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## Improvement in Inspection Methods for Aeroengine

 Components using Computer Linked Electro Optical
## Techniques.

## by

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## A Doctoral Thesis

## Submitted in partial fulfilment of the requirements for the award of

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## SUMMARY

An improved technique for inspecting aero engine turbine blades was developed. The technique increased the area of the blade that was examined and presented the difference information in an easily assimilated form without increasing the overall inspection time above that of present methods. To achieve this an optical contouring technique was used to measure the blade shape and this was linked to a computer to enable the dimensional information to be processed quickly.

Two holographic methods of surface contouring, two wavelength and two refractive indices, and a moiré fringe shadow technique were studied on the basis of forming part of a routine inspection system. The moire fringe method was found to be the most suitable technique and a modified version of moiré contouring was developed for use in conjunction with image scanning and computer processing.

Methods of comparing the shape of a component with that of a master were studied. To obtain sufficient accuracy the optically measured shapes were compared rather than the direct contour fringe patterns. This technique relied upon the use of a computer to process the data.

A method of generating the shape of the component from the contour fringe information was formulated together with a method of comparing the shape of the component with a master. The results of the comparison were presented in the form of the overall differences in two orthagonal directions and an angular rotation plus details of any abnormal localized differences superimposed on the overall values. The contour fringe information was read in to the computer at a rate of 1000 words per second with the aid of a slow scan television interface.

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A
a
$d_{s}$
dz
$E$
$\boldsymbol{F}, \mathbf{f}^{\mathbf{f}}$
$h_{1}$
$h_{2}$

I

J

K
k
$\ell$

M

N
$P \quad$ pressure
p period of moiré grid.

R radial distance of a point on the image from the hologram.
s. speckle brightness.
$\mathrm{T} \quad$ transmission function.
U distance of the object from the lens.
V . distance of the image from the lens.
$W_{n}(s) \quad$ probability of speckle brightness $s$.
( $x, y, z$ ) co-ordinates.
$z(x, y)$ depth of component.
$\alpha \quad$ angle of illumination of moiré grid.

## $\Delta x$

## $\Delta z$

$\theta$
rec
$\theta_{r e f}$
$\lambda$
$\rho$
$\sigma^{2}$
$\phi$
$\psi$
angle of observation of moire system.
gradient of the linear region of the Hurter Driffield curve. spread of moiré grid shadow.
contour depth interval.
angle of reconstruction beam.
angle of reference beam.
angle of illuminating beam.
angle of observation.
wavelength of light.
gas density.
phase of optical wavefront.
angular increment of spline fit.

INTRODUCTION.
The dimensional inspection of components together with the measurement of such characteristics as vibration and strain with the aid of optical techniques has greatly increased in the last decade. This is principally a result of the use of television systems and the development of the laser.

Holography made possible the measurement of complex deflections produced by static loads and the visualization of vibration mode and amplitude contour patterns for real engineering components. The potential advantages of holography for this type of deflection visualization over measuring techniques in current use prompted a study of the engineering applications of holography with particular reference to aeroengine components, Holographic methods of vibration analysis were studied by Hockley $[1,2]$ and a self-gontained unit for applying holographic vibration analysis developed. Improvements to the basic holographic technique for vibration analysis in this context have been made including a phase modulation system to extend the amplitude range of holography from 25 miorons to 500 microns, Hockley $[3,4]$.

The use of holography as a method of non-destructively testing honeycomb structures in aeroengines has been studied, Hockley [5].

The ability of a holographic system to record truly threedimensional information about the surface of a component gives rise to the possibility of comparing two similar companents to obtain the dimensional differences between them. The ability to obtain accurate comparisons between complex components is important in the aeroengine industry particularly since small differences can produce significant changes in engine performance and when the components are being operated very close to the limits of the materials. Work on holographic inspection by Archbold [6] demonstrated that holography could be used to
measure dimensional differences on small accurately made components, provided that the differences were of the order of 1 micron. To achieve this the component had to be located to an accuracy of 0.2 microns in the six degrees of freedom.

Although the resolution of this system was considerably greater than that required in the manufacture of components such as turbine blades, the fundamental ability to measure these types of differences was extremely important. Consequently a system capable of comparing components with a measuring accuracy of approximately 25 microns (1 $\times 10^{-3}$ inches) as suggested by Hockley [1] would form a significant contribution in both the manufacture and testing of aeroengine components, This is particularly relevant to turbine blades because they have a complex shape and require detailed and accurate inspection over the aerofoil surface.

The cost of manufacture of each component is high because of their complexity and the relatively small quantities of each type that are made. Subsequently component wastage is expensive. Improved inspection methods that are accurate and can be used in corrective procedures to reduce the rejection rate are always of interest to the industry.

In addition to production inspection there is an important requirement for a system capable of measuring the complex distortions of turbine blade aerofoils that have been produced by tests of the blades under engine running conditions.

A possible method of desensitizing holography would be the use of optical contouring, Hildebrand and Haines [7], to generate surface contour fringes of equal depth with respect to a given reference plane. Such a system was thought feasible on turbine blades if contours of the order of 250 microns depth ( $1 \times 10^{-2}$ inches) were formed on the blade and the dimensional differences measured to an accuracy of one tenth of the contour depth interval.

Because of the complexity of the tolerance limits and the need to develop an automatic system some form of computation would be required to manipulate the difference information. A computer system introduces the prospect of comparisons being made using computed design data rather than a physical master. This is a significant feature because the majority of turbine blades are designed by computer techniques.

The need for a fast and accurate method of dimensional inspection or analysis of components together with the potential of holography as a method of inspection prompted the research described in this thesis.

The work presented describes the investigation into the feasibility of the use of optical contouring techniques as a means of comparing two. similar components. Two holographic contouring techniques were studied, two wavelength contouring, Hildebrand and Haines [7] and a two refractive index method, Shiotake et al。 [8] together with a moiré shadow contouring technique, Meadows et al. [9]. The three methods were compared with respect to their use on a routine basis in an industrial environment. The moiré technique was found to be the most suitable. A simplified version was developed for use in conjunction with an image scanning system.

A method of comparing the contour fringe patterns of two similar components with the aid of a computer was formulated to provide concise and easily assimilated information on the dimensional differences.

In Chapter I the present techniques used for dimensional inspection are described and the fundamental approach used to develop optical contouring as a means of improving the inspection techniques is discussed.

In Chapter II the methods of generating optical contours are discussed together with an evaluation of their relative merits and the choice of the most appropriate system for adoption for this work.

In Chapter III the possible methods of comparing one component with
a master are considered. The methods of interrogating the contour pattern to obtain dimensional information within the computer and means of manipulating the dimensional information of the component to be inspected with respect to the master component shape are described in Chapter IV, the objective being to develop a method of describing the overall differences between the two components plus any abnormal localized differences that occur.

The experimental television system used to form the interface between the optical contours and computer is described in Chapter V together with some experimental results. The conclusions and recommendation with respect to this research are presented in Chapter VI.

## Chapter I. <br> Dimensional Measurement Techniques.

1.1 INTRODUCTION.

In this chapter the two principal methods of inspection of turbine blade aerofoils, profile following and point gauging are described, together with an analysis of the future requirements of aeroengine turbine blade inspection.

### 1.2 PRESENT INSPECTION TECHNIQUES.

1.2.1 Units of Measurement.

Throughout this thesis both the M.K.S. and the pound-force, inch, second system of units will be used. Although it is more appropriate to discuss optical parameters in the M.K.S. system the dimensions of the aeroengine components are still defined in inches within the Company. Hence, when referring to turbine blades and the appropriate optical contour depth used to measure their dimensions, the inch unit will be used.
1.2.2 Profile Following. (Calliperscope)

This technique consists of a self-contained machine into which the turbine blade is clamped and a pair of spring loaded pincer-like probes are drawn across the blade aerofoil in a chordwise direction. One probe is in contact with the convex surface and the other the concave surface. These probes are mechanically connected to a similar pair of slave probes that follow the movement of the probes on the blade. The outline of the slave probes are projected with a large degree of magnification ( $x \neq 00$ ) on to a soreen on which the blade outline and tolerance envelope is drawn. The position of the probes relative to the tolerance envelope are noted by the operator as the probes are drawn across the blade to form the inspection process. The blades are inspected across three positions on the aerofoil, centre of the aerofoil and towards the root and tip.

The calliperscope is simple in design and is easy to use. It is operated by non-specialist female personel. The blade type under inspection can be conveniently changed by modifying the clamping arrangement and renewing the master profile drawings on the viewing screen. It can be used to inspect more difficult shaped aerofoil sections than the point gauging system because only two probes are in contact with the blade at any one time as opposed to eighteen for the point contact gauge. In the point gauging system there are difficulties in getting the probes physically close enough together on the small radius curves of the concave surface of some turbine blades. However, the calliperscope is somewhat slower than point gauging.

The calliperscope does have a number of disadvantages. Only three sections of the aerofoil are examined. The sections are only measured with respect to the overall tolerance envelope. No measurements or observations are made to indicate whether the blade is overall thicker than the nominal shape, either twisted or bent within the envelope, or has a surface undulation or roughness effect. The whole of the decisionmaking process relies on the human operator, who is subjected to outside distractions and fatigue, resulting in lack of concentration and possible errors of judgement.

To improve the inspection technique the objective of this research was to increase the area of the blade that was inspected by covering up to 20 sections and to reduce or eliminate the decision-making responsibilities carried by the operator by the use of a computer to analyse the difference information. The overall differences can be quantised by the computer into twist and bending effects plus localised differences. Using this information it is anticipated that it will be possible to reduce the rejection rate. For example, a blade may have the correct aerofoil section but it may be bent so that it gives an out of
tolerance condition on the calliperscope. This may, however, be an acceptable condition aerodynamically and the computer can be programmed to pass this condition. The computer output can either be a go/no-go indication or present the difference information in an easily assimilated form.

### 1.2.3 Point Gauging.

The system provides a series of point measurements of the dimensions on the whole of the blade including the fir tree root and shroud. This usually involves 18 point measurements on the aerofoil. A mechanical lever type probe is used to measure the blade and a transducer is used to measure the position of the probe lever so as to calculate the actual position of the probe head. The principal technique used to form the measuring transducer was air gauging which has now been superceded by a differential transformer transducer system. The differential transformer is more compatible with electronic instrumentation.

For air gauging the needle of a small needle valve is attached to the arm of the mechanical probe and air is fed through the valve and the effective back pressure caused by the valve measured by a manometer. This back pressure is a measure of the position of the valve needle, hence probe position. There are a number of variations to this system.

The differential transformer type transducer gives a voltage output proportional to the position of the mechanical probe of the transducer. The probe forms the core of the transformer which slides within the centre of the two windings, hence altering the coupling efficiency between the primary and secondary windings. A particular r.m.s. voltage is applied to the primary from an oscillator and the output voltage from the secondary gives the position of the transducer probe. The use of the transformer type transducer increased the flexibility of the measuring system and a number of probes can be scanned in turn and an overall
go/no-go system used. Also where there are particular errors outside the tolerance band the error on any probe can be measured individually to allow more discretion to be used by the operator as to whether this component has to be rejected or not.

The point gauging system is automated to a high degree and is operated by unskilled personel. It is, however, limited to 18 point measurements per blade and when the type of blade being inspected is changed the whole of the mechanical probe assembly must be changed, at a cost of approximately £1,000.

### 1.2.4 Disadvantages.

Both of these systems take measurements only at specific points or along lines on the blade, leaving a large proportion of the surface unexamined. Hence, the blade could be out of tolerance over a significant proportion of its surface and still comply to the inspection requirements, which could affect its performance. More scans and points can be used, but this will greatly increase the complexity and time involved in the inspection process and consequently the cost. The point gauging system is also difficult to use on turbine blade aerofoils because the curvature of the aerofoil section is high and it is extremely difficult to set the probes in the correct measuring position at the correct angle. Consequently, in the majority of cases the calliperscope is used.

As with all mechanical probes, wear on the points of the measuring probes caused by routine use is a problem requiring frequent adjustment or replacement and a small change in the actual measurement during use.
1.3 FUTURE REQUIREMENTS.

### 1.3.1 Use of Computers.

Manufacturing tolerances on turbine blades are being reduced because the aerodynamic performance of the blades becomes more critical as the engine efficiency increases. As an example of the changing trend
in manufacturing tolerances, the overall tolerance envelope for the majority of turbine blades has been reduced from 0.010" to 0.007". Ripples or undulations in the surface finish across the direction of the airflow are important and limits are imposed on the amplitude and frequency of this phenomenon. These factors affect the boundary layer conditions, which are becoming increasingly important as surface film cooling is being used to achieve higher gas working temperatures. Consequently, more information is required about the shapes and manufacturing tolerances of the blades being produced. This requires more point measurements, making the mechanical techniques impractical. Hence, other forms of measuring transducers must be used that are either optical or purely electronic. To cope with the increased information in a reasonable time some form of computerized analysis is required or the time penalty per blade using present techniques will be too great.

The point gauging system takes approximately 15 seconds to measure one blade giving 18 points of measurement but the number of points cannot be greatly increased above this value. The calliperscope requires approximately 45 seconds per blade for three sections. If this is increased to 10 sections this will take approximately 4 minutes allowing some time for the operator to perform some general analysis of the data. The time involved plus the burden placed on the operator to handle a large amount of visual data will make this system impractical when compared to an estimated 40 seconds per blade for an electric-optical computer linked system that includes automatic data manipulation.

The use of computers in dimensional inspection is being applied and studied within the manufacturing industries both for basic inspection and control and monitoring of machine tool performance as reviewed by Carlisle and Bugden [10]. Examples of these include tube thickness gauging, lathe control, rod diameter measurements, and precision profile
measurements. The use of computers to store and manipulate dimensional data has been demonstrated by Watkins [11] and Aveyard [12]. In the system described by Watkins [11] five measurements were taken on a small mass-produced component and processed by the computer to give a go/no-go situation. The computer was also used to store and analyse the general trends of the errors to improve manufacture procedure. Aveyard [12] described a basic computer linked system which was used on turbine blades. This used a mechanical probe/transducer system to measure 30 points on the aerofoil of the turbine blade, 15 on the concave surface and 15 on the convex, and the errors compared to the master component calibration was given in a paper printout form. The systems described above still only measure a small number of points and compare this to specific information without processing the differences to determine the overall difference, such as twisting or bending. For the quantity of information provided these techniques are expensive, £10,000 for 30 points, and still require the mechanical probe retooling and setting if the component is changed, cost $£ 1,000$.

The Ferranti-Cordax 3000 inspection machine [13] represents a further development in so far as it is commercially available and takes 112 point measurements. This still uses a mechanical transducer to take the measurements and only provides point by point differences from the norm. It is principally used on large objects such as shadow masks for colour cathode-ray tubes and the overall inspection cycle takes 15 minutes.

The present computer linked inspection machines have a number of limitations with their application in industry. They are expensive, slow, and only perform a small amount of data manipulation. Thus they are not adequate for turbine blade inspection. Blade inspection requires a technique which is capable of examining the blade aerofoils
at a rate greater than 1 per minute. The surface must be examined in detail with at least 50 measuring points per scan for at least 10 scans with an overall measuring accuracy of $0.001^{\prime \prime}$.

Small computers have a data input rate of approximately 1000 words per second, which means that in the present inspection machines the computer is being under-utilized since the data input rate is much lower at approximately 10 words per second. This is limited by the rate at which the mechanical measuring probe can be manipulated.

To increase the data input rate into the computer it is proposed to use an optical contouring system with a slow scan television interface between the optical system and the computer. This would be capable of a data input rate of 1000 words per second into the computer. With the increased quantity of data and data input rate the computer would be used more effectively and it would be possible to inspect the blade in less than 1 minute and present the dimensional differences in an easily assimilated form. In such a device the component could be changed. without.retooling the measuring system and the computer used to serve several measuring units.

A fundamental difference and advantage of any optical/computer system of inspection over the present methods is the possibility of comparing a component to computer generated design information rather than a physical master. Turbine blades are now designed by computer and this data is used to make drawings from which a master blade is made. The manufacture of these blades is difficult as they have complex profiles and are to a large extent hand-made to an accuracy of $\pm .002^{\prime \prime}$. A relatively large number of them have to be made as the blade designs are frequently changed in the development of an aeroengine. Consequently the direct inspection from computer data would eliminate the need for a master blade to set up the inspection machines.

### 1.3.2 Dimensional Analysis of Development Components.

As well as the need for optical inspection techniques in the inspection of turbine blades during manufacture there is a requirement in the dimensional measurement of blades that are under development tests. As the result of the need to continually develop the aeroengine to improve the overall efficiency, research into improving turbine systems has to be made. Subsequently turbine blades are being subjected to higher temperature conditions and higher overall stresses to increase engine thrust and efficiency. During this work there is a need to measure the distortions and resultant damage produced by these temperature and stress effects. At present the only method of obtaining this information is by the standard metrology techniques using micrometer gauges. This is laborious, slow and costly if the measurement is required over a large number of points. Because of these difficulties this type of measurement is seldom used, so that in a number of examples very little actual distortion information is used within this research. Consequently there is a requirement for a form of comparison of a component before and after it has been subjected to high thermal and mechanical stress conditions to measure the resultant distortion. The distortions are of the order of between 25 and 500 microns ( 0.001 to $0.02^{\prime \prime}$ ) and the detailed surface finish of the component changes during the test.

Direct holographic interferometry is not feasible in this case because the overall distortions of 25 to 500 microns are too large and result in higher spatial frequency fringes than can be resolved by the observing optics.

This specific requirement and other similar measuring problems increase the need for a method for obtaining a detailed measurement of the three-dimensional shape of a component.

### 1.3.3 Economic Considerations.

The basic cost of an electro-optical contouring system with a form of computerized data handling would be more expensive than present systems but would be more efficient. In the long term the overall cost of a computer linked system would be the same or less than the present calliperscope. The computer would be used on a time sharing basis to run several optical units. Each unit is expected to be faster than the equivalent calliperscope or point gauging unit. To change the type of blade to be examined would require reprogramming the computer to provide the new shape for comparison, whereas with a system such as the point gauging the whole of the mechanical probe unit has to be substantially modified or renewed at the cost of the order of $£ 1,000$. In addition, the cost of small computers is decreasing as the state of the technology advances and so makes their use more attractive.

For the examination of components subjected to thermal stressing, the actual ability to measure distortions for comparison with the theoretical predictions represents a substantial saving in time and an increase in practical knowledge of the behaviour of the component.

Chapter II.
Methods of Contour Generation.

### 2.1 INTRODUCTION.

In this chapter the results of the investigation into the holographic and moire fringe methods of generating surface contours are described. Two holographic methods of surface contouring were studied, two wavelength and, two refractive index. The former was first developed by Hildebrand and Haines [7] and the latter by Shiotake [8]. The main emphasis was placed on the examination of the feasibility of the use of these techniques as part of a routine inspection system. To enable the dimensional data to be automatically processed the contour information should be in a form where it could be accurately fed into a computer by the use of an electro-optical interface.

Holographic contouring was found not to be ideally suited for this purpose and a moiré shadow contouring technique as described by Meadows [G] was examined. A modified version of moiré contouring was developed for use in conjunction with image scanning and computer processing.

### 2.2 SPECIFICATION OF REOUIREMENTS FOR OPTICAL BLADE INSPECTION.

The requirements of the dimensional analysis system for turbine blades imposed a number of limitations and requirements upon the method of generating depth contours.
a) The size of the turbine blades to be inspected vary from an aerofoil length of 2 inches up to 7 inches with a depth variation across the blade width of between 0.2 and 1.0 inches.
b) The contour depth interval should be comparatively large, greater than or equal to $0.010^{\prime \prime}$, so as to keep the number of contours across the object to a reasonable number.
c) : Required measuring accuracy is $\pm .001^{\prime \prime}$ indicating that the component shape or differences must be measured to an accuracy of
$1 / 10$ contour fringe interval or greater.
d) The contour technique must provide the information as quickly as possible with a minimum of adjustment or photographic processing.
e)

The contoured images must be of sufficient intensity to enable a television camera to be used as an optical to electronic information interface.
f) The contour fringe intensity should have a sufficiently low noise level to enable the computer linked measuring system to comply with condition (c).
g) The comparison information must be clearly presented in a form easily assimilated by the operator, either directly as a simple optical fringe form, on a television monitor to minimize eye strain, or as a form of printout, either paper or C.R.T., denoting the general differences, bending, twisting plus details of localized differences.
h) The overall system should be simple and capable of operation on a routine basis by non-specialist personnel.

### 2.3 PRINCIPLES OF HOLOGRAPHY.

The detailed analysis of the formation of holograms, the coherence requirements of the illuminating source and the factors affecting the quality and resolution of the hologram have been previously documented by the author in the thesis [1] and will not be discussed in this presentation.

When a hologram plate is accurately repositioned back into the original system used to form the hologram and illuminated with the original coherent reference beam, the wavefront diffracted by the hologram is identical to the wavefront scattered from the object's surface. Since the two waves are indistinguishable they may be interchanged and hence it is possible to compare the reconstructed wavefront with the scattered wave-
front originating from the object at some later time. Any difference between the object resulting, for example, from an applied stress and its original reconstruction is shown as interference fringes similar to those produced by standard interferometry. This is the principle upon which holographic interferometry is based, Haines and Hildebrand [14].

In a similar manner two holograms may be recorded on the same photographic plate representing the object at different instances in time and the resultant reconstruction will show any displacement of the object that has occurred between the two exposures in terms of fringes. This is referred to as double exposure holography.
2.4 TWO TAVELENGTH HOLOGRAPHIC CONTOURING.

Holographic interferometry may also be used to generate depth contours at much larger intervals than is possible by previous methods. There are three techniques, double source, double wavelength and double refractive index. The double source technique, as shown in fig. 1, was basically a fringe projection technique. It was not able to accommodate a convoluted surface, as shown in fig. $1 b$, and consequently was unsuitable for turbine blade analysis. The other two were found to be suitable for turbine blade inspection and were considered in detail. The two wavelength contouring technique is based upon the interference between two images of the same object formed holographically by using two different wavelengths of laser light. The contour depth is a function of the two wavelengths used according to the relationship

$$
\begin{equation*}
\frac{\lambda_{1} \cdot \lambda_{2}}{2\left|\lambda_{1}-\lambda_{2}\right|} \tag{1}
\end{equation*}
$$

and is thus precisely defined from a knowledge of $\lambda_{1}$ and $\lambda_{2}$. The system is limited to a laser that will operate at different wavelengths and with sufficient coherence for holography, e.g. the argon ion, or krypton
lasers and some of the latest dye lasers that are optically pumped by an argon ion laser. With the continual development of dye lasers it is expected that it will be possible to generate contours of around 250 micron depth interval.

### 2.4.1 Contour Fringe Formation.

The two wavelength contouring technique was first discussed by Hildebrand and Haines [7] and further developed by Zelenka and Varner [15]. The optical telescope system used to generate the contour fringes is shown in fig. 2. An image of the object with unit magnification is relayed to the hologram plate, $H$, by a telescope. ( $L_{1}$ and $L_{2}$ ) The object is illuminated by a plane wave propagated in the same direction as the axis of the telescope by means of the beamsplitter B.S.1. Zelenka considered the effects of two wavelengths of light upon the magnification and position of the reconstructed image in space. A holographic image of an object taken at wavelength $\lambda_{1}$ and then reconstructed at wavelength $\lambda_{2}$ will change its position in space along the axis of the telescope and also change its lateral magnification. If the object is also illuminated at wavelength $\lambda_{2}$ there will be a difference in position of the holographically reconstructed image and the telescope image. The difference is shown as interference fringes.

For a small portion of the hologram formed at wavelength $\lambda_{1}$ using a plane reference wave and then reconstructed by a plane reference wave at wavelength $\lambda_{2}$ the radial distance from a point on the hologram to a point on the recorded image $R_{1}$ is imaged at a distance $R_{1}^{\prime}$ when reconstructed. The relation between these radii is a function of the two wavelengths given by

$$
\begin{equation*}
R_{1}^{\prime}=\left(\lambda_{1} / \lambda_{2}\right) R_{1} \tag{2}
\end{equation*}
$$

Because the telescope is used to form an image of the object on the
hologram plate the radial distances are kept to a minimum.
The hologram plate acts as a complex diffraction grating in two dimensions to reconstruct the image when illuminated with the reference beam and so obeys the normal diffraction grating equations. Thus if the hologram is recorded in one wavelength and illuminated at a different wavelength with the reference beam: still at the same angle, the image is slightly displaced by the change in the angle of the wavefront diffracted by the hologram. This effect changes both the lateral magnification and position of the image. To overcome this phenomenon and keep the two images exactly superimposed the reference beam angle must comply with the equation.

$$
\begin{equation*}
\sin \theta_{\mathrm{rec}}=\left(\lambda_{2} / \lambda_{1}\right) \sin \theta_{\mathrm{ref}} \tag{3}
\end{equation*}
$$

where $\theta_{\text {ref }}$ and $\theta_{\text {rec }}$ are the angles of the reference and reconstruction beams respectively as shown in fig. $3 a$.

To obtain the contour fringes the apparent displacement or difference along the optical axis of the telescope must be less than its depth of field. The depth of field for a single lens, fig. 3b, is given by Born and Wolf [16].

$$
\begin{equation*}
\mathrm{d} z=4 \lambda\left(\frac{\mathrm{~V}}{\mathrm{D}}\right)^{2} \tag{4}
\end{equation*}
$$

$\lambda=$ wavelength of light used.
$V=$ distance of image from the lens.
$D=$ diameter of the lens aperture used.

Thus for the telescope the depth of field on the object is given by

$$
\begin{equation*}
\mathrm{d} z=4 \cdot \lambda \cdot\left(\frac{\mathrm{~F}}{\mathrm{D}}\right)^{2} \tag{5}
\end{equation*}
$$

$F=$ focal length of objective lens, which equals that of the eye piece for unit magnification.

Using this oriterion Zalenka showed that provided that a small aperture was used (F.No. $\geqslant 40$ ) contours of planes perpendicular to the optical axis were formed with a depth interval.

$$
\begin{equation*}
\Delta z=\frac{\lambda_{1}-\lambda_{2}}{2\left|\lambda_{1}-\lambda_{2}\right|} \tag{6}
\end{equation*}
$$

### 2.4.2 Limitations.

The use of the unit magnification telescope to relay the image of the object on to the hologram formed a fundamental limitation on the system in so far as the depth of field would be small. The resolution of a telescope with unit magnification is defined in terms of the minimum separation between two point sources in the object plane that can be resolved by the telescope and is given by Born and Wolf [16].

$$
\begin{equation*}
\text { Minimum resolvable separation }=1.22 \lambda(F / D) \tag{7}
\end{equation*}
$$

The telescope resolution is also defined in terms of the reciprocal of the minimum resolvable separation and denoted as the resolution limit. This is quoted in terms of cycles/mm.

Resolution limit $=\frac{0.82}{\lambda}\left(\frac{D}{F}\right)$

A phenomenon common to coherent optical systems and known as a speckle pattern affects the subjective appearance of any object illuminated with coherent light. This gives the object an appearance of being covered by a series of speckles rather than the uniform appearance of an object illuminated with incoherent light. The speckle effect occurs because interference takes place between a large number of
randomly phased disturbances generated by the light being scattered off the optically rough surface of the object, as discussed by Burch [17, 18]. The size of these speckles is defined as equal to that of the minimum resolvable separation of the optical system that is used to observe the speckles.

$$
\begin{equation*}
\text { Speckle size }=1.22 \lambda(F / D) \tag{9}
\end{equation*}
$$

The variation of the apparent local intensity distribution across the image that was produced by the speckles was examined by Burch [18]. This is described in terms of the probability of the relative brightness of the speckle being between the values $s$ and $s+d s, W(s) d s$. The variation is also an indication of the degree of contrast or granularity of the speckle pattern and is given by

$$
\begin{equation*}
\text { Variance }=\sigma^{2}=\int_{0} s^{2} W(s) d s-1 \tag{10}
\end{equation*}
$$

Burch [17] discussed the speckle brightness probability function for a finite number of $n$ equal disturbances which was represented in terms of infinite integrals of zero-order Bessel functions.

$$
\begin{equation*}
W_{n}(s)=n / 2 \int_{0}^{\infty} z J_{0}\left(z \sqrt{x_{0} n}\right) \cdot\left[J_{0}(z)\right]^{n} d z \tag{11}
\end{equation*}
$$

This function varies with the number $n$ disturbances. When the aperture $D$ of the observing optics is reduced the number of disturbances n. that can be distinguished is also reduced. Burch [17] showed that at the resolution limit of the optics where $n=2$ the probability function is principally at zero or twice the average brightness of the image. Hence from equation 11. good contrast speckles are formed whose size is determined by equation 9.

The depth of focus of the telescope required to observe the majority of turbine blades varied from 5 mm to 25 mm . The corresponding aperture settings on the telescope were F. 50 and F.115. These produced minimum resolvable separations of 0.03 mm and 0.06 mm . Under these F. Number conditions the speckle diameters of 0.03 to 0.06 mm produced an obtrusive speckle pattern on the contoured image and degraded the overall quality of the image.

The object size was limited to less than that of the effective diameter of the objective lenses used in the telescope. The amount of light from the object transmitted through the telescope to the hologram was limited by the small aperture, which increased the exposure time of the hologram.

The requirement that the reference beam angle should be rotated through an angle when moving from the recording to reconstruction mode, to keep the two images exactly superimposed, involved an adjustment to the reference beam which must be made by the operator. The angle of rotation was of the order of 3 mrad for a reference beam angle of $23^{\circ}$ using wavelengths of 476.5 nm and 472.7 nm . Varner [19] used a diffraction grating set perpendicular to the optical axes of the telescope and so in a plane parallel to that of the hologram. This was illuminated by the collimated reference beam at normal incidence and the overall position of the grating arranged so that the light from one of the diffraction orders was projected on to the centre of the hologram.

### 2.4.3 Experimental Resuits.

A two wavelength laser with the correct wavelength separation to produce 250 micron ( $0.01^{\prime \prime}$ ) contours was not available. The tests were carried out using a Coherent Radiation Ltd 2 W argon laser with power outputs of 150 mW each at wavelengths 501.7 nm and 496.5 nm to give 15 micron ( $\div .5 \times 10^{-3}$ n ) contour. depth interval and 250 mW at 476.5 nm
and 30 mW at 472.7 nm for 30 micron contours.
A telecentric optical system was used as shown in fig. 2 with an adjustable mirror used to change the reference beam incident angle on the hologram for the two wavelengths. A coin was used as an initial test object and this represented an irregular and complex shaped surface with a depth variation of approximately $0.01^{\prime \prime}$ so as to produce a reasonable number of contour fringes. A turbine or compressor blade was not initially used at these contour intervals, as the $0.30^{\prime \prime}$ change of shape represented 300 to 600 fringes which were beyond the resolution capabilities of the optical system.

Initially live fringe holographic interferometry was used to observe the contour fringes. This involved taking a hologram with one wavelength, developing the hologram, relocating it in the original position and then illuminating the interferometer with the second wavelength. The telescope and holographic images were observed as the reference beam angle was changed. This produced little success as only very low contrast fringes were obtained. This was primarily caused by the inherently low diffraction efficiency of the hologram (5\%). Consequently the reconstructed image was extremely weak even though the reference beam intensity was increased to compensate. Photography of the resultant contour images was just possible.

Some spurious fringes were formed in the holographic interferometer by emulsion shrinkage on developing and drying the hologram. Additional fringes were produced by misalignment when relocating the hologram even though a kinematic design of plate holder was used, as described by Hockley [1]. To overcome this an in situ processing liquid gate hologram plate holder similar to the type described by Biedermann [20] was made and used, which improved the overall quality of the contour fringes.

The required accurate adjustment of the reference beam between recording the hologram at wavelength $\lambda_{1}$ and reconstructing the processed hologram at $\lambda_{2}$ greatly detracted from the usefulness of the system as a routine method. To overcome this a diffraction system in the reference beam was used as discussed in section 2.4.2. The experimental system used is shown in fig. 2. The grating was lined at $30,000 \mathrm{c} / \mathrm{inch}$ with a first order diffraction angle at $36^{\circ} 12^{\prime}$ at 501.7 nm with a change of angle by $26^{\prime}$ to $35^{\circ} 54^{\prime}$ at 496.5 nm . To obtain good quality contour fringes using the single exposure holographic interferometer extreme care was required to set up the diffraction grating at the correct attitude to within a fraction of a minute of arc. This was achieved by using a rigorous setting up procedure.

The double exposure method of generating the contours was also used in which the hologram was exposed in two stages illuminated at different wavelengths and reconstructed with one of then. The contour fringes were frozen on to the reconstructed image. This method eliminated the image intensity mismatch and the tests proved that it was possible to obtain repeatable and reasonably good fringes although the same rigorous setting up procedure was needed. Contours at both 30 and 15 microns were obtained. Fig. 4a shows a photograph of the contoured image of the coin, contoured at 30 micron intervals. The coin was tilted slightly with respect to the optical axis of the contouring system as shown by the horizontal fringes. Fig. 4 b shows a contoured reconstruction of a test object with four angular slopes at $1^{\circ}, 2^{\circ}, 3^{\circ}$ and $5^{\circ}$. These slopes were clearly defined by the contours.

Part of a small compressor blade was contoured but the fringes were only formed over a small portion of its chord due to the curvature producing a large number of 30 micron contours, and the observable depth over which the contours occurred was approximately 1.2 mm .

The overall system was generally inefficient in the use of the available light, due to the use of the beamsplitter in the telescope for normal incident illumination and the F.No. 50 optics needed for blades, but this was a factor in both holographic systems. It was not possible to use the beam transmitted through the beamsplitter as the reference beam, due to the effects of the fringes produced by internal reflections within the mirror even though an anti-reflection coating was used on the second surface.

Hence summarizing, the system advantages were:-
a) Any shaped object could be considered.
b) Contour depth intervals were accurately repeatable.
c) Simple to operate although complex to set up.

Disadvantages:-
a) Contour depth not variable; 30 microns was the maximum depth obtainable using an argon ion laser, although tunable dye lasers are now becoming available.
b) The object size must be less than the usable diameter of the telescope objective lens.

### 2.5 TWO REFRACTIVE INDEX CONTOURING.

Two wavelength contouring was found to be feasible but the need for a two wavelength laser would increase the cost above that of standard holographic systems. The two refractive index method of contouring, however, could be operated using normal single wavelength lasers, thus reducing the overall cost of the system.

This method principally involves the double exposure method of taking holograms as discussed in section 2.3 and the contour fringes are formed by the interference between two images formed in different: refractive index media. The object is placed in an enclosed tank with a window through which it can be observed. A standard holographic system
as shown in fig. 5a can be used to observe the object. The first exposure of the double exposed hologram is formed with the object immersed in a medium of refractive index $n_{1}$. The tank is then replenished with a second medium of refractive index $n_{2}$ for the second exposure of the hologram. The refractive index media can either be a gas or a liquid. The effect of the different refractive index media within the immersion tank is to change the effective wavelength of the light. The wavelength of light $\lambda_{1}$ in any medium with refractive index $n_{1}$ is related to its absolute wavelength in vacuo, $\lambda$, by the function

Similarly

$$
\begin{equation*}
\lambda_{1}=\lambda / n_{1} \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
\lambda_{2}=\lambda / n_{2} \tag{13}
\end{equation*}
$$

The two images reconstructed by the processed hologram have consequently been formed at different wavelengths and the images interfere to produce depth contours as described by equation 1 in section 2.4, where the wavelength difference is defined by the refractive index change.

The accuracy and repeatability of the contour depth interval is dependent upon the accuracy with which the refractive index is changed. The basic setting up procedure is less complex than that of the two wavelength technique.

### 2.5.1 Two Refractive Index Contour Fringe Formation.

This contouring technique was first introduced by Shiotake [8] and later discussed by Zelenka and Varner [21]. The contour hologram is recorded as described in section 2.5. On reconstruction of the hologram the two images produced in the tank, using different refractive indices, will be in different positions relative to the tank's glass window, and will interfere to produce difference contours as in the two wavelength contouring process. The optical path length differences between the
resultant images are only affected by the two refractive indices within the tank $n_{1}$ and $n_{2}$. When the holographic reconstruction is imaged onto the plane of a viewing system the complex amplitudes of the two images will be of the form

$$
\begin{equation*}
A_{I m}=A_{1} \exp \left[i \phi_{1}\right]+A_{2} \exp \left[i \phi_{2}\right] \tag{14}
\end{equation*}
$$

where $A_{1}$ and $A_{2}$ are the amplitudes directly related to the brightness of the two images. These are usually made equal by exposure control. The phase terms are dependent upon the optical path length in each optical medium. For an immersion tank system as shown in fig. bb the phase terms $\phi_{1}$ and $\phi_{2}$ for a point ( $x, y$ ) on the object a distance $z(x, y)$ from the inner surface of the window.

$$
\begin{align*}
& \phi_{1}=k_{1} z(x, y)\left(\cos \theta_{11}+\cos \theta_{12}\right)  \tag{15}\\
& \phi_{2}=k_{2} z(x, y)\left(\cos \theta_{21}+\cos \theta_{22}\right) \tag{16}
\end{align*}
$$

$k_{1}$ and $k_{2}=$ the wavenumbers in the refractive index media $n_{1}, n_{2}$. $\theta_{11}, \theta_{21}, \theta_{12}$ and $\theta_{22}$ are as defined in fig. 5b.

The incident and observing angles are related by Snells Law.

and $\frac{\sin \theta_{12}}{n_{1}}=\frac{\sin \theta_{22}}{n_{2}}$

The resultant intensity of the image is

$$
\begin{equation*}
I=\left(A_{I m}\right)^{2}=2 A^{2}\left[1+\cos \left(\phi_{1}-\phi_{2}\right)\right] \tag{19}
\end{equation*}
$$

In practice the angles of illumination and observation are made to be equal or close to zero hence equation 19 becomes

$$
I=2 A^{2}\left[1+\cos \left(2\left(k_{1}-k_{2}\right) z(x, y)\right)\right]
$$

Substituting for the wavenumber $k_{1}$ and $k_{2}$

$$
I=2 A^{2}\left[1+\cos \left(\frac{4 \pi}{\lambda}\left(n_{1}-n_{2}\right) z(x, y)\right)\right]
$$

This function represents plane contours parallel to the inner surface of the window with an interval

$$
\begin{equation*}
\Delta z=\frac{\lambda}{2\left|n_{1}-n_{2}\right|} \tag{20}
\end{equation*}
$$

If this is considered in terms of effective wavelength with the two refractive media

$$
\lambda_{1}=\lambda / n_{1}
$$

and

$$
\lambda_{2}=\lambda / n_{2}
$$

Then equation 20 becomes

$$
\begin{equation*}
\Delta z=\frac{\lambda_{1} \cdot \lambda_{2}}{2\left|\lambda_{1}-\lambda_{2}\right|} \tag{21}
\end{equation*}
$$

which is identical to that of the two wavelength contouring system. To obtain plane contours the incident and observing angles are kept equal to zero and to achieve this a concentric telescope holographic system is used. In this case the hologram is placed in the Fourier transform plane of lens $L_{1}$, where the telescope aperture is located. At the Fourier plane the optical information is distributed in the
form of a spatial frequency spectrum with zero frequency on the optical axis of the telescope and the spatial frequency increasing radially from the axis. Under these conditions the maximum quantity of useful information is transmitted through the optical system on to the hologram for a given aperture diameter. This leaves the image plane free for normal observation and photography.

### 2.5.2 Refractive Index Media.

The media used to change the refractive index can be a ges or a liquid. Since the technique is for use on turbine blades on a routine system a liquid which will wet the blades and get contaminated by dirt from the blade is not suitable. Hence a gas with a high refractive index must be used. Freon 12 or Arcton 12 (Dichlorodifluoromethane $\mathrm{CCl}_{2} \mathrm{~F}_{2}$ ) has suitable properties and is reasonably inexpensive. The refractive index at $20^{\circ} \mathrm{C}$. and a pressure of 760 millimetres of mercury is 1.001055 at the mercury green line 546.2 nm. , Horvath [22]. This is related to the density of the gas by the Gladstone-Dale equation, Partington [23].

$$
\begin{equation*}
(n-1)=\rho_{0} k \tag{22}
\end{equation*}
$$

$n=$ the refractive index
$\rho=$ gas density
$k=a$ constant.

For the change of refractive index with gas pressure the ideal gas equation gives the change of density with pressure at constant temperature as

$$
\begin{equation*}
\frac{\rho_{2}}{\rho_{1}}=\frac{P_{2}}{\overline{P_{1}}} \tag{23}
\end{equation*}
$$

$\rho_{1}, \rho_{2}=$ the densities at pressures $P_{1}$ and $P_{2}$.

In a practical system care must be taken to keep the general surroundings at a constant temperature and ensure that the temperature within the pressure chamber remains constant when being pressurized to avoid and change in the pressures caused by temperature variation. This will impose some environmental control on the overall system and impose a maximum rate at which the chamber can be pressurized.

The change in refractive index with respect to increase in pressure is given by the equation

$$
\begin{equation*}
n_{2}-n_{1}=\left(n_{1}-1\right)\left(\frac{P_{2}}{P_{1}}-1\right) \tag{24}
\end{equation*}
$$

Therefore the effective contour depth is

$$
\begin{equation*}
\Delta z=\frac{\lambda}{2}\left[\left(n_{1}-1\right)\left(\frac{P_{2}}{P_{1}}-1\right)\right]^{-1} \tag{25}
\end{equation*}
$$

Hence for an increase in pressure from one to two atmospheres the contour depth at 514.5 nm is $0.24 \mathrm{~mm}\left(0.0094^{\prime \prime}\right)$. So the pressure increase from atmospheric pressure needed to produce 0.25 mm contour is just less than one atmosphere.

Fig. 6 shows a graph of the pressure increase required to produce various contour depth intervals.

Similarly the refractive index change from vacuum to Freon 12 at one atmosphere produces a contour depth interval of 0.24 mm .

### 2.5.3 System Limitations.

As a telecentric hologram recording system was used to obtain plane contours the limitation on the available depth of field, resolution and object size as discussed for the two wavelength technique, section 2.4 .2 , were applicable to this system. However, the image size limitation could be relaxed if a single observing lens system was used and the angle of
observation increased from near zero provided that a correction for the change in contour depth interval was applied as the angle was increased. 2.5.4 Experimental Study.

The telecentric hologram system was used as shown in fig. 2 and discussed by Zelenka and Varner [21]. The blade was placed in an enclosed chamber with a 9 inch internal diameter and a $10^{\prime \prime}$ diameter $\times 0.75^{\prime \prime}$ thick armour plate glass window. A strong chamber was used to allow air to be pressurized up to four atmospheres to produce a refractive index change of $1.013 \times 10^{-3}$ for $0.01^{\prime \prime}$ depth interval contours. Freon $12\left(\mathrm{CCl}_{2} \mathrm{~F}_{2}\right)$ was the gas used rather than air as this had a significantly higher refractive index change with pressure. Two principal methods of using the freon to change the refractive index were used. Firstly the chamber was evacuated and the refractive index difference between vacuo and the freon at just above atmospheric pressure was used to produce the $0.01^{\prime \prime}$ contours. Secondly without the need for a vacuum pump the chamber was flushed with freon for a short period to expel the air and the refractive index difference between atmospheric pressure and at a pressure of 15 psig used to give $0.01^{\prime \prime}$ contours. This was the simpler and faster of the two methods for a routine system and was used for most of this work. Fig. 6 shows a graph of the required pressure difference above atmospheric required to produce various depth contours.

The telescope lenses used in this system were the same as used for the double wavelength system to obtain a direct comparison between the techniques. The chamber was angled slightly with respect to the optical axis of the telescope to avoid the specular reflection from the chamber window. This did not affect the contours as they were referenced with respect to the chamber window but did slightly decrease the effective depth of field of the telescope due to the skew of blade. Both the
holographic interferometer and double exposure techniques were used with excellent results with various depth contours, an example of which is. shown in fig. 7.

An aperture setting of F. No. 40 and above was required to produce contour fringes over most of the turbine blade. At these apertures the limited resolution and laser speckle effect gave the contour fringes a definite granular appearance. The contours were observable but the accuracy with which the exact location of the maximum and minimum of the fringes was greatly reduced. Fig. 8a shows the appearance of $0.010^{\prime \prime}$ contour fringes of the concave surface of a turbine blade (1:1 magnification) with an aperture of F.No. 40. The fringes were visible only over part of the surface and towards the trailing edge the spatial frequency of the fringes were getting towards the limit of the optics due to the speckle effect. Decreasing the aperture to F.No. 80, fig. 8b, showed the increasing prominence of the speckle effect and that the fringes towards the trailing edge could not be resolved. A microdensitometer scan across the negative of fig. 7., along the blade chord, fig. 9, illustrated the effects of the speckie noise. It was extremely difficult to locate the positions of the maximum and minimum intensity points of the fringes. The spurious peaks and troughs produced by the speckles would make automatic reading of such information difficult.

The high spatial frequency of the contours towards the trailing edges of the blade in fig. 8 and the limiting effect of the optics showed that contour depths of greater than $0.01^{\prime \prime}$ depth must be used to enable the whole surface to be analysed. Hence to retain a $\pm 0.001^{\prime \prime}$ dimensional measuring accuracy the contour positions must be known more accurately.

It was found difficult to obtain good contrast contours on the normal finish of a turbine blade because of the specular reflection effect of the surface.

The overall system was simple to operate but the need to change pressures between exposures and the time taken to load and remove the blade from the chamber would be extremely inconvenient for a routine system.

## Advantages.

a) Simple optical system.
b) Any contour depth could be achieved.

## Disadvantages.

a) Required a chamber to enclose the object.
b) Access to the component was limited.
c) Size of component was limited by the diameter of the telescope objective.
d) Contour depth repeatability depended on the accuracy of the gas pressure adjustment.

### 2.6 MOIRE FRINGE CONTOURS.

The holographic contouring techniques produced good contoured images of turbine blades but for any electro-optical measuring system had a distinct disadvantage created by the speckle pattern moire superimposed on the contour fringe intensity function as illustrated in fig. 9. This made the accurate location of the contour fringe peaks and troughs by an intensity measuring device difficult and the measurement between these values to better than a $20 \%$ accuracy impossible. This raised serious doubts as to the suitability of holographic contouring for the envisaged inspection system as an electro-optical measuring device must be used to link the optical system to a computer to cope with the increased number of measuring points being used, as discussed in chapter $I$.

To overcome this problem and to simplify the overall concept of the contour generation system the alternative moire fringe contouring technique was examined. This method of contour generation had been
known for a number of years, Theocaris [24] and was re-introduced in the literature in 1970 as a measuring technique with the evolution of contour holography and its applica'ions by Meadows [9], Takosaki. [25] and All.an [26]. The first application of moire techniques to any form of component inspection was by Burch, in 1968 [27] who used a combined holographic and moiré system to form contour fringes on a master component and compared the production component with the contoured holographic image of the master. This was a complex system and suffered greatly from the problems of the low diffraction efficiencies of holograms and was not developed into a practical system.

Moiré contouring had a number of basic advantages over holography in so far as it was a real time system, and did not require a photographic process before the contour information was available. Nor was a laser required, as a white light source would be used. However, the minimum contour depth interval obtained would be limited by diffraction effects. 2.6.1 Moiré Contour Fringe Formation.

The theoretical analysis of moiré fringe systems for strain measurement has been well documented by Theocaris [24] and others. The formation of moiré contour fringes has been recently described by Meadows [9]. However, the basic formation of moire fringe contours are discussed here in detail in order to introduce the concept of the fringe projection/image scan method of generating shape information.

The moiré contouring technique operates by comparing two grid patterns. A particular moiré technique is used where a shadow of a moiré, grid is projected on to the surface of the object, fig. 10a, and its image is observed through the same moire grid. In fig. 10a the grid is illuminated by a collimated beam of incoherent light, making an angle $\alpha$ with the normal to the grid. The shadow of this is viewed through the grid at an angle $\beta$ with the normal to the grid. The light from the source is modulated by the grid to produce the shadow lines on the object.

Let the grid have a sine wave modulation function so that the light transmitted through the grid is modulated by the function.

$$
\begin{equation*}
T(x)=\frac{1}{2}(1+\ln 2 \pi x / p) \tag{0.0}
\end{equation*}
$$

$x=$ the position of any point across the grid
$p=$ the period of the grid.
For a point ( $x, y, z$ ) on the object, fig. 10a, the light illuminating it is from the point ( $x-z \tan \alpha, y, 0$ ) on the grid. Assuming that the object's surface is near to a pure Lambertian diffuse surface where the light scattered from all parts of the surface is scattered by equal amounts in all directions, i.e, the amount of the source I scattered in any direction is a constant $K$, the light scattered from the surface is

$$
\begin{equation*}
I_{0}(x, y, z)=\frac{I_{\cdot K}}{2}[1+\sin [(2 \pi / p)(x-z \tan \alpha)]] \tag{27}
\end{equation*}
$$

This function is demodulated by the action of viewing the function through the original grid.

In this case the intensity function passes back through the grid at the point $(x+z \tan \beta, y, 0)$ fig. 10a, and hence is modified by the function

$$
T\left(x_{1}+z \tan \alpha, y, 0\right)=[1+\sin [(2 \pi / p)(x+z \tan \beta)]] \ldots(28)
$$

The resultant intensity reaching the observer is

$$
\begin{align*}
I_{I}(x y z)= & \frac{I K}{2}\left[1+\sin \left(\frac{2 \pi}{p}\right)(x-z \tan \alpha)+\sin \left(\frac{2 \pi}{p}\right)(x+z \tan \alpha)\right. \\
& -\frac{1}{2} \cos \left(\frac{2 \pi}{p}\right)(2 x+z(\tan \beta-\tan \alpha)) \\
& +\frac{1}{2} \cos \left(\frac{2 \pi z}{p}\right)(\tan \alpha+\tan \beta) \quad \ldots \ldots(29) \tag{29}
\end{align*}
$$

From this equation the only term that is solely dependent upon the
depth $z$ is the last term.

$$
\frac{I K}{4} \cos \left(\frac{2 \pi z}{p}\right)(\tan \alpha+\tan \beta)
$$

This produces the depth contours with a contour depth interval

$$
\begin{equation*}
\Delta_{\mathbf{z}}=\frac{\dot{\mathbf{p}}}{(\tan \alpha+\tan \beta)} \tag{30}
\end{equation*}
$$

These contours represent planes of equal depth parallel to the surface of the moire grid. In the majority of practical cases the direction of observation will be perpendicular to the moiré grid, i.e. $\beta=0$. Hence the contour depth interval is given by

$$
\begin{equation*}
\Delta_{z}=\frac{p}{\tan \alpha} \tag{31}
\end{equation*}
$$

The other terms in equation 29 are dependent upon $x$ and produce noise at approximately the same spatial frequency as the moire grid.

In a practical system the illuminating source and point of observation are a finite distance from the grid so that the angles $\alpha$ and $\beta$ vary across the object. In this case consider a point of illumination a distance $h_{1}$ above the grid and a point of observation a distance $h_{2}$ above the grid that are separated by a distance $d$, as shown in fig. 10 b . The apparent frequency of the shadow of the moire grid cast on to the object is dependent both on the distance $z$ of the point from the grid and the height of the illuminating source above the grid. By similar triangles the apparent grid frequency $p^{\prime}$ becomes

$$
\begin{equation*}
p^{\prime}=\frac{\left(h_{1}+z\right)}{h_{1}} p \tag{32}
\end{equation*}
$$

This modifies the intensity modulation function at the object's surface to

$$
\begin{equation*}
I \propto \frac{1}{2}\left(1+\sin \left(2 \pi h_{1} x /\left(h_{1}+z\right) p\right)\right) \tag{33}
\end{equation*}
$$

Meadows [9] showed that in this case if the point of illumination and observation are at the same distance from the grid $h$ contours are produced whose depth interval is dependent upan the depth being measured.

$$
\begin{equation*}
z_{2}-z_{1}=\frac{N_{2} p h}{d-p N_{2}}-\frac{N_{1} p h}{d-p N_{1}} \tag{34}
\end{equation*}
$$

where $\mathbb{N}_{1}$ and $\mathbb{N}_{2}$ are the fringe number.

If the distance $h$ was much greater than $z_{1}$ this simplifies down to

$$
\begin{equation*}
z_{2}-z_{1}=\frac{N p h}{d} \tag{35}
\end{equation*}
$$

where $N$ is the number of fringes between the two points.
In practice it is less difficult to make square wave moire grids than sine wave grids. For the square wave transmission grid the intensity modulation function can be described in the form of a Fourier series.

$$
\begin{equation*}
T(x)=\frac{1}{2}\left[1+\frac{4}{\pi} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n} \sin \frac{2 \pi n x}{p}\right] \tag{36}
\end{equation*}
$$

In this case if $h>z$ contour depth interval is still

$$
\begin{equation*}
\Delta z=\mathrm{Nph} / \mathrm{d} \tag{37}
\end{equation*}
$$

and the intensity function of the contour fringes is of the form, Meadows [9].
$I$ (contours) $=c\left[1+\frac{2}{\pi^{2}} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi}{p}\left(\frac{n d z}{n+z}\right)\right]$

This is a triangular intensity function as described by Rektorys [28]. 2.6.2 Limiting Parameters.

A practical system for generating moiré contours using a square wave
grid is shown in fig. 11. The overall size of the object is Iimited to the size of the grid. The quality of the resultant contours are also dependent upon tho qua.ity and accuracy of the grid lines so as to retain the higher order terms of equation 36. For the inspection technique and contour depth of $0.01^{\prime \prime}$, grids of spatial frequencies of the order of . 100 cycles/inch are required. The limits on the performance of system to produce good quality contours depend upon depth of focus of imaging optics, diffraction effects and penumbra.

### 2.6.2.1 Depth of Focus.

As with the holographic methods of generating contours the depth of focus of the imaging optics was the principal limitation on the system. In this case both the moire shadow fringes on the surface of the object and moire grid must be within the depth of focus of the optical system. Beyond these limits the definition of both sets of grids diminish, which would reduce the contrast in the modulation of the function forming contour fringes. As in section 2.3.2 the depth of field is defined as

$$
\mathrm{dz}=4 \lambda(V / D)^{2}
$$

Consequently, the component had to be placed as close to the grid as possible to obtain the maximum benefit of the depth of field of the system. This in fact was a disadvantage but not a significant one for turbine blades, which could be placed close to the grid.

### 2.6.2.2 Diffraction Effects.

The spread function produced by diffraction at the edge of the interface between maximum and minimum transmission of the lines of the square wave can be described by Fresnel diffraction. To a first approximation the position of the first maximum of intensity beyond the geometrical edge of the interface is given by Longhurst [29].

$$
\begin{equation*}
x=(b(r+b) 2 \lambda / r)^{\frac{1}{2}} \tag{39}
\end{equation*}
$$

$r=$ the distance of source from the grid.
$b=$ the distance from the grid of the surface on which the shadow is formed.

Examining this function for the laboratory contouring system, where the grid was illuminated at $45^{\circ}$ with a point source .71 m . from the grid and the shadow of the grid falling on the object placed 12 mm behind the grid, the spread in the position of this maximum with illumination wavelength variation from .4 to .7 microns was approximately $1 \times 10^{-3}$ inches. This also approximately represented the distance by which the light intensity spread into the geometrical shadow of the grid line. For a moire grid of 100 cycles/inch this represented a spread function operating over $20 \%$ of the half period and so produced a marked blur on the shadow fringes. For grids of higher spatial frequencies the diffraction effects became even more significant and prohibit the use of contour depths of less than $0.01^{\prime \prime}$ on turbine blades.

### 2.6.2.3 Penumbra Effect.

The penumbra effect is defined as the extent of the blurring or spread $\Delta x$ of the geometrical shadow of an edge of a line on a moiré grid at a distance $b$ behind it when the grid is illuminated by a source of a finite diameter $d_{s}$ situated a distance $r$ in front of the grid. The region over which this function extends into the shadow from the geometrical edge is given by simple geometry as

$$
\begin{equation*}
\text { Extent of spread } \Delta x=\frac{b d_{s}}{r} \tag{40}
\end{equation*}
$$

The effect of the penumbra effect was to decrease the size of the dark shadow of the grid and reduce the clarity of the grid line edges and this reduced the overall definition of the contour fringes.

For a small diameter source such as a high pressure mercury arc with a diameter of 1.0 mm the penumbra effect would be small but for a source such as a quartz halogen bulb with an overall filament diameter of 6 mm this effect would become significant. For the laboratory moiré contouring system where $r=.71 \mathrm{~m} \quad \mathrm{~b}=.012 \sqrt{2} \mathrm{~m}$ from incident angle of $45^{\circ}$ $\Delta x=0.02 \mathrm{~mm}$ for the arc source and 0.12 mm for the quartz halogen source. The penumbra effect was negligible for the arc source with the 100 cycles/inch grid but for a quartz halogen source the shadow was considerably blurred which resulted in low contrast contour fringes. 2.6.3 Moiré Contouring Experimental Results.

The experimental evaluation of moiré contouring was carried out using a 100 W high pressure mercury arc light source, with a source diameter of approximately 1.0 mm . Moiré contour fringes were produced by the optical system shown in fig. 11 using turbine blades as test objects. Their aerofoil lengths varying from 40 mm to 80 mm . The quality of the moiré contours was heavily dependent upon the quality of the moiré grids. The initial results using an inferior quality master moiré grid gave very poor contrast fringes. A new $10^{\prime \prime} \times 14^{\prime \prime} 50$ cycles/inch master grid was obtained and secondary grids taken from the master using Agfa Gevaert 10556 high resolution ( $3000 \mathrm{c} / \mathrm{mm}$ ) holographic plates with a high photographic gamma. These were made with spatial frequencies from $50 \mathrm{c} /$ inch to $150 \mathrm{c} /$ inch.

Using the ${ }_{\lambda}^{\text {ef }}$ grids the quality of the moire contours greatly increased. An example is shown in fig. 12. The contour depth interval is $0.02^{\prime \prime}$ using a 50 cycles/inch moiré grid with a $45^{\circ}$ incident angle. The contoured shape of the blade was clearly visible and the grid lines were
set parallel to the blade chord to:- a) minimize the variation of the blades surface to the incident illumination to obtain unfform scattering intensity of the light across the blade, b) eliminate the erid noise when scanning along the blade chord as discussed in section 2.7, and c) to prevent allasing between the contour fringes and the grid lines towards the trailing edge of the blade when the contour fringes approach the same spatial frequency as the grid lines.

Although the depth contours in fig. 12 were well defined over most of the blade the image of the moire grid lines impaired the clarity of the contour fringes towards the trailing edge where the spatial frequency of the fringes was relatively high and made detailed analysis difficult. To overcome this effect Takosaki [25] suggested moving the moire grid in its own plane at right angles to the grid lines during the formation of the photographic image. This was later discussed by Allen [26] in greater detail. This had the effect of averaging out the grid line fringes as the grid was moved, whereas the contour fringes remained stationary and unaffected, thus increasing the overall olarity of the contour fringes. For the square wave grid the movement was linear. The effects of moving the grid in its own plane to average out the grid fringes showed a marked improvement in the quality of the contour fringes. In this case the grid was set parallel to the chord of the blade and a camera aperture of F. 32 was used. Fig. 13a shows the contours at $0.02^{\prime \prime}$ depth interval plus grid lines and in fig. 13 b the grid was moved linearly in its own plane by approximately $0.25^{\prime \prime}$ during the 2 sec . exposure time. The improvement in the clarity of the fringes was quite discernable.

A rig was made to enable the contour depth intervals to be set accurately. The Hg arc, moire grid and camera were tied rigidly together by the rig with the axis of observation of the camera, set
perpendicular to the moiré grid to form the contour planes perpendiculer to the axis of observation. The moiré grid to camera axis was set with the aid of a laser and then permanently clamped. The Hg arc was set on an optical bench rail at right angles to the camera axis. The arc slid along the rail to enable the illuminating angle to be accurately set by triangulation. A lens was used to relay a real image of the arc to any point along the arc to grid line so that the point source and camera aperture were in the same plane parallel to the grid.

### 2.7 FRINGE PROJECTION/IMAGE SCAN CONTOURING.

For the inspection system envisaged in this research the optical information had to be converted into an electronic signal by some form of interface. In the case of this work a slow scan television system was used. In such a system the television line scan would be as described in section 2.6 .1 and would traverse the intensity function described by equation 29 across the moire grid lines as well as the contour fringes. Hence the output signal would include the spurious grid fringes as well as the required contours. The intensity function described by equation 29 applied to a turbine blade is illustrated by a microdensitometer trace across the negative of a moire contoured blade shown in fig. 14a. The intensity modulation produced by the grid lines made a detailed analysis of the contour fringes extremely difficult.

The investigation into methods of eliminating this grid noise when the contoured image was scanned led to the development of the fringe projection/image scanning method of generating shape information. The theoretical analysis of this technique is described on the basis of a mechanism for eliminating the grid line noise on an image soan system. The fringe projection/image scanning system is a development of
moire shadow contouring which eliminates both the grid line noise and the need to have a moire grid physically close to the object. Animage of the moiré grid is projected directly on to the object as shown in fig. 15. The grid lines formed on the surface of the object are distorted by the shape of the object. The image of the object with the superimposed moiré grid lines is scanned in a direction parallel to the lines of the projected grid by a small aperture. This generates a light intensity distribution equivalent to that obtained by scanning the normal moire shadow system in a direction pafallel to the grid lines. The aperture size used to scan the resultant image is equivalent to half the period of the projected grid at the object plane of the viewing optics. In effect the scanning aperture replaces the analyser grid of the moire shadow contouring system.

### 2.7.1 Elimination of grid lines on a scanning system.

As shown in fig. 14 a the intensity modulation produced by the grid lines when the contour pattern was scanned makes detailed measurement of the contour fringes difficult. This could be reduced by moving the grid In its own plane at right angles to the grid lines while exposing the photographic image as suggested by Takosaki [25] and Allen [26] and discussed in section 2.6.6. The resultant microdensitometer trace across a contoured negative taken with the grid moving, fig, 14b, showed that the contour fringes were then well defined and the effects of the grid lines were greatly reduced.

This form of averaging would be possible with photography but not practical for a television system which would scan the resultant image at a high speed and would have a short exposure time (< . 02 secs). To blur
the grid fringes under these conditions the grid must be moved or vibrated by an amplitude greater than the period of the grid at a frequency of 50 Hz or greater, depending upon the integration time of photo cathode.

To overcome this problem, consider the effect of scanning the moire contour system in the direction parallel to the grid lines with a finite sized detector.

The total energy incident on the detector surface in the image plane of the optical system is proportional to the intensity of the image times the effective area of the object being observed.

Hence,

$$
\begin{equation*}
\text { Total Energy } \propto I_{\text {object }} x \text { Area } \tag{41}
\end{equation*}
$$

Consider now the effect of examining the light scattered from the surface of the object by a finite sized slit extending, or being traversed, in the y direction.

For a point $(x, y, z)$ on the object a distance $z(x, y)$ from the modulating square wave grid the intensity scattered from this point is given by equation 30 , and the symbols are as for fig. $10 a$.

$$
I_{0}=\frac{C}{2}\left[1+\frac{4}{\pi} \sum_{\substack{n=1 \\ n}}^{\infty} \frac{1}{n} \sin \frac{2 \pi m}{p}(x-z(x, y) \tan \alpha)\right]
$$

The energy emitted from this point is dependent upon the area over which it is integrated in the ( $x, y$ ) plane.
i.e. Energy $=\iint I_{0} d x \cdot d y$

$$
\begin{align*}
= & \frac{c}{2} \int_{y_{1}}^{y_{2}} \int_{x_{1}}^{x_{2}}\left[1+\frac{1}{\pi} \sum_{\substack{n=1 \\
n \text { odd }}}^{\infty} \frac{1}{n} \sin \frac{2 \pi n}{p}(x-z(x, y) \tan \alpha)\right] d y \cdot d x \\
E & =\frac{c}{2}\left[y\left[x-\frac{2 p}{\pi^{2}} \sum_{\substack{n=1 \\
n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi n}{p}(x-z(x, y) \tan \alpha)\right]_{x_{1}}^{x_{2}}\right]_{1}^{y_{2}} \tag{43}
\end{align*}
$$

This is a triangular wave form similar to that of equation 38 and illustrated in fig. $16 a$ and does not contain any of the spurious fringe terms associated with equation 29. To obtain the maximum signal and contrast of the fringes the sum of the series must be a maximum and as near to the value of the left hand term as possible when integrated between the limits $x_{1}$ and $x_{2}$.

This occurs when phase difference of the series is $\pi$ between the limits $x_{1}$ and $x_{2}$. That is over a slit equal to half the period of the grid. Hence integrating over the limits $x_{1}=x-p / 4, x_{2}=x+p / 4$.

$$
\begin{equation*}
E_{p / 2}=\frac{c}{2} \Delta y\left[\frac{p}{2}+\frac{4 p}{\pi^{2}} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n^{2}} \sin \frac{2 \pi n}{p}(x-z(x, y) \tan \alpha)\right] \tag{44}
\end{equation*}
$$

For a slit size less than $\mathrm{p} / 2$ the contour fringe function becomes more rounded towards a combination between a triangular and square wave. For a slit width above $\mathrm{p} / 2$ the contrast reduces to zero at p , i。e.

$$
\begin{equation*}
E_{p}=\frac{C \Delta y}{2} p \tag{45}
\end{equation*}
$$

As the width is varied above $p$ some contrast returns to some extent with a secondary maximum at $3 p / 2$ then decreases to zero at $2 p$ and so on, as shom in fig. 16 b.

If the area of examination is now scanned in the $y$ direction, $x$ being constant, the intensity function is purely dependent upon the value of $z(x, y)$ so as to generate the depth contour fringes without any of the spurious grid line fringes.

For the standard moiré shadow contouring system, fig. 11, the moiré grid is still superimposed on the contoured image as shown in fig. 12, and so an integrating slit length greater than $\mathrm{p} / 2$ can be used as the Integration region is 0 to $p / 2, p$ to $3 p / 2$.... $(n-1) p$ to $(2 n-1) p / 2$ as show in fig. 16 c , rather than over the full period of the grid. This increases the total energy observed without decreasing the fringe contrast. The limit upon the extent of the slit length is dependent on the rate at which $z$ changes with respect to $x$.

This shows that it is possible to eliminate the noise produced by the moiré grid by scanning the contoured image in the direction parallel to the moiré grid lines.

### 2.7.2 Fringe Projection/Image Scan Method of Contouring.

The above section described a method of eliminating the grid line noise from the moire shadow contouring system. However, the implications of equation 44 are even greater than just forming a method of noise reduction in so far as it is not necessary to view the moire shadow cast onto the object through the original grid. The scanning aperture in effect acts as the moire analyser to generate the depth contour information. Hence if the slit width used on any scanning device is less than or equal to $\mathrm{p} / 2$, as for example with a television system, the grid in front of the object can be removed and the basic moire grid projected on to the surface of the object.

The modified contouring system is shown in fig. 15 a where the grid is projected by the lens system and the object observed in the normal Way. The structure of the moire fringes projected on to a turbine blade is shown in fig. 17. The curved nature of the fringes gives a clear indication of the shape but detailed analysis of this fringe pattern would be difficult by purely visual means. Consequently this technique is only applicable to a scanning system.

### 2.7.3 Advantages of Fringe Projection/Image Scanning.

The fringe projection/scanning technique reduces many of the practical limitations on the moire contouring technique. The principal advantage is then reduction in the depth of field requirement which in the standard moire contouring system requires that the shadow fringes on the object together with the grid in front of the object must be within the depth of field of the observing optical system. The modified requirement is that the moire grid projected on to the object's surface must be in focus by both the fringe projection system and observing system over the whole of the depth of the object's surface. This now relies solely on the depth of the object as with any photographic system rather than object depth plus object to grid distance.

The diffraction and penumbra effects of the shadow fringes are reduced because the grid is imaged directly on to the object and the required depth of field is less than that for the moiré shadow technique. The quality of the projected fringes relies heavily on the quality of projector lens, principally the ability of the lens to overcome chromatic aberration and to produce a flat focal plane.

The system eliminates the requirement to have a moire grid physically close to the object and so it is possible to measure the shape of surfaces in restricted places where it would not be possible to put a moiré grid in front of the surface. This extends the use of the technique in the
aeroengine field to parts of engine casings and similar components. The size of object that can be analysed is also increased as the magnification of the projection system is variable over a wide range. Providing the projected fringes are of a suitable spatial frequency and the object can be imaged on to the television system and the element scanned is less than $\mathrm{p} / 2$ on the object depth contours will be obtained. It should be possible to examine components several feet in diameter with a suitable increase in the contour depth interval.

### 2.7.4 Visual Observation of Contour Fringes with Fringe Projection.

As discussed in section 2.7.2, it is not possible with the basic fringe projection system to obtain an accurate visual impression of the shape of the object as only the projected fringes are visible, fig. 17. However, in some cases not necessarily connected with computer linked dimensional measurements it may be desirable to observe the contour fringes visually.

This is achieved by placing a straight line moiré grid in the image plane with the lines parallel to the projected grid and of the same spatial frequency as the effective projected grid frequency on the image. The visual effect is the same as in the standard moire contouring technique. This is moving towards the more classical moire fringe techniques, Theocaris [24] but to obtain depth information rather than strain information. Fringe projection has recently been used to measure deflection and vibration amplitude by Der-Hovanesian [30] and Brooks [31]. The system shown in fig. 15 can be modified to produce visual contours by putting a demodulation grid in the image plane instead of the T.V. system and using a relay lens to view the resultant contours. Such a modification with a projected moire grid would overcome the depth of field of the standard moiré contouring system, fig. 11. However, the use of this would be limited in so far as the demodulation grid would
have to be changed every time the projection grid was changed or the magnification of either the projection or observing systems was changed.

### 2.7.5 Experimental Kesuits.

The quality of the fringes projected on to the turbine blade determined the ultimate quality of the depth contours obtained when scanning across the blade. To utilize the depth of field advantage of this system to its full extent the focal plane of the projected fringes must be parallel to the object as shown in fig. 15a. To achieve this the actual grid used in the projection system must be angled with respect to its image as shown in fig. 15 b . This would result in a variation in the image magnification across the moire grid and so to obtain a projected image with uniform fringe spacing across it the spatial frequency of the master grid must vary to compensate. Hence a special grid must be used.

Consider the diagram shown in fig. 15b. where $U$ is the distance of the master grid from the projection lens, $V$ is the image to lens distance, $M_{0}$ is the image magnification on the axis of the optical system and $\ell$ is the distance along the image plane from the axis.

The image magnification (M) at any point $\&$ along the image plane is given by the equation

$$
\begin{equation*}
M=M_{0}-\frac{\ell \sin \theta}{f} \tag{46}
\end{equation*}
$$

where $f$ is the focal length of lens used.
The resultant magnification at the master grid is $1 / M$ and for any point $l$ along the grid is given by

$$
\begin{equation*}
\frac{1}{M}=\frac{1}{M_{0}}\left(1+\frac{\ell^{\prime} \cos \beta \tan \theta}{f^{\prime}}\right) \tag{47}
\end{equation*}
$$

The angle $\theta$ and $\beta$ are as show in fig. 19.

To achieve the correct angular position of master grid to produce the angled image plane the master grid angle must conform to the equation

$$
\begin{equation*}
\tan \beta=\tan \theta\left(\frac{V}{M_{0} f}-1\right) \tag{48}
\end{equation*}
$$

Thus if the magnification is $1: 1 \tan \beta=\tan \theta$.
The required master grid was conveniently made using Agfa Gevaert 10E56 photographic plates by photographing a linear grid, positioned in the image plane of the projected fringes, with the fringe projection optics. The photographic plate was angled to comply with equation 48. The lens used for the projection system was of good optical quality to minimize the aberrations produced bycthereffect of the angled object and Image.

The resultant projection fringes were of good quality, fig. 17, over the whole of the aerofoil section of the turbine blade. The advantages of boing able to focus the projected fringes directly on to the blade ace shown by the ability to obtain clear fringes across the whole of the convex surface of the blade which has a depth of approximately $1.0^{\prime \prime}$ towards the blade tip.

The contour fringes obtained by scanning the blade image with a suitable aperture is shown in fig. 18. The fringes were comparatively noise-free, particularly in the region of the shape direction inversion, and the intensity function was triangular. These results compared favourably with the fringes obtained by the standard moire contouring technique.
2.8 COMPARISON OF CONTOURING TECHNIQUES.

The comparison between the methods of contour generation described. in this chapter was made with the view towards the application of
optical contouring as a routine means of production inspection and dimensional distortion analysis. Consequently the system must be accurate, reliable, require little attention while in operation and be fast. The overall inspection system must be capable of producing the difference information in a visual form such as on a television monitor or be used in conjunction with an on line computer, Hence the quality information generated by scanning the image must be good.

The holographic techniques suffered from the need of a photographic process before the contour information was available but they could not be prematurely condemned on these grounds. It would be possible to eliminate the need for a photographic process in these systems by the use of alternative storage media such as Lithium Niobate crystals, Chen [32], or thermoplastic materials; Lin [33] and Bellamcy [34]. These techniques enable: holograms to be recorded and processed within a few seconds. An alternative process would be to reduce the spatial frequency of the information so that the 'hologram' could be imaged on to a good quality television camera and recorded on a standard video tape or disc recorder. This technique has been developed by Leendertz and Butters [35, 36] in the U.K, and Macovski [37] in the U.S.A.. All these systems would be attractive alternatives to photographic processing and should be available as usable techniques in the near future, Work carried out by the author on the use of the Laser Speckle Pattern/T.V. System described by Butters [38] used for vibration analyses indicated that the technique had a definite potential. The use of video storage was not studied, as no suitable equipment was available.

From the investigation carried out on the double wavelength technique the contouring results were reasonable using the double exposure technique. (The comparison of the systems with respect to measuring accuracy, equipment required and ease of operation on a routine
basis is summarised in Table I.) Although difficult to set up, it would be simple to operate on a routine basis and the handing of the component would be straightforward using a kinematio relocation mounting for the blade with a $0.001^{\prime \prime}$ accuracy. It was restricted in the available contour depths but for a routine application variable contour depth intervals would not be needed, so that a dye laser with the required wavelength separation would be used. The object size was Iimited to the diameter of the telescope objective lens. The double refractive index technique geve excellent results; the contour depth was variable over a wide range but the repeatability depended upon the accuracy of setting the gas pressure differential. It had the same size and depth of field limitation as the double wavelength technique. For routine application the major disadvantages were the need for a pressure chamber, the use of a gas which would be expensive if a large number of components were tested and the time involved loading the component into and out of the chamber and adjusting pressure between exposures. These disadvantages precluded, the double refractive index method as a routine system.

In comparing holographic and moiré fringe contouring the moire had a number of advantages. The information content was the same, although they had different intensity distribution functions, but the moiré system was more flexible in the range of components to which it could be applied. It would be considerably cheaper, not requiring a leser, simple to operate and produced a contoured image instantly without requiring any photographic processing. The moire system was limited to contour intensity of $0.010^{\prime \prime}$ and above by diffraction effects but, because of the shape and depth of the blades, contour intervals of above $0.01^{\prime \prime}$ would be required.

The principal factor in the comparison of the techniques was the fringe intensity distribution when the image was scanned, as any
automatic inspection system would have an optical to electronic interface in the form of a television system. The effects of the speckle pattern structure on the holographic image produced a considerable quantity of noise on any scan signal, fig. 9. This made the true location of the centres of the fringes difficult. This was particularly true of the small apertures that had to be used to provide sufficient depth of field. The scan across a moiré contoured blade along the direction of the grid lines produced a fringe intensity with very little spurious noise, fig. 18, which was more appropriate for automation analysis.

The holographic and laser speckle pattern techniques would be more applicable to shapes that did not have such large depths in relation to the required inspection accuracy. The required measuring acouracy would be of the order of $\pm .25$ times the contour interval and an overall visual impression of the difference between the two objects would be required rather than a computer linked analysis.

For the reasons stated above the moiré contouring was selected as the most suitable system for the application to aeroengine component dimensional inspection and was used for the remainder of this research.

# Chapter III. <br> Methods of Obtaining Comparative Information. 

3.1 INTRODUCTION.

In this chapter the possible methods of comparing the shapes of two turbine blades are examined. There were three principal methods, direct optical subtraction of the two contour patterns, subtraction of the two contour patterns by electronic (computer) methods and subtraction of the shapes produced by the contours using a computer. These methods were compared with respect to the accuracy with which the differences were measured and their suitability for use on a routine basis.

The overall difference between an uncompleted blade and its master was normally less than $+0.02^{\prime \prime}$ and the tolerance envelope on a completed blade is $+0.007^{\prime \prime}$. Hence the comparison system must be capable of accurately discerning small differences between the blades. This would be particularly relevant when discussing the direct optical subtraction technique. The difference information must be fed into a computer via an electro-optical interface to enable the difference information to be processed quickly and presented to the inspector in a clear form.

The comparison of the blade shapes rather than the direct subtraction of the contour fringe patterns was found to be the most accurate and the most compatible with computer processing.
3.2 DIRECT OPTICAL CONTOUR SUBTRACTION.

This method is the most simple and direct method of generating the difference between two sets of contours and the technique is fundamentally the same as the well-known moire measuring devices. The contoured image of the master object is formed in a standard moiré contouring system, fig. 11. The image is photographed on a high resolution photographic plate, processed, and the resultant negative relocated into the same position at which it was taken. This forms the master grid of the moire $\Rightarrow$ subtraction system as shown in fig. 11. The resultant image transmitted
through this master grid is observed by a second camera, with lens $L_{2}$, to form the resultant moire subtraction. This lends itself to two possible systems. Firstly the moire difference fringe pattern is displayed directly on to a television monitor screen. The differences are then read directly off the screen or the position of the component is adjusted to eliminate or minimize the moiré fringes and the required movement gives the overall shape differences. The latter method becomes difficult when the full six degrees of freedom of movement are utilized. In the second technique the computer is used to obtain the dimensional differences from the resultant moiré pattern and apply suitable fit calculations to calculate the overall differences. On a practical basis the second technique was preferred where the computer would be used to calculate the dimensional differences.

For turbine blades the contours of equal depth lie in the direction parallel to the blade axis as can be seen in fig. 12 and the inspection requirements are that sections parallel to the chord and so normal to the blade axis are taken and measured. This is a result of the techniques used in the design of the blades. Hence the analysis is taken in scans along the chord of the blade and so acrossithemajority of contours on the blade. The difference in depth of the two blades in the $Z$ direction of the co-ordinate system produced by blade twist, bending, etc. is shown as a difference in spatial frequency of the contours across the blades as the section is scanned rather than a skew effect of the contours with the spatial frequency remaining the same, as in normal moiré strain measurements. This makes the visualization of the dimensional differences more difficult than conventional moiré techniques where the grid frequencies are kept constant. The formation of the difference fringes formed by direct optical subtraction is discussed in greater detail in the next section.

### 3.2.1 Difference Fringe Generation.

For the contoured blade shown in fig. 12 the contour fringe intensity distribution for the moire shadon contours, neglecting the grid fringes which can be eliminated as shown in section 2.7.1, is given by the triangular intensity function equation 38.

$$
I_{M}(x, y)=c_{M}\left[1+\frac{2}{\pi^{2}} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi n}{p}\left(\frac{d z(x, y)}{h+z(x, y)}\right)\right]
$$

This is used to expose a high resolution photographic plate to form the master grating.

The transmission of the negative $T$ is related to the optical density of the photographic plate $D$ by

$$
\begin{equation*}
\log \frac{1}{T}=D \tag{49}
\end{equation*}
$$

The optical density of the emulsion of the negative is given by the Hurter-Driffield (H \& D) curve of optical density versus log (Exposure)

$$
\begin{equation*}
D=\log \left(\frac{1}{T}\right)=\gamma(\log E-\log K) \tag{50}
\end{equation*}
$$

$\begin{aligned} \gamma \text { (photographic gamma) }= & \text { the gradient of the linear region of } \\ & (H \& D) \text { curve. }\end{aligned}$
$E=$ the energy used to expose the film.
$\log K=$ the projected intercept of the linear region of the curve in to the log (Exposure) axis.

$$
\begin{align*}
E & =\text { (Incident intensity) } \times \text { time } \propto \text { Intensity } \\
\therefore T & =K^{\gamma} E^{-\gamma} \propto E^{-\gamma} \propto I^{-\gamma} \tag{51}
\end{align*}
$$

Hence the transmission of the master negative is proportional to the intensity of the master contour pattern to the power - $\gamma$. Gamma is a Punction of the film used and the developing process.

Hence for a point $\left(x_{1}, y_{1}\right)$ on the image representing the point ( $x, y$ ) on the object the transmission is given by

$$
\begin{equation*}
T_{M}\left(x_{1}, y_{1}\right)=C_{M}\left[1+\frac{2}{\pi^{2}} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi n}{p}\left(\frac{d z(x, y)}{h+z(x, y)}\right)^{-\gamma}\right. \tag{52}
\end{equation*}
$$

The contour pattern of the production component is modified by the master negative by the above transmission function and forms an image in the plane shown in fig. 11.

$$
\text { Let } \begin{aligned}
z_{1}(x, y)= & \text { the depth of the master objeot } \\
z_{2}(x, y)= & \text { the depth of the production objeot } \\
z_{1}(x, y)-z_{2}(x, y)= & \text { the dimensional difference between the } \\
& \text { two objects. }
\end{aligned}
$$

The production contour pattern on the image plane of the master negative has the intensity of the form

$$
I_{P}\left(x_{1} y_{1}\right)=C_{P}\left[1+\frac{2}{\pi^{2}} \sum_{\substack{m=1 \\ m \text { odd }}}^{\infty} \frac{1}{m^{2}} \cos \frac{2 \pi m}{p}\left(\frac{d z_{2}(x, y)}{h+z_{2}(x, y)}\right)\right]
$$

Therefore the transmitted intensity through the master negative is

$$
\begin{equation*}
I_{\text {Image }}\left(x_{1} y_{1}\right)=I_{P}\left(x_{1} y_{1}\right) \times\left(I_{p 1}\left(x_{1} y_{1}\right)\right)^{-\gamma} \tag{53}
\end{equation*}
$$

$=K\left[1+\frac{2}{\pi^{2}} \sum_{\substack{m=1 \\ m \text { odd }}}^{\infty} \frac{1}{m^{2}} \cos \frac{2 \pi m}{\cdot p}\left(\frac{d z_{2}(x, y)}{h+z_{2}(x, y)}\right)\right] x$

$$
\begin{equation*}
\left[1+\frac{2}{\pi^{2}} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi n}{p}\left(\frac{d z_{1}(x, y)}{n+z_{1}(x, y)}\right)\right]^{-\gamma} \tag{54}
\end{equation*}
$$

$K$ is a constant. Letting the photographic $\gamma=1$ (a low contrast negative) to simplify the equation and expanding the transmission function by a Taylor series and neglecting terms squared and above the equation becomes of the form

$$
\begin{align*}
I_{\text {Image }}\left(x_{1} y_{1}\right)= & K\left[1+\frac{2}{\pi^{2}} \sum_{\substack{m=1 \\
m \text { odd }}}^{\infty} \frac{1}{m^{2}} \cos \frac{2 \pi m}{p}\left(\frac{d z_{2}(x, y)}{h+z_{2}(x, y)}\right)\right] x \\
& {\left[1-\frac{2}{n^{2}} \sum_{\substack{n=1 \\
n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi n}{p}\left(\frac{d z_{1}(x, y)}{h+z_{1}(x, y)}\right)\right] } \tag{55}
\end{align*}
$$

The argument of the cosine functions can be further simplified by expanding $(h+z(x, y))^{-1}$ as a Taylor series.

$$
\begin{equation*}
(h+z(x, y))^{-1}=\frac{1}{h}\left[1-\frac{z(x, y)}{h}+\frac{z(x, y)^{2}}{h^{2}}\right] \tag{56}
\end{equation*}
$$

Putting this into the cosine argument it becomes

$$
\frac{2 \pi n d}{p h}\left(z(x, y)-\frac{z(x, y)^{2}}{h}+\frac{z(x, y)^{3}}{h^{2}}\right.
$$

$$
\begin{equation*}
\div \frac{2 \pi \mathrm{nd} z(\mathrm{x}, \mathrm{y})}{\ddot{\mathrm{p}} \mathrm{~h}} \tag{57}
\end{equation*}
$$

Therefore multiplying out equation 55 then becomes

$$
\begin{align*}
& I_{\text {Image }}\left(x_{1}, y_{1}\right)=K\left[1+\frac{2}{\pi^{2}} \sum_{\substack{\text { min }=1 d d}}^{\infty} \frac{1}{\mathrm{~m}^{2}} \cos \frac{2 \pi m \alpha}{\mathrm{ph}} z_{2}(x, y)\right. \\
& -\frac{2}{\pi^{2}} \sum_{\substack{n=1 \\
n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi n . d}{p h} z_{1}(x, y) \\
& -\frac{4}{\pi^{4}} \sum_{\substack{n=1 \\
n \text { odd modd }}}^{\infty} \sum_{\substack{m=1 \\
n^{2}}}^{\infty} \log _{n} \frac{2 \pi n, d}{p h}, y z_{1}\left(x_{i-1}, y\right) \frac{1}{m^{2}} \cdot \\
& \cos \frac{2 \pi m d}{p h} \quad z_{2}(x, y) \tag{58}
\end{align*}
$$

Expanding the last term of the equation

$$
\begin{aligned}
I_{\text {Image }}(x, y)= & K\left[1+\frac{2}{\pi^{2}} \sum_{\substack{m=1 \\
m \text { odd }}}^{\infty} \frac{1}{m^{2}} \cos \frac{2 \pi m a}{p h} z_{2}(x, y)\right. \\
& -\frac{2}{\pi^{2}} \sum_{\substack{n=1 \\
n \text { odd }}}^{\infty} \frac{1}{n^{2}} \cos \frac{2 \pi n d}{p h} z_{1}(x, y)
\end{aligned}
$$

$$
\begin{align*}
& -\frac{2}{\pi^{4}} \sum_{\substack{n=1 \\
n \text { odd } m \text { odd }}}^{\infty} \sum_{m=1}^{\infty} \frac{1}{n^{2}} \frac{1}{m^{2}} \cos \frac{2 \pi d}{p h}\left(n z_{1}(x, y)+m z_{2}(x, y)\right) \\
& -\frac{2}{\pi^{4}} \sum_{n=1}^{\infty} \sum_{\substack{m=1 \\
n \text { odd } m \text { odd } \\
n \neq m}}^{\infty} \frac{1}{n^{2}} \frac{1}{m^{2}} \cos \frac{2 \pi d}{p h}\left(n z_{1}(x, y)-m z_{2}(x, y)\right) \\
& \left.-\frac{2}{\pi^{4}} \sum_{n=1}^{\infty} \sum_{n^{n}}^{\infty} \frac{1}{n^{4}} \cos \frac{2 \pi d}{p h} n\left(z_{1}(x, y)-z_{2}(x, y)\right)\right]
\end{align*}
$$

From this equation the term solely dependent upon $z_{1}-z_{2}$ is the last expression in the equation. Hence the resultant intensity distribution that gives the direct difference information is of the form

$$
\begin{equation*}
f\left(z_{1}-z_{2}\right)=1-\frac{2}{\pi^{4}} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n^{4}} \cos \frac{2 \pi \alpha n}{p h}\left(z_{1}(x, y)-z_{2}(x, y)\right) \tag{60}
\end{equation*}
$$

The two basic contour patterns of the two shapes also modulate the intensity distribution as can be seen from the second and third terms of the equation to form a carrier grid frequency which is amplitude modulated to form the difference contours. This consequently makes interpretation of the difference pattern more difficult.

The direct measurement of the difference in depth (z) is possible using direct optical subtraction. The overall visual impression of the difference can thus be obtained from the moire pattern as with the


Computer Simulated Shapes for Moire Differencing
standard moiré technique.

### 3.2.2 Computer Simulated Optical Subtraction.

To obtain an insight into the intensity distribution of equation 59 and to examine the accuracy with which the difference information $z_{1}-z_{2}$ could be measured from the resultant fringe pattern a computer model of the subtraction of two contour patterns was formed. A Conversational Programming computer system (C.P.S.) was used with a teleprinter terminal linked direct into an IBM 360 computer system.

In the program a cosine contour intensity distribution $\left(1+\cos \frac{2 \pi z}{\mathrm{Delta}}\right)$ was used rather than the triangular wave function series to reduce the computation. Two contour patterns were generated of a linear shape, with $0.010^{\prime \prime}$ contours (Delta $=.01$ ) but each with a different angle. As $x$, see sketch increased the difference in depth increased. One contour formed the master negative through which the second, production, pattern was transmitted using equation 53 and an effective photographic gamma $=1$. The resultant intensity distribution of the fringes for increasing $x$, the distance along the object, is shown in fig. 19. At $x=0$ the depths $z_{1}$ and $z_{2}$ were equal. The difference moire fringe was seen as the amplitude modulation of the carrier fringe pattern formed by the shape contours given by the 2 nd and 3 rd terms of equation 59:0 The difference contours were formed by joining the peaks of the shape fringes.

The moire differences were calculated by fitting a cosine function to the modulation using the contour peaks as reference points. From fig. 19 it could be seen that the difference fringe was well defined and the moire fringe formed a symmetrical modulation as the difference increased in a linear form. Using this computer method it was possible to form the moiré difference fringe intensity distribution and it was possible to calculate the dimensional difference for any given value of x. The difference value calculated from the moire fringes was within
$\pm .001^{\prime \prime}$ of the actual value of the difference which was within the predicted accuracy of the system.

The linear shapes represented the ideal case but was not, however, representative of the case of a turbine blade which had a curved shape where the frequency of the shape contour fringes was continually changing. To simulate the curved surface a solid sphere was used as the master object and an ellipsoid as the production object. The axis of observation (the $z$ axis) was the same as the major axis of the ellipsoid and a diameter of the sphere. The length of the minor axis of the ellipsoid (the $x$ axis of the co-ordinate system) was $1.8^{\prime \prime}$ compared to $2.0^{\prime \prime}$ of the major axis and diameter of the sphere. The ellipsoid was symmetrical about the $x$ axis. Computer scans of the moire subtraction image were taken along the $x$ axis at $y=0$ and other values of $y$ to form three dimensional information. Part of the computed moire pattern as scanned along the $x$ axis $(y=0)$ is shown in fig. 20a. In this case dimensional difference increased in a non-linear form as $x$ increased, fig. 20b, and the resultant moiré difference pattern was less well defined, particularly in the minimum intensity region. Calculations to find the dimensional differences from the fringe pattern were possible but the accuracy decreased around the minimum intensity region from $\pm .001$ to $\pm .0015^{\prime \prime}$.

As the rate of difference increased as $x$ increased, fig. 20b, the number of contours or carrier fringes that defined the moire difference decreased, making accurate measurement of the difference more difficult. As $x$ increased further a state was reached where the difference fringes approached the frequency of one of the contour patterns and aliasing occurred. These calculations showed that it was possible to measure to the dimensional difference to an accuracy of between $\pm .001$ and $\pm .0015$ for any part of an object by analysing the intensity distribution of the resultant moiré fringes. This method did, however, rely upon having
sufficient carrier fringes to define the shape of the difference fringe. At least three carrier fringes were required to define the shape of the difference fringe. This restricted the rate of change of the dimensional difference $\left(z_{1}-z_{2}\right)$ to less than one third of the maximum slope of one of the contoured surfaces. Also there must be a dimensional difference over the component greater than one contour depth to enable the contour difference fringe to be fully defined. The measurement accuracy of this technique depended directly upon the accuracy of the intensity values of the peaks. Consequently the accuracy would be impaired by any noise on the fringe intensity distribution such as non-uniform surface scattering, or illumination and noise caused by surface marks on the object.

### 3.2.3 Experimental Verification.

The experimental work carried out on these effects showed that the accurate analysis was difficult and could only be carried out under particularly favourable conditions. Fig. 21 a shows subtraction of two contour patterns of a flat metal bar that has been deflected. The bar was loaded at one corner to produce a twisting as well as a bending effect so that deflection could be seen more easily.

The deflection was most clearly indicated by the lines of minimum contour fringe visibility, i.e. the two superimposed contours were in phase, representing a zero or $n$ times $\Delta / 2$ deflection. $\Delta$ is the contour depth interval. It was difficult to accurately locate the regions where the deflection was $\left(n+\frac{1}{2}\right) \Delta / 2$, the contours out of phase, and subsequently practically impossible to estimate by eye a deflection between these two positions.

As shown by equation 59 and the computer results measurements of less than half a contour depth relied on actual intensity measurements. The eye is not capable of judging relative intensities of the form generated by equation 59 to any degree of accuracy. It is good at
estimating distances (width etc.) and changes in colour but has a logarithmic response to intensity and operates on a local reference system. Hence it is not possible to accurately estimate the intensity variation of the contour fringe peaks shown in fig. 21a, to measure the deflection between the moiré in-phase and out of phase condition.

From the intensity information of such a system, fig, 21b, obtained by scanning the image with a photodetector, the deflection of the plate between the in-phase and out of phase conditions of the two contour patterns could be estimated with little difficulty. Consequently any system that displayed the moire deflection or difference fringe pattern directly to an operator for visual interpretation would be limited to an accuracy of $\pm .25$ fringes.

Microdensitometer traces of the intensity distribution across the negative of fig. 21a, shown in fig. 21b, illustrate the measuring difficulties. The moiré deflection frequency was approximately a quarter of the contour frequency and it could be seen that the moire difference pattern was not very well defined. The total modulation depth was less than $40 \%$ of that of the contour fringes. For a measuring accuracy and resolution of $\Delta / 10$ the intensity profile of the moire would be required to be known to within 15 or $20 \%$ although in this case the moire difference fringe was only defined by 3 or 4 points per contour interval deflection. The measuring problem was alleviated to some extent by small deflection so that the moire difference was defined by more contour fringes. But, as shown in fig. 22, for conditions where large deflections or dimensional differences were involved, the spatial frequency of the moire difference approached the frequency of one of the contour patterns, thus measurement became extremely difficult.

### 3.2.4 Discussion of System Performance.

If the dimensional difference was equal to several contour depths
a number of difference fringes would be seen and it would be possible to measure the difference to between $\pm .5$ to $\pm .25$ of the contour depth by direct visual inspection. For accuracies above this an image intensity measuring technique must be used.

For the particular application to turbine blades the maximum difference expected between a master and production component was $0.02^{\prime \prime}$, which represented between one and two difference fringes. The required measuring accuracy was 0.1 times the contour depth. Two typical examples of these measuring conditions are shown in fig. 23. These photographs were formed by double exposure to add the two patterns, for convenience, rather than one observed through the other, but the final result of the moiré pattern was the same. A $180^{\circ}$ phase change has effectively been applied to one contour pattern. Fig. 23 shows a turbine blade bent in the $z$ direction and the moire difference where the contours were outuof phase could be clearly seen. Accurate assessment of the difference at any region across the chord of the blade, apart from the out of phase region, was extremely difficult by either visual inspection or fringe intensity scans. For the case of the blade being twisted about its own axis the visual interpretation of this effect was also difficult because the moire difference fringes were parallel to the contour fringes and there are a large number of contour fringes which tended to lead to visual confusion. Accurate assessment of this deflection was impossible. The intensity scan would give better results but because the scan technique relied upon accurate relative intensity measurements of the contour fringes the noise produced by the practical factors of surface finish, etc., would greatly reduce the accuracy of this method.

Finer contour fringes < $0.01^{\prime \prime}$ produced a greater visual effect of the differences but could not be used because of the depth of the blade and the diffraction effect limitations produced by the finer fringes.

### 3.2.5 Discussion of Overall Technique.

Although the system was attractively simple the results showed that for a practical application it had a fundamental disadvantage in that it relied upon the accurate measurement of the relative intensities of the resultant carrier fringes to estimate sub-contour differences. The computer simulation showed that the technique was capable of 0.1 fringe accuracy under ideal conditions but in a practical case of a blade the scattered light intensity could vary with surface finish across the blade or be affected by grease or dust marks. Also it relied on the linearity of the readout system. In this case the readout system would be a television camera which under the prevailing low light level conditions expected from the moire subtraction technique would notcobe operating's inca region where the response wouldobelinear.ide thot.

The form of the difference information moire pattern when displayed on a monitor would be complex, as seen in fig. 23 , and would be quite different from the accepted form of a moiré pattern. Because of the complexity of the patterns caused by the carrier fringes it would not be possible to use this method as a direct visual indication of shape difference either for direct assessment or an object adjustment method in which the operator adjusted the position of the object to minimize the number of moiré fringes. Some of the carrier fringes could be eliminated by electronic low pass filtering of the vidicon output but since the shape of a blade would be complex and the spatial frequency of the contours varied across the chord of the blade satisfactory electronic filtering would be difficult. Even then visual judgement of relative fringe intensity to measure sub-fringe differences would be difficult and inaccurate.

With reference to the information processing aspect of this technique this method did not indicate the direction of the difference
information whether a point on the production object was closer to or further, away from the master.

An additional practical limitation of the technique was formed because the contoured image was passed through the master negative to obtain the difference information. Consequently, much of the available light would be attenuated by the master negative resulting in a very low light level final image on the vidicon. Hence very sensitive vidicons would be needed to obtain reasonable signal levels at the normal T.V. scan rates. Alternatively, slow scan rate or storage systems have to be used which increases the complexity of the apparatus required. Slow scanning systems would have to be used for computer input because of the limiting digitizing rates available but even then the light levels would be very low.

The limitations of this technique are summarized as follows:-
a) the complexity of the resultant fringe pattern,
b) reliance upon relative intensity measurements to obtain accurate sub-fringe information,
c) limitation upon the rate of increase of the shape difference,
d) the practical problems of very low light levels.

These limitations resulted in the conclusion that it was not a suitable technique for application to either visual or computer based shape difference subtraction for the applications to turbine blades,

### 3.3 ELECTRONIC FRINGE SUBTRACTION.

This system was identical to direct optical subtraction but used an electronic method of subtracting the two contour patterns. The master contour image must be stored optically or electrically and assessed for subtraction from the production contour fringes. Since the electronic fringe subtraction also relied on relative measurement of intensity it would be prone to the same limitations and disadvantages as direct
optical subtraction and would be somethat more complex. For these reasons the technique would not be suitable for the envisaged inspection system.

### 3.4 ELECTRONIC/COMPUTER SHAPE SUBTRACTION.

Shape subtraction with optical contouring could only be achieved with the aid of a computer to produce any form of automatic system capable of routine application. It would compare actual dimensional shapes rather than optical information relative to the shape. Hence it would be a direct comparison for a large number of points. The principle of operation was that the shape co-ordinates of the master object were taken for a number of plane sections through the blade aerofoil parallel to the chord and stored within the computer. These were compared to the shape data of the same section of a production or distorted blade. The computer would be used to generate the difference data at any given number of points and used to manipulate this data to obtain the best fit between the two shapes to give the overall difference in the form of a difference in two directions plus an angle. The manipulation of the data would be one of the main assets of this technique over the present inspection systems. The accuracy of the shape comparison technique would be better than the contour subtraction method as it does not rely to such a great extent on absolute intensity information as the direct optical subtraction system did.

As all the data reduction in this technique wauld be done by the computer it would be more complex than the optical subtraction but considerably more versatile. By building up dimensional difference information in a number of planes along the blades axis and relating the planes it would be possible to produce the three-dimensional differences between the blades.

It would be possible to compare objects direct with computer design data and if linked into the Company's main computer complex would have
automatic access to the design data. Hence this would eliminate the requirement for a physically made master for comparison, as was still needed by the present system.

The format of the output from the computer could be varied according to the requirement of the application, such as in the production application this could be the general difference between the blades, such as thickness bending, twisting plus detailed results of areas with localized differences above a particular limit. For a distortion the format may be required to be compatible to a theoretical computer programme.

An additional use of computer facilities could be used to store the values of the general differences between blades to obtain statistical data to analyse trends which could be used to improve production techniques.

Before studying the available methods of converting the optical information into computer co-ordinate data, or the methods of handing shape data within the computing the reliability and accuracy of the contour shape information was assessed.
3.6.1 Contour Shape Information.

The shape contour intensity distribution function as a component is scanned is given by the equation

$$
I(x, y) \propto \Delta y\left[\frac{p}{2}+\frac{4 p}{\pi^{2}} \sum_{\substack{n=1 \\ n \text { odd }}}^{\infty} \frac{1}{n^{2}} \sin \frac{2 \pi n}{p}(x-z(x, y) \tan \alpha)\right]
$$

The clarity of this function depended upon the area of the scanning detector. The extent of this in the $x$ direction would be fixed at $p / 2$ to obtain maximum fringe contrast. Thus the controlling factor would be the width of the slit in the $y$ direction. Consequently this must be
as small as possible. The overall accuracy of the system depended upon the accuracy with which the peaks and troughs of the contour fringes could be located and how faithfully the practical signal follows the theoretioal intensity function. The most appropriate method of examining these factors was experimentally.

### 3.4.2 Mechanical Scan System Accuracy.

The scan system developed for these tests was simple but an extremely effective device. The contoured image formed on the camera image plane in the system shown in fig. 11 was mechanically scanned by a photomultiplier with a small slit, approximately $0.015 \times 0.005^{\prime \prime}$. The position of the slit provides the $X$ and $Y$ co-ordinates of each point with respect to the blade and the contours give the $z$ co-ordinate.

The photomultiplier was clamped in a lathe bed worm-driven slide unit, the worm being driven by a variable speed geared electric motor. The position of the Photomultiplier tube (P.M.T.) with respect to the $Y$ co-ordinate was set manually by adjustment of a second lathe slide unit to which the first assembly was clamped, fig. 24. The P.M.T. was scanned in the X direction by the electric motor and its position with respect to the scan was monitored by the voltage output of a 10 turn potentiometer geared to the electric motor drive. The potentiometer output was fed to the $X$ direction of an XY plotter and the P.M.T. output voltage applied to the $Y$ direction. This generated an intensity versus scan position plot of the contour pattern.

The blade was set in a horizontal position in the moiré contouring rig (section 2.7.3) with the aerofoil surface facing the camera, as close to the moire grid as possible. The grid lines were in the vertical direction and thus the angle of incidence of the grid illuminating beam in the horizontal plane. The P.M.T. was scanned vertically parallel to the grid lines to eliminate the grid noise. The slit length
of $0.015^{\prime \prime}$ covered 3 complete periods of the grid in the image plane (100 lines per inch with a 0.5 image magnification).

With this system the scan was only recorded when the P.M.T. was moving in the upward direction so that it was loaded by gravity against the drive scredre to overcome the slackness within the mechanical drive.

The photomultiplier had some degree of high frequency noise on its output in addition to the spurious noise caused by dust particles on the blade and grid which produced a random noise effect on the contour intensity trace, fig. 28. This type of noise decreased the accuracy of the location of the contour maxima and minima and could not be tolerated for computer analysis as it would result in spurious fringe counts. Consequently a low frequency pass filter was put on to the P.M.T. output calculated to transmit only the frequencies below the maximum contour frequency for the particular scan rate of the P.M.T. which was approximately $1 \mathrm{scan} /$ minute. This technique worked well and gave a good noisefree trace, show in fig. 25a, which was also suitable for computertu analysis.

An examination showed that for repeat scans of the same part of the blade the $X$ direction scan position repeatability was within $\pm .002^{\prime \prime}$. The $Z$ direction measuring repeatability and accuracy of the contour system was within $10 \%$ of the contour depth for contours between $0.020^{n}$. and $0.010^{\text {i }}$ : depth intervals.

The turbine blades were kinematically mounted at the fir tree root and it was possible to achieve a relocation accuracy of $\pm 0.001^{\prime \prime}$ towards the blade tip. The overall performance of the scanning system illustrated that the measuring system was sufficiently accurate for the present measuring requirements on the turbine blades.

Dimensional measurement by manually plotting the shape from the contour information, using the maxima and minima values, showed that the
system was consistent and accurate. An example of this work is shown in figs 26, 27. This was an example of the comparison of a turbine blade that had been run in a test rig under engine conditions and had been thermally fatigued. This was compared to a new blade of the same type. The contour depth was $0.018^{\prime \prime}$ with measuring accuracy of $\pm .0015^{\prime \prime}$. The comparison of scans across three chord sections along the blade aerofoil, two of which are shown in figs 26 and 27 , showed that there was little difference between the standard and run blades. There was a progressively increasing difference in the aerofoil positions towards the tip of the blades, indicating that the engine run blade had been bent backwards by approximately $0.010^{\prime \prime}$ at the uppermost scan. The engine run blades were also thicker by an average of $0.006^{\prime \prime}$. This agreed with the effects of a carbon and oxidization coating across the engine run blade which was measured to be between $0.002^{\prime \prime}$ and $0.005^{\prime \prime}$ thick. This layer was shown by the definite kink in the aerofoil shape, fig. 27, where part of it had broken away from the convex surface.

### 3.5 DISCUSSION.

The investigation into the methods of comparing two components to find the dimensional difference has shown that direct shape comparison would be the most suitable method of obtaining the difference information. The moiré contouring method was capable of producing the shape to an accuracy of $10 \%$ of the contour depth interval used.

For the dimensional inspection of turbine blades using optical contouring techniques a computer linked system should be used to cope with the increased quantity of data. Although this appeared a complex procedure, it would have the increased potential of being accurate to within $0.001^{\prime \prime}$ over the depth of the object and would have the ability to manipulate the difference information to calculate the overall differences and present the information in a form easily assimilated by the operator.

The overall system envisaged for the computer linked inspection is shown in block diagram form in fig. 28. The optical to electronic information interface would be a slow scan television camera which would provide 20 line scans of 500 information points per scan in a total time of 10 seconds. This information would be fed into a small computer for analysis and the difference information would be displayed on a Cathode Ray Tube or printed on paper. From this information the operator would either accept or reject the blade, unless the computer did the decision making for him, or specify what remedial action would be required on the blade. This system would be basically a passive system as far as the operator was concerned, as no adjustments would be required from him and so operator errors would be eliminated.

# Chapter IV <br> Information Processing by Computer. 

### 4.1 INTRODUCTION.

The methods used to process the optical contour information within the computer are described in this chapter. There were two separate objectives in the computing. First, to obtain an array of shape co-ordinates within the computer memory that was accurate and reliable. Secondly, to compare this information with the design or master data so as to calculate the overall differences together with any abnormally large localized errors.

The system envisaged for the computer linked inspection has been discussed in section 3.5. The equipment used to investigate the computing aspect of this work was different from that of the overall system and is also shown in fig. 28. The information from the television system was fed into a computer type digital tape recorder. The magnetic tape was then fed directly into the IBM 360 computer system for analysis and the final output was in a printed form. This system was dictated by the lack of the availability of a small computer. However, a digital tape recorder was acquired to enable the blade data to be put into the IBM 360 system. The equipment was sufficient to investigate the feasibility of the technique.

### 4.2 SHAPE FORMATION FROM CONTOUR FRINGES. PROGRAMME OUTLINE.

The shape formation computer programme was required to calculate the shape of the aerofoil surface of a turbine blade from a signel generated from a scan across the contoured image. Fig. 29a shows the intensity profile of the contours on the convex surface of the blade taken by a scan along the direction of the chord. The shape represented. by the contour pattern is shown in fig. 29b.

The programme used to obtain the shape information consisted of four main sub-routines:-
a) Location of intensity maximum and minimum values for each half fringe.
b) Calculation of object shape from intensity data between the intensity maximum and minimum values of each half fringe.
c) Counteraction against noise on the contour intensity signal.
d) Location of shape inversion region.

The first two sub-routines were the main part of the programme and could together calculate the basic shape of the component. As the computer had to process a real electronic signal from the optical to electronic information interface the noise effects produced on this signal had to be taken into account in the third sub-routine. The final sub-routine was a method used to locate when the direction of the shape changed as it was being scanned' as shown in fig. 29 b .

The programme is explained in greater detail in a block, or flow diagram form in fig. 30. The four main sub-routines are shown by the dotted lines. During the majority of the programme only the maximum and minimum location and shape calculation would be used. The computer was used to locate each half fringe and defined the maximum and minimum intensity values and their positions with respect to the scan and then calculated the shape for the measuring points between these values.

The noise generated on the intensity signal was principally produced. by surface effects on the blade, i.e. scratches, dirt, rather than electronic noise. Hence the noise was inherent in any practical system for blade inspection and had to be taken into account by the computer, otherwise spurious intensity maximum and minimum values were indicated. The noise effects were only found to be critical where there were few contour fringes, i.e. where the surface was nearly parallel to the $\dot{r} e f e r e n c e$ plane of the contouring system. Two typical intensity scans across a contoured turbine blade are shown in fig. 31 and these illustrate the noise effects that are produced on the fringe intensity.

The noise correction sub-routine was only applied over the region shown in fig. 31.

The point where the direction of the shape changed from going away from the observer to coming towards the observer, or vice versa, as the scan proceeded, occurred in the same region as the noise effects. Hence in the region of the scan the input information was examined to find the inversion point. Once located and the direction parameters changed accordingly, this sub-routine was locked out of the main programme.

Both the noise correction and shape inversion sub-routines were of necessity complex, to accommodate all the variations by which the noise and inversion point indication occurred and to ensure that false shape information did not occur. Although the sub-routines were complex only a small part of each one was used for each individual scan of the blade, so that the actual calculations performed by the computer were kept to a minimum,

The contour fringe intensity data was in the form of a digitized value of the intensity for a given position along the scan. The distance between each intensity value was equal or nearly equal depending upon the scan system used, fig. $32 \mathrm{a}, \mathrm{b}$, and the position of each point along the scan was known. The object of the information processing system was to obtain as much data as possible and as accurately as possible. Information could always be rejected later on in the process if it was not required. The amount of information used depended on the application and the component being investigated.

The contour fringes formed a series of maxima and minima with a dimensional depth separation of half the contour depth interval, fig. 29. The computer system could be used just to count these and record their positions along the scan. This would generate the shape but would only provide 20 to 50 points across the blade with an uneven distribution
along the scan. There would be little detailed information in the areas where the blade's surface is nearly parallel to the reference grid as shown in fig. 32c. This would reduce the spatial resolution of the measurements to below a tolerable limit. Hence information was required between the fringe maxima and minima.

Another factor that directly affected the components' programming was the requirement to utilize the minimum quantity of storage within the computer. The system would ultimately be used in conjunction with a small on-line computer which would have only a restricted storage capacity. Increasing the storage requirements of the computer system would be expensive and its subsequently high cost would make the overall inspection device less attractive for industrial application.

The absolute values of the intensityeat the peaks and tranghsiculto of the contour pattern and the overall intensity level varied across the blade, as shown in fig. 31. These overall fluctuations meant that it was not possible to calibrate distance (contour depth) with respect to any given value of absolute intensity.

The computer was used as a data processing or sorting system rather than a pure calculating machine and its logic was used to sort the incoming data before actually calculating any depth values. The computer program was written in the language PL/1 in a punched card format for use on an IBM 360 system. The use of the IBM 360 computer dictated slightly different procedures to what would be used with a small on-line computer, so that the maxim of keeping strictly to an on-line system could not be adhered to.

The method used to put the data into the TBM 360 system operated most efficiently if all the data of one scan was loaded in one step and stored as an array of a known number of values and then this array could be interrogated by the programme.

### 4.3 CONTOUR MAXIMUM AND MINTMUM LOCATION.

The information input of the contour fringes was in the form of an intensity or voltage value, V , and a given voltage representing the position $X$ along the scan, VX.

The values of the intensity voltage were compared in groups of three to locate the maximum and minimum values using the procedure of an 'IF' statement.

IF $V(I)>V(I-1) \& V(I)>V(I+1)$ THEN GO TO (Max. value)

IF $V(I)<V(I-1) \& V(I)<V(I+1)$ THEN GO TO (Min. value)
where $I$ is the integer counter within a D0-LOOP.
For an on-line system this would be of the form

IF $V(2)>V(1) \& V(2)>V(3)$ THEN GO TO .......

$$
V(2)<V(1) \& V(2)<V(3) \text { THEN GO TO -..... }
$$

and at the end of each comparison the group would be up-dated to include the next point, i.e.

$$
V(1)=V(2), V(2)=V(3), V(3)=V
$$

When a maximum or minimum was located, its position in the array was located and fringe intensity stored. The computer programme is shown in Appendix 1.
4.4 BLADE SHAPE FORMATION.

The maximum and minimum location procedure was continued until both a maximum and minimum value were located. From the known values of these maximum and minimum intensity values a triangular waveform was fitted to the intensity distribution to calculate the depth $z$ for any intermediate
intensity value. The relative value of the position of the intermediate intensity point in the waveform which was proportional to the depth was of the form
$R=[V(I)-$ MMIN $][$ WMAX-VMIN $]$
$R=$ the incremental value between 0 and 1 for any intermediate position.

VMAX and VMIN $=$ the intensities of the maximum and minimum points of fringe.

In some practical cases when the contour fringes were soanned the diffraction effects etc. spread the moire shadow edges and the finite widh of the scanning detector distorted the triangular intensity. These effects broadened the peaks and troughs of the fringes, so that a $\sin ^{2} V$ function was more representative of the curve. To fit this curve to the intensity distribution between the maximum and minimum values, an intermediary intensity function was used of the form

$$
\begin{equation*}
\text { AINT }=[V(I)-V M I N] /[\text { WMAX }-V M I N ~] \tag{63}
\end{equation*}
$$

This varied from a value of 0 to 1 as $V(I)$ varied between the maximum and minimum limits.

The incremental position was then given by the expression

$$
\begin{equation*}
R=\sin ^{-1}(\sqrt{\text { AINT }}) \tag{64}
\end{equation*}
$$

To find this value an iteration process was used by setting up an angle, calculating the value of the sine and comparing this to the root of the actual contour intensity function AINT.

This calculation was of the form

```
    C = SQRT(AINT);
    K;J=0;
    P = 1;
ASIN : YEW = J/2**K ;
    P=P + YEW
        S = SIN(.7854*P);
            IF C-S < .001 & C-S > -.001 THEN GO TO. BSIN ;
            IF C-S > 0. THEN J = 1;
            IF C-S < 0 THEN J = -1 ;
            K = K+1 ;
            GO TO ASIN ;
BSIN : R = P/2;
```

$R$ was the solution of the iteration. The IF statement determined the accuracy to which the iteration was operated.

To form the component shape over the whole of the scan the peaks and troughs were counted by the normal counting procedure of

$$
\text { COUNT }=\operatorname{COUNT}+1
$$

The calculation of the total depth also required knowledge of whether the fringe under investigation was going from trough to peak or vice versa. For this an indicator was used.
$Q=1$ for min to max (intensity increase with increasing VX)
$Q=-1$ for max to min (intensity decrease with increasing $V X$ )

Finally in the depth calculation the direction of depth represented by increasing contours must be established. Whether the surface was coming towards or away from the observing system. This could not be established from the contour fringes and was preset by the computer from knowledge of the expected shape of the component. Since the system could be used to
compare shapes the basic knowledge of the expected shape would be known.

This indicator was of the form

```
Y = 1 for increasing depth z, object going away from the
            observer.
Y = -1 decreasing depth Z.
```

The surface on a turbine blade, however, would be curved and this direction could change, which was indicated by a change in a constant from $\mathbb{W}=1$ to $W=-1$.

The resultant depth of the surface was given by
$Z()=(S T A R T+(W * Y *(\operatorname{COUNT}+R))) * D E L T A * .5$
for $Q=1$ and
$Z()=\left(S T A R T+\left(\mathbb{W} Y^{*}(\operatorname{COUNT}+1-R)\right)\right) *$ DELTA *. 5
for $Q=-1$.
DELTA $=$ the contour depth of the fringes.
START $=$ the number of contour half fringes the first point was from the reference plane.

The position of the point analysed along the blades in the $X$ direction was calculated from either the number of the point in the array or from the $X$ direction voltage $V X$ and a previous calibration. The position of any point EX along the scan was given by

EXX ()$=\left(\operatorname{WIDTH}^{*}(X M A X-V X())\right) /(X M A X-X M I N)$
where WIDTH was the actual width in the object plane represented by a voltage change of $X M A X-X M I N$. The reference point $X=0$ was formed at the position represented by a voltage of XMAX.

During the blade scan the computer operation was of the form where
it located the position of the maximum and minimum contour intensity, noted the position of these points in the array, calculated the intensity shape vilue $Q$, then proceeded to calculate the values of the depth $Z$ between these points. It then continued to the next maximum and minimum pair.

### 4.5 NOISE EFPECTS.

Sections 4.3 and 4.4 discussed the basic outline of $\lambda^{\boldsymbol{a}}$ shape generation process which worked well under idealized conditions. However, the output signal from the optical to electronic interface was a practical signal and consequently had some noise produced by both the interface system and optical surface effects from the object.

The noise on the contour fringe intensity scan produced spurious spikes on the signal which were located by the computer as fringe maximum and/or minimum values and produced additional fringe counts. This was eliminated by a process which examined the values of the maximum and minimum voltages and rejected them if the difference between them was less than a given value. This worked well over the regions where the fringes were closely spaced and for spurious maximum and minimum indications that occurred close to the true maximum or minimum voltage values. However, much of the noise that affected the calculations occurred when the fringes were widely spaced, as shown in fig. 31.

The effects of the noise over this region was reduced by an averaging method. This region was approximately known for each scan. Let it lie between the values VX = XLIM1 and XLIM2.

When the scan reached this region a series of average values of the intensity voltage was calculated.

$$
\begin{equation*}
\operatorname{VAV}()=\frac{\sum_{n=1}^{m} V()}{m} \tag{68}
\end{equation*}
$$

$m$ was set between 4 and 9 .

This average was taken at every 3rd or 5th information point and was stored in a separate array, as shown in appendix 1.

Tr ise average values were then interrogated in blocks of three to find the fringe maxima and minima and then the depth calculated in the normal way.

By going into this averaging routine, it was possible to miss a true maximum or minimum value close to the entry and exit points of the sub-routine. A principal cause of this was that these parameters could only be detected from the second value of the averaging array to the penultimate value since the location depended upon the IF statement

IF $\operatorname{VAV}(A A)>\operatorname{VAV}(A A-1) \& \operatorname{VAV}(A A)>\operatorname{VAV}(A A+1)$

An overlapping procedure was used to overcome this problem but it introduced a possibility of one peak or trough being located twice to produce an additional count. This situation was prevented by comparing the number of the point in the input array at which the peak was found in the main programme to the number of the point in the input array at which the peak occurred in the averaging sub-routine. If they coincided within the increment between consecutive averaging points the fringe count was rejected. The point was still used to form a maximum and minimum point pair.

The averaging sub-routine was not infallible in rejecting the noise. It has been found when running the programme that spurious counts occur when noise peaks occur on the main intensity shape between the maximum and minimum regions on the averaged signal.

If these occurred on an increasing intensity slope from a minimum to maximum fringe the noise would produce a spurious maximum, then a spurious minimum and then the intensity would continueito increase. This characteristic was located by examining the intensity of the points after
the spurious maximum value using the IF statement

IF $\operatorname{VAV}(A A+1)>\operatorname{VAV}(A A+2) \& \operatorname{VAV}(A A+2)>\operatorname{VAV}(A A+3)$

THEN GO TO (MAX Calculation) ; ELSE GO TO (Reject point) ..... (70) Equation .70 defined that unless the points after the location of the maximum point continued to decrease over the next two points as would occur in a true maximum the point would be rejected as being spurious. For the spurious point on the negative intensity slope the rejection procedure was similar.
4.6 DIRECTION INVERSION PROCEDURE.

The shape of both the concave and convex surfaces of the turbine blade involved a change of direction of the gradient of the surface with respect to the reference plane, fig. 31. The change was not directly indicated by the contour fringes themselves, but only by the reduction in the spatial frequency of the contours in the region where the surface was nearly parallel to the reference plane. It would be possible to locate this region by measuring the relative fringe spatial frequency and locating the minimum. This would not be a satisfactory method as it required knowledge of the maximum-minimum intensity pair beyond the pair over which the slope was being calculated. Hence it needed a large amount of storage within the computer as these maximum and minimum points in this region would be well separated. The increased storage would prohibit the use of this technique on a small computer.

There were two techniques that were used in conjunction with each other to locate the inversion region. These were sub-fringe location and shape gradient measurements.

### 4.6.1 Sub-Fringe Location.

The fringe intensity distribution of the scans taken along the chord of the blade, fig. 29 , shows the inversion region as a decrease in the
spatial frequency of the contours with a subsequent increase beyond the actual inversion point. In the majority of cases the point of inversion would not occur at a contour depth of $Z=n^{*} D E T T A / 2$ where either a fringe maximum or minimum occurred but at a depth arbitrarily spaced between these values. In these cases the intensity distribution would show either a sub-maximum or sub-minimum, an example of which is shown in fig. 31b. The location of the sub-fringe gives an accurate indication of the inversion point.

When a turbine blade was being inspected the approximate region of the inversion point would be known to within 0.2 inches from the master shape information. It also occurred within the region of the noise averaging process. Hence the region of which the inversion point was expected could be set. The region was set principally to reduce the computing effort used to locate this point. A sub-fringe test was carried out when the programme was in the process of locating the maximum or minimum points for the next half-fringe step.

If a maximum was located its value was compared to the previous maximum and minimum values by the equation

$$
\begin{equation*}
\text { SUBMAX }=(\mathrm{VAV}(A A)-\mathrm{VMIN}) /(\text { VMAX-VMIN }) \tag{71}
\end{equation*}
$$

IF SUBMAX <. 75 THEN (Sub-fringe is located)

As seen above, a sub-fringe was defined if the relative height of the peak was less than $75 \%$ of the previous half-fringe parameters. This factor was used to allow for the variation of the half-fringe intensity excursion as the blades scanned, as seen in fig. 31. The factor could be varied according to the uniformity of the fringe contrast across the blade. The higher this factor was, the more accurate the distinction between a normal maximum and a sub-fringe. A similar process was used for a sub-minimum.

SUBMIN $=(\operatorname{VAV}(A A)-\operatorname{VMIN}) /(V M A X-V M I N)$
IF SUBMIN $>.25$ THEN (Śub-fringe)

Once the sub-fringe was located the fringe counting factor was stopped. An indicator was also used to stop the system testing for another subfringe once it had been located as at times it was possible to get a false location if the SUBMAX and SUBMIN limits were set too close to 1 and 0 respectively.

For a sub-maximum the computer then continued to look for a true maximum and minimum. It then went on to locate the next minimum. This minimum had the same depth value as the previous value and the submaximum region increased beyond the depth at these minimum values. The component's shape was then plotted between these two values.

On completing this the next half-fringe pair was found where the intensity was increasing to a maximum value. In this case inversion had already occurred and so before the shape was calculated a number of parameters were modified.

$$
\begin{aligned}
& W=-1 \\
& S T A R T=S T A R T+C O U N T-1 \\
& \text { COUNT }=0
\end{aligned}
$$

W changed the direction of the value of the depth $Z$ with increase in contour fringes.

The START parameter, the factor which related the shape to the reference plane, was updated by adding the value of (COUNT-1) to it. This fixed the inversion point with respect to the reference plane. COUNT was then set to zero so that it could operate in the normal way of fringe counting but now the depth was going in the opposite direction. The equation of the component shape was of the form

$$
Z()=(S T A R T+(W * Y *(C O U N T+R))) * D E L T A * \cdot 5
$$

Hence the depth beyond the inversion region was subtracted from the depth calculated from the updated START parameter.

A similar process was adopted for the sub-minimum fringe location. Details of both procedures are shown in Appendix 1. Many of the printout statements (PUT EDIT) were not required in the final programme but were used for fault locations in the development of the programme and have been included in Appendix 1 to make the programme more easily followed.

### 4.6.2 Gradient Inversion Location.

This procedure calculated the inversion point when it occurred in the region of a full maximum or minimum contour region. The actual inversion point always occurred at either an intensity maximum or minimum, including sub-fringes, because of the depth function symmetry at the depth inversion. Consequently for inversion at a true maximum or minimum it was sufficient to be able to say whether or not the inversion was going to occur at the end of any particular half-fringe set of calculations. As with the sub-fringe effect the inversion point in test was only carried out over a specific region of the blade.

Gradient inversion location was formed by examining the rate of change of depth $Z$ of the points immediately preceding the actual point being plotted. This could be achieved calculating the gradient between the point being plotted and its immediate predecessor in the form

Gradient $D Z=(Z(L)-Z(L-1)) /(E X(L)-E X(L-1))$

This was not sufficiently reliable under practical conditions due to the noise and slight inaccuracies within the system which caused DZ to equal zero or go negative prematurely.

To overcome this a more detailed estimation of the gradient of the
shape was taken by operating a straight line fit to the blade shape for a number of points preceding the one being plotted by the method of least squares, Brownlee [39].





This was operated for the 6 points before the point that was being plotted. The gradient $D Z$ was compared to a given limit GRAD and if DZ < GRAD then the inversion point would occur at the end of the halffringe being plotted. (GRAD was always positive.) When this occurred a number of parameters were reset as with the sub-fringe inversion location.

```
W=-1
START = START+COUNT
COUNT = 0
```

After this normal shape plotting was resumed.
The same method would be used if the inversion point occurred at either a maximum or minimum intensity point.

There were occasions, if the limit GRAD was set too high, when the noise in the intensity input signal triggered the inversion procedure prematurely and as the program continued a sub-fringe value would be
subsequently located. In the event of this happening on the IBM 360 system the programe was made to restart with the knowledge that the inversion point would be formed by a sub-fringe. For a smalle, computer the programme would be made to return to the point where the premature inversion occurred and then recalculate the shape from that point knowing that the inversion would be in a sub-fringe form.
4.7 SYSTEM REFINEMENTS.

The shape plotting programme operated from the first maximum to minimum fringe pair which left the shape of the part of the blade up to extrapolation this area unknown. This was overcome by in which a straight line fit was applied to the shape plotted from the first few contours by the method of least squares and then this was extended backwards to the edge of the blade. The edge was defined by the position in which the intensity voltage went above a given threshold value as the scan went from a dark background to the blade.

On the concave surface of the turbine blade aerofoil there was a second direction inversion region close to the leading edge. In this case the point of inversion was located by measuring the spacings between the half fringe peaks and troughs and locating the region of the maximum spacing. This was not particularly effective because of the rapid change of depth in this region and the decrease in intensity signal level as the edge was approached as seen in fig. 31. An approximation method was used in which it was assumed that the direction inversion occurred at a particular position from the leading edge and the inversion procedure was operated accordingly.

Other possible improvements to the technique depend upon the application for which the system was designed. It would be possible for the computer to control the scanning interface during the scan but this would greatly increase the complexity of the system. A simple method of
controlling the number and density of the point calculations as the scan was operated would appear more practical so that the detail of the analysis of different parts of the blade could be controlled. Considering the routine application of the overall inspection system a computer must be used to control the whole inspection operation. The computer would indicate when the system was ready to accept the blade, check that the blade was loaded correctly, trigger the scanner to operate, analyse the blade and state when the cycle had been completed.

### 4.8 THREE-DIMENSIONAL ANALYSIS.

The turbine blade aerofoils were three-dimensional and the detailed analysis required a shape profile at a number of positions along the axis of the blade to build up the three-dimensional information. The threedimensional link between the scans could be achieved by two methods; scanning in the direction at right angles to the normal scan and counting fringes or by having a physical marker at each scan position which relates the first point on the blade to a given reference plane.

The former method would mean scanning the blade parallel to the axis and across the grid lines of the contouring grid which would introduce a noise factor but this could be reduced by either accepting information points that occur only in between the grid lines, or just accepting the values of the intensity peaks, which would be modulated by the contour fringes. This would relate a particular value across each scan and so modify the set parameter START to a derivation of this value. By this means three-dimensional information could be built up.

The second method would perform the same function and could be produced by using a number of arms from a reference plane that were spring loaded against the surface of the blade as shown in fig. 33. The thickness of the levers was made to be equal to one contour depth interval.

### 4.9 TURBINE BLADE DIMENSIONAL REFERENCES.

The reference system used on turbine blades was defined by a point ' P ' on or just below the bottom of the fir tree root through which the see sketeh.
axis of the blade runs ${ }_{\wedge}$. This point was also on a line that ran parallel to the fir tree root in a plane normal to the blade's axis. This line was defined as datum 'D'. The tip of the blade was defined by a point on the face of the shroud that would be visible in the contouring system. This point was a specific distance from the axis of the blade measured normal to the plane of datum D. The aerofoil was defined with respect ( $\mathrm{A} E$ ) to a plane ${ }_{\lambda}$ parallel to the blade's axis and set at a specific angle, rotated about the point $P$ and normal to the blade axis. The angle was dependent upon the blade design.

For inspection by optical contouring the turbine blade mounted in a kinematic mount on the base of the fir tree root to locate the reference point $P$ and the datum line $D$. The axis of the blade was located by the reference point on the shroud by a point in this position on the mounting jig. The whole of the blade was then held in position kinematically with the blade's axis in a horizontal plane normal to the direction of observation.

The dimensional depth of the aerofoil was related to the reference point on the shroud by the mechanical probes as shown in fig. 33. The reference plane of the optical system was not necessarily the one which defined the aerofoil on the drawing as the blade was rotated about its axis to present the most suitable contour pattern on the aerofoil for the contouring system. This angle was set for each type of blade. It would be a simple arithmetical process within the computer to relate the aerofoil dimensions to the reference plane set by the optical system from that defined on the drawing.

The blade would be rotated about its axis through $180^{\circ}$ to inspect

the other surface of the aerofoil in a similar manner.
The basic concept of this analysis programe was straightforward but the logic within the programme to permit analysis of a practical intensity scan signal with all the possible variations and factors involved increased the overall complexity. However, all the control indicators were necessary to ensure that the programme would function correctly for the number of practical variations that occur. Details of the use of this programme is given in section 5.3.

### 4.10 OBJECT SHAPE COMPARISON.

It has been established that direct shape subtraction was the most satisfactory method of comparing two objects for this method of inspecting turbine blades. A number of comparison methods were investigated. These were direct depth subtraction, co-ordinate system manipulation and sectional moment effects.

### 4.10.1 Depth Subtraction.

This provided identical information to direct optical contour subtraction as discussed in section 3.2 but with greatly increased accuracy. The co-ordinate array of a scan across the reference object could be of the form RZ() and REX() and the corresponding scan across the production object would be of the form $Z()$, EX( ). The values of the distancesalong the scan $\operatorname{REX}($ ) and $E X($ ) did not necessarily correspond because of the variations that occurred in the rate of scan across the blade and the total number of point measurements taken with the mechanical scan system as discussed in section 404 .

For this subtraction method for each value of $\operatorname{EX}(I)$ on the production shape the nearest value of $\operatorname{REX}($ ) was located and the exact Value of $\mathrm{RZ}($ ) corresponding to the scan position $E X(I)$ was found using an extrapolation of the form
$R E F Z=R Z(U)-\left((\operatorname{REX}(U)-\operatorname{EX}(I)) \times \frac{(R Z(U)-R Z(U-1))}{(\operatorname{REX}(U)-\operatorname{REX}(U-1))}\right.$
where $U$ was the number of the point in the reference array that is the nearest value to the value $\operatorname{EX}(\mathrm{I})$ in the production array.

The difference between the shapes was given by the array

$$
\begin{equation*}
\operatorname{ERROR}(I)=Z(I)-\operatorname{REFZ} \tag{79}
\end{equation*}
$$

This gave the point by point difference between the shapes. The difference normal to the actual surface of the blade was given by applying a cosine correction to the value of $\operatorname{ERROR}(I)$. The value of cosine was calculated from the shape of the surface with respect to the scan direction $Z$.

A linear fit to the values within the ERROR(I) array by the method of least squares gave the general trend of the difference but from this information it was not possible to obtain an accurate assessment of the overall difference between the object and its reference shape. There was not sufficient information to isolate the dimensional difference possibilities because no account had been taken of the differences in the direction of the scan. Ideally the overall difference should be given as the displacements in the $x$ and $z$ directions plus an angle of rotation.

### 4.10.2 Co-ordinate System Manipulation.

This technique was a more appropriate means of providing the data for the overall difference as a co-ordinate shift plus an angle of rotation. In this case the changes to the co-ordinate system of the production component to superimpose its shape on to that of the reference shape would be calculated. To achieve this, consider a point $(x, z)$ on the co-ordinate system $x, z$, fig. 34a. This system is displaced by an amount ( $p, q$ ) with respect to the reference system ( $X Z$ ) and is also rotated by an angle $\alpha$ with respect to the reference. The point $P(x, z)$ on the ( $x z$ ) system is equivalent to the point ( $x, z$ ) on the reference
co-ordinate system whose values are given by

$$
\begin{align*}
& X=x \cos \alpha-z \sin \alpha+p  \tag{80}\\
& Z=x \sin \alpha+z \cos \alpha+q \tag{81}
\end{align*}
$$

In the case of the shape of a turbine blade this had to be applied in two stages. Firstly a point on both the reference shape and production shape must be identified as identical points, fig. 34b. Such a point could be the starting point on each blade, on the trailing or leading edge. These two points would be superimposed by solving the equations

$$
\begin{align*}
& \text { REX }=E X+p  \tag{82}\\
& R Z=Z+q \tag{83}
\end{align*}
$$

for $p$ and $q$.
The production shape array would then be modified by the effects of $p$ and $q$. The angle $\alpha$ would be found by an iteration process of estimating an approximate angle through which the co-ordinate system should be rotated. Then the effects of the rotation would be examined and a better estimate of the angle made. This iteration loop would be continued until the required accuracy was achieved. The iteration function could be made to converge rapidly to minimise the number of iterations but the method has a number of fundamental faults.
(a) It was solely dependent upon the integrity of the two points at the edge of the blade for the superimposition of the two shapes. In practice these points would not always be in the same position with respect to the main part of the blade. This would be particularly true of blades that have been affected by thermal fatigue where the trailing edge may have been grossly distorted. There would be no other reference points on
the blade that would be suitable unless the blade was deliberately marked. Marking the blade would not be practical for a production system.
(b) To gain more accurate information and improve on the fit obtained by superimposing the end points, additional iteration processes would be involved for both the $x, z$ comordinate shifts and angle.
(c) As iteration processes were involved it would be liable to take up more computer time than a direct approach and hence increase the overall inspection time.

The practical problems of the position of the end points with respect to the main section of the blade and the number of iteration processes involved made this technique impractical in comparison to the sectional centroid technique. 4.10.3 Sectional Centroid Superimposition.

At any given scan position along the axis of the turbine blade aerofoil both the concave and convex surfaces could be scanned and linked together within the computer to form the complete section of the blade. For the whole sectional area the basic shape of the reference section and the corresponding section on the production blade would be the same. The actual shape differences would be small as in practice the production surface would be only of the order of $0.020^{\prime \prime}$ larger than the reference surface at any one point for a blade section around $1.0^{\prime \prime}$ wide and up to 0.301 maximum sectional thickness. Under these conditions the fundamental assumption could be made that the Centroids of both sections would remain in the same relative positions with respect to their surfaces. Hence for a blade section that had an overall displacement with respect to the reference section plus some localized differences the overall displacement could be calculated by superimposing its centroid on to that of the reference surface. The required co-ordinate shift would
give the overall difference between the two in the $x$ and $z$ directions. The angular displacement between the two could be calculated from the angular difference between the principal axes of each of the two surfaces.

In addition to these parameters the comparison of the area of the production and reference sections would give an indication of the integrated size difference between them.

To examine these properties in greater detail, consider the turbine blade section in fig. 35a. The co-ordinate system is the depth Z plotted against the position along the scan $X$ as derived from the practical scanning system. Both the concave and convex surfaces were formed to give the complete section. This could be achieved by suitable manipulation of the surface data within the computer. The concave surface is represented by curve $z_{1}$ and $z_{2}$ represents the convex surface.

The area of the blade section is given by taking individual elements in the $x$ direction, Massey [40] and is given by the equation:-

$$
\begin{align*}
\text { Area } & =\sum_{n=1}^{m} \frac{1}{2}\left[\left(z 2_{n}+z 2_{n+1}\right)-\left(z 1_{n}+z 1_{n+1}\right)\right]\left[x_{n+1}-x_{n}\right] \\
& =\sum_{n=1}^{m}(\text { Area } d x)_{n} \tag{84}
\end{align*}
$$

where $m$ is the number of elements that the surface is divided into. This can also be expressed as a sum of a series of elements in the $z$ direction.

$$
\begin{align*}
\text { Area. }= & \sum_{n=1}^{m} \frac{1}{2}\left[\left(x 2_{n}+x 2_{n+1}\right)-\left(x 1_{n}+x 1_{n+1}\right)\right]\left[z_{n+1}-z_{n}\right] \\
& \sum_{n=1}^{m}\left(\text { Area }_{d z}\right)_{n} \tag{85}
\end{align*}
$$

The position of the centroid of this area with respect to each of the two axes is calculated by taking the first moment of each element about the respective axis and dividing by the total area.


$$
\bar{z}=\frac{\sum_{n=1}^{m}\left(\text { Area }_{d z}\right)_{n} \cdot \frac{1}{2}\left(z_{n}+z_{n+1}\right)}{\sum_{n=1}^{m}\left(A r e a_{d z}\right)_{n}}
$$

Using the above equation the centroid of both the reference and production section can be found with respect to the co-ordinate system of the scanning device.
, The effective radius of gyration about the two axes are given by equations

$$
R X^{2}=\frac{\sum_{n=1}^{m}\left(\operatorname{Area}_{d x}\right)_{n} \cdot\left[\frac{1}{2}\left(x_{n}+x_{n+1}\right)\right]^{2}}{\sum_{n=1}^{m}\left(\operatorname{Area}_{d x}\right)_{n}}
$$

and

$$
R z^{2}=\frac{\sum_{n=1}^{m}\left(\text { Area }_{d z}\right)_{n} \cdot\left[\frac{1}{2}\left(z_{n}+z_{n+1}\right)\right]^{2}}{\sum_{n=1}^{m}\left(\text { Area }_{d z}\right)_{n}}
$$

The equivalent to the product of inertia over this area about the origin of the co-ordinate system would give the relation of the angular orientation of the surface with respect to the co-ordinate system. This would produce a radius of gyration of the form, Timoshenko [41],

$$
R H Z^{2}=\frac{\sum_{n=1}^{m}\left(A r \& a_{d z}\right)_{n} \cdot\left[\frac{1}{4}\left(x_{n}+x_{n+1}\right)\left(z_{n}+z_{n+1}\right)\right]}{\sum_{n=1}^{m}\left(A r e a_{d z}\right)_{n}}
$$

The radius of gyration of the surface about any line parallel to the respective axes and a distance $k$ from it can be calculated by the parallel axes theorem.

$$
\begin{equation*}
R X^{2}=k^{2}+R X_{k}^{2} \tag{91}
\end{equation*}
$$

Hence using this theorem the radius of gyration about the lines parallel to the axes through the centroid is given by

$$
\begin{align*}
& \mathrm{XXG} G^{2}=R X^{2}-\bar{X}^{2}  \tag{92}\\
& R Z G^{2}=R Z^{2}-\bar{Z}^{2} \tag{93}
\end{align*}
$$

Similarly the radius of gyration of the product of inertia can be transferred from the origin to the centroid by the equation

$$
\begin{equation*}
R H X Z G^{2}=\left(R H X Z^{2}-\bar{X} \cdot \bar{z}\right) \tag{94}
\end{equation*}
$$

The total radius of gyration of the section about the centroid is:-

$$
\begin{equation*}
R_{G}=\left(R X G^{2}+R Z G^{2}\right)^{\frac{1}{2}} \tag{95}
\end{equation*}
$$

If the product of inertia was taken for a series of co-ordinate systems rotated about the origin the value of the radius of gyration will vary between a maximum and minimum value to form an ellipse, known as the ellipse of inertia. The position of the radius where the value would be either a maximum or a minimum would be known as the principal axis of inertia and occurs when the product if inertia with respect to this co-ordinate system is equal to zero. Hence the principal axes form axes of symmetry of the cross section.

The angle, $\phi$, of the principal axes with respect to the comordinate system xz is given by Timoshenko [36].

$$
\begin{equation*}
\tan 2 \phi=2 \cdot I_{x z} /\left(I_{x}-I_{z}\right) \tag{96}
\end{equation*}
$$

where $I_{x}$ and $I_{z}$ are the inertia values about the $x$ and $z$ axes respectively and $I_{x z}$ is the product of inertia.

Referring this to the computed values the angle calculated at the centroid of the surface becomes

$$
\begin{equation*}
\tan 2 \phi=\frac{2(R H X Z G)^{2}}{\left(R Z^{2}-R X 6\right)} \tag{97}
\end{equation*}
$$

By trigonometiry

$$
\begin{equation*}
\tan \phi=\left(1+\cot ^{2} 2 \phi\right)^{\frac{1}{2}}-\cot 2 \phi \tag{98}
\end{equation*}
$$

If the angular position of the principal axes of both the production and reference surfaces are found the angular difference between these axes gives the actual angular difference between the two surfaces, fig. 36.

Hence the angular difference is given by

$$
\begin{equation*}
\tan \alpha=\frac{\tan \phi_{\text {prod }}-\tan \phi_{\text {ref }}}{1+\tan \phi_{\text {prod }} \tan \phi_{\text {ref }}} \tag{99}
\end{equation*}
$$

where $\phi_{\text {ref }}$ and $\phi_{\text {prod }}$ are the angles of the principal axes of the reference and production surfaces respectively.

This angle plus the co-ordinate corrections found by superimposing the centroids of the two sections would give sufficient information to describe the overall difference between any component and its master shape. To compare the shape of the production blade to its master in greater detail the co-ordinate system of the production blade should be modified according to the overall differences to superimpose the production on to master shapes within the same co-ordinate system.

To modify the production system the effect of the linear differences between the shape centroids was considered first. This formed an intermediate co-ordinate syंstem whose origin was displaced with respect to the production co-ordinate system by ( $\mathrm{P}_{\text {int }}, Q_{\text {int }}$ )

$$
\begin{align*}
& P_{\text {int }}=\bar{x}_{\text {ref }}-\bar{x}_{\text {prod }}  \tag{100}\\
& Q_{\text {int }}=\bar{z}_{r e f}-\bar{z}_{\text {prod }}
\end{align*}
$$

( $\bar{x}_{r e f}, \overline{\bar{z}}_{\text {ref }}$ ) and ( $\bar{x}_{\text {prod }}, \overline{\bar{z}}_{\text {prod }}$ ) $=$ the co-ordinates of the reference on production centroids respectively.

The modified co-ordinates of the production shape ( $\mathrm{f}_{\text {int }}, \mathrm{z}_{\text {int }}$ ) were given by

$$
\begin{equation*}
\left(x_{i . n t}=x_{\text {prod }}+P_{\text {int }}, z_{\text {int }}=z_{\text {prod }}+Q_{\text {int }}\right) \tag{101}
\end{equation*}
$$

The displacement of the origin of the intermediate co-ordinate system produced by the angular rotation $\alpha$ about the centroid was given by

$$
\begin{align*}
& P=\bar{x}_{r e f}-\bar{x}_{r e f} \cos \alpha+\bar{z}_{r e f} \sin \alpha  \tag{102}\\
& Q=\bar{z}_{r e f}-\bar{x}_{r e f} \sin \alpha-\bar{z}_{r e f} \cos \alpha  \tag{103}\\
& \left(\bar{x}_{\text {int }}=\bar{x}_{\text {prod }}+p_{i n t}=\bar{x}_{r e f}\right)
\end{align*}
$$

Combining the effects of the intermediate translation and the rotational effect the co-ordinates of the production shape were modified by the equations

$$
\begin{aligned}
\operatorname{XMOD}() & =\left[E X()+P_{i n t}\right] \cos \alpha+\left[z()+Q_{i n t}\right] \sin \alpha-P \\
\operatorname{ZMOD}() & =-\left[E X()+P_{i n t}\right] \sin \alpha+\left[z()+Q_{i n t}\right] \cos \alpha-Q \quad \ldots(104)
\end{aligned}
$$

Having established the overall differences between the production and reference blade sections and modified the production co-ordinate data accordingly the localized differences could be calculated by direct co-ordinate subtraction.

This technique could also be applied to $\frac{a d j u s t}{j u s t}$ the shape curve of either the concave or convex surface of the blade, if the curve was considered as an area or line with an infinitely small width. In this case the length of the curve would be equivalent to the area of the blade section. Thus

$$
\begin{equation*}
\text { Curve length }=\sum_{N=1}^{M} d s_{N} \tag{106}
\end{equation*}
$$

where ds is the length of each of the m elements into which the curve is divided.

Using the present computer programme nomenclature for the Nth point in the shape co-ordinate array the yalue of ds between two adjacent points would be:-

$$
\begin{equation*}
\mathrm{ds}_{\mathrm{N}}=\left[(\mathrm{Z}(\mathrm{~N})-\mathrm{Z}(\mathrm{~N}-1))^{2}+(E X(N)-E X(N+1))^{2}\right]^{\frac{1}{2}} \tag{107}
\end{equation*}
$$

Taking the first moment about the Z axis to calculate the centroid

$$
\begin{aligned}
& \bar{x}=\frac{\sum_{N=1}^{M} d s_{N}-\frac{1}{2}(E X(N)+E X(N+1))}{\sum_{N=1}^{M} d s_{N}} \\
& \text { Also } \quad \bar{z}=\frac{\sum_{N=1}^{M} d s_{N} \cdot \frac{1}{2}(Z(N)+Z(N+1))}{\sum_{N=1}^{M} d s_{N}}
\end{aligned}
$$

Similarly the second moments about the coordinate system axes


and the radius of the product of inertia would be:-


Using these values and the procedure for calculating the rotational and displacement differences that have been previously discussed it would be possible to calculate the change in co-ordinate position of any point on the blade's surface, within the scan, or any point within the XZ plane.

The use of the centroid location technique for the individual shape curves enabled the problems discussed in the co-ordinate system manipulation method section 4.10 .2 to be overcome.

### 4.10.4 Three-Dimensional Assessment.

Using the above methods of analysis it would be possible to build up an array of the values of the difference in the $X$ and $Z$ direction and the angular difference for each of the sections of the blade that had been scanned. Considering each of these parameters individually a linear equation fit could be applied to estimate the general trend along the axis of the blade as shown in fig. 35b. Or if more detailed information was required a spline fit could be applied to all the points in one array. In a spline fit a line would be drawn between the first two points in the graph or array. A similar line would be drawn through the second and third points. The spline would be defined as the co-ordinate of the second point in the array plus the angle between the line through the first two points and the line through the second and third points as shown in fig. 35b. The next pair of points would be considered and the values calculated and these parameters are continued for all the points
in the array. The first point in the spline curve would be the co-ordinates of the first point plus the angle of the line through the first and second points with respect to the axis. Such a function would enable the deviation from the general trend to be shown clearly.

If a spline fit-was constructed for the overall deflection in the $Z$ direction of each section scanned the variation in the values of the angle as defined in fig. 35 b , would indicate the overall distortion of the blade. If $\psi$ was approximately zero for all the sections, excluding the first, this would indicate that the aerofoil was bent backwards about a point towards the blade root and the overall aerofoil was not distorted, whereas if was other than zero the blade had a processive bend or curved structure. The type of bending effect on the blade would be particularly interesting when the system was being used to examine thermally stressed blades. The blade twist could be analysed in a similar manner, to measure whether the twist was progressive or not.

A more rigorous and usual spline fitting procedure would be the application of the cubic spline, Ahlberg [42]. This would apply a smoothing out procedure between the discrete points on the deflection curve by applying a polynomial equation to the points. This would have particular advantages in calculating the first and second derivatives of the deflection to form the strain and bending moments, and therefore could be effectively used for the examination of a blade that had been thermally or mechanically stressed. The cubic spline has been used in holographic strain measurement by Brandt [43].

The type of analysis in three dimensions primarily depends on the application.

The hypothesis stated above was examined in greater detail using a theoretical model on the computer. A shape was formed mathematically to
represent the blade section. This shape was displaced by known amounts and the point by point co-ordinates recalculated. The original shape represented the reference object and the displaced shape represented the production object. Using the area, first and second moments of the area, as described above, the distortion parameters were calculated. These values were comparedoto the actual dintsplaceinentst thatchadibeen appliedto thetoriginal shapesilied to tho 1

An elliptical shape was used as this could be generated conveniently within the computer using the equation

$$
\begin{equation*}
\frac{(x-p)^{2}}{a^{2}}+\frac{(y-q)^{2}}{b^{2}}=1 \tag{113}
\end{equation*}
$$

where $a=1, b=0.4, p=1.0, q=0.6$ to form the ellipse in approximately the same position as that of a blade section. The centre of the ellipse ( $p, q$ ) was displaced and the ellipse was tilted through a small angle, and these parameters were used to calculate the new point by point co-ordinates of the now displaced shape. .

The centroid and second moments calculated for the reference shape with respect to the two axes of the reference system and the components of the radius of gyration about the centroid were calculated. The program used is shown in Appendix 2.

The same set of parameters was calculated for the production surface, using interpolation technique when calculating the values of the area elements, Appendix 2. From these results the displacement was calculated and compared with the actual displacement applied. This was also applied to a line forming one half of the elliptical surface. The results of this examination are shown in Table 2, and Appendix 2.

The calculated linear and angular displacements for the single curve of half of the ellipse agreed exactly with the applied distortion
for all the cases considered. There were, however, some slight differences between the calculated and applied distortion when the whole area of the ellipse was considered. These were due to the errors built up in calculating the surface area and second moments of the displaced. shape when using a finite number of area elements. The calculations showed that these methods gave the overall dimensional differences in the form of co-ordinate system change as predicted in section 4.10.

## Chapter V. <br> Experimental Results and Television Interface.

### 5.1 INTRODUCTION.

In this chapter the experimental work carried out on the contour image scanning system and the results obtained from the computer using contour intensity information is described. The slow scan television camera interface and digital recorder system that were used to increase the input rate of the information into the computer and the results obtained are also described.
5.2 IMPROVEMENT OF MOIRE CONTOUR FRINGES.

The objective of this work was to improve the quality of the contour fringe intensity traces that were obtained when the contoured image was scanned. Both the moiré shadow and fringe projection image scan techniques were studied and develóped.

Much of the initial work was carried out using these techniques before the television interface was developed and a mechanical scan system was used, This has been described in section 3.4.2, and is shown in fig. 24. The quality of the contour fringe intensity profiles obtained by scanning parallel to the moire grid lines was improved by decreasing the width of the slit, normal to the direction of the scan, to a minimum < . $005^{\prime \prime}$ for an image to object magnification of approximately 0.95 in the observing camera. The slit length was approximately $0.020^{\prime \prime}$. The limit on these parameters was the noise produced on the photomultiplier as the gain was increased to accommodate the smaller signal levels. However, on a typical turbine blade aerofoil section with a slit some $\cdot 20 \mathrm{~mm}$ wide and 40 mm long it is possible to obtain good quality contours at $0.015^{\prime \prime}$ depth interval. At 0.010" intervals the

- spatial frequency of the fringes was high and could not be effectively resolved by the photomultiplier slit. Further examination of the variation of fringe quality with depth interval on the turbine blade
showed that if the contour depth interval was increased to the region of $0.015^{\prime \prime}$ to $0.020^{\prime \prime}$ the quality of the fringe intensity profile greatly increased and a true triangular waveform was obtainable, an example of which is shown in fig. 37. Since the intensity function followed the theoretical function more closely and the maximum and minimum intensity values of the contours were more clearly defined, it was possible to form the interfringe depth values to an accuracy of $5 \%$ of the contour depth interval and hence retain the measuring accuracy of $\pm 0.001^{\prime \prime}$ and in some cases improve on it.

Thus for the dimensional analysis of turbine blades whose total depth varjie from $0.2^{\prime \prime}$ to $1.0^{\prime \prime}$ the most accurate depth information six would be obtained by using a contour depth that provided a good quality fringe intensity distribution that followed the theoretical triangular intensity function rather than generating contours with a small depth interval which became difficult to resolve and the intensity distribution became distorted by diffraction and penumbra effects.

Further examination of the fringe projection/scanning method of contour generation, using the photomultiplier mechanical image scanning system, confirmed that this was a viable and attractive technique. It was possible to make good quality moiré grids with a varying spatial frequency across the grid to comply with the requirements discussed in section 2.7.6. When projected, the grid lines were focused"im:a plane at $45^{\circ}$ to the optical axis of the projection system. The blade was placed in the focal plane of the projected fringes and the contouring system formed as shown in fig. 15. It was possible to obtain good quality contour fringe scans with contour depth intervals down to $0.010^{\prime \prime}$. The photomultiplier aperture size was found to be most effective and produce better contrast fringes if it was less than half the period of the projected fringes. Reducing the aperture size takes
into account the cosine effect of the projected fringe width produced by the slope of the surface of the object.

Very good quality intensity scans were produced, as shown in fig. 38, with a contour interval of $0.010^{\prime \prime}$. The maximum and minimum points of the fringes were well defined and the intensity distribution was triangular. The overall variation in the intensity of the fringe peaks across the blade was less than that of the moiré shadow contouring technique. Also the clarity of the high spatial frequency fringes was better than that of the moire shadow contouring. The image scanned in fig. 38 is shown in fig. 17.

In addition to the improvements in contour fringe quality obtained by the fringe projection system it had the purely practical advantage in handling the turbine blade in so far as the blade was not close to the actual moiré grid. This meant that more space would be available for the operator in which to handle the blade when it was being loaded and rotated to inspect both surfaces.

An investigation into the range of surface finishes which would produce reasonable contour fringes showed that the surface must have good light scattering rather than reflecting properties. The most efficient surface finish was matt white paint which could be easily applied from a spray can. The majority of turbine blade surface finishes would be suitable including the yapour blasted surface on a completed turbine blade and the blackened surface of a blade that had been run in an engine.

Examining both the moire shadow and fringe projection methods of contouring for use in conjunction with the television interface on a routine basis, the fringe projection system was found to be the more suitable. In the television system the size of the integrating area of the scanning spot for a television tube with a resolution of 800 lines
across the frame of $0.75^{\prime \prime}$ was approximately $0.0005^{\prime \prime}$. For a turbine blade an object to vidicon image demagnification of the order of 5:1 was required to enable the whole of the aerofoil section to be observed. Thus for 100 lines/inch spatial frequency of the fringes on the object (for $0.010^{\prime \prime}$ contour depth interval) the width of half the grid period on the vidicon was $0.001^{\prime \prime}$. This was twice the size of the scanning spot. Hence under these conditions the fringe projection would be the ideal form of contouring system. The moire erid in the moiré shadow technique presented a possibility of not being able to obtain any contour fringes if the scanning line position coincided with the dark section of the moire grid. Also because the spot size was less than the half period of the moiré grid the grid was not acting as any form of demodulation system and so was redundant. The alignment of the scan direction parallel to the grid lines was critical in both contouring systems. If the alignment was not correct for the moire shadow method, contour information would only be obtained during part of the scan. The effect on the fringe projection system was less crucial as misalignment added a varying depth error on to the contour fringes. From these observations it was concluded that the moire fringe projection system was the most suitable for use with the television interface.
5.3 COMPONENT TOTAL DEPTH EFFECTS.

The effects of the variation of the contour depth interval with the depth being plotted, fringe number, were investigated for the blades that were examined during the experimental work. This was basically a study of to what extent it could be assumed that the depth contour interval was constant for any contour fringe number, i.e. that the contour depth for the Nth fringe was given by equation 35 , section 2.6.1.

$$
z=\frac{\mathrm{N} \cdot \mathrm{p} \cdot \mathrm{~h}}{\mathrm{~d}}
$$

If $h$ was not much greater than the depth $z$ the value of $z$ from the reference plane was given by equation 34.

$$
z=\frac{N p h}{d-N p}
$$

For the laboratory contouring system $\mathbf{h}=35.5^{\prime \prime}$.
The depth excursion of the majority of turbine blades that would be examined by the contouring system was less than $0.50^{\prime \prime}$. This represented $a \pm 0.25^{\prime \prime}$ depth excursion from the image plane of the projected grid, fig. 15a, on the fringe projection/image scan contouring system. The blade under examination was always positioned so that the image plane of the projected grid was approximately midway along the total depth excursion of the blade. Hence both positive and negative values of N were used. This made best use of the depth of focus of the fringes either side of the image plane.

For a contour depth interval of $0.010^{\prime \prime}$ generated using a 100 cycles/. inch moiré grid, $p=0.01^{\prime \prime}, d=35.6^{\prime \prime}$, the depth represented by the +25 th fringe on the uncorrected equation $z=0.250^{\prime \prime}$ and for the corrected equation $z=0.251^{\prime \prime}$. The resultant error was $0.001^{\prime \prime}$ which was within the measuring accuracy of the overall system. Hence it would not be necessary to correct the depth value against fringe number for the majority of turbine blades. If the depth excursion was increased to $\pm .50^{\prime \prime}$ the error would be $0.005^{\prime \prime}$ and so the correction equation would be required.

The requirement to use the corrected equation for calculating the fan
depth $z$ was highlighted when a model $\lambda_{\lambda}$ blade was examined for a development department within the Company. This was approximately 12 " long and had a maximum depth excursion of $2.5^{\prime \prime}$ over the area of interest.

A nominal contour interval of $0.040^{\prime \prime}$ was used to examine the blade. The grid frequency was 50 cycles/inch. $h=35.5, d=17.77$. For the
fringe number $N=+30$ representing maximum fringe number encountered, the uncorrected depth was $z=1.200^{\prime \prime}$ and the corrected depth was $z=1.240^{\prime \prime}$. The difference was $0.040^{\prime \prime}$ which in terms of a percentage error was $3.3 \%$ but in terms of the actual depth dimension this was a large error for a system with an expected 0.004 " measuring accuracy. Hence in this case the corrected value of depth had to be used.

It was not difficult to locate the image plane of the projected moire grid with respect to the blade and to keep note of the number of the fringe with respect to the reference plane that was being measured. A second reference plane was formed which was $\mathbb{N}$ fringes in front of the imaged grid with respect to the observing optics, i.e. the fringe number $N$ and the depth value $z$ were negative. The second reference plane was a physical marker. A spring loaded lever was made to run from the second reference plane to the edge of the blade nearest to the observer. This was similar to the reference location probes shown in fig. 33. By counting the fringes from the second reference plane to the blade the fringe number at any part of the blade was known.

Another factor was found to affect the accuracy of the contouring system when the laboratory system was used to study objects with a large depth such as the model fan blade and this was variation in the lateral magnification of the image produced by the depth values 2. These produced errors in the scan distance along the blade, i.e. distance $x$. This effect was the same as those that caused the non-linear effects in the production of the grids for the fringe projection technique, section 2.7.6.

Consider the conventional single lens imaging system that would represent the camera used to image the contour object on to the photemultiplier scanning plane similar to that shown in fig. 15b.
$U=$ the distance of the image plane of the moire grid on the object from the camera lens.
$V=$ the distance of the image from the camera lens.
$M_{0}=$ the image magnification on the optical axis.
$M_{0}=\frac{V}{U}$

The distance $U$ would be modified by the shape of the object.

Hence

$$
U^{\prime}=U+Z
$$

Under these conditions the distance $X$ of the point on the object from the axis of the optical system corresponding to a distance $x$ from the optical axis on the scanned image is given by the function

$$
\begin{equation*}
X=\frac{X}{M_{0}}\left(1+\frac{\dot{Z}}{\tilde{U}}\right) \tag{114}
\end{equation*}
$$

For the example of the model blade the $X$ direction error on the object along the scan from the optical axis for the 30 th depth fringe was $3.3 \%$. This represented a $0.030^{\prime \prime}$ error in the blade in the $X$ direction at its edge, which was approximately 1 " from the optical axis.

This direction error would not be significant in the majority of cases on the measurement of turbine blades as this would represent an X directional error of $0.003^{\prime \prime}$ which would be less than the effective integrating area on the object of the scanning system. However, when components were inspected the likelihood of these two factors affecting the accuracy of the results should be considered to define whether corrective procedures were necessary.

During the manual plotting of the model blade and other similar components these correction factors were conveniently applied by the use of a small computer programme on the C.P.S. teleprinter system. The
fringe number and position along the resultant scan were typed in and then $X$ and $Z$ dimensions of the blade calculated and printed out. These corrections could also be incorporated into the main shape generation programme for use on deep objects.

The shape measurement made by contouring agreed with the manufacturing specification. Specific parts of the blade were also contoured using a difference direction of observation to reduce the overall dimensional depth and increase the measuring accuracy from $0.004^{\prime \prime}$ to $0.001^{\prime \prime}$. Details of the results cannot be included because of the advanced nature of this component.
5.4 SHAPE GENERATION BY COMPUTER.

The optical information to electronic interface used for this work was based on the photomultiplier scanning device described in section 3.4.2. This was used in conjunction with the moiré shadow method of contour generation. The photomultiplier output was digitized with the aid of a data logger and then put on to punched paper tape for input into the IBM 360 computer system.

The photomultiplier tube output and $X$ direction potentiometer voltage were fed into two channels of an Ampex FM tape recorder taking between 3 and 6 minutes to scan the blade. The time limit was set by the data logger system. The tape recorder was used for convenience rather than moving the moire rig to the data logger or vice versa.

The two output channels of the tape recording were replayed into the data logger via a switching system. The data logger system converted the input voltage to a frequency proportional to the voltage and counted the number of cycles over a given integration time. The number of cycles representing the voltage was digitized and punched on to paper tape. It had a speed of 2 points per second, the limitation being the paper tape punch, but it had a buffer store which enabled it to measure
two points rapidly, hold one while it punched the first, then punch the second.

Using this property, a switching device was constructed which every second fed two samples of the tape recorder output in quick succession into the data logger. Hence this system enabled the two channels of the contouring system to be read at second intervals resulting in between 180 and 360 co-ordinate pairs per blade scan for analysis. This number of points was sufficient to accurately represent the contour traces.

The system was initially examined using a test object with a Iinear slope set at $10^{\circ}$ to the contouring reference plane. This produced evenly spaced contours over the whole of the test surface. Both a triangular waveform fit and a $\sin ^{2} V$ fit were applied to the contour fringe intensity profile within the computer. The resultant shape co-ordinates derived by the computer produced a good linear shape when plotted out but the triangular waveform showed a larger deviation about the straight line than the $\sin ^{2} V$ fit. The integrating area used on the photomultiplier was relatively large and so reduced the intensity function of the contour fringes to an approximate $\sin ^{2} V$ function. Fig. 39 shows the error between the $10^{\circ}$ slope as plotted by the computer compared to a purely theoretical $10^{\circ}$ slope plotted against the distance along the scan using a contour depth interval of $0.020^{\prime \prime}$. The error increased with distance across the scan due to an inaccuracy in the calibration of the fringe depth that was used by the computer. However, the main area of interest was the deviation of the computed points from the general error curve due to the errors in the fringe intensity fit applied within the computer. This had a maximum of $\pm .001$ " representing an error of 0.05 times the contour interval and was better than initially predicted for the system.

Application of the system to turbine blades produced a clear, well-
defined set of shape co-ordinates for scans across blades at several positions along the blade axis. An example of the computer co-ordinate printout is shown in Table 3. A contour depth interval of $0.016^{\prime \prime}$ was used together with a $\sin ^{2} V$ intensity function. The overall spread of the shape co-ordinates was within $\pm .001^{\prime \prime}$. An example of the calculated shapes of two scans across the concave surface of the blade is shown in fig. 40.

To examine the dimensional differencing capabilities of the technique a turbine blade was rotated by a small amount about its axes between consecutive photomultiplier scans. Two separate angles of rotation were used, 44 minutes and 1 degree 28 minutes, to simulate approximately one and two contour fringes difference at the trailing edge of the blade. The contour data was transferred from paper tape to a magnetic tape storage unit within the computer facility for ease of handing on repeated computer runs. The computer programe was extended to use the first scan as the master shape and store the information and the second and third scans were compared to the master data. Because of the nature of the recording and digitizing system, each digitized point in the scan, say the nth, did not have the same scan position as the corresponding point, the nth, in the subsequent scan. Thus, the positions along the scan were exactly matched by extrapolation between the point co-ordinates on the master object.

The shapes of the reference scan and the scan with the blade rotated through $1^{\circ} 28^{\prime}$ is shown in fig. 41. The direct $Z$ direction dimensional differences between the reference scan and a rotated scan is shown in fig. 42.

A straight line fit by the method of least squares was applied to the error values. The errors did not have a linear trend across the blade as shown by fig. 42 because of the cosine effect of the differences
with respect to the normal to the surface of the blade and the $Z$ direction. Fig. 42 also illustrates the information that would be obtained from any direct optical moiré subtraction system.

The method of calculating the co-ordinates of the centroid of the blade shape for a scan across either the concave or convex surface of the blade and the angle of the principal axes of the shape as discussed in section 4.7 .3 was applied for the blade rotation. This was done for a number of scans at various positions along the axis of the blade.

An example of the computer output used to calculate the angle of rotation from the basic shape information is shown in Appendix 3. The first output array was that of the reference surface, i.e. the blade with zero angular rotation applied. The scan was taken across the convex surface in the chordwise direction at approximately the centre of the aerofoil. The second data array was for the blade rotated by 44'. From the two data arrays the difference in the positions of the two centroids and the angles of the principal axes were calculated as described in chapter 4. The overall difference was given as three parameters, $X$ direction difference, $Z$ direction difference and angular difference. The values of the various parameters such as the centroid position of the radii of gyration were also printed out mainly for information rather than direct use. The computer output of the overall differences illustrated that it was possible to obtain these values direct and they are the only values that are required to describe the overall differences between the shapes. For a production system the co-ordinate system of the production blade would be modified according to section 4.10 .3 using these difference values and then a point by point subtraction done to investigate local differences. In Appendix 3 similar calculations were performed for a second angular rotation of $1^{\circ} 28^{\prime}$. Table 5 shows the values of the angular rotation of the blade calculated from different
scans in other positions on the blade aerofoil. The computed values of the angular rotation agreed with the applied value to within $\%$. These results showed that the method of calculating the general shape differences by the centroid displacement and principal axes methods operated as predicted for practical objects. It was not possible to investigate these techniques for complete blade sections and other types of blade displacement because of the difficulty of arranging for all the recording and digitizing equipment to be available at one time and the time involved for other departments in such an operation. Further investigations were made using the television interface and magnetic tape input into the computer. These are discussed in section 5.6.

### 5.5 SLOW SCAN TELEVISION INTERFACE.

The objective of manufacturing a television optical to electronic information interface was to speed up the computer processing times to those that are acceptable to a routine inspection system, i.e. to enable the whole inspection process to be undertaken in less than one minute: In the system envisaged for automatic inspection, the computation would be done by a small on-line computer such as a PDP8. This would be capable of handling an input speed of greater than 1000 ten bit bytes per second and thus, to form an efficient system, the optical data must be capable of being obtained at approximately the same rate. These speeds obviously precluded any form of mechanical scanning devices, which also would have the disadvantage of moving parts. Consequently, a television system would be the most probable interface device for this application.

At normal scan rates of 25 frames/second with 625 lines/frame, the information output rate for a line of 800 points of information would be 12.5 MHz. This was three orders of magnitude above the rate at which the computer could read the data and well above the maximum rate to which standard Analogue to Digital converters would operate. The information could be
read out by observing the different points in sequential frames, but this would slow the system down below the required input rate. Consequently, the most efficient way of obtaining this information would be by slow scan television techniques.

The overall computer analysis system is shown in fig. 28 and due to industrial circumstances it was not possible to use a small on-line computer and so a digital magnetic tape recorder was used as a temporary storage medium between the television system and the IBM 360 computer system. Fig. 43 shows the layout of the television interface. The input data was of the form of a digitized light intensity value at each of 500 equispaced points across one line scan. This was obtained for up to 20 lines within one complete frame. The positions of these lines within the frame could be set manually. The speed of this system was set at 1000 points of information/second and was limited by the input rate of the tape recorder.

### 5.5.1 Television Camera.

The selection of the most suitable vidicon tube for this system was critical. The tube had to have a high spatial resolution, good spectral response in the visible and compatible with an Hg arc source, flexibie slow scan facilities and low noise. A survey of available vidicon tubes, as summarized in Table 4, concluded that the Thomson CSF 9892 one inch'slow scan vidicon tube was the most suitable. The tube's main attributes were as follows:-
a) High resolution 1000 IV lines in the tube centre under maximum electrode voltage conditions. The system uses electromagnetic deflection and focusing rather than electrostatic focusing and deflection, which increases the power requirements on the vidicon control circuitry over those of the more common electrostatic slow scan tubes, but the high resolution warrants its use.
b) Peak spectral response is at 430 nm and covers the whole visible
spectral band. The overall sensitivity is high, to enable the system to accommodate the low light levels when scanning dull black objects.
c) The effective photographic gamma, $\gamma$, over the light input region of 0.05 to Lux-seconds is between 0.9 and 1.0

The photographic gamma in this case is the gradient of the graph of the log of the output signal versus the log of the optical exposure on the photocathode (intensity $x$ time) similar to the Hurter-Driffield curve for photographic emulsions as discussed in section 3.2.1. When $\gamma=1$ the output signal is directly proportional to the optical exposure, and hence optical intensity for a given exposure time. Thus the triangular intensity function of the contour fringes produces a triangular output signal from the vidicon and hence no mathematical signal corrections are required.

### 5.5.2 Vidicon Scanning.

The vidicon deflection and focusing parameters were governed by the normal deflection and focusing coils surrounding the tube. These coils were supplied with the correct voltages by power amplifiers designed to operate at the slow scan rates rather than normal video scan speeds. The vidicon was scanned in the $X$ direction in 500 incremental steps with a linear spacing to give 500 video signal output values for digitization and storage.

The $X$ deflection coil voltage was controlled by the following system, referring to fig. 43. The control logic sent a pulse to the scan position unit to indicate a required shift to the next position. This pulse operated a nine bit binary counter and increased the count by one. The binary counter could vary from 0 to 499. A digital to analogue converter produced a voltage proportional to the counter value which was amplified and fed into the vidicon deflection coil. This scanned the vidicon accordingly in the required 500 steps.

When the X deflection counter reached a value of 499 and was then instructed to move to the next position, the counter emitted a pulse into the $Y$ deflection logic and then set itself to zero ready to start the next line. The $Y$ deflection consisted of a similar five bit counter which counted the line number and supplied the $Y$ deflection coils with the appropriate voltage via a second digital to analogue converter.

The scan system operated for 20 scans unless stopped at a particular line and point number that could be set manually in the control logic. The position of the line scans within the television frame could be set manually. There was an indicator on the control logic console which indicated the line and point number being interrogated during the scan.

### 5.5.3 Digitizing and Recording System.

The digitizing and control system was linked to a Racal Thermionic T.D.L.10,000 digital tape recorder. The control logic system, fig. 43, instructed the $X, Y$ scan to move to the next position, allowed a time for the system to settle and then instructed the vidicon output Analogue to Digital converter to operate. The A/D converter was a 10 bit $B C D$ system. On completion of the conversion the control logic instructed the Mixer Level change unit to record the digitized signal. The mixer level change unit put the signal on to the tape recorder in the form of three separate digits, hundreds, tens and units. This was recorded in binary form on four channels of the recorder. On recording the first digit, the recorder informed the control logic that it was ready to receive the next digit. The control logic then instructed the mixer unit and recorder to present and record the second digit. The same control loop continued to record the third digit. On completion the scan system was moved to the next position to record the next intensity value.

The limiting factor on the speed of this system was the tape recorder, which took $0.3 \mathrm{~m} . \mathrm{sec}$. to record each digit, i.e. $1 \mathrm{~m} . \mathrm{sec}$. per video signal word.

The system had an additional facility for putting normal data on to the tape in the form of a 5 digit number to identify the run. Data such as contour depth interval, START parameter, inversion limits were fed into the computer in card form.

The television system could be scanned without recording information, intermittently or continuously at a range of scan speeds. This was used for focusing the camera, general setting up and wiping the remaining signal off the photo cathode before recording optical data on the tape recorder.

### 5.5.4 Television System Development.

This section is subdivided into two sections, the development of the tape recorder and then the television camera, described in chronological order. The digital tape recorder and the procedures involved to read the magnetic tape into the computer for the main contouring program were examined first since the whole television system depended upon the ability to get the intensity information into the computer.

### 5.5.4.1 Digital Tape Recorder.

The process of recording information on to the tape recorder and then reading this into the computer suffered from a number of difficulties, some technical, but the majority were administrative or communication problems with the Company's Computer Centre. The Computer Centre within the Company is one of the largest in Europe and serves the other Divisions as well as the Derby Engine Division, plus other Companies who buy the services of the computer. Hence the computer facilities are organized to accommodate a large throughput of computer programs on a continuous production-line
basis and their data input handling capabilities are organized accordingly. The magnetic tape and disc facilities are run entirely by the Computer Centre, which involves both reading and writing information on to the magnetic tapes. Consequently the Computer Centre was not readily organized to accept magnetic tape that had been written outside the computer area. The taper normally used by the Computer Centre has an automatic magnetic labelling system, by which the computer automatically verifies that the operator is using the correct tape for the program. Also the computer information is set outin a series of blocks of a given length. The information on the tapes is read into the computer a block at a time. A block can be up to 30,000 characters long, although the usual size is 800 . The computer reads the block and then checks that there are no errors within this block and that the format is correct before continuing to the next. This is principally a means of reading data quickly into the computer to minimize time wasted by data errors. If a fault is located the complete block of data is rejected and the next block is examined or the program abandoned. The action depends upon the instructions formed within the program. Both the automatio labelling and standard block sizes are not convenient for use on the television system. The block size used for the television system varied from 5 bytes to 1500 bytes and so it was not possible to define a standard block size.

A computer program run on the Company's IBH 360 system has a series of cards before the start of the progran, defined as control cards, which inform the computer and its operator the type of program that is being used. In detail these state:-
a) the program language, P.L./1 or Fortran,
b) the type of program, test or production,
c) where the program is located, i.e. on cards or stored in a magnetic disc or tape,
d) additional facilities required, such as access to other programs or data on disc, tape or card outputs,
e) the number of any magnetic tape or tapes or discs or paper tape reels being used that contain input data and the format of the blocks of data.

All these control cards have to be correct before the program will run on the computer. A control and program set up sequence was devised that enabled the magnetic tape from the television system to be read into the computer. The tape was defined as unlabelled and a manual verification procedure used by the operator to check that the correct tape was being used for the program instead of the automatic verification. It was also found that the data could be read in blocks of varying lengths if the undefined blocklength control statement was used.

An additional limitation was found in the use of magnetic tape information in so far as it was only possible to run the program on the night shift operation of the computer because of other commitments during the daytime. These were caused by higher priority programs and direct linkages to the computing facilities of the other Divisions of the Company. This meant that only five computer runs could be made per week, which restricted the rate of progress on the development of the television system when using the computer.

The initial attempts to read information into the computer in .the form of a constant voltage fed directly into the analogue to digital converter, fig. 43, in place of the video signal resulted in the computer readout system completely stopping without printing any characters. This was traced to the $f$ act that the magnetic tape being
used was relatively old and had become magnetically noisy and produced spurious magnetized areas which could not be decoded by the computer as a character. This was rectified by the use of a new tape.
, The main problem was found to be the binary format used on the tracks of the tape recorder to form the characters. Since in the normal circumstances magnetic tapes are both written on and read by IBM peripheral equipment the actual format of character in relation to the individual tape tracks :1s: not required for normal operation, and consequently the format was not generally known within the Computer Centre. This presented the main communication problem with the Centre.

The magnetic tape used by the recorder was standard $\frac{1}{2}$ " wide computer tape. The tape contained nine parallel tracks on to which the information was fed as shown in fig. 45 a with a character density of 800 bytes per inch. The information from each track was staggered, as shown in fig. 45a. This enabled each track to be read separately and gave a clear indication of the beginning and end of each byte of information to ensure that the information was read accurately into the computer.

Two data formats were used for 9 track recorders with the IBM 360 system. These were Hexadecimal or Character, as shown in fig. 45 b . Only the character format could be used with a program written in the P.L./1 language. Each of these nine tracks operate at the binary logic level of 0 or 1 . For numerical characters in this format the tracks 0, 1, 2 and 3 are held at logic level 1 and tracks 4, 5, 6, 7 for the numerical character from 0 to 9 in the binary form. To form an alphabetic character from $A$ to $I$ the tracks 0 and 1 are held at level 1 and tracks 2 and 3 are held at 0. Tracks 4, 5, 6, 7 form
the binary number from 1 to 9 , forming the letters from $A$ through to I. The first track on the tape, denoted as 'P', formed a parity track. With this I.B.M. system an odd parity format was used which stated that the number of tracks on the tape for each character which were set at a logic level of 1 must be odd. If this applied, the parity value was 0 . If the number of tracks at level 1 was even, the parity was set at 1 to make the total number odd.

Another requirement in the format of the digitized data was the formation of definite blocks of data rather than a continuous stream, as discussed earlier in this section. This was a distinctive magnetic mark on the tape put on by the tape recorder and called an interblock gap. This was automatically applied at the end of each line of the television scan and so each data block consisted of 500 readings of the video signal, each of 3 digits forming blocks of 1500 bytes each.

In addition to the television video data a facility for putting a five digit number onto the tape manually was made as discussed in section 5.5.3. At the end of this data an interblock gap (I.B.G.) was put on to the tape, forming a block of some 5 bytes.

Using the above format it was possible to record data on to the tape recorder and read it off with the aid of the computer. There were some occasions when the recorder would either miss a digit or form some uncodable character on the tape. This was caused by some crosstalk within the Control Logic unit but was soon rectified.

Having established that it was possible to read data from the television $A / D$ converter into the computer the control of this data by the computer was studied and formulated as shown in fig. 45 c . The television scan data was preceded by a set of manual data. The first digit of the manual data block was wired up with the logic
levels on tracks 2 and 3 at zero to form an alphabetic character which was set to 'A'. The remaining four digits were used as a means of identifying the television scan. The television then recorded the video intensity data for one complete frame of 20 lines. After the television scan had been recorded a second set of manual data was recorded with the first of the 5 characters set at 'B'. This completed the recording of one television frame. The next frame was repeated by the same procedure, starting with the character ' $A$ ' and ending with ' $B$ '. At the end of the recording a block of manual data beginning with $' C$ ' was recorded. After the insertion of the manual data containing ' $A$ ' at the beginning of each run an I.B.G. was automatically added but at the end of the scan the manual data was followed by an I.B.G. which had to be inserted by a manual operation. Any number of television frames could be recorded by following the loop as shown in fig. 45c. Each magnetic tape contained 1200 ft of tape with a packing density of 800 characters per inch and so the storage capacity was $11.5 \times 10^{6}$ characters. Each complete scan consisted of 30,000 characters and hence 300 complete scans could be recorded. On normal test runs between 3 and 30 frames only were used.

At the end of a complete recording a 'Tape Mark' was recorded control signal by means of a facility on the recorder. This $\lambda^{\text {was }}$ used to inform the computer that that was the end of the data on the tape, Otherwise the computer would run through the rest of the tape looking for data, which Them greatly increased computer time and inconvenienced the computer operators.

The main computer program was made to read the tape from the beginning and locate the first letter 'A'. By doing this the magnetic tape was set by the program at the beginning of the first
television scan and hence its exact position was known. The next four digits were read to form an identification of the scan. Following this two DO-LOOPS were used to read and store the 500 points of each of the 20 television lines. The next character immediately after the video data was the alphabetic character ' $B$ ' which was set manually. If the character ' $B$ ' occurred in the correct place on the tape the television scan data had been recorded and read correctly and so the program was made to proceed with the shape calculations. If the character was not a 'B' but was a numerical character something was wrong with the information, such as some of the characters missing, and the whole of the information to this frame was rejected and the computer was instructed to look for the next ' $A$ ' and hence the next television frame, as shown in fig. 45 c . This formed a basic check on the incoming information.

Whenever the alphabetic character ' $C$ ' was located which signified the end of the recorded data the computer program was shut down.

This overall format was verified by using the manual data input and a sawtooth waveform applied to the $A / D$ converter to simulate the video signal. Tests showed that this system operated satisfactorily when the magnetic tape input was controlled by the moiré contouring computer program.

### 5.5.4.2 Television Camera.

The construction and development of the slow scan television camera to the design discussed in section 5.5 .1 revealed a fundamental problem when the camera was driven in conjunction with the digital tape recorder to record the image intensity. The output current of the video circuit was extremely low in the region of several nano amps. This current was not sufficient to produce a
reasonable signal above the ambient noise level. The scan rate under these circumstances was 1 ms . per point for 500 points per line, resulting in a frame time of 312 seconds for a normal T.V. raster, which ${ }_{\lambda} 7500$ times slower than the normal television framing rate.

The faceplate of the vidicon tube was coated with a thin layer of photo-conductive material, fig. 46a, whose conductivity was dependent upon the level of optical illumination at that point. The backplate between the tube window and the photo-conductive layer was set at a positive voltage of approximately 30 volts. The photoconductive layer (photocathode) was scanned with the electron beam and its surface brought to the cathode potential which was less than the backplate potential of +30 V . The vidicon was then exposed to the image. The illumination modified the conductivity of the individual elements of the photocathode to produce a flow of current across the layer at each element. This modified the potential distribution on the photoconductive layer, i.e. potential increased with increasing illumination. The electron beam was then rescanned across the photocathode to return the photocathode to the cathode potential and the current flow. in the backplate circuit when the elements of the photocathode were recharged form ${ }_{\lambda}$ the video output signal.

The charging current was basically the same as that for a normal capacitor and had an exponential decay with time as shown in fig. 46b. When the electron beam was charging a single element of the tube for approximately 1 ms . as with the contouring television system the averaged current output would be extremely low which corresponded to the effects observed when testing the system. The scan rate was determined by the recording rate of the tape recorder and could not be increased.

The only means to overcome this problem was to use a buffer store so that the vidicon could be scanned faster and the output signal stored in the buffer and then read into the tape recorder at the normal rate. This meant redesigning and rebuilding a major part of the data handiing system. A 500 times 12 bit shift register store was used as the buffer storage system.

The buffer storage enabled the point by point analysis time to be reduced from 1 ms . to $25 \mu \mathrm{~s}$. This limit was set by the time required for the analogue to digital converter to operate. Even the $25 \mu \mathrm{~s}$. per point was a relatively long time and was equivalent to a scan time of 7.8 seconds, which gave a specified signal to noise ratio of $10: 1$. During the $25 \mu \mathrm{~s}$. taken for the $\mathrm{A} / \mathrm{D}$ converter to operate the video signal still had a rapid decay. However the operation of the converter was on a sample and hold basis where the input signal was integrated for the first 40 nan seconds of the cycle and then this voltage was held while it was digitized as shown in fig. 46c. Thus the output signal was sampled before it had decayed by any appreciable amount and a reasonable signal was obtained.

Experimental tests on the completed television camera showed that the vidicon tube was extremely sensitive and operated at the Iow light levels dictated by the fringe projection system with the camera's 85 mm focal length lens set at f.16. However it was not possible to achieve a reasonable optical resolution across the vidicon tube under either internal or external scan conditions. On continuous internal scan the resolution was limited to approximately 100 television lines across the face of the vidicon where the tube specification was 800 television lines.

Internal scan conditions were set by frequency of the $X$
deflection and $Y$ deflection ramp generators and these produced a line scan in 1 ms . and some 400 lines per frame in a continuous scanning mode. The video output was displayed on an oscilloscope. The external scan was controlled by the Logic Control Unit with a line scan time of 12.5 ms .

The resolution of 100 lines was extremely low for the camera particularly for the internal scan conditions although for the external scan mode the 500 point by point sampling and aliasing effects reduced the predicted resolution of a continuous lined grid to approximately 150 lines. Neglecting the 500 discrete sampling points and displaying the video output signal from the cemera directly on to an oscilloscope the resolution was still found to be approximately 100 television lines.

The lack of resolution was attributed to the spot size of the scanning electron beam being larger than anticipated. This was verified by the fact that it was not possible to produce contour fringes on a turbine blade when the fringe projection system was used to illuminate the blade for $0.020^{\prime \prime}$ contours. Hence the spot size was greater than half the period of the projected fringes imaged on to the vidicon. If the lack of resolution had been caused by such effects as frequency limiting on the video amplifiers the signal would have made some attempt to follow the contours as they were at an effective spatial frequency of less than 100 television lines. The spot size assumption was re-affirmed by the fact that contours could be seen on the blade when the moiré shadow contouring technique was used. The spot size was estimated to be approximately $0.004^{\prime \prime}$ dia. on the vidicon surface which represented a spot of approximately $0.020^{\prime \prime}$ dia. on the object with the camera's 5:1 image demagnification factor.

Adjustments were made to the electrical operating parameters of the vidicon tube including the alignment and electrical focusing coils but the resolution could not be improved.

Apart from the lack of resolution the camera operated as anticipated. It was possible to store data on the tube and then record the digitized intensity information on the tape recorder. The overall operation of the camera and digitizing system was verified using the moiré shadow contouring technique with a contour depth of approximately $0.030^{\prime \prime}$. A typical digitized line scan that was produced by this system for computer processing is shown in Table 6. An analogue plot of this data is shown in fig. 47. As seen in fig. 47 the depth contours were visible but the intensity modulation produced by the contours was low and there was a significant noise level on the overall signal. The television camera was not sufficiently reliable to be used in conjunction with the main computer program to continue the research.

After a number of unsuccessful attempts to improve the television camera, including consultations with the U.K. representatives of the tube manufacturers, the vidicon tube designers in France were contacted. They confirmed that a spot enlarging effect would occur when the tube was run in a slow scan mode and hence degrade the tube resolution.

The effective spot enlargement was due to the relatively long time that the scanning electron beam was incident on any one spot on the face of the photocathode. When the electron beam was scanning a line and replacing the charge lost on the face of the photocathode, the electrons migrated radially from the point of incidence of the electron beam. Hence the electron beams charging permeated influence along the surface of the photoconductive
layer as well as directly through the layer. This produced an effective enlargement of electron beam at the photocathode. The slower the electron beam scanning rate, the more time the charge had to spread and hence the greater the spot size. At the scanning rate used when digitizing the image intensity information the $0.004^{\prime \prime}$ spot size that was observed was considered normal by the tube's design engineers.

Consequently it would not be possible to improve the resolution of the slow scan television interface when operating it in this particular slow scan mode.

To achieve optimum resolution the vidicon must be operated at the normal scanning rate of 25 frames per sec. at 625 lines. Thus a line would be scanned in $64 \mu \mathrm{~s}$. and the time available to digitize each of the 500 points on the line scan would be $0.12 \mu \mathrm{~s}$. This would require an $A / D$ converter that could operate at 8 MHz . There are commercially available $A / D$ converters that operate up to 4 MHz or 15 MHz such as the Micro-Consultants Ltd. AN-DI 802 VID. 10 bit converter at 4 MHz and the AN-DI-RAD-A 8 bit converter at 15 MHz .

There appeared no fundamental reason why it should not be possible to run the television system at close to the normal framing rate in conjunction with the digital tape recorder. The system would be controlled by the camera rather than the logic control unit in so far as the camera would operate in the standard continuous mode at the full freme resolution of 625 lines and the interface unit would be instructed by the camera when to digitize and store the nth line. When this line was recorded on the tape the next line (i.e. ( $n \times 30$ ) th line of the IV frame) would be processed until the complete 20 lines were recorded.

Digitizing a single T.V. line fast would be preferable to processing the 500 points in sequence frame by frame as this would take a long time, 20 seconds/line, and there would be possible exposure time and illuminating intensity variations from frame to frame that would reduce the accuracy of the contour intensity function across the blado.

From this work it was concluded that the achieved resolution of the television interface of 100 television lines was not sufficient but that it would be possible to increase the resolution to an acceptable value by further electronic development. However it would not be possible to carry out this development within a sufficiently short time to be included in this thesis.

To continue the research into the optical contouring technique and to enable a continuing influx of turbine blades and research aerofoil sections from other departments of the Company to be examined the logic control unit and the digital tape recorder were linked to the photomultiplier mechanical scanning system. The control logic unit was slowed down to digitize the photomultiplier output signal every 120 ms . so as to complete the 500 point cycle in 60 seconds. The photomultiplier was arranged to scan across the blade in just less than 60 seconds. The photomultiplier mechanical scan speed was constant to within $0.4 \%$ and was sufficient to assume that each of the 500 data points were equi-spaced along the length of the scan.

This interface system worked extremely well and enabled the contour intensity information to be recorded onto the digital tape recorder in exactly the same format as used for the full television system including the manual data inserts at the beginning and end of each frame, fig. 45c. The scans across the blade using the
fringe projection system were similar to those shown in fig. 38 and were noise free apart from the inherent optical noise produced by dirt on the surface of the blades. An example of the digitized data fed into the computer for analysis is shown in Table 7. For normal testing purposes three scans per aerofoil surface or frame were used although for some components five scans were taken. These limits were used to keep the data recording time and computer program time to a minimum to conserve processing expenditure.

### 5.6 TEST RESULTS.

A number of tests were carried out using the photomultiplier mechanical scan and magnetic tape recorder system in addition to the experimental results described in section 5.4 to extend the experimental confirmation of the computer linked optical contouring technique. In addition to these tests the interface system was used to measure the aerofoil shape of a number of blades for potential users of the technique within the Company.

For this experimental work a number of modifications were made to the main computer program from that shown in Appendix 1.

It was not possible to simplify the shape generation program to any great extent as the surface noise effects produced by the dirt on the surface of the blade was still prevalent and had to be accommodated. The program was made more efficient and its size reduced slightly by subdividing the inversion parameter change and restart parameter change sections into separate procedures or subroutines which were then called when required within the main progrem.

To accommodate the complete frame of information from the television oamera or photomultiplier of up to 20 lines some of the parameters were defined as two-dimensional arrays. These were principally the input data, shape data $Z$ and $R Z$ and the centroid and principal axis data of both the
individual surface line shapes and the whole sections. Each of these arrays had to be defined separately at the start of the program to generate the correct storage areas within the computer. However when using the program a varying number of lines per frame were used to conserve computer costs and to enable the extent of these arrays to be varied easily a computing procedure known as a controlled variable array was used.

In this case the array was defined in a declare statement as controlled:-

DCL ( $\left.\mathrm{V}(*, *), \mathrm{Z}\left(*,{ }^{*}\right), \mathrm{RZ}\left({ }^{*},{ }^{*}\right)\right)$ CONTROLLED ---

The number of lines to be used within the actual program was obtained from a single data card and denoted as 'LINES'. Then the main arrays were set up by an allocation statement
allocate $\mathrm{V}($ LINES, 500$), \mathrm{Z}$ (LINES, 170) , R2(LINES, 170) ;

The values of the second dimension of the array, 500 , and 170 , were the number of points within the line scan and the number of points at which the dimensional information was calculated.

Using this system the computer storage could be set up for any particular experimental run by the use of only one number on a data card. This saved having redundant computer storage space within the program and hence reduced costs as the storage requirements form part of the running expenses. An example of the use of the controlled variable for a onedimensional array can be seen at the beginning of Appendix 1.

For the dimensional shape calculations of the blade aerofoil a number of data points less than 500 across the aerofoil would be adequate. The 500 measuring points were still required to define the contour intensity with sufficient accuracy. For these tests a number of
approximately 150 was considered a reasonable number giving a data point every $0.010^{\prime \prime}$ for an average blade.

This was achieved by taking every third point of the scan for the depth calculation. This was most conveniently done by using the 'FLOOR' function within the computer. This was preferable to the use of a normal counting system as it was not known where within the scan the blade surface started or where the contour peaks were located but the number of the point within the $Z$ array gave the $X$ co-ordinate.

The 'FLOOR' function gave the integer part of any value, i.e. $\operatorname{FLOOR}(1.33)=1$. Hence the depth calculation section of the main program, Appendix 1, (statement 258) was modified as follows:-

CALA : DO L = NN TO MM;

$$
\begin{align*}
& A L=L / 3 ; \\
& B L=F L O O R(A L) ; \\
& \text { IF BL } 7=A L \text { THEN GO TO CALEND; }  \tag{115}\\
& \text { ELSE DO; (Calculate } Z \text { dimension) }
\end{align*}
$$

Hence the IF statement defined that if the number $L$ was not completely divisible by 3 the $Z$ calculation was not performed.

Using this technique it was possible to divide the 500 point array by any number to give the required number of shape data points by changing divisor in the 'AL' statement.

The retro-plotting technique discussed in section 4.7 to complete the dimensional shape of the blade right up the edges were included and this worked well.

The overall methods of calculating the monents of complete sections of the blade were simplified from those used on the mathematical model as shown in Appendix 2. These parameters were calculated by a closed loop method in so far as the blade section was a completely enclosed and
self-contained area. The calculation started at one point, i.e. the trailing edge of the blade on the convex surface as shown in fig. $35 a$ and traversed around the closed loop in a clockwise sense. The area and moments of element enclosed between the curve between the two element points and the co-ordinate systems axes were calculated rather than the element of the blade section. As the elements were taken around the closed loop the values became negative for the elements on the concave surface giving only the parameters for the area within the closed loop. This overcame the difficulty in matching the $X$ and $Z$ co-ordinates of the two surfaces when calculating the $d X$ and $d Z$ element moments as shown in Appendix 2.

Consider the blade section in fig. 35a where the closed loop calculation starts at the point $A$ and proceeds round the loop in a clockrise direction to return to $A$. For an element between the nth and $(n+1)$ th points the area under the curve with respect to the $X$ axis is given by

$$
\begin{equation*}
\text { Area }=\frac{1}{2}\left(z_{n}+z_{n+1}\right) \cdot\left(x_{n+1}-x_{n}\right) \tag{116}
\end{equation*}
$$

Proceeding round the loop along the convex surface of the blade $X_{n+1}>X_{n}$, hence the area elements have a positive value and along the concave surface $X_{n+1}<X_{n}$ and the elements are negative. Hence summing the whole of the loop defined by $N$ points the area within the loop was obtained.

$$
\begin{equation*}
\text { Section Area }=\frac{1}{2} \sum_{n=1}^{N-1}\left(z_{n}+z_{n+1}\right) \cdot\left(x_{n+1}-x_{n}\right) \tag{117}
\end{equation*}
$$

The first and second moments of the section about both the $X$ and $Z$ axes and the product of inertia were found using this technique.

The manual data inserted on the magnetic tape before each frame was
used to identify the frame by a number and to inform the computer on the type of scan being taken, as shown by the use of the REF parameter in Appendix 1. In this case the parameter REF had four discrete values:-

```
REF = 0 ----- Convex surface, Reference data
REFR = 1 ----- Concave surface, Reference data
REF = 2 m-m- Convex surface, Production data
REF = 3-m--- Concave surface, Production data
```

Using these identifiers it was possible to control the program to set up the parameters of the reference blade and then those of the production components and compare the production centroid location and angle of the principle axis to the reference blade's parameters to find the overall differences as described in section 4.10.

The experimental results obtained using a 3" long turbine blade and the fringe projection contouring gave a dimensional accuracy of $\pm 0.0015^{\prime \prime}$ for a $0.015^{\prime \prime}$ contour depth interval. An example of the analogue trace of a scan that was digitized and fed into the computer is shown in fig. 48. The noise in the region of the inversion point was optical noise produced on the surface of the blade rather than electronic noise within the photomultiplier and signal amplifiers. This was similar to the scans produced in the previous. experimental work as was illustrated in fig. 38. The computer analysis and printout of a typical line scan across the convex surface of the blade is shown in Appendix 4. The photomultiplier output was amplified to give an analogue voltage from 0 to 10 volts into the $A / D$ converter and when digitized and recorded formed a number from 0 to 999. The digitized photomultiplier output was printed at the beginning of the computer program as seen in Appendix 4. A number of printout statements were included in the main program for fault location purposes. These printout statements give an indication of the operation of the program.

The full program was tested by rotating the turbine blade through a small angle about the blade's axis. This was the most stringent test that involved the whole shape program as the angle of rotation was calculated from the difference in the angle of the principal axis of the reference and production surfaces rather than any direct $X$ and $Z$ direction shifts that only had a real effect upon the location of the edges of the blade with respect to the scan and the 'START' parameter.

The results for the comparison of the convex surface of the blade rotated through a small angle gave similar results to those discussed in section 5.4 and Table 5. This confirmed the overall accuracy of the calculated rotation to the actual applied rotation to be better than $10 \%$ as had been previously obtained. An example of the results obtained are shown in Table 8. The accuracy obtained at these small angles was particularly good since the direct effect of a shape error of $\pm .0015^{\prime \prime}$ at the edges of the blade would produce a 0.2 degree error about the centroid of the surface resulting in a $20 \%$ error for a $1^{\circ}$ rotation.

Three scans were taken across both the convex and concave surfaces of the blade, in the region of the blade root, centre and tip. A set of scans was taken on the convex and concave surfaces of the blade to set up the reference parameters for both the line shapes of the surfaces and the completed sections that were formed by fitting the convex and concave surfaces together. Similar scans were taken for the blade rotated through a number of small angles to form the production blade data. The angular differences between the production and reference shapes were calculated. A sample of the results obtained is shown in Table 8. Small angles were used of less than $3^{\circ}$ as these
were considered to be within the possible range expected on production components.

The results show a good agreement between the calculated angles for the various blade sections for both the line shapes and sectional shapes within an accuracy of better than $10 \%$. The agreement between the calculations of the blade's sectional areas for the reference and production conditions shown in Table 8 indicated that the sectional parameter calculations were consistent and acourate.

The results obtained and described in this section and section 5.4 demonstrated that the oomputer linked optical contouring operated as predicted to give the overall difference between the aerofoil surfaces of a turbine blade in a simple form of two orthogonal linear shift parameters and the angular difference. The accuracy of the shape information was within the predicted one tenth of the contour depth used.

More detailed analysis of the differences between the blades could be obtained by straightforward manipulations of the data within the computer as discussed in section 4.10 since all the basic information to do this was available.

The ultimate type of result obtained would depend upon the application.

## Chapter VI. <br> Discussion and Conclusions.

### 6.1 INTRODUCTION.

This research has demonstrated that computer Iinked optical contouring was a feasible technique for inspecting the aerofoil sections of aero engine compressor and turbine blades. The system was accurate to within $\pm .001^{\prime \prime}$ for a turbine blade that had a dimensional depth of between 0.3 and 1.0 inches. The system was capable of being used as an automatic inspection system on a routine basis. The information produced by this system was as accurate as the present Calliperscope and Point Gauging methods and produced more detailed information over a greater area of the blade than the latter methods. In this chapter the concept of the overall system is discussed together with the possible ways of utilizing the technique. Finally, the conclusions drawn from this research are formed.

### 6.2 DISCUSSION.

The information output from the computer for this technique could be made to suit the particular epplication and produced data on two levels, (a) the overall differences between the components and (b) details of the localized differences after condition (a) had been applied to the shape being inspected.

It was not possible to produce a purely visual"inspection system for turbine blades using contouring techniques such as a pure moiré fringe comparison on a television monitor because of the complexity of the resultant moire pattern and the limited amount of data that it provided. Hence the computer linked system must be used as described.

The teohnique could also be applied to the accurate measurement of complex distortion of turbine blades produced by engine condition type
tests. Information of these distortions to this accuracy and detail had not previously been possible and hence the application of the optical contouring system to this type of analysis greatly increased the potential uses of the technique. The computer aspect of the device enabled dimensional difference data to be presented in a form compatible to any theoretical computer program that was being used to study the blade.

### 6.2.1 Contour Generation.

Both the moiré shadow and moiré fringe projection/scanning techniques produced good results suitable for turbine blade inspection. The fringeprojection was more appropriate for use with the television interface and computer. It did not, however, produce any visible impression of contour fringes. The intensity distribution obtained in practice followed the theoretical triangular intensity function extremely well and from some scan examples examined it was possible to measure the shape funotion to better than $10 \%$ of the contour depth, resulting in an increase in measuring accuracy by 2 to 3 times.

The fringe projeotion/television system would be well suited to routine application since both the projection optics and the television camera would be well away from the blade being inspected. Hence they would not be liable to be damaged while loading and unloading the blade and there would be sufficient room to construct a kinematio mounting system for the blade that also rotated the blade through $180^{\circ}$ to inspect both the concave and convex surfaces of the blade. If operating in a particularly harsh environment, the optics could be completely hermetically sealed from their surroundings.

### 6.2.2 Television Interface.

The overall concept of the television interface with the magnetic tape recorder worked well and it was possible to record 20 television

Ines within approximately 10 seconds as originally predicted.
The basic limitation of the interface was the lack of resolution when the television system was operated in a slow scan mode. This was a function of the vidicon tube as discussed in seotion 5.5.4.2. The resolution could be improved by increasing the scan speed up to that of the normal scanning rate of 25 frames per second of 625 lines per frame and performing the digitization of one line within $64 \mu \mathrm{~s}$. with the aid of a. fast $A / D$ oonverter and a buffer store. These modifications were technically possible and formed an electronic development task rather than fundamental research into the system, As discussed in section 5.5.4.2 it was not possible to carry out these modifications before the completion of this thesis.

The work carried out on the television system showed that it was the correat technique to be used to feed the contour fringe data into a computer either by direct access into a small on line computer or, as was done for this research, fed into the Company's main computer complex Via a digital tape recorder. The system had the spaed required to obtain the data within 10 seconds and the recording rate of the tape recorder formed the limiting factor rather than the television system. Hence there was room for possible improvements in these speeds as computer technology improved.

The system would be particularly suitable for routine application as it rould be compact and easy to maintain.

### 6.2.3 Information Output.

The present output format of the data using the IBM 360 computer was In a paper printout form providing both the general differences between the components and the localized differences formed after the inspected oomponents shape co-ordinates have been modified by the overall difference parameters. The exact form of this output depended upon the application;
in some cases of inspection it could be a direct go nomgo statement, whereas other cases would require details of the localized differences.

The locallzed differences could be presented in a paper printout form in three ways.
(a) Direat list of co-ordinates and errors.
(b) Qutline of blade with errors tabulated in relevant position.
(c) Error contour map.

Method (a) would be straightforward. Method (b) would give a direct imperssion of the location and values of the errors. A basic outline of the blade would be formed on the printout by the computer, The area within the outline would be divided into a matrix and at any point where the efror was greater than a given value, defined by the tolerance envelope, the actual error would be printed. Points where the error was within the tolerance envelope woula be left blank. Hence one coula obtain a visual impression of the errors over the whole blade, The error contour map (c) would be a slight simplification of (b). Here for a given value of the erfor a number or character mould be printed. The whole matrix would be scanned and the error printed out. This could either be of a line contour form, where a character would be printed if the error was exactly say, $2,4,6,8, x 10^{-3}$ inches, or an area contour, Where a given character would be printed for any error between 0 and $2 x$ $10^{-3}$ inohes, or between 2 and $4 \times 10^{-3}$ inches and so on. Both would give a good visual impression of the effors. The area contour printout would be the most appropriate method for general use.

If used on a rautine production Inspection basis, the printout could be displayed on a C.R.T. display to give a fast, clear and easily observable display without the operator having to handle a sheet of paper.

For distortion measurements the difference information could be
stored directly on a disc or magnetic tape for easy access for subsequent computer programs as well, as a printout form for normal analysis. 6.2.4 Routine Application.

One of the main attributes of the computer linked inspection system was the ability to compare an object to design data rather than requiring a physically made master object, This represented a considerable saving in the cost of manufacture of the master component. Also, the master had a tolerance envelope of $\pm .002^{\prime \prime}$ in its manufacture, which would reduce the accuracy of any inspection system. This saving would bocome increasingly important in the development of any aero engine, when the blade design was being constantly improved and the blades manufactured In small quantities,

All the turbine and pompressor blade vare designed with the aid of the computer. The detailed drawings and speaffications wece produced from the computer data. The basic and detailed design information mis stored on either disc or tape within the I.B,M. 360 system, This design information for any blade could be easily accessed mithin the computer complex, given the diso or tape number and access code. Hence the preoise shape information would be readily available and could be made compatible to the contour inspection system by some simple arithmetic manipulation and then used as the reference information. Thus the cost of changing the blade that pas being inspeoted would be reduced to reprograming the computer to obtain the correct reference data as opposed to a complete change in the mechanical probe of the point gauging system that was used at prosent.

To reduce the cost and increase the effiaiency of the optical contouring system the digitizing and computer system would be used on a time sharing basis by a number of inspection units, between 3 and 5 units, each with their opn fringe prajection and television interface. To
operate this complex the whole system wauld be controlled by the computer phere the operaton loads the component and presses a button to indicate the system is ready. The computer would check the mounting of the blade, soan the contoured image, process the data, and finally present the data and indicate that the inspection cycle was complete.

## 6,2.5 Additional Applioations,

In addition to the two basio applipations in production inspection and blade distortion measurement, the technique had been applied to a number of dimensional analysis megsurements of aeno engine components. These measurements principally used the fringe projection/scenning system. Some blade measurements were done using the mairé shadiow contouring method. The applications included the dimensionel analysis of approxImately fifty fesearch blade aerofoil sections to provide the overall dimensional shape and some modified sets of nozzle guide venes. The ability to project the grid fringes direatly on to the object's surface proved a particularly valuable asset for measurement on inaccessible surfaces. The measurements were mainly performed using manual shape plotting. These appliaations are now being continued and are being extended.

The practical applications showed that the technique could be used on a number of diverse components and situations and so increased the averall potyential of the system. A number of patential applications of the system inoluded the dimensional analysis of oomponent dies to provide a more detailed analysis then used at present by clock gauging. The results qould be oompared to the computer desten data. Also included would be the measurement of sections of small engine oasings. The range of object size could be extended using the fninge projection system to large objeats, enging casings, provided that the contour depth interval and orerall object depth was kept within the fesolution limits of the
television camera, Using this principle, there pould be a number of applications for the technique outside the aero ongine industry, Some applfcations wquld require modification to the system, particularly the contour programing aspect to accommodate the points of depth inversion that ocour, The principal objectives for the development of the system on some oomponents would be to simpifify the contour analysis system so as to operate mainly on the contour peaks and troughs and only use the contour intermediate calculations where negessary so as to use a minimum of gomputer storage space. These parameters depend upon the component and the amount of detailed information requirea.

## 6,3 CONCLUSIONS.

This programme of research has investigated the impfovement in inspection methods for aero engine components using a computer linked optical contouring technique and from this work the following conolusions have been shom,

The computer linked optical contouring technique was a practical method of improving the inspection of gompnessor and turbine blades aerofoil sections. This technique significantiy increased the amount of data that could be obtained on the aerofoils and achieved a measuring acouracy of $\pm, 001^{n}$. The dimensional data obtained within the pomputer could be processed to calculate the differenoe between tro components in the form of an overall difference in two orthogonal difections and an angle of rotation plus any localized differences as required by the particular application. The information could be presented to an operator in an easily assimilated form.

The use of a computer to process the dimensional information would relifeve the operator of much of the burden of handling a large amount of optical datha and in some pases the decision-making responsibilities. The computerized inspection system would be partioularly corapatible
with existing design and production technology as much of the design work was computer orientated.

The overall system could be used for either inspection of turbine blades during production or to measure the distortions of blades caused by thermal fatigue in engine tests.

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TABLE 1. Comparison of the Optical Contouring Technigues examined for Turbine Blade Inspection.

| Technique | Contour <br> Depth <br> Interval <br> (inches) | Measuring Accuracy (Fraction of contour depth) | Fringe Contrast | Light <br> Source | Contour <br> Recording <br> Medium | Ease of Operation | Comments | Conclusions |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Double <br> Wavelength <br> Illumination <br> (Argon Laser) | $\begin{gathered} .0005- \\ .001 \end{gathered}$ | $1 / 4$ | Reasonable | Argon <br> Laser | Photo Plate | Good | ```Contours too fine for blades. System difficult to set up.``` | Not suitable for use with an optical to |
| $\begin{aligned} & \text { Double wave-* } \\ & \text { length. } \\ & \text { (Dye laser) } \end{aligned}$ | $2 \times 10^{-5}$ -.1 | 1/4 | Reasonable | Dye <br> Laser | Photo Plate | Good | Requires accurate setting up. | computer interface. |
| Double <br> Refractive <br> Index | . 001 - | $1 / 4$ | Good | Argon <br> Laser | Photo Plate | Slow | Good results. Loading workpiece and gas too slow. | Not applicable for routine work. |
| $\begin{array}{\|l} \text { L.S.P. } \\ \text { Video } \\ \text { Recording } \end{array}$ | $2 \times 10^{-5}$ -.1 | 1/4, | Fair | Dye <br> Laser | Video tape/disc | Unknown | Low spatial resolution. | Shows good potential |
| $\begin{aligned} & \text { Moire } \\ & \text { Fringe } \end{aligned}$ | . 01 -. 1 | 1/20 | Good | $\begin{aligned} & \mathrm{Hg} \\ & \mathrm{Arc} \end{aligned}$ | Not required | Real time | Equipment simple | Suitable for visual observation and mechanical scanning. |
| Projected Moiré Fringes | . 01 - . 1 | 1/20 | Very <br> Good | $\begin{aligned} & \mathrm{Hg} \\ & \mathrm{Arc} \end{aligned}$ | Not required | Real time | Very good results | Suitable for use with image scanning. |

* Estimated performance from experimental work on associated methods.

TABLE 2. Computer Calculated Displacements of
a Simulated (Elliptical) Object.

| Applied Displacement for the Point ( $1,0.6$ ) on the reference surface. |  |  | Calculated Displacement of the production surface (With respect to the point $(1,0.6)$ on the reference surface.) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| : |  |  | Surface Line |  |  | Surface Section |  |  |
| x Displ. | y Displ. | Angle | x Displ. | y Displ. | Angle | x Displ. | y Displ. | Angle |
| 0.000 | 0.000 | $5^{\circ} 0^{1}$ | 0.000 | 0.000 | $5^{\circ} \quad 1^{\prime}$ | 0.001 | 0.0005 | $4^{\circ} 51^{\prime}$ |
| 0.100 | 0.100 | $5^{\circ} 0^{\prime}$ | 0.100 | 0.100 | $5^{\circ} \quad 1^{\prime}$ | 0.102 | 0.100 | $4^{\circ} 52^{\prime}$ |
| 0.100 | 0.100 | $3^{\circ} 0^{\prime}$ | 0.100 | 0.100 | $3^{\circ} 0^{\prime}$ | 0.100 | 0.102 | $2^{\circ} 581$ |
| 0.100 | 0.050 | $3^{\circ} 01$ | 0.100 | 0.049 | $3^{\circ} 01$ | 0.100 | 0.051 | $2^{\circ} 58^{\prime}$ |
| 0.100 | 0.100 | $1^{\circ} 01$ | 0.100 | 0.100 | $1^{\circ} 0^{\prime}$ | 0.100 | 0.100 | $1^{\circ} 1^{\prime}$ |
| 0.100 | 0.050 | $1^{\circ} 00^{\prime}$ | 0.100 | 0.050 | $1^{\circ} 0{ }^{\prime}$ | 0.100 | 0.0505 | $1^{\circ} 1^{\prime}$ |
| 0.100 | 0.100 | $0^{\circ} 30^{\prime}$ | 0.100 | 0.100 | $0^{\circ} 30^{\prime}$ | 0.100 | 0.100 | $0^{\circ} 30^{\prime}$ |









0.37794
0.37803
0.37812
0.37709
0.37587
0.37462
0.37462
0.37444
0.37237
0.37231
0.37230
0.37139
0.37070
0.37045
0.37014
0.36914
0.36799
0.36612
0.36508
0.36464
0.36331
0.35987
0.35841
0.35653
0.35598
0.35498

TABLE 4. Slow Scan Vidicon Comparison.

| M'facturing Company | Type No. | Focusing \& deflection system. | Spectral <br> Response <br> Peak (nm) | Maximum Resolution (Centre) | Modulation Depth at 400 TV Lines (Slow Scan Rate) | Typical <br> Light <br> Input <br> (Lux-S) | Dark <br> Current ( nA ) | \% age Signal <br> Remaining <br> After 10 secs | Cost | Additions <br> Required |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { English } \\ & \text { Electric } \end{aligned}$ | V8034 | Electromagnetic | 610 | 700 | 50 | 2.7 | 8.0 | 90 | $\mathfrak{E 1 8 0}$ | Focusing \& deflection coils $£ 50$. |
| EMI | 9745 | Electro- <br> static | 490 | 600 | 40 | $\frac{1}{\text { (Estimated) }}$ | 10.0 | Not specified | $\div$ \&100 | None |
| R.C.A. | 4500 | Electromagnetic | 610 | 700 | 50 | 2.5 | 8.0 | 95 | - | $\begin{aligned} & \mathrm{Coils} \mathrm{~s} \\ & £ 50 \end{aligned}$ |
| ThomsonCSF | TH9892 | Electromagnetic | 430 | 1000 | $\begin{gathered} 50 \text { at } \\ 500 \mathrm{TV} \text { lines } \end{gathered}$ | 1.0 | 0.6 | 99 | £250 | $\begin{aligned} & \text { Coils } \\ & £ 50 \end{aligned}$ |
| Westinghouse | WL4384 | Electrostatic | 400 | 700 | 40 | 1.0 | 0.2 | 95 | £320 | None |
| Westinghouse | WX5111 | Electromagnetic | 400 : | 700 | 40 | 1.0 | 0.2 | 95 | - | Coils |

[^1]TABLE 5. Example of computed values for the centroid shift
for a rotation applied to the blade.

| AEROPOIL | SCAN POSITION | APPLIED ROTATION | COMPUTE | VALUES OF DIFFERENCES THE CENTROIDS. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Surface <br> Scanned |  | (degrees) ${ }^{\text { }}$ | X SHIFT | Z SHIFT | ANGULAR <br> DIFFERENCE <br> (degrees) |
| Convex | Centre | 0.73 | 0.000 | 0.001 | - 0.70 |
| Convex | Centre | 1.47 | - 0.005 | 0.005 | 1.50 |
| Concave | Centre | 0.73 | 0.001 | $-0.003$ | 0.78 |
| Concave | Centre | 1.47 | - 0.001 | -0.005 | 1.54 |
| Concave | Bottom | 0.73 | 0.006 | - 0.005 | 0.69 |
| Concave | Bottom | 1.47 | 0.009 | - 0.008 | 1.42 |

TABLE 6. Computer Printout of a Digitized Line Scan Recorded by the
Television Interface System.


TABLE 7. Computer Printout of a Digitized Line Scan Recorded by the
Photomultiplier/Maenetic Tape Recorder System.


## TABLE 8. Blade Angles of Rotation Calculated from Contour Information.

| Rotation applied to blade. (degrees) | Angular rotation calculated for the convex surface. <br> - (degrees) |  |  | Angular rotation calculated for the concave surface. (degrees) |  |  | Angular rotation calculated for the whole blade section. (degrees) |  |  | Blade Section Area$\text { (inches }{ }^{2} \text { ) }$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pm .05$ | Towards root | Centre | Towards tip | Towards root | Centre | Towards tip | Towards root | Centre | Towards tip | Towards root | Centre | Towards tip |
| 0 <br> Reference | - | - | - | - | - | - | - | - | - | 0.143 | 0.123 | 0.105 |
| 1.00 | 1.05 | 0.99 | 0.94 | 1.07 | 0.98 | 0.92 | 1.05 | 0.90 | 0.95 | 0.148 | 0.123 | 0.107 |
| 1.50 | 1.48 | 1.60 | 1.46 | 1.61 | 1.55 | 1.43 | 1.50 | 1.47 | 1.47 | 0.141 | 0.129 | 0.104 |
| 2.00 | 2.05 | 1.99 | 2.03 | 2.01 | 1.93 | 1.85 | 2.04 | 1.96 | 1.91 | 0.151 | 0.132 | 0.113 |
| 2.50 | 2.65 | 2.51 | 2.48 | 2.43 | 2.56 | 2.59 | 2.45 | 2.49 | 2.65 | 0.144 | 0.128 | 0.108 |


a) Schematic diagram of the optical arrangement.

b) Example of two source contouring (depth interval $=1.5 \mathrm{~mm}$.)

PIG. 1. HOLOGRAM SYSTRM FOR THE GENERATICN OF DEPMH CONTOURS BY THE TWO SOURCE METHOD.


FIG. 2. TELECENTRIC IMAGING ARLANGLAENT FOR OPTICAL CONTOURING.

a) Reference and Reconstruction Beam Angles for Two Navelength Contouring.

b) Depth of Focus on a Single Lens Imaging System. FIG. 3.

a)

COIN

b) $\quad \cdots \quad$ TEST OBJECT

a) Schematic diagram for the optical arrangement

b) Optical path arrangement for the object within an enclosed chamber.


FIG. 6. CONTOUR DEPTH INTERVAL IN 'FREON 12' VERSUS PRESSURE INCREASE.
电



FIG. 7. DOUBLE REFRACTIVE INDEX CONTOURS ON A
SMALL TURBINE BLADE (DEPTH INTERVAL $=0.010)$.


A. SOURCE \& OBSERVER AT INFINITY

B. FINITE SOURCE \& OBSERVER DISTANCES

FIG. 10. MOIRÉ FRINGE CONTOURING


FIG. 11. MOIR: Shadow contouring and mage subtraction sustim.


FIG. 12. MOIRE SHADON DEPTH CONTOURS ON A TURBINE BLADE.

(i) GRID STATTONMRY

(b) GRID KOVING

## a) Grid stationary

(Grid lines
parallel to
blade axis.)
b) Grid moving in the direction of scan when the negative
.was formed.


FIG. 14. MICRODENSITOMETER TRACES ACROSS A NEGATIVE OF A CONTOURED TURBINE BLADE (MOIRE SHADOW CONTOURS).

a) Optical Arrangement.

b) Production of a Moiré Grid Imace with a Linear Spatial Frequency at an angle $\theta$ to the axis.

c) Intensity function - Grid in Observing Optics.


FIG. 17. FRINGES PROJECTED ON TO A TURBINE BLADE.
ZQUIVALENT CONTOUR DEPTH $=0.010^{\prime \prime}$
(PHOTOGRAPH MAGNIFICATION $=2.5: 1$ )


FIG. 18. DEPTH CONTOUR SCAN USING MOIRE PROJECTED FRINGES.


FIG. 19. COMPUTER SIMULATED MOIRÉ DIFFERINCE FRINGES FOR TWO LINBAR SLOPES.

FRINGE

- 190

(A) COMPUTER GENERATED SUBTRACTED MOIRE FRTNGA PATTERN. (ELIMPTICAL OBJECT)

(B) MOIRE DIFFERENCE PLOT.

FIG. 20. COMPUTZR SIMULATED MOIRÉ PATTERN ANALYSIS OF ELLIPTICAL OBJECTS.

## WHWN Wh

(A) Moiré pattern of subtracted contour fringes for a deflected plate.

(B) Microdensitometer trace of fringe intensity.

FIG. 21. MOIRE FRINGE SUBTRACTION.
(CONTOUR FREQUENCY TO MOIRE FRINGE FREQUENCY
RATIO $* 4: 1$ )

(A) Noiré pattern of subtracted contour fringes for a deflected plate.

(B) Microdensitometer trace of fringe intensity.

FIG. 22. MOIRÉ FRINGE SUBTRACTION.
(CONTOUR FREOUENCY TO MOIRÉ FRINGE FREQUUNCY RATIO $\div 2.5: 1$ )


ricc. 24.
MECHANICAL PHOTOMULITPLIER SCANNING UNIT FOR MOIRÉ CONTOURING.

(A) With low pass filtering.

(B) Without filtering.

FIG. 25. THE USE OF LOT PASS ELECTRONIC FILTERING TO ELIMINATE NOISE ON THE CONTOURING SIGNAL.


INCHES


FIG. 27. BLADE COMPARISON BEFORE AND AFTVR BNGINE RUNS (ABOVE CHTRE).


FIG. 28. BLOCK DIAGRAM OF THE OVERALL DIMENSIONAI INSPECTION SYSTEM.

a) Intensity scan across a contoured image of the blade. (Contour interval $=0.016^{\prime \prime}$ )


FIG. 29. SHAPE OF THE CONVEX SURFACE OF A TURBINE BLADE CALCULATED FROM THE CONTOUR FRINGES.


FIG. 30. BLOCK DIAGRAM OF SHAPE GEIERATION COMPUTER PROGRAMAE.
.Shape Direction Inversion Point

(A) Scan concave surface.

Noise Correction Region


(a) Mechanical Scan.


Information at each point
V.

Point number
(Scan divided into 500pts Point lio. gives EX )
(b) Slow Scan Television.

(c) Maximum and Minimum Point Location.

FIG. 32. AVAILABLE TYPES OF DIGITIZED IHFORMATION FROM THE CONTOUR FRIIGESS.


FTG: 33. MECHANICAL REPERENCE PLANE LOCATION FOR
VARIOUS PARTS OF THE TURBINE BLADE.


FIG. 34a. CHANGE OF CO-ORDINATE AXES.


FIG. 34b. BLADE SHAPE FITTING BY ITERATION.


FIG. $35^{\prime}(a)$. HOMENTS OF A BLADE SECTION.

Overall
Difference
EX, 2 or Angle


Scan position along Blade Axis.

FIG. $35(\mathrm{~b})$. LINEAR AND SPLINE FITS TO DATA.


ON THE REFERHVE CENTROID



FIG. 37. TYPICAL GOOD QUALITY MOIRE SHADOW CONTOURS ACROSS A TURBINE BLADE. (CONTOUR DEPTH $=0.02^{\prime \prime}$ )


FIG. 38. TYPICAL GOOD QUALITY CONTOUR FRINGES USING THE FRINGE PROJECTION/SCAN TECHNIQUE. (CONTOUR DEPTH INTERVAL $=0.010^{\prime \prime}$ )


FIG. 39. COMPUTED ERROR PLOT ON A $10^{\circ}$ SLOPE. CONTOUR INTERVAL $=0.02$ ".


FIG. 40. COMPUTER CALCULATED SHAPE ACROSS TWO SECIIONS OF A BLADE. CONCAVE SURPACE.


FIG. 41. COMPUTER CALCULATED SHAPES FOR THE MASTER AND ROTATED BLADE CONCAVE SURFACE.


FIG. 42. COMPUTED SHAPE DIFFERENCES MEASURED ALONG THE 2 AXIS.
T.V. CAMERA


SIGNAL

FIG. 43. TELEVISION INTYRFACE FUNCTION - BLOCK DIAGRAM


FIG. 44. TELEVISION INTERFACE UNIT FOR MOIRE CONTOURING.


Tape
widh
a) Recording Format.

| Track No. | P | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hexadecimal Character | 1/0 | 0 | 0 | 0 | 0 | B.C.D. Number <br> B.C.D. Number Number |  |  |  |  |
| Numerical Character, | 1/0 | 1 | 1 | 1 | 1 |  |  |  |  |  |
| Alphabetic Letters | 1/0 | 1 | 1 | 0 | 0 |  |  |  |  |  |
| A to I. |  |  |  |  |  |  |  |  |  |  |

b) Recording Code.


FIG. 45. mangilic tape recording poriats.

a) Operation of Vidicon at the Photocathode.

b) Iffective Signal Decay with Tire.

c) Samples Taken by $1 / D$ Converter.

FIG. 46. TELEVISION CMERA CPRRGTION.




FIG. 48. AN EXAIPLE OF A SCAN RECORDED BY THE PHOTOMULTIPLIER/MAGNETIC TAPE RECORDER SYSTEM FOR COMPUYGR ANALYSIS (CON'SOUR DEPTH INITERVAL 0.015").

SOURCE LISTING

```
HOLO
/* APPENDIX ONE: BLAD
A010S749
- PROC OPTIONS (MAIN) 
A025S749
    DCL MP(20) FLOAT ;
A033S749
OCL ASTART FIXED BINARY
    DCL (ISUB1,ISUB2) FIXED BINARY ;
    DCL APO FIXED BINARY ;
    OCL APOINT FIXED BINARY ;
    DCL Y FIXED BINARY ;
    DCL (POINTS,START, COUNT,AA,AV,Q,T,AVSTAT) FIXED BINARY : A045S749
    DCL (IMAX,IMIN) FIXED BINARY ;
    DCL (VX(*),V(*),Z(*),EX(*)) CCNTROLLED ;
    DCL (REX(*),RZ(*)) CONTROLLED ;
    DCL RUN CHAR(10) EXT ;
    DCL (LIMI,LIM2) FLOAT :
    DCL IDENT CHAR(9) EXT ;
    DCL D CHAR(1) EXT ;
    OCL INDIC CHAR(1)
    DCL VAV(*) CONTROLLED;
    DCL CARDST (30) CHAR (80):
    Y = 1 ;
    OCL CARD CHAR (80) EXT ;
    DCL TEST CHAR (9) DEFINED CARD POSITION (1) :
    DCL BW CHAR (20) DEFINED CARD POSITION (60);
    DCL INPT FILE INPUT ;
    DCL ERRI CHAR (21) INIT ('FIRST CHARACTER NOT A') ;
    DCL ERR2 CHAR (14) INIT ('ERROR IN BLOCK') ;
    ON CONV GO TO REPEAT :
    ON ENDFILE (SYSIN) GO TO P2 ;
    II = OB
```



```
IF D = 'A' THEN GO TO ABLK ;
A034S749
A041S749
A042S749
A043S 749
A045S749
A046S749
A047S 749
A048S749
A049S749
A050S
A050S749
A051S749
A052S749
A053S749
A055S749
A056S749
A057S749
A058S749
A059S749
A060S749
A061S749
A061S749
A062S749
A0635749
A064S749
A064S749
A065S749
A066S749
A101S749
```

10
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
10
810
010 VDATA


| 61 | 1 | 0 |  |
| :--- | :--- | :--- | :--- |
| 62 | 1 | 0 |  |
| 63 | 1 | 0 | ERROR1 |
|  |  |  |  |
| 64 | 1 | 0 |  |
| 65 | 1 | 0 | AGIN |
| 66 | 1 | 0 |  |
| 67 | 1 | 0 |  |
| 68 | 1 | 0 |  |
| 69 | 1 | 0 |  |
| 70 | 1 | 0 |  |
| 71 | 1 | 0 | P1 |
| 72 | 1 | 0 |  |
| 73 | 1 | 0 |  |
| 74 | 1 | 0 | P2 |
| 75 | 1 | 0 | LOOK |
| 76 | 1 | 1 |  |
| 77 | 1 | 1 |  |
| 78 | 1 | 2 |  |
| 79 | 1 | 2 |  |
| 80 | 1 | 2 |  |
| 81 | 1 | 1 |  |
| 82 | 1 | 0 |  |

STMT LEV NT

| 85 | 1 | 0 |  | PUT EDIT ('XMAX =', XMAX, 'XMIN = ', XMIN)(SKIP(2), X(20), A, $F(6,0), X(30), A, F(6,0))$; | $\begin{aligned} & \text { A166S } 749 \\ & \text { A167S749 } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 86 | 1 | 0 |  | PUT EDIT ('COMPONENT WIDTH', WIDTH) (SKIP(2), X ${ }^{\text {(15), }} \mathrm{A}, \mathrm{F}(9,5)$ ); | A168S 749 |
| 87 | 1 | 0 |  | PUT EDIT ('DIRECTION INDICATOR $=1, \gamma)(\operatorname{SKIP}(4), X(9), A, F(3,0))$; | AEIOS 749 |
| 88 | 1 | 0 |  |  | AE20S749 |
|  |  |  |  | 'XLIM2 $=1, \mathrm{XLIM} 2)(\operatorname{SKIP}(3), X(10), A, X(10), A, F(7,0), X(10), A$, | AE30S 749 |
|  |  |  |  | $F(7,0))$ | AE40S 749 |
| 89 | 1 | 0 |  | PUT EDIT ('RUN TYPE $0=$ REF $1=\operatorname{SUBTRACT}$ REF $=\boldsymbol{1}$,REF) (SKIP(2), X(5), A,F(3,0)): | AE50S 749 |
| 90 | 1 | 0 |  | IMAX $=1$, | AE60S 749 |
| 91 | 1 | 0 |  | IMIN = 1; | A180S 749 |
| 92 | 1 | 0 |  | I SUB1 $=1$; | A182S 749 |
| 93 | 1 | 0 |  | I SUB2 $=1$; |  |
| 94 | 1 | 0 |  | ISMAX, ISMIN $=0$; | A188S 749 |
| 95 | 1 | 0 |  | ISUBF $=0$ : |  |
| 96 | 1 | 0 |  | $A V=1$; | A1905749 |
| 97 | 1 | 0 |  | $A A=1 ;$ | A195S749 |
| 98 | 1 | 0 |  | CDUNT $=-2$; | A 2005 749 |
| 99 | 1 | 0 |  | IPK,IAPK $=5$; | A 2015749 |
| 100 | 1 | 0 |  | $W=1$; |  |
| 101 | 1 | 0 |  | IIND $=0$; | A 2075749 |
| 102 | 1 | 0 |  | ASTART = START ; | A 2085749 |
|  |  |  |  | /* START OF MAIN PROGRAMME */ | A 2095749 |
|  |  |  |  | /* TO FIND MAX \& MIN */ | A 2105749 |
| 103 | 1 | 0 | CONTOR | : DO I = 2 TC (POINTS-1) ; | A 2205749 |
| 104 | 1 | 1 |  | IF IINO $=1$ THEN GO TO PEAK ; | A225S749 |
| 105 | 1 | 1 |  | IF VX(I) > LIMI \& VX(I) < LIM2 THEN GO TO AVCAL; | A 2305749 |
| 106 | 1 | 1 | PEAK | : IF V(I) $>V(I-1) \& V(I)>V(I+1)$ THEN GO TO LAB2; | A 2405749 |
| 107 | 1 | 1 |  | IF V(I) < V $(I-1) \& V(I)<V(I+1)$ THEN GO TO LAB1; | A 2505749 |
| 108 | 1 | 1 |  | GO TO LAB4; <br> /* AVERAGE CALCULATION */ | A 2605749 |
|  |  |  |  | /* AVERAGE CALCULATION */ | A300S 749 |
| 109 | 1 | 1 | AVCAL | : IF. $A V=1$ THEN AVSTAT $=1-6$; | A310S 749 |
| 110 | 1 | 1 |  | IF $A V=1$ THEN $1=1-6$; | A 3155749 |
| 111 | 1 | 1 |  | $V A V(A V)=(V(I-3)+V(I-2)+V(I-1)+V(I)+V(I+1)+V(I+2)$ | A 3205749 |


| 112 | 1 | 1 |
| :--- | :--- | :--- |
| 113 | 1 | 1 |
| 114 | 1 | 1 |
| 115 | 1 | 1 |
| 116 | 1 | 1 |
| 117 | 1 | 1 |
| 118 | 1 | 1 |
| 119 | 1 | 1 |
| 120 | 1 | 1 |
| 121 | 1 | 1 |
| 122 | 1 | 1 |
| 123 | 1 | 1 |
| 124 | 1 | 1 |
| 125 | 1 | 1 |
| 126 | 1 | 1 |
| 127 | 1 | 1 |
| 128 | 1 | 1 |
| 129 | 1 | 1 |
| 130 | 1 | 1 |
| 131 | 1 | 1 |
| 132 | 1 | 1 |
| 133 | 1 | 1 |
| 134 | 1 | 1 |
| 135 | 1 | 1 |
| 136 | 1 | 1 |
| 137 | 1 | 2 |

```
    +V(I+3))/7 : A330S749
    I=1+2;
    AV = AV+1:
    IF AV < }6\mathrm{ THEN GO TO AVCAL ;
    6 THEN GB TO AVCAL : A3755749
    IF VX(I) > LIM2 THEN GO TO AVLP: A3SOS749
    IF VX(I) < LIMI THEN GO TO AVLP ; A 385S749
    GO TO AVCAL
    - AA = AA+1 .
    I=(AA-1)*2+AVSTAT ;
    IF AA = AV - I THEN IIND = 1. . A 392S749
    IF AA = AV- 1 THEN I = I - 6 A A 393S749
    IF AA = AV-1 THEN GO TO LAB4;
    1* TO FIND MAX AND MIN OF AVERAGE VALUES *// A397S749
    IF VAV(AA) > VAV(AA-1) & VAV(AA) > VAV(AA+1) THEN GO TO A400S749
    AVMAX ; A410S749
    IF VAV(AA) < VAV(AA-1) & VAV(AA) < VAV(AA+1) THEN GO TO A420S749
    AVMIN
    GO TO AVLP ; A440S749
    A430S749
    /* INVESTIGATION FOR SPURIOUS MAX PRDDUCED BY NOISE */ A442S749
AVMAX : IF AA+3 > AV THEN GO TO AVMAX2 ; A446S749
    IF AA< 3 THEN GO TO AVMAX2 ; A4465749
    IF VAV(AA+1) > VAV(AA+2) & VAV(AA+2) > VAV(AA+3) THEN GO TO A447S749
AVMAXI ; ELSE GO TO AVEND; A ; A48S749
AVMAX1 : IF VAV(AA-1) > VAV(AA-2) & VAV(AA-2) > VAV(AA-3) THEN GO TO A448S749
AVMAX2 ; ELSE GO TO AVEND;
/* SUBMAX LOCATION */
A4495749
A449S749
AVMAX2 : IF ISUBF = 1 THEN GO TO SMAX ; A450S749
IF VX(I) > XLIM2 THEN GO TO SMAX ;
IF VX(I) < XLIMI THEN GO TO SMAX;
SUBMAX = CVAV(AAA-VMIN)/IVMAX-VMIN):
/* INVERSION ERROR CORRECTION */
A452S749
AF1OS749
IF SUBMAX < .75 & ISMIN = 1 THEN DO ;
AF15S749
PUT EDIT I'SUBMAX FRINGE ALSO FOUND SUB =', SUBMAX,'AA =',
AF20S749
AA) (SKIP(10),X(10),A,F(6,3),X(20),A,F(3,0));
A 3305749
A360S749
A370S749
A375S749
A380S 749
A385S 749
A386S749
AVLP
: \(A A=A A+1\).
A3905749
A391S749
A392S749
A3935749
A395S749
A3975749
A400S749
A410S749
A420S 749
A430S 749
A440S749
A442S 749
A4465749
A446S749
47S749
A448S749
A449S749
A450S749
A450S749
A451S749
A452S749
AF10S 749
AF20S 749
AF25S 749
```

| 138 | 1 | 2 |  |  | PUT EDIT ('PROGRAM TO RESTART ISUBI =', ISUB1) (PAGE,SKIP,X(30),A,F(4,0)): | $\begin{aligned} & \text { AF30S } 749 \\ & \text { AF } 35 S 749 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 139 | 1 | 2 |  |  | IMAX,IMIN = 1 ; | AF40S749 |
| 140 | 1 | 2 |  |  | ISUB2, AV $=1$; | AF415749 |
| 141 | 1 | 2 |  |  | $A A, W=1 ;$ | AF42S 749 |
| 142 | 1 | 2 |  |  | I SMIN, I SMAX $=0$; | AF45S 749 |
| 143 | 1 | 2 |  |  | ISUBF, IIND $=0$; | AF46S 749 |
| 144 | 1 | 2 |  |  | COUNT $=-2 \ldots$; | AF50S749 |
| 145 | 1 | 2 |  |  | $\mathrm{I}=2$; | AF55S 749 |
| 146 | 1 | 2 |  |  | START $=$ ASTART ; | AF60S 749 |
| 147 | 1 | 2 |  |  | GO TO LAB4 ; | AF65S 749 |
| 148 | 1 | 2 |  |  | END ; | AF70S 749 |
| 149 | 1 | 1 |  |  | IF SUBMAX < 75 THEN DO ; | 1454S 749 |
| 150 | 1 | 2 |  |  | I SUB1 $=-1$; | A455S 749 |
| 151 | 1 | 2 |  |  | IMIN $=-1$; | A456S 749 |
| 152 | 1 | 2 |  |  | I SUBF = 1 ; | A457S 749 |
| 153 | 1 | 2 |  |  | PUT EDIT ('SUB FRINGE PEAK ', 'SUB =', SUBMAX,'AA =', AA) (SKIP(5), X(10), $A, X(10), A, F(6,3), X(10), A, F(3,0)):$ | A 4585749 |
| 154 | 1 | 2 |  |  | $I S M A X=1 ;$; | $\begin{aligned} & \text { A460S749 } \end{aligned}$ $\text { A461S } 749$ |
| 155 | 1 | 2 |  |  | GO TO AVEND ; | A 4625749 |
| 156 | 1 | 2 |  |  | END ; | A464S 749 |
| 157 | 1 | 1 | SMAX | : | VMAX $=$ VAV(AA) $;$ | A 4685749 |
| 158 | 1 | 1 |  |  | IF IPK-I $<2 \& I-I P K<2$ THEN COUNT $=$ COUNT - 1 ; | A468S 749 |
| 159 | 1 | 1 |  |  | IAPK = I ; | A 4685749 |
| 160 | 1 | 1 |  |  | IF (VMAX-VMIN) < DIFMIN \& COUNT > O.THEN GO TO AVEND ; | A469S 749 |
| 161 | 1 | 1 |  |  | COUNT = COUNT +1 ; | A470S749 |
| 162 | 1 | 1 |  |  | GO TO AMAX ; | A480S 749 A 4815749 |
| 163 | 1 | 1 | AVMIN | : | IF AA+3. $>$ AV THEN GO TO AVMIN2 ; | A481S749 |
| 164 | 1 | 1 |  |  | IF AAく 3 THEN GO TO AVMIN2; | A4815749 |
| 165 | 1 | 1 |  |  | IF VAV $(A A+1)<\operatorname{VAV}(A A+2) \& \operatorname{VAV}(A A+2)<\operatorname{VAV}(A A+3)$ THEN GO TO | A482S 749 |
| 166 | 1 | 1 |  |  | AVMIN1 : ELSE GO TO AVEND : | A483S 749 |
| 167 | 1 | 1 | A VMIN1 | : | IF VAV $(A A-1)<\operatorname{VAV}(A A-2) \& \operatorname{VAV}(A A-2)<\operatorname{VAV}(A A-3)$ THEN GO TO | A483S 749 |
| 168 | 1 | 1 |  |  | AVMIN2 ; ELSE GO TO AVENO ; $\because \sim$ \% | A484S749 |

```
/* SUBMIN LOCATION */
A484S749
AVMIN2 : IF ISUBF:= 1 THEN GO TO SMIN ; A485S749
IF VX(I) > XLIM2 THEN GO TO SMIN ; A486S749
    IF VX(I) < XLIMI THEN GO TO SMIN ; A 487S749
    SUBMIN = (VAV(AA)-VMIN)/(VMAX-VMIN); /*:SUBFRINGE*/A492S749
    /* INVERSION ERROR CORRECTION */ AGIOS749
    IF SUBMIN > . 25 & ISMAX = 1 THEN DO ;: AG15S749
    PUT EDIT &'SUBMIN FRINGE ALSD FOUND SUB =',SUBMIN,'AA =', AG2OS749
    AA) {SKIP(10),X(10),A,F(6,3),X(20),A,F(3,0)); AG25S749
    PUT EDIT ('PROGRAM TO RESTART ISUBI =',ISUBI) AG30S 749
    (PAGE,SKIP,X(30),A,F(4,0)); AG35S749
    IMAX,IMIN = 1 A AG4OS749
    IMAX,IMIN =1; ; 
    AA,W = 1 AG42S749
    ISMIN,ISMAX = 0 AG45S749
```



```
    COUNT = -2. ; AG50S749
    I = 2 AG55S749
    START = ASTART ; AG60S749
    GO TO LAB4 ;
    END ;
    IF SUBMIN > . 25 THEN DO ;
        ISUBI = -1 ;
        IMAX = -1;
        I SUBF = 1;
        PUT EDIT ('FRINGE SUBMINN:',SUBMIN =',SUBMIN,'AA =',AA) A498S749
        (SKIP(5),X(10),A,X(10),A,F(6,3),X(10),A,F{3,0)); A500S749
        ISMIN = 1 ;
        GO TO AVEND;
        END ;
SMIN :VMIN = VAV(AA); A508S749
    IF IPK-I < 2 & I -IPK < 2 THEN COUNT = COUNT - 1; 
    IAPK = I ;
IF (VMAX-VMIN) < DIFMIN & COUNT > O THEN GO TO AVEND ; A509S749
    AG65S749
    IF VX(I) > XLIM2 THEN GO TO SMIN ;
    I = 2 ; 'ASTART ;
AG70S 749
```



```
        ^495S749
        A496S749
        A497S749
        A501S749
        END ; 
        A502S749
A504S749
A508S 749
A508S749
```

| 179 | 1 |  |
| :--- | :--- | :--- |
| 170 | 1 | 1 |
| 171 | 1 | 1 |
| 172 | 1 | 1 |

17311
17412
17512
$176 \quad 1 \quad 2$
$177 \quad 1 \quad 2$
17812
17912
$180 \quad 1 \quad 2$
18112
$1821 \quad 2$
183 1 2
$\begin{array}{lll}185 & 1 & 2\end{array}$
186 1 1
18712
188

| 191 | 1 | 2 |
| :--- | :--- | :--- |
| 192 | 1 | 2 |
| 193 | 1 | 2 |
| 194 | 1 | 1 |
| 195 | 1 | 1 |
| 196 | 1 | 1 |
| 197 | 1 | 1 |


| 198 | 1 | 1 |  |
| :--- | :--- | :--- | :--- |
| 199 | 1 | 1 |  |
| 200 | 1 | 1 | LAB1 |
| 201 | 1 | 1 |  |
| 202 | 1 | 1 |  |
| 203 | 1 | 1 |  |
| 204 | 1 | 1 |  |
| 205 | 1 | 1 | AMIN |
| 206 | 1 | 1 |  |
| 207 | 1 | 1 |  |
|  |  |  |  |
| 208 | 1 | 1 |  |
| 209 | 1 | 2 |  |
| 210 | 1 | 2 |  |
| 211 | 1 | 2 |  |
| 212 | 1 | 2 |  |
| 213 | 1 | 2 |  |
| 214 | 1 | 2 |  |
| 215 | 1 | 1 |  |
| 216 | 1 | 2 |  |
| 217 | 1 | 2 |  |
| 218 | 1 | 2 |  |
| 219 | 1 | 2 |  |
| 220 | 1 | 2 |  |
| 221 | 1 | 2 |  |
| 222 | 1 | 2 |  |
| 223 | 1 | 1 |  |
| 224 | 1 | 1 |  |
| 225 | 1 | 1 | LAB2 |
| 226 | 1 | 1 |  |
|  |  |  |  |

```
    COUNT = COUNT+1 ;
    GO TO AMIN :
: COUNT = COUNT+1 ;
    IF IAPK-I< < & I-IAPK < 2 THEN COUNT = COUNT - 1: A535S749
    IPK = I ;
    VMIN = V(I);
    PUT EDIT !VMIN = ',YMIN,'I = ',I,'COUNT =1,COUNT)(SKIP,
    COUNT = , COUNT)(SKIP
    X(10),A,F(6,0),X(10),A,F(5,1),X(15),A,F(4,1));
    : IF IMAX = -1 THEN N=M:
    IF IMIN = -1 THEN N = I;
    IF IMIN = 1 THEN M = I ;
    /* PARAMETERS RESET FOR INVERSION AT A SUBMIN */ A570S749
    IF IMAX = -1 THEN DO ;
        W=-1 ;
        START = START + COUNT - 1;
        COUNT = 0;
        IMAX = 1 ;
        PUT EDIT {'INVERSION POINT AT SUBMIN START =',START)
        (SKIP(5),A;F(4,0));
        END ;
    /* PARAMETERS RESET FOR INVERSION AT A MAX */
    IF ISUB2 = -1 THEN DO ;
START = START + COUNT ;
    COUNT = 0 ;
    W=-1;
    I SUB1 = -1;
    I SUB2 = 1;
    PUT EDIT ('INVERSION POINT AT MAX START =',START)
    (SKIP(5),X(20),A,F(4,0));
    END ;
IF COUNT < O THEN GO TO LAB4;
GO TO LAB3 :
- COUNT = CDUNT+1 ;
IF IAPK-I < 2&I-IAPK < 2 THEN COUNT = COUNT - 1;
A510S 749 A520S749 A530S 749 A535S749 A537S 749 A540S749
A550S 749
A560S749
A562S 749
A565S 749
A570S749
A571S749
A572S749
A573S749
A574S 749
A575S749
A575S 749
A5775749
A578S 749
A579S749
A580S 749
A580S749
A581S749
A582S 749
A583S 749
A584S749
A585S 749
A586S749
A587S749
A588S 749
: COUNT \(=\) COUNT+1 ;
A5895 749
IF IAPK-I \(<2\) \& I-IAPK \(<2\) THEN COUNT = COUNT - 1 ;
A590S 749
A600S 749
A605S749
```

LAB 2

| 227 | 1 | 1 |  |
| :--- | :--- | :--- | :--- |
| 228 | 1 | 1 |  |
| 229 | 1 | 1 |  |
|  |  |  |  |
| 230 | 1 | 1 | AMAX |
| 231 | 1 | 1 |  |
| 232 | 1 | 1 |  |
|  |  |  |  |
| 233 | 1 | 1 |  |
| 234 | 1 | 2 |  |
| 235 | 1 | 2 |  |
| 236 | 1 | 2 |  |
| 237 | 1 | 2 |  |
| 238 | 1 | 2 |  |
|  |  |  |  |
| 239 | 1 | 2 |  |
| 240 | 1 | 1 |  |
| 241 | 1 | 2 |  |
| 242 | 1 | 2 |  |
| 243 | 1 | 2 |  |
| 244 | 1 | 2 |  |
| 245 | 1 | 2 |  |
| 246 | 1 | 2 |  |
| 247 | 1 | 2 |  |
| 248 | 1 | 1 |  |
| 249 | 1 | 1 | LAB3 |
| 250 | 1 | 1 |  |
| 251 | 1 | 1 |  |
| 252 | 1 | 1 |  |
| 253 | 1 | 1 |  |
| 254 | 1 | 1 |  |

```
    IPK = I
    VMAX = V(I) ;
    X(10),A,F(6,0),X(10),A,F(5,1),X(10),A,F(5,1); A630S749
:IF IMIN =-1 THEN M = N ; A632S749
    IF}\cdotIMAX = -1 THEN M = I ;
    IF IMAX = 1 THEN N = I ;
    /* PARAMETERS RESET FOR INVERSION AT A SUBMAX */ A641S749
    IF IMIN = -1 THEN DO ;
        W=-1;
        START = START + COUNT - 1;
        COUNT = 0 ;
        IMIN = 1;
        PUT EDIT ('INVERSION POINT AT SUBMAX START =',START) ^6475749
        (SKIP(5),X(20),A,F(4,0)): A648S749
        END ; A649S749
    /* PARAMETERS RESET FOR INVERSION AT A MIN */ A65OS749
    IF ISUB2 = -1 THEN DO ;
    START = START + COUNT ;
        COUNT = 0;
        W=-1 ;
        ISUB1 = -1 ;
        ISUB2 = 1;
        PUT EDIT ('INVERSION POINT AT MIN START:=',START)
        (SKIP(5),X(20),A,F(4,0));
        END ;
    IF COUNT < O THEN GD TO LAB4 ;
    /* NOISE INVESTIGATION */
: IF (VMAX-VMIN) < DIFMIN THEN COUNT = COUNT-2 ;
IF (VMAX-VMIN) < DIFMIN THEN GO TO AVEND;
IF N > M THEN GO TO LABA :
NN=N;
MM = M ;
Q = -1:
A607S749
A610S 749
Abss
A632S 749
A635S749
A640S749
A6415749
A642S 749
A643S 749
A644S 749
A645S 749
A646S749
A6475749
A648S749
A650S749
A6505749
A651S749
AS51S749
A652S749
A653S 749
A654S749
A655S 749
A656S749
A657S749
^658S749
A6595749
A659S 749
A660S 749
A670S 749
A680S749
A690S 749
A 7005749
```



STMT LEV NT

| 285 | 1 | 2 |
| :--- | :--- | :--- |
| 286 | 1 | 2 |
| 287 | 1 | 2 |
| 288 | 1 | 2 |
| 289 | 1 | 3 |
| 290 | 1 | 3 |
| 291 | 1 | 3 |
| 292 | 1 | 3 |
| 293 | 1 | 3 |
| 294 | 1 | 3 |
| 295 | 1 | 2 |
| 296 | 1 | 1 |
| 297 | 1 | 1 |
| 298 | 1 | 0 |
| 299 | 1 | 0 |
|  |  |  |
| 300 | 1 | 0 |
| 301 | 1 | 0 |
| 302 | 1 | 0 |
| 303 | 1 | 1 |
| 304 | 1 | 1 |
| 305 | 1 | 1 |
| 306 | 1 | 1 |
| 307 | 1 | 1 |
| 308 | 1 | 1 |
| 309 | 1 | 1 |
| 310 | 1 | 1 |
| 311 | 1 | 1 |
| 312 | 1 | 1 |

DSUM=((EX(L)-DEXB)*Z(L)+(EX(L-1)-DEXB)*Z(L-1)+(EX(L-2)-DEXB)BB13S749
$* Z(L-2)+(E X(L-3)-D E X B) * Z(L-3)+(E X(L-4)-D E X B) * Z(L-4)) ;$ BB14S 749
$D S Q U=((\operatorname{EX}(L)-D E X B) * * 2+(E X(L-1)-D E X B) * * 2+(E X(L-2)-D E X B)$, BB15S749
$* * 2+(E X(L-3)-D E X B) * * 2+(E X(L-4)-D E X B) * * 2)) ; \quad$ BB16S 749
$\mathrm{DZ}=\mathrm{DSUM} / \mathrm{DSQU}$;
BB17S 749
IF DZ < GRAD \& DZ > -GRAD THEN DO ; BB2OS 749

```
        I SUB1 \(=-1\);
```

        \(\begin{array}{ll}\text { ISUB2 }=-1 ; & \text { BB40S749 }\end{array}\)
        ISMIN \(=1\); BB44S749
        ISMAX \(=1\) B BB46S749
        PUT EDIT ('INVERSION POINT EX \(=1, E X(L)\) (SKIP(5), X(20), BB48S749
        A,F(9,5)) ; BB49S749
        ENO:
        BB50S749
    : END CALA : B115S749
AVEND : IF AA < AV-I THEN GO TO AVLP.;
LAB4 : END CONTOR ;
PUT EDIT ('POINTS NUMBER', EX VALUE','Z VALUE') B132S749
B120S749
(PAGE, $X(20), A, X(20), A, X(20), A) ;$
PUT EDIT (\{T,EX(T),Z(T) DO T = 1 TO POINTS)
B133S749
B134S 749
(SKIP,X(25),F(4,0),X(24),F(9,5),X(20),F(9,5)); B135S749
/* CALCULATION OF CENTROID \& RADII OF GYRATION */ POIOS749
PLIN = 0 ;
$X M 1, X M 2, Z M 1, Z M 2, H 1=0 ;$
LICAL
: DOT $=12$ TO (POINTS - 12 );
DS $=(E X(T+1)-E X(T)) * * 2+(Z 12$
PO60S749
IF DSQ $=0$ THEN GO TO LICEND ;
DS = SQRT(DSQ) ;
PLIN = PLIN + DS ;
XM1 $=X M 1+(.5$ *DS*(EX(T+1) $+E X(T))) ;$
$X M 2=X M 2+(.25 * D S *(E X(T+1)+E X(T)) * * 2)$;
$Z M 1=Z M 1+(.5 * D S *(Z(T+1)+Z(T))) ;$
$Z M 2=Z M 2+(.25 * D S *(Z(T+1)+Z(T)) * * 2) ;$
$H 1=H 1+(.25 * D S *(E X(T+1)+E X(T)) *(Z(T+1)+Z(T))) ; \quad P 220 S 749$
LICEND
: END LICAL :



## ATTRIBUTE AND CROSS-REFERENCE TABLE

```
CCL NO. ICENTIFIER ATTRIBUTES AND REFERENCES
```

8 AA AUTOMATIC ALIGNED BINARY FIXED $(15,0)$
97, 118, 118, 119, 120, 121, 122, 123, 123, 123, 123, 124, 124, 124, 124, 126, 127, 128, 128, 128
$128,130,130,130,130,135,137,141,153,157,163,164,165,165,165,165,167,167,167$,
167, 172, 174, 178, 190, 194, 296
34 ABLK /* STATEMENT LABEL CONSTANT */
65
AGIN $\quad$ * STATEMENT LABEL CONSTANT */
AINT AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
260, 261, 261, 261, 262
230 AMAX $/ *$ STATEMENT LABEL CONSTANT */
205
AMIN $\quad 1$ : STATEMENT LABEL CONSTANT */
ANGL AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
347, 349
APO AUTOMATIC ALIGNED BINARY FIXED(15,0)
50, 51
APOINT AUTOMATIC ALIGNED BINARY FIXED(15,0)
51, 52
ASIN /* STATEMENT LABEL CONSTANT */
273

```
DCL NO. IDENTIFIER ATTRIBUTES AND REFERENCES
ASTART AUTOMATIC ALIGNED BINARY FIXED(15,0)
    102, 146, 183
AV AUTOMATIC ALIGNED BINARY FIXED(15,0)
    96, 109, 110, 111, 113, 113, 114, 120, 121, 122, 126, 140, 163, 177, 296
AvCAL /* STATEMENT LABEL CONSTANT */
    105, 114, 117
AVEND /* STATEMENT LABEL CONSTANT */
    129, 131, 155, 160, 166, 168, 192, 197, 250
AVLP /* STATEMENT LABEL CONSTANT */
    115, 116, 125, 296
avmax /* STATEMENT LABEL CONSTANT */
    123
AVmaX1 /* STATEMENT LABEL CONSTANT */
    128
AVmax2 /* STATEMENT LABEL CONSTANT */
    126, 127, 130
AvMin /* STATEmENT LABEL CONSTANT */
    124
AvMINI /* STATEMENT LABEL CONSTANT */
        165
AVMIN2 /* STATEMENT LABEL CONSTANT */
        163, 164, 167
AVSTAT AUTOMATIC ALIGNED BINARY FIXED(15,0)
```

DCL NO. IDENTIFIER ATTRIBUTES AND REFERENCES

274 BSIN | /* STATEMENT LABEL CONSTANT */ |
| :--- |
| 268,272 |

BW $\quad$| DEFINED CARD POSITION(60) UNALIGNED CHARACTER(20) |
| :--- |

C AUTOMATIC ALIGNED DECIMAL/* SINGLE */ FLOAT(6) 262, 268, 268, 269, 270

CALA $\quad$ : STATEMENT LABEL CONSTANT */ 255

CALEND $/ *$ statement label constant */ 280, 281, 282, 283

CARD EXTERNAL STATIC UNALIGNED CHARACTER(80) 21, 22. 76

CARDST (30) AUTOMATIC UNALIGNED CHARACTER(80) 71, 76

CARDTOT AUTOMATIC ALIGNED DECIMAL $/ *$ SINGLE 75 */ FLOAT(6)
CONTOR $/ *$ STATEMENT LABEL CONSTANT */

COUNT AUTOMATIC ALIGNED BINARY FIXED(15,0)
98, 144, 158, 158, 160, 161, 161, 181, 195, 195, 197, 198, 198, 200, 200, 201, 201, 204, 210, 211 216, 217, 223, 225, 225, 226, 226, 229, 235, 236, 241, 242, 248, 249, 249, 276, 278

D EXTERNAL STATIC UNALIGNED CHARACTER(I)

```
    29, 30, 32, 33, 60, 61, 64, 65,67,69
```

| ******** | DELTA | AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6) <br> $53,82,276,278$ |
| :--- | :--- | :--- |
|  |  |  |
| ******** | DELTAA |  |
|  | AUTOMATIC ALIGNED DECIMAL <br> 37,53 |  |

DEXB AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
284, 285, 285, 285, 285, 285, 286, 286, 286, 286, 286
DIFMIN AUTOMATIC ALIGNED DECIMAL/* SINGLE */ FLOAT(6)
DIFTAN AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLDAT(6)
345, 346, 346, 349
DS AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
305, 306, 307, 308, 309, 310, 311
DSQ AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT (6)
303, 304, 305
DSGU AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
286, 287
DSUM AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
285, 287
DTAN AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
D2 AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
287, 288, 288
/* APPENDIX ONE: BLADE SHAPE FROM CONTOUR FRINGES */
CCL NO. IDENTIFIER ATTRIBUTES AND REFERENCES


```
DCL NO. IDENTIFIER ATTRIBUTES AND REFERENCES
```

LABC $\quad{ }_{275}^{*}$ STATEMENT LABEL CONSTANT */
LABD $\quad \underset{277}{*}$ STATEMENT LABEL CONSTANT */

| LAB1 | 107 |
| :--- | :--- |

LAB2 /* STATEMENT LABEL CONSTANT */
106
LAB3 /* STATEMENT LABEL CONSTANT */
224
LAB4 /* STATEMENT LABEL CONSTANT */
108, 122, 147, 184, 223, 248
LICAL /* STATEMENT LABEL CONSTANT */
LICEND /* STATEMENT LABEL CONSTANT */
304
LIM1 AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
37, 82, 105, 116
LIM2 AUTOMATIC ALIGNED DECIMAL $/ *$ SINGLE */ FLOAT(6)
37, 82, 105, 115
LOOK /* STATEMENT LABEL CONSTANT */
AUTOMATIC ALIGNED BINARY FIXED(15,0)

ECL NO. IDENTIFIER ATTRIBUTES AND REFERENCES

| 8 | $Q$ | AUTOMATIC ALIGNED BINARY FIXED(15,0) 254, 258, 275 |
| :---: | :---: | :---: |
| \#******** | R | AUTOMATIC ALIGNED DECIMAL /* SINGLE */ fLOAT(6) 274, 276, 278 |
| ******** | RADX | AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6) 314, 317, 318, 329 |
| ******** | RADZ | aUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6) 316, 321, 322, 330 |
|  | REF | AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6) 40, 42, 44, 47, 39, 332 |
| 32 | REPEAT | ** STATEMENT LABEL CONSTANT */ 26, 33 |
| 11 | REX | (*) CONTROLLED ALIGNED DECIMAL /* SINGLE */ FLOAT(6) 43, 48 |
| ******** | RTAN | automatic aligned decimal /* Single */ float(6) 339, 345, 345 |
| 12 | RUN | EXTERNAL STATIC UNALIGNED CHARACTER(10) 35, 36 |
| ********* | RXBAR | automatic aligned decimal /* Single */ float (6) 333, 343 |
| ******** | RXRAD | AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT 6 (6) 335 |
| ********* | RXRG | AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6) |

```
DCL NO. IDENTIFIER ATTRIBUTES AND REFERENCES
    SUBMAX AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
    135, 136, 137, 149, 153
    SUBMIN AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
        172, 173, 174, 186, 190
SYSIN EXTERNAL FILE
    27, 71
SYSPRINT EXTERNAL FILE STREAM OUTPUT PRINT
    31, 36, 63,64, 67, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 137, 138, 153, 174, 175, 190, 204,
    213, 221, 229, 238, 246, 293, 298, 299, 328, 329, 330, 331, 340, 348, 349
T AUTOMATIC ALIGNED BINARY FIXED(15,0)
    57, 58, 58, 299, 299, 299, 299, 302, 303, 303, 303, 303, 307, 307, 305, 308, 309, 309, 310, 310,
    311; 311; 311, 311
TEST DEFINED CARD POSITION(1) UNALIGNED CHARACTER(9)
    77
10
17 VAV (*) CONTROLLED ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
    52, 111, 123, 123, 123, 123, 124, 124, 124, 124, 128, 128, 128, 128, 130, 130, 130, 130, 135, 157
    165, 165, 165, 165, 167, 167, 167, 167, 172, 194, 350
VDATA / ** STATEMENT LABEL CONSTANT */
VMAX AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
    135, 157, 160, 172, 197, 228, 229, 249, 250, 260
```



```
CCL NO. ICENTIFIER
    ATTRIBUTES AND REFERENCES
```

******** ZRG AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
324, 327, 330, 338
ZROOT AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
322, 323, 323, 323, 324
ZSHIFT AUTOMATIC ALIGNED DECIMAL /* SINGLE */ FLOAT(6)
344, 348

AGGREGATE-LENGTH TABLE


STORAGE REQUIREMENTS

| BLOCK, SECTION OR STATEMENT | type | BLOCK LENGTH | (HEX) | DSA SIZE | (HEX) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| HOLO | PROCEDURE BLOCK |  |  | 2520 | 908 |
| 28 | ON UNIT |  |  | 488 | 1E8 |
| 27 | ON UNIT |  |  | 488 | 1 E8 |
| HOLO | DICTIONARY | 11452 | 2CBC |  |  |
| HOLO | LINK EDIT STUB | 320 | 140 |  |  |
| HOLO | STATIC INTERNAL | 728 | 208 |  |  |

```
WARNING DIAGNOSTICS
```

IEN0023I W $31,36,63,64,67,78,79,82,83,84,85,86,87,88,89,137,138,153,174,175,190,204,213,221$, $229,238,246,293,298,299,328,329,330,331,340,348,349$ NO 'FILE' OR 'STRING' OPTION SPECIFIED FOR 'PUT' STATEMENT. 'FILE(SYSPRINT)' ASSUMED.
IENOO22I W 71 NO 'FILE' OR 'STRING' OPTION SPECIFIED FOR 'GET' STATEMENT. 'FILE(SYSINJ' ASSUMED.
end of Compiler diagnostic messages


| 1 |  |  |
| :---: | :---: | :---: |
| 2 | 1 |  |
| 3 | 1 |  |
| 4 | 1 |  |
| 5 | 1 |  |
| 6 | 1 |  |
| 7 | 1 |  |
| 8 | 1 |  |
| 9 | 1 |  |
| 10 | 1 |  |
| 11 | 1. |  |
| 12 | 1 |  |
| 13 | 1 |  |
| 14 | 1 |  |
| 15 | 1 |  |
| 16 | 1 |  |
| 17 | 1 |  |
| 18 | 1 |  |
| 19 | 1 |  |
| 20 | 1 |  |
| 21 | 1 |  |
| 22 | 1 | 1 |
| 23 | 1 | 1 |
| 24 | 1 | 1 |
| 25 | 1 | 1 |

SIMCAL

DATA

SCAL
/* APPENDIX TWO: SIMULATED CONTOUR SHAPE FITTING*/
: PROC OPTIONS (MAIN) :
A040S749
DCL X(250), RY1 (250), RY2(250), PY1(250), PX21250); AO60S749
DCL PX1(250), PY2(250), YO(250), RDX1(250), RDX2(250);
DCL EX(250), EY1(250),EY2(250) ;
DCL PYA (250), XA (250), XB(250) ;
; A080S749 ACL A120S749
DCL (U.T) FIXED BINARY ;
DCL R FIXED BINARY ;
$K K=0$; A1 205749 A145S749
GET EDIT $(A, B, P, Q)(X(14), F(6,4), X(14), F(6,4), X(14), F(6,4), A 150 S 749$
X(14),F(6,4)) ;
A1605749
GET EDIT (SR,TR,AL)(X(14),F(6,4),X(14),F(6,4),X(34),F(6,5)) A170S749
/* INPUT \& DISTORTION PARAMETERS $\times(14), F(6,4), X(34), F(6,5)) ; A 180 S 749$
PUT EDIT ('INPUT PARAMETERS', $A=1, A, B=1, B, P=1, P, \quad A 190 S 749$

'Y DIRECTION TR = ', TR) (PAGE,SKIP(5), X(30), A,SKIP,X(10),
$A, F(9,5), X(20), A, F(9,5), S K I P(3), X(10), A, F(9,5), X(20)$,
$A, F(9,5), S K I P(5), X(20), A, S K I P(2), X(10), A, F(9,5), X(20), A$, $F(9,5))$; A220S749 A240S749 A260S749 A280S749

PUT EDIT('ANGLE OF ROTATION =', AL)(SKIP(2),X(10), A,F(9,5)); A300S749
$B L=57.3066 * A L$;
$\begin{array}{ll}B L & =57 \cdot 3066 * A L \\ \text { PUT EDIT ('ANGLE OF ROTATION IN DEGREES }=, ~ B L) & A 320 S 749\end{array}$
(SKIP(2), X(10), A,F(9,5));
$Z X=P * C O S(A L)-Q * \operatorname{SIN}(A L) ;$
$Z Y=P * S I N(A L)+Q * \operatorname{CDS}(A L) ;$
$S R=P-Z X+S R$;
$T R=Q-Z Y+T R$;
PUT EDIT ('SR =', SR,'TR =', TR) (SKIP(2), X (10), A,F(9,5), $X(30), A, F(9,5))$;
/* TO CALCULATE REFERENCE AND PRODUCTON SHAPE */
$\mathrm{I}=0$;
: DO XK $=0$ TO 2.00 BY. 01 ;
$I=I+1$;
$X(I)=X K$;
$\Delta A=(1-((X(1)-P) / A) * * 2) ;$
IF $A A<0$ THEN GO TO SCEND;

A3015749
A340S749 A360S749 A4005749 A410S749 A4205749 A4305749 A440S749 A450S749 BO 205749 B0255749 B0305749 B0355749 B0405749 B060S749 B080S749

| 53 | 1 | 1 |
| :--- | :--- | :--- |
| 54 | 1 | 1 |
| 55 | 1 | 1 |
| 56 | 1 | 1 |
| 57 | 1 | 1 |
| 59 | 1 | 1 |
| 60 | 1 | 1 |
| 61 | 1 | 1 |
| 62 | 1 | 1 |
| 63 | 1 | 1 |
| 64 | 1 | 1 |
| 65 | 1 | 1 |
| 66 | 1 | 1 |
| 67 | 1 |  |
| 68 | 1 |  |
| 69 | 1 |  |
| 70 | 1 |  |
| 71 | 1 |  |
| 72 | 1 |  |
| 73 | 1 |  |
| 74 | 1 |  |
| 75 | 1 |  |
| 76 | 1 |  |
| 77 | 1 |  |
| 78 | 1 |  |
|  |  |  |
| 79 | 1 |  |
| 80 | 1 |  |
| 81 | 1 |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

```
    RYMI = RYM1 + (.5*DS*(RY2(R) +RY2(R+1))); K220S749
    RYM2 = 2YM2 +( - 25*DS*(RY2(R)+RY2(R+1))**2); K240S749
    RH1=2HI + (.25*DS*(X(R+1)*X(R))*(RY2(R)+RY2(R+1))); K260S749
```



```
    IF PDSQ = O THEN GO TO LICEND ;
    PDS = SQRT(PDSQ) ;
    PLIN = PLIN + PDS;
    PXM1 =PXM1 + (.5*PDS*(PX2(R+1)+PX2(R)));
    PXM2 =PXM2 + (. 25*PDS*(PX2(R+1)+PX2(R))**2)
    PYM1 =PYM1 +(.5*PDS*(PY2(R+1)+PY2(R)));
PYM2 =PYM2 + (.25*PDS*(PY2(R+1) +PY2(R))**2) ; K440S749
    PH1 = PH1 +(.25*DS*(PX2(R+1)+PX2(R))*(PY2(R+1)+PY2(R))); K450S749
    K4605749
RLXB = RXMI/RLIN;
RXAD = RXM2/RLIN;
RLYB = RYMI/RLIN;
RYAD = RYMZ/RLIN;
XRADL = SQRT(RXAD);
XRG = SQRT(RXAD-RLXB**2);
YRADL = SQRT(RYAD) ;
YRG = SQRT(RYAD-RLYB**2) ;
RHXY = RHI/RLIN;
RHBAR = (RHXY - RLXB*RLYB)
RTAN2 = 2*RHBAR/(XRG**2-YRG**2);
PUT EDIT ('REF LINE','LENGTH =',RLIN,'XBAR =',RLXB,
YYBAR =',RLYB)(PAGE,SKIP(10),X(20),A,SKIP,X(10),A,F(9,5),
X(20),A,F(9,5),X(20),A,F(9,5));
PUT EDIT ('RAD OF G X SQ =',RXAD,'XRADL =',XRADL,
*XRG =',XRG)(SKIP(5),X(10),A,F(9,5),X(20),A,F(9,5),X(20),
A,F(9,5));
PUT EDIT ('RAD OF G Y SQ =',RYAD,'YRADL =',YRADL,
'YRG =',YRG)(SKIP(2),X(10),A,F(9,5),X(20),A,F(9,5),X(20),
A,F(9,51);
PUT.EDIT I'CROSS PRODUCT RHXY =',RHXY,'RHBAR =',RHBAR,
k3205749
K360S749
LICAL;
```

K220S749 K240S749 K3005749 K3205749 K340S749 K360S749 K 380 S749 K400S749 $K 420 S 749$
$K 440 S 749$ K450S749 K4605749 K500S749 K520S749 K540S749 K560S749 K580S749 K600S749 K6205749 K6405749 K642S749 K644S749 K646S749 K660S749 K6805749 K700S749 K7205749 K740S749 K760S749 K780S749 K800S749 K820S749 K830S749
LICEND
82
83
84
85
86
87
88
89
90
91



[^2]
9
94
95





```
'TWICE ANGLE TAN-1 = ', RTAN)(SKIP(5),X(10),A,F(9,5),X(20), K832S749
```

'TWICE ANGLE TAN-1 = ', RTAN)(SKIP(5),X(10),A,F(9,5),X(20), K832S749
A;F(9,5),SKIP,X(10),A,F(9,5)); K834S749
A;F(9,5),SKIP,X(10),A,F(9,5)); K834S749
PLXB = PXM1/PLIN;
PLXB = PXM1/PLIN;
PXAD = PXM2/PLIN;
PXAD = PXM2/PLIN;
PLYB = PYMI/PLIN;
PLYB = PYMI/PLIN;
PYAD = PYM2/PLIN ;
PYAD = PYM2/PLIN ;
XPADL = SQRT(PXAD); ;
XPADL = SQRT(PXAD); ;
XPG = SQRT(PXAD - PLXB**2);
XPG = SQRT(PXAD - PLXB**2);
YPADL = SQRT(PYAD) ;
YPADL = SQRT(PYAD) ;
YPG = SQRT(PYAD - PLYB**2) ;
YPG = SQRT(PYAD - PLYB**2) ;
PHXY = PHI / PLIN ;
PHXY = PHI / PLIN ;
PHBAR = (PHXY - PLXB*PLYB) ;
PHBAR = (PHXY - PLXB*PLYB) ;
PTAN2 = 2*PHBAR/(XPG**2-YPG**2);
PTAN2 = 2*PHBAR/(XPG**2-YPG**2);
L020S749
L020S749
PUT EOIT ('PROD LINE','LENGTH =',PLIN,'PXBAR =', PLXB,
PUT EOIT ('PROD LINE','LENGTH =',PLIN,'PXBAR =', PLXB,
,PYBAR = ', PLYB)(PAGE,SKIP(10),X(20), A,SKIP,X(10),A,F(9,5),
,PYBAR = ', PLYB)(PAGE,SKIP(10),X(20), A,SKIP,X(10),A,F(9,5),
X(20),A,F(9,5),X(20),A,F(9,5)) ;
X(20),A,F(9,5),X(20),A,F(9,5)) ;
PUT EDIT ('RAD OF G Y SQ =', PXAD,'XPADL =', XPADL,
PUT EDIT ('RAD OF G Y SQ =', PXAD,'XPADL =', XPADL,
'XPG =',XPG)(SKIP(5),X(10),A,F(9,5),X(20),A,F(9,5),X(20), L280S749
'XPG =',XPG)(SKIP(5),X(10),A,F(9,5),X(20),A,F(9,5),X(20), L280S749
A,F(9,5));
A,F(9,5));
A,F(9,5)';
A,F(9,5)';
(SKIP,X(10),A,F(9,5),X(20),A,F(9,5),X(20),A,F(9,5)); L340S749
(SKIP,X(10),A,F(9,5),X(20),A,F(9,5),X(20),A,F(9,5)); L340S749
PUT EDIT ('CROSS PRODUCT PHXY =',PHXY,'PHBAR =',PHBAR, L360S749
PUT EDIT ('CROSS PRODUCT PHXY =',PHXY,'PHBAR =',PHBAR, L360S749
'TWICE ANGLE TAN-1 =',PTAN2)(SKIP(5),X(10),A,F(9,5),X(20), L370S749
'TWICE ANGLE TAN-1 =',PTAN2)(SKIP(5),X(10),A,F(9,5),X(20), L370S749
A,F(9,5),SKIP,X(10),A,F(9,5)) ;
A,F(9,5),SKIP,X(10),A,F(9,5)) ;
/* TO CALCULATE OVERALL DIFFERENCES */
/* TO CALCULATE OVERALL DIFFERENCES */
XSHIFT = PLXB-RLXB ;
XSHIFT = PLXB-RLXB ;
YSHIFT = PLYB-RLYB ;
YSHIFT = PLYB-RLYB ;
DFTN = (PTAN2-RTAN2)/(1+PTAN2*RTAN2);
DFTN = (PTAN2-RTAN2)/(1+PTAN2*RTAN2);
DFTN = (PTAN2-RTAN2)/(1+PTAN2*RTAN2); L500S749
DFTN = (PTAN2-RTAN2)/(1+PTAN2*RTAN2); L500S749
DFT = SQRT(1+1/(DFTN**2))-1/DFTN ; L592S749
DFT = SQRT(1+1/(DFTN**2))-1/DFTN ; L592S749
DANGL = 57.3066*DFT ; L594S749
DANGL = 57.3066*DFT ; L594S749
PUT EDIT ('LINE CENTROID') (SKIP(4),X(20),A): L596S749
PUT EDIT ('LINE CENTROID') (SKIP(4),X(20),A): L596S749
PUT EDIT ('SHIFT PARAMETERS','XSHIFT =',XSHIFT,'YSHIFT =', L600S749
PUT EDIT ('SHIFT PARAMETERS','XSHIFT =',XSHIFT,'YSHIFT =', L600S749
YSHIFT)(SKIP(2),X(20),A,SKIP,X(10),A,F(9,5),X(20),A,F(9,5));L640S749
YSHIFT)(SKIP(2),X(20),A,SKIP,X(10),A,F(9,5),X(20),A,F(9,5));L640S749
PUT EDIT ('ANGULAR DIFFERENCE IN DEGREES =',DANGL)(SKIP(5), L680S749

```
PUT EDIT ('ANGULAR DIFFERENCE IN DEGREES =',DANGL)(SKIP(5), L680S749
```

| 105 | 1 |  | $X(10), A, F(9,5)) ;$ | 1.7005749 |
| :---: | :---: | :---: | :---: | :---: |
| 106 | 1 |  | $P P=R L X B-R L X B * C O S D(D A N G L)+R L Y B * S I N D(D A N G L)$ $Q Q=R L Y B-R L X B * S I N D(D A N G L)-R L Y B * C D S D$ | L8005749 |
| 107 | 1 |  |  | L8205749 |
| 108 | 1 |  |  | 18405749 18505749 |
| 109 | 1 |  | $\mathrm{CXSH}=\mathrm{XIN}+\mathrm{XSHIFT}-\mathrm{P}$; |  |
| 110 | 1 |  | CYSH $=$ YIN + YSHIFT - Q ; | $L 8605749$ 18805749 |
| 111 | 1 |  | PUT EDIT ('EFFECTIVE SHIFT OF SECTIONAL CENTROID', | L8905749 |
|  |  |  | ' XSHIFT $=$ ', CXSH, 'YSHIFT $=1$, CYSH) | L900S749 |
|  |  |  | (SKIP(2), X(10), A, SKIP,X(10), A,F(9,5), X(20), A,F(9,5)) ; | L9105749 |
|  |  |  | /* SHAPE FITTING BETWEEN REF \& PROD: COMPLETE SECTIONS*/ | B400S749 |
| 112 | 1 |  | IF KK = 1 THEN GO TO PROCAL ; | B450S749 |
| 114 |  |  | ** TO CALCULATE MNTS ONREF OBJECT DX ELEMENTS */ | B500S749 |
| 114 115 | 1 |  | RAREA, RMNT1, RMNT2 $=0$; | B520S749 |
| 116 | 1 |  | DO IA $=1$ TO 200 | B5305749 |
| 117 | 1 | 1 | DELR $=(1$ YY2 (IA) +RY2(IA+1) $)$ | B540S749 |
| 118 | 1 | 1 | RAREA $=$ RAREA + DELR ; | B560S749 |
| 119 | 1 | 1 | RMNT1 $=$ RMNT1 $+($ OELR* $(X(I A)+.055))$ | B6005749 |
| 120 | 1 | 1 | RMNT2 $=$ RMNT $2+(D E L R *((X(I A)+.005) * * 2)) ;$ |  |
| 121 | 1 | 1 | $X R X Y=X R X Y+(.5 * D E L R *(X(I A)+.005) *(R Y 2(I A)+R Y I(I A))\} ;$ | B620S749 B630S749 |
| 122 | 1 | 1 | END ; | B6405749 |
| 123 | 1 |  | XBAR = RMNTI/RAREA ; | B6605749 |
| 124 | 1 |  | RAD2 $=$ RMNT2/RAREA ; | B680S749 |
| 125 | 1 |  | XRAD2 $=$ SQRT(RAD2) ; | B7005749 |
| 126 | 1 |  | $X R A D G=S Q R T(R A D 2-X B A R * * 2) ;$ | 87205749 |
| 127 | 1 |  | $X R=X R X Y / R A R E A$; | B720S749 |
| 128 | 1 |  | PUT EDIT ('ELLIPSE SECTIONS: SHIFT PARAMETER CALCULATIONS') (PAGE,SKIP(3), X(30), | B7305749 |
| 129 | 1 |  |  | 87355749 $B 7405749$ |
|  |  |  | 'RAD OF G SQ =', RAD2,'XRAD2 =', XRAD2,'XRADG =', XRADG) (SKIP(5), X(10), A, F(9,5), SKIP(2) | $B 7405749$ $B 7605749$ |
|  |  |  |  | B7805749 |
| 130 | 1 |  | SKIP (2), A,F(9,5), X(20), $A, F(9,5), X(10), A, F(9,5))$; | B8005749 |
| 130 | 1 |  | PUT EDIT ('CROSS PRODUCT = ', XR)(SKIP(2), X(10), A,F(9,5)) ; | B820S749 |



| 167 | 1 |  |  |
| :--- | :--- | :--- | :--- |
| 168 | 1 |  |  |
|  |  |  |  |
|  |  |  |  |
| 169 | 1 |  |  |
| 170 | 1 |  |  |
|  |  |  |  |
| 171 | 1 |  | PROCAL |
| 172 | 1 |  |  |
| 173 | 1 |  |  |
| 174 | 1 |  |  |
| 175 | 1 | 1 |  |
| 177 | 1 | 1 |  |
| 178 | 1 | 1 | MAXI |
| 179 | 1 | 1 |  |
| 180 | 1 | 1 |  |
| 181 | 1 | 1 |  |
| 182 | 1 |  |  |
| 183 | 1 |  |  |
| 184 | 1 |  | PDX |
| 185 | 1 |  |  |
| 186 | 1 | 1 |  |
| 187 | 1 | 1 |  |
| 188 | 1 | 1 | LOPI |
| 189 | 1 | 1 |  |
| 191 | 1 | 1 |  |
| 192 | 1 | 1 |  |
| 194 | 1 | 1 |  |
| 195 | 1 | 1 | DCAL |
| 197 | 1 | 1 |  |

```
    RT2 = 2*RHB/(XRADG**2-YRADG**2) ;
    PUT EDIT ('DY ELEMENTS AREA =',YRAR,'C OF G YBAR',YBAR,
    'RAD OF G SQ =',RYD2,'YRAD2 =',YRAD2,'YRADG =',YRADGI C440S749
    (SKIP(9),X(10),A,F(9,5),SKIP,X(10),A,F(9,5),X(10),A,F(9,5),C460S749
    SKIP,X(10),A,F(9,5),X(20),A,F(9,5)); C480S749
    PUT EDIT ('TWICE ANGLE OF PRINCIPAL AXIS =',RT2) (SKIP(5), C490S749
    X(10),A,F(9,5));
    PUT EDIT ('CROSS PRODUCT =',YR)(SKIP(2),X(10),A,F(9,5)); C520S749
    1* PRODUCTION COMPONENT */ C5, C5S749
    /* TO CALCULATE MNTS */ DO25S749
:KK = 1 ; 0030S749
1* DX ELEMENTS */
CMIN = 9 ;
CMAX = 0 ; }\quad0.
DO MM = 1 TO 199 BY 3 ; }\quad\mathrm{ DO45S749
IF MM > 190 THEN GO TO MAXI ; DO45S749
IF MM > 190 THEN GO TO MAXI ; DO45S749
XMIN = MIN(PXI(MM),PXI(MM+1),PXI(MM+2),CMIN); DO46S749
: XMAX = MAX(PXI(MM),PX1(MM+1),PX1(MM+2),CMAX), D048S749
M, PX1(MM+1),PX1(MM+2),CMAX)
END ;
L = 0;
PARX,PMNT1,PMNT2 = 0 ;
PYMT1,PYMT2,XPXY = 0;
: DO XX = XMIN TO XMAX BY .O1:
L = L + 1 ;
EX(L) = XX ;
K,U =1 ;
: IF PXI(U) < EX(L) & PXI(U+1) > EX(L) THEN GO TO DCAL ;
U=U+1 ;
IF U >= 202 THEN GO TO PDXEND ;
GO TO LOP1 ;
: IF (PXI{U)-PX1(U-1))=0 THEN GO TO PDXEND
EY1(L)= PY1(U)-((PX1(U)-EX(L))*(PY1(U)-PY1(U-I))/
00405749
D040S749
CMIN = XMIN ; PMM, PXI(MM+1),PXI(MM42),CMAXI DO48S749
; DO47S749
D049S749
D049S749
D0505749
0055S749
D060S749
D070S749
00805749
00805749
D0905749
D100S749
D1205749
D140S749
D160S749
D1805749
D2005749
0220S749
0240S749
```

| 198 | 1 | 1 | LOP 2 |
| :--- | :--- | :--- | :--- |
| 200 | 1 | 1 |  |
| 201 | 1 | 1 |  |
| 203 | 1 | 1 |  |
| 204 | 1 | 1 | FCAL |
| 206 | 1 | 1 |  |
| 207 | 1 | 1 |  |
| 209 | 1 | 1 |  |
| 210 | 1 | 1 |  |
| 211 | 1 | 1 |  |
| 212 | 1 | 1 |  |
| 213 | 1 | 1 | PDXEND |
| 214 | 1 | 1 |  |
| 215 | 1 |  |  |
| 216 | 1 |  |  |
| 217 | 1 |  |  |
| 218 | 1 |  |  |
| 219 | 1 |  |  |
| 220 | 1 |  |  |
|  |  |  |  |
|  |  |  |  |
| 221 | 1 |  |  |
| 222 |  |  |  |
| 223 | 1 |  |  |
| 224 | 1 |  |  |
| 225 | 1 | 1 |  |
| 227 | 1 | 1 |  |
| 228 | 1 | 1 |  |
| 229 | 1 | 1 |  |



```
stmt level nest
```

| 230 | 1 | 1 |
| :--- | :--- | :--- |
| 231 | 1 | 1 |
| 232 | 1 | 1 |
| 233 | 1 | 1 |
| 234 | 1 |  |
| 235 | 1 |  |
| 236 | 1 |  |
| 237 | 1 |  |
| 238 | 1 | 1 |
| 239 | 1 | 1 |
| 240 | 1 | 1 |
| 241 | 1 | 1 |
| 242 | 1 | 1 |
| 244 | 1 | 1 |
| 246 | 1 | 1 |
| 248 | 1 | 1 |
| 249 | 1 | 1 |
| 251 | 1 | 1 |
| 253 | 1 | 1 |
| 254 | 1 | 1 |
| 256 | 1 | 1 |
| 257 | 1 | 1 |
| 259 | 1 | 2 |
| 260 | 1 | 2 |
| 261 | 1 | 2 |
| 262 | 1 | 1 |
| 263 | 1 | 1 |
| 264 | 1 | 1 |
| 265 | 1 | 1 |
| 266 | 1 | 1 |

```
    YMAX = MAX(PYZ2(M),PY2(M+1),PY2(M+2),BMAX); E085S749
    AMAX = Y1MAX ; EO87S749
    BMAX = YMAX ;
        END :
        J = 0 ;
        PARY,PYMT1,PYMTZ =0 ;
        YPXY = 0 ;
DO PYE = YMIN TO {YMAX+.005\ BY .005 ; E2OOS749
    J = J+1 ;
    PYA(J) = PYE +.0001 ; E240S749
    KA,KB = 1 ;
    ISHIFT,JSHIFT = 0 ;
        IF PYA(J) > Y1MAX THEN GO TO TLJP
```



```
IF PYI(KA)> YIMAX THEN GOTO TLJP ; E290S749
    IFPYI(KA) < PYA(J) & PYI(KA+1) >PYA(J) THEN GO TO MCAL ; E300S749
    IF PYI(KA) > PYA(J) & PY1(KA+1) < PYA(J) THEN GO TO MCAL ; E310S749
    KA = KA + 1 ;
```



```
    IF KA >2OO THEN GO TO YCALE ; E360S749
    GO TO SLOP ; E37OS749
: IF ISHIFT = 1 THEN GO TO NCAL ; E380S749
    XA(J)=PX1(KA)-((PY1(KA)-PYA(J))*(PX1(KA)-PX1(KA-1)) E400S749
    /(PY1(KA)-PY1(KA-1)); E420S749
    IF J = I THEN DO ; E430S749
    XB(J)=XA(J); E432S749
    GO TO YCALE ;
    END ;
    ISHIFT = 1;
    KA = KA + 2;
    GO TO SLOP ;
: XB(J) = PX1(KA)-{(PY1(KA)-PYA(J))*(PX1(KA)-PX1(KA-1))
    /(PY1(KA)-PY1(KA-1))):
    GO TO ARCALS;
PCAL: XB(J) = XA(J):
TLOP : IF PYZ(KB) < PYA(J) & PYZ(KB+1) >PYA(J) THEN GO TO RCAL : E580S749
```

STMT LEVEL NEST

| 270 | 1 | 1 |
| :--- | :--- | :--- |
| 272 | 1 | 1 |
| 273 | 1 | 1 |
| 275 | 1 | 2 |
| 276 | 1 | 2 |
| 277 | 1 | 2 |
| 278 | 1 | 1 |
| 280 | 1 | 1 |
| 281 | 1 | 1 |
| 283 | 1 | 1 |
| 284 | 1 | 1 |
| 286 | 1 | 1 |
| 287 | 1 | 1 |
| 288 | 1 | 1 |
| 289 | 1 | 1 |
| 290 | 1 | 1 |
| 292 | 1 | 1 |
| 293 | 1 | 1 |
| 294 | 1 | 1 |
| 295 | 1 | 1 |
| 296 | 1 | 1 |
| 297 | 1 | 1 |
| 298 | 1 |  |
| 299 | 1 |  |
| 300 | 1 |  |
| 301 | 1 |  |
| 302 | 1 |  |
| 303 | 1 |  |
| 304 | 1 |  |
| 305 | 1 |  |

RCAL : IF JSHIFT = 1 THEN GO TO XCAL;

```
```

IF PY2(KB) >PYA(J) \& PY2(KB+1) < PYA(J) THEN GO TO RCAL ; E585S749

```
```

IF PY2(KB) >PYA(J) \& PY2(KB+1) < PYA(J) THEN GO TO RCAL ; E585S749
KB = KB + 1 ; SHTFT = THEN DO E600S749
KB = KB + 1 ; SHTFT = THEN DO E600S749
IF KB > 200 \& JSHIFT = 1 THEN DO ; E622S749
IF KB > 200 \& JSHIFT = 1 THEN DO ; E622S749
XB(J) =2.095; E624S749
XB(J) =2.095; E624S749
GO TO ARCALS ;
GO TO ARCALS ;
END ;
END ;
IF KB >200 THEN GO TO YCALE;
IF KB >200 THEN GO TO YCALE;
GO TO TLOP ;
GO TO TLOP ;
XA(J)=PX2(KB)-((PY2(KB)-PYA(J))*{PX2(KB)-PX2(KB-1)) E660S749
XA(J)=PX2(KB)-((PY2(KB)-PYA(J))*{PX2(KB)-PX2(KB-1)) E660S749
/(PY2(KB)-PY2(KB-1))):
/(PY2(KB)-PY2(KB-1))):
IF ISHIFT = 1 THEN GO TO ARCALS ;
IF ISHIFT = 1 THEN GO TO ARCALS ;
JSHIFT = 1 ;
JSHIFT = 1 ;
KB = KB + 2;
KB = KB + 2;
GO TOTLOP ;
GO TOTLOP ;
: XB(J) =P X2(KB)-((PY2(KB)-PYA(J))*(PX2(KB)-PX2(KB-1))
: XB(J) =P X2(KB)-((PY2(KB)-PYA(J))*(PX2(KB)-PX2(KB-1))
/(PY2(KB)-PY2(KB-1)));
/(PY2(KB)-PY2(KB-1)));
ARCALS : IF J = 1 THEN GO TO YCALE ;
PYDEL =. OO25*(XB(J) +XB(J-1)-XA(J)-XA(J-1));
PYDEL =. OO25*(XB(J) +XB(J-1)-XA(J)-XA(J-1));
PARY = PARY + PYDEL ;
PARY = PARY + PYDEL ;
PYMT1 = PYMT1 + {PYDEL*(PYA(J)-.0025));
PYMT1 = PYMT1 + {PYDEL*(PYA(J)-.0025));
PYMT2 = PYMT2 + (PYDEL*(PYA(J)-.0025)**2);

```
PYMT2 = PYMT2 + (PYDEL*(PYA(J)-.0025)**2);
```

```
    E624S749
```

    E624S749
    E628S749
    E628S749
    E629S749
    E629S749
    E630S749
    E630S749
    E640S749
    E640S749
    YPXY = YPXY + (.5*PYDEL*(PYA(J)-.0025)*(XA(J)+XB(J))); E860S749
YPXY = YPXY + (.5*PYDEL*(PYA(J)-.0025)*(XA(J)+XB(J))); E860S749
: END YCAL :
: END YCAL :
YPBAR = PYMT1/PARY;
YPBAR = PYMT1/PARY;
PRAD =PYMT2/PARY ;
PRAD =PYMT2/PARY ;
YRAD = SQRT(PRAD);
YRAD = SQRT(PRAD);
YPADG = SQRT(PRAD-YPBAR**2) ;
YPADG = SQRT(PRAD-YPBAR**2) ;
YP = YPXY/PARY ;
YP = YPXY/PARY ;
PHB = (XP - XPBAR*YPBAR );
PHB = (XP - XPBAR*YPBAR );
PT2 = 2*PHB/(XPADG**2-YPADG**2);
PT2 = 2*PHB/(XPADG**2-YPADG**2);
PUT EDIT |'PRODUCTION DY FLEMENTS','AREA =',PARY, F120S749
PUT EDIT |'PRODUCTION DY FLEMENTS','AREA =',PARY, F120S749
F120S749
F120S749
'C OF G =',YPBAR,'PRAD SQ =',PRAD,'R OF G =',YRAD,'YPADG = ',F140S7449
'C OF G =',YPBAR,'PRAD SQ =',PRAD,'R OF G =',YRAD,'YPADG = ',F140S7449
YPADG)(SKIP(10),X(10),A,SKIP,X(10),A,F(9,5),X(20),A,
YPADG)(SKIP(10),X(10),A,SKIP,X(10),A,F(9,5),X(20),A,
F160S749

```
F160S749
```

YCALE


|  |  | attribute and cross-reference table |
| :---: | :---: | :---: |
| DCL No. | IDEVTIFIER | Attributes and references |
|  | A | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 9,11,24,142 \end{aligned}$ |
|  | AA | ```AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 24,25,27``` |
|  | AL | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECINAL, FLOAT(SINGLE) } \\ & 10,12,13,15,15,16,16,30,30,31,31,32,32,33,33,38,38,39,39 \end{aligned}$ |
|  | AMAX | ```AUTOMATIC,ALIGNED,DECIMAL,FLDAT(SINGLE) 223,229,231``` |
|  | AMIN | $\begin{aligned} & \text { AUTOMATIC, ALIGNED,DECIMAL,FLOAT(SINGLE) } \\ & 222,227,228 \end{aligned}$ |
|  | ANG | aUtomatic, aligned, decimal, float(single) 40,41 |
|  | ANGL | $\begin{aligned} & \text { AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) } \\ & 311,313 \end{aligned}$ |
| 293 | ARCALS | STATEMENT labEL CONSTANT 266,276,285 |
|  | B | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 9,11,27,135 \end{aligned}$ |
|  | BL | aUtomatic,aligned, decimal, float(Single) $13,14$ |
|  | BYAX | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) |


| IDENTIFIER | ATtRIBUTES AND REFERENCES |
| :---: | :---: |
|  | 223,230,232 |
| $\operatorname{cox}$ | $\begin{aligned} & \text { AUTOMATIC,ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 38,41 \end{aligned}$ |
| cGY | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 39,41 \end{aligned}$ |
| CMAX | AUTOMATIC, AL IGNED, DEC IMAL, FLOAT(SINGLE) 173,179,180 |
| CMIN | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) $172,177,178$ |
| cos | GENERIC,BUILT-IN FUNCTION <br> 15,16,30,31,32,33,38,39 |
| COSD | $\begin{aligned} & \text { GENERIC,BUILT-IN FUNCTION } \\ & 105,106,107,108 \end{aligned}$ |
| CXSH | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 109,111 |
| CYSH | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 110,111 \end{aligned}$ |
| DA | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT (SINGLE) } \\ & 135,136,140,141,141,142 \end{aligned}$ |
| DANGL | AUTOMATIC,ALIGNED, DECIMAL, FLOAT(SINGLE) $101,104,105,105,106,106,107,107,108,108$ |
| DATA | statement label constant 314 |


| DCL NO. | IDENTIFIER | ATTRIBUTES AND REFERENCES |
| :---: | :---: | :---: |
| 195 | DCAL | STATEMENT LABEL CONSTANT 190 |
| $\therefore$. | DELR | AUTOMATIC, ALIGNED,DECIMAL, FLOAT(SINGLE) 117,118,119,120,121 |
|  | DFT | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) $100,101$ |
|  | DFTAN | $\begin{aligned} & \text { AUTOMATIC, ALIGNED,DECIMAL, FLOAT(SINGLE) } \\ & 309,310,310 \end{aligned}$ |
|  | DFTN | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 99,100,100 |
|  | DRDOT | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 142,143,144 |
|  | DS | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 47,50,51,52,53,54,55,65 |
|  | DSQ | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 46,47,48 |
|  | DTAN | AUTOMATIC, ALIGNED,DECIMAL,FLOAT(SINGLE) 310,311,312 |
| 132 | DYCAL | Statement label constant |
| 145 | DYEND | statement label constant $138$ |
| 4 | EX | (250) AUTOMATIC,ALIGNED, DEC IMAL, FLOAT |

EYI

EY2

FCAL

FINISH







## ATTRIBUTES AND REFERENCES

```
187,189,189,197,198,198,206,211,212,213
```


## (250)AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE)

 197.209.209,213(250)AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) $206,209,209,213$

STATEMENT LABEL CONSTANT 199

STATEMENT LABEL CONSTANT
AUTOMATIC, ALIGNED, BINARY,FIXED(15,0) $20,22,22,23,24,28,29,30,30,30,31,31,31,32,32,32,33,33,33$

AUTOMATIC, ALIGNED, BINARY,FIXED(15,0) $116,117,117,117,117,119,120,121,121,121$

AUTOMATIC, ALIGNED, BINARY, FIXED(15,0) $131,133,133,134,135,143,144$

AUTOMATIC,ALIGNED, BINARY, FIXED (15,0) $148,149,149,149,149,152,157,158,159,159,159$

AUTOMATIC, ALIGNED, BINARY,FIXED(15,0) $241,249,254,262,284$

AUTOMATIC, ALIGNED, BI NARY, FIXED $(15,01$
$234,238,238,239,242,244,244,246,246,256,256,257,259,259,265,265,267$ $267,268,268,270,270,275,283,283,289,289,290,292,292,292,292,294,295$ 296.296 .296

AUTOMATIC, ALIGNED, BINARY, FIXEO $(15,0)$

PCL NO.
IDENTIFIER



$K B$



LICAL
LICEND

LOP1

LOP2

KA

198

ATTRIBUTES $\triangle N D$ REFERENCES
$241 \cdot 273 \cdot 281 \cdot 286$

AUTOMATIC,ALIGNED,BINARY,FIXED(15,0)
$188,198,198,200,200,201,204,204,206,206,206,206,206,206$

AUTOMATIC, ALIGNED, BINARY, FIXED(15,0)
$240,244,244,246,246,248,248,249,251,256,256,256,256,256,256,263,263$ $265,265,265,265,265,265$

AUTOMATIC, ALIGNED, BINARY,FIXED(15,0)
$240,268,268,270,270,272,272,273,278,283,283,283,283,283,283,287,287$ 289.289 .289 .289 .289 .289

AUTOMATIC, ALIGNED, BINARY, FIXED(15,0) 8,112,171

AUTOMATIC,ALIGNED, BINARY,FIXED(15,0)
$182,186,186,187,189,189,197,197,198,198,206,206,207,209,209,209,209$ 211.212.213.213.213

STATEMENT LABEL CONSTANT

STATEMENT LABEL CONSTANT
49,58
STATEMENT LABEL CONSTANT 194

STATEMENT LABEL CONSTANT 203

AUTOMATIC, ALI GNED, BI NARY, FIXED $(15,0)$ $224,225,227,227,227,229,229,229,230,230,230$

ATTRIBUTES AND REFERENCES

MAX

MAXI

MCAL

MIN

MM

NCAL
$p$

PARX

PARY

PCAL

PDS

PDS2

GENERIC, BUILT-IN FUNCTION 179,229,230

STATEMENT LABEL CONSTANT

- 176

STATEMENT LABEL CONSTANT 245,247

GENERIC, BUILT-IN FUNCTION 177,227

AUTOMATIC, ALIGNED, BINARY,FIXED(15.0) 174,175,177,177,177,179,179,179

STATEMENT LABEL CONSTANT 255

AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 9,11,15,16,17,24,38,39,107,108,109,143,144
AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 183,210,210,215,216,219,220

AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 235,293,293,298,299,302,305

STATEMENT LABEL CONSTANT 250

AUTOMATIC,ALIGNED, DECIMAL, FLOAT(SINGLE) 59,60,61,62,63,64

AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE)


IDEVTIFIER
pp

```
PRAD AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE)
    299,300,301,305
    AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE)
    216,217,218,220
statement label constant
113
AUTOMATIC, ALIGNED,DECIMAL,FLOAT(SINGLE)
304,309,309
AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE)
92,96,99,99
(250)AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE)
30,37,40,40,177,177,177,179,179,179,189,189,195,195,197,197,197,256
```

256,256,265,265,265
(250)AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE)
$32,37,56,56,61,61,62,62,65,65,198,198,204,204,206,206,206,283,283$
283,289,289,289

AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 83,86,87,94

AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 209,210,211,212,213

AUTOMATIC, ALIGNED, DECIMAL, FLDAT(SINGLE) 43,61,61,82

| OCL NO. | IDENTIFIER | ATTRIBUTES AND REFERENCES |
| :---: | :---: | :---: |
|  | P XM2 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 43,62,62,83 |
| 2 | PY1 | $\begin{aligned} & \text { (250)AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 31,37,40,40,197,197,197,227,227,227,229,229,229,244,244,246,246,256 \\ & 256,256,265,265,265 \end{aligned}$ |
| 3 | PY2 | (250)AUTOMATIC, ALIGNED,DEC IMAL, FLOAT(SINGLE) <br> $33,37,56,56,63,63,64,64,65,65,206,206,206,230,230,230,268,268,270$ 270,283,283,283,289,289,289 |
| 5 | PYA | $\begin{aligned} & \text { (250)AUTOMAT IC, AL I GNED, DEC IMAL, FLOAT(SINGLE) } \\ & 239,242,244,244,246,246,256,265,268,268,270,270,283,289,294,295,296 \end{aligned}$ |
|  | PYAD | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 85,88,89,95 |
|  | PYDEL | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 292,293,294,295,296 |
|  | PYE | AUTOMATIC, ALIGNED, DECIMAL,FLOAT(SINGLE) 237.239 |
|  | PYM1 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 43,63,63,84 |
|  | PYM2 | AUTOMATIC,ALIGNED, DECIMAL, FLOAT(SINGLE) 43,64,64,85 |
|  | PYMT1 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 184,235,294,294,298 |
|  | PYMT2 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) |


| IDENTIFIER | ATTRIRUTES AND REFERENCES |
| :---: | :---: |
|  | 184,235,295,295,299 |
| Q | AUTOMATIC,ALIGNED, DECIMAL, FLOAT(SINGLE) 9,11,15,16,18,28,29,38,39,107,108,110,135 |
| QQ | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 106,108 \end{aligned}$ |
| R | $\begin{aligned} & \text { AUTOMATIC, ALI GNED, BI NARY, FIXED }(15,0) \\ & 45,46,46,46,46,51,51,52,52,53,53,54,54,55,55,55,55,56,56,56,56,61, \\ & 62,62,63,63,64,64,65,65,65,65 \end{aligned}$ |
| RAD2 | AUTOMATIC, ALIGNED,DECIMAL, FLOAT(SINGLE) 124,125,126,129 |
| RAREA | AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 114,118,118,123,124,127,129 |
| RCAL | STATEMENT LABEL CONSTANT 269,271 |
| ROXI | (250)AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 143,149,149,159 |
| RDX2 | (250)AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 144,149,149,159 |
| RH1 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 44,55,55,75 |
| RHB | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 166,167 |
| RHBAR | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) |

bCL NO.

| IDENTIFIER | ATTRIBUTES AND REFERENCES |
| :---: | :---: |
|  | 76,77,81 |
| RHXY | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 75,76,81 |
| RLIV | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) $42,50,50,67,68,69,70,75,78$ |
| RLXB | AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 67,72,76,78,97,105,105,106 |
| RLYB | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 69,74,76,78,98,105,106,106 |
| RMNT 1 | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 114,119,119,123 |
| RYNT2 | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 114,120.120,124 |
| RT2 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 167,169,309,309 |
| RTAV | AUTOMATIC, ALIGNED,DECIMAL, FLOAT(SINGLE) 81 |
| RTAN2 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 77,99,99 |
| RXAD | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 68,71,72,79 |
| RXM1 | AUTOMATIC,ALIGNED, DECIMAL,FLOAT(SINGLE) $42,51,51,67$ |




| IDENTIFIER | ATTRIBUTES AND REFERENCES |
| :---: | :---: |
|  | 188,189,189,191,191,192,195,195,197,197,197,197,197,197 |
| $x$ | (250)AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) $23,24,30,31,32,33,37,46,46,51,51,52,52,55,55,119,120,121$ |
| XA | (250)AUTOMATIC, ALIGNED,DECIMAL, FLOAT(SINGLE) 256,259,267,283,292,292,296 |
| XB | (250)AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 259,265,267,275,289,292,292,296 |
| XBAR | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 123,126,129,166,307 |
| XCAL | STATEMENT LABEL CONSTANT 282 |
| XIN | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT (SINGLE) } \\ & 107,109 \end{aligned}$ |
| XK | AUTOMATIC,ALIGNED, DECIMAL, FLOAT(SINGLE) 21,23 |
| XL | AUTOMATIC,ALIGNED, DECIMAL,FLOAT(SINGLE) 132,134 |
| XMAX | AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 179,180,185 |
| XMIV | AUTOMATIC,ALIGNED, DECINAL,FLOAT(SINGLE) 177,178,185 |
| XP | AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 219,221,303 |


|  | $x$ |
| :---: | :---: |
|  | XA |
|  | XB |
|  | XBAR |
|  | XCAL |
|  | XIN |
|  | XK |
|  | XL |
|  | XMAX |
|  | XMIV |
|  | $X P$ |


| IDENTIFIER | ATTRIBUTES AND REFERENCES |
| :---: | :---: |
| XPADG | AUTOMATIC, ALIGNED, DECIMAL, FLOAT (SINGLE) 218,220,304 |
| XPADL | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 86,94 |
| XPBAR | AUTOMATIC, ALIGNED,DECIMAL, FLOAT(SINGLE) 215,218,220,303,307 |
| XPG | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 87,92.94 |
| XDXY | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 184,213,213,219 |
| XR | AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) $127,130,166$ |
| $X R A D$ | AUTOMATIC,ALIGNED, DECIMAL,FLDATISINGLEI 217,220 |
| XPAD2 | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 125,129 |
| XRADG | AUTOMATIC, ALIGNED, DECIMAL,FLOAT(SINGLE) 126,129.167 |
| XRADL | AUTOMATIC, ALIGNED, DECIMAL,FLOAT(SINGLE) 71,79 |
| XRG | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 72,77,79 \end{aligned}$ |

IDENTIFIER
XROOT
XRXY
XSHIFT
$Y X$
YIM
YIMAX
YBAR
YCAL
YCALE
YD
YDELR
YEL

## ATTRIBUTES AND REFERENCES

AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 27.28.29

AUTOMATIC, ALIGNED, DECINAL, FLOAT(SINGLE) $115,121,121,127$

AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 97.103,109

AUTOMATIC, ALIGNED, DECIMAL,FLOAT(SINGLE) 185,187

STATEMENT LABEL CONSTANT 226

AUTOMATIC, ALIGNED, DECIMAL,FLDAT(SINGLE) 229.231.242

AUTOMATIC,ALIGNED, DECIMAL,FLOAT (SINGLE) $161,164,166,168,308$
statement label constant
STATEMENT LABEL CONSTANT 252,260,279,291
(250)AUTOMATIC,ALIGNED, OECIMAL,FLOAT (SINGLE) $134,135,157,158,159$

AUTOMATIC, ALIGNED,DECIMAL,FLOAT(SINGLE) 155,156,157,158,159

AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 149.150 .155

| IDENTIFIER | $\triangle T T R I B U T E S ~ A N D ~ R E F E R E N C E S ~$ |
| :---: | :---: |
| YELA | statement label constant |
| YELEND | STATEMENT LABEL CONSTANT 153 |
| YIN | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 108,110 |
| YMAX | $\begin{aligned} & \text { AUTOMATIC, ALIGNED,DECIMAL, FLOAT (SINGLE) } \\ & 230,232,237 \end{aligned}$ |
| YMIV | $\begin{aligned} & \text { AUTOMATIC, AL IGNED, OECIMAL, FLOAT (SINGLE) } \\ & 227,228,237 \end{aligned}$ |
| YP | $\begin{aligned} & \text { AUTOMATIC, ALIGNED,DECIMAL, FLOAT(SINGLE) } \\ & 302,306 \end{aligned}$ |
| YPADG | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 301,304,305 |
| YPADL | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 88,95 |
| YPBAR | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 298,301,303,305,308 |
| YPG | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 89,92,95 |
| YPXY | AUTOMATIC,ALIGNED,DECINAL, FLOAT(SINGLE) 236,296,296,302 |
| $Y R$ | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) |


| IOENTIFIER | ATTRIBUTES AND REFERENCES |
| :---: | :---: |
|  | 165,170 |
| YRAD | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 300,305 |
| YRAD 2 | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 163,168 \end{aligned}$ |
| YRADG | AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 164,167,168 |
| YRADL | AUTOMATIC,ALIGNED,DECIMAL,FLOAT(SINGLE) 73,80 |
| YRAR | AUTOMATIC,ALIGNED,DECIMAL, FLOAT(SINGLE) 146,156,156,161,162,165,168 |
| YRG | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT (SINGLE) } \\ & 74,77,80 \end{aligned}$ |
| YRMVT1 | $\begin{aligned} & \text { AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) } \\ & 146,157,157,161 \end{aligned}$ |
| YRMNT2 | AUTOMATIC, ALIGNED,DECIMAL,FLOAT(SINGLE) 146,158,158,162 |
| YRXY | AUTOMATIC, ALIGNED, DECIMAL, FLOAT(SINGLE) 147,159,159,165 |
| YSHIFT | AUTOMATIC,ALIGNED, DECIMAL, FLOAT(SINGLE) 98,103,110 |
| ZX | AUTOMATIC,ALIGNED, DECIMAL,FLOAT(SINGLE) 15,17 |

AUTOMATIC,ALIGNED, DECIMAL, FLOAT (SINGLE)
16.18
agGregate length table

## STATEMENT NO. IDENTIFIER

EX
EY1
EY2
P×1
$\mathrm{P} \times 2$
PY1
PY2
PYA
RDXI 1000
RDX2 1000
RY1 1000
RY2 1000
$x$ 1000
XA 1000
$X B$
YD

1000
LENGTH IN BYTES
1000
1000
1000
1000
1000
1000
1000
1000

1000

Storage requirements.
the storage area for the procedure labelled simcal is 16924 bytes long.
THE PROGRAM CSECT IS NAMED SIMCAL AND IS 17524 BYTES LONG.
the static csect is named *simcala and is 2283 bytes long.
*STATISTICS* SOURCE RECORDS $=\quad 358$, PROG TEXT STMNTS $=\quad 315$, OBJECT BYTES $=17524$

COMPILER DIAGVOSTICS.

## WARNINGS.

| IEMO227I | NO FILE/STRING OPTION SPECIFIED IN ONE OR MORE GET/PUT STATEMENTS. SYSIN/SYSPRINT HAS BEEN |
| :--- | :--- |
| IEMO764I | ASSUMED IN EACH CASE. |
| IEM3898I | ONE OR MORE FIXED BINARY ITEMS OF PRECISION 15 OR LESS HAVE BEEN GIVEN HALFWORD STORAGE. THEY |
| COMPILER CORE REQUIREMENT EXCEEDED SIZE GIVEN. AUXILIARY STORAGE USED. |  |

END OF DIAGNOSTICS.
AUXIlIARY Storage will not be used for dictionary when size = 122 K

COMPILE TIME
ELAPSED TIME
. 31 MINS
. 90 MINS

```
F128-LEVEL LINKAGE EDITOR OPTIONS SPECIFIED MAP,OVLY,SIZE={172K,6K)
    VARIABLE OPTIONS USED - SIZE=(176128,6144)
IENS201
IEWO461 IHEUPAB
IEW2461 IHEUPBB
I EW9461 IHEDNCA
IEWO46I IHEDNBA
IEWO461 IHEVSAA
IENO461 IHEVCAA
IEWO461 IHEDBNA
I EN0461 IHEVSBA
IEW0461 IHEVSFA
IEW0461 IHEDDOD
IEND461 IHEVPCA
IEN0461 IHEVPDA
IEWO461 IHEVPFA
IEW0461 IHEVPGA
IEW0461 IHEVPHA
IEN0461 IHEVFBA
IEWO461 IHEVKBA
IEWO461 IHEVKCA
IEW0461 IHEVKFA
IEW0461 IHEVKGA
IEN0461 IHEUPAA
IENO461 IHEUPBA
IEW3461 IHEM91A
IEW2461 IHEM91B
IEWO461 IHEM91O
IEN0461 IHETERA
```

name origin length seg. no. name location name location name location name location

| NAME |  | ORIGIN | LENGTH | SEG. NO. | NAME | LOCATION | NAME | LOCATION | NAME | LOCATION | NAME | LOC ATION |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I HEMAIN |  | 4068 | 4 | 1 |  |  |  |  |  |  |  |  |
| IHENTRY |  | 4070 | C | 1 |  |  |  |  |  |  |  |  |
| SYSIN |  | 4080 | 38 | 1 |  |  |  |  |  |  |  |  |
| IHESPRT |  | 4 DB 8 | 38 | 1 |  |  |  |  |  |  |  |  |
| IHEDIA | * | 4DFD | 338 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEDIAA | 4DFO | IHEDIAB | 40F2 |  |  |  |  |
| I HEDOA | * | 5128 | 23 A | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEDOAA | 5128 | IHEDOAB | 5124 |  |  |  |  |
| IHEDOB | * | 5368 | 144 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEDOBA | 5368 | IHEDOBB | 536A | IHEDOBC | $536 C$ |  |  |
| IHEIOP | * | 54 BO | 1 EB | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEIDPA | 5480 | IHEIOPB | $54 \mathrm{B2}$ | IHEIOPC | $54 \mathrm{B6}$ |  |  |
| IHEIOX | * | 5640 | 14 C | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEIOXA | 5640 | IHEIOXB | 56A2 | IHEIOXC | 5646 |  |  |
| I HESAP | * | 57 FO | AB8 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHESADA | 57 FO | IHESAPC | 580 A | IHESAPD | 5812 | I HE SAPA | 581 A |
|  |  |  |  |  | IHESAPB | 5822 | IHESADF | 582A | IHESADB | 5832 | I HESADE | 583 A |
|  |  |  |  |  | IHESAFC | 5842 | IHESAFA | 584A | IHESAFB | 5852 | I HE SAFD | 585 A |
|  |  |  |  |  | IHESARA | 5862 | IHESAFQ | 586A | IHESARC | 6026 | IHESADD | 6114 |
|  |  |  |  |  | IHESAFF | 614 E |  |  |  |  |  |  |
| IHEIOA | * | 6248 | 164 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEIOAA | 6248 | IHEIOAB | 62AA | IHEIOAC | 62AC | I HE IOAD | 62AE |
|  |  |  |  |  | IHEIOAT | 6398 |  |  |  |  |  |  |
| IHEIOB | * | 6418 | $1 E 4$ | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEIOBA IHEIOBE | $\begin{aligned} & 6418 \\ & 6438 \end{aligned}$ | IHEIOBB IHEIOBT | $\begin{aligned} & 6420 \\ & 6524 \end{aligned}$ | IHEIOBC | 6428 | IHE IOBD | 6430 |
| IHEMXS | * | 6600 | 54 | 1 |  | - |  |  |  |  |  |  |
|  |  |  |  |  | IHEMXSX | 6600 | IHEMXSN | 660C |  |  |  |  |
| IHESNS | * | 6660 | 140 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHESNSK | 6660 | IHESNSC | 666C | IHESNSZ | 668 E | IHESNSS | 669 A |
| IHESQS | * | 6740 | A4 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHESQSO | 67 A0 |  |  |  |  |  |  |
| IHEDMA | * | 6848 | F8 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEDMAA | 6848 |  |  |  |  |  |  |
| IHEVFA | * | 6940 | 16 C | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVFAA | 6940 |  |  |  |  |  |  |
| I HEVFC | * | 6 AB0 | 26 | 1 |  |  |  |  |  |  |  |  |


| NAME |  | ORIGIN | LENGTH | SEG. NO. | NAME | LOCATION | NAME | LOCATION | NAME | LOCATION | NAME | LOCATION |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | IHEVFCA | $6 \triangle B 0$ |  |  |  |  |  |  |
| IHEVFD | * | 6AD8 | 66 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVFDA | 6AD8 |  |  |  |  |  |  |
| IHEVFE | * | 6840 | 1 C | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVFEA | 6840 |  |  |  |  |  |  |
| I HEVPA | * | 6860 | 1 E0 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVPAA | 6B60 |  |  |  |  |  |  |
| IHEVPB | * | 6040 | 142 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVPBA | 6 D 40 |  |  |  |  |  |  |
| I HEVPE | * | 6EE8 | 260 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVPEA | 6EE8 | - |  |  |  |  |  |
| I HEVOB | * | 7158 | 494 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVQBA | 7158 |  |  |  |  |  |  |
| IHEVSC | * | 75F0 | AC | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVSCA | 75F0 |  |  |  |  |  |  |
| IHEDCN | * | 7640 | $21 E$ | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEDCNA | 7640 | IHEDCNB | 7642 |  |  |  |  |
| IHEERR | * | 7800 | 729 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEERRD <br> IHEERRE | $\begin{aligned} & 78 C 0 \\ & 7 F 56 \end{aligned}$ | IHEERRC | 78CA | IHEERRB | 7804 | I HEERR A | 780E |
| IHEIOD | * | 7FFO | 29A | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEIODG | 7FFO | IHE IODP | 7FF2 | IHEIODT | 80EA |  |  |
| IHEIOF | * | 8290 | 2 DC | 1 |  | - 8200 |  |  |  |  |  |  |
|  |  |  |  |  | IHEIOFB <br> IHEITAA | $\begin{aligned} & 8290 \\ & 854 \mathrm{E} \end{aligned}$ | IHEIOFA | 8292 | IHEITAZ | 852E | IHEITAX | 853A |
| IHEOCL | * | 8570 | 554 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | RROCLA | 8570 | IHEOCLB | 8572 | IHEOCLC | 8574 | 1 HEOCLD | 8576 |
| I HEVQC | * | 8AC8 | 268 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEVQCA | 8AC8 |  |  |  |  |  |  |
| IHEBEG | * | 8030 | 80 | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEBEGN | 8030 | IHEBEGA | 8070 |  |  |  |  |
| IHEPRT | * | 8080 | $2 C 8$ | 1 |  |  |  |  |  |  |  |  |
|  |  |  |  |  | IHEPRTA | 8 BBO | IHEPRTB | 8DB2 |  |  |  |  |
| IHESIZ | * | 9078 | c | 1 |  |  |  |  |  |  |  |  |
| IHETAB | * | 9088 | C | 1 | IHESIZE | 9078 |  |  |  |  |  |  |
|  |  |  |  |  | IHETABS | 9088 |  |  |  |  |  |  |



```
A=1.00000
INPUT PARAMETERS
P=1.00000 Q = 0.60000
SHIFT PARAMETERS
X OIRECTION SR = 0.10030
YDIRECTION TR = 0.10000
ANGLE OF ROTATION = 0.01745
ANGLE OF ROTATION IN DEGREES = 1.00000
SR=0.11062
TR=0.08264
```

|  | REF X | Y1 |
| :--- | :---: | :---: |
| 1 | 0.05000 | 0.63000 |
| 2 | 0.01000 | 0.54357 |
| 3 | 0.02000 | 0.52040 |
| 4 | 0.03000 | 0.50276 |
| 5 | 0.04000 | 0.48800 |
| 6 | 0.05000 | 0.47510 |
| 7 | 0.06000 | 0.46353 |
| 8 | 0.07000 | 0.45298 |
| 9 | 0.08000 | 0.44323 |
| 10 | 0.09000 | 0.43416 |
| 11 | 0.10000 | 0.42564 |
| 12 | 0.11000 | 0.41762 |
| 13 | 0.12000 | 0.41001 |
| 14 | 0.13000 | 0.40278 |
| 15 | 0.14000 | 0.39588 |
| 16 | 0.15000 | 0.38929 |
| 17 | 0.16000 | 0.38297 |
| 18 | 0.17000 | 0.37689 |
| 19 | 0.18000 | 0.37105 |
| 25 | 0.19000 | 0.36543 |
| 21 | 0.20000 | 0.36000 |
| 22 | 0.21000 | 0.35476 |
| 23 | 0.22000 | 0.34969 |
| 24 | 0.23000 | 0.34478 |
| 25 | 0.24000 | 0.34003 |
| 26 | 0.25000 | 0.33542 |
| 27 | 0.26000 | 0.33096 |
| 28 | 0.27000 | 0.32662 |
| 29 | 0.28000 | 0.32241 |

$Y 2$
0.60000
0.65643
0.67960
0.69724
0.71200
0.72490
0.73647
0.74702
0.75677
0.76584
0.77436
0.78238
0.78999
0.79722
0.80412
0.81071
0.81703
0.82311
0.82895
0.83457
0.84000
0.84524
0.85031
0.85522
0.85997
0.86457
0.86904
0.87338
0.87759
$P \times 1$
0.10015
0.11114
0.12154
0.13184
0.14210
0.15232
0.16252
0.17271
0.18288
0.19303
0.20318
0.21332
0.22345
0.23357
0.24369
0.25381
0.26391
0.27402
0.28412
0.29422
0.30431
0.31440
0.32449
0.33457
0.34465
0.35473
0.36481
0.37488
0.38495
PY1
0.68255
0.62631
0.60331
0.58585
0.57127
0.55854
0.54715
0.53677
0.52720
0.51830
0.50997
0.50211
0.49468
0.48763
0.48091
0.47449
0.46834
0.46245
0.45678
0.45133
0.44608
0.44101
0.43612
0.43139
0.42681
0.42238
0.41809
0.41392
0.40989

| $P \times 2$ | PY2 |
| :---: | :---: |
| 0.10015 | 0.68255 |
| 0.10917 | 0.73914 |
| 0.11876 | 0.76249 |
| 0.12845 | 0.78030 |
| 0.13819 | 0.79523 |
| 0.14797 | 0.80830 |
| 0.15776 | 0.82005 |
| 0.16758 | 0.83077 |
| 0.17740 | 0.84069 |
| 0.18724 | 0.84994 |
| 0.19709 | 0.85863 |
| 0.20695 | 0.86683 |
| 0.21682 | 0.87461 |
| 0.22669 | 0.88201 |
| 0.23657 | 0.88908 |
| 0.24645 | 0.89585 |
| 0.25634 | 0.90234 |
| 0.26623 | 0.90859 |
| 0.27613 | 0.91460 |
| 0.28603 | 0.92040 |
| 0.29593 | 0.92600 |
| 0.30584 | 0.93142 |
| 0.31575 | 0.93666 |
| 0.32566 | 0.94174 |
| 0.33558 | 0.94667 |
| 0.34550 | 0.95145 |
| 0.35542 | 0.95609 |
| 0.36534 | 0.96060 |
| 0.37527 | 0.96498 |


| 30 | 0.29000 | 0.31832 | 0.88168 | 0.39502 | 0.40597 | 0.38519 | 0.96925 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31 | 0.30000 | 0.31434 | 0.88566 | 0.40509 | 0.40217 | 0.39512 | 0.97340 |
| 32 | 0.31000 | 0.31048 | 0.88952 | 0.41516 | 0.39848 | 0.40505 | 0.97744 |
| 33 | 0.32000 | 0.35672 | 0.89328 | 0.42522 | 0.39489 | 0.41499 | 0.98137 |
| 34 | 0.33000 | 0.30306 | 0.89694 | 0.43528 | 0.39141 | 0.42492 | 0.98521 |
| 35 | 0.34000 | 0.29949 | 0.90051 | 0.44534 | 0.38802 | 0.43486 | 0.98894 |
| 36 | 0.35000 | 0.29603 | 0.90397 | 0.45540 | 0.38473 | 0.44479 | 0.99259 |
| 37 | 0.36000 | 0.29265 | ก. 90735 | 0.46546 | 0.38153 | 0.45473 | 0.99614 |
| 38 | 0.37000 | 0.28936 | 0.91064 | 0.47552 | 0.37842 | 0.46468 | 0.99960 |
| 39 | 0.38000 | 0.28616 | 0.91384 | 0.48557 | 0.37539 | 0.47462 | 1.00297 |
| 43 | 0.39000 | 0.28304 | 0.91696 | 0.49562 | 0.37244 | 0.48456 | 1.00627 |
| 41 | 0.40000 | 0.28000 | 0.92000 | 0.50567 | 0.36958 | 0.49451 | 1.00948 |
| 42 | 0.41000 | 0.27704 | 0.92296 | 0.51572 | 0.36679 | 0.50445 | 1.01262 |
| 43 | 0.42000 | 0.27415 | 0.92585 | 0.52577 | 0.36408 | 0.51440 | 1.01568 |
| 44 | 0.43000 | 0.27134 | 0.92866 | 0.53582 | 0.36145 | 0.52435 | 1.01866 |
| 45 | 0.44000 | 0.26860 | 0.93140 | 0.54587 | 0.35888 | 0.53430 | 1.02157 |
| 46 | 0.45000 | 0.26593 | 0.93407 | 0.55591 | 0.35639 | 0.54425 | 1.02442 |
| 47 | 0.46000 | 0.26333 | 0.93667 | 0.56596 | 0.35396 | 0.55421 | 1.02719 |
| 48 | 0.47000 | 0.26080 | 0.93920 | 0.57600 | 0.35160 | 0.56416 | 1.02990 |
| 49 | 0.48000 | 0.25833 | 0.94167 | 0.58604 | 0.34931 | 0.57412 | 1.03254 |
| 53 | 0.49000 | 0.25593 | 0.94407 | 0.59608 | 0.34708 | 0.58407 | 1.03512 |
| 51 | 0.50000 | 0.25359 | 0.94641 | 0.60612 | 0.34492 | 0.59403 | 1.03763 |
| 52 | 0.51000 | 0.25131 | 0.94869 | 0.61616 | 0.34281 | 0.60399 | 1.04008 |
| 53 | 0.52000 | 0.24909 | 0.95091 | 0.62620 | 0.34077 | 0.61395 | 1.04248 |
| 54 | 0.53000 | 0.24693 | 0.95307 | 0.63623 | 0.33879 | 0.62391 | 1.04481 |
| 55 | 0.54000 | 0. 24483 | 0.95517 | 0.64627 | 0.33686 | 0.63387 | 1.04709 |
| 56 | 0.55000 | 0.24279 | 0.95721 | 0.65630 | 0.33499 | 0.64383 | 1.04930 |
| 57 | 0.56000 | 0.24080 | 0.95920 | 0.66633 | 0.33318 | 0.65380 | 1.05147 |
| 58 | 0.57000 | 0.23887 | 0.96113 | 0.67637 | 0.33142 | 0.66376 | 1.05357 |
| 59 | 0.58000 | 0.23699 | 0.96301 | 0.68640 | 0.32972 | 0.67373 | 1.05562 |
| 60 | 0.59000 | 0.23517 | 0.96483 | 0.69643 | 0.32807 | 0.68370 | 1.05762 |
| 61 | 0.60000 | 0.23339 | 0.96661 | 0.70646 | 0.32647 | 0.69366 | 1.05957 |
| 62 | 0.61000 | 0.23167 | 0.96833 | 0.71649 | 0.32493 | 0.70363 | 1.06146 |
| 63 | 0.62000 | 0.23001 | 0.96999 | 0.72651 | 0.32343 | 0.71360 | 1.06331 |
| 64 | 0.63000 | 0.22839 | 0.97161 | 0.73654 | 0.32199 | 0.72357 | 1.06510 |
| 65 | 0.64000 | 0.22682 | 0.97318 | 0.74657 | 0.32059 | 0.73354 | 1.06684 |
| 66 | 0.65000 | 0.22530 | 0.97470 | 0.75659 | 0.31925 | 0.74351 | 1.06853 |
| 67 | 0.66000 | 0.22383 | 0.97617 | 0.76661 | 0.31795 | 0.75349 | 1.07018 |
| 68 | 0.67000 | 0.22241 | 0.97759 | 0.77664 | 0.31671 | 0.76346 | 1.07178 |
| 69 | 0.68000 | 0.22103 | 0.97897 | 0.78666 | 0.31551 | 0.77344 | 1.07333 |


| 70 | 0.69000 | 0.21971 | 0.98029 | 0.79668 | 0.31435 | 0.78341 | 1.07483 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 71 | 0.70000 | 0.21842 | 0.98158 | 0.80670 | 0.31325 | 0.79339 | 1.07628 |
| 72 | 0.71000 | 0.21719 | 0.98281 | 0.81672 | 0.31219 | 0.80336 | 1.07769 |
| 73 | 0.72000 | 0.21600 | 0.98400 | 0.82674 | 0.31117 | 0.81334 | 1.07905 |
| 74 | 0.73000 | 0.21486 | 0.98514 | 0.83676 | 0.31020 | 0.82332 | 1.08037 |
| 75 | 0.74000 | 0.21376 | 0.98624 | 0.84678 | 0.30928 | 0.83330 | 1.08165 |
| 76 | 0.75000 | 0.21270 | 0.98730 | 0.85680 | 0.30840 | 0.84328 | 1.08288 |
| 77 | 0.76000 | 0.21169 | 0.98831 | 0.86681 | 0.30756 | 0.85326 | 1.08406 |
| 78 | 0.77000 | 0.21072 | 0.98928 | 0.87683 | 0.30677 | 0.86324 | 1.08520 |
| 79 | 0.78000 | 0.20980 | 0.99020 | 0.88684 | 0.30602 | 0.87322 | 1.08630 |
| 80 | 0.79000 | 0.20892 | 0.99108 | 0.89686 | 0.30531 | 0.88321 | 1.08736 |
| 81 | 0.80000 | 0.23808 | 0.99192 | 0.90687 | 0.30465 | 0.89319 | 1.08837 |
| 82 | 0.81000 | 0.20729 | 0.99271 | 0.91688 | 0.30403 | 0.90318 | 1.08934 |
| 83 | 0.82000 | 0.20653 | 0.99347 | 0.92689 | 0.30345 | 0.91316 | 1.09026 |
| 84 | 0.83000 | 0.20582 | 0.99418 | 0.93690 | 0.30292 | 0.92315 | 1.09115 |
| 85 | 0.84000 | 0.20515 | 0.99485 | 0.94691 | 0.30242 | 0.93313 | 1.09199 |
| 86 | 0.85000 | 0.20453 | 0.99547 | 0.95692 | 0.30197 | 0.94312 | 1.09280 |
| 87 | 0.86000 | 0.25394 | 0.99606 | 0.96693 | 0.30156 | 0.95311 | 1.09356 |
| 88 | 0.87000 | 0.20339 | 0.99661 | 0.97694 | 0.30119 | 0.96310 | 1.09428 |
| 89 | 0.88000 | 0.20289 | 0.99711 | 0.98695 | 0.30086 | 0.97309 | 1.09495 |
| $9)$ | 0.89000 | 0.20243 | 0.99757 | 0.99695 | 0.30057 | 0.98308 | 1.09559 |
| 91 | 0.90000 | 0.20201 | 0.99799 | 1.00696 | 0.30032 | 0.99307 | 1.09619 |
| 92 | 0.91300 | 0.20162 | 0.99838 | 1.01696 | 0.30011 | 1.00306 | 1.09674 |
| 93 | 0.92000 | 0.20128 | 0.99872 | 1.02697 | 0.29995 | 1.01305 | 1.09726 |
| 94 | 0.93000 | 0.20098 | 0.99902 | 1.03697 | 0.29982 | 1.02305 | 1.09774 |
| 95 | 0.94000 | 0.20072 | 0.99928 | 1.04698 | 0.29973 | 1.03304 | 1.09817 |
| 96 | 0.95000 | 0.20050 | 0.99950 | 1.05698 | 0.29969 | 1.04304 | 1.09857 |
| 97 | 0.96000 | 0.20032 | 0.99968 | 1.06698 | 0.29968 | 1.05303 | 1.09892 |
| 98 | 0.97000 | 0.23018 | 0.99982 | 1.07698 | 0.29972 | 1.06303 | 1.09923 |
| 99 | 0.98000 | 0.20008 | 0.99992 | 1.08698 | 0.29979 | 1.07302 | 1.09951 |
| 109 | 0.99000 | 0.20002 | 0.99998 | 1.09698 | 0.29991 | 1.08302 | 1.09974 |
| 101 | 1.00000 | 0.20000 | 1.00000 | 1.10698 | 0.30006 | 1.09302 | 1.09994 |
| 132 | 1.01000 | 0.20002 | 0.99998 | 1.11697 | 0.30026 | 1.10302 | 1.10009 |
| 103 | 1.02000 | 0.20008 | 0.99992 | 1.12697 | 0.30049 | 1.11302 | 1.10021 |
| 104 | 1.03000 | 0.20018 | 0.99982 | 1.13697 | 0.30076 | 1.12301 | 1.10028 |
| 105 | 1.04000 | 0.20032 | 0.99968 | 1.14696 | 0.30108 | 1.13301 | 1.10032 |
| 105 | 1.05000 | 0.20050 | 0.99950 | 1.15696 | 0.30143 | 1.14302 | 1.10031 |
| 107 | 1.05999 | 0.20072 | 0.99928 | 1.16695 | 0.30183 | 1.15302 | 1.10026 |
| 108 | 1.06999 | 0.20098 | 0.99902 | 1.17694 | 0.30226 | 1.16302 | 1.10018 |
| 109 | 1.07999 | 0.20128 | 0.99872 | 1.18694 | 0.30274 | 1.17302 | 1.10005 |


| 117 | 1.08999 | 0.20162 |
| :--- | :--- | :--- |
| 111 | 1.09999 | 0.20200 |
| 112 | 1.10999 | 0.20243 |
| 113 | 1.11999 | 0.20289 |
| 114 | 1.12999 | 0.20339 |
| 115 | 1.13999 | 0.20394 |
| 115 | 1.14999 | 0.20452 |
| 117 | 1.15999 | 0.20515 |
| 118 | 1.16999 | 0.20582 |
| 119 | 1.17999 | 0.20653 |
| 123 | 1.18999 | 0.20729 |
| 121 | 1.19998 | 0.20808 |
| 122 | 1.20998 | 0.20892 |
| 123 | 1.21998 | 0.20980 |
| 124 | 1.22998 | 0.21072 |
| 125 | 1.23998 | 0.21169 |
| 126 | 1.24998 | 0.21270 |
| 127 | 1.25998 | 0.21375 |
| 128 | 1.26998 | 0.21485 |
| 129 | 1.27998 | 0.21600 |
| 133 | 1.28998 | 0.21719 |
| 131 | 1.29998 | 0.21842 |
| 132 | 1.30998 | 0.21970 |
| 133 | 1.31998 | 0.22103 |
| 134 | 1.32998 | 0.22240 |
| 135 | 1.33997 | 0.22383 |
| 136 | 1.34997 | 0.22530 |
| 137 | 1.35997 | 0.22681 |
| 138 | 1.36997 | 0.22838 |
| 139 | 1.37997 | 0.23000 |
| 143 | 1.38997 | 0.23167 |
| 141 | 1.39997 | 0.23339 |
| 142 | 1.40997 | 0.23516 |
| 143 | 1.41997 | 0.23698 |
| 144 | 1.42997 | 0.23886 |
| 145 | 1.43997 | 0.24079 |
| 146 | 1.44997 | 0.24278 |
| 147 | 1.45997 | 0.24483 |
| 148 | 1.46996 | 0.24693 |
| 149 | 1.47996 | 0.24908 |
|  |  |  |

0.99838
0.99800
0.99757
0.99711
0.99661
0.99606
0.99548
0.99485
0.99418
0.99347
0.99271
0.99192
0.99108
0.99020
0.98928
0.98831
0.98730
0.98625
0.98515
0.98400
0.98281
0.98158
0.98030
0.97897
0.97760
0.97617
0.97470
0.97319
0.97162
0.97000
0.96833
0.96661
0.96484
0.96302
0.96114
0.95921
0.95722
0.95517
0.95307
0.95092
1.19693
1.20692
1.21691
1.22690
1.23689
1.24688
1.25686
1.26685
1.27684
1.28682
1.29681
1.30679
1.31677
1.32676
1.33674
1.34672
1.35670
1.36668
1.37666
1.38664
1.39661
1.40659
1.41656
1.42654
1.43651
1.44649
1.45646
1.46643
1.47640
1.48637
1.49634
1.50630
1.51627
1.52624
1.53620
1.54617
1.55613
1.56609
1.57605
1.58601

| 0.30325 | 1.18303 | 1.09988 |
| :---: | :---: | :---: |
| 0.30381 | 1.19303 | 1.09968 |
| 0.30441 | 1.20304 | 1.09943 |
| 0.30504 | 1. 21304 | 1.09914 |
| 0.30572 | 1. 22305 | 1.09881 |
| 0.30644 | 1.23306 | 1.09844 |
| 0.30720 | 1.24306 | 1.09803 |
| 0.30800 | 1.25307 | 1.09758 |
| 0.30885 | 1.26308 | 1.09708 |
| 0.30973 | 1.27309 | 1.09655 |
| 0.31066 | 1.28310 | 1.09597 |
| 0.31163 | 1.29311 | 1.09535 |
| 0.31264 | 1.30313 | 1.09469 |
| 0.31370 | 1.31314 | 1.09398 |
| 0.31479 | 1.32315 | 1.09323 |
| 0.31594 | 1.33317 | 1.09244 |
| 0.31712 | 1.34318 | 1.09160 |
| 0.31835 | 1.35320 | 1.09072 |
| 0.31962 | 1.36322 | 1.08980 |
| C. 32094 | 1.37323 | 1.08883 |
| 0.32230 | 1.38325 | 1.08781 |
| 0.32371 | 1.39327 | 1.08675 |
| 0.32517 | 1.40329 | 1.08565 |
| 0.32667 | 1.41331 | 1.08449 |
| 0.32822 | 1.42334 | 1.08329 |
| 0.32982 | 1.43336 | 1.08205 |
| 0.33146 | 1.44338 | 1.08075 |
| 0.33315 | 1.45340 | 1.07941 |
| 0.33490 | 1.46343 | 1.07802 |
| 0.33669 | 1.47346 | 1.07657 |
| 0.33853 | 1.48348 | 1.07508 |
| 0.34042 | 1.49351 | 1.07353 |
| 0.34237 | 1.50354 | 1.07194 |
| 0.34437 | 1.51357 | 1.07029 |
| 0.34642 | 1.52360 | 1.06858 |
| 0.34853 | 1.53363 | 1.06683 |
| 0.35069 | 1.54366 | 1.06501 |
| 0.35291 | 1.55370 | 1.06315 |
| 0.35518 | 1.56373 | 1.06122 |
| 0.35751 | 1.57377 | 1.05924 |

153

| 1.48996 | 0.25130 |
| :--- | :--- |
| 1.49996 | 0.25358 |
| 1.50996 | 0.25592 |
| 1.51996 | 0.25832 |
| 1.52996 | 0.26079 |
| 1.53996 | 0.26332 |
| 1.54996 | 0.26592 |
| 1.55996 | 0.26859 |
| 1.56996 | 0.27133 |
| 1.57996 | 0.27414 |
| 1.58996 | 0.27703 |
| 1.59996 | 0.27999 |
| 1.60995 | 0.28303 |
| 1.61995 | 0.28614 |
| 1.62995 | 0.28935 |
| 1.63995 | 0.29263 |
| 1.64995 | 0.29601 |
| 1.65995 | 0.29948 |
| 1.66995 | 0.30304 |
| 1.67995 | 0.30670 |
| 1.68995 | 0.31046 |
| 1.69995 | 0.31432 |
| 1.70995 | 0.31830 |
| 1.71995 | 0.32239 |
| 1.72995 | 0.32660 |
| 1.73995 | 0.33093 |
| 1.74994 | 0.33540 |
| 1.75994 | 0.34000 |
| 1.76994 | 0.34476 |
| 1.77994 | 0.34966 |
| 1.78994 | 0.35473 |
| 1.79994 | 0.35997 |
| 1.80994 | 0.36540 |
| 1.81994 | 0.37102 |
| 1.82994 | 0.37686 |
| 1.83994 | 0.38293 |
| 1.84994 | 0.38925 |
| 1.85994 | 0.39584 |
| 1.86994 | 0.40273 |
| 1.87994 | 0.40996 |
|  |  |


| 1.59597 | 0.35991 | 1. 58380 | 1.05719 |
| :---: | :---: | :---: | :---: |
| 1.60593 | 0.36236 | 1.59384 | 1.05509 |
| 1.61589 | 0.36487 | 1.60388 | 1.05292 |
| 1.62584 | 0.36745 | 1.61392 | 1.05070 |
| 1.63580 | 0.37009 | 1.62396 | 1.04840 |
| 1.64575 | 0.37280 | 1.63400 | 1.04605 |
| 1.65570 | 0.37557 | 1.64404 | 1.04362 |
| 1.66565 | 0.37841 | 1.65409 | 1.04113 |
| 1.67560 | 0.38133 | 1.66413 | 1.03856 |
| 1.68555 | 0.38431 | 1.67418 | 1.03593 |
| 1.69550 | 0.38737 | 1.68423 | 1.03322 |
| 1.70545 | 0.39050 | 1.69428 | 1.03043 |
| 1.71539 | 0.39372 | 1.70433 | 1.02757 |
| 1.72534 | 0.39701 | 1.71438 | 1.02462 |
| 1.73528 | 0.40039 | 1. 72444 | 1.02160 |
| 1.74522 | 0.40385 | 1.73449 | 1.01849 |
| 1.75516 | 0.40740 | 1.74455 | 1.01528 |
| 1.76509 | 0.41104 | 1.75461 | 1.01199 |
| 1.77503 | 0.41477 | 1.76467 | 1.00861 |
| 1.78496 | 0.41861 | 1.77473 | 1.00512 |
| 1.79490 | 0.42254 | 1.78479 | 1.00154 |
| 1.80482 | 0.42658 | 1.79486 | 0.99785 |
| 1.81475 | 0.43073 | 1.80492 | 0.99405 |
| 1.82468 | 0.43499 | 1.81499 | 0.99013 |
| 1.83460 | 0.43938 | 1.82506 | 0.98610 |
| 1.84453 | 0.44389 | 1.83514 | 0.98194 |
| 1.85445 | 0.44853 | 1.84521 | 0.97765 |
| 1.86436 | 0.45330 | 1.85529 | 0.97322 |
| 1.87428 | 0.45823 | 1.86537 | 0.96864 |
| 1.88419 | 0.46331 | 1.87545 | 0.96391 |
| 1.89410 | 0.46855 | 1.88554 | 0.95902 |
| 1.90401 | 0.47396 | 1.89563 | 0.95395 |
| 1.91391 | 0.47956 | 1.90572 | 0.94870 |
| 1.92381 | 0.48536 | 1.91582 | 0.94325 |
| 1.93370 | 0.49137 | 1.92592 | 0.93759 |
| 1.94360 | 0.49762 | 1.93602 | 0.93170 |
| 1.95348 | 0.50411 | 1.94613 | 0.92555 |
| 1.96337 | 0.51088 | 1.95624 | 0.91913 |
| 1.97324 | 0.51794 | 1.96636 | 0.91242 |
| 1.98312 | 0.52535 | 1.97648 | 0.90536 |


| 193 | 1.88993 | 0.41756 | 0.78244 | 1.99298 | 0.53312 | 1.98662 | 0.89794 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 191 | 1.89993 | 0.42559 | 0.77441 | 2.00284 | 0.54132 | 1.99675 | 0.89009 |
| 192 | 1.90993 | 0.43410 | 0.76590 | 2.01269 | 0.55000 | 2.00690 | 0.88175 |
| 193 | 1.91993 | 0.44317 | 0.75683 | 2.02253 | 0.55924 | 2.01705 | 0.87286 |
| 194 | 1.92993 | 0.45291 | 0.74709 | 2.03236 | 0.56916 | 2.02722 | 0.86330 |
| 195 | 1.93993 | 0.46345 | 0.73655 | 2.04217 | 0.57988 | 2.03740 | 0.85293 |
| 198 | 1.94993 | 0.47502 | 0.72498 | 2.05196 | 0.59161 | 2.04760 | 0.84154 |
| 197 | 1.95993 | 0.48790 | 0.71210 | 2.06174 | 0.60467 | 2.05783 | 0.82883 |
| 198 | 1.96993 | 0.50264 | 0.69736 | 2.07148 | 0.61958 | 2.06808 | 0.81427 |
| 199 | 1.97993 | 0.52026 | 0.67974 | 2.08117 | 0.63737 | 2.07839 | 0.79683 |
| 205 | 1.98993 | 0.54337 | 0.65663 | 2.09076 | 0.66065 | 2.08879 | 0.77390 |
| 201 | 1.99993 | 0.59515 | 0.60485 | 2.09986 | 0.71260 | 2.09969 | 0.72229 |
| 202 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |

ELLIPSE SHIFT PARAMETERS
CGX $=1.10000$
CGY $=0.70000$
ANG $=0.01503$

```
    REF LINE
LENGTH = 2.29583
```

RAD OF GXSQ $=1.39430$
RAD OF GY SQ $=0.80423$

CROSS PRJOUCT RHXY $=0.88878$
TWICE ANGLE TAN-1 $=0.00000$
$X R A D L=1.18081$
YRADL $=0.89679$

YBAR $=0.89002$
$\mathrm{XBAR}=0.99794$

RHBAR $=0.00060$

```
        PROD LINE
LENGTH = 2.29583
PXBAR = 1.09288
PYBAR = 0.98994
RAD OF G Y SQ = 1.59267
XPADL = 1.26201
YPADL = 0.99610
XPG = 0.63110
YPG = 0.11059
CROSS PRODUCT PHXY = 1.08922
CROSS PRODUCT PHXY = 1.08922
PHBAR = 0.00734
    LINE CENTROID
SHIFT PARAMETERS
XSHIFT = 0.09494 YSHIFT =0.09992
ANGULAR DIFFERENCE IN DEGREES = 1.00122
EFFECTIVE SHIFT OF SECTIONAL CENTROID
XSHIFT =0.10001 YSHIFT = 0.10000
```

```
SURFACE AREA = 1.25622
C OF G XBAR = 1.00006
OFGSQ=1.24988 XRAD2 = 1.11798 XRADG = 0.49977
CROSS PRODUCT = 0.60004
DY ELEMENTS AREA = 1.25597
C OF G YBAR 0.60001 RAD OF G SQ = 0.39995
YRAD2 = 0.63242
    YRADG = 0.19985
```

TWICE ANGLE OF PRINCIPAL AXIS $=-0.00004$
CROSS PRODUCT $=0.59988$

```
PRODUCTION SHAPE DX ELEMENTS
AREA = 1.25512
RAD2 = 1.45963 R OG G = C OFG = 1.10029
CROSSPRODUCT = 0.77394
```

Production dy elements
AREA $=1.25741$
PRAD SQ $=0.53005$
R OF G $\stackrel{C}{=}$ OF $G=0.72805=0.70005$
YPADG $=0.19996$

## SHIFT PARAMETERS

$X$ OIRECTION $=0.10024$
$Y$ DIRECTION $=0.10004$
TAN OF ANGULAR DIFFERENCE $=0.01761$

ANGULAR DIFFERENCF IN DEGREES $=1.00909$
$A=1.00000$
INPUT PARAMETERS
$B=0.40000$
$p=1.00000$
$Q=0.60000$

SHIFT PARAMETERS
$X$ DIRECTION SR $=0.10000$
Y DIRECTION TR $=0.10000$
ANGLE OF ROTATION $=0.00873$
ANGLE OF ROTATION IN DEGREES $=0.50029$
$S R=0.10528$

```
TR = 0.09129
```

|  | REF X | Y1 | Y2 | PXI | PYI | $\mathrm{P} \times 2$ | PY2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.00000 | 0.60000 | 7. 60000 | 0.10004 | 0.69127 | 0.10004 | 0.69127 |
| 2 | 0.01000 | 0.54357 | 0.65643 | 0.11053 | 0.63493 | 0.10955 | 0.74778 |
| 3 | 0.02000 | 0.52040 | 0.67960 | 0.12073 | 0.61185 | 0.11934 | 0.77104 |
| 4 | 0.03000 | 0.50276 | 0.69724 | 0.13089 | 0.59429 | 0.12919 | 0.78877 |
| 5 | 0.04000 | 0.48800 | 0.71200 | 0.14101 | 0.57962 | 0.13906 | 0.80361 |
| 6 | 0.05000 | 0.47510 | 0.72490 | 0.15113 | 0.56681 | 0.14895 | 0.81660 |
| 7 | 0.06000 | 0.46353 | 0.73647 | 0.16123 | 0.55533 | 0.15884 | 0.82826 |
| 8 | 0.07000 | 0.45298 | 0.74702 | 0.17132 | 0.54486 | 0.16875 | 0.83890 |
| 9 | 0.08000 | 0.44323 | 0.75677 | 0.18140 | 0.53521 | 0.17867 | 0.84873 |
| $1)$ | 0.09000 | 0.43416 | 0.76584 | 0.19148 | 0.52622 | 0.18859 | 0.85789 |
| 11 | 0.10000 | 0.42564 | 0.77436 | 0.20156 | 0.51779 | 0.19851 | 0.86649 |
| 12 | 0.11000 | 0.41762 | 0.78238 | 0.21163 | 0.50985 | 0.20844 | 0.87461 |
| 13 | 0.12000 | 0.41001 | 0.78999 | 0.22169 | 0.50234 | 0.21837 | 0.88230 |
| 14 | 0.13000 | 0.40278 | 0.79722 | 0.23175 | 0.49519 | 0.22831 | 0.88962 |
| 15 | 0.14000 | 0.39588 | 0.80412 | 0.24181 | 0.48838 | 0.23825 | 0.89660 |
| 16 | 0.15000 | 0.38929 | 0.81071 | 0.25187 | 0.48187 | 0.24819 | 0.90328 |
| 17 | 0.15000 | 0.38297 | 0.81703 | 0.26193 | 0.47564 | 0.25814 | 0.90969 |
| 13 | 0.17000 | 0.37689 | 0.82311 | 0.27198 | 0.46966 | 0.26808 | 0.91585 |
| 19 | 0.18000 | 0.37105 | 0.82895 | 0.28203 | 0.46390 | 0.27803 | 0.92178 |
| 23 | 0.19000 | 0.36543 | 0.83457 | 0.29208 | 0.45837 | 0.28798 | 0.92749 |
| 21 | 0.20000 | 0.36000 | 0.84000 | 0.30213 | 0.45303 | 0.29794 | 0.93301 |
| 22 | 0.21000 | 0.35476 | 0.84524 | 0.31217 | 0.44787 | 0.30789 | 0.93834 |
| 23 | 0.22000 | 0.34969 | 0.85031 | 0.32221 | 0.44289 | 0.31784 | 0.94349 |
| 24 | 0.23000 | 0.34478 | 0.85522 | 0.33226 | 0.43807 | 0.32780 | 0.94849 |
| 25 | 0.24000 | 0.34003 | 0.85997 | 0.34230 | 0.43341 | 0.33776 | 0.95332 |
| 26 | 0.25050 | 0.33542 | 0.86457 | 0.35234 | 0.42889 | 0.34772 | 0.95802 |
| 27 | 0.26000 | 0.33096 | 0.86904 | 0.36238 | 0.42451 | 0.35768 | 0.96257 |
| 28 | 0.27000 | 0.32662 | 0.87338 | 0.37241 | 0.42026 | 0.36764 | 0.96700 |
| 29 | 0.28000 | 0.32241 | 0.87759 | 0.38245 | 0.41614 | 0.37760 | 0.97129 |


| 0.29000 | 0.31832 |
| :--- | :--- |
| 0.30000 | 0.31434 |
| 0.31000 | 0.31048 |
| 0.32000 | 0.30672 |
| 0.33000 | 0.30306 |
| 0.34000 | 0.29949 |
| 0.35000 | 0.29603 |
| 0.36000 | 0.29265 |
| 0.37000 | 0.28936 |
| 0.38000 | 0.28616 |
| 0.39000 | 0.28304 |
| 0.40000 | 0.28000 |
| 0.41000 | 0.27704 |
| 0.42000 | 0.27415 |
| 0.43000 | 0.27134 |
| 0.44000 | 0.26860 |
| 0.45000 | 0.26593 |
| 0.46000 | 0.26333 |
| 0.47000 | 0.26080 |
| 0.48000 | 0.25833 |
| 0.49000 | 0.25593 |
| 0.50000 | 0.25359 |
| 0.51000 | 0.25131 |
| 0.52000 | 0.24909 |
| 0.53000 | 0.24693 |
| 0.54000 | 0.24483 |
| 0.55000 | 0.24279 |
| 0.56000 | 0.24080 |
| 0.57000 | 0.23887 |
| 0.58000 | 0.23699 |
| 0.59000 | 0.23517 |
| 0.60000 | 0.23339 |
| 0.61000 | 0.23167 |
| 0.62000 | 0.23001 |
| 0.63000 | 0.22839 |
| 0.64000 | 0.22682 |
| 0.65000 | 0.22530 |
| 0.66000 | 0.22383 |
| 0.67000 | 0.22241 |
| 0.68000 | 0.22103 |
| 0 |  |


| 0.88168 | 0.39249 |
| :---: | :---: |
| 0.88566 | 0.40252 |
| 0.88952 | 0.41255 |
| 0.89328 | 0.42259 |
| 0.89694 | 0.43262 |
| 0.90051 | 0.44265 |
| 0.90397 | 0.45268 |
| 0.90735 | 0.46271 |
| 0.91064 | 0.47274 |
| 0.91384 | 0.48276 |
| 0.91696 | 0.49279 |
| 0.92000 | 0.50282 |
| 0.92296 | 0.51284 |
| 0.92585 | 0.52287 |
| 0.92866 | 0.53289 |
| 0.93140 | 0.54291 |
| 0.93407 | 0.55294 |
| 0.93667 | 0.56296 |
| 0.93920 | 0.57298 |
| 0.94167 | 0.58300 |
| 0.94407 | 0.59302 |
| 0.94641 | 0.60304 |
| 0.94869 | 0.61306 |
| 0.95091 | 0.62308 |
| 0.95307 | 0.63310 |
| 0.95517 | 0.64312 |
| 0.95721 | 0.65313 |
| 0.95920 | 0.66315 |
| 0.96113 | 0.67317 |
| 0.96301 | 0.68318 |
| 0.96483 | 0.69320 |
| 0.96661 | 0.70322 |
| 0.96833 | 0.71323 |
| 0.96999 | 0.72324 |
| 0.97161 | 0.73326 |
| 0.97318 | 0.74327 |
| 0.97470 | 0.75328 |
| 0.97617 | 0.76330 |
| 0.97759 | 0.77331 |
| 0.97897 | 0.78332 |


| 0.41213 | 0.38757 | 0.97547 |
| :--- | :--- | :--- |
| 0.40824 | 0.39753 | 0.97954 |
| 0.40446 | 0.40750 | 0.98349 |
| 0.40079 | 0.41747 | 0.98734 |
| 0.39722 | 0.42743 | 0.99108 |
| 0.39374 | 0.43740 | 0.99473 |
| 0.39036 | 0.44737 | 0.99829 |
| 0.38707 | 0.45734 | 1.00175 |
| 0.38387 | 0.46731 | 1.00513 |
| 0.38076 | 0.47728 | 1.00842 |
| 0.37773 | 0.48726 | 1.01162 |
| 0.37477 | 0.49723 | 1.01475 |
| 0.37190 | 0.50720 | 1.01780 |
| 0.36910 | 0.51718 | 1.02077 |
| 0.36638 | 0.52715 | 1.02367 |
| 0.36373 | 0.53713 | 1.02649 |
| 0.36115 | 0.54710 | 1.02925 |
| 0.35863 | 0.55708 | 1.03194 |
| 0.35619 | 0.56706 | 1.03456 |
| 0.35381 | 0.57704 | 1.03711 |
| 0.35149 | 0.58702 | 1.03960 |
| 0.34924 | 0.59699 | 1.04203 |
| 0.34705 | 0.60697 | 1.04440 |
| 0.34492 | 0.61695 | 1.04670 |
| 0.34284 | 0.62694 | 1.04895 |
| 0.34083 | 0.63692 | 1.05114 |
| 0.33887 | 0.64690 | 1.05327 |
| 0.33697 | 0.65688 | 1.05534 |
| 0.33513 | 0.66686 | 1.05736 |
| 0.33334 | 0.67685 | 1.05933 |
| 0.33160 | 0.68683 | 1.06124 |
| 0.32992 | 0.69681 | 1.06310 |
| 0.32828 | 0.70680 | 1.06491 |
| 0.32670 | 0.71678 | 1.06666 |
| 0.32517 | 0.72677 | 1.06837 |
| 0.32369 | 0.73676 | 1.07002 |
| 0.32226 | 0.75674 | 1.07163 |
| 0.32088 | 1.07319 |  |
| 0.31954 | 1.07470 |  |
| 0.31825 | 1.07616 |  |
| 0 | 0.77670 |  |
| 0 | 0 | 0 |


| 70 | 0.69000 | 0.21971 | 0.98029 | 0.79333 | 0.31701 | 0.78669 | 1.07757 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 71 | 0.70000 | 0.21842 | 0.98158 | 0.90334 | 0.31582 | 0.79668 | 1.07894 |
| 72 | 0.71000 | 0.21719 | 0.98281 | 0.81335 | 0.31467 | 0.80667 | 1.08026 |
| 73 | 0.72000 | 0.21600 | 0.98400 | 0.82336 | 0.31357 | 0.81666 | 1.08154 |
| 74 | 0.73000 | 0.21486 | 0.98514 | 0.83337 | 0.31251 | 0.82665 | 1.08277 |
| 75 | 0.74000 | 0.21376 | 0.98624 | 0.84338 | 0.31150 | 0.83664 | 1.08396 |
| 76 | 0.75000 | 0.21270 | 0.98730 | 0.85339 | 0.31053 | 0.84663 | 1.08510 |
| 77 | 0.76000 | 0.21169 | 0.98831 | 0.86340 | 0.30961 | 0.85662 | 1.08620 |
| 78 | 0.77000 | 0.21072 | 0.98928 | 0.87341 | 0.30873 | 0.86661 | 1.08725 |
| 79 | 0.78000 | 0.20980 | 0.99020 | 0.88341 | 0.30789 | 0.87660 | 1.08826 |
| $8)$ | 0.79000 | 0.20892 | 0.99108 | 0.89342 | 0.30710 | 0.88659 | 1.08923 |
| 81 | 0.80000 | 0.20808 | 0.99192 | 0.90343 | 0.30635 | 0.89659 | 1.09016 |
| 82 | 0.81000 | 0.20729 | 0.99271 | 0.91343 | 0.30564 | 0.90658 | 1.09104 |
| 83 | 0.82000 | $0.2) 653$ | 0.99347 | 0.92344 | 0.30498 | 0.91657 | 1.09188 |
| 84 | 0.83000 | 0.20582 | 0.99418 | 0.93345 | 0.30435 | 0.92656 | 1.09268 |
| 85 | 0.84000 | 0.20515 | 0.99485 | 0.94345 | 0. 30377 | 0.93656 | 1.09343 |
| 86 | 0.85000 | 0.20453 | 0.99547 | 0.95346 | 0.30323 | 0.94655 | 1.09415 |
| 87 | 0.86000 | 0.20394 | 0.99606 | 0.96346 | 0.30273 | 0.95655 | 1.09482 |
| 83 | 0.87000 | 0.20339 | 0.99661 | 0.97347 | 0.30227 | 0.96654 | 1.09545 |
| 89 | 0.88000 | 0.20289 | 0.99711 | 0.98347 | 0.30186 | 0.97654 | 1.09605 |
| 90 | 0.89000 | 0.20243 | 0.99757 | 0.99347 | 0.30148 | 0.98653 | 1.09660 |
| 91 | 0.90000 | 0.20201 | 0.99799 | 1.00348 | 0.30115 | 0.99653 | 1.09711 |
| 92 | 0.91000 | 0.29162 | 0.99838 | 1.01348 | 0.30085 | 1.00652 | 1.09757 |
| 93 | 0.92000 | 0.20128 | 0.99872 | 1.02348 | 0.30060 | 1.01652 | 1.09800 |
| 94 | 0.93000 | 0.20098 | 0.99902 | 1.03348 | 0.30039 | 1.02652 | 1.09839 |
| 95 | 0.94000 | 0.20072 | 0.99928 | 1.04349 | 0.30021 | 1.03652 | 1.09874 |
| 96 | 0.95000 | 0.20050 | 0.99950 | 1.05349 | 0.30008 | 1.04651 | 1.09905 |
| 97 | 0.96000 | 0.20032 | 0.99968 | 1.06349 | 0.29999 | 1.05651 | 1.09931 |
| 98 | 0.97000 | 0.20018 | 0.99982 | 1.07349 | 0.29993 | 1.06651 | 1.09954 |
| 99 | 0.98000 | 0.20008 | 0.99992 | 1.08349 | 0.29992 | 1.07651 | 1.09973 |
| 100 | 0.99000 | 0.20002 | 0.99998 | 1.09349 | 0.29995 | 1.08651 | 1.09988 |
| 101 | 1.00000 | 0.20000 | 1.00000 | 1.10349 | 0.30002 | 1.09651 | 1.09998 |
| 102 | 1.01000 | 0.20002 | 0.99998 | 1.11349 | 0.30012 | 1.10650 | 1.10005 |
| 103 | 1.02000 | 0.20008 | 0.99992 | 1.12349 | 0.30027 | 1.11650 | 1.10008 |
| 104 | 1.03000 | 0.20018 | 0.99982 | 1.13348 | 0.30046 | 1.12650 | 1.10007 |
| 105 | 1.04000 | 0.20032 | 0.99968 | 1.14348 | 0.30068 | 1.13650 | 1.10001 |
| 106 | 1.05000 | 0.20050 | 0.99950 | 1.15348 | 0.30095 | 1.14650 | 1.09992 |
| 107 | 1.05999 | 0.20072 | 0.99928 | 1.16348 | 0.30126 | 1.15650 | 1.09979 |
| 108 | 1.06999 | 0.20098 | 0.99902 | 1.17347 | 0.30161 | 1.16651 | 1.09961 |
| 109 | 1.07999 | 0.20128 | 0.99872 | 1.18347 | 0.30200 | 1.17651 | 1.09940 |


| 110 | 1.08999 | 0.20162 | 0.99838 | 1.19347 | 0.30242 | 1.18651 | 1.09915 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 111 | 1.09999 | 0.20200 | 0.99800 | 1.20346 | 0.30289 | 1.19651 | 1.09885 |
| 112 | 1.10999 | 0.20243 | 0.99757 | 1.21346 | 0.30340 | 1.20651 | 1.09852 |
| 113 | 1.11999 | 0.20289 | 0.99711 | 1. 22345 | 0.30395 | 1.21652 | 1.09814 |
| 114 | 1.12999 | 0.20339 | 0.99661 | 1. 23345 | 0.30454 | 1.22652 | 1.09772 |
| 115 | 1.13999 | 0.20394 | 0.99606 | 1.24344 | 0.30518 | 1.23652 | 1.09727 |
| 115 | 1.14999 | 0.20452 | 0.99548 | 1.25343 | 0.30585 | 1.24653 | 1.09677 |
| 117 | 1.15999 | 0.20515 | 0.99485 | 1.26343 | 0.30656 | 1.25653 | 1.09623 |
| 11.3 | 1.16999 | 0.23582 | 0.99418 | 1.27342 | 0.30732 | 1. 26654 | 1.09565 |
| 119 | 1.17999 | 0.20653 | 0.99347 | 1.28341 | 0.30812 | 1. 27654 | 1.09502 |
| 123 | 1.18999 | 0.20729 | 0.99271 | 1.29340 | 0.30896 | 1.28655 | 1.09436 |
| 121 | 1.19998 | 0.20808 | 0.99192 | 1.30340 | 0.30984 | 1. 29655 | 1.09365 |
| 122 | 1.20998 | 0.20892 | 0.99108 | 1.31339 | 0.31077 | 1.30656 | 1.09290 |
| 123 | 1.21998 | 0.20980 | 0.99020 | 1.32338 | 0.31173 | 1.31657 | 1.09211 |
| 124 | 1.22998 | 0.21072 | 0.98928 | 1.33337 | 0.31274 | 1.32657 | 1.09127 |
| 125 | 1.23998 | 0.21169 | 0.98831 | 1.34336 | 0.31380 | 1.33658 | 1.09039 |
| 126 | 1.24998 | 0.21270 | 0.98730 | 1.35335 | 0.31490 | 1.34659 | 1.08947 |
| 127 | 1.25998 | 0.21375 | 0.98625 | 1.36334 | 0.31604 | 1.35660 | 1.08850 |
| 128 | 1.26998 | 0.21485 | 0.98515 | 1.37333 | 0.31723 | 1.36661 | 1.08749 |
| 129 | 1.27998 | 0.21600 | 0.98400 | 1.38332 | 0.31846 | 1.37661 | 1.08643 |
| 130 | 1.28998 | 0.21719 | 0.98281 | 1.39331 | 0.31973 | 1.38662 | 1.08533 |
| 131 | 1.29998 | 0.21842 | 0.98158 | 1.40330 | 0.32105 | 1.39663 | 1.08418 |
| 132 | 1.30998 | 0.21970 | 0.98030 | 1.41328 | 0.32242 | 1.40664 | 1.08299 |
| 133 | 1.31998 | 0.22103 | 0.97897 | 1.42327 | 0.32384 | 1.41665 | 1.08175 |
| 134 | 1.32998 | 0.22240 | 0.97760 | 1.43326 | 0.32530 | 1.42866 | 1.08046 |
| 135 | 1.33997 | 0.22383 | 0.97617 | 1.44324 | 0.32681 | 1.43668 | 1.07913 |
| 135 | 1.34997 | 0.22530 | 0.97470 | 1.45323 | 0.32837 | 1.44669 | 1.07774 |
| 137 | 1.35997 | 0.22681 | 0.97319 | 1.46321 | 0.32997 | 1.45670 | 1.07631 |
| 138 | 1.36997 | 0.22838 | 0.97162 | 1.47320 | 0.33163 | 1.46671 | 1.07483 |
| 139 | 1.37997 | 0.23000 | 0.97000 | 1.48318 | 0.33333 | 1.47672 | 1.07330 |
| 143 | 1.38997 | 0.23167 | 0.96833 | 1.49317 | 0.33509 | 1.48674 | 1.07172 |
| 141 | 1.39997 | 0.23339 | 0.96661 | 1. 50315 | 0.33689 | 1.49675 | 1.07009 |
| 142 | 1.40997 | 0.23516 | 0.96484 | 1.51314 | 0.33875 | 1.50677 | 1.06840 |
| 143 | 1.41997 | 0.23698 | 0.96302 | 1.52312 | 0.34066 | 1.51678 | 1.06667 |
| 144 | 1.42997 | 0.23886 | 0.96114 | 1.53310 | 0.34263 | 1.52680 | 1.06488 |
| 145 | 1.43997 | 0.24079 | 0.95921 | 1. 54308 | 0.34465 | 1.53681 | 1.06303 |
| 145 | 1.44997 | 0.24278 | 0.95722 | 1.55307 | 0.34672 | 1.54683 | 1.06113 |
| 147 | 1.45997 | 0.24483 | 0.95517 | 1.56305 | 0.34885 | 1. 55685 | 1.05918 |
| 148 | 1.46996 | 0.24693 | 0.95307 | 1.57303 | 0.35104 | 1.56686 | 1.05716 |
| 149 | 1.47996 | 0.24908 | 0.95092 | 1.58301 | 0.35329 | 1. 57688 | 1.05509 |


| 153 | 1.48996 |
| :--- | :--- |
| 151 | 1.499996 |
| 152 | 1.50996 |
| 153 | 1.51996 |
| 154 | 1.52996 |
| 155 | 1.53996 |
| 156 | 1.54996 |
| 157 | 1.55996 |
| 158 | 1.56996 |
| 159 | 1.57996 |
| 165 | 1.58996 |
| 161 | 1.59996 |
| 162 | 1.60995 |
| 163 | 1.61995 |
| 164 | 1.62995 |
| 165 | 1.63995 |
| 166 | 1.64995 |
| 167 | 1.65995 |
| 163 | 1.66995 |
| 169 | 1.67995 |
| 170 | 1.68995 |
| 171 | 1.69995 |
| 172 | 1.70995 |
| 173 | 1.71995 |
| 174 | 1.72995 |
| 175 | 1.73995 |
| 175 | 1.74994 |
| 177 | 1.75994 |
| 179 | 1.76994 |
| 179 | 1.77994 |
| 187 | 1.78994 |
| 181 | 1.79994 |
| 182 | 1.80994 |
| 183 | 1.81994 |
| 184 | 1.82994 |
| 185 | 1.83994 |
| 186 | 1.84994 |
| 187 | 1.85994 |
| 188 | 1.86994 |
| 189 | 1.87994 |
|  |  |

0.25130
0.25358
0.25592
0.25832
0.26079
0.26332
0.26592
0.26859
0.27133
0.27414
0.27703
0.27999
0.28303
0.28614
0.28935
0.29263
0.29601
0.29948
0.30304
0.30670
0.31046
0.31432
0.31830
0.32239
0.32660
0.33093
0.33540
0.34009
0.34476
0.34966
0.35473
0.35997
0.36540
0.37102
0.37686
0.38293
0.38925
0.39584
0.40273
0.40996
0.94870
0.94642
0.94408
0.94168
0.93921
0.93668
0.93408
0.93141
0.92867
0.92586
0.92297
0.92001
0.91697
0.91386
0.91065
0.90737
0.90399
0.90052
0.89696
0.89330
0.88954
0.88568
0.88170
0.87761
0.87340
0.86907
0.86460
0.86000
0.85524
0.85034
0.84527
0.84003
0.83460
0.82898
0.82314
0.81707
0.81075
0.80416
0.79727
0.79004
1.59299
1.60297
1.61294
1.62292
1.63290
1.64288
1.65285
1.66283
1.67280
1.68278
1.69275
1.70273
1.71270
1.72267
1.73264
1.74261
1.75258
1.76255
1.77252
1.78248
1.79245
1.80241
1.81238
1.82234
1.83230
1.84226
1.85222
1.86218
1.87214
1.88210
1.89205
1.90200
1.91196
1.92191
1.93185
1.94180
1.95174
1.96168
1.97162
1.98156
1

| 0.35559 | 1.58690 | 1.05296 |
| :---: | :---: | :---: |
| 0.35796 | 1.59692 | 1.05077 |
| 0.36039 | 1.60694 | 1.04852 |
| 0.36288 | 1.61696 | 1.04620 |
| 0.36543 | 1.62698 | 1.04382 |
| 0.36805 | 1.63700 | 1.04138 |
| 0.37074 | 1.64702 | 1.03886 |
| 0.37349 | 1.65704 | 1.03628 |
| 0.37632 | 1.66707 | 1.03363 |
| 0.37922 | 1.67709 | 1.03091 |
| 0.38219 | 1.68711 | 1.02811 |
| 0.38524 | 1.69714 | 1.02524 |
| 0.38836 | 1.70716 | 1.02229 |
| 0.39157 | 1.71719 | 1.01925 |
| 0.39486 | 1.72721 | 1.01614 |
| 0.39823 | 1.73724 | 1.01294 |
| 0.40170 | 1.74727 | 1.00965 |
| 0.40525 | 1.75730 | 1.00627 |
| 0.40890 | 1.76733 | 1.00280 |
| 0.41264 | 1.77736 | 0.99923 |
| 0.41649 | 1.78739 | 0.99556 |
| 0.42044 | 1.79743 | 0.99178 |
| 0.42451 | 1.80746 | 0.98789 |
| 0.42868 | 1.81749 | 0.98389 |
| 0.43298 | 1.82753 | 0.97976 |
| 0.43740 | 1.83757 | 0.97552 |
| 0.44196 | 1.84760 | 0.97114 |
| 0.44665 | 1.85764 | 0.96662 |
| 0.45149 | 1.86768 | 0.96196 |
| 0.45648 | 1.87773 | 0.95714 |
| 0.46163 | 1.88777 | 0.95216 |
| 0.46696 | 1.89781 | 0.94701 |
| 0.47247 | 1.90786 | 0.94167 |
| 0.47819 | 1.91791 | 0.93613 |
| 0.48411 | 1.92796 | 0.93038 |
| 0.49027 | 1.93801 | 0.92440 |
| 0.49667 | 1.94806 | 0.91817 |
| 0.50335 | 1.95812 | 0.91166 |
| 0.51034 | 1.96818 | 0.90485 |
| 0.51765 | 1.97824 | 0.89771 |


| 193 | 1.88993 | 0.41756 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 191 | 1.89993 | 0.42559 | 0.78244 0.77441 | 1.99149 | 0.52534 | 1.98831 | 0.89020 |
| 192 | 1.90993 | 0.43410 | 0.77441 | 2.00142 | 0.53345 | 1.99838 | 0.88226 |
| 193 | 1.91993 | 0.44317 | 0.76590 0.75683 | 2.011134 | 0.54205 | 2.00845 | 0.87384 |
| 194 | 1.92993 | 0.45291 | 0.75683 0.74709 | 2.32127 | 0.55121 | 2.01853 | 0.86486 |
| 195 | 1.93993 | 0.46345 | 0.73655 | 2.03118 | 0.56103 | 2.02861 | 0.85521 |
| 196 | 1.94993 | 0.47502 | 0.72498 | 2.04109 2.05098 | 0.57166 | 2.03870 | 0.84475 |
| 197 | 1.95993 | 0.48790 | 0.71210 | 2.05098 2.06087 | 0.58331 | 2.04880 | 0.83327 |
| 198 | 1.96993 | 0.50264 | 0.69736 | 2.06087 2.07074 | 0.59629 | 2.05891 | 0.82047 |
| 199 | 1.97993 | 0.52026 | 0.67974 | 2.07074 | 0.61112 | 2.06904 | 0.80582 |
| 200 | 1.98993 | 0.54337 | 0.65663 | 2.09038 | 0.62882 | 2.07919 | 0.78829 |
| 201 | 1.99993 | 0.59515 | 0.60485 | 2.09038 | 0.65201 | 2.08939 | 0.76527 |
| 202 | 0.00000 | 0.00000 | 0.00000 | 2.09993 | 0.70388 | 2.09984 | 0.71358 |
|  |  |  |  | . 00000 | 0.00000 | 0.00000 | 0.00000 |

ELLIPSE SHIFT PARAMETERS
CGX $=1.10000$
CGY $=0.70000$
ANG $=0.00631$

## REF LINE <br> LENGTH $=2.29583$

RAD OF G X SQ $=1.39430$
RAD DF GY SQ $=0.80423$

CROSS PRDDUCT RHXY $=0.88978$ TWICE ANGLE TAN-1 $=0.00000$
$X B A R=0.99794$
YBAR $=0.89002$
$X R A D L=1.18081$
YRADL $=0.89679$

RHBAR $=0.00060$

```
LENGTH= PROD LINE
LENGTH = 2.29583
PXBAR = 1.09541
```



```
RAD DF G YSQ = 0.99221
YPADL = 0.99610
CROSS PRDDUCT PHXY = 1.08841 PHBAR = 0.00397
THICE ANGLE TAN-1 = 0.02054
    LINE CENTROID
    SHIFT PARAMETERS
XSHIFT = 0.09747 YSHIFT = 0.09997
ANGULAR DIFFERENCE IN DEGREES = 0.50017
EFFECTIVE SHIFT OF SECTIONAL CENTROID
XSHIFT = 0.10000 YSHIFT = 0.10000
```

PYBAR $=0.98999$
$X P G=0.63118$
$Y P G=0.11012$

PRODUCTION SHAPE DX ELEMENTS
AREA $=1.25512$

RAD $=1.45960$$\quad$ R DG $G=$| $C O F G=1.10026$ |
| :--- |
| CROSSPRODUCT $=0.77202$ |$\quad$ XPADG $=0.49003$

PRODUCTION DY ELEMENTS
$A R E A=1.25740 \quad$ C OF G $=0.70003$
$P R A D S Q=0.52997 \quad Y P A D G=0.19982$
CROSSPRODUCT $=0.77193$

SHIFT PARAMETERS

```
\(X\) DIRECTION \(=0.10020\)
Y DIRECTION \(=0.10002\)
TAN OF ANGULAR DIFFERENCE \(=0.00867\)
```

ANGULAR DIFFERENCE IN DEGREES $=0.49668$
IHE $140 I$ FILE SYSIN - END OF FILE ENCOUNTERED IN STATEMENT 00009 AT OFFSET +OOOO2 FROM ENTRY POINT SIMCAL
input constants

```
POINTS =222.0
    DELTA = 0.016
```

LIMI $=10900$
LIM2 $=12300$
DIFMIN $=100$ NOISE LIMIT
START PARAMETER $=0.00$
XMAX $=2150 \quad$ XMIN $=14900$
CDMPDNENT WIDTH 1.19000
DIRECTION INDICATOR $=1$
$x$ oirection inversion region
XLIM1 $=11000$
XLIM2 $=12500$
RUV TYPF $n=$ REF $\&=$ SUBTRACT REF $=0$

POINTS NUMBER | 1 |
| :---: |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| 13 |
| 14 |
| 15 |
| 16 |
| 17 |
| 18 |
| 19 |
| 20 |
| 21 |
| 29 |
| 23 |
| 24 |
| 25 |
| 26 |
| 27 |
| 28 |
| 29 |
| 30 |
| 31 |
| 32 |
| 33 |
| 34 |
| 35 |
| 36 |
| 37 |
| 38 |
| 39 |
|  |

EX VALUE
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
1.16667
1.1 .6247
1.15789
1.15323
1.14893
1.14445
1.13960
1.13447
1.12961
1.12457
1.11963
1.11496
1.10936
1.10525
1.10021
1.09573
1.09013
1.08491
1.07912
1.07305
1.06745
1.06195
1.05551
1.04953
1.04421
1.03787
1.03264
1.02741
1.02247
1.01724
1.01291
1.00707
VALUE
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00012
0.00801
0.01612
0.02075
0.02401
0.03187
0.02401
0.03212
0.04001
0.04361
0.04812
0.05601
0.05836
0.06412
0.06800
0.07201
0.07678
0.08012
0.08486
0.08801
0.09216
0.09612
0.09870
0.10189
0.10401
0.10706
0.10962
0.11212
0.11353
0.11516
0.11672
0.11828

| 1.00193 | 0.12001 |
| :--- | :--- |
| 0.99773 | 0.12117 |
| 0.99288 | 0.12273 |
| 0.98831 | 0.12387 |
| 0.98392 | 0.12508 |
| 0.97963 | 0.12669 |
| 0.97505 | 0.12799 |
| 0.97029 | 0.12799 |
| 0.96525 | 0.12801 |
| 0.96040 | 0.12801 |
| 0.95517 | 0.12966 |
| 0.95013 | 0.13089 |
| 0.94481 | 0.13198 |
| 0.93949 | 0.13275 |
| 0.93427 | 0.13355 |
| 0.92876 | 0.13444 |
| 0.92307 | 0.13511 |
| 0.91765 | 0.13584 |
| 0.91 .159 | 0.13566 |
| 0.90655 | 0.13559 |
| 0.90020 | 0.13572 |
| 0.89404 | 0.13571 |
| 0.88863 | 0.13586 |
| 0.88312 | 0.13559 |
| 0.87687 | 0.13531 |
| 0.87127 | 0.13502 |
| 0.86604 | 0.13451 |
| 0.86007 | 0.13403 |
| 0.85540 | 0.13347 |
| 0.85027 | 0.13278 |
| 0.84513 | 0.13208 |
| 0.84019 | 0.13137 |
| 0.83477 | 0.13072 |
| 0.82945 | 0.12972 |
| 0.82497 | 0.12825 |
| 0.81975 | 0.12787 |
| 0.81433 | 0.12592 |
| 0.80976 | 0.12356 |
| 0.80425 | 0.12216 |
| 0.79903 | 0.1 |


| 80 | 9．79389 | 0.12055 |
| :---: | :---: | :---: |
| 91 | 9．78885 | 0.11999 |
| 82 | 9.78353 | 0.11778 |
| 83 | 0.77812 | 0.11602 |
| 84 | 0.77168 | 0.11412 |
| 85 | 0.76627 | 0.11187 |
| 86 | 0.76011 | 0.10969 |
| 87 | 0.75460 | 0.10720 |
| 88 | 0.74900 | 0.10399 |
| 39 | 0.74284 | 0.10340 |
| 90 | 0.73677 | 0.10069 |
| 91 | 0.73052 | 0.09831 |
| 92 | 0.72445 | 0.09587 |
| 93 | 0.71867 | 0.09325 |
| 94 | 0.71307 | 0.09081 |
| 75 | 0.70635 | 0.08799 |
| 96 | 0.70093 | 0.08615 |
| 97 | 7．69552 | 0.08377 |
| 98 | 0.68945 | 0.08100 |
| 99 | 0.68479 | 0.07987 |
| 100 | 0.68049 | 0.07708 |
| 101 | 0.67527 | 0.07491 |
| 102 | 0.67153 | 0.07199 |
| 103 | 0.66621 | 0.07133 |
| 104 | 0.66099 | 0.06848 |
| 195 | 0.65641 | 0.06572 |
| 106 | ？． 65156 | 0.06387 |
| 107 | 0.64680 | 0.06041 |
| 108 | 0.64195 | 0.05762 |
| 109 | 0.63700 | 0.05599 |
| 110 | 0.63215 | 0.05208 |
| 111 | 0.62683 | 0.04787 |
| 112 | 0.62095 | 0.04625 |
| 113 | 0.61647 | 0.04261 |
| 11.4 | 0.61105 | 0.03999 |
| 315 | ？． 60415 | 0.03516 |
| 116 | 0.59836 | 0.03187 |
| 117 | 7． 59276 | 0.02777 |
| 118 | O． 58669 | 0.02399 |
| 119 | 0．58ワ53 | 0.02027 |


| 120 | 0.57465 |
| :--- | :--- |
| 121 |  |
| 112 | 0.56905 |
| 123 | 0.56336 |
| 124 | 0.55720 |
| 125 | 0.55160 |
| 126 | 0.54600 |
| 127 | 0.54040 |
| 128 | 0.53527 |
| 129 | 0.53051 |
| 130 | 0.52547 |
| 131 | 0.52071 |
| 132 | 0.51632 |
| 133 | 0.51184 |
| 134 | 0.50680 |
| 135 | 0.50269 |
| 136 | 0.49793 |
| 137 | 0.49243 |
| 138 | 0.48804 |
| 139 | 0.48300 |
| 140 | 0.47824 |
| 141 | 0.47357 |
| 142 | 0.46853 |
| 143 | 0.46377 |
| 144 | 0.45799 |
| 145 | 0.45323 |
| 146 | 0.44744 |
| 147 | 0.44119 |
| 148 | 0.43484 |
| 149 | 0.42933 |
| 150 | 0.42317 |
| 151 | 0.41748 |
| 152 | 0.41179 |
| 153 | 0.40637 |
| 154 | 0.40021 |
| 155 | 0.39405 |
| 156 | 0.38873 |
| 157 | 0.36699 |
| 158 |  |

0.01587
0.01291
0.00799
0.00581
0.00153
$-0.00012$
$-0.00498$
$-0.00801$
$-0.01169$
$-0.01612$
$-0.01795$
$-0.02094$
$-0.02401$
$-0.02747$
$-0.03212$
-0.03397
$-0.03788$
$-0.04001$
-0.04447
$-0.04812$
$-0.05184$
$-0.05601$
$-0.05986$
$-0.06412$
$-0.06786$
$-0.07201$
$-0.08012$
-0.08181
-0.08801
$-0.09255$
$-0.09612$
$-0.10266$
$-0.10401$
$-0.11212$
$-0.11500$
$-0.12001$
$-0.12383$
$-0.12812$
$-0.13186$
$-0.13601$

| 160 | 0.36176 | -0.13973 |
| :--- | :--- | :--- |
| 160 | 0.35663 | -0.14412 |
| 162 | 0.35243 | -0.14711 |
| 163 | 0.34757 | -0.15201 |
| 164 | 0.34235 | -0.15567 |
| 165 | 0.33768 | -0.16012 |
| 166 | 0.33236 | -0.16411 |
| 167 | 0.32732 | -0.16801 |
| 168 | 0.32209 | -0.17259 |
| 169 | 0.31715 | -0.17612 |
| 170 | 0.31248 | -0.18243 |
| 171 | 0.30650 | -0.18401 |
| 172 | 0.30165 | -0.19212 |
| 173 | 0.29605 | -0.19545 |
| 174 | 0.29008 | -0.20001 |
| 175 | 0.28439 | -0.20812 |
| 176 | 0.27823 | -0.21072 |
| 177 | 0.27272 | -0.21601 |
| 178 | 0.26693 | -0.22284 |
| 179 | 0.26059 | -0.22412 |
| 180 | 0.25489 | -0.23201 |
| 181 | 0.24855 | -0.23759 |
| 182 | 0.24323 | -0.24012 |
| 183 | 0.23679 | -0.24801 |
| 184 | 0.23128 | -0.25403 |
| 185 | 0.22596 | -0.25612 |
| 186 | 0.22017 | -0.26401 |
| 187 | 0.21429 | -0.26680 |
| 188 | 0.20907 | -0.27212 |
| 189 | 0.19496 | -0.27673 |
| 190 | 0.19423 | -0.28001 |
| 191 | 0.18965 | -0.28653 |
| 192 | 0.18424 | -0.28812 |
| 193 | 0.17948 | -0.29601 |
| 194 | 0.17435 | -0.30077 |
| 195 | 0.16949 | -0.30412 |
| 196 | 0.15960 | -0.31201 |
| 197 | 0.15447 | -0.31540 |
| 198 |  | -0.32012 |
| 199 |  | -0.32801 |

CURVE LENGTH $=1.29902$
$X B A R=0.50605$
ZBAR $=-0.04392$

RAD OF G $X S Q=0.45545$
$X R A D=0.67487$
$X R G=0.31649$

RAD OF G Z SQ $=0.02830$
ZRAD $=0.16822$
$Z R G=0.16239$

CROSS TERM $=0.02003$
HBAR $=0.04621$
TWICE ANGLE OF P AXES $=1.252 ? 1$
ref data set

IDENT $=008720724$
RUN $=B A C C-1-160$

CORRECT CARD AND TAPE FOUND II $=10$
RUNB_CEN_BACK_D1DEFL

NOIIV7กdINVW GIOYINヨコ ヨ0 ヨ7dWマXヨ

ヨヨyH1 XIONJddV

```
            POINTS =303.0
                            DELTA = 0.016
```

$\operatorname{LIM1}=10900$
LIM2 $=12300$

```
                NOISE LIMIT
    DIFMIN = 100
        START PARAMETER = 1.00
        XMAX = 2150 XMIN = 14900
        COMPONENT WIDTH 1.19000
    OIRECTION INDICATOR= = 1
        x DIRECTION INVERSION REGION
        XLIM1 = 11000
XLIM2 = 12500
RUV TYPE O = REF 1= SUBTRACT REF = l
```

POINTS NUMBER | 1 |
| :---: |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 3 |
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| 11 |
| 12 |
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| 14 |
| 15 |
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| 21 |
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| 23 |
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| 28 |
| 29 |
| 30 |
| 31 |
| 32 |
| 33 |
| 34 |
| 35 |
| 36 |
| 37 |
| 38 |
| 39 |
|  |
|  |

| EXVALUE | Z VALUE |
| :--- | ---: |
| 0.00000 | 0.00000 |
| 0.00000 | 0.00000 |
| 0.00000 | 0.00000 |
| 0.00000 | 0.00000 |
| 0.00000 | 0.00000 |
| 0.00000 | 0.00000 |
| 0.00000 | 0.00000 |
| 0.00000 | 0.00000 |
| 1.17180 | 0.00812 |
| 1.16807 | 0.01601 |
| 1.16424 | 0.01766 |
| 1.1 .6116 | 0.02412 |
| 1.15687 | 0.02927 |
| 1.15276 | 0.03199 |
| 1.14940 | 0.02412 |
| 1.14548 | 0.03201 |
| 1.14147 | 0.03695 |
| 1.13745 | 0.04012 |
| 1.13363 | 0.04801 |
| 1.12971 | 0.04945 |
| 1.12597 | 0.05612 |
| 1.12243 | 0.05728 |
| 1.11832 | 0.06401 |
| 1.11468 | 0.06634 |
| 1.11076 | 0.07081 |
| 1.10731 | 0.07212 |
| 1.10292 | 0.07769 |
| 1.09993 | 0.08001 |
| 1.09601 | 0.08378 |
| 1.09237 | 0.08812 |
| 1.08836 | 0.09019 |
| 1.08425 | 0.09284 |
| 1.08061 | 0.09601 |
| 1.07679 | 0.09830 |
| 1.07277 | 0.10073 |
| 1.06885 | 0.10412 |
| 1.06521 | 0.10931 |
| 1.06129 |  |
| 1.05793 | 0.10410 |
|  | 0 |


| 1.05364 | 0.11201 |
| :---: | :---: |
| 1.04972 | 0.11327 |
| 1.34561 | 0.11504 |
| 1.04169 | 0.11664 |
| 1.03749 | 0.11828 |
| 1. 23385 | 0.12012 |
| 1. 12965 | 0.121 .62 |
| 1.92564 | 0.12333 |
| 1.02153 | 0.12485 |
| 1.01771 | 0.12603 |
| 1. 11323 | 0.12711 |
| 1.00931 | 0.12801 |
| 1.00501 | 0.12946 |
| 1.90137 | 9.13052 |
| 0.99773 | 0.13158 |
| 0.99353 | 0.13252 |
| 0.98980 | 0.13341 |
| 0.98551 | 0.13437 |
| 0.98243 | n. 13612 |
| 0.97785 | 0.13644 |
| 0.97440 | 0.13728 |
| 0.97020 | 0.13805 |
| 0.96703 | 0.13881 |
| 0.96283 | 0.13952 |
| 0.95937 | 0.14023 |
| 0.95527 | 0.14083 |
| 0.95200 | 0.14154 |
| 0.94761 | 0.14209 |
| 0.94407 | 0.14262 |
| 0.94117 | 0.14312 |
| 0.93697 | 0.14372 |
| 0.93287 | 0.14330 |
| 0.92895 | 0.14491. |
| 0.92503 | 0.14462 |
| 0.92139 | 0.14448 |
| 0.91756 | 0.14426 |
| 0.91355 | 0.14481 |
| 0.90972 | 0.14505 |
| 0.90617 | 0.14501 |
| 0.90216 | 0.14500 |


| 80 | 0.89787 | 0.14505 |
| :---: | :---: | :---: |
| 81 | 3.89395 | 0.14504 |
| 82 | 0.88984 | 0.14479 |
| 83 | 0.88639 | 0.14454 |
| 84 | 0.88200 | 0.14440 |
| 85 | 0.87817 | 0.14420 |
| 86 | 0.87397 | 0.14330 |
| 87 | 0.87033 | 0.14328 |
| 88 | 0.86576 | 0.14387 |
| 89 | 0.86231 | 0.14322 |
| 90 | 0.85820 | 0.14266 |
| 91 | 0.85540 | 0.14208 |
| 92 | 0.85017 | 0.14144 |
| 93 | 0.84691 | 0.14083 |
| 94 | 0.84187 | 0.14031 |
| 95 | 0.83869 | 0.13962 |
| 96 | 0.83515 | 0.13884 |
| 97. | 0.83011 | 0.13819 |
| 98 | 0.82647 | 0.13775 |
| 99 | 0.82236 | 0.13703 |
| 100 | 0.81883 | 0.13587 |
| 101 | 0.81471 | 0.13431 |
| 102 | 0.81116 | ก. 13353 |
| 103 | 0.80761 | 0.13264 |
| 104 | 0.80360 | 0.13153 |
| 1.5 | 0.79987 | 0.13044 |
| 106 | 0.79651 | 0.12920 |
| 107 | 0.79259 | 0.12799 |
| 108 | 0.78820 | 0.12677 |
| 109 | 0.78465 | 0.12570 |
| 110 | 0.78045 | 0.12441 |
| 1.11 | 0.77691 | 0.12303 |
| 112 | 0.77271 | 0.12153 |
| 113 | 0.76925 | 0.11987 |
| 114 | 0.76543 | 0.11853 |
| 115 | 0.76123 | 0.11711 |
| 116 | 0.75871 | 0.11584 |
| 117 | 0.75404 | 0.11431 |
| 118 | 0.75003 | 0.11199 |
| 119 | 0.74629 | 0.11152 |


| 120 | 0.741 .91 |
| :--- | :--- |
| 121 | 0.73920 |
| 172 |  |
| 123 | 0.73528 |
| 124 | 0.73136 |
| 125 | 0.72707 |
| 126 | 0.72315 |
| 127 | 0.71857 |
| 128 | 0.71521 |
| 129 | 0.71101 |
| 130 | 0.70719 |
| 131 | 0.70261 |
| 132 | 0.69907 |
| 133 | 0.69487 |
| 134 | 0.69104 |
| 135 | 0.68731 |
| 136 | 0.68320 |
| 137 | 0.67919 |
| 138 | 0.67508 |
| 139 | 0.67116 |
| 140 | 0.66733 |
| 141 | 0.65360 |
| 142 | 0.65987 |
| 143 | 0.65604 |
| 144 | 0.65156 |
| 145 | 0.64895 |
| 146 | 0.64475 |
| 147 | 0.64064 |
| 148 | 0.63756 |
| 149 | 0.63373 |
| 150 | 0.62944 |
| 151 | 0.62571 |
| 152 | 0.61889 |
| 153 | 0.61497 |
| 154 | 0.61105 |
| 155 | 0.60695 |
| 156 | 0.50349 |
| 157 | 0.59603 |
| 158 | 0.59211 |
| 159 |  |

0.10966 0.10791 0.10634 0.10387 0.10375 0.10166
0.09962 0.09791 0.09599 0.09452 0.09316
0.09147
0.08966 0.08787
0.08536 0.08353
0.08178
0.07999
0.07876
0.07680
0.07487
0.07187
0.07016
0.06807
0.06639
0.06399
0.06255
0.06031
0.05787
0.05587
0.05278
0.05051
0.04799
0.04594
0.04350
0.03987
0.03853
0.03555
0.03250
0.03199


| 200 | 0.43437 |
| :--- | :--- |
| 201 | 0.43055 |
| 202 | 0.42635 |
| 203 | 0.42 .280 |
| 204 | 0.41860 |
| 205 | 0.41533 |
| 206 | 0.41123 |
| 207 | 0.40731 |
| 208 | 0.40329 |
| 209 | 0.30965 |
| 210 | 0.39564 |
| 211 | 0.39125 |
| 212 | 0.38789 |
| 213 | 0.38388 |
| 214 | 0.37865 |
| 215 | 0.37501 |
| 216 | 0.37156 |
| 217 | 0.36699 |
| 218 | 0.36260 |
| 219 | 0.35971 |
| 220 | 0.35495 |
| 221 | 0.35149 |
| 222 | 0.34757 |
| 223 | 0.34328 |
| 224 | 0.34011 |
| 225 | 0.33609 |
| 226 | 0.33245 |
| 227 | 0.32872 |
| 228 | 0.32433 |
| 229 | 0.31995 |
| 230 | 0.31743 |
| 231 | 0.31267 |
| 232 | 0.30968 |
| 233 | 0.30473 |
| 234 | 0.30137 |
| 235 | 0.29708 |
| 236 | 0.29381 |
| 237 | 0.28989 |
| 238 | 0.28065 |
| 239 |  |
|  |  |
| 20 |  |
| 20 | 0 |

$-0.08316$ $-0.08641$ $-0.08812$ $-0.09262$ -0.09601 $-0.09914$ $-0.10281$ $-0.10412$ $-0.10805$ $-0.11 .201$ $-0.11481$ $-0.11786$ $-0.12012$ $-0.12436$ $-0.12801$ $-0.13123$ $-0.13612$ $-0.13681$ $-0.14062$ $-0.14401$ $-0.14661$ $-0.15009$ $-0.15212$ $-0.15606$ $-0.16001$ -0.16259 -0.16591 $-0.16812$ $-0.17278$ $-0.17601$ $-0.1793 n$ $-0.18412$ $-0.18647$ $-0.19024$ $-0.19201$ $-0.19697$ $-0.20012$ $-0.20375$ $-0.20801$
$-0.21145$

| 240 | 0.27739 |
| :---: | :---: |
| 241 | 0.27309 |
| 242 | 0.26973 |
| 243 | 0.26507 |
| 244 | 0. 261133 |
| 345 | 0.25723 |
| 246 | 0.25275 |
| 247 | 0.24892 |
| 248 | 0.24519 |
| 249 | 0.24164 |
| 250 | 0.23753 |
| 25! | 0. 23343 |
| 25? | 0.22913 |
| 253 | 0.22456 |
| 254 | 0.22092 |
| 255 | 0.21681 |
| 256 | 0.21271 |
| 257 | 0.20860 |
| 258 | 0.20412 |
| 259 | 0.20029 |
| 260 | 0.19637 |
| 261 | 0.19329 |
| 252 | 0.18909 |
| 2.63 | 0.18461 |
| 264 | 0.18163 |
| 265 | 0.17789 |
| 2.56 | 0.17425 |
| 267 | 0.17024 |
| 268 | 0.16697 |
| 269 | 0.1 .6296 |
| 270 | 0.15820 |
| 271 | O. 15493 |
| 272 | 0.15167 |
| 273 | 0.14728 |
| 274 | 0.14336 |
| 275 | 0.14065 |
| 276 | 0.13701 |
| 277 | 0.13253 |
| 278 | 0.12824 |
| 279 | 0.12516 |

-0.21612
-0.21923
-0.22401
-0.22473
-0.22969
-0.23212
-0.23732
-0.24001
-0.24430
-0.24812
-0.25137
-0.25601
-0.25881
-0.26412
-0.26616
-0.27201
-0.27354
-0.27834
-0.28012
-0.28566
-0.28801
-0.29267
-0.29612
-0.30052
-0.30401
-0.30786
-0.31212
-0.31531
-0.032001
-0.32255
-0.32812
-0.033056
-0.33601
-0.33916
-0.34412
-0.344741
-0.35201
-0.35562
-0.36012
-0.36319

| 280 | 0.12115 |
| :--- | :--- |
| 281 | 0.11657 |
| 282 | 0.11265 |
| 283 | 0.10883 |
| 284 | 0.10556 |
| 285 | 0.10108 |
| 286 | 0.09725 |
| 287 | 0.09315 |
| 288 | 0.08904 |
| 289 | 0.08540 |
| 290 | 0.08111 |
| 291 | 0.07672 |
| 292 | 0.07355 |
| 293 | 0.06953 |
| 294 | 0.00000 |
| 295 | 0.00000 |
| 296 | 0.00000 |
| 297 | 0.00000 |
| 298 | 0.00000 |
| 299 | 0.00000 |
| 300 | 0.00000 |
| 301 | 0.00000 |
| $30 ?$ | 0.00000 |
| 303 | 0.00000 |

-0.36801
-0.37223
-0.37612
-0.38028
-0.38401
-0.38934
-0.39212
-0.39641
-0.40001
-0.40562
-0.40812
-0.41 .256
-0.41601
-0.42387
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00600
0.00000
0.00000

CURVE LENGTH $=1.34285$

```
XBAR = 0.59603
ZBAR = -0.04531
RAD BF G X SQ = 0.46053
XRAD = 0.67962
XRG = 0.32446
RAD OF G Z SQ = 0.03151
ZRAD = 0.17750
ZRG = 0.17162
CROSS TERM = 0.02292
HBAR = 0.04992
TWICE ANGLE OF P AXES = 1.31686
```

DIFFERENCE PARAMETERS
XSHIFT $=0.0$ OOn2 $\quad$ ZSHIFT $=0.00139$

DIFTAN $=0.02440$

AVGULAR DIFFERENCE $=0.69954$

IDENT $=009720724$
RUN $=\mathrm{BACC}-1-160$

CJRRECT CARD AND TAPE FOUND II $=9$

RUN9_CEN_BACK_D2DEFL

HOLOGRAPHIC CONTOURING PROGRAMME
INPUT CONSTANTS

```
        POINTS =234.0 DELTA = 0.016
        LIMI= 10OOn LIM2 = 12300
        NOISE LIMIT
        DIFMIN = 100
        START PARAMETER = 3.00
        XMAX = 2150 XMIN = 14900
        COMPONENT WIDTH 1.190OO
    DIRECTION INDICATOR = 1
        X DIRECTION INVERSION REGION
        XLIMI = 11000
        XLIM2 = 12500
RUN TYPE O = REF 1 = SUBTRACT REF = 1
```

| POINTS | NUMBER |
| :---: | :---: |
|  | 2 |
|  | 3 |
|  | 4 |
|  | 5 |
|  | 6 |
|  | 7 |
|  | 8 |
|  | 9 |
|  | 10 |
|  | 11 |
|  | $1 ?$ |
|  | 13 |
|  | 14 |
|  | 15 |
|  | 16 |
|  | 17 |
|  | 18 |
|  | 19 |
|  | 20 |
|  | 21 |
|  | 22 |
|  | 23 |
|  | 24 |
|  | 25 |
|  | 26 |
|  | 27 |
|  | 28 |
|  | 29 |
|  | 30 |
|  | 31 |
|  | 32. |
|  | 33 |
|  | 34 |
|  | 35 |
|  | 36 |
|  | 37 |
|  | 38 |
|  | 39 |

EX VALUE
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
1.1 .8085
1.17563
1.171 .52
1.16657
1.1 .6135
1.15640
1.15145
1.14669
1.14165
1.13671
1.13157
1.12635
1.12159
1.11655
1.11104
1.10637
1.10096
1.09611
1.09069
1.08565
1.08033
1.07520
1.06979
1.06409
1.05887
1.05355
1.04823
1.04291
1.03796
$z$ value
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.02412
0.02798
0.02880
0.03201
0.04012
0.04801
0.04879
0.05612
0.06401
0.06645
0.07212
0.07702
0.08001
0.08612
0.08812
0.09341
0.09601
0.10037
0.10412
0.10675
0.10959
0.11201
0.11470
0.11700
0.11975
0.12012
0.12291
0.12508
0.12688

| 40 | 1.03283 | 0.12801 |
| :--- | :--- | ---: |
| 41 | 1.02760 | 0.12984 |
| 42 | 1.02228 | 0.13120 |
| 43 | 1.01724 | 0.13258 |
| 44 | 1.01220 | 0.13403 |
| 45 | 1.05744 | 0.13612 |
| 46 | 1.00296 | 0.13650 |
| 47 | 0.99764 | 0.13728 |
| 48 | 0.99391 | 0.13803 |
| 49 | 0.98887 | 0.13911 |
| 50 | 0.98420 | 0.14002 |
| 51 | 0.97944 | 0.14086 |
| 52 | 0.97449 | 0.14166 |
| 53 | 0.96992 | 0.14242 |
| 54 | 0.96432 | 0.14322 |
| 55 | 0.95928 | 0.14552 |
| 56 | 0.95396 | 0.14527 |
| 57 | 0.94864 | 0.14497 |
| 58 | 0.94313 | 0.14457 |
| 59 | 0.93809 | 0.14483 |
| 60 | 0.93287 | 0.14546 |
| 61 | 0.92745 | 0.14572 |
| 62 | 0.92195 | 0.14569 |
| 63 | 0.91616 | 0.14575 |
| 64 | 0.91065 | 0.14566 |
| 65 | 0.90477 | 0.14523 |
| 66 | 0.89945 | 0.14479 |
| 67 | 0.89329 | 0.14429 |
| 68 | 0.88769 | 0.14317 |
| 69 | 0.88237 | 0.14291 |
| 70 | 0.87677 | 0.14311 |
| 71 | 0.87989 | 0.14313 |
| 72 | 0.86567 | 0.14211 |
| 73 | 0.86007 | 0.14124 |
| 74 | 0.85456 | 0.14030 |
| 75 | 0.84915 | 0.13923 |
| 76 | 0.84383 | 0.13819 |
| 77 | 0.83393 | 0.13587 |
| 78 |  | 0.13500 |
| 79 | 0 |  |


| 0.82376 | 0.13308 |
| :--- | :--- |
| 0.81891 | 0.13156 |
| 0.81368 | 0.13002 |
| 0.80827 | 0.12799 |
| 0.80313 | 0.12683 |
| 0.79763 | 0.12502 |
| 0.79203 | 0.12300 |
| 0.78680 | 0.11987 |
| 0.78111 | 0.11900 |
| 0.77551 | 0.11678 |
| 0.77037 | 0.11461 |
| 0.76459 | 0.11199 |
| 0.75880 | 0.11027 |
| 0.75292 | 0.10798 |
| 0.74760 | 0.10528 |
| 0.74144 | 0.10387 |
| 0.73565 | 0.10056 |
| 0.72996 | 0.09812 |
| 0.72408 | 0.09599 |
| 0.71857 | 0.09344 |
| 0.71325 | 0.09114 |
| 0.70775 | 0.08787 |
| 0.70243 | 0.08616 |
| 0.69608 | 0.08355 |
| 0.69160 | 0.08073 |
| 0.68665 | 0.07999 |
| 0.68133 | 0.07712 |
| 0.67676 | 0.07483 |
| 0.67237 | 0.07187 |
| 0.66836 | 0.06956 |
| 0.66379 | 0.06681 |
| 0.65875 | 0.06399 |
| 0.65417 | 0.06128 |
| 0.64951 | 0.05842 |
| 0.64456 | 0.05587 |
| 0.63933 | 0.05209 |
| 0.63429 | 0.04799 |
| 0.62897 | 0.041066 |
| 0.62421 |  |
| 0.61880 |  |
| 0 | 0.03987 |
| 0 |  |


| 120 | 0.61357 | 0.03506 |
| :---: | :---: | :---: |
| 121 | 0.60825 | 0.03199 |
| 122 | 0.60293 | 0.02803 |
| 123 | 0.59733 | 0.02387 |
| 124 | 0.59136 | 0.02072 |
| 125 | 0.58585 | 0.01599 |
| 126 | 0.58063 | 0.01453 |
| 127 | 0.57521 | 0.01019 |
| 128 | 0.56952 | 0.00787 |
| 129 | 0.56401 | 0.00314 |
| 130 | 0.55860 | -0.00001 |
| 131 | 0.55309 | -0.00375 |
| 132 | 0.54787 | -0.00812 |
| 133 | 0.54283 | -0.01087 |
| 134 | 0.53788 | -0.01442 |
| 135 | 0.53265 | -0.01601 |
| 136 | 0.52799 | -0.02045 |
| 137 | 0.52295 | -0.02412 |
| 1.38 | 0.51 .921 | -0.02733 |
| 139 | 0.51427 | -0.03117 |
| 140 | 0.50979 | -0.03201 |
| 142 | 0.50652 | -0.03581 |
| 147 | 0.50223 | -0.04012 |
| 143 | 0.49821 | -0.04259 |
| 144 | 0.49476 | -0.04587 |
| 145 | 0.49103 | -0.04801 |
| 146 | 0.48683 | -0.05233 |
| 147 | 0.48272 | -0.05612 |
| 148 | 0.47768 | -0.05939 |
| 149 | 0.47311 | -0.06401 |
| 150 | 0.46769 | -0.06795 |
| 151 | 0.46284 | -0.07212 |
| 152 | 0.45771 | -0.07645 |
| 153 | 0.45276 | -0.08001 |
| 154 | 0.44595 | -0.08548 |
| 155 | 0.44184 | -0.08812 |
| 156 | 0.43605 | -0.09601 |
| 157 | 0.43055 | -0.09789 |
| 158 | 0.42495 | -0.10412 |
| 159 | 0.41981 | -0.10752 |

0.41412
0.40899
0.40292
0.39732
0.39172
0.38621
0.38089
0.37557
0.37035
0.36605
0.36083
0.35579
0.35112
0.34627
0.34169
0.33703
0.33199
0.32732
0.32247
0.31752
0.31201
0.30707
0.30175
0.29596
0.29157
0.28607
0.28037
0.27449
0.26843
0.26283
0.25751
0.25181
0.24584
0.24024
0.23483
0.22895
0.22353
0.21849
0.21289
0.20739
$-0.11201$
$-0.11639$
$-0.12012$
$-0.12521$
$-0.12801$
$-0.13431$
$-0.13612$
$-0.14264$
$-0.14401$
$-0.15062$
$-0.15212$
$-0.15911$
$-0.16001$ $-0.16812$ $-0.16928$ $-0.17601$ $-0.17869$ $-0.18412$ $-0.18792$ $-0.19201$ $-0.19894$ $-0.20012$ $-0.20801$ $-0.21266$ $-0.21612$ $-0.22401$ $-0.22806$
$-0.23212$ $-0.24001$ $-0.24251$ $-0.24812$ $-0.25601$ -0.25641 -0. 26412 $-0.26898$ $-0.27201$ $-0.28012$ $-0.28353$ $-0.28801$ -0. 29612

| 200 | 0.20291 |
| :--- | :--- |
| 201 | 0.19740 |
| 202 | 0.19264 |
| 203 | 0.18788 |
| 204 | 0.18321 |
| 205 | 0.17808 |
| 206 | 0.17379 |
| 207 | 0.16893 |
| 208 | 0.16361 |
| 209 | 0.15913 |
| 219 | 0.15372 |
| 211 | 0.14877 |
| 212 | 0.14401 |
| 213 | 0.13916 |
| 214 | 0.13356 |
| 215 | 0.12843 |
| 216 | 0.12292 |
| 217 | 0.11779 |
| 218 | 0.11237 |
| 219 | 0.10649 |
| 220 | 0.10071 |
| 221 | 0.09501 |
| 222 | 0.08932 |
| 223 | 0.08335 |
| 224 | 0.07793 |
| 225 | 0.07261 |
| 226 | 0.00000 |
| 227 | 0.00000 |
| 228 | 0.00000 |
| 229 | 0.00000 |
| 230 | 0.70000 |
| 231 | 0.00000 |
| 232 | 0.00000 |
| 233 | 0.00000 |
| 234 | 0.00000 |

$-0.29825$
$-0.30401$
$-0.31012$
$-0.31212$
$-0.32001$
$-0.32348$
$-0.32812$
$-0.33601$
$-0.33916$
$-0.34412$
$-0.35201$
$-0.35486$
$-0.36012$
$-0.36801$
$-0.37042$
$-0.37612$
$-0.38401$
$-0.39212$
$-0.39306$
-0.40001
$-0.40812$
$-0.41291$
-0.41601
$-0.42412$
$-0.43201$
$-0.43987$
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000
0.00000

```
CIJRVE LENGTH = 1.3481.8
XBAR = 0.60059 ZBAR = -0.04916
RAD OF GXSQ = N.46543
RAD OF G Z SQ = 0.03328
ZRAD = 0.18243
ZRG = 0.17569
CROSS TERM = 0.02204 HBAR = 0.05156
TWICE ANGLE OF P AXES = 1.39632
```


## difference parameters

XSHIFT $=-3.00454 \quad$ ZSHIFT $=0.00524$

DIFTAN $=0.05243$

AVGULAR DIFFERENCE $=1.50227$



|  |  |  | LINE N | NUMBEF. | $=1$ | $\varepsilon$ | 10 | 8 | 5 | 2 | 11 | 10 | 11 | 8 | 2 | 2 | 3 | 5 | 11 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | 44 | 74 | 110 | 170 | 253 | 282 | 228 | 162 | 151 | 232 | 358 | 488 | 612 | 674 | 674 | 639 | 575 | 499 | 441 | 378 | 319 |
| 289 | 280 | 282 | 295 | 328 | 382 | 455 | 528 | 593 | 652 | 708 | 751 | 748 | 715 | 672 | 609 | 538 | 460 | 375 | 314 | 249 | 208 |
| 170 | 168 | 182 | 208 | 257 | 322 | 388 | 462 | 522 | 581 | 640 | 692 | 746 | 777 | 798 | 811 | 819 | 822 | 798 | 759 | 719 | 69 |
| 659 | 628 | 575 | 551 | 515 | 475 | 451. | 415 | 388 | 358 | 334 | 305 | 275 | 268 | 253 | 244 | 231 | 217 | 211 | 202 | 188 | 185 |
| 186 | 184 | 185 | 177 | 186 | 195 | 200 | 202 | 210 | 210 | 208 | 217 | 228 | 228 | 245 | 251 | 255 | 259 | 288 | 300 | 322 | 332 |
| 348 | 380 | 400 | 415 | 455 | 477 | 515 | 528 | 557 | 577 | 622 | . 658 | 688 | 722 | 751 | 768 | 775 | 788 | 791 | 777 | 759 | 738 |
| 715 | 675 | 668 | 638 | 608 | 563 | 528. | 475 | 408 | 343 | 299 | 252 | 228 | 195 | 180 | 152 | 140 | 138 | 148 | 173 | 222 | 268 |
| 307 | 352 | 415 | 478 | 528 | 594 | 639 | 694 | 728 | 735 | 740 | 708 | 659 | 611 | 541 | 477 | 401 | 333 | 271 | 222 | 182 | 152 |
| 148 | 162 | 195 | 237 | 294 | 359 | 445 | 515 | 589 | 655 | 705 | 709 | 699 | 668 | 608 | 535 | 471 | 397 | 328 | 251 | 208 | 177 |
| 180 | 205 | 248 | 309 | 393 | 480 | 568 | 628 | 669 | 695 | $68 ?$ | 638 | 580 | 502 | 417 | 345 | 274 | 222 | 188 | 188 | 211 | 267 |
| 339 | 428 | 515 | 53.3 | 650 | 658 | 655 | 606 | 535 | 451 | 348 | 268 | 212 | 200 | 217 | 280 | 358 | 451 | 529 | 593 | 630 | 620 |
| 571 | 497 | 428 | 348 | 282 | 228 | 208 | 222 | 262 | 330 | 418 | 500 | 562 | 602 | 593 | 555 | 479 | 410 | 335 | 268 | 222 | 210 |
| 250 | 308 | 282 | 455 | 522 | 571 | 571 | 528 | 455 | 368 | 283 | 238 | 227 | 251 | 305 | $3 \in 8$ | 445 | 508 | 548 | 538 | 505 | 439 |
| 368 | 310 | 253 | 238 | 255 | 311 | 380 | 438 | 488 | 518 | 504 | 455 | 388 | 319 | 259 | 242 | 255 | 301 | 368 | 428 | 477 | 498 |
| 469 | 417 | 351 | 278 | 268 | 275 | 305 | 368 | 422 | 475 | 482 | 468 | 415 | 358 | 305 | 268 | 271 | 311 | 358 | 412 | 448 | 457 |
| 440 | 388 | 338 | 295 | 272 | 288 | 331 | 375 | 422 | 432 | 428 | 388 | 341 | 295 | 275 | 282 | 317 | 362 | 402 | 408 | 388 | 358 |
| 320 | 279 | 273 | 292 | 331 | 375 | 408 | 410 | 388 | 345 | 315 | 284 | 285 | 311 | 345 | 381 | 395 | 382 | 355 | 315 | 288 | 295 |
| 328 | 358 | 388 | 388 | 378 | 348 | 315 | 295 | 298 | 319 | 355 | 375 | 368 | 342 | 309 | 288 | 288 | 304 | 328 | 355 | 362 | 351 |
| 322 | 298 | 277 | 288 | 310 | 330 | 35 C | 335 | 315 | 291 | 277 | 288 | 308 | 340 | 350 | 338 | 319 | 298 | 268 | 242 | 209 | 188 |
| 160 | 145 | 132 | 138 | 158 | 191 | 208 | 215 | 21.3 | 208 | 235 | 189 | 182 | 148 | 115 | 75 | 38 | 28 | 20 | 8 | 14 | 11 |
| 8 | 10 | 5 | 8 | 8 | 4 | 3 | 8 | 11. | 2 | 5 | 8 | 5 | 5 | 5 | 8 | 4 | 10 | 11 | 4 | 11 |  |
| 5 | 8 | 3 | 5 | 11 | 8 | 5 | 11 | 8 | 10 | 5 | 11 | 8 | 4 | 11 | 8 | 2 | 5 | 5 | 8 | 3 | 4 |

LINE REAC \& STORED

COMPLETE FRAME QEAD \& STORED

| 80 | 4 | 753. 00000 |
| :---: | :---: | :---: |
| 82 | 5 | 690.42847 |
| 84 | 6 | 620.28564 |
| 86 | 7 | 550.57129 |
| 88 | 8 | 481.42847 |
| 90 | 9 | 415.42847 |
| 92 | 10 | 360.85693 |
| 94 | 11 | 311.57129 |
| 96 | 12 | 272.85693 |
| 98 | 13 | 242.71423 |
| 100 | 14 | 220.85713 |
| 102 | 15 | 202.85713 |
| 104 | 16 | 191.57143 |
| 106 | 17 | 184.42856 |
| 108 | 18 | 187.57143 |
| 110 | 19 | 193.57143 |
| 112 | 20 | 201.57143 |
| 11.4 | 21 | 210.71428 |
| 116 | 22 | 220.85713 |
| 118 | 23 | 233.14285 |
| 120 | 24 | 250.57143 |
| 122 | 25 | 274.28564 |
| 124 | 26 | 300.57129 |
| 126 | 27 | 338.57129 |
| 128 | 28 | 378.85693 |
| 130 | 29 | 427.14282 |
| 132 | 30 | 478.14282 |
| 134 | 31 | 533. 10000 |
| 136 | 32 | 592.14282 |
| 138 | 33 | 653.57129 |
| 140 | 34 | 712.00000 |
| 142 | 35 | 754.71411 |
| 144 | 36 | 772.71411 |
| 146 | 37 | 763.28564 |
| 148 | 38 | 734.71411 |
| 150 | 39 | 638.71411 |
| 152 | 40 | 630.71411 |
| 154 | 41 | 555.42847 |
| 156 | 42 | 460.57129 |
| 158 | 42 | 361.85693 |




|  | $V M A X=$ | 668 |
| :---: | :---: | :---: |
| R = | 0.00000 |  |
| R | 0.31458 |  |
| $R=$ | 0.83333 |  |
|  | VMIN = | 200 |
| $0=$ | 0.97222 |  |
| R | 0.53632 |  |
| $\mathrm{R}=$ | 0.02564 |  |
|  | VMAX $=$ | 630 |
| ? | 0.18605 |  |
| R | 0.76512 |  |
|  | VMIN $=$ | 208 |
| $2=$ | 0.97630 |  |
| $\mathrm{R}=$ | 0.52133 |  |
| R | 0.04739 |  |
|  | VMAX = | 602 |
| $\mathrm{R}=$ | 0.12706 |  |
| R | 0.74112 |  |
|  | VMIN $=$ | 215 |
| R | 0.97674 |  |
| P. | 0.50388 |  |
| R | 0.01809 |  |
|  | VMAX $=$ | 571 |
| $\mathrm{R}=$ | 0.26124 |  |
| R | 0.86236 |  |
|  | VMIN = | 227 |
| $\mathrm{R}=$ | 0.87500 |  |
| R | 0.16279 |  |
|  | VMAX $=$ | 548 |
| $\mathrm{R}=$ | 0.07477 |  |
| R | 0.67913 |  |
|  | VMIN $=$ | 238 |
| $R=$ | 0.96774 |  |
| R | 0.41935 |  |
| Q | 0.00000 |  |
|  | VMAX $=$ | 518 |
| D | 0.00000 |  |
| $R=$ | 0.50714 |  |
| $R=$ | 1.00000 |  |
|  | VMIN = | 242 |

```
BL= I=242.0}782=0.0150
BL=79Z = 0.01264
BL = 802 = 0.00875
BL= 81L= = . C0729
BL=82Z=0.00402
BL=83Z=0.00019
            I =257.0
BL=.84Z=-0.00140
BL = 85Z = - 0.00574
            I =2\epsilon5.0
BL=86Z=-0.00768
BL=87L=-0.01109
BL=882=-0.01464
            I =272.0
BL=89Z = - 0.01603
BL=90Z=-0.02056
        I =280.0
BL=91L=-0.02267
BL=92L=-0.02622
BL=93Z=-0.02986
            I =296.0
BL = 942 = -0.03196
BL = 95Z = -0.03647
            I =293.0
BL = 96Z = -0.03844
BL}=972=-0.0437
            I =299.0
BL=98Z=-0.04556
BL=99Z=-0.05009
        I =306.0
BL=100Z=-0.05274
BL=101Z=-0.05685
BL= 102Z = -0.06000
        I = 312.0
    BL=102Z = - 0.060C0
    BL=103Z = - 0.06380
    BL=1042=-0.06750
```

COUNT = 8.0

```
COUNT = 8.0
    COUNT =9.0
    COUNT =9.0
COUNT = 10.0
COUNT = 10.0
    COUNT =11.0
    COUNT =11.0
COUNT = 12.0
COUNT = 12.0
    COUNT =13.0
    COUNT =13.0
COUNT = 14.0
COUNT = 14.0
    CCUNT =15.0
    CCUNT =15.0
COUNT }=16.
COUNT }=16.
    COUNT =17.0
    COUNT =17.0
COUNT = 18.0
COUNT = 18.0
COUNT =19.0
```

```
COUNT =19.0
```

```
242
\begin{tabular}{|c|c|c|c|c|}
\hline \(\mathrm{R}=\) & 1.00000 & & \(B L=1042=-0.06750\) & \\
\hline \(\mathrm{R}=\) & 0.52899 & & \(B L=1052=-0.07103\) & \\
\hline \(R=\) & 0.00000 & & \(B L=1062=-0.07500\) & \\
\hline & \(V \mathrm{MAX}=\) & 498 & \(I=324.0\) & COUNT \(=20.0\) \\
\hline \(R=\) & 0.00000 & & \(B L=1062=-0.07500\) & \\
\hline \(R=\) & 0.49219 & & \(B L=1072=-0.07869\) & \\
\hline \(R=\) & 1.00000 & & \(3 L=1082=-0.08250\) & \\
\hline & VMIN = & 268 & \(\mathrm{I}=329.0\) & COUNT \(=21.0\) \\
\hline 只 \(=\) & 1.00000 & & \(B L=108 Z=-0.08250\) & \\
\hline \(R=\) & 10.36087 & & \(3 \mathrm{~L}=1092=-0.08729\) & \\
\hline & VinAX \(=\) & 482 & \(1=335.0\) & COUNT \(=22.0\) \\
\hline \(p=\) & 0.02271 & & \(B L=1102=-C .09025\) & \\
\hline \(\mathrm{R}=\) & 0.71963 & & \(3 L=1112=-0.09540\) & \\
\hline & VMIN = & 268 & \(I=340.0\) & COUNT \(=23.0\) \\
\hline \(\mathrm{R}=\) & 0.93458 & & \(B L=1122=-0.09799\) & \\
\hline R \(=\) & 0.17290 & & \(B L=113 Z=-0.10370\) & \\
\hline & VMAX \(=\) & 457 & \(I=346.0\) & COUNT \(=24.0\) \\
\hline \(\mathrm{R}=\) & 0.22751 & & \(B L=1142=-0.10671\) & \\
\hline R \(=\) & 0.75238 & & \(B L=115 Z=-0.11214\) & \\
\hline & VMIN = & 272 & \(1=351.0\) & COUNT \(=25.0\) \\
\hline \(R=\) & 0.52703 & & \(B L=1162=-0.11530\) & \\
\hline \(F_{1}=\) & 0.00000 & & \(B L=1172=-C .12000\) & \\
\hline & VMAX \(=\) & 432 & \(I=356.0\) & COUNT \(=26.0\) \\
\hline \(R=\) & 0.00000 & & \(B L=1172=-C .12000\) & \\
\hline R \(=\) & 0.64375 & & \(B L=1182=-0.12483\) & \\
\hline & VMIN = & 275 & \(I=361.0\) & COUNT \(=27.0\) \\
\hline \(R=\) & 0.97452 & & \(B L=119 Z=-0.12769\) & \\
\hline \(\mathrm{R}=\) & 0.12739 & & \(B L=120 Z=-0.13404\) & \\
\hline & VMAX \(=\) & 408 & \(1=366.0\) & COUNT \(=28.0\) \\
\hline \(\mathrm{R}=\) & 0.31579 & & \(B L=1212=-0.13737\) & \\
\hline \(R=\) & 1.00000 & & \(B L=1222=-C .14250\) & \\
\hline & VMIN = & 273 & \(\mathrm{I}=371.0\) & COUNT \(=29.0\) \\
\hline \(R=\) & 1.00000 & & \(B L=122.2=-0.14250\) & \\
\hline \(R=\) & 0.34815 & & \(B L=123 L=-0.14739\) & \\
\hline & VMAX \(=\) & 410 & \(\mathrm{I}=376.0\) & COUNT \(=30.0\) \\
\hline \(R=\) & 0.13869 & & \(B L=1242=-0.15104\) & \\
\hline \(R=\) & 0.98540 & & \(B L=125 L=-0.15730\) & \\
\hline & VMIN = & 284 & \(\mathrm{I}=380.0\) & COUNT \(=31.9\) \\
\hline \(R=\) & 0.48413 & & \(B L=1262=-0.16137\) & \\
\hline & VMAX \(=\) & 395 & \(I=385.0\) & COUNT \(=32.0\) \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline \multirow[t]{3}{*}{R} & 0.00901 & \\
\hline & 0.87387 & \\
\hline & VMIN = & 288 \\
\hline \multirow[t]{2}{*}{\(R\)} & 0.62617 & \\
\hline & \(V M A X=\) & 388 \\
\hline \(R\) & 0.07000 & \\
\hline \multirow[t]{2}{*}{\(R\)} & 1.00000 & \\
\hline & VMIN = & 295 \\
\hline \(R=\) & 1.00000 & \\
\hline \multirow[t]{2}{*}{R} & 0.56989 & \\
\hline & \(V M A X=\) & 375 \\
\hline \(R=\) & 0.03750 & \\
\hline \multirow[t]{2}{*}{F.} & 1.00000 & \\
\hline & VMIN = & 288 \\
\hline \(R=\) & 1.00000 & \\
\hline \multirow[t]{2}{*}{R} & 0.24138 & \\
\hline & \(\mathrm{VAX}=\) & 362 \\
\hline \(p=\) & 0.21622 & \\
\hline \multirow[t]{2}{*}{P} & 1.00000 & \\
\hline & VMIN = & 277 \\
\hline \(R=\) & 1.00000 & \\
\hline \multirow[t]{2}{*}{R} & 0.2 .4706 & \\
\hline & \(V M A X=\) & 350 \\
\hline \multirow[t]{2}{*}{Q} & 0.45205 & \\
\hline & VMIN = & 277 \\
\hline \(\mathrm{R}=\) & 0.79452 & \\
\hline \multirow[t]{2}{*}{R} & 0.0 COOO & \\
\hline & VMAX \(=\) & 350 \\
\hline \(R=\) & 0.00000 & \\
\hline \multirow[t]{2}{*}{R} & 0.86301 & \\
\hline & VMIN = & 1.32 \\
\hline \(R=\) & 0.85780 & \\
\hline R & 9. 50459 & \\
\hline \multirow[t]{2}{*}{R} & 0.12844 & \\
\hline & VMAX \(=\) & 215 \\
\hline R = & 0.07229 & \\
\hline \(\mathrm{p}=\) & 0.91566 & \\
\hline
\end{tabular}
\(R=0.00901\)
\(R=0.62617\)
    VMAX =
    \(=0.07000\)
VMIN =
\(R=1.00000\)
VMAX
    \(=0.03750\)
    - . 0000
    \(=1.0 \mathrm{COOO}\)
    \(=0.24138\)
        \(\mathrm{V} \operatorname{MAX}=\)
\(R=0.21622\)
\(\mathrm{F}=1.00000\)
    \(\mathrm{VMIN}=\)
\(R=1.00000\)
    \(V M A X=\)
    VMIN =
    \(=0.79452\)
\(R=0.0 C 000\)
    VMAX =
\(R=0.00000\)
\(\mathrm{Q}=0.86301\)
    VMIN =
\(R=0.85780\)
\(R=0.50459\)
\(R=0.12844\)
    \(\mathrm{VMAX}=\)
    \(p=0.91566\)
\(B L=1272=-0.16507\)
\(B L=1282=-0.17155\) \(1=389.0\)
\(B L=1292=-0.17530\) I \(=393.0\)
\(B L=130 Z=-0.18052\)
\(B L=1312=-0.18750\) \(\mathrm{I}=398.0\)
\(B L=1312=-0.18750\)
\(B L=1322=-0.19073\) \(1=402.0\)
\(B L=1332=-0.19528\)
\(B L=1342=-0.20250\) \(\mathrm{I}=406.0\)
\(B L=1342=-0.20250\)
\(B L=135 Z=-0.20819\)
I \(=411.0\)
\(3 L=1362=-0.21162\)
\(B L=1372=-0.21750\) \(\mathrm{I}=415.0\)
\(B L=1372=-C .21750\)
\(B L=1382=-0.22315\) \(1=419.0\)
\(B L=1392=-0.22839\) \(1=423.0\)
\(B L=140 Z=-0.23404\)
\(B L=1412=-C .24000\) \(1=427.0\)
\(B L=1412=-0.24000\)
\(B L=142 Z=-0.24647\) I \(=437.0\)
\(B L=1432=-0.24857\)
\(B L=1442=-0.25122\)
\(B L=145 Z=-0.25404\) \(1=442.0\)
\(B L=1462=-0.25554\)
\(B L=1472=-0.26187\)
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\begin{tabular}{lr}
0.63828 & -0.13404 \\
0.64419 & -0.13737 \\
0.65010 & -0.14250 \\
0.65601 & -0.14739 \\
0.66192 & -0.15104 \\
0.66783 & -0.15739 \\
0.67374 & -0.16137 \\
0.67965 & -0.16507 \\
0.68556 & -0.17155 \\
0.69147 & -0.17530 \\
0.69738 & -0.18052 \\
0.70329 & -0.18750 \\
0.70920 & -0.19073 \\
0.71511 & -0.19528 \\
0.72102 & -0.20250 \\
0.72693 & -0.20819 \\
0.73284 & -0.21162 \\
0.73875 & -0.21750 \\
0.74466 & -0.22315 \\
0.75057 & -0.22839 \\
0.75648 & -0.23404 \\
0.76239 & -0.24000 \\
0.76830 & -0.24647 \\
0.77421 & -0.24857 \\
0.78012 & -0.25122 \\
0.78603 & -0.25404 \\
0.79194 & -0.25554 \\
0.79785 & -0.26187 \\
0.80376 & -0.26496 \\
0.80967 & -0.26805 \\
0.81558 & -0.27115 \\
0.82149 & 0.00000 \\
0.82740 & 0.00000 \\
0.83331 & 0.00000 \\
0.83922 & 0.00000 \\
0.84513 & 0.00000 \\
0.85104 & 0.00000 \\
0.85695 & 0.00000 \\
0.86286 & 0.00000 \\
0.86877 & \\
0.000
\end{tabular}
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161
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163

\(K A=6\)\(\quad 164\)
\[
0.00000
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0.00000
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\begin{tabular}{ll}
0.16548 & 0.07415 \\
0.17139 & 0.07368 \\
0.17730 & 0.07312 \\
0.18321 & 0.07225 \\
0.18912 & 0.07127 \\
0.19503 & 0.07025 \\
0.20094 & 0.06896 \\
0.20685 & 0.06778 \\
0.21276 & 0.06769 \\
0.21867 & 0.06733 \\
0.22458 & 0.06654 \\
0.23049 & 0.06547 \\
0.23640 & 0.06383 \\
0.24231 & 0.06166 \\
0.24822 & 0.06037 \\
0.25413 & 0.05970 \\
0.26004 & 0.05989 \\
0.26595 & 0.05815 \\
0.27186 & 0.05592 \\
0.27777 & 0.05382 \\
0.28368 & 0.05257 \\
0.28959 & 0.05147 \\
0.29550 & 0.04917 \\
0.30141 & 0.04656 \\
0.30732 & 0.04505 \\
0.31323 & 0.04437 \\
0.31914 & 0.04218 \\
0.32505 & 0.03910 \\
0.33096 & 0.03750 \\
0.33687 & 0.03609 \\
0.34278 & 0.03314 \\
0.34869 & 0.03050 \\
0.35460 & 0.02953 \\
0.36051 & 0.02683 \\
0.36642 & 0.02346 \\
0.37233 & 0.01964 \\
0.37824 & 0.01627 \\
0.38415 & \\
0.39006 & \\
0.39597 & 0.01264 \\
& \\
\hline
\end{tabular}
\begin{tabular}{llr}
80 & 0.40183 & 0.00875 \\
81 & 0.40779 & 0.00729 \\
82 & 0.41370 & 0.00402 \\
83 & 0.41961 & 0.00019 \\
84 & 0.42552 & -0.00140 \\
35 & 0.43143 & -0.00574 \\
86 & 0.43734 & -0.00768 \\
87 & 0.44325 & -0.01109 \\
88 & 0.44916 & -0.01464 \\
89 & 0.45507 & -0.01603 \\
90 & 0.46098 & -0.02056 \\
91 & 0.46689 & -0.02267 \\
92 & 0.47280 & -0.02622 \\
93 & 0.47871 & -0.02986 \\
34 & 0.48462 & -0.03196 \\
95 & 0.49053 & -0.03647 \\
96 & 0.49644 & -0.03844 \\
97 & 0.50235 & -0.04378 \\
38 & 0.50826 & -0.04556 \\
99 & 0.51417 & -0.05009 \\
100 & 0.52008 & -0.05274 \\
101 & 0.52599 & -0.05685 \\
102 & 0.53190 & -0.06000 \\
103 & 0.53781 & -0.06380 \\
104 & 0.54372 & -0.06750 \\
105 & 0.54963 & -0.07103 \\
106 & 0.55554 & -0.07500 \\
107 & 0.56145 & -0.07869 \\
108 & 0.56736 & -0.08250 \\
109 & 0.57327 & -0.08729 \\
110 & 0.57918 & -0.09025 \\
111 & 0.58509 & -0.09540 \\
112 & 0.59100 & -0.09799 \\
113 & 0.59691 & -0.10370 \\
114 & 0.60282 & -0.10671 \\
115 & 0.60873 & -0.11214 \\
116 & 0.61464 & -0.11530 \\
117 & 0.62055 & -0.12000 \\
118 & 0.63237 & -0.12483 \\
119 & & 0
\end{tabular}
\(F I N=150\)
POINTS NUMBER 1 \begin{tabular}{r}
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\end{tabular}
EX VALUE
-0.06501
-0.05910
-0.05319
-0.04728
-0.04137
-0.03546
-0.02955
-0.02364
-0.01773
-0.01182
-0.00591
0.00000
0.00591
0.01182
0.01773
0.02364
0.02955
0.03546
0.04137
0.04728
0.05319
0.05910
0.06501
0.07092
0.07683
0.08274
0.08865
0.09456
0.10047
0.10638
0.11229
0.11820
0.12411
0.13002
0.13593
0.14184
0.14775
0.15366
0.15957
ZALUE
0.00000
0.00000
0.00000
0.00000
0.00000
0.01765
0.02162
0.02559
0.03116
0.03661
0.03817
0.04194
0.04483
0.04524
0.04779
0.05092
0.05254
0.05433
0.05734
0.05949
0.06016
0.06177
0.06406
0.06601
0.06722
0.06718
0.06844
0.06956
0.07095
0.07217
0.07317
0.07397
0.07443
0.07478
0.07498
0.07509
0.07480
0.07467
0.07444
```

    INPUT CONSTANTS
    NUMBER DF LINES INN FRAME = 1
DELTA = 0.01500
POINT X DIRECTION INTERVAL = 0.00197
NOISE COMPENSATION REGION XLIMI $=0.55600$
XLIM2 $=0.72500$
XLIM3 $=0.12500$

|  | $\begin{aligned} & \text { TS }= \\ & \text { VMAX }= \end{aligned}$ | 282 |
| :---: | :---: | :---: |
|  | VIIIN = | 151 |
| F. $=$ | 0.58779 |  |
|  | VMAX = | 674 |
| $R=$ | 0.15488 |  |
| R | 0.88145 |  |
|  | VMIN = | 280 |
| $\mathrm{R}=$ | 0.91117 |  |
| $R=$ | 0.40863 |  |
| R | 0.02284 |  |
|  | VIIAX $=$ | 751 |
| $\mathrm{p}=$ | 0.03185 |  |
| R | 0.37155 |  |
|  | 0.78981 |  |
|  | VMIN = | 168 |
| $\mathrm{P}=$ | 0.99485 |  |
| R | 0.75643 |  |
| $R$ | 0.35 .506 |  |
|  | 0.06861 |  |
|  | VMAX = | 822 |
| $R=$ | 0.02141 |  |
| $R=$ | 0.23547 |  |
| P. | 3.54128 |  |
| \% | 0.80122 |  |
| $R$ | 0.96330 |  |
| ? | 1.00000 |  |
|  | 74 |  |
|  | 76 |  |
|  | 78 |  |

```
COUNT = -1.0
        COUNT = 0.0
COUNT = 1.0
    COUNT =2.0
COUNT = 3.0
    COUNT = 4.0
```

COUNT $=5.0$


[^0]:    The work described in this thesis was carried out by the author at the Advanced Research Laboratory, Rolls-Royce (1971) Limited, Derby, during the period October 1969 to September 1973. The slow scan television and digital tape recorder interface was designed and built by the Electronic Services Section of the Advanced Research Laboratory in co-operation with the author.

[^1]:    * Listed in alphabetical order.

[^2]:    

