Torsional vibration energy harvesting through transverse vibrations of a passively tuned beam

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Abstract: The paper highlights the potential of harvesting vibration energy from mechanical systems in the form of electrical power to activate remote electronic devices. The principal idea is based upon the resonant response of a lightweight oscillator subjected to applied external excitation, coupled with an electrodynamic transducer (e.g. piezoelectric material, inductive coils). As far as the mechanical system is concerned, the aim is to maximize the harvested energy when an attachment vibrates with relatively high amplitudes. This means that the system natural frequency should be close to the expected dominant frequency of the applied (host) vibrations. However, in practice the dominant vibration frequency varies either within a limited range due to system uncertainties or across a large band due to the fundamental operation of the host structure, such as in rotational power transmission systems with speed variations. Recently, the introduction of nonlinearities has been proposed in order to compensate for small-scale frequency shifts. Nevertheless, in most cases one cannot fully bypass the necessary tuning effects, attributed to linear stiffness components in system dynamics. In this paper, a rotational vibration energy harvester is outlined, based upon a beam attachment, coupled with an electromagnetic transducer. The stiffening effect due to centrifugal action is utilized in order to passively tune the attachment to the dominant frequency of the rotational host structure. A reduced order model of the harvester is presented and its power extraction potential is assessed.

1. Introduction

Vibration energy harvesting is an emerging field, converting the environmental vibration energy of a host structure or machine for usable electric output [1]. This energy can then potentially be used to power low consumption electronic devices, such as wireless sensors. Common energy harvesters initially comprised linear oscillators, coupled with an electrodynamic element, such as piezoelectric patches or permanent oscillatory magnets in the proximity of a coil of thin wire. The main drawback of this technique is the limited frequency response of such oscillators [2]. Briefly, these harvesters need to be tuned to a certain frequency in order to resonate with the host's vibrations. However, mechanical systems often operate across a wide range of frequencies leading to mistuning of the harvesting devices. Consequently, their efficiency in terms of power output dramatically reduces when frequency variations occur.

In recent years, the introduction of non-linearity has broadened the frequency band response of mechanical oscillators [3-4]. Many researchers have employed the transverse vibrations of a beam, attached to a host structure to exploit its nonlinear response. A recent review is provided by Wei and Jing [5]. Erturk and Inman studied a distributed beam model for piezoelectric energy harvesting [6]. They also studied experimentally a bimorph cantilever beam for enhanced harvesting capabilities [7]. Recently, significant progress has been made with bi-stable energy harvesters, a vast majority of which concern transverse vibrations of buckled beams [8].

An overwhelming majority of reported studies have considered the translational vibration problems in this regard, largely neglecting the opportunities with torsional systems that abound in powertrains and rotor dynamic applications. One can conceive the design of a piezo-generator, based on torsional stressing of its active elements [9-10]. Furthermore, the piezoelectric cantilever design has been used for either torsional vibratory modes with piezoelectric attachments, or in bending modes, extending radially from a shaft subjected to torsional oscillations [11].

In this paper, a vibration energy harvester based on transverse beam vibrations is proposed for rotational systems. Torsional vibrations in rotor systems often experience large scale frequency shifts attributed to changing operating speed. The proposed concept aims to passively tune the beam element of the harvester to the speed-dependent torsional vibrations. This is accomplished through induced centrifugal action acting upon an attached beam as an agent for passive tuning. The design strategy is presented and the harvested power is predicted numerically.

2. System modelling

A disc (D) is assumed to rotate with angular speed $\omega = 2\pi n/60$, as shown in Fig. 1. A thin beam of length L, width a and thickness b is attached to the disc with one end connected to its rotational axis via the support S1. The other end is connected at an eccentric location on the disc via the support S2, such that the beam extends radially from the disc centre. These supports can be one of clamped, pinned or roll independent from each other. The beam has modulus of elasticity E, density ρ and it is positioned with its width b perpendicular to the plane of the disc, such that it can experience transverse vibrations along the disc's plane. The distance of a point P on the beam from the axis of rotation is denoted by x, whereas the beam deflection in the transverse direction is denoted by $w(x)$. A magnet is fixed at an arbitrary point P on the beam via a massless, rigid link. A coil of thin wire is also fixed onto the disc, at a position such that the magnet lies within the coil. Torsional vibrations of the disc D cause transverse beam vibrations with respect to its rotating plane. The vibratory response of the beam entails relative motion between the magnet and the coil, thus inducing a coil voltage due to electromagnetic coupling, k_c. Then, an electric load with an electric resistance equal to the coil's, $R_l = R_c$, is connected at the

coil ends, completing the circuit. The harvested energy is usually taken as the energy that the load consumes. This also holds for the harvested power as well. The described system corresponds to an elemental vibration energy harvester for the purpose of demonstrating the utility of the proposed concept.
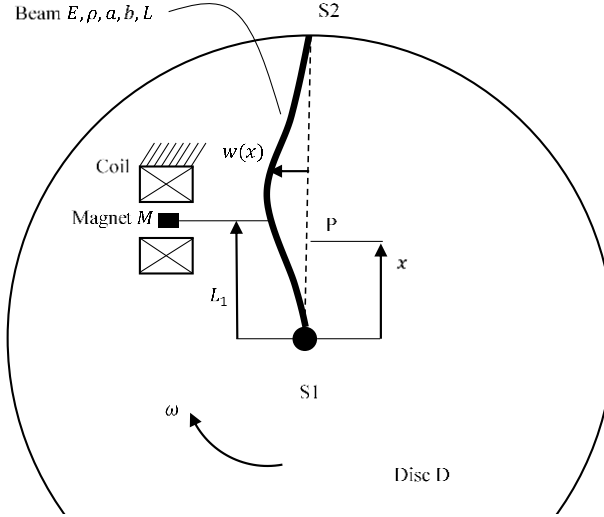

Figure 1. Sketch of the proposed torsional vibration energy harvester.

It is assumed that the beam is sufficiently thin to follow the Euler-Bernoulli beam theory. Therefore, considering an axial load $P(x)$, the transverse vibrations of the beam are described by the following spatio-temporal differential equation:

$$m_1(x)\ddot{w} + c_1\dot{w} + \frac{\partial}{\partial x}[P(x)w'] + EIw'''' - \frac{EA}{2L}w''\int_0^L (w')^2 dx = -m_1(x)\ddot{z}, \tag{1}$$

where $m_1(x) = [\rho A + M\delta(x - L_1)]$, $A = ab$ is the beam cross-sectional area, c_1 is the viscous damping coefficient, I is the second area moment of inertia of the beam cross-section and z denotes the disc's vibration response. Using a single mode approximation such that $w(x,t) = q(t)\varphi(x)$, where $\varphi(x)$ is the first mode shape, one arrives at:

$$m\ddot{q} + c\dot{q} + kq + k_3q^3 = -f_m\ddot{z}, \tag{2}$$

where

$$m = \int_0^L \rho A\varphi^2\, dx + M\varphi^2(L_1), \tag{3}$$

$$c = \int_0^L c_1\varphi^2\, dx, \tag{4}$$

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$$k = EI \int_0^L (\varphi'')^2 \, dx + \int_0^L P(x) \, (\varphi')^2 \, dx, \tag{5}$$

$$k_3 = -\frac{EA}{2L} \int_0^L \varphi'' \varphi \, dx \int_0^L (\varphi')^2 \, dx, \tag{6}$$

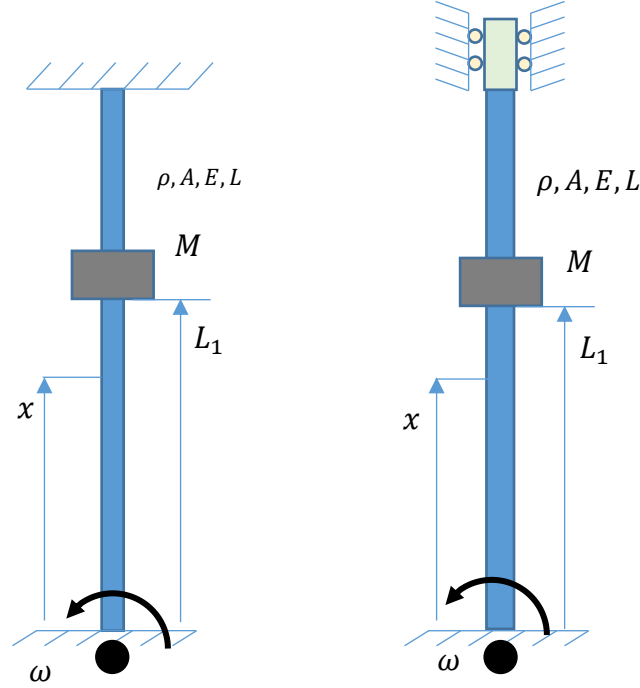$$f_m = \int_0^L \rho A \varphi \, dx + M\varphi(L_1), \tag{7}$$

Figure 2. Two cases of boundary conditions: (a) clamped-clamped; (b) clamped-roll.

3. Passive tuning of the beam

The axial load in the proposed concept is as a result of the centrifugal action on the beam and the lumped mass, M. This is heavily dependent on the boundary conditions. In this paper, two cases of beam support are considered: (i)- clamped-clamped (C-C) and (ii)- clamped-roll (C-R), as shown in Fig. (2). The axial load depends on the position of the lumped mass, L_1, representing the magnet and its fixture, and on the disc's rotational speed. Fig. (3) shows the axial load across the span of the beam with respect to the position of the mass, L_1 and the rotation speed, n. Note that for C-C boundaries the load is tensile when $x < L_1$ and compressive elsewhere, whereas for the C-R supports the load is always tensile. This is caused by the axial reaction of the outer clamp, contrary to the inability of the roll support to contribute in the axial direction.
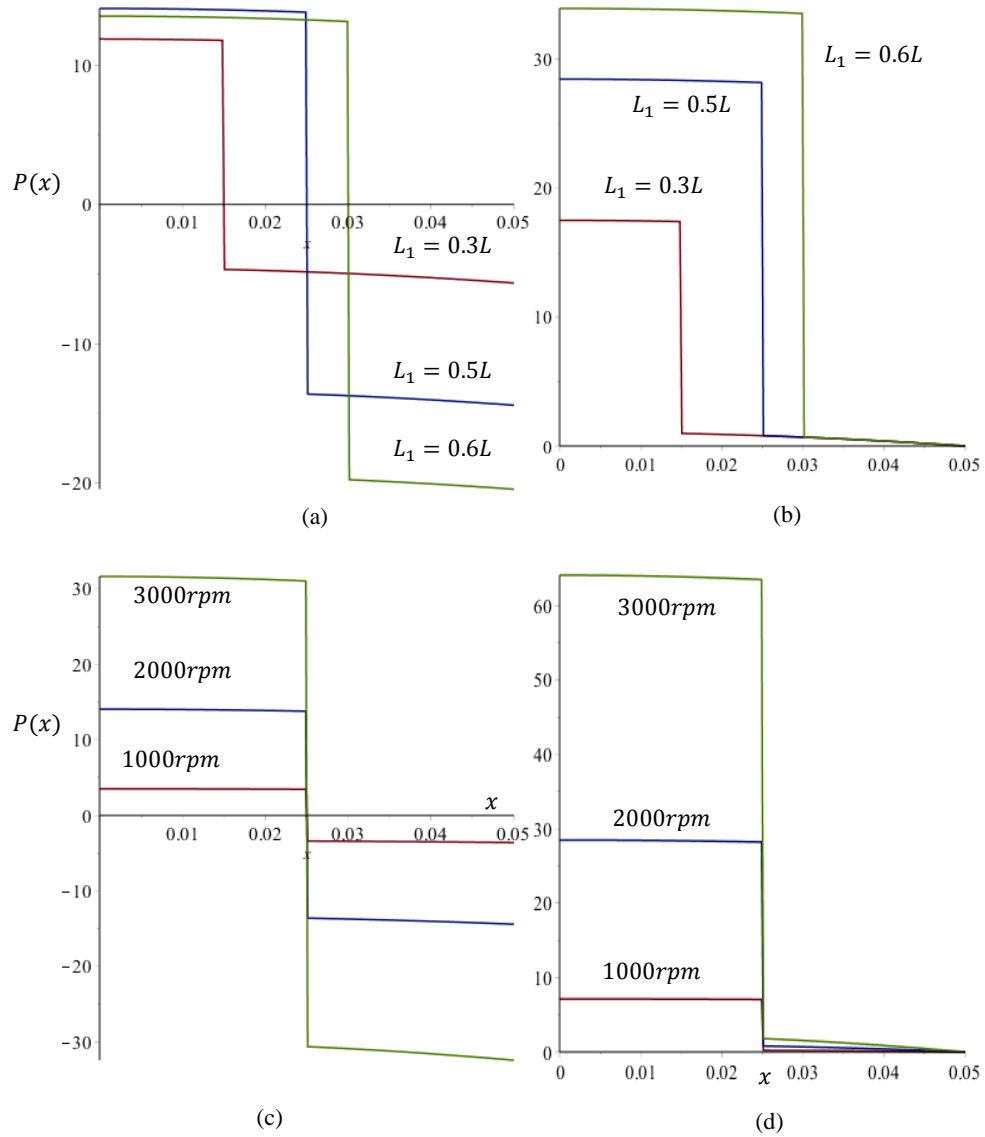
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Figure 3. Axial load acting on the beam along its length. The left column corresponds to C-C support and the right column to C-R. The top figures depict the influence of the position L_1 at 2000 rpm and the bottom figures show the influence of the rotation speed n for $L_1 = 0.5L$.

The axial load variation induces the beam modal frequencies to vary accordingly. Following the reduced model in Eq. (2)-(7), the natural frequency can be calculated as:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, \tag{8}$$

As an example we consider a beam with the following characteristics: $E = 200\ GPa, L = 50mm, b = 5mm, t = 0.5mm, \rho = 7810\ kg/m^3, M = 0.100\ kg$. Fig. (4) shows the variation of the beam's first modal frequency with respect to the rotational speed of the disc, n. As the torsional vibrations in most rotational systems directly correspond to the system rotational speed, one can potentially passively tune the beam to the frequency of the speed-dependent vibrations, subjected to proper design provisions and *a priori* knowledge of the expected system response. Therefore, resonant conditions may be maintained in a wide-band of frequencies, promoting efficient vibration energy harvesting.
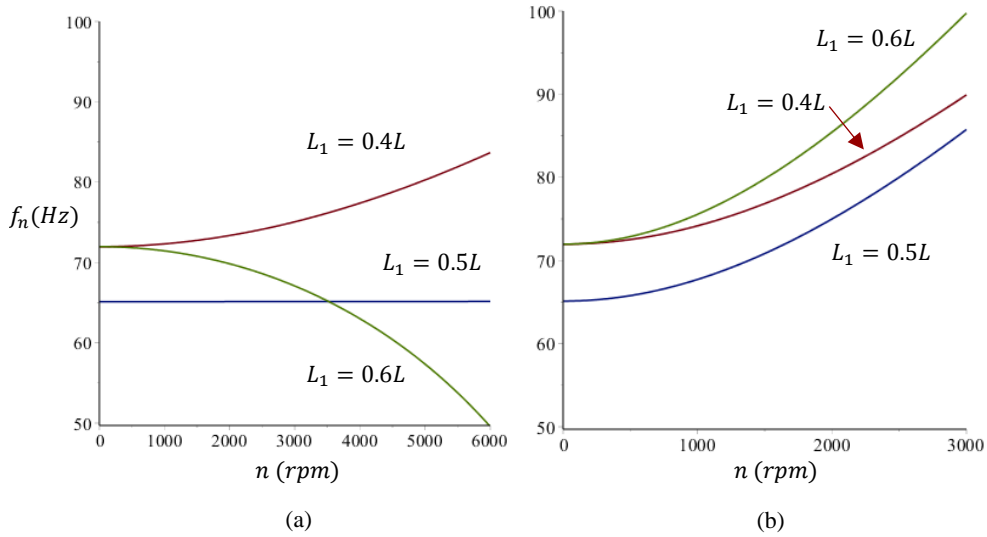

Figure 4. Dependence of the model's modal frequency for varying speed and different positions of the mass M; (a) C-C boundaries; (b) C-R boundaries.

Fig. (4) shows the frequency variations for the two considered boundary condition options. Depending on the desired variation of the system's natural frequency, one can choose the proper boundary conditions to passively tune the beam to higher or lower frequencies. Note that with the C-C supports, if the mass is positioned at the middle of the beam no tuning would be achieved. This is

consistent with Fig. (3c), which shows that the tensile load is opposing symmetrically the compressive load.

As a second example, consider the case of torsional vibrations in automotive powertrains. In such systems, vibrations unfold with a frequency directly linked to the system's rotational speed; specifically to the engine's firing frequency. In order to achieve efficient vibration energy harvesting, resonant conditions must be satisfied across a wide operating frequency range, which is nearly impossible with typical linear harvesters. The design proposed herein requires the modal frequency to closely match the expected vibration frequency. Consider then the following beam: $E = 200 \, GPa, L = 80 \, mm, b = 15 \, mm, t = 0.2 \, mm, \rho = 7810 \, kg/m^3, M = 0.100 \, kg$, which is clamped at one side and supported by a roller at the other. Fig. (5) shows the variation of the beam's first modal frequency with the system's rotational speed. The magnet is fixed at $L_1 = 0.575L$ in order to ensure good coincidence with the excitation frequency. It is observed that the modal frequency is tuned such that it remains close to the excitation frequency. In this manner, the harvester's response is expected to reside close to the resonant region.
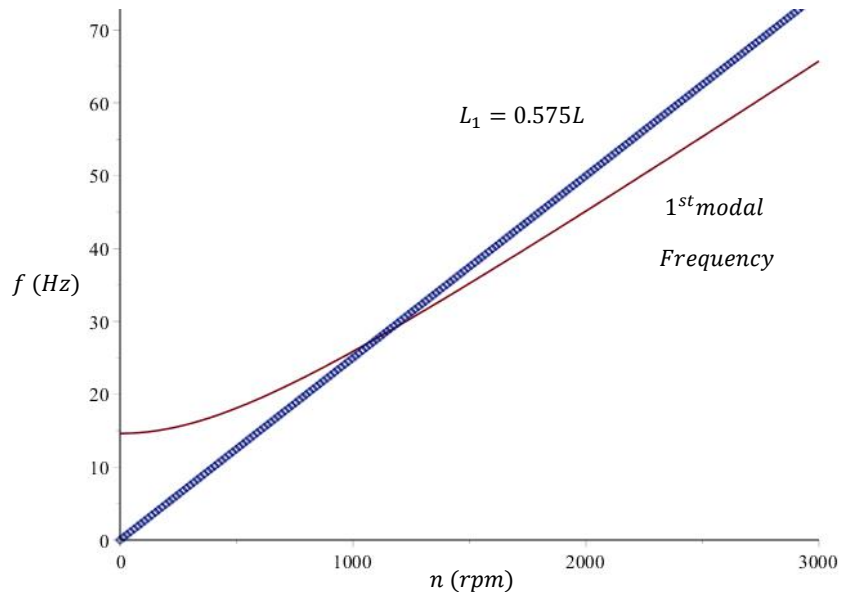

Figure 5. Variation of the C-R beam's first modal frequency across a wide range of frequencies (solid line), plotted with the expected dominant frequency of the host's torsional vibrations (circles).

4. Harvested power

Including the electric circuit in the equations of motion introduces an additional damping term which dissipates energy from the beam and so, $c_1 = c_m + c_e$, where $c_e = k_c^2/2R_l$. Furthermore, the harvested power is given by:

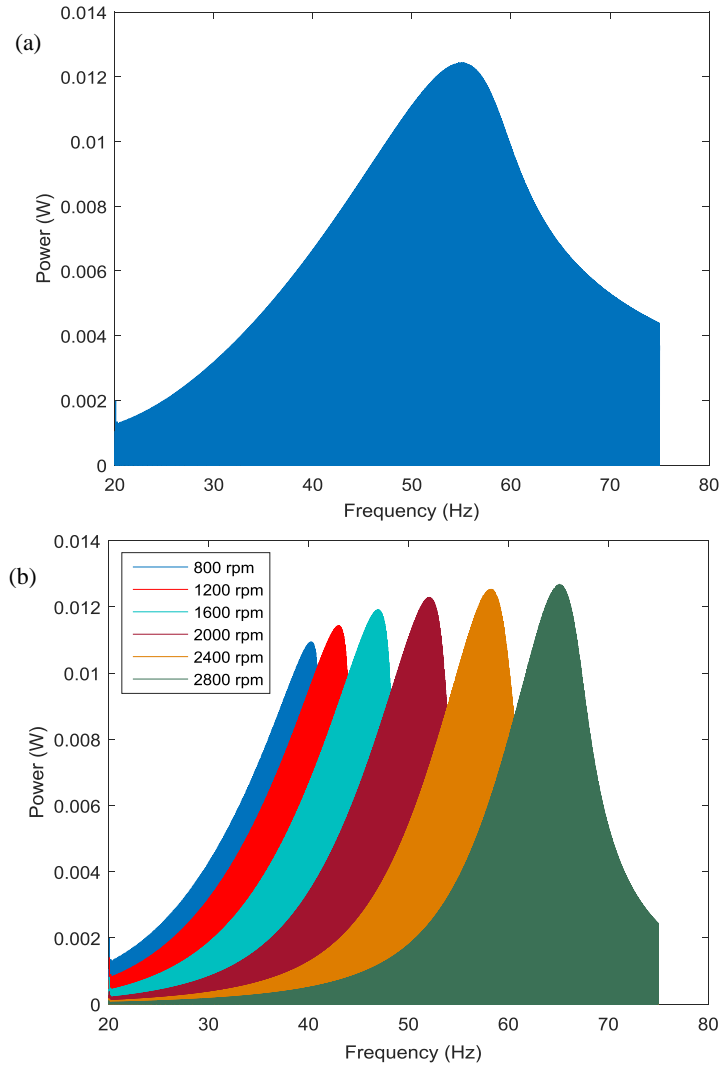$$P_l = \frac{k_c^2}{2R_l} \dot{w}^2(L_1),$$ (8)

Figure 6. Comparison of the power output; (a) harvester tuned by centrifugal effect; (b) harvesters tuned at different shaft speeds.

Adopting these formulae for the assumed beam in Fig. (5), selected for the purpose of demonstration $k_c = 11\,Tm, R_l = 50\,\Omega$, then the harvested power with the proposed passively tuned concept – Fig. (6a) – is compared with several non-tunable harvesters with set frequencies in the range of interest in Fig. (6b). One can readily observe that the passively tuned harvester demonstrates a far wider frequency response as opposed to its narrow-band counterparts tuned at specific frequencies.

5. Conclusions

In this paper, a concept for harvesting vibration energy from rotating systems is presented. The concept utilizes the dependence of a thin beam's natural frequency on externally applied axial loads. A means of passively tuning the harvester to the host's torsional vibrations through the utilization of the centrifugal action is proposed. It is shown that the proposed device can target a wide frequency range offering resonant vibration energy harvesting over a large range of operating conditions of the host structure.

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