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# Motion Compensation for Image Compression ( Pel Recursive Motion Estimation Algorithm) 

A doctoral thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

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## Abstract

## Pel-Recursive Motion Compensation Techniques for Video Compression

In motion pictures, there is a certain amount of redundancy between consecutive frames These redundancies can be exploited by using interframe prediction technıques To further enhance the efficiency of interframe prediction, motion estimation and compensation, various motion compensation techniques can be used There are two distinct techniques for motion estimation block matching and Pelrecursive Block matching has been widely used as it produces a better signal to noise ratio or a lower bit rate for transmission than the Pel-recursive method

In this thesis, various Pel-recursive motion estimation techniques such as steepest descent gradient algorthm have been considered and sımulated Netravali's algorithm was one of the early algorthms which was implemented and simulated to evaluate the performance of the Pel-Recursive technique compared with the Block Matching approach The performance of the gradient method was further enhanced by adaptively selecting the convergence factor (modified gradıent) A second algorithm was developed and simulated to produce further improvements

A hybrid system mcorporating both the block matching and the Pel-recursive approaches was developed and sımulated This combination exhibits even further ımprovement over existıng technıques.

These methods were then applied to various herarchical hybrid based video coding techniques such as the ITU-T H 263 standard The arm was to reduce the overall bit rate required to transmit video sıgnals

## ACKNOWLEDGEMENTS \& DEDICATION

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It is a pleasure to thank all members of staff in the Department of Electronic and Electrical Engineerng at the University of Loughborough for therr assistance with this research project.

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Finally, a special note of thanks goes to my friends and my colleagues, Dr. H Sar1, Dr R Naımı-mohasses, Dr. K. Shahhamzeı for all their valuable discussions, assistance, help and encouragement.

Above all I would like to thank my mother and father for their unconditional love and support throughout many years of life devotion and dedication. This work is a small token of my immense love and gratitude to them both, whom I dedicate this thesis to.

Special thanks are extended to Islam Rahematullah and Platinum Rahematullah for bemg a supportive friend for me in the course of my PhD.

## Acronyms and Abbreviations

| AC | Alternative Current. |
| :---: | :---: |
| ATD | Absolute Temporal Difference |
| BBMC | Block-Based Motion Compensation |
| BMMC | Block-Matching Motion Compensation |
| BT | Britısh Telecommunications |
| CCIR | International Radio Consultative Committee |
| CCITT | International Telegraph and Telecommunication Consultative Committee (see ITU) |
| CD-ROM | Compact Disc Read-Only Memory |
| CIF | Common Intermediate Format |
| Codec | Coder-decoder |
| CRT | Cathode Ray Tube |
| DAT | Digital Audıo Tape |
| DC | Direct Current |
| DCT | Discrete Cosine Transform |
| DPCM | Differential Pulse Code Modulation |
| HDTV | High Defintion Television |
| IS | International Standard |
| ISDN | Integrated Systems Digıtal Network |
| ISO | International Standardisation Organisation |
| ITU | International Telecommunication Union |
| ITU-T | International Telecommunication Union Telecommunication |
|  | Standardisation Sector |
| MC | Motion Compensation |


| MCP | Motıon Compensated Prediction |
| :--- | :--- |
| ME | Motıon Estimation |
| Modem | Modulator-demodulator |
| MPEG | Moving Pıcture coding Experts Group |
| NICAM | Near Instantaneous Companded Audıo Multıplex |
| NTSC | Natıonal Televısion System Committee |
| PAL | Phase Alternatıng Lıne |
| PSNR | Peak Signal to Noise Ratı. |
| QCIF | Quarter Common Intermedıate Format. |
| SUB-QCIF | Sub-Quarter Common Intermediate Format |
| MAD | Mean Absolute Difference. |
| MSD | Mean Squared Difference |
| PSNR | Peak Sıgnal to Noise Ratıo |
| PSTN | Publıc Switched Telephone Network |
| QCIF | Quarter Common Intermedıate Format |
| SAC | Syntax-based Arithmetic Coding |
| Sub-QCIF | Sub-Quarter Common Intermediate Format |
| VLC | Varıable Length Code or Variable Length Codıng |
| VLSI | Very Large Scale Integration |
| VOD | Video on Demand |
| x, y | spatıal co-ordınates in the pixel domain |

## AUTHOR'S PUBLICATON

[1] H Gharavı, and H Reza-Alıkhanı, " Pel-Recursıve Motion Estımation for video compression", IEE, Electronics Letters, Vol. 37, No. 21, pp 1285 - 1286, October 2001.

# A Pel-Recursive Motion Estimation Algorithm <br> H Gharavı and H Reza-Alıkhanı 


#### Abstract

This paper presents a new pel recursive motion estımation algorthm for video coding applications. The derivation of the algorithm is based on Recursive Least-Squares (RLS) estımation that mınımızes the mean square predıction error A companson with the modified Steepest-descent gradient estimation technique algorithm shows significant improvement in terms of mean-square predıction error performance.


Introduction: Netravalı and Robbins [1] developed a pel recursive spatio-temporal steepest-descent gradient technique in which the displacement of a pel (picture element) was predicted from previously transmitted information Since then vanous algorithms have been proposed to improve the performance of pel recursive motion estrmation (PRME) technıques The most important contribution was the modification of the steepest-descent algonthm developed by Walker and Rao [2] In this paper we present a simple but very efficient PRME algorithm that sıgnificantly outperforms the modified steepest-descent technique

Proposed Algorithm: For the sake of our analysis, we assume the translational movement of an object is in a plane parallel to the camera and illumination is uniform We also assume the effect of uncovered background to be negligible Under these assumptions, let $S(x, y, t)$ denote the monochrome intensities at point ( $x, y$ ) of a moving object in the image plane where its translational movement is at a constant velocity of $v_{x}$ and $v_{y}$ We can show that after $\Delta t$ second (one frame period), the object moves to a new location where we can show,

$$
\begin{equation*}
\mathrm{S}(\mathrm{x}, \mathrm{y} \mathrm{t}+\Delta \mathrm{t})=\mathrm{S}\left[\left(\mathrm{x}+V_{\mathrm{x}} \Delta \mathrm{t}\right),\left(\mathrm{y}+V_{\mathrm{y}} \Delta \mathrm{t}\right) \mathrm{t}\right] \tag{1}
\end{equation*}
$$

After expanding the field in a power series in $\Delta t$ and neglecting the higher order terms, the frame difference can be shown as,
$S(x, y: t+\Delta t)-S(x, y \cdot t)=\frac{\partial}{\partial x} S(x, y t) d x+\frac{\partial}{\partial y} S(x, y t) d y$
where $d_{x}$ and $d_{y}$ correspond to the horizontal and vertical components of the motion vector $D$ Assuming $\frac{\partial}{\partial x} S(x, y t)$ and $\frac{\partial}{\partial y} S(x, y t)$ are known for each $x, y, t$, and defining ED, LD, and FD as the magnitude of the element, line, and frame difference at point $n$, from (3), we can write,

$$
\begin{equation*}
\mathrm{FD}=\Phi_{\mathrm{n}}{ }^{T} \mathrm{D} \tag{3}
\end{equation*}
$$

where $\Phi n=\left[\begin{array}{l}\frac{\partial}{\partial x} S(x n, y n t) \\ \frac{\partial}{\partial y} S(x n, y n t)\end{array}\right]=\left[\begin{array}{l}E D \\ L D\end{array}\right]$

From (4) the frame difference (FD) measurement is,

$$
\begin{equation*}
\xi_{\mathrm{n}}=\Phi_{\mathrm{n}} \mathrm{~T} \overline{\mathrm{D}}+\text { noise } \tag{5}
\end{equation*}
$$

where $\overline{\mathrm{D}}=[\overline{\mathrm{d}}(\mathrm{x}), \overline{\mathrm{d}}(\mathrm{y})]^{\mathrm{T}}$ is the motion vector estimate

For a cluster of M moving pels, the least-squares estımate of D , after carrying out the mınımization, can be shown as,

$$
\begin{equation*}
\sum_{n=1}^{m} \Phi_{n} \xi_{n}=\bar{D} \sum_{n=1}^{m} \Phi_{n} \Phi_{n}^{T} \tag{6}
\end{equation*}
$$

For, $\eta=\frac{1}{M} \sum_{n=1}^{M} \Phi_{n} \xi_{n} \quad$ and $R=\frac{1}{M} \sum_{n=1}^{M} \Phi_{n} \Phi_{n}^{T}$
the estumated motion vector from (6) is obtained as,

$$
\begin{equation*}
\overline{\mathrm{D}}=\mathrm{R}^{-1} \eta \tag{8}
\end{equation*}
$$

For recursive estrmation of $\eta$ and $R$, we can write

$$
\begin{align*}
& \eta_{1}=\eta_{1-1}+\Phi_{n} \xi_{n} \\
& R_{1}=R_{1-1}+\Phi \Phi{ }_{n}^{T} \tag{9}
\end{align*}
$$

Based on the so-called matrix inversion lemma, the inverse of $R_{1}$ can be obtained

$$
\begin{equation*}
\mathrm{R}_{\mathrm{t}}^{-1}=\mathrm{R}_{\mathrm{t}-1}^{-1} \cdot \frac{\mathrm{R}_{\mathrm{l}}^{-1} \Phi \Phi{ }_{n}^{\mathrm{T}} \mathrm{R}_{\mathrm{l} \cdot-1}^{-1}}{1+\Phi_{n}^{\mathrm{T}} \mathrm{R}_{\mathrm{t}-1}^{-1} \Phi_{\mathrm{n}}} \tag{10}
\end{equation*}
$$

as,

From (8), (9), and (10),

$$
\begin{equation*}
\overline{\mathrm{D}}_{1}=\overline{\mathrm{D}}_{\mathrm{i}-1}-\frac{\mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{\mathrm{r}-1}^{-1} \Phi_{\mathrm{n}}}\left(\Phi^{\mathrm{T}} \overline{\mathrm{D}}_{1-1} \cdot \xi\right) \tag{11}
\end{equation*}
$$

In the above equation, the term in the right hand side bracket can be replaced by what is known as the Displaced Frame Difference, DFD Thus,

$$
\begin{equation*}
\bar{D}_{1}=\bar{D}_{1-1} \cdot \frac{\mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}}}\left[D F D\left(\mathrm{x}, \mathrm{y}, \bar{D}_{\mathrm{D}-1}\right)\right] \tag{12}
\end{equation*}
$$

To avoid matrix inversion at each iteration, (12) can be simplified by ignoring the $x$ and $y$ cross terms in calculating $\phi_{n}$ and $R$ Thus, from (4) and (7),

$$
\begin{align*}
& \Phi_{\mathrm{n}}(\mathrm{x})=E D \quad \text { and } \quad \Phi_{\mathrm{n}}(\mathrm{y})=L D  \tag{13}\\
& \mathrm{R}(\mathrm{x})=\frac{1}{\mathrm{M}} \sum_{\mathrm{m}} E D_{m}^{2} \text { and } \mathrm{R}(\mathrm{y})=\frac{1}{\mathrm{M}} \sum_{\mathrm{m}} L D_{m}^{2}
\end{align*}
$$

Applying (13) to (12), the components of the motion displacement estimates are,
$\overline{\mathrm{d}}_{\mathrm{l}}(\mathrm{x})=\overline{\mathrm{d}}_{\mathrm{l}-1}(\mathrm{x})-\frac{E D}{\frac{1}{\mathrm{M}} \sum E D^{2}+E D^{2}}\left\{D F D\left[\mathrm{x}, \mathrm{y}, \overline{\mathrm{d}}_{i-l}(\mathrm{x})\right\}\right.$
$\overline{\mathrm{d}}_{1}(\mathrm{y})=\overline{\mathrm{d}}_{\mathrm{l}-1}(\mathrm{y})-\frac{L D}{\frac{1}{\mathrm{M}} \sum L D^{2}+L D^{2}}\left\{D F D\left[\mathrm{x}, \mathrm{y}, \overline{\mathrm{d}}_{-l}(\mathrm{x})\right]\right\}$

Simulation Results: The computation involved in (14) is performed recursively. At each iteration the estımated motion displacement is applied to measure a new DFD This would first require obtannng the location of the displaced pel on the previous frame, based on the estimated components of motion displacement Since the motion estimates are expected to be non-integer, the luminance value of the displaced pel is predicted by a two dimensional interpolator which uses the four corners of the surrounding pels in a two dimensional grid. In our expenments, the DFD is measured at two locations with reference to the current pel, the pel above ( 1 e, previous line), and the previous pel along the same line The average of the two DFD's (with equal weightings) is then used to update the displacement estumates

The ED and LD in (14) were also measured using the interpolated luminance values from the displaced previous frame For $\Sigma E D^{2}$ and $\Sigma L D^{2}$ the summation includes the luminance values of five interpolated neighboring pels from the previous frame Two video sequences, known as "Salesman" and "Suzie," were used to evaluate the performance of the proposed algorthm The format of both sequences was based on the CIF (Common Intermediate Format 352 -pels by 288 -lines and 30 frames/s) In addition, for the sake of comparison, we have sımulated the Walker-Rao algorthm
[2] The simulation results of both schemes, in terms of mean square prediction error (in dB), are shown in Figures 1 and 2 for the "Salesman" and "Suzie" sequences, respectively In these figures, we have also included the results of interframe prediction without motion compensation ( 1 e , frame difference) The number of iterations for both schemes was 3 The above algorthm was applied to those pels whose frame difference exceeds a predefined threshold (ie $|\mathrm{FD}|>9$ ) In addition, these results were obtained using the second previous frame for prediction ( 1 e , skıpping one frame). Looking at these figures, it is clear that the proposed scheme significantly reduces the motion compensated prediction error. In terms of subjective comparısons, Figure 2 presents the motion compensated prediction error
mages between frames 49 and 51 of the "Suzie" sequence In these images, relatively darker or lighter patches represent the degree of maccuracies in estimating the components of the motion displacement Comparing the two images confirms the supenor performance of the proposed scheme over the modified steepest-descent algonthm, particularly in regions where the motion activities are relatively high

Conclusion: This paper proposes an efficient pel-recursive estimation technique for motion tracking and coding of moving images. The proposed algorithm has been compared with the modified steepest-descent gradient algorithm The results indicate a considerable reduction in the prediction error, particularly in regions where the motion activities are relatively high

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[1] A N. Netravalı and J D Robbins, "Motion compensated television coding- part I", Bell Syst Tech J, Vol 58, pp 631-670, Mar 1979
[2] D R Walker and K r Rao, "Improved Pel-recursive motion-estımation", IEEE Trans Commun, Vol COM-32, No 10, 1984, pp 1128-1134


Figure 1 Mean square error performance using the second previous frame for prediction
(a) Salesman sequence, (b) Suzie sequence

(a)

(b)

Figure 2: Motion compensated prediction error images for Suzie sequence
(a): Walker \& Rao scheme (b) Proposed scheme.

## Pel-recursive motion estimation algorithm

H Gharavi and H Reza-Alıkhanı

A new pel-recursive motion estimation algorithm for video coding applications is presented The derivation of the algonthm is based on recursive least-squares estimation that minimises the meansquare prediction error A comparison with the modified steepestdescent gradient estimation technique algonthm shows significant improvement in terms of mean-square prediction error performance

Introduction Netravalı and Robbins [1] developed a pel-recursive patio-temporal steepest-descent gradient technique in which the displacement of a pel (picture element) was predicted from previpusly transmitted information Since then various algorithms have peen proposed to improve the performance of pel-recursive notion estimation (PRME) techniques The most important conribution was the modification of the steepest-descent algorithm leveloped by Walker and Rao [2] In this Letter we present a simle but very efficient PRME algorithm that significantly outperorms the modfified steepest-descent techmique

Proposed algorthm For the sake of our analyss, we assume the anslational movement of an object is in a plane parallel to the amera and illumination is uniform We also assume the effect of ncovered background to be negligble Under these assumptions, et $S(x, y, t)$ denote the monochrome intensities at point $(x, y)$ of a hoving object in the image plane where its translational movehent is at a constant velocity of $v_{x}$ and $v_{y}$ We can show that after $t$ second (one frame period), the object moves to a new location here we can show

$$
\begin{equation*}
S(x, y \quad t+\Delta t)=S\left[\left(x+v_{x} \Delta t\right),\left(y+v_{y} \Delta t\right) \quad t\right] \tag{1}
\end{equation*}
$$

fter expanding the field in a power series in $\Delta t$ and neglecting the igher-order terms, the frame difference can be shown as

$$
\begin{equation*}
S(x, y t+\Delta t)-S(x, y t)=\frac{\partial}{\partial x} S(x, y t) d_{x}+\frac{\partial}{\partial y} S(r, y t) d_{y} \tag{2}
\end{equation*}
$$

here $d_{x}$ and $d_{y}$ correspond to the horizontal and veitical compoents of the motion vector $D$ Assuming $\partial / \partial x S(x, y t)$ and $\partial / \partial v$ $(x, y t)$ are known for each $x, y, t$, and defining $E D, L D$, and $D$ as the magnitude of the element, line, and frame difference at ont $n$, from eqn 3 , we can write

$$
\begin{equation*}
F D=\Phi_{n}^{T} D \tag{3}
\end{equation*}
$$

here

$$
\Phi_{n}=\left[\begin{array}{l}
\frac{\partial}{\partial x} S\left(x_{n}, y_{n}\right.  \tag{4}\\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial y} S\left(x_{n}, y_{n}\right. \\
t)
\end{array}\right]=\left[\begin{array}{l}
E D \\
L D
\end{array}\right]
$$

rom eqn 4 the frame difference ( $F D$ ) measurement is

$$
\begin{equation*}
\xi_{n}=\Phi_{n}^{T} \bar{D}+\text { nolse } \tag{5}
\end{equation*}
$$

here $\bar{D}=[d(x), d(y)]^{T}$ is the motion vector estimate
For a cluster of $M$ moving pels, the least-squares estumate of $D$,
ter carrying out the minumisation, can be shown as

$$
\begin{gather*}
\sum_{n=1}^{m} \Phi_{n} \xi_{n}=\tilde{D} \sum_{n=1}^{m} \Phi_{n} \Phi_{n}^{T}  \tag{6}\\
\eta=\frac{1}{M} \sum_{n=1}^{M} \Phi_{n} \xi_{n} \quad \text { and } \quad R=\frac{1}{M} \sum_{n=1}^{M} \Phi_{n} \Phi_{n}^{T} \tag{7}
\end{gather*}
$$

estimated motion vector from eqn 6 is obtained as

$$
\begin{equation*}
\bar{D}=R^{-1} \eta \tag{8}
\end{equation*}
$$

r recursive estimation of $\eta$ and $R$, we can write

$$
\begin{align*}
\eta_{2} & =\eta_{2-1}+\Phi_{n} \xi_{n} \\
R_{2} & =R_{2-1}+\Phi_{n} \Phi_{n}^{T} \tag{9}
\end{align*}
$$

Based on the so-called matrix inversion lemma, the inverse of $R_{t}$ 1 be obtamed as

$$
\begin{equation*}
R_{\imath}^{-1}=R_{\imath-1}^{-1}-\frac{R_{\imath-1}^{-1} \Phi_{n} \Phi_{n}^{T} R_{z-1}^{-1}}{1+\Phi_{n}^{T} R_{\imath-1}^{-1} \Phi_{n}} \tag{10}
\end{equation*}
$$

From eqns 8-10

$$
\begin{equation*}
\bar{D}_{\imath}=\bar{D}_{\imath-1}-\frac{R_{\imath-1}^{-1} \Phi_{n}}{1+\Phi_{n}^{T} R_{\imath-1}^{-1} \Phi_{n}}\left(\Phi^{T} \bar{D}_{\imath-1}-\xi\right) \tag{11}
\end{equation*}
$$

In the above equation, the term in the right-hand-side bracket can be replaced by what is known as the displaced frame difference (DFD) Thus,

$$
\begin{equation*}
\bar{D}_{\imath}=\bar{D}_{\imath-1}-\frac{R_{\imath-1}^{-1} \Phi_{n}}{1+\Phi_{n}^{T} R_{\imath-1}^{-1} \Phi_{n}}\left[D F D\left(x, y, \bar{D}_{\imath-1}\right)\right] \tag{12}
\end{equation*}
$$

To avord matrix inversion at each iteration, eqn 12 can be simplified by ignoring the $x$ and $y$ cross terms in calculating $\phi_{n}$ and $R$ Thus, from eqns 4 and 7,

$$
\begin{align*}
\Phi_{n}(x) & =E D \quad \text { and } \quad \Phi_{n}(y)=L D \\
R(x) & =\frac{1}{M} \sum_{m} E D_{m}^{2} \quad \text { and } \quad R(y)=\frac{1}{M} \sum_{m} L D_{m}^{2} \tag{13}
\end{align*}
$$

Applying eqn 13 to eqn 12 , the components of the motion displacement estımates are

$$
\begin{align*}
& \bar{d}_{2}(x)=\bar{d}_{1-1}(x)-\frac{E D}{\frac{1}{M} \sum E D^{2}+E D^{2}}\left\{D F D\left[x, y, \bar{d}_{2-1}(x)\right]\right\} \\
& \bar{d}_{2}(y)=\bar{d}_{1-1}(y)-\frac{L D}{\frac{1}{M} \sum L D^{2}+L D^{2}}\left\{D F D\left[x, y, \bar{d}_{\imath-1}(x)\right]\right\} \tag{14}
\end{align*}
$$

Simulation restlts The computation involved in eqn 14 is performed recursively At each iteration the estimated motion displacement is applied to measure a new DFD This would first require obtaining the location of the displaced pel on the previous frame, based on the estimated components of motion displacement Since the motion estimates are expected to be non-integer, the luminance value of the displaced pel is predicted by a twodimensional interpolator which uses the four corners of the surrounding pels in a two-dimensional grid In our experiments, the DFD is measured at two locations with reference to the current pel, the pel above ( 1 e previous line), and the previous pel along the same line The average of the two DFDs (with equal weightings) is then used to update the displacement estimates

The $E D$ and $L D$ in eqn 14 were also measured using the interpolated luminance values from the displaced previous frame For $\Sigma E D^{2}$ and $\Sigma L D^{2}$ the summation includes the luminance values of five interpolated neighbourng pels from the previous frame


Fig 1 Mean square error pelformance using second prevous frame for prediction
a Salesman sequence
$b$ Suzle sequence

-     - no motion compensation
-- Walker-Rao algorithm
-A- proposed algorthm
Two video sequences, known as 'Salesman' and 'Suze', were used to evaluate the performance of the proposed algorthm The format of both sequences was based on the common intermediate format (CIF) 352 pels by 288 lines and 30 frames/s) In addition, for the sake of comparison, we have smulated the Walker-Rao algorithm [2] The simulation results of both schemes, in terms of mean-square prediction error (in dB), are shown in Figs $1 a$ and $b$ for the 'Salesman' and 'Suzie' sequences, respectively We have
also included the results of interframe prediction without motion compensation (i e frame difference) The number of iterations for both schemes was three The above algorthm was applied to those pels the frame difference of which exceeds a predefined threshold ( $\mathrm{e}|F D|>9$ ) In addition, these results were obtaned using the second previous frame for prediction ( 1 e skipping one frame) It is cledr that the proposed scheme significantly ieduces the motion compensated prediction error In terms of subjective companisons, Fig 2 piesents the motion compensated prediction error mages between frames 49 and 51 of the 'Suzie' sequence In these images, relatively darker or lighter patches represent the degree of inaccuracies in estimating the components of the motion displacement Comparing the two mages confirms the superior performance of the proposed scheme over the modified steepest-descent algorithm, particularly in regions where the motion activities are relatively high


Fig 2 Motion compensated piechction erior mages for Suzie sequence
$a$ Walker-Rao algorithm
$b$ Proposed algorithm
Conchusion We have proposed an efficient pel-recursive estimation technique for motion tracking and coding of moving mages The proposed algorithm has been compared with the modified steep-est-descent gradient algonthm The iesults indicate a considerable reduction in the piediction error, particulaily in regions where the motion activitics are relatively high
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## Montgomery residue number systems

## B J Philhps

The Montgomery residue number system (MRNS) for long wordlength arthmetic is introduced MRNS, a modification of the residue number system (RNS), represents a long integer as a sct of smailer Montgomery residues Long integer addition, subtraction and multuptication can then be performed using hardware-eflicient Montgomery operations applied mdependently to each of the residues An MRNS hardware architeclure sultable for incorporation on a mucroprocessor data path is also proposed

Background Residue number systems (RNS) have long been considered an efficient means of performing long word-length addition, subtraction and multiplication [1] Recent efforts have succeeded in reducing the cost of RNS modular multiplication and
reggnted interest in RNS, especially for the implementation of public-key cryptosystems [2] The Montgomery residue number system (MRNS) described in this Letter is a modification to RNS that permits the use of hardware-efficient Montgomery multiplication and reduction [3]

Resulue number systems In RNS a number $X$ is represented by its residues modulo a set of co-prome moduli $\left\{m_{k-1}, \quad, m_{1}, m_{0}\right\}$ We write $X=\left(x_{k-1}| | x_{1} \mid x_{0}\right) R N S\left(m_{k-1}|\quad| m_{1} \mid m_{0}\right)$ whene $x_{s}=X \bmod m_{t}$ $=\langle\mathrm{X}\rangle_{m_{t}}$ The dynamic range of the RNS (the number of different values that can be represented) is given by $M=\Pi_{i=0}^{k-1} m_{1}$
Addution, subtiaction and multiplication can be performed within RNS by operating on each of the $k$ residues independently

$$
\begin{aligned}
X+Y= & \left(\left\langle x_{k-1}+y_{k-1}\right\rangle_{m_{k-1}} \mid\right. \\
& \left.\left.\left|\left\langle x_{1}+y_{1}\right\rangle_{m_{1}}\right|\left\langle r_{0}+y_{0}\right\rangle_{m_{0}}\right)_{R N S\left(m_{k-1} \mid\right.}\left|m_{1}\right| m_{0}\right) \\
X-Y= & \left(\left\langle x_{k-1}-y_{k-1}\right\rangle_{m_{k-1}} \mid\right. \\
& \left.\left.\left|\left\langle x_{1}-y_{1}\right\rangle_{m_{1}}\right|\left\langle x_{0}-y_{0}\right\rangle_{m_{0}}\right)_{R N S\left(m_{k-1} \mid\right.}\left|m_{1}\right| m_{0}\right) \\
X \times Y= & \left(\left\langle x_{k-1} \times y_{k-1}\right\rangle_{m_{k-1}} \mid\right. \\
& \left.\left|\left\langle x_{1} \times y_{1}\right\rangle_{m_{1}}\right|\left\langle x_{0} \times y_{0}\right\rangle_{m_{0}}\right)_{R N S\left(m_{k-1}\left|i m_{1}\right| m_{0}\right)}
\end{aligned}
$$

Montgomety restdues As discussed in subsequent Sections, Montgomery's reduction method [3] is an altenative to full modular reduction with advantages for hardware mplementations For now, let us concentrate on the mathematical formulation of Montgomery reduction and begin by defining a Montgomery residue $\bar{x}_{t}$ thus $\bar{x}_{t}=x_{i} l_{1} \bmod m_{1}$ Montgomery reduction is the function $M R_{m_{t} r_{t}}\left(x_{t}\right)=x_{t} r_{t}^{-1} \bmod m_{i}$ so that $M R_{m_{t} r_{i}}\left(\overline{x_{t}}\right)=x_{t} \bmod m_{t}$ The Montgomery residue $\overline{x_{i}}$ is unque for each residue $x_{i}$ provided $r_{1}>$ $m_{t}$ and $m_{t}$ and $r_{t}$ are co-pıme numbers [3] Therefore, for every repiesentation within a residue number system, there is an equivalent representation in the Montgomery residue number system thus

$$
\begin{aligned}
X & =\left(x_{k-1} \mid\right. & & \left.\left.\left|x_{1}\right| x_{0}\right)_{R N S\left(m_{k-1} \mid\right.}\left|m_{1}\right| m_{0}\right) \\
& =\left(\overline{x_{k-1}} \mid\right. & & \left.\left.\left|\overline{x_{1}}\right| \overline{x_{0}}\right)_{M R N S\left(m_{k-1} \mid\right.}\left|m_{1}\right| m_{0}\right)
\end{aligned}
$$

MRNS operations The Montgomery sum, difference and product functions can be defined as

$$
\begin{aligned}
& M S_{m_{2}, r_{2}}\left(\overline{x_{2}}, \overline{y_{2}}\right)=\overline{x_{2}}+\overline{y_{2}} \bmod m_{2} \\
& M D_{m_{1}, r_{1}}\left(\overline{x_{2}}, \overline{z_{2}}\right)=\overline{x_{2}}-\overline{y_{2}} \bmod m_{2} \\
& M P_{m_{1}, r_{2}}\left(\overline{x_{2}}, \overline{y_{2}}\right)=M R_{m_{2}, r_{1}}\left(\overline{x_{2}} \times \overline{y_{2}}\right)=x_{2} y_{2} r_{i} \bmod m_{i}
\end{aligned}
$$

Note that if $z_{t}=x_{1}+y_{2} \bmod m_{1}$ then $\overline{z_{t}}=M S_{m_{t} r}\left(\overline{x_{l}}, \overline{y_{i}}\right)$, if $z_{1}=$ $x_{t}-y_{1} \bmod m_{t}$ then $\overline{z_{1}}=M D_{m_{1} r_{1}}\left(\overline{x_{i}}, \overline{\jmath_{1}}\right)$, and if $z_{t}=x_{t} \times y_{t} \bmod$ $m_{t}$ then $\bar{z}_{t}=M P_{m_{t} r_{1}}\left(\overline{x_{t}}, \overline{y_{t}}\right)$ Also note that the Montgomery sum and difference functions are identical to full modular addition and subtraction but that the product function makes use of Montgomery reduction
Using these functions addition, subiraction and multiplication can be performed directly on numbers in MRNS representation

$$
\begin{aligned}
& X+Y=\left(M S_{m_{\lambda-1}, r_{k-1}}\left(\overline{x_{k-1}}, \overline{y_{k-1}}\right) \mid\right. \\
& \left.\left.\left|M S_{m_{1}, r_{1}}\left(\overline{x_{1}}, \overline{y_{1}}\right)\right| M S_{m_{0}, r_{0}}\left(\overline{x_{0}}, \overline{y_{0}}\right)\right)_{M R N S\left(m_{\lambda-1} \mid\right.}\left|m_{1}\right| m_{0}\right) \\
& X-Y=\left(M D_{m_{k-1}, r_{\lambda-1}}\left(\overline{x_{k-1}}, \overline{y_{k-1}}\right) \mid\right. \\
& \left.\left.\left|M D_{m_{1}, r_{1}}\left(\overline{x_{1}}, \overline{y_{1}}\right)\right| M D_{m_{0}, r_{0}}\left(\overline{x_{0}}, \overline{y_{0}}\right)\right)_{M R N S\left(m_{k-1} \mid\right.}\left|m_{1}\right| m_{0}\right) \\
& X \times Y=\left(M P_{m_{k-1}, r_{k-1}}\left(\overline{x_{k-1}}, \overline{y_{k-1}}\right) \mid\right. \\
& \left.\left.\left|M P_{m_{1}, r_{1}}\left(\overline{x_{1}}, \overline{y_{1}}\right)\right| M P_{m_{0}, r_{0}}\left(\overline{x_{0}}, \overline{y_{0}}\right)\right)_{M R N S\left(m_{k-1} \mid\right.}\left|m_{1}\right| m_{0}\right)
\end{aligned}
$$

Convertmg to and fiom MRNS Conversion between MRNS and RNS can be straightforwardly accomplished by converting each of the $k$ residues using $x_{t}=M R_{m_{1}, r_{t}}^{\prime}\left(\overline{x_{t}}\right)$ or $\widetilde{x}_{t}=M P_{m_{t} r_{t}}\left(x_{t}, r_{t}^{2} \bmod \right.$ $m_{1}$ ) Note that in the latter equation $r_{1}^{2}$ mod $m_{1}$ may be pie-computed

It is also possible to convert directly to MRNS using a sum of pre-computed residues approach If we take an $n$-bit number $X$ in a multu-precision form as $w$-bit words

$$
X=\sum_{j=0}^{n / v-1} X_{j} 2^{j \times w}
$$

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## Chapter 1

## Introduction

### 1.1 General Overview and Background

0f all the technological achievements in the 20th century, broadcast television has assumed a dominant role and has shown a great usage and effect in our everyday life to such an extent that today in the U.S. there are more homes that contan a television set than have telephone service

Television has perhaps had the greatest effect on our everyday lives. For many people, a television set is an obscure box in the corner of their living room - providing education, entertainment, and etc. Children are now said to be addicted to it and there is no doubt that the nature of leisure time activities has radically changed over the past thirty years to accommodate television. Telecommunications systems have also invaded our home People can now hold a telephone conversation as comfortably as they would do face to face Going
further it became possible to combine pictures and sound and transmit them via telephone lines for video conferencing.

The telecommunications system has been able to take advantage of new technology enabling modern digital network to become avalable to everyone Recent advances in mobile communications have shown high potential for telephones to be associated with individuals rather than their home and offices.

But the evolution in television and telecommunications systems has followed different paths Since the introduction of color television in the 1950's, there have been no significant changes to the mechanism of picture transmıssion and display. The difficulty in modifying the television signal that is broadcasted for local distribution is that the television receiver would almost certainly need to be modified or replaced. The difficulty of achieving this with an invested base of over $\$ 10$ billion is staggering

In this country, the 625 line format has been with us for a long time and for many people, their perception of improvements in the quality of television has been assisted by advances in associated audio reproduction, particularly since the advent of NICAM digital stereo The telecommunications system, on the other hand, has been able to take advantage of new technology to provide a modern, digital network, available to the users. So it is natural that in thinking of television transmıssion we immediately think of the sıgnal that is broadcast into the home. More efficient encoding of this sıgnal would free valuable spectrum space.

There is a large amount of point-to-point transmission of picture material taking place today in addition to UHF/VHF broadcastıng. For example, each of the four U.S television networks has a distribution system spanning the whole of the continental United States, international satellite links transmit live programs around the world. Video-conferencing services are receiving increasing attention. Satellites are transmitting to earth a continuous stream of weather photographs and earth-resource pictures, and there are a number of important military applications such as the control of remotely piloted vehicles and so on

Taking account of this background, it is perhaps surprising that the concept of combining pictures and sound into a single PSTN channel for video conferencing has taken so long to evolve. The essential difficulty is that bandwidth is limited in the services provided by telephone companies on the basis that to transmit speech, only 4 kHz is required for acceptable quality. Broadcast quality digital television, on the other hand, in comparison with a digitized speech signal at $64 \mathrm{~kb} / \mathrm{s}$, requires over $100 \mathrm{Mbits} / \mathrm{s}$ to supply pictures Even existing terrestrial channels allocated for television cannot accommodate this amount of data. Consequently, video compression and coding appear to be the best approach to the problem, until someone provides a mass communications system in which bandwidth is not a limitation Further more efficient coding of picture material for these applications provides the opportunity for significantly decreasing transmıssion costs, these costs can be quite large. The am of efficient coding is to reduce the required transmission rate of a given picture quality so as to yield a reduction in transmission costs.

Some early efforts in picture coding used analog coding techniques and attempted to reduce the required analog bandwidth, giving rise to the term "bandwidth compression". Complex manıpulations of the sıgnal are today much more easıly done by first sampling and digitizing the signal and then processing the signal in the digital domain rather than using analog techniques

Ideally, one would like to take advantage of any structure (both geometric and statistical) in a picture signal to increase the efficiency of the encoding operation Also the coding process should take into consideration the resolution (amplitude, spatial, and temporal) requirements of the receiver, $i$ e., the television display and very often the human viewer [1].

International co-operation has proved important in the development of video codec algorithms. Under the auspices of the CCITT, now known as the International Telecommunication Union (ITU), a recommendation was published in 1990, describing the framework of a video codec intended for use on the ISDN system on channels of 64 kb .ts $/ \mathrm{s}$ Its primary concern is the removal of redundancy, which occurs within and between picture
frames Intraframe coding can be used to compress a single frame and redundancy is sald to be present where the picture comprises of groups of adjacent equal value picture elements, or pixels. Similarly, where pixel values have not changed over time, interframe coding can remove temporal redundancy. Only changes in picture content need to be supplied to the decoder and, as a result, an efficient mechanism of picture coding is developed.

In most cases, however, video codecs are said to be lossy, since additional processing tends to lower the resolution and introduce errors This sald, provided certain requirements of quality are kept, most users are unable to detect coding errors and those who do will probably be able to tolerate them.

The implementation of video codecs has also been limited by the technology avarlable. Where real-tıme processing is required, compression and coding must be performed at high speed - a requirement that VLSI technology has recently appeared to be able to satısfy A new generation of software video codecs is being proposed in current ITU recommendations, to work on the growing number of personal computers connected to the PSTN by a modem As the processes are refined and the technology is improved, video conferencing codecs will become less expensive and more widely available. Whether they become more popular is, however, a different matter. It took many years for televisions and telephones to get into most homes and wariness about seemg the person the user is talking to may, for some while, make the videophone something the public feels it can do without

### 1.2 Aims, Motivations, Objectives and the Scope Of The Research

This thesis examines the current state of video technology and assesses different aspects of video compression. Further it goes into developing new ways of motion estımation The combined new proposed algorithm with block matching is to contribute a higher performance to the existing algonthm which in time could perhaps given rise to an alternative standard.

### 1.3 Structure and outline of this Thesis

Chapter two provides the reader with an insight into contemporary techniques of video compression. Although motion compensation is a very wide-ranging topic, chapter 2 concentrates on the principles of DPCM and block matching motion compensation This chapter also considers the ISO/MPEG standard and comes up to date with the latest H 263 recommendation for very low bitrate video codecs, using the framework of the H 263 algorthm.

Chapter three Analyses the state of the art techniques of another class of image compression known as pel-recursive motion compensation with focus on the pel-recursive Wiener-based displacement estimation algonthm.

Chapter four Investigates the novel techniques of displacement estimation algorithm in comparison to existing techniques.
Chapter five shows experimental results illustrating the performance of a few applications applying the proposed idea and method to some degree.
Chapter six examines the novel idea of combining the two different classes of mage compression, the block matching motion estimation and pel-recursive motion estımation, into a Hybrid system.
Chapter seven concludes the thesis with a summary and provides conclusions drawn from this work. Also suggestions for further work are made, particularly in the area of image compression, expressing the trade-off between quality and compression complexity which could outhne and open up further avenues of research

# Chapter 2 

## Review of contemporary techniques

### 2.1 Introduction

0ver fifty years have passed since the introduction of broadcast television in the United Kingdom. However, it is only recently that the concept of using moving pictures for interactive video and multimedia has received interest, as the costs of transmitting a television signal over anything other than short distances have proved prohibitive We have been limited to sending mainly stlll images over the public telephone network, mainly due to restriction in the bandwidth available to most users

It seems paradoxical that whilst the technology of digital television has advanced in remarkable leaps in recent years, we still have no efficient, widespread means of sending high quality video over the telephone network for the purposes of videotelephones One of the fundamental costs of colour television is the bandwidth required to transmit a channel of sound and pictures The five terrestrial channels allocated in the United Kingdom have equivalent digital bandwidths from 12 to 24Mbits/s, which would be insufficient to carry sound, chrominance (colour) and luminance (brightness) signals without any form of compression It is the scarcity of space in the radio-frequency spectrum that has limited the extent of broadcast television

In its uncompressed state, conventional broadcast-quality digital television requires bit rates of typically 166 megabits per second - well over that available for the $2 \mathrm{Mb} / \mathrm{s}$ link Integrated Services Digital Network (ISDN) [2] channel and it is not economical to use $1550 \mathrm{r} 622 \mathrm{Mb} / \mathrm{s}$ links. Given this primary constraint, contemporary research has focused on the compression of video images, allowing transmission of low resolution images over digital networks. In most cases, compression is easy to achieve, removing spatial and temporal redundancies naturally occurring in sequences of images.


Figure 2.1.1 A frame of Suzie, demonstrating picture redundancies.

Consider the image of figure 2.1.1. This could be regarded as typical of a videoconferencing scene, where during the conversation, most of the picture will not change other than, say, the lips, eyes and occasional hand or head movements. This feature can be used to good effect, such that only information about differences that have occurred will need to be sent to the recipient. This process is called interframe coding and is ideal for the low level of temporal changes, associated with videoconferencing. Interframe coding is based on the fact that there exists a large amount of frame-to-frame correlation in moving images, which is also called temporal correlation. It is also possible to extract information about differences between spatially adjacent pixels at a given instant in time. This process is called intraframe coding and efficiently compresses large areas of consistent colour and shade (the plain background
in this example). Boundanes are easy to detect, where significant changes in lummance and chrominance occur Interframe and intraframe coding are two methods of redundancy compression that have been used to good effect in the development of videoconferencing hardware for transmıssion over telecommunications channels

The intrinsic effect of redundancy coding is not to reduce picture quality significantly, or to affect spatial resolution However, subsequent processing of the difference information can take place, where useful information can be described as those aspects of the image that convey meaning to the human viewer, even if that is only a small proportion of the image content The contrast sensitivity function [3] allows understanding of the human ability to detect spatial and temporal detail Assuming the human eye can resolve down to two minutes of an arc, it can take in the equivalent of a million pixels of information without moving By moving the eye, but not the head, the field of view is at least an order of magnitude greater We know the head is likely to remain stationary whilst a person is doing something specific, but the eyes are moving continuously. If we assume that to represent the colour and luminance of a pixel, 12 bits are required, over 100 million bits of information are needed to represent the user's static scene

Consideration of these factors gives an understanding of the essential nature of video compression algonthms. It is necessary to take a picture, which under normal circumstances would require extensive data representation, and code it to the constrants of, telecommunications network, whilst mantaning an mage satısfactory to the human perception

At an early stage, the international telecommunications communtty identified the need for close collaboration to ensure the adoption of a system which could be applied in all countries and make videotelephony available to a world market. Even though a European standard specification did emerge in the 1980's [4], for a $2 \mathrm{Mbits} / \mathrm{s}, 625$ line, 25 frames per second PAL system, demand in North America required plans using the 525 line, 30 frames per second NTSC system Subsequently, the conversion between these standards was regarded as the focal point of international co-operation and under the auspices of the Organızation now known as the International Telecommunication

Union * (the ITU), a videophone algonthm was recommended, meeting the needs of the new ISDN systems and working for all bit rates between 64 kb ts/s and $2 \mathrm{Mbits} / \mathrm{s}$

The resulting ITU-T Recommendation, H 261 [5][6], forms the basis of the international development of videoconferencing systems using the new ISDN networks being installed throughout the world However, many concepts used are equally applicable to other areas of video codec design, such as high-definition dıgital television (HDTV), where an increased amount of picture data is to be transmitted within the constraints of existing terrestrial bandwidths

## * The International Telecommunication Union was formed from an amalgamation of the CCITT and the CCIR

### 2.2 Review of Image Compression

In image transmission and storage, dıgital techniques instead of analog are increasingly used due to the rapid growth in the use of digital computers, and the declining cost of digital processing and transmitting equipments This is also because the digital transmission and storage system has many inherent advantages over the analog system, such as easy processing, processing flexibility, easy and random access in storage, higher sıgnal-to-noise ratio (SNR), possibility of errorless transmıssion etc. However, images, whether digital or analog, contan a large amount of information and require wideband channels for transmission and big memory for storage, especially digital images. For example, a 4 MHz television signal sampled at Nyquist rate with 8 bit samples could require a transmitting bandwidth of 64 MHz . Therefore it is highly desirable to compress mage data for transmission and storage A lot of techniques for digital image compression [7] [8] have been developed.

The statistical properties of images are the main reasons that images can be compressed The statistical property upon which intraframe coding techniques rely is the high correlation between neighboring pixels This means that adjacent pixels are usually sımilar to one another and the magnitude of a pixel may be estimated from the values of the pixels around it Most mages, even farrly active mages which contan a large
amount of spatial detail, have quite high values of correlation For example, in moving images, the background is likely to remain stationary in successive frames The correlation in one frame of an image or successive frames are image data redundancies which can be reduced without apparent degradation of image quality

Image compression techniques can be classified into two classes, namely information lossless and information lossy techniques The former is able to reconstruct the original image without any loss of information, whereas the later introduces some distortion in the reconstructed image and cannot recover the original image exactly which can not be perceived by human eyes. Lossless and lossy compression techniques are used in different applications For example, medical images often require completely lossless compression because any slight distortion may result in wrong diagnosis In other applications, such as entertanment, education etc, the reconstructed ımages need not necessanly be exactly the same as the ongmal ones and lossy compression techniques are then widely used The lossless techniques normally reach lower compression ratio while the lossy techniques can reach higher compression ratio.

### 2.3 Transform Coding Technique

One of the most effective image compression techniques is transform coding The basic motivation and fundamental principle behind transform coding [9] [10] is to transform the image from the data domain to a frequency domain by an energy preserving unitary transform In the frequency domain, the image pixels are decorrelated and the energy is concentrated on a few coefficients so that the high frequency coefficients and the coefficients with less energy can be removed without any visual effect on the reconstructed image, since they play less important roles in the image reconstruction The transform could be applied to the entire image but implementation problems make this impractical First, the amount of the memory and the computation required increase proportionally to $\mathrm{M}^{2}$, where M is the image dimension. Second, because of the elımınation of unımportant coefficients, small transform size often leads to more sıgnificant degradation than a large size A typical approach is to divide the image into a number of rectangular blocks or sub-images, normally the input image is partitioned into NxN (eg $8 \times 8$ or $16 \times 16$ ) blocks (sub-images), and then an unitary transform is appled to each sub-image A block size 8 by 8 has been adopted for most video coding
standards mainly to reduce the transformation complexity as well as better exploitation of image redundancies between the neıghboring blocks

After the transformation, actual image compression is achieved by quantzing the transform coefficients. If all the coefficients are quantized and coded, the compression ratio is quite small. It has been pointed out that the important characterstic of the transform is that most of the energy of the image is packed into a small number of low frequency coefficients and the coefficients with less energy or the high frequency coefficients play less important roles in the image reconstruction To achieve hagher compression, one possibility is to use a mask covering an area of low frequency coefficients and to discard the remaining coefficients, 1 e . set the remaining coefficients to zero Only those coefficients in the mask are quantized and coded Considerable compression can be achieved depending on the size of the mask used in this method This technique is known as zonal coding The only problem with the zonal coding approach is a blurring effect as a result of the elimınation of higher frequency components Another possibility is to use a threshold on each transform coefficient and set the coefficients which are below the threshold value to zero The remaining non-zero coefficients together with their address information are quantized and finally entropy coded efficiently by coding schemes such as, Huffrnan coding [11], anthmetic coding [12] or combining Variable Length Coding (VLC) and runlength coding For better subjective image quality, the quantizer in all cases should be designed to optımize the reconstructed image quality for a given number of bits

The encoded image is transmitted through the channel (or stored) An inverse operation is performed at the receiver end. A number of orthogonal transforms can be used in the transform coding and most of them are linear transformations

Transform coding has a good immunity to channel noise. In transform coding, a code error in transmission only influences the corresponding block and has no effect on the succeeding blocks because this error is distributed by the reverse transform over the entire block. Visually, a code error in the transform coding is less visible than that in predictive coding However, the transform coding has some defects First, since the mage is divided into blocks, block to block correlation is not employed in the
transform Furthermore, artificial blocking segments the image arbitranly without considering its contents Second, in transform coding at low bit rate, sometimes so called blocking effects are apparent in the reconstructed mage Blocking effects are perceived in the reconstructed image as visible discontinuities between adjacent blocks This is especially visible around the boundaries of moving objects and, still background This is caused by the improper coding of the transform coefficients, such as elımınating too many coefficients or due to coarse quantization Finally, transform coding needs more operations and memory than predictive coding This is improved due to the rapidly decreasing cost of digital hardware and computer memory, and this may no longer be a disadvantage

### 2.3.1 The Karhunen-Loeve Transform (KLT) Technique

The Karhunen-Loeve transform [10] is an optımal linear transformation in the sense that it completely decorrelates the data and maximızes the amount of energy compacted into the lowest order coefficients However, it is not certan that the KLT is the absolute optımum transform since it does not consider other factors such as the human visual system Additionally, the transform matrix depends on the image data, 1 e the transform matrix is different for different mage data Thus, the KLT transform matrices are also transmitted and stored along with the coded data. Furthermore, the amount of computation in the transform matrix generation is very large and the KLT has no fast transformation algorthm associated with it.

Because of the computation complexity, the large storage requirement and dependence on the input images, the KLT is seldom used in practice but it is employed in theoretical studies of image coding It gives an indication about the upper bound computational efficiency of what other transformations should attempt to reach for decorrelating data samples.

### 2.3.2 The Discrete Fourier Transform (DFT) Technique

The discrete Fourier transform [10] is naturally applied to image coding because of its widespread use in other signal processing fields and the fact that it has efficient computational algonthms and fast implementation It is the only complex transform used in data coding schemes The DFT is not convenient for general use due to the
necessity to evaluate both real and imaginary components, which require a large number of operations and large storage

### 2.3.3 The Walsh-Hadamard Transform (WHT) Technique

The Walsh-Hadamard transform [10][13] is the simplest transform among vanous types of orthogonal transforms The elements in the transform matrix are etther 1 and -1 , and the only multuplication needed is that of the final scaling operation However, it is too simple to compact energy well

### 2.3.4 The Discrete Cosine Transform (DCT) Technique

The Discrete Cosine Transform (DCT), which is an information lossless technique [10][14][15] was proposed by Ahmed et al 1974 It is one of an extensive family of sinusoidal transforms At that time, there was increasing interests in the class of orthogonal transforms, such as the discrete Fourier transform, the Hadamard transform, in the general area of digital signal processing, such as image coding, pattern recognition etc. It is known that the KLT is the optimal transform with respect to performance measure, but it needs a large amount of computation and has no fast transform Compared with other orthogonal transforms, the DCT has the best all-around performance with respect to efficient computation and acceptable perceptual quality for a given compression rate. It also has correlation reduction capability, good energy compaction and fast computational properties [16] It is a widely used transformation for image compression for example in JPEG still-mage compression standard Therefore, researchers tried to develop a transform which is close to the performance of the KLT and has fast algorthms To fill the role, the discrete cosine transform was proposed

The two-dimensional discrete cosine transform of a data sequence $X(x, y)$ is defined as

$$
F(u, v)=\frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} C(u) C(v) X(x, y) \cos \left[\frac{(2 x+1) u \pi}{2 N}\right] \cos \left[\frac{(2 y+1) v \pi}{2 N}\right]
$$

Where $x, y=0,1, \ldots \ldots, \mathrm{~N}-1$

The inverse two-dimensional discrete cosine transform is defined as.
$X(x, y)=\frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} C(u) C(v) F(u, v) \cos \left[\frac{(2 x+1) u \pi}{2 N}\right] \cos \left[\frac{(2 y+1) v \pi}{2 N}\right]$

Where $u, v=0,1, \quad ., \mathrm{N}-1$

It has been shown that the performance of the DCT is nearly identical to the KLT transform for blocks of reasonably large size [17] Furthermore, the empincal evidence shows that even for blocks of small size the performances of the DCT and the KLT are close It also has correlation reduction capability, minimum block distortion, superb high energy compaction and fast computational properties [18] [19] DCT is widely used transformation for mage compression for example in JPEG still-image compression standard

Since the computation effort for DCT is quite large for time critical applications, fast versions of the DCT[20] [21] have been proposed Though speed performance is improved by the fast algonthms, the fast algonthms still require a large amount of computation DCT can be hardware implemented by digital signal processor to achieve high speed at reasonable cost

### 2.3.5 Hybrid (Transform / DPCM) Coding system

Hybrid coding [23] - [25] is a kınd of technıque which combines transform codıng and predictive coding together to generate a better coding scheme It takes the advantages of transform coding and predictive coding and overcomes their short comings to a certan degree Generally, hybrid codıng performance lies between transform coding and predictive coding This technique removes the spatial redundancies, which normally exist between the neighbouring pixels within a two dimensional mage array Hybrid coding is less sensitive to channel errors than predictive coding

Typically, in hybrid coding, a two-dimensional image is unitarly transformed to obtain a sequence of one dimensional sequences Each of these sequences is then coded independently by a one dimensional predictive coding technıque, such as the DPCM

### 2.3.6 Differential Pulse Code Modulation (DPCM)

In PCM time discrete, amplitude discrete, representation of the sample is made without removing much statistical or perceptual redundancy from the signal The time discreteness is provided by sampling the signal generally at the Nyquist rate, amplitude discreteness is provided by using a sufficient number of quantization levels so that the degradation due to quantization is not easily visible In DPCM, the sample to be encoded is predicted from the encoded values of the previously transmitted samples and only the prediction error is quantized for transmission Such an approach can be made adaptive etther by changing the prediction or quantization or by not transmitting the prediction error whenever it is below a certain threshold, as in conditional replenishment.

In basic predıctive coding systems [26]-[28] (Fig. 23.6 1) in their simplest form, DPCM uses the coded value of the previously coded horizontal information (pel) that has been transmitted as the prediction However, more sophisticated predictors, use the previous line (two-dımensional Predictor) as well as previous frame of information (interframe predictor) The error resulting from the subtraction of the prediction from the actual value of the sample is quantised into a set of discrete amplitude levels. These levels are represented as binary words of etther fixed or variable length and sent to the channel
coder for transmission The predictive coder has three basic components. 1) predictor, 2) quantizer, 3) entropy coding


Figure 2.3.6.1 Block dagram of a DPCM transmitter and receiver.

1-Predıctors for DPCM coding can be classified as linear or nonlinear depending upon whether the prediction is a linear or a nonlinear function of the previously transmitted sample values. A further division can be made depending upon the location of the previous elements used: one-dimensional predictors use previous elements in the same Inne, two-dimensional predictors use elements in the previous lines as well, whereas interframe predictors use picture elements also from the previously transmitted frames Predıctors can be fixed or adaptıve Adaptıve predıctors change their characterıstics as a function of the data, whereas fixed predictors maintain the same characteristics independent of the data. As an example of adaptive predıctıon, see Habıbı [29] for predictors which use different numbers of picture elements within a frame.

The set of predictors from which a predictor is selected are usually linear and are chosen such that each one of them will give a small prediction error if the signal was correlated in a certain manner. In Graham's predictor [30]-[32], either the previous line or the previous element is used for prediction, and the switching is done by the surrounding line and element differences. Several extensions have been made to this basic
philosophy However, the results have not been very encouraging in terms of the mean square error or the entropy of the prediction error In some cases the rendering of certain types of edges can be remarkably improved by these adaptive predictors Another vanation [33] in adaptıve prediction is to use a weighted sum of several predictors, where the weights are switched from element to element and are chosen by observing certann characteristics of already transmitted neighboring pels The same calculation can be performed at the receiver and, therefore, the predictor switching information does not need to be transmitted Such technıques have been considered for gray level signals [34].

The more successful adaptive predictors for frame-to-frame coding are the ones that take into account the motion of objects These are based on the notion that, if there are objects moving in the field of view of a television camera and if an estumate of their translation is avarlable, then more efficient predictive coding can be performed by taking the differences of elements with respect to elements in the previous frame that are appropriately spatally translated. Such prediction has been called motion compensated prediction [35] [36] Its success obviously depends upon the amount of translational motion of objects in real television scenes and the ability of an algonthm to estumate translation with the accuracy that is desirable for good prediction One set of technıques developed [37] [38] obtain an estımate of translation in a block of pels, whereas techniques developed by Netravali et al [39]-[41], recursively adjust the translational estimate at every pel or at every small block of pels. Another approach to motion compensation is adaptive linear prediction by using elements in both the present and the previous frame (or field), which surround the element being encoded, and adapting the coefficients to minmmize an intensity error function [42]. Such an approach is implementationally difficult and requires transmission of coefficients of the predictors

In scenes with higher detail and motion, field difference prediction does better than frame difference prediction [43] As the motion in the scene is increased further, intrafield predictors do better [44] This is largely because for higher motion, there is less correlation between the present pel and either the previous field or the frame pels

For the same reason, predictions such as element or line difference of frame or field differences perform better than frame or field difference for higher motion

2- Quantization DPCM schemes achieve compression, to a large extent, by not quantizing the prediction error as finely as the original signal itself. Several methods of optımızing quantizers have been studied Most of the work on systematic procedures for quantizer optimization were taken from studies of DPCM coding, in which the approximate horizontal slope of the input signal is quantized Three types of degradations can be seen due to the improper design of the quantizer of a DPCM coder These are referred to as granular noise, edge busyness and slope overload as shown in Fig 2362 If the inner levels (for small magnitudes of differential signal) of the quantizer are too coarse, then the flat areas are coarsely quantized and have the appearance of random noise added to the picture On the other hand, if the dynamic range ( 1 e , largest representative level) of the quantizer is small, then for every high contrast edge it takes several samples for the output to follow the input, resulting in slope overload, which appears similar to low-pass filtering of the image For edges whose contrast changes somewhat gradually, the quantizer output oscillates around the signal value and may change from line to line, or frame to frame, giving the appearance of a busy edge Quantizers can be designed purely on a statistical basis or by using certann psychovisual measures


Figure 2.3.6.2 An intuitive classıfication distortion due to DPCM coding.
(Adapted from digital picture)

It had been realized for some time that for a better picture quality, quantizers should be designed on the basis of psycho-visual criteria. However, the debate [45] [46] continues on what is a good criterion to use, and expectedly so, considering the complexities of the human visual system
3.- Entropy coding is the last stage in which shorter code word are assigned to the more frequent occurring symbols, therefore mınimizing the average length of the binary representation of the input information [47] The average information rate is given by entropy (measured in bits) .-
$H(S)=-\sum_{t=1}^{N} p\left(s_{t}\right) \log _{2} p\left(s_{t}\right)$

Where there are N input symbols $s_{1}, s_{2}, s_{3}, \quad, s_{N}$ with probabilities $p\left(s_{1}\right), p\left(s_{2}\right)$,

$$
p\left(s_{3}\right), \quad, p\left(s_{N}\right)
$$

And the average codeword length which is the average number of bits required is given by -
$R(S)=\sum_{i=1}^{N} l_{i} p\left(s_{i}\right)$

Where $l_{1}, l_{2}, l_{3}, \quad, l_{N}$ are the word length for the code words

Run Length coding (RLC) was first considered for black and white images The run length is found by counting the number of consecutive black and white pixels along each line, as an example for a horizontal line along an image as illustrated in figure
2.363 is 7 black-run, 3 white-run, 4 black-run, 4 white-run Where 0 and 1 represents black and white pixels respectively

## 000000011100001111

Figure 2.3.6.3 An example of runtength coding.

This runlength coding method has been further developed as two-dimensional Variable Length Coding (2D VLC) [48] so that colour images can be encoded The image is encoded as an EVENT. Each EVENT contans RUN and LEVEL.

EVENT = (RUN, LEVEL)
Where RUN is the number of successive zeros preceding the quantised coefficient LEVEL is the non zero value for the quantised coefficient

Finally, 3D VLC [48] is developed to improve the coding efficiency. In this approach, each EVENT contans LAST, RUN, LEVEL The LAST event is represented by the

End of Block (EOB) which indicates that no more zero coefficients are encoded for this block

EVENT = (LAST, RUN, LEVEL $)$

Where LAST $=0 \quad$ there are more non zero coefficients in this block.
LAST $=1 \quad$ this is the last non zero coefficient in this block
RUN is the number of successive zeros preceding the quantised coefficient

LEVEL is the non zero value of the quantised coefficient

The limitation of this method is the complexity of constructing the codebook However, it is very efficient in terms of coding and has been adopted as part of the ITU-T H. 263 Coding Standard [49].

Another problem with the use of variable length codes is that the output rate from the source coder changes with local picture content In order to send such a sıgnal over a constant bit rate channel, the source coder output has to be held temporanly in a buffer which can accept inputs at a non unform rate and can be read out to the channel at a uniform rate

### 2.4 Block Matching

In block matching motion estimation the coding (current) frame is partitioned into small non-overlapping blocks of size $\mathrm{m} \times \mathrm{n}$ (where often $\mathrm{m}=\mathrm{n}$ ), assuming that all the pixels within each of the non-overlapped block have the same displacement vector It is assumed that the motion is purely translational The motion vector for each block is estımated by searching through a larger block (search window of sıze $m+2 u \times n+2 v$ ), centered at the same location on the previous frame, for the best matching block (figure 24 1). For the minımum error, set by a criteria, the motion vector is therefore taken from this location


Previous frame
Figure 2.4.1 Block Matching search window.

The matching of the blocks can be quantıfied according to various criteria including Sum Absolute Difference (SAD), Sum Squared Difference (SSD), and Pel Difference Classification (PDC), etc

These criteria are outlined as followed--
Sum Absolute Difference (SAD)
$S A D(x, y)=\sum_{x=0}^{m-1} \sum_{y=0}^{n-1}|s(l, J, k)-s(l-x, j-y, k-1)|$

Sum Squared Difference (SSD)
$S S D(x, y)=\sum_{x=0}^{m-1} \sum_{y=0}^{n-1}\left[s(l, J, k)-s(l-x, J-y, k-1]^{2}\right.$

## Pel Difference Classification (PDC)

In the Pel Difference Classification method [50], each pixel in the block is classified as A matching or mismatching pixel

$$
\begin{align*}
& \mathrm{T}(\mathrm{i}, \mathrm{j}, \mathrm{x}, \mathrm{y})=1,  \tag{Eqn.2.4.3}\\
& \\
& \text { if } \mid s(l, J, k)-s(l-x, J-y, k-1 \mid \leq t \\
& \\
& \text { otherwise }
\end{align*}
$$

## Where $t$ is a selected threshold

$\mathrm{T}(\mathrm{i}, \mathrm{j}, \mathrm{x}, \mathrm{y})$ is the binary representation of pixel difference and its value of etther one or zero corresponds to a matching or mismatching pixel, respectively

The numbers of matching pixels are given by $G(x, y)$, which can be defined as follows.

$$
\begin{equation*}
G(x, y)=\sum_{x=0}^{m-1} \sum_{y=0}^{n-1} T(t, j, x, y) \tag{Eqn244}
\end{equation*}
$$

Where $G(x, y)$ is the number of matching pixels that exist between the current block and the block on the previous reference frame that was shifted by $\imath$ pixels and $J$ lines The largest value of $G(x, y)$ is found by searching through a search window. This gives the best match Thus
$G m\left(d_{x}, d_{y}\right)=\max [G(x, y)]$

Where $i, j$ are the spatial coordmates,
$x, y$ are the motion vector spatial coordinates,
$d_{x}, d_{y}$ are the components of the best estimated displacement vector,
$k \quad$ is the time reference for the current frame,
$k-1 \quad$ is the time reference for the previous frames,
$(1, j, k)$ is the intensity of the current frame,
$s(l, j, k-l)$ is the intensity of the previous frame.

The performance of PDC from prediction matching point of view is better than the other methods $1 \mathrm{e} . \mathrm{SAD}$, SSD . Etc In this method, the matching process is reduced to a binary level which consequently simplifies the computational complexity, as described by Gharavi [50] in 1990. However, the SAD method has been adopted as an international standard because of its simplicity

### 2.4.1 Half pixel interpolation

A half pixel searching window is created by a bilinear interpolation technique [51][52] (Figure 2.4.1.1). Matching is now done by first using the integer searching window and then using half pixel searching to find the best block match. This method has the advantage of producing more accurate prediction than the integer pixel block matching method However, this method requires extra computational complexity to create the half pixel searching window. Therefore, for each of the reference blocks the search begins with an integer pixel block first. Then the motion vector for the best match is used to carry out further half pixel searching. This searching will carry on until the best block match is found

| $\mathrm{X}_{\mathrm{a}}$ | $\mathrm{O}_{\mathrm{b}}$ | $\mathrm{X}^{\mathrm{B}}$ | X |
| :---: | :---: | :--- | :--- |
|  |  |  | Integer pixel position |
|  |  |  |  |
| $\mathrm{O}_{\mathrm{c}}$ | $\mathrm{O}_{\mathrm{d}}$ |  | Half pixel position |
|  |  |  | $\mathrm{a}=\mathrm{A}$ |
| C |  | $\mathrm{D}=(\mathrm{A}+\mathrm{B}) / 2$ |  |
| X |  | D | $\mathrm{c}=(\mathrm{A}+\mathrm{C}) / 2$ |
|  |  | $\mathrm{~d}=(\mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{D}) / 4$ |  |

Figure2.4.1.1 Half pixel prediction

Anyhow, it was based on the previous models that all the current existing international standards (1 e. H 261, H 263, MPEG-1, MPEG-2, etc) for video compression were built up

Graphical representation for block matching with half pel accuracy shows on average less than 005 dB improvement over traditional block matching without half pel accuracy. To justify the argument two well known sequences of "Suzie" and "Salesman" have been employed and graphs of the Average Mean Square of prediction Error have been plotted for sequences consisting of 20 frames (see Figures 2.4.1.2 and 24.1.3). The graphs have also been produced for different frame skips, as it is
frequently used in different application. e g. video conferencing and so on In general the graphs shows that using block matching with half pel accuracy contribute very little improvement over block matching without half pel accuracy consideration But it still widely used, for example as an optional feature in H 263

a) No frame skıp comparison

b) One frame skıp comparison

c) Two frame skip comparison

Figure 2.4.1.2 Suzie comparison with previous frame reconstructed.

a) No frame skip comparison

b) One frame skip comparison

c) Two frame skıp companson.

Figure 2.4.1.3 Salesman comparison with previous frame reconstructed

### 2.5 H. 263

### 2.5.1 Introduction

An outline block diagram of the H. 263 codec, the videoconferencing codıng standard is given in Figure 2.5.1.1.


Figure2.5.1.1 H. 263 block diagram of the viedeo codec

The H. 263 algortthm [53][54], (which evolved from H.261[3]) is broadly based on its predecessor, recommendation H.261. However, there are some changes in the basic implementation and optional processes are avalable to improve the interframe prediction (Fıgure 2.5.1.2).


T Transform coding
Q Quantizer
P Picture Memory with motion compensated variable delay

## CC Coding control

p Flag for INTRA/INTER
t Flag for transmitted or not
qz Quantizer indicator
q Quantizing index for transform coefficients
v Motion vectors
Figure 2.5.1.2 H 263 Draft recommendation encoder block diagram

The H261 was onginally devised to standardise the transmission of audio visual services and in particular the transmission of videophone and videoconference data The whole idea is that the H 261 has a fixed bit rate of $\mathrm{P} \times 64 \mathrm{kbits} / \mathrm{s}$ (where $\mathrm{P}=1 \quad 30$ ) whereas the H 263 which has a capability of achieving lower varrable bit rate and is targeted for extensive deployment of any future video services The entire issue of this recommendation centers on bandwidth compression of the video sıgnals The reason is that the video signal has a bandwidth of 3 Mhz (using regular television signal) as compared to 34 khz of voice bandwidth This is a ratio of $1: 1265$ Therefore in order to transmit the video signal through telephone line, its bandwidth has to be grossly reduced

The H 261 operates on pıctures based on a Common Intermediate Format (CIF) which has been derived from 525 and 625 line television standards It uses a hybrid of Discrete Cosıne Transform (DCT) and Differentral Pulse Coded Modulation (DPCM) and can acheeve transmission rates between 16 kbps and 2 Mbps .

The H 263 also uses a hybrid of DCT and DPCM but has an improved performance when compared with H 261 One of the man reasons for this is that half pixel precision is used for motion compensation whereas full pixel precision is used in H 261 . However the H 261 algonthm incorporates a spatial low-pass filter in the encoder feedback loop, which has been omitted from H 263 It has been shown that the pixel interpolation function involved in the half-pixel motion compensation process has the effect of lowpass filtering, without the need for a specific spatial function to remove high frequency noise caused by the quantisation of transform coefficients and also evident at the boundares of blocks in the motion compensation process. The recommendation can also be applied to a wider range of picture formats and allows variable bit rates to be used therefore increasing the possible uses for the package, for example it further supports QCIF, sub-QCIF, 4CIF, 16CIF ( Table 2511 ) resolutions which are more appropriate to the low bit rate environment

| Picture <br> Format | number of pixels <br> for luminance (x) | number of pixels <br> for luminance (y) | number of pixels <br> for chrominance (x) | number of pixels <br> for chrominance (y) |
| :--- | :---: | :---: | :---: | :---: |
| sub-QCEF | 128 | 96 | 64 | 48 |
| QCIEF | 176 | 144 | 88 | 72 |
| CIEF | 352 | 288 | 176 | 144 |
| 4 CIIF | 704 | 576 | 352 | 288 |
| 16CIIF | 1408 | 1152 | 704 | 576 |

Table 2.5.1.1 ITU-T H 263 picture formats

The compressed ITU-T H. 263 video bit stream contans four layers which is the same as ITU-T H 261. From top to bottom the layers are Picture, Group Of Blocks, Macroblock, and block. Each picture frame is parttioned into $8 \times 8$ image blocks. A MacroBlock (MB) consists of 4 lumınance blocks (Y), 2 chromınance blocks ( $\mathrm{C}_{\mathrm{b}}$ \& $\mathrm{C}_{\mathrm{r}}$ ) As shown in Figure 2.51 .3


Figure 2.5.1.3 Macroblock structure

However, the Group Of Block (GOB) arrangement for the picture formats are different from ITU-T H 261. A Group Of Block (GOB) comprises of a Kx16 lines, depending on picture format ( i e. $\mathrm{K}=1$ for sub-QCIF, QCIF, and CIF, $\mathrm{K}=2$ for 4 CIF, $\mathrm{K}=3$ for 16CIF). Each GOB is divided into Macroblocks (Table 2.5.1.2) Sımılarly each Macroblocks is divided into blocks.

| Picture Format | number of group of block <br> (GOB) for picture | number of macroblock (MB) <br> a group of block (GOB) |
| :--- | :---: | :---: |
| sub-QCEF | 6 | 8 |
| QCIEF | 9 | 11 |
| CIEF | 18 | 22 |
| 4 CIIF | 18 | 88 |
| 16 CIIF | 18 | 352 |

Table 2.5.1.2 Group of Block and Macroblock arrangement

Macroblocks for colour video sequence comprise $16 \times 16$ pixels luminance, plus two corresponding $8 \times 8$ chrominance blocks Vectors can take the form of one per macroblock (Figure 2.5.1.4), or on a block basis, where four vectors per macroblock would exist. The latter forms part of the Annex F "Advanced Prediction Mode" of H 263

$\times$ Luminance Sample
O Chrominance Sample

-     - Block edge

Figure 2.5.1.4 Positioning of block luminance and chrominance samples

The H 263 algorithm has been demonstrated as a versatıle low bit rate video coding procedure, taking account of the growing populanty of home personal computers connecting to PSTN by a modem, having bitrates of $144 \mathrm{kbits} / \mathrm{s}$ or $288 \mathrm{kbits} / \mathrm{s}$ where "software codecs", using the processing power of a contemporary personal computer can do away with the need for an expensive custom receiver

A number of additional optional functions have been included in the H263 recommendation in order to improve the interframe prediction performance.

- Unrestricted Motion Vectors
- Syntax Based Arithmetic Coding
- Advanced Prediction
- PB-frames

All of these four are optional and can be selected when runnıng the H 263 sımulation software.

Graphical Figures 2515-2517 are to show subjectively how well block matching base algorthm behaves for a sequence with relatively a fast motion. Very heavy on prediction error means block matching base algorithms may not perform as they would, if the motion were not relatively so fast These graphs are done using a software program very simılar to H 361 (called motion D) on "Car" sequence.

Motion D is a laboratory version software utilizing block matching base algorithms Motion D is meant to be quite versatile It has all the features of the H 261 . It can also be employed for non-standard picture sizes such as the Car sequence ( 720 by 576 pels) used in the thesis


Figure 2.5.1.5 PCM of previous frame for car.
(clean frame)

Figure 2.5.1.6 PCM of present frame for car. (clean frame)



Figure 2.5.1.7 Prediction error for the two successive car frames with MC, with previous frame clean.

### 2.5.2 Unrestricted motion vectors Mode (Annex D)

In the default prediction mode of H.263, the search for motion vectors can only take place inside the normal picture. In the Unrestricted Motion Vector mode, this requirement is removed and motion vectors are allowed to point outside the picture. The edge pels are used as a prediction for the "not existing" pels. To do this, edge pixel values are extrapolated in the x and y directions as appropriate, producing a virtual search window for the current block to search outside the normal picture boundaries (figure2.5.2).


Figure 2.5.2 Extrapolation for Unrestricted Motion Vectors

With this mode a significant gain is achieved and the image prediction is improved particularly where there is motion involving objects entering or leaving the scene, or there is movement along the edge of the picture, especially for the smaller picture formats. Additionally, this mode includes an extension of the motion vector range so that larger motion vectors can be used. This is especially useful in case of camera movement, where the camera itself is moving in a pan (panning situations). This mode is optional as it does not improve the prediction for static camera and central objects (which would be common in videoconferencing).

### 2.5.3 Syntax-based Arithmetic Coding Mode SAC (Annex E)

SAC is a variant of Arithmetic Coding [55], used in place of the traditional Variable Length Code for mınımum redundancy serial transmission The optımum length of Variable Length Codes is derived from the entropy of the data which tends to be noninteger Syntax-based Arthmetic Coding is an algonthm which encodes the symbols into a fractional number [56]

The implementation of SAC is, however, rather complex and it is impossible to recognize individual symbols in an encoded bit stream. Recovery from errors is difficult and it has a low tolerance to error, since SAC does not resynchronise after a few false symbols, as Varrable Length Codes do The SNR and reconstructed frame will be the same, but generally fewer bits will be produced

### 2.5.4 Advanced Prediction Mode (Annex F)

This option means that Overlapped block motion compensation (OBMC) [57] [58] is used for P-frames Four motion vectors instead of one per macroblock, that is four 8 x 8 vectors instead of one $16 \times 16$ vector are used for some of the macroblocks in the picture, which tends to provide a smoother prediction ımage and a better spatial quality at the decoder It is necessary that this mode operates in conjunction with the Unrestricted Motion Vector Mode (Annex D), to make a consistent prediction from the availability of extrapolated lumınance and chrominance pixels

The four $8 \times 8$ pixel luminance blocks in some of the macroblock allow a better representation of motion to be made, albeit at the price of a greater data over head It is therefore the responsibility of the implementing organisation to decide the value of this additional motion data

### 2.5.5 PB Frames Mode (Annex G)

This algorthm allows for the use of forward and bi-directionaly predicted frames. That is two pictures are being coded as one unit called as PB-frame (Figure 25 5) The name PB comes from the name of picture types in MPEG where there are P-pictures and Bpictures A PB-frame consist of one P-picture ( $P$-frame) which is predicted from the last decoded P-picture and one B-picture ( $B$-frame) which is predicted from both the previous decoded P-picture and the P-picture currently being decoded. Motion vectors can be used from the P-frames to generate predictions for the B-frames. This last picture is called a B-picture, because it is bi-directionally predicted from the past and future Ppicture For relatively simple sequences, the framerate can be doubled with this mode without increasing the bitrate by much. Additional vectors may also be transmitted as an optional mode, which effectively doubles the temporal resolution of the image with only a small increase in the coded video data rate. However, this tends to produce a less satisfactory prediction in sequences having very fast or complex motion, that is with a lot of motion or low initial frame rates Never the less, the PB-frame does not work as well as the B-frame in MPEG because there is no separate bi-directional vectors in ITUT H. 263 The advantage of ITU-T H. 263 over MPEG is that it requires much less overhead which is useful in low bit rate transmission


Figure 2.5.5 PB frame Arrangement

For H 263 hierarchy flow diagram and H 263 programming function description refer to appendices A 1 and A 2 respectively.

### 2.6 Further developments in the standard bodies

The orignal available videophone standard is the ITU-T H 261 [59]-[63] ITU-T Recommendation H 261 defines a video coding scheme for digital audıovisual services by the ITU-T Study Group XV Two bit-rates which have been established for Integrated Services Digital Networks (ISDN) and are of interest for image transmission are called the B-channel of $64 \mathrm{kbits} / \mathrm{s}$ and the H0-channel of $384 \mathrm{kbits} / \mathrm{s}$ The development of ITU-T H 261 went through many stages However, by late 1989, the final CCITT recommendations were made for the range of $64 \mathrm{kbits} / \mathrm{s}$ up to $1920 \mathrm{kbits} / \mathrm{s}$ Therefore, ITU-T H 261 is also known as a $p \times 64$ codec, where $p$ is between 1 and 30 Similar to ITU-T Recommendation, the algorthms specified by the Moving Picture coding Experts Group (MPEG) [64] employ a degree of both loss-less and lossy coding technıques. However, whilst the H. 261 algorthm is specifically designated as the framework of video codecs working on ISDN channels of $p \times 64 \mathrm{kbits} / \mathrm{s}$, the scope of MPEG is more wide-ranging

In the late 1980's an obvious relationship began to emerge between personal computers, digital storage on mexpensive media (such as CD-ROM) and the sale of video entertanment and educational software As the result of that the Motion Picture Expert Group (MPEG) was formed in 1988 to establish a standard for the compression of dıgital audio and video storage and later on for transmissions. The MPEG-1 [65] [66] is the first phase video compression standard The prmary objective of MPEG was to produce a compression algorithm for storage media having a through put of $1-15$ $\mathrm{Mbits} / \mathrm{s}$, with other goals of up to 60Mbits/s Whilst the direct application of CD-ROM was an obvious one, the bref of MPEG was to produce a standard that would apply to other storage techniques and applications This scheme is well suited to a wide range of applications such as, Compact Disk Read-Only Memory (CD-ROM), Digital Audıo Tape (DAT), Cable Television (CATV), telecommunication networks, and digıtal video
broadcastıng MPEG has also been applied to the compression of video for the purposes of Video-on-Demand [67] and for HDTV.

The MPEG-1 video coding algorithm [70] resulted from the requirements of CDROM and was greatly influenced by formulation of the ITU-T H 261 algorithm The development and evaluation of the algonthm was performed at bit rates in the region of $1 \mathrm{Mbits} / \mathrm{s}$ and video resolutions of 352 pixels x 288 lines, 25 frames per second, for PAL and 352 pixels x 240 lines, with an average of 2997 frames per second for the NTSC system These rates are not fixed and can be varied according to the requirements of different applications

The essential difference of MPEG-1, compared with H 261, is that, by the nature of the application to CD-ROM, random access is required This allows the end user to arbitranly choose any point in the video sequence from which to start viewing the moving images To acheve this, MPEG-1 has a number of frames which are encoded on their own and without any reference to other frames in the sequence, which are referred to as key frames and occur typically once in every twelve frame. As a result, MPEG-1 deliberately forces intraframe coding on some frames, whilst the majonty are formed as an interframe prediction with reference to temporally adjacent frames

The presence of regularly occurring intraframe coding is one of the reasons why MPEG1 is unsuitable for real-tıme codıng in audiovisual communications The time taken to process and transmit an intraframe coded frame is considerably higher than for interframe difference data, causing considerable variations in the quantity of bits per frame If the I-frames were to be taken as prımary start frames for an interframe sequence, they would have to be encoded with minimal losses, rendering the availability of data for the subsequent interframe coding relatively low in a given time perrod

One of the essentral differences between MPEG-1 and the H 261 algorthm is the way in which interframe predictions are made. H261 is primarily an interframe coding algorthm using the previous frame as the main prediction source for the generation of the next frame However, since MPEG-1 applies mainly to pre-recorded video
sequence, subsequent frames can also be used to make a better prediction of the current frame

Motion vectors used by MPEG have a greater range than would be required for video conferencing applications, since the nature of a wide range of video comprises more interframe motion than would be anticipated in a typical head-and-shoulders scene

Subsequent work on MPEG standards has considered the application of the algorithm for data rates of up to $40 \mathrm{Mb} 1 \mathrm{ts} / \mathrm{s}$ MPEG-2 [69] has been adopted for direct satellite broadcasting in Europe and by the US Advanced Television Committee (FCC) for HDTV It is effectively the same as MPEG-1, except that interlace scanning can be retained and interframe delays are less, resulting in a picture of improved quality

The MPEG-1 standard was published in 1993 as ISO/IEC 11172 (Coding of moving pictures and associated for digital storage up to about $15 \mathrm{Mbits} / \mathrm{s}$ ) [68] Part 1 of this standard describes the system, which includes information about the synchronization and multiplexing of video and audio streams. Parts 2, 3 and 4 describe video, audio and conformance testing respectively

The MPEG-2 [70] is the second phase of video compression standard which is aimed at coding above $2 \mathrm{Mbits} / \mathrm{s}$ Preparation of the MPEG-2 standard started in 1991 and provides a solution for applications that are not successfully covered by MPEG-1 The next phase of video compression standard, MPEG-3 was dropped in July 1992 A text identical to that of MPEG-2 was published as ITU-T Recommendation H 262. Recently, the MPEG-2 standard has been approved by the Advanced Television System Committee (ATSC) as a Digital High Definition Television (HDTV) [71] [72] Standard in the United States.

Formulation of a new MPEG-4 [73] Standard was begun at the MPEG meeting in Brussels in September, 1993. A draft specification is drawn in 1997 The primary target of this standard is very low bit rate applications The MPEG-4 standard supports a wide range of applications such as videophone over analogue telephone lines, sign language
captioning, mobıle audiovisual communications and interactive multimedia communications.

H 263 is also better than MPEG-I/MPEG-2 for low resolutions and low bitrates H 263 is less flexible than MPEG, but therefore requires much less overhead Another difference is again the negotrable options in H.263. MPEG has B-frames, but H. 263 has PB-frames which are almost as good for moderate amounts of movement, but require much less overhead H 263 has overlapped block motion compensation, motion vectors outside the picture and syntax-based anthmetic coding These options are not in MPEG at all Note that it is only possible to use H 263 at certain resolutions SQCIF , QCIF, CIF, 4CIIF and 16CIF, if you follow the standard H 263 software can be changed to run at every resolution divisible by the macroblock size 16, but the bitstreams generated will not be legal H 263 bitstreams in this case

## Chapter 3

## Pel-recursive techniques

### 3.1 Background

Motion compensation techniques predict the frame-to-frame (or field-to-field) motion of an object point and then access the intensity value from the previous frame (or field). The assumption is that predicting the motion and accessing the intensity values from the previous frame (or field) results in a better prediction of the intensity values than tryıng to predict the intensity values directly Previous work [74]-[81] [37]-[40] has shown that motion estımation techniques do improve the prediction of the intensity values in the images.

There have been basically two approaches to motion estimation - block-matching and pel recursive techniques [39] [78] [79]. In block-matching, a block of intensity values in a frame is compared with blocks of intensity values in the previous frame untıl a best match is determined. From this an interframe displacement vector (how much the block has
moved between frames) for the whole block can be estimated for the frame being transmitted Poor estimates result if all sample points in the block do not move the same way. Using the pel recursive approach a displacement is determined for each pel value. This technıque allows for amore exact estımation of the intensity value and has the abılity to handle scale changes (zooming, dilatıng, movement perpendicular to the image plane).

In, both block matching and pel recursion the prediction can be backward or forward, i.e., the displacement can be determined from previously transmitted information only (backward) or from past values and the current value (forward). Forward prediction requires explicit transmission of information about the displacement value, backward does not. The advantage of the forward technique is that the presumably better estimate or the displacement vector reduces the error in the intensity prediction. The majonty of the previous approaches have used backward prediction, applying backward prediction leads to 1 ) reduced bit rates, 2) lower computational requirements. or 3 ) faster prediction or estımation techniques

The pioneering work in detecting motion in interframe coders was done by estimating the speed (magnitude, but not the direction) by dividing the sum of the frame differences in a moving area by the sum of the element differences in that moving area [75] It was assumed that a speed of half a pel per frame was relatively slow, while a speed of four pels per frame was seldom exceeded. The results were obtained using a fixed camera and a moving object, it was also claimed that the technique could be applied to a panning camera and a moving object. Later the technique was extended to estimate velocity, 1 e. determine the direction of motion [37] Further pioneenng work in the area of motion compensated technıques were done by Cafforio and Rocca [38] [76]. Their work was more theoretıcal.

The proposed techniques [75] [37] required an estimate of the motion velocity to be sent. Netravalı and Robbins [39] [40] [77] developed a pel recursive spatio-temporal gradient technique in which the displacement of a pel was predicted from previously transmitted information. Thus since both transmitter and receiver could predict the motion vector, it did
not have to be sent. If an error correction needed to be sent for the predicted brightness then only an address and the difference value had to be transmitted. They used a 35 level symmetric quantizer, as a result of which the coder performance was only slightly affected by the quantizer. Previous field intensities were used for interpolation. They found that a rather simple interpolator is sufficient. Therr algorithm was able to reduce the data transmission rate by up to $50 \%$.

The next algonthm developed was called gan compensation [82]. It should be noted that gain compensation has some inherent motion tracking ability. Separate displacement compensation and gain compensation reduce the bit rate; together they reduced it even more, especially for the cases in which separately they produce minımal reduction Some further theoretical work was done on the implications and constraints of the assumptions which were being made in the motion compensated algonthms

Snyder et al [83] [84] investigated the assumption that frame differences can be expanded as a Taylor senes. Followed by Horn and Schunck [85] [86] who segmented the image into moving and stationary regions.By building on the work of Horn and Schunck, Nagel [87] developed a motion estımation technıque which can be seen [88] to do a good job of predicting the motion in a scene containing translational motion. No attempt has been made to apply these technıques[83] - [88] to information bandwidth compression. This is mainly because the resultıng system of equations is very computationally expensive

Thompson and Barnard [89] reported on ways of estimating and interpreting motion They discussed spatio-temporal gradıent techniques; feature point matching (pattern matching) was determined to be too computationally expensive.

Robbins and Netravalı [90] investıgated spatial subsampling in motion compensated coders. Spatial subsampling is a common way of preventing buffer overflow, in the presence of high or complex motion although motion estımation is degraded somewhat. The bit rate was reduced by $50 \%$, the same percentage as in conditional replenishment
coders. They were able to confine the blurring inherent in subsampling to the moving areas by an adaptive interpolation technique although the reduction factor was the same, the motion compensated algorithm produced better quality reconstruction than the conditional replenıshment algorithm

Prabhu and Netravalı [91] [92] developed a motion compensated algorithm to compress and transmit component color sequences. The first investigation involved predicting each component separately. Three predictor schemes were evaluated•-1) use only the previous frame, 2) switch the predictor between previous frame and displaced previous frame, and 3) switch the predictor between previous frame, displaced previous frame, and an intraframe predictor. They ultimately concluded that one predictor (the third one) could be used to predict both the luminance and the chrominance component. The luminance information was used to switch the predictor.

Ishiguro and Innuma [78] gave a brief overview of the existing motion Compensated bandwidth compression techniques. They divided the techniques into pel recursive, and pattern matching. Given the pattern matching approach, the choice of backward or forward detection implies that the transmitter and receiver both determine the motion prediction from common information (previously transmitted data). In forward detection, the block about to be transmitted is translated and a motion vector determined. This motion vector must be sent as well as the block of error correcting values. The assumption in forward detection is that the error values are smaller and thus require less bandwidth to be transmitted, leaving room for the motion vector. This type of pattern matching technique was actually implemented in a production system [93] by NEC. It is interesting to note that it uses pattern matching technique since other researchers had stated that a pattern matching techntque would be too computationally intensive [39] [79] and since the spatio-temporal gradient method had received more favorable consideration in the literature [94] - [97]

All the algonthms discussed so far have in effect modelled the motion in the sequence as purely translational Huang and Tsa1 [95] pointed out that if rotation of object is to be
considered, and then pattern matching technique requires a three dımensional data space with an increase in processing bandwidth, indicatıng a spatı-temporal gradient approach would be more feasible.

Paquin and Dubois [79] investigated spatio-temporal gradient algorithm which employed motion compensated prediction. Although they obtaned an algonthm sımılar to that of Netravalı and Robbins [39] [40], they started from a slightly different perspective and with slightly different asumptions The displacements were estımated on a field basis. Therr maxımum allowable displacement was 10 pels per field while Limb and Murphy [75] assumed 4 pels per frame would seldom be exceeded. They were primarily interested in determining trade-offs between accuracy and computational complexity for interpolator and the estımator.

### 3.2 Motion compensated image sequence compression

In video conferencing applications, correlation between consecutive frames is significantly high due to the limited amount of motion. This correlation can be exploited more efficiently by taking into consideration the displacements of moving objects in the coding process. Thus in any motion compensated coding scheme, the coding performance depends heavily on the accuracy of the motion estimation

There are instances when the DPCM technique cannot successfully code a segment of an image sequence because motion is a major cause of interframe differences. Motion Compensation (MC) can be used to improve the efficiency of the predictive coding algorthm

If translation of a moving object is avalable, a more efficient prediction can be estımated using elements in the previous frame(s) that are appropriately spatally displaced. This type of prediction is called Motion Compensated Prediction. Furthermore, motion can be a complex combination of translation and rotation Transitional motion is relatıvely easily
estımated and has been used for motion compensated codıng, depending on the amount of translation motion in the scene and the ability of an algonthm to estimate translation with the accuracy that is necessary for a good prediction.

The main problem is developing a good algorthm used for motion estımation Various algonthms which have been successfully used in coding application include Block Matching, Pel Recursive, and Gain motion compensated estimation.

Block matching is widely used in coding applications but has its own limitations and weaknesses due to looking at displacement over a block as a whole, which is perhaps a trade-off, 1 e a less accurate estimation producing less coding which in turn gives higher compression It is not a good idea to trade off the accuracy of estımation for the motion for some over head or perhaps come up with a different algonthm which could take care of the mentioned problem.

The pel recursive method for displacement of motion compensation can overcome the above problem In this method, we look at every pel by pel estımatıng the displacement vector for every single pel resulting in motion estimation for every single pel rather than every block of pels, therefore higher accuracy is acheved but with the cost of more overhead.

Among the many different algorthms, the one by Netravali [37] [39] [40] [77] [98] - [100] is looked at in more detall.

### 3.3 Initial estimation of displacement vector

For simplicity the algonthm used for motion estimation in interframe coding follows the assumptions below (these are true for most algorithms for motion estımation in interframe coding)

I - Translation movement of an object is in a plane which is parallel to the camera plane.
II - Illumination is spatially and temporally uniform
III - Occlusion of one object by another, and also uncovered background are neglected.
Under these assumptions, the monochrome intensities $b(z, t)$ and $b(z, t-\tau)$ of two consecutive frame are related by
$b(z, t)=b(z+D, t-\tau)$
Where $\tau$ is the time between two frames, D is the two dimensional translation vector of the object during the time interval $[\mathrm{t}-\tau, \mathrm{t}$ ], and z is the two dimensional vector $[\mathrm{x}, \mathrm{y}]$ ' of spatial position. Using Eqn (3.3.1) we can write the frame difference signal FDIF $(z, t)$ as
$\operatorname{FDIF}(\mathrm{z}, \mathrm{t}) \Delta \mathrm{b}(\mathrm{z}, \mathrm{t})-\mathrm{b}(\mathrm{z}, \mathrm{t}-\tau)=\mathrm{b}(\mathrm{z}, \mathrm{t})-\mathrm{b}(\mathrm{z}+\mathrm{D}, \mathrm{t})$

For small D, using Taylor's expansion about z (assuming D to be small)
$\operatorname{FDIF}(\mathrm{z}, \mathrm{t})=-\mathrm{D}^{`} \nabla_{\mathrm{z}} \mathrm{b}(\mathrm{z}, \mathrm{t})+$ higher order terms in D
Where $\nabla_{z}$ is the spatial gradient with respect to $z$.
Assuming that the translation of the object is constant over some moving area $R$ and neglecting higher order terms in D .
$\hat{D}$, the mınımum mean square estımate of D can be obtained by mınimızing

$$
\begin{equation*}
\sum_{R}\left[\operatorname{FDIF}(z, t)+D^{`} \nabla_{z} b(z, t)\right]^{2} \tag{Eqn3.3.4}
\end{equation*}
$$

with respect to D , therefore

$$
\begin{align*}
& \hat{D}=-\left[\sum_{R} \nabla_{z} \mathrm{~b}(\mathrm{z}, \mathrm{t}) * \nabla_{z} \mathrm{~b}(\mathrm{z}, \mathrm{t})\right]^{-1} * \\
& {\left[\left[\sum_{R} \operatorname{FDIF}(\mathrm{z}, \mathrm{t}) * \nabla_{z} \mathrm{~b}(\mathrm{z}, \mathrm{t})\right]\right.} \tag{Eqn3.3.5}
\end{align*}
$$

$\nabla_{z} b(z, t)$ can be approximated as
$\nabla_{z} \mathrm{~b}(\mathrm{z}, \mathrm{t})=\left[\begin{array}{l}E D I F(z) \\ \operatorname{LDIF}(\mathrm{z})\end{array}\right]$
Where EDIF is a horizontal element difference and LDIF is a vertical line difference given by
$E D I F(z)=1 / 2[b(z+\Delta x, t)-b(z-\Delta x, t)]$
$\operatorname{LDIF}(z)=1 / 2[b(z+\Delta y, t)-b(z-\Delta y, t)]$

Using Eqn (3 3 8)

$$
\begin{align*}
& \hat{D}=-\left[\begin{array}{ll}
\sum E D I F^{2}(z) & \sum E D I F(z) * L D I F(z) \\
\sum E D I F(z) * L I D F(z) & \sum L D I F^{2}(z)
\end{array}\right]^{-1} * \\
& {\left[\begin{array}{l}
\sum F D I F(z, t) * E D I F(z) \\
\sum F D I F(z, t) * E D I F(z)
\end{array}\right] }
\end{align*}
$$

$\triangleq \quad$ denotes by definition
prime ` denotes therr transpose
consider $\mathrm{D}, \mathrm{Z}, \nabla$ to be column vectors of size ( 2 x 1 )

An sumption is made to convert the above matrix into diagonal one, that is:-

$$
\begin{equation*}
\sum_{R} E D I F(z) * \operatorname{LDIF}(z) \approx 0 \tag{Eqn3310}
\end{equation*}
$$

Then
$\hat{D}=-\left[\begin{array}{l}\frac{\sum F D I F(z, t) * \operatorname{EDIF}(z)}{\sum E D I F^{2}(z)} \\ \frac{\sum F D I F(z, t) * \operatorname{LDIF}(z)}{\sum L D I F^{2}(z)}\end{array}\right]$
in order to proceed with simulation. Moving area segmentation was defined by considering the moving pels That is if the frame difference for the considering pel is less than a threshold value, the pel is considered or classed as moving pel which is chosen in relation with camera noise.

Using Eqn (3 3.11), the initial estımate of local displacement was provided by simulation, giving good results where we were not on the edge of the moving area in the scene.

Careful consideration should be given, in order to estımate intial displacement vectors accurately enough, as these are highly dependant on the implementation of the moving area pels. Therefore the moving area pel should not be classed as a moving pel if the left, right, and upper neighboring pels are not moving pels (and vice versa)

### 3.4 INTERPOLATION

Having estimated an initial value for the displacement vector, the intensity of the pel displaced by $\hat{D}$ is estimated by means of an interpolating technique and the following formula is used

$$
\begin{equation*}
I=I_{D}+\hat{D}_{x}\left(I_{A}-I_{D}\right)+\hat{D}_{Y}\left(I_{B}-I_{D}\right)+\hat{D}_{X} * \hat{D}_{Y}\left(I_{D}+I_{C}+I_{A}+I_{B}\right) \tag{Eqn34.1}
\end{equation*}
$$



Figure 3.4.1 : Two dimensional linear interpolation.
Displacement $D$ is decomposed into integral part $D_{I}$ and non-integral part $D_{F}$

### 3.5 PEL RECURSIVE MOTION VECTOR ESTIMATOR

Now we define the Displaced Frame Difference (DFD) as follow -

$$
\begin{equation*}
D F D(z, \hat{D})=b(z, t)-b(z-\hat{D}, t-\tau) \tag{Eqn35.1}
\end{equation*}
$$

In practice , the $D F D, D F D(z, \hat{D})$, hardly ever becomes exactly zero for any value of $\hat{D}$, because - I ) there is observation noise, II )there is occlusion (covered / uncovered background problem), III ) errors are introduced by the interpolation step in the of noninteger dısplacement vectors, and IV ) scene illumination may vary from frame to frame. Therefore, it is generally aim to minımize the absolute value or the square of the $D F D$.

Pel recursive displacement estımators tries to minımıze recursively $[\operatorname{DFD}(z, \hat{D})]^{2}$ at each moving area pel using a steepest descent algonthm thus

$$
\begin{equation*}
\hat{D}_{k}=\hat{D}_{k-1}-\frac{\varepsilon}{2} * \nabla_{\hat{D}_{k-1}}\left[D F D\left(z, \hat{D}_{k-1}\right)\right]^{2} \tag{Eqn35.2}
\end{equation*}
$$

Where $\nabla_{\hat{D}}$ is the two dimensional gradient operator with respect to $\hat{D}$.Using Eqn. (3.5.1) therefore

$$
\begin{equation*}
\hat{D}_{k}=\hat{D}_{k-1}-\varepsilon * D F D\left(z_{n}, \hat{D}_{k-1}\right) * \nabla_{z} b\left(z_{n}-\hat{D}_{k-1}, t-\tau\right) \tag{Eqn3.5.3}
\end{equation*}
$$

Thus, the new value for $\hat{D}$ is the old value plus an update term.
where $\varepsilon$, the convergency parameter is some positive scalar, known as the step size. The step size $\varepsilon$ is critical for the convergence of the iterations, because if step size is too small, we move by a very small amount each time, and the iterations will take too long to converge On the other hand, if it is too large the algonthm may become unstable and oscillate about the minimum In the above method, the step size is usually chosen heurstically

The above algorithm can be extended by computing the displaced frame differences at many picture elements in order to estımate $D$

The steepest descent algorithm is used to minimize a weighted sum of the squared displaced frame differences at some previously transmitted neighboring pel, thus

$$
\begin{equation*}
\hat{D}_{k}=\hat{D}_{k-1}-\frac{\varepsilon}{2} * \nabla_{\hat{D}_{k-1}}\left[\sum_{j=0}^{p} W_{\jmath}\left(\operatorname{DFD}\left(z_{n-\jmath}, \hat{D}_{k-1}\right)\right)^{2}\right] \tag{Eqn3.5.4}
\end{equation*}
$$

Where $W_{t} \geq 0$ and $\sum_{j=0}^{p} W_{J}=1$

Usıng Eqn (3 3 1) therefore

$$
\begin{equation*}
\hat{D}_{k}=\hat{D}_{k-1}-\varepsilon *\left[\sum_{j=0}^{p} W_{J} \operatorname{DFD}\left(z_{n-j}, \hat{D}_{k-1}\right) * \nabla z b\left(z_{n-\jmath}-\hat{D}_{k-1}, t-\tau\right)\right] \tag{Eqn3.5.5}
\end{equation*}
$$

Where $\nabla_{z}(0)$ can be approximated by finite differences as before
Now recursively $\hat{D}$ is updated using Eqn (341) and Eqn (355). For each step, the update term seeks to improve the estimate of $\hat{D}$. The ultimate goal is minimization of the magnitude of the predıction error $D F D$. If a pel at location $Z_{a}$ is predicted with $\hat{D}_{k-1}$ to have intensity $b\left(z-\hat{D}_{k-1}, t-1\right)$, resulting in a prediction error of $\operatorname{DFD}\left(\mathrm{z}, \hat{D}_{k-1}\right)$ the prediction should attempt to create a new estımate, $\hat{D}_{k}$ such that .-

$$
\left|D F D\left(z, \hat{D}_{k}\right)\right| \leq\left|D F D\left(z, \hat{D}_{k-1}\right)\right|
$$

### 3.6 Implementation and Experimental results

In order to implement and sımulate the previously mentioned technıque, calculation of line and element differences in addition to the displacement frame difference are the most crucial and should be given the most concern. In the experıment, different ways of
implementation for each parameter should be examined e.g.:- interpolated, none interpolated, averaged using different causal supports (Figure 3.6.1), etc. Epsilon, $\varepsilon$, the convergency parameter is recommended to be 1 / 1024 [77]. Further more, not every pel need to be motion compensated, therefore some kind of masking should be employed e,g . where frame difference, $|F D I F| \leq$ threshold; no prediction is needed. This is classıfied as non-moving area.

X X X X X X X<br>X X X X X X X<br>X X X X O

Figure 3.6.1 : An example of a second order causal support.

For the experıment a good result is produced having $\varepsilon=0.9$ and using a 3 by 2 causal support (Figure 3.6.2) for line and element differences. The maxımum permitted update term was chosen to be limited to 4 . Absolute Frame Difference, $|F D I F| \leq 9$ for a nonmoving area. This is done for two different sequences, Suzie and Salesman.

X X X X<br>X X O

Figure 3.6.2 : causal support.

Figures 3.63 to 3.6 .6 show and indicate the validity of the theory behind the pel-recursive motion estimation. In each of the figures, the graphs represent the energy of the error for the situations in which there is no motion compensation and where the motion is compensated using pel-recursive motion estimation It also looked at different frame skips, that could be frequently used in video conferencing and so on From the graphical and pictorial results (Figure 3.6.3-3.6.6) it can be seen that pel-recursive motion compensation does very good job and shows, high dB reduction in transmittable error from the DPCM loop

a) No frame skip comparison.

b) One frame skip comparison.

c) Two frame skip comparison.

Figure 3.6.3 Suzie comparison after three iteration with previous frame clean.

a) PCM frame of Suzie (previous frame)


Prediction error with no MC for Suzies in $\mathrm{a} \& \mathrm{~b}$

b) PCM frame of Suzie (present frame)

d) Prediction error for Suzies in a \& b (gradient)

Figure 3.6.4 Prediction error comparison for two successive frame of Suzie after three iteration with previous frame clean.

a) No frame skip comparison.

Salesman Prediction Error Comparison

b) One frame skip comparison.

c) Two frame skip comparison.

Figure 3.6.5 Salesman comparison after three iteration with previous frame clean.

a) PCM frame of Salesman (previous frame)


Prediction error with no MC for Salesman in a \& b

b) PCM frame of Salesman (present frame)

d) Prediction error for Salesman in a \& b (gradient)

Figure 3.6.6 Prediction error comparison for two successive frame of Salesman after three iteration with previous frame clean.

a) no frame skip comparison.

b) One frame skip comparison.

c) Two frame skıp comparison.

Figure 3.6.7 Suzie comparison after three iteration with previous frame reconstructed, with half pel accuracy on block matching, and system resetting to zero for each pel.

a) No frame skip comparison.

b) One frame skıp comparison

c) Two frame skıp comparison

Figure 3.6.8 Salesman comparison after three iteration with previous frame reconstructed, with half pel accuracy on block matching, and system resetting to zero for each pel.

### 3.7 Improved pel-recursive motion compensation

We now consider further improvements for pel-recursive Motion compensation [101] [106]. Consider the basic algorithm (Eqn 35.3 or 3.5 .5 ) for the intensity function at an object edge The condition requirng the largest vector corrections or updates factor are when $|D F D|$ is large and $|\nabla b|$ is small. Conversely, if $|D F D|$ is small and $|\nabla b|$ is large, as could exist at an object edge, the vector correction must be small. For the affirmation algorthms to work, $E$ must be chosen to allow for the case where the correction or update must be small. This gives nise to

$$
\begin{equation*}
\varepsilon=1 / 2 * \frac{1}{\left(\mid \nabla_{2} b\left(z_{n-\jmath}-\hat{D}_{k-1}, t-\tau\right)\right)^{2}} \tag{Eqn3.71}
\end{equation*}
$$

Or

$$
\begin{equation*}
\varepsilon=1 / 2 * \frac{1}{\sigma^{2}+\left(\left|\nabla_{z} b\left(z_{n-\jmath}-\hat{D}_{k-1}, t-\tau\right)\right|\right)} \tag{Eqn3.7.2}
\end{equation*}
$$

and

$$
\begin{equation*}
\left\{\nabla_{z} b\left(z_{n-\jmath}-\hat{D}_{k-1}, t-\tau\right)\right\}^{2}=\left\{\nabla_{x} b\left(z_{n-\jmath}-\hat{D}_{k-1}, t-\tau\right)\right\}^{2}+\left\{\nabla y b\left(z_{n-\jmath}-\hat{D}_{k-1}, t-\tau\right)\right\}^{2} \tag{Eqn3.7.3}
\end{equation*}
$$

Where $\sigma$ is recommended to be of the order of 10 [104], which takes account for $|\nabla b|$ becomıng small or zero.

### 3.8 Helpful implementation details and constrains

Implementation and simulation of algonthms needs many sensible constraints and restrictions in order to show that the algorthms even work. Some of them are as follows -
a) - If $|D F D| \leq$ threshold, the correction term or update of Eqn 3.5 .3 or 3.5.5 is zero
b) :- If $|D F D|>$ threshold $|\nabla b|$ is not zero, the update term is calculated. When $\mid$ update-term $\mid<1 / 16$, the update term is recommended to be assigned to the value of $\pm 1 / 16$.
c) :- If $|D F D|>$ threshold and if $|\nabla b|$ is zero, then the update term again is zero.
d) :- If $\mid$ update-term $\mid$ exceeds 2 , the update term is recommended to be assigned to the value of $\pm 2$.

It can be seen that as the $|\nabla b|$ or $\mid$ gradient $\mid$ becomes large, the update term decreases, and vice versa.

Further, some of the restrictions implemented and applied for simulation are as follow -
a) - Use $\hat{D}_{k}$ displacement obtained for the previous pel. Predict the current pel by obtaining a pel value from the previous frame at the offset $\hat{D}_{k}$ from the current pel location z .
b) - If $|D F D| \leq$ threshold, transmit zero If $|D F D|>$ threshold and $|F D I F| \leq$ threshold, transmit a reset to set $\hat{D}_{k}=0$ If $|D F D|$ and $|F D I F|>$ threshold, transmit $D F D$.
c) :- If $|D F D| \leq$ threshold, use $\hat{D}_{k}$ as obtained from the previous pel, i.e, $\hat{D}_{k}=\hat{D}_{k-1}$. And if $|D F D|>$ threshold and $|F D I F|<$ threshold, set $\hat{D}_{k}=0$.

### 3.9 Implementation and Experimental results

Once again in the experimental work, different ways of implementing each parameter should be examined e g. $\cdot$ interpolated, non-interpolated, averaged using different causal supports (Figure 361 ) For consistency in comparison the threshold value chosen for the experimental work was 9 for frame difference and 20 for displacement frame difference and 3 by two causal support as before The maxımum update limit was chosen to be 3 . These restrictions give rise to the graphical and pictorial result in (figures 39.1 - 3.9 6)

Each figure depicts the graphical representation of the basic state of the art gradient algonthm for $\varepsilon$, the convergence factor, to be non-adaptıve and adaptıve as first and second gradient Looking at the result from the same sequences of Suzie and Salesman, it can be easily noticed that having $\varepsilon$, the convergence factor, as a variable shows quite substantial improvement over the basic algorithm and reduces the energy of the error

a) No frame skıp comparison

b) One frame skip comparison.


Figure 3.9.1 Suzie comparison after three iteration with previous frame clean.

a) PCM frame of Suzie (previous frame)

b) PCM frame of Suzie (present frame)


Prediction error with no MC for Suzie in $\mathrm{a} \& \mathrm{~b}$

d) Prediction error for Suzie in a \& b (gradient)

Figure 3.9.2 Prediction error comparison for two successive frame of Suzie after three iteration with previous frame clean.

a) No frame skip comparison.

b) One frame skip comparison.

c) Two frame skip comparison.

Figure 3.9.3 Salesman comparison after three iteration with previous frame clean.

a) PCM frame of Salesman (previous frame)


Prediction error with no MC for Salesman in a \& b

b) PCM frame of Salesman (present frame)

d) Prediction error for Salesman in $\mathrm{a} \& \mathrm{~b}$ (gradient)

Figure 3.9.4 Prediction error comparison for two successive frame of Salesman after three iteration with previous frame clean.

a) No frame skıp comparison

b) One frame skip comparison.

c) Two frame skip comparison.

Figure 3.9.5 Suzie comparison after three iteration with previous frame reconstructed, with half pel accuracy on block matching, and system resetting to zero for each pel.

a) No frame skip comparison

b) One frame skip comparison.

c) Two frame skıp comparison

Figure 3.9.6 Salesman comparison after three iteration with previous frame reconstructed, with half pel accuracy on block matching, and system resetting to zero for each pel.

# Chapter 4 

## A New pel-recursive technique For MOTION COMPENSATED IMAGE SEQUENCE COMPRESSION

### 4.1 Background

One of the main developments in image coding in recent years is the application of mathematical models describing the motion of objects

For applications in dynamic scene analysis in a sequence of moving images, ie. television pictures, a moving object generates frame-to-frame luminance changes These luminance changes can be used in order to estımate the parameters of a mathematical model that
describes the displacement and movement of the object. For instance, consider a simple moving edge as in Figure 411.


Figure 4.1.1 Illustration of displacement estimation.
The dashed line indicates the position of the edge in the previous frame

$$
\begin{equation*}
\hat{D}_{x}=\hat{d} x=\sum_{M}|F D I F| / \sum_{M}|E D I F| \tag{Eqn4.1.1}
\end{equation*}
$$

Where $M_{1 s}$ the moving area which is generally defined by frame differences greater than a given threshold

For the $y$ direction, similar principle applies, therefore
$\hat{D}_{y}=\hat{d} y=\sum_{M}|F D I F| / \sum_{M}|L D I F|$

Motion models can also be used for improving the efficiency of predictive and interpolative television coding techniques. Because of the real-time computing requirements or VLSI architecture implementation [107], only relatively simple and easily realizable models which consider the translational component of motion have been worth while investigating

The x component of the displacement estimate $\hat{D}_{x_{i}}$, for a few different mathematical models can be summarized below [108] :-

## First model

$$
\hat{D}_{x_{i}}=\hat{D}_{x_{i-1}}+\varepsilon^{*} \frac{\partial}{\partial x} R_{S_{k} s_{k-1}}\left(z, \hat{D}_{i-1}\right), \quad \varepsilon=1 / 1024 \text { (recommended) }
$$

(Eqn 4.1.3) [39], [109]
Where

$$
\begin{equation*}
R_{s_{k} s_{k-1}}(z, D)=\mathrm{E}\left[s_{K}(z) \bullet s_{K-1}(x-d x, y-d y)\right] \tag{Eqn4.1.4}
\end{equation*}
$$

Its simplified update term (the difference between the present and previous estimation) is
$\mathrm{U}_{t}=-\frac{1}{2} \varepsilon^{*} \nabla_{\hat{D}_{i}} \sum_{j \in M} W_{j}\left[D F D\left(z, \hat{D}_{l-1}\right)\right]^{2}$
(Eqn 4.1.5)
Where $W_{i} \geq 0$ and $\sum_{j \in M} W=1$

A quicker update can be achieved by increasing the constant convergence factor, $\varepsilon$ However, this also implies a decrease of the achievable estımation accuracy which is limited by $\varepsilon$

## Second model

$$
\begin{equation*}
\hat{D}_{x_{t}}=\hat{D}_{x_{t-1}}-\frac{\frac{\partial}{\partial x} R_{S_{k} S_{k-1}}\left(z, \hat{D}_{t-1}\right)}{\frac{\partial^{2}}{\partial x^{2}} R_{S_{k} S_{k-1}}\left(z, \hat{D}_{t-1}\right)} \tag{Eqn4.1.6}
\end{equation*}
$$

Its simplified update term is
$\mathrm{U}_{t}=-\frac{\sum_{M} \frac{\partial}{\partial d x}\left[D F D\left(z, \hat{D}_{i-1}\right)\right]}{\sum_{M} \frac{\partial^{2}}{\partial d x^{2}}\left[D F D\left(z, \hat{D}_{i-1}\right)\right]}$

## Third model

$$
\begin{equation*}
\hat{D}_{x_{t}}=\hat{D}_{x_{t-1}}+\frac{\frac{\partial}{\partial x} R_{S_{k} s_{k-1}}\left(z, \hat{D}_{t-1}\right)}{\left|\frac{\partial^{2}}{\partial x^{2}} R_{S_{k-1}} s_{k-1}(z, 0)\right|+\eta^{2}}, \quad \eta=10 \tag{Eqn418}
\end{equation*}
$$

The correction term, $\eta$ is introduced to avord problems which would occur in areas of nearly constant lummance where $\frac{\partial s_{k-1}}{} / \partial x$ is small and prevents the overshoots

Its simplified update term is
$\mathrm{U}_{t}=-\frac{\frac{1}{2} * \sum_{M} \frac{\partial}{\partial d x}\left[D F D\left(z, \hat{D}_{t}\right)\right]}{\sum_{M}\left[\frac{\partial}{\partial x} s_{k-1}\left(x-\hat{d} x_{i}, y-\hat{d} y_{t}\right)\right]^{2}+\eta^{2}}$

## Fourth model

$\hat{D}_{x_{t}}=\hat{D}_{x_{t-1}}-\frac{\frac{\partial}{\partial x} R_{S_{k} S_{k-1}}\left(z, \hat{D}_{t-1}\right)}{\frac{1}{2}\left[\frac{\partial^{2}}{\partial x^{2}} R_{S_{k} S_{k-1}}\left(z, \hat{D}_{t}\right)+\frac{\partial^{2}}{\partial x^{2}} R_{S_{K} S_{k}}(z, 0)\right]}$

Its simplified update term is

$$
\begin{equation*}
\mathrm{U}_{1}=-\frac{\frac{1}{2} * \sum_{M} \frac{\partial}{\partial d x}\left[D F D\left(z, \hat{D}_{i}\right)\right]}{\frac{1}{2} * \sum_{M}\left[\frac{\partial}{\partial x} s_{k-1}\left(x-d x_{i}, y-d y_{i}\right)+\frac{\partial}{\partial x} s_{k}(z)\right] \frac{\partial}{\partial x} s_{k}(z)} \tag{Eqn4.1.11}
\end{equation*}
$$

Simılarly, the x component of the displacement estimate $\hat{D}_{y y}$, can be deduced and follows the same format.

Companng the above displacement estimation algorithms shows that these algorithms only differ in the denominator of the update term. The previous algorithm gives rise to faster convergence respectively, while the first algorithm gives slower convergence (virtually damped).

Block matching algorithms work on finding the best matching block by comparıson where as pel recursive algonthms work on finding pel by pel estimation. As a good figure of judgment one would expect superiority by pel recursive algorithms over block matching algorithms But experimental results proved otherwise.

In spite of the expectation one might have had for existing pel recursive algorithms somehow, block matching algorthm have shown better performance in digital image compression As a result of this, block matching algorithms dominate compression applicatıons e.g.: JPEG, MPEGs, H.263, and so on. This opportunity has given rise to research to improve the performance of block based algorithms further in many different applications. Less research has been directed in the area of pel recursive algorithms causing the pel recursive base algorithms to be left even further behind.

Of the four existing pel recursive algorithms, the first two were simulated as a bench mark for comparison between pel recursive algonthms and block matching algorthms, (in chapter 3).

As an educated guess one mıght suggest that the problem may not be with the pel recursive scheme in general. A better pel recursive algorithm may prove that the pel recursive algorithms perhaps should do better than block matching algorithms.

In the light of the above argument a new algorithm is proposed and simulated to show its validity (see the following sections).

### 4.2 A new algorithm,

Motion compensated image sequence compression (algorithm)

For the sake of the analysis, it is assumed that the translational movement of an object is in a plane parallel to the camera and illumination is uniform. It is also assumed that the effect of uncovered background is negligible Under these assumptions, let $S(x, y, t)$ denote the monochrome intensities at point ( $\mathrm{x}, \mathrm{y}$ ) of a moving object in the rmage plane where it's translational movement is at a constant velocity of $\mathrm{v}_{\mathrm{x}}$ and $\mathrm{v}_{\mathrm{y}}$. It can be shown that after $\Delta \mathrm{t}$ second (one frame period), the object moves to a new location where it can be shown,
$S(x, y . t+\Delta t)=S\left[\left(x+v_{x} \Delta t\right)\right],\left[\left(y+v_{y} \Delta t\right): t\right]$

After expanding the field in a power series in $\Delta t$ and neglecting the higher order terms, the frame difference can be shown as,
$S(x, y t+\Delta t)-S(x, y: t)=\frac{\partial}{\partial x} S(x, y: t) d x+\frac{\partial}{\partial y} S(x, y: t) d y$
where $d_{x}$ and $d_{y}$ correspond to the horizontal and vertical components of the motion vector D Assuming $\frac{\partial}{\partial x} S(x, y, t)$ and $\frac{\partial}{\partial y} S(x, y: t)$ are known for each $x, y, t$, and
defining EDIF, LDIF, and FDIF as the magnitude of the element, line, and frame difference at point $n$, from (42.2), we can write,
$\mathrm{FDIF}=\Phi_{n}^{\mathrm{T}} \mathrm{D}$

Where $\Phi_{n}=\left[\begin{array}{l}\frac{\partial}{\partial x} S(x n, y n t) \\ \frac{\partial}{\partial y} S(x n, y n: t)\end{array}\right]=\left[\begin{array}{l}E D I F \\ \\ L D I F\end{array}\right]$
From Eqn(4 24 ) the frame difference (FDIF) measurement is,
$\zeta_{n}=\Phi_{n}^{\mathrm{T}} \bar{D}+$ noise
where $\overline{\mathrm{D}}=[\overline{\mathrm{d}}(\mathrm{x}), \overline{\mathrm{d}}(\mathrm{y})]^{\mathrm{T}}$ is the motion vector estumate.
Now let
$\mathrm{y}=\left(\zeta_{n}-\Phi_{n}^{\mathrm{T}} \alpha\right)^{2}+$ noise
For the least-squares of $\alpha$ to be minimized, gives
$\frac{\partial}{\partial \alpha} \mathrm{y}=-2 \Phi_{n}^{\mathrm{T}}\left(\zeta_{n}-\Phi_{n}^{\mathrm{T}} \alpha\right)=0$
That is
$\zeta_{n}=\Phi_{n}^{\mathrm{T}} \alpha$
Or multıplying each side or equation by $\Phi_{n}$
$\Phi_{n} \zeta_{n}=\Phi_{n} \Phi_{n}^{\mathrm{T}} \alpha$
For a cluster of $M$ moving pels, the least-squares estimate of $D$, can be shown as,

$$
\begin{equation*}
\sum_{n=1}^{m} \Phi_{n} \zeta_{n}=\left(\sum_{n=1}^{m} \Phi_{n} \Phi_{n}^{\mathrm{T}}\right) \bar{D} \tag{Eqn426}
\end{equation*}
$$

For, $\eta=\frac{1}{M} \sum_{n=1}^{M} \Phi_{n} \zeta_{n}$ and $\mathrm{R}=\frac{1}{M} \sum_{n=1}^{M} \Phi_{n} \Phi_{n}^{\mathrm{T}}$
the estımated motion vector from Eqn(4.2.6) is obtained as,

$$
\begin{equation*}
\bar{D}=\mathrm{R}^{-1} \eta \tag{Eqn428}
\end{equation*}
$$

For recursive estımation of $\eta$ and $R$, we can write

$$
\begin{align*}
& \eta_{t}=\eta_{t-1}+\Phi_{n} \zeta_{n}  \tag{Eqn429}\\
& \mathrm{R}_{t}=\mathrm{R}_{t-1}+\Phi_{n} \Phi_{n}^{\mathrm{T}} \tag{Eqn4210}
\end{align*}
$$

Based on the so-called matrix inversion lemma, which is :-
$\left(\mathrm{A}+\mathrm{XBX}^{-\mathrm{T}}\right)^{-1}=\mathrm{A}^{-1}-\mathrm{AX}\left(\mathrm{B}^{-1}+\mathrm{X}^{-\mathrm{T}} \mathrm{A}^{-1} \mathrm{X}\right)^{-1} \mathrm{X}^{-\mathrm{T}} \mathrm{A}^{-1}$
Substitute as follows -
$\mathrm{A}=\mathrm{R}_{t-1}^{-1}$
$B=I \quad$ Unit Matrix
$\mathrm{X}=\Phi_{n}$

That is
$\left(\mathrm{R}_{t-1}+\Phi_{n} \mathrm{I} \Phi_{n}^{-\mathrm{T}}\right)^{-1}=\mathrm{R}_{t-1}^{-1}-$

$$
\mathrm{R}_{t-1} \Phi_{n}\left(\mathrm{I}^{-1}+\Phi_{n}^{-\mathrm{T}} \mathrm{R}_{t-1}^{-1} \Phi_{n}\right)^{-1} \Phi_{n}^{-\mathrm{T}} \mathrm{R}_{t-1}^{-1}
$$

In the above equation, the term in the left hand side bracket can be replaced, using Eqn(4 2 10), therefore
$\mathrm{R}_{t}{ }^{-1}=\mathrm{R}_{t-1}{ }^{-1}-\mathrm{R}_{t-1} \Phi_{n}\left(\mathrm{I}^{-1}+\Phi_{n}{ }^{-\mathrm{T}} \mathrm{R}_{t-1}{ }^{-1} \Phi_{n}\right)^{-1} \Phi_{n}{ }^{-\mathrm{T}} \mathrm{R}_{t-1}{ }^{-1}$

The term $\left(\mathrm{I}^{-1}+\Phi_{n}^{-\mathrm{T}} \mathrm{R}_{t-1}^{-1} \Phi_{n}\right)^{-1}$ is scalar, Therefore the inverse of $\mathrm{R}_{1}$ can be obtanned as,
$\mathrm{R}_{t}^{-1}=\mathrm{R}_{i-1}^{-1}-\frac{\mathrm{R}_{i-1}^{-1} \Phi_{\mathrm{n}} \Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{i-1}^{-1}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{t-1}^{-1} \Phi_{n}}$

Multıplying each side of the Eqn (42.11) by $\eta_{1}$ and using Eqn (4.2.9)
$\mathrm{R}_{t}^{-1} \eta_{t}=\mathrm{R}_{t-1}^{-1}\left(\eta_{t-1}+\Phi_{n} \zeta_{n}\right)-\frac{\mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}} \Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{\mathrm{t}-1}^{-1}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{\mathrm{i-1}}^{-1} \Phi_{n}}\left(\eta_{t-1}+\Phi_{n} \zeta_{n}\right)$

Simplifying the above, therefore -
$\mathrm{R}_{t}^{-1} \eta_{t}=\mathrm{R}_{t-1}^{-1} \eta_{t-1}+\mathrm{R}_{t-1}^{-1} \Phi_{n} \zeta_{n}-\frac{\mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}} \Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{t-1}^{-1}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{\mathrm{l}-1}^{-1} \Phi_{n}} \eta_{t-1}-$

$$
\frac{\mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}} \Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1} \Phi_{n}} \Phi_{n} \zeta_{n}
$$

Using Eqn(4.2.8)and simplifying further, That is

$$
\bar{D}_{t}=\bar{D}_{i-1}-\frac{\mathrm{R}_{t-1}^{-1} \Phi_{\mathrm{n}}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1} \Phi_{n}} \Phi_{n}^{\mathrm{T}} \mathrm{R}_{t-1}^{-1} \eta_{i-1}+\left(\mathrm{R}_{t-1}^{-1} \Phi_{n}-\right.
$$

$$
\frac{\mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}} \Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1}}{\left.1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1} \Phi_{n}\right) \zeta_{n}, ~}
$$

Using Eqn(4.2 8) and simplifying the above further, That is

$$
\begin{array}{r}
\bar{D}_{t}=\bar{D}_{t-1}-\frac{\mathrm{R}_{1-1}^{-1} \Phi_{\mathrm{n}}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1} \Phi_{n}} \Phi_{n}^{\mathrm{T}} \bar{D}_{t-1}-\frac{\mathrm{R}_{i-1}^{-1} \Phi_{\mathrm{n}}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{i-1}^{-1} \Phi_{n}}\left(1+\Phi_{n}^{\mathrm{T}} \mathrm{R}_{t-1}^{-1} \Phi_{n}-\right. \\
\left.\Phi_{n}^{\mathrm{T}} \mathrm{R}_{t-1}^{-1} \Phi_{n}\right) \zeta_{n}
\end{array}
$$

Finally it gives rise to
$\bar{D}_{t}=\bar{D}_{t-1}-\frac{\mathrm{R}_{t-1}^{-1} \Phi_{n}}{1+\Phi_{n}^{\mathrm{T}} \mathrm{R}_{i-1}^{-1} \Phi_{n}}\left(\Phi^{\mathrm{T}} \bar{D}_{t-1}-\zeta\right)$

In the above equation, the term within the brackets can be replaced by what is known as the Displaced Frame Difference, DFD. Thus,

$$
\begin{equation*}
\bar{D}_{t}=\bar{D}_{t-1}-\frac{\mathrm{R}_{1-1-1}^{-1} \Phi_{n}}{1+\Phi_{\mathrm{n}}^{\mathrm{T}} \mathrm{R}_{1-1}^{-1} \Phi_{n}}\left[D F D\left(\mathrm{x}, \mathrm{y}, \bar{D}_{t-1}\right)\right] \tag{Eqn42.13}
\end{equation*}
$$

To avoid matrix inversion at each iteratıon, Eqn (4.2.13) can be sımplıfied by ignoring the $x$ and $y$ cross terms in calculating $\phi_{n}$ and $R$. Thus, from Eqn (4.2.4) and Eqn (4.2.7),

$$
\begin{equation*}
\Phi_{n}(\mathrm{x})=E D I F \quad \text { and } \quad \Phi_{n}(\mathrm{y})=L D I F \tag{Eqn4.2.14}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{R}(\mathrm{x})=\frac{1}{M} \sum_{m} E D I F_{m}^{2} \quad \text { and } \quad \mathrm{R}(\mathrm{y})=\frac{1}{M} \sum_{m} L D I F_{m}^{2} \tag{Eqn4.215}
\end{equation*}
$$

Applying Eqn(4 2.14) to Eqn(4 2.13), the components of the motion displacement

$$
\begin{align*}
& \bar{d}_{t}(\mathrm{x})=\bar{d}_{t-1}(\mathrm{x})-\frac{E D I F}{\frac{1}{M} \sum E D I F^{2}+E D I F^{2}}\left\{D F D\left[\mathrm{x}, \mathrm{y}, \bar{d}_{t-1}(\mathrm{x})\right]\right\}  \tag{Eqn4.2.16}\\
& \bar{d}_{t}(\mathrm{y})=\bar{d}_{t-1}(\mathrm{y})-\frac{L D I F}{\frac{1}{M} \sum L D I F^{2}+L D I F^{2}}\left\{D F D\left[\mathrm{x}, \mathrm{y}, \bar{d}_{t-1}(\mathrm{x})\right]\right\}
\end{align*}
$$

According to the developed new algorithm for motion compensated image sequence compression [112].

$$
\begin{equation*}
\hat{D}_{x}=\hat{D}_{x-1}-\frac{E D I F}{\sum_{R} E D I F^{2}+E D I F^{2}} *\left\lfloor\varepsilon^{*}(E D I F)\left(\hat{D}_{x-1}\right)-F D I F\right] \tag{Eqn4.2.18}
\end{equation*}
$$

or

$$
\begin{equation*}
\hat{D}_{x}=\hat{D}_{x-1}-\frac{E D I F}{\sum_{R} E D I F^{2}+E D I F^{2}} *\left\lfloor\varepsilon^{*}(E D I F)\left(\hat{D}_{x-1}\right)+D F D\right\rfloor \tag{Eqn4219}
\end{equation*}
$$

Which can also be sımplified as

And simılarly

$$
\begin{equation*}
\hat{D}_{y}=\hat{D}_{y-1}-\frac{L D I F}{\sum_{R} L D I F^{2}+L D I F^{2}} *\left\lfloor\varepsilon^{*}(L D I F)\left(\hat{D}_{y-1}\right)-F D I F\right\rfloor \tag{Eqn42.21}
\end{equation*}
$$

or

$$
\begin{equation*}
\hat{D}_{y}=\hat{D}_{y-1}-\frac{L D I F}{\sum_{R} L D I F^{2}+L D I F^{2}} *\left\lfloor\varepsilon^{*}(L D I F)\left(\hat{D}_{y-1}\right)+D F D \mid\right. \tag{Eqn4222}
\end{equation*}
$$

Which can also be simplified as

$$
\begin{equation*}
\hat{D}_{y}=\hat{D}_{y-1}-\frac{L D I F}{\sum_{R}^{L D I F^{2}+L D I F^{2}} *\left[\varepsilon^{*} L D I F * D F D\right]} \tag{Eqn4.2.23}
\end{equation*}
$$

Where $\varepsilon$, the convergency parameter to control the rate of convergence, is recommended (experimentally) to be in the region of 0.98 to 1.00 .

The above recursion to update $\hat{D}_{k}$ is carried out only in the moving area of the current frame, 1.e., for those pels where

$$
\begin{equation*}
\sum_{j=-p}^{+p}\left|b\left(z_{k+\mu}, t\right)-b\left(z_{k+j}, t-\tau\right)\right| \geq \text { Threshold } \tag{Eqn4.224}
\end{equation*}
$$

Otherwise
$\hat{D}_{x}=\hat{D}_{x-1}$ and $\hat{D}_{y}=\hat{D}_{y-1}$

The threshold, Threshold, is pre-selected. It should be noted that the choice of the Threshold is mainly based on camera noise, light variation, and so on. In this thesis for the sequences used, a figure of 8 to14 out of 256 intensity levels was chosen. A poor chore of the Threshold figure, far off from the true value will cause errors

Recursively $\hat{D}_{x}$ and $\hat{D}_{y}$ are updated using Eqn (4.2.20) and Eqn (4.2.23) where for each step, the update term attempts to improve the estımate of $D$. The ultımate goal is the minimization of the magnitude of prediction error, $D F D$ If a pel at location $Z_{a}$ is predicted with $\hat{D}_{x-1}$ and $\hat{D}_{y-1}$ having intensities of $b\left(z-\hat{D}_{x-1}, t-1\right)$ and $b\left(z-\hat{D}_{y-1}, t-1\right)$ respectively and results in a prediction error of $\operatorname{DFD}\left(\mathrm{z}, \hat{D}_{k-l}\right)$ the prediction should attempt to create new estımations, for $\hat{D}_{x}$ and $\hat{D}_{y}$ such that :-

$$
\begin{equation*}
D F D\left(\mathrm{z}, \hat{D}_{k}\right)\left|\leq\left|D F D\left(\mathrm{z}, \hat{D}_{k-1}\right)\right|\right. \tag{Eqn42.26}
\end{equation*}
$$

ie the prediction error is reduced.

The predictor is based on intensities in the previous frame and current frame

### 4.2.1 Interpolation

Using affirmation for the new algorithm to get the first estimated value for the displacement vector, the intensity of the pel displaced is estımated by means of an interpolation technique. For consistency with the pel-recursive motion estımation shown in pervious chapter and for simplicity, the same algorithm can be used, that is the following formula -

$$
\begin{equation*}
I=I_{D}+\hat{D}_{x}\left(I_{A}-I_{D}\right)+\hat{D}_{Y}\left(I_{B}-I_{D}\right)+\hat{D}_{X} * \hat{D}_{Y}\left(I_{D}+I_{C}+I_{A}+I_{B}\right) \tag{Eqn42.11}
\end{equation*}
$$

Finally the displacement vector $D$ is decomposed into two parts, the integral part and the non-integral part $D_{F}$

### 4.3 Implementation and Experimental results

As far as implementation and simulation are concerned, the great importance of the work lies in the calculation of the components defining the formula, i.e. such as line, element, and displacement frame differences. For these, different ways and techniques can be examıned; e g : interpolation, non-interpolation, averaged using different causal supports (Figure 4.3.1), etc. Not every single pel needs to be motion compensated, therefore a masking mechanısm should be utilized eg $\cdot$ - where frame difference, $|F D I F| \leq$ threshold, no prediction is needed. This is classed as a non-moving area.

X X X X X X X<br>X X X X X X X<br>X X X X O

Figure 4.3.1 : A second order causal support
The computation involved in Eqn (4.2.20) and Eqn (4.2.23) is performed recursively. At each iteration the estimated motion displacement is applied to measure a new DFD. This would first require obtaining the location of the displaced pel on the previous frame, based on the estimated components of motion displacement. Since the motion estimates are expected to be non-integer, the luminance value of the displaced pel is predicted by a two
dimensional interpolator which uses the four corners of the surrounding pels in a two dimensional grid. In our experiments, the DFD is measured at two locations with reference to the current pel; the pel above (1.e., previous line), and the previous pel along the same line. The average of the two DFDs (with equal weightings) is then used to update the displacement estımates.

In this thesis for simplicity, the non-interpolated averaged 3 by 2 causal support (Figure 4.3.2) is used for line and element differences and displacement frame difference as normal (a pel value of a frame - the interpolated pel value of previous frame), with the convergency parameter, $\varepsilon=0.98$. The maxımum update limit for consistency proposes was chosen to be $|F D I F| \leq 9$ for non-moving areas This is done for two different sequences, "Suzie" and "Salesman".

$$
\begin{array}{llll}
\mathrm{X}(1) & \mathrm{X}(2) & \mathrm{X}(3) & \mathrm{X}(4) \\
\mathrm{X}(5) & \mathrm{X}(6) & \mathrm{O}
\end{array}
$$

Figure 4.3.2 : causal support.

It should be noted that as for causal support concerns, the previous pel value of $X(6)$ and the last two previous line pel values of $X(3)$ and $X(4)$ have the most importance in order to estimate any of element , line, frame or displace frame difference (see Figure 43 2)

Further constraint or limitation on the predictor can be used to augment the prediction strategy, the following rule (Eqn 43.1 ) can be used to switch or move adaptively between them on a pel by pel basis.

$$
\begin{equation*}
\sum_{J=-m}^{+m} w_{i}\left|F D I F\left(z_{k+J}\right)\right| \geq \sum_{J=-m}^{+m} w_{i}\left|D F D\left(z_{k+J}, \hat{D}_{k}\right)\right| \tag{Eqn4.3.1}
\end{equation*}
$$

Figures 433 to 4.3 .11 show and indicate the validity of the new pel-recursive motion estimation algorithm. The graphical and pictorial results are compared for the existıng and the new algorithms. In the pictorial results (Figures 4.34 and 4.3.6) it can clearly, but subjectively be seen that an improvement occurs in reducing the prediction error for two different successive frames of "Suzıe" and "Salesman".

In the Figures 433 to 4.3.9, the clean frames (the PCM value of pels in the frame) were used In Figures 4.3.10 and 43.11 , reconstructed frames (as in most codecs, the clean frame is not avalable in the decoder, therefore the predicted quantized reconstructed frame is used throughout) were used.

a) No frame skip comparison

b) One frame skip comparison.

c) Two frame skip comparison.

Figure 4.3.3 Suzie comparison after three iterations with the previous frame clean.


d) Prediction error for Suzies in a \& b
(for the proposed)

Figure 4.3.4 Prediction error comparison for two successive frames of Suzie after three iterations with previous frame clean.

a) No frame skip comparison.

b) One frame skip comparison.

c) Two frame skip comparison.

Figure 4.3.5 Salesman comparison after three iterations with the previous frame clean.

a) PCM frame of Salesman (previous frame)

c) Prediction error for Salesman in a \& b (for the gradient)

b) PCM frame of Salesman (present frame)

d) Prediction error for Salesman in a \& b
(for the proposed)

Figure 4.3.6 Prediction error comparison for two successive frame of Salesman after three iterations with the previous frame clean.


Figure 4.3.7 Suzie companson for 20 iterations with previous frame clean.


Figure 4.3.8 Salesman comparison for 20 iterations with the previous frame clean

a) No frame skip comparison

Figure 4.3.9 Car comparison after three iterations with the previous frame clean.

a) No frame skip comparison.

b) One frame skıp comparison.

c) Two frame skip companson.

Figure 4.3.10 Suzie comparison after three iterations with the previous frame reconstructed.

a) No frame skıp comparison

b) One frame skıp comparison.

c) Two frame skip comparison.

Figure 4.3.11 Salesman comparison after three iterations with the previous frame reconstructed.

### 4.4 Results and summary

As was expected the outcome of the use of the proposed algorithm over the existing modified steepest gradient algorithm is that the new pel recursive algorithm has proven to produce a better result than the existing ones The Figures 433 to 4311 indicate the statement regardless to whether clean or reconstructed frames are employed.

In the Figures $433,4.35,4.310$, and 4.3.11; the comparisons were done for different frame skips. This is mainly to show that the proposed algorithm does always have better performance over the existing ones. In real time practical applications one may have to use sequences with different frame skips, especially in situations where we are dealing with sequences consisting of bigger or larger frames.

As can be seen from the Figures 4.3 3, 4.3 5, and 4.3.7-4.3.11; there has been a great improvement of 15 dB , over the existing pel recursive algonthm This is achieved by the proposed pel recursive algorithm. This is quite a substantial improvement when compared with the case when no motion compensation is employed. The graphs in Figures 4.3.4 and 4.3.6 ( c and d sections) depict that the proposed pel recursive algorithm should result in a good prediction error in comparison with the existing pel recursive algorithm.

Strictly speaking, the proposed pel recursive gradient has quite fast convergency, therefore fewer iterations will be needed. In spit of the fast convergency which is acceptable by most applications, it should not be overlooked that in some sequences little more convergence can be obtain by increasing the number of iterations (see Figures 43.7 and 4.38 )

In some cases we might deal with sequences with very fast motion, where even block matching motion compensation often results in a poor compression; the proposed pel recursive algorithm can show good improvement in reducing the prediction error

Never the less, the result from the proposed algorithm can not compete with the old block matching scheme as it is; this would require a better pel recursive algonthm to be developed in the future.

Finally looking at Figures 4.3.4(c) and 4.3.4(d) and Figures 4.36 (c) and $436(\mathrm{~d})$, in these images, relatıvely darker or lighter patches represent the degree of inaccuracies in estımating the components of the motion displacement. Comparing the two images 4.3.4(c) and 4.3.4(d) and also Figures 4.3.6(c) and 4.3.6(d) confirm the superior performance of the proposed scheme over the modified steepest-descent algonthm, particularly in regions where the motion activities are relatively high.

## Chapter 5

## Application for hierarchical system

pel recursive motion compensation has not yet been able to replace block matching motion compensation in hierarchical systems (e g. H.263, MPEGs, and so on); In the light of this, this chapter, looks at an application developed in view of a paper by Bierlıng [113].

### 5.1 Overview

Block matching is a widely used displacement estimation method, and can easily be implemented in hardware. Using block matching, a displacement vector is obtained by matching a rectangular measuring window, consisting of a certain number of neighboring picture elements, with a corresponding measuring window within a search area, placed in the next successive or the next preceding image. The match is achieved by searching the spatial position of the extremum of a matching criteria (e $\mathrm{g}: \mathrm{MAD}$, the mean absolute
displaced frame difference) The resulting displacement vector is then taken to be the motion vector for all the picture elements inside the measuring window.

The basic assumption of the displacement estımation technıques used is that neighboring picture elements have the same motion parameters. It is not possible to obtain a displacement estımate for every isolated picture element of a block

The known block matching techniques provide fairly good results for motion compensation prediction in general, as their computation and the complexity are low and the prediction error is remarkably small when using the achieved motion compensation However, the match obtained by block matching is an optımum only in the sense of a minımum MAD, the mean absolute displaced frame difference; but frequently it does not correspond to the true motion of the objects.

The reliability of the displacement estımate depends on the size of the chosen measurng windows, in conjunction with the present amount of motion. The estimate tends to be unreliable, if small measurng windows are used and the displacement is large. The smaller the measurng window, the higher is the probability that there are blocks (and hence will be selected by the matching criteria) in the corresponding search area, contannng a more similar or identical pattern of picture elements, although there is no correspondence in the sense of motion. Therefore, large measuring windows are required in order to cope with large displacement. Thus, the known block matching techniques fail frequently as a result of using a fixed measuring window size [113]

In order to take into account the above problem, a hierarchical block matching for displacement estimation was suggested by Bierling [113]. The hierarchical structure uses distınct sizes of measuring windows at different levels of the hierarchy The estımator starts with large measurng windows at the highest level. From one level to the next level of the hierarchy, the size of the measuring window is decreased The displacement estimate is obtained recursively, i.e. at each level of the hierarchy, the resulting estımate serves as an inital guess for the next lower level. The first hierarchy levels serve to provide a reliable
estimate of the major part of a large displacement, whereas the last levels serve to estımate the remaining part of the displacement accurately. Figure 5.1 shows the principle of hierarchical displacement estimation for the example of three levels. A displacement vector between two successive frame of images is achieved as the sum of three estimates, using three different measuring window sizes [113]. The second herarchy level starts motion compensation using the results of the first level, i.e. the search points of the search procedure are displaced by the estimate of the first level, and carries on the same way through the rest of the levels recursively.


Figure 5.1 Principle of hierarchical displacement estımatıon for three hierarchy levels

### 5.2 Application

In order to reduce the computational effort resulting from large windows, sub-sampling inside the measurng window can be performed [113]. This raises the idea of applying the same method of sub-sampling to pel-recursive, in particular for the proposed pel-recursive method. If the task is proven satisfactory, this can perhaps be used to have an affect of final tuning on the displacement which is estımated by block matching.

Looking at the example from another angle the performance of any hierarchical codec can be improved by introducing pel-recursive motion compensation. In view of this let allow and investigate if pel-recursive motion compensation can be active side by side in the presence of block matching motion compensation. This may possibly have some benefit for codecs standards like H 263, MPEGs, or any other.

Basically the way the method is structured is as follow:-

1) applying block matching motion estimation on two successive image frames, and producing displacement vectors (i e.- for each block of $16 \times 16$ pels)
2) Passing the images through a low pass filter in order to have them down sampled, that is to shrink the images ( $1 \mathrm{e}:-$ by $16 \times 16$ ). Two dimensional Q.M F (quadrature Mirror Filter) can be used as a crude substitution for the low pass filter. A further rough substitution can be achieved by takıng the intensity of the first DCT coefficient (DC coefficient) for each block (1.e - block of $16 \times 16$ pels); which is really the average intensity of pels in each block.
3) Allocatıng each block matching displacement vector as the motion vector for every pel of the down sampled images.
4) Apply the pel-recursive motion estimation algorithm on the down sampled (or shrinked) images by taking the motion vectors as the intial iterative estımation of pel-recursive estimation
5) The resulting motion vectors are to be the final tuning on block matching motion estımation displacement vectors

As for the experimental results concerned in this thesis; the above procedure is applied to images with block sizes $8 \times 8$ Figures 5.2 .1 to 5.2 .6 shows the graphical result for two sequences of "Suzıe" and "Salesman". As it can be seen the outcome is not very promising.

Finally the above procedure could also be carried out for any other block sizes 1 e a block of $4 \times 4$....Etc.

One of the drawbacks of the above method is that due to statistical randomness of sub sampled images, which causes an estimation of the error for each pixel, there is some possibility of uncontrolled overshoot as can be seen from the graphs in the figure 52.4 . This is mainly due to the situation that intial motion vector is independently estimated for every pels.

a) No frame skip companson

Figure 5.2.1 Suzie block recursive comparison after three iterations with previous frame clean, without half pel accuracy on block matching.

a) No frame skıp comparison.

Figure 5.2.2 Salesman block recursive comparison after three iterations with previous frame clean, without half pel accuracy on block matching.

a) No frame skip comparison.

Figure 5.2.3 Suzie comparison after three iterations with previous frame reconstructed, with half pel accuracy on block matching, and initial motion vectors set to zero.

a) No frame skip comparison.

Figure 5.2.4 Salesman comparison after three iterations with previous frame reconstructed, with half pel accuracy on block matching, and initial motion vectors set to zero.

a) No frame skip comparison.

Figure 5.2.5 Suzie block recursive companson after three iteration with previous frame reconstructed, with half pel accuracy on block matching.

a) No frame skıp comparison.

Figure 5.2.6 Salesman block recursive comparison after three iterations with previous frame reconstructed, with half pel accuracy on block matching.

# Chapter 6 

## Combined block matching and pel-recursive techniques

In the application of pel-recursive motion compensation, even the propsed pelrecursive as well as the modified pel-recursive steepest descent gradient did not show a promising performance when used as block recursive algorithm (refer to chapter five) In view of the situation that has ansen, it is a good idea to investigate the possibility of combining the two estımator techniques, pel-recursive and Block matching, in such a manner that block matching can assist the pel-recursive approch to form a Hybrid system. Here we have to investigate further the possibility of developing a hybrid system from block matching and pel-recursive systems

### 6.1 Local versus Global Minima

Steepest descent is probably the simplest numerical optimization method. It updates the present estımate of the location of the mınımum in the direction of the negative gradient, called the steepest descent direction Recall that the gradient vector points in the direction
of the maximum. That is, in one dimension (function of a single variable), its sign will be positive on an "uphill" slope Thus, the direction of steepest descent is in the opposite direction.

The descent gradient approach however suffers from a serious drawback'- the solution depends on the initial point. If we start in a "valley", it will be stuck at the bottom of that valley, even if it is a "local" minimum (Figure 6.1.1). Because the gradient vector is zero or nearly zero, at or around a local minimum, the updates become too small for the method to move out of a local mınimum. One solution to this problem is to initialize the algorithm at several different starting points, and then pick the solution that gives the smallest value of the criterion function However, this method usually requires significantly more processing time


Figure 6.1.1 Demonstrative Graphical sketch of local and global minima.

### 6.2 Hybrid system

As has been seen from the papers [39] [110] [114] [111] and through the simulation (chapter 3) none of the algorithms by Netravalı and Robbıns [39], Newton-Raphson [110], Caffero and Rocca [114], or Bergmann [111] give full convergence for every pel to produce a perfect estimation for motion. In addition, some pels converge to unsatisfactory figures and sometımes become unstable leading to the conclusion that the algorthms suffer from some form of instability.

The aim of compression is based on the idea that it is possible to find displacement or motion vectors for each pel so as to have a minımum error image signal Going through an iterative process (1.e. steepest descend algorithm), it is not necessanly true that one can find an area of a global minimum, therefore we face a situation where one lands on a local mınımum and perhaps ultimately gets to the actual local mınımum or goes into oscillation and becomes unstable.

In spite of all the above, the algorithm by Netravali and Robbins [39] has shown convergence with less overshoot in relation to the other three algonthms [115], with the cost of a high number of iterational computations for estimation of the displacement vectors. If we define the stability constrain criteria as

$$
\begin{equation*}
\left|D-\hat{D}_{l-1}\right|\langle | D-\hat{D}_{l} \mid \tag{6.1.1}
\end{equation*}
$$

the algorithm by Netravalı and Robbins shows better stability as it requires that the update vector be always directed towards and not opposite to the actual displacement

It has been seen that the initial estimation of displacement vectors has a great effect on determinıng final motion estimation by the iterative process of the steepest descent algorithm or the proposed algorithm. As can be seen from Figure 6.1.1, if the intial estimation of displacement vectors are not well chosen, when the steepest descent algorithm is applied, after a few iteratıons, one can have a situation where a local mınımum is estimated instead of the global mınımum estımation.

In view of the affirmation argument, in order to estimate the displacement vectors with more accuracy, virtually for every single pel and particularly the pels where therr motion vector happened to be situated on local mınımum instead of global mınımum, will not be estimated correctly. So the prediction error associated with these pels will not be accurately estımated. Therefore, to overcome this inaccurate estımation it is possible to suggest that an easy and simple solution would be to chose the intial displacement vector by a different mechanism. Havıng chosen the right intial displacement vector, then the motion vector resulting from first stage can be feed back into the iterative processing of the pel-recursive system This led to the idea of the hybnd system. Relating the above technique to the problem in this thesis, block matching motion estımation is combined with pel-recursive motion estımation to form a hybrid system As for the experimental results, the block matching algorithm is applied to a sequence of a moving images, producing motion vectors for every block of the image and therefore a higher signal to noise ratio.

One of the drawbacks of motion estimation using block matching is that displacement is estimated as one estimation for each block, for example; a block of 16 by 16 pels. This should not necessarily apply to every pixel of the block as some pels may not be moving pels e g.-- blocks contaınıng edges. This also may cause a blocking effect which is one of the drawbacks of the method used.

One needs to transmit a displacement estımation for each block as well as the number of blocks with no motion estimation. This causes more overhead to be transmitted resulting in transmission of a higher number of bits per second.

The Netravalı algorithm and the modified algonthm were employed to investigate the advantage and disadvantage of the motion estumation by the pel recursive method. It has been seen that the Netravalı algorithm itself suffers from some major defects eg:- lack of divergence and stability which manifests itself through certain pels.

The pel-recursive algonthms try to munmize the prediction error by locking into etther a local or the global minımum. As the algorithm iteratively tries to force the error to a mınımum value, which is determıned by the onginal motion displacement estimation, one should note that the lack of convergence or stability caused by being in the vicinity of a wrong mınımum may give rise to a local mınimum instead of global minimum. This may be the main problem associated with pel-recursive algonthms in general.

Combinıng Block Matching motion estımation and pel recursive motion estimation in a complex manner has shown some improvement of the signal to norse ratio of Block Matching with no extra cost on the overhead, producing new publishable results which stıll can be improved further. This actually means that, the block matching does the main displacement estimation and the pel-recursive does the fine tuning on each pel.

One of the advantages of this method is that it does not require any extra overhead in transmission because it does not need to transmit any extra information for the motion estimated than is needed for the block matching technique.

In this thesis, for example, by employing H. 263 and using block matching without $1 / 2$ pel accuracy; the energy was measured to be 20.52 dB for an image in a sequence And also employing H 263 and using block matching with $1 / 2$ pel accuracy, the energy was reduced by a factor of 0.04 dB to a figure of 20.48 dB . This is also to justify the obvious which is, using block matchıng with $1 / 2$ pel accuracy is more advance than without $1 / 2$ pel accuracy. This is the one of the main advantages of H 263 over H 261 (H261 does not have $1 / 2$ pel accuracy feature)

Using block matching (by employing H 263 with $1 / 2$ pel accuracy) and pel recursive motion estimation combined as a hybrid system reduces the energy of the error substantially. Employing the new pel recursive motion estimation would further reduce the energy of the error.

### 6.3 Implementation and Experimental results

In order to show the outcome resulting from the hybrid system, motion vectors were estimated using the standard traditional method of block matching H. 263 with half pel accuracy was employed to generate these displacement vectors. Having estumated the motion vectors for every individual block (for example, block of $16 \times 16$ pel), they are assigned to be the inital estımation for the pel recursive motion estimation for final tunng of the estimations.

As for implementation of the simulation, great accuracy was needed when calculating components representing the pel recursive formula, such as line, element, displacement frame differences, and so on. For this, different ways and techniques can be utilized and examined; e.g . interpolated, not interpolated, averaged using different causal supports, and etc. It should be noted that not every single pel is to be motion compensated, therefore a masking mechanısm needs to be used, e.g.: where frame difference, $|F D I F| \leq$ threshold; no prediction is needed (non-moving area).

In this thesis in order to be uniform throughout the algonthms implementation and simulation for calculation of line and element differences, non-interpolated averaged 3 by 2 causal support (Figure 6.3.1) is used. For displacement frame difference the interpolated pel value of previous frame is subtracted from the average pel intensities of $X(3)$ and $X(6)$ of present frame. As far as the proposed algorithm is concerned, different convergency parameters have been used; that is where the absolute value of an element or line difference is less than 11 , the convergency parameter $\varepsilon=0.8$, other wise $\varepsilon=0.7$. Here one can have a good educated view that, this is a reasonable indication of improvement by attempting to have the convergency parameter adaptıve.

The maximum update limit for consistency purposes were chosen to be $|F D I F| \leq 9$ for non-moving areas. The results show that further constraint or limitation is needed to accomplish a better estimation of motion vectors. As an example, where motion vectors squared are less than or equal to 1 , not to update the motion vectors. It should also be mentioned that more constraint or limitation or toggling of the predictor could result in
better tuning of the motion vectors. This is done for three different sequences, Suzie, Salesman, and Car.

$$
\begin{array}{llll}
X(1) & X(2) & X(3) & X(4) \\
X(5) & X(6) & O
\end{array}
$$

Figure 6.3.1 : A 4 by 2 causal support.

Figures 6.3 .2 to 6.3 .7 show and indicate the validity of the new pel-recursive motion estımation algorithm [112] in comparison with the existing pel-recursive motion estimation algorithm. The graphical results provide comparison for existing and the new algorithms.

Consider the motion estımation using a pel recursive motion estımation. Experimentally it has been shown that it produces an average improvement of over 0.5 dB in signal to noise ratıo.

a) No frame skip comparıson

b) One frame skip comparison.

c) Two frame skip comparıson.

Figure 6.3.2 Suzie companson after three iterations with the previous frame reconstructed with half pel accuracy on block matching.

a) No frame skip companison

b) One frame skip comparison.

c) Two frame skip comparison

Figure 6.3.3 Salesman comparison after three iterations with the previous frame reconstructed, with half pel accuracy on block matching

a) No frame skip companson.

Figure 6.3.4 Car comparison after three iterations with the previous frame clean, without half pel accuracy on block matching.

Setting the initial motion vectors to zero which almost in effect is turning off the block matching motion estımator will produce sımılar results to the one generated by the new proposed pel-recursive motion estimation. This is another justification of the obvious, that the proposed method of pel-recursive motion estımation in general has not being able to compete with block matching motion estimation as it can be seen from the Figures 63.5 637

a) No frame skip comparison

b) One frame skıp comparison


Figure 6.3.5 Suzie companison after three iterations with the previous frame reconstructed, with half pel accuracy on block matching, and initial motion vectors set to zero

a) No frame skıp comparison.

b) One frame skip comparıson.

c) Two frame skip comparison.

Figure 6.3.6 Salesman comparison after three iterations with the previous frame reconstructed, with half pel accuracy on block matching, and intial motion vectors set to zero.

a) No frame skıp comparıson.

Figure 6.3.7 Car comparison after three iterations with the previous frame clean, without half pel accuracy on block matching, and initial motion vectors set to zero.

### 6.4 Conclusions

According to the results of Figures 6.3 .2 and 6.3 .3 whenever the hybrid system is used, a substantial improvement in compression is being achieved A greater improvement is shown through using the hybrid system with the proposed algorithm

Figure 6.3.4 indicate that for a fast moving image like the "Car" sequences ( 1 e Figures 2515 and 2.5.16) the hybrid system does show some improvement over the traditional block matching method.

If the initial value motion vectors are set to zero as obtained from the block matching part of the hybnd system (disregarding the effect of block matching from the system), then the situation of pel-recursive versus block matching will arise. Figures 635 and 6.36 show the result when there is no intial value estimator present.

# Chapter 7 

## Conclusions and Further works

TThis chapter presents a general view of the results described in this thesis and summaries the contribution of new knowledge for the implementation of steepest gradient pel-recursive motion estimation. A further discussion is also developed based on an example in chapter 5 , utilizing the new pel recursive algorthm to a certan degree. It goes further to discus the effect of involving block matching motion estumation in pel-recursive motion estumation to form a hybrid system

### 7.1 Conclusion

As an educated guess or rule of thumb one may suggest that the pel-recursive steepest decent gradient should perform better than block matching in estımatıng motion for sequences of moving images. The pel-recursive algorithms look at images pel by pel, where as block matching algonthms consider a block of an ımage as a whole However, experimental results have shown otherwise to the extent that the pel-recursive approach could not get be used in international standards for low bit transmission e.g.:- H 261, H 263, MPEGs and so on. Figures 2.4.1.2 and 2.4.1.3 and Figures 3.6.3-3 68 and Figures 36.3-36.8 and Figures 3.9.1-3.9 6 show a good indication of this.

Figures 2.41 .2 and 2.41 .3 in general depicted that in block matching motion estimation energy of the errors are very low, that is with very high signal to noise ratio, when compared with the situation in which there is no motion estimation present. These results have been generated using H 263 for different situations, estımating motion with and without half pel accuracy. In almost all codec standards, the motion estımator is designed on the basis of a block matching motion algorthm

In chapter 3 the state of the art of existing pel-recursive algorithms have been implemented and simulated for situations where the previous frame has been clean or a reconstructed image. Figures 3.6.3-3.68 depicted that pel-recursive motion estimation shows a good low value of average energy for the error image signal, but still energy of the error is higher than for block matching. Figures $391-3.9 .6$ indicate that even where there is some improvement by making $\varepsilon$, the convergency factor, adaptıve, the error mage signal is not low enough as far as block matching is concemed, even though the error sıgnals of the images are lower than the case where $\varepsilon$ is not considered to be adaptıve.

Taking a step further, a new algorithm for pel-recursive motion estimation has been proposed, implemented and simulated as detailed in chapter 4. The graphical and pictorial results show and indicate that the average error mage signal resulting from the new algorithm is much lower than that for the existing pel-recursive algorthm. Figures 4.21 -

427 compare the existing state of the art with the new algorithm for motion estimation These justify that the new algorithm is working and produces a better result than existing ones. However, the average error resulting from new algorithm on its own is not lower than that for the case of block matching.

Chapter 5 shows a crude example of employing the new pel-recursive algorithm. To see whether or not pel-recursive in general can contribute further improvement into international standards such as H. 263 or any other hierarchical codecs, side by side of block matching motion compensation, the new gradient is blocked recursively are applied to sequences of images and detaled in chapter 5, resulting the graphs in Figures 5.2.1-5.2.7. As can be seen the graphical results are not very promısing in comparison to the situation if one would just use the block matching technique.

Considering the above case, it shows that it would not be feasible to have both the pelrecursive motion compensation and block matching motion compensation present as separate elements of estımator in codecs. Then, the thesis moves into new work by combining the two some what different algorithms of block matching and pel-recursive into a hybrid system. The resulting hybrid system does show the average error image sıgnal levels to be lower, when compared with the old block matching algorithm by at least a 05 dB (for comparison, introducing half pel accuracy into block matching results in an 0.05 dB improvement over block matching on its own, for, the average error image sıgnal) Figures 6.3.2 and 6.3.3 depict the variation of average error signals for two different sequences, justifying the improvements of the hybrid system over block matching or pel-recursive on therr own.

It should be noted that the above conclusions in this thesis are based generally on experiments which were performed for about 20 frames of two different well known sequences of moving images ("Suzie" and "Salesman")

### 7.2 Suggestions for further research work and recommendations

Since block matching motion compensation has become standardized, research and work on pel recursive motion compensation has not been given any significance and has virtually stopped. This may be due to the better performance of block matching over pel-recursive for motion compensated image compression.

Now hybrid motion compensation can be employed for image compression, resulting in a more advanced performance than each of the two aforementioned motion compensation technıques, "block matching and proposed pel-recursıve". This could open a new door for research and development in image compression areas investigating the usage of pelrecursive algorithms or as hybnd systems.

There are many papers relating to the development and further development into block matching techniques since the allocation of the standards. As a starting point a good example of the application suggestion is given in chapter five. In general the developments which were already applied to block matching could be applied to the hybrid system With reference to the these discussions, it is certain that there are many methods and developments, whether small or large, applicable for block matching motion compensation where pel- recursive and block matching motion compensation could work together A good example of this is the application suggestion given in chapter five. The block recursive motion compensation idea was developed with reference to a paper for block matching motion compensation [116].

In pel-recursive work, in general, the predictors used are based on intensities in the previous frame but not previous frames or calculated displaced previous frame. Of course, as for further work, other predictors can be employed in order to augment the prediction strategy. This in turn should enhance the system performance.

In chapter 3; it has been shown that pel-recursive motion estimation can improve its performance by makıng $\varepsilon$, the convergency factor, adaptive. In the onginal pel-recursive
motion algorithm, $\varepsilon$ was chosen to be a fixed variable and this has been improved to be fully adaptıve as "modified pel-recursive motion algorthm". In chapter $4 ; \varepsilon$, the rate convergency controller, is recommended to be a constant variable But as can be seen from chapter 4 , a better result is produced by setting different values for different conditions. In turn this suggests that making $\varepsilon$ adaptive should improve the performance of the proposed motion estimation algorithm. In view of this, it would be sensible to conduct further work toward improvement of the proposed algorithm

Finally, further work can be carned out in view of the example in chapter 5. This can basically be done to improve the performance of a herarchical codec This is done firstly by applying block matching motion estimation on image frames, and obtaining displacement vectors (ie. - for each block of $16 \times 16$ pels). The images are then passed through a low pass filter in order to have them down sampled, that is to shrink the images (i.e.:- by $16 \times 16$ pels). Various methods can be employed for this, for example, Two dimensional Q.M.F (quadrature Mirror Filter) can be used as a crude substitution for the low pass filter. A further rough substitution can be achieved by taking the intensity of first DCT coefficient (DC coefficient) for each block (i e - block of $16 \times 16$ pels); which is really the average intensity of pels in each block. Taking the value of each block matching displacement vector as the motion vectors for every pel of the down sampled images. Apply the pel-recursive motion estımation algorithm on the down sampled (or shrunk) ımages by takıng the motion vectors as the initial iterative estimation of the pel-recursive estimation (the current pels for estimating predictive frames is also to be used). This can be looked at as fine tuning on the block matching motion estımation dısplacement vectors. As for any transmission concern, there will be no further extra over head to be transmitted.

## Appendices

## Appendices

## A. 1 Hierarchy flow diagram of $\mathbf{H .} 263$

The Hierarchy c files flow diagram for H 263 are as shown in figure A.1.1


Figure A.1.1 Hierararchy flow diagram of H.263.
main c The first routine call It acts on the input command line (tmn) and set parameter accrdingly.
coder $\mathrm{c} \quad$ Performs all the encoding processes. Activated by main.c.
det c Performs the function of Discrete Cosine Transform. Initialized by main c and activated by coder c.
pred c Relates prediction of PB frames and Advanced Prediction mode. Activated by coder.c

| quant c | Sets all the quantisation index during encoding. Controled by coder c |
| :---: | :---: |
| ratectrl c | Organizes the control coding with control parameter as the quantisation index The control parameter is generic within ratectrl.c and doses not have any influence on quant.c. |
| mot_est c | Performs the motion estimation in the encoding process. Activated by coder c . |
| countbit c | its functionality is to count the bits during encoding process. activated by mann c and coder.c. |
| sac.c | Performs the Syntax Based Anthmetic coding when this obtion is selected Activated by countbit.c. |
| stream c | Handels all of the bit level stream commands. |
| huffman c | Performs the function of huffman encoding routınes. Activated by main c. |
| IO c | Contains the memory management for the component files. Actıvated by mann.c. |
| snr c | Processes signal to noıse ratıo for every frame. Activated by main.c. |

## A. 2 Programming function discription

The programming functions as they appear in H .263 software, are described as follow :

## main.c

| int Next TwoPB (--) | decides whether or not to code the next two |
| :--- | :--- |
|  | mages as PB. |
| void Help 0 | help. |
| vord AdvancedHelp () | help. |
| vord PrintResult (-) | prints results of bits in logn file. |
| vord PrintSNR (--) | print snr of luminance and chrominance in log |
|  | file. |

## coder.c

void CodeOneOrTwo (--)
PıctImage *CodeOnelntra (--) codes one intra 1 mage.
int *MB_Encode (--) performs dct and quantisation of macroblocks
int MB_Dncode (--) reconstruction of quantised det coded macroblocks.
void FillLumBlock (--)
void FillChromBlock (--)
void ZeroMBlock (--)
void ReconImage (--)
void MotionEstımatePicture (--)
void MakeEdgeImage (--)
void Clıp (--)
fills the luminance of one block of PictImage.
fills the chrominance of one block of PictImage.
fills one MB with zeros.
put together reconstructed image. find integer and half pel motion estimation. copy edge pels for use with unrestricted motion vector mode.
clips reconstructed data 0-255.
det.c

| int Dct (--) | perform dct on an $8 \times 8$ block and zıgzag <br>  <br> scanning of coefficients |
| :--- | :--- |
| int idct (--) | descans zigzag scanning coefficients and <br>  <br> vord init_idctref (--) |
| perform inverse dct on 64 coefficients. |  |
| vord idctref (--) | intıate the inverse dct reference |
|  | inverse dct reference. |

pred.c
MB_Structure *Predict_P (--)predict P macroblock in advance or normal mode

MB_Structure *Predict_B (--) predict B macroblock in PB frame prediction MB_Structure *MB_Recon_B (--) reconstruct the B macroblock in PB frame prediction
void FindForwLumPredPB (--) find the forward Luma prediction in PB frame.
vord FindBıDırLumPredPB (--)
void FindBiDirChrPredPB (--)
vord FindBıDıLLimıts (--)
void FindBıDirChromaLimits (--)
void BiDirPredBlock (--)
void DoPredChrom_P (--)
void FindHalfPel (--)
void FindPred (--)
vord FindPred OBMC (--)
MB_Structure *MB_Recon_P (--)
vord ReconLumBlock_P (--)
void ReconChromBlock_P (--)
vord FindChromBlock_P (--)
int ChooseMode (--)
int ModifyMode (--)
find the bi-dir Luma pred in PB frame find the $b_{1}$-dir Chroma pred in PB frame find the bi-dır limits find the bi-dir chroma limits. find the bi-dir prediction block. perform the chrominance pred for P frame. find the optimum half pel prediction. find the prediction block. find the OBMC prediction block reconstruct MB after quantisation for P images.
reconstruct one block of luminance data. reconstruct chrominance of one block in $P$ frame.
find chrominance of one block in P frame. choose coding mode. modify coding mode.

## quant.c

void Quant (--) quantiser for SIM3.
void Dequant (--) dequantiser for SIM3.

## ratectrl.c

| void InitıalizePıctureRate (-) | compute the target bitrate and target frame |
| :--- | :--- |
|  | rate for the current picture being coded |
| int UpdateQuantızer (--) | generate a new quantiser step size base on bits |
|  | spent until current macroblock and bits spent |
| int UpdatePictureRate (-) | from the previous picture. |
|  | updates buffer content and determine frame |
|  | skip. |

## mot_est.c

void MotionEstımation (--)
unsigned char *LoadArea (--)
int SAD_Macroblock (--)
int SAD_Block (--)
int SAD_MB_integer (--) void FindMB (--)

## countbit.c

void CountBitsMB (--) void Count_sac_BitsMB (--)
int CountBitsSlice (--)
vord CountBitsCoeff ( -- )
void Count_Sac_BitsCoeff ( - )
1nt CodeTcoef (--)
int FindCBP (--)
int CountBitsPicture (--)
sac.c
int AR_Encode (--)
arithmetic
int encoder_flush (--)
void bit_in_psc_Layer (--)
int indexfn (--)
estımate all motion vector for one MB. fill array with a square of image data. fast way to find the SAD of one vector. fast way to find the SAD of one vector. fast way to find the SAD of one vector. pick out one field of one MB.
count bits use for MB informatiom. count bits use for MB informatiom using sac models modified from CountBitsMB. count bits use for slice (GOB) informatiom count bits use for coefficients count bits use for sac models. encode an AC coefficient using the relevant sac model
find the CBP for a macroblock.
count the number of bits needed for picture header.
encode a symbol using syntax based coding
completes anthmetic coding before stream, or before any fixed length code are transmitted. inserts a bit into output bitstream and avoid picture start code emulation by stuffing a one bit
index into frequency cumutative frequency tables or escape code.

## stream.c

vord mwopen (--)
vord mwclose (--)
int Zeroflush (--)
void mputv (--)
long mwtell (--)
void mwseek (--)
opens a bit stream for writing.
close the write bitstream and flushes the remaning byte with " 1 ", consistent with -1 returned on EOF.
flushes out the rest of the byte with zeros and return number of bits written to bitstream (kol)
put a $n$ bits to the stream from byte $b$ return the position in bits of the write stream. seek to a specific bit position on the write stream.
initialized VLC tables. free the VLC tables. construct an encoder huffman with a designated table size. This table size n , is used for the lookup of huffman values and must represent the largest positive huffman value. loads an array into an encoder table. The array is grouped into triplts and the first negative value signals the end of the table. print out 256 elements in a nice byte ordered fashion.
encode a symbol according to a designated encoder huffman table out to the stream. It return the number of bits written to the stream and a zero on error.

IO.c
unsigned char *ReadImage (--) reads one qcif image from disk

PictImage *FillImage (--)
void WriteImage (--)
PictImage *InitImage (--)
void FreeImage (--)
char *StripName (--)
fills $\mathrm{Y}, \mathrm{C}_{\mathrm{b}}$ and $\mathrm{C}_{\mathrm{r}}$ of a PictImage struct.
write PictImage struct to disk
allocates memory for structure of 42.0 image
free memory allocated.for structure of 420 1mage.
remove character behind ".", and in front of (including) the last " $/$ ".
compare two image files using SNR.

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