

HOW CAN TRUMP WIN?

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***Abstract.** In this paper, the McCulloch-Pitts model built on an artificial neuron is first introduced briefly, followed by a modified model – the coupled network model to describe social opinion network in period of the presidential election. To illustrate the new model, its formalism and analytical results on fixed points will be stated step by step. Then, we investigate the dependence on the ratio of the initial conditions so that we could find out more on relationship between current information and preference on final results. Finally, U.S. election campaign in 2016 will be examined comprehensively including support rates, possible preference, time series analysis, and period analysis. Besides mathematical research, we also take real-life activities into consideration. For example, Trump used Twitter to help his view spreading and take advantage of the underlying uncertainty to some extent.*

1. Introduction

The early study of complex system was raised from field of cognitive science. An artificial neuron model was introduced by Warren McCulloch, a neuroscientist, and Walter Pitts, a logician, in 1943.^[1] It is called McCulloch-Pitts neural model, also known as linear threshold gate.^[2] The model describes the procedure of a neuron accepting signals and then spreading them. It tried to understand how the brain could produce highly complex patterns by using many basic neuro tangled together. For one artificial neuron, the transfer equation is

$$\sigma_k = \theta \left(\sum_{i=0}^m \omega_{ki} x_i \right), \quad (1)$$

which is the weighted sum of inputs according to ω_{ki} after transferring by function σ . For two interacting nodes along discrete time steps, we can deduce similar evolution equations from Eq.(1),

$$\sigma_k^A(t+1) = \left(1 - \sigma_i^B(t) \right) \theta \left(\sum_{i=0}^m \omega_{ki} \sigma_i^A(t) \right),$$

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and

$$\sigma_k^B(t+1) = (1 - \sigma_i^A(t)) \theta \left(\sum_{i=0}^m \omega_{ki} \sigma_i^B(t) \right),$$

where we suppose, there are m inputs towards each node and only one input between two nodes A and B .

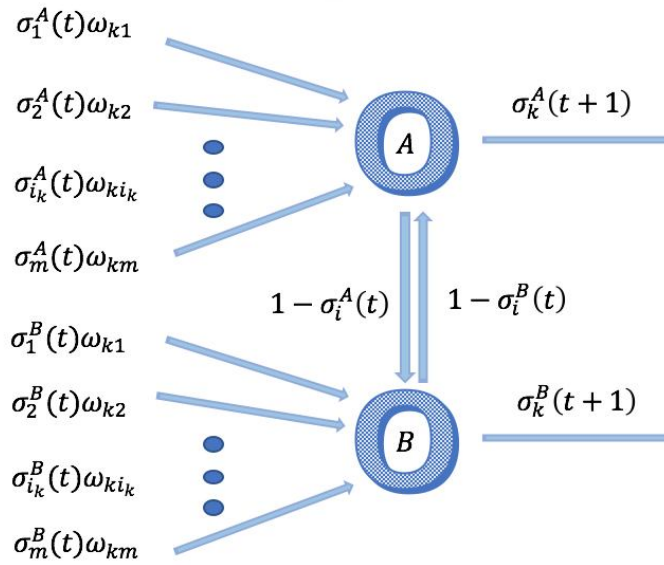


Figure 1. Illustration on Evolution of two interacting nodes A and B at discrete time step t to time step $t + 1$. There are two nodes A and B . At time t for each node, m input signals from m different randomly chosen nodes apart from nodes A and B influence the node at time t . Meanwhile, those two nodes affect each other. At time step $t + 1$, each node outputs its own signal, which is ready to influence others at the following time.

During the work of investigating the multi-agent social chaotic networks, the interacting agents shows an interesting phenomenon that in a whole system, parts of the system might suggest macroscopic collective properties similar to the local microscopic interactions. These have some similarity to statistical mechanics' situation. In order to complete this property more precisely in terms of mathematical description, the division of the whole system is needed. For example, each community could be regard as a set of social infrastructures and such networks could consist of its subnetworks interfering with each other^[3]. In coupled opinion networks, we examine a system, here, which is the USA population in practice, consisting of N agents who hold their own opinion on choosing A or B .

2. Case Study

The coupled opinion network model stated above shows an overview prospective. Now we shall look at a particular case, 2016 U.S. presidential election (Trump vs. Hillary, see, Figure 2), which is one outcome of all the possible phases generating from the model. In 2016, as we all know that Mr Donald Trump has been elected as the 45th president of United States; I believe we still remember the intense and fierce campaign competition between Donald Trump and Hillary Clinton. In the next step, we are going to find out the interaction between Donald Trump and Hillary Clinton during the campaign, based on the real polling data analysis. Finally, we focus on the polling data to find out how they are interact according to the practical situation we combine analysis results and real life environment, to prove what we assumed. Especially the chat networks played a significant role in this election campaign, see, the Figure 2.

Direct and Indirect reciprocity between two competitors

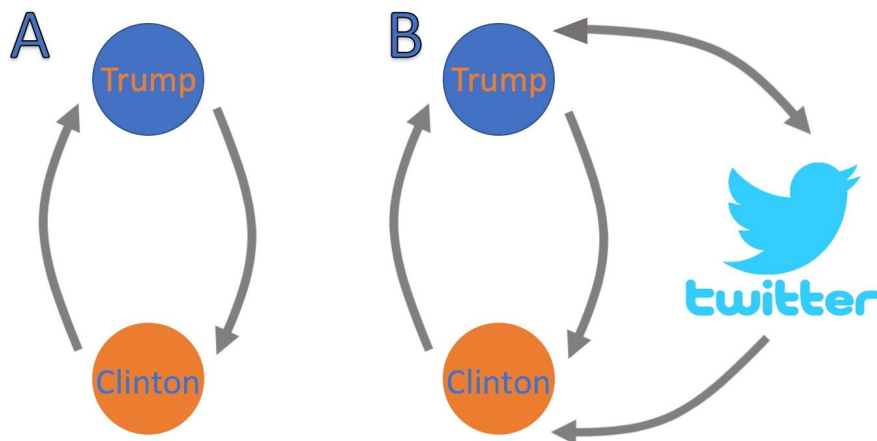


Figure 2. Direct (*A*) and indirect (*B*) reciprocity between two competitors roughly shows above. Situation *A* describes a traditional way that Trump and Clinton (or two networks) interact. In situation *B*, with the appearance of Twitter, Trump posted enough tweets so that he can obtain benefit from such behaviour. At meantime, the increasing of Trump's support rate somehow implies decreasing support towards Clinton to some extent.

First stage, during the voting progress, we assume that all the US citizenship only have two option to choose, support Trump or Clinton. The opinion polls data on a day-to-day basis has been collected from different states, where after that for each day it was averaged over all states. The polls data is presented in Figure 3.

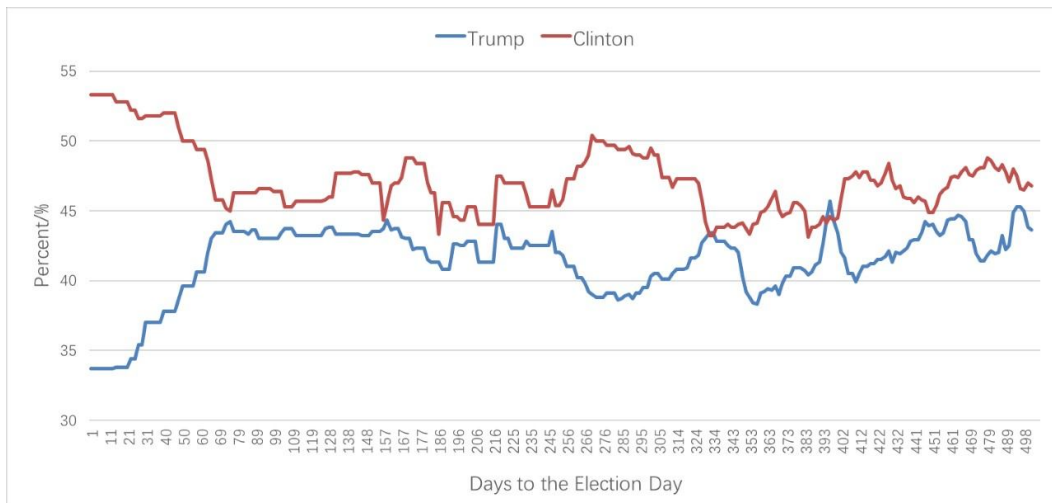


Figure 3. Shows the Opinion poll data from the general election of presidential campaign in the U.S.A. The x -axis represents number of days accumulate from July 2015 to November 2016. The y -axis indicates the polling support rates in percentage for Trump (Blue) and Clinton (Red). Data collected from web sited of realclearpolitics.com [12].

From the first glance of curves in Figure 3 which illustrates the opinion polls data from U.S. president election campaign in 2016, random fluctuations are obvious. In Figure 3, the red upper curve is for Hillary Clinton, while the blue lower curve is for Donald Trump. It is hard to find out the chaotic behaviour and related underlying information just from this figure; we will investigate later in this section.

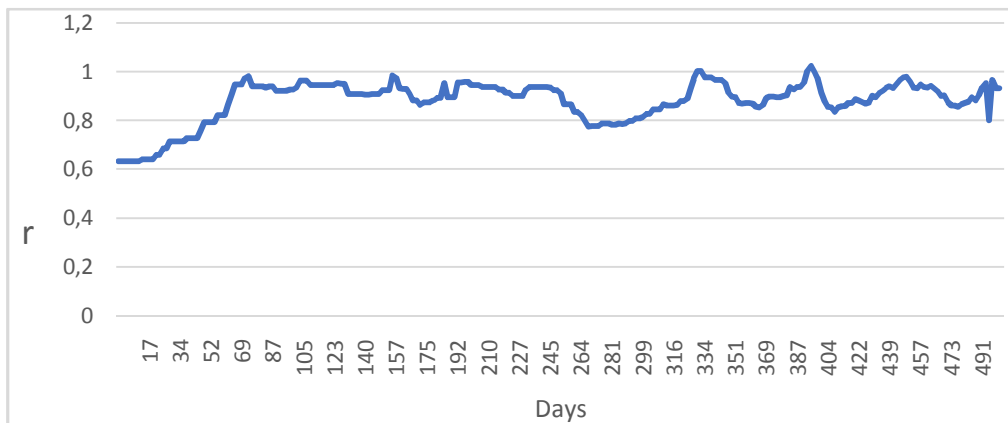


Figure 4. Ratio r of the two magnetizations m^{Trump} and $m^{Hillary}$, where we regard each time spots as the initial value of both magnetizations in the systems. We then check how the ratio r (i.e. the initial status) affects ultimate result at any time when the coupled system begins evolving.

While we look at the poll data collected, the election result is expected to predict in advance. Based on the analysis in section 4 in this article, we compute the ratio $r = m^{Trump} / m^{Hillary}$ and draw Figure 4. Then we could notice that the ratio r is almost inside the range of 0.8 to 1.0. According to the paper, the acceptance rates in this circumstance for both parties should be around 50%, which implies a tie situation. In fact, the result of 2016 USA election, Clinton gained 48.5% votes while Trump had 46.4% votes. There is only a difference of 2.1%.

To make a comparison with our theory by meanings of time series analysis on data of supportive rates, Figure 3 and Figure 4 indicating the autocorrelation are drawn for Trump and Hillary series, respectively. The Autocorrelation function is given by

$$A(k) = \frac{\sum_{k=1}^{N-k} (m(t) - \bar{m})(m(t+k) - \bar{m})}{\sum_{k=1}^N (m(t) - \bar{m})^2}.$$

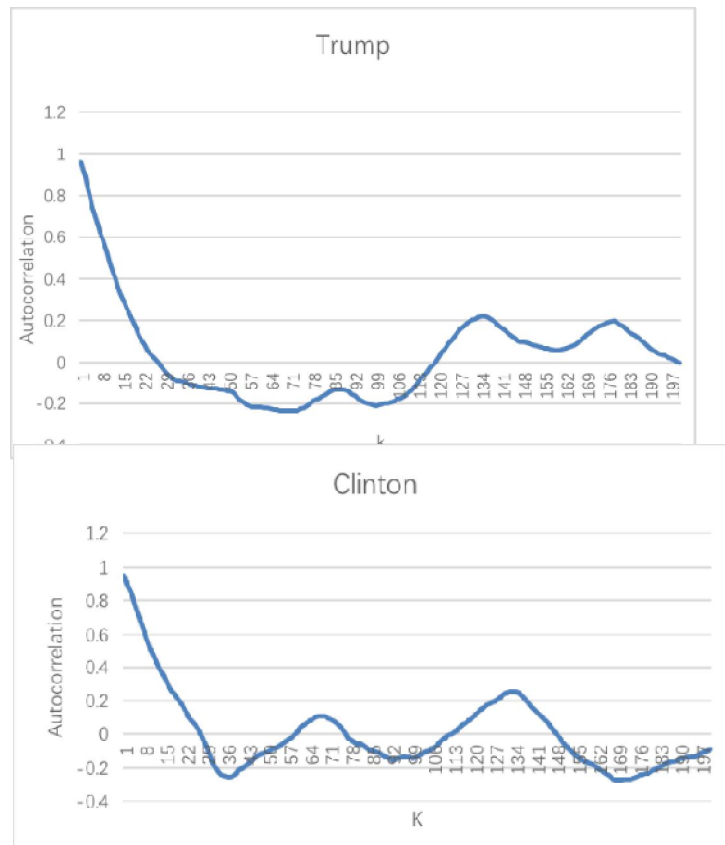


Figure 5. Autocorrelation Functions of Support rates of Trump (Left) and Hillary (Right). K is the lag number. In the left graph, the autocorrelation is negative until k increases to around 118. While in the right graph, the curve goes up and down of the x -axis in a roughly period.

From the distribution of campaign results and autocorrelation figure we can see that some fluctuations, which represent many uncertainties. This is why so hard to predict the results of campaign from phenomenon. But where are the uncertainties come from? As we know that all the residents of United States have authority to vote, this is the popular vote. In general, some states have traditional personal political preference; it is hard to change their mind to support different parties. So the key point the win the campaign is to gain the support from the states without traditional personal political preference, this is why there are uncertainties during the campaign. In other way, the uncertainties is kind of good opportunities for presidential candidates, if use it properly, will use it properly, will receive “positive” feedback, vice versa obtain “negative” outcome.

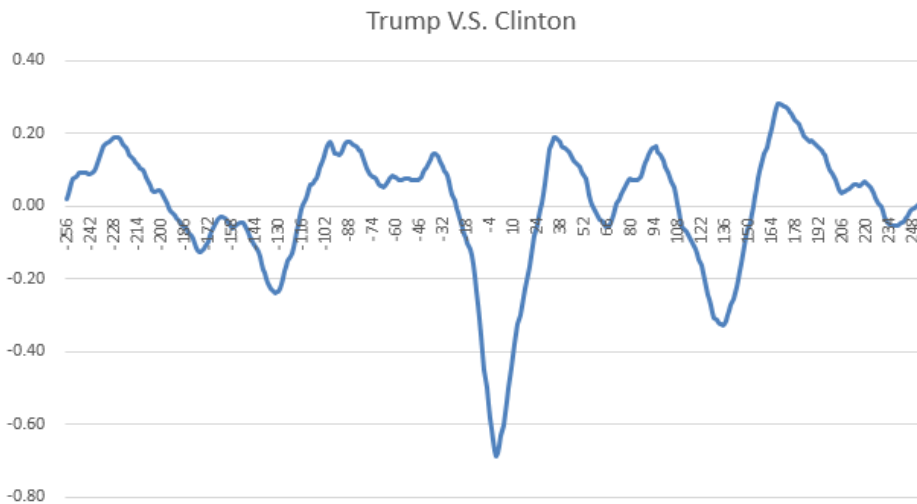


Figure 6. Cross-correlation diagram between support rates of Trump and Hillary. At the lag number $k = 0$, the cross correlation reaches its peak. According to observation, we would like to find out its period using method of Fourier transformation to see more closely of its property and interesting conclusion if lucky.

Using similar method as the front, we can calculate the cross-correlation coefficients between the two samples and draw the result in Figure 5. Series X and Y represent support rates of Trump and Clinton, respectively. Once we have function of covariance function

$$\gamma_{XY}(\tau) = E[(X_t - \mu_X)(Y_{t+\tau} - \mu_Y)],$$

we could calculate cross-correlation given by

$$\rho_{XY}(\tau) = \frac{1}{\sigma_X \sigma_Y} E[(X_t - \mu_X)(Y_{t+\tau} - \mu_Y)] = \frac{1}{\sigma_X \sigma_Y} \gamma_{XY}(\tau),$$

where we suppose the series X and Y represent support rates of Trump and Hillary, respectively, and σ_X, σ_Y are standard derivations for series X and Y . Additionally, due to the uniform distribution of data of each day's support rate, $E[]$ means the average, where $E[X] = \frac{1}{N} \sum_{t=1}^N X_t$ for series X .

Intuitively, it is a time series with a certain regularity and periodicity, which means there is a certain correlation between the data of Trump and Clinton. Actually, it describes the degree of correlation between the random signal of Trump and Clinton at any two different moments $t1, t2$.

Time series can be regarded as mixture by sine waves and cosine waves of different frequencies^[5]. With the help of Fourier transform, we can convert a function to a series of periodic functions, which transforms time series data into frequency sequence data. The periodogram is originally used to detect and estimate the amplitude of the sinusoidal component hidden in the noise whose frequency is known. Here, we will consider the possibility that the unknown periodic component is implicit in the sequence. For the time series, the Fourier series can be expanded as:

$$x_t = \alpha_0 + \sum_{j=1}^k (\alpha_j c_{jt} + \beta_j s_{jt}) + e_t \quad (1)$$

Where $c_{it} = \cos(2\pi f_i t), s_{it} = \sin(2\pi f_i t), N$ represents the number of observations, k represents the number of periodic components, f_i represents frequency and e_t the error term. Since $\sum_{i=1}^N c_{it}^2 = \sum_{i=1}^N s_{it}^2 = \frac{N}{2}$ and all terms in (1) are mutually orthogonal at $t=1, \dots, N$, the LSE of α_0 and (α_i, β_i) are

$$\begin{aligned} \alpha_0 &= \bar{x} \\ a_i &= \frac{2}{N} \sum_{t=1}^N x_t c_{it} \\ b_i &= \frac{2}{N} \sum_{t=1}^N x_t s_{it} \end{aligned}$$

Where $i=1, 2, \dots, k$. Therefore, the periodogram consists of $k = (N-1)/2$ values:

$$I(f_i) = \frac{N}{2} (a_i^2 + b_i^2), i = 1, 2, \dots, k \quad (2)$$

Where $I(f_i)$ represents the intensity at frequency f_i . In this way, given a stationary sequence with N observations and frequency f , if we can

get the repeated implementation of them from the random process, a series of a_f, b_f and the population $I(f)$ could be constructed. Thus, we could calculate the mean value of $I(f)$

$$E[I(f)] = 2[E[c_0] + 2 \sum_{k=1}^{N-1} E[c_k] \cos(2\pi f k)].$$

Where c_k represents the estimate of covariance function and it can be proved that

$$I(f) = 2 \left[c_0 + 2 \sum_{k=1}^{N-1} c_k \cos(2\pi f k) \right], 0 \leq f \leq \frac{1}{2}.$$

Where $I(f)$ is called sample spectrum.

When N is very large, it is proved that the estimate of the coefficient of the auto-covariance tends to the theoretical covariance γ_k

$$\lim_{N \rightarrow \infty} E[c_k] = \gamma_k.$$

When N tends to infinity, take the limit of (14), the power spectrum $p(f)$ is defined as follows

$$p(f) = 2 \left[\gamma_0 + 2 \sum_{k=1}^{N-1} \gamma_k \cos(2\pi f k) \right].$$

By varying the frequency, we can get the image of power spectral and find the cycle of the time series.

First, with the help of Eviews 8, we can prove that the cross-correlation sequence is smooth.

Null Hypothesis: CORRELATION has a unit root
 Exogenous: Constant
 Lag Length: 2 (Automatic - based on SIC, maxlag=18)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-5.136690	0.0000
Test critical values:		
1% level	-3.442945	
5% level	-2.866988	
10% level	-2.569733	

*Mackinnon (1996) one-sided p-values.

Figure 7. Main figures in Periodic analysis.

Then, after using software SPSS to conduct the spectral analysis, we can get the spectrum analysis chart as follows

Cross-correlation frequency cycle diagram

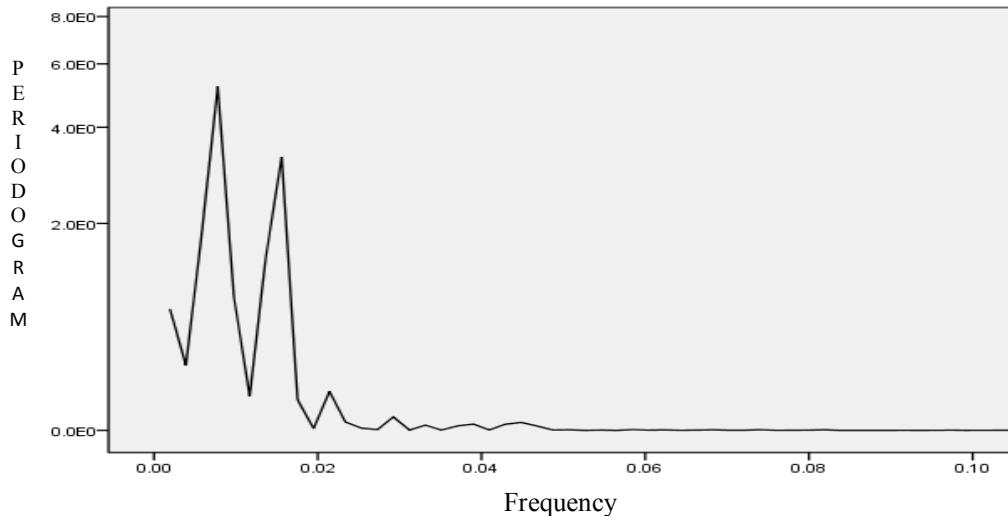


Figure 8. Cross correlation frequency cycle diagram. The diagram has two significant local maximum points. Then it almost goes to zero. In order to determine the period, what we need is the peak point $f = 0.008$.

As can be seen from the graph, the spectrum achieves its peak value at $f = 0.008$. Hence, the true frequency of the sequence is $f * N = 0.008 * 513 = 4.104$ and the cycle equals to $1/f = 0.243$. Finally, we can calculate the cycle of the cross-correlation is $0.243 * 513 = 124$. Since the quarter of one period 31 is around one month, the candidates may take signification actions once a month.

3. Modification after use of Twitter

Uncertainty does not suggest backwards if one could know well of its underlying information. The winning of Trump took advantage of this unpredictable property. In our modified model, the counterpart of the stochastic element is the influential strength of which Trump used Twitter. For example, he posted his views and potential acts through Twitter to gain more support among voters who are active in social network, especially youngsters. As a social method, Twitter plays an increasingly important role for candidates to approach supporters and be against competitors in U.S. president election campaign, especially for the republican Trump the

businessman in real estate. If there's anything that goes some way to explaining Trump's popularity in the midst of his quasi-fascistic views that reached a nadir with his call to ban all Muslims from entering the United States, it is his social media prowess.^[10] Trump has more than 5.5 million Twitter followers and 4.5 million Facebook fans. It means a lot to Trump. He is also a skilled live-tweeter. He knows that live-tweeting a popular event is an opportunity to engage with a wide audience in real time. Dan Pfeiffer, Obama's highly-regarded former digital and social media guru, has said Trump is "way better at the internet than anyone else in the GOP which is partly why he is winning"^[10].

From the basic model, threshold h^A and h^B may not stay constant as we supposed in former section. It will be more interesting if we regard the threshold as a combination of a random element and a constant that are all from external world. They also variate according to time. Let the threshold for node i at time t in network A and B be

$$h^A(t, i) = h_0 + \sum_{\alpha=1}^{N(t)} h_i^\alpha(t), \quad (17)$$

and

$$h^B(t, i) = h_0 - \sum_{\alpha=1}^{N(t)} h_i^\alpha(t), \quad (18)$$

respectively, where $N(t)$ is the number of tweets that Trump posted, $h_i^\alpha(t)$ randomly shows the strength of slice α th of Twitter towards the supporting opinion of node i to network A which has opposite effect to network B and h_0 is the deterministic and identified constant threshold. No one can say for sure that how can one tweets attract or disappoint people and then influence the original-holding opinion. So, the model becomes

$$x_i^A(t+1) = (1 - x_i^B(t)) \Theta(h_i^A(t) - h^A(t, i)), \quad (19)$$

and

$$x_i^B(t+1) = (1 - x_i^A(t)) \Theta(h_i^B(t) - h^B(t, i)), \quad (20)$$

By this way, the effect of the use of Twitter would emerge both in model and in reality.

We will then take deeper sight of collected data trying to find out some potential relationships in two ways: observation and time series analysis.

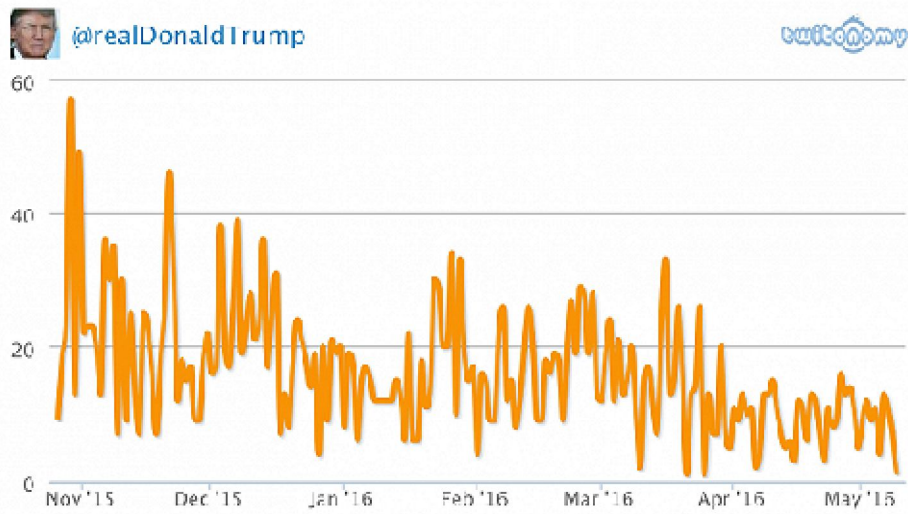


Figure 9. The figure above shows the number of tweets Trump posted per day from time period between November 2015 and May 2016. The *x*-axis shows the time periods, and the *y*-axis represents the number of tweets. The data collected from Twitonomy [13].

First, direct from the observation, from 28th Oct. 2015 to 12th May. 2016, Trump has posted 3200 Twitter messages, in average 16.16 per day [11]. He Twittered 33 posts on 16th Mar. which is the most from Feb. 2016 for celebrating and appreciating of Missouri and other six states in the first selection of Trump even under the five cities on 15th Mar.

Observing Figure 1 and Figure 7 at same time, we could notice that, on Nov. 2015, the support rate of Trump was low at 33% compared to 53% from Hillary. However, that figure kept increasing rapidly till Nov. and the twitter frequency was quite high even peaked at 58 per day. Then the distribution began fluctuating within a narrow range while the post frequency also fluctuated at about 18 per day. This fluctuation lasted about 5 months till May 2016.

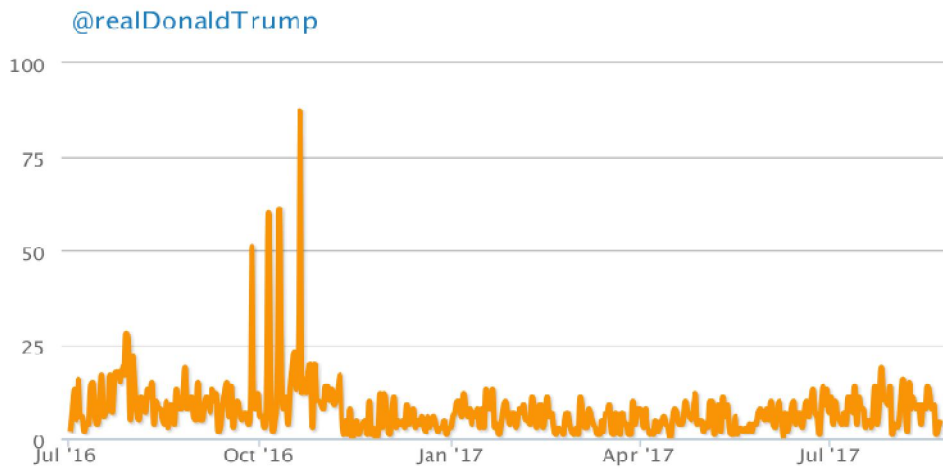


Figure 10. The figure above shows the number of tweets Trump posted from the time period from July 2016 to July 2017. The x-axis shows the time periods, and the y-axis represent the number of tweets. The data collected from Twitonomy [13].

Then on Sep. to Nov. 2016, which is several months just before the U.S. election date, to save the disadvantaged position, Trump tweets far more than ever, from about 20 jumping to 60 and more, even 87 tweets on 20th Oct. 2016.

We focus on a certain special time period which is from August 2016 to November 2016, just before the general election ending. In other way to say this period is the fiercest and strongest competition during the whole campaign.

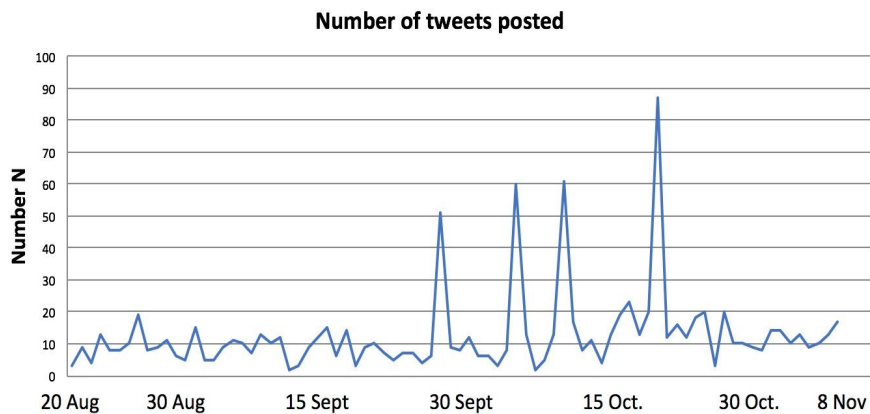


Figure 11. The figure above shows the number of tweets Trump had posted from 19th August 2016 to 8th November 2016. Data indicating tweets frequency are abstracted from original data poll from end of August till the Election Day in 2016.

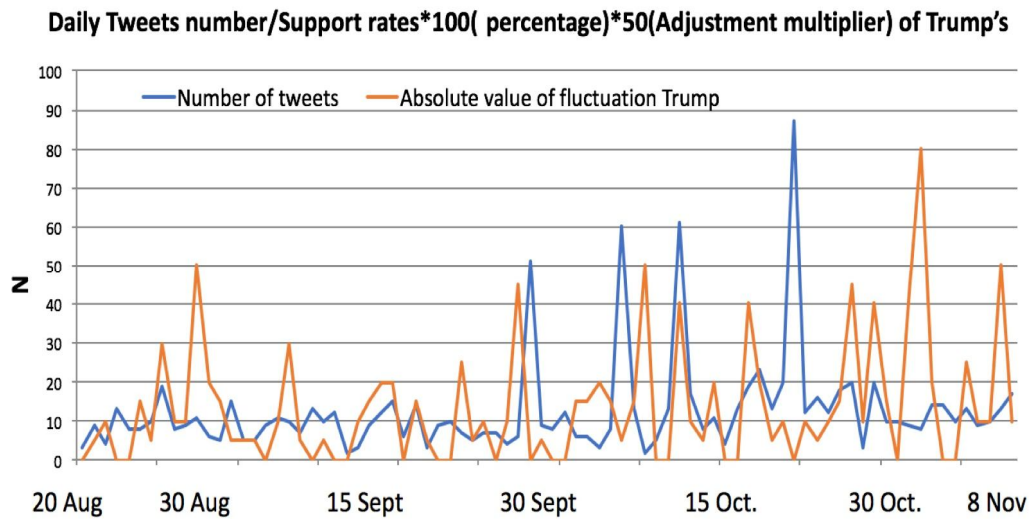


Figure 12. Numbers of tweets and the fluctuation form Trump's support rates of the general election. Figure 12 above shows the relationship between numbers of tweets and the fluctuation of the general election. The orange line indicates the fluctuation of general election, along with the curve in figure 11, we have the fluctuation in numbers, then magnify all the numbers by certain quantity 50 to adjust figures in a proper range.

We now first look more inside for Trump. The redline represent fluctuation of supporting rate. Here, we take all absolute value to focus on the changes rather than changing directions. Above all, we can see that the effect of tweets sometimes negative, sometimes positive, especially the trend of fluctuation during the time in October, the supporting is getting down, meantime by post large number of tweets, the supporting increase in end of October. This is indicating that the twitter, the social network has significant influence during the campaign; even though not always positive results, but it is breaking the traditional campaign machine.

Intuitively, we will also take sight to the potential effect on Trump's competitor. Figure 13 gives the corresponding information from Clinton's data pool. It is important to compare Twitter's, an innovative tool, impacts on both candidates. In later this section, same analysis method will be used on two situations, which making the comparison more direct.

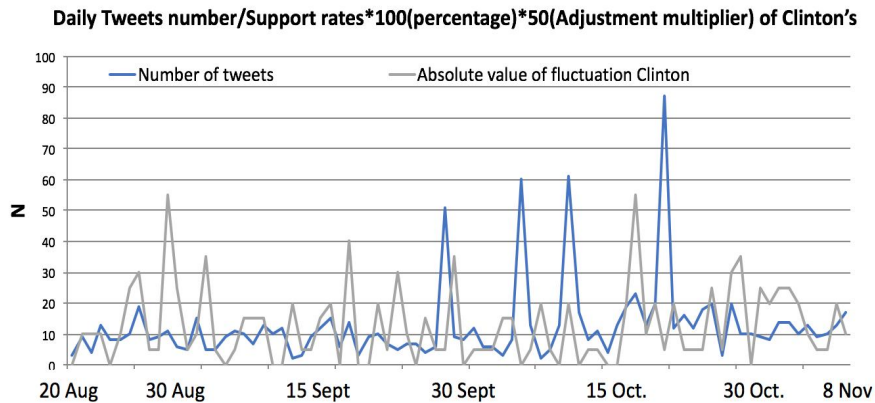


Figure 13. Numbers of tweets and the fluctuation form Clinton’s support rates of the general election. Figure 13 above shows the relationship between numbers of tweets and the fluctuation of the general election. The grey line indicates the fluctuation of general election, along with the curve in figure 11, we have the fluctuation in numbers, then magnify all the numbers by certain quantity 50 to adjust figures in a proper range.

Using method of time series analysis and Fourier spectral period analysis, we obtain the cross-correlation and cross correlation frequency cycle diagrams (Figure 14, Figure15, respectively).

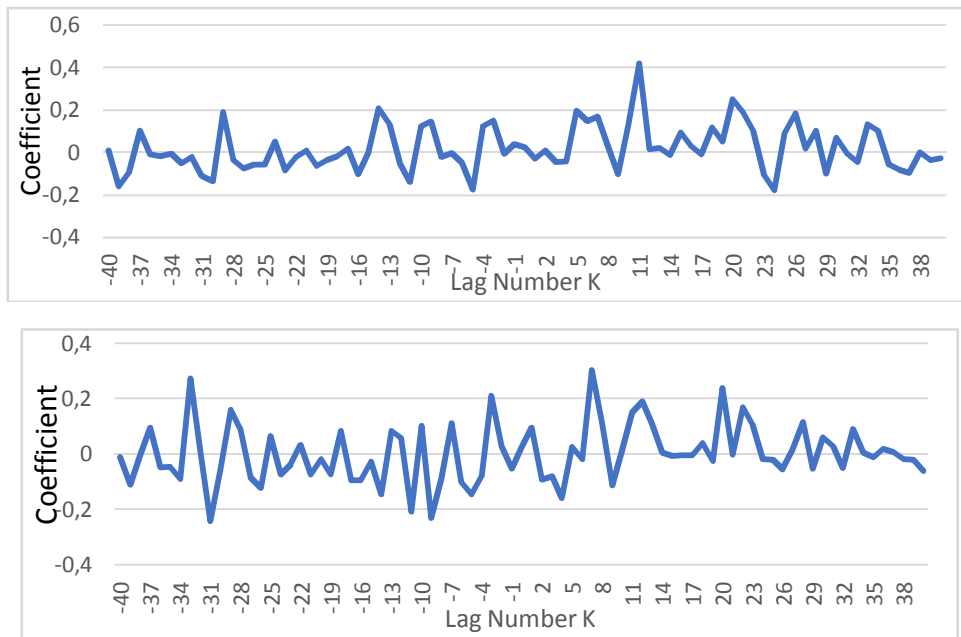


Figure 14. Cross-Correlations between Difference and tweets daily for Trump (upper) and Clinton (Lower), respectively. Both graph show pieces of mess. There may be not much good conclusions to have till now. Surprising findings will be showed in the following step.

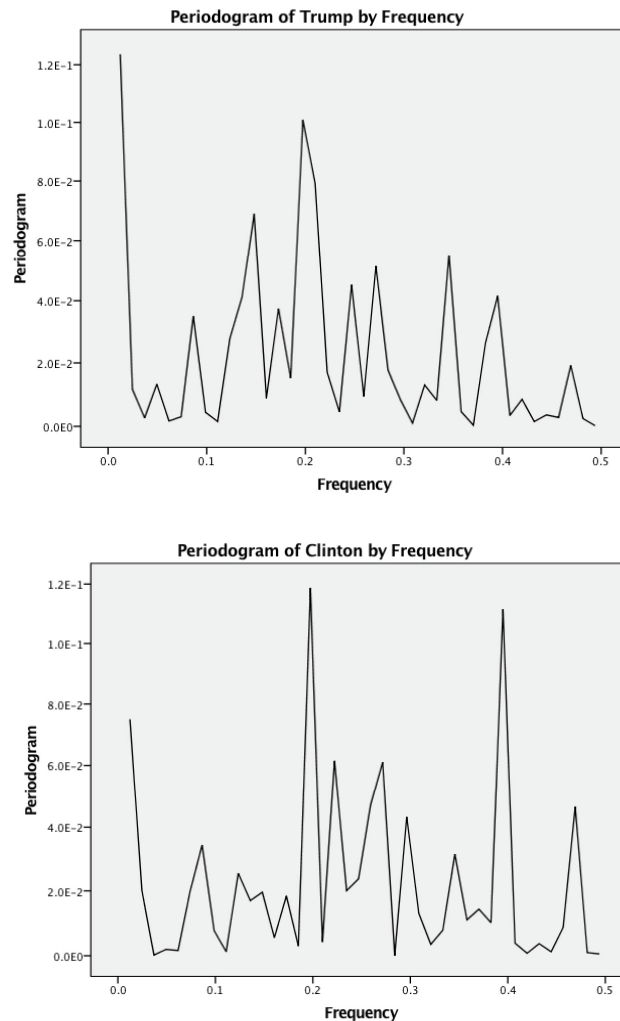


Figure 15. Cross correlation frequency cycle diagram daily for Trump (Left) and Clinton (Right). Frequency cycle diagrams are getting from spectral analysis to find out the period through cross correlation and Fourier function transformation. In the left graph, there is only one peak point that much taller than others. While in the right graph, there are two. However, in our analysis, only peak matters. These two curves gave us one same peak point $f=0.2$.

As can be seen from the graphs, the spectrum achieves its peak value at $f=0.2$ in each graph. Hence, the periods for two would be $1/f=5$. Things change every one and half days, which indicates a quite rapid reaction to twitter posting. However, those two graphs do not show any significant difference in terms of period, since the periods we obtain are the same. As the periods are the same, in other way to say, the fluctuation

change in the same time. This indicate both of them are effected during Trump post the tweets.

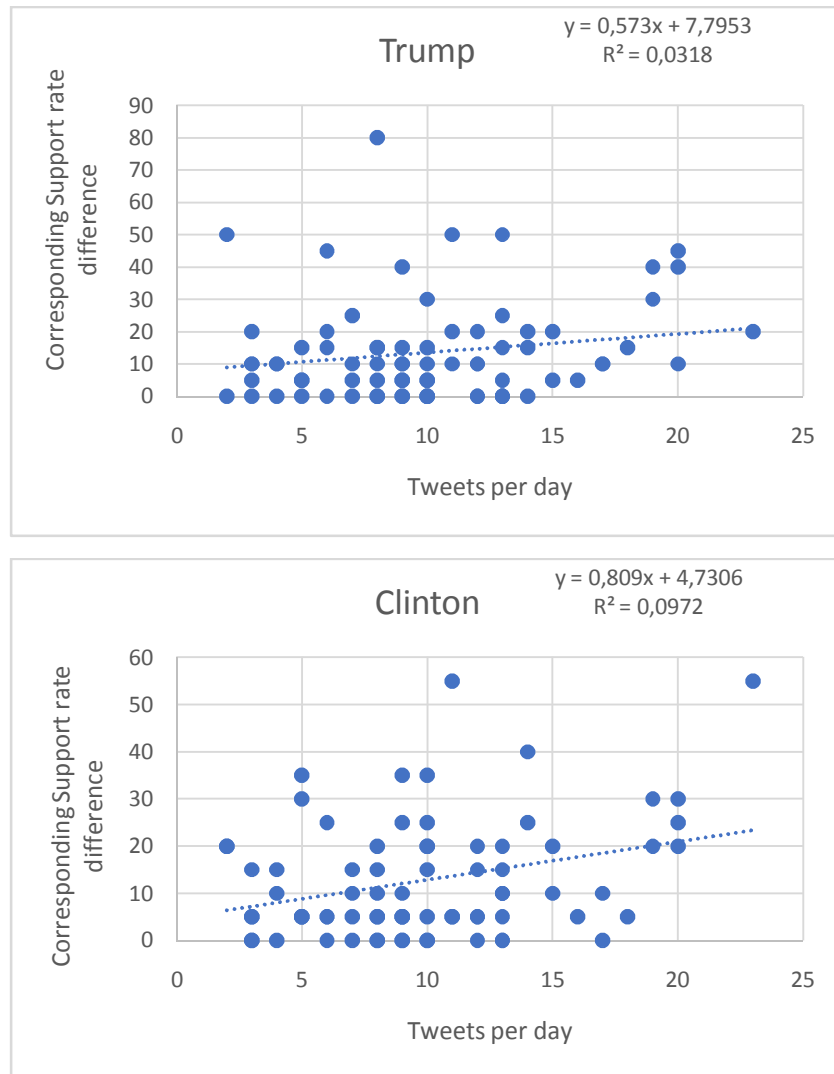


Figure 16. Distribution of support rate differences along corresponding to the number of tweets per day. (Trump: upper; Clinton: lower). First, points (x = tweets per day, y = support rate) from 19th August 2016 to 8th Nov ember 2016 for Trump and Clinton are plotted in coordination respectively. Then linear regression is done on both with regression function $y = 0.573x + 7.7953$ and $y = 0.809x + 4.7306$. If comparing the multipliers of x , we could find whether tweets influenced Trump more or Clinton more.

Linear regression analysis may tell something. Obviously, 0.573 is less than 0.809. We could say that there are more effects on Clinton than

Trump through Trump's twitter. On other words, using twitter is not only a boost of Trump's view and opinion, but also an impair to Clinton's.

We should say that Donald Trump try a new way to use the opportunity of uncertainty. It is Twitter, a social networking and news service platform. Every Twitter register user can post message 'so do Donald Trump' he post many messages during the campaign. These messages represent what Donald Trump is real would like to share, and also he become more and more popular during the campaign.

People always pay more attention on new things, which with loads of passion, enthusiasm and actively. Donald Trump did well in this point; share more about him in various ways. In other way to say, because of his high exposure rate, most people of unconcern about political know Donald Trump is in the campaign. When this group of people had been asked to vote, Donald Trump is the first candidate pop-up in their mind. Stand to reason that Donald Trump received more support from the group people of unconcern in political.

4. Conclusion

In this paper, the coupled opinion network has been developed from traditional uncoupled network, which pervasively known as McCulloch-Pitts neural model or linear threshold gate. The main analysis, which based on the model with linear threshold, shows the mechanism of opinion spread and the dependence on the ratio between the initial figures of two networks. To illustrate this point in case of 2016 U.S. president election, Figure 4 shows that the ratio always lies between 0.8 to 1.0 from which we could give no solid prediction on result of the election. Supporting rate distribution in figure 3 clearly describes little difference between votes to Trump and to Clinton at the very end of the general election.

For insider look at the rates' distribution, time series analysis plays an important role. Autocorrelation and cross-correlation imply the possible underlying period. Using Fourier transformation function spectral analysis on the cross-correlation, the period is four-month length in which things seem to change every month.

Various factors impact the public's opinion every moment in which they expose themselves of abundant information world. Nowadays, the fast and the most efficient meanings to spread views and communicate with others are via the internet. The U.S. president candidate Donald Trump

took advantage of Twitter, one of the most popular online news and social networking service where users post and interact with messages, called “tweets”. As The New York Times said, Election Day was a reminder of Twitter’s influence in media and the distribution of information [14]. In America, the immediacy and speed of Twitter is unmatched by any other network such as Facebook who reaped the benefit of news breaking on Twitter.

Figure 2 uses simple graphical representation of direct and indirect mutual promotion or interference. To see how strong Twitter is as the main tool for Trump’s success in the election through mathematical language, we first modified the model by adding the stochastic term, indicating the effect on opinion change and decision making, into the external constant threshold. Put differently from previous comparison method, we now compare the Twitter’s potential influence on Trump and Clinton in that only examining effect on Trump’s support is not sufficient to show the extent. To find out whether Twitter influence more on Trump or on Clinton, we do some data analysis work. After correlation analysis, the correlation coefficient between Trump’s support absolute fluctuation and tweets number and coefficient between Clinton’s support absolute fluctuation and tweets number are surprisingly exact the same. Still surprisingly, after Fourier transformation spectral analysis, both periods are 5 days which is a considerably small period compared with 4 months in former case. It again emphasizes how quick Twitter can be. The difference emerges in linear regression analysis where the number of tweets affects more on Clinton than on Trump.

Once the first sign from Trump began stirring on Twitter, it quickly began to mushroom. Using Twitter was not only bringing strength to Trump as he wished, but also sending potentially backward to Clinton in this nationwide competition.

The further work would be obtaining analytical result of stochastic threshold coupled network model. Additionally, based on this model, there must be many enough phenomenons that can be described in mathematical language, and then we can also have a deeper insight of those underlying the surface observations. Taking the Christianity spread in some areas (rich or poor) of China as an example, where there exists an original religion, we would ask: How the missionaries managed to do that? Or, why they cannot finish their mission?

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