must receive a bonus in favourable states. However, this bonus payment interferes with the firm's ex post employment decision by raising the marginal cost of employing an additional unit of labour above the disutility of employment. The profit maximising firm makes its ex post employment decision by equating the marginal cost of employment to the marginal product, but since the marginal cost schedule is steeper in an inefficient contract than in the first best contract, the firm will hire less labour in states for which the bankruptcy constraint is binding.

Additionally, Farmer also showed that since the degree of (artificial) concavity of the firm's utility function varies as the bankruptcy constraint becomes more or less binding, then its employment contracts will be less efficient when the value of outside opportunities for workers increases, whereupon the firm becomes less able to guarantee factor payments. In the same way, variations in interest rates may manifest themselves as variations in the incidence of layoffs, because with rising interest rates the firm is forced to pay a higher expected return to its factors of production, being the sum of the reservation utilities of workers and investors.

Farmer (1985) had established a link between credit and labour markets, in which it is asserted, in common with Kahn and Scheinkman, that worker underinsurance is a result of the limitation placed on firms by bankruptcy constraints to make payments in low states. This assertion has been challenged by Tsoulouhas (1996). Typically, firms are assumed to be risk neutral since they have access to credit markets, and can therefore offer complete insurance to risk averse workers. Access to credit markets therefore means that for workers who commit to ex post employment with the firm, the firm can offer full insurance by shifting the variability in payments to creditors, even though limited liability restricts the aggregate (workers plus creditors) payments a firm can make in low states. However, for workers who do not commit to ex post employment with the firm, the distribution of pay across states must adjust to the ex post arbitrage opportunities available. Since the pay of workers in high states who can quit ex post must increase to stop a quit, given that the firm would not pay a rent ex ante to workers, pay in low productivity states decreases. As a result, the existence of outside sources of credit, which are important in shifting risk away from immobile workers has no effect when workers are mobile. It is therefore asserted instead, that it is worker mobility which leads to underinsurance of workers when firms have access to credit markets.

2.3.6 Bankruptcy[74]

The legislative intent of bankruptcy laws, such as Chapter 11, is to avoid the liquidation of financially distressed but efficient firms, and to liquidate distressed inefficient firms (Mooradian (1994)). This is an issue of efficient resource allocation and the debate about the effect of Chapter 11, which specifically allows for a renegotiation between equity and debt holders over the allocation of claims for a firm, throws up two competing views. The first view is that bankruptcy laws exacerbate overinvestment, where in the extreme, managers may reorganise when liquidation is efficient. The second view is that Chapter 11 enhances efficiency by inhibiting the inefficient liquidation of firms in default.

Mooradian (1994) introduced asymmetric information, where creditors cannot observe firm efficiency, into a model of public debt restructuring. Mooradian showed that *without* a collective reorganisation mechanism like Chapter 11, inefficient firms pool with efficient firms, where both are equally likely to continue or liquidate. It is further shown that Chapter 11 imposes a cost on pooling that efficient firms do not incur, which induces

[74] In this subsection we present only a brief review detailing some of the issues of the bankruptcy debate, since continuation issues form no part of the contribution made by this thesis

voluntary filing for bankruptcy by inefficient firms and consequently enables efficient firms to continue when they would otherwise be liquidated.

An alternative dissemination of the information structure is provided by Berkovitch and Israel (1999), where in addition to fundamental information such as intrinsic firm efficiency, importance is given to strategic managerial information, which allows the manager to determine the chance with which an investigation by creditors will successfully identify firm efficiency. Optimal bankruptcy laws can then be derived, each depending in a different way upon the quality of both fundamental and strategic information endowed to managers, in which creditors' information is utilised and the use by managers of strategic information is minimised.

Since a manager chooses his effort level whilst anticipating both the possibility of the firm entering financial distress and the resolution of distress as governed by the existing bankruptcy law, Berkovitch et al (1998) extend the definition of optimal bankruptcy laws to include optimal ex ante incentives of managers to supply effort, whilst also achieving an ex post efficient allocation of resources. Berkovitch et al. (1998) identify that directly affecting the structure of the bargaining process between owner-managers and investors in directing the assets of the firm to their highest value use in bankruptcy, for example by mandating a first move advantage to the owner-manager and enforcing some minimum delay before counter offers may be considered, can effect implementation of the optimal resolution of financial distress whilst achieving the first-best incentives for effort[75]

The significance of not giving debt holders absolute priority in renegotiation, but instead giving the owner some bargaining power in default, is also illustrated by Heinkel and Zechner (1993). These authors also permit owner-managers to make effort choices which

[75] Legros and Mitchell (1995) suggest that there is a trade-off between efficient resource allocation and incentive effects, since a liquidation policy has a disciplinary effect in which a manager with any given productivity venture will exert more effort when faced with the threat of liquidation than when assured of a bailout

are individually utility maximising, but consider how new information about future cash flows is obtained over time. A conflict of goals may arise between owner-managers and creditors in which intermediate cash flows revealing an expectation of final default on debt payments, may give managers an incentive to conceal poor prospects in order to pay themselves a dividend. This can also lead to false declarations of expected default in order to achieve debt forgiveness, thereby rendering less binding the restrictions on the ability of managers to pay themselves dividends. When cash flows are positively serially correlated, this problem is mitigated by the issue of debt with a risky intermediate debt payment, which acts as a signal of future default. When flows are less highly correlated, owner-managers can be induced to truthful revelation by being given sufficient bargaining power in default, such that the value of the owner's position after renegotiation exceeds the benefits of concealing expected future default with inefficient continuation.

Finally, that bankruptcy rules which are identical across firms automatically embody the risk that firms that should be liquidated are bailed out and vice versa, as discussed in the literature above, is well known. However, these inefficiencies may even exist when regulators can tailor bankruptcy rules to each specific firm, given particular firm specific beliefs about efficiency (Legros and Mitchell (1995)).

CHAPTER 3

INVESTMENT, HIDDEN PRODUCTIVITY AND LIMITED LIABILITY PROFIT SHARING CONTRACTS

3.1 Introduction

The characterisation of optimal financial arrangements between asymmetrically informed borrowers and lenders has been studied for two polar cases, *ex ante* and *ex post* informational asymmetry Ex ante information asymmetry arises when the borrower is privately informed ex ante about the probability distribution of ex post profits which can be costlessly observed by the lender An ex post informational asymmetry implies that investors can only observe ex post profits (or realised state) with a cost. Prior to contracting, the borrower and lender are symmetrically informed about the probability distribution of ex post profits.

The literature on the effect of information asymmetries upon investment (see section 3.2) has examined the impact of both types of information asymmetry, ex ante and ex post, in the context of debt contracts. Debt contracts, in which the lender is sole residual claimant when project returns are less than the promised payment, are limited liability contracts. This is because, the borrower is liable to pay the fixed amount only when project returns are sufficient to meet this requirement. Upon default, when project returns are less than the promised amount, the borrower is not liable for the shortfall.

The aim of this chapter is to explore the impact of providing a floor to the pay that an informed borrower receives when it is a profit sharing contract rather than a debt contract which is used to reward capital. The borrower, who contributes no capital himself, is informed about the marginal productivity of capital prior to formalisation of the profit sharing agreement that he commits to with the lender. Concerned about the opportunity that the borrower may have to overinvest capital, the investor designs the profit sharing contract to allocate a proportion of the realised cost of capital to the borrower. However, neither the borrower nor the investor know the actual unit cost of capital supplied by the investor at the time of contract acceptance. Therefore, in order to placate concerns of the borrower that the realised cost of capital may turn out to be very high, the investor provides a floor to the pay that the borrower is allocated from ex post profits.

This problem is one of bilateral information asymmetry, since both the investor and the borrower are required to make declarations about private information which is separately endowed to each of them. The information asymmetry of the borrower is also *ex ante*, in the sense that his private information about the marginal productivity of capital induces a probability distribution over ex post profits, where uncertainty is about the future realisation of the unit cost of capital.

The chapter is set out as follows. In section 3.2 we provide a detailed review of the literature on optimal lending arrangements between asymmetrically informed borrowers and lenders with limited liability constraints. From the review it will become apparent that the literature concentrates on the discussion of the use of debt contracts in determining the significance of information asymmetries between borrower and lender, and that no such analogous contributions exist to examine the use of profit sharing contracts with a pay floor for at least one of the participants. In section 3.3 we introduce the model. In sections 3.4, 3.5 and 3.6 we present and discuss the analysis. In section 3.7 we collect concluding remarks.

3.2 Asymmetric information and investment: A detailed review

Studies of *ex post* informational asymmetries, in which lenders can only observe ex post profits (or realised state) with a cost, but for which no information asymmetry exists ex ante, can be traced back to Townsend (1979), who explained the existence of debt contracts as a result of costly state verification procedures. If a contract is contingent on an event (the realised state), then it must be known whether or not that event has occured. The range of possible contingent contracts is limited to those states which are easily verified by both parties to the contract. By characterising a contract to specify for which states an asymmetrically informed borrower must provide state verification (at cost to himself) to the lender, and the amount to be transferred to the lender for each state, Townsend was able to show that the optimal incentive compatible contract, in which the borrower truthfully declares the realised state when called to do so, is one for which the verification states are a lower interval of possible state realisations. Such verification obtains for debt contracts upon default, where the lender must verify the realised state in order to precisely extract the residual surplus, which is necessarily less than the promised fixed payment. Verification is absent for all higher states for which no default occurs and the promised fixed payment can be made.

Gale and Hellwig (1985) also derive a debt contract as the optimal contract form by endogenising a binary observation decision for each state However, they concentrate their study of ex post asymmetry on how the cost of observation and the asymmetry in information gives rise to an inefficiency. This inefficiency causes a market failure for the provision of credit. It is shown that given a fixed opportunity cost of investment, diminishing returns to investment ensure that as the level of investment increases beyond some point, the distribution of profits shifts to the left. Due to both diminishing returns and the structure of bankruptcy costs, the point at which this shift starts to occur is less than the first-best level of investment which obtains when the lender has the same information ex ante as the borrower. Thus it is optimal to reduce investment some way below the first-best level in order to reduce the probability of bankruptcy and consequently its associated cost.

In Gale and Hellwig (1985), credit is rationed in the sense that borrowers cannot borrow all of the capital that they would like to given the quoted interest rate. This is in contrast to the credit rationing result of Williamson (1987), in which all would-be borrowers are identical, ex ante, but some receive loans while others do not. Williamson's model relies on monitoring costs to produce this result, as it contains none of the features that produce rationing in the other models so far discussed. Williamson was able to characterise two equilibrium types, a rationing equilibrium and a no rationing equilibrium. By assuming that borrower utility (all parties are risk neutral) is concave in the interest rate, when demand for credit is greater than the available supply, the equilibrium interest rate (which maximises the lender expected utility) ensures that some entrepreneurs (borrowers) are denied credit. Those entrepreneurs who do not receive loans can offer no contract that will bid loans away from those who receive them or that draw more lenders into the credit market This is the case since the equilibrium interest rate maximises the expected utility of the lenders. Offering to pay a higher interest rate implies a higher probability of default, with larger expected monitoring costs for the lender. This increase in monitoring costs exceeds the increase in expected payments to the lender which result from the higher interest rate.

The issue of credit rationing (when demand for credit exceeds supply) also motivates much of the discussion of the effects of *ex ante* information asymmetry upon investment These studies attempt to ascertain the conditions which support the existence of either an under or over investment problem, where the degree of investment is made with reference to the first-best level. For these studies, characterisation of the ex ante probability distribution of ex post profits invokes either differing risk for the same expected return (as in Stiglitz and Weiss (1981)) or differing expected returns (as in DeMeza and Webb (1987)).

Without the possibility that high quality loan applicants can signal their characteristics to potential lenders, Stiglitz and Weiss (1981) consider a pooling equilibrium in which borrowers seek to fund projects of different risk but equal expected return, where lenders are unable to distinguish between loan applicants because loan amounts are equal for all potential borrowers. The truncation of borrower payoffs for low states afforded by debt contracts ensures that for projects which vary according to a mean preserving spread in risk considered by Stiglitz and Weiss, the expected return for higher risk loan applicants who debt finance their projects will be greater. Raising the interest rate to reduce excess demand in a non-cleared loan market will then tend to force the preferred loan applicant with a lower risk project out of the pool. Instead of raising the interest rate to reduce the demand that it meets, lenders must therefore ration credit.

The intuition for the result in Stiglitz and Weiss comes from the way in which loan contracts can induce a conflict of goals between borrower and lender. Consider instead a borrower who can choose his character to be from a range of possible risk types. The borrower has an incentive to shift to high risk projects in order to maximise the value of insurance provided by the limited liability of a debt contract. Anticipating this, lenders may demand a higher interest rate to offset the higher risk (Parigi (1992) shows that this moral hazard is mitigated in a multi-period setting when lenders offer a performance related interest rate, which conditions the current period interest rate on the realised project return of the previous period). This type of asymmetric information problem is analogous to that considered by Akerlof (1970) for the second hand car market. When an uninformed buyer cannot distinguish high quality from low quality products, the discount that the buyer requires given his uncertainty forces the sellers of high quality products from the market In order to prevent the disappearance of low risk loan applicants from the market for loans by an increase in interest rates (analogous to the demand for discounts by buyers in Akerlof's problem), credit must instead become rationed as shown by Stiglitz and Weiss.

The criteria that differentiate high from low quality projects, whether it be risk or mean return, also define what a lender considers to be the marginal project. The importance of this definition has been exposed by DeMeza and Webb (1987). The latter study again considers a pooling equilibrium but instead derives an over-investment result in contrast to the under-investment result of Stiglitz and Weiss. For both studies the equilibrium, in which the terms of all loan contracts are homogeneous, involves entrepreneurs with high success probability projects subsidising their low success probability counterparts. However, in DeMeza and Webb the marginal project financed has the lowest success probability whereas in Stiglitz and Weiss the marginal project financed has the highest success probability. For both cases it may be stated that the presence of an ex ante information asymmetry between borrower and lender introduces an inefficiency in the market for loans, the quantitative result of which depends upon how borrowers in the pool are different.

For the studies of the effects of the use of debt contracts in establishing results concerning the supply of credit so far mentioned, the importance of an implicit assumption made therein has been highlighted by Innes (1993b) Innes exposed an underlying assumption of monotonicity. A debt contract awards the lender a payoff which is monotone non-decreasing. This "monotonic contract" constraint can be motivated either by a requirement that investors never have an incentive to sabotage the firm or by an ability of entrepreneurs to costlessly revise their profit reports upwards (with hidden borrowing, for example). Assuming limited liability for both the borrower[1] and the lender[2], and that projects differ in quality in the sense of the monotonic likelihood ratio property[3], Innes showed that a pooling equilibrium would again result with or without a monotonicity constraint on the contract form when investor capital was fixed across quality types. With the constraint, borrowers pool and those with project quality

[1] The borrower cannot allocate more than the realised profit to investors

[2] Investors have limited liability in that their loss is limited to their initial investment

[3] The monotonic likelihood ratio property implies that, for any given profit level, higher quality implies relatively greater probability weight on all higher profit levels Under MLRP, if higher quality leads to greater weight on some profit levels below X, it must also induce a proportionally greater increase in probability weight on all higher profit levels

less than some threshold do not invest in their projects. Without the constraint, a new type of contract form is possible in which lenders receive the entire profit if it is below a certain level and nothing otherwise. This non-monotonic contract[4] form minimises the incentives of low quality borrowers to pool with high quality types, and all high quality types who would previously have accepted a debt contract, defect to the new contract form. Further, if investment capital can vary across quality types, then any variable investment equilibrium contains either a debt contract (if constrained monotone) or the new non-monotonic contract, but both a pooling and a separating equilibrium are possible. If a separating equilibrium obtains, high quality borrowers can use their investment choice to signal their type, but at a cost (which manifests as either over or under investment, see Milde and Riley (1988) below). However, high quality types can reduce these investment signalling costs by choosing the payoff function which minimises the incentive of low quality types to masquerade. The payoff function chosen in the separating equilibrium is the new non-monotonic contract form if the monotonicity constraint is relaxed.

The importance of Innes' (1993b) paper was to characterise a contract form other than a debt contract in the presence of limited liability constraints that would minimise the costs of high quality loan applicants in signalling their type, and therefore best mitigate the adverse selection problem faced by investors. Permitting the amount of borrowings to vary between borrower types was crucial to the signalling process. Milde and Riley (1988) had earlier examined a variable investment equilibrium characterising a separating equilibrium without rationing. Generalising previous work by Jaffee and Russell (1976)[5], Milde and Riley (1988) demonstrated that lenders will screen loan applicants by offering

[4] Innes refers to this contract as a "live-or-die" contract since if the lender receives a return, he receives the entire surplus and the borrower receives nothing (and vice versa).

[5] The key insight in Jaffee and Russell (1976) is that the amount of risk "purchased" by the uninformed lender is dependent on the size of the loan. Therefore, even a perfectly competitive lender will not be indifferent as to loan size Jaffee and Russell then show that there may be no "competitive" Nash equilibrium in such a world

them a schedule of loan amounts and corresponding interest rates[6]. The symmetric information benchmark in Milde and Riley shows that, given a standard debt contract and project risk which varies according to investment amount, the Pareto optimal loan contract awards projects with less than the loan capital which is individually optimal. This is because the marginal increase in interest rate that a lender requires for a marginal increase in loan amount reaches a point, below the loan amount which is individually optimal for the borrower, where it starts to increase faster than the marginal increase in interest rate that the borrower is willing to pay for a marginal increase in loan amount. This no rationing equilibrium (in which loan demand equals loan supply) also obtains when information is asymmetric. Thus in the competitive equilibrium considered, the emphasis is on the way in which higher quality loan applicants (lower risk) will signal their type when information is asymmetric and the way in which this mechanism is determined by how the ex ante uncertain state of nature affects productivity. The production function is taken to be either multiplicative or additive in uncertainty. For multiplicative uncertainty, the marginal increase in interest rate that a loan applicant is willing to accept in order to receive a larger loan is greater for higher quality projects (those with lower risk). In the case of additive uncertainty, the reverse is true. Therefore higher quality applicants can signal their type in either case by their greater or lesser willingness respectively to pay a larger interest rate for a larger loan, leading to a stable multiple contract separating equilibrium without rationing.

From the review above, it is apparent that the debt contract proliferates the literature which characterises optimal financial arrangements between asymmetrically informed borrowers and lenders with liability constraints. Whilst it is true that this contract form is shown to be optimal for ex post asymmetries in very general settings for which state observation is costly to the lender (Townsend (1979), Gale and Hellwig (1985), and Williamson (1987)), it is not apparent from ex ante asymmetry studies that there is

6 Milde and Riley assumed that no profitable (from the lenders viewpoint) pooling contract Pareto dominates the Pareto efficient set of separating contracts

sufficient (if any) reason to impose this contract form[7]. Instead of imposing a debt contract to reward investor capital, which necessarily limits the liability of the borrower, we could equally impose a profit sharing contract providing the borrower with a pay floor, and address similar issues, such as whether an ex ante information asymmetry causes over or under investment In the next section we present a model which we will use to examine the implications of providing a pay floor when a profit sharing rule allocates the ex post return from investment instead of a debt contract.

3.3 The Model

We present the model in two stages. In the first stage we present the production technology and the profit sharing contract, whereupon we briefly discuss the form of the profit sharing contract between investor and borrower, since it contains features which are important to this analysis as well as to some other economic issues (see 3 3.1 below). In the second stage we introduce the game played by the investor and the borrower.

3.3.1 Production technology and Contract form

Let $\theta \in [\underline{\theta}, \overline{\theta}]$ be a multiplicative productivity parameter, such that for capital investment $K(r,\theta)$, which also depends on unit capital cost $r \in [\underline{r}, \overline{r}]$, there exists a production technology $H(K(r,\theta))$ which generates revenue $\theta H(K(r,\theta))$ in productivity state θ. $H(K(r,\theta))$ is increasing in $K(r,\theta)$ at a decreasing rate, i.e. $H'(K(r,\theta)) > 0$ and $H''(K(r,\theta)) < 0$. For per unit cost of capital r, and productivity state θ, the profit from the venture is $\theta H(K(r,\theta)) - rK(r,\theta)$.

[7] The notable exception to this being Innes (1993b), whereby specifically assuming a monotonicity constraint and that quality types differ according to MLRP, establishes the debt contract as an optimal contract form that minimises the cost of adverse selection when ex ante information asymmetry obtains

In allocating realised profit between the investor and the borrower, let $\alpha(r,\theta)\in[0,1]$ denote the deductibility of capital costs for unit capital cost r and productivity state θ. Then we can define the share base, $S(r,\theta)$, being the realised revenue net of deductible capital costs, as

$$S(r,\theta) \equiv \theta H(K(r,\theta)) - \alpha(r,\theta) r K(r,\theta) \tag{1}$$

If $\pi\in(0,1)$ is the proportion of the realised share base which is allocated to the borrower, then the ex post allocation of the borrower, $\varphi(r,\theta)$, is given by

$$\varphi(r,\theta) \equiv \pi(\theta H(K(r,\theta)) - \alpha(r,\theta) r K(r,\theta)) \tag{2}$$

and the agent receives *zero* base wage.

The ex post allocation of the investor, $\rho(r,\theta)$, is therefore given by

$$\rho(r,\theta) \equiv (1-\pi)\theta H(K(r,\theta)) - (1-\alpha(r,\theta)\pi) r K(r,\theta) \tag{3}$$

A profit sharing contract for unit cost of capital r and productivity state θ is defined to be a pair $(\alpha(r,\theta), K(r,\theta))$[8], where we assume that the share ratio π is exogenously determined. A contract menu M is then a set $\{\alpha(r,\theta), K(r,\theta)\}$ for all $\theta\in[\underline{\theta},\bar{\theta}]$ and for all $r\in[\underline{r},\bar{r}]$.

From (1) it is immediate that $\alpha(r,\theta) = 0$ would mean that the surplus generated from investment is allocated according to a pure revenue sharing contract, whilst $\alpha(r,\theta) = 1$

[8] We include $K(r,\theta)$ as an element of the profit sharing contract for productivity state θ and unit capital cost r, since in addition to the rule $(\alpha(r,\theta),\pi)$ allocating ex post profit, where π is exogenous and therefore excluded from the definition of the profit sharing contract, the investment capital determines the borrower pay

would divide the realised surplus according to a pure profit sharing contract. Thus the borrower may share none, some or all of the capital costs, depending upon the deductibility of this cost from the share base.

The motivation for modelling the profit sharing contract to explicitly include the deductibility of capital costs from the share base is that the pay of the borrower is directly affected by the actual cost of capital[9]. Therefore, by an appropriate choice of $\alpha(r,\theta)$, the investor, who supplies the entire investment capital, may bring the incentives of the borrower more in line with his own objectives

We have also deduced that the deductibility of capital costs from the share base impacts at least two more economic problems (see Appendix I for proofs). Drawing from the work of Ross (1973), the first of these examples concerns linear profit sharing contracts and the possibility of achieving Pareto efficiency when investment decisions are made under conditions of uncertainty. It is found when capital costs are not deductible from the share base, that the attainment of Pareto efficiency and the simultaneous utility maximisation of a risk averse principal and risk averse agent by the use of linear share contracts is precluded. In contrast the exact opposite is true when these costs are fully deductible. The second example concerns the effect of the deductibility of capital costs on investment when the investment decision is an unverifiable action delegated entirely to the agent. It is shown that complete deductibility generates a problem of under-investment whereas non-deductibility creates a problem of over-investment[10].

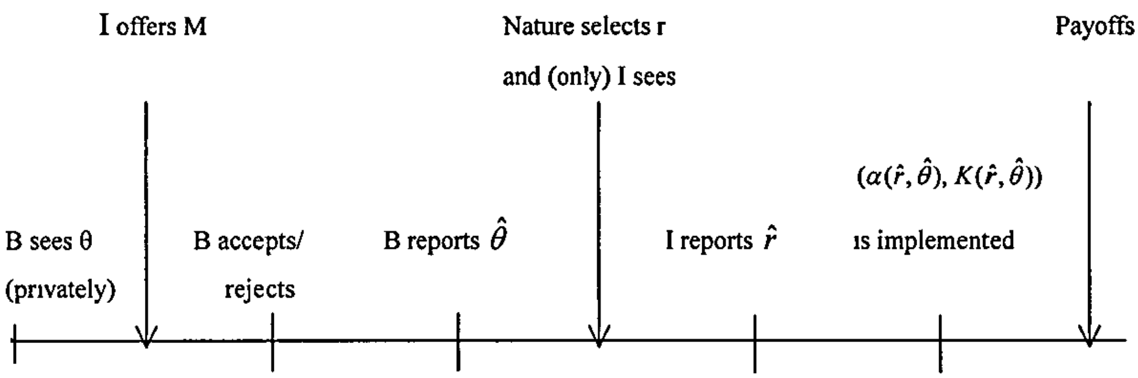
[9]The pay of the borrower is also indirectly affected by the unit capital cost via the dependence of investment capital $K(r,\theta)$ on r It is also important to observe that the only requirement of r is that there exists a cost $rK(r,\theta)$ which is to be apportioned between the investor and the borrower This cost need not therefore be an opportunity cost of capital but is any cost in direct proportion to the capital stock (e g operating costs such as utilities in direct proportion to the scale of plant and machinery)

[10] Note that remote investment, where an investor is unable to inspect capital equipment, is an example of a delegated investment decision which cannot be observed However, the capital stock decision is in reality more likely to be observable.

Some mention of the deductibility of capital costs from the share base is made in the literature on profit/revenue sharing. Most notably, and with symmetrically distributed information, Michaelis (1997) uses a bargaining model to derive conditions under which profit and revenue sharing are equivalent. In his model a firm and union bargain the pay parameters (which are a base wage and share ratio) and in anticipation of this the firm unilaterally determines the investment in capital stock. Michaelis' contribution came about in order to show for the assumptions common to the literature on the effects of profit sharing on employment in a unionised labour economy (see for example Pohjola (1987,1990), Jackman (1988), Hoel and Moene (1988), Palokangas (1992)), that revenue and profit sharing are equivalent, thus making arbitrary the definition of the share base with respect to the deductibility of capital costs. However, for our purposes the deductibility of capital costs is an essential element of the contract menu.

3.3.2 The Game

The essence of the investor's problem lies in the information structure of the environment. There are two key elements First, the realised unit cost of capital (r) is never observed by any party other than the investor. Second, the borrower has private information concerning the productivity state (θ) which is known to the borrower prior to the formalisation of the agreement with the investor. Let us first describe the timing of the game and specifically the revelation of information by reference to the following time-line:



where M = $\{ \alpha(\hat{r},\hat{\theta}), K(\hat{r},\hat{\theta}) \}$ for all $\hat{\theta} \in [\underline{\theta},\overline{\theta}]$ and for all $\hat{r} \in [\underline{r},\tilde{r}]$ is a contract menu (to be derived), and I, B respectively denote the investor and borrower.

The timing of the game is therefore as follows[11]:

First, the borrower acquires private information about the productivity state θ. Second, the investor announces the terms of the contract menu. At this stage both borrower and investor have the same beliefs concerning the possible costs of capital. Third, the borrower then either accepts or rejects the contract menu. Fourth the borrower reports productivity state $\hat{\theta}$. Fifth, nature (randomly) selects the unit cost of capital. Sixth, the investor reports realised cost of capital $\hat{r}$. Then the contract $(\alpha(\hat{r},\hat{\theta}), K(\hat{r},\hat{\theta}))$ is implemented given the reported productivity state and unit cost of capital, after which the investor and the borrower each receive their respective payoffs.

A real world example of when the dynamics of information revelation in our model describes how an investor comes to commit his funds, is where capital investment is deferred to some future date, before which data captured by r is not yet available The delay between contract menu offer/acceptance and the revelation of the true cost of capital to the investor, is reflected by the fact that both investor and borrower have identical knowledge at the outset as to the ex ante probability distribution of ex post unit cost of capital. At the time of revelation of the true unit cost of capital, the returns of alternative investment opportunities forecast at the time of contract menu offer/acceptance which would possibly present themselves as alternatives at the time of actually committing funds, will be known by the investor. It is then that the investor has an incentive to misrepresent such opportunities to the borrower.

[11] Bilateral information asymmetry is discussed in Maskin and Tirole (1990,1992) In their analysis, the principal is informed at the time that he offers a contract to the agent However, as seen from the game studied here, the agent (borrower) not the principal (i e the participant who designs and offers the contract) is informed at the time that the principal (investor) offers the contract menu This point is taken up in 3.6

It is also worth noting that given the borrower was either not willing to commit his own capital, or had insufficient capital to provide the investment capital advanced by the investor, the opportunities available to the investor may in real life be very different to those available to the borrower. To this extent, that knowledge of the true opportunity cost of capital becomes privately endowed only to the investor is consistent with the establishment of the need for the borrower to raise capital in the first instance.

To describe the information structure formally, let $R(r|\theta)$ be the cumulative conditional probability distribution of r given productivity state $\theta \in [\underline{\theta}, \overline{\theta}]$. Then $R_1(r|\theta)$ represents the partial derivative of $R(r|\theta)$ w.r.t. its first argument r. $R_1(r|\theta)$ is the associated conditional density function, and has strictly positive support on $r \in [\underline{r}, \overline{r}]$. To associate high productivity states θ with low costs of capital we would require $R_2(r|\theta) \geq 0$ i.e. higher θ would mean higher $R(r|\theta)$ at any given r, and thus make low r values more probable. However, for the present analysis we will assume that the productivity state θ and the unit cost of capital r are *independent* and note that correlating θ and r allows possible extension to the current analysis. Additionally, let $F(\theta)$ be the (unconditional) distribution function of $\theta \in [\underline{\theta}, \overline{\theta}]$ and let $F_1(\theta)$ be the associated density.

In the next section we derive the optimal contract menu when the investor provides no floor to the pay of the borrower. This will provide a benchmark solution for a subsequent analysis of the effects of the provision of a pay floor for the borrower.

3.4 Optimal contract menu: No pay floor

Before we proceed to derive the optimal contract menu when there is no pay floor for the borrower, a few points are worthy of a brief mention.

Firstly we will invoke the revelation principle in its nested form. That is, in designing the optimal contract menu, we will invoke the revelation principle once at the time that the investor reports realised cost of capital $\hat{r}(r) = r$ after the productivity state is reported by the borrower, and again for the earlier report by the borrower of the realised productivity state $\hat{\theta}(\theta) = \theta$

Secondly, implicit in the revelation of information by the investor is that once the borrower has (truthfully) revealed the actual productivity state, the investor will truthfully report the realised unit cost of capital. If the investor were not to commit to the contract menu that he offered the borrower, then in anticipation of this the borrower might not truthfully reveal θ earlier on in the game, and our analysis would be inappropriate

Thirdly, we derive the optimal contract menus with and without a pay floor using incentive compatibility (i.e. truth-telling) constraints which are locally true. However, since we apply these constraints in a global sense, it is necessary to provide justification for this approach. In order to elucidate the methodology of the derivation of the optimal contract menus, we derive necessary and sufficient conditions for globally applying locally true incentive compatibility constraints in Appendix II. We will again discuss this point when we have arrived at the optimal contract menus.

3.4.1 Derivation of the optimal contract menu

We will construct the investor's problem in stages Firstly consider the incentive compatibility constraint for the investor. Let $\rho(r,\hat{r};\hat{\theta})$ denote the payoff for the investor when the true cost of capital and productivity state are r and θ respectively, and the reported values are $\hat{r}$ and $\hat{\theta}$ respectively (which then means that it is the contract $(\alpha(\hat{r},\hat{\theta}),K(\hat{r},\hat{\theta}))$ which defines actual investment and the allocation rule dividing ex post profits). Then, recalling (3),

$$\rho(r,\hat{r};\hat{\theta}) = (1-\pi)\theta H(K(\hat{r},\hat{\theta})) - (1-\alpha(\hat{r},\hat{\theta})\pi)rK(\hat{r},\hat{\theta}) \qquad (4)$$

We use a technique applied in the derivation of the envelope theorem[12] to derive the investor's local incentive compatibility constraint. For fixed θ and $\hat{\theta}$, by the chain rule

$$d\rho(r,\hat{r};\hat{\theta}) = \frac{\partial\rho(r,\hat{r};\hat{\theta})}{\partial\hat{r}}d\hat{r} + \frac{\partial\rho(r,\hat{r};\hat{\theta})}{\partial r}dr \qquad (5)$$

Evaluating (5) at $\hat{r} = r$ we see that the first term of (5) must be zero since $\hat{r}(r) = r$ locates the local maximum of $\rho(r,\hat{r};\hat{\theta})$ w.r.t. $\hat{r}$ for incentive compatible contracts. Writing $\rho(r,r;\hat{\theta})$ as $\rho(r,\hat{\theta})$ we see that

$$\frac{\partial\rho(r,\hat{\theta})}{\partial r} = \frac{\partial\rho(r,\hat{r},\hat{\theta})}{\partial r}\Big|_{\hat{r}=r} \qquad (6)$$

[12] See for example Dixit (1990)

From (4) and (6) therefore

$$\frac{\partial \rho(r,\hat{\theta})}{\partial r} = -(1-\alpha(r,\hat{\theta})\pi)K(r,\hat{\theta}) \tag{7}$$

Integrating (7) and using (4) again we see that

$$\rho(r,\hat{\theta}) = \rho(\underline{r},\hat{\theta}) - \int_{\underline{r}}^{r}(1-\alpha(s,\hat{\theta})\pi)K(s,\hat{\theta})ds$$

$$= (1-\pi)\theta H(K(r,\hat{\theta})) - (1-\alpha(r,\hat{\theta})\pi)rK(r,\hat{\theta}) \tag{8}$$

Any contract menu satisfying the first equality of (8) will then (locally) assure the borrower that once the investor has learned r he will implement the contract corresponding to that realisation of the unit cost of capital.

The second equality of (8) then gives the deductibility of capital costs from the share base $\alpha(r,\hat{\theta})$ as a function of the optimal invested capital $K(r,\hat{\theta})$ for reported productivity state $\hat{\theta}$. Thus we can turn our attention to the optimal investment schedule $K(r,\theta)$ alone in order to know the optimal contract menu M.

We will now derive the incentive compatibility constraint for the borrower. Consider the payoff that the borrower receives At the time that the borrower reports the productivity state, neither the investor nor the borrower knows the unit cost of capital as it has not yet been realised. Both investor and borrower are however assumed to both have common knowledge of the cumulative distribution of r given by R(r). Therefore at the time the borrower comes to report the realised productivity state, the borrower is motivated by his *expected* utility where the expectation is taken w.r.t. the distribution of r. We are assuming[13] that the investor can commit to eventually report the unit cost of capital truthfully when it becomes realised and therefore only need consider the distribution of r since $\hat{r} = \hat{r}(r) = r$ i.e. the future $\hat{r}$ and r are assumed to coincide so that the only uncertainty we need concern ourselves with is about r.

Let $\varphi(r,\hat{\theta})$ be the borrower utility when unit cost of capital is reported truthfully, and the borrower reports productivity state $\hat{\theta}$ when the true productivity state is θ. Then

$$\varphi(r,\hat{\theta}) = \pi(\theta H(K(r,\hat{\theta})) - \alpha(r,\hat{\theta})rK(r,\hat{\theta})) \tag{9}$$

Let $U^a(\theta,\hat{\theta})$ [14] be the expected utility of the borrower when the reported productivity state is $\hat{\theta}$ and true productivity is θ. Then $U^a(\theta,\hat{\theta})$ is given by

$$U^a(\theta,\hat{\theta}) = \int_{\underline{r}}^{\bar{r}} \varphi(r,\hat{\theta})R_1(r)dr \tag{10}$$

By the chain rule

$$dU^a(\theta,\hat{\theta}) = \frac{\partial U^a(\theta,\hat{\theta})}{\partial\hat{\theta}}d\hat{\theta} + \frac{\partial U^a(\theta,\hat{\theta})}{\partial\theta}d\theta \tag{11}$$

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[13] Without loss of generality (the revelation principle)

[14] In this analysis we assume that effort is observable, the costs of which are written into the reservation utility. We also assume implicitly that the production function is separable in effort and capital, the marginal productivity of effort therefore being independent of θ, with equal effort supplied by all borrower types If this were not the case, the starting point in extending the current analysis to incorporate hidden actions would be $U^a(\theta,\hat{\theta},e) = \varphi(\theta,\hat{\theta},e) - Q(e)$ where the last term is effort disutility Applying the chain rule still gives (11) because effort is maximal for the borrower See Chapter 6 for an analysis of adverse selection with moral hazard due to unobservable effort

Evaluating (11) at $\hat{\theta} = \theta$ we see that for incentive compatible contracts the first term of (11) must be zero since $\hat{\theta}(\theta) = \theta$ locates the local maximum of $U^a(\theta,\hat{\theta})$ Therefore writing $U^a(\theta,\theta)$ as $U^a(\theta)$,

$$\frac{dU^a(\theta)}{d\theta} = \left.\frac{\partial U^a(\theta,\hat{\theta})}{\partial\theta}\right|_{\hat{\theta}=\theta} \qquad (12)$$

From (9), (10) and (12) we see that

$$\frac{dU^a(\theta)}{d\theta} = \int_{\underline{r}}^{\bar{r}} \pi H(K(r,\theta))R_1(r)dr \qquad (13)$$

Integrating (13) gives

$$U^a(\theta) = U^a(\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\int_{\underline{r}}^{\bar{r}} \pi H(K(r,s))R_1(r)drds \qquad (14)$$

where the limits of integration are $r \in [\underline{r},\bar{r}]$ and $s \in [\underline{\theta},\theta]$. From (14), since the second term is non-negative, the participation constraint of the borrower can be written

$$U^a(\underline{\theta}) \geq 0 \qquad (15)$$

provided $K(r,\theta)$ is non-decreasing in θ[15]. Explicit inclusion of the participation constraint in the investor's maximisation problem can therefore be omitted since $U^a(\underline{\theta})$ is a constant.

We now turn to the investor's maximisation problem. The expected utility of the investor is given by

[15] We check that this is true by reference to the optimal contract menu (to be derived) which gives the dependence of K(r,θ) on θ

$$\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \rho(r,\theta)F_1(\theta)R_1(r)drd\theta \tag{16}$$

In Appendix III we show that the investor's expected utility can equivalently be written as

$$\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} (\theta H(K(r,\theta)) - rK(r,\theta))F_1(\theta)R_1(r)drd\theta - \int_{\underline{\theta}}^{\bar{\theta}} U^a(\theta)F_1(\theta)d\theta \tag{17}$$

and that $$\int_{\underline{\theta}}^{\bar{\theta}} U^a(\theta)F_1(\theta)d\theta = U^a(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \pi \frac{F(\theta)}{F_1(\theta)} H(K(r,\theta))F_1(\theta)R_1(r)drd\theta \tag{18}$$

Therefore the investor's expected utility (ignoring the constant $U^a(\bar{\theta})$) is

$$\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} (\theta H(K(r,\theta)) - rK(r,\theta) + \pi \frac{F(\theta)}{F_1(\theta)} H(K(r,\theta)))F_1(\theta)R_1(r)drd\theta \tag{19}$$

Pointwise optimisation[16] of (19) w.r.t. $K(r,\theta)$ gives the first order condition for the optimal investment schedule[17] $K(r,\theta)$ as

$$\left[\theta + \pi \frac{F(\theta)}{F_1(\theta)}\right] H'(K(r,\theta)) - r = 0 \quad \forall\theta \in [\underline{\theta},\bar{\theta}], \forall r \in [\underline{r},\bar{r}] \tag{20}$$

[16] Pointwise optimisation of the integral in (19) means that if K(r,θ) maximises the integrand in (19) over each infinitesimal unit of area drdθ in the (r,θ) plane, then the integral, being the summation of each maximised integrand over the (r,θ) plane, will also be maximised The first order condition (20) is formally established (as (A65) with $r_c = \bar{r}$) using an optimal control approach in Appendix IV

[17] Note that the results in this chapter are robust (at least) to an additive uncertainty with zero mean, i e where profit is instead θH(K(r,θ)) – rK(r,θ) + ε, where the expectation of ε is zero, since all agents are assumed risk neutral

Remark 1: In deriving the first order condition for $K(r,\theta)$ in (20), we have globally applied incentive compatibility constraints (7) and (13), which are locally true conditions In Appendix II we show that the necessary and sufficient conditions which justify this approach are that $K(r,\theta)$ is non-decreasing in θ and $g(r,\theta)$ is non-increasing in r, where $g(s,\theta) \equiv (1-\alpha(s,\theta)\pi)K(s,\theta)$.

We confirm the integrity of our analysis, in which these constraints were not included, by ensuring that these conditions are satisfied by the solution, as given by (8) and (20).

Differentiating (8) w.r.t r yields $g_r(r,\theta)r = (1-\pi)\theta H'(K(r,\theta))K_r(r,\theta)$. However, differentiating (20) w.r.t. r gives $K_r(r,\theta) < 0$. Therefore, $g_r(r,\theta) < 0$, and $g(r,\theta)$ is non-increasing in r.

Differentiating (20) w.r.t. θ yields $K_\theta(r,\theta) = -\frac{H'()}{H''(\cdot)} d\ln\phi(\theta)$ [18]

Therefore, providing $\pi \frac{d}{d\theta}\frac{F(\theta)}{F_1(\theta)} \geq -1$[19], the necessary and sufficient condition that $K(r,\theta)$ is non-decreasing in θ is also satisfied.

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[18] $\phi(\theta) \equiv \theta + \pi\frac{F(\theta)}{F_1(\theta)}$

[19] A sufficient condition for this to be true is that the risk ratio $F(\theta)/F_1(\theta)$ is non-decreasing This condition is satisfied for many practically relevant distributions such as the uniform, normal and exponential This requires that the density function not increase "too quickly" with increasing θ

3.4.2 Discussion

From (20), we may draw some interesting insights into the effects of the information which was privately, and separately revealed to the investor and the borrower.

The first order condition (20) tells us that if the true cost of capital is privately revealed to the investor, then provided the investor can commit (see below) to the terms of the contract menu after the borrower has revealed the true productivity state, it is only the information which is private (at least initially) to the borrower which introduces a distortion into the optimal investment schedule[20]. We may deduce this fact by reference to the first best solution, for which all private information is instead public.

The first best investment schedule satisfies the following first-order condition

$$\theta H'(K(r,\theta)) - r = 0 \qquad \forall \theta \in [\underline{\theta},\bar{\theta}], \forall r \in [\underline{r},\bar{r}] \quad (21)$$

where, knowing the true productivity state θ[21], without having to rely on a truthful revelation of this parameter by the borrower, the investor chooses $K(r,\theta)$ to simply maximise profit $\theta H(K(r,\theta)) - rK(r,\theta)$.

The distortion[22] in the optimal investment schedule causes an *overinvestment* for all productivity states except for the lowest $\underline{\theta}$, due to the extra term $\pi F(\theta)/F_1(\theta)$ in the coefficient of $H'(K(r,\theta))$ in (20), recalling that $H''(K(r,\theta)) < 0$ and $F(\underline{\theta}) = 0$.

[20] By investment schedule we mean a map from the reported productivity state and unit cost of capital, to the capital stock decision, i e capital to be invested

[21] The unit cost of capital would also be publicly observable

[22] By distortion we mean an investment schedule which is not first best and does not therefore maximise profits

The distortion in optimal investment when the investor cannot observe θ may be viewed as an accommodation of the fact that the borrower would have an incentive to attempt to secure greater pay by understating productivity if he was offered the full information solution when in fact the productivity state was private[23]. Thus, in order to overcome this problem, the investor does not award the borrower the profit maximising contract, but one instead for which the pay of the borrower is increased through the award of an information rent. This information rent recognises the "temptation" in the asymmetric information case to pass off the true productivity state as some other lower state, and awards (just[24]) sufficient pay to the borrower to truthfully reveal the actual productivity state[25].

The reason that there is no distortion for the lowest productivity state $\underline{\theta}$, is that for this state there are no other lower states which the borrower may use to his advantage when called to report the actual state. However, the borrower does receive an information rent[26] for all productivity states other than the lowest The information rent is greater for higher productivity states because the potential gain from reporting the true state as one lower than itself increases as the number of states lower than the true state increases

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[23] Consider the full information solution For the two productivity states θ and $\hat{\theta}$, where $\theta > \hat{\theta}$, the borrower receives his reservation utility equal to $U^a(\hat{\theta},\hat{\theta}) = \int_{\underline{r}}^{\bar{r}} \varphi(r,\hat{\theta}) R_1(r)dr = U^a(\theta,\theta) = \int_{\underline{r}}^{\bar{r}} \varphi(r,\theta) R_1(r)dr$. However, if the investor offered the same contracts when information was in fact asymmetric, then the borrower would report the lower state $\hat{\theta}$ when the true state was θ as well as when it was $\hat{\theta}$. To see this, $U^a(\theta,\hat{\theta}) - U^a(\theta,\theta) = U^a(\theta,\hat{\theta}) - U^a(\hat{\theta},\hat{\theta})$ $= \int_{\underline{r}}^{\bar{r}} (\varphi(r,\theta,\hat{\theta}) - \varphi(r,\hat{\theta},\hat{\theta}))R_1(r)dr = \int_{\underline{r}}^{\bar{r}} \pi(\theta-\hat{\theta})H(K(r,\hat{\theta}))R_1(r)dr > 0$.

[24] The information rent is just sufficient in the optimal solution to elicit truthful reporting by the borrower.

[25] This is what makes the contract menu incentive compatible w r t. truthful information reporting by the borrower

[26] This information rent equals $\int_{\underline{\theta}}^{\theta}\int_{\underline{r}}^{\bar{r}} \pi H(K(r,s))R_1(r)drds$

Consequently, the higher the true productivity state θ, the greater is the distortion of investment away from its efficient level in order to minimise the information rent that the borrower receives.

The effect of the information which is privately revealed to the investor can be seen by considering the set of feasible contract menus from which the investor is able to choose. In the absence of a public observation of the unit cost of capital, the borrower will only be willing to accept a contract menu for which the investor would have no incentive to misreport the true capital costs. These are the contract menus which satisfy the investor's incentive compatibility constraint, as given by (8). Therefore, requiring the investor to truthfully reveal his private observation of the unit cost of capital creates a welfare loss (see O. Hart (1983) for a discussion of this point) by restricting the space of feasible mechanisms (contract menus), and not by creating a distortion in the optimal investment schedule.

This latter point leads us to an important assumption of the analysis. We have assumed that the investor commits to the contract menu which the borrower accepts. However, it is interesting to note the sequential order of events in the game The borrower (truthfully) reports the actual productivity state after which capital is invested. However, were the investor able to use the optimal contract derived above to elicit truthful reporting of the actual productivity state and then *replace* this contract by one for which investment was efficient (i e capital only invested up to the point where marginal return equals marginal cost), additional gains from investment, which are to be shared between the investor and borrower, would be available (for contract renegotiation and credibility see Baron (1989), Hart and Tirole (1988) and Laffont and Tirole (1990)). This arrangement would be beneficial to both parties to the contract and would therefore (at least at a first glance) provide a superior equilibrium. The problem that arises however with such an arrangement is one of credibility. Were the borrower to be aware that the original contract would be torn up and replaced by a new one once he had truthfully declared the productivity state, the initial incentive to truthfully report the realised productivity state would vanish and we would be back to square one whereby the investor is effectively offering the borrower the symmetric information contract when in fact information about the productivity state was not publicly observable.

3.5 Optimal contract menu: With a pay floor

We now turn to consider the case of a borrower for which the investor provides a pay floor. In the next sub-section we extend the analysis of the previous section, and then discuss the results.

3.5.1 Derivation of the optimal contract menu

Suppose for any possible (r,θ) pair that the minimum payoff that the borrower can receive is -L[27], where L is a constant. This means that for each realisation of the productivity state θ there exists an $r \in [\underline{r}, \bar{r}]$ such that for unit cost of capital realisations greater than this critical r value the borrower payoff is a constant -L (the floor level). Since the profit is increasing in the productivity state θ and decreasing in the unit cost of capital r, we expect that this critical value of r is increasing in θ, i.e. the more favourable is the productivity state to investment the higher must be the unit cost of capital above which the borrower's liability becomes limited to -L.

[27] The analysis which proceeds is independent of the sign of L. However, it is preferable to think of L as non-positive, since the ex post profit is known prior to the sinking of capital Assuming that the an investor would sink funds into a venture which he knows to be loss making undermines the assumption that there is no renegotiation after the borrower has accepted the contract menu

This way of modelling limited liability for the borrower can now be used to derive the incentive compatibility constraint with a pay floor at -L. Define the critical value of unit cost of capital r_c at which the pay floor of a type θ borrower reporting type $\hat{\theta}$ becomes binding, by the following equation

$$\varphi(r_c,\theta,\hat{\theta}) = \pi(\theta H(K(r_c,\hat{\theta})) - \alpha(r_c,\hat{\theta})r_c K(r_c,\hat{\theta})) = -L \qquad (22)$$

For some fixed π and L, (22) explicitly defines r_c as a function of θ and implicitly as a function of $\hat{\theta}$ through K(r, $\hat{\theta}$) and α(r, $\hat{\theta}$). Equation (22) also states that $r_c = r_c(\theta, K, \alpha)$.

Once the borrower has seen θ he reports $\hat{\theta}$. At this stage the expected utility of the borrower is $U^a(\theta,\hat{\theta})$ where

$$U^a(\theta,\hat{\theta}) = \int_{\underline{r}}^{r_c} \varphi(r,\theta,\hat{\theta})R_1(r)dr + \int_{r_c}^{\bar{r}} - L.R_1(r)dr$$

$$= \int_{\underline{r}}^{r_c} \varphi(r,\theta,\hat{\theta})R_1(r)dr + [R(r_c) - R(\bar{r})]L \qquad (23)$$

Differentiating (23) w.r.t. θ gives

$$\frac{\partial U^a(\theta,\hat{\theta})}{\partial\theta} = \varphi(r_c,\theta,\hat{\theta})R_1(r_c)\frac{\partial r_c}{\partial\theta} + \int_{\underline{r}}^{r_c} \frac{\partial\varphi(r,\theta,\hat{\theta})}{\partial\theta}R_1(r)dr + R_1(r_c)\frac{\partial r_c}{\partial\theta}L \qquad (24)$$

But from (22) the first term on the right hand side of (24) is $-L.R_1(r_c)\frac{\partial r_c}{\partial\theta}$.

Therefore

$$\frac{\partial U^a(\theta,\hat{\theta})}{\partial\theta} = \int_{\underline{r}}^{r_c} \frac{\partial\varphi(r,\theta,\hat{\theta})}{\partial\theta}R_1(r)dr \qquad (25)$$

Differentiating $\varphi(r,\theta,\hat{\theta}) = \pi(\theta H(K(r,\hat{\theta})) - \alpha(r,\hat{\theta})rK(r,\hat{\theta}))$ (9) w.r.t. θ and setting $\hat{\theta}$ equal to θ in (25) gives the borrower's incentive compatibility constraint via (12) as

$$\frac{dU^a(\theta)}{d\theta} = \int_{\underline{r}}^{r_c} \pi H(K(r,\theta))R_1(r)dr \tag{26}$$

For incentive compatible contracts, from (22) with $\hat{\theta}$ equal to θ, r_c is given by

$$\varphi(r_c,\theta) = \pi(\theta H(K(r_c,\theta)) - \alpha(r_c,\theta)r_c K(r_c,\theta)) = -L \tag{27}$$

Integrating (26) gives

$$U^a(\theta) = U^a(\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\int_{\underline{r}}^{r_c} \pi H(K(r,s))R_1(r)drds \tag{28}$$

where the limits of integration are $r \in [\underline{r}, r_c]$ and $s \in [\underline{\theta}, \theta]$. Thus, since the second term in (28) is non-negative provided $K(r,\theta)$ is non-decreasing in θ[28], the participation constraint of the borrower can be replaced by

$$U^a(\underline{\theta}) \geq 0 \tag{29}$$

The participation constraint of the borrower is again excluded from hereon since $U^a(\underline{\theta})$ is a constant.

Now in exactly the same way as we derived (20), from (26)

$$\int_{\underline{\theta}}^{\bar{\theta}} U^a(\theta)F_1(\theta)d\theta = U^a(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{r_c} \pi \frac{F(\theta)}{F_1(\theta)} H(K(r,\theta))F_1(\theta)R_1(r)drd\theta \tag{30}$$

[28] See footnote no 15

We now write the investor's expected utility as (see (17))

$$\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{r}}^{\overline{r}}(\theta H(K(r,\theta)) - rK(r,\theta))F_1(\theta)R_1(r)drd\theta - \int_{\underline{\theta}}^{\overline{\theta}}U^a(\theta)F_1(\theta)d\theta \qquad (31)$$

Substituting (30) into (31) then gives the investor's expected utility (ignoring the constant $U^a(\overline{\theta})$) as

$$\int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{r}}^{r_c}(\theta H(K(r,\theta)) - rK(r,\theta) + \pi\frac{F(\theta)}{F_1(\theta)}H(K(r,\theta)))F_1(\theta)R_1(r)drd\theta$$
$$+\int_{\underline{\theta}}^{\overline{\theta}}\int_{r_c}^{\overline{r}}(\theta H(K(r,\theta)) - rK(r,\theta))F_1(\theta)R_1(r)drd\theta \qquad (32)$$

Pointwise optimisation[29] of (32) w r.t $K(r,\theta)$ gives the first order condition for the optimal investment schedule $K(r,\theta)$ ∀θ∀r as

$$\int_A\left[\left(\theta+\pi\frac{F(\theta)}{F_1(\theta)}\right)H'(K(r,\theta)) - r\right]R_1(r)dr + \int_{A^c}[\theta H'(K(r,\theta)) - r]R_1(r)dr$$
$$+\pi\frac{F(\theta)}{F_1(\theta)}H(K(r_c,\theta))R_1(r_c)\frac{\partial r_c}{\partial K} = 0 \qquad (33)$$

where $r_c = r_c(\theta,K,\alpha)$ from (27), and A, A^c are the ranges of r defined by $A = \{r \mid r \in [\underline{r}, r_c(\theta,K,\alpha)]\}$ and $A^c = \{r \mid r \in (r_c(\theta,K,\alpha),\overline{r})\}$ respectively.

[29] A formal derivation of the first order condition (33) using an optimal control approach is given in Appendix IV.

In order to observe what the first order condition (33) states, invoke as a benchmark the optimal investment schedules for each of two extremes, being a pay floor that is always binding and the opposite case of a pay floor which never binds.

From (33), if the pay floor never binds, $A^c = \{\varnothing\}$, $r_c(\theta, K, \alpha) = \bar{r}$ (a constant), and therefore

$$\left[\theta + \pi \frac{F(\theta)}{F_1(\theta)}\right] H'(K_{nb}(r,\theta)) - r = 0 \qquad \forall\theta\forall r \quad (34)$$

Alternatively, if the pay floor always binds, then $A = \{\varnothing\}$, $r_c(\theta, K, \alpha) = \underline{r}$, and

$$\theta H'(K_{ab}(r,\theta)) - r = 0 \qquad \forall\theta\forall r \quad (35)$$

Since the critical unit capital cost varies with investment, such that the third term in (33) is non-zero[30], the optimal investment schedule $K(r,\theta)$ given by (33) takes values for each r and θ which are between the investment levels corresponding to the schedules which are optimal when the pay floor always binds ($K_{ab}(r,\theta)$) and when the pay floor never binds ($K_{nb}(r,\theta)$), i.e. $K_{ab}(r,\theta) \le K(r,\theta) \le K_{nb}(r,\theta)$ ∀θ∀r

Diagrammatically, Figures 3.1 and 3.2 sketch the variation of $K(r,\theta)$ with r and θ respectively[31] (the unbroken curve).

[30] Since the third term in (33) is non-zero, a solution to (33) which is unadmissible is an investment rule given by (34) for $r \in A$ and a rule given by (35) for $r \in A^c$, leading to a discontinuity in $K(r,\theta)$ at $r = r_c$

[31] The shapes of the sketches for the benchmark cases (pay floor always or never binds) are derived in Appendix V

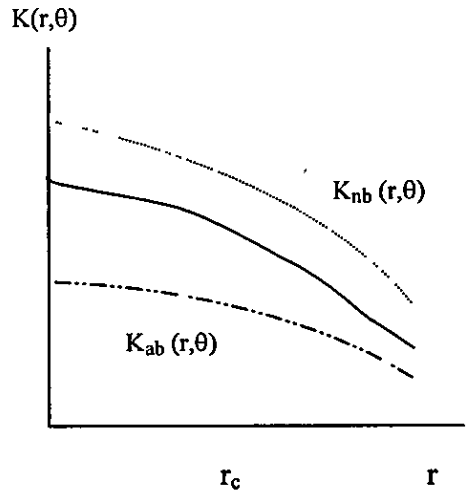

Figure 3.1

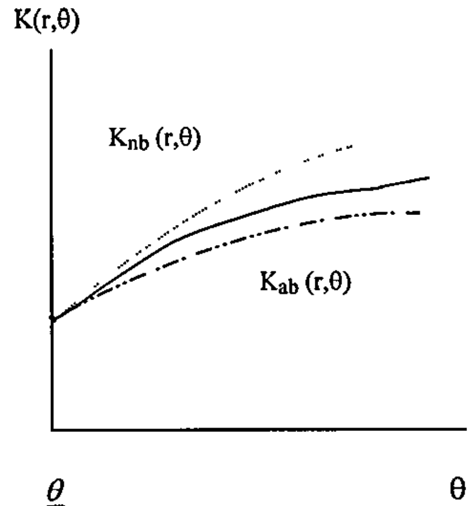

Figure 3.2

For a monotonic optimal investment schedule given by (33), we also note the following remark.

Remark 2: The necessary and sufficient conditions for the global applicability of locally true incentive compatibility constraints (7) and (13) are $K_\theta(r,\theta) \geq 0$ and $K_r(r,\theta) \leq 0$ (see Remark 1 for the equivalence of conditions $K_r(r,\theta) \leq 0$ and $g_r(r,\theta) \leq 0$) From Figures 3.1 and 3 2 we see that these monotonicity conditions are satisfied for the optimal investment function with a pay floor which is sometimes ($\exists r_c < \bar{r}$) binding.

3.5.2 Discussion

The optimal investment schedule given by (33) has an intuitive economic rationale which derives from the way in which the incentive compatibility constraint of the borrower (26) introduces distortions into the investment schedule. In the case of unlimited liability where the pay floor is never binding, the borrower will participate ex ante[32] (provided (15) is satisfied), and receive pay which depends on the actual productivity state for all possible $r \in [\underline{r}, \bar{r}]$, irrespective of the actual realised unit cost of capital.

However, when the liability of the borrower is limited for $r \in [r_c, \bar{r}]$ realisations, the pay of the borrower is fixed and equal to –L over this range of r. If the borrower knew ahead of time that he were to receive a fixed payoff, then ex ante he would have no incentive to report $\hat{\theta} \neq \theta$ (here as well as for all the previous analysis there is an implicit assumption that unless the borrower has an incentive to untruthful revelation he will report truthfully). Since the borrower is uncertain as to the future realisation of r, this will only

[32] Note that it is more accurate to describe borrower participation as ex post w r t θ but ex ante w r t r

apply to the borrower in an expectations sense over the range of possible r for which his payment would be fixed, given his knowledge of the actual θ.

Since the investor introduces distortions away from the efficient investment level by overinvesting in order to award the borrower an information rent which (just) induces the borrower to truthfully reveal the actual productivity state (when his pay varies with the productivity state), the investor can reduce the distortions necessary for incentive compatibility if there exist possible future realisations of unit capital costs for which given the actual productivity state θ, the pay of the borrower would be constant.

In fact, were the pay floor of the borrower to always bind for all possible realisations of unit capital cost, then no information rents are awarded ex ante given that the borrower would be unable to benefit from his private information. In this case the borrower would receive the floor pay with certainty and truthfully reveal θ, with the first best (θ public) investment schedule prevailing (as given by (35)).

In summary since there exist some unit capital cost realisations for which the borrower expects to receive a fixed payment, the incentive compatibility constraint will not precipitate a distortion in the investment schedule for this range of future outcomes. This is because the optimal investment schedule minimises the information rents that the borrower can command, but rents exist only to induce truthful reporting when the borrower can gain by misrepresenting the true productivity state. Reducing the sensitivity of the pay of the borrower to the productivity state, which is private to the borrower prior to the contract, therefore serves to reduce the possible gains from misrepresentation and consequently the inefficiencies introduced into the investment schedule.

Another important point should be mentioned. Introducing a pay floor for the borrower has reduced the distortionary affect upon the optimal investment schedule for all realisations of unit cost of capital. This creates a total welfare enhancement. However, the welfare enhancement is second order in the sense that it relates to the first order condition for optimal investment (c.f. (34) and (35)). The reduction in expected utility for the investor from insuring the borrower against a pay below the floor level, will exceed his share of welfare gain from the second order improvement in the efficiency of investment, since providing the insurance creates a first order loss for the investor, if the floor is binding. Overall, introducing the pay floor therefore reduces the expected utility of the investor.

To be more precise, let ρ_f and ρ_{nf} denote the expected utility of the investor with and without a pay floor. Denote $K(r,\theta)$ the solution to the first order condition (33) and $K_{nb}(r,\theta)$ the solution to first order condition (34). Abbreviating $R_1(r)F_1(\theta)drd\theta$ by $d\tau$, and the profit for productivity state θ, unit capital cost r, and investments K and K_{nb} by X[33] and X_{nb} respectively, gives (see (2) and (3))

$$\rho_f = \int_{\Sigma_\theta}\int_{A}((1-\pi)X - \pi rK(1-\alpha))d\tau + \int_{\Sigma_\theta}\int_{A^c}(X+L)d\tau \tag{36}$$

and

$$\rho_{nf} = \int_{\Sigma_\theta}\int_{\Sigma_r}((1-\pi)X_{nb} - \pi rK_{nb}(1-\alpha))d\tau \tag{37}$$

Subtracting (37) from (36) gives (with slight manipulation) $\rho_f - \rho_{nf}$ equal to

$$\int_{\Sigma_\theta}\int_{A}((1-\pi)(X-X_{nb}) - \pi r(1-\alpha)(K-K_{nb}))d\tau + \int_{\Sigma_\theta}\int_{A^c}(X-X_{nb})d\tau - \int_{\Sigma_\theta}\int_{A^c}(-L-\varphi_{nf})d\tau \tag{38}$$

where $\varphi_{nf} \equiv \pi(X + rK(1-\alpha)) = \pi(\theta H - \alpha rK)$

Observe that $K_{ab}(r,\theta) \le K(r,\theta) \le K_{nb}(r,\theta)$, and that $K_{ab}(r,\theta)$ (as given by (35)) maximises profit Therefore, the first and second terms in (38) are positive, since profit

[33] $X(\theta,r,K) = \theta H(K) - rK$

falls as capital investment is increased beyond first best, i.e. $X > X_{nb}$ and $K < K_{nb}$. The first and second terms in (38) are also the investor's share of the second order gain in welfare from providing a floor which reduces the (ex post) inefficiency in investment.

The sum in the braces of the last term (38) is positive The sum of these terms is the difference between the floor pay –L, and the pay that the borrower would receive conditional on $r \in [r_c, \bar{r}]$ without the floor. Since –L exceeds the pay of the borrower for all $r \in [r_c, \bar{r}]$ without the floor, the sum of the terms in braces is positive. Hence, the last term in (38) is the first order loss in investor utility through providing a floor.

Since the introduction of a pay floor for the borrower reduces the expected utility of the investor, given that both the investor and the borrower have identical precontractual beliefs about the unit cost of capital, we predict that the investor would not offer a pay floor unless he received a compensatory subsidy for doing so. However, were the investor to have (relatively) superior beliefs to the borrower concerning the probability distribution of r, we might be tempted to think that the investor could devise a mechanism whereby he would benefit from this superior information.

In the next section we examine the issues raised if the investor has superior precontractual information, before collecting concluding remarks.

3.6 Signalling phenomena

Throughout the discussion of investment and asymmetric information in this chapter, we have assumed that prior to the design of the contract mechanism[34] the investor has identical beliefs to the borrower concerning the probability distribution of unit capital

[34] which is a direct revelation mechanism since it induces the borrower to reveal the actual marginal productivity of capital by the way in which he chooses between contracts in the contract menu

costs. It is however more realistic to assume that, even if the investor were uncertain about what the realised unit (opportunity) cost of capital would be, that he would at least maintain superior beliefs[35] in comparison to those of the borrower[36].

Before concluding this chapter, we assess the implications of admitting signalling phenomena into the game between the investor and the borrower, i.e. the investor has precontractual information not endowed to the borrower which may be signalled by the contract menu offered[37].

In order to focus on the determinant features of a game which admits this phenomenon, also suppose (temporarily) that the marginal productivity of capital is public and that the realised unit cost of capital (which is ex ante uncertain) is observed symmetrically by both the investor and the borrower[38]. The investor's type is defined by the (superior) beliefs which he holds about the probability distribution of unit capital costs.

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[35] For example, the investor and the borrower share the same expected value of r, but the investor is correctly able to assign greater probability to values of r close to the mean relative to the probability mass that the borrower assigns to the same range of values

[36] It is trivial to show (by following exactly the procedure presented in this chapter) that if the beliefs of the investor and the borrower about r are given by $R^{I}(r)$ and $R^{b}(r)$ respectively, then the risk ratio term in the optimal contract menu is replaced by the adjusted term $\frac{F(\theta)}{F_1(\theta)}\frac{R_1^{b}(r)}{R_1^{I}(r)}$. However, a contract menu which includes this term (implicitly in K) for a borrower with beliefs given by $R^{b}(r)$ will immediately signal $R^{I}(r)$

[37] The beliefs of the borrower about the type of investor that he faces which result following (Bayesian) updating with information signalled by the contract menu are referred to as "interim" beliefs

[38] As seen in 3.4 2, the only effect of the investor's private observation of the realised unit capital cost is that the set of feasible mechanisms is reduced with no distortionary effect on investment The symmetric observation of realised unit capital costs therefore removes obfuscation of the key issues discussed in this brief digression

Maskin and Tirole (1992) examine a three stage game in which the investor's private information is either an argument of the borrower's utility function or of the probabilities that the borrower assigns to the variables entering his utility function[39]. In this ("common values") game, the investor offers a contract which can be either accepted or rejected by the borrower. If accepted, the contract is executed, in which each party carries out the contracted actions (which are observable) after which allocations are made according to the contract. The borrower has *no* private information.

The conclusion reached by Maskin and Tirole (1992) is that the investor cannot gain by withholding private information at the contract offer stage, and may not even be able to secure the payoff that he would receive were his information made public at that time.[40] An example of this type of scenario is that considered by Spence (1973,1974), in which a highly productive employee (acting as principal[41] by offering a contract to a potential employer) may be forced to invest in wasteful signalling[42] activity (achieving education that does not enhance his productivity) in order to be set apart from less efficient potential employees This is a common values example because the private information of the employee relates to his productivity which directly affects the payoff of the employer.

Further, if the borrower also has private information, the conclusions (and analysis) in Maskin and Tirole (1992) remain unaffected[43]. The intuitive reason for this is the following. From Maskin and Tirole (1990), if different investor types were to pool at the

[39] The opposite case studied in Maskin and Tirole (1990) is that of "private" values, in which the investor's private information is *neither* an argument of the borrower's utility function *nor* of the probabilities that the borrower assigns to the variables entering his utility function

[40] This result is in contrast to the private values case in which the investor can do strictly better than if his information were public at the contract offer stage Instead, the investor may prefer to conceal his private information at the contract offer stage and reveal it during the execution of the contract

[41] the economic actor who designs and offers a contract

[42] The distinction between signalling and screening is that in the former the informed party offers the contract and in the latter it is the uninformed party offers the contract (Maskin and Tirole (1992) p 29)

[43] This is only true when the preferences of the investor and borrower are linear in each of their allocations

contract offer stage by offering the same contract menu (thereby withholding information about their type), then the participation and individual rationality constraint for each type of borrower would only have to be satisfied in expectation over the different types of possible investor which the borrower faces, where the distribution is w.r.t. the (Bayesian) updated beliefs of the borrower[44]. It is therefore possible for the case of a risk averse investor and borrower, that the investor could raise his utility above the full-information[45] level (where the incentive compatibility and participation constraints of the borrower must each hold separately for each type of investor) by violating some constraints, as long as these violations are offset by the violation of the constraints for some other types of investor[46]. However, if instead both investor and borrower are risk neutral w r.t. monetary transfers, the shadow prices of the incentive compatibility and participation constraints of the borrower are equal for each type of investor, in which case the investor neither gains nor loses if the borrower has private information, since there is no gain available from "trading slack" on these constraints

Lastly, we can draw an analogy between the results of this chapter and the signalling case of Maskin and Tirole (1992) We concluded above that when the informational advantage of the investor exists at the contract stage, then (for the "common values" case) the investor may not be able to secure his full-information payoff[47]. In the bi-lateral information game studied in this chapter, we have also arrived at this result by concluding that when the investor must commit to a mechanism which induces him to truthfully reveal the unit capital costs when they become realised, that a welfare loss results given the restriction in the space of feasible mechanisms, with no contributory

[44] Which equate to the precontractual beliefs given pooling by different investor types.

[45] In which the investor truthfully declares his type from the outset

[46] Maskin and Tirole (1990) suggest that in essence, different types of investor trade "slack" with one another one type accepts some slack on the participation constraint, whereas another type does exactly the opposite

[47] This result may now be seen to essentially derive from a conflict which arises between investors of different types In the Spencian education model (1973, 1974), high productivity employees do not wish to be pooled with their low productivity counterparts.

distortion in investment. This result is in accordance with that of Maskin and Tirole (1992), albeit that the timing of the arrival of information is different.

We now turn to collect concluding remarks.

3.7 Concluding remarks

In this chapter we undertook an investigation of the effects of awarding a pay floor for a borrower who could privately observe a multiplicative productivity parameter that determined the capital invested by an investor with whom the borrower shared revenue net of some proportion of the costs of capital

We found that in the absence of a floor, a problem of overinvestment arose, because the borrower was awarded an information rent in order to induce him to truthfully reveal the actual marginal productivity of capital. This information rent caused capital to be invested past the point at which its marginal product equalled its marginal cost. However, it was shown that introducing a floor to the pay of the borrower could be expected to mitigate this problem at the expense of a net welfare loss to the investor.

Notwithstanding the significant differences[48] between limited liability profit sharing contracts and debt contracts, some comparison can be made with the literature on ex ante

[48] An example of this can be seen from the following observation For a debt contract, the variability in pay that borrower and lender may receive occur over mutually exclusive ranges of the possible profit outcomes Absent bankruptcy, the lender receives a fixed pay whilst the borrower is sole residual claimant to the outcome net of the fixed financing cost Contingent on bankruptcy, the lender is sole residual claimant whilst the borrower receives a constant zero payoff However, for limited liability sharing contracts, the range of outcomes over which payoffs to each party are variable are not mutually exclusive Absent bankruptcy *both* lender and borrower necessarily receive variable pay, the lender no longer receiving a fixed fee but instead sharing in profit.

information asymmetries when loan size may vary between types[49]. Specifically, Milde and Riley (1988) show that higher quality loan applicants, with greater marginal productivity of capital, can signal their type to competitive[50] lenders by accepting larger loans than would be the case if their type was publicly observable[51]. This accords with the result derived in this chapter in which capital is overinvested. However, the impact of a pay floor is to alleviate overinvestment, and therefore any positive relation between these results is essentially due to similarity in the information structure of the models in which these problems are analysed

Additionally, Milde and Riley (1988) state that increasing the borrowing cost per unit of capital at all loan sizes will decrease the loan size for all types. For a profit sharing contract, the analogous change would be a decrease in the share ratio (π), thereby reducing the surplus allocated to the borrower for all outcomes in which the pay floor does not bind. Since overinvestment is increasing in the share ratio (see (33) and (34)), increasing the cost of loanable funds by reducing the share ratio will decrease investment when the pay floor is not binding[52]. This observation is therefore a positive comparison.

Lastly, we mention an important point concerning information which is private to the borrower. In the problem we considered in this chapter, we assumed that the utility of the borrower was separable in money and effort, and absorbed the cost of effort into the reservation utility. The separability assumption together with an endowment of private information that directly affected the profit available for allocation, meant that the

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[49] Type refers to θ in both studies

[50] In our study, lenders are non-competitive as they may retain a positive (expected) surplus

[51] Note that the loan cost schedule in Milde and Riley (1988) specifies a per unit capital cost which is increasing in loan size for all types

[52] Note that the appropriate comparison here is between the share contract in which the pay floor is non binding, and a debt contract in which the promised payment can be made. This allows a comparison of the affects of an increase in the cost of loanable funds upon the surplus retained by the borrower for each contract type.

inefficiencies introduced by information private to the borrower were affected by the imposition of a floor to borrower pay.

However, consider instead a problem[53] in which the only information which is private to the borrower is about his effort disutility[54]. The investor must design a contract menu which is a schedule of borrower pay and associated effort (observable) for each type[55] of borrower. The inefficiency which is introduced attempts to limit the information rent that borrowers with low effort disutilities may earn by being able to pass themselves off as high effort disutility types if the full information contract is offered when types are privately known.

However, introducing a floor to borrower pay in this example would not affect the inefficiency caused by the private information of the borrower, since the incentive compatibility constraint is unaffected[56] by the allocation of ex post profit to the borrower[57].

In contrast, were utility to be non-separable in money and effort, in which case the utility of a given amount of money that the borrower enjoys also depends on the effort that he expends, then the imposition of a floor to agent pay would affect the inefficiencies[58] created by information about effort disutility being privately endowed to the borrower.

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[53] this problem is analysed in Macho-Stadler and Perez-Castrillo (1997)

[54] i e a parameter k where the disutility of effort e is kQ(e) and Q(e) is increasing in e at an increasing rate

[55] as given by k

[56] specifically, if $\varphi(k,\hat{k}) = w(\hat{k}) - kQ(e(\hat{k}))$, where $w(\hat{k})$ is the allocation of ex post profits to the borrower for type $\hat{k}$, then $d\varphi(k,\hat{k}) = \frac{\partial\varphi(k,\hat{k})}{\partial k}dk + \frac{\partial\varphi(k,\hat{k})}{\partial\hat{k}}d\hat{k}$ and $\frac{d\varphi(k)}{dk} = -Q(e(k))$ for truthful revelation The incentive compatibility constraint is therefore unaffected by the form of $w(\hat{k})$.

[57] For comparison also note a causality relation, in which imposing a floor (ceiling) to borrower effort induces a floor (ceiling) to borrower pay

[58] whereupon the borrower receives more than the value of his marginal product.

Separability of utility between money and effort is therefore a key assumption in understanding the impact of pay floors upon the economics of information asymmetry.

Appendix I

Deductibility of capital costs from the share base

The motivation for using the deductibility of capital costs from the share base will be illustrated by two examples. The first invokes consideration of the Pareto efficiency of linear sharing contracts when the investor and agent are both risk averse and the second shows how the deductibility of capital costs from the share base might lead us to expect to encounter either an over or under investment problem when investment decisions are delegated.

(A) Pareto efficiency and risk averse participants

We follow the analysis of decision making under uncertainty developed by Ross (1973) Consider a risk averse investor and a risk averse agent with state independent utility functions $\Omega^p(\cdot)$ and $\Omega^a(\cdot)$ respectively, where $\Omega^{p\prime}(\cdot) > 0$, $\Omega^{p\prime\prime}(\cdot) < 0$, $\Omega^{a\prime}(\cdot) > 0$, and $\Omega^{a\prime\prime}(\cdot) < 0$.

Let $H(\varepsilon, K)$ be a production function depending on the ex ante uncertain productivity state ε and capital stock K, where $H_\varepsilon(\varepsilon, K) > 0$ and $H_K(\varepsilon, K) > 0$ and production output has unit value. Denote $E(\cdot)$ the expectation operator w r t. the equivalent subjective probability beliefs held by both agent and investor. Also denote $\varphi(\cdot)$ a fee schedule for the agent (i.e. a mapping of production value net or gross of capital costs to agent remuneration).

If r is the unit cost of capital and the capital cost is fully deductible from the share base, then the optimal choice of investment for the investor and agent is given by the solution to the following single variable maximisation problem:

Investor: $\max_{K} \quad E\Omega^{p}[H(\varepsilon, K) - \varphi(H(\varepsilon, K) - rK) - rK]$ (A1)

Agent: $\max_{K} \quad E\Omega^{a}[\varphi(H(\varepsilon, K) - rK)]$ (A2)

If the same investment decision can maximise the well-being of both agent and investor then we equate the first order conditions from (A1) and (A2) such that

$$E\Omega^{p\prime}(\cdot)[H_K - \varphi'(\cdot)(H_K - r) - r] = E\Omega^{a\prime}(\cdot)[\varphi'(\cdot)(H_K - r)] = 0 \qquad \text{(A3)}$$

Ross then states (for an isomorphic problem) that for (A3) to be true for all possible fee schedules we must have

$$\Omega^{p\prime}(\cdot)[H_K - \varphi'(\cdot)(H_K - r) - r] = \Omega^{a\prime}(\ )[\varphi'(\cdot)(H_K - r)] \qquad \forall\varepsilon \qquad \text{(A4)}$$

Now specify that the fee schedule is Pareto efficient such that the schedule maximises a linear combination of investor and agent expected utility. Then

$$\max_{\varphi(\cdot)} \quad E[\Omega^{p}(H(\varepsilon, K) - \varphi(H(\varepsilon, K) - rK) - rK) + \beta\Omega^{a}(\varphi(H(\varepsilon, K) - rK)\ ]$$

$$\Rightarrow \qquad \Omega^{p\prime}(\cdot) = \beta\Omega^{a\prime}(\cdot) \qquad \text{(A5)}$$

where β is a constant.

Therefore, if the investment decision maximises the expected utility of both agent and investor and the fee schedule is Pareto optimal, then (A4) and (A5) yields

$$[H_K - r][\beta(1 - \varphi') - \varphi'] = 0 \qquad \forall\varepsilon \qquad \text{(A6)}$$

From (A6) we see that when the investor and agent share the cost of capital and $[H_K - r] \neq 0$, then a linear fee schedule is pareto optimal when the investment decision maximises the expected utility of both agent and investor (since $\beta(1 - \varphi') - \varphi' = 0$ implies that φ' = constant and $\varphi(W) = aW + b$, where a and b are constants).

Consider next that instead of the cost of capital being fully deductible from the share base (i.e. the share base is the production output value minus the cost of capital) that the investor alone pays the cost of capital (i e. the share base is now the production output value only) Then the first order conditions derive from

$$\text{Investor:} \quad \max_K \; E\Omega^p(H(\varepsilon, K) - \varphi(H(\varepsilon, K)) - rK) \qquad \text{(A7)}$$

$$\text{Agent:} \quad \max_K \; E\Omega^a(\varphi(H(\varepsilon, K)) \qquad \text{(A8)}$$

such that if the investment decision maximises both the agent and investor expected utility then

$$\Omega^{p\prime}(\cdot)[H_K - \varphi'(\cdot)H_K - r] = \Omega^{a\prime}(\cdot)[\varphi'(\cdot)H_K] \qquad \forall\varepsilon \qquad \text{(A9)}$$

Again, the Pareto efficiency condition for the fee schedule is (A5) (with the arguments being given by $H(\varepsilon, K) - \varphi(H(\varepsilon, K)) - rK$ for the investor and $\varphi(H(\varepsilon, K))$ for the agent) which together with (A9) yields

$$H_K[\beta(1 - \varphi') - \varphi'] - r\beta = 0 \qquad \forall\varepsilon \qquad \text{(A10)}$$

From (A10) it is clear that $\varphi' \neq$ constant, such that non-deductibility of the capital cost from the share base has precluded the attainment of Pareto efficiency by linear sharing contracts when the expected utilities of the agent and investor are both maximised by the same investment decision.

(B) Over/Under-investment

We next consider the simple case of the investment decision which maximises the expected utility of a risk neutral investor and a risk averse agent. Suppose that the production function is separable and multiplicative in uncertainty such that $H(\varepsilon, K) \equiv g(\varepsilon)H(K)$, $g'(\varepsilon) > 0$, $H'(K) > 0$, and $H''(K) < 0$. Suppose also that $\alpha \in [0, 1]$ is the degree of deductibility of capital costs from the share base Then the expected utility of investor and agent is given by

Investor: $$E(1 - \pi)g(\varepsilon)H(K) - (1 - \alpha\pi)rK \quad \text{(A11)}$$

Agent: $$E\Omega^{a}(\pi(g(\varepsilon)H(K) - \alpha rK)) \quad \text{(A12)}$$

where we assume that the agent receives a share $\pi \in (0, 1)$ of revenue net of a proportion α of capital costs rK (for simplicity and without loss of generality we take the base wage as nil). Maximising (A11) and (A12) w.r.t. K we see that the utility maximising investment level for the investor and agent respectively is given by the implicit solution to the following:

Investor: $$H' = \frac{(1 - \alpha\pi)r}{(1 - \pi)\bar{g}} \quad \text{(A13)}$$

Agent: $$H' = \frac{\alpha r E\Omega^{a\prime}[\pi(g(\varepsilon)H(K) - \alpha rK)]}{E\Omega^{a\prime}[\pi(g(\varepsilon)H(K) - \alpha rK)]g(\varepsilon)} \quad \text{(A14)}$$

where $\bar{g} \equiv Eg(\varepsilon) > 0$. Consider the following limiting cases:

(a) Fully deductible cost of capital: $\alpha = 1$

In this case (A13) gives the investment decision that the investor would make implicitly from

$$H' = r/\overline{g} \tag{A15}$$

For the agent from (A14),

$$H' = \frac{rE\Omega^{a'}[\pi(g(\varepsilon)H(K)-rK)]}{E\Omega^{a'}[\pi(g(\varepsilon)H(K)-rK)]g(\varepsilon)} \tag{A16}$$

But

$$\begin{aligned} E[\Omega^{a'}[\pi(g(\varepsilon)H(K)-rK)]g(\varepsilon)] &= E\Omega^{a'}[\pi(g(\varepsilon)H(K)-rK)]Eg(\varepsilon) \\ &\quad + Cov[\Omega^{a'}[\pi(g(\varepsilon)H(K)-rK)], g(\varepsilon)] \\ &< E\Omega^{a'}[\pi(g(\varepsilon)H(K)-rK)]Eg(\varepsilon) \end{aligned}$$

since for a (strictly) risk averse agent $\Omega^{a''}(\cdot) < 0$ and the covariance term is negative.

Therefore (A16) becomes

$$H' > r/\overline{g} \tag{A17}$$

Comparing (A15) and (A17) we conclude that since $H''(K) < 0$, when the cost of capital is fully deductible from the share base and the agent chooses the investment decision, then there exists a problem of *under-investment* from the view point of a risk neutral investor (principal).

(b) Non-deductible cost of capital: $\alpha = 0$

In this case the share base is simply the production output value $g(\varepsilon)H(K)$. From (A13) the investment decision of the investor is given implicitly by

$$H' = \frac{r}{(1-\pi)\bar{g}} > 0 \tag{A18}$$

For the agent given that $\Omega^{a\prime}(\cdot) > 0$ and $g(\varepsilon) > 0\ \forall\varepsilon$, then $\alpha = 0$ gives

$$H' = 0 \tag{A19}$$

Since there exists diminishing marginal returns to investment (by assumption), (A19) establishes that the agent will commit (without limit if possible) the entire capital made available by the investor for investment purposes. From (A18) and (A19) we also conclude that when the cost of capital is not deductible from the share base and the agent chooses the investment decision, then there exists a problem of *over-investment* from the view point of a risk neutral investor (principal).

From the above illustrations we see that the deductibility of capital costs has a significant role to play when considering the (Pareto) efficiency of linear sharing contracts as well as upon the way in which an agent may be motivated differently towards the level of investment compared to an investor on whose behalf he is acting.

Appendix II

Global applicability of locally true incentive compatibility constraints

In the following analysis[59], we derive the conditions which justify the global use of incentive compatibility constraints which are, of themselves, only locally true. The results obtain with or without a pay floor for the borrower.

For a bilateral information asymmetry, we need to consider the incentive compatibility constraints of both the borrower and the lender.

Borrower incentive compatibility

The global incentive compatibility constraint of the borrower is

$$U^a(\theta,\theta) \geq U^a(\theta,\hat{\theta}) \qquad \forall\theta \in [\underline{\theta},\overline{\theta}], \forall\hat{\theta} \in [\underline{\theta},\overline{\theta}] \qquad \text{(A20)}$$

From (10) $$U^a(\theta,\hat{\theta}) = \int_{\underline{r}}^{\overline{r}} \varphi(r,\theta,\hat{\theta}) R_1(r)dr \qquad \text{(A21)}$$

Since (from (9)) $$\varphi(r,\theta,\hat{\theta}) = \pi(\theta H(K(r,\hat{\theta})) - \alpha(r,\hat{\theta})rK(r,\hat{\theta})) \qquad \text{(A22)}$$

and therefore $$\varphi(r,\hat{\theta},\hat{\theta}) = \pi(\hat{\theta} H(K(r,\hat{\theta})) - \alpha(r,\hat{\theta})rK(r,\hat{\theta})) \qquad \text{(A23)}$$

Subtracting (A23) from (A22) and substituting into (A21) gives

[59] in which we draw from the survey work of Baron (1989)

$$U^a(\theta,\hat{\theta}) = U^a(\hat{\theta},\hat{\theta}) + \int_{\underline{r}}^{\bar{r}} \pi(\theta-\hat{\theta})H(K(r,\hat{\theta}))R_1(r)dr \qquad \text{(A24)}$$

From (A20), the global incentive compatibility constraint can now be written

$$U^a(\theta,\theta) - U^a(\hat{\theta},\hat{\theta}) \geq \int_{\underline{r}}^{\bar{r}} \pi(\theta-\hat{\theta})H(K(r,\hat{\theta}))R_1(r)dr \qquad \text{(A25)}$$

Similarly, $$\varphi(r,\hat{\theta},\theta) = \pi(\hat{\theta}H(K(r,\theta)) - \alpha(r,\theta)rK(r,\theta)) \qquad \text{(A26)}$$

and $$\varphi(r,\theta,\theta) = \pi(\theta H(K(r,\theta)) - \alpha(r,\theta)rK(r,\theta)) \qquad \text{(A27)}$$

Therefore $$U^a(\hat{\theta},\theta) = U^a(\theta,\theta) + \int_{\underline{r}}^{\bar{r}} \pi(\hat{\theta}-\theta)H(K(r,\theta))R_1(r)dr \qquad \text{(A28)}$$

and since $U^a(\hat{\theta},\hat{\theta}) \geq U^a(\hat{\theta},\theta)$ from (A20),

$$U^a(\theta,\theta) - U^a(\hat{\theta},\hat{\theta}) \leq \int_{\underline{r}}^{\bar{r}} \pi(\theta-\hat{\theta})H(K(r,\theta))R_1(r)dr \qquad \text{(A29)}$$

Combining (A25) and (A29) gives

$$\int_{\underline{r}}^{\bar{r}} \pi(\theta-\hat{\theta})H(K(r,\hat{\theta}))R_1(r)dr \leq U^a(\theta,\theta) - U^a(\hat{\theta},\hat{\theta}) \leq \int_{\underline{r}}^{\bar{r}} \pi(\theta-\hat{\theta})H(K(r,\theta))R_1(r)dr \qquad \text{(A30)}$$

From (A30) we immediately[60] see that globally true incentive compatibility constraint (A20) implies that $K(r,\theta)$ is non-decreasing in θ. Thus, that $K(r,\theta)$ is non-decreasing in θ is a necessary condition for global incentive compatibility.

We now consider sufficiency. We will show that $K(r,\theta)$ non-decreasing in θ is also sufficient.

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[60] since H'(K) > 0

From (14), $$U^a(\theta,\theta) = U^a(\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\int_{\underline{r}}^{\bar{r}} \pi H(K(r,s))R_1(r)drds \tag{A31}$$

and $$U^a(\hat{\theta},\hat{\theta}) = U^a(\underline{\theta}) + \int_{\underline{\theta}}^{\hat{\theta}}\int_{\underline{r}}^{\bar{r}} \pi H(K(r,s))R_1(r)drds \tag{A32}$$

Substituting (A31) into (A32) gives

$$U^a(\hat{\theta},\hat{\theta}) = U^a(\theta,\theta) + \int_{\theta}^{\hat{\theta}}\int_{\underline{r}}^{\bar{r}} \pi H(K(r,s))R_1(r)drds \tag{A33}$$

Now substituting (A33) into (A24) gives

$$U^a(\theta,\hat{\theta}) = U^a(\theta,\theta) + \int_{\theta}^{\hat{\theta}}\int_{\underline{r}}^{\bar{r}} \pi H(K(r,s))R_1(r)drds + \int_{\underline{r}}^{\bar{r}} \pi(\theta-\hat{\theta})H(K(r,\hat{\theta}))R_1(r)dr \tag{A34}$$

Rewriting $(\theta-\hat{\theta})H(K(r,\hat{\theta}))$ as $\int_{\hat{\theta}}^{\theta} H(K(r,\hat{\theta}))ds$ in (A34) gives

$$U^a(\theta,\theta) - U^a(\theta,\hat{\theta}) = \int_{\theta}^{\hat{\theta}}\int_{\underline{r}}^{\bar{r}} \pi[H(K(r,\hat{\theta})) - H(K(r,s))]R_1(r)drds \tag{A35}$$

Therefore, by inspection of (A35) we deduce that if $K(r,\theta)$ is non-decreasing in θ, then the incentive compatibility constraint (A20) is indeed global[61]. Consequently, the incentive compatibility constraint (13) (or equivalently (14)) which is only locally true, will apply globally if and only if $K(r,\theta)$ is non-decreasing in θ.

[61] We require the right hand side of (A35) to be non-negative, in which case the integrand of (A35) must be non-negative, and therefore K(r,θ) must be non-decreasing in θ To see why, if $\hat{\theta} \geq \theta$ then the integrand is non-negative, and if $\theta \geq \hat{\theta}$ then the integrand is nonpositive but the direction of the integral is reversed, so the integral is non-negative.

Investor incentive compatibility constraint

First we deduce the necessary condition. The global incentive compatibility constraint for the investor to truthfully reveal the true unit cost of capital when it is realised, for *all* possible values of the unit cost of capital given some $\theta \in [\underline{\theta}, \overline{\theta}]$ is

$$\rho(r,r;\theta) \geq \rho(r,\hat{r};\theta) \qquad \forall r \in [\underline{r},\overline{r}], \forall \hat{r} \in [\underline{r},\overline{r}] \quad \text{(A36)}$$

By following exactly the same procedure used for the incentive compatibility constraint of the borrower, from the definition of investor pay (4), we deduce that

$$\rho(r,r) - \rho(\hat{r},\hat{r}) \geq (\hat{r}-r)(1-\alpha(\hat{r},\theta)\pi)K(\hat{r},\theta) \qquad \text{(A37)}$$

and

$$\rho(r,r) - \rho(\hat{r},\hat{r}) \leq (\hat{r}-r)(1-\alpha(r,\theta)\pi)K(r,\theta) \qquad \text{(A38)}$$

Therefore

$$(\hat{r}-r)(1-\alpha(\hat{r},\theta)\pi)K(\hat{r},\theta) \leq (\hat{r}-r)(1-\alpha(r,\theta)\pi)K(r,\theta) \qquad \text{(A39)}$$

and a necessary condition for global incentive compatibility is that $(1-\alpha(r,\theta)\pi)K(r,\theta)$ is *non-increasing* in r[62]. In similitude with the case of the borrower, the necessary condition also proves to be sufficient, as will now be demonstrated.

Let $g(s,\theta) \equiv (1-\alpha(s,\theta)\pi)K(s,\theta)$ Then from (8)

$$\rho(r,r;\theta) = \rho(\underline{r},\theta) - \int_{\underline{r}}^{r} g(s,\theta)ds = (1-\pi)\theta H(K(r,\theta)) - g(r,\theta)r \qquad \text{(A40)}$$

[62] note that we are considering incentive compatibility for the investor and need only concern ourselves with the dependence of K(r,θ) on the unit cost of capital r. Any results about the global relevance of the incentive compatibility constraint of the investor w r t r apply equally for all values of θ spanning its support.

and $$\rho(\hat{r},\hat{r};\theta) = \rho(\underline{r},\theta) - \int_{\underline{r}}^{\hat{r}} g(s,\theta)ds = (1-\pi)\theta H(K(\hat{r},\theta)) - g(\hat{r},\theta)\hat{r} \qquad \text{(A41)}$$

But $$\rho(r,\hat{r};\theta) = (1-\pi)\theta H(K(\hat{r},\theta)) - g(\hat{r},\theta)r \qquad \text{(A42)}$$

Combining (A41) and (A42) gives

$$\rho(r,\hat{r};\theta) = \rho(\hat{r},\hat{r};\theta) - g(\hat{r},\theta)(r-\hat{r})$$

$$= \rho(\underline{r},\theta) - \int_{\underline{r}}^{\hat{r}} g(s,\theta)ds - g(\hat{r},\theta)(r-\hat{r}) \qquad \text{(A43)}$$

$$= \rho(r,r;\theta) + \int_{\underline{r}}^{r} g(s,\theta)ds - \int_{\underline{r}}^{\hat{r}} g(s,\theta)ds - g(\hat{r},\theta)(r-\hat{r}) \qquad \text{(A44)}$$

where the last equality (A44) follows from (A40).

Rearranging (A44) yields

$$\rho(r,r;\theta) - \rho(r,\hat{r};\theta) = \int_{r}^{\hat{r}} g(s,\theta)ds - (\hat{r}-r)g(\hat{r},\theta)$$

$$= \int_{r}^{\hat{r}} [g(s,\theta) - g(\hat{r},\theta)]ds \qquad \text{(A45)}$$

Hence, if g(s,θ) is non-increasing in s, then[63] the left hand side of (A45) is non-negative, i.e. $(1-\alpha(s,\theta)\pi)K(s,\theta)$ non-increasing in s is also a sufficient condition for global incentive compatibility.

[63] for example when $\hat{r} > r$, $g(s,\theta) \geq g(\hat{r},\theta)$ for all $s \in [r,\hat{r}]$ and vice versa

Appendix III

(1) $\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \rho(r,\theta)F_1(\theta)R_1(r)drd\theta$:

By definition $\rho(r,\theta)+\varphi(r,\theta)=\theta H(K(r,\theta))-rK(r,\theta)$. Therefore the investor's expected utility is

$$\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \rho(r,\theta)F_1(\theta)R_1(r)drd\theta = \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \left(\theta H(K(r,\theta)) - rK(r,\theta) - \varphi(r,\theta)\right)F_1(\theta)R_1(r)drd\theta$$

Now $\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \varphi(r,\theta)F_1(\theta)R_1(r)drd\theta = \int_{\underline{\theta}}^{\bar{\theta}} F_1(\theta)\left(\int_{\underline{r}}^{\bar{r}} \varphi(r,\theta)R_1(r)dr\right)d\theta$

$$= \int_{\underline{\theta}}^{\bar{\theta}} F_1(\theta)\left(\int_{\underline{r}}^{\bar{r}} \varphi(r,\hat{\theta})R_1(r)dr\Big|_{\hat{\theta}=\theta}\right)d\theta$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} F_1(\theta)U^a(\theta)d\theta$$

where the last equality is from (10). Therefore the investor's expected utility is

$$\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} (\theta H(K(r,\theta)) - rK(r,\theta))F_1(\theta)R_1(r)drd\theta - \int_{\underline{\theta}}^{\bar{\theta}} U^a(\theta)F_1(\theta)d\theta$$

(3) $\int_{\underline{\theta}}^{\bar{\theta}} U^a(\theta)F_1(\theta)d\theta = U^a(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \pi \frac{F(\theta)}{F_1(\theta)} H(K(r,\theta))F_1(\theta)R_1(r)drd\theta$:

Integrating $\int_{\underline{\theta}}^{\bar{\theta}} U^a(\theta)F_1(\theta)d\theta$ by parts gives

$$\int_{\underline{\theta}}^{\bar{\theta}} U^a(\theta)F_1(\theta)d\theta = [F(\theta)U^a(\theta)]_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} \frac{dU^a(\theta)}{d\theta} F(\theta)d\theta$$

$$= F(\bar{\theta})U^a(\bar{\theta}) - F(\underline{\theta})U^a(\underline{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{r}}^{\bar{r}} \pi H(K(r,\theta))R_1(r)F(\theta)drd\theta$$

where the second equality follows from (13). The result is immediate upon rewriting the integrand in the second equality, and substituting the values of zero and unity for the cumulative distribution of θ at its supports.

Appendix IV

Derivation of the optimal investment schedule from first principles using an optimal control approach

Let the expected utility of the principal be

$$W[U^a, K] \equiv \int_{\Sigma_\theta} \int_{\Sigma_r} Y(U^a, K, r, \theta) dr d\theta \qquad \text{(A46)}$$

where $\Sigma_\theta = \{\theta \mid \theta \in [\underline{\theta}, \bar{\theta}]\}$, $\Sigma_r = \{r \mid r \in [\underline{r}, \bar{r}]\}$ From (26), the expected utility of the type θ borrower, $U^a(\theta)$, is determined by $K(r,\theta)$ according to the constraint

$$\frac{dU^a}{d\theta} = \int_A g(K, r, \theta) dr \qquad \text{(A47)}$$

where $g(K,r,\theta) \equiv \pi H(K) R_1(r)$, and $A = \{r \mid r \in [\underline{r}, r_c(\theta, K, \alpha)]\}$ From (A47) it is true that

$$\int_{\Sigma_\theta} \eta(\theta) \left[\int_A g dr - U^{a'} \right] d\theta = 0 \qquad \text{(A48)}$$

for all possible functions $\eta(\theta)$, where $\eta(\theta)$ is also the shadow price of constraint (A47), and $j(\theta)' \equiv \frac{dj(\theta)}{d\theta}$. Adding (A46) and (A48) yields

$$W[U^a, K] \equiv \int_{\Sigma_\theta} \left[\int_A (Y + \eta g) dr + \int_{A^c} Y dr - \eta U^{a'} \right] d\theta \qquad \text{(A49)}$$

where $A^c = \Sigma_r \setminus A$.

Now define the Hamiltonian

$$\widetilde{H} = \widetilde{H}(U^a, K, \eta, \theta) \equiv \int_A (Y + \eta g)dr + \int_{A^c} Y dr \qquad \text{(A50)}$$

Then
$$W[U^a, K] \equiv \int_{\Sigma_\theta} \left[\widetilde{H} - \eta U^{a'} \right] d\theta \qquad \text{(A51)}$$

Suppose that $K_o(r,\theta)$ and $U_o^a(\theta)$ denote the optimal pair of functions. Consider the variations

$$K(r,\theta) = K_0(r,\theta) + \beta\mu(r,\theta) \qquad \text{(A52)}$$

and

$$U^a(\theta) = U_0{}^a(\theta) + \beta\xi(\theta) \qquad \text{(A53)}$$

where $\mu(r,\theta)$ and $\xi(\theta)$ are arbitrary functions, and β is a constant. Then

$$W[U_o{}^a + \beta\xi, K_o + \beta\mu] = \int_{\Sigma_\theta} \left[\widetilde{H}(U_o{}^a + \beta\xi, K_o + \beta\mu, \eta, \theta) - \eta(U_o{}^{a'} + \beta\xi') \right] d\theta \quad \text{(A54)}$$

In order that $K_o(r,\theta)$ and $U_o{}^a(\theta)$ are the solutions to the problem, we require that the first variation of the functional $W[U^a, K]$, denoted $\delta W[U^a, K]$, is zero at $\beta = 0$, i.e. if

$$\delta W[U^a, K] \overset{def}{=} \frac{d}{d\beta} W[U_o{}^a + \beta\xi, K_o + \beta\mu]\Big|_{\beta=0} = 0$$

then from (A54) we require

$$\delta W[U_o{}^a, K_o] = \int_{\Sigma_\theta} \left[\frac{\partial \widetilde{H}(U_o^a, K_o, \eta, \theta)}{\partial U^a} \xi + \frac{\partial \widetilde{H}(U_o^a, K_o, \eta, \theta)}{\partial K} \mu - \eta\xi' \right] d\theta = 0 \quad \text{(A55)}$$

Integrating the last term in (A55) by parts gives

$$-\int_{\Sigma_\theta} \eta\xi' d\theta = -[\eta\xi]_{\underline{\theta}}^{\overline{\theta}} + \int_{\Sigma_\theta} \xi\eta' d\theta \tag{A56}$$

Therefore, substituting (A56) into (A55) gives

$$\delta W[U_o{}^a, K_o] = \int_{\Sigma_\theta} \left[\left[\frac{\partial \tilde{H}}{\partial U^a} + \eta' \right] \xi + \frac{\partial \tilde{H}}{\partial K} \mu \right] d\theta - [\eta\xi]_{\underline{\theta}}^{\overline{\theta}} = 0 \tag{A57}$$

If we now impose the transversality condition $[\eta\xi]_{\underline{\theta}}^{\overline{\theta}} = 0$, then since the functions ξ and μ are arbitrary, we demand that

$$\frac{\partial \tilde{H}(U^a, K, \eta, \theta)}{\partial U^a} = -\eta' \text{ and } \frac{\partial \tilde{H}(U^a, K, \eta, \theta)}{\partial K} = 0 \quad \forall\theta \tag{A58}$$

where the first order conditions are evaluated at the optimal functions $K_0(r,\theta)$ and $U_o{}^a(\theta)$.

Now, from (A50)

$$\tilde{H} = \int_A (Y + \eta g) dr + \int_{A^c} Y dr = \int_{\Sigma_r} Y dr + \int_A \eta g dr \tag{A59}$$

Specifically, $$\int_{\Sigma_\theta} \int_{\Sigma_r} Y(U^a, K, r, \theta) dr d\theta = \int_{\Sigma_\theta} \left[\int_{\Sigma_r} X(K, r, \theta) R_1(r) dr - U^a \right] F_1(\theta) d\theta \tag{A60}$$

where profit $X(K,r,\theta) \equiv \theta H(K) - rK$, so that

$$\int_{\Sigma_r} Y(U^a, K, r, \theta)dr = \left[\int_{\Sigma_r} X(K, r, \theta)R_1(r)dr - U^a\right]F_1(\theta) \tag{A61}$$

Also, from (26), $$g(K, r, \theta) \equiv \pi H(K)R_1(r) \tag{A62}$$

Substituting (A61) and (A62) into (A59) gives

$$\widetilde{H} = \left[\int_{\Sigma_r} (\theta H(K) - rK)R_1(r)dr - U^a + \int_A \pi \frac{\eta(\theta)}{F_1(\theta)} H(K)R_1(r)dr\right]F_1(\theta) \tag{A63}$$

From (A58) and (A63), $\dfrac{\partial \widetilde{H}(U^a, K, \eta, \theta)}{\partial U^a} = -\eta'$ yields $-F_1(\theta) = -\eta'(\theta)$, so that

$$\eta(\theta) = F(\theta) \tag{A64}$$

taking the constant of integration to be zero.

Also, recalling that $A = \{r \mid r \in [\underline{r}, r_c(\theta, K, \alpha)]\}$, using Liebnitz's rule to differentiate the Hamiltonian w.r.t. K in (A63), applying the first-order condition $\widetilde{H}_K(U^a, K, \eta, \theta) = 0$ from (A58), and noting that $F_1(\theta) > 0 \;\; \forall\theta$ by assumption,

$$\int_A \left[\left(\theta + \pi \frac{F(\theta)}{F_1(\theta)}\right) H'(K(r, \theta)) - r\right] R_1(r)dr + \int_{A^c} [\theta H'(K(r, \theta)) - r] R_1(r)dr$$
$$+ \pi \frac{F(\theta)}{F_1(\theta)} H(K(r_c, \theta))R_1(r_c) \frac{\partial r_c}{\partial K} = 0 \tag{A65}$$

where we have used $\eta(\theta) = F(\theta)$ from (A64).

Appendix V

Let $J(Y) \equiv X$ where $Y \equiv H'(X)$. Then $J'(H'(X))H''(X) = 1$ so that $J'(\cdot) < 0$ given that $H''(X) < 0$ Also $J'(H'(X))H'''(X) + J''(H'(X))[H''(X)]^2 = 0$ so that $J''(\cdot) < 0$ assuming that $H'''(X) < 0$

Now let $\phi(\theta) \equiv \theta + \pi \dfrac{F(\theta)}{F_1(\theta)}$ in (34), where we assume that the risk ratio increases in θ faster than $-\pi^{-1}$ s.t. $\phi'(\theta) > 0$. Then $K_{nb}(r,\theta)$ is given explicitly (suppressing the subscript *nb*) by

$$K(r,\theta) = J\left[\frac{r}{\phi(\theta)}\right] \tag{A66}$$

Differentiating (A66) w.r.t. r and θ yields

$$K_r(r,\theta) = \frac{1}{\phi(\theta)} J'\left[\frac{r}{\phi(\theta)}\right] < 0 \tag{A67}$$

$$K_{rr}(r,\theta) = \frac{1}{\phi^2(\theta)} J''\left[\frac{r}{\phi(\theta)}\right] < 0 \tag{A68}$$

$$K_\theta(r,\theta) = -\frac{r\phi'(\theta)}{\phi^2(\theta)} J'\left[\frac{r}{\phi(\theta)}\right] > 0 \tag{A69}$$

$$K_{\theta\theta}(r,\theta) = \frac{r\phi'^2(\theta)}{\phi^4(\theta)}\left[2rJ'\left[\frac{r}{\phi(\theta)}\right] + J''\left[\frac{r}{\phi(\theta)}\right]\right] \leq 0 \tag{A70}$$

$$K_{r\theta}(r,\theta) = -\frac{\phi'(\theta)}{\phi^2(\theta)}\left[J'\left[\frac{r}{\phi(\theta)}\right] + \frac{r}{\phi(\theta)} J''\left[\frac{r}{\phi(\theta)}\right]\right] > 0 \tag{A71}$$

In summary, $K_{nb}(r,\theta)$ is increasing in θ and decreasing in r. It is concave in both θ and r, and the slope of $K_{nb}(r,\theta)$ w r.t. r is increasing in θ.

CHAPTER 4

DICHOTOMOUS INCENTIVE CONTRACTS, UNOBSERVABLE EFFORT AND LIMITED LIABILITY

4.1 Introduction

In this chapter we examine dichotomous incentive schemes aimed at dealing with the problem of moral hazard in a single-period principal-agent framework when agent actions (here chosen prior to the realisation of some uncertain event) are not observable by the principal. By dichotomous we mean that the incentive contract awards the agent a realised pay derived according to one of two possible rules. Which rule obtains is conditional on some criteria, the fulfilment of which is ex ante uncertain. A simple example of a dichotomous contract might be a base wage plus a fixed bonus component of pay conditional upon the attainment of some performance target.

Research in this area has primarily focused on whether these type of contracts can implement a first-best effort[1] level. A first-best effort level is that effort which maximises the expected utility of the principal subject to participation by the agent (see Holmstrom (1979) and Mirrlees (1974)). An incentive contract will usually not implement a first-best effort level when actions are unobservable since the agent is free to make an effort choice which optimises his own utility and not necessarily that of the principal. This latter additional constraint leads to the optimal contract only implementing a second-best effort. The intuitive reason for this is that whilst the agent and principal may well both be motivated to enjoy the returns from the venture, it is only the agent who bears the cost (in terms of effort disutility) in generating these returns. Risk sharing considerations may

[1] Effort and action will be used interchangeably.

also serve to exacerbate the problem, for example when the principal is risk neutral and the agent risk averse there is a conflict between incentives and insurance for the agent (see 2.2.1.1). This leads to a lack of goal congruence between principal and agent and consequently the supply of an effort level which is not first-best

Intuitively we expect that if the principal can impose arbitrarily large penalties for an inferred effort level below that of first-best, then he might be able to enforce the first-best effort choice. This is indeed the case and has been shown by Gjesdal (1976)[2]. What is important in understanding this result is that the outcome (e g. revenue or profit) is the result of an ex ante uncertain state of nature together with the agent effort. The punishment is triggered when, given the realised outcome, the effort of the agent must have been below the first-best assuming that the worst possible state of nature has occurred. Thus it is with certainty that the principal is able to know that the agent has shirked. This element of certainty together with the dichotomous contract can force the agent ex ante to choose the first-best effort choice for sufficiently large penalties. This bears a direct relation to the fact that when the principal has complete information about the agent effort and can observe it with certainty, then a forcing contract (legally enforceable given observability and verifiability of effort) can be used to achieve the first-best

However, a practical problem arises in that there may exist institutional constraints such as bankruptcy laws and limited liability constraints that mean that the threat of a penalty otherwise sufficient to induce the first-best effort may not be viable and may therefore not eliminate the incentive problem. In extending the work of Gjedal (1976), Lewis (1980) has shown that the use of a lump-sum penalty of any size (not necessarily that which would have been large enough to eliminate the incentive problem completely without liability constraints) based on some measure of performance like output that

[2] Osband (1987) emphasises how achievement of the first-best by applying unbounded punishments depends on whether the support for the output distribution is stationary or moves with the effort supplied by a risk averse agent. For stationary support (e g Holmstrom (1979)) the first best is approached only asymptotically, in contrast to the case of moveable supports (e g. Lewis (1980))

varies continuously with the effort of a strictly risk averse agent can improve a contract by reducing the incentive for the agent to supply an (unobservable) effort which is not first-best. In other words the use of lump-sum bonuses or penalties in incentive contracts is Pareto-improving but may not necessarily implement the first-best effort choice by the agent if there are limited liability constraints

Mehta (1993) has studied the effect that imposing bounds on the rewards/punishments has upon the attainment of efficiency in incentive contracting. In Mehta's model, an efficient incentive contract is one that minimises the risk premium paid to the agent for bearing risk in an incentive contract that induces the agent to supply some fixed level of effort rather than to supply none at all. By making some rather strong assumptions (the distribution of outcomes is binomial) Mehta is able to determine conditions (involving the effect of effort upon the probability of successful outcomes) under which the imposition of liability constraints does not impair the attainment of efficiency when the principal uses a dichotomous incentive contract. The economics of his result is that to maintain efficiency using dichotomous contracts with limited instead of unlimited liability, the inference from the realised outcome about effort must be stronger.

However, Mehta's study lacks generality. An earlier study by Innes (1990) stressed the importance of the monotonicity properties of the *principal's* allocation from ex post profits when the agent is instead risk neutral (in order to abstract from risk sharing considerations). Innes was able to show when bi-lateral limited liability obtains (the principal requires a minimum rate of return on capital invested) that a monotone nondecreasing fee schedule precludes the attainment of the first-best effort level, whereas when there is no such monotonicity property a first-best effort level may be elicited from the agent (depending upon the technology and the extent of external investment required by the agent). Of critical importance in deriving this result is the monotonic likelihood ratio property (MLRP). This implies that for any given profit level, increased effort leads to relatively greater probability weight on all higher profit levels (MLRP implies first order stochastic dominance but not vice versa). With any contract that delivers an allocation to the principal that is strictly increasing in some region, some of the benefits of the marginal effort supplied by the agent are shared with the principal. Thus since the agent still bears the total cost of effort, he will choose an effort that is less than first best. However, in the absence of a monotonic contract constraint the situation is different. Innes considers a contract which delivers the principal an allocation which is non-monotonic, whereby profit is entirely allocated to the principal if it is less than some threshold level and is completely awarded to the agent otherwise. By giving the agent the entire share of profits in some high-profit states, the share of marginal-effort benefits captured by the principal in lower-profit states is offset by principal losses from a higher probability of zero payoff. By appropriate choice of the critical profit level above which the principal gets nothing, the agent will lose none of the marginal benefits (at the first-best effort choice) and first-best efficiency may prevail.

Following on from Innes' work, Park (1995) and Kim (1997) considered instead limited liability for only the agent in determining general existence conditions for contracts that induce the agent to supply the first-best effort level. Kim highlights the importance of a limited liability constraint (which sets a lower bound to the payoff the agent receives of zero) in that it precludes the use of a fixed rent contract. As is well known, in the absence of such a constraint when the agent is risk neutral a fixed-rent contract is found to be optimal. This is because by allowing the agent to be sole residual claimant the principal will induce the agent to supply the first-best effort level and can extract the expected surplus in excess of the agent reservation utility via the fixed rent paid by the agent. Being risk neutral the agent does not suffer utility loss from bearing the entire risk of the venture and there is therefore no conflict between insurance and incentives (see 2.2.1.1). However, when the liability of the agent is limited such a contract may not be feasible since for some low profit outcomes the profit net of the fee may leave the agent with a payoff less than zero. Assuming MLRP Kim derives a necessary condition that when satisfied ensures a first-best effort level for a contract in which the agent and principal proportionally share the output, but for which the agent receives a lump-sum bonus when the output exceeds a predetermined target He also shows that if this condition does not hold, then there exists no contract that will achieve the first-best effort since the bonus contract is amongst the most efficient contractual forms when the agent has limited liability.

The form of dichotomous contracts that we consider are those whose trigger depends on the output side of the production process. An alternative form of dichotomous contract may depend on the imperfect monitoring of productive input supplied by the agent (effort) as in Harris and Raviv (1979), who show that such monitors are Pareto improving. Other literature on the effect of limited liability on the attainment of the first-best effort level assumes that the agent sees the state of nature prior to choosing effort. Sappington (1983a) shows that in such a case a first-best outcome is not possible. His results, however, do not hinge crucially on the presence of the agent's limited liability, but on the presence of the agent's private information. The agent's private information obtained before choosing an effort makes his strategy set much bigger. This restricts the principal's contractual ability, causing the preclusion of the first-best outcome. Finally, Lawarree and Audenrode (1996) also assuming the agent to be risk neutral and to see the state of nature prior to choosing effort have shown in the presence of limited liability and imperfect output observability, when there are two agent types who differ only in productive efficiency, that both agents receive a positive rent and that even the most efficient agent will not produce at his first-best effort level.

In this chapter we examine dichotomous incentive schemes which award the agent a contingent share of the outcome. This share ratio takes one of two values depending on whether or not the realised outcome exceeds some pre-specified target. In contrast to those schemes considered by Kim (1997) and Park (1995), for which a risk neutral agent is awarded a fixed bonus for target achievement which is permitted to be as large as necessary to elicit socially optimal effort, these schemes place an upper bound on such an inducement. Consequently, for some technologies, the assumption of risk neutrality prohibits the supply of a first-best effort level. This permits a meaningful discussion of risk aversion, and whether and how risk aversion will impact the inducement of the agent to supply the required effort.

4.2 The Model

We consider a principal-agent model in which the principal offers an agent a contract for the supply of productive effort in a risk bearing venture on a take it-or-leave it basis. The supply of effort is costly to the agent and the principal is unable to observe agent effort. Since effort cannot be a contracted variable, the principal's problem is to design an incentive scheme which induces the agent to make an effort choice which is socially optimal (first-best), and which awards the agent according to the publicly observable realised outcome.

Denote the (non-negative) outcome by $X = X(e,\varepsilon)$, where e is effort supplied by the agent prior to the realisation of ex ante uncertain state of nature $\varepsilon \in [\varepsilon_0, \varepsilon_1]$. ε has (strictly positive) density function $G_1(\varepsilon)$ and cumulative distribution $G(\varepsilon)$. Production technology specifies $X_e(e, \varepsilon) > 0$, $X_\varepsilon(e, \varepsilon) > 0$[3] and $X_{ee}(e, \varepsilon) \leq 0$, $X_{e\varepsilon}(e, \varepsilon) \geq 0$, $X_{\varepsilon\varepsilon}(e, \varepsilon) \leq 0$[4] $\forall e,\varepsilon$. The agent is taken to have separately additive utility in money and effort[5].

The principal chooses a target outcome X_c. For realised outcomes above or equal to X_c, the agent is awarded a proportion $\pi \in (0,1)$ of the outcome, and for realised outcomes below X_c, the agent is awarded a proportion $\lambda\pi$ of the outcome where $\lambda \in [0,1]$.
Formally

$$\varphi(X) = \begin{cases} \lambda \pi X & if \quad X < X_c \\ \pi X & if \quad X \geq X_c \end{cases} \tag{1}$$

[3] The marginal product of effort and state of nature is non-negative

[4] The marginal product of effort is non-increasing in effort and non-decreasing in realised state, and production is non-decreasing in the state with non-increasing returns i e effort and productivity state are substitutes, each with decreasing returns to scale.

[5] This means that agent risk preferences w r t money are independent of effort.

which is illustrated in a plot of $\varphi(X)$ versus X below.

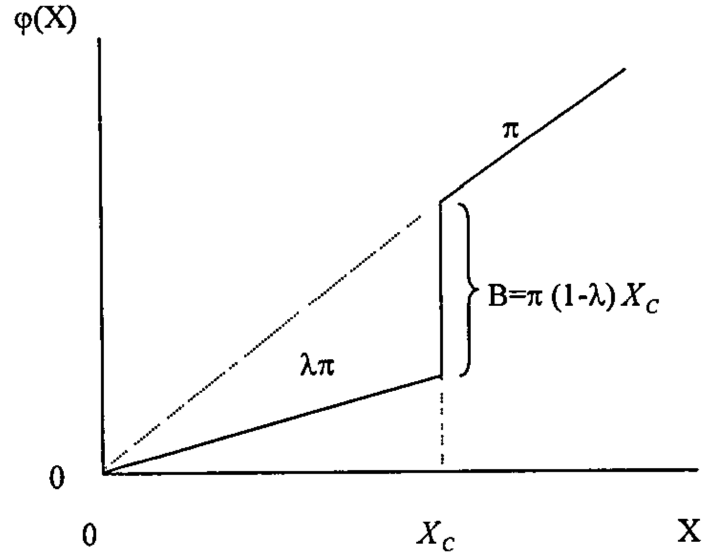

Figure 4.1

From Figure 4.1 we may note the limitation for B, the discontinuous increase in agent allocation for $X \geq X_C$. The maximum value of B (at $\lambda = 0$) is πX_C, and the agent allocation is bounded above by the linear share line with gradient π through the origin. In Kim (1997) and Park (1995), the share ratio is continuous and the agent receives a contingent bonus which does not restrict the pay of the agent to values contained on or below the straight line in Figure 4.1 of gradient π (see 4.3 for further comparison).

Depending on the performance target X_C which is specified in the contract between the principal and the agent, the principal may either know with certainty or remain uncertain, as to whether the agent has supplied less than the first-best effort For example, if $X_C = \inf X(e^*,\varepsilon) = X(e^*,\varepsilon_0)$, which is the lowest possible outcome given that the agent supplies the first-best effort, then the principal punishes the agent by allocating the ex post realised outcome according to the reduced share ratio $\lambda\pi$ only when he is completely certain that the agent supplied less than the first-best effort e^*. If the target outcome is set above $X(e^*,\varepsilon_0)$ then punishment is more severe in the sense that the agent may receive the reduced share when he had actually supplied the requisite amount of effort.

Intuitively the lower the effort e supplied by the agent, the greater must be the realised state ε in order to achieve the target outcome, and vice versa. This is formalised by defining $\bar{\varepsilon}(e)$ by the following equivalence:

$$X_C \equiv X(e,\bar{\varepsilon}(e)) \tag{2}$$

From (2) we see that $\bar{\varepsilon}(e)$ is the realised state of nature which just ensures that the agent achieves the target outcome for a given effort e. For a fixed value of X_c, by differentiating (2) totally and applying the chain rule,

$$\frac{d\bar{\varepsilon}(e)}{de} = -\frac{X_e(e,\bar{\varepsilon}(e))}{X_\varepsilon(e,\bar{\varepsilon}(e))} < 0 \tag{3}$$

The expected utility of the agent when supplying effort e is U^a where

$$U^a = \int_{\varepsilon_0}^{\bar{\varepsilon}(e)} \Omega^a(\lambda\pi X(e,\varepsilon))dG(\varepsilon) + \int_{\bar{\varepsilon}(e)}^{\varepsilon_1} \Omega^a(\pi X(e,\varepsilon))dG(\varepsilon) - Q(e) \tag{4}$$

Q(e) is the disutility of effort e, where $Q'(e) > 0$ and $Q''(e) \geq 0$, such that effort disutility is increasing in effort at a non-decreasing rate. $\Omega^a(W)$ is the utility of money W, where $\Omega^{a\prime}(W) > 0$ and $\Omega^{a\prime\prime}(W) \leq 0$, such that utility of money is increasing in money at a non-increasing rate. Finally, the first-best effort level e^* is given by the solution to

$$\int_{\varepsilon_0}^{\varepsilon_1} X_e(e^*,\varepsilon)dG(\varepsilon) - Q'(e^*) = 0 \tag{5}$$

whereby the marginal product of effort equals its marginal cost, and the first-best allocation is one in which the agent supplies effort e^* and receives (in expected value) his reservation utility $\underline{U}$.

4.3 A comparison of schemes

Before formally analysing contingent share ratio schemes, we undertake a comparison of the expected payoffs of lump-sum bonus schemes of the type considered by Kim and Park, with the contingent share ratio scheme in (1).

For a lump-sum bonus scheme in which the agent is awarded a (linear) share $\lambda\pi$ of ex post profits for all outcomes, and receives a lump-sum B_{LS} if the outcome is no less than X_C, the expected payoff is

$$\int_{\varepsilon_o}^{\varepsilon_1} \lambda\pi X(e,\varepsilon)dG(\varepsilon) + \int_{\underline{\varepsilon}(e)}^{\varepsilon_1} B_{LS}dG(\varepsilon) \qquad (6)$$

From (4), for a payoff which awards a (linear) share $\lambda\pi$ of ex post profits for outcomes less than X_C, and a (linear) share π of ex post profits otherwise, the expected payoff is

$$\int_{\varepsilon_o}^{\varepsilon_1} \lambda\pi X(e,\varepsilon)dG(\varepsilon) + \int_{\underline{\varepsilon}(e)}^{\varepsilon_1} (1-\lambda)\pi X(e,\varepsilon)dG(\varepsilon) \qquad (7)$$

Rewriting (7) gives the expected payoff for the contingent share ratio scheme as

$$\int_{\varepsilon_o}^{\varepsilon_1} \lambda\pi X(e,\varepsilon)dG(\varepsilon) + \int_{\underline{\varepsilon}(e)}^{\varepsilon_1} B_{SR}dG(\varepsilon) + \int_{\underline{\varepsilon}(e)}^{\varepsilon_1} (1-\lambda)\pi(X(e,\varepsilon) - X_c)dG(\varepsilon) \qquad (8)$$

where $B_{SR} \equiv (1-\lambda)\pi X_C$ = difference in share ratio × performance target.

By comparing (6) and (8) we can immediately identify some important differences between the two schemes. The first difference to note is that the lump sum bonus in the case of contingent share ratio contracts may not be determined by the principal independently of other contract parameters. Its value is instead derived from the performance target and the difference in share ratio which obtains for outcomes no less than, and below, the target outcome.

A second, and essential difference, derives from the way in which bonus schemes create incentive pressure. The rationale for awarding an agent a lump-sum performance bonus is that none of the marginal benefits of effort are shared with the principal for this element of pay. In contrast, when the agent shares the outcome in some fixed proportion, the agent is allocated a pay which is less than the value of his marginal product, and as a result is less motivated to provide effort. For contingent share ratio schemes, increasing the lump-sum bonus by reducing λ creates two effects. The first is to increase incentive pressure by increasing the lump-sum bonus. The second, and offsetting effect, is to reduce incentive pressure by reducing the marginal share (λπ) of outcomes which are less than the performance target. The overall effect of reducing λ is therefore ambiguous and depends on the production function, the distribution of exogenous uncertainty, and the performance target (see 4.5).

However, in the case of lump-sum bonuses whose value is chosen explicitly, as opposed to being implicitly determined by other contract parameters, increasing the bonus unambiguously increases incentive pressure[6].

[6]From (6), $U^a = \int_{\varepsilon_o}^{\varepsilon_1} \lambda\pi X(e,\varepsilon)dG(\varepsilon) + \int_{\bar{\varepsilon}(e)}^{\varepsilon_1} B_{LS}dG(\varepsilon) - Q(e)$ Therefore, differentiating the FOC we obtain $\frac{\partial}{\partial B_{LS}}\frac{\partial U^a}{\partial e} = -G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} > 0$.

A third and more subtle difference relates to the choice of performance target by the principal. For contingent share ratio schemes, increasing the lump-sum element of pay through increasing the performance target also reduces the probability of achieving the lump-sum element of pay, for a given level of effort. However, for lump-sum schemes, the bonus element is a constant, independent of the performance target. Therefore, in contrast to contingent share schemes, increasing incentive pressure through increasing the lump-sum bonus is not reduced by an associated decrease (for the same level of effort) in the probability of target performance achievement.

A final and obvious difference seen by comparing (6) and (8) is that extra incentive pressure exists in contingent share schemes from an additional share $(1-\lambda)\pi$ of outcomes in excess of the performance target (the last term in (8)), in comparison to the lump-sum scheme given by (6).

Having introduced the basic model we now formalise the discussion of incentive pressure through the use of contingent share ratio incentive contracts. We start by considering a general (money) risk preference for the agent and then go on to examine how increasing the risk aversion of the agent from zero (risk neutrality) to higher levels of risk aversion impacts the use of contingent share ratio schemes.

4.4 General (money) utility preference

The agent chooses his effort in order to maximise his expected utility. The first order condition is obtained from (4) by applying Liebnitz's rule[7] to give

$$\frac{\partial U^a}{\partial e} = \left[\Omega^a(\lambda\pi X(e,\bar{\varepsilon}(e))-\Omega^a(\pi X(e,\bar{\varepsilon}(e))\right]G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de}$$

$$+\int_{\varepsilon_o}^{\bar{\varepsilon}(e)}\lambda\pi X_e(e,\varepsilon)\Omega^{a'}(\lambda\pi X(e,\varepsilon))dG(\varepsilon)+\int_{\bar{\varepsilon}(e)}^{\varepsilon_1}\pi X_e(e,\varepsilon)\Omega^{a'}(\pi X(e,\varepsilon))dG(\varepsilon)-Q'(e)$$

$$=0 \qquad (9)$$

However, in order that the agent supply an effort level at least equal to e^*, we require that $\frac{\partial U^a}{\partial e}$ evaluated at e^* is non-negative, i e

$$\left[\Omega^a(\lambda\pi X(e^*,\bar{\varepsilon}(e^*))-\Omega^a(\pi X(e^*,\bar{\varepsilon}(e^*))\right]G_1(\bar{\varepsilon}(e^*))\frac{d\bar{\varepsilon}(e^*)}{de}$$

$$+\int_{\varepsilon_o}^{\bar{\varepsilon}(e^*)}\lambda\pi X_e(e^*,\varepsilon)\Omega^{a'}(\lambda\pi X(e^*,\varepsilon))dG(\varepsilon)+\int_{\bar{\varepsilon}(e^*)}^{\varepsilon_1}\pi X_e(e^*,\varepsilon)\Omega^{a'}(\pi X(e^*,\varepsilon))dG(\varepsilon)-Q'(e^*)$$

$$\geq 0 \qquad (10)$$

Additionally, if φ(X) (see (1)) is to achieve a first-best allocation then in addition to (10) it must also give the agent exactly his reservation utility, i.e.

$$\int_{\varepsilon_o}^{\bar{\varepsilon}(e^*)}\Omega^a(\lambda\pi X(e^*,\varepsilon))dG(\varepsilon)+\int_{\bar{\varepsilon}(e^*)}^{\varepsilon_1}\Omega^a(\pi X(e^*,\varepsilon))dG(\varepsilon)=Q(e^*)+\underline{U} \qquad (11)$$

---

[7] see Stephensen (1973) p182.

Combining (10) and (11) we arrive at a necessary condition for φ(X) to implement the first best allocation:

$$\frac{\dfrac{\partial E\Omega^a(\varphi(X(e^*,\varepsilon)))}{\partial e}}{E\Omega^a(\varphi(X(e^*,\varepsilon)))} \geq \frac{Q'(e^*)}{\underline{U}+Q(e^*)} \qquad (12)$$

where E(·) is the expectation operator over all states of nature.

This condition has an intuitive economic explanation. The left hand side is the proportional rate of change of expected utility of money w.r.t. effort at $e = e^*$. The right hand side is the proportional rate of change of disutility of effort w r.t. effort at $e = e^*$. We therefore see that if an incentive scheme is to achieve the first-best allocation then at the first-best effort, the incentive pressure caused by the marginal effect of effort on money must be no less than the pressure to supply less effort due to effort disutility[8].

We note that the crucial feature for the bonus share scheme to elicit the first-best effort is that the agent's payoff increases discontinuously at some point. While the bonus share contract necessarily forces the agent to give up some of the marginal benefit of effort through sharing the outcome, thereby reducing the agent's effort, this effort-reducing incentive is offset by the effort-enhancing incentive created by a jump in agent payoff for achieving the target. The jump B in agent payoff at $X = X_C$ is seen graphically in Figure 4.1.

From (12) we see that essential to an optimal response of increased effort by the agent in answer to the use of the threat of a possible reduction in share ratio, is that the left hand side of (12) be decreasing in λ, i.e. greater punishments elicit greater effort at least until the optimal response of the agent is to supply e^*.

[8] We can also interpret (12) as an elasticity expression

If the maximal expected utility of the agent (the expected utility when effort is utility maximising) is decreasing in λ, then so also will be the proportional rate of change of expected utility w r t. effort. Therefore, critical to φ(X) as a scheme to induce an increase in effort towards first-best is that

$$\frac{\partial}{\partial\lambda}\left[\frac{\partial E\Omega^{a}(\varphi(X(e,\varepsilon))}{\partial e}\right]<0 \quad \text{for} \quad e\leq e^{*} \tag{13}$$

We therefore differentiate the first order condition (9), which defines maximal effort e(λ) as a function of λ (for given π), in order to learn how the agent responds to a change in λ Instead of differentiating (9) directly and collecting terms in de(λ)/dλ, a simpler method to derive de(λ)/dλ is to notice that the first order condition can be expressed as

$$\left.\frac{\partial U^{a}}{\partial e}\right|_{e=e(\lambda)}\equiv\Gamma(\lambda,e(\lambda))=0 \tag{14}$$

where Γ(λ, e(λ)) is given by the differentiated expression in (9). Differentiating (14) w.r t. λ gives

$$\frac{\partial\Gamma(\lambda,e(\lambda))}{\partial\lambda}+\frac{\partial\Gamma(\lambda,e(\lambda))}{\partial e(\lambda)}\frac{de(\lambda)}{d\lambda}=0$$

and therefore

$$\frac{de(\lambda)}{d\lambda}=-\left[\frac{\partial\Gamma(\lambda,e(\lambda))}{\partial\lambda}\right]\left[\frac{\partial\Gamma(\lambda,e(\lambda))}{\partial e(\lambda)}\right]^{-1} \tag{15}$$

The denominator of (15) is $\frac{\partial\Gamma(\lambda,e(\lambda))}{\partial e(\lambda)}=\left.\frac{\partial^{2}U^{a}}{\partial e^{2}}\right|_{e=e(\lambda)}<0$ since the expected utility of the agent is assumed concave in effort in order that the maximal effort of the agent is unique. Therefore, the sign of de(λ)/dλ is the sign of the numerator of (15), i.e.

$$\frac{de(\lambda)}{d\lambda} \stackrel{s}{=} \frac{\partial}{\partial\lambda}\left[\frac{\partial U^a}{\partial e}\right] \text{ at } e = e(\lambda) \tag{16}$$

where $\stackrel{s}{=}$ means equal in sign.

Partial differentiation of (9) w r t. λ gives[9]

$$\frac{de(\lambda)}{d\lambda} \stackrel{s}{=} X_C \Omega^{a'}(\lambda\pi X_C) G_1(\bar{\varepsilon}(e)) \frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_0}^{\bar{\varepsilon}(e)} \left[X_e \Omega^{a'}(\lambda\pi X) + \lambda\pi X X_e \Omega^{a''}(\lambda\pi X) \right] dG(\varepsilon) \tag{17}$$

where e = e(λ), the maximal effort for the agent with contingent proportional share ratio reduction λ.

Equation (17) gives us an important result. It tells us whether the effort of the agent, which is individually maximal, will increase or decrease as the share ratio which obtains when the performance target is not reached is varied. As will become clear in the next subsection, depending on the technology and the performance target specified, for a risk neutral agent there exist conditions for which the threat of a lower share ratio will reduce rather than increase the maximal effort.

[9] It should now be clear why we have used a states space model viz-a-viz an induced distribution model, in order to consider risk aversion For an induced distribution ($\tilde{G}(X \mid e)$) model, the first-order condition for a risk averse agent is $U_e^a = \int_{\underline{X}}^{X_c} \Omega^a(\lambda\pi X) d\tilde{G}_e(X \mid e) + \int_{X_c}^{\bar{X}} \Omega^a(\pi X) d\tilde{G}_e(X \mid e) - Q'(e) = 0$. From (16), the sign of the change in maximal effort w r t λ is given by differentiating the first-order condition w r t λ However, for an induced distribution model this only generates a first-order derivative for $\Omega^a()$ For a further comparison of the two modelling approaches see Appendix II, which derives a states space representation of MLRP.

4.5 Risk neutrality

To see why in general an ambiguity exists as to the sign of the change of maximal effort with λ, observe from (17) that for a risk neutral agent ($\Omega^{a\prime}(\lambda\pi X) = 1$ and $\Omega^{a\prime\prime}(\lambda\pi X) = 0$),

$$\frac{de(\lambda)}{d\lambda} \overset{s}{=} \frac{\partial}{\partial e} \int_{\varepsilon_0}^{\bar{\varepsilon}(e)} X(e,\varepsilon)dG(\varepsilon) \qquad (18)$$

Since the upper limit of the integral is decreasing in effort, whilst the integrand is increasing in effort, the overall sign of the rate of change of the expected outcome over the range of states for which the performance target is not achieved is ambiguous.

In order to remove this ambiguity and to render the impact of risk aversion interesting, we assume a technology which precludes the use of schemes such as $\varphi(X)$ (given by (1)) as devices used to elicit greater effort when the agent is risk neutral, such that $\frac{de(\lambda)}{d\lambda} > 0$.

The following assumptions are sufficient to ensure that $\frac{de(\lambda)}{d\lambda} > 0$ for a risk neutral agent.

Assumption 1:[10] $$\frac{d}{de}\left(G_1(\bar{\varepsilon}(e)) \frac{d\bar{\varepsilon}(e)}{de} \right) \leq 0 \qquad \forall e$$

---

10 $\frac{d}{de}\frac{d\bar{\varepsilon}(e)}{de} = \frac{[-X_\varepsilon X_{ee} + X_e X_{e\varepsilon}] - [X_\varepsilon X_{e\varepsilon} - X_e X_{\varepsilon\varepsilon}]\frac{d\bar{\varepsilon}(e)}{de}}{X_\varepsilon^2}$ evaluated at $(e, \bar{\varepsilon}(e))$.

Observe that the first term in the numerator is non-negative, since $X_{ee}(e,\varepsilon) \leq 0$ and $X_{e\varepsilon}(e,\varepsilon) \geq 0$ Also, the second term is non-negative since $X_{e\varepsilon}(e,\varepsilon) \geq 0$ and $X_{\varepsilon\varepsilon}(e,\varepsilon) \leq 0$ Therefore, $\frac{d}{de}\frac{d\bar{\varepsilon}(e)}{de} \geq 0$

Assumption 2:
$$X_C < \frac{\int_{\varepsilon_o}^{\bar{\varepsilon}(e^*)} X_e(e^*, \varepsilon) dG(\varepsilon)}{G_1(\bar{\varepsilon}(e^*)) \left| \frac{d\bar{\varepsilon}(e^*)}{de} \right|}$$

To see why these assumptions are sufficient to positively sign (18) (in which case the maximal effort decreases as λ decreases), (for clarity) define Z(e) by the following equivalence,

$$Z(e) \equiv \frac{\partial}{\partial e} \int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X(e,\varepsilon) dG(\varepsilon) = X_C G_1(\bar{\varepsilon}(e)) \frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_e(e,\varepsilon) dG(\varepsilon) \qquad (19)$$

where the second line follows from Liebnitz's rule and (2). Then

$$\frac{dZ(e)}{de} = X_C \frac{d}{de}\left(G_1(\bar{\varepsilon}(e)) \frac{d\bar{\varepsilon}(e)}{de} \right) + X_e(e,\bar{\varepsilon}(e)) G_1(\bar{\varepsilon}(e)) \frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_{ee}(e,\varepsilon) dG(\varepsilon)$$
$$< 0 \qquad (20)$$

where the sign of (20) follows from (3), Assumption 1, and diminishing returns to effort. Since $Z'(e) < 0$, if $Z(e^*) > 0$, then $Z(e) > 0$ for $e \leq e^*$. Therefore, to ensure that $Z(e) > 0$ for all $e \leq e^*$, we assume that $Z(e^*) > 0$, which is Assumption 2.

For the remainder of this chapter we make Assumptions 1 and 2 such that the threat of share ratio reduction contingent on an outcome less than the target profit, will actually decrease rather than increase the effort of a risk neutral agent.

Intuitively, we might expect that schemes such as φ(X), which fail as a means to elicit greater effort when the agent is risk neutral (given Assumptions 1 and 2), will be more likely to induce an increase in effort when the agent is instead risk averse. This is because uncertainty as to the fee schedule adds an extra layer of discomfort to a risk averse agent who would already suffer a drop in welfare compared to his risk neutral counterpart, due to ex ante uncertainty about the ex post outcome, from which his reward is allocated.

In the following analysis we find that this is indeed the case, but of importance to the principal is the actual degree of risk aversion of the agent. We start by introducing (Arrow-Pratt) measures of risk aversion, and then identify the importance of matching the actual degree of risk aversion of an agent to a suitable dichotomous share contract.

4.6 Risk aversion

Relative risk aversion $\chi_r(W)$ and absolute risk aversion $\chi_A(W)$ are defined as follows[11]:

$$\chi_r(W) \equiv -\frac{\Omega^{a''}(W)W}{\Omega^{a'}(W)} \tag{21}$$

$$\chi_A(W) \equiv -\frac{\Omega^{a''}(W)}{\Omega^{a'}(W)} \tag{22}$$

where $\Omega^a(W)$ is the utility of money W.

In order to illustrate these concepts, consider an investor who is endowed with some initial wealth. The investor must decide how much of his initial wealth he wishes to

[11] See Huang and Litzenberger (1988) pp20-23

invest in some risky asset, the balance of which earns a risk free rate of return. If the investor has constant relative risk aversion, that is his degree of relative risk aversion is invariant w.r.t. wealth W, then as his wealth changes, the proportion of wealth invested in the risky asset would stay the same. This must therefore imply that the absolute dollar amount of wealth invested in the asset increases as his wealth increases, i e. that his absolute risk aversion decreases with wealth, where absolute risk aversion is a measure used to describe how the absolute dollar investment of initial wealth in the risky asset varies as wealth changes.

Since constant relative risk aversion implies decreasing absolute risk aversion, where the latter preference appeals to real attitudes towards risk, we will assume it henceforth. Additionally, constant relative risk aversion, though not critical to the following discussion, will provide simplification.

4.7 Critical relative risk aversion

If the principal is to viably use the threat of a possible reduction in share ratio as a means to elicit greater effort, then it is vital that such a threat should be credible[12]. In the following analysis, we identify three important ranges of (constant relative) risk aversion. These are $\chi\in[0, \chi(0)]$, $\chi\in(\chi(0), \chi(1)]$, and $\chi\in(\chi(1), \infty)$ (definitions of $\chi(0)$ and $\chi(1)$ proceed from the following analysis). For risk aversions in the lower interval, threats of punishment are counterproductive and only tend to reduce effort (as seen for the case of risk neutrality). For risk aversions in the upper range, threats of punishment (however severe) elicit greater effort Lastly, for agents with risk aversion in the middle range, reducing λ below unity will, for λ values above some critical value $\lambda_C\in(0,1)$, decrease the effort of the agent. Only for reductions in λ below this critical value, will the threat of punishment elicit greater effort

[12] In the sense that it does not induce the agent to lower effort levels

We derive these results by recalling (17),

$$\frac{de(\lambda)}{d\lambda} \stackrel{s}{=} X_C\Omega^{a'}(\lambda\pi X_C)G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_o}^{\bar{\varepsilon}(e)}\left[X_e\Omega^{a'}(\lambda\pi X) + \lambda\pi XX_e\Omega^{a''}(\lambda\pi X)\right]dG(\varepsilon) \tag{23}$$

Consider, $$\frac{d}{dW}\left[W\Omega^{a'}(W)\right] = \Omega^{a'}(W) + W\Omega^{a''}(W) = \Omega^{a'}(W)[1-\chi_r(W)] \tag{24}$$

where the second equality follows from the definition of relative risk aversion (21)

Substituting (24) into (23), gives

$$\frac{de(\lambda)}{d\lambda} \stackrel{s}{=} X_C\Omega^{a'}(\lambda\pi X_C)G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_e\frac{d}{d(\lambda\pi X)}\left[\lambda\pi X\Omega^{a'}(\lambda\pi X)\right]dG(\varepsilon)$$

$$\stackrel{s}{=} X_C\Omega^{a'}(\lambda\pi X_C)G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_e\Omega^{a'}(\lambda\pi X)[1-\chi_r(\lambda\pi X)]dG(\varepsilon)$$

$$\stackrel{s}{=} X_C\Omega^{a'}(\lambda\pi X_C)G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} + (1-\chi)\int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_e\Omega^{a'}(\lambda\pi X)dG(\varepsilon) \tag{25}$$

where the first equality follows from (24) and the second from the assumption that the agent has constant relative risk aversion χ.

We now proceed to use (25) to establish the sign of the rate of change of maximal effort with λ at the extremum values of λ, being zero and unity. This allows us to identify the critical values of relative risk aversion ($\chi(0)$ and $\chi(1)$) for which the rate of change of effort with λ changes sign at either of the extremum values of λ.

From (25) it immediately follows that for $\lambda = 0$,

$$\left.\frac{\partial e}{\partial \lambda}\right|_{\lambda=0} \stackrel{s}{=} \Omega^{a\prime}(0)\left[X_C G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} + (1-\chi)\int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_e dG(\varepsilon)\right]$$

$$\stackrel{s}{=} X_C G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} + (1-\chi)\int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_e dG(\varepsilon) \qquad (26)$$

since $\Omega^a(W)$ is increasing for all W. Therefore,

$$\left.\frac{\partial e}{\partial \lambda}\right|_{\lambda=0} > (<)\ 0 \qquad \text{if} \qquad \chi < (>)\ \chi(0) \qquad (27)$$

where $\chi(0) = 1 - \dfrac{X_C G_1(\bar{\varepsilon}(e))\left|\dfrac{d\bar{\varepsilon}(e)}{de}\right|}{\int_{\varepsilon_o}^{\bar{\varepsilon}(e)} X_e dG(\varepsilon)}$ and $e = e_0$ (the maximal agent effort when $\lambda = 0$)[13].

In exactly the same way, we derive $\left.\frac{\partial e}{\partial \lambda}\right|_{\lambda=1}$ whereby

$$\left.\frac{\partial e}{\partial \lambda}\right|_{\lambda=1} > (<)\ 0 \qquad \text{if} \qquad \chi < (>)\ \chi(1) \qquad (28)$$

[13] Note that $\chi(0)$ and $\chi(1)$ are defined implicitly since maximal effort and the money utility functions depend upon the degree of relative risk aversion By (twice) integrating (21), the money utility function for an agent with relative risk aversion less than one is $\Omega^a(W)=W^{1-\chi}/(1-\chi)\ \forall W \geq 0$.

and $\chi(1) = 1 - \dfrac{X_C \Omega^{a'}(\pi X_C) G_1(\bar{\varepsilon}(e)) \left| \dfrac{d\bar{\varepsilon}(e)}{de} \right|}{\int_{\varepsilon_0}^{\bar{\varepsilon}(e)} X_e \Omega^{a'}(\pi X) dG(\varepsilon)}$ where e = e_1 (the maximal $\lambda = 1$ effort).

We can also order the values of $\chi(0)$ and $\chi(1)$ as given by the following proposition.

Proposition 1: $\chi(0) < \chi(1)$[14].

Proof. See Appendix I.

What Proposition 1, (27) and (28) tell us is that for agents with relative risk aversion $\chi < \chi(0)$, reducing λ will reduce effort irrespective of how close λ is to zero. For agents with $\chi = \chi(0)$, the benefit of greater effort from reducing λ will just appear only for the maximum possible contingent reduction in share ratio, being $\lambda = 0$. For all other values of λ above zero, agents with $\chi < \chi(0)$ will supply less effort as a result of a possible reduction in share ratio if the performance target is not reached

For $\chi > \chi(1)$, the degree of constant relative risk aversion is sufficient in the sense that the threat of a proportional reduction in share ratio for outcomes less than the performance target will elicit greater effort for all values of λ on its support. For $\chi = \chi(1)$, whilst the threat of λ equal to zero elicits greater effort, the threat of λ infinitesimally below unity only just induces the agent to supply greater effort.

[14] Agents are assumed to have relative risk aversion less than one in all of the proceeding analysis for $\chi(0)$ and $\chi(1)$ For relative risk aversion greater than one, maximal effort is trivially decreasing in λ for all $\lambda \in [0,1]$, as seen from (25)

For $\chi(0) < \chi < \chi(1)$, the threat of λ equal to zero induces greater effort, but the threat of λ just below unity is counterproductive to the principal in that it reduces effort Consequently, there exists a critical contingent proportional reduction in share ratio, λ_C, for which punishment threats less severe than λ_C induce less effort (are counterproductive), whilst punishment threats more severe than λ_C increase effort.

The following table summarises the effects of a change in λ upon the maximal effort of the agent, where plus (minus) indicates a decrease (increase) in effort with a decrease in λ, being the sign of the change in maximal effort with λ:

RANGE	$\left.\frac{\partial e}{\partial \lambda}\right\|_{\lambda=0}$	$\left.\frac{\partial e}{\partial \lambda}\right\|_{\lambda=1}$	COMMENT
$\chi < \chi(0)$	+	+	Counterproductive
$\chi(0) \leq \chi < \chi(1)$	-	+	$\lambda_C \in (0,1)$ exists
$\chi(1) < \chi$	-	-	Sufficient aversion

Table 4.1

The existence of some critical contingent proportional reduction in share ratio for risk aversion χ in the range $\chi(0) \leq \chi < \chi(1)$ is given by Proposition 2.

Proposition 2: If $\chi(0) \leq \chi < \chi(1)$, there exists some $\lambda_C \in (0,1)$ where for $\lambda \leq \lambda_C$, (maximal) effort is non-increasing[15] in λ, and for $\lambda > \lambda_C$, (maximal) effort is increasing in λ.

[15] Recall that $y = y(x)$ is increasing in x if $dy/dx > 0$ and decreasing in x if $dy/dx < 0$.

It must be noted that the value of λ_C depends on the precise value of χ for $\chi(0) \leq \chi < \chi(1)$, with λ_C close to unity for χ just below $\chi(1)$, and λ_C close zero for χ just above $\chi(0)$.

Finally, from (25), for constant relative risk aversion not less than one, $\frac{de(\lambda)}{d\lambda} < 0$ $\forall\lambda \in [0,1]$, given (3). Therefore, from (10), (11) and (12) a necessary condition for a first-best allocation[16], where $\lambda = 0$ induces the greatest effort is

$$\frac{-\Omega^a(\pi X(e^*, \bar{\varepsilon}(e^*))G_1(\bar{\varepsilon}(e^*))\frac{d\bar{\varepsilon}(e^*)}{de} + \int_{\bar{\varepsilon}(e^*)}^{\varepsilon_1} \pi X_e(e^*, \varepsilon)\Omega^{a'}(\pi X(e^*, \varepsilon))dG(\varepsilon)}{\int_{\bar{\varepsilon}(e^*)}^{\varepsilon_1} \Omega^a(\pi X(e^*, \varepsilon))dG(\varepsilon)} \geq \frac{Q'(e^*)}{\underline{U} + Q(e^*)} \tag{29}$$

4.8 Concluding remarks

In this chapter we examined the use of dichotomous incentive contracts when agents are risk averse and have limited liability. The dichotomous contracts which were the subject of this chapter conditioned the ex post allocation of the agent on the realisation of an ex ante uncertain outcome via a linear sharing rule, as well as on whether or not the outcome was no less than some pre-specified performance target.

[16] It is worth stressing that limited liability does not necessarily preclude a first-best allocation when the agent sees ε after supplying effort (29) However, limited agent liability *does* preclude the first-best when the agent sees ε prior to supplying effort (Sappington (1983a))

Typically, limited liability and risk aversion are considered substitutes in the analysis of incentive schemes, since both assumptions impede the use of franchise contracts, either by rendering them unenforceable through liability limitations, or suboptimal due to the allocation of risk which results from their use.

However, in this chapter we have exposed the importance of the precise degree of risk aversion when technology considerations and too low a performance target preclude the use of a dichotomous incentive contract for risk neutral agents, in which the (linear) profit sharing ratio is contingent on the outcome. It was found that there exists a critical proportional reduction in share ratio for agents with (constant relative) risk aversion in some range, for which less severe threats of share ratio reduction will be counterproductive to the supply of effort by the agent. We also derived a necessary condition for the attainment of the first best allocation (first best effort and the agent achieves his reservation utility in expectation) for a risk averse agent with relative risk aversion greater than one.

In the next chapter we extend the basic model of contingent share ratio contracts to permit a capital contribution by the agent. By requiring the agent to commit some of his own capital as a substitute in part for the capital committed by the principal to the venture, we formalise the notion that agent commitment in the sense of capital contribution permits less severe threats of punishment in order to induce the agent to greater effort levels. As such, the relationship we will examine is one of capital partnership, but is still considered an agency since effort is unilaterally supplied by the agent. In formalising this argument, we derive sufficient conditions for a Pareto improvement. We then briefly illustrate how an imperfect monitoring technology augments the use of dichotomous incentive schemes

Appendix I

Proposition 1: $\chi(0) < \chi(1)$

Proof. The following observations are relevant for $e_0 > e_1$[17]:

$$\bar{\varepsilon}(e_0) < \bar{\varepsilon}(e_1) \tag{A1}$$

$$X_e(e_0,\varepsilon) < X_e(e_1,\varepsilon)\ \forall\varepsilon \tag{A2}$$

$$\Omega^{a'}(\pi X(e_0,\bar{\varepsilon}(e_0))) \le \Omega^{a'}(\pi X(e_0,\varepsilon))\ \ \forall\varepsilon\in[\varepsilon_0,\bar{\varepsilon}(e_0)] \tag{A3}$$

$$\Omega^{a'}(\pi X(e_0,\varepsilon)) \le \Omega^{a'}(\pi X(e_1,\varepsilon))\ \forall\varepsilon \tag{A4}$$

(A1) follows from (3), (A2) from diminishing returns to effort, (A3) from risk aversion and $X_\varepsilon > 0$, and (A4) from risk aversion together with $X_e > 0$.

From (27) and (28), consider

$$\frac{G_1(\bar{\varepsilon}(e_0))\left|\dfrac{d\bar{\varepsilon}(e_0)}{de}\right|}{\int_{\varepsilon_0}^{\bar{\varepsilon}(e_0)} X_e(e_0,\varepsilon)dG(\varepsilon)} \ge \frac{G_1(\bar{\varepsilon}(e_0))\Omega^{a'}(\pi X_C)\left|\dfrac{d\bar{\varepsilon}(e_0)}{de}\right|}{\int_{\varepsilon_0}^{\bar{\varepsilon}(e_0)} X_e(e_0,\varepsilon)\Omega^{a'}(\pi X(e_0,\varepsilon))dG(\varepsilon)} \qquad \text{(by (2) and (A3))}$$

[17] If maximal effort at $\lambda = 0$ is less than for $\lambda = 1$, then there is no rationale for the use of punishment threats to elicit greater effort $e_0 \ge e_1$ is consistent with the proposition

$$> \frac{G_1(\bar{\varepsilon}(e_0))\Omega^{a'}(\pi X_C)\left|\frac{d\bar{\varepsilon}(e_0)}{de}\right|}{\int_{\varepsilon_o}^{\bar{\varepsilon}(e_1)} X_e(e_1,\varepsilon)\Omega^{a'}(\pi X(e_1,\varepsilon))dG(\varepsilon)} \qquad \text{(by (A1), (A2) and A(4))}$$

$$\geq \frac{G_1(\bar{\varepsilon}(e_1))\Omega^{a'}(\pi X_C)\left|\frac{d\bar{\varepsilon}(e_1)}{de}\right|}{\int_{\varepsilon_o}^{\bar{\varepsilon}(e_1)} X_e(e_1,\varepsilon)\Omega^{a'}(\pi X(e_1,\varepsilon))dG(\varepsilon)} \qquad \text{(by Assumption 1)}$$

Then from the definitions of $\chi(0)$ and $\chi(1)$ (see (27) and (28)), $\chi(0) < \chi(1)$.

Q.E.D.

Appendix II

MLRP in the states space model

Suppose a production technology X(e,ε) for effort e and exogenous uncertainty ε, where ε ∈[ε_0, ε_1] is described by a distribution function G(ε). An induced distribution approach synthesises the production function and exogenous uncertainty to arrive at a model specifying a distribution $\widetilde{G}(X \mid e)$ of outcomes X∈[X_0, X_1].

For an induced distribution model, MLRP is $\frac{\partial}{\partial X}\frac{\widetilde{g}_e(X \mid e)}{\widetilde{g}(X \mid e)} > 0$, or equivalently

$$\frac{\partial}{\partial X}\frac{\partial}{\partial e}\ln \widetilde{g}(X \mid e) > 0 \qquad \forall X \in [X_0, X_1] \quad \text{(A5)}$$

where $\widetilde{g}(X \mid e)$ is the probability density function for distribution $\widetilde{G}(X \mid e)$.

This means that for any outcome X, an increase in effort leads to relatively greater probability weight on all higher profit levels (Milgrom (1981)).

Now consider the states space model.

$$\text{Prob } (X \in [Y, Y+dY] | e) = \text{Prob. } (X > Y|e) - \text{Prob. } (X > Y+dY|e)$$

$$= \int_{\bar{\varepsilon}(Y,e)}^{\varepsilon_1} G_1(\varepsilon)d\varepsilon - \int_{\bar{\varepsilon}(Y+dY,e)}^{\varepsilon_1} G_1(\varepsilon)d\varepsilon$$

$$= G_1(\bar{\varepsilon}(Y,e))\bar{\varepsilon}_1(Y,e)dY$$

where the subscript denotes differentiation w.r.t. the first argument.

Therefore, from (A5) and Prob. (X∈[Y,Y+dY]|e) = $\tilde{g}(Y\,|\,e)\,\mathrm{d}Y$ (by definition), the equivalent states space MLRP assumption is

$$\frac{\partial}{\partial X}\frac{\partial}{\partial e}\ln G_1(\bar{\varepsilon}(X,e))\bar{\varepsilon}_1(X,e) > 0 \qquad \forall X\in[X_0, X_1] \quad \text{(A6)}$$

CHAPTER 5

DICHOTOMOUS INCENTIVE CONTRACTS, CAPITAL CONTRIBUTION AND ADDITIONAL SIGNALS

5.1 Introduction

As we have seen in the previous chapter, dichotomous incentive schemes will only achieve a first-best allocation, wherein the agent receives his reservation utility in expectation whilst supplying the first-best effort, when the proportional rate of change of expected utility of money w r.t. effort is no less than the proportional rate of change of disutility of effort w r.t. effort evaluated at the first-best effort level (Chapter 4, (12)).

In this chapter we explore ways in which the incentive pressure created by dichotomous incentive schemes can be supplemented either through capital contribution by the agent, or through conditioning agent pay on an additional imperfect, but informative, signal

5.2 Agent capital contribution

We have so far considered a principal-agent relationship in which the agent supplies unobservable effort in return for a share of the outcome, where for all possible realisations of an exogenous uncertainty variable ε, the outcome is non-negative. The liability of the agent was necessarily therefore limited to zero

Without a liability constraint for a risk neutral agent, the principal would make the agent sole residual claimant and extract the entire surplus with a fixed rent contract (see

2.2 1.1). However, given that such a contract would lead to negative payoffs in low productivity states, scope was created to study alternative contract forms designed to induce the supply of the first-best effort, when instead the pay of the agent was constrained to be non-negative. The alternative contract form studied was a linear share contract in which the share ratio could increase discontinuously upon the attainment of a target outcome specified ex ante. Such dichotomous contracts which provide a risk neutral agent with a jump in payoff at some pre-specified outcome may, given a suitable technology and performance target (see 4.5 of previous chapter), increase the effort supplied by the agent in comparison to contracts in which the (linear) share ratio of the agent is constant for all possible outcomes

In order to abstract from risk sharing considerations and to examine the manner in which capital contribution or the incorporation of additional signals may provide increased incentive pressure, throughout this chapter we assume that a suitable technology and performance target exist for which the threat of a lower share ratio for outcomes less than the performance target elicits greater effort from a risk neutral agent than would otherwise be supplied

The basic rationale for exploring capital contribution by the agent is as follows. If the money liability of the agent is limited to the capital contributed by the agent gross of opportunity costs, then the underlying[1] share ratio of the agent may be permitted to increase, *whilst increasing* λ, the proportional reduction in share ratio, so as to maintain the utility of the agent (net of effort costs) in *expectation* in respect of a contribution. This may create extra incentive pressure if the sensitivity of the maximal effort level of the agent for compensating changes (which preserve total utility) in threat level and capital contribution are different. A second possible source of (Pareto) improvement may also be generated due to the difference in the opportunity costs of capital through substituting the

[1] By underlying we mean the share ratio which obtains upon achievement of the performance target

agent's capital with that of the principal at the *margin*[2]. We therefore wish to explore changes in maximal effort when both the underlying share ratio and the threat of a possible reduction are adjusted to reflect a capital contribution by the agent, in order to increase the utility of the principal whilst holding the agent at his reservation amount[3].

It is also true of agency relationships common to real world economics, that capital partnership contracts often admit the supply of effort by only a subset of participants, with so-called sleeping partners supplying only capital and not effort. It is precisely the moral hazard which arises from unobservable effort in capital partnerships with at least one sleeping partner that we wish to capture in this chapter.

In the next section we present the model, which is an extension of the model of Chapter 4, after which we derive sufficient conditions for a capital contribution by the agent to generate a Pareto improvement.

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[2] If the principal faces an increasing marginal opportunity cost of capital, then for large scale projects it is feasible that the marginal opportunity cost of an incrementally small amount of extra capital, may at some level of existing contribution by the principal, be less for the agent than for the principal

[3] The only related moral hazard literature with limited agent liability of which we are aware is Brander and Spencer (1989), who consider the choice of firm debt level by an owner-manager who can substitute his own capital Lower equity levels decrease debt security and raise the required premium for a given level of borrowing Brander and Spencer further show that the substitution of borrowed funds for equity investment induces less effort and output from the firm. This is because the probability of firm bankruptcy increases with the level of borrowing (other things equal) thereby reducing the range of states of nature in which the owner receives a return from his effort

5.2.1 The Model

Let total fixed investment I[4] be the sum of capital E contributed by the agent, and capital I - E contributed by the principal. The return (net of initial investment) that the agent and the principal can earn if they invest their wealth in the next best opportunity available to them is $V^a(E)$[5] and $V^p(I - E)$. Revenue net of all costs except capital costs is X(e, I, ε), where e is agent effort and ε is the realised state of nature.

The expected utilities of the agent and principal, U^a and U^p, are given by

$$U^a = \int_{\varepsilon_0}^{\bar{\varepsilon}(e)} \lambda\pi(E)X(e,\varepsilon)dG(\varepsilon) + \int_{\bar{\varepsilon}(e)}^{\varepsilon_1} \pi(E)X(e,\varepsilon)dG(\varepsilon) - Q(e) - E - V^a(E) \qquad (1)$$

$$U^p = \int_{\varepsilon_0}^{\bar{\varepsilon}(e)} (1-\lambda\pi(E))X(e,\varepsilon)dG(\varepsilon) + \int_{\bar{\varepsilon}(e)}^{\varepsilon_1} (1-\pi(E))X(e,\varepsilon)dG(\varepsilon) - (I-E) - V^a(I-E)$$

(2)

where $X_C = X(e,\bar{\varepsilon}(e))$, and both the principal and agent are risk neutral. Note that we have suppressed the dependence of $\bar{\varepsilon}(e,I)$ and X(e, I, ε) on I, since we assume that total investment is fixed. Also, the underlying share ratio of the agent π(E), being the share ratio which obtains if the outcome X(e, ε) is no less than the performance target, explicitly depends on the capital contributed by the agent

[4] We assume investment is fixed in order to abstract from scale effects

[5] For the agent this is better thought of as a borrowing cost

5.2.2 Pareto improvements

We are now ready to derive sufficient conditions for a Pareto improvement through capital contribution by the agent The approach taken in this section is to derive a relationship between compensating changes in E and λ which maintain the total expected utility of the agent at the reservation amount, as simplified by the first order condition for which agent effort is maximal. The corresponding change in the expected utility of the principal can then be derived. Pareto improvements are admissible when this change is positive.

The first-order condition of the agent is obtained by differentiating (1) w.r.t e to give

$$\frac{\partial U^a}{\partial e} =$$

$$-(1-\lambda)\pi(E)G_1(\bar{\varepsilon}(e))X_C\frac{d\bar{\varepsilon}(e)}{de}+\pi(E)\left[\int_{\varepsilon_0}^{\bar{\varepsilon}(e)}\lambda X_e(e,\varepsilon)dG(\varepsilon)+\int_{\bar{\varepsilon}(e)}^{\varepsilon_1}X_e(e,\varepsilon)dG(\varepsilon)\right]-Q'(e)$$

$$=0 \qquad (3)$$

From (3), the first-order condition may be solved to give the maximal effort as a function of λ and E (for fixed I and X_C), i e e = e(λ,E).

We can therefore express the expected utility of the agent as a function of λ, E, and e(λ, E), whereby the explicit dependence of utility on λ and E is given by (1), and the implicit dependence of utility on λ and E through maximal effort e(λ, E) is given by (1) and the first-order condition (3) (as used to express the dependence of maximal effort on λ and E), i e.

$$U^a = U^a(\lambda, E, e(\lambda, E)) \qquad (4)$$

If we now consider *compensating* changes in λ and E such that introduction of capital by the agent leaves the expected utility of the agent unchanged, then the relationship between dλ and dE is given by differentiating (4) totally to yield

$$dU^a = \left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} d\lambda + \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} dE \qquad (5)$$

where we have used the first-order condition (3) to set the derivative of U^a w r.t. effort equal to zero. Setting dU^a equal to zero we derive

$$0 = \left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} d\lambda + \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} dE \qquad (6)$$

Now consider the utility of the principal. For maximal agent effort e = e(λ, E), from (2), the utility of the principal may be expressed as

$$U^p = U^p(\lambda, E, e(\lambda, E)) \qquad (7)$$

The change in principal utility for compensating changes in λ and E is obtained by total differentiation of (7) to yield[6]

$$dU^p = \left[\left.\frac{\partial U^p}{\partial e}\right|_{\lambda,E} e_\lambda + \left.\frac{\partial U^p}{\partial \lambda}\right|_{e,E}\right] d\lambda + \left[\left.\frac{\partial U^p}{\partial e}\right|_{\lambda,E} e_E + \left.\frac{\partial U^p}{\partial E}\right|_{e,\lambda}\right] dE \qquad (8)$$

Substituting (6) into (8) for dE > 0 gives

[6] Note here that the derivative of the principal's expected utility w r t effort is positive, whereas the derivative of the agent's expected utility w r t effort is zero evaluated at the same (maximal) effort level

$$dU^p \stackrel{s}{=} \left.\frac{\partial U^p}{\partial e}\right|_{\lambda,E}\left[\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} e_E - \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} e_\lambda\right] + \left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E}\left.\frac{\partial U^p}{\partial E}\right|_{e,\lambda} - \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda}\left.\frac{\partial U^p}{\partial \lambda}\right|_{e,E} \tag{9}$$

where we have used the fact that $\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} = \int_{\varepsilon_0}^{\bar{\varepsilon}(e)} \pi(E)X(e,\varepsilon)dG(\varepsilon) > 0$ from (1) in order to sign dU^p. Now, from (1) and (2),

$$\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} = -\left.\frac{\partial U^p}{\partial \lambda}\right|_{e,E} \tag{10}$$

and for fixed total investment I,

$$\left.\frac{\partial U^p}{\partial E}\right|_{e,\lambda} + \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} = V^{p\prime}(I-E) - V^{a\prime}(E) \tag{11}$$

Substituting (10) and (11) into (9) for $dE > 0$ gives

$$dU^p \stackrel{s}{=} \left.\frac{\partial U^p}{\partial e}\right|_{\lambda,E}\left[\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} e_E - \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} e_\lambda\right] + \left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E}\left[V^{p\prime}(I-E) - V^{a\prime}(E)\right] \tag{12}$$

Hence, given that $\left.\frac{\partial U^p}{\partial e}\right|_{\lambda,E} \geq 0$ for $e \leq e^*$ and $\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} > 0$, sufficient conditions for a Pareto improvement by substitution of agent for principal funds are

Sufficient Condition 1: $$V^{p\prime}(I-E) > V^{a\prime}(E) \tag{13}$$

Sufficient Condition 2: $$\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} e_E \geq \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} e_\lambda \tag{14}$$

It is also possible to alternatively express condition (14). Define the following equivalence

$$\left.\frac{\partial U^a}{\partial e}\right|_{e=e(\lambda,E)} \equiv \Gamma(\lambda, E, e(\lambda, E)) = 0 \tag{15}$$

where $\Gamma(\lambda, E, e(\lambda, E))$ is equivalent to (3) when e = e(λ,E).

Differentiating (15) totally gives $\left.\frac{\partial \Gamma}{\partial \lambda}\right|_{e,E} d\lambda + \left.\frac{\partial \Gamma}{\partial E}\right|_{e,\lambda} dE + \left.\frac{\partial \Gamma}{\partial e}\right|_{\lambda,E} de(\lambda, E) = 0$, i.e.

$$\left(\left.\frac{\partial \Gamma}{\partial \lambda}\right|_{e,E} + \left.\frac{\partial \Gamma}{\partial e}\right|_{\lambda,E} e_\lambda\right) d\lambda + \left(\left.\frac{\partial \Gamma}{\partial E}\right|_{e,\lambda} + \left.\frac{\partial \Gamma}{\partial e}\right|_{\lambda,E} e_E\right) dE = 0^{7} \tag{16}$$

Combining (6) and (16) gives for dλ ≠ 0 (or dE ≠ 0)

$$\left.\frac{\partial \Gamma}{\partial e}\right|_{\lambda,E} \left[\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} e_E - \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} e_\lambda\right] = -\left[\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} \left.\frac{\partial \Gamma}{\partial E}\right|_{e,\lambda} - \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} \left.\frac{\partial \Gamma}{\partial \lambda}\right|_{e,E}\right] \tag{17}$$

Now substituting (17) into (12) and noting that $\left.\frac{\partial \Gamma}{\partial e}\right|_{\lambda,E} < 0$ (a concavity assumption required to ensure that the maximal effort of the agent is a unique interior maximum), Sufficient Condition 2 may *equivalently* be written

[7] Note that we cannot trivially set the quotients of dλ and dE equal to zero in (16), since dλ and dE are related to each other by (6)

Sufficient Condition 2:

$$\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,\Gamma} \left.\frac{\partial \Gamma}{\partial E}\right|_{e,\lambda} \geq \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} \left.\frac{\partial \Gamma}{\partial \lambda}\right|_{e,E} \tag{18}$$

In order to interpret the second sufficient condition, we must sign the comparative static effects

5.2.3 Comparative static effects

Using (1), (3) and (15), we may state explicitly the following comparative static effects:

$$\left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} = \pi'(E)\left[\int_{\varepsilon_0}^{\bar{\varepsilon}(e)} \lambda X(e,\varepsilon)dG(\varepsilon) + \int_{\bar{\varepsilon}(e)}^{\varepsilon_1} X(e,\varepsilon)dG(\varepsilon)\right] - 1 - V^{a'}(E) \tag{19}$$

$$\left.\frac{\partial \Gamma}{\partial \lambda}\right|_{e,E} = \pi(E)\left[G_1(\bar{\varepsilon}(e))X_C \frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_0}^{\bar{\varepsilon}(e)} X_e(e,\varepsilon)dG(\varepsilon)\right] \overset{s}{=} e_\lambda < 0 \tag{20}$$

where the sign of (20) follows from assuming that the threat of a reduction in share ratio contingent on an outcome less than the performance target increases the effort of a risk neutral agent (see Chapter 4, (18) and (19)). Also from (3) and (15)

$$\left.\frac{\partial \Gamma}{\partial E}\right|_{e,\lambda} = \frac{\pi'(E)}{\pi(E)} Q'(e) \tag{21}$$

We assume henceforth that

$$U_E^a\Big|_{e,\lambda} < 0 \text{ and } \pi'(E) > 0 \tag{22}$$

The reason for doing so is as follows. Since $U_\lambda^a\Big|_{e,E} > 0$ (from (1)), in order for there to exist a utility conserving trade-off (see (6)) between capital contribution (dE > 0) by the agent and lighter punishment threats (dλ > 0), we require $U_E^a\Big|_{e,\lambda} < 0$. This trade-off is motivated by the idea that a capital contribution by the agent induces greater alignment of the goals of the agent and principal, in the sense that the relationship takes on the substance of a partnership, in which the principal is a sleeping partner who supplies no effort, whilst retaining the form of a principal-agent relationship in which the principal retains all bargaining power

Also, recognising the potential loss of capital by the agent, the principal awards the agent an underlying share which is increasing in the capital contributed by the agent, i e. $\pi'(E) > 0$[8].

Summarising the comparative static effects,

$$\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} > 0,\ \left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} < 0,\ \left.\frac{\partial \Gamma}{\partial E}\right|_{e,\lambda} > 0, \text{ and } \left.\frac{\partial \Gamma}{\partial \lambda}\right|_{e,E} < 0 \tag{23}$$

We can now interpret (14) (or equivalently (18)).

[8] It is also worth noting that (22) permits a non-trivial fulfillment of the second sufficient condition (18), in that if capital contribution and reduced punishment threats both increase utility, with $\pi'(E) > 0$, then the left hand side of (18) would be positive whilst the right hand side of (18) would be negative

5.2.4 Interpretation

From (6),

$$\left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda} = -\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} \left.\frac{d\lambda}{dE}\right|_{\underline{U}} \tag{24}$$

where $\left.\frac{d\lambda}{dE}\right|_{\underline{U}}$ is the tangent of the iso-utility line of the agent in (E, λ) space for compensating changes in λ and E[9].

Substituting (24) into (14) gives

$$\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} e_E \geq -\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} \left.\frac{d\lambda}{dE}\right|_{\underline{U}} e_\lambda \tag{25}$$

From (23), since $\left.\frac{\partial U^a}{\partial \lambda}\right|_{e,E} > 0$ and $e_\lambda < 0$, (25) becomes

$$\left.\frac{d\lambda}{dE}\right|_{\underline{U}} \leq \frac{e_E}{|e_\lambda|} \tag{26}$$

However, the iso-maximal effort lines for the agent are defined by e(λ, E) = c, where c is some constant effort. Therefore, since $e_\lambda d\lambda + e_E dE = 0$ for constant maximal effort,

$$\left.\frac{d\lambda}{dE}\right|_{e(\lambda, E)=c} = \frac{e_E}{|e_\lambda|} \tag{27}$$

Combining (26) and (27), the second sufficient condition reduces to

[9] This is the marginal rate of substitution between λ and E, i e $-U^a_E/U^a_\lambda$

$$\left.\frac{d\lambda}{dE}\right|_{\bar{U}} \leq \left.\frac{d\lambda}{dE}\right|_{e(\lambda,E)=c} \tag{28}$$

Hence, the sufficient conditions for a Pareto improvement are

$$V^{p'}(I-E) > V^{a'}(E) \quad \textbf{and} \quad \left.\frac{d\lambda}{dE}\right|_{\bar{U}} \leq \left.\frac{d\lambda}{dE}\right|_{e(\lambda,E)=c} \tag{29}$$

The first sufficient condition in (29) states that an improvement from capital cost savings is possible if the marginal opportunity cost of capital for the agent is less than the marginal opportunity cost of capital for the principal at the investment levels for each, being E and I – E respectively.

The second sufficient condition in (29) is illustrated diagrammatically in Figure 5.1

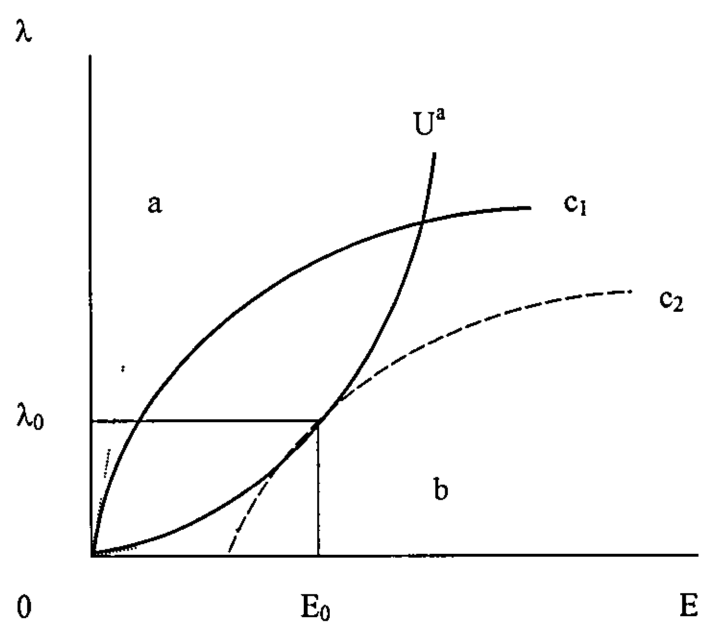

Figure 5.1

In Figure 5.1, an initial equilibrium is assumed to exist, without the introduction of capital by the agent, with a dichotomous contract which specifies the maximum threat of $\lambda = 0$ at (0,0), and an agent who supplies effort c_1. Oa represents the gradient of the iso-maximal effort line for effort c_1 at (0,0), whilst Ob represents the gradient of the iso-utility line of the agent at (0,0). The iso-utility line of the agent OU^a is assumed convex[10]. Two concave iso-maximal effort lines are shown for efforts c_1 and c_2, where $c_2 > c_1$. Agent utility is increasing to the northwest, whilst the utility of the principal is increasing to the southeast

An improvement is possible through the substitution of capital E_0 by the agent and the use of a reduced threat of share ratio reduction, λ_0 Moving along the iso-utility line of the agent, this involves a change in contract parameters from (0,0) to (E_0,λ_0), with the agent instead supplying effort c_2. Since iso-effort line c_2 lies to the south-east of iso-effort line c_1, the utility of the principal is increased whilst maintaining the utility of the agent. This improvement is only possible if the ovoid area contained within OU^a and OC_1 is non-empty. Non-emptiness of this region obtains if the gradient of line Oa is greater than line Ob, which is the second sufficient condition in (29).

Requiring a contribution of capital by the agent as a substitute for the capital invested by the principal, with an associated increase in the expected share of ex post outcome, is one way of increasing the incentive of the agent to provide effort, whilst possibly (see (29) for sufficient conditions) generating a Pareto improvement

In the next section, we consider the way in which information about the effort supplied by the agent, in addition to the inference available from the realised outcome, can be used by the principal in obtaining (or moving closer to the attainment of) the first-best allocation.

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[10] A convexity expression for agent utility is derived in Appendix I

5.3 Additional signals

We have assumed so far that the only inference available to the principal concerning the effort of the agent is the publicly observable outcome X(e,ε). Consequently, the incentive contract could only be conditioned on the realised outcome observed ex post. However, in reality it is possible that there may exist other signals about agent effort that facilitate improvements to the incentive contract. An example of this is direct monitoring or supervision, in which a noisy signal about agent effort may be available. Holmstrom (1979) states that when an informative signal is costlessly obtained and administered into the contract, that it has positive value no matter how noisy it is, and that a contract which uses the signal will strictly Pareto dominate a contract in which the signal is not used.

We will illustrate the use of additional signals for dichotomous contracts in which the agent is again precluded from receiving negative pay, by reference to lump-sum bonus contracts instead of contingent share ratio contracts, since the latter needlessly obfuscate the conclusion reached in this section

Let there exist a publicly observable signal y(e,μ) which is increasing in the effort supplied by the agent and the ex post realisation of an ex ante uncertain random variable μ. For simplicity assume that ε and μ are independent, and that μ may be described by a distribution function Ψ(μ) with support $[\mu_0,\mu_1]$ Consider an incentive contract φ(X,y) where

$$\varphi(X,y)=\begin{cases}0 & if \quad X<X_c \\ B(y(e,\mu)) & if \quad X\geq X_c\end{cases} \tag{30}$$

Incentive contract φ(X,y) is a simple bonus contract in which the agent receives a lump-sum payment B(y(e,μ)) contingent upon a realised outcome not less than X_C The expected utility of the risk neutral agent is then U^a where

$$U^a = \int_{\mu_0}^{\mu_1}\int_{\varepsilon_0}^{\varepsilon_1} \varphi(X,y)dG(\varepsilon)d\Psi(\mu) - Q(e) \tag{31}$$

Substituting (30) into (31) with $X_C \equiv X(e,\bar{\varepsilon}(e))$ we get

$$U^a = (1 - G(\bar{\varepsilon}(e)))\int_{\mu_0}^{\mu_1} B(y(e,\mu))d\Psi(\mu) - Q(e) \tag{32}$$

Differentiating (32) w.r.t. e to obtain the first-order condition, the necessary condition for the incentive scheme given by (30) to elicit the first-best allocation ($e^*, \underline{U}$) for an agent with reservation utility $\underline{U}$ is

$$\frac{\int_{\mu_0}^{\mu_1} B'(y(e,\mu))y_e(e,\mu)d\Psi(\mu)}{\int_{\mu_0}^{\mu_1} B(y(e,\mu))d\Psi(\mu)} - \frac{G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de}}{1-G(\bar{\varepsilon}(e))} \geq \frac{Q'(e)}{\underline{U}+Q(e)} \tag{33}$$

evaluated at $e = e^*$.

From (33) we see that if the incentive contract can be conditioned on the ex post observation of signal y, then the incentives to supply effort can be increased The first term denotes the incentive pressure created by conditioning the lump-sum bonus on the ex post observation of signal y If $B'(y) > 0$, and $y_e(e,\mu) > 0$ then the first term is positive. Thus when the lump-sum bonus is increasing in a signal y which itself is increasing in agent effort, extra incentive pressure is created in addition to the incentive to achieve the target outcome, as given by the second term on the left hand side of (33). Incentive pressure therefore derives from two effects, the incentive to increase the lump-sum transfer to the agent through extra effort, and the incentive to supply sufficient effort in order to achieve the target and to therefore obtain the lump-sum bonus.

An example of the significance of extra information concerning unobservable agent effort can be seen from the following proposition

Proposition 1· If $G_{11}(\varepsilon) \geq 0$ and there exists some X_C^L for which (33) is binding with $B'(y) = 0$, then there exists some $X_C^{L-} < X_C^L$ for which (33) is binding with $B'(y) > 0$

Proof. See Appendix II

The importance of Proposition 1 is seen as follows. Typically (as noted by Park (1995)), if the first-best allocation is achievable by the use of a lump-sum bonus contract, then there will exist a range of performance targets for which this remains true. As shown in the *proof* of Proposition 1, if the density function ($G_1(\varepsilon)$) is non-decreasing in ε, then the incentive pressure (being the proportional rate of change of expected money utility w.r.t. effort) will be increasing in the target outcome Subsequently, if the first-best allocation is not achievable for the greatest possible target outcome ($X(e^*,\varepsilon_1)$), then no incentive scheme can elicit the first-best allocation, given that (pure) lump-sum bonus contracts ((30)) are the most efficient dichotomous contract form (Park (1995)).

However, from Proposition 1, if there exists an additional signal y from which information concerning the agent effort supplied can be inferred, then by conditioning the incentive contract on this signal, the necessary condition for the existence of an incentive scheme that achieves the first best allocation (33) becomes less restrictive in the sense that the lower bound of target outcomes which achieve the first-best is reduced. This provides a clear advantage to the use of dichotomous contracts that use an informative signal over those that disregard such signals when available, when the first-best allocation is not attainable for the latter contracts in the absence of the use of additional signals

Lastly we note that the use of a lump-sum bonus contract to illustrate the benefit of an additional informative signal about agent effort is without loss of generality. In fact, for any incentive contract, provided the expected pay of the agent is increasing in signal y, the agent has an incentive to increase y through greater effort, whenever the resulting utility from the share of the marginal product value for the agent exceeds his marginal effort cost. This means that a fixed bonus with target outcome decreasing in the observation of y may also be used to increase agent effort, or even a contract with a mix of signal dependent target and signal dependent bonus elements

We now conclude this chapter by collecting some final remarks.

5.4 Concluding remarks

The aim of this chapter was to consider two variations to the use of dichotomous incentive contracts. In the first we extended the contingent share ratio model of Chapter 4 to permit a substitution of capital by the agent, thereby deriving sufficient conditions for a Pareto improvement In the second, we illustrated how the admission of additional publicly observable signals as contract variables in the simplest of dichotomous contracts, the pure lump-sum bonus contract, could expand the available set of contracts which achieve a first-best allocation.

An important aspect of incentive contracts for risk averse agents is the uncertainty faced concerning the end of period income. As mentioned in Chapter 1, fixed wage contracts, which provide no mitigation of moral hazard problems, are in reality the basic contract paradigm. In contrast, profit sharing contracts reduce the moral hazard problem, but may create enough risk that the income premium required by the agent to achieve the reservation utility may be sufficiently large as to impede the efficient use of these contracts.

However, an important advantage of profit sharing contracts over fixed wage contracts is not captured in a single-period setting. Notwithstanding the benefit of incentive effects, profit sharing contracts induce a greater survival probability for firms when the principal-agent relationship is repeated. This is because the revenue necessary to break-even in each period is less than that of a corresponding wage firm by the amount of wages paid to agents. Therefore, the pay premium required by risk averse agents for bearing risk is less if the single-period relationship is repeated due to a greater expected time over which pay is received[11].

Some natural questions which arise from this idea are, for example, how many repetitions, or what inter-period earnings retention policy, are required before the income risk is reduced sufficiently that the agent is awarded no more than his reservation amount? An alternative way to address these questions is to instead derive a measure of threshold risk aversion, such that with the premium savings available from repeating the profit sharing relationship in comparison to repetition of a single-period wage agreement, a risk averse agent is just indifferent between the two contract types.

In Appendix III a multi-period model is developed which captures this issue, and a methodology is established to answer the above questions

In the next chapter, we synthesise much of the discussion of preceding chapters by considering a problem which combines both adverse selection and moral hazard due to the unobservability of effort. This chapter will therefore not only provide an understanding of problems in which the distribution of information disadvantages the principal to an even greater extent in comparison to pure moral hazard or pure adverse selection, but also permits a drawing of the thesis towards a natural pinnacle prior to concluding in Chapter 7.

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[11] This also assumes that the reservation utility is not available to an agent for a duration of time which is the difference between the expected time to bankruptcy of the wage and profit sharing firms

Appendix I

The isoutility line of the agent is given by

$$\left.\frac{d\lambda}{dE}\right|_{\bar{U}} = -\frac{\left(\partial U^a/\partial E\right)_{e,\lambda}}{\left(\partial U^a/\partial\lambda\right)_{e,E}} \tag{A1}$$

Using the operator $$\frac{d}{dE}=\frac{d\lambda}{dE}\left.\frac{\partial}{\partial\lambda}\right|_E+\left.\frac{\partial}{\partial E}\right|_\lambda \tag{A2}$$

differentiate (A1) totally w.r.t. e, using the FOC (3), the independence of the order of differentiation w.r.t E and λ (given (1)) of the cross (second order) partial derivative of U^a w r.t E and λ, and the fact that $U^a_{\lambda\lambda}$ equals zero (from (1)), to give

$$\left.\frac{d^2\lambda}{dE^2}\right|_{\bar{U}} \stackrel{s}{=} -\frac{\partial U^a}{\partial\lambda}\frac{\partial^2 U^a}{\partial E^2}+2\frac{\partial U^a}{\partial E}\frac{\partial^2 U^a}{\partial\lambda\partial E} \tag{A3}$$

where it is understood that partial differentiation w r t. E (λ) holds constant λ (E). From (1)

$$\left.\frac{\partial^2 U^a}{\partial E^2}\right|_{e,\lambda} = \pi''(E)\left[\int_{\varepsilon_0}^{\bar{\varepsilon}(e)}\lambda X(e,\varepsilon)dG(\varepsilon)+\int_{\bar{\varepsilon}(e)}^{\varepsilon_1}X(e,\varepsilon)dG(\varepsilon)\right]-V^{a''}(E) \tag{A4}$$

$$\frac{\partial^2 U^a}{\partial\lambda\partial E}=\int_{\varepsilon_0}^{\bar{\varepsilon}(e)}\pi'(E)X(e,\varepsilon)dG(\varepsilon)>0 \tag{A5}$$

Therefore, since $\left.\frac{\partial U^a}{\partial\lambda}\right|_{e,E}>0$ and $\left.\frac{\partial U^a}{\partial E}\right|_{e,\lambda}<0$ (by assumption), the convexity of the agent's isoutility line is ambiguous.

Appendix II

Proposition 1: If $G_{11}(\varepsilon) \geq 0$ and there exists some X_C^L for which (33) is binding with B′(y) = 0, then there exists some $X_C^{L-} < X_C^L$ for which (33) is binding with B′(y) > 0.

Proof: If we can show that the proportional rate of change of expected money utility w.r.t. effort (the left hand side of (33) with B′(y) = 0) is increasing in target outcome, then we will have proved the proposition From the definition of X_C,

$$X_C^L \equiv X(e^*, \bar{\varepsilon}(e^*)) \qquad \text{(A6)}$$

Keeping e^* constant (this is defined by Chapter 4, (5)) and differentiating (A6) totally gives

$$\frac{\partial}{\partial X_C^L} = \frac{1}{X_\varepsilon} \frac{\partial}{\partial \bar{\varepsilon}(e)} \qquad \text{(A7)}$$

where e and ε are at $(e^*, \bar{\varepsilon}(e^*))$. Also, at $(e^*, \bar{\varepsilon}(e^*))$, from (Chapter 4, (3))

$$\frac{\partial \bar{\varepsilon}(e^*)}{\partial e} = -\frac{X_e}{X_\varepsilon} \qquad \text{(A8)}$$

Using (A7) to differentiate (A8) w.r t. X_C^L gives

$$\frac{\partial}{\partial X_C^L} \frac{\partial \bar{\varepsilon}(e^*)}{\partial e} = -\frac{1}{X_\varepsilon} \frac{\partial}{\partial \bar{\varepsilon}(e)} \frac{X_e}{X_\varepsilon} = -\frac{1}{X_\varepsilon} \frac{X_\varepsilon X_{e\varepsilon} - X_e X_{\varepsilon\varepsilon}}{X_\varepsilon^2} \leq 0 \qquad \text{(A9)}$$

From (33) with B′(y) = 0, the proportional rate of change of expected money utility w.r.t effort at e^* is

$$-\frac{G_1(\bar{\varepsilon}(e^*))\frac{\partial\bar{\varepsilon}(e^*)}{\partial e}}{1-G(\bar{\varepsilon}(e^*))} \tag{A10}$$

Differentiating (A10) w.r.t. X_C^L gives

$$\frac{\partial}{\partial X_C^L}-\frac{G_1(\bar{\varepsilon}(e^*))\frac{\partial\bar{\varepsilon}(e^*)}{\partial e}}{1-G(\bar{\varepsilon}(e^*))}\overset{s}{=}-(1-G(\cdot))\frac{\partial}{\partial X_C^L}\left(G_1(\cdot)\frac{\partial(\cdot)}{\partial e}\right)+G_1(\cdot)\frac{\partial(\cdot)}{\partial e}\frac{\partial}{\partial X_C^L}(1-G(\cdot))$$

$$=-(1-G(\cdot))\left(G_1(\cdot)\frac{\partial}{\partial X_C^L}\frac{\partial(\cdot)}{\partial e}+\frac{\partial(\cdot)}{\partial e}\frac{\partial}{\partial X_C^L}G_1(\cdot)\right)-G_1(\cdot)\frac{\partial()}{\partial e}G_1(\cdot)\frac{\partial(\cdot)}{\partial X_C^L}$$

$$\overset{s}{=}-(1-G(\cdot))\left(G_1(\cdot)\frac{\partial}{\partial()}\frac{\partial()}{\partial e}+\frac{\partial(\cdot)}{\partial e}\frac{\partial}{\partial(\cdot)}G_1(\cdot)\right)-G_1(\cdot)\frac{\partial(\cdot)}{\partial e}G_1(\cdot) \tag{A11}$$

where $(\cdot)$ means $\bar{\varepsilon}(e^*)$ and the second $\overset{s}{=}$ follows from (A7) and $X_\varepsilon > 0$. From (A7), (A9) and $X_\varepsilon > 0$, the first term in the braces in (A11) is non-positive, and the second term in the braces is non-positive for $G_{11}(\varepsilon) \geq 0$ since $\frac{\partial\bar{\varepsilon}(e^*)}{\partial e}<0$ Also, the final term in (A11) is positive since $\frac{\partial\bar{\varepsilon}(e^*)}{\partial e}<0$ and the density $G_1()$ is strictly positive. Therefore, for $G_{11}(\varepsilon) \geq 0$, $\frac{\partial}{\partial X_C^L}-\frac{G_1(\bar{\varepsilon}(e^*))\frac{\partial\bar{\varepsilon}(e^*)}{\partial e}}{1-G(\bar{\varepsilon}(e^*))}\geq 0$.

Q.E.D.

Appendix III

Income risk and the acceptability of profit sharing contracts by risk averse agents in a multi-period relationship

One-period Model

Consider a firm, in which there are N *identical* agents, each of which are compensated equally for a single period contract of duration t. If each agent receives a wage rate w, then the uncertain profit of the firm at time t is X, where

$$X = Y - Nwt \qquad \text{(A12)}$$

Y is the uncertain revenue, net of all costs excluding the fixed wage bill, generated in time t. All profit for the period is distributed at the period end. We assume that the stochastic variable Y is a *normal distribution* such that

$$Y = vt + \sigma t^{1/2} \varepsilon \qquad \text{(A13)}$$

v is the revenue drift rate, σ^2 is the revenue variance rate, and ε is a normal distribution with mean zero and unit variance (i.e. $\varepsilon \sim ND(0, 1)$) The expected profit for the period EX is then given by

$$EX = (v - Nw)t \qquad \text{(A14)}$$

where $v > Nw$.

For the single and multi-period relationship, the principal acts as guarantor, thereby securing a trade credit facility. The liability of each agent is limited to zero, whilst the principal bears all losses.

Multi-period Model

Consider a finite time horizon T divided into n equal intervals of duration t, where T is the contract period for each agent The start time of the m th period is denoted t_{m-1}, where $m = 1, 2, ..., n$. Profit in each period (which is after wages) is realised at the period end, at which time the firm retains a proportion $\eta \in [0, 1]$ of its cumulative earnings to date and carries the retained earnings into the next period

Provided that the cumulative earnings to date (denoted Z) are greater than zero, where agents receive pay via profit sharing, the firm pays out $(1 - \eta)\pi$ of its cumulative earnings to date to the agents, and $(1 - \eta)(1 - \pi)$ of its cumulative earnings to date to the principal at the end of each period, for period ends other than the contract expiry date At the contract expiry date, if the cumulative earnings to date are greater than zero, then the cumulative earnings are entirely distributed to the principal and agents in accordance with the sharing ratio

If the cumulative earnings to date at a period end other than the contract expiry date are less than zero but not less than a bankruptcy limit –C, then no profit share is paid to either the principal or the agents, and for agents who are paid via fixed wages, each agent receives the fixed wage, the firm carrying forward the trading deficit into the next period. Conditional on survival to the contract expiry date, all trading deficits are made good at that time. If however the cumulative earnings are less than or equal to –C at any period end, the firm is bankrupt and all losses are made good at that time.

The cumulative earnings Z_{m-1} at the end of the m-1 th period, are given by

$$\begin{aligned} Z_m &= Y_m - Nwt + \eta Z_{m-1} \\ &= \beta t + \sigma t^{1/2}\varepsilon_m + \eta Z_{m-1} \end{aligned} \tag{A15}$$

where Y_m is the revenue (net of all costs except wages) generated in the m th period, ε_m is a normal distribution with mean zero and unit variance, $Z_0 = 0$ and $\beta \equiv v - Nw$.

For $\eta > 0$, (A15) establishes a recursive expression for the survival probability of the firm past time t_m, $P_\eta(Z_m \geq -C \mid Z_{i<m} \geq -C)$, where Z_m is the cumulative earnings up to and including time t_m, $m \geq 1$, $Z_{i<m}$ denotes $Z_0, Z_1, .., Z_{m-1}$, and $P_\eta(Z_m \geq -C \mid Z_{i<m} \geq -C)$ is equal to

$$(\sigma\sqrt{2\,pi\,t})^{-m} \int_{-C}^{\infty}\int_{-C}^{\infty}\int_{-C}^{\infty} \exp - \frac{1}{2\sigma^2 t}\sum_{j=1}^{m}(Z_j - \beta t - \eta Z_{j-1})^2 dZ_1 dZ_2 \;\; dZ_m \tag{A16}$$

This relationship is derived by observing that the probability that $Z_m \in (\tilde{X}_m, \tilde{X}_m + d\tilde{X}_m)$ conditional upon $Z_i \in (\tilde{X}_i, \tilde{X}_i + d\tilde{X}_i)$ for i = 0, 1 ,2, . ., m-1 (denoted i < m) is given by

$$(\sigma\sqrt{2\,pi\,t})^{-m} \exp - \frac{1}{2\sigma^2 t}\sum_{j=1}^{m}(\tilde{X}_j - \beta t - \eta\tilde{X}_{j-1})^2 d\tilde{X}_1 d\tilde{X}_2 \;\; d\tilde{X}_m \tag{A17}$$

and then integrating (A17) over the intervals of $\tilde{X}_i$, where i = 1, 2, , m-1 and $\tilde{X}_i \geq -C$

Conditional Expected Utility

We assume that the expected utility from a multi-period contract is the sum of the discounted conditional expected utility in each future period, where the condition is survival of the firm into each future period in which the contracting parties expect to receive utility from the contract, and the discount factor translates future utility to its present value

Thus, if $\xi(\Phi(Z_m))$ is the utility of wealth $\Phi(Z_m)$ receivable at time t_m by a contracting party, and $\delta(t_m)$ is the discount factor assigned to future utility receivable at time t_m, then the total expected utility of an n-period contract is

$$\sum_{m=1}^{n} E[\xi(\Phi(Z_m)) \mid Z_{i<m} \geq -C]\delta(t_m) \qquad \text{(A18)}$$

where using (A16), $E[\xi(\Phi(Z_m)) \mid Z_{i<m} \geq -C]$ is

$$(\sigma\sqrt{2\,pi\,t})^{-m} \int_{z_m=-\infty}^{\infty} \int_{z_{m-1}=-C}^{\infty} \int_{z_1=-C}^{\infty} \xi(\Phi(Z_m)) \exp - \frac{1}{2\sigma^2 t} \sum_{j=1}^{m} (Z_j - \beta t - \eta Z_{j-1})^2 dZ_1 dZ_2 \; dZ_m$$

(A19)

Methodology

Denote a *single* period contract which awards a fixed wage w and no profit share $(w, 0)^s$, and a corresponding pure profit sharing contract with no fixed wage element $(0, \pi)^s$ The agents are assumed risk averse, whilst the principal is risk neutral. We then fix w and π such that the expected *pay* for each contract party from either $(w, 0)^s$ or $(0, \pi)^s$ is equal This means that the agent prefers $(w, 0)^s$, whilst the principal is indifferent between $(w, 0)^s$ and $(0, \pi)^s$, since in contrast to the (risk neutral) principal, each (risk averse) agent requires a pay premium for bearing risk in the profit sharing contract over the pay of the fixed wage contract, in order to be indifferent between the two contract types.

The essential feature that we wish to capture is that over a multi-period horizon, the enhanced survival probability of a profit sharing firm over that of a wage firm implies that a multi-period contract which is the repetition of the single period contract in which the principal is indifferent between pure wage and pure profit sharing, creates a preference by the principal for profit sharing Therefore, since the principal expects to receive utility that is increasing in his share of profits, this sharing ratio may be *reduced* for the multi-period contract in order to maintain his indifference between a pure wage and a pure profit sharing agreement

Additionally, the enhanced survival chances of the profit sharing firm over a multi-period horizon may be sufficient for agents to come to prefer a multi-period pure profit sharing contract which is the repetition of the single period pure sharing agreement, over that of the corresponding pure wage agreement. If the expected benefit from enhancement in survival probability alone is not sufficient to persuade agents to accept a pure multi-period sharing contract which is a repetition of the single period contract, then provided that the expected utility of agents is increasing in their sharing ratio, the sharing ratio of the agents can be increased by reducing that of the principal up to a limit where the principal is just indifferent between the two (multi-period) contract types

However, the degree of risk aversion of the agents is of critical importance. Even with an incremental transfer of sharing ratio from the principal to the agents, it may be the case that agents are too risk averse to come to prefer sharing contracts It is this threshold level of risk aversion that we wish to derive.

Critical risk aversion

Let $Y^{+(-)}$ denote the greater (lesser) of Y and zero. By performing a Taylor expansion of uncertain pay about its mean value, the expected utilities of contracts $(w, 0)^s$ and $(0, \pi)^s$ for each agent and the principal are

Agents:	$(w, 0)^s$:	$\Omega^a(wt)$
	$(0, \pi)^s$:	$\Omega^a(EW(0)) + (1/2)\,\Omega^{a''}(EW(0))E(W(0) - EW(0))^2$

where $W(0) \equiv (\pi / N)Y^+$, $EW(0) = (\pi / N)EY^+$ and

Principal:	$(w, 0)^s$:	βt
	$(0, \pi)^s$:	$vt - (\pi / N)EY^+$

The value of the profit sharing ratio $\tilde{\pi}$ which establishes indifference between $(w, 0)^s$ and $(0, \pi)^s$ for the principal is derived by setting βt equal to $vt - (\pi / N)EY^+$, i e.

$$\tilde{\pi} = \frac{Nwt}{EY^+} \tag{A20}$$

Suppose now that the relationship is repeated n times. The principal will now prefer $(0, \tilde{\pi})^m$ to $(w, 0)^m$ (where m denotes multiple repetitions of the single period contract). To restore the indifference of the principal between $(0, \tilde{\pi})^m$ and $(w, 0)^m$, $\tilde{\pi}$ (the total share ratio of the agents) is increased to $\tilde{\pi}^*$, where $\tilde{\pi}^*$ is derived from equating the expected utilities of the principal between $(0, \tilde{\pi}^*)^m$ and $(w, 0)^m$, i e

$$\sum_{m=1}^{n} E[(1 - \tilde{\pi}^*)(1 - \Delta(m)\eta)Z_m^{0+} + J_m^0 \mid Z_{i<m}^0 \geq -C]\delta_p(t_m)$$

$$= \sum_{m=1}^{n} E[(1 - \Delta(m)\,\eta)Z_m^+ + J_m \mid Z_{i<m} \geq -C]\delta_p(t_m) \tag{A21}$$

where $Z_m^0 = vt + \sigma t^{1/2}\,\varepsilon_m + \eta Z_{m-1}^0$, $Z_m = \beta t + \sigma t^{1/2}\,\varepsilon_m + \eta Z_{m-1}$, $Z_0^0 = 0$, $J_m^0 \equiv (Z_m^0 + C)^- - C(Z_m^0 + C)^- / (Z_m^0 + C)$, $\Delta(m) = 1$ for $m \neq n$, $\Delta(n) = 0$, and the principal discounts future utility at time t_m by $\delta_p(t_m)$ By rearranging terms, (A21) gives $\tilde{\pi}^*$ explicitly

The n-period utilities of the agents are:

$(w, 0)^m$: $$\sum_{m=1}^{n} E[\Omega^a(wt) \mid Z_{i<m} \geq -C]\delta_a(t_m)$$

$(0, \tilde{\pi}^*)^m$. $$\sum_{m=1}^{n} E[\Omega^a[(\tilde{\pi}^*/N)(1-\Delta(m)\eta)Z_m^{0+}] \mid Z_{i<m}^0 \geq -C]\delta_a(t_m)$$

where $\delta_a(t_m)$ is the discount factor by which each agent discounts utility at time t_m

Now assuming that each agent has *constant absolute risk aversion* χ_A, and denoting the expectation of the utility of pay W_m at t_m conditional on event τ_m, $E[\Omega^a(W_m)|\tau_m] = \Omega^a(E[W_m|\tau_m]) - (1/2)\chi_A\Omega^{a''}(E[W_m|\tau_m])E[(W_m - E[W_m|\tau_m])^2|\tau_m]$, the threshold risk aversion making each agent indifferent between $(w, 0)^m$ and $(0, \tilde{\pi}^*)^m$ is

$$\frac{\sum_{m=1}^{n} [\Omega^a(E_c^0 \tilde{W}_m^0) - E_c \Omega^a(wt)]\delta_a(t_m)}{1/2 \sum_{m=1}^{n} \Omega^{a'}(E_c^0 \tilde{W}_m^0) E_c^0[(\tilde{W}_m^0 - E_c^0 \tilde{W}_m^0)^2]\delta_a(t_m)} \qquad \text{(A22)}$$

where $\tilde{W}_m^0 \equiv (\tilde{\pi}^*/N)(1 - \Delta(m)\eta)Z_m^{0+}$, and subscript c denotes expectation conditional on survival, with the superscript 0 denoting zero base wage.

Remark

The methodology used to derive (A22) is certainly more general than the assumptions used to build the model. The most obvious limitation of this model is the use of absolute returns, in which the revenue generated in each period is independent of the cumulative earnings at the beginning of the period Other simplifications include a constant drift (v) and variance (σ^2) rate, a constant earnings retention policy between periods, and the assumption of a normal distribution for period returns.

CHAPTER 6

DICHOTOMOUS LIMITED LIABILITY PROFIT SHARING CONTRACTS WITH UNOBSERVABLE EFFORT AND HIDDEN PRODUCTIVITY

6.1 Introduction

The moral hazard problems considered thus far in the thesis have *separately* examined the impact of imposing a floor to the pay of an agent who has either been endowed with private information about the marginal productivity of a project for which he appeals to a financier to invest venture capital (Chapter 3), or is able to supply effort which cannot be observed or at least verified by the investor (Chapters 4 and 5).

The aim of this chapter is to synthesise the problems explored in Chapters 3, 4, and 5 in order to examine a situation in which a borrower supplies unobservable effort (moral hazard) and is privately informed as to the marginal productivity of capital prior to the formalisation of a contract (adverse selection)

In the next section we review some of the key literature in information economics which provides insight into issues which arise in problems that combine both moral hazard and adverse selection[1].

[1] Another example of a mixed problem (taken from Picard (1987)) includes an insurer who is unable to identify low and high risk individuals, and who also cannot observe the level of care taken once insurance is purchased

6.2 Moral Hazard and Adverse Selection

In the literature which examines mixed moral hazard and adverse selection problems, where the principal and agent are risk neutral[2], the central message is that the optimal solution does not necessarily entail welfare losses when compared to the optimal pure adverse selection contract in which effort is observable and agent type is *apriori* hidden (see review article by Guesnerie et al (1988)).

In focusing concentration on the design of contract mechanisms which implement (or at least approximately implement) the optimal pure adverse selection fee schedule and associated effort levels for each agent type when effort is unobservable, Picard (1987) derives an optimal menu[3] of reward schedules where agent pay is the sum of a fixed (type contingent) fee[4], and a transfer which is either linear or quadratic in the difference between the actual outcome and that which is expected given the agent's type[5]

To appreciate the intuition of optimal fee schedules which are linear, consider the situation of a market for the services of agents where a principal designs a contract menu which exploits the competition among potential agents whilst inducing them to reveal their types[6] (McAfee and McMillan (1987)) The marginal disutility of effort is lower for

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[2] In the following discussion, all contracting parties are risk neutral unless otherwise stated

[3] Caillaud et al (1986) (and Melumad and Reichelstein (1986)) consider an alternative implementation approach to the use of a family of reward schedules They examine the conditions which permit implementation via a single reward schedule A necessary condition that this reward schedule must satisfy is that its expectation (over exogenous uncertainty) for a given effort must be equal to the function which maps type dependent effort to its associated reward (also by type) in the menu of contracts, such that each approach implements the same outcome However, informational requirements are usually stronger in single schedule mechanisms, possibly as a result of which this technique is far less prevalent in the literature

[4] which can be positive or negative

[5] When the mapping from agent effort to reward for each type is convex, a linear fee schedule is optimal For non-convex mappings, the quadratic schedule implements the incentive compatible allocations

[6] See the revelation principle discussed in 2 2 2 1

higher agent types and potential agents are asked to report their type, after which the winning agent chooses an effort not observed by the principal. For a contract which is linear in output to be optimal for the principal, incentive compatibility requires that the share component of the agent's fee schedule must increase with agent efficiency[7] This is because it does not pay a potential agent with low ability to claim that he is more able than he really is, given that he would then be penalised by a contract which highly gears his pay to a level of output which is expected to be lower in comparison to his more efficient counterparts Further, as a result of a more highly geared contract, more efficient agents are induced to work harder, with the most able agent supplying a socially optimal effort and retaining his entire marginal output[8].

In McAfee and McMillan (1987), a trade-off exists in that the greater the diversity in the efficiency of potential agents, the less successful is the contract in inducing effort. In a similar framework[9] for which agents are instead risk averse and precontractually endowed with private information about expected production costs, McAfee and McMillan (1986) consider the trade-offs of an incentive contract awarded to the lowest (cost) bidding agent, which makes rewards dependent both on the bid and on realised costs[10] These authors identify an effect only present in the mixed adverse selection and moral hazard bidding model Bidding competition amongst agents increases (bids decline) as their share of cost over/under runs decreases. For example, if their share is

[7] The sharing provision in a contact both screens the potential agents and elicits effort

[8] The fully deductible/franchise contract is therefore only optimal for the most efficient agent, but is otherwise totally inefficient For all other agents, by lowering the induced level of effort below the first-best (full-information) solution which obtains when types are precontractually observable, the principal captures some of the information rents which less efficient agents (who would otherwise pass themselves off as being more efficient) can achieve

[9] Important differences include the assumption of the existence of a symmetric Nash equilibrium in which agents bid in exactly the same *way* given their true expected costs, i e a bid function which is the same for all agents The approach in McAfee and McMillan (1986) is to impose a contract which is linear in cost over/under runs, and derive the optimal share ratio given symmetric behaviour by agents who each submit bids which maximise their expected utilities as weighted by the probability of their bid being the lowest

[10] This type of contract is evident for example in publishing rights, whereby payments to an author include a fixed sum equal to the bid, plus royalties

one, each agent must bid to cover his entire cost, thereby constraining the bid to be high However, with a share ratio close to zero, each agent can effectively ignore his costs in making a bid, thereby forcing agents to bid lower and increasing the competition among them Essentially, the need to increase bidding competition by allocating agents a lower share of ex post costs acts together with insurance in the trade-off with incentives[11]

Optimal incentive contracts which are linear in cost over/under runs have also been reported by Laffont and Tirole (1986) in a model of firm regulation without bidding in which a central planner procures a public good that provides a consumer surplus. Production cost depends on an intrinsic cost parameter (private to the firm), (unobservable) cost reducing effort, and a random variable with zero mean The planner optimally pays a fixed sum and reimburses a fraction of the costs. This fraction is inversely related to the fixed transfer[12] and increases with the firm's announced expected costs, as (perfectly) signalled by its choice from a contract menu specifying production quantity and transfer to the firm by type, with the transfer also dependent on (observable) realised cost[13]

The optimality of linear or quadratic schemes for problems which mix moral hazard and adverse selection relies crucially on assuming that agents are risk neutral. In common with the case of pure moral hazard for which agent type is publicly observable, linear or

[11] as a result of which, even under risk neutrality, corner solutions (values at either end of a support) for the share ratio are never optimal

[12] The most efficient firm chooses a fixed-price contract, with the less efficient firms opting for an incentive contract The fixed transfer increases with the proportion of total costs that the firm is willing to share

[13] In deriving these results, Laffont and Tirole (1986) apply an unusual approach They consider a restricted class of possible deviations from an equilibrium $(\beta, e(\beta))$ in which, after having untruthfully announced its intrinsic cost efficiency (as β'), a firm supplies a cost reducing effort $(e(\beta|\beta'))$ which is different from the effort associated with the announcement which would be optimal for the planner, were the announcement to have been true $(e(\beta'))$, i e $e(\beta|\beta') = e(\beta) + \beta' - \beta$ The set $(\beta', e(\beta|\beta'))$ is referred to as the concealment set They show that the linear contract rules out deviations for this restricted class and then show that the solution makes deviations outside of the concealment set also unprofitable for the firm

quadratic incentive contracts induce inefficiencies[14] under an assumption of risk aversion[15] At variance with linear or quadratic schemes, dichotomous incentive contracts which impose large penalties for outcomes below a pre-specified performance target may implement the optimal pure adverse selection allocations and effort[16] when agents are instead risk averse.

Zou (1992) examines dichotomous contracts for which risk averse agents, who differ according to their effort disutilities, self-select a contract (an effort/fee schedule pair) parametised by agent type When the distribution of output has a fixed support and output realisations permit a sufficiently accurate inference of agent effort[17], moral hazard may be approximately[18] eliminated Alternatively, if the lower bound of the output distribution strictly increases with agent effort, then the moral hazard element of the relationship can be entirely eliminated.

To appreciate why moral hazard is completely eliminated in the case of movable supports, we observe from Zou (1992) that the performance target is set equal to the lower bound of the support evaluated at the effort level which is optimal in the pure adverse selection problem for that agent type. Since outcomes below this lower bound are a perfect signal that the agent has supplied less than the required effort, threats of

[14] due to the premium for bearing risk required by the agent (see 2 2 1 1)

[15] Note again that McAfee and McMillan (1986) *impose* a linear share contract and derive the share ratio which maximises the principal's utility when agents are risk averse There is no assertion that a linear contract is optimal

[16] These schemes can also be used when agents are risk neutral and can be credibly threatened with punishment for low realised profit outcomes However, for these schemes to apply, the principal is supposed to exactly know the entire set for the support of the (bounded) stochastic component in the production function In contrast, linear or quadratic schemes are less informationally demanding (Picard (1987))

[17] such as for a normal distribution, where the likelihood ratio diverges to $-\infty$ when the output approaches the lower bound of the distribution

[18] i e the optimal dichotomous scheme asymptotically implements the best pure adverse selection outcome

sufficiently large penalties force[19] the agent to supply the desired effort and achieve the performance target Additionally, and most importantly for risk averse agents, by awarding the agent a fixed allocation (dependent only on type) for outcomes above the performance target, which is equal to the agent's pay in the pure adverse selection problem[20], all income risk is removed, thereby eliminating the cost which would otherwise accrue given the risk aversion of the agent

The results in Zou (1992) discussed above rely on the credibility of the principal imposing severe penalties on the agent for outcomes below the performance target Clearly, when the liability of the agent is instead limited, such schemes become unenforceable However, given that the principal can never obtain a strictly greater expected utility than that which he would obtain if effort were instead observable[21], a methodology has been applied[22] in Zou (1992) by which we may attempt to synthesise the topics in the previous chapters of this thesis. After we present the model in the following section, we will introduce the mixed adverse selection and moral hazard game by discussing the methodology that will allow us to derive the necessary condition for implementation of the optimal pure adverse selection outcome by a contingent share ratio scheme in which borrower income cannot be less than zero

[19] the utility of income must tend to $-\infty$ as income tends to $-\infty$

[20] In the pure adverse selection problem, agent pay is fixed per type, since there is no requirement to provide incentives given that effort is observable and therefore enforceable Any fee schedule which conditions the pay of agents on realised profit imposes unnecessary costs due to risk aversion

[21] This is shown formally in Zou (1992) (Corollary 1) The intuition is that the best solution to a constrained optimisation problem cannot be improved upon by increasing the number (or severity) of constraints

[22] That the optimal pure adverse selection outcome represents the benchmark (or first-best) outcome of the combined moral hazard and adverse selection was earlier shown (inter alia) by Laffont and Tirole (1986), Picard (1987), Guesnerie et al (1988), and Caillaud et al (1988)

6.3 The Model

In this section we introduce a model which synthesises a variant of each of the models in Chapter 3, 4, and 5 We then discuss the hybrid model with specific reference to the original specifications, prior to a subsequent discussion of the methodology used to derive and implement the pure adverse selection solution using a contingent share ratio scheme

Let $\theta \in [\underline{\theta}, \overline{\theta}]$ be a multiplicative productivity parameter[23], such that for capital investment $K(\theta)$, effort $e(\theta)$, and ex ante uncertain state of nature $\varepsilon \in [\varepsilon_0, \varepsilon_1]$, there exists a production technology $H(K(\theta),e(\theta),\varepsilon)$ which generates revenue $\theta H(K(\theta),e(\theta),\varepsilon)$ $H(K(\theta),e(\theta),\varepsilon)$ is increasing in ε, and increases in $K(\theta)$ and $e(\theta)$ at a decreasing rate, i e $H_i(.,.,.) > 0$ for $i = 1,2,3$, $H_{ii}(, ,) < 0$ for $i = 1,2$, where subscripts denote partial differentiation w r t the i^{th} argument of $H(, ,)$ The certain and constant publicly observable marginal opportunity cost of capital is r, and profit is $\theta H(K(\theta),e(\theta),\varepsilon) - rK(\theta)$.

23 An interesting model in which agent type θ is a probability distribution $G^{\theta}(\varepsilon)$ of exogenous uncertainty ε is applied in Sappington (1983b) Sappington determines whether the outcome of a principal-agent relationship will be ex-post efficient when the agent's information is initially better than that of the principal, in the sense that the agent alone knows the actual distribution of uncertainty, and uncertainty ε is resolved prior to the supply of unobservable effort The analysis concludes that although the risk-neutral agent's superior information will often lead the principal to induce inefficient outcomes intentionally, such will not always be the case This means that the standard result of inefficiency in all states except the highest when the agent's information is initially perfect (the agent knows the actual value of ε) does not carry over to the case of imperfect information (the agent knows only the distribution of uncertainty) Sappington also separately derives conditions sufficient to ensure either an efficient or inefficient outcome

Further, the assumptions of continuous or discrete distributions $G^{\theta}(\varepsilon)$ are shown to qualitatively differ The reason for this is as follows Since the principal will sometimes find it optimal to induce inefficient outcomes for some distributions in order to reduce the rents that must be awarded the agent in more productive environments, it is advantageous to be able to manipulate payoff differentials more freely However, the conditions under which it will be possible to do so are stronger when $G^{\theta}(\varepsilon)$ is continuous than when discrete because the incentive compatibility constraints are less restrictive in the latter case

Since a pay floor at zero is imposed for the agent, we assume that profit is non-negative for all θ and ε. The investor has prior beliefs about borrower types captured by cumulative distribution F(θ), and the cumulative distribution of ε is G(ε) The disutility of borrower effort e(θ) is Q(e(θ)), where Q'(e(θ)) > 0 and Q''(e(θ)) > 0.

Features which exist in the isolated models and not in the hybrid case are the following. In the pure adverse selection problem in Chapter 3, sorting (separation) of borrowers was achieved by parametising a set of investment and deductibility of capital cost pairs on a reported message variable $\hat{\theta}$. Importantly for moral hazard due to unobservable effort in the present context, we instead achieve separation of borrowers using a set of investment and effort pairs, and set the deductibility of capital cost (α(θ)) equal to one for all types of borrower.

A feature of the hybrid model not present in the isolated case, is a production function which explicitly includes effort as a factor of production In contrast in Chapter 3, effort was assumed equal for all borrowers, the disutility of which was written into their (equal) reservation utilities, with an implicit assumption of separability between the dependence of the production function on borrower effort and investor capital As such, it is true in general for the production function[24] in the present context that the marginal productivity of capital is not only determined by θ, but also by effort e(θ) and a random variable ε

We now introduce the game played between the investor and borrower, which explains in detail the methodology applied in solving the mixed problem

[24] unless we make simplifying assumptions For example, if the production function is $a\theta H(K(\theta)) + b\widetilde{H}(e(\theta)) + \varepsilon$, where a and b are constants, then θ is the marginal productivity of capital, but not effort

6.4 The Game

An investor is faced with a borrower who is privately informed about his type θ and is required to supply productive effort e(θ) in generating uncertain profits θH(K(θ),e(θ),ε) – rK(θ). If the investor could observe effort, then he could offer the borrower a simple contract menu which induces the borrower to truthfully reveal his type The contract menu would be a set of effort and investment pairs {e(θ), K(θ)} parametised on a message variable which is the borrower's reported type, where the relationship between e(θ) and K(θ) is determined by the type θ incentive compatibility constraint. From the revelation principle (2.2 2 1) this is without loss of generality, and the investor's problem reduces to identifying the contract menu from amongst the family of menus which induce truthful reporting, that menu which delivers him the greatest expected utility Self-selection of one of these pairs by a borrower not only determines the recommended effort, but also the expected pay of the borrower, since the borrower is depicted to share a proportion $\pi \in (0,1)$ of ex post realised profit.

Strictly speaking, in a pure adverse selection context in which effort is observable, there is no gain to the investor in conditioning the pay of the borrower on ex post profits, since effort is enforceable and contracts need not provide incentives If the borrower were risk averse, unnecessary losses result for the investor by conditioning the pay of the borrower on ex post profits This observation provides justification for the modelling simplification admissible in Zou (1992), where a contract menu $\{\tilde{e}(\theta), \tilde{\varphi}(\theta)\}$ is offered to an agent with utility $\Omega^a(\tilde{\varphi}(\hat{\theta}))$-$Q(\tilde{e}(\hat{\theta}),\theta)$. Agents reporting truthfully receive fixed pay $\tilde{\varphi}(\theta)$, and differ according to their marginal disutility of effort as captured by θ

In the context of the pure adverse selection problem in this chapter, where there is no moral hazard, the borrower's realised utility is instead $\varphi(\theta,\hat{\theta})$-$Q(e(\hat{\theta}))$, where $\varphi(\theta,\hat{\theta})$ = $\pi(\theta H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon) - rK(\hat{\theta}))$. It is clear that a contract menu such as {e(θ), K(θ)} which induces truthful type revelation is equivalent to $\{e(\theta), \tilde{\tilde{\varphi}}(\theta)\}$ for a *risk neutral* borrower provided that the fixed pay $\tilde{\tilde{\varphi}}(\theta)$ equals the expectation of $\varphi(\theta)$ (identical to $\varphi(\theta,\theta)$) taken over the distribution of ε *for investment* $K(\theta)$, i e $\tilde{\tilde{\varphi}}(\theta) = E_\varepsilon \pi X(\theta,\varepsilon)$ where $X(\theta,\varepsilon) \equiv \theta H(K(\theta),e(\theta),\varepsilon) - rK(\theta)$ In the analyses of the proceeding sections, we impose agent pay to be a proportional allocation of ex post profits, and merely observe in passing the simplification available when examining pure adverse selection problems for which pay schemes may be permitted to reward agents with fixed allocations which do not vary with the realisation of ex post profit.

Turning now to the introduction of moral hazard into the problem faced by the investor, at issue is how the investor can maximise his expected utility through the design of a contract mechanism when the the borrower chooses his effort to maximise his own utility. A crucial observation in answering this question derives from the impact of moral hazard on the incentive compatibility constraints in the mixed problem.

It transpires that moral hazard does not affect the distortionary influence of the borrower's precontractual private information[25] Therefore, the compound incentive problem, that of truthful revelation of private information and the procurement of greater effort from the borrower, distil into two separate problems As such, the investor offers an expanded contract menu The contract menu is the set of effort and investment pairs for each borrower type which would be optimal in the pure adverse selection problem, and in addition, a type contingent dichotomous scheme The effort and associated investment pair chosen by the borrower induces truthful type revelation, and the contingent share ratio scheme precipitated from the choice of effort/investment pair, attempts to induce the borrower to supply the recommended level of effort. In order for the expanded contract menu to achieve the outcome of the pure adverse selection

[25] To see this note that if the expected utility of a borrower of type θ supplying effort e and reporting his type to be $\hat{\theta}$ is $U^a(\theta,\hat{\theta},e)$, then (local) incentive compatibility means that $U^a_{\hat{\theta}}(\theta,\hat{\theta},e)\Big|_{\hat{\theta}=\theta} = 0$ and moral hazard ensures $U^a_e(\theta,\hat{\theta},e) = 0$ Therefore, $dU^a(\theta,e) = U^a_\theta(\theta,\hat{\theta},e)\Big|_{\hat{\theta}=\theta}$ However, this is the (local) incentive compatibility constraint without moral hazard (for effort e)

problem, the contingent share ratio scheme must also leave the borrower (and therefore the investor) with exactly the same expected utility

Lastly, for the contingent share ratio scheme to award the borrower the same expected utility as he would achieve in the pure adverse selection problem, there must be no chance that the borrower can be punished by a lower share ratio if he had actually supplied the recommended amount of effort The performance target is therefore set equal to the profit which would obtain were the borrower to supply the recommended effort and the lowest possible value of ε were to be realised The share ratio which obtains for outcomes no less than the performance target is equal to the share ratio in the pure adverse selection problem Therefore, if the scheme induces the borrower to supply the recommended effort, his expected utility is the expectation of his share of ex post profit taken over the *entire* support of $\varepsilon \in [\varepsilon_0, \varepsilon_1]$, with a share ratio equal to π

We may now summarise the game for the mixed problem. The investor offers the borrower a contract menu $\{e_A(\theta), K_A(\theta), \lambda(\theta), X_c(\theta)\} \forall\theta$ on a take-it-or-leave-it basis, where $\{e_A(\theta), K_A(\theta)\} \forall\theta$ are the incentive compatible effort and investment pair for each type of borrower in the pure adverse selection problem The set $\{\lambda(\theta), X_c(\theta)\} \forall\theta$ define a contingent share ratio scheme with proportional share ratio reduction $\lambda(\theta)$ and profit target $X_c(\theta)$ The share ratio of the borrower which obtains if realised profit is no less than the performance target is π, and for realised profit less than the performance target, the share of ex post profits allocated to the borrower is $\lambda(\theta)\pi$ The target $X_c(\theta)$ equals the lowest possible profit if the borrower supplies effort $e_A(\theta)$. If accepted, the borrower selects a contract intended for his type from the menu The investor then sinks capital $K_A(\theta)$, after which the borrower supplies effort $e_A(\theta)$, being induced to do so by the incentive pressure created by the associated contingent share ratio scheme State of nature ε is then realised from $\varepsilon \in [\varepsilon_0, \varepsilon_1]$, after which allocations are made to the investor and to the borrower from realised profits

In the next section we derive the optimal contract menu for the pure adverse selection problem, after which we turn to examine the conditions for which a contingent share ratio scheme implements this solution in a combined moral hazard and adverse selection setting

6.5 The Pure Adverse Selection Problem

Recall from 6 2 that the best outcome possible for the investor in the mixed problem is to design a contract mechanism which is incentive compatible and induces a borrower to implement the investment and effort pair which would be optimal were effort to be observable. In this section we derive the contract menu which is optimal in the pure adverse selection problem

6.5.1 Incentive compatible mechanisms

Suppose that the pay of a borrower of type θ who declares his type to be $\hat{\theta}$ for state of nature ε is $\varphi(\theta,\hat{\theta},\varepsilon)$, where $\varphi(\theta,\hat{\theta},\varepsilon) = \pi X(\theta,\hat{\theta},\varepsilon) = \pi(\theta H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon) - rK(\hat{\theta}))$

The expected utility of a type θ borrower declaring $\hat{\theta}$ is then $U^a(\theta,\hat{\theta})$ where

$$U^a(\theta,\hat{\theta}) = \int_{\varepsilon_0}^{\varepsilon_1} \pi X(\theta,\hat{\theta},\varepsilon)dG(\varepsilon) - Q(e(\hat{\theta}))$$

$$\equiv E_\varepsilon \pi X(\theta,\hat{\theta},\varepsilon) - Q(e(\hat{\theta})) \qquad (1)$$

where the (unconditional) expectation operator is over stochastic state of nature ε.

Global incentive compatibility requires that

$$U^a(\theta,\theta) \geq U^a(\theta,\hat{\theta}) \qquad \forall\theta \in [\underline{\theta},\overline{\theta}], \forall\hat{\theta} \in [\underline{\theta},\overline{\theta}] \quad (2)$$

where (given suitable a monotonicity condition, see Appendix I) we can instead replace the global incentive compatibility constraint of (2) with the locally true conditions

$$\left.\frac{\partial U^a(\theta,\hat{\theta})}{\partial\hat{\theta}}\right|_{\hat{\theta}=\theta} = 0 \quad \text{and} \quad \left.\frac{\partial^2 U^a(\theta,\hat{\theta})}{\partial\hat{\theta}^2}\right|_{\hat{\theta}=\theta} < 0 \qquad (3)$$

where the second condition in (3) ensures that the first condition is both necessary and sufficient to ensure that the local stationary point $\hat{\theta} = \theta$ is a maximum for a type θ borrower[26]

Now differentiating $U^a(\theta,\hat{\theta})$ totally (by the chain rule), setting $\hat{\theta} = \theta$, and applying (3), we derive

$$\frac{dU^a(\theta)}{d\theta} = \left.\frac{\partial U^a(\theta,\hat{\theta})}{\partial\theta}\right|_{\hat{\theta}=\theta} \qquad (4)$$

Substituting (1) into (4) now yields

$$\frac{dU^a(\theta)}{d\theta} = E_\varepsilon \pi H(K(\theta), e(\theta), \varepsilon) \qquad (5)$$

where $U^a(\theta,\theta)$ is written as $U^a(\theta)$, and (5) may be integrated to give

$$U^a(\theta) = U^a(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} E_\varepsilon \pi H(K(s), e(s), \varepsilon) ds \qquad (6)$$

[26] Note that by differentiating $U^a_{\hat{\theta}} = 0$ totally, and then setting $\hat{\theta} = \theta$, the second order condition may also be written $\left.U^a_{\theta\hat{\theta}}\right|_{\hat{\theta}=\theta} > 0$

Since all borrowers except type $\underline{\theta}$ have an incentive to pass themselves off as lower θ types if the investor were to offer the full information (observable type) contract menu, borrowers are awarded an information rent in the adverse selection problem[27](the second term in (6)) which is increasing in type (θ)

6.5.2 The optimal pure adverse selection contract menu

We are now ready to derive the optimal pure adverse selection contract menu, denoted $\{e_A(\theta), K_A(\theta)\}\forall\theta\in[\underline{\theta},\overline{\theta}]$. The investor's expected utility $U^p(\theta)$ for a type θ borrower who self-selects a contract from an incentive compatible menu $\{e_A(\theta), K_A(\theta)\}$ is given by

$$U^p(\theta) = \underset{\varepsilon}{E}(\theta H(K(\theta),e(\theta),\varepsilon) - rK(\theta)) - \underset{\varepsilon}{E}\varphi(\theta,\varepsilon)$$

$$= \underset{\varepsilon}{E}\theta H(K(\theta),e(\theta),\varepsilon) - rK(\theta) - U^a(\theta) - Q(e(\theta)) \qquad (7)$$

where (7) follows from (1) with $\hat{\theta} = \theta$ The investor designs a contract menu which maximises the expectation of the utility $U^p(\theta)$ he receives from a type θ borrower over the distribution of possible types that he faces, subject to incentive compatibility and participation by the borrower From (6), this latter constraint is satisfied for all types if $U^a(\underline{\theta}) \geq \underline{U}$, where $\underline{U}$ is the reservation utility of each borrower type[28].

The solution will satisfy the incentive compatibility constraint of the borrower if $U^a(\theta)$ in (7) is given by (5) or equivalently (6) Therefore, taking the expectation of $U^p(\theta)$ over the distribution of types, the expected utility $\underset{\theta}{E}U^p(\theta)$ of the investor is given by

[27] see discussion in 3 4 2 , and particularly footnote no 23

[28] Assuming that the reservation utility for all types of borrower is equal implies that borrowers can only extract rents from their privately endowed productivity information if they contract an agreement with the investor Outside of this relationship their information has no value

$$\int_{\underline{\theta}}^{\overline{\theta}}[E_{\varepsilon}\theta H(K(\theta),e(\theta),\varepsilon)-rK(\theta)-U^{a}(\theta)-Q(e(\theta))]dF(\theta) \tag{8}$$

In order to simplify (8), we integrate $E_{\theta}U^{a}(\theta)$ by parts, whereby

$$\int_{\underline{\theta}}^{\overline{\theta}}U^{a}(\theta)dF(\theta)=F(\overline{\theta})U^{a}(\overline{\theta})-F(\underline{\theta})U^{a}(\underline{\theta})-\int_{\underline{\theta}}^{\overline{\theta}}\frac{dU^{a}(\theta)}{d\theta}F(\theta)d\theta$$

$$=U^{a}(\overline{\theta})-\int_{\underline{\theta}}^{\overline{\theta}}\pi E_{\varepsilon}H(K(\theta),e(\theta),\varepsilon)F(\theta)d\theta \tag{9}$$

and the second line in (9) follows from incentive compatibility (5) Substituting (9) into (8) we derive the expected utility (ignoring $U^{a}(\overline{\theta})$ which is a constant) as

$$\int_{\underline{\theta}}^{\overline{\theta}}\left[\left(\theta+\pi\frac{F(\theta)}{F_{1}(\theta)}\right)E_{\varepsilon}H(K(\theta),e(\theta),\varepsilon)-rK(\theta)-Q(e(\theta))\right]dF(\theta) \tag{10}$$

Now maximising (10) by pointwise optimisation w r t e(θ) and K(θ), the optimal pure adverse selection contract menu $\{e_A(\theta), K_A(\theta)\}$ is given for all $\theta\in[\underline{\theta},\overline{\theta}]$ by the first order conditions[29]

$$\left(\theta+\pi\frac{F(\theta)}{F_{1}(\theta)}\right)\int_{\varepsilon_{o}}^{\varepsilon_{1}}H_{1}(K_{A}(\theta),e_{A}(\theta),\varepsilon)dG(\varepsilon)-r=0 \tag{11}$$

[29] Given the general form of production function $H(K_A(\theta),e_A(\theta),\varepsilon)$, signing $e_A'(\theta)$ and $K_A'(\theta)$ is ambiguous However, by differentiating (11) and (12) w r t θ, assuming $H_{12}(\,)=0$, and $\phi'(\theta)>0$, where $\phi(\theta)\equiv\theta+\pi F(\theta)/F_1(\theta)$, $K_A'(\theta)>0$ Further, if $Q''(e_A(\theta))-\phi(\theta)H_{22}(\,)>0$ (as for example if $Q(e)=e^{\omega}$, where ω is only just greater than one), then $e_A'(\theta)>0$

$$\left(\theta + \pi \frac{F(\theta)}{F_1(\theta)}\right) \int_{\varepsilon_0}^{\varepsilon_1} H_2(K_A(\theta), e_A(\theta), \varepsilon) dG(\varepsilon) - Q'(e_A(\theta)) = 0 \qquad (12)$$

We are now ready to establish the necessary conditions for implementation of the solution to the pure adverse selection problem when moral hazard obtains, and effort is instead *unobservable*.

6.6 Adverse Selection with Moral Hazard

Having derived the optimal pure adverse selection contract menu $\{e_A(\theta), K_A(\theta)\} \forall \theta \in [\underline{\theta}, \overline{\theta}]$, we are now ready to derive the conditions under which a contingent share ratio scheme can implement effort $e_A(\theta)$ after a borrower selects $(e_A(\theta), K_A(\theta))$ from the menu

Let the target profit be $X_C(\theta) = X(K_A(\theta), e_A(\theta), \varepsilon_0)$, where $X(K_A(\theta), e_A(\theta), \varepsilon) \equiv \theta H(K_A(\theta), e_A(\theta), \varepsilon) - rK_A(\theta)$. Denote the critical realisation $\bar{\varepsilon}(e(\theta))$ of ex ante uncertain variable ε for effort $e(\theta)$ and investment $K_A(\theta)$ of a θ type borrower such that

$$X_C(\theta) \equiv X(K_A(\theta), e(\theta), \bar{\varepsilon}(e(\theta))) \qquad (13)$$

From (13), if type θ borrower supplies effort $e(\theta)$, for $\varepsilon \geq \bar{\varepsilon}(e(\theta))$, $X \geq X_C(\theta)$, and for $\varepsilon < \bar{\varepsilon}(e(\theta))$, $X < X_C(\theta)$. Define a contingent share ratio scheme such that the share ratio of the borrower is π for $\varepsilon \geq \bar{\varepsilon}(e(\theta))$, and $\lambda(\theta)\pi$ for $\varepsilon < \bar{\varepsilon}(e(\theta))$. Then the expected utility $U^a(\theta, e(\theta))$ of a type θ borrower who truthfully declares his type and supplies effort $e(\theta)$ is

$$\int_{\varepsilon_o}^{\bar{\varepsilon}(e(\theta))} \lambda(\theta)\pi X(K_A(\theta),e(\theta),\varepsilon)dG(\varepsilon) + \int_{\bar{\varepsilon}(e(\theta))}^{\varepsilon_1} \pi X(K_A(\theta),e(\theta),\varepsilon)dG(\varepsilon) - Q(e(\theta)) \qquad (14)$$

Differentiating (14) w r t. $e(\theta)$, a necessary condition for the borrower to supply an effort no less than $e_A(\theta)$ is $U_e^a(\theta,e_A(\theta)) \geq 0$. This is because if $U_e^a(\theta,e(\theta)) > 0$ for $e(\theta) < e_A(\theta)$, the utility of the borrower can be increased by increasing effort to $e_A(\theta)$ Also, for a borrower who supplies effort $e(\theta) \geq e_A(\theta)$ thereby receiving expected utility $E_\varepsilon \pi X(K_A(\theta),e(\theta),\varepsilon)-Q(e(\theta))$, there is no incentive to provide effort greater than $e_A(\theta)$ since the marginal benefits and none of the associated costs of effort are shared with the investor.

For $\{e_A(\theta), K_A(\theta)\}$ given by (11) and (12), the type θ borrower therefore chooses an incentive compatible contract $(e_A(\theta), K_A(\theta))$, supplies effort $e_A(\theta)$, and receives expected utility $E_\varepsilon \pi X(K_A(\theta),e_A(\theta),\varepsilon)-Q(e_A(\theta))$, provided

$$\int_{\varepsilon_o}^{\bar{\varepsilon}(e_A(\theta))} \lambda(\theta)\pi X_e(K_A(\theta),e_A(\theta),\varepsilon)dG(\varepsilon) + \int_{\bar{\varepsilon}(e_A(\theta))}^{\varepsilon_1} \pi X_e(K_A(\theta),e_A(\theta),\varepsilon)dG(\varepsilon)$$

$$-(1-\lambda(\theta))\pi X_C(\theta)G_1(\bar{\varepsilon}(e_A(\theta)))\frac{d\bar{\varepsilon}(e_A(\theta))}{de} \geq Q'(e_A(\theta)) \qquad (15)$$

Also note however from (13) that $X_C(\theta) = X(K_A(\theta),e_A(\theta),\varepsilon_0) = X(K_A(\theta),e(\theta),\bar{\varepsilon}(e(\theta)))$ implies that $\bar{\varepsilon}(e_A(\theta)) = \varepsilon_o$ Therefore, (15) simplifies to

$$\int_{\varepsilon_o}^{\varepsilon_1} \pi X_e(K_A(\theta),e_A(\theta),\varepsilon)dG(\varepsilon) - (1-\lambda(\theta))\pi X_C(\theta)G_1(\varepsilon_o)\frac{d\bar{\varepsilon}(e_A(\theta))}{de} \geq Q'(e_A(\theta))$$

(16)

From (6) (with $U^a(\underline{\theta}) = \underline{U}$), the expected type θ borrower utility is $U^a(\theta) = \underline{U} + \int_{\underline{\theta}}^{\theta} E_{\varepsilon}\pi H(K(s), e(s), \varepsilon)ds$ Therefore, for a borrower supplying effort $e_A(\theta)$, from (14) with $\bar{\varepsilon}(e_A(\theta)) = \varepsilon_o$, the optimal pure adverse selection allocation obtains if

$$\int_{\varepsilon_o}^{\varepsilon_1} \pi X(K_A(\theta), e_A(\theta), \varepsilon)dG(\varepsilon) = \underline{U} + Q(e_A(\theta)) + \int_{\underline{\theta}}^{\theta} E_{\varepsilon}\pi H(K_A(\theta'), e_A(\theta'), \varepsilon)d\theta' \quad (17)$$

Finally, dividing (16) by (17) we obtain the necessary condition for contract menu $\{e_A(\theta), K_A(\theta), \lambda(\theta), X_c(\theta)\}\forall\theta$ to implement the best pure adverse selection outcome[30],

$$\frac{\int_{\varepsilon_o}^{\varepsilon_1} X_e(K_A(\theta), e_A(\theta), \varepsilon)dG(\varepsilon) - (1-\lambda(\theta))X_C(\theta)G_1(\varepsilon_o)\frac{d\bar{\varepsilon}(e_A(\theta))}{de}}{\int_{\varepsilon_o}^{\varepsilon_1} X(K_A(\theta), e_A(\theta), \varepsilon)dG(\varepsilon)} \geq$$

$$\frac{Q'(e_A(\theta))}{\underline{U} + Q(e_A(\theta)) + \int_{\underline{\theta}}^{\theta} E_{\varepsilon}\pi H(K_A(\theta'), e_A(\theta'), \varepsilon)d\theta'} \quad (18)$$

where $e_A(\theta)$ and $K_A(\theta)$ are derived from (11) and (12)

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30 For threats of a lower share ratio $\lambda(\theta)\pi$ to induce effort greater than e (< $e_A(\theta)$), we assume that $\frac{\partial}{\partial\lambda(\theta)}\frac{\partial E_{\varepsilon}\varphi(\theta, e)}{\partial e} < 0$ From (14), with X(θ,e) identical to θH(K(θ),e(θ),ε) – rK(θ), this reduces to $X_C(\theta)G_1(\bar{\varepsilon}(e))\frac{d\bar{\varepsilon}(e)}{de} + \int_{\varepsilon_o}^{\bar{\varepsilon}(e)} \theta H_2(K_A(\theta), e, \varepsilon)dG(\varepsilon) < 0$ Note that with $X_C(\theta)$ equal to the lowest possible profit when the borrower supplies effort $e_A(\theta)$, this expression reduces to $X_C(\theta)G_1(\bar{\varepsilon}(e_A(\theta)))\frac{d\bar{\varepsilon}(e_A(\theta))}{de} < 0$ which is necessarily true

6.7 Discussion

Some interesting features of the necessary condition (18) exist in comparison to the equivalent pure moral hazard condition (Chapter 4, (29), for a risk neutral borrower, $\lambda > 0$, and $X_C = X(e^*, \varepsilon_0)$) The pure moral hazard condition is

$$\frac{\int_{\varepsilon_o}^{\varepsilon_1} X_e(e^*, \varepsilon)dG(\varepsilon) - (1-\lambda)X_C{}^* G_1(\varepsilon_o)\frac{d\bar{\varepsilon}(e^*)}{de}}{\int_{\varepsilon_o}^{\varepsilon_1} X(e^*, \varepsilon)dG(\varepsilon)} \geq \frac{Q'(e^*)}{\underline{U} + Q(e^*)} \qquad (19)$$

For the contingent share ratio contract in the mixed problem to implement the best pure adverse selection outcome, the target profit had to be set equal to the profit which obtains if the borrower supplies the recommended effort ($e_A(\theta)$) for the lowest realisation of exogenous uncertainty (ε_o). However in the pure moral hazard case, the target profit can be set in excess of the corresponding lowest profit (i e. the profit which obtains for effort e^* and realised uncertainty ε_0)[31] This difference reflects a restriction in the space of dichotomous incentive schemes[32] which implement the effort level most preferred by the investor, when there is an increase in the severity of the information asymmetry between the borrower and the investor

[31] Note that the first best effort in the pure adverse selection problem will only equal the first best effort in the pure moral hazard problem for the lowest value θ borrower ($\underline{\theta}$) To see this, observe that from (12), the optimal pure adverse selection contract menu equates the (expected) marginal product and marginal costs of effort only for $\theta = \underline{\theta}$ (since $F(\underline{\theta}) = 0$)

[32] Recall from chapters 4 and 5 that in general there may exist a range of target profits for the pure moral hazard case that induce the agent/borrower to supply the socially optimal effort

A second interesting feature is the last term in the denominator of the right hand side of (18), which is an information rent that increases with borrower type. Clearly, irrespective of the incentive contract that the investor designs to overcome the problem of moral hazard due to the unobservability of effort, the precontractual endowment of valuable private information to the borrower introduces inefficiencies, since except for the lowest borrower type, the investor cannot keep borrowers at their reservation utility

At first glance this consideration seems to suggest that since the investor is resigned with probability one to award the borrower an information rent, a consequent increase in the denominator of the right hand side of (18) renders this condition weaker than (19) However, the effort most preferred by the investor for the mixed problem exceeds that of the pure moral hazard problem for all but the lowest borrower type Consequently, the weakening of (18) in comparison to (19) due to the allocation of a rent to the borrower is offset by a required effort in the mixed case which exceeds its corresponding value for the pure moral hazard problem. As a result, the ordering of the strengths of conditions (18) and (19) is in general ambiguous

Lastly, a feature of the model of adverse selection in Chapter 3 not specifically alluded to in the mixed model is the welfare improving effect of the liability constraint. In this respect, ex ante uncertainty as to whether the pay floor becomes binding in the mixed problem derives from exogenous uncertainty ε instead of the opportunity cost of capital r Notwithstanding that the optimal pure adverse selection outcome depends on the level of the pay floor of the borrower, the issue for the mixed problem is whether the associated contingent share ratio contract can implement this solution. Varying the pay floor will therefore serve to 'move the goal posts' when we permit a greater range of borrower payoffs, such as in a model which allows for the sharing of both profits and losses

However, observe that a trade-off exists when we extend the range of payoffs over which the pay of the borrower varies with profit Decreasing the pay floor increases the wedge between the first-best pure moral hazard effort and the first-best pure adverse selection effort, by increasing the information rent that higher borrower types can command (see

3 5 2) A second effect of lowering the minimum pay floor is however to better facilitate the use of threat-based incentive mechanisms aimed at implementing the optimal pure adverse selection outcome, since an associated widening of the pay differential for the borrower is thereby rendered possible, between outcomes in which the realised profit is less than, or greater than or equal to, the target profit

We now collect concluding remarks.

6.8 Concluding remarks

In this chapter we examined a combined problem of moral hazard and adverse selection, where a borrower endowed with precontractual information about the production function of a venture wholly financed by an investor, supplies hidden effort and is awarded pay in direct proportion to realised profits. To limit the costs of his ignorance, the investor can do no better than to offer a contract menu which elicits truthful (type) revelation by the borrower about the production function, and which motivates the borrower to supply the recommended level of effort for his type It was possible to determine the optimal truth revealing contract menu independently of the moral hazard problem arising from hidden effort, and a necessary condition (18) for a contingent share ratio contract was derived for which the optimal pure adverse selection (observable effort) solution could be implemented.

As discussed in Chapter 4, contingent share ratio schemes may be less effective in inducing an agent to supply effort than pure-lump sum schemes This inefficiency arises from the offsetting incentive effects of the threat of a lower share ratio, which decreases incentive pressure by allocating the agent a reduced share of all outcomes below the target level, whilst increasing incentive pressure through a greater jump in agent pay for outcomes no less than the target At the expense of invoking the use of (relatively less efficient) contingent share ratio schemes, we have therefore synthesised the key components of the isolated information problems examined in the previous chapters, in order to derive a more holistic view

Appendix I

Global applicability of locally true incentive compatibility constraints

We will now show in outline[33] that H(K(θ),e(θ),ε) non-decreasing in θ is both a necessary and sufficient condition for locally true incentive compatibility to obtain globally

(1) Necessity.

From (2), global incentive compatibility requires

$$U^a(\theta,\theta) \geq U^a(\theta,\hat{\theta}) \qquad \forall\theta\in[\underline{\theta},\overline{\theta}],\ \forall\hat{\theta}\in[\underline{\theta},\overline{\theta}] \quad \text{(A1)}$$

Using (1),

$$U^a(\theta,\hat{\theta}) = U^a(\hat{\theta},\hat{\theta}) + \int_{\varepsilon_o}^{\varepsilon_1} \pi(\theta-\hat{\theta})H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon)dG(\varepsilon) \quad \text{(A2)}$$

Therefore, (A1) may be expressed as,

$$U^a(\theta,\theta) \geq U^a(\hat{\theta},\hat{\theta}) + \int_{\varepsilon_o}^{\varepsilon_1} \pi(\theta-\hat{\theta})H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon)dG(\varepsilon) \quad \text{(A3)}$$

Similarly,

$$U^a(\hat{\theta},\theta) = U^a(\theta,\theta) + \int_{\varepsilon_o}^{\varepsilon_1} \pi(\hat{\theta}-\theta)H(K(\theta),e(\theta),\varepsilon)dG(\varepsilon) \quad \text{(A4)}$$

and therefore, (A1) may also be expressed as

$$U^a(\theta,\theta) \leq U^a(\hat{\theta},\hat{\theta}) + \int_{\varepsilon_o}^{\varepsilon_1} \pi(\theta-\hat{\theta})H(K(\theta),e(\theta),\varepsilon)dG(\varepsilon) \quad \text{(A5)}$$

Combining (A3) and (A4) yields

$$\int_{\varepsilon_o}^{\varepsilon_1} \pi(\theta-\hat{\theta})H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon)dG(\varepsilon) \leq \int_{\varepsilon_o}^{\varepsilon_1} \pi(\theta-\hat{\theta})H(K(\theta),e(\theta),\varepsilon)dG(\varepsilon) \quad \text{(A6)}$$

If global incentive compatibility obtains, then a necessary condition is that condition (A6) is satisfied, implying that H(K(θ),e(θ),ε) is non-decreasing in θ

(2) Sufficiency

From locally true incentive compatibility condition (6),

$$U^a(\theta,\theta) = U^a(\underline{\theta}) + \int_{\underline{\theta}}^{\theta}\int_{\varepsilon_o}^{\varepsilon_1} \pi H(K(s),e(s),\varepsilon)dsdG(\varepsilon) \quad \text{(A7)}$$

and

$$U^a(\hat{\theta},\hat{\theta}) = U^a(\underline{\theta}) + \int_{\underline{\theta}}^{\hat{\theta}}\int_{\varepsilon_o}^{\varepsilon_1} \pi H(K(s),e(s),\varepsilon)dsdG(\varepsilon) \quad \text{(A8)}$$

Substituting (A7) into (A8) gives

$$U^a(\hat{\theta},\hat{\theta}) = U^a(\theta,\theta) + \int_{\theta}^{\hat{\theta}}\int_{\varepsilon_o}^{\varepsilon_1} \pi H(K(s),e(s),\varepsilon)dsdG(\varepsilon) \quad \text{(A9)}$$

Substituting (A2) into (A9), rewriting $(\theta-\hat{\theta})H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon)$ as $\int_{\hat{\theta}}^{\theta} H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon)ds$

yields

$$U^a(\theta,\theta) - U^a(\theta,\hat{\theta}) = \int_{\theta}^{\hat{\theta}}\int_{\varepsilon_o}^{\varepsilon_1} \pi[H(K(\hat{\theta}),e(\hat{\theta}),\varepsilon) - H(K(s),e(s),\varepsilon)]dsdG(\varepsilon) \quad \text{(A10)}$$

which together with (A1) immediately yields the sufficiency result

[33] See proof in Chapter 3, Appendix II for more details

CHAPTER 7

CONCLUSION

7.1 Summary of results and areas for future research

The aim of this thesis was to explore the implications of imposing a minimum pay constraint for an agent who is endowed with private information either before, during or both before and during the execution of a profit sharing contract with a principal The motivation for examining the affect of pay floors in the context of asymmetric information derives from having identified that this area is relatively under-researched

For a problem of adverse selection, in which there exists a precontractual information asymmetry, limited agent (borrower) liability has been researched only in the context of debt contracts (see 2 3.4), with no analogous examination of the affects of limited agent liability in profit sharing agreements In Chapter 3 we established an innovative model in which knowledge of the production function was private to the agent before the principal invested capital, and in which importance was attributed to the proportion of capital costs allocated to the borrower as a contractual variable used to sort agents by type We found that the imposition of a pay floor for the agent created an overall welfare improvement and reduced the extent to which the investor was required to overinvest capital as part of a strategy intended to minimise the cost to the investor of his precontractual ignorance We also established when information is bi-laterally asymmetric, where the investor is privately endowed with information which is valuable to the agent, that a welfare loss results with no associated inefficiency in the optimal investment schedule

The welfare enhancing effect of a minimum pay floor derived in Chapter 3 is extremely important to the understanding of the affects of precontractual information asymmetry in profit sharing agreements. The analogous insight for problems of moral hazard, in which effort supplied during the execution of a contract is unobservable to the principal, is that varying the pay of an agent with realised profit creates incentives to provide effort and mitigates the problem of moral hazard, albeit that it is a reduction in the variability of pay for some range of profits in which the pay floor binds, that creates a benefit for profit sharing agreements with a pay floor

Further research into the affects of pay floors in profit sharing agreements could consider alternative model specifications, such as an additive (viz-a-viz multiplicative) privately endowed information parameter determining the production function, or better still, extend the discussion by admitting a general production function and/or risk aversion[1]. However, we conjecture that the basic intuition revealed using the model in Chapter 3 will remain unaltered and will not be affected by relaxing the assumptions made therein at the expense of greater modelling complexity.

A second, and no less important contribution made by the thesis was the examination of incentive contracts used to motivate agents to supply greater (unobservable) effort, when limited agent liability renders incredible the threat of severe or potentially unlimited punishment in the event of low profit outcomes A commonly discussed incentive contract in the literature (see 2 3.2) is a bonus scheme in which an agent receives a lump-sum increase in pay if realised profit exceeds a prespecified performance target. However, we identified that no published literature exists to examine the case of a (piecewise linear) profit sharing contract in which the profit sharing ratio itself is contingent on some outcome

In Chapter 4 we found that the effective use of these contracts is sensitive to the production technology as well as to the ex ante beliefs which obtain concerning exogenous uncertainty, in a manner not true of their lump-sum bonus counterparts The reason for this stems from the contrary incentive effects of a widening share ratio differential Increasing the difference between the share ratios by decreasing the share ratio which obtains for profits below the performance target increases incentives to provide effort through a greater lump-sum pay element, but decreases incentives due to a

[1] see Zou (1992) for an insight into the effects of agent risk aversion in problems of adverse selection

reduced agent share of profit for all outcomes below the target. This general ambiguity for the case of a risk neutral agent motivated an investigation of the effects of assuming agents to instead be risk averse, wherein we identified three important ranges of (constant relative) risk aversion. Most importantly we identified a middle range for which there exists a critical reduction in share ratio for outcomes below the target, such that the effective use of contingent share ratio schemes relies on the use of threats which are sufficiently severe in relation to this critical value

Principal-agent models used to examine moral hazard usually assume that the agent supplies no capital (see 2.2.1 and 2 3 2) However, real-world profit sharing schemes may require a (nominal) contribution of capital by an agent as a way of showing commitment to the success of a venture through the supply of productive effort Therefore, the main theme of Chapter 5 was to consider an extension of the contingent share ratio incentive contract explored in Chapter 4, in order to examine a capital partnership agreement in which capital contributors other than the agent are sleeping partners who supply no effort. We were able to derive sufficient conditions for which a Pareto improvement would be available by the relaxation of punishment threats which reduce the ex post profit share of the agent for low profit outcomes, in favour of (marginal) capital substitution by the agent

In reality, the isolated issues of moral hazard and adverse selection will coexist as an integrated problem of information asymmetry. For example an entrepreneur may be privately endowed with productivity information which determines the risk of return for would-be investors, but would also (characteristically) provide effort which is only privately observable.

As such, Chapter 6 examined the interaction of these problems, and how the solutions which obtain in the isolated cases are affected by the existence of additional and separate information problems We determined a necessary condition for establishing the best possible outcome available to an investor who faces a combined problem of moral hazard due to unobservable effort, with adverse selection arising from (ex ante) privately endowed productivity information The most interesting feature of the combined problem, in which we were able to synthesise the essential analysis from preceding chapters, was that irrespective of the (type dependent) incentive scheme used to implement the effort level most preferred by the investor for the particular productivity of the project in which he invests, the investor will almost surely[2] bear a cost attributed to his ex ante ignorance concerning the productivity of the venture In this regard, the first-best outcome is manifestly different from the case of pure moral hazard. However, it is also the case that moral hazard due to the unobservability of effort does not introduce additional inefficiencies into the optimal investment schedule Further, lowering the minimum pay floor creates countervailing effects As established in Chapter 3, a lower pay floor extends the range of profit outcomes over which the pay of the agent varies with realised profit, and therefore increases the (ex post) inefficiencies in investment through greater information rents However, in contrast, lower pay floors also increase the feasible pay differential between agent pay which obtains for realised profit above or below the performance target, thereby rendering more effective the use of punishment schemes as a way to elicit greater agent effort

In the introductory chapter, we (partly) motivated the use of (piecewise linear) profit sharing contracts by the notion that awarding effort through (only) a fixed wage may in some way be a result more of inertia than strict economic rationale, since although both the micro and macro economic virtues of profit sharing remain contentious, we have exposed valuable features in profit sharing agreements hitherto not mentioned by the literature.

To be more specific, as we draw this thesis to a close, it is informative to present a holistic view of the effects of awarding an agent a (linear) share of ex post profits with a pay floor, in comparison to a pure fixed wage agreement On the one hand, fixed wage agreements create no adverse selection problem, *if* entrepreneurs receive a wage which is independent of the productivity of their ventures. However, fixed wage agreements engender the worst possible problem of moral hazard, since theoretically at least, an

[2] with probability one

agent has no incentive to provide effort. In contrast, piecewise linear limited liability profit sharing agreements bring about a problem of adverse selection through the variability in agent pay from an (ex ante) uncertain profit This problem is mitigated by the pay floor. Also, since profit sharing contracts are necessarily incentive schemes, motivation is provided for the agent to supply a non-zero effort Therefore, overall, these contracts represent a middle ground when considering the implications of both moral hazard and adverse selection In contrast, wage agreements highly favour the mitigation of the effects of either one or the other of these information asymmetries, but cannot simultaneously reduce the extent of both problems

An interesting area for further work is therefore to formalise the above argument in order to benchmark the efficiency of piecewise linear limited liability profit sharing contracts in relation to fixed wage agreements, perhaps for a variety of assumptions including for example the probability distribution of ex ante uncertainty and the risk aversion of all contracting parties

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