Cognitive Predictors of Children's Development in Mathematics Achievement: A Latent Growth Modeling Approach

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Research Highlights

1. A constellation of domain-general and mathematics-specific cognitive abilities contributed to children’s starting level of mathematical success in the middle of grade 1.
2. Specifically, IQ, WM capacities, counting skills and both nonsymbolic and symbolic approximate comparison explained unique variance in children’s initial status in mathematics achievement.
3. However, symbolic approximate addition not only predicted initial status, but it was also the only predictor of children’s individual growth rates in mathematics achievement
4. Future research needs to place more focus on the predictors of children’s individual growth, i.e. their intra-individual change in general mathematics achievement

Abstract

Research has identified various domain-general and domain-specific cognitive abilities as predictors of children's individual differences in mathematics achievement. However, research into the predictors of children's individual growth rates, i.e., between-person differences in within-person change, in mathematics achievement is scarce. We assessed 334 children's domain-general and mathematics-specific early cognitive abilities and their general mathematics achievement longitudinally across four time-points within the 1st and 2nd grade of primary school. As expected, a constellation of multiple cognitive abilities contributed to the children's starting level of mathematical success. Specifically, latent growth modeling revealed that WM abilities, IQ, counting skills, nonsymbolic and symbolic approximate arithmetic and comparison skills explained individual differences in the children's initial status on a curriculum-based general mathematics achievement test. Surprisingly, however, only one out of all the assessed cognitive abilities was a unique predictor of the children's individual growth rates in mathematics achievement: their performance in the symbolic approximate addition task. In this task, children were asked to estimate the sum of two large numbers and decide if this estimated sum was smaller or larger compared to a third number. Our findings demonstrate the importance of multiple domain-general and mathematics-specific cognitive skills for identifying children at risk of struggling with mathematics and highlight the significance of early approximate arithmetic skills for the development of one's mathematical success. We argue the need for more research focus on explaining children's individual growth rates in mathematics achievement.

Keywords: Mathematical Cognition, Mathematics Education, Approximate Number System, Symbolic Number Processing, Working Memory, IQ.

Introduction

Human cognition is incredibly variable. Naturally, mathematical cognition is no exception. Why is it that mathematics is fun and easy for some children, whereas for others it is a constant struggle, which can follow them even up to adulthood? Our understanding of the cognitive factors underlying the development of mathematics achievement is gradually progressing. The past decades, mathematical cognition research has been championing the role of various cognitive factors; domain-general, i.e., abilities that are important for all school-subjects (Baddeley & Repovs, 2006; Colom, Escorial, Shih, & Privado, 2007; Cragg & Gilmore, 2014; Geary, 2011a; Passolunghi, Mammarella, & Altoe, 2008), as well as domain-specific cognitive factors, i.e., abilities that are important particularly for mathematics achievement (De Smedt, Noël, Gilmore, & Ansari, 2013; De Smedt, Verschaffel, & Ghesquière, 2009; Feigenson, Libertus, & Halberda, 2013; Gilmore, McCarthy, & Spelke, 2010; Holloway & Ansari, 2009; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Mazzocco, Feigenson, & Halberda, 2011; Xenidou-Dervou, Molenaar, Ansari, van der Schoot, & van Lieshout, 2017). In the present study, we addressed the question of which early cognitive factors – domain-general and/or mathematics specific – form the foundation that fosters the development of early mathematics achievement. We report a longitudinal study where we explored which cognitive factors uniquely contribute to the development of children's mathematics achievement and how. Particularly, we aimed to address the following questions: 1) Which cognitive factors influence the initial status of the development of a child's general mathematics achievement, and 2) Which ones predict children's' intra-individual change, i.e., their growth rate, in mathematics up until the end of grade 2.

Early Predictors of Inter-Individual Differences in Mathematics

A well-known theoretical framework in mathematical cognition research assumes that the human brain is equipped with the so-called “Approximate Number System” (ANS; Dehaene, 1997; Gallistel & Gelman, 1992), an evolutionary ancient, ontogenetic and phylogenetic mechanism for estimating and manipulating quantities. It assumes that our innate approximate number sense guides the process of learning numerical symbols and mathematics development (Piazza, 2010; but see also Leibovich & Ansari, 2016; Reynvoet & Sasanguie, 2016). This theory is supported by studies with preverbal babies (Coubart, Izard, Spelke, Marie, & Streri, 2014; Izard, Sann, Spelke, & Streri, 2009; Xu & Spelke, 2000) and even animals being capable of discriminating quantities (Agrillo, Piffer, & Bisazza, 2011; Cantlon, 2012; Flombaum, Junge, & Hauser, 2005). In children, the ANS is typically assessed with tasks where they are asked to compare the magnitudes of nonsymbolic numerosities (e.g., dot arrays). Two well-known measures are the nonsymbolic approximate comparison and the nonsymbolic approximate arithmetic task (Barth, Beckmann, & Spelke, 2008; Gilmore, Attridge, De Smedt, & Inglis, 2014; Gilmore, McCarthy, & Spelke, 2007; Hyde, Khanum, & Spelke, 2014; McNeil, Fuhs, Keultjes, & Gibson, 2011; Park & Brannon, 2013; Xenidou-Dervou, van Lieshout, & van der Schoot, 2014). Performance in these tasks is characterised by the so-called ratio effect: accuracy drops as the ratio of the quantities to be compared approaches one.

It has been hypothesised that nonsymbolic magnitude processing skills are a key cognitive factor in the development of mathematics achievement as some studies have found an association between nonsymbolic processing skills and children’s mathematics achievement (e.g., Gilmore et al., 2010; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011). However, many other studies have not found such a relationship, and findings regarding nonsymbolic processing skills are mixed (for reviews see

De Smedt et al., 2013; Reynvoet & Sasanguie, 2016). On the other hand, symbolic processing skills – i.e., where the non-symbolic stimuli are replaced by their corresponding Arabic symbols – have been consistently demonstrated to robustly predict mathematics achievement (for reviews see De Smedt et al., 2013; Reynvoet & Sasanguie, 2016). Thus, we hypothesised that symbolic magnitude processing skills – symbolic approximate comparison and symbolic approximate addition – would be important predictors for the development of children's mathematics achievement.

But one may wonder: From what age can children actually perform approximate arithmetic with symbolic stimuli, i.e., Arabic digits? On the basis of Piagetian and neo-Piagetian theories, children would not be expected to conduct computational estimations before reaching eight years of age (Case & Sowder, 1990). However, Gilmore et al.'s (2007) seminal study demonstrated that children as young as kindergarteners not only perform above chance level in nonsymbolic magnitude comparison and arithmetic tasks, but also in the corresponding symbolic approximate tasks. Performance in these approximate symbolic tasks seemed to also be ratio-based. Given the fact that simple addition, in the form of "a + b = c", where an exact response is required, is a laborious process, which can takes years to master (Case & Sowder, 1990; Hamann & Ashcraft, 1985), Gilmore et al.'s (2007) findings were striking. Essentially, what they showed is that symbolic *approximate* arithmetic – in the form of "a + b" vs. "c", "Which is larger?" – even with large numbers such as "58", is possible as early as the kindergarten stage (see Figure 1B for an example). This ability is also known in the literature as "computational estimation" (Case & Sowder, 1990; Dowker, 2003).

There is now strong evidence that children's early ability to compare the magnitude of numbers (symbolic comparison), is a robust predictor of their mathematics achievement (for a review see De Smedt et al., 2013). But the predictive role of children's early symbolic approximate arithmetic skills is unclear. Since Gilmore et al.'s (2007) study, few have addressed the role of this early ability and how it relates to the development of mathematics achievement. What we do know, however, is that by the time children start formal schooling, most of them have also developed counting skills, which form the basis for starting to understand simple exact addition and subtraction (see Geary, 2011b), and are predictive of their mathematics development (Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009; LeFevre et al., 2006; Passolunghi, Vercelloni, & Schadee, 2007).

The aforementioned abilities are all *domain-specific predictors of mathematics*, i.e., – as the term implies – they are particularly important for the school subject of mathematics. But certain cognitive abilities are important for all school-subjects, not just mathematics. These are known as *domain-general* cognitive predictors. IQ – fluid intelligence, in particular – refers to the ability of using mental operations such as identifying relations, drawing inferences, transforming information, to solve novel problems (Primi, Ferrão, & Almeida, 2010), and is thus predictive of mathematics achievement (Alloway & Alloway, 2010). Working Memory (WM: Baddeley & Hitch, 1974), on the other hand, is an attention driven multicomponent cognitive construct. This construct refers to a system that stores and processes elements in an online manner when performing cognitive tasks (Baddeley, 2003; Repovs & Baddeley, 2006). Three of its components are particularly relevant for mathematics achievement: 1) the Visuospatial Sketchpad (VSSP), where a limited amount of visual and spatial elements are stored for a short amount of time, 2) the Phonological Loop (PL), which instead stores, also for a short period, phonological elements, and 3) the Central Executive (CE), which monitors, controls and regulates the workings of the other two components, and is activated when visual, spatial or phonological elements need to be manipulated. Since solving mathematical problems requires processing both verbal and visuospatial information, which often require multi-step solution procedures, it is no surprise that WM plays a fundamental role in mental arithmetic and mathematics achievement in general (for reviews see Cragg & Gilmore, 2014; DeStefano & LeFevre, 2004) and has been shown to be a strong longitudinal predictor of various mathematical skills (De Smedt, Janssen, Bouwens, Verschaffel, Boets, & Ghesquière, 2009; Gathercole, Tiffany, Briscoe, & Thorn, 2005; Hornung, Schiltz, Brunner, & Martin, 2014; Passolunghi et al., 2007), independently of IQ (Alloway & Alloway, 2010). Given that mathematics is a complex skill, it is also no surprise that IQ and Working Memory (WM) capacities have both been established by now as primary domain-general cognitive predictors of mathematics achievement, already early on in development.

From Inter-individual Differences to Intra-individual Change

Most studies addressing individual differences in mathematics achievement so far focused on the cognitive predictors of children's inter-individual differences, i.e., why some children perform better or worse than others, by examining which variables predict average mathematics achievement at a concurrent or future time-point. From the aforementioned literature, it is evident that a complex interplay amongst both mathematics-specific and domain-general capacities contributes to variation in children's mathematics achievement. However, little focus has been placed on the predictors of children's intra-individual change, i.e. what predicts children's individual growth rates in mathematics achievement, not just their average mathematics achievement at a specific time-point. Latent growth modelling, which was used in the present study, is a statistical method that permits the estimation of inter-individual variability in intra-individual rates of change over time (see Curran, Obeidat, Losardo, 2010; Grimm, Ram, & Hamagami, 2011). It takes into account the fact that no-one is really "average"; performance changes over time, but not necessarily in the same way or in the same rate for all children. Some may grow fast, while others may demonstrate slower or little developmental change; growth trajectories vary on the basis of many different characteristics, which vary from person to person. In the present study, our aim was to identify which cognitive characteristics explain differences in children's individual developmental change in mathematical achievement.

Studies on the topic of predictors of growth in mathematics are scarce. Jordan, Kaplan, Ramineni, and Locuniak's (2009) longitudinal study demonstrated that early number competencies and counting skills in kindergarten, as well as their growth from kindergarten to first grade predicted third grade mathematics achievement variation. Similarly, Toll, Van Viersen, Kroesbergen, and Van Luit (2015) followed kindergarteners for two and a half years and found that individual growth in nonsymbolic and symbolic comparison skills predicted future maths achievement at the end of grade 1. These results indicate that improved performance in early mathematics-specific abilities across time has a positive influence on their future mathematics achievement. But again, in these studies the dependent variable was children's average mathematics achievement at a specific time-point. Thus, the question remains: Which cognitive factors (domain-specific or domain-general) predict individual growth in mathematics achievement?

In a 3 year-long study Dulaney, Vasilyeva, and O'Dwyer (2015) assessed 4.5 year-olds' verbal intelligence, short-term memory and attention and explored whether these abilities predicted children's growth in mathematics achievement from grade 1 to grade 3. Their results demonstrated that these domain-general cognitive abilities only predicted children's early differences in mathematics but not growth. Fuchs, et al. (2010)'s longitudinal study, on the other hand, included a larger range of domain-general predictors as well as two mathematics-specific predictors assessed at the beginning of grade 1. In this case, different types of mathematics achievement abilities - procedural calculations and word problems – were assessed, but only twice: in the fall and spring of grade 1. Using latent change scores in a multiple regression approach, Fuchs, et al. (2010) showed that different constellations of domain-general and numerical cognition skills predicted change within grade 1 in different types of mathematics. Unfortunately, however, their mathematics specific predictors were quite limited and primarily focused on symbolic skills (transcoding and symbolic number line estimation). Lastly, Geary (2011a) addressed the question of predicting children's mathematics growth by conducting a comprehensive five-year longitudinal study. He assessed children's arithmetic skills from kindergarten through to grade 5 and administered to them a large test battery assessing mathematics-specific and domain-general abilities at the end of grade 1. Using multilevel modeling, this study showed that both mathematics-specific as well as domain-general abilities predicted inter-individual differences in initial level and growth rate from grade 1 to grade 5. Specifically, whereas a variety of domain-general and domain-specific abilities predicted children's starting point in kindergarten (intercept predictors), only the Central Executive measure of WM, symbolic number line processing, and addition strategies were unique predictors of children's growth in arithmetic.

The aforementioned studies comprised the first step in identifying predictors of children's growth in mathematics. The present study takes this type of research a step further. We assessed children before the start of their formal schooling on a wide variety of domain-general and mathematics-specific measures, including for the first time the relatively recently acknowledged cognitive factors of magnitude processing and approximate arithmetic (nonsymbolic and symbolic), which - as described earlier - are assumed to be fundamental for the development of mathematics achievement. Furthermore, we used a more comprehensive dependent measure for assessing general mathematics achievement, i.e., the Cito tests (Janssen, Scheltens, & Kraemer, 2005; Janssen, Verhelst, Engelen, & Scheltens, 2010). In mathematical cognition research, one notices that, often, arithmetic measures, i.e., tasks including only computational tasks such as addition, subtraction, etc., are treated as measures of mathematics achievement. In other words, arithmetic is sometimes viewed as a synonym of mathematics achievement. However, general mathematics achievement assessment in school includes much more besides just arithmetic, e.g., solving mathematical problems in verbal and pictorial contexts. The Cito tests are national curriculum-based mathematics tests, which school staff in the Netherlands use to monitor children's progress. They influence plans and decisions made for children who lag behind to receive extra support. At the end of primary school, Cito tests are used to identify which children should attend higher-or lower-level variants in the Dutch secondary educational system. Thus, performance on these tests play a paramount role in Dutch children's academic development.

The Present Study

We conducted a 3-year long longitudinal study beginning when the children were in kindergarten (around 5 years of age). We assessed their performance on an IQ measure, various domain-general and mathematics-specific WM measures covering all components of WM (Baddeley & Repovs, 2006), their counting skills, and their nonsymbolic and symbolic magnitude comparison and arithmetic abilities. In previous studies, we focused on explaining these children's inter-individual differences in kindergarten mathematics achievement (Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013) and their general mathematics achievement at the end of grade 2 (Xenidou-Dervou et al., 2017). But, as described earlier, in the Netherlands - from the middle of grade 1 and on – children are assessed by school staff on a curriculum based standardised test, known as the Cito mathematics tests. Thus, we had children's Cito scores from the middle of grade 1, end of grade 1, middle of grade 2 and end of grade 2. We, therefore, repurposed these kindergarteners' data on the various cognitive abilities to address the question: Which of the domain-specific and domain-general cognitive factors, assessed before the middle of grade 1, predict children's initial status and individual mathematics achievement growth rates from mid-grade 1 up until the end of grade 2? Given the strong and consistent predictive role that IQ, WM and symbolic magnitude processing appear to play in mathematical cognition and its development, we expected that these abilities would predict both children's initial status as well as their growth rates in general maths achievement.

Method

Participants

This study is part of a larger longitudinal collaborative project titled "The MathChild project". In this larger project, we assessed various cognitive abilities of 444 kindergarteners from 25 mainstream schools in the Netherlands, whom we followed up until grade 2. In the present study, we included all the children who completed all the tests of interest, including the Cito mathematics tests up to the end of grade 2. Our current sample included 334 kindergarteners, M_{age} = 5.59 (*SD* = .35), 148 girls. Dropouts were primarily due to family relocations or repetition of a grade. The children were approximately eight years old when they were assessed by the school staff on the last Cito test used in this study. All participants spoke Dutch and 96.4% held the Dutch nationality. They were sampled from middle- to high-SES environments: 33.8% of the children's mothers and 26.3% of their fathers had received middle-level applied education (in the Dutch Educational system: MBO), whereas 42.2% of their mothers and 45.8% of their fathers had acquired higher levels of education (in the Dutch Educational system: HBO and higher levels).

Procedure

Children were tested individually in quiet rooms within their schools by trained experimenters on all tasks except for the IQ and mathematics achievement test. The IQ test was administered in group-settings in their last kindergarten year (see measurement timeline in Appendix A). The mathematics achievement tests (Cito) were administered by school staff at the middle and end of grade 1 and middle and end of grade 2 (four time-points). The rest of the data comprise a subset of tasks administered to the children by the experimenters as part of the collaborative project. This data was acquired across two testing sessions lasting approximately 30 minutes each at the end of the kindergarten year or the beginning of grade 1 (see Appendix A). All of our experimenters used the same elaborate protocol for the administration of the tasks at each measurement time-point. Parts of the kindergarten data, i.e., the data on the IQ, Word and Digit Recall tasks, have been reported in previous articles (Xenidou-Dervou et al., 2013, 2017), as well as the Cito ability scores (Friso-van den Bos, Kroesbergen, et al., 2015; Xenidou-Dervou et al., 2017). In those articles, we addressed other research questions and made use of different statistical analyses.

Tasks

Apart from the paper and pencil IQ test, and the mathematics achievement tests, all other tests were computerised, presented with E-prime 1.2 (Psychological Software Tools, Pittsburgh, PA, USA) on HP Probook 6550b laptops.

IQ. Raven's Coloured Progressive Matrices (Raven, Raven, & Court, 1998) were used to assess children's nonverbal fluid intelligence. This test is suitable for children aged 5 through 11 years old. It entails visual patterns of increasing difficulty. In each pattern, a piece is missing and the child is asked to identify this missing piece from a set of six pieces to complete the pattern's design. The outcome measure entailed the raw accuracy scores.

Working Memory (WM). We used a set of six tasks translated and adapted from the Automated WM Assessment battery (AWMA; Alloway, 2007; Alloway, Gathercole, Willis, & Adams, 2004) to assess children's capacity on all three components of WM, i.e., the Visuospatial Sketchpad (VSSP), the Phonological Loop (PL), and their interaction with the Central Executive (CE). Since we were interested in examining all aspects of WM, we included both mathematics-related WM tasks, i.e., tasks that included digits, as well as tasks entailing elements not directly related to mathematics such as words. For each task, instructions were read aloud to the child by the trained experimenters and each task started with a short practice session.

Visuospatial Sketchpad (VSSP). The *Cross Matrix* assessed children's VSSP capacity. This task was identical to the Dot Matrix of the AWMA, only the dots were now replaced with crosses to avoid any overlap with our nonsymbolic tasks, which included dots. A trial included a 4x4 matrix where a cross appeared and disappeared. Participants were awarded one point when recalling correctly the location where the cross had appeared in the matrix. After four correct trials, the child automatically advanced to the next level of difficulty. With each increasing level of difficulty, one extra cross would appear (levels ranged from 1 up to 5 crosses). From the 2nd level and on, the child had to recall correctly both the location as well as the order of the locations where the crosses had appeared in the matrix. After three incorrect responses within one level of difficulty the task terminated automatically. The sum of correct responses comprised the outcome measure.

Phonological Loop (PL). The PL component of WM was assessed with two tasks. In the *Word Recall Forward* children heard a series of highly frequent, unrelated Dutch one-syllable words, which they had to recall in the presented order. A response was registered as correct if the child recalled the words correctly and in the correct order. The task started with one word and could go up to the level of five words. The *Digit Recall Forward* task was similar, the only difference was that now children heard and recalled digits (1-9) instead of words. Task progression rules were similar to those of the VSSP task. The sums of correct responses comprised the outcome measures.

Central Executive (CE). The CE component of WM was assessed with three tasks, which differed on the basis of the type of the to be maintained and manipulated information (phonological not mathematics-specific, phonological mathematics-specific, or visuospatial) that needed to be processed in one's CE. The *Word Recall Backwards* task was similar to the aforementioned Word Recall Forward task, only this time children had to recall the presented series of words in the reversed order. This task started with a series of two words and could go up to the level of seven words. Similarly, the *Digit Recall Backwards* task comprised of digits instead of words. Lastly, the *Odd One Out* task assessed children's capacity to manipulate visuospatial information, i.e., the interaction of the CE with the VSSP. In this task, the child saw three shapes inside three boxes presented one next to each other and had to identify the odd-one-out shape by pointing out the correct box on the screen. After the odd-one-out shapes had been identified, their locations had to be recalled in the same order as presented. The task entailed five levels of difficulty with each level including an extra shape. Progression rules were similar to the Cross Matrix task.

Counting. We used the four counting subscales from the A version (20 items) of the Early Numeracy Test-Revised (ENT-R; Van Luit & Van de Rijt, 2009). This test is suitable for children aged 4-7 years old. The four scales assess children's counting skills and their ability to apply this knowledge. Specifically, the scales address the following skills: 1) Using number words, i.e., counting forwards and backwards up to maximally twenty; 2) Structured counting, i.e., counting when pointing at objects; 3) Resultative counting, i.e., counting without pointing; 4) General understanding of numbers and using the counting system in everyday life. A correct response was recorded with one point.

Magnitude Processing.

Nonsymbolic Approximate Addition. This task was an adapted version of the nonsymbolic approximate addition task used by Barth and colleagues (Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; Barth, La Mont, Lipton, & Spelke, 2005) and Xenidou-Dervou et al. (2014). The task entailed 6 practice and 24 test trials. In each trial, a cartoon image of a girl on the top left side of the screen (Sarah) and an image of a boy on the top right side of the screen (Peter) appeared and the following sequential steps took place (Figure 1A): 1) a set of blue dots appeared on the top left side of the screen next to Sarah and dropped down; 2) These were then covered by a grey box; 3) Next, another set of blue dots dropped down into the box. 4) At this stage both sets of blue dots were hidden behind the box. 5) Then, a set of red dots appeared next to Peter on the right side of the screen and dropped down. The child was asked to answer as correctly and as fast as possible the question "Who got more dots, Sarah or Peter?" Essentially the child had to estimate the sum of the two blue dot-sets and compare that with the red set. The large amount of dots and the fast interchange of events made it impossible for the children to count or add the dots. They responded by pressing the blue or red response box situated in front of them. Each animated event lasted 1300 ms and between them there was an interval of 1200 ms. Numerosities ranged from 6 up to 70. The child could respond from the moment the red dots appeared on the screen and had a maximum of 7000 ms to register their response. There were three ratio differences between the sum of the blue sets of dots and the red set of dots: 4:7, 4:6, 4:5 (easy, middle and difficult ratio, respectively) and eight trials for each ratio. In half of the trials the sum of the blue dots set was larger than the red set of dots, whereas the reversed occurred for the other half of the trials. Trial order was randomised and between each trial there was an interval of 300 ms. To control for children's responses being reliant on the physical features of the dots in half of the trials, dot size, total dot surface area, total dot contour length and density correlated positively with numerosity and array size correlated negatively with numerosity. In the other half of trials these relations were reversed (Barth et al., 2006; Gilmore et al., 2010; Xenidou-Dervou et al., 2014). Dot sets were designed with MATLAB 7.5 R2007 b. The resulting outcome measure was the total number of correct responses (0-24).

Symbolic Approximate Addition. This task was identical to its nonsymbolic counterpart only this time the dot-sets were replaced with their respective Arabic numerals (Figure 1B). A complete list of all the trials included in these tasks can be found in Xenidou-Dervou et al., 2013).

Nonsymbolic Approximate Comparison. This task was similar to the nonsymbolic approximate addition task with the key difference that this time the child saw only one set of blue dots (the sum of the previously mentioned two blue sets). This time the child had to compare the magnitude of the single blue set of dots with the comparison red set.

Symbolic Approximate Comparison. The symbolic approximate comparison task was similar to its nonsymbolic counterpart with the difference that the dots were now replaced with the respective Arabic numerals. A complete list of all the trials included in these tasks can be found in Xenidou-Dervou et al. (2013).

- Figure 1 about here -

Symbolic Exact Addition. This was an adapted version of Jenks, De Moor, and van Lieshout's (2009) arithmetic ability addition task. In this task, children were asked to respond as correctly and as fast as possible to a series of addition problems, which asked for an exact response in the form of $a + b = c$, where $a \neq b$ and $a, b > 1$. There was one practice trial and 10 trials with simple ($c < 10$) and 5 items with harder addition problems ($10 < c < 16$). Each problem remained visible on the screen until the child provided a verbal response to the experimenter, who at that instance would press the space bar to stop the trial and recorded the child's answer (Xenidou-Dervou et al., 2013).

General Mathematics Achievement. Children's general mathematics achievement was assessed with the Cito ability scores (see Janssen et al., 2010). The national Cito mathematics tests are administered by school staff twice every academic year to all children starting from the middle of Grade 1 and onwards. They are administered in each academic year in January and June to monitor children's academic progress in mathematics. We acquired children's scores at four time-points: middle of grade 1, end of grade 1, middle of grade 2, and end of grade 2. The tests entail grade-appropriate mathematics problems that increase in difficulty over the grades. They include primarily word problems, which cover a wide range of mathematics topics, such as measurement (weight, time, length), proportions, numbers and number relations, arithmetic (addition, subtraction, multiplication, division) and complex applications such as more than one operation per problem. Raw test scores are converted to normed ability scores provided by the publisher, which typically increase across the primary school years.

Results

Table 1 depicts descriptive statistics on the predictor variables. To examine children's average developmental trajectory of mathematics achievement, we conducted a Repeated Measures ANOVA with Time (four time points) as the within-subjects factor. Since the assumption of sphericity was violated, degrees of freedom were corrected using Greenhouse-Geisser estimates. As expected, there was a main effect of Time, $F(2.87, 1002.12) = 752.66$, $p < .001$, $\eta_p^2 = .68$. As seen in Figure 2, children's general mathematics achievement appears to show a linear increase across the four time points. Tests of polynomials indicated that there was indeed a significant linear effect, $F(1, 349) = 1952.08$, $p < .001$, $\eta_p^2 = .85$, and a smaller significant cubic effect, $F(1, 349) = 13.22$, $p < .001$, $\eta_p^2 = .37$, the quadratic component was not significant.

- Table 1 about here –

- Figure 2 about here -

Latent Growth Modelling

First, we built an unconditional growth model, i.e., without predictors, which would identify an appropriate growth curve that would accurately and parsimoniously depict development on the individuals' level. We initially built a linear growth curve model. This model included an intercept (initial status) and a slope (growth) latent factor. The initial status factor is a constant for any individual across time; therefore, the factor loadings were fixed to 1. The growth latent factor encapsulates the developmental slope of an individual's mathematics achievement, i.e., the slope of the line indicated by the four measurement time-points. We initially hypothesised a linear growth across time and thus fixed the factor loadings accordingly assuming that growth in mathematical achievement was assessed over approximately equally spaced four different occasions. Thus, the first factor loading (Mathematics achievement assessed in the middle of grade 1) was fixed to 0 to represent initial status and the other three time-points had fixed factor loadings 1, 2 and 3 respectively. The unconditional linear growth model demonstrated good fit based on the CFI (.981), TLI (.977) and SRMR (.037) fit indices, but its RMSEA value (.093) was not within the acceptable range (< 0.08). Inspecting Figure 2 and the repeated measures results regarding the polynomial, one notices that children's average growth was not perfectly linear, particularly after the end of grade 1. Consequently, we ran a non-linear latent growth model where the growth factor loadings for middle of grade 2 (Mathematics_3) and end of grade 2 (Mathematics_4) were freely estimated (see Figure 3). This approach is often referred to as *latent basis growth modeling* and it is one of the preferred methods in child development to model non-linear growth (Grimm et al., 2011). This model demonstrated an acceptable fit to the data – Table 2 depicts the unconditional model's fit statistics and the corresponding fit criteria (Hu & Bentler, 2009; Schermelleh-Engel & Moosbrugger, 2003). Unstandardized model results indicated that the estimated factor loadings for Mathematics_3 and Mathematics_4 were 1.735 (SD = 0.09; p = < .001) and 2.834 (SD = 0.16; p = < .001) respectively. Apparently, the source of the misfit of the previous linear unconditional model's RMSEA value were the factor loadings of the third and fourth time-point, but as evidenced from the best-fitting non-linear latent growth model, the estimated factor loadings of Mathematics_3 and Mathematics_4 did not differ much from linearity (1.735 ≈ 2 and 2.834 ≈ 3). Nevertheless, we sustained the best-fitting non-linear basis latent growth model (Table 2, Figure 3). As expected, there was a negative correlation between the initial status and the growth factor, r = -.28, p = .004, which suggests that the lower a child's intercept was, the steeper his or her mathematics achievement growth was over time.

- Figure 3 about here –

Having identified the best-fitting unconditional model, we subsequently ran the conditional model by including the predictors[1] (Figure 4). This model demonstrated a very good fit to the data (Table 2). As expected, also in this model, initial status correlated negatively with the slope factor, $r = -.46$, $p < .001$. Table 3 depicts the standardised regression coefficients of the various domain-general and domain-specific skills predicting children's initial status and individual developmental growth in general mathematics achievement. With respect to the *initial status*, results showed that, as expected, children's IQ and their capacity on all three components of WM, i.e., the VSSP (Cross Matrix), the PL (Digit Recall Forward), and the CE (Word Recall Backwards) were significant predictors. Also, as expected, children's counting skills as well as their nonsymbolic and symbolic magnitude comparison and arithmetic skills were unique significant predictors of children's initial status. However, only one ability was identified as a significant predictor of children's *mathematics achievement growth* (i.e., of the growth factor in Figure 4), that was symbolic approximate addition (Table 3). None of the other hypothesised predictors explained variance in the growth latent factor. The conditional model explained 52% ($SE = 0.05$, $p < .001$) of the variance in the initial status factor but only 11% ($SE = 0.05$, $p = .038$) of the variance in the growth factor.

- Table 2 about here –

- Figure 4 about here –

- Table 3 about here –

[1] See Appendix B for the correlations between the predictors. Covariances amongst the predictors were accounted for in our conditional model as depicted in Figure 4.

One may argue that the reason why the symbolic approximate addition measure significantly predicted mathematics growth was because it entailed an “arithmetic” component, i.e. the addition component, and of course the dependent measure as a general mathematics achievement test (i.e., the Cito) also contains several arithmetic problems with Arabic numerals. Notice that the symbolic approximate addition measure had the form of “a+b” vs. “c”, “Which was larger”? Children were, thus, only asked to give an approximate response, i.e., make an estimation – the child did not need to mentally compute the calculation, i.e., provide an exact numerical response. However, arithmetic problems administered with the Cito ask for the exact solution, i.e., have the form of: “a + b = ?”. Nevertheless, to be certain that our symbolic approximate addition task was not acting as a type of autoregressor, we added to the conditional model an additional predictor, which was certainly such a type of an autoregressor for arithmetic: children’s performance on the symbolic exact addition task. Naturally, results showed that performance on the exact addition task was a significant predictor of children’s growth[2] (β = -.25, *SE* = 0.10, *p* = .009) but, more importantly, symbolic approximate addition remained the only other unique predictor of children’s growth, β = .30, *SE* = 0.10, *p* = .003, even after controlling for symbolic exact addition. Thus, the addition component of the symbolic approximate addition task does not account on its own for its significant predictive positive relation with growth in mathematical achievement.

Discussion

[2] Note that, as expected, exact addition significantly predicted also the initial status latent factor, β = .38, *SE* = 0.05, *p* < .001. This explains the negative regression coefficient with the growth factor, i.e., the better children were in arithmetic to start with, the less space they had to grow (a pattern which would be expected by an autoregressor).

The present study examined which cognitive factors predict the development of children's general mathematics achievement at four developmental time-points across grades 1 and 2 (middle of grade 1, end of grade 1, middle of grade 2, end of grade 2). Before or at the start of formal schooling, we assessed the children on various domain-general and mathematics-specific skills. Latent growth modelling analyses demonstrated that multiple domain-general and domain-specific abilities were significant unique longitudinal predictors of children's initial status, i.e. the level of their general mathematics achievement in the middle of grade 1. Specifically, as expected, we found that the children's WM capacities, their IQ score and their counting skills were all unique predictors of the children's starting point in mathematics achievement. In addition, their performance on both the nonsymbolic as well as the symbolic magnitude processing measures also explained unique variance – beyond all domain-general capacities. However, only one of these skills – symbolic approximate addition – was a unique predictor of the children's individual developmental growth in mathematics achievement from grade 1 to grade 2. Despite the wide range of early predictors that were assessed in this study, a large percentage of the variance in children's individual mathematics developmental growth remained unexplained. So far, research has focused on the predictors of children's inter-individual differences in mathematics achievement. However, every child's developmental trajectory is different. The present findings highlight the importance of further addressing developmental predictors also on the intra-individual change level. To our knowledge, this is the first study to examine the unique contribution of this wide variety of domain-general and domain-specific cognitive factors to children's individual developmental growth in general mathematics achievement. Below we discuss our findings in more detail, as well as the implications for cognitive and educational psychology research.

In Figure 4, the initial status latent factor represents the children’s initial status in mathematics achievement. To understand this better, imagine a racetrack; the point at which each runner starts the race from is somewhat analogous to the concept of the initials status factor. So, the LGM analyses could tell us which of the assessed cognitive factors influenced how far or behind a child’s starting point is in the “race” for mathematics achievement. In this respect, our findings replicated and extended past findings (Dulaney et al., 2015; Fuchs et al., 2010; Geary, 2011a). Specifically, comparing our results to that of Geary’s (2011a), which is the study most comparable to ours, we also found that IQ, the VSSP (assessed with the Cross Matrix) and the CE component of WM (Word Recall Backwards), as well as counting abilities (ENT-R) were unique predictors of children’s initial status in mathematics achievement (Table 3). Beyond Geary’s (2011a) findings[3], we found that the Phonological Loop, a component of WM (assessed with the Digit Recall Forward), was also a significant predictor of children’s initial status. But, performance on the Odd One Out, Word Recall Forward and Digit Recall Backwards did not explain unique variance in the initial status latent factor (Table 3). However, this is not surprising given that each WM component was measured with two tasks, and because the model accounted for their covariance, only one representative of each component of WM rose as a significant predictor. Intuitively, one may expect that the WM tasks that entailed numbers (digit recall tasks) would be better predictors than their domain-general counterparts (word recall tasks), but that was not the case for the CE component predictors. This is probably due to our dependent measure (Cito tests), which includes many mathematical word problems.

The more innovative aspect of our study concerned the role of children’s performance on the magnitude processing measures (nonsymbolic and symbolic approximate comparison and approximate addition tasks) on mathematics achievement, having accounted

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[3] Note that in Geary’s (2011a) final model IQ and the Phonological Loop measures were not included as predictors

for domain-general capacities (IQ and WM). Interestingly, nonsymbolic approximate comparison was a significant predictor of children's initial status in mathematics achievement, beyond all domain-general capacities, as well as symbolic number processing skills. This finding appears to support the assumption that nonsymbolic processing skills, i.e., the ANS, may be an important cognitive foundational underpinning for the development of mathematics achievement (Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010). So far, findings regarding the relationship between nonsymbolic skills and mathematics achievement have been mixed; some seem to find this relationship and others do not (for reviews see De Smedt et al., 2013; Feigenson et al., 2013; for a meta-analysis see Schneider et al., 2017). This inconsistency may lie on methodological issues, such as the type of tasks used and the age of the participants (Xenidou-Dervou et al., 2017). An alternative explanation, however, could be that this relationship between the nonsymbolic task and children's initial status in mathematics achievement may be an artefact of the inhibitory control demands that this type of task entails (Clayton & Gilmore, 2014; Gilmore et al., 2013). In the present study, although we assessed and controlled for the children's CE WM capacities, which in childhood correlate highly with inhibition (e.g.,Van der Ven, Kroesbergen, Boom, & Leseman, 2012), we did not assess and control for their inhibition skills per se. Future studies should address this limitation. Nevertheless, our results also verified past findings rendering symbolic magnitude processing skills as consistent and robust predictors of children's general mathematics achievement (for a review see De Smedt et al., 2013; Schneider et al., 2017). Both symbolic approximate comparison as well as symbolic approximate addition were unique predictors of the initial status latent factor of mathematics achievement (Table 3). In sum, our results demonstrated that all these championed cognitive underpinnings of mathematics achievement – domain-general and mathematics-specific – are unique predictors of children's starting point in early mathematics achievement, even after accounting for their covariance.

The most interesting part of the present study, however, regarded identifying the predictors of children's individual growth rate (i.e., their intra-individual change) in mathematics achievement across grades 1 and 2. As mentioned earlier, it is of primary importance given how each child develops in his or her own way – some faster, others slower. We sought to answer the question: Which cognitive mechanisms predict a child's particular growth rate in mathematics? Surprisingly, despite the wide range of predictors included in this study, only one cognitive factor uniquely predicted growth: symbolic approximate arithmetic. This is an ability that we use in our daily lives, e.g., when quickly estimating how much we will pay at the counter. It often works as a type of monitoring mechanism, e.g., if we have bought two books, which cost € 23.99 each, and the cashier asks for € 67.98, then we know something has gone wrong. As mentioned earlier, this is an ability, which appears very early in development, around 5 years of age (Gilmore et al., 2007), well before children are actually able to do exact arithmetic with large numbers in the form of "a + b = c" (Case & Sowder, 1990). The fact that symbolic approximate arithmetic – otherwise known as computational estimation – was the only predictor of growth signifies that research must identify its cognitive underpinnings and invest effort in investigating the effect of fostering or enhancing its development.

Until quite recently most research in this field focused primarily on understanding children's computational estimation skills from 10 years of age and above (Dowker, 2003; Ganor-Stern, 2016; Lefevre, Greenham, & Waheed, 1993; Lemaire, Lecacheur, Farioli, 2000). Actually, there are large individual differences even in adults' computational estimation explained by the large variability in the types of strategies that they employ (Dowker, 2003). Estimation strategies become more sophisticated with development, for example, Ganor-Stern (2016) suggests that at 10 years of age the most common strategy employed is the “sense of magnitude”, i.e., an intuitive sense of magnitude without any calculation, whereas adults use the “approximate calculation strategy”, i.e., rounding either one or two operands, calculating the result, and comparing that to the reference number. But, at 10 years of age estimation has already been partially established. The present study’s findings suggest that future research should address the emergence of symbolic approximate arithmetic already at five years of age, for which at the moment we know little about. What we do know is that symbolic approximate arithmetic appears to be underlined by different cognitive mechanisms compared to symbolic exact arithmetic (Xenidou-Dervou, van der Schoot, & van Lieshout, 2015) – they are correlated but distinct cognitive abilities (see Dowker, 2003). Actually, at the young age of five years, symbolic approximate arithmetic predicts children’s exact arithmetic (Xenidou-Dervou et al., 2013), and it correlates with WM abilities and children’s performance on nonsymbolic processing tasks (Xenidou-Dervou et al., 2013; Xenidou-Dervou et al., 2014). But why is it that symbolic approximate arithmetic is the only predictor of children’s growth in mathematics achievement, even beyond WM capacities and exact arithmetic? Perhaps what makes it special is that it is a type of monitoring mechanism tailored for arithmetic – a sense of magnitude, telling one’s self whether their answer is within an acceptable range or not. Children at this age may have insufficient knowledge to provide an exact answer for such problems, but they have some sense of number magnitudes (at least up to 10) and basic arithmetic principles, which may be the guide of their estimation performance. Dowker (1997) suggests that several skills underlie computation estimation performance at five years of age. Future research should specify the mechanisms that cultivate the emergence of symbolic approximate arithmetic skills and foster its development.

It should be noted that it is possible of course that our findings are affected by our sample's characteristics, especially the cultural background. We know from past publications that symbolic approximate arithmetic – the only significant predictor of growth in the present study – is affected by language, i.e., the way numbers are named in a given language (Xenidou-Dervou, Gilmore, van der Schoot, & van Lieshout, 2015). The way two-digit numbers are named in Dutch (first the unit and then the decade, known as the inversion property) imposes extra WM load on children (Xenidou-Dervou et al., 2015). These cognitive demands may drive part of the strength of symbolic approximation as a predictor. Furthermore, differences in educational systems (in some countries like in the UK for example, formal education starts earlier), and home numeracy may affect results. Therefore, cross-cultural studies are rendered necessary to identify the predictors of children's growth independent of cultural background.

Geary (2011a), the most comparable study to our present study, had found that CE, number line processing, addition retrieval and addition decomposition were significant predictors of growth in arithmetic from Grade 1 up to Grade 5, but had not investigated the contribution of the ANS, symbolic magnitude and approximate addition processing skills. Geary's (2011a) outcome variable only tapped into arithmetic abilities, whereas our general mathematics achievement test included many different problems, including also problems with number lines. Therefore, we avoided including predictors that are measuring skills which are already included in the outcome variable – i.e., Cito tests (e.g., exact addition or number line skills). But, on the basis of Geary's findings we expected that WM would be a significant predictor of growth in general mathematics achievement. Unexpectedly, this was not the case – none of our WM variables predicted individual growth in general mathematics achievement. One possible explanation is based on past findings, which show that reliance on WM capacities increases with age (Friso-van den Bos, Van der Ven, Kroesbergen, & Van

Luit, 2013). Geary's study addressed growth up to grade 5, whereas our study only examined growth up to grade 2. Perhaps the WM demands of the Cito tests until the end of grade 2 were not yet high enough to deem WM as a significant predictor of growth. Alternatively, perhaps symbolic approximate arithmetic, which on its own encompasses WM demands (Xenidou-Dervou, Gilmore, et al., 2015; Xenidou-Dervou, van der Schoot et al., 2015), depicts indirectly the effects of WM on growth in general mathematics achievement. It would be interesting for future research to follow children's growth up to the end of primary school to identify the cognitive factors that contribute to their growth across the entire developmental stage of primary school years.

A striking finding of this study is the fact that, despite the wide range of early cognitive factors that we assessed, and even though our latent growth model explained sufficiently variance (52%) in the initial status factor (i.e., mathematics achievement in middle of Grade 1), it only explained 11% of the variance in children's individual mathematics growth rates. The cognitive factors that we assessed are the ones that are most championed within the field of mathematical cognition for setting the foundations of children's mathematics achievement. Given the well-documented shortcomings of mathematics education in Western societies (e.g., Ker, 2016), it is imperative that we identify the underpinnings of children's individual growth rates in mathematics achievement. In the "race" analogy used earlier, we must strive to identify the factors that make one run faster or slower than others. Based on our findings, symbolic approximate addition is one of those influential factors, but unarguably it is not sufficient on its own. Future research should also examine the contribution of non-cognitive factors in predicting mathematics achievement growth, such as socio-economic status, home numeracy, mathematics anxiety, language and other student- teacher or schooling-related factors (e.g., Galindo & Sonnenschein, 2015; Göbel, Moeller, Pixner, Kaufmann, & Nuerk, 2014; Ker, 2016; Ramirez, Gunderson, Levine,

& Beilock, 2013). So far, the present study's results suggest that a constellation of multiple domain-general and domain-specific cognitive abilities should be used as screening tools to identify children at risk for difficulties in mathematics. Particularly, symbolic approximate addition can also be an indicator for a child's growth rate in mathematics. Also, future studies should examine if training children's symbolic approximate addition skills – in the form of "a+b vs. c"; "Which is larger?" – could improve a child's growth rate in mathematics achievement.

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Table 1.

Descriptive Statistics (Means and SDs) on the Predictor Variables.

Predictors	*M (SDs)*
IQ	21.83 (4.86)
Word Recall Forward	14.13 (2.41)
Word Recall Backwards	5.18 (1.78)
Digit Recall Forward	14.28 (2.30)
Digit Recall Backwards	4.69 (1.61)
Cross Matrix	13.12 (2.91)
Odd One Out	11.31 (2.60)
Counting skills	15.87 (2.33)
Nonsymbolic Approx. Comparison	17.39 (2.97)
Nonsymbolic Approx. Addition	16.44 (2.45)
Symbolic Approx. Comparison	18.94 (3.55)
Symbolic Approx. Addition	16.32 (3.33)

Table 2.

Fit Statistics on the Unconditional and Conditional (i.e., with Predictors) Latent Growth Models (LGMs) and the Corresponding Fit Criteria.

	Fit Indices						
Models	χ^2	df	χ^2/df	CFI	TLI	RMSEA	SRMR
Unconditional	7.769*	3	2.59	0.994	0.988	0.067	0.026
Conditional	41.79*	27	1.55	0.991	0.982	0.040	0.014
Fit Criteria							
Acceptable fit			≤ 5.0	≥ 0.90	≥ 0.90	< 0.08	≤ 0.10
Good fit			$0 \leq \chi2/\text{df} \leq 2$	≥ 0.95	≥ 0.95	< 0.05	$0 \leq \text{SRMR} \leq 0.05$

Note. χ^2 = chi-square value; df =degrees of freedom; χ^2/df = chi-square by degrees of freedom ratio; CFI = Comparative Fit Index; TLI = Tucker–Lewis Index; RMSEA = Root Mean Square Error of Approximation; SRMR = Standardized Root Mean Square Residual.
*$p \leq .05$

Table 3.

Standardized Regression Coefficient Results for each Predictor in the Conditional Model (Figure 4).

	Latent Factors			
	Initial Status		Growth	
Predictors	β	*SE*	β	*SE*
Domain-general				
IQ	**.15****	0.05	- .02	0.10
Cross Matrix	**.12***	0.05	- .06	0.10
Odd One Out	.09	0.05	- .01	0.10
Word Recall Forward	.02	0.06	- .11	0.12
Word Recall Backwards	**.12***	0.06	- .07	0.10
Digit Recall Forward	**.13***	0.06	.10	0.12
Digit Recall Backwards	.02	0.05	.06	0.09
Domain-specific				
Nonsymbolic Approx. Comp.	**.12***	0.05	- .05	0.10
Nonsymbolic Approx. Add.	.07	0.05	- .04	0.10
Symbolic Approx. Comp.	**.13***	0.05	.08	0.09
Symbolic Approx. Add.	**.10***	0.05	**.26****	0.10
Counting skills	**.23*****	0.05	- .15	0.09

Note. Bold figures indicate significant predictors, *** $p \leq .001$, ** $p \leq .01$, * $p \leq .05$.

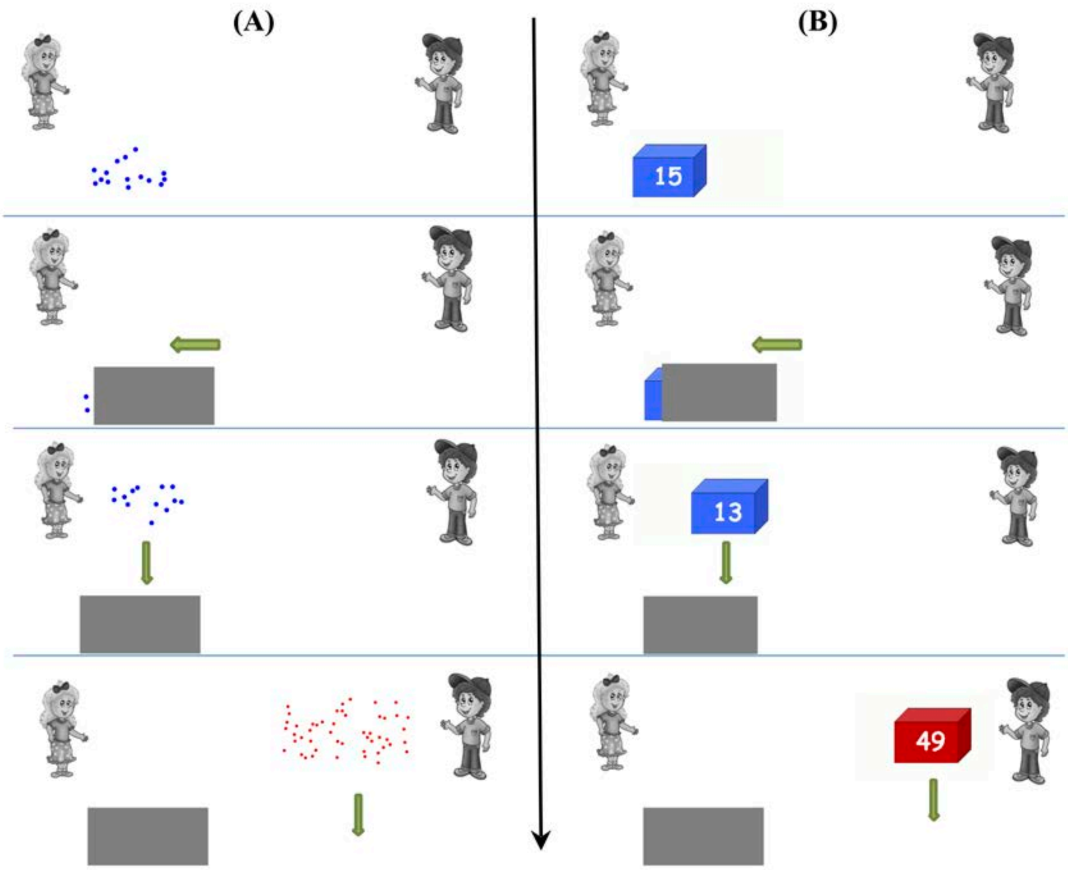
Figure Captions

Figure 1. Nonsymbolic (A) and symbolic (B) approximate addition example trials (from Xenidou-Dervou, Gilmore, et al., 2015)

Figure 2. Average mathematics achievement development across the four time points.

Figure 3. Graphical representation of the best fitting unconditional LGM. Note: Mathematics_i = mathematics achievement at one of the four time-points.

Figure 4. The conditional model (i.e., predictors included). Note: CM = Cross Matrix; OOO = Odd One Out; WRF = Word Recall Forward; WRB = Word Recall Backwards; DRF = Digit Recall Forward; DRB = Digit Recall Backwards; NS = Nonsymbolic; S = Symbolic; Comp = Comparison; Mathematics = General Mathematics Achievement



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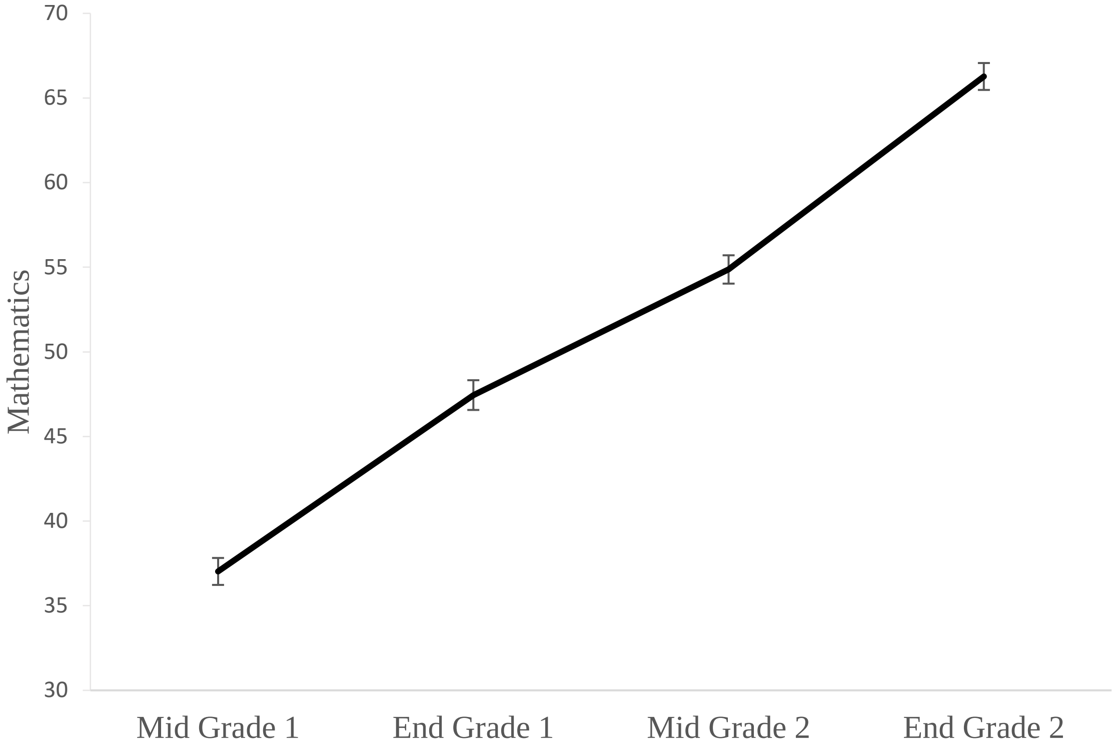

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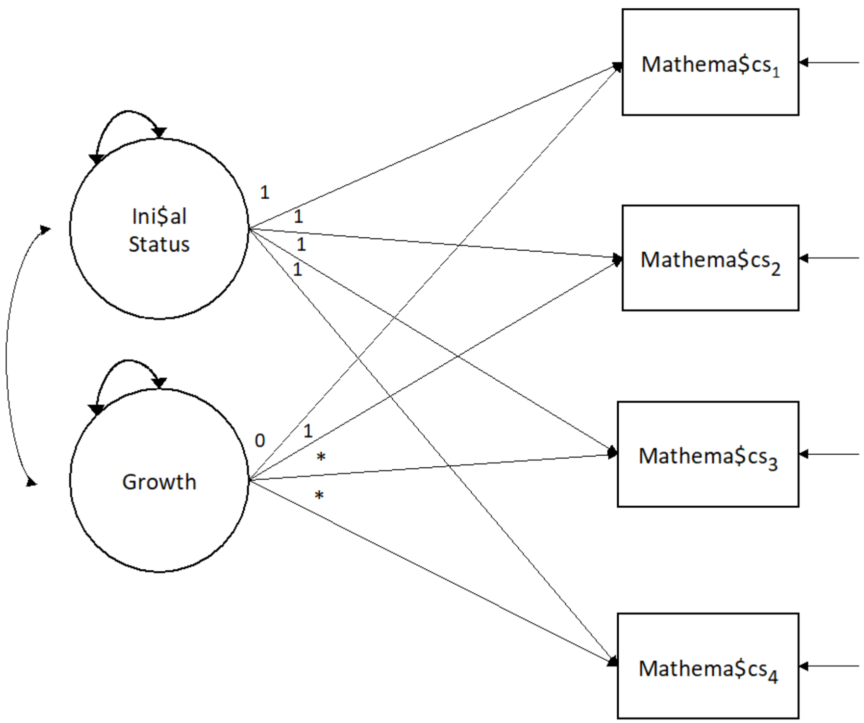

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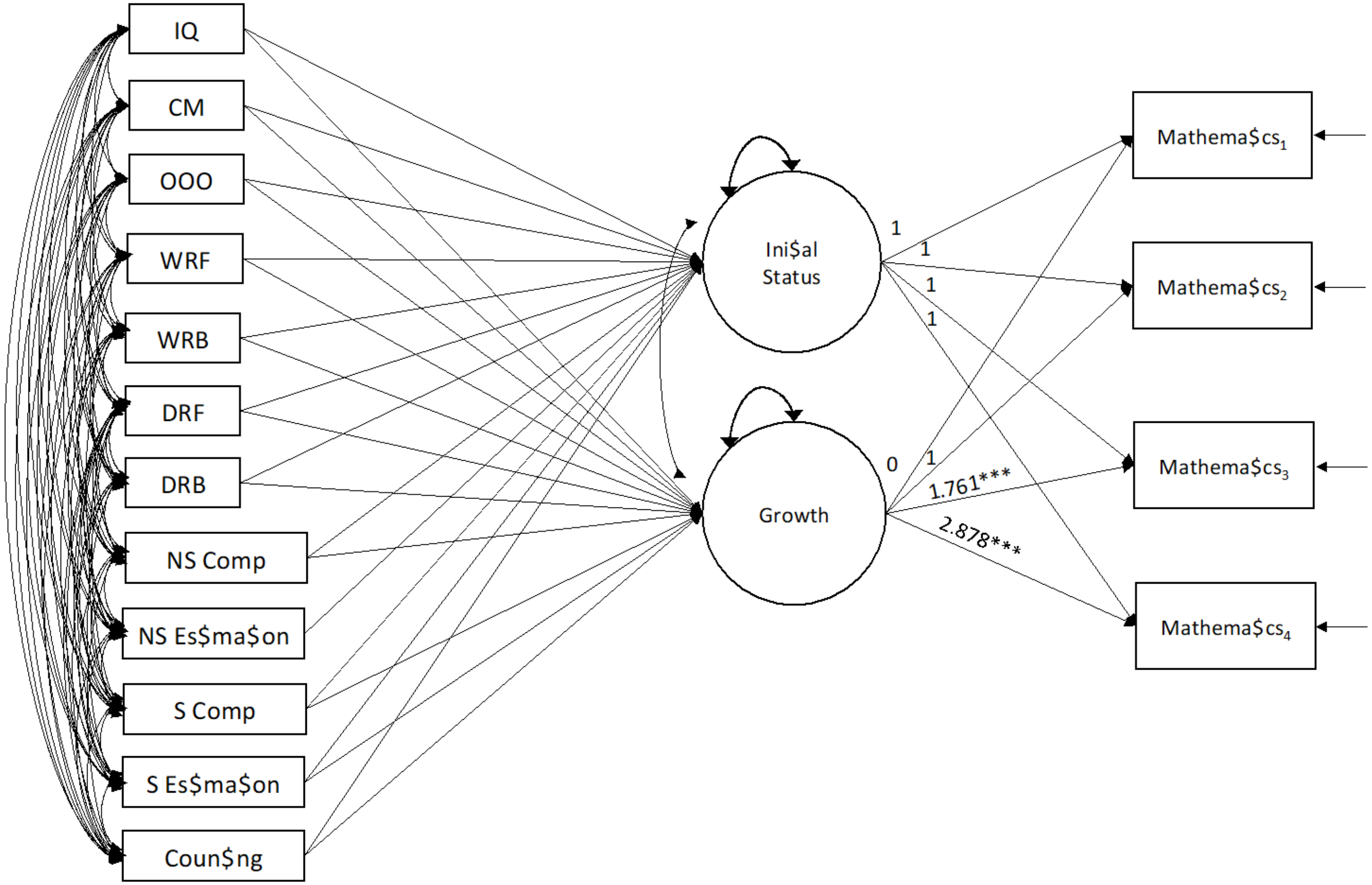

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