

"THE PROBABILITY OF DETECTING AND TRACKING RADAR TARGETS IN CLUTTER AT LOW GRAZING ANGLES" BY

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A DOCTORAL THESIS

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30 \text { September } 1982
$$

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## CERTIFICATE OF ORIGINALITY

This is to certify that $I$ am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a higher degree.
"It is probably safe to say that clutter will
never be understood completely because there
are so many variables to control ..........."
TOMLINSON
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## PREFACE

This report was compiled as a part-time study during the author's appointment as Guided Weapon Specialist at the Department of Air Warfare at Royal Air Force College Cranwell. Although sponsored by the Ministry of Defence the results and opinions are those of the author and should not be taken as official.

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## ABSTRACT

Modern military acquisition and tracking radars are required to operate against aircraft and missiles specifically designed to have minimal radar cross section (RCS) and which fly at very low level to take maximum advantage of terrain screening.

A model for predicting system performance is necessary for a range of terrain types in varying precipitation and seasonal cultural conditions. While the main degradation is from surface clutter and denial of sightline due to terrain and other local obstructions, several other factors such as multipath propagation, deliberate jamming and even operator performance contribute to the total model. The possibility that some raders may track obscured targets, however briefly, by using the diffraction path, is of particular interest.

Although this report critically examines each of the contributory factors in order to select optimum values for inclusion in an overall computer prediction model; a new surface clutter model is specifically developed for sloped terrain using actual clutter measurements. The model is validated by comparison with an extensive survey of worldwide clutter results from both published and unpublished sources.

Certain constraints have been necessary to restrict the study to a manageable size, while meeting the requirements of the sponsors. Attention is therefore focussed upon performance prediction for typical mobile tracking radar systems designed for operation against small RCS low level targets flying overland.
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## Chapter 1

$$
\begin{array}{ll}
\tau \\
\text { GRAZING ANGLE }= & \text { Radar pulse width (pulse duration) secs. } \\
& \text { The angle between the local horizon at the } \\
& \text { target and the radar beam direction. }
\end{array}
$$

## Chapter 2

| $\lambda$ | Radar wavelength (m). |
| :--- | :--- |
| $\mathrm{d}, \mathrm{R}$ | Distance or range to target (m). |
| $\mathrm{h}_{1}$ | Height of radar transmitter (m). |
| $r_{e}$ | Earth's radius (km). |
| $\mathrm{h}_{2}$ | Height of Target (m). |
| $4 / 3$ Earth's Radius $=8494.7 \mathrm{Km}(4587 \mathrm{nmls})$. |  |

Chapter 3

| ${ }^{\text {f }}$ | Radar Transmitter frequency ( Hz ). |
| :---: | :---: |
| D | Particle diameter (cm). |
| $\rho$ | Density of water. |
| v | Optical visibility (feet). |
| M | Liquid water content of cloud or fog (g.m ${ }^{-3}$ ) |
| $\alpha$ | Atmospheric Attenuation Coefficient. |
| R | Range (target, obstacle or rain) (m). |
| m | Complex refractive index (water). |
| D | Diameter of rain drop. |
| $\zeta$ | Radar reflectivity of rain ( $\mathrm{m}^{3} . \mathrm{m}^{-3}$ ). |
| Z | Rain reflectivity factor. |
| p | Precipitation rate ( $\mathrm{mm} . \mathrm{hr}^{-1}$ ). |
| $\theta_{\text {A }}$ | Aerial 3dB Beamwidth, Azimuth (deg). |
| $\theta_{E}$ | Aerial 3aB Beamwidth, Elevation (deg). |
| $\emptyset_{A}, \emptyset_{E}$ | Angular departure from beam axis in Azimuth or Elevation (rads). |
| $\sigma_{R}$ | Beam rain filled echoing area ( $\mathrm{m}^{2}$ ). |
| K | Windshear coefficient ( m sec ${ }^{-1} \cdot \mathrm{~m}^{-1}$ ). |
| C | Speed of light (m. $\mathrm{s}^{-1}$ ). |
| S | Spectrum of wind velocity. |
| fa | Doppler frequency (Hz). |
| fw | Doppler freq at mean wind velocity ( Hz ). |
| sd | Standard deviation. |
| n | Rainfall frequency per 10 years. |
| $t$ | Rainfall duration (hours). |
| $\wedge$ | Total rainfall in time $t$ (inches). |

Chapter 4
$\sigma_{m}$
$\mu$
$A(t)$
$R(t)$
$\phi(t)$
$\beta$
$\tau$
$a_{k}$

Median RCS $m^{2} . m^{-1}$ (or average where shown). Spatial mean value of $m$.
)
) Amplitude, Range and Phase components
$\beta$
$2 \pi / \lambda$
$a_{k}$
Radar pulse duration.
Lumped amplitude term


## Chapter 5

| $V$ | Wind Velocity (knots). |
| :--- | :--- |
| $V_{0}$ | Wind Velocity (ms |
| $\beta$ | Azimuth angle relative to beamwidth. |
| $\theta_{2}$ | 2-way half power beamwidth (rads). |
| $\sigma_{S}$ | Chaff RCS per dipole. |
| $\Sigma_{0}$ | Chaff volume reflectivity density $\left(\mathrm{m}^{2} \cdot \mathrm{~m}^{-3}\right)$. |
| N | No of chaff dipoles. |
| E | Chaff Dispersal efficiency. |
| $\mathrm{p}(\mathrm{f})$ | Clutter power spectrum as a function of frequency. |

## Chapter 6

Standard symbols are listed with equations (1-13).

| $\sigma_{A V}$ | Average target RCS $\left(m^{2}\right)$. |
| :--- | :--- |
| $\sigma$ | Quoted target RCS $\left(m^{2}\right)$. |
| $L_{o} / \lambda$ | Characteristic target length. |
| $\Delta \theta / \Delta t$ | Rate of change of target aspect. |

## Chapter 7

| $E_{S}, E_{I}, E_{O}$ | Scattered and incident intensity from target. |
| :--- | :--- |
| $\alpha$ | Diffraction angle. |
| $v$ | Dimensionless Fresnel-Kirchoff parameter. |
| $A(v)$ | Diffraction loss rate. |
| $r(m)$ | Fresnel zone radius. |
| $a_{m}$ | Diffraction loss. |
| $a_{0}$ | Free space loss. |


| $R$ | Range beyond diffraction ridge. |
| :--- | :--- |
| $R_{c}$ | Radius of curvature of diffracting edge. |
| $R^{I}$ | First Fresnel zone radius. |
| $P_{t g t}$ | Power at target. |
| $\alpha$ | Curvature factor. |

## Chapter 8

```
a., a Earth's radius 6370 Km, effective earth's radius,
N Refractivity.
N
(Remaining symbols dimensioned on diagrams).
```


## Chapter 9



## Chapter 10

$h_{t} \quad$ Average height terrain clutter patch (m).
$h_{t_{x}}$ Height of radar aerial.
(Remaining symbols dimensioned on diagrams).

## Chapter 11

| $P_{\text {DET }}$ | Probability: |
| :--- | :--- |
| $P_{\text {TL }}$ | Detection. |
| $P_{M X}$ | of obtaining minumum track length. |
| $P_{O E}$ | $:$ of missile success. |
| $P_{R}$ | $:$ of operator (efficiency) performance. |
| $P_{R}$ | $:$ of system availability (Readiness). |


| $m$ | Weibull shape parameter (replaced by b). |
| :--- | :--- |
| $\lambda$ | Weibull scale parameter (replaced by c). |
| x | Signal level. |
| $(\mathrm{n})$ | Gamma function. |
| $\mathbb{N}$ | Echo amplitude level. |
| P | Echo amplitude level. |
| m | Ratio of constant power to random power. |
| S | Standard deviation. |
| $\mu$ | Median value of x. |
| X | Normally distributed variable. |
| Y | Lognormally distributed variable. |
| Y | Median value of Y. |

Annex B
Received echo from target (watts).
Peak transmitted power (watts).
Peak Aerial Gain.
Radar cross section (RCS) of target $\left(m^{2}\right)$.
Range to target (m).
Effective aerial capture area ( $m^{2}$ ).
Radar wavelength (m).
Combined system losses.
Radar pulsewidth (sec).
Aerial azimuth 3 aB beamwidth (deg).
Grazing angle characteristic.
Scattering RCS pwer unit area ( $\mathrm{m}^{2}$ )
Grazing angle (defined at Chap 1) (deg).
Aerial elevation 3 AB beamwidth (deg).
Height difference due to surface
Specular reflection.
Smith Factor
Annex C

| $E$ | $=$ Electronic field at receiver (target) from unit source. |
| :--- | :--- |
| $S_{1} C$ | $=$ Fresnel integrals of argument. |
| $\theta$ | $=$ Diffraction angle. |
| $v$ |  |
|  | $=\theta \sqrt{d o / \lambda}$ |
| $k$ |  |
| $d_{0}$ | $=2 \pi / \lambda$ |
|  |  |
|  |  |

## Annex E

Linear tracking error ( m ).
Power ratio of direct and indirect signal.
Tracking error for target.
Displacement of peak of aerial beam relative to equal signel line.
Angular distance between real and image target. Required signal.
Interfering signal (multipath).

| $G_{S}$ | Specular power gain ratio. |
| :--- | :--- |
| (Remaining symbols specified in Annex text or in diagrams). |  |
| poi | Fresnel reflection coefficient for element i |
| pos | Fresnel reflection coefficient for specular reflection. |
| T | Tracking condition. |
| $\bar{T}$ | Non tracking condition. |
| q | I- probability of success. |
| $\mathrm{V}_{\mathrm{t}}$ | Threshold voltage. |
| N |  |
|  | Number of signals integrated. |

## Annex $F$ and Appendix 1 to Annex $F$

| $A_{1}$ to $A_{9}$ | Terrain spot heights in $3 \times 3$ grouping. |
| :--- | :--- |
| a to f | Terrain slope, aspect and convexity coefficients. |
| $\hat{r}, \hat{\mathrm{n}}, \lambda, \mu, \gamma$, ) | Direction cosine designation. |
| $\mathrm{l}, \mathrm{m}, \mathrm{n}$ n <br> (All other symbols listed in text or on diagrams).  <br> $\psi$ Actual terrain/radar energy grazing angle. <br> $\theta$ Mean terrain gradient. <br> s Observable (in shadowed) slope. |  |

Annex G

| $V_{m}$ | Velocity of missile (m. $s^{-1}$ ). |
| :--- | :--- |
| $P_{E}$ | Probability of detection becoming an engagement. |
| $P_{f a}$ | Probability of false alarm. |
| $P_{b}$ | Probability of detection during a single <br> $D_{f}$ |
| $P_{s}$ | Diffraction burst of pulses. |
| $\mathrm{E}_{\mathrm{f}}$ | Probability of sightine falling on target. |

## CHAPTER 1

## THE NATURE OF RADAR CLUITTER

## INTRODUCTION

1. This report considers radar performance prediction when operating at low grazing angles with the horizon - such that radar beam illumination of the ground ineyitably occurs, resulting in unwanted clutter echoes. These clutter signals diminish the probability, or even totally prevent the radar from detecting the wanted signal from aircraft and missiles.
2. Minimisation of interference effects, based on a knowledge of the expected clutter, is possible to a certain degree at the radar design stage. However, it is also necessary to be able to assess the probability of detecting and tracking a target of given radar cross section (RCS) for an existing radar when the target is at very low altitude over variable terrain, or water. Probability of overall success clearly depends upon the likelihood of encountering competing clutter, the time for which such clutter persists before the target moves to a more advantageous position (where the target signal overrides the clutter signal) and the reaction time of the associated command and control or missile system which is to make use of the target tracking data. Hence a statistical analysis is necessary which takes into account the very large numbers of variables involved. (Annex A).
3. To make a complete assessment for a particular radar type and location it is first necessary to analyse the terrain profile to obtain sightline data to the target. Secondly to investigate the corresponding surface characteristics beneath the target, and finally to assess degradation of signals due to volume clutter including cloud, rain, snow etc and the
effects of deliberate clutter such as chaff (electronic countermeasures). Several techniques are used to reduce clutter effects but even these may have only minimal effect in the scenario in question, A general selection of parameters to minimise clutter are set out at Table 1 below:

|  | EFFECTIVE AGAINST |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PARAMETERS | GROUND CLUTITER | WEATHER CLUTHER | CHAFF (ECM) | SEA CLUTTTER | ANGELS |
| LOW RF |  | X |  |  |  |
| NARROW AE BW | X | X |  | X | X |
| SHORT PUUSE <br> (RESOLUTION CEIL) | X | X |  | X | X |
| STC |  |  |  | X | X |
| MII | X | (x) | X | (x) |  |
| LOG RX/FTC | X | X |  | X |  |
| CIRCULAR POLARISATION |  | X | (x) | X |  |
| FREQ DIVERSITY/ AGILITY |  | X | ( | X |  |

Table 1-Clutter Reduction Techniques
Notes: 1. X Effective in limiting clutter
2. (X) Effective in some cases

FORMS OF CLUTMER
4. In differentiating between surface-distributed and volume-distributed clutter the situation can be initially described by geometry, detailed at Annex B. In particular it is seen that the illuminated surface area
and volume vary with range and pulse duration, as well as with the radar aerial depression angle. The existence of clutter returns overland from long ranges is significantly dependent upon the height of the scanned terrain, since hills at short range will often shadow any possible signal returns (see Fig lb) from targets further away. However, this shadowing effect may be limited in azimuth and will therefore depend critically upon line of sight terrain screening as the radar aerial is incrementally scanned in azimuth. On the contrary, measurements taken at sea will be more or less uniformly distributed over the surface. Here the surface clutter echo strength will be directly related to the area of the resolution cell, in contrast to ground clutter which varies from place to place within the cell, and is not therefore proportional to the resolution cell size (Warden $\{1\}$ and Riley $\{2\}$ ). It is however convenient to use the echoing area per resolution cell $\left(\sigma_{0}\right)$ as a standard; explained at $p$ 4-82. This allows direct comparison with the target echoing area in studies of the probability of detection. At sea, multipath signal phenomena (see Fig la) are quite probable, whereas this effect is possible overland, but far less likely.

## RESEARCH PARAMETERS

5. Detection predictions are required for radars having the typical parameters listed below. Monostatic radars are the main interest, although some bistatic work has been done (mainly in the USA) and this may be referred to, where applicabla. The following main characteristics are adhered to throughout the study:
a. Radar Wavelength $<3 \mathrm{~cm}(10 \mathrm{GHz}$ up to 18 GHz$)$
b. Pencil or Fan aerial beams, with sharp beamwidths and mounted on masts up to 30 m high.
c. Small radar resolution cells (15ns $<\tau<2 \mu s$ ).
d. Small target radar cross section ( $0.05 \mathrm{~m}^{2}$ minimum).
e. Tracking type radars, as distinct from surveillance radars.

## CLUTYIER VARIABLES

6. Variables contributing to the complex overall extent of clutter in any particular radar system include:

## a. Topographical Features

1. Terrain Type - Snow, Desert, Forest, Urban, Water etc.
2. Seasonal Variations - Defoliation, surface water content, surface motion.
3. Terrain Profile - Hills, undulating, flat etc.

## b. Radar Characteristics

1: Resolution cell size - dependent on puise duration (PD) and radiated beamwidth (BW).
2. Radio Frequency (RF), polarisation of transmit and receive aerials and distribution of radiation.
3. Aerial induced fluctuations.
4. Grazing angle of radar beam with surface, also known

```
        as 'depression' or 'incidence' angle. (But see later definitions).
```

c. Atmospheric and Propagation Effects
> 1. Diffraction, Reflection and Refraction, including multipath.
2. Air temperature, water vapour absorption.
3. Rain/precipitation, chaff attenuation and backscatter.

## CLUTMER STATISTICAL DISTRIBUTIONS

7. Weibull, Ricean, Rayleigh and Gaussian statistical distributions are used in clutter research and although stated in the main text, since some are uncomon, they are detailed at Annex A. Spatial and temporal characteristics of the variables at para 6 above are summarized at table 2 and each is investigated fully in the following chapters. (except sea clutter).

RELIABILITY AND REPEATABILITY OF CLUTYTER MEASUREMENTS AND ASSESSMENTS
8. Despite the considerable number of clutter research programmes over many years it is unfortunate that few have used identical test parameters, particularly in relation to those areas critical to this study. Probabilities obtained even day to day have varied significantly and a large number of research workers have reported that the available database may be inadequate at present for conclusive and repeatable relationships to be stated. While there are considerable shortcomings in most clutter models, worse still there appears to be no standardised approach discernable in the many papers read during this study.

TABLE 2 CLUTTER MODEILING PARAMETERS

| CLUTIER TYPE | INPUT PARAMETERS | MODELLING DESCRIPTION |
| :---: | :---: | :---: |
| RAIN BACKSCAITER | Rainfall Rate, Type, Rain Frequency, Radar Frequency (RF). | Spatial distribution of Reflectivity Spectrum and Probability of occurence. |
| CHAFF BACKSCATTER | Amount and Type, Dissemination Mode, RF. | Spatial and Temporal distribution of Reflectivity, Spectrum. |
| SEA BACKSCATTER | Sea State, Incidence Angle, RF and radar polarisation. | Reflectivity (average) and Clutter Signal distribution. |
| LAND BACKSCATTER | Land type, Incidence Angle, RF, Pulse length. | Distribution of reflectivity Spectrum of motion. |
| REFLECIIONS/MULIIPATH | Geometry, RF, Surface Roughness, Polarisation | Coherent propagation loss. Diffuse scattering intensity. |
| AIMOSPHERIC/RAIN AITENUATION | RF, Range, Rain Rate, Type and Frequency, Atmospheric Characteristics. | Propagation loss. Attenuation rates. |
| REFRACIION AND DIFFRACTION | Geometry, RF. | Propagation losses, path patterns. |
| JAMMING | Type. | Modulation, Signal ratios and thresholds. |

9. Recently, Allan \{ 3 \} in particular, surveyed a number of clutter prediction papers and found both anomalies and inadequacy of data; concluding that "a completely general analytical method is probably an impossibility". The added complications of terrain screening, chaff or atmospheric propogation effects were outside the scope of Allan's paper, as with many other research studies. On consideration it was felt necessary to include all these factors in a more comprehensive study. Meanwhile, it was discovered (by chance) that an ongoing study at MIT (USA) has similar terms of research, hence the author and Dr Briggs $\{4\}$ have been able to exchange information on clutter research reports. Despite the wide resources of MIT in producing a world-wide clutter bibliography of some 300 items, the author's investigations at Cranwell have resulted in the addition of another 70 to 80 reports to the MIT list. (at PARTII)

## REYIEW OF CLUTYER RESEARCH

10. In preparation for the construction of a model, a comprehensive reyiew of radar clutter and associated literature was a time consuming initial requirement. However, with respect to the radar parameters required for this study, the search revealed that many reports used parameters very widely dispersed from those of interest. Nevertheless several hundred papers were filtered for information.
11. Extensive descriptions of clutter are given in several standard texts $\{5,6,7\}$. However, they are usually intended for the radar student requiring a general grasp of the problem and they invariably avoid the difficulty of assessing practical system performance. That there is a serious lack of data and that researchers, for their own specific purposes, have embarked on measurement programmes with a range of
different averaging criteria - even though ostensibly repeating research done by others - all serves to cast doubt on the integrity of conclusions drawn from comparison studies. Hence, published sets of results for (apparently) the same test conditions may not correlate and it is clear that the phenomenon of surface clutter in radar receivers remains poorly understood and poorly predictable.
12. During the study, discussions and correspondence with MIT (USA), RSRE (Malvern) and UK Industry has proved rewarding. It is clear that the respective Departments of Defence have a particular interest at present in this topic, but both now realise the enormity of the task if all unknowns in the clutter parameter matrix are to be found. The use of the best published research is therefore essential, since no organisation could afford to undertake the vast range of measurements necessary to build a complete picture from the beginning. On the other hand, confidence in a model will only be acheived if a spread of repeatable results is identified, and with the shortage of data, certain assumptions and judgements based upon sound reasoning must be used if any sort of useable algorithm is to be attempted. It is found that research papers fall, in general, into 3 main types:
a. Short radar range, scientifically orientated reflectivity experiments, specifically radiated narrow-beam energy against small but highly homogeneous clutter patches eg snow, crops, concrete or regular vegetations. Usually the radiating and receiving aerials are stationary.
b. Intermediate range measurements using large clutter patches, trees, fields, etc, again using narrow beamwidths, and often exploring stationary single resolution cells.
c. Surveillance radar measurements, usually involving $360^{\circ}$ azimuth coverage with clutter sources varying from flat to rough terrain and containing both diffuse scatterers such as crops, weeds etc and point scatterers of the isolated type such as pylons, and water towers.
13. Since a complete clutter detection model should embrace all possible variables which degrade the radar performance it is appropriate at this point to differentiate between the amplitude, phase and other fluctuations leading to the clutter statistics received as signals from outside the receiver, and including all the effects mentioned so far including RF spectrum and frequency agility effects; contrasted with the characteristics of the clutter signal processing circuits which are inside the radar. Published reports often attempt modelling while accounting for several variables but ignore others pertinent to the circumstances of a practical scenario. For example a seaborne radar model would not normally need to account for sightline screening since, unlike on land, the only screening at sea is the longer range horizon limiting. On land there is the infinite variability of the terrain to contend with and so the modelling task becomes daunting; and even worse if the model is to cover both sea and land mixed. Investigations have shown that the simple categorisation of landscape into broad types is not suitable, and it has been well established that detailed categorisation is necessary. Indeed it is possible that reliable radar prediction under all conditions may be denied until clutter descriptions become more elaborate.
14. The Canadian Soil Survey Committee adopted a hierarchical classification scheme in 1976 \{ 8 \} which allocates 10 first level classes (eg undulating, rolling, level etc); second level modifiers (eg eroded) and other levels specifying coverage slope, local relief and so on. Any type of terrain can be described by the system. The USA also have a land use and cover
classification system. However a truly comprehensive scheme would necessitate a means of describing, for example, the natural, cultural and man-made vertical obstructions which tend to cluster along roads and field edges; and seasonal effects. United Kindom and European terrain data bases are also considered in the investigations at Chapter 4.

## PROBLEM AREAS

15. The foregoing paragraphs generally highlight the shortcomings in the raw material to insert into a clutter model. Coupling this with the lack of a widely applicable analysis method, one approach would be to use homogeneous clutter, using perhaps 8 or 10 surface classifications and to assume the entire resolution cell "footprint" contains one type of scatterer. An extreme alternative would involve detailed mathematical representation of the reflectivity of every scatterer within the cell, taking for example, in the limit, grass-blades to be dipoles with associated phase and amplitude behaviour. After investigations a method is developed (and justified in later chapters) as the most reasonable practical approach, given the limitations described above.
16. Chaff and Electronic Jamming. One aspect, so far not expanded upon, is the use of deliberate radar clutter to reduce the probability of detection, ie the dispersion of "chaff" within the resolution cell by military targets or the radiation of interfering or misleading (deception) signals. In both cases a serious degradation of radar performance may result in tracking disturbance or break-lock on a target of interest.
17. Electronic jamming may not emanate from the target being tracked but from another source which is giving countermeasures support, ie a 'stand-off' jamming aircraft. These emissions may enter the tracking radar through the main beam or sidelobes. Chaff interference is also considered and it is of interest that statistically chaff and rain backscatter characteristics are similar.
18. Diffraction and Terrain Slope. Surface obstacle diffraction and clutter from sloped terrain are of particular importance to the prediction of performance of low grazing angle tracking radars. Since few practical measurements have been published on these topics, they seemed to be worthwhile areas for detailed study.

## MINIMUM AIMS OF THE STUDY

19. As a minimum the author sets out, it is believed uniquely, to summarize in one document:
a. A reasonably detailed method of assessing the performance to be expected from a tracking radar and associated missile system when deployed on a pre-surveyed site, by computer modelling.
b. A simpler method of performance prediction for a system deployed anywhere within a geographical area where terrain data may be available only.in general form and where radar performance data is perhaps limited.

In both cases either the specific radar and relevant system parameters are known or can be varied to observe the effects, for example, with and without interference.

## BIBLIOGRAPHY

20. It should be stated that the bibliography, because of the nature of the study, is unusually extensive for a report of this type. Items listed are not all cross-referenced in the main text but it is considered necessary to include the entire reading list for completeness as a new consolidated clutter reference. Readers will find a proportion of the bibliography repeated in the MIT list $\{4\}$ as mentioned at para 9 above.

## DEFINITION

21. For the purpose of this study 'clutter' is generally taken to mean all effects which impede or degrade detection and hence tracking. It maybe caused by the type or condition of the surface, hindrance or disturbance of propagation due to volume clutter, atmospheric effects, or by jamming or target manoeuvre.

## ORIGINAL RESEARCH

22. It has already been mentioned that a single prediction document is probably unique. To achieve the aims at para 19 above, each of the contributory factors to tracking radar performance (relevant to the radar characteristics described at para 5) have been examined in detail:
a. To select the best method of representing terrain data for prediction purposes.
b. To select the best model to describe clutter in all it's forms with particular emphasis on those aspects where measurements or results are scarce or of doubtful value.

As a result, indeed as expected, surface clutter modelling proved to be the weakest link in the overall prediction model. To overcome this shortcoming, first an extensive survey of existing measurements was made to bring all known models together for comparison. But, most importantly, a new model is developed from raw radar measurements and critically compared with the existing real or interpolated models.

## PRESENTATION

23. Perhaps unusually, so that the report should meet the requirements of the sponsors as an easily readable reference for both the scientific and nonscientific reader, results, reasoning and models are stated in the appropriate chapter with a sumary at each chapter end. Expanded detail is in the Annexures; hence the original research on the new clutter model over sloped terrain is detailed at Annex $F$, with the resulting model contrasted with others in the clutter chapter (Chap 4). All aspects are brought together at Chapter 11, for the overall performance prediction model, with detailed examples at Annexures $G$ and $H$.


FIG ia MULTIPATH


FIG ib SHADOWING

## CHAPTER 2

## TERRAIN MODELIING

## INIRRODUCTION

1. Chapter $I$ considered the various sources of radar clutter in general, however, a more precise examination of the system geometry pertaining to ground clutter is necessary for the planning of a meaningful terrain model. With the exception of those occasions where obstacle diffraction occurs (thus possibly allowing radar tracking when a direct optical sight line does not exist), a sight line from the radar aerial to the target is normally essential. Radar aerial and target heights and positions are used in a simple geometrical calculation in conjunction with terrain, building and obstacle data to check for sight-line blockage.
2. It is also necessary to consider Earth's curvature, refraction and reflection effects, hence the terrain model is built up in several stages. Diffraction effects are complex and they are considered separately at Chapter 7

EARTH'S SURFACE MODEL
3. The purpose of the surface model is to determine:
a. Unobstructed Surface Sight Lines. Using contour heights from Ordinance Surrey maps, it is possible to test for the existence, or otherwise, of a tracking sight line between any 2 points at any altitude on the terrain. Accurate terrain data is thus a basic requirement for the geographical area in which the radar is to operate. Manual production of this type of data base is tedious, but several agencies have produced terrain data which is suitable for clutter studies. Terrain data base methods are outlined in paras 16-19 below.
b. Surface Obstructions. Super-position of a surface culture and obstacle array upon the terrain array and which describes the mean height of all surface obstacles and types of reflecting surface, enables blocked sight lines to be identified and reflectivity to be accurately modelled.
c. Tracking Times. Low level target tracking may be periodically interrupted by terrain or obstacle screening. The time elapsing from a sight line being first established until the sight line ceases to exist (as the target again becomes obscured:), may be critical in the case of missile fire control systems or aircraft on a landing approach. Tracking times are, of course, a function of aircraft velocity as well as obstructions. (Also see Annex E).
d. Clutter Levels. Undulations in terrain may present a situation where a target can be seen but the underlying ground or obstacles are in shadows and hence clutter returns in the main beam are not possible, (examples of this are noted later). Clutter may be received by side lobes (if the side lobe suppression is poor) from terrain not in the resolution cell currently being searched.

## SIGHT-ITNE

4. Given the radar aerial height and site position within the terrain array, together with the target height, track, and the position as it enters the array, the sight lines are calculated and then clutter criteria is applied. Radar cross section modelling of the target itself is of course necessary for signal comparison purposes and the signal from the target should itself fluctuate realistically to allow for glint (scintillation). If weather degradation is to be included, the attenuation due to precipitation can also be applied. The typical effects of weather on radar are considered at Chapter 3. Other assumptions made for the model are:
a. Initially the targets will fly straight lines and constant altitude and would not be planned; for example, to take deliberate advantage of terrain screening.
b. Ground tracks can transit across the terrain in any direction, including passing overhead the radar.
c. No wind drift is incorporated and hence the attitude aspect of the target with respect to the ground radar station would be predictable within the limits of the scintillation required.
d. Radial ground tracks, with a subsequent period when targets are within the radar's minimum tracking range are simulated.
e. When RCS is modelled on a crossing target maximum signal value is assigned when the target is at a tangent to the radar normal.
f. When RCS is modelled on an approaching target RCS is modelled to fluctuate statistically about a mean.

## TERRAIN SPOT HEIGHTS

5. Terrain matrix array spacing of 500 metres was used initially for the Malvern area, by interpolating, as necessary, between contours. 250 metre and 125 metre matrix spacing is also available for areas of UK and Germany. A maximum useful radar tracking range of 25 km is assumed against a low level target flying at altitudes between 30 metres and 150 metres (clearance above ground level (agl)). A 50m matrix spacing was used for slope studies.

## EARTH'8 CURVATURE AND INTER-VISIBILITY

6. Earth's curvature is a basic limitation in considering inter-visibility at significant ranges over the earth's surface. Approximating the Earth's radius as 6400 km , there is a finite distance within which 2 points are inter-visible depending on their height. A general description follows which ignores refraction but gives an approximation of the range of values used in the model. Refraction is considered separately at Chapter 8 , where $\mathrm{re}_{\mathrm{e}}$ is replaced by $k$ re, $(k \approx 4 / 3)$; with the resulting extension of the radar horigon.

7. Point $A$ can just see point $B$ using a tangential sight line. Maximum range $R=d_{1}+d_{2}$, where $d_{1}=\sqrt{\left(h_{1}+r_{e}\right)^{2}-r_{e}^{2}}$ and $d_{2}=\sqrt{\left(h_{2}+r_{e}\right)^{2}-r_{e}^{2}}$
If $h$ is negligible in both cases compared with the earth's radius $r$, then $R=\sqrt{2 r_{e} h_{1}}+\sqrt{2 r_{e} h_{2}}$
$=\sqrt{2 \times 6400 \times 10^{3}}\left(\sqrt{h_{1}}+\sqrt{h_{2}}\right)$
$\approx 3.6\left(\sqrt{h_{1}}+\sqrt{h_{2}}\right) \mathrm{km}$
Example unobstructed maximum sight line ranges are given below at Table 1 .
Targets and radars at various altitudes are assumed to be over a "smooth earth" and with no refraction or diffraction,

| Radar <br> Aerial Ht <br> $(\mathrm{m})$ | Target <br> Ht <br> $(\mathrm{m})$ | Maximum <br> Sightructed <br> (km) |
| :---: | :---: | :---: |
| 5 | 30 | 27.76 |
| 5 | 60 | 35.93 |
| 10 | 30 | 31.10 |
| 10 | 60 | 39.26 |
| 30 | 30 | 39.4 |
| 60 | 60 | 47.6 |

TABLE 1
MAXTMUM UNOBSTRUCTED SIGHT LINE RANGES FOR GIVEN RADAR AND TARGET HEIGHTS
corrections, using the simplified method above:
8. Table 2 gives approximated grazing angles to the surface (assuming the target to be at ground level) for various aerial heights at fixed ranges. (See figure 1). Bracketed figures are for the $4 / 3$ earth correction, which is necessary for the more exact results required in subsequent chapters.

| Aerial <br> Ht (m) | Grazing Angle for Target <br> Horizon Range <br> (Degrees) |  |  |
| :---: | :---: | :--- | :--- |
|  | 5 km | 10 km | 15 km |
| 5 | $0.05(0.04)$ | $0.03(0.02)$ | $0.01(-)$ |
| 15 | $0.17(0.15)$ | $0.08(0.05)$ | $0.05(-)$ |
| 20 | $0.22(0.21)$ | $0.11(0.08)$ | $0.07(0.02)$ |
| 30 | $0.34(0.32)$ | $0.17(0.14)$ | $0.11(0.06)$ |

TABLE 2
GRAZING ANGLE FOR TARGETS AT FIXED
RANGES FOR GIVEN RADAR AERIAL MAST HEIGHTS

## EFFECT OF AERIAL BEAMWIDTHS

9. Table 3 gives the approximate range to the surface when grazing at low angles for a spread of typical 3aB beamwidths. (See figure 2).

| Vert 3aB <br> BEAMWIDTH <br> (Deg) | TARGET RANGE <br> AERIAL |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 30 | 20 | 15 | 5 |
| 0.2 | 17.18 km | 21.46 km | 8.59 km | 2.86 km |
| 0.4 | 8.59 km | 5.73 km | 4.30 km | 1.43 km |
| 0.6 | 5.73 km | 3.82 km | 2.86 km | 0.95 km |
| 0.8 | 4.28 km | 2.86 km | 2.15 km | 0.71 km |
| 1.0 | 3.43 km | 2.29 km | 1.72 km | 0.57 km |

TABLE 3
RANGE TO SURFACE FOR 3 AB BEAMWIDTHS AND GIVEN AERIAL MAST HEIGHTS
10. If a radar is designed for tracking rather than surveillance, it's vertical beamwidth is narrow, hence targets tracked at the higher angles enable the radar to be well clear of the ground clutter. As target altitudes are reduced eventually the beam will collect ground clutter. For tracking radars a 'rule of thumb' for radars at ground level, is to assume the radar beam is well clear of ground clutter if the difference in elevation between the target and the ground is $>0.7$ times the vertical beamwidth. For example a tracking radar with a $1^{\circ}$ elevation beamwidth and target at 100 metres altitude will be free of clutter if the target is closer than 8 km ; but at 30 metres altitude the target must be at range 2.4 km or closer. The geometry is shown at figure 3.

## RESOLUTION CELLS

11. The simulation model calculates the cell size and associated surface area, "footprint" taking into account the diverging radar beam.

| Pulse Width | Resolution Cell <br> Length (m) |
| :--- | :---: |
| 13 ns | 1.95 |
| 15 ns | 2.25 |
| 20 ns | 3.00 |
| 40 ns | 6.00 |
| $200 \mathrm{~ns}(0.2 \mu \mathrm{~s})$ | 30.00 |

## TABLE 4 <br> RESOLUTION CELI LENGTH FOR GIVEN PULSE WIDTHS

CLUTTER REFLECTORS WITHIN RESOLUTTON CELLS
12. Within each resolution cell the reflecting surface type can be determined from the obstacle matrix (see para $3 b$ above). This is achieved by adding a terrain identifying factor into the terrain data and extracting this value each time to assess the surface clutter reflectivity likely to occur. A scale of identifiers is used as follows; with typical median tackscatter values (for $d=3 \mathrm{~cm}$ ):

1 Swamp/Marsh. (-4odB)
2 Discrete (prominent isolated reflectors)
3 Water (Landlocked) ( - GodB)
4 Grassland/Cropped fields ( -35 dB )
5 Buildings (continuous) ( $-23 d B$ )
6 Buildings (scattered) ( $-30 d B$ )
7 Forest/Trees (-3ノdB)
A roughness factor and hence reflectivity (backscatter) value is then allocated, so that clutter versus target signal levels can be studied. (Using more precise models at (hap 4 paras 22-34).
13. Homogeneity of Terrain Within a Cell. It is seen that the spatial distribution of scatterers within a resolution cell will significantly alter the received backscatter. This problem is necessarily addressed again at Chapter 4, as it is of prime importance in the determination of a statistical clutter distribution. From the purely geometrical viewpoint the area of the surface 'footprint' can be calculated from the resolution cell length and diverging aerial beamwidth at the given range. It is however clear that really accurate predictions from modelling will only be possible with an enhanced terrain descriptive system in contrast to the very basic framework shown at para 12. Terrain spot heights will also be required on a finer matrix spacing of 90,75 or even 30 metres (US National Cartographic Centre Grid).

SLOPE(TILT) OF SURFACE WITHIN RESOLUTION CELL
14. Backscatter is not only dependent upon the resolution cell area, but also to some extent on the aspect or 'tilt' of the cell area or 'facet' to the incident wavefront. The terrain model must calculate the cell area and cell slope to provide the clutter subroutine with information to enable adjustment of the signal/clutter ratio. It is of interest that the clutter and target aspects presented by a particular resolution cell to a radar receiver at a monostatic site are quite different from that in a bistatic system, where the aspect illuminated by the transmitter is not the same as that presented to the receiver; however this is outside the scope of the study.
15. Francois \{9\} states that the average slope of terrain has only a second order effect on clutter patch locations and terrain masking. Adgie $\{10\}$ in conversation, states that slope probably has a limited effect on
backscatter. It is clear that little, if any, serious work has been done on the dependence of backscatter on slope for any of the terrain types of interest. As this seemed a fruitful area in which to make some basic research, raw measurements taken at RSRE and BAe have been obtained and correlation runs computed for terrain slope using a prepared terrain data base. The method and results are detailed at Chapter 10, \&Annex F. App I.

## TERRAIN DATA BASE

16. Paragraph 3 introduced the concept of a terrain data matrix, and the necessity for an adequate descriptive system for the total surface features was further outlined at Chapter 1. It remains to consolidate the options available in the representation . of surface obstacles and to state the reasons for selecting the method used here.
17. In the past many studies used an approach which categorised large areas of terrain and obstacles according to average type - and used statistical descriptions for terrain having like backscaiter characteristics. Degradation was then determined over given flight profiles. This approach is not considered adequate (but see Chapter 11 for approximate predictions).
18. Another alternative for representing obstacles, but also rejected, is briefly described here to show its disadvantages. A string of profile co-ordinates is produced to describe the edge profiles of each obstacle. Each obstacle perimeter is therefore represented by a group of points produced by approximating all obstacles into a series of straight lines. Obscuration is then determined by geometric considerations as to whether the intended sightline and obstacle edge lines intersect. An underlying basic culture data base using a matrix or lattice method would still be
necessary, but the storage requirement for obstacles would naturally be a variable quantity for each geographical area, depending on it's constituent surface features. Generation of this type of data is especially tedious since every obstacle must be catergorised by its extremities and assigned an obstacle height and type to be added to the underlying terrain spot heights. Data production is tedious because it is a difficult process to automate.
19. An added complication which may arise with large obstacles when using this method is shown at figure 4. Sightlines are confirmed by checking for obscuration at the obstacle boundaries $\mathrm{Bl}, \mathrm{B} 2$, and special arrangement's would be required to ensure the situation at figure 4 did not exist. Theresby \{11 \} states that the "co-ordinate string" method in fact has the potential. for a more accurate obstacle representation and indeed Hunting Engineering Ltd have used this approach for models of limited geographical extent. The penalties, apart from the relatively larger volume of data preparation, impinge upon timing overheads, extra software, retrieval, and storage requirements. Theresby \{11\}estimates a total storage requirement for a 20 km by 20 km area to be 4 times as great as matrix methods of obstacle representation.

## SELECHED METHOD

20. Figure 5 shows an example terrain matrix at $500 \times 500$ metre spacing. Terrain spot heights are known at each intersecting point of the matrix. The figures bracketed represent the various terrain factors (see para 12). Reference to figure 6 shows the same area as fig 5 , but with forest areas (F) and built-up areas (B) shown outlined. In allocating terrain factors two anomalies will be seen when the figures are compared. One of the spot values (70) (at the South East Corner) is assigned terrain factor 6, although
it appears outside the $B$ area boundary. This illustrates the difficulty of sharply delineating towns or villages when discrete obstructions tend to cluster, for example, along roads in the suburbs.
21. A second anomaly is in the large forest area to the west where 95 (7) would appear to be the correct value but 95 (4) is used. This situation can arise when a significant open area (clearing), often several hundred square metres in area is surrounded by trees. As a result it is seen that since this particular forest does not embrace any other intersections (with the matrix spacing used $X=Y=500$ metres) the forest area would not be represented correctly in the model. Interpolations made for the remainder of the area would be inaccurate since tree height would not be incorporated. Surface objects such as the more specific vertical reflectors eg towers, culture and buildings can therefore only be represented if the matrix is fine. Much data has been produced in the past using old maps in which the contour accuracy may be in doubt, and there is an urgent need to digitise data directly from stereo photos.

## INTERVISIBILITY AND SCREENING DIAGRAMS

22. Mobile radar systems unfortunately suffer from target and terrain masking which is site-specific, and although a geometric model, given sufficient data, can predict the positions of probable clutter patches, clutter strength from within a patch is far more difficult to predict (see Chap 4).
23. If the target is assumed to be at zero altitude then clutter patch masking predictions are the same as predictions of target masking at zero altitude (see para 30).
24. Whether the clutter actually prevents detection or causes break-track depends on the clutter strengths, clutter rejection capability of the system and relative radar cross section of the target. Investigations by Briggs and Billingsley $\{4\}$ have revealed in the past that insufficient data is available to support an accurate low grazing angle model.
25. Clutter Predictions. Francois \{9 \} has researched the sensitivity of clutter prediction using the geometry of aerial and target height. In particular it was found that on examination of some 20 sites, coverage in geometrical prediction was "rarely in good quantatitive agreement with the spherical earth". Further, the radar site must be in good fit; or very near to the best fit plane with the terrain data. Plots showing the sensitivity of coverage to the aerial and target heights typically take the form shown at figure 7. These are clearly site-specific, however it is possible to predict (for the type of terrain prevailing in a general geographic area), a probability that unmask will occur out to a given range for a given target height and radar height. In these assessments an expected percentage of the $360^{\circ}$ scan will be denied due to terrain screening.
26. Once a clutter prediction has been made it is further modified in practice by smearing due to the convolution of the clutter within the appropriate resolution cell (see also dependence of clutter on aerial motion - Chapter 5).
27. Actual and geometrically - predicted clutter maps have, on occasion, proved successful and useful, but uncertainties in terrain spot height, data base, culture variations and propagation effects unfortunately tend to degrade results.

## INTERVISIBILITY PLOTS

28. Figures 8 and 9 show typical intervisibility plots (using program SLINE.FOR) for two aerial heights. Targets were at 70 metres altitude in both cases. The percentage masking at a given range is plotted at Figure 10. Terrain spot height only was used to produce these results and a far more serious effect follows when the terrain surface culture and obstructions are included. (See also Annex E).
29. A first approach to the production of realistic screening diagrams ignores the effects of microwave radar energy partially penetrating vegetation, diffraction effects, or multipath which causes angular errors. It is assumed that target range-gating will always be used by modern tracking radars, hence only clutter from a range close to that of the target has to be considered.' This implies a clutter problem only when the surface beneath an aircraft is illuminated by the radar. Since a sightline may not exist to this area beneath the target due to terrain or obstacle screening, there will be many occasions when clutter cannot be received.
30. A simple way to check those cells in which clutter is obscured is to place the target at zero altitude and test for the existence of a sightine. A "Clutter Visibility" map can be drawn and combined with the terrain screening map to produce an overall map where the clear areas represent positions where the target can be seen but the ground underneath cannot. On flat ground, near to the radar, the ground is likely to be seen, since the probability of a sightline is high. Hence at close ranges the target will be in a clutter region. Readers of the short paper $\{10\}$ could misinterpret the significance of this situation and it should be noted that paper was produced initially for an optical visibility study. At close target
ranges the radar beam elevation angle is likely to increase, (taking narrow tracking beams clear of the surface clutter), while simultaneously at shorter ranges the returned target signal will be greater due to the shortened two-way transmission path length and will better compete with any remaining noise or clutter.
31. Figures 11 and 12 show typical (max range 30 km ) screening diagrams for fairly flat terrain for aircraft at 100 metres altitude and zero (notionally) altitude respectively. The example diagram at Fig $13\{10\}$ indicates where targets can be seen but the co-located clutter cells cannot. It is seen that many (clear) areas exist, particularly in the NE quadrant, where a high probability of successful and uninterrupted detection and tracking will exist. (Fig $13=$ fig 11 tnegatwe of fig 12 )
32. Figures 14 and 15 compare the probabilities of target visibility (for targets at 100 m altitude) and clutter averaging. Om and 10 m above terrain spot heights. Adgie's paper naturalily assumes the same clutter from all ranges, since from the optical point of view all obstacles are the same. Chapter 4 investigates radar clutter levels in detail. Figure 14 shows the typical trend for fairly flat terrain where the upper curve probability falls almost linearly withlarge, compared with Fig 15, where hilly terrain cause the corresponding curve to fall rapidly as the closest ground cover to the radar on some azimuths causes the inevitable loss of sightline. The measurements at fig 14 correspond with the plan diagram at fig 11. Two lower curves at figs 14 and 15 representing the combined effects (with and without the 10 m tree cover) indicate:
a. As expected the probability of detection is lower in hilly terrain (but not necessarily the probability of obtaining a required crossing track length in the same terrain - See Annex E).
> b. Best visibility is at intermediate range (ie the highest probability of target sightline coincident with the lowest probability of clutter beneath the target).
c. The difference in detection probability made by a 10 m coverage of trees is small. The differences are plotted at figure 16 where it can be seen that they are remarkably constant out to about 26 km on flat terrain and out to $14-16 \mathrm{~km}$ on hilly ground.

It should of course be stressed that results are site dependent, but the trend perhaps indicates that constant height ground cover (ie over large tree covered areas) does not reduce the overall detection probability by as much as was expected, particularly in the intermediate ranges;important for example in surface to air missile tracking scenarios. (See Fig 16).
35. Dependent on the radar type, 2 km may be an impractically short range for tracking purposes, since although the minimum range of the radar may be less than $2 k m:$ high speed targets at very short range present an extremely high sightline rate which may well exceed the lock-follow rate of the associated tracker control loop.

## CHAPTER SUMMARY

36. An earth's surface model has been investigated in order to set up:
a. Sight lines to the target.
b. Corresponding sightline to clutter regions below the target.
matrix method of spot heights and culture identification system selected for this project. The necessity of allowing for ground footprint resolution cell "tilt" (or slope) has been recognised and some ariginal work has been done on this in a later chapter. The problems of intervisibility have been outlined and shown to be site-specific. Clutter prediction research in the USA has been investigated and this confims the necessity for fineness of terrain descriptive data to enable a realistic clutter prediction map to be produced. This approach is of course recognised as a means only of identifying the existence of a probable clutter patch in any particular position, and not of the signal nature of the clutter itself; these aspects are examined in detail in Chapter 4.
37. Example intervisibility plots have been used to highlight the difficulty of predicting masking, even in a most arbitrary manner. Particularly sensitive variables in relation to mobile radars will therefore be:
a. The necessity to operate at any time in terrain which varies from hilly to smooth.
b. Nearby obstacles which are fixed (ie poles, pylons etc)which block sectors, but which may not appear on maps.
c. The inability (or inadvisability) of the radar to move to a better position from the clutter viewpoint under battle conditions.
38. Thus it is seen that a realistic assessment of the probability of a radar obtaining a sightline to a target at a given range and altitude when sited at some arbitary position that has not been painstakingly surveyed
is indeed difficult, and this does not yet include the many other factors covered in later chapters concerning the ability of the radar to detect and track the target successfully when the clutter backscatter is competing with the target echo.
39. Computer Program. Details of a terrain sightline computer program are briefly described at AnnexD.
40. Observed Track-Lengths. A useful aspect of terrain survey data is the ability to predict, for a given radar position, the unmasked sectors (shown at Figure 10), not in unmasked percentages but as probability of observing given track lengths. This is an important concept since tracking of any consequence can only take place if a sightline exists for a minimum period. Taken a stage further by the author, it is shown that this can be extended to a probability plot for total missile firing opportunities; by taking missile and target speeds and system reaction time into account. This is pursued as a separate part of the study, with an attempt to classify typical areas with deployed radars as 'high' or 'low' risk areas to an aircraft transitting through. (See also Annex E).


FIG 1 gRAZING ANGLE FOR TARGET ON SURFACE


FIG 2 RANGE TO SURFACE FOR GIVEN BW \& AE HEIGHT


FIG 3 CLUTTER BEAMWIDTH/ANGLE CRITERIA


FIG 4 OBSTACLE REPRESENTATION


FIG 5 geometry of pencil beam radar backscatter cells over terrain


FIG 6 MATRIX FOR TERRAIN DATA BASE



FIG 7 PERCENTAGE CIRCUMFERENCE UNMASKED



| RANGE | RADAR AERIAL HEIGHT(M) |  |
| :---: | :---: | :---: |
| OUT TO | 15 | 30 |
| 10 Km | $15 \%$ | $0 \%$ |
| 20 Km | $56 \%$ | $16 \%$ |
| 40 Km | $89 \%$ | $58 \%$ |



FIG 10(a) PERCENTAGE AREA MASKING OUT TO SPECIFIC RANGES

| RANGE <br> AT WHICH <br> MASKED | RADAR AERIAL HEIGHT(M) |  |
| :---: | :---: | :---: |
|  | 15 | 30 |
| 10 Km | $42 \%$ | $0 \%$ |
| 20 Km | $96 \%$ | $45 \%$ |
| 40 Km | $100 \%$ | $100 \%$ |



FIG 10(b) PERCENTAGE PERIMETER MASKING AT SPECIFIC RANGES


FIG 11 NORMAL SCREENING DIAGRAM


FIG 12 CLUTTER VISIBILITY DIAGRAM


MAXIMUM RANGE 30 Km
AIRCRAFT ALTITUDE 100M
RADAR ZERO ALTITUDE

FIG 13 COMBINED SCREENING DIAGRAM
(1) $P$ of seeingtarget
(2) $P$ of seeing target but not surface clutter
(3) $P$ of seeing torget but not clulter at 10 m .


FIG 14 average probability of target visibility (fairly flat terrain)


FIG 45 aVERAGE PROBABILITY OF TARGET VISIBILITY (HILIY TERRAIN)
(AFTER ADGIE)

fig ig reduction in sightune probability due to 10 m ground cover

## CHAPTER 3

## VOLUME CLUTTER - RAIN AND ATMOSPHERIC CLUTTER

GENERAL

1. Attempts are made to detect and track low level military aircraft in almost any conditions of weather, in contrast to civil operations, where airports may be below safe civil operating criteria if the visibility is degraded under conditions of intense rainfall, low cloud or fog.
2. The aim of this chapter is to provide realistic modelling values for rain and atmospheric clutter and to assess the atmospheric constraints on target detection for incorporation into the overall clutter model at chapter 11.
3. Effect of Operating Wavelength. For the purpose of the study the radar frequency is fixed at $>10,000 \mathrm{MHz}$; but where a choice is to be made for a radar operating in rain the preference would be for lower frequencies unless of course rain is to be deliberately detected for weather avoidance purposes. That the echoing area of rain increases dramatically with frequency is clearly seen from a simple example, by taking the specific echoing area of rain $\left(m^{2} m^{-3}\right)$ for 1 and 3 GHz band radars respectively (with two identical radars $1^{\circ}$ Beamwidth, $1 \mu \mathrm{sec}$ pulse length and 50 nm range), would give a ratio of $\approx 32$ to $1.1 \mathrm{~m}^{2}$, ie approximately 15 dB extra echoing area in favour of the 10 cm equipment. And for example, a difference of 20 dB (typically -73 dB and -53 dB ). is found respectively for $\lambda=3 \mathrm{~cm}$ and 9.3 cm in heavy rain at precipitation rate $\mathrm{p}=16 \mathrm{~mm} \cdot \mathrm{hr}{ }^{-1}$. Rain attenuation effects are also important and these are considered first, followed by the reflectivity of rain later in the chapter at para 14.
4. Rain. While the little atmospheric attenuation in good weather is due to gases and water vapour, precipitation in the form of rain, ice, hail or snow can significantly increase signal attenuation. Values can be calculated using Vie \{12\} ~ t h e o r y ~ a n d ~ i s ~ g i v e n ~ i n ~ g r a p h i c a l ~ f o r m ~ f o r ~ a ~ range of values in Skolnik P543/544 (note graphical error in Skolnik Fig 12.12).
5. For 3 cm radars calculated values are found to be different than those summarised by Nathanson \{13\} p 197 see figure 1, who presents measurements by a number of researchers, mostly at frequencies above $10 \mathrm{GHz}(3 \mathrm{~cm})$, and arrives at a mean curve which is fitted by the equation:
$\log A=1.85 \log \left(f \times 10^{-9}\right)-3.0$
Where $A=2$ wayatten $f=$ frequency Hz
dB. Km- ${ }^{-1}$ mm $\cdot h^{-1}$
Both sets of values are summarised at Table 1 below (for 10 GHz ). Hayes uses $0.00919 p^{1.16}$ and $1.6 p^{0.64} \mathrm{~dB} . \mathrm{Km}^{-1}$ for 9.4 GHz and 94 GHz respectively $\{54\}$.


## 6. Attenuation Modelling Values. Specific values for attenuation and

 reflectivity for rain and cloud for use in the overall model are considered in the summary to this chapter at para 37.7. Cloud and Fog. Water droplets are small compared with $\lambda$, and summing over 2 m $^{3}$ the Rayleigh approximation is used $\{14\}$.

$$
\begin{align*}
& \text { Attn }\left(\alpha B . \mathrm{km}^{-1}\right)=0.434 \left\lvert\, \frac{\pi^{2}}{\lambda} \quad\left(\Sigma D^{3}\right)\right. \text { Im }(-K) \mid  \tag{2}\\
& D=\text { Particle Diameter }(\mathrm{cm}) \\
& \text { Im }(-K)=\text { Imaginary part of }-K(0.0247 \text { for } \lambda=3.2 \mathrm{~cm}), \text { the } \\
& \quad \text { dielectric dependent factor. } \\
& \text { re-writing }(2) \\
& \text { Attn }\left(\text { (aB. } \mathrm{km}^{-1}\right)=0.434 \frac{6 \pi}{\lambda} \quad \frac{\mathrm{MIm}}{\rho} \quad(-\mathrm{K})  \tag{3}\\
& \because=\text { Liquid water content }\left(\mathrm{g} \cdot \mathrm{~m}^{-3}\right) \\
& \rho=\text { Density of water (taken as unity) } \\
& \lambda=\text { Wavelength }(\mathrm{cm})
\end{align*}
$$

8. Since it has been shown empirically that at $\lambda=3 \mathrm{~cm}$, Im varies as $\lambda^{-1}$, eqn (3) can be approximated within $5 \%$ to be:

$$
\begin{equation*}
\operatorname{Attn}\left(\mathrm{dB} \cdot \mathrm{~km}^{-1}\right)=\frac{0.438 \mathrm{M}}{\lambda^{2}} \tag{4}
\end{equation*}
$$

Together with $\bar{M}=1660 \bar{v}^{-1.43}$, where $\dot{\nabla}=$ optical visibility (feet) and $\bar{M}$ $=$ average moisture content (g.m ${ }^{-3}$ ) the one way attenuation curve at figure 2 is produced at $18^{\circ} \mathrm{C}$. (Two way attenuation variation with $R F$ is shown at Fig 3.)
9. It is seen that one way attenuation values at $\lambda=3 \mathrm{~cm}$ spread from about $0.10 \mathrm{db} \cdot \mathrm{km}^{-1}$ for heavy fog to $0.001 \mathrm{~dB} . \mathrm{km}^{-1}$ for light fog. These values decrease by more than a factor of 3 as the temperature varies over the range $0^{\circ} \mathrm{C}$ to $40^{\circ} \mathrm{C}$.
10. Figure 3a. presents two atmospheric attenuations curves interpolated for the radar parameters for this study, from which it is seen that within the low ranges and low grazing angles limits, there is an almost linear relationship. These were calculated from the US Central Radio Propogation Laboratory exponential reference atmosphere for refraction and the International Civil Aviation Organisation (ICAO) standard atmosphere for pressure - temperature values. The atmosphere is assumed to be regular and the one way attenuation factor $(F)$ is given by:

$$
\begin{align*}
& F=e^{-\alpha R} \text { or } e^{-2 \alpha R} \text { (for two-way) }  \tag{5}\\
& \alpha=\text { attenuation coefficient } \\
& R=\text { target range } \\
& \alpha=\frac{\text { one-way attenuation loss }}{\text { range }} \tag{6}
\end{align*}
$$

11. Attenuation coefficients for a $10 \mathrm{GHz}(3 \mathrm{~cm})$ radar with $0^{\circ}$ and $0.5^{\circ}$ elevation angles have been calculated at intervals from figure 3, and graphed at figure $4 a$ (1). It is seen that the curve of the attenuation coefficients is not linear with range. Using a constant value for the attenuation coefficient introduces an error that can be significant for high frequency radars at low grazing angles. Table 2 gives the one way attenuation losses for 10 GHz at $0.5^{\circ}$ grazing angle. Figg 3 b (from an alternative source) confirms the $10 \mathrm{GHz} 0^{\circ}$ and $0.5^{\circ}$ spot values.

| Range <br> $(\mathrm{nml})$ | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Loss dB <br> Per nml | 0.0250 | 0.0241 | 0.0230 | 0.0225 | 0.0220 | 0.0217 | 0.0214 | 0.0212 | 0.0208 | 0.0205 |

TABLE 2 - ONE WAY ATTEN, LOSSES 10 GHz AND $0.5^{\circ}$ ANGLE
12. To determine the attenuation coefficient a natural logarithm curve fitting technique was used with eqn (7) as the regression equation. Regression coefficients are $\alpha$, and $\alpha_{2}$.

$$
\begin{equation*}
\alpha=\alpha_{1}+\alpha_{2} \log _{e} R \tag{7}
\end{equation*}
$$

changing to attenuation loss from Fig 4.

$$
\begin{gather*}
R \alpha=R \alpha_{1}+R \alpha_{2} \log _{e} R-\cdots(8)  \tag{8}\\
L=L_{1}+L_{2} \log _{e} R-\cdots \text { attn loss for range } R \text { at } 0.5^{\circ}=0.0283-0.001972 \log _{e} R \text { (See figure 4a(2)) } \tag{9}
\end{gather*}
$$

13. The theoretical values are plotted at figure 5 using eqn (9) and compared with the values from figure 3. Conclusions as to the most reasonable values to use in the model are at para 37 below.

## ASSESSMENT OF RAIN REFLECTIVITY

14. Rain. The second effect of precipitation produces backscatter, or clutter. Surveillance radars are designed to detect targets in rainfall up to $15 \mathrm{~mm} \cdot \mathrm{hr}^{-1}$. Heavier rainfall is the exception and normally only occurs for a small percentage of the time and it's spatial extent is usually limited. Modification to the basic radar equation is necessary to take account of the reflectivity of rain. When viewed with linear polarisation the echoing area of a single raindrop whose diameter is very small compared with $\lambda$, is given by $\{15\} p 38$, as:
$\sigma_{i}=\frac{\pi^{5}}{\lambda^{4}}\left|\frac{m^{2}-1}{m^{2}+2}\right|^{2} d^{6}$ $\qquad$
$m=$ complex refractive index of water
$\mathrm{d}=$ diameter of raindrop
15. Up to a frequency of $10 \mathrm{GHz}(3 \mathrm{~cm})$ the raindrop size assumption holds but beyond 10 GHz in heavy rain the Mie scattering theory is required. Using a figure of 0.93 for $\lambda=3.2 \mathrm{~cm}\{18\}$, for $\left|\frac{m^{2}-1}{m^{2}+2}\right|^{2}$, the radar reflectivity $\sigma_{v}$ is the echoing area of unit volume of rain:

$$
\begin{equation*}
\sigma_{v}=(0.93) \frac{\pi^{5}}{\lambda^{4}} \sum d^{6} \tag{11}
\end{equation*}
$$

Quantity $\Sigma d^{6}$ is the reflectivity factor, normally denoted $Z$, and the relationship between precipitation rate $p\left(m m \cdot r^{-1}\right)$ and $Z$ is taken $\{18\}$ and \{16\} to be $\left(\mathrm{mm}^{6} \cdot m^{-3}\right)$ :
(a) Stratiform rain $Z=200 p^{1.6}$
(b) Orgraphic rain $\quad Z=31 p^{1.71}$
(c) Thunderstorm rain $Z=486 p^{1.37}$
16. Nathanson $\{13\}$ p200 and Barton $\{1\}$ p105 quote the value for stratiform rain as the most representative, and so this value is used here. Taking the value $Z$ and changing units in eqn (11):

$$
\begin{equation*}
\sigma_{v}=0.93 \frac{\pi 5}{\lambda 4}\left(200 \times 10^{-18}\right) \mathrm{p}^{1.6} \mathrm{~m}^{2} \cdot \mathrm{~m}^{-3} \tag{12}
\end{equation*}
$$

for $\lambda$ in metres and $f$ in Hz :

$$
\begin{equation*}
\sigma_{v}=5.69 \times 10^{-14} \frac{p^{1.6}}{\lambda 4} \text { or } 7.05 \times 10^{-48} \mathrm{p} 1.6 \mathrm{f}^{4} \ldots\left(\mathrm{~m}^{2} \mathrm{~m}^{-3}\right) \tag{13}
\end{equation*}
$$

In terms of $d B$ relative to $\mathrm{Im}^{2}$
$10 \log \sigma_{v}=-471.5+16 \log _{10} p+40 \log _{10} \rho$
Hence for $\lambda=3.2 \mathrm{~cm}$ the reflectivity in $d B$ relative to $\mathrm{Im}^{2}$ is at Table 3.

| Rainfall <br> Rate | $1 \mathrm{~mm} \cdot \mathrm{hr}^{-1}$ | $4 \mathrm{~mm} \cdot \mathrm{hr}^{-1}$ | $16 \mathrm{~mm} \cdot \mathrm{hr}^{-1}$ | $64 \mathrm{~mm} \cdot \mathrm{hr}^{-1}$ |
| :---: | :---: | :---: | :---: | :---: |
| dB | -72.6 | -63 | -53.4 | -43.7 |

Table 3 Radar Reflectivity for $\lambda=32 \mathrm{~cm}$ for Rain
17. According to Battan \{19\} $p 100$ it has been shown that on the average the calculated rain echo will be 1.4 dB greater than the measured value but it is not usual to make any allowance for this. Figure 6 shows radar reflectivity of rain for given radar frequency and precipitation rate, however variation in drop size causes minor variations regardless of rate.

ECHOING AREA OF RADAR PENCIL BEAM FILLED WITH RAIN
18. Since this study is concerned solely with monostatic pencil beam tracking radar performance(which are often mobile), it is assumed that the radar uses the same beam for both transmit and receive. It must also be assumed that the precipitation rate is uniform within the radar resolution cell. If the resolution cell is completely filled (worst case radar condition), and the polar diagram in both planes is rectangular, a first approximation of the echoing area of the rain in the beam is given by:

$$
\begin{equation*}
\sigma_{R}=\sigma_{V} \frac{\pi}{4} \cdot \theta_{A} \theta_{E}\left(\frac{\pi}{180}\right)^{2} R^{2} \frac{c \tau}{2} \quad\left(m^{2}\right) \tag{15}
\end{equation*}
$$

$\theta_{A}=$ azimuth $3 \partial B$ beamwidth (deg)
$\theta_{\mathrm{E}}=$ elevation 3 dB beamwidth (deg)
$\mathrm{R}=$ range of rain (metres)
19. A more exact result would be obtained by taking into account the variation in aerial gain over the beam cross section. If the polar diagram is assumed Gaussian in both planes the azimuth polar diagram power pattern is:

$$
\begin{equation*}
\exp \left|-\frac{1}{2}\left(\frac{1.665 \theta_{a}}{\theta_{\mathrm{A}}}\right)^{2}\right| \tag{16}
\end{equation*}
$$

and, the two-way pattern in terms of power is:
$\exp \left|-\frac{1}{2} \frac{\left(3.33 \theta_{a}\right)^{2}}{{ }^{\theta} \mathrm{A}}\right|$
$\begin{aligned} \theta_{a} & =\text { Angular departure in azimuth from the beam axis (radians) } \\ \text { \&below } \theta_{e} & =" \quad " \quad \text { elevation " }\end{aligned}$
20. Considering now a horizontal slice of beam with this pattern, of width $d \phi$ and a maximum value of power:

$$
\begin{equation*}
p_{(\text {slice })}=P d \phi \int_{-\infty}^{+\infty} \exp \left|-\frac{1}{2} \frac{\left(\frac{3.33 \theta_{a}}{\theta_{A}}\right.}{{ }_{A}}\right| d \theta \tag{18}
\end{equation*}
$$

$=\operatorname{Pd} \phi\left|\frac{\sqrt{2 \pi}}{3.33}\right| \theta_{A}=0.7527 \theta_{A} \operatorname{Pd} \phi$

Also, $P=\exp \left\lvert\,-\frac{1}{2}\left(\left.\frac{\left.3.33 \theta_{e}\right)^{2}}{\theta_{E}} \right\rvert\,\right.\right.$

Total Power $=0.7527 \theta_{\mathrm{A}} \int_{-\infty}^{+\infty} \mathrm{Pd} \phi=0.566 \theta_{\mathrm{A}} \theta_{\mathrm{E}}$

Compared with $\frac{\pi}{4} \theta_{A} \theta_{E}$ at eqn (15) above.
21. Beam Echoing Area. With linear polarisation the beam echoing area of the gaussian beam filled with rain is:

$$
\begin{align*}
& \sigma_{R}=\left(7.05 \times 10^{-48}\right) p p^{1.6} f^{4}(0.5666) \theta_{A} \theta_{E}\left(\frac{\pi}{180}\right)^{2} R^{2} \frac{C_{i}}{2} \quad\left(m^{2}\right)-(22) \\
& \text { or } \sigma_{R}=\left(1.82 \times 10^{-43}\right) \theta_{A} \theta_{E} R^{2} \tau p{ }^{1.6} f^{4} \tag{23}
\end{align*}
$$

If rain exists between the radar transmitter and the resolution cell, attenuation effects will make the resolution cell rain echoing area appear to be less (see attenuation effects at paras 4 and 5 above).
22. The case where a beam is partially filled with rain $\{20,16\}$
is not pursued, since only low level targets are of interest. For radar modelling it is customary to assume \{16\} that precipitation is constant below some arbitrary ceiling and zero above, hence with a pencil beam at Low grazing angles and short range it is reasonable to assume that only rain filled resolution cells are pertinent. For very low angles part of the beam may intercept the ground but the small effect of this is ignored.
23. An exception to this situation would exist if the resolution cell was just below the $0^{\circ} \mathrm{C}$ altitude level, where the so called "bright band" is situated and the reflectivity suddenly increases, because water has a greater reflectivity than snow and so the particles also change size and shape. Battan $\{19\}$ p 192 gives the increase in the bright band as

12-15 dB above that of snow 500 metres above, while the value of the rain at lower altitudes beneath the bright band may be 6-10 dB lower. Harrold \{21\} suggests 9 dB and 8 dB respectively. The $0^{\circ} \mathrm{C}$ level may occur at any altitude. It is assumed that there exists an exponential change of reflectivity above the $0^{\circ}$ level, and $\sigma_{\mathrm{v}} \alpha \exp \left(-0.6 \times 10^{-6} \mathrm{~h}\right.$ ) $-\cdots$ (24) $h=$ height above the freezing level (metres).

## POLARISATION EFFECTS

24. If in an ideal situation a perfectly spherical raindrop is illuminated by a circularly polarized wave, the reflected signal will have the opposite hand of polarisation and can be totally rejected on reception. In practice, raindrops are not perfectly spherical and it is not practicable to generate perfectly polarised waves, particularly over the whole of the beam. Rain rejection is not perfect although a significant degree of cancellation can be achieved. Warden \{2\}\} gives experimental results averaging 20 dB . Reiss et al \{ 23$\}$ using results taken over a year averages 16 dB and this is accepted as a typical figure (cancellation $=$ ratio of return using linear polarisation to the accepted part of the return with circuler polarisation).
25. Since raindrops can be regarded as oblate spheroids, optimised elliptical polarisation will give better cancellation than circular. However the optimum cancellation characteristics vary with range and the nature of the precipitation. This point is not pursued for the study in hand. On the average the backscatter for horizontal polarisation is larger than that for vertical polarisation.
26. Assuming the rain moved with the wind $\{24\} \mathrm{pp} 205-212$, the doppler spectrum arises from the resolved radial component of the wind velocity as it changes across the resolution volume. To this is added a component representing turbulence. The worst cases exist when looking up or down wind. Mean wind velocity and change of velocity with height (wind shear) are the main parameters. Assuming a two-way power pattern, Gaussian beam vertical polar diagram:

$$
\begin{align*}
& \text { Power }=\exp \left|-\frac{1}{2}\left(\frac{3.33 \theta}{\theta_{E}}\right)^{2}\right|  \tag{25}\\
& \theta_{E}=3 d B \text { beamwidth } \\
& \theta_{e}=\text { Angular departure in elevation from the beam axis (rads) }
\end{align*}
$$

27. If it is assumed that wind velocity changes uniformly with height and therefore uniformly with elevation angle, the standard deviation of velocity due to wind shear is $\{24\}$ :

$$
\begin{align*}
\text { s.d }(\mathrm{Vel}) & =\mathrm{KR} \frac{\theta_{\mathrm{E}}}{3.33} \frac{\pi}{180}  \tag{26}\\
\mathrm{~K} & =\text { Windshear coefficient m.sec }{ }^{-1} \mathrm{~m}^{-1} \\
\mathrm{R} & =\text { Range in metres } \\
& =\left(5.24 \times 10^{-3}\right) \mathrm{KR} \theta_{\mathrm{E}}^{-1}
\end{align*}
$$

28. The turbulent component is assumed to have a standard deviation of Im. $\sec ^{-1}$. So that the total standard deviation of velocity is:
$=\left\{1+\left(2.75 \times 10^{-5}\right)\left(\mathrm{KR} \theta_{E}\right)^{2}\right\}^{\frac{1}{2}} \quad \mathrm{~m} . \mathrm{s}^{-1}$
the corresponding standard deviation of the doppler spectrum is thus:

$$
\begin{align*}
& f_{\text {sd }}=\frac{2 f}{c}\left\{1+\left(2.75 \times 10^{-5}\right)\left(\mathrm{KR} \mathrm{\theta} \theta_{E}\right)^{2}\right\}^{\frac{1}{2}} \quad(\mathrm{~Hz})-  \tag{29}\\
& =\left(6.67 \times 10^{-9}\right)\left\{1+\left(2.75 \times 10^{-5}\right)\left(\mathrm{KR} \mathrm{\theta}_{E}\right)^{2}\right\}^{\frac{1}{2}} \mathrm{f} \quad \text { (Hz) }-\cdots(30)  \tag{30}\\
& \text { where } f \text { is the frequency in hertz. }
\end{align*}
$$

29. This spectrum is Gaussian in shape and centred on the frequency corresponding to the mean wind velocity in the resolution cell and can be written as:

$$
\begin{align*}
& S(f d)=\exp \left\{-\frac{1}{2}\left(\frac{f d-f w}{f_{s d}}\right)^{2}\right\}  \tag{3I}\\
& f d=\text { doppler frequency }(H z) \\
& f w=\text { doppler frequency corresponding to mean wind velocity (Hz) }
\end{align*}
$$

SHORT TERM FLUCTUATION OF RAIN ECHOES
30. As the rain echo is made up of contributions from a very large number of droplet scatterers the probability distributions of the envelope can be expected to be Rayleigh in characteristic, providing precipitation is constant. Warden $\{22\}$ has confirmed this experimentally. The rain echo therefore has the same distribution as thermal noise but with a much longer correlation time which can be significant when integration over the beamwidth of a scanning radar is considered. Any improvement in signal detectability as a result of integration will depend on the number of independent samples integrated. For thexmal noise this would be equal to the number of pulses integrated, but for rain clutter it can be considerably less.
31. Referring to the doppler spectrum (eqn 31). will result in arautocorrelation function of Gaussian Shape and $s d=\frac{1}{2 \pi f_{s d}} \quad$ Assuming a time equal to two standard deviations represents near complete decorrelation. This is typically of the order of 10 millisecs when $f_{s d}$ is near to it's lower limit, and this may be a substantial fraction of, or even exceed, the integration time.

## SPATIAL CONSIDERATIONS

32. Nathanson \{13\} p 217 states that the mean echoing area may change by as much as $\pm 10 \mathrm{~dB}$ over $\quad 10 \mathrm{nmls}$ under showery conditions and by as little as $\pm 1 \mathrm{~dB}$ in uniform rain. There is evidence $\{25\}$ from measurements at Cardington that considerably larger fluctuations can occur; 20 dB in 0.5 km on occasions. Nathanson also stated a fall in spatial correlation to 0.5 in 0.6 to 1.4 nmls in showers, and in 2 to 3 nmls in uniform rain.
33. Frequency Correlation of Rain. Nathanson \{13\} p 213 shows that a change of frequency by the reciprocal of the pulse length is sufficient to reduce the correlation to near zero.

FREQUENCY AND DURATION OF INTENSE RAINFALL
34. Bilham (26) quotes an empirical formula relating rainfall, its duration and frequency of occurence in the UK.

$$
\begin{equation*}
\log n=0.0952+\log _{10} t-3.55 \log _{10}(A+1.01) \tag{32}
\end{equation*}
$$

$n=$ number of occasions in 10 years
$t=$ duration in hours
$\Lambda=$ total rainfall in inches in time $t$
Re-written for $p=$ precipitation rate in ma/hour averanzd over time $t$.

$$
\begin{equation*}
\log _{10} n=0.0952+\log _{10} t-3.55 \log _{10} \frac{(\mathrm{pt}+2.54)}{25.4} \tag{33}
\end{equation*}
$$

This is plotted at figure 7 and relates to rainfall at a point on the ground.

RAIN OVER "SMOOTH EARTH"
35. Extensive small random scatterers over a smooth earth or sea can be considered to be uniformly distributed. With certain combinations of polarisation at low grazing angles (HH/WV) the relative radar cross section of the scatterers is enhanced by the smooth surface. This is shown by Long and Zehner $\{27\}$ to be as much as 7.8 dB larger at $\lambda=10 \mathrm{~cm}$ over the sea. It is not clear if this would effect multipath at $\lambda=3 \mathrm{~cm}$. The rain scatterers are assumed to extend at least several interference lobes in altitude above the earth's surface. As the depression (grazing) angle approaches zero specular reflection increases. Work reported upon in this field suggests that the problem is complex and that information is incomplete, no results have been found for $\lambda=3 \mathrm{~cm}$.

## CHAPTER SUMMARY

36. From the foregoing, extensive reading and by contrasting the findings of many reports, a number of main conclusions applicable to the radar parameters required have been selected. These are set out below as the basis for the rain and atmospheric clutter inputs to the overall model at Chapter 11.
37. Selected Values. Using as far as possible practical measurements from the sources quoted and including opinions from unpublished sources:
a. Rain Attenuation. Rain attenuation values used are those from Nathanson shown at Table 1 at para 5. The set of results graphed at figure 1 are considered the most representative and the curves show the important trends as both radar operating frequency and rainfall increase.
b. Cloud/Fog Attenuation. Modelling values for cloud and fog attenuation are relatively small compared with the other sources interfering with radar detection. However this value is included for completeness and under certain conditions cloud or fog attenuation effects may just take the radar system below detection threshold or introduce uncertainty. Values calculated from eqn (9) are used. These are plotted at Figure 5 and also tabulated for several values. Curve fitting for eqn 9 was done by computer program, correlation coefficient 0.996.
c. Rain Reflectivity. Rain reflectivity values at Table 3 are considered suitable.
d. Pencil Beam Rain Echoing Area. Resolution cells within pencil radar beams are always considered to be rain-filled, never partially filled because of the geometry of the situation. Aerial polar diagrams are taken to be Gaussian in power distribution.
e. Polarisation Effects. Figures from Reiss (see para 24) are taken to be statistically sound.
f. Doppler Spectrum of Rain. Total standard deviation of doppler shift due to wind effects are incorporated by using equations (30) and (31).
g. Rain Fluctuation and Spatial Extent. Probability meterological statistics for precipitation frequency, duration, short-term fluctuation and intensity are well documented. The model initially operates without reference to the statistics, by using fixed rain values for each target run.
38. Details of the programs used for signal and statistical analysis are briefly described at Annex D.
39. Anomalous Propagation. Finally, atmospheric conditions might exist to produce 'ducting', allowing the unexpected detection of low level targets at greater ranges than normal. Such conditions cannot be predicted overland with total accuracy; but are probable over water as 'evaporation ducts'. Overland there is a $35 \%$ probability of some ducting in Europe. Prediction can be enhanced by using radiosonde data, and by using software such as the Ferranti prediction programs. Ducting is not considered to be of interest for a low level tracking radar since 'ducting' ranges are likely to exceed missile system ranges. Ducting might however allow an off-site search radar to detect the target at greater range and thus direct a tracking radar onto a target at an earlier time.

FIG 1 ATTENUATION IN RAIN FOR PRECIPITATION RATE AND



FIG 2 ONE WAY RADAR ATTENUATION IN FOG AT $18^{\circ} \mathrm{C}$


FIG 3 ABSORPTION LOSS vE RADAR CARRIER FRECUENCY FOR DIFFERENT ANGLES OF ANTENNA ELEVATION


Km nm
FIG 3a ABSORPTION LOSS AS A FUNCTION OF RADAR RANGE AT $0^{\circ}$ AND $0.5^{\circ}$ GRAZING ANGLE AT $10 \mathrm{GHz}(3 \mathrm{~cm})$

FIG. 3b two-way attenuation loss for given grazing angle at ioghz


(2)


FIG 4a DETERMINATION OF L FOR 0.5 ${ }^{\circ}$ GRAZING ANGLE ( $(=0.0283-0.001972$ LOG $e$ RANGE). CORRELATION COEFICIENT 0.96)

(2)


FIG 4b determination of L FOR $0^{\circ}$ gRAZING ANGLE ( $L=0.0252-0.000018$ LOG $e^{\text {RANGE). }}$ CORRELATION COEFFICIENT 0.98)

FIG. 4 COEFFICIENT OF ATTENUATION - GRAZING ANGLE $0.5^{\circ}$ AT 10 GHz



FIG 5 COMPARISON OF THEORETICAL AND ACTUAL VAWES FOR $0^{\circ}$ AND $0.5^{\circ}$ GRAZING ANGLES

FIG 6 RADAR REFLECTIVITY OF RAIN : $\sigma_{v}$


FIG 7 FREQUENCY AND INTENSITY OF RAINFALL IN UK


# DEPENDENCE OF TERRAIN BACKSCATTER ON 

RADAR AND SURFACE PARAMETERS

INITRODUCTION

1. Non-uniform scatterers surrounding a radar cannot be easily described by a single coefficient, since the subject of radar energy scatter from terrain is complex. Standard texts often describe surface returns, which produce clutter, in a relatively simple way, but research into terrain response has been the subject of many detailed research reports in past years. It could perhaps be reasonably expected that the multiplicty of measurements taken over some 30 years (although each producing results pertinent to a particular requirement), would nevertheless leave few gaps. in the overall knowledge. This is not the case - and so an extensive survey of past surface clutter measurement programmes, and information from other sources has been made and summarised. Many of the clutter measurements made since World War II can be found at $\{28\}\{29\}\{30\}\{31\}$.

## RESEARCH ATMS

2. In order to assess radar performance with a reasonable degree of confidence, two main aims must be met:

> a. A description is required, in mathematical terms, for the expected clutter from any terrain radar resolution cell over which a target is flying, or from which clutter is received (eg sidelobes). The description should account for
the clutter dependence of the surface itself, radar grazing angle, resolution cell size, radar frequency, polarisation and spectra; since $\sigma_{0}=f(\theta, R, \tau, \Psi)$ etc.
b. An assessment of the degrading effect on radar detection, which a specific type of clutter is likely to have on a radar, given the various signal processing options which could be incorporated in the radar, together with its other parameters.
3. Taking 2(a) above, it is suggested that an ideal model should examine the terrain beneath each target resolution cell, by accessing a terrain culture data base for the area overflown. Predetermined reflectivity co-efficients or the reflectivity distribution should be used for the various types of terrain cover, suitably adjusted for the parameters at para 2(a) and further scaled after using local terrain spot heights to calculate slope, aspect angle. With terrain reflectivity as a function of aspect angle it should be possible to finally produce a single value for clutter power to represent the cell under investigation.
4. Initially each contributory clutter factor, in an ideal approach, should be separated from the others, proven experimentally and later made available for recombination with the other detection factors. Separation of the individual dependencies is however extremely difficult in the first place.


#### Abstract

5. The observable clutter values have rarely been collected in a useful manner for this purpose; since many researchers have usually collected clutter amplitude, temporal and polarisation characteristics separately but not simultaneously - thus not allowing best correlation to be investigated between the variables. Others have usually ignored terrain slope effects, or radar resolution cell size.


## CLUTTER DEPENDENCE

6. Surface clutter characteristics overall can be divided into two categories:
> a. Clutter-Processing Dependent. This grouping includes radar signal characteristics, such as amplitude fluctuation statistics, spectrum and frequency agility.
b. Clutter-Backscatter Dependent. Including previously
listed parameters, such as grazing angle, RCS terrain type, polarisation, RF and spatial distribution.
7. It is necessary to apportion the probability of detection and false alarm rate (FAR) factors correctly between the two groupings above. A simplified approach is then taken for the purpose of meeting the geometry and target parameters. Detection and FAR probabilities can be obtained from target and clutter fluctuation models - depending upon the effective numbers of statistically independent target clutter samples integrated by a postulated radar signal processing system.
8. Detection probability (high) and FAR (low) thresholds can then be established, based on clutter statistics and the desired FAR. Overall detection probability, above the mean integrated clutter level, can ther be computed, based on signal statistics.

## STATISTICAL DISTRIBUTTIONS

9. Three example statistical distributions $\{32\}$ are detailed at Annex A:
a. The exponential statistic (Wiebull with exponent parameter $=1$ ) is used when many independent scatterers are within a radar resolution cell.
b. A surface clutter (Ricean) distribution which is used in the case of a single dominant non-fluctuating scatterer (point specular) plus many smaller scatterers in the same resolution cell.
c. The Log-Normal distribution has a longer "tail", is applicable to modelling 'spiky' clutter, and which has also been shown $\{33$ \}, to give a reasonable description of scattering from randomly orientated shapes which can be represented as plates or cylinders.

$$
4-75
$$

10. Of the clutter-dependent parameters at para 2(b) above, terrain is the most significant. When observed by pulsed radar at low grazing angles most terrain is non-homogeneous and so a statistical approach is required, since the character of the surface, its slope (see Chapter 10) and consequently the backscatter coefficient, will vary almost from one resolution cell to the next. An overall probability density function (pdf) is required to describe the amplitude distribution. This will provide the probability that a resolution cell selected at random, within the terrain area, will contain clutter with a particular average of clutter power. The model at Chapter 11 will account for those cases where the surface is 'shadowed' using the sightline techniques described at Chapter 2 , and will also indicate if diffraction or refraction could take effect and possibly produce clutter from a cell in 'shadow'.
11. The typical radar resolution cell clutter footprint geometry assumes that any cell will contribute an average clutter value for the particular type of terrain dependent upon grazing angle, slope and the applicable pdf or coefficient of reflectivity.
12. It is seen that the amplitude probability for a single cell does not describe spatial distribution; since each cell is taken independently from within the overall area of radar tracking. Also adjacent cells could be 'shadowed' in hilly terrain, while at other times in a given area there will be extensive regions with the same surface reflectivity
characteristics and slope. Spatial distribution must be necessarily considered whenever a quick (overview) prediction is required for a given area of interest, since backscatter from adjacent cells will often be spatially correlated.

## PROBLEMS OF MODELLING THE TERRAIN AMPLITUDE PROBABILITY FUNCTION

13. It has recently $\{34\}$ been acknowledged that "more comprehensive and carefully controlled backscatter measurement programmes" are necessary at low grazing angles. A preliminary survey in this area by Allan has indicated both disparities and consistencies in an examination of a sample of results from the UK and USA. No attempt is made here to repeat Allan's summaries, but rather to extend his results to include several more sets of measurements which have now become available.

## FORMULATION OF STATISTICS

14. It is widely accepted that terrain clutter is the result of 2 basic mechanisms; the individual or specular reflections from strong point reflectors; and a Rayleigh distribution for diffuse clutter. The process can be developed $\{35\}$ in terms of the statistical properties of the scintillating returns from the elementary point scatterer, with the more complex distribution obtained by superposition of many point scatterers within the radar beam.
15. Type of Terrain. Investigations $\{36\}$ deduce as a general rule that the type of terrain is identified most markedly in the mean of the Normalised Radar Cross Section (NRCS) and in general clutter is neither Gaussian nor Log-normal. This has been demonstrated by means of a KOLMOGOROV-SMIRNOV test of the curaulative distribution of the NRCS \{37\}. The return from a point scatterer within the resolution cell will be of the complex form:

$$
Y(t)=\frac{A(t)}{R(t)} e^{i \phi(t)} e^{-i \omega_{0}(t)}
$$

## for $0 \leqslant t \leqslant T$

$\emptyset=2 \beta R(t)$ is the phase, where $\beta=2 \pi / \lambda$ (propagation factor)
$t$ is the observation time (dependent on aerial beamwidth)
$A(t)$ contains both the amplitude component of the 2 -way aerial radiation pattern and the intrinsic amplitude of the scatterer. $R(t)=\sqrt{\left(R_{0}^{2}+V^{2} t^{2}\right)}$ is the range to the scatterers at the extremities of the beam and $R_{o}$ is the range along the beam centreline. As the aircraft moves at velocity V this sets the observation time for a given range and beamwidth; for example, if mean $R_{0}=5 \mathrm{~km}, \mathrm{~V}=300 \mathrm{~m} \cdot \mathrm{~s}^{-1}$ and beamwidth $0.4^{\circ}$, then $t=0.058 \mathrm{sec}$. Generally Vt << $R_{o}$, giving an approximation of the return signal ("scintillating" linear FM signal due to aircraft velocity causing radar resolution cell motion across the point scatterer):

$$
\begin{aligned}
& Y(t)=\frac{A(t)}{R_{0}} e^{i\left(\phi_{0}+K t^{2}\right)} e^{-i \omega_{0} t} \\
& \phi_{0}=2 \beta R_{0} \\
& K=\beta V^{2} / R_{0}, \text { the scintillation rate }
\end{aligned}
$$

Assuming t>0, then representing terrain clutter as a superposition of many individual scintillating targets:

$$
\begin{equation*}
Y(t)=e^{-i \omega_{o} t} \sum_{k=1}^{n} a_{k} e^{i \phi_{k}+i K\left(t-t_{k}\right)^{2}} \text { for } 0<t<T \tag{3}
\end{equation*}
$$

16. Several assumptions are made above, since in practice, even with the observation time $T$, with a moving radar aerial the amplitudes and phases and the number of scatterers will be varying. All. amplitude effects at (3) are lumped into $a_{k}$. Figure 1 shows the general concept of evenly distributed scatterers within a resolution cell.
17. It is normal when following the 'point scatterer' approach to consider fluctuations from a single scatterer, using equation (1) where fluctuations about zero frequency for a linear frequency modulated signal are based upon:

$$
\begin{align*}
& Y(t)=A \operatorname{Cos}\left(K t^{2}+\varnothing\right)  \tag{4}\\
& 0 \leqslant t \leqslant T
\end{align*}
$$

It is shown \{37\} that as $K T^{2} \rightarrow \infty$ the spectrum of the scintillating signal tends to $\alpha$ constant, and this is assumed during the observation time. The characteristic function of equation (4) is obtained from:

$$
\begin{aligned}
& F(\varepsilon)=\int_{0}^{2 X_{0}} e^{i \varepsilon A \operatorname{Cos}\left[\left(X^{2} / K\right)+\bar{\emptyset}\right]} p(X) d x \ldots \ldots(5) \\
& \text { Where } X=K t \\
& \quad p(X)=\frac{1}{2 X_{0}}=\frac{1}{2 K T}
\end{aligned}
$$

18. By manipulation, reversing the order of integration, sumation and by changes in variables and, since by definition the pdf is the Fourier transform of the characteristic function then:

$$
\begin{equation*}
p(Y)=\left\{\pi^{-1}\left(A^{2}-Y^{2}\right)^{-\frac{1}{2}} \sum_{m=0}^{\infty}-(m) b_{m} \cos \left(m \operatorname{Sin}^{-1} \frac{Y}{A}+\frac{m \pi}{2}\right)\right\}(Y<A \tag{6}
\end{equation*}
$$

19. From (6) the mean value of the scintillating signal and the convergence of the distribution function are shown at $\{38\}$ in some detail. Finally, since in practical terms terrain clutter is the result of many point scatterers within the aerial beam, the mean, variance and other useful descriptive parameters can be obtained so that the statistics of the distribution envelope are evolved. Cumulative distributions at $\{39\}$ over city areas clearly show the specular nature of clutter from this type of target, however, one of the difficult areas in this report is the correlation of statistics found by one researcher at a specific location, with those of others at different locations. Nuch of the early work on the point scatterer formulation of the statistics of terrain clutter was by $\{40\}$, but more recent and extensive work using this technique \{41\} invariably recomends the necessity for many more measurements. In general \{42 \} concluded that the Gaussian distribution applied over homogeneous surfaces such as desert and farmland and lognormal (long tail) distributions would be likely over urban and moutainous areas. Many researchers used smooth surfaces to develop reflectivity models \{ 43\}, however $\{44\}$ states that the accompanying theories do not apply directly to the earths vegetation.
20. An alternative approach for the assessment of radar performance in clutter is the use of clutter simulation. However, the simple simulation of clutter as noise alone ignores the need for time, spatial correlation, or of frequency effects. Time correlation could perhaps be introduced in an appropriate way by numerical filtering of the random numbers used to simulate the noise, but frequency and spectral effects are complex. Andre et al $\{45\}$ recommends an 'open loop' approach to clutter simulations for basic performance prediction, with a 'closed loop' method preferred for detailed analysis. In the open loop case the sum of the signals from clutter and target signals (from clutter and target signal generators)are fed to the simulated radar receiver. The essential difference for the closed loop solution involves the simulation of a radar transmitter signal which is then processed to obtain signals for target and radar clutter which are mutually coupled to allow signal modification. Finally target and clutter signals are merged for processing by the simulated radar receiver.
21. Considerable effort has also been expended, $\{46\}$, in modelling clutter maps for other purposes, such as flight simulation, where the simulated airborne radar is 'looking down' for targets flying over a clutter-producing surface. In the main such simulations aim to evaluate system reaction to the clutter, Lognormal distributions ere often used, and since moving radar platforms are being simulated,
a measure of the rapidity of clutter variation is ideally incorporated. These simulations do not investigate the clutter itself and merely reproduce approximate (but nevertheless representative) visual effects for training purposes. The reader is directed to the reference for further general reading, but the technique cannot realistically contribute to this study.

## BACKSCATTER FROM VARIOUS TERRAIN TYPES

22. Although most terrain is composite in character, giving an observed wide dynamic range of land clutter distributions for the differing combinations of woods, fields, rocks, man-made objects and shadowed regions; the following paragraphs briefly consider individual terrain-type reflectivity characteristics, prior to investigations of the dependence of clutter upon the radar parameters $\tau$, RF, polarisation and $\psi$. Normalised RCS per unit surface area is used throughout:

$$
\sigma_{0}=10 \log _{10} \quad \frac{\text { Effective RCS Area }}{\text { Effective Illuminated Surface Area }}\left(\mathrm{m}^{2} \cdot \mathrm{~m}^{-2}\right)-(7)
$$

23. Several hundred sets of conditions would be necessary to specify all backscatter, with 8 or 10 different terrain type classifications. Some researchers \{ 47 \} have included an extra parameter to account for the practical inconsistencies of $\sigma_{0}$. For example, the large number of small scatterers which under normal circumstances would be labelled 'Rayleigh' in character are found in practice to occur on less than $50 \%$ of ocassions $\{48\}$. For the low grazing angles required in this report land backscatter amplitude distributions are often contaminated by shadowing due to trees, hills etc. Taking the extra
problems of backscatter coefficient variations with surface moisture content, and the past measurements taken with fixed and moving radar platforms (ie spatial average $v$ time average), it is seen that uncertainty can easily occur when attempting to survey findings and arrive at a reasonable model.
24. Trees/Forests. Electromagnetic radiation at 10 GHz or above does not significantly penetrate dense areas $\{49$ \}. Diffuse returns therefore come predominantly from the upper part of the tree canopy. Raising the aerial above trees and using pencil beam radar reduces clutter only for high flying aircraft \{ 50\}, but gives limited signal to clutter improvement for ultra-low flying aircraft. Aircraft which would otherwise suffer blocked optical sightline may therefore be observed subject to clutter limitations.
25. Researchers in the past \{51 \} p 221, have investigated backscatter from differing types of tree, ie pine, deciduous, under different moisture conditions and seasons of the year. The average RCS per unit area for trees seems to be about -2odB $\because$, with horizontal polarisation exceeding vertical by 1 or 2 dB. Evergreen (pine trees) tend towards a slightly lower RCS per unit area than deciduous ( 3 dB ), using $\lambda=10 \mathrm{GHz}\{53\}$. Clearly those trees which retain their foliage will not vary appreciably in reflectivity wịth the seasons. From reported data \{ 54\} a survey of amplitude returns from trees, using horizontal polarisation, with log-normal fit is given at equation (8). Contrary to \{52\} above vertically polarised values were 3 to 4 dB higher.

$$
\begin{equation*}
\sigma_{0}=-15+15 \log _{10} \frac{\psi}{25}-8 \log _{10} \frac{\lambda}{0.32} \mathrm{~dB} \mathrm{~m}^{2} \cdot \mathrm{~m}^{-2} \tag{8}
\end{equation*}
$$

No other parameters were included, however it is thought that (8) was derived for $\psi<25^{\circ}, \geqslant 95 \mathrm{GHz}$. The dependence upon $\psi$ and $\lambda$ is considered further at paras 47 and 68 .
26. Effects of Precipitation. Moisture probably contributes 5 dB extra reflectivity compared with dry trees $\{55\}$; snow and ice cover are separately examined at para 29 below.
27. Urban and City. Significant shadowing can be expected from buildings when operating at low grazing angles, but results must be analysed carefully. For example, Linnells results $\{56\}$ were obtained under conditions where perhaps reduced shadowing is probably because of his radar location on a high tower ( 30 m ). Median backscatter from urban and city areas at $\lambda=3 \mathrm{~cm}$ are likely to fall between-24-30 $d B$ (below $1 \mathrm{~m}^{2} \cdot \mathrm{~m}^{-2}$ ), for very low grazing angles. Katz $\{57\}$, and others $\{58\},\{59\}$, have also produced results for buildings. In general, $\{60\}$ concluded that the log-normal distribution is the best fit for reflectivity from buildings.
28. Flat Farmland and Cultivated Land. Linnell $\{61\}$ also obtained results for farmland; these ranged from $-33 d B$ in March to $-21 d B$ in August, for a spread of values for $\psi$, discussed below. As expected a maximum $\sigma_{0}$ occurred when the area contained fully grown crops. Also confirmed were other previously assumed conditions, such as that of ploughed ground giving a greater value before a rainstorm, since the surface is rougher in texture compared with values after the rain; the dielectric constant of soil being moisture dependent. The reader

$$
4-84
$$

is referred to $\{62\}$ for detailed information on the variation in average height and reflectivity of farm crops for given ground height above sea level in Germany. Land utilisation for certain crops is predominant in given geographical areas and in N Germany culture data is held to a grid spacing of $150 \mathrm{~m}(\mathrm{~N} / \mathrm{S}) \times 95 \mathrm{~m}(\mathrm{E} / \mathrm{W})$ in the German Military Geophysical Office Databank.
29. Snow and Ice. In some respects limited data is available concerning clutter directly from ice or snow, especially in those measurements which allow a comparison of the clutter plot from the same terrain both with and without snow cover. Some values obtained with an aerial height of $2 \mathrm{~m}, \mathrm{RF}$ 's at 10,35 and 94 GHz were made between 0.4 to $1^{0}$ grazing angle $\{63\}$, but with limitations in range and with the snow overlaying fresh-water ice rather than over trees or soil. Krason and Randig \{ 64\} made reflectivity coefficient measurements at 3 and 10 GHz for $\psi=0.5$ to 4 degrees using common terrain, and with leaves both on and off the trees. AT 9.405 GHz values were consistently shifted by +3 dB due to snow cover.
30. Results obtained using short pulse durations of $0.125,0.17$ and $0.10 \mu$ secs (Hoekstra and Spanogle), together with aerial beamwidths of $1.3,1.9$ and 0.38 degrees, are of particular interest here, since they are appropriate to high performance tracking radars. Unfortunately the 500 to 600 m range is not representative and the results could only be used if they stctrapolate satisfactorily to longer ranges. Further at short ranges it is thought possible that the clutter returns may come from beneath the dry snow cover, which varied in depth from 0 to 30 cm . Small amounts of free water in snow can significantly
affect the measured value, which changed due to this effect by about +10 dB in the case of Hoekstra.
31. Temporal Changes. More recently dramatic differences have been observed over short time periods. These may be as much as 10 dB in 30 minutes $\{65\}\{66\}$, and specifically occur when free water freezes, usually - though not exclusively, at night. Transition time is unequal between the two extremes as freezing generally takes longer than thawing. Hayes \{67\} observes that at least 0.15 m depth of snow is necessary to ensure no reflections from the underlying terrain, and that "calibrated data are insufficient to permit comparison with theoretical calculations".
32. Polarisation in Snow. Polarisation effects under normal conditions are again considered at para 64 below, however it is well established that horizontal aerial polarisation in snow gives approximately 10 dB more than vertical when the snow is dry, but this difference reduces when the snow is wet.
33. Reflectivity Models for Snow. The above comments are included here to show the uncertainty associated with selecting a suitable model, since it is proposed that the underlying snow which receives a proportion of energy (variable with RF), may refract the energy towards the normal; thus allowing backscatter to occur at a higher grazing angle $\Psi$. In practical terms there will be difficulty,
for example, in predicting the freeze-thaw cycles and possibly sporadic rain on variable-depth snow. It seems probable that a very general statistical clutter value for snow is the best to be hoped for. Other effects noted include evidence of returns from "blown snow" from hill tops \{68\} getting into sidelobes, and snow in forward scatter (at 35,95 and 140 GHz ) measurements producing as much as 25 dB variation in multipath signals, leading to serious angle tracking errors against horizon targets.
34. Tomlinson \{69\} obtained backscatter information for space-based radars for seven terrain types, and by regression analysis as a function of RF and $\psi$ obtained analytical models, for snow and other surfaces:

$$
\begin{equation*}
\sigma_{0}=A+B \psi+(C+D \psi) \log f \tag{9}
\end{equation*}
$$

Much larger resolution cells were used than is the case for low level tracking radars, and the applicability of the model calls for caution. However it is seen later that the values produced by this model equate reasonably well with those from other sources. The choice of an absolute value of $\sigma_{0}$ to be used for a particular assessment is much more of a problem than the gradient; for example, as the value of $\psi$ changes. Equation (9) above for snow computes with $A, B, C$ and $D$ as $-32.97,0.340,-1.797$ and 0.035 respectively. An 'adjustment factor ${ }^{\prime}$ of +2.9 dB is applicable.
35. Results $\{70\}$ as far back as 1969 indicated that clutter levels, as expected, must be a function of the area of the illuminated clutter patch and therefore dependent on $\tau$ and beamwiath. Measurements made at that time with long and short pulses transmitted alternately from the same radar gave differences in clutter levels of about 18 dB .
36. It seems that the effect of $\tau$ on clutter lies somewhere between two extremes. On the one hand with a very large number of scatterers, the power returned is proportional to pulse length. But with a very small number of scatterers the probability of any power level being returned is proportional to pulse length. A note on each of these conditions follows, before the results of various research papers and reports are discussed and a suitable model selected.
37. Many Scatterers. As an aerial is moved the short term clutter returns are assumed to be Rayleigh distributed with a mean value which varies slowly to give a lognormal distribution with sd about 20 dB , (ie lognormal running mean with superimposed Rayleigh for the difference between the clutter signal and the running mean). The scatterers are often located in patches so with $\tau$ reduced there is some probability that no scatterers are in the reduced resolution cell. It is assumed each large cell (if unshadowed) does contain some clutter.
38. On the average the scatterers in those cells containing them are more densely packed in the smaller cells (to give the same clutter levels). A situation can arise for very small values of $\tau$, where some cells return no power while others may return (since clutter is spatially distributed in patches),
proportionately more. . For example if $\tau$ is reduced arbitrarily by a factor of 10 and (say) half the new cells contain clutter then:

$$
\begin{equation*}
\log p=0.3 \log \frac{\tau_{1}}{\tau_{2}} \tag{10}
\end{equation*}
$$

p is the probability of the smaller (new) cell containing clutter. $\tau, \tau$ are the short and long values of $\tau$ respectively.

$$
\begin{equation*}
P=7 \log \frac{\tau_{1}}{\tau_{2}} d B \tag{11}
\end{equation*}
$$

$P$ is the power level returned by the small cell relative to the large cell.
39. Dodsworth $\{71\}$ proposed a deduction of the effect of a change of pulse length on the probability distribution of clutter. Using a numerical example where $p=0.5$ and $\frac{\tau_{1}}{\tau_{2}}=0.1$, giving $P=-7 \mathrm{~dB}$; for the small cell to have an RCS of OdB (ref $\mathrm{lm}^{2}$ ), the large cell must have a echoing area of $+7 d B$, and the probability that this is exceeded is $37 \%$. But $p=0.5$, hence the probability of the small cell exceeding $O d B$ is $18.5 \%$. The results are plotted at Figure 2 for 5 ratios of $\mathbb{T}_{1},{\underset{2}{2}}^{\tau}$. In modelling clutter $\{71$ \} chose a lognormal distribution for uniformly reflecting points expressed as a departure from the running mean of the clutter signal, ie the short term clutter component. Using an appropriate number of integrations and by adjustment to the pulse length $\tau$, the sd of the clutter signal increases as pulse length is reduced.
40. If it is assumed that the many clutter points from a large cell are more or less uniform and varying from cell to cell in a lognormal manner, this can be plotted with an arbitrary sd of 25 dB and replotted after $\tau$ is reduced by a factor of 10 , reducing all echoing areas by 10 dB as shown at figure 3. If only one echoing point exists within each large resolution cell and the same distribution applies as in the first curve above; and $\tau$ is now reduced by a factor of 10, a third curve results with a difference from the first of about 40 dB at the $5 \%$ level and 25 dB at the 1\% level. This gives the approximate result in para 39 above.
41. Relationship of $\tau$ With Wiebull Shape Parameter, In practice, for a given cell, ground clutter is not uniform, leading to a non-proportionate change in clutter when a resolution cell is shortened due to shadowing and other effects. Whereas a radar designer may wish to select a set of radar parameters and then find a suitable distribution - typical of the parameters, or alternatively to estimate the distribution change likely when $\tau$ alters; performance prediction of existing radar can only be based on the known parameters of the radar. $\{71\}$, using the pulse -length-beamwidth product has made empirical estimates of the effect of changing the resolution cell by factor $N$, on surface clutter distribution. It is established for a range of $\tau \times{ }^{\theta_{A}}$ of $2.5 \times 10^{-9}$ to $2.5 \times 10^{-7}$ radian seconds that a relationship exists between the size $\tau$ and the Weibull shape parameter;
42. In practical terms here, with an assumed pulse length $\tau=0.5 \mu \mathrm{sec}$ and $\theta_{\mathrm{A}}=2^{\circ}$, giving $0.0349 \times 0.5 \times 10^{-6}$ radian seconds $=1.7 \times 10^{-8}$. Given the Wiebull shape parameter relationship:

$$
\begin{equation*}
c=0.192-0.0764 \log \left(\theta_{A} \tau\right) \tag{12}
\end{equation*}
$$

From which scale parameter $b$ can be obtained An empirical method of estimating the clutter distribution for other resolution cell sizes is possible. (See also Annex A).
43. The existence of the Weibull distribution as being applicable to land clutter returns was probably first reported by Boothe \{ 72$\}$ in 1969. But again, like so many others since he took Linell's results presumably because they were almost the only ones available at that time which offered a spread of values. It will be shown at para 51 that certain characteristics of Linell's results differ significently from the majority taken elsewhere - although it must be recognised that this may in part be due to different terrain in Sweden. Also there is a general absence of available measurements from Continental Europe.
44. Boothe's Weibull values, based on Linell's results have been compared by the author here with 11 other sources, now available. Data is listed at Table 1, and correlation computations made between

|  |  | $\begin{gathered} \tau \\ (\mu \mathrm{s}) \end{gathered}$ | $\begin{gathered} { }^{\theta} \mathrm{A} \\ (\mathrm{~m} \cdot \mathrm{rad}) . \end{gathered}$ | $\begin{array}{r} \sigma_{\mathrm{m}} \\ (\mathrm{a})^{2} \end{array}$ | $\begin{aligned} & \left(\lg _{n^{2}}\right) \\ & \text { (CALC) } \end{aligned}$ | $\begin{gathered} \mathrm{b} \\ \text { (meas) } \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & \frac{\text { RILEY }}{5 \mathrm{GHz}} \\ & \{76\} \end{aligned}$ | 3.0 1.5 0.9 0.3 | 33.1 33.1 33.1 33.1 | -43 -46 -51 -51 | $\begin{aligned} & 0.343 \\ & 0.366 \\ & 0.382 \\ & 0.419 \end{aligned}$ | 2.7 3.6 4.2 4.3 |  |
| 2 | RIGDEN $5.75 \mathrm{GHz}^{2}$ | 0.015 | 33.1 | $-47$ | 0.518 | 3.0 |  |
| 3 | $\frac{\text { DODSWORTH }}{\{77\} 5 \mathrm{GHz}}$ | 3.5 0.5 | 8.7 8.7 | $\begin{aligned} & -80 \text { (Est) } \\ & -80 \text { (Est) } \end{aligned}$ | $\begin{aligned} & 0.382 \\ & 0.446 \end{aligned}$ | $\begin{aligned} & 7.7 \\ & 9.0 \end{aligned}$ | $\begin{aligned} & \text { Estimated } \\ & \sigma_{m} \end{aligned}$ |
| 4 | $\frac{\text { WARDEN et al }}{\{78\} 5 \mathrm{GHz}}$ | $\begin{aligned} & 5.0 \\ & 0.5 \\ & 5.0 \\ & 0.5 \end{aligned}$ | $\begin{aligned} & 8.7 \\ & 8.7 \\ & 8.7 \\ & 8.7 \end{aligned}$ | - 70 (Est) <br> - 70 (Est) <br> - 70 (Est) <br> - 70 (Est) | $\begin{aligned} & 0.370 \\ & 0.446 \\ & . .370 \\ & 0.446 \end{aligned}$ | $\begin{aligned} & 6.6 \\ & 9.5 \\ & 6.8 \\ & 5.9 \end{aligned}$ | Estimated $\sigma_{m}$ |
| 5 | $\frac{\text { DE LOOR et al }}{\{79\} 10 \mathrm{GHz}}$ | 0.5 | 31.4 | -28 -16 -14 | 0.404 <br> 0.404 <br> 0.404 | $\begin{aligned} & 9.5 \\ & 7.0 \\ & 9.5 \end{aligned}$ | APR <br> JUL <br> SEP |
| 6 | $\frac{\text { SURADS }}{\{80\} 10 \mathrm{GHz}}$ | 0.25 | 27.9 | $-34$ | 0.431 | 5.7 |  |
| 7 | $\begin{aligned} & \frac{\text { WARDENT }}{\{8\}\}} \\ & 5 \mathrm{GHz} \end{aligned}$ | $\begin{array}{r} 0.4 \\ 12.0 \\ 5.0 \\ \hline \end{array}$ | $\begin{gathered} 26.17 \\ 8.7 \\ 26.27 \end{gathered}$ | - 27 <br> - 70 (Est) <br> - 70 (Est) | $\begin{aligned} & 0.417 \\ & 0.348 \\ & 0.333 \end{aligned}$ | $\begin{gathered} 2.16 \\ - \\ 3.05 \end{gathered}$ | Estimated $\sigma_{m}$ |
| 8 | $\frac{\text { ERTCSON }}{\text { \{83\} }}$ | 1.0 | 57.5 | $\begin{aligned} & -25 \\ & -30 \end{aligned}$ | $\begin{aligned} & 0.361 \\ & 0.361 \end{aligned}$ | $\left\lvert\, \begin{gathered} \text { (not } \\ \text { avsil) } \end{gathered}\right.$ | $\begin{aligned} & \psi=5^{\circ} \\ & \psi=0.5^{\circ} \end{aligned}$ |
| 9 | $\frac{\text { APL }}{\{85\}} 5 \mathrm{GHz}$ | 0.34 | 34.9 | - 40 | 0.413 | 3.4 |  |
| 10 | $\frac{\text { NATHANSON }}{\{84\} 3 \mathrm{GHz}}$ | 2.0 | 26.1 | -46.25 | 0.364 | 3.9 |  |
| 11 | $\frac{\mathrm{APL}}{\{85\}} 8.8 \mathrm{GHz}$ | 0.25 | 20.0 | - 52 | 0.442 | 3.8 |  |
| 12 | $\begin{array}{r} \text { LINELL } \\ \{88\} 10 \mathrm{GHz} \mathrm{~b} \\ \mathrm{c} \\ \mathrm{~d} \end{array}$ | 0.17 | 24.4 | $-48$ <br> - 46 <br> - 36.4 <br> $-42$ | $\begin{aligned} & 0.524 \\ & 0.524 \\ & 0.524 \\ & 0.524 \end{aligned}$ | $\begin{aligned} & 3.3 \\ & 2.84 \\ & 3.76 \\ & 3.95 \end{aligned}$ | $\begin{aligned} & \psi=1.25 \mathrm{APR} \\ & \psi=1.25 \mathrm{MAY} \\ & \psi=0.7 \mathrm{NOV} \\ & \psi=0.7 \mathrm{MAR} / \mathrm{AU} \end{aligned}$ |

## NOTE

Selected Data - Serials 1 to 7 rural/farmland
Serials 8-11 (incl) Rural
Serial 12a, l2b rural
Serial 12c, 12d forest
c (calc uses Eqn (12)
${ }_{A}$, $\tau$. product and shape parameter $c$, and between $\theta_{A} \tau$ product and $\sigma_{m}$. Few values are still available at $\lambda=3 \mathrm{~cm}$. Detailed results are at Annex A, App 1.
45. Sea Clutter. Observations made in Japan in 1980 \{73\} relating sea clutter to Weibull, but at $\lambda=30 \mathrm{~cm}$, were made down to very low values of $\psi\left(0.13\right.$ to $\left.0.25^{\circ}\right)$. Sekine et al concluded that a Log Weibull relationship exists, and are currently checking this at $\lambda=3 \mathrm{~cm}$. Other relevant papers are by Shelerher \{74\} at about 24 GHz and RSRE \{75\} at 3 cm - all for sea clutter.
46. In view of the above conclusion in favour of Weibull which for temporal and small scale fluctuations has implications for CFAR arrangements - it was considered useful here to check some of the measurements taken by Dodsworth and others to see if they also exhibited Weibull for land backscatter. Results of the author's investigations into this are also at Annex A, App 1. False alarm rates are considered later in this chapter at para 85.

DEPENDENCE ON $\psi$
47. It has been clearly demonstrated $\{86\}$ that $\sigma_{0}$ increases rapidly as near grazing angles are reached, and as expected, $\sigma_{0}$ will also be higher at low grazing angles for rougher surfaces. $\sigma_{m}$, the median RCS, is used here for a brief investigation into the dependence of $\sigma$ on the grazing angle. Many reports use average RCS or $\gamma$ (see Annex B and para 49 below), caution should be exercised if comparisons are made.
48. While the quantity of measurements is now increasing, principly from space-based observation platforms' \{87\}, these are mostly taken at high grazing angles - usually down to about $\psi=20^{\circ}$. It is often difficult and imprecise to extrapolate the low angle significance of these measurements.
49. It is assumed that all targets of interest are in the near grazing zone - in which the use of the conversion $\gamma=\sin \psi / \sigma_{0}$ is of little use. As $\psi$ reduces, an appreciable rate of change in reflectivity seems to be initiated at values $10^{\circ}<\psi<15^{\circ}$ and unfortunately this corresponds to the lowest value of $\psi$ chosen by the majority of researchers in the past. Extrapolation difficulties can be seen from the general curve at Figure 4. This of course has limited the usuable results from which to evolve a model.
50. In Chapter 1, mention was made of the 'slope' or 'aspect angle' of terrain and the scant attention which appears to have been paid to this effect when measurements were taken. Clearly a change in terrain slope implies a change in $\psi$ for the particular resolution cell under investigation. This chapter confines investigations to selecting a model from those measurements already available. It is assumed that the values of $\psi$ are correct and the statistical spread of terrain slope within all the resolution cells scanned did not affect the measured RCS. The author's investigations into slope effects are considered separately at Chapter 10, \& AnnexF.Appl:
51. Trend of $\sigma_{m}$ with $\psi$ The well documented and widely quoted measurements of Linell \{88\}, together with as many others available with like (or near like) parameters were plotted by extracting $\sigma_{m}$ for variation of $\psi$. In all cases the results used were for rural, farmland, cultivated terrain and forest/woods. Many results were rejected. A few were interpolated, with care, into the lower values of $\psi\left(\right.$ eg $\left.15^{\circ}<\psi<70^{\circ}\right)$. The resulting plot at Figure 5 suggests the following conclusions for cultivated terrain:
a. A remarkable number of the curves give similar gradients which average approximately 1.25 dB per degree for $\psi>3^{\circ}$. Linell's results give a significantly different gradient.
b. There is a wide spread of absolute values of $\sigma_{0}$. However, it seems reasonable to expect this spread of values, taken in different countries, under variable conditions of moisture, wind, measurement accuracy, calibration differences and instrumentation (monitor losses).
c. The point at which the rate of change of reflectivity becomes more marked is around $\psi=2^{\circ}$. Below $2^{\circ}$ the slope could be reasonably be approximated by a second straight line with a gradient of approximately 5 dB per degree.
d. Linell's results (figure 5, curve 12) appear to come from a system which is far more sensitive to changes in $\psi$ than the others. It is not clear why this is so, but is may be a direct consequence of the 33 metre aerial height - and that a less shadowed area might provide a greater dynamic range of clutter levels.
52. A further point to consider is that of aerial gain towards a particular clutter patch. Some researchers mention this as part of their calibration process. Others, indeed few have not apparently corrected for this, or for "electrical tilt angle", sidelobe clutter, or variation of gain with range.
53. Forests and Woods. Figure 6, a similar plot for forest and wooded areas, is less explicit. A maximum of 0.5 dB per degree is taken as a reasonable value to use.

## SURVEY OF MODELS

54. Incorporation of $\psi$ into a set of model equations together with other parameters has been attempted by several researchers, but again these are often for higher grazing angles such as expected from space and using excessively large dimensions of resolution cell. Models investigated for similarity of results (in regression form) include:

$$
\begin{equation*}
\text { a. } \quad \sigma_{0}=-20+10 \log \psi / 25-15 \log \lambda d B m^{2} \cdot \mathrm{~m}^{-2} \tag{13}
\end{equation*}
$$

where $\psi$ is in degrees, $\lambda$ in cm .
b. $\quad \sigma_{0}=-15+15 \log \psi / 25-8 \log \frac{\lambda}{0.32} \mathrm{~dB} \mathrm{~m}^{2} \cdot \mathrm{~m}^{-2}$ where $\psi$ is in degrees, $\lambda$ is in metres.
c. $\quad \sigma_{0}=A+B \theta+C f d B m^{2} \cdot m^{-2}$ $\qquad$
where $\theta=90-\psi(\mathrm{deg}) \mathrm{f}$ is in GHz .
d. $\quad \sigma_{0}=-42.36+0.52 \psi+(24.93-0.3584) \log f d B \mathrm{~m}^{2} \mathrm{~m}^{-2}-(16)$
e. $\quad \sigma_{0}=A(\psi+C)^{B} \exp \left[-D /\left(1+0.035_{h}\right)\right] d B m^{2} \cdot m^{-2}$
where $\psi$ is grazing angle in radians $\sigma_{h}$ is RMS surface roughness ( cm )
f. $\quad \sigma_{0}=F_{s}^{4} 2 \times 10^{-\sigma_{f}} \sin \psi d B m^{2} \cdot m^{-2}$
where $F_{S}=$ spherical earth shadow factor
$\mathrm{f}=\mathrm{freq}(\mathrm{MHz})$
$F_{S}=2(\pi X)^{1} \cdot 5 \sum_{n=1}^{k} \exp \left(-j A_{n} X\right) f_{n}(h) d B m^{2} \cdot m^{-2}$
$F_{s}=1-0.465 \mathrm{X}$
for $x>1$
$x<1$
$X=R\left(2 \pi / \lambda^{2} r_{e}^{2}\right)^{1 / 3}$
Where $R=$ Range (metres)
$r_{e}=4 / 3$ earth radius
$A_{n}=$ set of complex constants
$\mathrm{n}=$ mode 1,2 ~-.-- $K$
$K=$ maximum of 40
$f_{n}(h)=$ height gain function.
g. $\left.\quad \sigma_{0}=\sigma_{0}^{1} /(1+R / R h)^{k}\right) d B m^{2} \cdot m^{-2}$
$\mathrm{R}=$ Range to clutter (km)
$R_{h}=$ Clutter horizon (km)
$\mathrm{k}=$ Constant (Value 4-12)
$\sigma_{0}^{1}$ (typical value as a constant $34 \mathrm{~dB} \mathrm{~m}^{2} \cdot \mathrm{~m}^{-2}$ )
h. $\sigma_{0}=-C 1+C 2 \log \left(\psi / \psi_{0}\right) C_{3} \log \left(\lambda / \lambda_{0}\right) d B m^{2} \cdot m^{-2}$

Typical values are: $\begin{array}{cc}\text { Cl } & 11.3 \\ \text { CL } & 26 \\ \text { CB } & 8\end{array}$

$$
\begin{align*}
& \psi_{0}=35^{\circ}  \tag{21}\\
& \psi_{0}=1 \mathrm{~m} \tag{22}
\end{align*}
$$

j. $\quad \sigma_{0}=\gamma_{t} f^{m} \operatorname{Sin}^{n} \psi d B^{2} \cdot m^{-2}$

Ens 21, 22. $C_{1}, C_{2}, C_{3}, \psi_{0}$ and $\gamma_{t}$ are terrain sensitive

$$
\begin{aligned}
& \text { Typical values are: } \begin{array}{rl}
\gamma_{t} & 2.1 \\
m & >0
\end{array} \\
& \text { n } 1.8 \\
& \text { c } 0.008
\end{aligned}
$$

(see comment on Barton's 'Unified model' at Annex F. App I. page F1-41)
$\{234\}$
55. Equation (13) above is an empirical formula based on statistical information. 10 dB extra should be added for foliated trees (dry) and 15 dB for wet trees. \{90\}. Fig 5 curve 11.
56. Equation (14) above \{91\} is taken as reasonable for Horizontal polarisation and more accurate at higher RFs than loGHz. Fig 6 curve 2.
57. Equation (15) is applicable over the range of frequencies $6-17 \mathrm{GHz}$ but to be used with caution at angles of $\psi<20^{\circ}$ \{92\}. Moore et al also include a general model with different coefficients. Referring to Figure 5, curve (6), the coefficients used for $A, B$ and $C$ were respectively $-7.09,-0.131$ and 0.315. While for Figure 6, curve (7) the results of Moore fit over Tomlinsons with negligible difference. Hence curves (3) and (7) are identical; with values of $-9.1,-0.12$, and 0.25 respectively.
58. Equation (16) contains the coefficients for forest plotted (3) at Figure 6 and is subject to an adjustment factor of $+0.91 d B\{93\}$. This equation format is the same for rural terrain (curve (1) at figure 5) but in this case the coefficients for equation (16) change to give:

$$
\begin{equation*}
\sigma_{0}=-23.61+0.994 \psi+(3.53+0.091 \psi) \log f \tag{23}
\end{equation*}
$$

and the 'adjustment factor' is $+0.79 \mathrm{~dB} .\{94\}$ goes further to discuss snow, desert, terrain and sea, with models for each type. His main objective was to obtain models for space based radars and so detailed measurements at low grazing angles were not required.
59. Equation (17) \{95\} is the first equation to incorporate RMS surface roughness, presumably to indirectly quantify RF in the model. Plots at Figs 5 and 6 (curves 14,15 and 8,9 respectively) use the stated empirical constants for $A B C$, as $0.079,1.5$ and 0.012 for rural and $0.019,0.64,0.002$ for forests. There is insufficient data to compute constant $D$ in both cases, although this is stated as 2.3 for soil, sand or rocks.
60. Equation (18), the FTD model \{96\} is based on generalised site geometry for $\psi=0.17$ to $0.05^{\circ}$, but not validated above 2.8 GHz , until present measurements at MIT are completed. Curve 16 at figure 5 shows this result using a $K$ of 3 .
61. Equation (20) \{97\} describes clutter as range dependent remaining constant up to the radar horizon. Beyond Rh the clutter decreases at 10 kdB per decade of range. It is not included on the curves at Figs 5 or 6.
62. Equations (21) and (22). Both developed by Georgia Institute of Technology, are included for completeness but have unfortunately not been validated at low values of $\psi$ and are not included at Fig 5 or 6 .
63. From the results examined, replotted and recalculated where necessary to fit the required parameters, it is concluded that the effect of $\psi$ on $\sigma_{0}$ is such that the median ( $\sigma_{m}$ ) backscatter increases linerly with $\psi$. in the range approx $0.5^{\circ}$ to $10^{\circ}$, but below $0.5^{\circ}$ the variance is likely to increase quite markediy. Values selected for the model here are considered at the Chapter Summary.
64. The main cause for polarisation sensitivity of backscatter is multipath reflections, hence polarisation effects are of concern only over relatively smooth surfaces. At $\lambda=3 \mathrm{~cm}$ (or greater RF), and very low values of $\psi$, the surface is not considered smooth in terms of the Rayleigh Roughness Criterion.
65. Linear Polarisation. For practical purposes, over general terrain a few $d B$ difference may exist between $\sigma_{\mathrm{HH}}$ and $\sigma_{\mathrm{VV}}$ linear polarisations; with horizontal being the higher. This has been well supported with a good spread of measurements over 9 different surfaces at Ohio State University \{98\}, by Cosgriffe et al, and is reprinted in Barton's textbook \{99\} pp 165-286 for easy reference. For general terrain it is proposed to neglect small differences at low grazing angles in the model at Chapter 11 , and for this reason polarisation was ignored in comparing the effects of $\psi$ and $\tau$ from the various sources earlier in this chapter. At the lowest values of $\psi$, where multipath surfaces exist, a maximum of 10 dB should be applied for horizontal polarisation.
66. Cross and Circular Polarisation. Cross and circular polarisation are of interest here in performance prediction, since the 52 techniques
may be employed to reduce clutter returns compared with the wanted aircraft signals. Few new polarisation results have become available since Allan's recent summary, except Tomlinson \{100\}, who reinforces earlier findings. The following conclusions apply:
a. For linear (plane) polarisation with cross-polar reception, the backscatter is likely to be up to 10 dB lower in the orthogonal plane than in the parallel plane. For isolated dominant reflectors (eg pylons), this difference may be over 20 dB .
b. At the frequencies in use and low values of $\psi$, lower than Brewster's angle ( $20^{\circ}$ for earth, $5^{\circ}$ to $10^{\circ}$ for sea), the sense of circular polarisation is probably not reversed.
67. The reader is cross referred to remarks on polarisation change for raindrop rejection at Chapter 3, and reminded that the RCS of aircraft may be reduced, (typically by 3 to 5 dB ), with the samesense circular polarisation; compared to perhaps 7 dB with crossed linear polarisation. Finally, polarisation effects are usefully considered in the following papers: Ament \{101\}, Rider \{102\}, Reiss et al \{103\}. Gent et al \{104\}, Brindley \{105\}, Daley et al \{106\}, Goodyear \{107\}, Linell \{88\}, Katz and Spetner (for $\psi>10^{\circ}$ ) \{108\}.
$\sigma_{0}$ DEPENDENCE ON RF
68. It has previously been stated that backscatter for snow covered terrain is difficult to predict, because of penetration, and it is
here that can be seen an analogous situation in attempting to isolate the effects of RF on backscatter. The dielectric properties of the surface are clearly all-important, since earth, like snow, is penetrated to an extent by microwaves and the actual electromagnetic roughness of the surface may not be visually apparent. Since the dielectric constant of terrain is also a function of $\lambda$, that which is seen as 'smooth' by a particular wavelength will be seen as rough by a shorter wavelength. An upwards change in RF therefore implies a change from 'smooth' to 'rough' if the change is such that:

$$
\begin{equation*}
\Delta h \sin \psi>\frac{\lambda}{8} \tag{24}
\end{equation*}
$$

where $\Delta h=r m s$ height of surface irregularities $\psi=$ Grazing Angle (after Rayleigh).
69. However, as stated above, microwaves will penetrate the surface (typically 1 to 10 cm \{106\}) dependent on the conditions - which might vary from one resolution cell to the next - even for the same surface material. And so it is seen that a general tendency can be concluded rather than absolute values. Long $\{86\}$ surveyed results in this area and states "the totality of experimental results do not yield agreement". It is probably reasonably to state that the wavelength dependance of $\sigma_{0}$ can be expressed generally in terms of $\lambda^{-n}$ (normally $0<n<1$ ). Classical interference effects (see Long $\{86\}$ pp 219-220) can in principle cause o to vary as fast as $\lambda^{-4}$ at grazing incidence, but this is for the ideal surface, and is perhaps applicable at sea.
70. It should also be noted that 'roughness', as viewed along the radar beam will depend on $\psi$, as in equation (24) at para 68, Once again because of the shortage of measurements available at low $\psi$, reliable data relating $\sigma, f$ and $\psi$ cannot be used to produce a model of adequate validity. Since this project involves RF's of 10 GHz (or above) it is assumed that all (land) surfaces are 'rough', and indeed this would be the case for measurements used here from all the sources used in earlier paragraphs.

## DISTRIBUTION AND CORRELATION OF SPATIAL AND TEMPORAL CLUTTER

71. Distribution. A composite scattering model where the probability density function $P_{\left(\sigma_{0}\right)}=f(x . y, t)$ is considered by $\{109\}$, who derive pdf's for use at sea, built-up areas, forest and rural conditions, measurements show time variations to be exponentially distributed. If the required value is $P_{t}$ then:

$$
\begin{equation*}
P_{\sigma_{0}}=\int_{-\infty}^{\infty} P_{t}(\sigma \mid m) P_{s}(m) d m \tag{25}
\end{equation*}
$$

$m$ is the average $\sigma_{0}$, taine into account local terrain slope in the resolution ceil. If the surface is flat (facet tilt zero - see Chapter 10), $P_{s}(m)=d(m-\mu)$. This is to be expected at sea with many independent scatterers. on gland if a $\log$ normal distribution is assumed then:

$$
\begin{aligned}
& P\left(\sigma_{0}\right)=\frac{10}{\sqrt{2 \pi} \log _{e} 10 s \sigma_{0}} \exp \left[-\frac{100}{2 s^{2}}\left(\log \sigma_{0}-\log \sigma_{o m}\right)^{2}\right] \\
& \text { with median } \sigma_{o m}=\mu \exp \left[-\frac{1}{2}\left(\frac{\log _{\mathrm{e}} 10}{10}\right)^{2} \mathrm{~s}^{2}\right]
\end{aligned}
$$

$$
\begin{align*}
& P_{t}\left(\sigma_{0} \mid m\right)=\left\{\frac{1}{m} \exp \left(-\frac{\sigma_{0}}{m}\right)\right\} \sigma_{0} \geqslant 0  \tag{26}\\
& =\left\{\begin{array}{l}
0
\end{array}\right\} \sigma_{0}<0-\infty-\infty-\infty \\
& P\left(\sigma_{0}\right)=P_{t}\left(\left.\sigma_{0}^{\sigma_{0}}\right|_{\mu}\right)\left\{\frac{1}{\mu} \exp \left(\frac{-\sigma_{0}}{\mu}\right)\right\} \sigma_{0} \geqslant 0 \cdots-\cdots(28) \\
& \left\{\begin{array}{l}
0 \\
0
\end{array} \sigma_{0}<0-\infty-\infty-\infty-\infty\right.
\end{align*}
$$

$P_{s}(m)$ is the spatial variation pdf
$\mu \quad$ is the spatial mean value of $m$
72. Correlation. The few published data on spatial correlation of land clutter are usually concerned with scanning (rotating) search radars. Results may not be applicable at all times to the pencil beam tracking radars under investigation here. With a circular scanning pattern the clutter components change continuously since clutter elements are regularly entering and leaving the illuminated surface footprint. For a narrow beam tracking radar this would occur most markedly for crossing target flown past at a velocity and range to produce a high sightline rate; reaching a peak rate at the tangential point. Radially or near radically approaching or receding targets could cause less effect.
73. In areas where large single man made objects occur, giving predominant specular returns, the probability of spatial correlation is less likely between adjacent resolution cells, but in normal terrain or forest, spatial correlation is likely to be higher, providing adjacent
cells are similarly tilted to the incident surface illumination. Seek Igloo \{110\} confirms distributions tending towards log-normal as sampled terrain becomes more homogeneous, although other recent research has shown a distribution falling somewhere between log-normal and contaminated normal. Earlier work by Dodsworth \{111\} isolated 'Past' and 'slow' components as the radar aerial scans the clutter surface. 'Fast' components are found to fluctuate with the median value equal to the running mean, while 'slow' conponents reflect the majority features of the terrain and are regarded as the running mean.
74. Strong clutter tends to occur in patches, giving good spatial correlation, sloped terrain giving the strongest values. For a presurveyed radar site position a terrain data base of the type proposed in Chapter 2 can indicate with fair accuracy the likelihood of positions of clutter patches. At the shorter (millimetric)wavelengths a good indication can be gained from large scale ordinance survey or more particularly vertical photographs of the area.
75. Since clutter is not evenly distributed in practice and it has been shown experimentally (see paras 38 to 40 above) that a change in resolution cell size does not bring about a proportional change in clutter, it is clear that as the radar beam scans with a fixed resolution cell size, (set by $\tau$ and $\theta_{A}$ ) the loss or gain of surface reflectors for part of the resolution cell, due to aerial rotation, will incraase or reduce the number of clutter producing elements in the cell and have a temporary effect as though actual resolution cell size is changing. With very large resolution cells (not usually applicable to the tracking radars), they become more likely to contain partly man-made and partly natural reflectors. In a mobile battle situation
there is some reason to suppose a higher probability that man-made objects will appear in tracker resolution cells since many vehicles are likely to be dispersed in the same area. However this will be dependent on local terrain screening conditions for a ground based radar. If a log-normal distribution is assumed, this will strictly only be applicable to a fraction of the resolution cells in an area (since many are ahadowed), or for only parts of cells - if the cells are large. This approach is confirmed at \{Il2\} where the cell values aggregated would produce a threshold which is applied to every cell.
76. Spatial Clutter Decorrelation. Autocorrelation factors derived for RF changes (frequency agility) have been researched at \{113\} where it is proposed that the autocorrelation function of clutter may be periodic, with increasing pulse to pulse RF change. Conditions will be expected to vary with $\tau$ and the number of dominant scatterers in the resolution cell, although $\{114\}$ found that decorrelation times of clutter were not appreciably affected by changes in $\tau$. Autocorrelation lengths investigated by Tomlinson \{115\} over several terrain types show almost like variations in autocorrelation coefficient irrespective of terrain type at ranges greater than approximately 4 km .

## CLUTYTER PATCH LENGTH STATISTICS

77. It will be shown in succeeding chapters that the factors affecting an overall effectiveness prediction model for a given tracking radar located at a known geographical position are closely related. No single aspect can be taken in isolation without considering the others. Although this report first attempts to separate these factors for more detailed examination before bringing
them finally together as a complete model, it is difficult to ignore the closely related topic of 'probability of obtaining a given track length' at this early point in the report. The importance of observable track length can be seen from Annex $E$, but the overall requirement for a certain system must include the probability of maintaining signal detection above the set threshold for the duration of the observable track length. Taking this a stage further, it concerns the probability of maintaining track under these conditions. Probability of holding radar track, loosing track or gaining a new track is also considered at Annex E.
78. Spatial clutter statistics can be presented in various ways:
a. Probability of clutter exceeding a given track length.
b. Probability of clutter patch separations exceeding given lengths.
c. Probability of exceeding set threshold levels.
d. Probability of clutter variations with range.
79. Clutter Patch Lengths and Discrimination. Two reports by the SHAPE Technical Centre $\{10$ \{ilf on clutter in Europe, together with Rigden \{il 8 in the UK and Briggs \{119) in the USA, have been considered, the results are interpolated, and re-presented in different forms at Figs 7 to 10. Clutter patches vary in length from a few metres up to 1400 m , although of course at varying signal levels. UK figures for a specific site $\quad$ 20) show that clutter $>\mathrm{Im}^{2}$ does not exceed about 30 m length while strong
levels such as $10 \mathrm{~m}^{2}$ are limited to patthes about 6 m in length. It is not clear whether the available US results are wrt $1 \mathrm{~m}^{2}$; however the main point of interest is the similarity of distributions at Fig 7. When replotted (plot not included) on log-normal graph paper these give sensibly straight lines over most of the patch lengths for $0.1<P<0.7$.
80. Clutter can be reduced by using pulse length discrimination. For the UK site about $75 \%$ of the clutter exceeds $0.1 \mathrm{~m}^{2}$ (Fig 8) and is in patches longer than 30 m - these could be removed simply by setting the appropriate thresholds. Fig 9 compares UK and European measurements.
81. If the probability of clutter exceeding given equivalent reflecting areas can be plotted from a knowledge of the terrain, this, together with the earlier date and data on clutter patch separations could lead to a model for radar tracking conditions by assessing the statistical opportunities when tracking can take place for given track lengths. These would of course be site specific assessments.
82. For tracking in clutter to be successful (as opposed to intermittent detection) the two cases are essentially:
a. Statistical likelihood of clutter patch separation such that the target may be tracked with no clutter present (ie target track held for a minimum time period).
and b. Those occasions where the target can be (additionally) tracked where the clutter level, though present, is negligibly
low - or can be processed out in the receiver.

In both a. and b. above, conditions must exist with a sightline to the target (see Chap $2-$ Screening) and a sightline to the clutter(in case b. abovel.
83. It is further proposed that from the statistics for a particular terrain area, an examination of track length unscreened, clutter patch length and the distribution of resolution cell slope facets, could produce a prediction for example "when a target enters an area Type A (eg flat terrain with $30 \%$ vegetation cover up to say lom high) with known target velocity and altitude, with missile and radar type ' $X$ ' deployed, there will be a $20 \%$ probability of the system obtaining a firing opportunity in which a complete engagement could occur". Further it might be possible to vary the prediction to take account of the higher probability expected where the radar system is deployed in a premeditated manner on a previously surveyed (optimum) site. For example a higher probability would be expected from a presuryeyed site in undulating terrain - since the probability of obtaining a target sightline is more likely as the target cannot maintain a set altitude clearance over terrain which undulates with a fast period. These points are considered further at Annex E and Chapter 10.
84. Variation of Clutter with Range. The SHAPE reports also express clutter probability in terms of range, the median values of which are replotted at Figure 10 (from fig 9 in \{121\} and Fig 4 a in (t22), converted in each case to give relative echoing area by applying a $R^{4}$ correction by taking the average range in each interval. The UK and USA results are also shown for comparison. It is seen that the median clutter
value reduces almost linearly with range for all sites and although the individual values are site-specific, the rate of change of clutter Level with range varies between approximately $0.5 \mathrm{~dB} . \mathrm{Km}^{-1}$ to $1.5 \mathrm{~dB} . \mathrm{Km}^{-1}$.

## FALSE ALARM RATES

85. It is not the intention here to repeat receiver processing options, such as MPI, which are well covered in many standard texts. However a brief mention of false alarm rates is appropriate.
86. At the receiver input will be a combined signal of noise, clutter and wanted target; from which the receiver will adjust the ratios to separate the target from the other unwanted signals. The distribution of the noise envelope at the detector input is given by the Rayleigh distribution. In weak signal conditions (ie wanted signal near noise level) the action of a detector is square law and the distribution of the signal envelope modified by the square law action will be:

$$
\begin{equation*}
P_{n}(v) d v=\frac{1}{2 a \sigma^{2}} \exp \left(-\frac{v}{2 a \sigma^{2}}\right) d v \tag{30}
\end{equation*}
$$

where $2 a \sigma^{2}$ is the mean value; a is a constant. $v$ is the detector output voltage. If the noise envelope exceeds a threshold $V_{t}$ a false alarm with probability $P_{f a}$ is given:

$$
\begin{equation*}
P_{f a}=\frac{1}{2 a \sigma^{2}} \int_{k n}^{\infty} e-\frac{v}{2 a \sigma^{2}} \tag{31}
\end{equation*}
$$

$=e^{-k n}$ (normalised threshold for noise) $=\frac{V_{t}}{2 e \sigma^{2}}$

A threshold is chosen to give a tolerable false alarm rate eg $10^{-6}$ and the probability of detecting the presence of a signal (or the probability that signal + noise exceeds the threshold $V_{t}$ ) is then:

$$
\begin{align*}
& P_{d}=\frac{1}{2 a \sigma^{2}(1+\bar{x})} \int_{k_{s}}^{\infty} \exp \left(-\frac{v}{2 a \sigma^{2}(1+\bar{x})}\right) d_{v}  \tag{32}\\
& =\exp ^{-k}
\end{align*}
$$

Where $\mathrm{k}_{s}$ is a normalised threshold (signal + noise) $=\frac{\mathrm{V}_{\mathrm{t}}}{2 a \sigma^{2}(1+\bar{x})}$
$\bar{x}$ is the mean signal to noise power ratio at the receiver input.

$$
k_{s}=\frac{k_{n}}{(1+\bar{x})} \quad\left(k_{n}\right. \text { defined on pll3) }
$$

hence $P_{f a}=e^{-k} n$
$P_{d}=e^{-k_{s}}$

$$
\begin{equation*}
\text { and } \log _{2} \mathrm{~d}=\frac{\log _{2} P_{\mathrm{fa}}}{(1+\bar{x})} \tag{35}
\end{equation*}
$$

87. For example if $P_{f a}=10^{-6}$ and $S / N=10$ then on the basis of a single echo:

$$
4-112
$$

$$
\begin{aligned}
P_{d}=\frac{10^{-6}}{1+10} & =-1.26 \\
\text { and } P_{d}=\exp -1.26 & =0.284(\text { ie } 28.4 \%)
\end{aligned}
$$

88. Assuming a number of successive pulses $N$ are integrated, each having crossed the threshold $V_{t}$ then:

$$
\begin{equation*}
P_{f a}=\frac{N^{N}}{(N-1)!} \int_{k_{n}}^{\infty} x^{N-1}\left(\exp -N_{x}\right) d x \tag{36}
\end{equation*}
$$

Where $k_{n}=\frac{v_{t}}{2 a \sigma^{2}} ; x$ is the value at any instant.

The solution to this integral is tabulated by standard methods as the incomplete Gamma function, of which a solution is:

$$
\begin{equation*}
P_{f a}=e^{-N x}\left(1+N_{k}+\frac{\left(N_{k_{n}}\right)^{2}}{2!} \cdots \cdots \cdots \cdot\left(\frac{\left(N_{n}\right)^{N-1}}{(N-1)}\right)\right. \tag{37}
\end{equation*}
$$

and the probability of detection after integrating $\mathbb{N}$ samples of the signal noise is:

$$
\begin{equation*}
P_{d}=\frac{N^{N}}{(N-1)}!\int_{k_{s}}^{\infty} x^{N-1} \exp -N x d x \tag{38}
\end{equation*}
$$

A solution is:
89. Assuming $N=2, P_{f a}=10^{-6}$ and $x=10$ then $P_{d}=55 \%$, compared with $28.4 \%$ when $\mathbb{N}=1$ (see para 87 above).
90. Figure 11 shows $P$ when $N=8$ for a radar operating on a number of frequencies $\left(N_{f}\right)$, where the probability density function of $N$ integrated pulses is:

$$
\begin{equation*}
P(x)-N^{N} \frac{x^{N}-1}{(N-1)!} \exp \left(-N_{x}\right) \tag{40}
\end{equation*}
$$

for which the general case is

$$
\begin{equation*}
P(x)=\frac{a^{b}}{(b+1)!} x^{b-1} \exp (-a x) \tag{41}
\end{equation*}
$$

$\mathrm{a}, \mathrm{b}$ are constants where if $\mathrm{b}=1$ and $\mathrm{a}=\frac{1}{\mathrm{x}}$ then

$$
\begin{equation*}
p(x)=\frac{1}{x} \exp \frac{x}{x} \tag{42}
\end{equation*}
$$

a and b are deduced from:

$$
a=\frac{N_{f}(1-\bar{x})}{N_{f}-1+\left(1+N_{f} \bar{x}\right)} 2 ; b=\frac{N_{f}(1+\bar{x})^{2}}{N_{f}-1+\left(1+N_{f} \bar{x}\right)} 2^{2}
$$

91. For further details the readeris referred to Swerling \{123\}, \{89\} or Marcum \{124\}, to Chapter 6, para 14, and to Annex E, para 29.
92. Ideally an adaptive value of $\mathrm{V}_{\mathrm{t}}$ is required to take account of the variations in clutter received from each resolution cell since the probability of detection is a function of false alarm probability and signal to noise

$$
4-114
$$

ratio as dictated by the statistical model. The overall probability of detection is found from:

$$
\begin{equation*}
\text { (overall) } P_{d}=\int_{0}^{\infty} P_{d} P(c) d C \tag{43}
\end{equation*}
$$

93. If $\mathbb{T}_{r}$ and $C_{r}$ are received Target and Clutter power respectively; and $\mathbb{N}$ is noise power (referred to receiver input) and if $I$ is the improvement factor then, from (35):-

$$
\begin{equation*}
\log _{e} P_{d}=\frac{\log _{e} P_{f a}\left(C_{r}+I!\right)}{1+I T_{r}} \tag{44}
\end{equation*}
$$

From (43) above, the overall probability of detection in clutter will be obtained:

$$
\begin{equation*}
\text { (overall) } \left.P_{d}=\int_{0}^{\infty} P_{f a}\left(1 /\left(1+I T_{r} / C_{r}+I N\right)\right)\right) \quad P(C) d C \tag{45}
\end{equation*}
$$

and with $C_{r}$ (assuming a log-normal pdf):

$$
\begin{equation*}
\left.P\left(c_{r}\right)=\frac{1}{\sqrt{2 \pi} \sigma C_{r}} \quad \exp \cdot\left(-\log _{e}\left(C_{r} / C_{m}\right)\right)^{2} / 2 \sigma^{2}\right) \tag{46}
\end{equation*}
$$

Where $C_{m}$ is the median of clutter power $C r$.
Combining together (46) and (45):

$$
\begin{array}{r}
\text { (overall) } \left.P_{d}=\int_{0}^{\infty} P_{f_{a}}\left(1 /\left(1+I T_{r} / C_{r}+I N\right)\right)\right) \\
\frac{\exp \left(-\left(\log _{C}\left(C_{r} / c_{m}\right)\right)^{2} / 2 \sigma^{2}\right) d C_{r}}{\sqrt{2 \pi} \sigma C_{r}} \tag{47}
\end{array}
$$

94. Some assumptions have been made here concerning the false alarm probability since the work by Marcum and Swerling are based on a constant false alarm rate whereas (as at para 92) this will not strictly be the case. However any prediction model will necessarily operate within constraints; since many modern radars will have MTI filters the basic eqns above will not always apply directly but will be subject to certain assumptions of clutter residue characteristics after passing through the MTI filter compared with receiver thermal noise levels. similarly there may be receiver nonIinearities which introduce changed statistics, however the above equations assume no receiver limiting. Pursuance of the relative performance of limiting circuitry is beyond the scope of this report. Thes $R_{0} / d$ seltings for Rayleigh, Ricean and Log-Narmal clulter are at Figs12-14.: CHAPIER SUMMARY
95. At the outset of this research it was decided to include many terrain types from a wide variety of sources to provide a broad basis for a general prediction model; rather than basing the conclusions on a few models albeit with more precise values which may be site-specific. A model is thus sought which is both simple and contains adequate statistical information to give reasonable integrity for a generalised prediction.
96. In the past it may be that excessive importance has been attached, to distributions and curve fitting to clutter prediction statistics. For acceptable false alarm rates the signal to clutter ratio must be very high, hence only the tails of distributions are of real interest. Predictions at these extremes may be based on excessive interpolation.
97. It is clear that land statistics are less easily related to the surface than sea clutter statistics and that low grazing angles produce variable statistics which are much affected by 'shadowing'. As the grazing angle increases the shadowing effect diminishes and the s.d. decreases.

## Rural Terrain

98. By careful examination of the plots of clutter values (figure 5) for grazing angle, and by rejecting the space-based results $\{94\}\{92\}$ and $\{88\}$ respectively shown at Figure 5 as curves (1) (2) and (12), a model is proposed for rural terrain as follows:

$$
\begin{equation*}
\sigma_{m}=A+B \psi \tag{48}
\end{equation*}
$$

By regression analysis $A=-32.22, B=1.017$, with correlation coefficient - 0.99. This is plotted at Figure 5 at curve (17). It is seen that the model forms ${ }_{A}^{a}$ reasonable median of the world-wide results surveyed. A gradient of $1.25 \mathrm{~dB} /$ degree is taken for $\psi>3^{\circ}$ and $5 \mathrm{~dB} /$ deg for $\psi<3^{\circ}$

## Forest Terrain

99. Similarly the coefficients proposed for forest are $A=-331$ $B=+1.625$. This is plotted at figure 6, curve (10). This proposal equates well with the model at eqn (8). An adjustment of 3 dB is necessary for vertical polarisation or with snow cover, and 5 dB for wet trees (see also para 101 below). Both rural and forest results are based upon analysis of worldwide data.

$$
4-117
$$

100. The general trend for low $\psi$ and homogeneous terrain is that the median backscatter coefficient increases linearly withfrequency for most terrain types, out to the horizon; and more variably thereafter (seealso Page F1-41). Anew model is proposed for $K$ Band at App 1 to Annex $F$.

## Snow Cover

101. Due to temporal, snow depth, water content and polarisation variations a reliable backscatter model is probably impossible to assess especially as these parameters may vary from one resultion cell to another in any but the most homogeneous conditions. Up to 10 dB should be added if conditions of free-water exist due to partial thawing and re-freezing.

## Pulse Length

102. Median $\sigma_{0}$ is taken to vary with $\tau$ as suggested by Dodsworth but (inconclusively) related to the Weibull shape parameter as investigated at Appendix 1 to Annex A.

## Simulation of Clutter

103. Computer simulation of site specific data is possible within reasonable limits if a precise digital data matrix is available. In general however a digital landmass data base does not provide precise terrain screening information for vegetation. Therefore sightline information would be unreliable for an unknown site.

New Backscatter V Grazing Angle Model Based on Measurements at K Band:
104. The model developed by the author from raw radar measurements is also plotted at Figure 5 (Curve 18). Details of the analysis method are at Annex F, Appendix 1 . They confirm the general model, although at a higher value of RF. When the data was taken as an entity it did not exhibit the reversal of $\sigma_{0}$ at low grazing angles reported by some other researchers. It is on interest that the values obtained clearly plot as a Weibull distribution and statistical tests show they are definitely not log-normal. 105. Selective Analysis: when the K Band data (at Annex F, Appendix 1) was examined critically, and outlying values from specular reflectors and probable sidelobe leakage removed; it became apparent that the clutter values did in fact rise at very low grazing angles. This confirms the reports mentioned above. Possible causes of this phenomena, including the possibility of terrain measurement errors, are at Annex $F$, Appendix 1 , with many of the results. In the course of this analysis considerable care was taken in matching the measured backscatter to the terrain matrix and hence to the surface gradient concerned.
106. The results obtained compare favourably with Barton's latest unified clutter model proposed at this frequency, but not apparently - supported by published measurements at present.


FIG 1 POINT SCATTERER CLUTTER REPRESENTATION


FIG 2 effect of pulse lengit ratios on clutter probability distribution

FIG 3 EFFECT OF PULSE LENGTH ON CLUTTER


FIG 4 DEPENDENCE OF $\sigma_{0}$ ON $\Psi$


FIG 5 VARIATION IN $\sigma_{0}$ WITH $\Psi$ FOR RURAL, FARMLAND \& CULTIVATED TERRAIN AT ( $\lambda=3 \mathrm{~cm}$ )

fig 6 variation in $\sigma_{0}$ with $\Psi$ for trees, forest at ( $\lambda=3 \mathrm{~cm}$ )

fig 7 comparison of probablity that clutter patch lengths EXCEED RADIAL LENGTHS L - RURAL TERRAIN


FIG 8 PROBABILITY DISTRIBUTION OF CLUTTER REFLECTING AREA
RADAR CROSS SECTIONS FOR UK RURAL TERRAIN
（after Rigden）（118）
(LOGNORMAL)


FIG 9 PROBABILITY OF EXCEEDING REFLECTING AREA FOR SPECIFIC UK AND EUROPEAN SITES


## FIG 10 MEDIAN CLUTTER RETURN LEVEL (ABOVE MINIMUM DETECTABLE SIGNAL) FOR RANGE



FIG 11 PROBABILITY OF DETECTION FOR GIVEN $S / N\left(N=8, n=10^{6}\right)$


FIG 12 threshold setinggs above mean for rayleigh clutter


FIG 13 THRESHOLD SETTINGS FOR RICEAN CLUTTER

fig 14 THRESHOLD SETtINGS fOR LOG-NORMAL CLUTTER

## CHAPTER 5

## NON STATIONARY CLUTITER

WIND, AIR TURBULENCE, CHAFF AND BIRDS

## WIND EFFECT ON GROUND SCATTERERS

1. Since the re-radiation of electromagnetic energy from ground objects (and chaff) moved by the wind can have significant effects on backscatter, a survey was made to isolate practical wind or turbulence parameters for incorporation into the detection model. Twigs, branches, grass, crops etc all oscillate in the wind and when illuminated with centimetric (and especially millimetric) radar energy will contribute almost all possible electrical phases to the overall backscatter signal. Hayes \{125\} has shown a directly increasing relationship in signal fluctuation rate with increasing windspeed, as would be expected; but researchers have generally found a lack in correlation for measurements of tree fluctuations for yarying polarisations; in particular when using a pencil beam at 3 cm wavelength. Radar observations on foliage have generally yielded Rayleightype statistics (See Annex A), with many researchers concluding that the ground echo from a resolution cell is likely to contain a sensibly steady component plus fluctuating echoes caused by oscillating surface motion; thus modifying the components into a Ricean distribution (also see Annex A).
2. Land and sea doppler spectra and surface radar cross section of foliage are all windspeed dependent. Kerr \{126\} confirmed ground echo amplitude to be peaked at a value near the amplitude of the constant component. As the wind increased, the ratio of the clutter emanating from the moving component increased, compared with the steady component of the overall signal. When the steady to moving ratio $\mathrm{m}^{2}$ is small, ie <1,
there is little difference between equations (9) and (10) at Annex A. As $\mathrm{m}^{2}$ increases, the distribution approaches a Gaussian shape centred about the ratio of the steady echo component. This effect has been well established at $\lambda=3 \mathrm{~cm}$ to show variation with surface culture \{127\} with various polarisations, with and without snow cover, with small grazing angles and at different times of the year.
3. Sensitivity at Certain Windspeeds. Hayes and Walsh \{128\} found an abrupt increase in fluctuation rate comprising positive to negative reversals in slope and vice versa near windspeeds of $10 \mathrm{mph}\left(44 \mathrm{~ms}^{-1}\right)$, and that leaves and twigs are likely to be in constant, rather than intermittent, motion at $8-12 \mathrm{mph}$. Other researchers (Barlow, Fishbein, Graveline, Kerr and Ritenbach) agree that the spectra are more complex than the basic Gaussian distribution. As an example, in wooded areas the sd of clutter using Gaussian values would be 25 Hz in 11 GHz . Wind-produced clutter is part of the overall clutter characteristic, where, for example, the Rayleigh characteristic is often seen in the homogeneous clutter of urban areas and very rough terrain having high intensity tails tending towards the lognormal.
4. Motion of Radar Beam. Additionally, wind spectra are found to be broadened by the motion of a radar beam \{129\}, but this is more appropriate to radars on moving platforms such as aircraft and it is not thought to be applicable to this particular study; since it is assumed that a low-level tracking radar will be stationary when tracking even though in a mobile radar system some associated acquisition radars may have limited ability to acquire on the move. When tracking, the land-based radar tracker beam only moves slowly, compared with aircraft speeds.

## SURVEY

5. Table 1 surmarizes a survey of 30 years work on windspeed effects

TABLE 1 SURVEY OF EFFECTS OF WIND ON SURFACE CLUTTER

| Serial | Year | Researcher | Scatterer | ( cm ) | $\begin{aligned} & \text { Wind } \\ & \text { Velocity } \\ & \text { (MPH) } \end{aligned}$ | Pol | Distribution | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1949 | BARLOW * | Woods, Sea, Rain, Chaff | 30 | - | - | See para 5 |  |
| 2 | 1951 | KERR * | Woods, Sparse, Rocky | 9.2 3.2 1.25 | 25-50 Gusts | - | Gaussian (Approx) |  |
| 3 | 1956 | IVEY* et al | Deciduous and Coniferous | - | Moderate | HEV |  | Also at 35 GHz |
| 4 | 1957 | HAYES et al | " " " | 3 | $0-7$ $2-15$ | //8X $/ 18 \mathrm{X}$ | See para 5 | 1.860 Pencil Beam PW $0.25 \mu \mathrm{sec}$ |
| 5 | 1959 | HAYES et al | Deciduous | 3 | - | All | Rayleigh |  |
| 6 | 1963 | I.INNELL | Forest and Cultivated | 3 | - | Variable | $\begin{aligned} & \text { Lognormal (approx) } \\ & \text { and Rayleigh } \end{aligned}$ | Grazing Angles $0.7^{\circ}, 1.25^{\circ} \text { and } 5^{\circ}$ |
| 7 | 1967 | $\begin{aligned} & \text { GUIMARD * } \\ & \text { et al } \end{aligned}$ | Unknown | 3 | - | H\&V | Ricean | Also $P, L \& C$ Bands down to $5^{\circ}$ |
| 8 | 1967 | $\begin{aligned} & \text { FISHBEIN * } \\ & \text { et al } \end{aligned}$ | Deciduous | 3 | 10 | H | See para 5 |  |
| 9 | 1968 | DALEY et al | Unknown | 3 | - | HH\&VV | Rayleigh (see also Valenzuela) | Also P, L \& C Bands |


| Serial | Year | Researcher | Scatterer | $\begin{gathered} \lambda \\ (\mathrm{cm}) \end{gathered}$ | $\begin{gathered} \text { Wind } \\ \text { Velocity } \\ \text { (MPH) } \end{gathered}$ | Pol | Distribution | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1974 | $\begin{aligned} & \text { Rosenbaum * } \\ & \text { et al } \end{aligned}$ | Forest | 23 | Variable |  | Rayleigh |  |

Note 1. //ex represent parallel and cross-polarisations
2. "All" includes circular polarisation with //\&X polarisations
*. Details of this reference in Part II bibliography
(made on $\lambda=3 \mathrm{~cm}$ wherever possible), however few measurements are available at the lower grazing angles necessary for this study. The results of Hayes (serial 4) and Linnell (serial 6) most nearly use parameters of particular interest. Linnell \{130\} used a 25 metre resolution-cell radar mounted on a 30 metre tower with a vertical beamwidth of $30^{\circ}$ and horizontal beamwidth 1.40. Results included $15-17 \mathrm{~dB}$ standard deviation at $0.7^{\circ}$ grazing angle and an approximately lognormal distribution. Fishbein et al (serial 8) produced a relationship which gave good agreement with measured power spectra but for deciduous foliage only and horizontal polarisation:

$$
\begin{equation*}
p(f)=\frac{1}{1+\left({ }^{f} / f_{c}\right)^{3}} \tag{1}
\end{equation*}
$$

Where $f_{c}=1.33 e^{0.1356 V} \quad v=$ Windspeed (knots)
6. Wind Effect at Sea. Wind effects on the surface at sea also cause significant radar signal pertubations but since this report is only concerned with radar tracking overland, values for wind effects at sea are not required for the model.

## CHAFF CHARACTERISTICS

7. Chaff is a feature of the modern military electronic countermeasures scenario and it is not the intention of this study to examine the possible disturbance effects of chaff on the radar tracking function when streamed or rapidly bloomed, but only as a clutter source. Statistical characteristics of chaff are similar to rain and therefore demand similar signal processing requirements to minimise degradation of radar performance; dependent on RF and the spectral width of the clutter so caused. The instantaneous position of chaff within the radar tracking beam is dependent upon windshear, windspeed and air turbulence. Windshear occurs when the radial windspeed varies
vertically through a radar beam. Dodsworth \{131\} in a note on windshear refers to $\{132\}$ which shows windshear to be largely indepentent of altitude. Ref $\{133\}$ gives typical windshear values of 1 or $2 \mathrm{~m} \sec ^{-1} \mathrm{~km}^{-1}$ of altitude. A typical MTI canceller can be made to eliminate the mean effect of wind velocity within limits.
8. Beam Broadening, Turbulence and Chaff Fall-Velocity distribution can all be considered as producing independent Spectra, but if all effects are sumed, $\{134\}$, a Gaussian variance distribution can be taken as a good fit. Beam Broadening is a wind effect (small compared with windshear or turbulence) with a typical sd $\sigma_{\text {beam }}=0.42 V_{0} \theta_{2} \sin \beta$, where $V_{0}=$ Wind Velocity, $\theta_{2}=2$ way half power beamwidth (rads) and $\beta=$ azimuth angle relative to wint directionat centre of theam.
9. Chaff Dispersion. Once dispensed, chaff will disperse under the influence of the local turbulence. Windshear rates in the USA appear to be more severe than those in Europe, perhaps as high as $5 \mathrm{~m} \mathrm{sec}^{-1} \mathrm{~km}{ }^{-1}$ in altitude. This is contrasted with a typical maximum chaff fall-rate of $0.7 \mathrm{~m} \mathrm{sec}{ }^{-1}$ for 3 cm wavelength chaff. Under turbulent conditions it has been shown \{135\} that chaff under the influence of eddy transport speeds can exceed mean wind speeds $\{136\}$, and this causes a considerable problem in assessment. Haddow \{137\}, concluded that the time-distance movement of eddy carried chaff cannot be quantified with any degree of accuracy under all conditions. Two aspects of chaff must however be considered - attenuation and backscatter.
10. Chaff Attenuation. The total RCS of dispersed chaff seen within the radar resolution cell will naturally depend upon a sightline, and, as described in earlier chapters, this may be intermittent, clear with no underlying clutter, or additional to underlying surface clutter. To completely obscure a target, ie to prevent energy reaching the target or
returning to the radar from the target, it can be shown that an exceptionally dense chaff cloud would be necessary. For 2-way attenuation of a uniform chaff cloud of thickness $D$ and chaff dipole density of $N$ per unit volume, then:

$$
\text { attn }=e^{-\bar{\sigma}} \operatorname{ND}\left(\begin{array}{ll}
\text { as } & a  \tag{2}\\
\text { factor }
\end{array}\right)
$$

Where $\bar{\sigma}_{s}$ is the average radar cross section per dipole. The product $\bar{\sigma}_{\mathrm{s}} \mathrm{N}$ is the volume reflectivity density $\Sigma \sigma$ in $\mathrm{m}^{2}$ per unit volume. Expressing this in $d B$ per metre

$$
\begin{equation*}
\text { 2-way attn }\left(\mathrm{dB} . \mathrm{m}^{-1}\right)=-4.34(\Sigma \sigma) \tag{3}
\end{equation*}
$$

Where $\Sigma \sigma$ is in units $\mathrm{m}^{2} \mathrm{~m}^{-3}$. A heavy chaff cloud may comprise a chaff reflectivity density of approximately $3000 \mathrm{~m}^{2} \mathrm{~nm}^{-3}$ (corresponding to $475 \times 10^{-9} \mathrm{~m}^{2} \mathrm{~m}^{-3}$ ). Therefore to attenuate a radar return by 3 dB would require a chaff cloud of thickness 800 nautical miles ( 1500 km ); clearly an impracticable situation!.
11. Chaff Backscatter. Although signal attenuation due to chaff could occur momentarily under certain conditions when the chaff is self-dispensed and providing the dipole spacings are for a short period of the order a wavelength apart, significant volume attenuation is not a factor of consequence compared with that of backscatter. As implied at para 7 above, chaff may be dispensed by military aircraft so as to bloom rapidly within the radar resolution cell in the hope of breaking tracking capability or at the very least to disturb tracking accuracy by forcing the radar boresight to move to a different tracking centroid. Success or otherwise depends upon many factors in the radar system, such as tracking loop time constants,
resolution cell size, deceleration of the chaff, causing range gate pull-off and the effects caused upon velocity gates, and several other factors $\{138\}$.
12. After dispersion the chaff dipoles are randomly distributed by turbulence and researchers $\{139\}$, $\{140\}$, have discovered as many as six modes of fall when chaff of mixed characteristics is dispensed. Vakin and Shustor \{141\} suggest 2 main fall modes, one predominantly horizontal and the other vertical. In the absence of shadowing and clumping effects the idealised RCS of a number of dipoles $N$ is:

$$
\begin{equation*}
\sigma_{\text {total }}=0.18 \lambda^{2} \mathrm{~N} \tag{4}
\end{equation*}
$$

Howeyer the chaff may not be cut to precisely the radar transmission frequency (particularly with frequency agile radars), all dipoles may not contribute ideally and the chaff material will have some finite conductivity. More recent measurements \{142\} state that the RCS based on $0.14 \lambda^{2}$ is more likely. RCS varies with dipole thickness as well as length, and maximum RCS can be approximated for practical purposes as:

$$
\begin{equation*}
\sigma_{\text {total }}=0.14 \lambda^{2} \mathrm{EN} \tag{5}
\end{equation*}
$$

$E$ is the dispersal efficiency (ie a scattering efficiency factor) which may vary between 0.3 and 0.6 . Actual RCS acheived per unit weight of dispensed chaff is of course also dependent upon the chaff type.
13. The use of MII is likely to cancel most of the chaff spectral effects, where low frequency clutter over several KHz may be eliminated by a notch filter. If the entire chaff cloud was subjected to wind gusting in the same direction as an aircraft flying it could be evident in more than one
resolution cell, but it is of diminishing importance when using MII; and as explained above could not shield radar energy from reaching a target at greater range.

## BIRD ECHOES

14. Backscatter from birds can cause clutter at the very low altitudes relevant to this study. Bird clutter (also known as "angels") is briefly explored under the following headings:
a. Height and Velocity distribution.
b. Bird Radar Cross Section and distribution within a population.
c. Radar resolution cell, polarisation effects and spatial density.

Limited measurements were found at $\lambda=3 \mathrm{~cm}$, but the results of several papers at other RF's are in reasonable agreement.
15. Height and Velocity Distribution. Results from several researchers at different geographical locations show that $80 \%$ of all birds are encountered below altitudes of 250 metres and velocities spread between 10 and $25 \mathrm{~m} \mathrm{sec}{ }^{-1}$.
26. Bird RCS and Distribution. Mean RCS per single (medium sized) bird is unlikely to exceed $10 \mathrm{~cm}^{2}$ (pigeon at $\lambda=3 \mathrm{~cm}$ ), and in isolation will not be confused with an aircraft RCS. However flocks of birds very close together can reach clutter proportions. The distribution of echoing areas will naturally depend upon the proportion of birds of various sizes in a. particular location, but in general will be lognormal.
17. Resolution Cell and Radar Polarisation. Effects of resolution cell size investigated by several workers, eg, \{143\}, \{144\}, were in some cases made using a pencil beam tracking radar, but with longer pulse lengths than applicable here. Although there is no accurate prediction of the effect of varying resolution cell size on bird echoes, a smaller cell size would split up larger groups of birds into perhaps a number of adjacent cells, reducing the observed RCS from the flock. Minimisation of bird returns by using circular polarisation has been shown $\{145\}$ to be non productive since the clutter reduction obtained is approximately the same as for the wanted aircraft targets and hence target filtering is not acheived.
18. Spatial Density. Several researchers have attempted to quantify the density of bird clutter echoes likely to be present within a PPI search area (assuming these are not filtered out by the signal processing). Averaged over one year in UK, the probability of one bird echo per $\mathrm{km}^{2}$ is slightly less than $0.5 \%$. However, a typical PPI may typically scan $1000 \mathrm{~km}^{2}$ on each $360^{\circ}$ sweep, and so the probability of some bird activity at most locations is high. With a target tracking radar following at target sight-line rate (or almost stationary for closing or receding targets), birds may enter, leave, or pass through the resolution cell of interest at any time.

## CHAPTER SUMMARY

19. Factors selected for incorporation in the overall prediction algorithm from this chapter are:
a. Wind Effects. Values researched by Hayes are used in conjunction with the surface cover discussed at Chapter 4. Hayes uses $F=\frac{1}{1+\left(\frac{f}{9}\right)^{3}}$ for 9.4 GHz and $\frac{1}{1+\left(\frac{f}{35}\right)^{2}}$ at 95 GHz , giving, for a windspeed of 12 kts a half-power value of 9 Hz .
b. Chaff Attenuation and Backscatter. Equations (2) and
(3) are used for modelling radar signal attenuation due to dispersed chaff between target and radar. Backscatter is incorporated using equation (5), for non-MII radars only.
c. Bird Clutter - 'Angels'. Bird clutter can be expected in wooded locations and may cause significant signal returns at any time, but more particularly so in migratory periods and at sunrise and sunset. However, it is assumed that once a target is correctly range-gated and velocity-gated by a narrow-beam tracking radar with good discrimination (and since total tracking periods are likely to last for no more than 60 secs for really low level fast targets); then 'angel' effects are minimal for the tracking radar itself. It should be noted that overall system effectiveness may be reduced if 'angels' clutter degrades an area search radar's performance to the extent that "hand-on" to the associated tracking radar is delayed or prevented.

## CHAPIER 6 <br> RADAR SYSTEM AND TARGET DETECTION PARAMETERS

1. Preceding chapters have show that the probability of successful detection and tracking very low - altitude targets is dependent on a great many variables. Additional to the very basic requirement of a direct sightline (or a set of fortuitous diffraction conditions), together with the imposition of clutter - and even jamming signals - the end result is finally dependent upon the radar system characteristics and the given target response. This chapter sumarises the radar system parameters considered and their relationship in the radar equations used in the model. Some parameters, when varied slightly, become critical; since the very nature of the study involves targets which are likely to be often on the threshold of detection.

RADAR SYSTEM
2. Within the radar system, account must be taken of the radiation pattern, the transmitter waveform characteristics and the signal processing of the target and clutter returns in the radar receiver. Equations to describe clutter and target power received, receiver noise and jamming effects are fairly standard, however many basic texts generalise certain losses which have been considered here in more detail. Specialised references such as \{146\} and \{147\} give adequate relationships for such topics as aerial motion and jamming. It has been necessary to include the whole range of parameters for a complete model, but it is not the intention to investigate every parameter in detail. Once the model vas completed, further investigations, based on the model, were made into diffraction and terrain slope effects (Chapters 7 and 10 ).
3. Radar system parameters considered, together with typical values are shown below. A sample calculation for this system is shown at Annex $G$.

|  | Example System |  |
| :---: | :---: | :---: |
|  | TRACK-WHILE SCAN (TWS) |  |
| $\begin{aligned} & \text { a. Aerial Gain (Mainlobe) } \\ & \text { (dB). } \end{aligned}$ | 2290 | 33.6 dB |
| $\begin{aligned} & \text { b. Aerial Gain (Sidelobes) } \\ & \text { (dB) } \end{aligned}$ |  | - 20 dB |
| $\begin{aligned} & \text { c. Peak Transmitter Fower } \\ & \text { (Kw) } \end{aligned}$ | 150 |  |
| $\begin{aligned} & \text { d. Operating Frequency } \\ & (\mathrm{GHz}) \end{aligned}$ | $10 \mathrm{GHz}(3 \mathrm{~cm})$ | - 15.2 dBm |
| e. Receiver Noise Figure (aB) | 8 | 9.03 dB |
| f. System Losses ( dB ) | - | 12 dB |
| g. Pulse Duration ( $\mu \mathrm{sec}$ ) | 1 |  |
| h. Azimuth \& Elevation | $2 \times 9$ |  |
| j. Aerial Polarisation | H |  |
| k. Integration Improvement $\text { in } S \text { N ratio } d B$ | - |  |
| 1. Radar Aerial Height above datum (m) | 20 |  |
| m. Radar Type (eg MTI, PD) | PD |  |
| n. Aerial Radiation Pattern | - |  |
| p. PRF (Hz) | 12000/10750 |  |
| q. Rotation (scan) | 60 rpm |  |
| $\begin{aligned} & \text { r: } \\ & \left(\mathrm{m}^{2}\right) \end{aligned}$ | 0.05 | $-13 \mathrm{dBm}^{2}$ |
| s. Signal Processing | - |  |


| t. Range: Accuracy (m) | $\pm 30$ ( $20 \%$ of, | esolutiong) cele) |
| :---: | :---: | :---: |
| u. Velocity Resolution | $\pm 15$ |  |
| $\left(\mathrm{m} \cdot \mathrm{~s}^{-1}\right)$ |  |  |
| $\begin{aligned} & \text { V. Azimuth Resolution } \\ & (\mathrm{deg}) \end{aligned}$ | $\pm 0.5$ |  |
| w. Frequency Agility | - |  |
| $\begin{aligned} & \text { x. Tracker mode } \\ & \text { (eg monopulse) } \end{aligned}$ | TWS |  |
| y. Transmitter Character- <br> istics | 4 bursts of 10 pulse per scan |  |

RECEIVED TARGET, CLUTYER AND JAMMING POWERS
4. At the receiver, target, clutter, attenuation and jamming powers are largely dependent on statistical distribution dependent upon cross section and on PRF, pulse duration, transmitted power, and multipath. The overall signal/noise ratio is given by:

$$
\begin{aligned}
& \text { where } \operatorname{Sig}_{(\text {tgt })}=\text { Signal from target } \\
& F_{(\text {attn 2) }}=2 \text { way attenuation factor } \\
& F \text { (mult) }=2 \text { way multipath effect } \\
& N_{R X} \quad=\text { Receiver Noise } \\
& \mathrm{Sig}_{(c l t)}=\text { Surface Clutter Signal } \\
& \mathrm{Sig}_{(\mathrm{wtr})}=\text { Clutter Signal (Atmospheric and Weather) } \\
& \operatorname{Sig}_{(j a m)}=\text { Jamming Signal from Target } \\
& F_{(\text {attn } 1)}=1 \text { way attenuation factor }
\end{aligned}
$$

5. Only the self-screening jamming signal (assumed to be noise) in the main beam is considered. Later it will also be seen that the numerator at (1) can be modified to allow for diffraction and the denominator adjusted to incorporate a factor for terrain clutter variation with slope.
6. Using the standard (unmodified) radar equation (pulsed radar) the received $\mathrm{S} / \mathrm{N}$ ratio is:

$$
\begin{equation*}
\frac{\mathrm{S}}{\mathrm{~N}} \quad=\frac{\mathrm{P}_{\mathrm{T}} \cdot{ }^{G} \mathrm{~T} \cdot{ }^{2} \lambda^{2} \sigma_{t} \cdot n \Sigma_{i}(n)}{(4 \pi)^{3} \cdot K \cdot T 0 \cdot b \cdot N F \cdot R^{4} \cdot L} \tag{2}
\end{equation*}
$$

where

| R | $=$ | Target range (m) |
| :---: | :---: | :---: |
| $\mathrm{P}_{\mathrm{T}}$ | $=$ | Peak Transmitter Power (watts) |
| L | $\pm$ | System Losses (but see para 7 ) |
| b | $=$ | Receiver noise bandwidth Hz (eg 30.8 $\mathrm{dB}^{\mathrm{Hz}}$ for 1200 Hz ) |
| $\sigma_{t}$ | $=$ | RCS of target $\left(m^{2}\right)$ |
| S | $=$ | Minimum detectable signal (watts) |
| K | $=$ | Boltzmans Constant (-204 dBW) |
| To | $=$ | Temperature ( ${ }^{\circ} \mathrm{K}$ ) ( $290^{\circ}$ ) |
| NF | $=$ | Noise Figure |
| $\lambda$ | $=$ | Wavelength (m) |
| $n \Sigma_{i}(\mathrm{n})$ | $=$ | Integration Improvement Factor |
| $\mathrm{G}_{\mathrm{T}}$ | = | Aerial Power Gain (Receiver or Transmitter) |

7. Losses. System losses included as L, in this general form, can also be more exactly specified, according to conditions. The following losses are applicable, as appropriate to paragraphs 6 above and paragraphs 9 and 13 below:
$\mathrm{L}=$ All losses, both transmit, receive, propagation and beam pattern factor losses.
$I_{T}=$ All transmitter losses eg waveguide, feeder, radome, TR-switching.
$I_{R}=$ All receiver losses eg waveguides etc as above for $L_{T}$.
$I_{p}=$ Beam shape and pattern lobing eg tracking radar cross-over losses.
$I_{a}=\quad$ Two way absorption or atmospheric propagation losses.
$I_{C} \quad=\quad$ Collapsing losses.
$L_{s}^{+}=$Signal processing losses applicable to jamming.

NOTE L'a and L'p are the one-way losses applicable to jamming.
8. G, the aerial power gain, must be modified according to the aerial radiation pattern. For example, if the (fairly common) cosine distribution is used $G_{T}^{2}$ becomes $\left(G_{0} \operatorname{Cos}^{2}(\pi \sigma / 2 \theta)\right)^{2}$ where $G_{0}$ is the onaxis power gain and $\theta$ the one way $3 d B$ beamwidth.
9. Received Surface Clutter. Taking the basic equation the surface clutter power $C_{p}$ at the receiver input is:

$$
\begin{equation*}
C_{p}=\frac{P_{T} \lambda^{2} L_{F} G_{T}^{2} \sigma_{0} A_{c}}{(4 \pi)^{3}} \frac{R_{c}^{4}}{\left({ }^{4}\right.} \tag{3}
\end{equation*}
$$

The illuminated surface clutter area $A_{c}$ is:

$$
\begin{equation*}
A_{c}=R_{c} \theta_{A} \frac{C T}{2} \tag{4}
\end{equation*}
$$

but at very low grazingangles where $\tan \psi<\frac{2 R_{c} \operatorname{Sin} \theta / 2}{C \tau / 2}$
the clutter area is modified

$$
\begin{equation*}
A_{c}=R_{c} \theta_{A} \frac{C_{\tau}}{2} \operatorname{Sec} \psi \tag{6}
\end{equation*}
$$

$\left(\operatorname{Tan} \psi<\frac{\theta_{A}{ }^{R} c}{c \tau / 2}\right)$ See Annex Balso,
$\operatorname{giving}_{C_{p}}=\frac{P_{T} \lambda^{2} \quad L_{F} G_{T}^{2} \sigma_{0} \theta_{A} C \tau \quad \operatorname{Sec} \psi}{2 R_{C}^{3}}$
or $C_{p}=\frac{P_{T} G_{T} L_{T} L_{a} I_{R} A_{e}\left(\frac{C \tau}{2}\right) \theta_{A} \sigma_{0}}{(4 \pi)^{2} R_{c}^{3} \operatorname{Cos} \psi}$

This modifies the basic $\mathrm{S} / \mathrm{C}$ ratio (target/clutter ratio)

$$
\begin{equation*}
\frac{S}{C}=\frac{\sigma_{t}}{\sigma_{0} \frac{C \tau}{2} \theta_{A} R_{C}} \quad \text { into } \frac{L_{S} L_{P} L_{F} \cos \psi \sigma_{t}}{R(C \tau / 2) \theta_{A} \sigma_{0}} \tag{9}
\end{equation*}
$$

where $\sigma_{0}=$ Average surface clutter per unit area $\left(m^{2}\right)$

$$
\bar{\sigma}=\sigma_{0} R_{c} \theta_{A} \frac{C_{\tau}}{2}
$$

$\theta_{A}=$ Azimuth $3 d B$ beamwidth, $A_{e}=$ Effective Aerial Aperture
$L_{F}=$ Loss factor in clutter receive chain (not necessarily the same as $L$ in eqn (2)) (non-dimensional factor)
$C_{p}=$ Received clutter power (watts)
$R_{c}=$ Range of clutter cell
$\bar{\sigma}=$ Average clulter RCS
$\sigma_{t}=T_{\text {arget }} R C S$
10. Detection Range in Clutter. It is of ten convenient to assess the detection range in clutter simply by re-arranging (9). Calculations for an example radar system are included at Annex $G$.
11. Received Volume Clutter. Volume clutter is a combination of backscatter, attenuation and chaff (see chaps $3 \& 5$ ).

$$
\begin{align*}
\text { For tain: }  \tag{10}\\
\qquad \operatorname{Sig}_{(w t r)}=\frac{0.93 P_{T} G_{T} C \tau \pi^{4} Z}{128 \lambda^{2} R^{2}}
\end{align*}
$$

where $Z=200 p^{1.6}$ (See Chap 3).

It is assumed that only the single resolution volume containing the target is contributing volume clutter. Skolnik \{148\} produces a composite expression incorporating both attenuation and backscatter from the two terms at (12):

$$
\begin{equation*}
\left(\frac{S}{C}\right)=\frac{P}{N_{r}}=\frac{K_{1} \sigma_{t}}{R^{4} N_{r}} \tag{11}
\end{equation*}
$$

where $P=$ received echo power from target

$$
K_{1}=\frac{\frac{p r^{2}}{2} \lambda^{2}}{(4 \pi)^{3}}
$$

$$
N_{r}=\text { receiver noise power }
$$

and

$$
\begin{equation*}
\frac{S}{N}=\frac{R_{r}}{N_{r}+N_{c}}=\frac{K_{1} \sigma_{t} \exp (-2 \propto R)}{R^{4}\left(N_{r}+N_{c}\right)} \tag{12}
\end{equation*}
$$

where $\mathbb{N}_{C}=$ Clutter backscatter power

$$
\alpha=\text { Attenuation coefficient }
$$

12. Chaff Clutter. Chaff backscatter is modelled at equation (5) Chapter 5.
13. Received Jamming Clutter. Although the foregoing clutter sources are almost always present, jamming will only apply to specific situations, and so the computer model can be initialised to include or ignore the jamming segment, as necessary. If the radar is modern, and assumed to have minimum sidelobes, with the mainlobe on the target all noise jarming energy enters along, or close to the mainbeam axis. Allowing for all losses the equation for signal to jamming noise ratio is:

$$
\begin{equation*}
\frac{s}{J}=\frac{P_{T} G_{T} L_{T} L_{p}^{\prime}{ }_{p}{ }^{\prime}{ }_{c}{ }^{L^{\prime}}{ }_{s}\left(\sigma_{t}\right)}{4 \pi B_{N} R_{J}^{2}} \frac{\left(P_{J} G_{J}\right)}{} \tag{13}
\end{equation*}
$$

where $R_{J}\left(=R_{t}\right)=$ Range to jarmer (ie target)
$G_{J}=$ Jammer Aerial Gain
$P_{\mathcal{J}} \quad=\quad$ Power of jammer per unit bandwidth (watts for Hz )
$B_{N} \quad=$ Noise bandwidth of receiver (before detection)

Since only self screening jamming is considered $R_{J}$ and $R_{t}$ are equal:

## TARGET CHARACTERISTICS

14. Targets are generally taken to comprise a predominant (steady) signal
re-inforced by many small reflectors (ie Ricean distribution of reflectors). Target fluctuations are taken to be independent scan to scan and based upon Swerling Type 3 detection probability.

$$
\begin{equation*}
p(\sigma)=\frac{4 \sigma}{\sigma_{A V}^{2}} \exp \left(\frac{-2 \sigma}{\sigma_{A V}}\right) \tag{14}
\end{equation*}
$$

for $\sigma \geqslant 0$ ( $=0$ elsewhere)
where $\sigma_{A V}$ is the average target $\operatorname{RCS}\left(m^{2}\right)$ and $\sigma$ the instantaneous RCS. All relevant priority targets are 'aspect sensitive', as shown at fig l, where a $0.05\left(\mathrm{~m}^{2}\right)$ RCS target head-on can produce an enormous RCS on the beam (crossing target). Since the overall model detection probability is roughly the probability (excluding sightline blocking) that the target return signal will cross a detection threshold with a sufficient s/C ratio; it is seen that RCS can be a critical parameter. Because of the uncertainty of the instantaneous value of RCS, present when an aircraft is ostensibly in straight and level flight (and even more variable when the aircraft is deliberately manoeuvring), target RCS must be considered statistically. A Rayleigh distribution for larger targets has been found suitable by Ament et al \{149\} but aircraft and missiles of small RCS tend towards higher order chi-square functions. Typical radar cross sections for small aircraft range from $1.2 \mathrm{~m}^{2}$ (head-on) to 20 to $60 \mathrm{~m}^{2}$ (beam-on), giving a median of 1.3 to $5 \mathrm{~m}^{2}$ over $360^{\circ}$ and all roll plane aspects. For the purpose of the model 0.05 to $1 \mathrm{~m}^{2}$ has been used for head on targets and $4 \mathrm{~m}^{2}$ for beam targets. It is further assumed that the targets of interest are designed with profiled structures to minimise RCS and may comprise dielectric panels and possibly radar absorbent coating for a proportion of the observed echoing skin area (see also Chap 4 and Annex E).
15. RCS Spectra. Turbine and/or Propeller, and airframe spectra investigations are outside the scope of this report. Briefly, the airframe spectrum is due to the relative motion between target scattering points, and although an RCS range (eg 1.3 to $5 \mathrm{~m}^{2}$ ) was easily selected for assessment purposes, the selection of a suitable airframe spectrum (due to random and systematic changes) is far more difficult. According to $\{150\}$ the width of the airframe spectrum has the relationship:

$$
\begin{equation*}
\Delta f=K\left(\frac{I_{0}}{\lambda}\right)\left(\frac{\Delta \theta}{\lambda t}\right) \tag{15}
\end{equation*}
$$

With smaller targets likely to have a greater random motion than large aircraft. Further spectra information is available from the reference \{151\}. Measurement of the rate of change of target aspect $(\Delta \theta / \Delta t)$ is complex although the factor $L_{o} / \lambda$, the characteristic length of the target, is more readily available. $K$ is a proportionality constant.
16. Frequency Agility. Frequency agile radars have improved performance against fluctuating targets since the probability is reduced that the target will be at an aspect angle which gives a very low RCS or a null. Frequency agility can also reduce range and tracking errors caused by target glint and multipath (see Chap 9). Improvements in detectability of several $d B$ have been measured when using frequency agility \{152\} at 10 cm wavelength. At the same time frequency agility can be used to decorrelate distributed clutter echoes (see Chap 4). The model incorporates an allowance, if required, to improve detection probability for frequency agile systems.
17. Relationships stated in the chapter for Received Target power, Surface Clutter power, Jamming power, Signal/Noise ratio, Volume Clutter and Fluctuating target characteristics are incorporated in the model.
18. Radar Cross Section. Experimental distributions made by the US Applied Physics Laboratory indicate no simple solution for RCS modelling of all aircraft aspects. Much of the uncertainty in modelling RCS lies in the observation time used to obtain the distribution. Although the Rayleigh distribution is suitable for large aircraft, RCS modelling is necessarily a coarse procedure. Cumulative detection curves can be used if detection is required on an approaching target before it reaches a certain range. Missile targets have larger mean to median ratios; a log-normal distribution is more accurate in this case than Rayleigh. Equation (14) is used, for example, based on an average RCS with $\sigma_{t}$ varied using a random number generated in the model to simulate target glinting. Distributions, Rayleigh or Log-Normal, are selected according to target type and appropriately for fixed FF or frequency agile radars, ie change of Swerling case.
19. Although the RCS of future aircraft will be reduced by careful design, stealth - low reflectivity coating, perhaps to lower than OdBSm; the range of terrain RCS (per $\mathrm{m}^{2}$ ) may vary between - 30 dB to possibly +30 AB with -15 dB as a typical average (see Chapter 4). A clutter signal may be present even when clutter from the target range gate is
masked but enters due to sidelobe clutter reception. Experience has shown that parameters such as target doppler, beamwidth (azimuth resolution) or range resolution may not be sufficient to separate targets from clutter, particularly at low level. The fluctuating target RCS can therefore be critical in the detection and tracking process since a tactical aircraft RCS may be of the order 10 to 20 dBSm .
20. Fluctuating Target. The problem of fluctuating target returns is closely related to FAR (see Chapter 4) and further considered at Annex E, where it is shown that a target fluctuating with low amplitude peaking is more easily detected at short range, while a more excessively fluctuating signal is more easily detected at longer range.


FIG 1 RADAR CROSS SECTION (RCS) - CRUISE MISSILE TYPE TARGET

## CHAPTER 7

## DIFFRACTION

1. Computer-aided and manual literature searches have revealed several comprehensive reports covering diffraction of data links at UHF and VHF, but with very limited research at microwave link frequencies. No detailed reports could be found on low level tracking radar diffraction, indeed practical prediction algorithms are thought not to exist. As recently as 1980 a report from the Lincoln Laboratory, MIT, \{154\}, stated "diffraction of radar transmissions over terrain obstacles has not received as much attention as refraction"; and, "diffraction has effects which should worry military mission planners' (military context of planning low level terrainrouting profiles to avoid detection). Also, "some obstacle problems remain unsolved - the debate continues over the proper way to estimate losses over terrain obstacles". To complete the radar performance prediction algorithm a detailed inyestigation is clearly necessary into diffraction effects.

## DIFFRACTION PARAMETERS AND AIMS

2. There are several approaches to the theory of diffraction, including extended waye theory. The following research aims were selected:
a. Research the nature of diffraction in practical terms.
b. Determine the criteria under which diffraction is likely to enhance low level tracking.
c. Consider the substitution of terrain with cylinders or baffles for diffraction modelling purposes.
d. Consider diffraction effects with reference to modifications by reflection and multipath.
e. Determine diffraction path radar power values on outward and return paths.
f. Generate a diffraction subroutine for the main radar performance prediction algorithm.
g. Produce a subroutine capable of scanning a land area; given the terrain data base, and determining a general probability of diffraction from the nature of the surface profile for given target altitudes.

## KNIFE-EDGE DTFFRACTION

3. Assuming that the local terrain does not support reflection, knife edge diffraction approximations are often used with the geometry shown at figures 1 and 2. With the radar transmitter near the earth's surface, and the target airborne (unlike the data-link case); the diffraction angle can be considered at figure 2 as fixed, while distance $d_{2}$ and hence $R$ are reduced. This has the effect of moving the target upwards on the figure to the dotted position, changing $d_{2}$ to $d_{2}^{1}$ and $R$ to $R^{1}$. From Fig 1 , if the radius of the assumed diffraction edge (cylinder in practice) is large compared with $\lambda$, then:

$$
\begin{equation*}
\frac{\mathrm{E}_{\mathrm{S}}^{2}}{\mathrm{E}_{\mathrm{I}}^{2}}=\frac{1}{2 \pi \mathrm{kR} \mathrm{\alpha}_{1}^{2}}=\frac{\mathrm{R}}{2 \pi \mathrm{kh}^{2}} \tag{1}
\end{equation*}
$$

Where $\mathrm{k}=\frac{2 \pi}{\lambda}, \mathrm{E}_{\mathrm{S}}=$ Scattered intensity from the target and $\mathrm{E}_{\mathrm{I}}=$ Incident intensity. $R$, $h$ and of are shown in the figure. However, both figures are essentially the same as explained above.
4. The geometry at figure 2 is used to derive the further approximations at para 5 below, but at this stage it is necessary to discard the negative knif'e edge situation (ie diffraction ridge below radar horizon) since a direct sightline would exist simultaneously; hence detection capability would not be significantly impaired. Nevertheless, it is recognised that if the diffracted ray received via a negative knife edge exceeded the signal strength of the direct ray, an angle tracking error could occur.
5. Taking $v$ as the dimensionless parameter of the Fresnel - Kirchoff diffraction formula (see Annex C) then:

$$
\begin{equation*}
v=+2 \sqrt{\frac{\Delta r}{\lambda}} \tag{2}
\end{equation*}
$$

Where $\Delta r=r_{1}+r_{2}-R$ (or $R^{1}$ if the airborne target is used)

$$
\begin{align*}
& \text { or } v=+\sqrt{\frac{2 R \propto \beta}{\lambda}}  \tag{3}\\
& \text { or } v=+h_{t} \sqrt{\frac{2 R}{d_{1} \partial_{2} \lambda}} \tag{4}
\end{align*}
$$

Where $h_{t}$ is the obstacle height
$\infty, \beta$ and $\theta$ are in radians
$\lambda, R, d_{1}$ and $d_{2}$ are in consistent units.
6. Comparison of the free space and diffracted fields to obtain the diffraction loss ratio $A(v)$ gives :

$$
\begin{equation*}
A(v)=-20 \log _{10}|F(v)| d B \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
\text { Where } F=\frac{1-1}{2} \int_{V}^{\infty} \exp \left(J \pi u^{2} / 2\right) d u \tag{6}
\end{equation*}
$$

as v becomes large and positive $|F(\mathrm{v})| \rightarrow 2^{\frac{0.5}{2 \pi \mathrm{v}}}$
and $A(v)$ becomes approx $-20 \log _{10}\left[\begin{array}{ll}\left(\frac{d_{1}+d_{2}}{d_{1} d_{2}}\right)^{2} & \frac{\lambda_{2}^{2}}{2 \pi \theta}\end{array}\right] d B$
$\theta$ is the angle of diffraction.
7. For a single edge diffraction obstacle Deygout \{155\} uses a criteria to characterise the diffraction path to check that the first Fresnel Zone, of radius $r$, is not obstructed. Assuming $\lambda \ll h_{t}<\frac{d_{1}}{10}$ and $\lambda \ll h_{t}<\frac{d_{2}}{10}$, at a frequency $f(\mathrm{MHz})$ separated by $R$ kilometers, the free space loss is:

$$
\begin{equation*}
a_{0}=32.5+20 \log _{10} f+20 \log _{10} R(\mathrm{~dB}) \tag{8}
\end{equation*}
$$

Expressing the diffraction loss as a function of $\frac{h_{t}}{r}$

Where $r(m)=548 \sqrt{\frac{d_{1} d_{2}}{f\left(d_{1}+d_{2}\right)}}$ using $f(m H z), d_{1} d_{2}(k m)$
for $h_{t}>r, a_{m}=20 \log _{10}\left(\frac{h_{t}}{r}+16\right)$

Where $a_{m}=$ diffraction loss; total loss $=a_{0}+a_{m}$
8. When $\frac{h_{t}}{r}$ is used the results differ by $\sqrt{2}$ (see equation for $v$ below). For comparison purposes in the use of $v$ fig 3 shows the diffraction loss while figure 4 shows diffraction loss $a_{m}$ against $\frac{h_{t}}{r}$ Again, using the diffraction loss ratio and including phase angle results and practical values for $v$ :

$$
\begin{equation*}
A(v)=-20 \log _{10}\left|\frac{E}{E}\right|=-20 \log _{10} a(v) \tag{10}
\end{equation*}
$$

Where $E=E_{0} a(v) \exp [-\rho \phi(v)]$
$\Phi(v)=$ Phase lag of diffracted field with respect to free space field. $E, E_{0}$ respectively diffracted and free space fielads $a(\mathrm{v})=\left|\frac{\mathrm{E}}{\mathrm{E}}\right|$
$\phi(v)=90 v^{2}$ (deg) ie, the phase difference in degrees attributable to the path length difference $\Delta r$.
9. Ref $\{156\}$ gives typical values for $v \geqslant 0, v \leqslant 0$ and $A(v)$ versus $v$ :

$$
\begin{align*}
& A(v)=12.953+20 \log _{10} v \text { for } v \geqslant 2.4  \tag{11}\\
& A(v)=6.02+9.11 v-1.27 v^{2} \text { for } 0 \leqslant v \leqslant 2.4  \tag{12}\\
& A(v)=6.02+9.0 v+1.65 v^{2} \text { for }-0.8 \leqslant v \leqslant 0 \tag{13}
\end{align*}
$$

and Larson \{157\} gives:

$$
\begin{equation*}
A(v)=6.0+11.28 v+4.28 \mathrm{y}^{2} \text { for }-1.4 \leqslant v \leqslant 0 \tag{14}
\end{equation*}
$$

10. Fig 3 shows the variation of $A(v)$ and phase shift with $v$, and Table 1 gives a selection of practical values for typical low level targets. In practice it is assumed that target altitudes vary between 30 m and 60 m and that the radar aerial will be no higher than 30 m AGL.

| Obstacle <br> $(\mathrm{Ht}(\mathrm{m})$ | $d_{1}$ <br> $(\mathrm{Km})$ | $d_{2}$ <br> $(\mathrm{Km})$ | v |
| :---: | :---: | :---: | :---: |
| 50 | 15 | 15 | 4.7 |
| 40 | 15 | 15 | 3.5 |
| 30 | 15 | 15 | 2.6 |
| 20 | 15 | 15 | 1.7 |
| 50 | 25 | 5 | 8.2 |
| 40 | 25 | 5 | 6.5 |
| 30 | 25 | 5 | 4.9 |
| 20 | 25 | 5 | 3.2 |
| 500 | 10 | 20 | 50.0 |
| 500 | 25 | 5 | 141.0 |
| 250 | 15 | 15 | 14.1 |

Table 1 Example Values - $v$ - for given Obstacle
Heights and Ranges

## INTERPRETATION

11. Where knife edge approximations are used, based on the relationship at Annex $C$, and the simplified criteria as used at paras 5 to 10 ; predictions have been found to be several $d B$ above the measured values \{158\}, \{159\} and \{160\}. However, when account is taken of the practical situation, ie rounded hillcrests, as is so often the case instead of idealised knife edges, $\{161\},\{162\},\{163\}$, and a rough conducting surface is present; then $\{164\}$ found general agreement with the conventional Fresnel-Kirchoff approach of ignoring the obstacle thickness. However $\{165\}$ states that an additional or "excess loss" is largely dependent upon the crest curvature, the angle of diffraction and wavelength, but almost independent of distance for a given angle of diffraction. The Fresnel integral is sometimes produced as a set of curves or tables.

## DIFFRACTION OVER ROUNDED HILLCRESTS

12. Up to $10,000 \mathrm{MHz}$ it is considered at $\{166\}$ that any rounded obstacle can be approximated by a knife edge, providing its radius of curvature $R_{c}$, satisfies:-

$$
R_{c}<\frac{\lambda}{\theta^{3}} \quad 2.10^{-3} \text { metres } \quad\left(\theta_{\text {in }} \text { rads }\right) ~(15)
$$

The geometry used for rounded hillcrests is shown at Figure 5, and \{167\}also suggested that the radius of curvature may be estimated by:

$$
\begin{equation*}
\text { Radius } \left.(m)=\frac{2 D_{S} d_{s t x} d_{s t g t}}{\theta\left(d_{s t x}^{2}+d_{s t g t}\right.}{ }_{\text {st }}\right) \tag{16}
\end{equation*}
$$

Where $D_{S}=$ distance between transmitter and target horizons

$$
\begin{aligned}
& \left(\text { ieD }_{S}=d-d_{L t x}-d_{L t g t}\right) \\
& d_{\text {stx }}=\text { distance between transmitter horizon and horizon ray } \\
& \text { intersection points. } \\
& d_{\text {stgt }}=\text { distance between target horizon and horizon ray intersection } \\
& \text { points. }
\end{aligned}
$$

A simplified solution $\{168\}$ for rounded hillcrests assumes each obstacle to be represented as a cylinder of radius equal to the radius of curvature at the obstacle top. The following parameters are used and marked where appropriate on Fig 5 .

$$
\begin{aligned}
& H_{1} \text { Obstacle height } \\
& R_{1}=\left[\lambda d_{1} d_{2} /\left(d_{1}+d_{2}\right)\right]^{\frac{1}{2}}=\text { First Fresnel zone radius } \\
& \left(d_{1}, d_{2}\right. \text { as at Fig 2.) }
\end{aligned} \quad \begin{array}{r}
\alpha=\lambda^{2 / 3} r_{1}^{1 / 3 / R}, \text { where } r_{1} \text { is the radius of curvature at the } \\
\text { top of the obstacle, } \alpha \text { is a curvature factor. }
\end{array}
$$

The main obstacle, assuming several lie on the path, is the one with the largest $\frac{H_{I}}{R_{I}}$ value. Figure 6 shows the relationship between $\frac{H_{1}}{R_{1}}$ and attenuation for various curvature factors.

## EFFECTS OF SLOPE INCLINATION AND ROUGHNESS

13. Practical implications of diffraction are considered later in the chapter, but slope inclination and roughness should be mentioned since waves incident upon diffraction ridges may be expected to suffer depolarisation due to these factors. Experiments by Carlson \{169\} revealed no appreciable complications (in diffraction effects) by terrain scattering, but his conclusion may only have been applicable to conditions pertaining locally at the time. Similarly it is difficult to quantify the effects of foreground scattering and also of interference diffraction signals at longer ranges.

EFFECTS OF FREQUENCY ON DIFFRACTION
14. Delaney $\{170\}$, has shown that lower radar frequencies are better than higher frequencies in terrain where diffraction is dominant. Other results show that the coverage of lower frequency radars in reflection-dominated terrain can be quite adequate if sufficient power is transmitted.

## RADAR POWER

15. In terms of radar power, the power at the target via the diffraction path will be:

$$
\begin{equation*}
P_{t_{g t}}=P_{T} G_{T} G_{R} \frac{\lambda^{2}}{64 \pi^{2} d, d_{1} d_{2} \theta^{2}} \tag{17}
\end{equation*}
$$

Troposcatter power is not considered here when using radars with narrow beams in the vertical, it could however be a contributory factor under other circumstances. Signals returned to the tracking radar are assumed to travel the same path in reverse and suffer therefore the same diffraction loss. CHAPTER SUMMARY
16. By careful interpretation of the few available results it is found that adequate diffraction loss calculations can be made - subject to the existence of the necessary site-specific target, radar and terrain data. However, it is possible that the existence of diffracting paths over mobile or small fixed obstacles, close to the radar site, cannot be accurately assessed unless diffraction measurements are made in situ. Precise obstacle positions will be unknown and will not, of course, be recorded in the terrain data base overlay. Indeed many such objects would not be included - such as isolated buildings, unless the data base was very finely spaced.
17. It is shown to be possible to predict the likelihood that diffractionpath tracking may take place; by incrementally testing the data base azimuth profile using equations (15 and 16) together with the necessary radar receiver sensitivity values, radar transmission and target parameters. A segment of the computer program at Chapter 11 was developed to produce a plan output plot of the first assessed diffraction ridge - behind which a target would not always be invulnerable to radar tracking. (See also page $F-10$ ).


FIG 1 BASIC DIFFRACTION GEOMETRY


FIG 2 KNIFE EDGE GEOMETRY


NOTE: FOR LARGE V (SAY >2), A(V) $=-20$ LOG $\sqrt{2 \pi T V}$ AND $\phi(V)=45^{\circ}$

FIG 3 PHASE LAG OF DIFFRACTED FIELD \& DIFFRACTION LOSS WITH VARIATION OF V OVER KNIFE EDGE


FIG 4 DIFFRACTION LOSS v OBSTACLE HT \& FRESNEL ZONE RATIO


FIG 5 geometry over rounded obstacle


FIG 6 DIFFRACTION OVER ROUNDED OBSTACLES


1. A number of standard texts and research papers are available on refraction and reflection, covering these phenomena in detail. However, some aspects are especially pertinent to the low level tracking case and so refraction and reflection are studied as a preliminary to the complete propagation model, which will finally include multipath and diffraction.

REFRACTION
2. Radar waves are bent primarily by water content in the atmosphere, which is normally denser at lower altitude. Two practical effects are considered here:
a. Radsr range may be considerably increased by refraction under certain conditions; where the system may be able to detect targets around the curvature of the earth.
b. Tracking radars in particular, may obtain a false target elevation angle by measuring the tracker dish boresight angle which is in fact not the true target sightline (Fig l).
3. Radar wave refractivity due to the variation in the velocity of wave propagation is given by\{172\} as:

$$
\begin{equation*}
(n-1) 10^{6}=N=\frac{77.6 p}{T}+\frac{3.73 \times 10^{5} e}{T^{2}} \tag{1}
\end{equation*}
$$

```
Where N = refractivity
    p = barometric pressure (mb)
    e = partial pressure of water (mb)
    n = atmospheric refractive index at zero altitude
    T = Temperature ( }\mp@subsup{}{}{\circ}\textrm{K}
```

The effective earth's radius is given by
$a=a_{0}\left[1-0.04665 \exp \left(0.005577 \mathrm{~N}_{s}\right)\right]^{-1} \mathrm{~km}$

Where $N_{S}=$ refractivity at surface of earth.
If $a_{o}$, the actual earths radius is taken as 6370 km ;

> for $N_{s}=301, \frac{a}{a_{0}}=\frac{4}{3}=k$, a good approximation to conditions in Europe.
4. Low Level Targets. Refraction effects can be significant at low grazing angles. For targets at 300 metres (or less), the $4 / 3$ earth correction is an adequate approximation. Since radar ranges are limited for this study, extended refraction (ducting) is not relevant. A representative refractivity model from Beari and Thayers $\{173\}$ is :

$$
\begin{equation*}
N=\dot{N}_{s} \exp \left[-c_{e}\left(h_{t g t}-h_{t x}\right)\right] \tag{3}
\end{equation*}
$$

Where

$$
\begin{aligned}
\mathrm{C}_{e} & =\ln \left(\mathrm{N}_{\mathrm{s}} / \mathrm{N}_{1}\right) \quad N_{l}=\text { refractivity at } 1 \mathrm{~km} \text { altitude } \\
\mathrm{h}_{\mathrm{tgt}} & =\text { altitude of target }(\mathrm{m}) \\
\mathrm{h}_{\mathrm{tx}} & =\text { altitude of radar aerial }(\mathrm{m})
\end{aligned}
$$

At the earth's surface a typical value for $n$ is 1.0003 , with a decrease rate of approximately $4 \times 10^{-8}$ per metre increase in altitude. Computed values for low level targets and various radar mast heights are at Table 1.

| Altitude (m) | Refractive <br> Index <br> $(\mathrm{n})$ |  |
| :---: | :---: | :---: |
| Target | Radar | 0 |
| 30 | 0 | 1.0003 |
| 60 | 4 | 1.0002 |
| 30 | 30 | 1.0003 |
| 30 | 30 | 1.0000 |
| 60 | 4 | 1.0006 |

- Table 1 Variation of Refractive Index with Radar and Target Altitude

5. Refraction Errors. From \{174\} and interpolation from CRPL National Bureau of Standards data for appropriate target altitudes and given radar grazing angles, the vertical error values were obtained at Table 2.

| GRAZING <br> Angle <br> (deg) | Angular Error (m rad) for Target <br> Altitude (m) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 m | 30 m | 60 m | 150 m | 300 m |
| 0 | 1.6 | 1.7 | 1.72 | 1.75 | 1.8 |
| 1 | 0.3 | 0.32 | 0.33 | 0.37 | 0.42 |
| 3 | 0.13 | 0.13 | 0.135 | 0.15 | 0.17 |
| 5 | - | - | - | - | 0.1 |

Table 2 Elevation Angular Error Due to Refraction

Taking, for example, the angular error and converting to altitude error at 17 km range a 300 m target at $1^{\circ}$ grazing would be measured with a
vertical error of 7.2 metres; or 0.5 metres for a similar target flying at 30 m . In each case the angular error is the angle between a straight line to the target and the apparent target elevation. Under refraction conditions the radar always measures a greater angle (ie greater altitude) of sightline than is actually the case.
6. Range Errors. Atmospheric refraction may also cause small errors in range, as shown at Table 3. To obtain the 2-way transmission path range errors the figures should be doubled.

| Grazing <br> (Degle | Range Error <br> (m) |
| :---: | :---: |
| 5 | 1.5 |
| 3 | 3 |
| 1 | 6 |
| 0 | 22 |

Table 3 Range Errors for Variation in Grazing Angle
7. The errors in both range and elevation angle, though small, are nevertheless present and may become significant where a tracking radar is being used in a commanded guided weapon system; since target position is degraded and eventual commanded miss distance may exceed the radius of effect of the weapon's warhead.

PROP
8. Spot terrain heights must be adjusted to allow for the effect of the average curvature of the earth's surface, as well as the refraction of the radar waves. Modified terrain height ( $y_{i}$ ) at any distance ( $x_{i}$ ) from the
radar location, taken along a great circle path is the height above a plane which is horizontal at the transmitter:

$$
\begin{equation*}
y_{i}=h_{s i}-\frac{x_{i}^{2}}{2 a} \tag{4}
\end{equation*}
$$

Where $h_{s i}$ is the unmodified terrain spot height above sea level ( $m$ ) $a=$ effective earths radius (km)

A proof of the validity of this approach is at \{175\}, and the geometry is shown at Figure 2 for a clear path.
9. Obstructed Sightline. When a path is obstructed the horizon ray grazing angles from the radar to the obstruction and from the target back to the obstruction are respectively given by:

$$
\begin{equation*}
\theta_{t x}=\frac{h_{1 t x}-h_{t x}}{R_{1} t x}-\frac{R_{1} t x}{2 a} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\theta_{\operatorname{tg} t}=\frac{h_{1_{\operatorname{tgt}}}-h_{\operatorname{tgt}}}{R_{1} t x}-\frac{R_{1_{\operatorname{tg} t}}}{2 a} \tag{6}
\end{equation*}
$$

Obstructed target sightline geometry is shown at Figs 3 and 4. Note that $\theta_{t x}$ and $\theta_{\text {tgt }}$ could be positve or negative - although a negative situation is unlikely to arise here since a levelled tracking radar does not usually depress below its nominal minimum tracking angle. Subsequent discussion of geometry will use the same notation as at (4) and (5) where:
${ }^{h_{1}}$ tx and $h_{\mathcal{L}_{\text {tgt }}}=$ the horizon range of terrain obstacles from transmitter and target respectively. $h_{t x}$ and $h_{t g t}=$ radar transmitter and target heights respectively.
10. The geometry for obstructed sightlines is only of significance for tracking radars if diffraction occurs. In this case a single 'knife-edge' formed by terrain must be located beneath point $X$ on fig 3 or 4. Multiple knife-edge diffraction is also considered at Chapter 7 ,
11. Unobstructed Sightlines. Figure 5 shows the situation where radars could be sited on a mast (at height $h_{t x}$ ) with a target flying at low level or nominally at surface level, (Point $Q$ ); or the target clutter on the surface (at point Q). It is assumed that targets will always be outside the Fresnel Zone since for $\lambda=0.03 \mathrm{~m}$ and for an aerial dimension $D=3 \mathrm{~m}$, $\frac{2 D^{2}}{\lambda} \simeq 600 \mathrm{~m} ;$ for $D=4 \mathrm{~m}$ the Zone boundary would be about 1 km .

## REFLECTION

12. Radar wave reflection theory is well covered in $\{176\}$ and $\{177\}$. It is of prime importance under conditions of approximately plane (flat) reflecting surfaces such as the sea and under more isolated cases overland at low grazing angles. The reflection coefficients for vertical and horizontal polarisation respectively are:

$$
\begin{equation*}
\bar{R}_{V}=R_{V} e^{j \emptyset V}=\frac{\bar{n}^{2} \operatorname{Sin} \psi-\sqrt{n^{2}-\operatorname{Cos} \psi}}{\vec{n}^{2} \operatorname{Sin} \psi+\sqrt{n^{2}-\operatorname{Cos} \psi}}=\frac{\operatorname{Sin} \psi-Z}{\operatorname{Sin} \psi+Z} \tag{7}
\end{equation*}
$$

where $\bar{n}^{3}$ is defined at Annex $B$ parall. $\quad \bar{n}$ = complex dielectric canst. of swface
$z=\frac{\sqrt{\pi^{2}-\operatorname{Cos}^{2} \psi}}{\pi^{2}}$
and $\bar{R}_{H}=R_{H} e^{J \varnothing H}=\frac{\operatorname{Sin} \psi-\sqrt{\bar{n}^{2}-\operatorname{Cos}^{2} \psi}}{\operatorname{Sin} \psi+\sqrt{n^{2}-\operatorname{Cos}^{2} \psi}} \quad \therefore=\frac{\operatorname{Sin} \psi-Z}{\operatorname{Sin} \psi+Z}$

$$
\Psi \text { measured perpendicular to surface }
$$

where $z=\sqrt{n^{2}-\operatorname{Cos}^{2} \psi}$

These equations may be approximated for low grazing angles for overland paths providing $\psi<0.1$ rads ( $5.7^{\circ}$ ) and $\mathrm{f} \gg 30 \mathrm{MHz}$, and are included in the computer programs described at Annex D. It is well established that horizontally polarised energy produces a greater reflection coefficient than vertically polarised energy. (See also Annex B page B-3).
13. A plane reflecting surface causes the continuous radar elevation coverage to break up into a lobed structure; where the approximate angle of the lowest lobe is approximately $\frac{\lambda}{4 \mathrm{~h}} \mathrm{tx}$ radians.

Where $h_{t x}=$ radar mast height (above local terrain level) for a 30 m mast at $\lambda=0.03 \mathrm{~m}$ the angle is 0.00025 rads and if the target elevation is less than a beamwidth direct ground reflected signals are received.
14. Multipath signal reception and reflected ray paths are synonomous, and curve fitting for terrain reflectivity profiles for smooth and rough earth situations using ray theory are considered at Chapter 9 . In general, ray theory calculations are valid out to the radar horizon where radar aerial heights are sufficient for the surface wave to be neglected and with the restriction on grazing angle $\Psi$ given by:
$\operatorname{Tan} \psi>\left[\frac{0.3}{2 \pi a f}\right]^{1 / 3}$
(Where $f$ is in $M H z$ and $a=$ effective earths radius in $k m$ ).
For $\lambda=0.03 \mathrm{~m}$ and $f=10 \mathrm{GHz} \operatorname{Tan} \psi$ must exceed 0.0009 giving $\psi>0.05^{\circ}$.
15. Smith $\{178\}$ contrasts ray and mode propagation theory and observes that propagation theory is incomplete in some areas. A detailed discussion of the theory is outside the scope of this report, however figure 6 shows
measurements which compare the two approaches. The full line uses ray theory and the dotted line mode theory. Millington \{179\} uses the oriterion $\psi>\left(\frac{\lambda}{2 \pi a}\right)^{1 / 3}$ to give the transition point (marked in Fig 6). Beyond this point as the horizon is approached the spreading of the rays due to earths curvature causes the ray theory to become unrealistic. For a radar mast height of 10 m at 3 cm wavelength it. is of interest that the transition occurs at about 12 km , hence ray theory is used here with some confidence for targets out to 15 km range.

## CHAPTER SUMMARY

16. Comments upon suitable factors to be incorporated in the radar performance prediction algorithm are:


#### Abstract

a. Refraction. Out to 30 km range curvature (refraction) effects are small. However, it will be shown that small vertical errors may become significant under combined refraction and diffraction conditions. The $4 / 3$ value for $k$ is reasonable, but more precise values can be used for $n$ from Table 1 and errors from Tables 2 and 3.


b. Refraction and Reflection Geometry. The approximations stated are used (see also Annex B), but these aspects are closely allied to the multipath and diffraction work covered in Chapters 7 and 9. For radar trackers associated with low level SAM systems 'Ray theory'is used.


FIG 1 TRACKING RADAR REFRACTION ERROR


FIG 2 MODIFIED TERRAIN PROFILE


FIG 3 PATH GEOMETRY


FIG 4 RANGE FROM RADAR TX


FIG 5 UNOBSTRUCTED SIGHTLINE


## MULTIPATH

INTRODUCTION

1. Target tracking at very low grazing angles may be disturbed by the presence of unwanted surface-reflected waves; giving rise to two main effects which have been recognized since the early days of radar \{180\}:
a. Signals arriving from spurious angles cause the radar tracker boresight axis to be driven off the real target sightline.
b. The direct signal is contaminated by additional
surface - reflected signals.
2. Surface reflections are usually classified as either 'specular' or 'diffuse', but here the objective is to consider practical means of incorporating multipath assessments into the computer model; rather than the detailed scattering processes. A brief survey of the effects of multipath on different tracker types is included. Minimisation of multipath at the design stage could be achieved by using narrower aerial beamwidths. This is not usually practicable with mobile systems since there is a limit to dish size, however there is a tendency for trackers to use higher RF's, giving some advantage in this respect. A number of techniques have been proposed $\{181\}$, but it is seen that the problem is mainly one of understanding terrain reflections rather than the hardware options available \{182\}. Indeed some techniques for reducing multipath may introduce other problems. One example of this $\{183\}$ is to
```
insert a screen ('barrier' or 'fence') to prevent reception of signals
from surface reflections. Although this can help at a pre-surveyed
and prepared site, the screen itself introduces a diffracting edge
with consequent interference with tracking results. The alternative is
to accept multipath and use other techniques to minimise the effects.
```


## RESEARCH AIMS

3. Specifically, the following aspects have been investigated:
a. Adjustment of $\mathrm{S} / \mathrm{N}$ for variable (indirect) path lengths when transmit and receive signals are subject to multipath.
b. To achieve (a), identify the conditions under which multipath is likely to occur overland.
c. Quantify uncertainties in elevation angle measurement due to multipath.
d. Estimate likelihood of multipath combining with diffraction edges.
e. Assess the probability of degradation of target tracking due to multipath as the target traverses a specific area of terrain.

Items 3 d and 3 e above are also dependent upon some factors considered at Chapters 4 and 8.
4. Many papers are found to address multipath phenomena from the inevitable occurence at sea, but few results or conclusions are available for overland operation. Several overland research reports, eg, \{184\} \{185\} unfortunately quote results not applicable to this study - since they are concerned only with relatively close-range targets over smooth approaches relevant to airport runway approach radars. Delaney $\{186\}$ has reported on the wider overland applications and it is clear that further data is required before really satisfactory assessment can be made. Delaney's model did not include target signal versus multipath clutter, but only the reflected signal for vertical angular errors.
5. The need for a multipath model which can be applied over general terrain with varying degrees of roughness has led to the development of a theory $\{187\}$ which describes the effects of scattering from the terrain between the source and the receiver. Determination of the point at which diffuse reflections predominate over specular is dependent upon surface roughness - the rougher the surface, the lower the elevation angle at which diffuse scattering dominates. ${ }^{4}$ separation of the resulting elevation errors has been the objective of $\{188\}\{189\}$ and others. A good survey of options can be found at \{190\} \{191\} on bistatic solutions, and $\{192\}\{193\}$ \{194\} \{195\} deal with other multipath compensation methods such as frequency agility, phased array processing and sidelobe reduction \{196\}. However, Barton \{197\} concludes that test data is extremely sparse in considering the arrival of diffuse multipath from angles other than the specular direction.

MULTIPATH GEOMETRY
7. Figure 1 shows the basic multipath geometry where illumination arriving
at the target via $\mathrm{R}_{1} \mathrm{R}_{2}$ will be:

$$
\begin{equation*}
E_{i}=\frac{G \rho}{R_{1}+R_{2}} E_{t} \tag{1}
\end{equation*}
$$

Where $G$ is the aerial voltage gain in the direction of the Specular point, $\rho$ is the surface reflection coefficient.
$E_{t}, E_{i}$ transmitted and incident field interiscties
The total illuminating field is the vector sum of the direct and indirect rays. Path lengths, direct and indirect are:

$$
\begin{align*}
& R_{\text {(Direct })}=\left\{R^{2}+\left(h_{2}-h_{1}\right)^{2}\right\}^{\frac{1}{2}} .  \tag{2}\\
& R_{\text {(Indirect })}=\left\{R^{2}+\left(h_{2}+h_{1}\right)^{2}\right\}^{\frac{2}{2}} \tag{3}
\end{align*}
$$

$h_{1}$ and $h_{2}$ are small in practice compared to $R$, hence taking the first 2 terms of the binomial expansion of each:

$$
\begin{align*}
& R_{D} \simeq R+\left(\frac{h_{2}-h_{1}}{2 R}\right)^{2}  \tag{4}\\
& R_{I} \simeq R+\left(\frac{h_{2}+h_{1}}{2 R}\right)^{2} \tag{5}
\end{align*}
$$

8. Path Difference. $R_{I}-R_{D}=\Delta R$

$$
\begin{equation*}
\Delta R=\frac{2 h_{1} h_{2}}{R} \tag{6}
\end{equation*}
$$

Phase difference $=\Delta \emptyset=\frac{2 \pi}{\lambda} \Delta R=\frac{2 \pi}{\lambda} \frac{2 h_{1} h_{2}}{R}$

To introduce a practical example here; given an aerial height of 7.5 metres at $\lambda=0.03 \mathrm{~m}$ and elevation angle of $0.5^{\circ}$, the multipath path-length difference will be of the order $5 \lambda(15 \mathrm{~cm})$.
9. Modified Signal. Interference from multiple lobes caused by the low grazing angle results in modified signal values. For smail angles of $\psi$ the pattern propagation factor (F) is:

$$
\begin{equation*}
F=1+\rho \mathrm{e}^{-j(2 \pi / \lambda) \Delta R+\emptyset} \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
E_{i}=E_{d}\left[1+e^{-\jmath(2 \pi / \lambda) \Delta R}\right] \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
=E_{d}\left[1+\rho e^{-\jmath(2 \pi / \lambda) \Delta R+\varnothing}\right] \tag{10}
\end{equation*}
$$

$\rho$ is the reflection coefficient
Ed is dinectly backscattered energy

For small grazing angles (lobe raised by $\beta=\frac{\lambda}{4 h_{t x}}$ ):

$$
\begin{equation*}
F(B)=\frac{4 \pi h_{t g t} h_{t x}}{\lambda R_{t g t}} \tag{11}
\end{equation*}
$$

Where $h_{t g t}, h_{t x}$ are target and transmitter heights. $R_{t g t}=$ Target Range Since the maximum range of a radar for detecting low flying targets is:

$$
\begin{equation*}
R_{\max }=8 \sqrt{\frac{4 \pi h_{t g t}^{4} h_{t x}^{4} P_{T}^{G} T^{2}}{\lambda^{2} S_{\min }}} \tag{12}
\end{equation*}
$$

If $\mathrm{R}_{\text {max }}>10 \pi \mathrm{~h}_{\mathrm{tgt}} \mathrm{h}_{\mathrm{tx}} / \lambda$
then $R_{\max }=\frac{4 \pi h_{t g t} h_{t x}}{\lambda} \sqrt[4]{\frac{P_{T} G_{T}{ }^{2} \lambda^{2} \sigma}{(4 \pi)^{3} S \min }}$
Where $\sqrt{\frac{P_{T} G^{2} \lambda^{2} \sigma}{(4 \pi)^{3} \text { Smin }}}$ is the maximum free space range

LOW LEVEL TRACKING
10. Fig 2 shows the multipath effect as a target reduces in altitude. At $A$ the target is well clear of the surface without multipath, at $B$ reflected energy enters the sidelobes causing oscillations of the aerial about a mean. Once the reflected energy enters the main beam at $C$ considerable angular uncertainty can arise. Figure 3 indicates the typical situation where the tracking boresight moves from real to image target angular displacement. In a practical situation where the target is assumed to be moving rapidly through the fluctuations the tracking stability will much depend upon the inertia of the tracking control loop. At other times the system will jump to the image and lose track. Elevation tracking errors are considered below at para 12.

## TRACKING MODES

11. As several tracking designs may be encountered in radar system assessment it is necessary here to take account of their individual vulnerability to multipath, briefly:
a. Monopulse. Standard monopulse uses Sum and Difference Channels to drive the aerial servo to zero error in the boresight. Under multipath conditions reflection signals also enter the Sum and Difference Channels. System vulnerable to multipath.
b. 'On-Boresight' Conical Scan. Derivation of the error voltage takes a significant time in contrast to monopulse. There remains the problem of boresight target motion during a typical scan period (eg $\frac{1}{30} \mathrm{sec}$ ). Vuinerable system to multipath.
c. Off Boresight. The boresight is held at a fixed angle some 0.7 X beamwidth above the horizon. The error voltage is taken as giving the target elevation below this angle. Some angular discrimination is achieved since the image signal is attenuated by being apprecially further off-boresight than the target. Since the aerial is fixed, it cannot move onto the image, the system having switched from closed-loop to openloop operation. The same technique can be used in both con-scan and monopulse systems. System resists elevation errors, but still susceptible to multipath clutter/noise.
d. Double-Null. Closed loop tracking is continued into the multipath region by generating an aerial pattern using monopulse, such that the difference function has 2 nulls equally dispersed about the horizon. Resistant to angular errors.
e. Quadrature Components. Three independent beams are used to make in-phase and quadrature measurements \{198\}. Resistant to angular errors.
f. Complex Indicated Angle Monopulse. Complex sum and difference signal can be processed to yield a complex indicated angle, and by combining this with more than one $R F\{199\}$ the real angle can be uniquely determined. A marginal improvement is claimed overland, with a factor of 2 or better at sea. Some resistance to elevation angular errors.
g. Frequency and Boresight Diversity. One complex indicated angle technique uses frequency and boresight diversity as a means of resolving ambiguities. The radar must have frequency agility or beam steering respectively. The principle relies on storing representative calibration spirals using 2 or more boresight angles. Calibrations can be made for specific sites or a generalised model used. Howard \{200\} \{201\} \{202\} surveys these techniques and claims good results under multipath conditions on $90 \%$ measurements with elevation error $\Sigma$ (rads) $0 \leqslant \Sigma \leqslant 0.5$.

## ELEVATION TRACKING ERRORS

12. Para 10 introduced the oscillatory nature of the signals received through multipath. It is possible to find the optimum target height for a given range when using a specific tracker control loop bandwidth. The Fiequency beat between divect and reflected signals is obtainable:

Since $F_{t}=\frac{2 h_{t x} h_{t g t} V}{\lambda R^{2}}$

Assuming $h_{t x}$ varies 5,15 and $30 \mathrm{~m}, F_{t}=1 \mathrm{~Hz}$ or 2 Hz , and $V$ is the target velocity - nominally 300 m sec ${ }^{-1}$ (ie Mach 1 at mean sea level) the following target table is produced for a spread of target altitudes:

|  | Target Altitude (m) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 30 | 50 | 70 | 100 |
| Tracker 2 Hz |  |  |  |  |
| Radar Ht (m) 5 | 1.22 km | 1.58 km | 1.87 km | 2.23 km |
| 15 | 2.12 | 2.73 | 3.24 | 3.87 |
| 30 | 3.00 | 3.87 | 4.82 | 5.48 |
| Tracker 1 Hz |  |  |  |  |
| Radar Ht (m) 30 | 4.24 | 5.47 | 6.48 | 7.74 |

Table 1 Target Range at which Tracker Bandwidth is Critical
13. If the error cyclic variations fall within the radar tracker bandwidth the practical result shown above will be that targets at ranges greater than those shown will be difficult to track in the presence of multipath, unless one of the compensatory systems such as off-boresight tracking is used. Equation (14) above is obtained from the derivative of the phase difference expression:

$$
\begin{equation*}
\phi \simeq(4 \pi / \lambda)\left({ }^{h_{t g t}}{ }^{h_{t x} / R}\right)+\phi_{R} \tag{15}
\end{equation*}
$$ where $\phi_{R}=$ ground reflection and other phase defferences in multiple paths.

$$
\begin{equation*}
\text { le. } F_{t}=|(1 / 2 \pi)(d \phi \mid d t)|=\left|2 h_{t g t} h_{t x} v / \lambda R^{2}\right| \tag{16}
\end{equation*}
$$

Example results are shown graphically at Figure 4.

## RMS ELEVATION ERROR

14. Evans \{203\} conducted tests at $\lambda=0.03 \mathrm{~m}$ overland ( $\rho=0.4$ ), using a $1^{\circ}$ beamwidth and shows $\pm 10^{\prime}$ elevation tracking error at $0^{\circ}$ target elevation. A suitable equation is derived, stated to be accurate down to
elevation angles of $0.5^{\circ}$ (for a two beam static split system in elevation). At $1^{\circ}$ elevation angle errors peaked at $\pm 3^{\prime}$. Targets were however at high level and long range. A graph of elevation tracking error is given by Barton \{204\} p 330 which illustrates these typical values, but his figures were obtained at " $C$ ' Band. The approximate $R M S$ error can be found \{205\} from:

$$
\begin{equation*}
\sigma_{\phi}=\frac{\rho \theta_{E}}{\left(8 G_{s}\right)^{\frac{1}{2}}} \tag{17}
\end{equation*}
$$

where $\rho$ is the coefficient of surface reflectivity $G_{s}$ is
$\frac{\text { main lobe gain }}{\text { side lobe gain }}$ (as a power ratio) taken at an angle 2 E below the beam axis ( $E$ is target elevation angle, $\theta_{E}$ is elevation beamwidth). Annex $E$ contains more detailed analysis of track errors for low level systems.

CONDITIONS FOR MULIIPATH
15. Propagation Path Length. With a ground-based tracking radar, where the aerial and terrain remain fixed, the target scattering properties are strongly aspect dependent (see Chap 6). If the range gate width is $T$ secs, then all scatterers contributing to the overall signal return must be located such that their return arrives in time interval $t$ such that:

$$
\begin{equation*}
\left(\frac{2 R}{c}-\frac{T}{2}\right)<t<\left(\frac{2 R}{c}+\frac{T}{2}\right) \text { and }\left(\frac{2 R}{c}-\frac{c T}{2}\right)<D<\left(\frac{2 R}{c}+\frac{c T}{2}\right) \tag{18}
\end{equation*}
$$

Hence $2 R+\frac{c T}{2}$, is the maximum path length a signal can travel and still remain in the range gate (see fig 5). Distance $=P S_{1}+S_{1} S_{2}+S_{2} P$ (max path D), hence:

$$
\begin{equation*}
=\left[R+\sqrt{h_{2}^{2}\left(R_{\max }-R_{G}\right.}\right)+\sqrt{R_{\max }^{2}+h_{1}^{2}} \tag{19}
\end{equation*}
$$

This shows that some scatterers beyond the target will be in the range gate. At low elevations $R \rightarrow R_{G}$ and the maximum range of a scatterer of interest is then $R_{\text {max }}=\left(R+\frac{c \tau}{4}\right)$

If $h_{1} \ll R_{\text {max }}$ then:

$$
\begin{equation*}
R_{\text {max }}=\frac{\left[\left(R+\frac{c T}{2}\right)^{2}-R^{2}-h_{2}^{2}\right]}{2\left(R+\frac{c T}{2}-R_{G}\right)} \tag{20}
\end{equation*}
$$

16. The foregoing is expanded upon at $\{206\}$, where the minimum range at which an unwanted scatterer could interfere is calculated. Clearly if the range gate duration was zero, the problem would be eliminated, hence the need for a small gate width is established for low level tracking systems.
17. Burk addresses the problem of power levels arriving at the aerial via the multipath. Assuming the target is scatterer $1\left(S_{1}\right)$ and the surface scatterer $2\left(\mathrm{~S}_{2}\right)$. Then (using the notation at Fig 5):

$$
\begin{equation*}
P_{(\text {at } t g t)}=\frac{P_{t} G_{R}}{4 \pi R^{2}} \tag{21}
\end{equation*}
$$

Overall Power scattered towards the surface scatterer $s_{2}$ (point $s$ ).

$$
\begin{equation*}
P_{\left(\text {at } S_{2}\right)}=P_{t_{g t}} \sigma_{t g t} \tag{22}
\end{equation*}
$$

Power $P_{2}$ arriving at $S_{2}$ is:

$$
\begin{equation*}
P_{2}=\frac{P_{t g t} \sigma_{\text {tgt }}}{4 \pi} q^{2} \tag{23}
\end{equation*}
$$

$q=\operatorname{dist} S_{1} t_{0} S_{2}$
Overall power re-radiated as scatter:

$$
\begin{equation*}
=\frac{P_{2} \sigma\left(S_{2}\right)}{4 \pi r^{2}} \tag{24}
\end{equation*}
$$

Power at radar receiver

$$
\begin{equation*}
P_{r}=\left(\frac{P_{t} G_{R} \sigma_{t g t}}{4 \pi R^{2}}\right)\left(\frac{\sigma_{2}}{4 \pi q^{2}}\right)\left(\frac{A e}{4 \pi r^{2}}\right) \tag{25}
\end{equation*}
$$

Where $A e$ is the effective receiver aerial aperture towards the surface scatterer. If multiple scatterers exist the problem becomes complex. Methods to obtain the incident and scattered total field are beyond the scope of this study but can be found at \{207\}.
18. Path Calculations. The method used at \{208\} is used for path length determination, see fig 6 . With the multipath angle very small $R_{T} \rightarrow$ $R_{1}+R_{2}$ and $h_{1}+2 R E+0$, shows that:

$$
R_{1}^{3}+\left[-\frac{3}{2} R T\right] R_{1}^{2}+\left[\frac{1}{2} R^{2}-R E\left(h_{1}+h_{2}\right)\right] R_{1}+\left[R E \cdot h_{1} \cdot R T\right]=0-(27)
$$

from which $R_{1}$, one of the three roots of (27) can be found, $R_{2}$ is then found from.

$$
\begin{equation*}
R_{2}=h_{2}+\sin \Psi \tag{28}
\end{equation*}
$$

The accuracy of $R_{2}$ decreases at long range usine equation (28), but it is satisfactory for the relatively short ranges in this study. An iterative process is used:

$$
\begin{equation*}
\operatorname{Cos}\left(\theta_{1}+\theta_{2}\right)=\frac{\left(R E+h_{1}\right)^{2}+\left(R E+h_{2}\right)^{2}-R_{T}^{2}}{2\left(R E+h_{1}\right)\left(R E+h_{2}\right)} \tag{29}
\end{equation*}
$$

Although the sum of $\theta_{1}+\theta_{2}$ is known, their individual values are not. In the first iteration let $\theta_{2}=\frac{1}{2}\left(\theta_{1}+\theta_{2}\right)$ and calculate $R_{1}$ :

$$
\begin{equation*}
\operatorname{Cos} \theta_{1}=\frac{\mathrm{RE}^{2}+\left(\mathrm{RE}+\mathrm{h}_{1}\right)^{2}-\mathrm{R}_{1}^{2}}{2 \mathrm{RE}\left(\mathrm{RE}+\mathrm{h}_{1}\right)} \tag{30}
\end{equation*}
$$

Hence $R_{1}=\sqrt{R E^{2}+\left(R E+h_{1}\right)^{2}-R E\left(R E+h_{1}\right) \operatorname{Cos} \theta_{1}}$

Using $R_{1}, \theta_{1}$ and $\Psi_{1}$ are found (Cosine Law).

$$
\begin{align*}
& \operatorname{Cos}\left(\frac{\pi}{2}-\Psi_{1}\right)=\frac{R E^{2}+R_{1}^{2}-\left(R E+h_{1}\right)^{2}}{2(R E)\left(R_{1}\right)}  \tag{32}\\
& \operatorname{Sin} \psi_{1}=\frac{\left(R E+h_{1}\right)^{2}-\left(R E^{2}\right)-\left(R_{1}^{2}\right)}{2 \operatorname{RE}\left(R_{1}\right)} \tag{33}
\end{align*}
$$

Hence $\mathrm{R}_{2}$ can be found:

$$
\begin{equation*}
R_{2}=\sqrt{R E^{2}+\left(R E+h_{2}\right)^{2}-2 R E\left(R E+h_{2}\right) \cos \theta_{2}} \tag{34}
\end{equation*}
$$

19. Testing $\psi_{1}$ against $\psi_{2}$, if $\psi_{1}<\psi_{2}$ the interval is re-defined for $\theta_{1}$ (as $\theta_{1}+\theta_{2}$ ). If $\psi_{1}<\psi_{2}$, re-define the interval 0 degrees to $\theta_{1}$ degrees. $\psi_{1}$ and $\psi_{2}$ are then recalculated with $\theta_{1}$ assumed to be half the new interval. An extremely accurate result is obtained in 31 iterations; in excess of the accurscy required here to determine the location of the surface specular point. The method reduces the error between the initially assumed $\theta_{1}$ and the actual $\theta_{1}$ by $\frac{1}{2^{N}}$; where $N$ is the number of iterations.

MONOPULSE RADAR TRACKING ERRORS
20. One agency has produced a desk-top computer program \{209\} which splits the multipath signal components into diffuse and specular; and assumes small angles over a "flat earth". For completeness the relevant equations to achieve the error calculation are shown at Annex E.

## CHAPTER SUMMARY

21. To meet the aims at para 3 the following items are incorporated in the model for low level tracking:
a. Multipath Conditions. After determination of the position of the probable specular point using equations (27) to (34), the algorithm (see programs at Annex D) examines the slope and surface material to assess whether multipath is likely.
b. Elevation Tracking Frrors. Equation (12) is used as a check to ascertain the target is within radar range; while equation (14) is used to find the optimum target range beyond which elevation tracking accuracy will probably become degraded.
c. Adjustment of Signal Levels. If multipath is assessed as likely, equation (1) is used to adjust $E_{i}$, incorporating the assessed $\rho$ from the terrain data base (see Annex B).
d. Multipath Coincident with Diffraction. Using the terrain data base the algorithm can produce a radial PPI-type plot for small azimuthal increments to indicate where diffraction edges could exist. Multiple diffraction edge assessment is complex and not thought to be particularly reliable at the higher RF's; especially since it is unlikely that more than one really significant diffraction edge will occur within the range brackets of interest here.

Further, double diffraction is only probable under limited conditions when plateaus between adjacent diffraction edges are sensibly smooth and horizontal. However, by using the same method as at (a) above, the fitting of a second specular point could be achieved by treating the first diffraction edge as the position of the radar transmitting source. The first diffracting edge would of course be considered in the first instance and would probably be the only edge relevant to short-range tracking systems.


FIG 1 MULTIPATH GEOMETRY


FIG 2 RECEDING TARGET - REDUCING IN ALTITUDE


FIG 3 TARGET TRACKING UNLOCK



FIG 5 MAXIMUM PATH PROPAGATION


## TERRAIN SLOPE - CLUTTTER

EFFECTS WITH AND WITHOUT TREE COVER

1. Sloping terrain implies a change in radar grazing angle since the surface resolution cell "tilt" as viewed by the radar, will vary. Land surface tilted away from the radar will be shadowed. Changes in slope gradient (1) and 'aspect angle' (2) may vary from cell to cell in areas of the roughest terrain, although spatially there is fairly high probability that adjacent cells may have the same slope and aspect in gently undulating conditions. If the period of undulation is less than the resolution cell length, the actual grazing angle could vary within the cell. Since clutter is strongly dependent upon $\Psi, \sigma_{0}$ can be expected to show significant variation with slope. Of the many radar research papers studied none considers slope in any detail; \{210\} and \{211\} mention that the 'slope effect' exists. The general geometry is shown at Figure 1. It is seen that the resolution cell 'footprint' on the surface, or 'facet', can be tilted at almost any angle depending on the local terrain aspect when viewed along the radar boresight. If gradient is zero, aspect is indeterminate. For mapping and geomorphological purposes Evans \{212\} proposes methods of slope representation and statistical terrain comparison; this is explained in some detail at Annex Fand converted to the radar slope and aspect situation found during measurement analysis by the author.
(1) Gradient is defined as the rate of change of terrain altitude with horizontal displacement (range $0-90^{\circ}$ ) ie, gradient is tangent to profile.
(2) Aspect angle is the compass azimuth angle (either with respect to the radar beam, or measured from North datum), along which the maximum gradient falls (range 0-360 )
2. For investigative purposes terrain with regular undulations could be approximated to a sinusoidal profile. Amplitude and period would then dictate the probability density function for any grazing angle and surface RCS could be computed for given values of $\theta_{E}$ and $\theta_{A Z}$ and $\tau$, as a function of amplitude and period. Gentle rolling hills for example may give an amplitude to period ratio of 0.05 (figure 2 a ), whereas very hilly terrain might produce a ratio of 0.1. Such a surface profile would have to be considered together with the existence of a sightline to the aircraft target. At the lowest grazing angles shadowing is of course at a maximum. In every case a sighting is assumed to the clutter patch with no intervening obstacles, apart from shadowing.
3. From Figure 3, the revised value of $\Psi_{1}$ (taken to be $\Psi^{l}$ ), towards a clutter patch is given by:

$$
\begin{equation*}
\left.\Psi^{1}=\sin -1 \quad-\frac{\left(h_{t}^{2}-h_{t} 2\right)}{2 R\left(\frac{4}{3}\right) r_{0}}-\frac{h_{t}-h_{t x}}{R}-\frac{R}{\frac{8}{3} r_{0}}\right] \tag{1}
\end{equation*}
$$

Where $h_{t}=$ height (average) of clutter patch (m)

$$
\begin{aligned}
& h_{t x}=\text { height of radar aerial (m) } \\
& \mathrm{R}_{\mathrm{tx}}=\text { range to clutter area (m) } \\
& \left.r_{0}=\text { Earths radius (nominal } 6500 \mathrm{~km}, 4587 \mathrm{n} . \mathrm{ml} .\right)
\end{aligned}
$$

4. Figure $2 b$ shows the pdf's for the probability that the actual grazing angle $\Psi^{l}$ falls between $\Psi$ and $\Psi+d \Psi$ for a nominal value $\Psi=5^{\circ}$ for amplitude/period ratios of 0.05 and 0.1 (respectively undulating \& hilly terrain) and allowing for masking effects. The radar is raised (or the terrain passes beneath an airbome radar platform) such that tho set value of 4 is $5^{\circ}$. By siting a mobile reader at any point the probability of acheiving the expected $\psi$ is very bow. (see ado ANNEX F APPENDIX aral
$10-206$
5. It has been observed \{213\} that as large areas of shadowing and hence facing slopes are illuminated, a pronounced 'knee' appears in the curves, for example where about $20 \%$ of the area was shadowed and $50 \%$ of the area sloped. Therefore the expected probability distribution is most likely to be contaminated in some way.
6. Providing radar reflectivity measurements are available it was realised that using terrain spot heights and a culture database, it should be possible to isolate and study those cells containing like foliage and with a particular slope - perhaps using $\Psi$ and $\Psi+d \Psi$ as a working range of aspect angles - to make correlation studies. Various errors such as aerial gain and propagation loss error should be taken into account, since these are site - specific, together with other relevant radar parameters present at the time.
7. It will be expected that the more heterogeneous the data becomes, then more areas are shadowed. However, if very large resolution cells are observed (large value of $\tau$ ), the probability distribution will take on a smooth transition from a small rate of change at the $50 \%$ percentile to a large rate of change at low percentiles. Detailed results of the author's correlation studies are at Appendix 1 to Annex $F$ for radar measurements taken by British Aerospace over varying terrain.

## PRACTICAL EFFECTS OF 'SLOPE'

8. Area. Equation 1 (above), does not however contain all the geometrical information necessary to define the slope and the associated radar footprint. If a very narrow (tracking) beam radar is considered several other effects are observed. Some examples are shown at Annex F, where the illuminated
plane dimensions can be less than the resolution cell dimensions with very steeply sloping terrain. Within the bounds of the aerial vertical beamwidth the surface footprint may exceed the resolution cell length $\tau$ if the plane is located at a maximum defined by the diagonally opposite edges of the resolution volume. As the terrain slope increases beyond $\alpha$ (CRIT) the surface footprint is reduced, and if it is assumed that the slope is centred on the point 0 , at the plane centre; then the illuminated plane is reduced in length and hence area at both ends by an amount dependent upon the slope. This extreme condition would occur in practice only in very rough terrain.
9. If small resolution cells are used (and ignoring the occasions when a specular reflector happens to be centred in the cell), it is assumed that the clutter from the cell centre gives a reflection typical of the whole cell. Surfaces tilted to the left or right similarly cause variations in radar footprint size.
10. Curvature. It is clear that many researchers have found it is inconvenient - or perhaps too tedious - to consider these effects in detail. Apart from the more obvious variations in radar footprint area caused by gradient, and calculated from the terrain altitude matrix, (shown at Annex $F$ ) the second derivative is that of curvature. In this study curvature is of interest; both convexity and concavity. Surface concavity is likely to produce radar shadowing, while profile convexity may be used to predict the curvature values necessary to test for the likelinood of diffraction in accordance with the criteria selected at Chapter 7 and also detailed in Annex $F$.
11. In an attempt to obtain correct terrain screening data (see Chap 2), it has been the normal practice in the worldwide clutter models studied, to add a set value for tree height to the terrain (contour) spot heights. Investigations by the author have shown that this approach is not strictly correct. For a given type of tree cover, measurements \{214\} \{215\} suggest that trees at the bottom of sloped terrain grow to a greater height than trees at the top of the slope. Nature's reasons for this phenomena are of no concern here, however the practice of adding a constant height for tree cover must result in slight inaccuracies in the calculated grazing angle of radar energy striking the tree canopy.
12. Tree growth rates vary with tree types as well as with altitude, further, all tree mensuration is made on a volume yield basis ( $\mathrm{m}^{3}, \mathrm{ha}^{-1}$ ). A brief examination of a forest of pine/spruce types - prevalent in larger quantities than deciduous in some parts of Europe - has shown a probable variation of the order $25 \%$ over a slope altitude change of 200 m. Interpolation of measurements provided by \{216\}, (assuming tree height (h) is approximately proportional to volume yield), leads to the conclusion that a nominal 20 m tree cover over the terrain can be expected to reduce to 19 m if the terrain rises 30 m ; ie, an approximate rate of reduction of tree cover of -lm per 30 m elevation increase. The relationship between yield and elevation is linear.
13. Translating this into practical significance means that grazing angle can only be approximate since the rate of change of tree height with terrain altitude also varies geographically. For example, the effect is more marked in the North. For precise clutter investigations under laboratory instrumentation conditions against a sloping forest area the effect should be noted as an extra input variable.

## GRAZING ANGLES

14. Nathanson \{217\} uses a simplification for depression angle $\alpha_{d}$ (see figure $3 a$ ), where $r_{e}=\frac{4}{3} r_{0}$ :

$$
\begin{equation*}
\alpha_{d}=\sin ^{-1} \quad\left[\frac{2 r_{e} h_{t x}+h_{t x}^{2}+R^{2}}{2 R\left(r_{e}+h_{t x}\right)}\right] \tag{2}
\end{equation*}
$$

then if $\frac{h_{t x}}{r_{e}} \ll 1$ and $\frac{h_{t x}}{2 r_{e} R} \ll\left(\frac{h_{t x}}{r_{e}}+\frac{R}{2 r_{e}}\right)$ an
approximation gives:

$$
\begin{equation*}
\alpha_{d}=\sin ^{-1}\left[\frac{h_{t x}}{R}+\frac{R}{2 r_{e}}\right] \tag{3}
\end{equation*}
$$

and similarly for grazing angle (assuming flat terrain) as at figure Bb

$$
\psi=\operatorname{Sin}^{-1}\left[\frac{h_{t x}}{R}\left(1+\frac{h_{t x}}{2 r_{e}}\right)-\frac{R}{2 r_{e}}\right]
$$

reducing to:

$$
\begin{equation*}
\Psi=\sin ^{-1}\left[\frac{h_{t x}}{R}-\frac{R}{2 r_{e}}\right] \tag{5}
\end{equation*}
$$

A plot of $\psi v$ Range for typical radar mast heights is at Figure 4.
15. Using the above equations and applying the error and hence slope variation in estimating true gradients over tree-tops, we get the situation where the appropriate gradient must be added to the result at Eqn (4) to get the true grazing angle; since Nathansons model is for the illumination to strike the surface at nominal sea level. It is usually the case that the terrain facet is not only sloped but raised above sea level (or above radar transmitter level for land-based radars). The geometry is shown at figure 5 and is assumed to have forest cover of varying depth as explained at paras 11-13 above.
16. Actual grazing angle, at which radar boresight energy strikes the sloped tree tops will be $\left(\psi^{I}+\alpha_{t}\right):-$
a. Sloped Terrain near Transmitter Level

$$
\begin{equation*}
\Psi^{I}=\sin ^{-1}\left[\frac{h_{t x}}{R}\left(1+\frac{h_{t x}}{2 r_{e}}\right)-\frac{R}{2 r_{e}}\right]+a_{t} \tag{6}
\end{equation*}
$$

b. Sloped Terrain below Transmitter Level. $\psi^{l}$ is greater ; the calculation is repeated but assuming the radar height to be at a greater height than the terrain thus:

$$
\Psi^{1}=\operatorname{Sin}^{-1}\left[\frac{h_{t x}-h_{t}}{R} \quad\left(1+\frac{h_{t x-} h_{t}}{2 r_{e}}\right)-\frac{R}{2 r_{e}}\right]+\alpha_{t}-\ldots(7)
$$

When $h_{t x}=h_{t}, \psi^{1}+\alpha_{t}$
17. If terrain height exceeds radar transmitter height (as will often be the case with ground based radars), the same equation can be used for $\psi^{I}$ by interchanging ${ }^{h_{t x}}$ and ${ }^{h_{t}}$ in Eqn (7) and subtracting from gradient:

$$
\psi^{1}\left(h_{t}>h_{t x}\right)=\alpha_{t}-\sin ^{-1}\left[\frac{h_{t .-}-h_{t x}\left(1+\frac{\left.h_{t}-h_{t x}\right)}{R}-\frac{R}{2 r_{e}}\right]-(8)(1)}{2 r_{e}}\right]
$$

18. With increasing height of tree cover and the radar height fixed, the grazing angle intersecting the terrain becomes shallower and as trees and terrain reduce in height grazing angle $\Psi^{l}$ approaches the value at Eqn (4).
19. In practice the difference in gradient, $\Delta \Psi^{1}$, is:-

$$
\begin{equation*}
\Delta \psi^{1}=\alpha_{t_{1}}-\alpha_{t_{2}}-2 \sin ^{-1}\left[\frac{h_{t}-h_{t x}}{R}\left(1+\frac{h_{t}-h_{t x}}{2 r_{e}}\right)-\frac{R}{2 r} .\right] \tag{9}
\end{equation*}
$$

For example, if the terrain rises 30 m (in a 1 in 4 gradient) then $X$ (horizontal distance) $=120 \mathrm{~m}$, Assuming tree cover of 20 m ( 19 m at top of slope), then $\Delta \psi^{1}=0.45^{\circ}$. Similarly for a terrain gradient of 1 in 10 the value of $\Delta \psi^{l} \approx 0.2^{\circ}$. It is seen therefore that the radar energy striking angle does vary significantly, and that the correct allowance should be made for tree cover hejght variations. This may explain why some results, such as those plotted at Chapter 4 Figure 6, exhibit such a wide spread of rate of change of median $\sigma_{0}$ at low values of $\psi$.

## POLARISATION WITH SLOPE

20. Hevenor $\{218\}$ made measurements which strongly indicate that slope in the field of incidence"influences the calculation of radar backscatter in an entirely different manner than the slope in a plane orthogonal to the plane of incidence for a given polarisation". His experiments concerned a slightly roughened surface, although an analysis of his results shows that they are probably applicable to the homogeneous surfaces presented by continuous forest cover or rural terrain, since the correlation period will be shorter than the roughness period.
21. It is clear that significant errors of up to 5 dB will occur, asmeasured $\{218\}$ if for example, single slope only is used. This may explain why, in the absence of computing in the past, many sets of raw results could not be fully reduced but were plotted and compared - such that often like has not been compared with like - leading to inaccurate conclusions.

## SUMMARY

22. A method of categorising aspect and slope of terrain which is 'tilted' to the incident radar energy is recomended at Annex $F$. Backscatter effects caused by terrain 'tilt', ie aspect and gradient, have been investigated using raw radar data on a specific site with controlled radar parameters. These results are detailed at Appendix $I$ to Annex F .
23. The computer program is also capable of separating resolution cell footprints of like 'slope' and 'aspect' and will plot these on a PPItype layout on hard copy. This information is then converted onto acetate overlays for comparison purposes on survey maps.
24. Probability statistics for slope for any grazing angle can be produced for a terrain area where the database of spot-height matrix and culture exists.
25. It is possible that backscatter from terrain sloped at exceptionally low angles may have little practical effect upon radars using modern anticlutter processing.
26. The majority of clutter is likely to fall in the first $5-10 \mathrm{n}$ miles of range, and within the amplitude range $0.1-100 \mathrm{~m}^{2}\left(\omega r t / \mathrm{m}^{2}\right)$ with only a small proportion in the range $100-1000 \mathrm{~m}^{2}$, although these values would increase in rough terrain. About $95 \%$ of clutter is typically $>30 \mathrm{~dB}$ above minimum detectable signal out to 5 nmls and $60 \%$ out to 15 nmls , however precise measurements should always allow for the changing grazing angle as the terrain slope varies. It is possible that Linell's (and others) results (see Chapter 4) would have been different if some slope effect had been incorporated.
27. Models. Equations shown in paras 14 to 19 are used, as appropriate, for investigations and the performance prediction algorithm. $3 d B$ boundaries are used for statistical analysis although it is naturally understood that the radar footprint spreads over a larger area in practice. It is further assumed that the radar aerial distribution is such that energy levels fall rapidly outside the 3 dB limits while energy distribution within the 3 dB volume is sensibly evenly spread.
28. Surface Reflectivity Reversal Phenomenon. The pdf's at figure 2 b are of particular interest since they clearly show the wide variation in actual grazing angle obtained in practice when illuminating undulating terrain. Several sets of worldwide research results have shown a hitherto unexplained reversal in the clutter values obtained at very low grazing angles (see Chapter 4, figure 4). It is the author's opinion that this could be partly explained by the pdf's shown. For example when nominally grazing at $\psi=5^{\circ}$, the probability of obtaining the expected grazing angle is extremely low. As the terrain gently undulates $\psi=5^{\circ}$ occurs on only $8 \%$ of occasions, while more hilly terrain reduces the probability to $4 \%$. The pdf's at Figure $2 b$ clearly
show that $\psi=11^{\circ}$ or $27^{\circ}$ would be obtained as the actual grazing angles on the majority of occasions, hence the radar reflectivity measured would be greater at these angles that the value to be expected at the shallower angles. There is, of course, zero (main beam) backscatter from the shadowed areas caused by surface undulations, unless diffraction occurs, and the pdf will depend upon the period of undu:lations - how many reflecting facets are contained within the resolution cell, their angles and amplitude.
29. Slope Correlation Studies: Some additional investigations have been made into the above proposals and detailed at Appendix 1 to Annex F. Actual grazing angles are shown to be larger than measured grazing angles in all cases when the suggestions at paragraph 28 above are applied to the author's terrain data base. Backscatter Distributions plotted, give a straight line on Wiebull paper, and detailed methods and discussion are at Appendix 1 to Annex F. Care was taken to Identify slope and aspect for every terrain facet and to correct slope for radar boresight. Correlation tests were carried out between those parameters which were likely to be in relationship, for grazing angles taken in steps of $2^{\circ}$ and $3^{\circ}$ and by median and mean filtering.

fig 1 general geometry - gradients \& aspect of terrain


FIG 2a TERRAIN EXAMPLE (RATIO 0.05) ROLLING $\mathrm{A} / \mathrm{r}$


FIG 2b ACTUAL PDF'S FOR $\Psi($ RATIOS $0.05 \& 0.1)$ FOR $\Psi=5^{\circ}$
(a)

(b)


FIG 3 gRAZING ANGLE v RADAR RANGE


FIG 4 gRAZING ANGLE v RADAR RANGE


FIG 5 VAriation in gradient due to tree cover

## CHAPTER 11

## SYSTEM PERFORMANCE ASSESSMENT MODEL

1. The general sequence of inputs necessary for the assessment model are shown at Fig 1. A more detailed procedure is at Fig 2, showing the interrelationship of optical and radar tracking with the terrain data and with certain operational factors which directly affect overall system assessment.
2. In applying the sequence, different circumstances may pertain, for example, assessments may be required for:
a. A general (and quickly produced) assessment, where reasonably accurate radar emission characteristics are known but with limited detail on the terrain obscuration affecting a deployed mobile system. The signal processing capability of the radar under these terrain conditions may also have to be assessed empirically.

OR b. Fully documented terrain data available from presurveyed sites to which a radar may be deployed and where details of the radar are known. Examples, which use typical figures are at Annexes $G$ and $H$.

## TERRAIN MODEL

3. Typical results for observable target track lengths in various types of terrain are shown at Annex E Fig 5a. It should be noted that the lower curve on this figure shows flat terrain with evenly distributed but not dense surface obscuration. A curve for flat terrain with sparse surface cover will usually approximate to the position of the higher curve on

Fig 5a and thus possess a higher probability of obtaining a given track length.

MISSILE SYSTEM MODEL
4. For a particular system reaction time, target altitude and target and missile velocity, the probability of obtaining minimum track length required at a given crossing range is calculated using the technique at Annex E paras l6-21. This is considered sufficiently accurate for prediction purposes without getting into the detail of missile trajectory shaping. Further factors which may affect a prediction might be included, for example, radars associated with point defence systems are more likely to engage radially approaching targets than those area coverage systems which will also engage crossing targets. Radial observable track lengths are often likely to be longer than the majority of crossing observable track lengths. The value of $P_{T_{L}}$ obtained from this process is not of course the tracking or missile success rate - merely the opportunity value for a particular area which meet the minimum track length requirements. In an optically controlled system the assessment sequence next moves into $P_{O E}$ and $P_{M X}$ as seen at Fig 2 (but see also paras 6 and 7 below).

## RADAR SYSTEM MODEL

5. In a radar tracking system the next step is to assess the probability of gaining and maintaining radar track within the observed track length periods. At times the sightline will include clutter plus target, while at other times clutter will be shadowed. Terrain may be flat or sloped hence backscatter model values proposed at Chapters 10, Chapter 4 and Appendix 1 to Annex F should be used; together with the known' or assessed radar
parameters. Propagation conditions should be incorporated, as necessary. It may be advantageous to work through the sequence under 'best case' and 'worse case' conditions to determine the 'spread' of performance to be expected. Ideally, System, Environmental and Statistical values should be taken into account each time with a range of possible values. Attempts have been made to simplify the 'paper' operation of the sequence at Annexes $G$ and $H$, however a modular computer program was also written in Fortran for analysis purposes for terrain and radar signals.

## SYSTEM AVAILABILITY AND OPERATOR PERFORMANCE

6. Adjustment of the predictions at the bottom of Figure 2 are often a matter of "military judgement" in an operational environment. A point of contention between manufacturers and the author in the past! In some cases fairly reliable figures may be available, eg MTBF, while in others, such as availability of spares for radars and associated equipments and reloads for missile systems may be more difficult to assess. The effect of iearning curves and operator climatic degradation are also part of the equation and cannot be ignored.

## SYSTEM PERFORMANCE IN COUNTERMEASURES

7. This is considered separately from the system performance in a benign environment because two aspects exist - that of inherent or incorporated automatic design features which minimise the effectiveness of countermeasures and that of operator involvement of reducing countermeasures effectiveness by his skill. The effect of target manoeuvre and chaff degradation in any given situation is applied as a reduction factor, for example, due to target
glinting or range gate disturbance leading to an increase guidance system miss-distance. (See Annex E Figs 1 to 4) and is at present often a matter of considered judgement rather than hard fact from trials because of the number of variables involved.

## APPROXIMATE PREDICTIONS

8. Approximate predictions can be obtained by applying empirical values, based on experience to the simple model $P_{D E T} \times P_{T I} \times P_{M X}$ with the extra factor $P_{O E}$ inserted as appropriate. Further adjustment may be necessary for multiple fire channel or refire situations where a second attempt is necessary if the initial tracking and the first missile fails - providing of course sufficient observable track length is available to accommodate target response analysis, refire reaction time and missile flyout time for a refire.

## HIGH AND LOW RISK TRACKING AREAS

9. Aircraft in transit, while encountering point defences at their destination - which they must radially approach, will inevitaibly be forced to transit through area defences en-route in both directions. Area defences may also be enhanced by other point defences,both mobile and fixed.
10. Area defences are likely to be on pre-determined sites in the main with higher $P_{T L}$; the value of $P_{T L}$ varying invalue with terrain cover and surface undulation. Sharp ridges in an area will enhance the possibility of tracking under diffraction conditions, and may slightly increase the system performance in these areas.


FIG 1 General sequence - prediction model


## CHAPIER 12

## SUMMARY OF RESEARCH AND RECOMMENDATIONS

GENERAL OBSERVATIONS

1. Sightline. It has been shown that the predominant factor in detecting and tracking a low level target successfully is the ability to obtain a sightline, ie, a minimum unscreened, or 'unmasked' track length. For most terrain this will only occur with certainty if the radar aerial is raised on a mast, clear of immediate obstacles and vegetation. In flat terrain this may be sufficient for all-round coverage, however in undulating and hilly terrain targets at longer range may remain obscured due to shadowing. A terrain data base allows an initial assessment of sightline probability in a given area to be made on a statistical basis. Precise sightline information can only be obtained by optical survey of the actual radar site and this will vary from season to season and with changes in local obstacles such as the proximity of mobile vehicles (or smoke in optical systems). In the past it has not been the normal practice to raise tracking radars on masts because of the difficulties of stabilising the radar beam under conditions of wind-gusting. Although raised tracking radars may now be (theoretically) possible, while maintaining accuracy by the use of mastmounted accelerometers and associated error correction by computer; radars with low aerial heights will be widespread for many years, hence the sightline prediction will be of continued importance, since all other radar tracking functions are dependent upon it.
2. Clutter. Given a sightline, clutter is of next importance since the radar beam will invariably strike the surface when directed at low level targets even if on a raised mast. It is then necessary to model the clutter levels expected at the low grazing angles as suggested at Chapter 4, and
extended with actual results at Annex $F$. At this point the significance of the prediction model depends on whether an offensive or defensive viewpoint is taken. The clutter model will enhance the apparent performance of a defensive radar if the clutter level is assessed as lower than is actually the case; or if the radar's clutter processing capability is over-assessed. Conversely if the clutter level is assessed as 'high' radar performance will be predicted as relatively 'poor', where even the best clutter processing may not enable a target to be separated from the clutter for tracking purposes. Example predictions at both extremes have been shown in the report ( $\boldsymbol{A}_{\mathrm{N} N E X H} \mathbf{H}$ ).
3. Validation of Models. Clutter models are difficult to validate because of the paucity of reliable measurements. It is the author's opinion that although a particular clutter model may be selected for practical purposes if for no other reason than to give a starting point for predictions - it would be wrong to assume that this can be much better than a reasonable estimate. No existing clutter model could be said to be really adequate or scientifically precise unless it is site-specific and radar-specific, resulting from on-site measurement in all weather and seasonal conditions. Much of the uncertainty is due to the very large number of variables which are so dependent on local conditions.
4. It is seen therefore that the scope for a study of this type could be almost open-ended, since, as more results of clutter research become available the conclusions can be influenced slightly - first one way and then the other. However clutter itself is just one part of the overall input required for a useful prediction model for a low level tracking radar.
5. Results from some systems and measurements studies have been made using search (acquisition) radars, where clutter is often measured over longer ranges, at different RF's and longer pulse durations (hence larger resolution cells); such results often indirectly affect the overall performance of a radar tracking system - since search radars are often used to 'hand-on' targets to associated tracking radars which operate in the same area. Both surface scanning and airborne radars, though providing clutter measurements are noted to suffer "smeared" clutter effects because of the speed of the radar beam's swift traverse from one resolution cell to the next. These effects have not been considered significant for stationary systems.
6. It is possible that too much is made of the clutter problem in isolation in the context of tracking radars, since in practical terms clutter is only of interest on those azimuths where a target sightline exists simultaneously with a sightline to the underlying clutter. Furthermore it is of interest only in those sectors where sufficient track length can be observed for a useful period of time (distance). Once within the "useful" part of the radar site's field of view the radar parameters and the many other variables such as chaff, multipath, diffraction and weather also become important.

SENSITIVITY TO PARAMETER CHANGES
7. The sensitivity of the overall prediction to variations in the individual parameters is of particular interest. Target altitude is a critical parameter in determination of observable track length probability ( $\mathrm{P}_{\mathrm{TL}}$ ). For example a reduction in height (seen in one practical case from 300 ft to 200 ft ) gave a $10 \%$ reduction in $\mathrm{P}_{\mathrm{TL}}$ - a much more significant effect in the overall result than say a change in target speed of $40 \mathrm{~m} \cdot \mathrm{~s}^{-1}$ which, in the same case, changed $P_{\text {TLL }}$ by $4 \%$. Similarly degradation due to target scintillation, multipath and the many other factors considered in the report cause only minor
changes in the overall prediction, varying to a greater or lesser extent due to circumstances.

## MODEL VALUES

8. Suitable model values, included in each chapter summary must be supplemented by an assessment of the signal processing capability of the particular radar. This may be difficult to assess, since clutter processing may be assisted by pulse to pulse frequency agility, polarisation agility, MTI, multiple channel operation with different processing in each channel, (such as ground clutter filtering and moving clutter filtering) noise or precipitation or chaff filtering. For the relatively short ranges for tracking, clutter is always assumed to be present at low grazing angles unless shadowing is present. A reduced effect will be felt if off-boresight modes are used (see Chap 9 para llc).
9. Assuming the clutter values to be averaged and taking figures from the extensive survey and measured values, a reliable median value for $\sigma_{0}$ (in flat terrain) is about $-25 \mathrm{dBm}^{2} \mathrm{~m}^{-2}$ with a standard deviation of 9 to $10 \mathrm{dBm}^{2} \mathrm{~m}^{-2}$. These values will vary slightly with changes in pulse duration, $R F$, polarisation etc, but are considered suitable for $I$ and $J$ band tracking radars. ( $10,000 \mathrm{MHz}$ to $18,000 \mathrm{MHz}$ ). As RF increases $\sigma_{\mathrm{m}}$ is likely to reduce to $-30 \mathrm{dBm}^{2} \mathrm{~m}^{-2}$. (sec also discussion at $A_{p p} \mid$ to $A_{n n e x} F$ )
10. From the purely radar aspects the overall $P_{D}$ used (Chap 4 eqn 47) must be converted to the probability of tracking. Given the observable track length probability for a given geo graphical area and assuming a percentage of shadowing during anobserved track length, multipath and other radar degradations are applied as reduction factors which will affect different tracking systems in
different ways. For example, a judgement must be made on the effect of short-term loss of sightline for optical or radar tracking as appropriate. Some systems will be able (by rate-aided tracking) to withstand narrow sectors of obscuration quite successfully. For some optical systems a slight adjustment will be necessary for the improved tracking of targets through deciduous trees when defoliation occurs in winter.

## CONCLUSIONS

11. The selected method of overall system performance prediction is suggested to be the most reliable approach available within the constraints set out at Chapter l, the limited worldwide results directly applicable to this study, and the large number of variables involved. It is thought that overall performance predictions will never be exact in the scientific sense, since apart from the measurable parameters there are also those human factors in an unknown operational environment. In addition the possibility of such occurances as electro-magnetic incompatibility, the variable and surprise effects of jamming and the largely unknown effectiveness of ECCM response all influence the results in practice.
12. Many related studies have been made but totally accurate predictions cannot be made for all the unknowns since even those results obtained from tracker radars and associated missile systems used in action in $N$ Vietnam, the Middle East or in the Falklands cannot be read-across to other geographical locations with any degree of reliability.

## RECOMMENDATIONS

13. Since raised tracker aerials will not totally overcome the problem of obscuration in hilly terrain, and since $P_{T L}$ is such a predominant factor in the
a. Further studies should be made into the distribution (density) of surface culture and buildings in those areas for which $P_{T L}$ is known, in order to search for possible correlation between $P_{\text {rLL }}$, undulation and surface coverage. The intention would be to assign a $\mathrm{P}_{\mathrm{TL}}$ to an area (of limited extent) by examination of accurate maps and stereo photographs. 'Good' or 'Bad' areas for deployment could possible be determined - or conversely safe or less safe areas to fly.
b. More clutter measurements are needed using carefully controlled conditions with as many radar parameters varied as possible. These should be made in areas typical of the intended deployment with particular attention to terrain slope measurement.
c. Some practical diffraction trials are required where an aircraft transits behind ridged terrain with and without tree cover and with accurate ridge profile measurements. Accurate instrumentation over a number of target runs would be necessary to compensate for target RCS fluctuations due to minor manoeuvres. Alternatively it might be possible to make diffraction measurements using a balloon-mounted reflector tethered behind the ridge.

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## DISIRIBUTIONS FOR FLUCIUATING RADAR SIGNALS

## THE WEIBULL DISTRIBUTION

1. The Weibull distribution (1951) is widely applicable and has a probability density function given by:

$$
\begin{equation*}
f(x)=m \lambda x^{m-1} \quad \exp \left[-\lambda x^{m}\right] \tag{I}
\end{equation*}
$$

$\qquad$
Variable $m$ and $\lambda$ are known as the shape and scale parameters respectively, and must be estimated from the data available. The function is re-written:

$$
\begin{equation*}
p(x)=\frac{c}{b}\left(\frac{x}{b}\right)^{c-1} \exp \cdot\left[-\frac{x}{b}\right]^{c} \tag{2}
\end{equation*}
$$

Where $b$ and $c$ are now the scale and shape parameters. The use of $\lambda$ is avoided here, since $\lambda$ is used for radar wavelength elsewhere.

When $c=2$ this distribution reduces to the Rayleigh Distribution (see below). From (2), the probability of a signal level $x$ being exceeded is

$$
\begin{aligned}
& \int_{x}^{\infty} p(x) d x=\int_{x}^{\infty}\left\{\frac{c}{b}\left(\frac{x}{b}\right)^{c-1}, \exp \left[-\left(\frac{x}{b}\right)^{c}\right]\right\} d t \\
&=\exp \cdot\left[-\left(\frac{x}{b}\right)^{c}\right]
\end{aligned}
$$

If $x_{m}$ is the median value then:

$$
\begin{align*}
& \exp .\left[-\left(\frac{x_{m}}{b}\right)^{c}\right]=0.5 \\
& \text { hence }\left(\frac{x_{m}}{b}\right)^{c}=-\ln 0.5=-(-0.69315) \\
& \text { and } b=\frac{x_{m}}{(0.69315) \frac{1}{c}} \tag{3}
\end{align*}
$$

2. The second moment of distribution is:

$$
\begin{align*}
& \int_{0}^{\infty} \frac{c}{b}\left(\frac{x}{b}\right)^{c-1} \cdot \exp \left[-\left(\frac{x}{b}\right)^{c}\right] x^{2} d t \\
& \quad=b^{2} \Gamma\left(1+\frac{2}{c}\right) \\
& \quad=\frac{2 b^{2}}{c} \Gamma\left(\frac{2}{c}\right) \tag{4}
\end{align*}
$$

Where $\Gamma(n)$ is the Gamma function.
3. If the Weibull parameters for a normalized distribution are $b_{0}$ and $c_{0}$ and new values are required after scaling for a different resolution cell size (by a factor $\mathbb{N}$ ), then:

$$
\begin{equation*}
\frac{1}{\mathrm{~N}} \quad \frac{2 b^{2}}{c} \Gamma\left(\frac{2}{c}\right)=\frac{2 b_{0}^{2}}{c_{0}} \Gamma\left(\frac{2}{c_{0}}\right) \tag{5}
\end{equation*}
$$

As the median value for the normalised distribution is:

$$
\begin{align*}
& x_{m_{0}}=(0.1)^{\frac{1}{2}}=0.31623 \\
& \text { from }(3), \quad b_{0}=\frac{0.31623}{(0.69315) \frac{1}{c}} \tag{6}
\end{align*}
$$

The median value of the new distribution is:

$$
\begin{align*}
x_{m} & =0.31623 \mathrm{~N} \\
\text { and } \quad b & =\frac{0.31623 \mathrm{~N}}{(0.69315)} \frac{1}{c} \tag{7}
\end{align*}
$$


substituting (7) in (5) gives

$$
\begin{equation*}
\frac{1}{\left((0.69315)^{\frac{1}{c}}\right)^{2}} \cdot \frac{2}{c} \cdot \Gamma\left(\frac{2}{c}\right)=\frac{10}{N} \cdot \frac{2 b_{0}^{2}}{c_{0}} \Gamma\left(\frac{2}{c_{0}}\right) \tag{8}
\end{equation*}
$$

(8) can be solved graphically to obtain a value for 2 hence $c$, and $b$ can then be found from (7).

## RAYLEIGH DISTRIBUTION

4. If the average returned signal level is essentially constant in time and there are a large number of statistically (independently positioned) scatterers, the probability of echo amplitude being between a level $P$ and an infinitesimally larger level $P+d P$ is given by the pdf:

$$
\begin{equation*}
W(P) d P=\frac{1}{\bar{P}} \exp \left(\frac{-P}{\bar{P}}\right) d P \tag{9}
\end{equation*}
$$

$\bar{P}$ is the average power.

## RICEAN (RICIAN) DISTRIBUTION

5. Also called the non-central Rayleigh density function. This distribution describes a received signal containing an essentially constant echo in addition to a Rayleigh-distributed fluctuation.
1 by making the median a suitable apparent echoing area (ego.1m²)set by the desired detection thresholef:2

$$
\begin{equation*}
W(P) d P=\left(1+m^{2}\right) e^{-m^{2}} e^{-P\left(1+m^{2}\right) / \bar{i}} I_{0}\left(2 m \sqrt{1+m^{2}} \sqrt{P /-}\right)(d P / \bar{P}) \tag{10}
\end{equation*}
$$

$\qquad$
$J_{0}$ is the Bessel function $\left(J_{0}(j x)=I_{0}(x)\right)$

$$
J_{0}=1-\frac{x^{2}}{2^{2}}+\frac{x^{4}}{2^{2} 4^{2}}-\frac{x^{6}}{2^{2} 4^{2} 6^{2}}+\cdots-\cdots-
$$

## NORMAL (GAUSSIAN) AND LOGNORMAL DISTRIBUTIONS

6. Large amplitude signal components may cause an appreciable departure from the Rayleigh or Ricean distributions. The Lognormal distribution can be used, and the normal distribution curve is:

$$
\begin{equation*}
f(x)=\int_{\infty}^{x} \frac{1}{s \sqrt{2^{\pi}}} \exp \cdot\left[-(x-\mu)^{2} / 2 s^{2}\right] \tag{11}
\end{equation*}
$$

where $s=$ standard deviation (to avoid $\sigma$, used elsewhere for target echoing area).

```
\mu}=\mathrm{ median value of }
    x = normally distributed variable
```

The pdf for the lognormal distribution can be obtained from (11) by using the transformation $x=\ln Y$ :

$$
\begin{equation*}
\text { giving } f(Y)=\int_{\infty}^{x} \frac{1}{Y_{s} \sqrt{2 \pi}} \exp \cdot\left[-\frac{1}{2 s^{2}}\left(\log _{R} \frac{Y}{Y_{m}}\right)^{2}\right] \tag{12}
\end{equation*}
$$

Where $Y=$ Lognormally distributed variable

$$
\begin{aligned}
Y_{m} & =\text { Median value of } Y \\
s & =\text { standard deviation of } \log _{e}\left(\frac{Y}{Y}\right)
\end{aligned}
$$

APPENDIX 1 TO
ANNEX A TO
"THE PROBABILITY OF
DETECTING AND TRACKING
RADAR TARGETS IN CLUTITER
AT LOW GRAZING ANGLES"
DATED 20 SEPTEMBER 1982

WeIbuLl shape parameter and $\sigma_{M}$ CORrelations

1. Investigations were made into:
a. Correlation between Beamwidth - Pulse Duration product and median backscatter $\sigma_{m}$.
b. Correlation between Beamwidth - Pulse Duration product and Weibull Shape Parameter ' C ', for each of the following:
(1) By applying Dodsworth's algorithm to the calculated values ( $\theta_{A} \Upsilon$ ) for the worldwide survey.
(2) By applying the author's model for 'C' derived by statistical analysis of the worldwide results by measuring slopes of all results replotted on Weibull paper.
2. It was also possible to compare measured results for ' $C$ ' (obtained from slope) with values predicted by the algorithms.
3. No account was taken of different $R F^{\prime}$ s. It was however noticed that some values were suspect or at extreme values (eg Serial 7 at Table 1 Chapter 4 page $4-92$ ). This was confirmed by computer plots where the correlations could be seen if extremes were deleted.' Several other sets of data became available in addition to those at Chapter 4 , and the total test was run with a reasonable selection of data from USA, UK and Continental Europe.
4. Correlation Matrix. Correlation results for $C$ with $\theta \tau$

5. Correlation between measured slope (2) and proposed models by Dodsworth (1) and the author (4) are not high. Correlation between $\theta_{A}{ }^{\tau}$ product and the models proposed by Dodsworth and the author are good. Correlation between the author's and Dodsworth's values are also high. It is noted that the correlations did not change significantly in most cases when Dodsworth's and Warden's UK figures (on which Dodsworth originally based his premise) were deleted and the correlation tests repeated. The figures shown in brackets on the correlation matrix are those using USA and European results other than RSRE results.
6. A poor correlation of 0.11 was found between $\theta_{A}{ }^{\tau}$ and $\sigma_{m}$ with $d B=39.7+0.24(\theta \tau) \times 10^{-9}$.
7. The Radar Cross Section (RCS) is defined as the area intercepting that amount of power, which, when scattered isotropically, produces an echo equivalent to that received from the object. An idealised received signal clutter power can be found using the radar range equation:

$$
\begin{equation*}
S_{R}=\frac{P_{T} G A e \sigma}{(4 \pi)^{2} R^{4}} \tag{1}
\end{equation*}
$$

or, since $G=\frac{4 \pi A e}{\lambda^{2}}$

$$
\begin{equation*}
S_{R}=\frac{P_{T} G^{2} \lambda^{2} \sigma}{(4 \pi)^{3} R^{4} L} \tag{2}
\end{equation*}
$$

Where $\mathrm{P}_{\mathrm{T}}=$ Peak Transmitted Power
$\mathrm{G}=$ Peak Aerial Gain
Ae $=$ Effective Aerial Capture Area
$R=$ Range to Target (assuming $R_{i}=R_{r}$, ie monostatic radar)
$\sigma=\operatorname{RCS}$
$L=$ Combined System Losses
$\lambda=$ Wavelength
2. Figure $B l$ shows the resolution cell geometry; from which it can be seen that the resolution cell range dimension $A B$ approaches $\frac{c \tau}{2}$ as $\psi \rightarrow 0 .{ }^{\theta} A$ is the azimuth $3 a B$ beamwidth. $\tau$ is the radar pulse length.
3. To express the clutter in terms of the scattering cross section per unit area $\left(\sigma_{0}\right)$ then:

$$
\begin{equation*}
\sigma=\sigma_{0} R \theta_{A} \frac{c \tau}{2} \sec \psi \tag{3}
\end{equation*}
$$

Eqn (3) is the result of introducing a characteristic $\gamma$ such that $\sigma_{0}=\gamma \operatorname{Sin} \psi$, thus removing the geometrical dependence of $\sigma_{0}$, and the other assumptions are made at para 4 below.
4. The area of illuminated terrain depends upon the grazing angle. If, for example a flat plate (ideal) reflector is used, and assuming sidelobes are minimal at practical ranges, the effective capture area and gain are dependent on crazing angle. The incident area will then be:

$$
\text { Area }=R \theta_{A} \frac{c \pi}{2} \operatorname{Tan} \psi
$$



Where $\theta_{A}$ and $\theta_{E}$ are respectively the $3 d B$ azimuthal and vertical beamwidths. A pencil beam (having small $\theta_{A}=\theta_{\mathrm{E}}$ ) produces an elliptical ground "footprint" such that the ellipse axis lengths are $R \theta_{A}$. and $R \theta_{E}$. Cosec $\theta$ (providing Tan ( $\left.\left.\theta_{A}\right) \theta_{A}\right)$. Area of footprint is hence $\left(2 R \operatorname{Tan} \frac{\theta_{A}}{2}\right)\left(\frac{c_{\tau}}{2} \operatorname{Sec} \psi\right)$ $\qquad$ If $\operatorname{Tan} \psi<2 R \sin \frac{\theta_{E}}{2} \frac{c \tau}{2}$ then (5) is used for area.
5. It is also assumed that the clutter scatterers are small in relation to $\lambda$,
the transmitted wavelength, in which case $\sigma$ varies as $\lambda^{4}$ (dependent on the Rayleigh Law). For general angles of depression the RCS (of a flat target perpendicular to the incident beam) is a function of the angle of incidence which varies rapidly if the wavelength is small compared with the target size. This is more complex at sea because dependence of reflectivity varies with both the sea state and the angle of arrival. The radar cross section variation with shape has been researched by RUCK \{219\}.

## SURFACE REFLECTION COEFFICIENTS

6. MulEipath and backscatter values are dependent upon the magnitude and spatial origin of surface ref lected waves. The effect of surface roughness in changing the relative proportions of amplitude and phase from specular towards diffused has been expressedas the specular scattering factor:

$$
\begin{equation*}
\left.\rho_{s}^{2}=\exp -\left[\frac{\left(4 \pi\left(\sigma_{h}\right)\right.}{\lambda} \sin \psi\right)^{2}\right] \tag{5}
\end{equation*}
$$

$\sigma_{h}$ is the rms height standard deviation relative to an idealised surface. Values such that $\frac{\sigma_{h}}{\lambda}$ sin $\psi>0.066$ significantly reduce the specular reflection (ie $\rho_{s}<0.7$ ). For most terrain the largest part of the non-specular energy appears as forward scatter, or is absorbed by the vegetation.
7. It has been shown \{220\}, using small scale diffuse reflection theory that a scatter lobe of width $4 \sigma_{\alpha}$ will be formed about an axis corresponding to the position of the specular reflector. ( $\sigma_{\alpha}$ is the rms slope of the illuminated facet). Viewed from the radar receiver this diffuse scattering "will appear to originate from a glistening surface surrounding the specular centre of the facet".
8. The diffuse scattering coefficient $\left(\rho_{\mathrm{d}}\right)$ depends upon the integration of the reflected power density $\eta_{d}$, together with the radar receiver aerial gain over the angular extent of the glistening surface:

$$
\begin{equation*}
p_{d}^{2}=\rho \eta d_{d}{ }_{r}^{d \Omega} \tag{6}
\end{equation*}
$$

Thus when $4 \sigma_{\alpha}>\theta_{E}$ (elevation Beamwidth), $\theta_{E}$ rather than the surface becomes the predominant factor in establishing the reflected fraction of diffuse power.
9. Magnitude of Reflection Coefficients. Terrain coefficient values near 1 give strong specular reflections. Reflection coefficients can be established over an interval from the relationship:

$$
\begin{equation*}
p=-\rho_{A} \rho_{B} \rho_{S} e^{2 \pi \Delta R / \lambda} \tag{7}
\end{equation*}
$$

Reflection coefficient $\rho$ is therefore dependent upon:

$$
\begin{aligned}
\Delta R= & \text { Difference in path length (direct and indirect) } \\
\rho_{A}= & \text { Absorption coefficient } \\
\rho_{B}= & \exp \left[-\left(\frac{4 \pi \sigma_{h} \sin \psi / \lambda}{2}\right)^{2}\right] \\
\rho_{S}= & 1+\operatorname{erf}(\alpha) \\
& 1+\operatorname{erf}(\alpha)+e^{-\alpha^{2}} / \sqrt{\pi} \alpha
\end{aligned}
$$

Where $\alpha=\frac{\sin \psi}{\sqrt{2} \sigma_{s}}$
B-4
$\rho_{S}$ is a correction to $\rho_{B}$ to account for shadowing in the Fresnel zone (Smith Factor) $\{221\} . \rho_{B}$ is known as the Bechmann-Spizzichino factor. $\sigma_{s}$ is the slope (facet) standard deviation (rads). of $=$ Leight sample standard deviation.
10. For the above conditions for strong specular reflection $\psi \leqslant 0,037 \lambda / \sigma_{h}$ and $\psi \geqslant 1.4 \sigma_{s} \cdot$ Taking $\sigma \leqslant 10_{\mathrm{mi}} \mathrm{a}_{\mathrm{d}}, \sigma_{\mathrm{h}}>10 \mathrm{~m}$ then strong specular reflection would be rare at $\lambda=0.03 \mathrm{~m}$.
11. Reference is made to Chapter 8 eqns (7) and (8), where $\bar{n}^{2}=\varepsilon_{r}-j 600 \lambda=\varepsilon_{r}-\frac{j 1.8010^{4}}{f_{M H z}}$
$\sigma=$ surface conductivity mho.m.m ${ }^{-2}$
$\varepsilon_{r}=$ relative permittivity

Typical values for insertion at Chapter 8 and Annex $E$ are given at Table 1:

| SURFACE | CONDUCTIVITY <br> $\sigma$ | DIELECTRIC <br> $\varepsilon_{r}$ | TYPACAL $\beta$ <br> for $\Psi<1^{\circ}$ |
| :--- | :---: | :---: | :---: |
| Dry, Flat | $1 \times 10^{-4}$ | 5 | 0.3 |
| Farmland Rural |  |  |  |
| Low Hills |  |  |  |
| Medium Hills | $5 \times 10^{-2}$ | 15 | $0.1-0.2$ |
| City | $5 \times 10^{-3}$ | 13 |  |
| Sea | $2 \times 10^{-3}$ | 5 |  |

Table 1. Table of Typical Values for Reflecting Terrain

$$
\text { Dotted length } \approx \mathrm{R}
$$


fig b1 resolution cell for low grazing angle

## FRESNEL-KIRCHOFF SINGLE KNIFE EDGE DIFFRACTION SOLUTION

1. Assuming small grazing angles, and using the previous notation at Chapter 7 figure 2:

$$
E=\frac{1}{2 d}[1-(1+\jmath)(C-j S)] \exp \left[-\mu k\left(d+d_{1} \alpha-d_{2} \beta\right)^{2} / 2 d\right]-(1)
$$

```
Where E = Electric Field at receiver (target) from unit source
```

$\mathrm{S}, \mathrm{C}=$ Fresnel integrals of argument
$\theta \quad=$ diffraction angle (see diagram), ie $\propto+\beta$
$y=\theta \sqrt{d_{0} / \lambda}$
$\mathrm{k}=2 \pi / \lambda$ (known as wavenumber)
$d_{0}=2 d_{1} d_{2} / d \quad d=d_{1}+d_{2}$
Simplified $\{222\}$ then (1) becomes:
$E=\frac{1}{2 \pi \theta} \sqrt{\frac{\lambda}{d d_{1} d_{2}}} \exp \left[-1\left(k\left(r_{1}+r_{2}\right)+\pi / 4\right)\right]$
2. This approximation is developed from \{223\} : ~

$$
\begin{equation*}
E=\int_{x_{0}}^{\infty} e^{-: x^{2}} d x \approx \frac{1}{2 x_{0}} e^{-j\left(x_{0}^{2}+\pi / 2\right)} \text { for } x_{0} \gg 1 \tag{3}
\end{equation*}
$$

$$
\begin{aligned}
\text { Thus } C(v)-\rho(v) & =\int_{0}^{v} e^{-\jmath \pi u^{2} / 2} d u \\
& \approx \frac{1}{\sqrt{2}} e^{-\jmath \pi / 4}-\frac{1}{\pi v} e^{-j \pi\left(v^{2}+1\right) / 2} \\
& \text { for } \sqrt{\pi / 2} v \gg 1
\end{aligned}
$$

3. For engineering purposes the approximation condition is:

$$
\begin{equation*}
\theta^{2} \geqslant 8 \lambda / \pi a_{0} \tag{6}
\end{equation*}
$$

Which is interpreted physically as requiring the receiver (target position in this report) to be deep in the shadow zone, or equivalently, that the first Fresnel zone on the obstacle path is well masked by the obstacle. This condition is satisfied by most microwave diffraction paths.
4. Although the above is included here for completeness, extended rigourous treatment of the wave theory of the solutions are often complex and cannot be directly applied to this practical case.

ANNEX D TO
"THE PROBABILITY OF
DETECTING AND TRACKING
RADAR TARGETS IN CLUTITER

AT LOW GRAZING ANGLES"
DATED Ko SEP 82

COMPUTER PROGRAMS AND FTOWCHARTS

1. It is not intended to include lengthy program listings or detailed flowcharts in the report, but to describe the programs and datafiles briefly.

DATA
2. The following data was available for clutter and other investigations either at the start of, or was generated as the project progressed:
a. Plotted results from the worldwide clutter survey, from which values could be interpolated for correlation studies for $\psi$, Weibull, etc.
b. Raw radar measurements taken at a set range over a known sector in the Malvern area (E J Dodsworth RSRE on his retirement).
c. A Malvern terrain data base (produced at Malvern) but with a larger matrix spacing than satisfactory for the particular work envisaged.
d. Two area data matrix (manually produced by the author) to a finer spacing for the project. (Malvern and Scottish).
e. Raw radar measurements in considerable quantity (unfortunately no tape available), from British Aerospace Stevenage. From these extensive listings, data was re-entered onto disc for the slope correlation studies.
f. Files generated for radar parameters.

PROGRAMS
3. A brief resume follows for each of the main programs (written in FORTRAN to run on the DEC 20 at Cranwell) which were used to calculate, process or plot results during the research. A number of smaller programs were also written to manipulate data bases in support.
a. SIJTNE.FOR Scans through $360^{\circ}$ in any increment and to any range, from any given site location within a terrain matrix (spot height) data base; for any target and radar site height and produces sightline (obscuration) data for plots of the type at Chapter $2(2-36)$. Incorporates height of surface obstacles eg trees, which it combines with the terrain/matrix data from the files at para $2 c$ and 2e above.
b. RADS.FOR Makes terrain data for a particular resolution cell match the corresponding backscatter signal. Calculates terrain slope, aspect, actual grazing angle and $\sigma_{m}$. Creates a new file containing all
data necessary for slope correlation studies. Also produces terrain profile data and through convexity calculations diffraction plot data files.
c. SNOISE.FOR Makes all radar propagation, range equation, pencil beam weather and surface clutter calculations. Including fluctuating target, main and sidelobe jamming and chaff jamming subroutines. Also includes multipath calculations and tracker range checks.
d. IRADAW.ALG/PAS Originally in ALGOL but now also in PASCAL and modified, this program flys the actual missile aerodynamic and control functions to produce a time readout of the missile trajectory. Apart from some interest in the tracker control functions it was decided for this report that a mean missile flyout range was adequate for the type of prediction envisaged.
e. PPI This plotter program produces a circular (PPI) radar type display for surface obscuration or diffraction plots of the type shown at Chapter $2(2-38)$.
4. STATPK. STATPK, the college statistical library, was used to produce correlation matrices, regression and all other statistical results.
5. Working flowcharts were made for each program, and hard-copy program and data listings were maintained for each revision. All outputs from STATPK were taken on hard copy for detailed analysis; these include regression plots, scatter plots, sorts, correlation matrices, histographs, bargraphs, frequency tables, Kolmorogov-Smirnov tests, and basic statistical measurements.

## LOW LEVEL TRACKING ERRORS AND TRACK LENGTH PROBABILITY

1. Probability Density Function for Tracking Error. An expression can be derived for the probability density function of the tracking error, in terms of target altitude $h_{t g t}$, linear error $\varepsilon$, and the power ratio of the direct signal reflected from the target compared to the multipath (surface reflection signal - see fig 1) $q_{s}^{2}$ :-

$$
\begin{equation*}
w(x)=\frac{2 q_{s}^{2}}{\sqrt{1+q_{B}^{2}}\left[\left(1+x^{2}\right)+q_{s}^{2}\left(1-x^{2}\right)\right]} \tag{1}
\end{equation*}
$$

$$
x=\varepsilon / h_{t g t}, \text { the relative centroid tracking error. }
$$

From Figure 2, the mean relative error $M_{\varepsilon}$ isgiven hy:

$$
\begin{align*}
& \frac{M_{E}}{h_{t g t}}=\int_{-\infty}^{\infty}: X W(x) d x=\frac{q_{g}^{2}-1}{q_{s}^{2}+1}  \tag{2}\\
& \text { and } M_{\varepsilon}=h_{t g t} \frac{q_{s}^{2}-1}{q_{s}^{2}+1} \tag{3}
\end{align*}
$$

$\qquad$
$\qquad$
2. Assuming the target maximum dimension (ie wingspan or fuselage length) is $s$, then the probability that the sightline will fall on the target during tracking is shown at Figure $3\{224\}$, from:

$$
\begin{equation*}
\mathrm{p}=\int_{\varepsilon_{2}}^{\varepsilon_{1}} \mathrm{w}(x) d x=\emptyset\left(\varepsilon_{1}\right)-\emptyset\left(\varepsilon_{2}\right) \tag{4}
\end{equation*}
$$

Where $\emptyset(\varepsilon)=\frac{\left(1+q_{B}^{2}\right) \varepsilon+\left(1-q_{s}^{2}\right)}{2 \sqrt{1+q_{B}^{2}\left[(1+\varepsilon)^{2}+q_{s}^{2}(1-\varepsilon)^{2}\right]}}$ $\qquad$
$\varepsilon_{1}=\frac{2 h_{\operatorname{tgt}}+s}{2 h_{\operatorname{tg} t}}$
$\varepsilon_{2}=\frac{2 h_{t g t}-s}{2 h_{t g t}}$
3. Figure 3 shows the relationship, where for a target flying at an altitude of 50 m , target size 10 m , linear error $\varepsilon$ of 10 m and $q_{\mathrm{g}}^{2}=10$, then the probability of the sightline falling on the target is approximately $10 \%$. At maximum tracking range it would be expected that to track correctly the system should remain within 10 m vertical error for $70 \%$ of the time. (Fig La)
4. Angular Tracking Error. Assuming the radar aerial receives 2 signals, ie direct and multipath, respectively $S_{1}=V_{1} \operatorname{Cos} \omega_{1} t, S_{2}=V_{2} \operatorname{Cos} \omega_{2} t$

If $\omega_{0}$ is the carrier frequency and $v_{1}$ and $V_{2}$ are the apparent approach (radial) velocities of the target and its image, then

$$
\omega_{1}=\omega_{0}\left[1+\left(\frac{v_{1}}{c}\right)\right] ; \quad \omega_{2}=\omega_{0}\left[1+\left(\frac{v_{2}}{c}\right)\right]
$$

Combining (6) above ( $S_{1}+S_{2}$ )

$$
\begin{align*}
& =v_{1} \operatorname{Cos} \omega_{1} t+v_{2} \operatorname{Cos} \omega_{2} t  \tag{7}\\
& =2 V_{1} v_{2} \frac{\operatorname{Cos}\left(\omega_{1}-\omega_{2}\right)}{2} t \operatorname{Cos} \frac{\left(\omega_{1}+\omega_{2}\right) t}{2} \tag{8}
\end{align*}
$$

5. Assuming a quadratic detector in the radar receiver then its output voltage is:

$$
\begin{equation*}
v_{t g t}=K\left[1+\frac{1}{2} \operatorname{Cos} 2 \omega_{1} t+\frac{1}{2} \operatorname{Cos} 2 \omega_{2} t+\operatorname{Cos}\left(\omega_{1}+\omega_{2}\right)+\operatorname{Cos}\left(\omega_{1}-\omega_{2}\right) t\right] . \tag{9}
\end{equation*}
$$

$K$ is a constant. A beat frequency $\omega_{1}-\omega_{2}$ causes disturbance in the tracking accuracy if the radial velocities $v_{1}$ and $v_{2}$ are close to the aerial scanning frequency $\Omega_{s c} i e \frac{2 \omega_{0}}{c}\left(v_{1}-v_{2}\right) \approx \Omega_{s c}$.
6. Multipath propagation causes the output from the radar tracker aerial to be:

$$
\begin{align*}
& =S_{1} L_{1}(\alpha)\left[1+m_{1}(\alpha) \cos \Omega_{s c} t\right] \cos \omega_{1} t+S_{2} L_{2}(\alpha, \Delta \alpha)\left[1+m_{2}(\alpha, \Delta \alpha)\right. \\
& \left.\quad \operatorname{Cos} \Omega_{s c}{ }^{t}\right] \operatorname{Cos} \omega_{2} t \\
& L_{1}(\alpha)=\frac{1}{2}\left[f\left(\alpha_{0}-\alpha\right)+f\left(\alpha_{0}+\alpha\right)\right]  \tag{11}\\
& L_{2}(\alpha \Delta \alpha)=\frac{1}{2}\left[1\left(\alpha_{0}-\alpha+\Delta \alpha\right)+f\left(\alpha_{0}+\alpha-\Delta \alpha\right)\right] \\
& m_{1}(\alpha)=\frac{1\left(\alpha_{0}-\alpha\right)-f\left(\alpha_{0}+\alpha\right)}{f\left(\alpha_{0}-\alpha\right)+f\left(\alpha_{0}+\alpha\right)} \tag{12}
\end{align*}
$$

$S_{1}$ is required signal. $S_{2}$ is interfering signal.
$m_{2}(\alpha, \Delta \alpha)=\frac{f\left(\alpha_{0}-\alpha+\Delta \alpha\right)-f\left(\alpha_{0}+\alpha-\Delta \alpha\right)}{f\left(\alpha_{0}-\alpha+\Delta \alpha\right)+f\left(\alpha_{0}+\alpha-\Delta \alpha\right)}$
$\boldsymbol{\alpha}_{0}=$ Displacement of aerial beam peak relative to equal signal line.
$\alpha$ a tiacking error for turget.
$\Delta \alpha=$ Angular separation between true target and image.

Signal detection and amplification at $F=\Omega_{s c}$, assuming $\omega_{1}-\omega_{2}=\Omega_{s c}$, the phase detector output voltage $S_{3}$ (zero at balanced condition) is:

$$
\begin{align*}
s_{3}= & s_{1}^{2} L_{1}^{2}(\alpha) m_{1}(\alpha)+s_{1}^{2} L_{1}^{2}(\alpha, \Delta \alpha) m_{2}(\alpha, \Delta \alpha)+s_{1} L_{1}(\alpha) S_{2} L_{2}(\alpha, \Delta \alpha)+\frac{3}{4} \\
& s_{1} S_{2} L_{1}(\alpha) L_{2}(\alpha, \Delta \alpha) m_{1}(\alpha)\left[m_{2}(\alpha, \Delta \alpha)\right] \tag{15}
\end{align*}
$$

Using equations (10) to (15) above, the angular radar tracker error is given by:

$$
\begin{align*}
& f^{2}\left(\alpha_{0}-\alpha\right)-f^{2}\left(\alpha_{0}+\alpha\right)+q_{n}\left[f^{2}\left(\alpha_{0}-\alpha+\Delta \alpha\right)-f^{2}\left(\alpha_{0}+\alpha-\Delta \alpha\right)\right]+ \\
& \frac{q_{n}}{4}\left\{f\left(\alpha_{0}-\alpha+\Delta \alpha\right)\left[f f\left(\alpha_{0}-\alpha\right)+f\left(\alpha_{0}+\alpha\right)\right]+f\left(\alpha_{0}+\alpha-\Delta \alpha\right)\right. \\
& \left.\left[f\left(\alpha_{0}-\alpha\right)+7 f\left(\alpha_{0}+\alpha\right)\right]\right\}=0 \tag{16}
\end{align*}
$$

$$
q_{n}=\frac{1}{q_{s}}
$$

7. Placing practical values into eqn (16), provides the relationship between tracking error and angular separation angle between target or image. If plotted with normalized error and separation over a range of $\Delta \alpha$ and $q_{n}$ values, it is seen that for conditions where the interfering
signal is comparable to the wanted signal $\left(q_{n} \approx 1\right)$ the effective target position is above the real target position; whilst for a swamping value of interference signal for very shallow grazing (ie $\Delta \alpha \rightarrow 0$ ) the effective target position is lower than the actual target position. Interference, causing inaccuracies may be minimised in practice by changing the scanning Prequency or narrowing the tracker receiver pass band. Alternatively the technique known as 'complex indicated angle' can be used to minimise the multipath effect \{225\} For accurate tracking it is stated \{226\} p 330 that for a $1^{\circ}$ beam width and 0.1 mrad measuring accuracy:

$$
\frac{\rho}{\sqrt{8 G_{s}}} \ll 0.005
$$

$\qquad$

A value of $\rho=0.3$ (typical for land) is used. $G_{B}=$ Specular Power Gain Ratio
8. Effect of Terrain Slope. It is of course possible that multipath reflections, (including tracker error), could come from a sloping patch of terrain, and for clutter and other effects, the relative height of the radar transmitter, slope of the terrain and target altitude must be used for calculations.

MULTIPATH TRACKING ERROR USING MONOPULSE TRACKER
9. For short ranges and assuming a flat earth the multipath tracking error of a monopulse radar is given by $\{227\}$ :

$$
\begin{equation*}
\Sigma(\text { mrads })=\frac{\theta_{E} \rho_{v}}{k_{m}} \sqrt{\frac{1}{2}\left(\sum_{i=1}^{n} I_{i}+I_{s}\right)} \tag{18}
\end{equation*}
$$

Where $I_{i}, I_{s}$ are the diffuse and specular components, (eqns $1 a_{1}, 2_{0}$ belbw)
$\theta_{E}$ the aerial beamwidth in elevation
$\rho_{v}$ vegetation absorption (commonly 0.1 to 0.3 for $\Psi=0.5<2 \mathrm{deg}$ ).
$k_{m}$ monopulse slope ( $\approx 2$ )
n number of depression angle elements summed
Eqn (18) then uses Eqns (19) to (26) below.
Separately $I_{i}$ and $I_{s}$ are given by

$$
\begin{align*}
& I_{i}=\Delta_{i}^{2} \rho_{o i}^{2} \eta_{d i} \Delta_{\theta}  \tag{19}\\
& I_{s}=\Delta_{s}^{2} \rho_{o i}^{2} \rho_{s}^{2} \tag{20}
\end{align*}
$$

$\qquad$

The Fresnel reflection coefficients are not repeated here, . $\eta_{\mathrm{di}}$ is the diffuse reflection density, $\Delta_{i}$ and $\Delta_{s}$ the difference pattern gains, $\Delta_{\theta}$ the width of element in depression angle, $\rho_{s}$ the specular scattering factor.
10. The difference channel illumination is assumed to be of the distribution $x \cos x$. For which

$$
\begin{equation*}
\Delta=\frac{6\left(\frac{\pi^{2}}{4}-U^{2}\right) \sin U-2 U \cos U}{\left(\frac{\pi^{2}}{4}-U^{2}\right)^{2}} \tag{21}
\end{equation*}
$$

$$
\begin{gathered}
\mathrm{U}=\frac{3.77 \theta}{\theta_{\mathrm{E}}} \quad \theta=\theta_{\mathrm{b}}+\psi_{1} \text { where } \theta_{\mathrm{b}} \text { is elevation of beam axis.(rads). } \\
\psi_{1}, \psi_{2} \text { grazing angle from radar to surface } \\
\text { and target to surface.(rads). }
\end{gathered}
$$

For the reflection coefficients it is taken that $\psi=\frac{\psi_{1}+\psi_{2}}{2}$ and the surface complex dielectric constant is calculated in the usual way to get $\rho_{0}$. Assume $\sin \psi \approx \psi \approx \operatorname{Tan} \psi . \rho_{0}>0.8$ for $\psi<2^{\circ}$ for bots $V \& H$ polarisations.
21. Diffuse power density per radian of depression angle is:

$$
\begin{equation*}
\eta_{d}=\frac{R}{R-x} \frac{\psi_{1}+\psi_{2}}{4 \sqrt{\pi} S_{0} \psi_{1}} \exp \left[-\left(\frac{\psi_{1}-\psi_{2}}{2 S_{0}}\right)^{2}\right] F_{d}^{2} z \tag{22}
\end{equation*}
$$

Where
$S_{0}$ is the RMS surface slope deviation
$\psi_{1}=h_{t x} / x$, and $\psi_{2}=h_{t g t} / R-x$ (rads)
$x=$ range from radar to surface
$R=$ radar range of target
$F_{d}=$ Roughness Factor (equ 23 below)
$z=$ Low grazing angle correction factor (see equs 25, 26)
12. Roughness Factor and specular scattering factor are given by:

$$
\begin{aligned}
& F_{d}=\sqrt{1-\rho_{s}} \text { where } \rho_{s}=\text { specular scattering factor } \\
& \text { hence } F_{d}^{2}=\sqrt{1-\rho_{s_{1}}} \cdot \sqrt{1-\rho_{s_{2}}} \\
& \\
& =F_{d_{1}} \cdot F_{d_{2}} \text { for } \psi_{1}, \psi_{2}
\end{aligned}
$$

$$
\rho_{s_{1}}, \rho_{s_{2}} \text { from equ(24) for } \psi_{1} \psi_{2} \text { respectively }
$$

$$
\begin{aligned}
\rho_{s}^{2} & =\exp \left[-\left(\frac{4 \pi h^{\prime}}{\lambda} \psi\right)^{2}\right] \\
& =\exp \left(-\psi^{2} / \psi_{c}^{2}\right) \\
\sigma_{h^{\prime}}^{\prime} & =\text { RMS surface height deviation corrected for shadowing } \\
\psi_{c} & =\text { critical angle for } \rho_{s}^{2}=\frac{1}{e}
\end{aligned}
$$

13. For low grazing angles 2 correction terms are used

$$
\begin{aligned}
& a=\psi_{1} / 2 S_{0}, \quad c=\psi_{2} / 2 S_{0} \\
& a^{1}=\min (a, c\rangle \quad c^{1}=\max \langle a, c\rangle
\end{aligned}
$$

(which is used depends on the smaller of the 2 angles).
Effective surface roughness is corrected for shadowing

$$
\begin{array}{rlrl}
\sigma_{h}^{\prime} & =\sigma_{h} 5 \sqrt{4 a^{\prime}} & & \text { where } 4 a^{\prime} \\
\text { or } \sigma_{h}^{\prime} & =1 \\
& =\sigma_{h} & & \text { where } 4 a^{\prime} \geqslant 1
\end{array}
$$

14. Low grazing angle correction for diffuse power density, where $b=$ $\psi_{c} / 2 S_{o}:$

$$
\begin{array}{ll}
z_{1}=\frac{8 \sqrt{1-\rho_{8 a}^{2}}}{\frac{1}{a}+4+3 a^{\prime}-\frac{b^{2}}{3 a^{\prime}}-b} & \text { for } b \leqslant 1+a^{\prime} \\
z_{2}=\frac{24}{\frac{2}{a^{\prime 2}}+\frac{9}{a^{\prime}}+12+5 a^{\prime}} & \text { for } b \geqslant 1+a^{\prime} \tag{26}
\end{array}
$$

$\qquad$

Where

$$
\begin{aligned}
& b=\frac{\psi_{c}}{2 S_{0}}=\frac{\lambda}{8 \pi a_{h}^{i} S_{0}} \\
& \rho_{\text {sa }}=\exp \left(-a^{2} / b^{2}\right)
\end{aligned}
$$

15. The specular error component is only present in the absence of nosediving ie for $\theta_{b}>0.7 \theta_{E}$ or for lower angles if $\rho_{O S} \rho_{s}<0.5$ ( $\rho_{O S}$ is Fresnel specular reflection coefficient). A calculator program for the above is available as a Texas Instruments master library module.

## TRACK LENGTH PROBABILITY

16. An initial survey or terrain data base may be used to produce (see Fig 5a) unscreened track lengths. This may be difficult in practice, but it is hoped that by examination of typical sites a pattern of probabilities might emerge so as to act as a starting point in predicting defensive performance or conversely offensive survivability when operating aircraft against these low level radar systems. Short periods of target obscuration might be considered negligible since modern systems, rate aided, may be able to track a target 'through' a narrowly obscured sector of, say, $1^{\circ}$; the exception is of course when the target is headed radially towards the radar along an obscured flight path.
17. Example. For explanation purposes an example is used:
a. Target Parameters. Velocity $300 \mathrm{~m} \mathrm{sec}{ }^{-1}\left(\mathrm{~V}_{\mathrm{tgt}}\right)$, Mean Kaiget range (crossing target) 3 km ( $\mathrm{R}_{\text {tgt }}$ ), Altitude 60 m ( $\mathrm{h}_{\text {tgt }}$ ).
b. Missile Parameters. Velocity $600 \mathrm{~m} \mathrm{sec}{ }^{-1}\left(V_{m}\right)$, Radar Site Reaction Time $15 \mathrm{sec}\left(t_{r}\right) . V_{m}$ is assumad conctant over unmasked time interval.
c. Obscuration. Probability of obtaining track length of $x$ metres is p .
18. For a given aircraft velocity a non-obscured target must be tracked on radar for $T$ secs. Useful tracks are dependent on range from the radar and $V$ tgt, since the geometry of a track at longer ranges (though perhaps visible for the same time as a track at shorter rangel may not allow an engagement to succeed because of the longer missile flight-time to reach the required range. This is especially so in a commanded missile system where missile and target must be observable at all times up to impact if they are to be tracked on radar and the appropriate guidance commands derived and transmitted to the missile. Using the example figures an approximation for crossing targets at mean range is:

$$
t_{f}=\frac{R_{t g t}}{V_{m}}=\frac{3000}{600}=\underline{5 \sec s} \text { and } T=t_{f}+t_{r}=20 \operatorname{secs}
$$

where $t_{f}+t_{r}$ is the minimum observation time required. An approximate track length $T_{c}$ necessary for an engagement for a target crossing at sensibly constant range is:

$$
T_{c}=V_{t g t}\left(t_{f}+t_{r}\right)=300 \times 20=6 \mathrm{Km}
$$

The probability of obtaining this track length is about 0.25 at Figure $5 b$. For a radially approaching target a close approximation is:
$T_{c}=t_{r} V_{t g t}+t\left(V_{t g t}+V_{m}\right)$ where $t>\frac{d}{V_{m}}$ given that
$d$ is the minimum possible impact range. ${ }^{[ }$If $d=600 \mathrm{~m}$, for this example
$t>1$, hence $t\left(V_{t g t}+V_{m}\right)>900$ and $T_{c} \approx 5.4 \mathrm{Km}$.
Accurate computations for system and target parameter change can be made from weapon trajectory programs written by the author, however the above method is adequate for manual predic tion purposes.

8
19. If the aircraft altitude is changed to $h_{t g t}=200 \mathrm{~m}$, the same timing calculations apply if the range is unchanged. However, a decrease in
obscuration due to the increased target altitude will cause the probability of obtaining the critical track length $T_{c}$ to (typically) rise to 0.7 for flat terrain. Table 1 shows the effect in similar terrain but with changes in target and missile parameters.

## TABLE 1 PROBABILITY OF OBTAINING MINTMUM TRACK LENGTH REQUIRED

Target : $V_{m}=600 \mathrm{~m} \mathrm{sec}^{-1}$ System Reaction 10 sec
Terrain: typical flat terrain with scattered clumps of trees.

|  | TARGET PARAMETERS |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} v_{\text {tgt }} \\ 260 \mathrm{~m} \mathrm{sec}-1 \\ (500 \mathrm{kts}) \end{gathered}$ |  | $\begin{gathered} v_{\text {tgt }} \\ 220 \mathrm{~m} \mathrm{sec}^{-1} \\ (420 \mathrm{kts}) \end{gathered}$ |  |
| mean Range to target Km | $\begin{gathered} h_{t g t} \\ 200(f t) \end{gathered}$ | $\begin{gathered} h_{t g t} \\ 300(f t) \end{gathered}$ | $200(f t)$ | $\begin{gathered} h_{t g t} \\ 300(f t) \end{gathered}$ |
| 1 | 0.62 | 0.75 | 0.66 | 0.77 |
| 3 | 0.37 | 0.52 | 0.41 | 0.55 |
| 5 | 0.22 | 0.37 | 0.26 | 0.40 |

20. As expected the probability figures are more sensitive to a change of target height, than of target velocity. It is of interest to note that doubling the missile speed (and using the same $t_{r}$ ) would marginally ( 0.01 ) increases the probability of engaging targets at close range, $t_{r}$ is more sensitive for close range targets. However, the higher missile speed increases probability values by 0.08 , ie almost $10 \%$ at 5 km range, with the example terrain used here.
21. In the above examples the radar aerial is almost at ground level and it is assumed the missile flies in a straight line (rather than the more
usual curvea trajectory). This approximation will make little difference for a general assessment, since it is assumed also that the target is being tracked at a mean range. In practice with the variation of ground tracks sometimes the target will be nearer at the beginning or end of an engagement if no evasive manoeuvres are used, the missile velocity will also vary, depending on range. Clutter is ignored for the moment. If the tracking aerial is raised above the immediate obscuration the situation will be changed significantly and in general at short ranges the target's only hope of evading a tracking situation is either by the inability of the radar to follow the high sightline rate, to separate the target from surface clutter, or by deliberately degrading the sybem by introducing noise or deception jamming and hard target manoeuvre.

## TRACKING ALGORITHM

22. It may seem from the foregoing in this Annex that radar detection theory has been temporarily forgotten. The picture is now completed by considering an example tracking algorithm as part of the overall detection process. Assuming a tracking situation (unobscured target - which may or may not last for minimum track length $T_{c}$ at para 18), then a statistical algorithm to separate genuine detection opportunities from false alarms can be used to detect an acceptable sequence of detections and a track is then declared. Markov chains can be used to study such sequences with the criterion that a tracking state should be held for a minimum number of time intervals and at a correct signals to noise ratio. The relationship between p (detection probability) and declared tracking status is derivable. Results are not only dependent upon the number of observations chosen when setting up the algorithm, \{228\} but the degree of correlation between individual target returns (hits) during the observation time interval.
23. Derivation of Algorithm. If the detection opportunities are taken in time sequence, as triplets, where $p=P$ (next digit is a 1 ), $q=P$ (next digit is a 0 ), then there are 8 -bit patterns. Initial and final states ( $i, j$ ) are plotted below; where $P_{i j}$ occurs in one change:


If two 1 states are required for tracking ( 2 our of 3 , ie $n=3, k=2$ ), then non tracking will be represented by the binary pattern for $0,1,2$ or 4 and tracking by $3,5,6$ or 7 . The above plot can be re-written, with $T$ and $\bar{T}$ representing tracking and non-tracking respectively. Although the example used here assumes 2 "hits" out of any successive observations (ie 2 signal returns out of 3 produce 1 states by crossing the detection threshold), other radars may use algorithms which use a larger number of observations ( $n$ ) and require more hits ( $k$ ). The values of the transition matrix at equation 35 will change accordingly. Performance prediction of a radar with unknown tracker processing characteristics will require exploration of a range of values for n and k . The technique used here is known as a 'sliding window' algorithm.

24. If each quadrant (ie $T \bar{T}, T T, \bar{T} T$ and $\overline{T T}$ ) is taken separately: (Joint probabilities)
a. $\bar{T}=P(T$ and $\bar{T})$ - Tracking to Non Tracking. Starting in a tracking state, ie 3, 5, 6 or 7. Moving from state 3 to state 1. $P$ (of being in state 1 at (time $t$ ) $\times P$ (transition from state 3 to state 1 at $t+1$ ).

$$
P_{3}, P_{3,1}=p^{2} q \times q=p_{q}^{2}{ }^{2}\left(P_{3} \times q\right)
$$

The only other non-zero term is $P_{5} \times P_{5,2}$

$$
=p^{2} q \times q=p^{2} q^{2}
$$

Combining ( $P_{3} P_{3,1}$ with $P_{5} P_{5,2}$ ) gives the transition $P(T \bar{T})$.

$$
\begin{aligned}
\therefore P(T \bar{T})= & \Sigma P_{i} P_{i j}=2 p^{2} q^{2} \\
& i, 3,5,6,7 \\
& j, 0,1,2,4
\end{aligned}
$$

b. $\bar{T} T=P(\bar{T}$ and $T$ ) - Non Tracking to Tracking. We have $P_{2}, P_{2,5}$ and $P_{4} P_{4,6}$

$$
\begin{equation*}
\therefore p(\overline{T T})=\Sigma p_{i} p_{j}=\left(p q^{2} \times p\right)+\left(p q^{2} \times p\right)=2 p^{2} q^{2} \tag{28}
\end{equation*}
$$

$\qquad$

$$
\begin{aligned}
& i, 0,1,2,4 \\
& j, 3,5,6,7
\end{aligned}
$$

c. $T T=P(T$ and $T)$ - Tracking Maintained. Six items are considered

$$
\begin{align*}
& P_{3} P_{3,5}+P_{5} P_{5,6}+P_{6}\left(P_{6,3}+P_{6,7}\right)+P_{7}\left(P_{7,3}+P_{7,7}\right) \\
& =\left(p^{2} q \times p\right)+\left(p^{2} q \times p\right)+\left(p^{2} q \times 1\right)+\left(p^{3} \times 1\right) \\
& =2 p^{3} q+p^{2} q+p^{3}=p^{2}(2 p q+q+p)=p^{2}(2 p q+1) \\
& \therefore P(T T)=\Sigma P_{i} P_{i j}=p^{2}(2 p q+1)  \tag{29}\\
& =i, 3,5,6,7 \\
&
\end{align*}
$$

d. $\bar{T} \bar{T}=P(\bar{T}$ and $\bar{T})$ - Non-Tracking Maintained

$$
\begin{aligned}
& P_{0}\left(P_{0,0}+P_{0,4}\right)+P_{1}\left(P_{1,0}+P_{1,4}\right)+\left(P_{2} \times P_{2,1}\right)+\left(P_{4} \times P_{4,2}\right) \\
& =\left(q^{3}+p q^{2}\right)+\left(p q^{2} \times q\right)+\left(p q^{2} \times q\right) \\
& =q^{3}+p q^{2}+2 p q^{3} \\
& \therefore P^{T} \bar{T}=\Sigma P_{i} P_{i j}=q^{2}(2 p q+1) \\
& \quad i, 0,1,2,4 \\
& \quad j, 0,1,2,4
\end{aligned}
$$

25. The transition matrix is that of conditional probability.
a. To get the entry for $\bar{T} T$, start in state $\bar{T}$ and finish in $T$

$$
\text { if } \begin{align*}
P(\bar{T}) & =P(\bar{T} \bar{T})+P(\bar{T} \bar{T}) \\
\text { and } P(\bar{T} / \bar{T}) & =\frac{P(\bar{T} T r}{P(\bar{I})} \quad(i e \text { probability of } T \text { given } \bar{T}) \\
& =\frac{2 p^{2} q^{2}}{2 p^{2} q^{2}+q^{2}}(2 p q+1) \tag{31}
\end{align*}
$$

b. Similarly for $P(T / T)$ then $P(\bar{T} / T)=\frac{P(\bar{T} T)}{P(T)}$

$$
\text { where } \begin{align*}
P(T / T) & =P(T T)+P(T T) \\
& =\frac{2 p^{2} q^{2}}{2 p^{2} q^{2}}+p^{2}(2 p q+1) \tag{32}
\end{align*}
$$

c. For $P(T / T)$ then,

$$
\begin{aligned}
\frac{P(T T)}{P(T)} & =\frac{P(T T)}{P(T I)+P(T I)} \\
& =\frac{p^{2}(2 q+1)}{2 p^{2} q^{2}+p^{2}(2 p q+1)}
\end{aligned}
$$

d. Finally for $P(\bar{T} / \bar{T})$ then,
$\frac{p(\bar{T} \bar{T})}{P(\bar{T})}=\frac{q^{2}(2 p q+1)}{2 p^{2} q^{2}+q^{2}(2 p q+1)}$

Giving the matrix, after cancellation:


From matrix above $P_{T, T}=1-P_{T M}$

$$
\begin{equation*}
=\frac{2 p q+1}{2 q^{2}+2 p q+1} \tag{36}
\end{equation*}
$$

26. The probability of obtaining a number of successive detections $N$ is:
(declarations)

$$
P_{T}(N)=\left(P_{T M P}\right)^{N} P_{T \bar{T}}
$$

$$
\begin{equation*}
P_{T(N)}=\left[\frac{2 p q+1}{2 q^{2}+2 p q+1}\right]^{N} \quad \frac{2 q^{2}}{2 q^{2}+2 p q+1} \tag{37}
\end{equation*}
$$

The mean track length (taken from the geometric distribution at (38)) and measured in terms of the number of successive detections is plotted at Fig 6.

$$
\begin{equation*}
\bar{L}=\frac{2 q^{2}+2 p q+1}{2 q}=1+\frac{p}{q}+\frac{1}{2 q^{2}} \tag{38}
\end{equation*}
$$

27. From the above, the probabilities of interest are; m summary (and subject to a continued sightime once a track is declared :-

$$
\begin{equation*}
P_{T}(\text { at any time })=p^{3}+3 p^{2} q \tag{39}
\end{equation*}
$$

$P_{\bar{T}} \quad($ no tracking $)=q^{3}+3 \mathrm{pq}^{2}$
$P_{\bar{T} T}$ (New track commencing) $=\frac{2 p^{2}}{2 p^{2}+2 p q+1}$
(given a sightline and at least 2 out of 3 successive threshold crossings in the sampling time frame)
PRACTICAL INTERPRETATION
28. Each detection is of course dependent on the signal/clutter ratio, both of which are fluctuating. In the first case this is due to target RCS variations (glint) and secondly as a function of the clutter level being simultaneously received. Swerling and Rayleigh distributions (229\}, and Heidbreder and Mitchell \{230\} show that lognormal distributions may be applicable to certain types of target.
29. The probability of an integrated group of $N$ signals (with noise samples) exceeding a threshold $\mathrm{V}_{\mathrm{t}}$ has been examined by Powell \{231\}. For the distribution (Swerling 3) this can be approximated with four degrees of freedom as:

$$
P_{N_{4}}=\left(1+\frac{2}{N \sigma}\right)^{N-2}\left[1+\frac{V_{t}}{1+\frac{N \sigma_{A V}}{2}}-\frac{2(N-2)}{N \sigma_{A V}}\right] \exp \left[-\frac{V_{t}}{1+\frac{N \sigma_{A V}}{2}}\right]-(42)
$$

and may be compared with the other Swerling distributions with two degrees of freedom (from $p(\sigma)=\frac{1}{\sigma} \exp .\left(-\frac{\sigma}{\sigma}\right)$ ):

$$
\begin{equation*}
P_{N_{2}}=\left(1+\frac{2}{\mathrm{~N} \sigma}\right)^{N-1} \exp \cdot\left(-\frac{V_{t}}{1+N \sigma_{\mathrm{AV}}}\right) \tag{43}
\end{equation*}
$$

30. At short ranges $N \sigma_{A V}{ }^{+\infty}$, therefore in equations (42) and (43) above $\mathrm{P}_{\mathrm{N}_{2}}>\mathrm{P}_{\mathrm{N}_{4}}$. At longer ranges $\mathrm{N} \bar{\sigma}$ will decrease. If strong signal peaks are received from time to time from targets at range they may nevertheless exceed $V_{t}$, even though the required target mean signals are below detection threshold. A critical or "crossover" range must exist, where $P_{N_{2}} \approx P_{N_{4}}$, at which this takes place. By further approximations taken over shorter ranges the "crossover range" may be deduced which shows that a low 'noise' target is more easily detected at short ranges, while a noisy, peaking or spiky target is more easily detected at long ranges. Using the range at which signal/noise ratio is $1(0 \mathrm{~dB}), \mathrm{P}_{\mathrm{N}_{2}}<\mathrm{P}_{\mathrm{N}_{4}}$ when $\mathrm{p}<1.256$ ie:

$$
\begin{equation*}
R=1.059\left(\frac{v_{t}-N+1}{N}\right)^{\frac{1}{4}} R_{(0 \mathrm{~dB})} \tag{44}
\end{equation*}
$$

31. With $F A R=10^{-6}$ and $N=1000$ the crossover range is $0.835 \mathrm{R}(\alpha B)$. Beyond this range the target signal fluctuations enhance the probability of detection' and hence tracking. If $R_{(O A B)}$ is approximately twice the detection range for $P_{\text {det }}=0.9$, then $R$ (crossover) is $\approx I .7 R\left(P_{d}=0.9\right)$.
It is seen therefore that a large signal variance does not always coincide with a high probability of detection.
32. Although this approach may be acceptable for certain aircraft targets, it is thought that they must exhibit fairly angular structural shapes, ie mutually orthogonal reflecting surfaces, in order to produce the large dynamic range of "spiked" returns. The technique is probably not applicable to small targets with smooothed profiles.
33. In all cases there is a crossover range beyond which fluctuations enhance $P_{d}$, but inside which fluctuations detract from $P_{d}$. When integration number $N$ is plotted against crossover rance, the following applies:
a. Low Integration Numbers e.E. Acquisition radars, crossover rance is low.
b. High Integration Numbers e.c. Tracking radars $p_{d}$ at corssover is at greater range $\left(0.75<\frac{\mathrm{R}}{\mathrm{R}_{0}}<0.90\right)$. Within this ronge a closing small. tarcet will be more difficult to track if it is "spiky".


FIG 1 REFLECTION CENTROID OF LOW FLYING AIRCRAFT


fig 3 PROBABILITY OF SIGHTLINE FALLING ON TARGET



FIG 5a PROBABILITY OF OBSERVING A GIVEN TRACK LENGTH


FIG 5h PROBABILITY OF OBTAINING A GIVEN TRACK LENGTH AT A given crossing range


FIG 6 PROBABILITY OF OBTAINING A GIVEN TRACK LENGTH EXPRESSED AS A NUMBER OF DETECTIONS

1. Reference is made to Figure 1, where the average gradients of a surfaceilluminated tilted 'facet' between adjacent matrix terrain spot heights ( $A B C D$ ) are:
a. In Range

$$
\begin{equation*}
\alpha_{R}=\frac{\alpha_{R_{1}}+\alpha_{R_{2}}}{2} \tag{1}
\end{equation*}
$$

b. In Azimuth
2. The simple facet shown can be taken as the inluminated area beneath the resolution cell for shallow surface gradients. Although it is realised that the radar energy will also strike the surrounding area, for the following statistical studies the $3 d B$ area is taken. Sidelobes are ignored since only sharp-beamed tracking radars are considered here. Facet range (along beam boresight) gradients will cause radar reflectivity to vary appreciably due to the changes in the illuminated surface area, as seen with the aspect variation at Figure 2. The critical condition for a reduced area in the range direction (as $\tau, \theta_{E}$ and $\theta_{A}$ vary) is given by (3) and (4). $R$ is taken as the radar range to the facet centre, $R^{1}$ and $R^{11}$ as range to the nearest and furthest cell edges respectively:

$$
\begin{equation*}
\alpha_{R(C R I T)}=\operatorname{Arctan} \quad \frac{\theta_{E} R}{\tau} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
\alpha_{\mathrm{AZ}(\mathrm{CRIT})}=\operatorname{Arctan} \frac{\theta_{\mathrm{E}} \mathrm{R}}{{\theta_{\mathrm{A}} \mathrm{R}}^{\operatorname{Arctan}} \frac{\theta_{\mathrm{E}}}{\theta_{\mathrm{A}}}, \text {. }} \tag{4}
\end{equation*}
$$

$\theta_{E}, \theta_{A}$ in rads, $r$ as a length
3. If the illuminated facet is assumed to be a flat plane (ie no significant undulations within the area bounds) its area can be calculated for any tilt angle within the volume of the resultion cell. Only those terrain facets which are tilted towards the source of radar energy are assumed to create backscatter. These are declared "illuminated" by the computer program. For this to occur $a_{R}$ must be a positive value or zero.
4. Non-Critical Slope Values. For a simple set of conditions (Fig 1) $\alpha_{R}<\alpha_{R \text { (CRIT) }}$ and $\alpha_{A Z}<\alpha_{A Z(C R I T)}$; the average dimensions of the sides of the illuminated area are $\tau / \cos \alpha_{R}$ and $R \theta_{A} / \cos \alpha_{A Z}$. More precisely, account should be taken of the accurate lengths of all four boundaries of the illuminated area by allowing for the slight beam divergence in azimuth; such that $R^{l} \theta_{A} / \cos \alpha_{A Z l}$ $<\mathrm{R}^{11}{ }_{\mathrm{AZ} 2}$

The basic shape is a regular quadilateral with maximum possible side-lengths for $\alpha_{A Z(C R I T)}>\alpha_{A Z} \geqslant 0, \alpha_{R(C R I T)}>\alpha_{R} \geqslant 0$ will be:

Rear ${ }^{R \theta} A_{A} / \cos \alpha_{A Z 2}$
Front $\quad \mathrm{Re}_{\mathrm{A}} / \cos \alpha_{\mathrm{AZ2}}$
Sides $\tau / \cos \alpha_{R 1}$ on $\tau / \cos \alpha_{R 2}$

$$
\begin{aligned}
& { }^{R} \theta_{A}<A B<\theta_{A}{ }^{\cos }{ }_{A Z}(C R I T 1) \\
& { }^{R} \theta_{A}<D C<\theta_{A}{ }^{\cos }{ }_{A Z(C R I T 2)} \\
& \tau<A D<\tau / \alpha_{R}(C R I T 1) \\
& \tau<B C<\tau / \alpha_{R}(C R I T 2)
\end{aligned}
$$

Giving average azimuth $X$ Range product area:

$$
\begin{equation*}
\text { Area }=\frac{\tau\left(\cos \alpha_{R 1}+\cos \alpha_{R 2}\right) R \theta_{A}\left(\cos \alpha_{A Z 1}+\cos \alpha_{A Z 2}\right)}{4 \cos \alpha_{R 1} \cos \alpha_{R 2} \cos \alpha_{A Z 1} \cos \alpha_{A Z 2}} \tag{7}
\end{equation*}
$$

5. Critical Slope Values. Figure 2 shows the effect of

$$
\left.\left.\alpha_{A Z}>\alpha_{A Z(C R I T}\right), \alpha_{R}<\alpha_{R(C R I T}\right) \text { and } \alpha_{A Z 1}<\alpha_{A Z 2}
$$

Calculation of illuminated areas for any condition thus becomes more difficult than the first case, since facet tilt either results in an irregular hexagon (figure 4) or a reduced area in which the plane does not cut either the range boundaries or the azimuth boundaries as at Figure 3. For calculative purposes the area at figure 4 is taken as a plane quadilateral $\mathrm{TD}^{1} \mathrm{QB}^{11}$ and the area within the resolution cell is defined by TUVQRS (ie TD ${ }^{1} Q^{l l}$ less the corner areas). Calculations are detailed at para 17 below.
6. Terrain data base interpolation can provide the spot heights $\mathbb{T D}^{1} \mathbb{Q B}^{I 1}$ if required. However, as will be shown below, these are not strictly necessary if facet gradient and aspect can be calculated from nearby data matrix points without the need for interpolation. In any event spot heights U, V, R, S cannot be obtained by simple interpolation.
7. Gradient and Curvature. Several techniques for the optimum calculation of surface parameters have been developed by geomorphological researchers as part of their studies for soil erosion, drainage and similar requirements. Such a study is considered at \{232\}, giving a demonstrably satisfactory method for estimating both surface gradient and curvature derivatives directly from the altitude matrix.
8. To apply this to radar here, using gradient to investigate clutter returns (as a function of facet slope and aspect) and curvature on a larger scale (for diffraction investigations); involves the inclusion of the eight nearest spot heights surrounding the centre of the required cell - which is the centre of the radar resolution cell "footprint".
9. The central spot height with the four nearest points define simple gradient, with the furthest four points additionally for curvature. It is assumed that the basic matrix dimensions are adequate to produce the required accuracy. It should be noted that gradient or curvature 'maps' produced by this method cannot be compared with others unless the matrix spacing is similarly defined.
10. Grid Definition. A nine-point altitude sub-matrix is defined at Figure 5a. By using the full quadratic a complete surface description can be obtained at equation (8). Gradient is more accurately calculated using 9 data points for the coefficients at eqns (9) to (14).

$$
\begin{equation*}
A=a x^{2}+b y^{2}+c x y+d x+e y+f \tag{8}
\end{equation*}
$$

Using the notation defined for the matrix of spacing ( $m$ ) the coefficients are calculated as follows:

$$
\begin{align*}
& a=\frac{A_{1}+A_{3}+A_{4}+A_{6}+A_{7}+A_{9}}{6 m^{2}}-\frac{A_{2}+A_{5}+A_{8}}{3 m^{2}}  \tag{9}\\
& b=\frac{A_{1}+A_{2}+A_{3}+A_{7}+A_{8}+A_{9}}{6 m^{2}}-\frac{A_{4}+A_{5}+A_{6}}{3 m^{2}} \tag{10}
\end{align*}
$$

$$
\begin{equation*}
c=\frac{A_{3}+A_{7}-A_{1}-A_{9}}{4 m^{2}} \tag{11}
\end{equation*}
$$

d. $\quad \frac{A_{3}+A_{6}+A_{9}-A_{1}-A_{4}-A_{7}}{6 m}$

$$
\begin{equation*}
e=\frac{A_{1}+A_{2}+A_{3}-A_{7}-A_{8}-A_{9}}{6 m} \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
f=\frac{3\left(A_{4}+A_{2}+A_{8}+A_{6}\right)-\left(A_{1}+A_{3}+A_{7}+A_{9}\right)+5 A_{5}}{9} \tag{14}
\end{equation*}
$$

Hence gradient, aspect and profile convexity are obtained respectively from:

$$
\begin{align*}
& \operatorname{grad}=\arctan \left(d^{2}+e^{2}\right)^{\frac{1}{2}} d e g  \tag{15}\\
& \text { or } \arctan (d \cos \theta+e \sin \theta)
\end{align*}
$$

$$
\begin{equation*}
\text { aspect }=\theta=\operatorname{Arctan} \frac{e}{d} \operatorname{deg} \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\operatorname{Conv}=\frac{-200\left(a d^{2}+b e^{2}+c e d\right)}{e^{2}+d^{2}\left(1+d^{2}+e^{2}\right)^{3 / 2}} \text { deg }(100 m)^{-1} \tag{17}
\end{equation*}
$$ (for 100 m matrix)

11. Frequency Distributions. Gradient frequency distributions were also considered by the author as a possible aid to the overall prediction process for a given area. Gradient steepness distribution shows an increase with altitude with moderately strong correlation, but in a non-linear way \{232\}. In general no single transformation is found to be universally valid \{233\}.
12. Therefore although it is possible to statistically summarize a surveyed area in terms of gradient frequency distribution, the results would be site-specific. However it seems quite possible that a frequency distribution for an area might be representative (within reasonable limits of judgement) of another area, unsurveyed, but with similar general characteristics.
13. Convexity distributions tend to be balanced by concavity (negative convexity) since the mean or medianconvexities tend to cancel $\{232\}$. Profile convexity has a weak positive correlation with altitude.
14. Gradient and Aspect Examples. Gradient calculations present no problems using the above method, but aspect values have orientations which depend upon simple rules developed below.
15. Aspect. It is seen that the actual aspect value can be defined with respect to North or with respect to the radar beam boresight. With respect to the beam boresight the relative direction of aspect depends upon the arithmetic sign combination of both numerator and denominator of eqn (16), as
seen at Table 2; where the terrain surface is in the quadrant 0 to $90^{\circ}$ (wrt North). Similar tables can be deduced for other quadrants. The slope reflectivity studies in this report were made using the $0-90^{\circ}$ quadrant. Actual angle "Aspect" $\pm(90-$ angle of Radar (wrt $\mathbb{N}))=0$ in this quadrant.

Table 2. Determination of Terrain Aspects

| Serial | e | d | Orientation |
| :---: | :---: | :---: | :---: |
| 1 | $+$ | 0 | $\downarrow$ |
| 2 | 0 | - | Negative Slope |
| 3 | 0 | + | 4 |
| 4 | - | 0 | \& Negative Slope |
| 5 | + | 1 | 1 |
| 6 | - | + | k |
| 7 | $+$ | + | - Towards Radar |
| 8 | - | - | $\nearrow \quad$ Negative Slope |
| 9 | 0 | 0 | Flat Terrain |

16. Serials 2, 4 and 8 produce radar shadowing(ie zero backscatter is assumed), aspect values close to serial 7 would be expected to give a maximum backscatter, serials 1 and 3 intermediate levels and serials 5 and 6 minimum levels. Absolute values of aspect (degrees) are measured as shown at figure 5b.
17. Calculation of Area - First Critical Case. The illuminated facet at figure 3 can be calculated as follows, since once aspect ( $\theta$ ) is known this also corresponds to the angle $\theta$ marked on the diagram. The conditions are $\alpha_{A Z}>\alpha_{A Z(C R I T)}$ and $\alpha_{R}<\alpha_{R(C R I T)}$ then:

$$
\text { Since } T S=\frac{\tau}{\cos \theta}, P T=\frac{R \theta_{E}}{\cos \left(90-\alpha_{A Z}\right)}
$$

$$
\begin{equation*}
\text { Area }=\frac{R \theta_{E}{ }^{\top}}{\operatorname{Cos} \theta \operatorname{Cos}\left(90-\alpha_{A Z}\right)} \tag{18}
\end{equation*}
$$

$R=$ mean range, as before.
18. Calculation of Area - Second Critical Case. Consideration was given to the use of Direction Cosines since the required area of a surface can be calculated with respect to its normal position.


$$
\begin{aligned}
\mathrm{dA}=\mathrm{dS} \cos \theta \quad & \text { Terrain } \hat{\mathrm{n}}=(1, \mathrm{~m}, \mathrm{n}) \\
& \text { Radar Beam } \hat{\mathrm{r}}_{1}=(\lambda, \mu, \gamma)
\end{aligned}
$$

$\operatorname{Cos} \theta=\hat{r}_{1} \hat{H}=(\lambda 1+\mu m+\gamma n)$ etc $\ldots \ldots$.

But to produce the required area a three-co-ordinate direction cosine system would be needed and the required terrain co-ordinates are not readily accessible

$$
F-8
$$

for the typical, but awkward shapes, as at figure 4. Hence in the long term it was simpler to use trigonometric methods.
19. If $\alpha_{A Z}>\alpha_{A Z(C R I T)}$ of if $<\alpha_{R(C R I T)}$ the side $T D^{1}$ (fig 4) will cut $A D$ and with $\alpha_{R}<\alpha_{R}$ (CRIT) then $Q D^{1}$ must cut $D C$. Hence the quadrilateral $T D^{1} Q B^{I l}$ is a terrain plane passing through the resolution cell. Points UVSR are positioned dependent upon $\alpha_{R}, \alpha_{A Z}$. It is points TUVQRS which define the illuminated area and which are not directly available from the spot height data base. The plan diagram accentuates the beam divergence, but for practical purposes, with small values of $\tau$, sides $U A, C R$ are assumed to be parallel. Aspect is calculated from the terrain matrix - it's direction is shown in the diagram (in direction $O B^{1 l}$ since this is the lowest terrain point). Gradient is available from eqn (15) and is angle $D^{1} B^{I l} D^{I I}, \alpha_{A Z}$ and $\alpha_{R}$ are computed as Arctan (eqn 12) and Arctan (eqn 13) respectively.
20. From the figure, if the centre of the illuminated terrain is also the centre of the resolution cell, then
$V Q=T S, S R=U V, \alpha_{A Z 1}=\alpha_{A Z}$, Area $D^{1} V=$ Area $S B^{11} R$.
$B^{I I} Q=T D$ and $T B^{11}=D_{Q}$.

Then $B^{11}{ }_{Q}=\frac{\tau}{\cos \alpha_{R}}$ and $T B^{11}=\frac{\theta_{A}}{\operatorname{Cos} \alpha_{A Z}}$

Quadrilateral Area $T D D^{1} Q^{11}=\frac{{ }^{\tau \theta} A^{R}}{\operatorname{Cos} \alpha_{R} \operatorname{Cos} \alpha_{A Z}}$

Since $\alpha_{R} \geqslant \alpha_{R(C R I T)}$ then $U D^{1} \operatorname{Cos} \alpha_{R_{1}}=\frac{t}{2}$.

$$
\begin{equation*}
\text { hence } U T=U D^{1}=B^{11} R=R Q=\frac{\tau}{2 \operatorname{Cos} \alpha_{R}} \tag{21}
\end{equation*}
$$

$$
\begin{equation*}
T A^{1}=R \theta_{E}-U T \operatorname{Sin} \alpha_{R}=T S \sin \alpha_{A Z} \tag{22}
\end{equation*}
$$

$$
\begin{equation*}
\text { giving TS }=\frac{\sin \alpha_{A Z} \operatorname{Cos} \alpha_{R}}{R \theta_{E} \operatorname{Cos} \alpha_{R}-\operatorname{Sin} \alpha_{R}} \tag{23}
\end{equation*}
$$

$\mathrm{TB}^{11}=\mathrm{SB}^{11.1}+\mathrm{TS}=\frac{\theta_{A} \mathrm{~A}^{\prime}}{\cos \alpha_{A Z}}$ from (19)
Hence Side $S B^{11}=\frac{\theta_{A} R}{\operatorname{Cos} \alpha_{A Z}}-\frac{\operatorname{Sin} \alpha_{A Z} \operatorname{Cos} \alpha_{R}}{R^{\theta}{ }_{E} \operatorname{Cos} \alpha_{R}-\tau \operatorname{Sin} \alpha_{R}}$

Area of both triangles is given by the product of eqn (24) and eqn (21). Subtraction from eqn (20) gives the radar illuminated area:

It is therefore possible to obtain the required area without direct knowledge of the terrain spot heights which define the illuminated area spot heights.

## TERRAIN CURVATURE APPLIED TO DIFFRACTION CONDITIONS

21. Reference is made to Chap 7 eqns (15) and (16), which define the criteria for approximating the terrain as a diffracting knife edge. The intention here is to use the convexity calculation at eqn (17) above to estimate the curvature - and then to test it against the criteria. By taking typical dispositions for target, obstacle and radar site together with the matrix values at figure 5 a, example calculations are:

From Fig 5a, coefficients are: $a=-0.00055$

$$
\begin{aligned}
& b=0.001833 \\
& c=0 \\
& d=\frac{5}{600} \\
& e=\frac{-30}{600} \\
& f=35.5
\end{aligned}
$$

hence from eqn (17) convexity $=0.356 \mathrm{deg}(100 \mathrm{~m})^{-1}$
22. The criteria for approximating an obstacle as a knife edge can be expressed from known obstacle radius or diffraction angle $\theta$. However, $\theta$ is normally available only if a complete set of conditions are known (ie range from radar to obstacle, and beyond to the target). Converting the above value to the rate of change of slope ( $\varnothing$ ) by taking the tangent at 2 successive matrix points; where $R$ is radius of curvature of obstacle $(m)$ :

$$
\begin{equation*}
R=\frac{180 \mathrm{~m}}{\phi \pi}=\frac{180 \times 100}{0.356 \pi}=\underline{16094 \mathrm{~m}} \tag{26}
\end{equation*}
$$

Clearly the rate of change of slope is insufficient to produce diffraction. $R$ is excessively large, as shown by the typical values at Table 1. Typical terrain profiles obtained from the terrain database used for slope reflectivity reported at Appendix 1 to this Annex had limited azimuth cover. In practice a full $360^{\circ}$ sweep would be required to produce an area assessement. If it is found that the correct conditions exist to aid tracking on a small \% of occasions then the probability of tracking is increased for the area in question eg $P_{T L}=P_{T L} x$ Diff Factor where Diff Factor $>1$. An example diffraction plot is at Figure 6.

Table 1 Typical Radar, Obstacle and Target Conditions

| Ser | Tgt Ht <br> $(\mathrm{m})$ <br> $\mathrm{h}_{\mathrm{tgt}}$ | Radar <br> $\mathrm{Ht}(\mathrm{m})$ <br> $\mathrm{h}_{\mathrm{tx}}$ | Obstacle <br> $\mathrm{Ht}(\mathrm{m})$ <br> $\mathrm{h}_{\mathrm{t}}$ | Obstacle <br> Rng Km <br> $\mathrm{d}_{1}$ | Target <br> Range $(\mathrm{m})$ <br> $\mathrm{d}_{1}+\mathrm{d}_{2}$ | Radius <br> $(\mathrm{m})$ <br> R |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 70 | 0 | 100 | 10 | 15 | 7.46 |
| 2 | 70 | 15 | 100 | 10 | 15 | 19.7 |
| 3 | 70 | 20 | 100 | 10 | 15 | 22.17 |
| 4 | 70 | 30 | 100 | 10 | 15 | 27.0 |
| 5 | 70 | 30 | 100 | 5 | 10 | 7.6 |
| 6 | 20 | 0 | 50 | 10 | 15 | 60.0 |
| 7 | 30 | 0 | 50 | 10 | 15 | 82.0 |
| 8 | 30 | 0 | 30 | 10 | 15 | 2221 |
| 9 | 40 | 0 | 30 | 10 | 15 | 480 |

Using the notation at Chap 7.

$$
\begin{align*}
& \alpha=\operatorname{Arctan} \frac{h_{t}-h_{t x}}{d_{1}}  \tag{27}\\
& \beta=\operatorname{Arctan} \frac{h_{t}-h_{t}}{\alpha_{2 t}} \tag{28}
\end{align*}
$$

Diffraction Angle $\theta=\alpha+\beta$
23. Retracing the steps to obtain $\theta$ for $R=16094$ gives $\theta=0.0888^{\circ}$. Using the scenario at Table 1 serial 8, the target would have to climb by approximately 7 metres (ie new target height 37m) if this larger terrain radius existed at the same obstacle range. The conditions at Serial 8 are such that the target would be just visible by direct sightline at an altitude of 45 m .


FIG 1 SURFACE SLOPE




FIG 4 RESOLUTION CELL GEOMETRY FOR $>$ AZ (CRIT) $\alpha<\alpha$ CRIT


Notes 1. Bracketed figures are example altitudes. 2. $\quad n=100 \mathrm{~m}$ (for example).

FIG 5a DEFINITION OF ALTITUDE SUB MATRIX


FIG 5h MEASUREMENT OF ASPECT


## STATISTICAL ANALYSIS OF RAW RADAR MEASUREMENTS

TO OBTAIN A DISTRIBUTED CLUTYER MODEL OVER
SLOPED TERRAIN
i. Full statistical analysis of the raw radar measurements commenced after initially correcting the range (see para 5 below), with the formation of a multivariable array containing some 20 k of measured and calculated values. All data reduction programs were written by the author in FORMRAN to run on the RAF College DEC 20 computer facility. Initial analysis was backed by a terrain data base interpolated (tediously) by hand from non-standard survey maps specially provided by the Mapping and Charting Establishment at Tolworth with a contour notation at 5 metre intervals. By photographic enlargement it was possible to interpolate matrix spot heights to within $\pm$ I metre or better, subject to the original accuracy of the contours. Using 50 metre grid spacing the data base produced extends for 9 square kilometres.
2. For calculative purposes, given the range and azimuth bearing of each radar discrete clutter signal, it was thus possible to extract the corresponding terrain data using the matrix method for terrain slope and

aspect detailed at Annex F. Illuminated radar footprint size could then be calculated, taking into account resolution cell size, slope and aspect and (by mean terrain height) the actual arrival angle of the radar wavefront. Generated data examples are at Table 1.
3. Clutter returns were processed radially (ie incrementally by increasing range at 5 metre steps) and incrementally in azimuth at either 0.3 or $0.4^{\circ}$ steps. The geographical area was chosen to include both flat and sloped terrain. For convenience this was divided into 4 sectors which contain predominately 'sloped' terrain and one sector which is mostly 'flat' - although it will be shown below that terrain is rarely flat in the scientific sense. Data was analysed by sectors and as an integral data bank. Figure la is a photograph, with 0 map showing the location of the measurements, at Figure lb.
4. Generally the extent of range measurements did not greatly exceed 7 km as indeed many would fall in shadow at this range: behind the hill ridges; as seen on the map. The terrain, (as viewed in the photograph along the boresight) contains scattered trees, but not in sufficient density to classify as 'forest'; for the purpose of backscatter studies the terrain is clearly 'rural'. There are a few features which may exhibit specular reflector characteristics in the form of small buildings - and possibly in places the railway line or associated fencing. It is difficult to specify how much of this is masked by the earth railway cuttings. However, much of the area of interest was at a higher angle of elevation than these reflectors - or excluded by the sharp beamwidth of the trials radar. It should be mentioned that some of the analysis was made by filtering out spurious values, taking these to be 'outliers' in the statistical sense.
5. It is re-iterated, at this point that by assuming $3 a B$ aerial beamwidth limits the backscattered energy is taken to be that only from the idealised surface "footprint" area. Clearly any sidelobe effects or backscatter, (for example from a large reflector just outside the $3 a B$ beamwidth) will contribute in some instances. However, it is proposed that although this instance can occur they are likely to do so with relatively low incidence. As can be seen, the area immediately ahead of the radar is clear of such obstacles. Further, the terrain chosen is more or less homogeneous at any given time, at the least for several 'footprints' dimension in both the range and azimuth directions. This was proved by making a correlation study between adjasent footprints in both alongboresight and across-boresight directions.

EXPERIMENTAL ERRORS AND LIMITATIONS
6. Although the raw radar signal measurements were not taken by the author, the prevailing conditions are known. Wind was light; weather fine and time of year - June. Other information suggested the possibility of range errors for which a correction would be necessary. Clearly any constant range error would cause the incorrect terrain data to be coupled to each backscatter reading - hence an incorrect grazing angle model could result.
7. At the outset the areas selected for distributed clutter analysis were especially chosen to avoid any major man-made specular reflectors such as pylons or metal buildings which might contaminate the statistical distributions. It is of course realised that such reflectors may occur when a tracking radar is deployed in practice. The absence of a really distinct reflector to act as a range calibrator in the sector of interest, resulted in an additional computation task - that of producing several additional data files, each with an iterative signal strength
shift relative to range along all azimuths, while simultaneously checking correlation values. These shifts were made in 5 metre increments both towards and away from the radar with correlation checks between several variables on each occasion. Although there was reason to believe that an error of up to 100 metres might exist, the plot at figure 2 most convincingly shows a range under-reading by only 5 metres. All data was therefore range-corrected by +5 m before statistical analysis commenced. Table 1 shows example data after processing to obtain ofor given terrain conditions.

ERRORS IN CALCULATE D TRUE GRAZING ANGLE
8. True grazing angle comprises 2 main components. The first is obtained by calculation of the terrain slope and aspect angles, with the remaining part determined by the angle of arrival of the wavefront from the radar. Slope and aspect are critically dependent upon the accuracy of the terrain data base as is the mean terrain height used in the determination of angle of arrival, always calculated at the centre of the radar footprint.
9. Raw signal amplitudes were corrected at source by a calibrated standard at the time of measurement. There may be slight propagation errors to apply, discussed at paras 26 \& 27 below.
10. On the majority of occasions terrain which appears to be flat will undulate slightly, giving rise to false grazing angles if the mean slope is used. Additionally, slope (and hence grazing angle) will depend on the accuracy of each of the spot heights representing a 'facet' on which the radar energy is impinging. Precise grazing angle will also depend upon range, earth's curvature, propagation, radar transmitter height; each of which is subject to small angular error. Additionally the surface culture varies, eg trees (see Chapter 10 page 209). There are 2 ways of considering the
magnitude of surface error. Either way it is clear that the effect of errors, as a proportion will be greatest at small grazing angles.
11. GENERAL SLOPE ERROR The magnitude of errors in tone grazing angle $\psi$ can be easily demonstrated by using the example at Figure 3 . A. sinusoidally varying surface with $A=7.5 \mathrm{~m}$ and $T=150 \mathrm{~m}$ is used to represent gently undulating terrain. Ignoring vegetation the surface gradient at point $P$ will be:

$$
\begin{equation*}
\phi=\tan ^{-1} \frac{d}{d \theta} A \sin \theta \tag{1}
\end{equation*}
$$

where $P$ is defined $y(P)=7.5 \sin \theta, x(P)=\frac{37.50}{90}$
hence at any point

$$
\begin{equation*}
\phi=\tan ^{-1} 0.314 \cos \theta \tag{2}
\end{equation*}
$$

12. Accurate grazing angles are therefore not only dependent upon the underlying mean terrain gradient, since the true value of $\psi$ can be significantly different than that obtained by pure facet geometry using spot heights, radar range and radar transmitter height.
13. Direct energy cannot reach point $R$, which is shadowed. Point $S$ is the highest point to which a direct energy path exists at a grazing tangent (i.e. $\psi=0^{\circ}$ ). If the underlying terrain is flat and the elevation angle $E$ is known, $\theta$, (which defines point $S$ ) will be given by:

$$
\begin{equation*}
\theta=\cos ^{-1} \frac{1-\tan E}{-0.314 \mid} \tag{3}
\end{equation*}
$$

$$
1 \_\quad 1
$$

If the terrain is sloped, as on Figure 3, point $S$ will move to a higher point on the curve.
14. For example, for $\mathrm{E}=2.5^{\circ}$ energy will reach S at $\theta=82^{\circ}$, with distance $O T=82 / 90 \times 37.5 \mathrm{~m}$. The surface distance. 0 S will be slightly longer and is the distance over which the wavefront is spread, depending upon the resolution cell dimension. At point $S$ the true grazing angle is zero. At point 0 the surface gradient is $17.4^{\circ}$, hence the value of $\psi$ is 14.9 . Thus, no single value of $\psi$ is correct for the surace illuminated. A mean could be taken (7.45 ) which does not compare favourably with the value using facet geometry $\left(\operatorname{Tan}^{-1} \frac{4 \mathrm{~A}}{\mathrm{~T}}=11.3\right)$.
15. PROBABILITY DENSITY FUNCTION. Only at $\theta=73.5^{\circ}$ will a true graze of $2.5^{\circ}$ be obtained. If it is assumed that $N$ parallel direct energy paths exist from the emitter to the surface then the probability of obtaining $2.5^{\circ}$ is 1 in this particular case -providing $N$ the resolution cell length $\leqslant 37.5 \mathrm{~m}$. If cell length is increased (due to larger $\tau$ ), shadowing beyond the point $V$ will eventually occur, as shown at Figure 4; where it is seen that the resolution footprint could embrace a number of sloped and shadowed areas, dependent on its length. By considering changes in the variables $A, T, \tau, E$ it is seen that the pdf will change. In the example shown energy arising at the surface between point 0 and $73.5^{\circ}$ will graze at a steeper angle than $2.5^{\circ}$; almost $90 \%$ of the energy is grazing at angles greater than $2.5^{\circ}$. This leads to the conclusion, that in general terrain observations it is likely that "the actual grazing angle from which backscatter is measured is higher in value than plane terrain geometry suggests, and would reasonably be expected to produce higher backscatter values". Practical results will of course depend also on the finer terrain texture and wavelength used.
16. For any grazing angle or terrain amplitude and period, mean surface gradient and mean true grazing angle, taken over N intervals with respect to mean terrain gradient, are respectively given by:

$$
\begin{equation*}
\left.\phi_{\mathrm{m}}=\frac{1}{\mathrm{~N}} \sum_{0}^{\mathrm{S}} \tan ^{-1} \frac{\{\mathrm{~d}}{\mathrm{d} \theta} A \sin \theta\right\} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\psi_{m}=\frac{1}{N} \int_{0}^{S}\left(\tan ^{-1} \frac{\{d}{d \theta} A \sin \theta\right\}-E\right) \tag{6}
\end{equation*}
$$

17. COMPARISON OF METHODS. A comparison of mean $\psi$ and mean surface gradients calculated from data base terrain spot heights (which assumes a plane facet between adjacent values) with the example method above show significant differences and confirms the hypothesis that backscatter values have almost certainly in the past been attributed to incorrect grazing angles. Ranges are those to give grazing angles of zero with the $\mathrm{A} / \mathrm{T}$ ratio shown.


Error values will clearly vary with terrain conditions and the above figures are used only to show that a more rigorous treatment is necessary in studying and representing terrain than is at first apparent; hence the importance placed on terrain studies at Chapters 2, 10 and Annex $F$. In practice; with typical ranges of $6 \mathrm{~km}, \mathrm{E}$ will
be very small unless the mean terrain height is about 250 metres above the radar.
18. It is seen that the method shown above for illustrative purposes for obtaining true grazing angle would be difficult to use for experimental measurements -since a sinusoidal approximation may not be representative. Further there are usually insignificant terrain data to set up accurate concavity and convexity profiles along any specific azimuth -especially with aspect changes and discontinuities with shadowing, However, the concept shows how easily errors can occur in mean grazing angle measurements within the radar footprint even under experimental conditions.
19. Effect of Matrix Errors. It was argued earlier that the matrix method of terrain representation was preferable for practical as well as experimental expediency, despite the greater overheads in digital storage requirements.
20. Returning to the 2 components of true grazing angle which change due to spot height errors, sensitivity of each is outlined below, taking a spot height error of $\Delta h_{t}$. True slope angle (as seen from the radar) is slope times $(\cos \theta)$, where $\theta$ is the aspect. Sensitivity of aspect change with $\Delta h_{t}$ is important since it is not constant, and depends significantly upon errors being across, rather than along, boresight. .-A small error across boresight will significantly change $\theta$, with error magnitudes proportionally greatest at low slope angles (both along and across), and least when near slope angles are highest.
21. The reader is referred to Annex $F$ (eqns 9 to 16) together with diagram 5 at page F17, for matrix slope and aspect methods; and to Chap 10 page 211 for the equations for beam boresight angles.
22. Beam Boresight Angular Errors. Referring to Chap 10 Eqn 8 and applying a. $\Delta h_{t}$ of $\pm I$ metre, each of the 3 terms, when examined, have differing significance. When $h_{t}-h_{t_{x}}$ is very small or zero, $\Delta h_{t}$ can impose a shadow situation, a small gradient where non existed before; or can change a shadowed situation into a flat terrain. Shadowed areas are detected and discarded by the computer program, whereas some shadowed areas should be included if errors exist; while other areas included should have been deleted. There is no reason to believe that instances of positive or negative errors predominate, and over the large amount of data they may well cancel for statistical purposes. The magnitude of beam boresight
errors, if they exist, was calculated to be approximately $\pm 0.01^{\circ}$ ie $\approx \sin ^{-1} \frac{\Delta h t}{R} \quad$ where $\Delta h_{t}= \pm 1$ and $R=6 \mathrm{Km}$. This holds at approximately an error of $.01^{\circ}$ for each metre of error in spot height at the centre of the footprint.
23. Slope Errors. Slope is calculated by using 8 of the 9 spot heights representing a group or 'facet' in which the illuminated footprint falls. The facet is dimensioned $100 \mathrm{~m} \times 100 \mathrm{~m}$ with heights at 50 metre intervals. Slope (given by Arctan $\sqrt{d^{2}+e^{2}}$ ), is thus susceptible to errors in e or $d$ or both. It is assumed that 1 , 2 or 3 errors may occur in any seti: of 9 since the spacing of contours (though at 5 metres) are spatially spread across the terrain such that it is considered that at least $80 \%$ of the interpolated figures are good, and probably more so. Three errors is taken as a worst case condition with the greatest effect of 3 errors when all 3 occur along the same 'edge', thus having greatest effect on e or d. Errors are approximately constant for changing slopes and are summarized at Table 2 .

TABLE 2 SLOPE ANGULAR ERROR DUE TO + IM SPOT HEIGHT ERROR

|  | Max Angular Error (Deg) |
| :--- | :---: |
| Single Error in 'Corner' | $\pm 0.20$ |
| Single Error (Centre-side) | $\pm 0.19$ |
| Two Errors (same side) | $\pm 0.38$ |
| Three Errors (same side) | $\pm 0.56$ |

24. Aspect Errors. Aspect $\theta=\operatorname{Arctan} \frac{e}{d}$, hence the change in ratio is important, with an increase in e relative to $d$ altering the aspect across boresight. The magnitude of aspect errors cannot be represented in simple form since a;change in any of 9 positions affect both slope and aspect. At extreme cases a 1 metre change in one corner can swing the aspect angle from $14^{\circ}$ to $8^{\circ}$ but fortunately this has little overall effect on true grazing ancle, as (slope $(\operatorname{Cos} \theta)$ ) does not change much for low angles of $\theta$. Aspect angles reached a maximum of $54^{\circ}$ during the experimental analysis for the terrain measured, and spot height errors could have produced this. value with an error of $10^{\circ}$. Hence $\operatorname{Cos} \theta$ would be represented as 0.58 where addition to the slope error of say $0.2^{\circ}$, described at Table 2 , causes the multiplier $\operatorname{Cos} \theta$ to be significantly in error.
25. Combined Errors. Assuming a boresight error of $0.01^{\circ}$ (para 22), slope errors as shown (Tabie 2) and typical aspect errors as discussed (at para 24), the combined errors can be $>0.25^{\circ}$ for a single error and exceed $0.5^{\circ}$ for 3 errors. In practice errors may have occurred in various senses and combinations, however if they inevitably act in the same direction on some occasions the result could be as shown at Table 3 .

TABLE 3 COMBINED ERRORS

| Correct | 1 | 2 | 3 |  |
| :--- | :---: | :---: | :---: | :---: |
| Slope (Deg) | 1.14 | 1.34 | 1.52 | 1.70 |
| Boresight (Deg) | 1.0 | 1.01 | 1.01 | 1.01 |
| Aspect (Deg) | $54^{\circ}$ | $44^{\circ}$ | $44^{\circ}$ | $40^{\circ}$ |
| Orror | Errors | Errors |  |  |
|  |  | 1.67 | 1.97 | 2.10 |

FI-12
26. Other Errors - Refraction. There is a slight change in refractive index between ground (radar) level and terrain surface levels which will give small angular pointing errors. As explained at Chap 8 the radar elevation angle is always slightly higher than the target at which it is aimed. However, for the purposes of clutter measurement here, elevation angle is not measured by the radar aerial position but by goemetry through radar range and terrain data base. Even though the radar energy is following a curved path the range measurement is assumed to be correct over such a short distance. Ray curvature will be slight and cause energy to impinge onto the surface at a slightly greater angle. The overall effect of refraction is taken to be negligible for the experimental readings.
27. Other Effects - Diffraction. Although the distributed clutter analysis was intended to be upon terrain backscatter from sloped and flat terrain, because of the extent of the area, flat terrain raised above sea level ie as small plateaus, appeared as a small negative slope during the analysis. On these occasions complete shadowing did not occur. They were detected by the program and flagged correctly on the data output. All fell in the range $0.0001^{\circ}$ to $1.2^{\circ}$ negative slope, but all produced backscatter, presumably due to diffraction effects. Backscatter values obtained did not indicate shadowing of the main lobe with residual side-lobe or wider beamwidth collection of back-scatter ( $>3 \mathrm{a} B$ ), but remained substantially similar in value to the forward sloping terrain values which immediately preceeded and followed the negative slope values. Insufficient of these were available to carry out a full statistical and diffraction analysis, however, a rough check has shown changes in backscatter median of the order of $10-14 \mathrm{~dB}$, and could reasonably correspond to the diffraction loss over the ridges.
28. Homogeneity of Terrain. It is reasonable to assume for rural terrain that a high spatial correlation value should be obtained from adjacent resolution cells (both in the range and azimuth directions). Spatial correlation was tested over a sample sector at azimuth increments of a beamwidth and in range for both 5 and 10 metre increments. Correlations were respectively 0.94 and 0.83 , indicating also a consistency of measurement of backscatter since values were measured at different times in practice - less so in range but significantly so in azimuth.
29. It was not considered necessary to validate terrain homogeneity further, althöugh even higher correlations would have been likely if adjacent cells with like grazing angle had been isolated and compared. As expected the terrain selected is truly representative of 'rurai', with scattered isolated specular reflectors within a broadly representative backscatter range of

$$
-20 \text { to } \quad-40 \mathrm{~dB} \cdot \mathrm{~m}^{-2} \mathrm{~m}^{2}
$$

30. The point should be made that sloping terrain does not necessarily imply a high grazing angle, since the actual arrival angle of the radar energy also depends on the relative radar transmitter height and elevation angle. Hence some of the lowest grazing angles are obtained at the crest of hills.
31. All radar measurements including calculated results for grazing angles, areas, ranges and aspect angles were thoroughly analysed in a number of ways:
a. As a total data base.
b. As a total package but rejecting outlying values.
c. By examination of means and medians at grazing angle class intervals.
d. By grazing angle steps by arbitrary division into $2^{\circ}$ and $3^{\circ}$ steps.
e. By contrasting data from different azimuth sectors.
f. By comparison of data from adjacent radar resolutions cells as described above at para 28.

In each case standard statistical methods were applied, hypotheses tested, correlations made and distributions plotted. The aim was to deduce a backscatter-grazing angle relationship which can be applied in practice.
32. Regressions. Straight-line regressions, with grazing angle as the independent variable were interpreted with caution, since treatment of all results as an entity (for this purpose) could lead to major inaccuracies. The author was aware that some researchers had found the $\sigma v \psi$ relationship to be a curve, at the lower values of $\psi$. One way to minimise this problem was to separate the data into sets based on grazing angle ie above and below say 2 or $3^{\circ}$, and thus obtain separate regressions from the two parts of the plot.
33. It was decided to analyse the data, which covered grazing angles up to $12^{\circ}$, as 4 sets of $3^{\circ}$ and also 6 sets of $2^{\circ}$. Because the number of observations is less in each set, care must be taken to ensure that those outlying results do not have undue influence on the results of the smaller data set.
34. By comparison of scattergraphs, bargraphs and histograms several important factors were noted and plots of cumulative frequenices were then made against Weibull and Lognormal distributions while KOLMOROV-SMIRNOV tests were also made using a standard computer routine. A quantity of relevant data is included here for future reference purposes. It should of course be stated that although the terrain statistics are site specific - they may nevertheless be reasonably applied to any other similar terrain and radar conditions.

## SECTOR RESULTS

35. Data was initially analysed in 5 azimuth sectors with statistical results shown at Table 4, and as an entity. Sections 1 to 4 each contain predominantly sloping, undulating or hilly backscatter measurements. Sector 5 is predominantly flat grassland at slightly less radar range. Table 5 summarises the regressions and correlations for Taile 4. All results are commented upon at para $52^{\circ}$ below, with reference to relevant plots.
36. Partition at $3^{\circ}$ Steps. It was observed that the deduction of any significant relationship was being distorted by a scatter of extreme values, caused presumably by scattered specular reflectors; although not obvious from the maps, but probably sufficiently reflective rocks, boulders and fence posts etc - to act as K band reflectors. Although it is appreciated that these objects will occur in rural terrain in practice in a somewhat random way, it was thought prudent to filter these peak values to expose the underlying trend.

TABLE 4 SUMMARY OF ALL DATA (NO FILTERTNG)

| PARAMETER | MEDIAN | MEAN | STD.DEV | MAX | MIN | STD.ERR |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma(\mathrm{dBs})$ | -25.49 | -25.99 | 12.14 | -5.26 | -59.85 | 0.27 |
| $\psi(\mathrm{deg})$ | 5.85 | 5.53 | 3.16 | 11.58 | 0.07 | 0.07 |
| Area (m$\left.{ }^{2}\right)$ | 636 | 622 | 63.85 | 721 | 473 | 1.43 |
| Aspect $\theta(\mathrm{deg})$ | 23.25 | 24.06 | 12.85 | 59.79 | 0.09 | 0.29 |
| Range (km) | 6.03 | 5.98 | 0.59 | 6.88 | 4.52 | 13.63 |
| Signal Received | $0.28 \mathrm{E}-2$ | $0.28 \mathrm{E}-1$ | 0.05 | 0.29 | $0.1 \mathrm{E}-5$ | $0.1 \mathrm{E}-2$ |

TABLE 5 CORRELATIONS AND REGRESSIONS

|  |  | SECTORS |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | OVERALU |
| $\frac{\pi}{\infty}$ | $\begin{gathered} \text { Correlation } \\ \begin{array}{c} \psi \vee \sigma \\ \theta \vee \sigma \end{array} \end{gathered}$ | $\begin{aligned} & 0.18 \\ & 0.18 \end{aligned}$ | $\begin{aligned} & 0.05 \\ & 0.06 \end{aligned}$ | $\begin{aligned} & 0.08 \\ & 0.37 \end{aligned}$ | $\begin{aligned} & 0.4 \\ & 0.67 \end{aligned}$ | $\begin{aligned} & 0.15 \\ & 0.39 \end{aligned}$ | $\begin{aligned} & 0.12 \\ & 0.35 \end{aligned}$ |
|  |  | $-38.06+.52 \psi$ | - $24.93-0.2 \psi$ | $-27.79+0.35 \psi$ | $-39.8+1.9 \psi$ | $-18.89+1.9 \psi$ | $\begin{aligned} & -31.6+0.53 \psi \\ & -24.6-0.008 \mathrm{R} \\ & -36.4+0.15 \theta \end{aligned}$ |
|  | Median $\sigma$ and s.d | $\begin{aligned} & -36.07 \\ & 9.1 \end{aligned}$ | $\begin{aligned} & -24.98 \\ & 9.8 \end{aligned}$ | $\begin{aligned} & -24.27 \\ & 12.2 \end{aligned}$ | $\begin{aligned} & -23.9 \\ & 12.9 \end{aligned}$ | $\begin{aligned} & -13.36 \\ & 10.18 \end{aligned}$ | $\begin{aligned} & -25.49 \\ & 12.14 \end{aligned}$ |

37. At the other extreme a random scatter of very small values were present. These are attributed to sidelobe reception and small backscatter levels from partially shadowed areas, perhaps partly due to diffraction in a few cases or spurious propagation effects. Those cells which were shadowed were detected by the computer program and excluded from the working data.
38. Several options were considered; peak and trough values could be arbitrarily discarded by inspection or a cut off could be imposed to exclude all but the majority of the data.
39. Data Selection. Scatter plots indicated that it would be reasonable to exclude peaks and troughs by using only those values falling between - 20 and - 40 dB . Statistical values were then recalculatea by taking results for each grazing angle. For example, the database was scanned for all like values of grazing angle and then relationships were considered by class interval, by reducing the data by taking the median and mean of each class. "Median filtering" (as suggested in some signal processing applications) could not be used since this would mean the imposition of an assumed rate of change in backscatter with grazing angle and thus defeat the objective of the analysis.
40. Reduction of the data in this way was justified for the following reasons:
a. Any particular backscatter value for a given grazing angle is the result of combining measurements (to get mean or median) taken at various azimuths, ranges and terrain heights. Since the terrain has been shown to be homogenecus: the individual values obtained should be more representative than an isolated backscatter value. It should also minimise signal measurement fluctuations and errors

|  | MEDIAN | MEAN | s.d | MAX | MIN | STD.ERR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0-3^{\circ}$ (a) $\psi$ (Deg) <br> (b) $\theta$ (Deg) <br> (c) $\sigma\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right)$ <br> (d) $\sigma$ ( dBs ) <br> (e) Area $\left(m^{2}\right)$ | $\begin{aligned} & 0.70 \\ & 27.65 \\ & 0.87 E-2 \\ & -20.58 \\ & 509 \end{aligned}$ | $\begin{aligned} & 1.34 \\ & 27.80 \\ & 0.49 \mathrm{E}-1 \\ & -21.55 \\ & 559 \end{aligned}$ | $\begin{aligned} & 1.00 \\ & 12.01 \\ & 0.61 E-1 \\ & 11.81 \\ & 85.12 \end{aligned}$ | $\begin{aligned} & 2.86 \\ & 58.30 \\ & 0.24 \\ & -6.09 \\ & 721 \end{aligned}$ | $\begin{aligned} & 0.07 \\ & 8.12 \\ & 0.15 E-4 \\ & -48.21 \\ & 473 \end{aligned}$ | $\begin{aligned} & 0.45 E-1 \\ & 0.53 \\ & 0.27 E-2 \\ & 0.53 \\ & 3.82 \end{aligned}$ |
| $3.6^{\circ}$ (a) $\psi($ Deg $)$ <br> (b) $\theta$ ( Deg ) <br> (c) $\sigma\left(m^{2} m^{-2}\right)$ <br> (d) $\sigma(d B s)$ <br> (e) Area $\left(\mathrm{m}^{2}\right)$ | $\begin{aligned} & 4.54 \\ & 22.41 \\ & 0.17 E-2 \\ & -27.59 \\ & 652 \end{aligned}$ | $\begin{aligned} & 4.52 \\ & 27.22 \\ & 0.14 E-1 \\ & -27.95 \\ & 651 \end{aligned}$ | $\begin{aligned} & 0.89 \\ & 15.61 \\ & 0.30 \mathrm{E}-1 \\ & 11.00 \\ & 33.79 \end{aligned}$ | $\begin{aligned} & 5.93 \\ & 59.79 \\ & 0.20 \\ & -6.89 \\ & 720 \end{aligned}$ | $\begin{aligned} & 3.07 \\ & 7.95 \\ & 0.12 E-4 \\ & -49.08 \\ & 523 \end{aligned}$ | $\begin{aligned} & 0.40 \mathrm{E}-1 \\ & 0.70 \\ & 0.14 \mathrm{E}-2 \\ & 0.49 \\ & 1.52 \end{aligned}$ |
| $6-9^{\circ}$ (a) $\psi(\mathrm{Deg})$ <br> (b) $\theta$ (Deg) <br> (c) $\sigma\left(m^{2} m^{-2}\right)$ <br> (d) $\sigma$ (dBs) <br> (e) Area $\left(\mathrm{m}^{2}\right)$ | $\begin{aligned} & 7.03 \\ & 23.95 \\ & 0.118-2 \\ & -29.4 \\ & 645 \end{aligned}$ | $\begin{aligned} & 7.4 \\ & 21.4 \\ & 0.2 \mathrm{E}-1 \\ & -28.2 \\ & 651 \end{aligned}$ | $\begin{aligned} & 0.87 \\ & 11.18 \\ & 0.4 \mathrm{E}-1 \\ & 12.46 \\ & 31.6 \end{aligned}$ | 8.73 <br> 44.18 <br> 0.19 <br> - 7.2 <br> 706 | $\begin{aligned} & 6.15 \\ & 1.2 \\ & 0.1 E-5 \\ & -59.8 \\ & 581 \end{aligned}$ | $\begin{aligned} & 0.03 \\ & 0.45 \\ & 0.17 E-2 \\ & 0.50 \\ & 1.27 \end{aligned}$ |
| 2-12 ${ }^{\circ}$ (a). $\psi$ (Deg) <br> (b) $\cdot \theta$ ( Deg ) <br> (c) $\sigma,\left(m^{2} m^{-2}\right)$ <br> (d) $\sigma$ ( $\alpha \mathrm{Bs}$ ) <br> (e) Area $\left(m^{2}\right)$ | $\begin{aligned} & 9.55 \\ & 20.02 \\ & 0.3 E-2 \\ & -24.5 \\ & 617 \end{aligned}$ | $\begin{aligned} & 9.9 \\ & 18.5 \\ & 0.27 E-1 \\ & -25.5 \\ & 618 \end{aligned}$ | $\begin{aligned} & 0.77 \\ & 8.75 \\ & 0.46 \mathrm{E}-1 \\ & 11.88 \\ & 21.5 \end{aligned}$ | 11.57 <br> 29.5 <br> 0.29 <br> - 5.2 <br> 679 | $\begin{aligned} & 9.17 \\ & 0.09 \\ & 0.01 E-4 \\ & -49.7 \\ & 560 \end{aligned}$ | $\begin{aligned} & 0.04 \\ & 0.49 \\ & 0.02 E-2 \\ & 0.66 \\ & 1.20 \end{aligned}$ |


|  | MEDIAN | MEAN | s.d | MAX | MLN | STD. ERR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0-2^{\circ}$ (a) $\psi(\mathrm{Deg})$ <br> (b) $\theta$ ( Deg ) <br> (c) $\sigma\left(m^{2} \cdot m^{-2}\right)$ <br> (d) $\sigma$ ( $\partial \mathrm{Bs}$ ) <br> (e) Area $\left(m^{2}\right)$ | $\begin{aligned} & 0.65 \\ & 27.65 \\ & 0.32 E-1 \\ & -14.9 \\ & 498 \end{aligned}$ | $\begin{aligned} & 0.64 \\ & 24.3 \\ & 0.64 E-1 \\ & -18.57 \\ & 524 \end{aligned}$ | $\begin{aligned} & 0.38 \\ & 9.9 \\ & 0.65 E-1 \\ & 11.06 \\ & 68.2 \end{aligned}$ | 1.5 <br> 58.3 <br> 0.24 <br> - 6.09 <br> 705 | $\begin{aligned} & 0.7 E-1 \\ & 8.1 \\ & 0.15 E-4 \\ & -48.11 \\ & 473 \end{aligned}$ | $\begin{aligned} & 0.21 E-1 \\ & 0.55 \\ & 0.36 E-2 \\ & 0.61 \\ & 3.80 \end{aligned}$ |
| $2-4^{\circ}$ (a) $\psi(\mathrm{Deg})$ <br> (b) $\theta$ (Deg) <br> (c) $\sigma\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right)$ <br> (d) $\sigma$ (dBs) <br> (e) Area $\left(\mathrm{m}^{2}\right)$ | $\begin{aligned} & 2.84 \\ & 27.28 \\ & 0.15 \mathrm{E}-2 \\ & -28.21 \\ & 658 \end{aligned}$ | $\begin{aligned} & 3.00 \\ & 35.2 \\ & 0.13 \mathrm{E}-1 \\ & -28.25 \\ & 645 \end{aligned}$ | $\begin{aligned} & 0.47 \\ & 15.38 \\ & 0.30 \mathrm{E}-1 \\ & 10.4 \\ & 63.2 \end{aligned}$ | $\begin{aligned} & 3.97 \\ & 59.70 \\ & 0.14 \\ & -8.42 \\ & 721 \end{aligned}$ | $\begin{aligned} & 2.26 \\ & 15.80 \\ & 0.19 E-4 \\ & -47.05 \end{aligned}$ <br> 511 | $\begin{aligned} & 0.26 E-1 \\ & 0.85 \\ & 0.17 E-2 \\ & 0.57 \\ & 3.49 \end{aligned}$ |
| $4-6^{\circ}$ (a) $\psi(\mathrm{Deg})$ <br> (b) $\quad \theta$ (Deg) <br> (c) $\sigma\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right)$ <br> (d) $\sigma$ ( dBs ) <br> (e) Area $\left(\mathrm{m}^{2}\right)$ | $\begin{aligned} & 4.87 \\ & 22.28 \\ & 0.21 E-2 \\ & -26.61 \\ & 644 \end{aligned}$ | $\begin{aligned} & 4.97 \\ & 22.54 \\ & 0.16 \mathrm{E}-1 \\ & -27.69 \\ & 642 \end{aligned}$ | $\begin{aligned} & 0.57 \\ & 12.45 \\ & 0.33 \mathrm{E}-1 \\ & 11.46 \\ & 33.9 \end{aligned}$ | $\begin{aligned} & 5.93 \\ & 45.39 \\ & 0.19 \\ & -7.21 . \\ & 711 \end{aligned}$ | $\begin{aligned} & 4.27 \\ & 7.95 \\ & 0.12 E-4 \\ & -49.08 \\ & 523 \end{aligned}$ | $\begin{aligned} & 0.31 E-1 \\ & 0.68 \\ & 0.18 E-2 \\ & 0.63 \\ & 1.87 \end{aligned}$ |


|  | MEDIAN | MEAN | s.d | MAX | MIN | STD.ERR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $6-8^{\circ}$ (a) $\psi(\mathrm{Deg})$ <br> (b) $\sigma$ (Deg) <br> (c) $\sigma\left(m^{2} m^{-2}\right)$ <br> (d) $\sigma$ ( dBs ) <br> (e) Area $\left(m^{2}\right)$ | $\begin{aligned} & 6.74 \\ & 23.25 \\ & 0.65 \mathrm{E}-3 \\ & -31.8 \\ & 672 \end{aligned}$ | $\begin{aligned} & 6.66 \\ & 20.43 \\ & 0.43 E-2 \\ & -31.7 \\ & 669 \end{aligned}$ | $\begin{aligned} & 0.26 \\ & 14.41 \\ & 0.10 \mathrm{~F}-1 \\ & 9.22 \\ & 23.69 \end{aligned}$ | 7.06 <br> 44.6 <br> 0.08 <br> - 10.6 <br> 706 | $\begin{aligned} & 6.15 \\ & 1.20 \\ & 0.16 \dot{E}-4 \\ & -47.7 \\ & 630 \end{aligned}$ | $\begin{aligned} & 0.74 E-1 \\ & 0.79 \\ & 0.57 E-3 \\ & 0.51 \\ & 1.31 \end{aligned}$ |
| 8-10 (a) $\psi(\mathrm{Deg})$ <br> (b) $\sigma$ (Deg) <br> (c) $\sigma\left(m^{2} m^{-2}\right)$ <br> (d) $\sigma$ (dBs) <br> (e) Area ( $\mathrm{m}^{2}$ ) | $\begin{aligned} & 8.70 \\ & 25.46 \\ & 0.62 E-2 \\ & -22.06 \\ & 624 \end{aligned}$ | $\begin{aligned} & 8.88 \\ & 22.60 \\ & 0.41 E-1 \\ & -24.01 \\ & 622 \end{aligned}$ | $\begin{aligned} & 0.49 \\ & 6.42 \\ & 0.53 E-1 \\ & 13.48 \\ & 19.12 \end{aligned}$ | $\begin{aligned} & 9.79 \\ & 29.52 \\ & 0.29 \\ & -5.26 \\ & 680 \end{aligned}$ | $\begin{aligned} & 8.02 \\ & 8.14 \\ & 0.10 \mathrm{E}-5 \\ & -59.80 \\ & 560 \end{aligned}$ | $\begin{aligned} & 0.24 E-1 \\ & 0.32 \\ & 0.26 E-2 \\ & 0.67 \\ & 0.95 \end{aligned}$ |
| 10-12 ${ }^{\circ}(\mathrm{a}) \quad \psi(\mathrm{Deg})$ <br> (b) $\sigma$ (Deg) <br> (c) $\sigma\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right)$ <br> (d) $\sigma$ ( dBs ) <br> (e) Area $\left(\mathrm{m}^{2}\right)$ | $\begin{aligned} & 10.55 \\ & 20.02 \\ & 0.51 E-2 \\ & -22.91 \\ & 606 . \end{aligned}$ | $\begin{aligned} & 10.77 \\ & 15.68 \\ & 0.35 E-1 \\ & -23.23 \\ & 609 \end{aligned}$ | $\begin{aligned} & 0.51 \\ & 8.45 \\ & 0.5 E-1 \\ & 11.33 \\ & 16.22 \end{aligned}$ | $\begin{aligned} & 11.57 \\ & 22.85 \\ & 0.20 \\ & -6.97 \\ & .655 \end{aligned}$ | $\begin{aligned} & 10.24 \\ & 0.9 \mathrm{E}-1 \\ & 0.14 \mathrm{E}-4 \\ & -48.58 \\ & 588 \end{aligned}$ | $\begin{aligned} & 0.43 E-1 \\ & 0.71 \\ & 0.42 E-2 \\ & 0.95 \\ & 1.37 \end{aligned}$ |

by taking a sample which is statistically large.
b. Since data originates from a number of different surface textures and inaccuracies in terrain slope calculations, as explained previously; the use of a large number of readings at each grazing angle is likely to minimise error.
c. The method is likely to result in the most representative practical values for $\sigma$ for all values of $\psi$.

RESULTS BY $2^{\circ}$ AND $3^{\circ}$ STEPS IN GRAZING ANGLE.
41. Tables 6 and 7 include as much statistical data as necessary for future reference. Figures 5 and 6 show other results graphically rather than as tables.
42. Observation. Parameters were tested for statistical distributions. For example, although aspect and grazing angle are site specific, these were tested in case of future cross reference of work in the same type of terrain. No recognisable distribution was found. This is to be expected since all negative slopes (shadowed terrain - as seen from the unique radar position) had been eliminated from the database. Therefore a general terrain analysis (as made by geographic surveyors) is.not applicable, since here the terrain-is viewed from a specific position.
43. Sample Spread of Grazing Angles, Areas and Aspect Angles. From the statistical viewpoint the author was well satisified by the spread of data in each parameter. The range of values was considered to well represent typical terrain. Although, as is expected, the data was not spread linearly, there were no significant gaps in the values or shortage of measurements at any particular point. In places a small spread of data did appear to be distributed normally, however, as seen below Weibull and Lognormal plots confirm the preference for using "Weibull" to represent this rural terrain.
44. Differences Between Means. It has been assumed that (although the $\psi \dot{\mathrm{v}} \sigma$ gradient may be less at $K$ Band than other frequency bands), the data would nevertheless exhibit an increase in backscatter for an increase in grazing angle. It was therefore necessary to test signal values obtained at different grazing angles to see if their differences could happen by statistical chance or whether differences could be attributed to a cause. An example comparison of means for $2-4^{\circ}$ and $8-10^{\circ}$, respectively $-28.25 d B$ and $-24.01 d B$ with sd's of 10.4 and 13.4 dB 's and 327 , 404 samples, is shown:

$$
\begin{gathered}
\text { Hence } \frac{10.4}{\sqrt{327}}=0.575, \frac{13.4}{\sqrt{404}}=0.666 \\
\text { Then } \sqrt{0.575^{2}+0.666^{2}}=0.880 \\
2 x \pm 0.880= \pm 1.76 \mathrm{~dB} \\
28.25-24.01=-4.24 \mathrm{~dB}
\end{gathered}
$$

Since 4.24 is not less than 1.76, the difference has not happened by chance but is due to a definite cause. Similarly, a test between means for $0-2^{\circ}$ with $8-10^{\circ}$ also proved significant. However, taking two means (originating from $8-10^{\circ}$ and $10-12^{\circ}$ ) from angles closer together the test failed - the difference in means could occur by chance. It is also observed that the difference in mean is likely to be less since the backscatter at $9^{\circ}$ is not expected to be greatly different from that at $11^{\circ}$.

## TOTAL DATA PACKAGE ANALYSIS

45. The results at Table 4 were obtained by taking all backscatter results (from all azimuths angles and grazing angles) and analysing as a complete data set, on the initial assumption that there were sufficient measurements to treat as a continuous result without significant gaps in the various parameters.

Distributions were plotted at Figures 7 a - and 7 b to test. for Weibull and Log Normal characteristics.
46. General Comments. Bargraphs produced by computer statistical package were used to examine the spread of data for each parameter. Observations are as follows:
a. Backscatter $\sigma($ dBs $)$. An unusual number of readings occurred at about -9 dBs . These were attributed to the rail-line or associated fencing, or both, as the obstacle could probably be seen at the lower grazing cycles (at the single range) over several azimuths. Backscatter plots well on Weibull paper, but a Kolmorov-Smirnov test gave a clearly negative result for all other distributions.
b. True. Grazing Angle $\psi$ (Deg). As expected, with the deliberate choice of low grazing angles, the histogram was strongly skewed towards the lower values. On examination the bargraph (not shown here) looks almost random. Indeed it would not be expected that the facets of a random piece of terrain, (illuminated by a radar operationally), should, for $\psi$, exhibit any particular distribution. For experimental purposes all shadowed terrain was excluded. Any distribution here would not be expected to accord with distributions made by geomorphological surveys, for example, for drainage purposes. On plotting the values, however, a good number of the data produced a log-normal distribution.
c. Illuminated 3 aB Area $\left(\mathrm{m}^{2}\right)$. Area is a function of resolution cell parameters and terrain slope. The concept is nominal since, as explained previously it assumes a fairly sharp cut-off of energy at 3 dB , whereas there will be backscatter also received from sidelobes or just outside the "footprint" area. With the tracking radar used here the sidelobe levels are extremely low. Further, any additional backscatter from the fringes of the nominal $3 a B$ footprint will most probably be reflected from an adjacent facet which is likely to be at the same angle or nearly so. Area bargraphs show two distinct peaks in the general distribution; one centred on $498 \mathrm{~m}^{2}$ and a second, more pronounced, centred on $640 \mathrm{~m}^{2}$. There is no obvious reason for this, other than the likelihood of a fall-off in area due to the large number of measurements taken at both azimuth extremes of Challoch Hill, ie the occurence of high aspect angles becomes larger because of the geographical location. Thus no particular significance is attached the the Area distribution.
d. Aspect $\theta$ (deg). Aspect values are distributed lognormally for part of the spread but are not at all Weibull. They are slightly skewed to the left and relatively few values fall below $8^{\circ}$. It is pertinent to comment again that aspect is essential to calculate the true grazing angle. The spread of aspect was therefore considered important and an analysis at $0.3^{\circ}$ class intervals showed that only a few intervals did not contain observations.
e. Range $\mathrm{R}(\mathrm{Km})$. Most measurements occurred at a range of 5 to 7 Km with a predominance around 6 Km , which is this radar's typical operational working range when acting as a low-level target tracker. The overall statistics do not give a strictly true picture here, since Sector 5 contributes about $20 \%$ of the readings taken mainly between 4.5 and 5 KM .
f. Backscatter Signal. An examination of the raw measurements before conversion to the decibel scale shows $80 \%$ to fail below 0.006 and a clear distribution of Weibull is even more strongly seen after the extreme values were removed as discussed. Because of the inclusion of the extremes observed on the scattergraph correlations can be significantly distorted. Correlation between $\psi$ and backscatter was weakly negative ( -0.11 ) and has little meaning when all data meaurements are lumped together.

## FILTERED DATA

47. Datia was next filtered to remove the outlying values < $-20 d B$ and< 40dB, and statistical tests repeated. Results and comments are as follows:
48. Effect of Reducing Data File. The complete backscatter measurements

|  | MEDIAN | MEAN | sd | MAX | MIN | STD.ERR |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\psi($ Deg $)$ | 5.90 | 5.57 | 3.02 | 11.54 | 0.07 | 0.10 |
| $\sigma \mathrm{dBm}^{2} \cdot \mathrm{~m}^{-2}$ | -29.10 | -29.30 | 5.53 | -20.06 | -39.93 | 0.18 |
| All Data no <br> Filter $\sigma(7)$ <br> $\psi$ |  |  |  |  |  |  |
| $\sigma$ | 5.85 | 5.53 | 3.16 | 11.57 | $0.70 \mathrm{E}-1$ | 0.07 |

TABLE 8b SUMMARY OF STATISTICAL RESULTS - UNFILTERED DATA-BASE
(WITH ALL RESULTS BELOW $\psi=3^{\circ}$ REMOVED)

|  | MEDIAN | MEAN | sd | MAX | MIN | STD.ERR |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\psi($ Deg $)$ | 6.81 | 6.98 | 2.22 | 11.54 | 3.07 | 0.06 |
| $\sigma \mathrm{dBm}^{2} \mathrm{~m}^{-2}$ | -27.63 | -27.53 | $11-89$ | -5.26 | -59.85 | 0.31 |

CORRELATION +0.15

TABIE OC SUMMARY OF STATISTICAL RESULTS DATA-BASE (WITH FILTERED SIGNALS AND ALL

$$
\text { BELOW } \psi=3^{\circ} \text { REMOVED) }
$$

|  | MEDIAN | MEAN | sd | MAX | MIN | STD.ERR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\psi$ Deg | 6.74 | 6.76 | 2.27 | 11.54 | 3.07 | 0.08 |
| $\sigma \partial B^{2} \mathrm{~m}^{-2}$ | - 29.20 | - 29.44 | $5.58{ }^{\circ}$ | - 20.07 | - 39.93 | 0.20 |

approximate rouchly to both Weibuli and Iagnormal for much of the range of values. After excluding the extreme values the total data exhibits a closer Weibull distribution, and if the values for $\psi<3^{\circ}$ are eliminated an even stronger Weibull fit exists. These are plotted at Figure 7 .
49. Results of the reduced data file are at Table $\delta$. Under these conditions a check on the statistical distribution of $\psi$ was also Weibull although this may have no significance, being site-specific. Median and mean of each $\psi$ class interval were also used, and these are plotted at figs $8 a$ and $8 b$ in comparison with the other results. In addition to the Weibull and Lognormal plots, the Kolmorov Smirnov Test indicated a. strong normal tendency.

## ANALYSIS IN $3^{\circ}$ GRAZING ANGLE STEPS

50. Data in four sets enabled the $0-3^{\circ}$ values to be analysised separately to ensure that any reversal of characteristics would be isolated from any influence of backscatter values from the higher angles. Data was analysised in 2 ways:
a. All data included (Identified as 03C, 36C, 69C, 912C).
b. Data reduced in each set by $a>-20<-40 d B$ filter (Identified as 03M, 36 M etc) and means applied for each class set. Class sets were determined, not by arbitrary equal. steps, but by grouping ail backscatter readings from terrain of the same grazing angle. For example, the mean was found of all the backscatter readings at $\psi=4.86^{\circ}$ and so on. Since $\psi=4 . \hat{0} 6^{\circ}$ occurred a large number of times at different ranges and on different azimuths (and aspect is accounted for - since true grazing angle is used), the backscatter mean is taken to be the most

TABLE 9 STATISTICAL RESULTS - REDUCED DATA BY FILTER,
MEANS AND MEDIANS OF CLASS INTERVALS

|  | MEDIAN | MEAN | s.d | MAX | MIN | STD.ERR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| FILTERED DATA <br> $\sigma(\mathrm{dBs})$ <br> $\psi(\mathrm{Deg})$ <br> Signal | $\begin{aligned} & -29.39 \\ & 6.52 \\ & 0.12 \mathrm{E}-2 \end{aligned}$ | $\begin{aligned} & -29.52 \\ & 6.12 \\ & 0.22 E-2 \end{aligned}$ | $\begin{aligned} & 5.56 \\ & 2.73 \\ & 0.24 E-2 \end{aligned}$ | $\begin{aligned} & -20.07 \\ & 11.57 \\ & 0.98 \mathrm{E}-2 \end{aligned}$ | $\begin{aligned} & -39.93 \\ & 0.07 \\ & 0.10 \mathrm{E}-3 \end{aligned}$ | $\begin{aligned} & 0.19 \\ & 0.09 \\ & 0.86 E-4 \end{aligned}$ |
| $\begin{aligned} & \text { MEANS OF CLASS INTERVALS } \\ & \sigma \text { (dBs) } \\ & \psi \text { (Deg) } \\ & \text { Signal } \end{aligned}$ | $\begin{aligned} & -29.72 \\ & 6.25 \\ & 0.20 \mathrm{E}-2 \end{aligned}$ | $\begin{aligned} & -29.77 \\ & 6.09 \\ & 0.21 E-2 \end{aligned}$ | $\begin{aligned} & 3.37 \\ & 3.23 \\ & 0.14 E-2 \end{aligned}$ | $\begin{aligned} & -21.91 \\ & 11.625 \\ & 0.68 \mathrm{E}-2 \end{aligned}$ | $\begin{aligned} & -36.54 \\ & 0.125 \\ & 0.25 E-3 \end{aligned}$ | $\begin{aligned} & 0.54 \\ & 0.53 \\ & 0.23 E-3 \end{aligned}$ |
| $\frac{\text { MEDIANS OF CLASS }}{\frac{\text { INTERVALS }}{\sigma(\mathrm{aBS})}}$$\psi($ Deg $)$ | $\begin{aligned} & -29.65 \\ & 6.125 \end{aligned}$ | $\begin{aligned} & -29.87 \\ & 6.00 \end{aligned}$ | $\begin{aligned} & 3.46 \\ & 3.21 \end{aligned}$ | $\begin{aligned} & -22.95 \\ & 11.875 \end{aligned}$ | $\begin{aligned} & -36.54 \\ & 0.25 \end{aligned}$ | $\begin{aligned} & 0.58 \\ & 0.54 \end{aligned}$ |
| $\frac{\text { FILTERED AND ALL }}{\frac{\text { BELOW } \psi=3^{\circ} \text { REMOVED }}{\sigma(d B s)}} \begin{gathered} \psi(D e g) \end{gathered}$ | $\begin{aligned} & -29.20 \\ & 6.74 \end{aligned}$ | $\begin{aligned} & -29.44 \\ & 6.76 \end{aligned}$ | $\begin{aligned} & 5.58 \\ & 2.27 \end{aligned}$ | $\begin{aligned} & -20.07 \\ & 11.54 \end{aligned}$ | $\begin{aligned} & -39.93 \\ & 3.07 \end{aligned}$ | $\begin{aligned} & 0.20 \\ & 0.08 \end{aligned}$ |
| $\frac{\text { UNFILTERED AND ALL }}{\text { BELOW } \psi=33^{\circ} \text { REMOVED }} \frac{\sigma(\mathrm{dBs})}{\psi(\mathrm{Deg})}$ | $\begin{aligned} & -27.63 \\ & 6.81 \end{aligned}$ | $\begin{aligned} & -27.53 \\ & 6.98 \end{aligned}$ | $\begin{aligned} & 11.89 \\ & 2.27 \end{aligned}$ | $\begin{aligned} & -46.9 \\ & 11.57 \end{aligned}$ | $\begin{aligned} & -5.2 \\ & 3.07 \end{aligned}$ | $\begin{aligned} & 0.31 \\ & 0.06 \end{aligned}$ |


|  | MEDIAN | MEAN | s.d | MAX | MIN | STD.ERR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \frac{0-3^{0}}{\psi(\mathrm{Deg})} \\ & \sigma\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right) \\ & \sigma(\mathrm{dBs}) \end{aligned}$ | $\begin{aligned} & 0.25 \\ & 0.26 \mathrm{E}-2 \\ & -27.75 \end{aligned}$ | $\begin{aligned} & 0.47 \\ & 0.21 E-2 \\ & -28.51 \end{aligned}$ | $\begin{aligned} & 0.50 \\ & 0.23 E-2 \\ & 5.35 \end{aligned}$ | $\begin{aligned} & 2.61 \\ & 0.88 \mathrm{E}-2 \\ & -20.52 \end{aligned}$ | $\begin{aligned} & 0.07 \\ & 0.11 E-3 \\ & -39.43 \end{aligned}$ | $\begin{aligned} & 0.08 \\ & 0.37 E-3 \\ & 0.85 \end{aligned}$ |
| $3-6^{\circ}$ <br> $\psi$ (Deg) <br> $\sigma\left(m^{2} m^{-2}\right)$ <br> $\sigma$ (dBs) | $\begin{aligned} & 3.95 \\ & 0.12 E-2 \\ & -29.1 \end{aligned}$ | $\begin{aligned} & 3.85 \\ & 0.22 E-2 \\ & -29.3 \end{aligned}$ | $\begin{aligned} & 1.00 \\ & 0.23 E-2 \\ & 5.46 \end{aligned}$ | $\begin{aligned} & 5.79 \\ & 0.98 \mathrm{E}-2 \\ & -20.08 \end{aligned}$ | $\begin{aligned} & 2.26 \\ & 0.10 \mathrm{E}-3 \\ & -39.88 \end{aligned}$ | $\begin{aligned} & 0.06 \\ & 0.14 E-3 \\ & 0.32 \end{aligned}$ |
| $\begin{aligned} & \frac{6-9^{0}}{\psi(\text { Deg })} \\ & \sigma\left(\mathrm{m}^{2} \mathrm{~m}^{-2}\right) \\ & \sigma(\mathrm{dBs}) \end{aligned}$ | $\begin{aligned} & 6.79 \\ & 0.96 E-3 \\ & -30.16 \end{aligned}$ | $\begin{aligned} & 6.71 \\ & 0.21 E-2 \\ & -29.83 \end{aligned}$ | $\begin{aligned} & 0.88 \\ & 0.24 E-2 \\ & 5.56 \end{aligned}$ | $\begin{aligned} & 0.98 E-2 \\ & 0.98 E-2 \\ & -20.07 \end{aligned}$ | $\begin{aligned} & 0.10 \mathrm{E}-3 \\ & 0.10 \mathrm{E}-3 \\ & -39.89 \end{aligned}$ | $\begin{aligned} & 0.05 \\ & 0.15 E-3 \\ & 0.35 \end{aligned}$ |


|  | MEDIAN | MEAN | s.d | MAX | MIN | STD.ERR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9-120 |  |  |  |  |  |  |
| $\psi$ ( Deg ) | 9.44 | 9.31 | 1.39 | 11.57 | 6.76 | 0.09 |
| $\sigma\left(m^{2} m^{-2}\right)$ | 0.12E-2 | 0.22E-2 | 0.24E-2 | 0.95E-2 | 0.10e - 3 | 0.16E-3 |
| $\sigma$ ( dBs ) | - 29.14 | - 29.57 | 5.62 | - 20.18 | - 39.93 | 0.37 |

practical value to represent the particular grazing angle of interest on each occasion. Results using this method are at Table 9 and plotted at figures $9 a, 9 b$ and 10. The same process was applied for raw signal strength. Values obtained for the means of the total data file are included for comparison.

## ANALYSIS IN $2^{\circ}$ GRAZING ANGLE STEPS

51. Six data sets identified as $02,24,46$ etc were analysed in the same way as the $3^{\circ}$ sets.

COMMENTS ON TABULATED DATA - TABLES 4 TO 9
52. Within the overall data sumnarised at Table 4 considerable fluctuation occurred, not obvious until seen by sector at Table 5 and subsequent analysis in $2^{\circ}$ and $3^{\circ}$ steps. For example, the variation in regression models ( $\psi \vee \sigma$ ) shows that quite different models are obtained from adjacent terrain sectors. These are inaccurate being distorted by extreme values and correlations are seen to vary widely.
53. Tables 6 and 7 reveal much more information about the general nature of the backscatter. The data in both tables is unfiltered, ie all measurements are included. Thus an examination and comparison of values (while showing trends as grazing angle varies) also highlights any unusual results due to extreme values falling within any particular step. The ratio of mean to median increased significantly as $\psi$ decreased. This finding is in accord with observations in the USA (Report "Seek Igloo"). It is not clear why the standard error for aspect $\theta$ is significantly different for $8-10^{\circ}$ at 0.32. Despite the various methods of data reduction, made to eliminate and
minimise undue influence of outlying values, it is of interest that $\sigma_{m}$ remains at approximately - 29 dB ; as seen in summary table 9 b

COMMENTS ON PIOTITED DATA - FIGURES 5-9
54. It is assumed that a Weibull distribution exists if plotted values fit between the $10 \%$ and $90 \%$ levels. Several important facts are confirmed by the distributions at Figure 5(a). It is irmediately apparent that a good Weibull fit exists, that the slope parameters are sensibly the same (slope parameter approx 2.57, shape parameter 0.39 ) for 5 of the 6 sets of data . The data set with a different slope ( $8-10^{\circ}$ ) warranted further investigation. Values were back plotted onto a large-scale map, since an unusually large number of high readings were found ( -8.5 to -9.6 dB , at $56.4^{\circ}$ to $56.7^{\circ}$ azimuth and 5750 to 5800 metres range). These were found to come from a rail line, on an embankment, and rail bridge over the trunk road $A 75$ between $E$ and $W$ Challoch. By removing these readings from an otherwise 'standard' set of results the distribution plot for $8-10^{\circ}$ moves towards the slope of the other data sets. This confirmed the contamination in this case and it was thought reasonable to make-this adjustment to the data. It should be noted that the median ( $50 \%$ Weibull) level fixes the position of the curves on the plot. These results most clearly demonstrate (confirmed again at figure 10) that backscatter values apparently increase at very low grazing angles - assuming of course that these angles are correctly measured (subject to terrain measurement errors discussed at para 25 above).
55. The lognormal plot at figure $5(\mathrm{~b})$ shows that the Log $N$ distribution could almost equally be used as a representative distribution. For $3^{\circ}$ steps the mean slope parameter is approx 2.8. As before, these distributions clearly show an increase in signal strength at the lower grazing angles.
56. Results at figure 7 were commented upon at paras 45-46., however it is noted that introduction of the signal filter changes the slope (as expected), but does not change the excellent alignment of the plotted readings on Weibull paper. If all data for $\psi<3^{\circ}$ is removed (postulating removal of data below the possible minimum in the $\sigma \vee \psi$ curve). Figure 7 (Curve *) also shows a very good fit with a slope parameter, identical to that at Figure 8 (means and medians).
57. The results at Figure 9 are for data taken in 4 sets with signal filter applied. Data was then separated into mean and median values by grazing angle class intervals. It is shown that a good Weibull fit exists (Figure 9(a)), for which the Weibull shape parameter is almost identical for the 4 sets ( 0.9 with slope parameter 1.1). Kolmorov-Smirnov tests indicated a strong tendency also to Lognormal, as seen at Figure 9(b).

SKEWNESS, VARIABILITY, REGRESSION AND CORRELATION
58. Data distribution were checked for skewness during the $\psi=2^{\circ}$ intervals, where the $\sigma(d B)$ skews were near zero on 2 occasions and negatively skewed on all others. A significantly higher coefficient of variation occurred below $\psi=2^{\circ}$ and also between $8-10^{\circ}$. In the latter case this is probably due to the problem at this angle recognised at para 54 above. It was also noted that both true grazing angle and backscatter values became more variable as $\psi$ reduced; markedly so for angles below $2^{\circ}$.
59. Regression. Regressions were made between $\sigma$ and $\psi$, however once it became clear that $\sigma$ increased for small values of $\psi$, it became clear that a straight line regression could not be used for the whole data base, but only to that portion to the right of the minimum. Reference to figure 10 shows that this minimum occurs at some point about $\psi=2^{\circ}$. The best model for this section of the curve probably lies between $\sigma_{a B}=-29.6+0.05 \psi$ and $-31.5+0.83 \psi \mathrm{dBs}$.
60. Correlations. Correlations between $\sigma$ and $\psi$ is shown to be positive (for $\psi>3^{\circ}$ ). The variability of data in some sets is such that without a greater number of samples a stronger correlation is unlikely.
61. General. Apart from the statistical values shown, a very large number of supporting calculations, computer file handling procedures an plots were made. It is not considered necessary to include bargraphs, scattergraphs, histograms or frequency tabulations, as this information appears on the various plots.
62. Weibull Relationships and Cell Size. Above $\psi=3^{\circ}$, as also found \{71\} (see para 39 page 4-89), the s.d increased as pulse length effectively decreased (ie illuminated area reduced due to surface tilting effects), however the empirical relationships between $\tau$ and the Weibull shape parameter could not be established. As seen from the consistency of the curves at Figure 5(a) the shape parameter is approximately constant despite fluctuations in cell size by as much as $16 \%$ from the standardised value. Neither could the shape parameter found be fitted into the tables, curves or relationship given by $\{71\}$ at $\lambda=10 \mathrm{~cm}$.

## CONCLUSIONS

63. It is proposed that the results here are statistically valid and that reasonable assumptions, with correct and careful procedures were used to expose the underlying trends in the backscatter data. Nore hypotheses were tested than have been shown; they are not included here if the results were inconclusive. For example a correlation matrix was made at each stage between all variables, together with all regressions and cumulative distributions; many were null or inadmissible relationships, or were site-specific.
64. The results compare well with other published findings as seen at Figure 5 Chapter 4; where the proposed model falls close to several others; although at a slightly shallower rate of change of $\sigma$ for $\psi$ at about 1.5 $d B$ per deg; down to the minimum. Since there are few other published results at $K$ Band, the results shown are inevitably contrasted with those from I or $J$ Band, or with other models which take account of frequency by postulation rather than actual measurement.
65. A most difficult area to be sure about is that at very low grazing angles, where, as found by some other researchers, $\sigma_{c}$ values inexplicably rise. It has already been postulated here that terrain slope angles are rarely what they appear to be, since even within a 50 or 100 metre square area terrain undulates appreciably. It may always be questioned as to the proportion of illumination actually impinging at the presumed mean angle. At shorter wavelengths this becomes even more relevant. A large part of a facet may be in shadow due to high frequency undulations of the surface eg in rows of crops, banks and hedegrows. In practice there will almost
always be a proportion of the cells containing (for the "rural" description), brick buildings, fences, metal farm buildings, vehicles and so on.

Therefore the contamination these produce cannot be ignored. Production of a 2 part clutter model may be possible, ie under lying trend plus an allowance for peaking.
66. It may be questioned whether there is some unknown mechanism which applies only at very low grazing angles and causes $\sigma_{c}$ to rise; or is the rise entirely due to inaccuracies in terrain angles? Consideration of this possibility comenced with a critical examination of possible terrain measurement errors. Correlation was definitely positive for $\psi>3^{\circ}$ and reversed at some point $\psi<3^{\circ}$. The variability of the data was such that a greater number of observations would be preferable to obtain stronger correlations. What errors were likely and where might the model fail? Every effort was made to minimise terrain errors and worst case errors would have to be present in large numbers to significantly change the underlying trend. Errors which did exist will have contributed to the fluctuations found.
67. . It is suggested that the model is accurate within the constraints of such a study, especially at $K$ Band wavelengths. There is a remarkable consistency in the culmulative distribution slopes when the data is analysed in arbitrary steps of 2 or $3^{\circ}$. Further, if it is assumed that errors do exist in $\psi$, causing values to be misplaced on the curve at Figure 10, an angular error far larger than the worst case calculated above would have to be applied in order to correcting reposition the backscatter reading elsewhere on the curve. It is concluded therefore that a rise in backscatter level does occur at very low grazing angles. It is nevertheless re-iterated
that true grazing angle errors can also be included, and that these may not be obvious without careful consideration.
68. For "rural" terrain it is suggested that the proposed model shouldbe applicable to similar tracking radars in similar terrain conditions, in the context of detectability and tracking of LLSAM systems.
69. Returning finally to 2 aspects, those of useful $\tau \theta_{A}$ product relationships and the mechanism causing $\sigma_{c}$ to rise at very low angles. First, a lot of effort was expended to try and apply Dodsworths proposal for scaling distributions to obtain new median values, and to compare shape parameters as suggested at Chapter 4 and Annex $A$, wothout success... It is clear that for meaningful Weibull conclusions a series of measurements must be made with a variable pulse duration radar at the same time and place for cell scaling relationships to be studied. In summary no conclusive evidence was found to link the shape parameter to the $\tau \theta_{A}$ product.
70. Secondly, the rise in $\sigma_{c}$ definitely occurs at low grazing angles, as distinct from low terrain slopes. Calculated grazing angles were as much as $3.6^{\circ}$ lower than slope angles, and many of the lower grazing angles were from terrain sloped at quite diverse angles. Above $\psi=3^{\circ}$ the ratio of $\psi$ to mean terrain slope increased almost linearly (from 0.78 at $\psi=3^{\circ}$ to 0.98 at $\psi=12^{\circ}$ ), but dropped abruptly to 0.63 at $\psi=1^{\circ}$. The implication is that as terrain slope increases, low values of $\psi$ are less likely. Greatest incident signal attenuation could be expected at the higher values of $\psi$, since the depth of penetration into surface cover is probably greater. It is proposed that below a critical angle (here about $2.5^{\circ}$ ), despite the fact that the rms surface roughness appears to be smoother in the general sense, $\psi$ reaches a value where the signal finds it difficult to propagate into the culture; since the majority of reflectors in 'rural' terrain are seen increasingly as vertical structures
(grass blades as long cylinders), as $\psi$ reduces. All other reflectivity factors, e.g. ground conductivity, relative dielectric constant, $\lambda$, etc. have not changed. Further, the mechinism would be more noticeable at higher frequencies, where absorption is higher. It is of interest that the experimental result here (Figure 10 and see Chapter 4, Figure 5) is generally below the other world-wide curves. Only Trebits at 95 GHz is 1ower.
71. Reliance of Findings to Overall Tracking Predictions:

Irrespective of the terrain slope it is confirmed that any target flying at near grazing angles could enter a region (unless the underlying ground is shadowed) where a rise in clutter values may occur to a level which may compete with the target RCS. Targets should fly such that an angle of $1^{\circ}$ or less occurs, typically 100 m altitude at 6 km range. This is easily achievable by manned aircraft and even more so by terrain following missiles with minimum RCS where grazing angles can be even lower.
72. Compatibility With Other Models: Within the past year Barton $\{234\}$ has proposed a unified clutter model for flat or rolling terrain, at a number of RF's. Although $\psi$ is implicit in his model out to $R_{1}=4 \pi h_{r}{ }^{1} \sigma h_{r}^{1} \lambda$, for both flat or rolling terrain, the model does not set out to relate $\sigma$ with $\psi$ in isolation. The short-range model, $\sigma=\gamma \sin \psi$ is plotted at Figure 10 in contrast to the $K$ Band results obtained here. $Y$ is a 'terrain constant' set at 0.04 for USA test-sites. It is observed that the $K$ Band results here are not scaleable to meet Barton's proposals by simple adjustment of $\gamma$; since the $\sigma V \psi$ gradient is shallower. Beyond $R_{1}$, out to the horizon, Barton used $\sigma=\gamma\left(h_{r}^{1 / R) F, ~ b a s e d ~ o n ~ p r o p o g a t i o n ~ f a c t o r ~}\right.$ $F=\frac{\left(R_{1}\right)}{R} 4 . h_{r} 1=h_{t_{x}}+{ }^{2} \sigma_{h_{2}}$ where $\sigma_{h}$ is the terrain surface standard
deviation. It may be concluded that although the Barton unified model is a good assessment for flat or rolling terrain, it may not be as applicable when a target is being tracked against a background of gradually rising terrain, since the filtered model $(-29.6+0.05 \psi)$ diverges significantly from Barton as $\psi$ increased. However, the median model over comparable ranges $(-30.8+0.875 \psi)$ confirms Barton's proposel within 1.5 dB or less, but it is only applicable for $\psi \leqslant 3$. Below $\psi=3^{\circ}$ the values of $\sigma$ rise at a rate which it would be inadvisable to quote as a general rule until more results become available to confirm the phenomenon and the mechanism becomes understood.


FIG laVIEW FROM RADAR TOWARDS SLOPED TERRAIN USED FOR CORRELATION STUDIES


fig a optimum correlation for error correction


FIG 3 undulations on sloped or flat terrain

fig 4. effect of moving resolution cell



FIG 5(b) CUMULATIVE PROBABILITY THAT VALUE IS
NOT EXCEEDED - $2^{0}$ STEPS (LOGNORMAL SCALE)


FIG 6(a) CUMULATIVE PROBABILITY THAT VALUE IS NOT EXCEEDED - 3 STEPS (WEIBULL SCALE)





FIG 8(a) CUMULATIVE PROBABILITY THAT VALUE IS NOT EXCEEDED - MEANS AND MEDIANS (WEIBULL SCALE)



FIG 9(a) CUMULATIVE PROBABILITY THAT VAIUE IS NOT EXCEEDED - MEANS OF $3^{\circ}$ STEPS (WEIBULL SCALE)



# ANNEX G TO <br> "THE PROBABILITY OF <br> DETECTING AND TRACXING <br> RADAR TARGETS IN <br> CLUTTER AT LOW GRAZING ANGLES" 

DATED

## APPLICATION OF MODEL TO EXAMPLE SYSTEM AND LOCATION

1. It is assumed that clutter is present in the resolution cell for the entire observable track length, taking the worst-case condition; although there will be occasions when clutter is 'shadowed'. Certain other initial assumptions are necessary, either assessed or postulated according to the situation. For this example calculation the basic assumption are:-
(a) Terrain: Rural almost flat with vegetation and buildings giving a terrain sightline response as at Annex E, Figure 5a, page E-25. It is assumed that the mean terrain slope produces $\sigma_{m}=-28 \mathrm{dBm}^{2} \cdot \mathrm{~m}^{-2}$ at 5 km range, and has a surface reflection coefficient of 0.3 .
(b) Missile System Parameters: Considered next (since it dictates the required radar track length) a Reaction time of 10 seconds and $\mathrm{V}_{\mathrm{m}}=600 \mathrm{~m} \cdot \mathrm{~s}^{-1}$.
(c) Target Parameters: Transitting target at $300 \mathrm{~ms}^{-1}$, and 60 m (200 ft) altitude and dimension maximum approximately 10 metres. RCS minimum $0.05 \mathrm{~m}^{2}$ (see page E-22). Crossing the site at a range of 5 km .
(d) Tracking Algorithm: A tracking algorithm of the type
described at Annex E (page E-12) is assumed.

(e) Radar Parameters: Radar parameters from the list at Chapter 6 (page 6-146) are used, but the aerial height is lowered to 4 m (i.e. not clear of obscuration). A diode mixer receiver is assumed on this occasion with a 1200 Hz filter bandwidth, 10 velocity gates and 90 range gates, i.e. 900 decisions per target pulse burst using a pulse-burst system of 4 bursts of 10 pulses, PRF's 12000 and
2. For 60 rpm the dwell-time will be 5.5 msec .
3. Other assumptions on ECM degradation etc. are made later in the sequence which follows.

## ASSESSED PROBABILITY VALUES

3. Track Length: A combination of parameters from paragraph 1(a), 1(b) and 1(c) above when applied to Table 1 and Figure 5a at Annex E gives

$$
P_{T L}=0.22
$$

It is important to remember that this is the minimum tracklength necessary for an engagement (see also paragraph 4 below).
4. Tracking: It is assumed that the equipment under consideration was designed to give a probability of detection (given a sightline) of a 0.9 overall. Applying this to the tracking algorithm at page E-18, where 2 in any successive 3 looks must cross the threshold to delare a track (2 out of 3 sliding window algorithm) hence:
(a) Probability of declaring a new track $\overline{\mathrm{P}_{\mathrm{TT}}}=0.57$
(b) Probability of maintaining track, once obtained $\quad \mathrm{P}_{\mathrm{TT}}=0.972$
Hence, (c) Probability of loosing track, once obtained

$$
P_{T T} \bar{T}=0.028
$$

5. Detection: Time between false alarms is assessed as 900 secs ( 15 mins). Transmission time of each burst of pulses is $10 \times 83.3 \mu$ (for PRF 12000) and $10 \times 93.02 \mu(f o r \operatorname{PRF} 10750$ ) ; totalling 5.28 in sec if 3 bursts of each are completed within the dwell time of 5.5 m.ser, i.e. at least 60 hits per antenna sweep. Probability of false alarm depends upon the number of decisions per second and the false alam interval.

$$
\text { No. of decisions }=\frac{900 \times 6}{5.28 \times 10^{-3}} \simeq 1 \times 10^{6} \text { and Pfa } \simeq 1 \times 10^{-9}
$$

The target is observed at least six times per antenna sweep, hence the probability of detection per burst for a $P_{D}$ of 0.9 is given by $P=1-(1-0.9)^{1 / 6}=0.32$. Using the standard curves at Figure 3, the required $\mathrm{S} / \mathrm{N}$ is 12.5 dB . Correcting for 10 pulses using Swerlings Integration Improvement factor add 0.6 dB for the Swerling 3 Case. 6. Use of Clutter Model at Annex F: Using the standard equations, the total receiver clutter reduction for a target to be detected at 5 km , for $q_{m}=-28 \mathrm{~dB}$, is $[-29.2 \mathrm{~dB}+(-12.5-0.6)]=-42.3 \mathrm{~dB}$. Worst case rain conditions give -24.1 dB , well within the radar's capability. In practice the radar may have a far better performance in rain due to circular polarisation. If the target is reduced in altitude so that the radar grazing angle $\Psi=<2^{\circ}$ and the clutter level rises (as shown in the research at Appendix 1 to Annex $F$ ) to a level of -18 dB at $\Psi=1^{\circ}$, the values above become:-

$$
[-39.2 \mathrm{~dB}+(-12.5+0.6)]=-52.3 \mathrm{~dB}
$$

If a clutter rejection capability is postulated for a system it is thus possible, by the insertion of the clutter model, to reverse the calculation process to detemine the detection performance, tracking performance and hence the total systen prediction.
7. Area Assessment: For a particular terrain, given target size and full terrain data, for a given site position it is possible to assess a percentage of occasions when the target may be lost in clutter. Many modern radars can process clutter to a high standard. It is assumed that in the terrain
in question targets are lost in clutter for only $2 \%$ of the time a sightline exists.

Clutter Factor 0.98

8. Chaff, Noise and Deception ECM. Chaff may pose a detection problem unless filtered by MII. In the basic detection mode, forward firing chaff from a radially approaching target can usually hide a target because of the large RCS produced. This would be a worst-case condition for a point defence tracking radar (failing MTI), l Kg of broadband chaff produces $660 \mathrm{~m}^{2}$ (since $\sigma_{\text {chaff }}=1365 \mathrm{~W} / \mathrm{f}_{\mathrm{GHz}}$ ). The reader is referred to Haddow \{137\} and subsequent reports on the technique of radar tracker break-lock as a distinctly different use of chaff. For example purposes here it is postulated that system radar performance is degraded to $50 \%$ of its undergraded value by the use of ECM, ie tracking error is increased to an unacceptable level $50 \%$ of the time

$$
E_{f}=0.5
$$

9. Tracking Errors. Para 4 above considers tracking probabilities given a sightline. Scanning radars will attract azimuth tracking errors ( $\varepsilon$ a _ _ $)$ Skolnik ( p ls Fig 5.16), while tracking radars will be $\sqrt{5 / 1}$ subject $\frac{\sqrt{T}}{\mathrm{~N}_{0}}$ errors discussed at $9-187$ and $9-193$. Range, refraction and elevation errors should be applied as appropriate Pages 8-174 to 8-176. These are also calculated by reference to Annex E -probability of sightline falling on target. For this example it is taken (page $\mathbb{E}-23$ ) that a target at 5 km can be tracked successfully when there is a surface reflection coefficient of 0.3 . From Annex $E$ (equations (4) and (5)) it is calculated that the probability of the sightline falling on the specified target is

$$
P_{S}=0.24
$$

A ratio of 10 was assumed for $q_{s}^{2}$ giving $=\phi\left(\varepsilon_{1}\right),\left(\varepsilon_{2}\right)$ as respectively 0.43 and 0.18 .
10. Diffraction. The effect of diffraction, which could enhance track lengths, is more difficult to assess in the absence of site - specific data. A computer program was written by the author to produce the example plot at Annex F Fig 6. Page Flo together with Chap 7 provide the criteria. For this example a diffraction factor of 1 is used.
$D_{f}=1 \quad$ (ie no enhancement).
11. Missile Performance. If an overall assessment of radar tracker system performance is to include a missile engagement, a
lethality figure must be included. This will depend on many factors including trials results under idealised and possible under countermeasure conditions. A figure $P_{k}=60 \%$ is used here for example purposes. $P_{M X}=0.6$
12. Operator Performance. In those systems where an operator is used several sources of degradation may occur which can seriously affect overall system performance but which are often difficult to quantify. Conditions may also change day to day and reflect, for example, on morale, fear, training standards, coldness if exposed or lack of confidence in the system (following possible earlier failures). The operator may be using a system but forced into the optical mode by the enemy jamming of his associated tracking radar; the system is thus already in a degraded or reversionary mode of operation. He may of course be assisted if the system is semi-automatic (ie SACLOS compared with CLOS). For this example operator efficienty is taken to be $70 \%$ ie daylight with good visibility $\therefore \mathrm{P}_{\mathrm{OE}}=0.7$
13. System Availability. If a system is mobile it may not be in an irmediately operational condition, while others in a similar location may be able to partially defend the air space during its redeployment. System equipment availability is calculated from MTBF and MTTR (see Fig 2 page 11-226), taken to be 75 and 4 respectively. With redeployments and reload availability to account for, overall probability of readiness is taken here as $70 \%$

this may of course degrade after a redeployment due to vibration, weather etc.
14. Target Re-engagement. If an engagement fails for one reason or another it may be possible, depending on target speed, sightline and available time to re-acquire and refire. There will always be a low probability of obtaining a larger track length than the minimum, but the refire reaction time ...is often much faster than the original reaction time. The re-establishment of tracking will naturally depend on clutter, operator skill or auto-system ability etc. To assist in the prediction process two nomograms are included at figs 1 and 2 (pages G 10 and G 11) Fig 2 is a standard multiple trial (engagement) nomogram to be used where required for salvo engagements. Fig 1 was deliberately produced as two 3rd order nomograms rather than a single (5th order Genus 1) nomogram so that either the track length covered by a target during missile flyout or tracklength flown during system reaction time can be read off a common scale. This means that the target velocity/reaction time grid on the nomogram can also be used to read off re-engagement track lengths.

## OVERALL PREDICTION

15. Results can be easily read from the simple multiplying nomogram at figure 4
a. Mobile System - Operator Controlled (No EOCM)
$P=P_{T L} \times P_{M X} \times P_{R} \times P_{O E}$
$\mathrm{P}=0.22 \times 0.7 \times 0.6 \times 0.7$
$P=0.06(6 \%)$ this result is shown on the nomogram
b. Mobile System - Radar/Automatic Fire Control

$$
P=P_{T H} \times P_{M X} \times P_{R} \times P_{T T} \times P_{S}
$$

additionally multiply by ECM, clutter and diffraction factors, as appropriate.

$$
\begin{aligned}
& P=0.22 \times 0.6 \times 0.7 \times 0.57 \times 0.24 \\
& \underline{P}=0.013(1.3 \%)-\text { this result is shown on the nomogram }
\end{aligned}
$$

16. This method assumes that tracking and hence engagement opportunities can always be used and that they apply to $360^{\circ}$ azimuth cover. There will be cases where $P_{T L}$ can be very much higher in value but only applicable to a limited sector in azimuth; advantage can, of course, only be taken from these sites if targets fly into the high $\mathrm{P}_{\mathrm{TL}}$ sector. The probability of targets entering these sectors then also becomes of
interest. However such siting is usually deliberate in order to protect a sector along which targets may be constrained to fly due to type or alignment of the intended surface target. It is suggested that another factor $P_{E}$, ie probability of converting a detection to an engagement might be incorporated.
17. The importance of $\mathrm{P}_{\mathrm{TL}}$ is re-iterated since in flat and gently undulating terrain it can be significantly improved by simply raising the tracker aerial clear of immediate obstructions. In the two cases at para 15 placing $P_{T L}$ to 1 imediately changes the results to $29 \%$ and $6 \%$ respectively. Similarly on a fixed site $P_{R}$ might be much better.
18. Interpretation. It is also necessary to remind the reader exactly what the results mean, since much misunderstanding of similar results has occurred in the past. The result ( $6 \%$ ) at para 15 a does not mean that $6 \%$ of all targets will be successfully engaged, however, $6 \%$ could ideally be engaged if every opportunity is taken since it is a statistical value. There may be slightly more - or far likely, less - opportunities in practice since it has been assumed that no target appears as a surprise and that targets are engaged (tracked) either approaching or crossing but not receding.
19. Further, improvements may occur in a radar tracked system if offsite assistance is given by other radars in the area; while on the other hand degradation will occur if electronic countermeasures or target manoeuvre is used to degrade the tracking function. Notice that with the tracking algorithm chosen there is a $3 \%$ probability of loosing a
successful track after obtaining it with a $90 \%$ detection probability once it became unmasked. Conversely there is a $97 \%$ probability that tracking will continue successfully subject to a sightline or diffraction path and no other degradation.

fig 1 NOMOGRAM FOR CALCULATION OF MINIMUM OBSERVED TRACKLENGTH NECESSARY FOR AN ENGAGEMENT

SSP $=1-(1-P)^{\frac{1}{n}}$



FIG 2 NOMOGRAM FOR MULTIPLE INDEPENDENT TRIALS
G-11


FIG 3a PROBABILITY OF DETECTION AS A FUNCTION OF THE SIGNAL-to-noise ratio and the probability of false alarm


FIG 3b ADDITIONAL SIGNAL-TO-NOISE REQUIRED TO ACHEIVE A PARTICULAR PROBABILITY OF DETECTION WHEN THE TARGET


## PART II

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