TRANSIENT BEHAVIOUR OF A GROUP OF INDUCTION MOTORS

by

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SYNOPSIS

The non-linear differential equations describing the transient behaviour of a group of induction motors are developed from the equations for a single machine. These equations enable the behaviour of each machine in the group to be investigated, either when the complete group is connected to a stiff supply or when the supply is weak and the machine behaviour is interactive. A numerical solution of the equations using a digital computer is used to predict the transient currents and torques of a 0.75 kW/1.5 kW motor group and of a 0.75 kW/1.5 kW/2.25 kW motor group, and the results obtained are compared with experimentally obtained results.

The initial stage of the investigation is concerned with the direct-on-line starting performance of the motors, and this is followed by a study of the transient behaviour of the situation when the machines are disconnected from the supply but their stators remain interconnected. The components of the transient currents and the common terminal voltage during disconnection are computed, and determined also by analytical methods, and are compared with experimental results for the two groups of motors investigated.

The investigation deals also with reconnection of a group of machines following a short interruption of the supply, and considers how the subsequent transients are affected by any currents which are still flowing in both the stator and the rotor

circuits. Plugging and star-delta starting switching conditions, and the transients which then result are investigated as examples of this kind of switching operation.

The computer program used throughout the investigation was written in Fortran IV. It was run on an ICL 1904A digital computer using a fourth-order Runge-Kutta method for numerical integration of the equations involved. A separate program was developed to determine the value of the step-length required to ensure a stable numerical solution throughout the duration of the program.

The original work described in this thesis may be summarized by the following points:-

i) The development of the non-linear differential equations describing the transient behaviour of a group of induction motors.

ii) The development of the digital computer solution of the non-linear equations simulating a group of induction motors.

iii) The experimental work performed on groups of induction motors to demonstrate their mutual interaction.

iv) The theoretical and experimental considerations of the phenomena which follow a variety of switching operations.

LIST OF PRINCIPAL SYMBOLS

i	instantaneous current, A
v	instantaneous voltage, V
R	resistance per phase, Ω
ℓ	leakage inductance per phase, H
M	mutual inductance, H
L	self inductance per phase, H
p	number of pole-pairs
T	instantaneous torque, Nm
J	moment of inertia, kg-m^2
D	operator d/dt
S	Laplace transform operator
Re	real part of a complex quantity
Im	imaginary part of a complex quantity
θ	angular position, elec. rad.
$\dot{\theta}$	rotational speed, elec. rad/sec.
ω	synchronous angular velocity, elec. rad/sec.
δ	instant of switching to a balanced supply
ψ	flux linkage, Wb-T
η	normalised speed

suffices 1, 2, ... and m denote the 1st, 2nd, ... and mth motor

suffices s and r denote stator and rotor

suffices d and q denote direct and quadrature axes

suffices p, n and z denote positive, negative and zero sequence instantaneous symmetrical components

Other symbols are defined as they occur.

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CHAPTER 1

INTRODUCTION

Almost all previous investigations of induction motor transients have been devoted to the behaviour of a single machine, when connected or reconnected to a stiff supply. Both theoretically and experimentally, attention has been paid to the current and torque transients following connection to the supply either of an electrically inert machine or of a machine with rotor current flowing.

The differential equations modelling a single 3-phase induction motor may be derived in several different ways.[1,2,3] If the speed of the motor changes, the basic equations are normally both non-linear and simultaneous, but they can be solved numerically with the aid of a digital computer or by analogue means. An analytical solution becomes possible only when the speed of the motor is assumed to remain unchanged during the transient period, thus making the differential equations linear. Lyon[2] has applied the method of instantaneous symmetrical component analysis to these linearised equations, to derive expressions for the transient currents and torque in terms of the roots of the associated characteristic equation, while Chidambara and Ganapathy[4] have used Lyon's basic equations to develop equations for the torque following different switching operations. They have also obtained an experimental curve for the transient torque developed following connection to the supply of an electrically and mechanically inert motor, using a capacitance strain gauge to provide an oscillographic trace of the transient torque. However, this curve serves only to compare the shapes of the theoretical and measured curves, and gives no idea of the order of accuracy with which the transients are predicted. Differential analyser solutions of the differential equations of Stanley[1] have been obtained by Gilfillan and Kaplan[5] and also by Maginniss and Schultz[6]. However, the solutions of the latter pair of authors are rather more general, in that they do not regard the speed as constant but assume a linear change in either the acceleration or the deceleration following sudden changes of terminal voltage and plugging.

An analogue simulation of the d,q equations for a small 2-phase servomotor and a large 223 kW/96.9 kW (300/130 hp), 3-phase 2-speed induction motor was given by Hughes and Aldred[7]. However, the only assessment of the accuracy of their simulation was by a comparison with experimental steady-state torque-speed characteristics measured over the entire speed range of the motor. These authors also investigated the starting transients of the large induction motor and the unbalanced starting performance of the servomotor.

A further solution for the transient stator and rotor currents for various switching circuit conditions was obtained by Rundenbürg[8], from the basic loop equations of the induction motor. Enslin, Kaplan and Davies[9] applied Rudenbürg's method to analyse the transient currents and flux in a machine and they discussed in detail the components of the torque produced on starting a 0.75 kW (1 hp) motor. They also compared theoretical results when the machine was switched to the supply at various speeds with oscillogramic recordings, obtained by measurement of the stator reaction on a microwave cavity. Wood, Flynn and Shanmugasunderan[10] presented the results of a systematic experimental investigation of a 3-phase 0.75 kW (1 hp) motor, and they included in their study the effect of non-simultaneous closure of the contacts of the supply switch. Experimental recordings of the transient torque were obtained by means of a d.c. excited 2-phase induction generator, directly coupled to the test motor and run as an accelerometer. In a continuation of this work, Slater, Wood, Flynn and Simpson[11] solved numerically the d,q equations of the induction motor, by means of a digital computer, for starting and following connection at 90% of synchronous speed, and they investigated these conditions for both simultaneous and non-simultaneous switching of the supply to a 3.75 kW (5 hp) motor and a 2.25 kW (3 hp) motor. In a parallel study, Smith and Sriharan[12,13] presented a comparison between computed and experimental results for the transient currents and torque of an induction motor following different switching operations. They took into account the effect of a coupled load and the flexibility of the connecting shaft, and they used a resistance strain-gauge for the measurement of the torque transmitted to the load. In continuing their work, Smith and Sriharan[14] investigated the influence of terminal capacitors on the transient currents and torque of an induction machine. They took into account the non-linear effect of magnetic saturation, by introducing a non-linear relationship between the peaks of the m.m.f. and the air gap flux-density distributions, and they expressed this relationship mathematically by a set of polynomial expressions. They obtained theoretical results from a digital-computer solution of the equations modelling the machine, and compared these with experimental results from a 5.6 kW (7.5 hp) motor, following disconnection and reconnection to the supply. The value of the capacitance considered was that required to correct the full-load power factor of 0.74 lag to unity, and reconnection transients with different values of capacitance were also obtained. Smith and Sriharan also obtained the transients when the machine operated as a capacitor-excited induction generator during disconnection, and when the capacitor used was of the value appropriate for capacitor braking. They compared their theoretical results for the transients obtained during the same switching and reswitching operations considered previously with experimental results. As a result of their work, they concluded that the p.f. correcting capacitors connected to the terminals of an induction motor increase the chance of subjecting the machine to severe reconnection transients, and that with unfavourable conditions of reconnection the transients are far more severe than those without capacitors. The severity of the transients showed that it is inadvisable to reconnect a capacitor-excited induction generator to the busbars, unless the interruption is of short duration, and even when the generator is not reconnected the build-up of voltage and the rise in magnetizing current following disconnection may be dangerous. Ramsden, Zorbas and Booth[15] applied the 3-phase/2phase transformation of Park[16] to the basic induction motor equations, in a consideration of the behaviour of an induction motor during fault conditions in the power system from which it was fed. They developed a computer program for their model of the induction motor, and used as input data the fluctuating supply voltage resulting from various fault conditions. With the induction motor modelled in this way, the equations are compatible with those already in existence in the analysis of synchronous machines, and make use of concepts developed in this context. The computer program they developed was used to predict the performance of induction motors supplied individually from a power-system on which a synchronous machine was running synchronously, and their predicted results were compared with laboratory experiments using several small induction motors. They were concerned with current, acceleration and speed during the fault condition and they adopted the technique used by previous authors of measuring the acceleration by means of a d.c. excited 2-phase induction generator.

In all of the previous investigations mentioned[1-15] there was no systematic study of the effects of parameter variations on the transient characteristics. The only consideration of this effect was a brief study by Slater, Wood, Flynn and Simpson[11], who investigated how the computed value of the greatest transient torque peak developed by an induction motor during starting was affected by separate ±10% changes in the machine resistances and leakage inductances. However, in a subsequent investigation using an experimental-design technique, Smith and Hamill[17] investigated fully the effect of variations in all the different winding parameters from their base values. They considered the transient torque and currents developed following a variety of switching operations, and derived equations establishing the significance of the various parameters in each operation.

Another feature not considered in the earlier publications was the effect of resistance and leakage induction variations with speed, consequent upon the use of deep-bar rotor conductors. This effect was investigated by Kalsi[18], who established theoretical formulae for calculating the transient torques and currents of a large motor following several switching operations. However, to obtain his formulae it was necessary to assume that the speed of the motor remained constant during the transient period, and they are therefore of somewhat limited applications.

The investigations mentioned so far[1-18] assumed that the supply to the motor remained stiff under all loading conditions, and that the terminal voltage was constant throughout the transient period. However, this restriction was lifted by Snider and Smith[19], who described a digital simulation of an induction-motor/synchronous generator combination, for which it is necessary to express the equations of both machines in phase quantities. They developed the overall system equations in this form, and described a digital-computer program for their numerical solution. The program was used to predict the current and torque transients following a variety of switching operations of a 5.6 kW motor to a 15 kVA generator, and also to investigate the associated change in the voltage of the connecting lines. This feature was also considered in a qualitative consideration by Stephen[20], who discussed the effect of system voltage depression caused by starting either a large induction motor or a large synchronous motor, and gave recommendations for choosing suitable machines and protective equipment for particular types of applications and system disturbances.

All the investigations mentioned above showed good agreement between experimental and computed results. Some authors predicted and measured the electromagnetic torque, and some the torque transmitted to the coupled load, but despite the simplifying assumptions made in the analysis, all the measuring techniques employed gave good agreement with the predicted results.

So far, only the performance of a single 3-phase induction machine has been considered, and the transient performance and the interactive effects involved in a group of motors connected to a common supply has not been given the same full attention. Nevertheless, this problem is of increasing practical importance, as groups of large motors are increasingly being switched together during operations such as busbar transfer of power station auxiliary equipment. During the transient conditions which arise as a consequence of such a switching operation, the effect of the impedance of the common transformer and line supplying the group of motors is of considerable significance, and Lewis and Marsh[21] have presented a simple study and an approximate analysis of this problem. They divided the time of supply interruption into two parts; the residual stator voltage being appreciable during the first of these and negligible during the second, when each motor in the group may be treated separately. In a much more detailed analysis, Humpage, Durrani and Carvalho[22], have represented the equations of interconnected synchronous-asynchronous-machine groups using Kron's concept[23,24,25] of a synchronously rotating reference frame, into which the equations of all the machines in the group are transformed. In particular, they studied the transient stability of each machine following short-circuit faults on the system. By neglecting in their analysis the terms in the stator-voltage equations corresponding to the transients associated with changing stator flux linkages, they succeeded in reducing the equations modelling each machine to two differential equations and two algebraic equations, from the four basic differential equations. This naturally introduced some error into their computed results, although this was only appreciable in the case of the induction motors. In a study restricted to consideration of only two machines, Kalsi and Adkins[26] have investigated a system consisting of an induction motor and a synchronous machine connected to a stiff supply through an impedance. They compared the transients measured on a model power system with the results of computations made on a very simplified mathematical model, for a variety of fault conditions. Their analysis, did, however, include a representation of the deep-bar rotor winding of the induction motor by two coils in each axis of the machine, as suggested in a previous publication[27].

In a very recent study restricted to induction machines only, Abdel-Hakim and Berg[28] have described a method of representing a group of such motors by a single-unit equivalent. Their representation is developed from the steady-state equivalent circuits of the individual machines, and includes also a single-unit equivalent for the overall inertia and mechanical load characteristics. The authors produced a computer program based on their equivalent machine, and used this to examine the dynamic performance of a group of motors following both sudden and gradual disturbances of the supply. However, it is clearly impossible to use the approach to investigate the response of the individual machines, and only the overall effect on the system can be obtained. This restriction does not appear to apply in the more recent work of Sriharan[29], who describes a computer program for the calculation of the transient currents, voltages, torques and speeds of each motor of a group supplied from a common transformer. However, the paper is only a computer program description and contains no details of the experimental approach followed nor of any comparison with experimental results.

1.1 The Scope of the Thesis

The work described in this thesis begins with the derivation of the d-q differential equations of a group of any number of induction motors connected to a common stiff or weak supply.

The impedance of the supply may be introduced either intentionally by the use of reduced voltage starters[30] or accidentally when the impedance is that of the transformer and connecting lines feeding the group of motors. In this case two approaches to the derivation of the equations are adopted. The first of these is by including the impedance of the supply in the matrix impedance of the group and the second is by considering the d-q equations of the group separately from the electric equations of the supply. In the latter approach, the supply equations are considered in phase rather than in d,q form, since this will reduce the 3-phase supply equations from six to only three equations. The system mechanical equations are also derived, on the assumption that each motor of the group is coupled to a mechanical load through a coupling shaft.

A digital computer program is developed for the numerical integration of the electrical and mechanical differential equations of the group of machines, and this is used to predict the behaviour of the group during transient conditions. The computer program is used for the computation of the behaviour of each machine of the group, when the supply is both stiff and non-stiff, and for several switching and reswitching operations. Although a numerical solution gives an overall measure of the transient performance, it does not provide any insight into the details of the components present in the transient. As an illustration of the way this detailed information may be obtained, an analytical solution is made for the currents, voltages and torques during disconnection of a two-machine group. The frequencies and time constants of the group are evaluated from the parameters of the two given machines.

Three special cases of reswitching the group to a stiff supply are investigated in depth; namely reswitching and plugging to the same supply and star-delta starting. To assess the theoretical work of the thesis, experimental measurements of the transients were made for various switching operations on a group of two motors and on a second group of three motors, and the experimental results are compared with corresponding predictions.

CHAPTER 2

MODEL OF A GROUP OF INDUCTION MOTORS

Even when the operating conditions of an induction motor change abruptly, the currents in the stator and rotor windings must still conform to the differential form of Kirchhoff's equations. In the application of these equations, certain idealizing assumptions are normally made concerning the mechanical and electrical characteristics of the machine. These assumptions may be summarized as follows:

i) the stator and rotor laminations are sufficiently thin for eddy current loss to be neglected, and the quality of the steel is such that hysteresis losses are also negligible,

ii) the inductances of the stator and rotor windings are independent of the current flowing, i.e. saturation is negligible,

iii) the squirrel-cage rotor can be represented by a 3-phase winding, and the windings of both the stator and the rotor are symmetrical,

iv) the mutual inductance between a stator winding and a rotor winding varies as the cosine of the angle between their magnetic axes, i.e. the m.m.f. and flux density produced by both windings is sinusoidal.

2.1 Induction Motor Equations in 3-Phase Quantities

The differential equations of an induction motor can be derived in many forms, but since these equations are normally both non-linear and simultaneous, they can only be solved numerically, with the aid of either a digital or an analogue computer. However, if the speed can be assumed to remain constant the differential equations are obtained in a linear form, so that an analytical solution becomes possible.

One way in which the differential equations of the motor can be derived is by using the method of instantaneous symmetrical component analysis, advocated in 2-phase form by Lyon[2], and thereby deriving expressions for the transient currents in terms of the roots of the characteristic equation associated with these equations. However an alternative approach, which has been used far more widely than the method of instantaneous symmetrical components, is to form the differential equations of a 2-phase (d,q) stationary axis model by deriving these from the original 3-phase model of the machine using the transformations of Stanley[1]. Many recent investigations have adopted this method of approach, although it must be realised that since the two sets of equations describe the same physical situation they are not independent, and that the d,q equations can be simply converted into instantaneous symmetrical components by an appropriate transformation. In this chapter the machine's equations in d,q form are developed.

On the basis of the assumptions of the previous section, the idealised model of the induction motor shown in Figure 2.1(a) may be formed. With upper-case letters referring to stator quantities, and lower-case letters to rotor quantities, the electrical equations of this model are:

$$\begin{bmatrix} v_A \\ v_B \\ v_C \\ v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R_A+L_A\,D & DM_{AB} & DM_{AC} & DM_{Aa} & DM_{Ab} & DM_{Ac} \\ DM_{BA} & R_B+L_B\,D & DM_{BC} & DM_{Ba} & DM_{Bb} & DM_{Bc} \\ DM_{CA} & DM_{CB} & R_C+L_C\,D & DM_{Ca} & DM_{Cb} & DM_{Cc} \\ DM_{aA} & DM_{aB} & DM_{aC} & R_a+L_a\,D & DM_{aB} & DM_{ac} \\ DM_{bA} & DM_{bB} & DM_{bC} & DM_{ba} & R_b+L_b\,D & DM_{bc} \\ DM_{cA} & DM_{cB} & DM_{cC} & DM_{ca} & DM_{cb} & R_c+L_c\,D \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \\ i_c \end{bmatrix} \qquad \ldots 2.1$$

in which the rotor quantities are assumed to be referred to the same number of turns as the stator quantities.

Equation 2.1 can be rewritten as:

$$[v] = [z][i]$$

where [z] is the impedance matrix of the machine, and [v] and [i] are column matrices of the machine voltages and currents respectively.

From assumption (iv) above, it follows that for the constant stator and rotor mutual inductances

$$M_{AB} = M_{BA} = M_{BC} = M_{CB} = M_{CA} = M_{AC} = \hat{M} \text{ Cos } \frac{2\pi}{3}$$

$$M_{ab} = M_{ba} = M_{bc} = M_{cb} = M_{ca} = M_{ac} = \hat{M} \text{ Cos } \frac{2\pi}{3}$$

and for the cosinusoidally variable stator to rotor mutual inductances

$$M_{Aa} = M_{Bb} = M_{Cc} = M_{aA} = M_{bB} = M_{cC} = \hat{M} \text{ Cos } \theta$$

$$M_{Ab} = M_{Bc} = M_{Ca} = M_{bA} = M_{cB} = M_{aC} = \hat{M} \text{ Cos } (\theta - \frac{2\pi}{3})$$

$$M_{Ac} = M_{Ba} = M_{Cb} = Mc_{A} = M_{aB} = M_{bC} = \hat{M} \text{ Cos } (\theta - \frac{4\pi}{3})$$

where $\hat{M}$ is the mutual inductance between a stator winding and a rotor winding when their magnetic axes coincide. The self inductance of a winding is the sum of the mutual inductance and the leakage inductance, i.e. for a stator phase

$$L_A = L_B = L_C = \hat{M} + \ell_s$$

and for a rotor phase

$$L_a = L_b = L_c = \hat{M} + \ell_r$$

where ℓ_s and ℓ_r are the leakage inductances of the stator and rotor phase windings respectively. Equations 2.1 are differential equations with some coefficients which depend on the angle θ. That is they are non-linear differential equations, so that a direct analytical solution is therefore impossible even when the speed is considered constant. To facilitate an analytical solution, it is necessary initially to reduce the six equations of 2.1 to four equations, whose coefficients are independent of θ. This is achieved firstly by obtaining the equations of an equivalent 2-phase machine, and then by reducing these to the equations of an equivalent stationary-axis model.

2.2 Transformation from 3-phase to 2-phase Equations

The transformation which relates the currents of the 2-phase machine shown in Figure 2.1(b) to those of the 3-phase currents of the original machine is

$$[i_{3ph}] = [C]\,[i_{2ph}]$$

where

$$[i_{3ph}] = \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_a \\ i_b \\ i_c \end{bmatrix}, \quad [i_{2ph}] = \begin{bmatrix} i_{us} \\ i_{vs} \\ 0 \\ i_{ur} \\ i_{vr} \\ 0 \end{bmatrix}, \quad [C] = \begin{bmatrix} C' & 0 \\ 0 & C' \end{bmatrix}$$

and

$$[C^{\backprime}] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & 0 \\ \frac{-\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{-\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} & 0 \end{bmatrix}$$

to maintain invariance of power during the transformation. Zero-sequence components of current cannot appear in the matrix for $[i_{2ph}]$, and it therefore follows that the 2-phase model cannot represent a 3-phase machine in which a neutral current is flowing. The condition of power invariance requires also that

$$[v_{2ph}] = [C_t]\,[v_{3ph}]$$

so that the original impedance matrix of Equation 2.1 is transformed to a new impedance matrix $[Z_{2ph}]$, by

$$[Z_{2ph}] = [C_t][Z][C]$$

where $[C_t]$ is the transpose of $[C]$. Applying the above transformation to Equation 2.1, we obtain

$$\begin{bmatrix} v_{us} \\ v_{vs} \\ v_{ur} \\ v_{vr} \end{bmatrix} = \begin{bmatrix} R_s+L_s\,D & 0 & MD\,Cos\theta & MD\,Sin\theta \\ 0 & R_s+L_s\,D & -MD\,Sin\theta & MD\,Cos\theta \\ MD\,Cos\theta & -MD\,Sin\theta & R_r+L_r\,D & 0 \\ MD\,Sin\theta & MD\,Cos\theta & 0 & R_r+L_r\,D \end{bmatrix} \begin{bmatrix} i_{us} \\ i_{vs} \\ i_{ur} \\ i_{vr} \end{bmatrix} \quad \ldots.\ 2.2$$

where

$$M = \frac{3}{2}\hat{M} \qquad R_s = R_A = R_B = R_C$$

$$L_s = M + \ell_s \qquad R_r = R_a = R_b = R_c$$

2.3 Transformation from 2-phase Rotating to Stationary Axes

It is clear that in Equation 2.2 some of the elements of the impedance matrix are still functions of the displacement angle θ, and an analytical solution of these equations still remains difficult. To overcome this, these equations now are transformed to those of the equivalent stationary axis or d,q model of the machine shown in Figure 2.1(c). If the transformation matrix from 2-phase rotating axes to d-q stationary axes is $[C^{``}]$, then

$$[i_{2ph}] = [C^{``}]\,[i_{dq}]$$

where, again to maintain power invariance during the transformation,

$$[v_{dq}] = [C_t^{``}]\,[v_{2ph}]$$

The currents and voltages of the d-q model are given by

$$[i_{dq}] = \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} \quad \text{and} \quad [v_{dq}] = \begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix}$$

and the impedance matrix of the d-q model is

$$[Z_{dq}] = [C_t^{``}][Z_{2ph}][C^{``}]$$

The transformation matrix between the currents of the 2-phase and the d-q machine models is, from Figures 2.1(b) and 2.1(c),

$$[C^{``}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \text{Cos}\theta & -\text{Sin}\theta \\ 0 & 0 & \text{Sin}\theta & \text{Cos}\theta \end{bmatrix}$$

which enables the electrical equations of the d-q model to be developed as

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix} = \begin{bmatrix} R_s+L_s\,D & 0 & MD & 0 \\ 0 & R_s+L_s\,D & 0 & MD \\ MD & M\dot{\theta} & R_r+L_r\,D & L_r\dot{\theta} \\ -M\dot{\theta} & MD & -L_r\dot{\theta} & R_r+L_r\,D \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} \qquad \ldots 2.3$$

which is the basis on which many studies of induction motor transients have been conducted.[10-14,17] The most notable feature of Equation 2.3 is that the elements of the impedance matrix are now all independent of θ, which enables an analytical solution to be obtained when the speed is considered constant.

2.4 General Electrical Equations of a Group of Induction Motors

Figure 2.2 shows a group of n induction motors connected in parallel to a supply of zero impedance through a common switch S which is assumed ideal, that is its contacts open and close simultaneously and they are capable of interrupting instantaneously whatever current is being carried.

The mathematical model which represents the group of motors, from which the transient behaviour of the individual machines can be investigated, is derived in this section from the d-q axis equations for a single machine developed in the previous section.

When the equations for all the motors of the group are referred to the same set of stationary axes, application of Equation 2.3 to the first motor of the group gives the equations for this motor as:

$$\begin{bmatrix} v_{sd_1} \\ v_{sq_1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{s_1}+L_{s_1}D & 0 & M_1 D & 0 \\ 0 & R_{s_1}+L_{s_1}D & 0 & M_1 D \\ M_1 D & M_1\dot{\theta}_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot{\theta}_1 \\ -M_1\dot{\theta}_1 & M_1 D & -L_{r_1}\dot{\theta}_1 & R_{r_1}+L_{r_1}D \end{bmatrix} \begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \end{bmatrix} \quad ..2.4$$

where the additional subscript 1 is used to indicate the parameters and conditions of this motor. We may notice that in squirrel-cage rotor v_{rd_1} and v_{rq_1} are necessarily zero, since the rotor is short-circuited, and in a wound-rotor machine they will also normally be zero. For the second motor, the differential equations referred to the same reference frame as the first motor are:

$$\begin{bmatrix} v_{sd_2} \\ v_{sq_2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{s_2}+L_{s_2}D & 0 & M_2D & 0 \\ 0 & R_{s_2}+L_{s_2}D & 0 & M_2D \\ M_2D & M_2\dot{\theta}_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot{\theta}_2 \\ -M_2\dot{\theta}_2 & M_2D & -L_{r_2}\dot{\theta}_2 & R_{r_2}+L_{r_2}D \end{bmatrix} \begin{bmatrix} i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \end{bmatrix} \quad \ldots 2.5$$

where the additional subscript 2 indicates the parameters and conditions of this motor. The equations of any motor of the group can thus clearly be written in the same general form, so that for the mth motor

$$\begin{bmatrix} v_{sd_m} \\ v_{sq_m} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{s_m}+L_{s_m}D & 0 & M_mD & 0 \\ 0 & R_{s_m}+L_{s_m}D & 0 & M_mD \\ M_mD & M_m\dot{\theta}_m & R_{r_m}+L_{r_m} & L_{r_m}\dot{\theta}_m \\ -M_m\dot{\theta}_m & M_mD & -L_{r_m}\dot{\theta}_m & R_{r_m}+L_{r_m}D \end{bmatrix} \begin{bmatrix} i_{sd_m} \\ i_{sq_m} \\ i_{rd_m} \\ i_{rq_m} \end{bmatrix} \quad \ldots 2.6$$

Since the motors are connected to the same power supply, the first and second terms in the voltage matrix of Equations 2.4, 2.5 and 2.6 are the d-q components of the supply voltage i.e.

$$v_{sd_1} = v_{sd_2} = v_{sd_3} = \ldots\ldots = v_{sd_m} = \ldots\ldots = v_{sd_n}$$

and

$$v_{sq_1} = v_{sq_2} = v_{sq_3} = \ldots\ldots = v_{sq_m} = \ldots\ldots = v_{sq_n}$$

On collecting the equations of the group into one matrix the general electrical differential equations for the n-motor system become as given in Equation 2.7, in which there is a total of 4n separate equations.

Equation 2.7 can be rewritten in an alternative form by considering the loops formed by the stator circuits of adjacent machines of the group. Thus, in Equation 2.8, the first four rows are the first four rows of Equation 2.7, but the first two rows of the second group of four are obtained by subtracting the fifth and sixth rows from the first and second rows of Equation 2.7. In each succeeding group of four rows, the first two are formed by subtracting the first pair of rows in the corresponding group of Equation 2.7 from the first pair in the group immediately above. The resulting matrix equation,(2.8), still contain 4n independent equations,

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ P \\ v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ \vdots \\ v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & & & & & & & & & \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & & & & & & 0 & & & \\
M_1D & M_1\dot{\theta}_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot{\theta}_1 & & & & & & & & & \\
-M_1\dot{\theta}_1 & M_1D & -L_{r_1}\dot{\theta}_1 & R_{r_1}+L_{r_1} & & & & & & & & & \\
& & & & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & & & & & \\
& & & & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & & & & & \\
& & & & M_2D & M_2\dot{\theta}_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot{\theta}_2 & & & & & \\
& & & & -M_2\dot{\theta}_2 & M_2D & -L_{r_2}\dot{\theta}_2 & R_{r_2}+L_{r_2}D & & & & & \\
& & & & & & 0 & & \ddots & & & & \\
& & & & & & & & & R_{s_n}+L_{r_n}D & 0 & M_nD & 0 \\
& & & & & & & & & 0 & R_{s_n}+L_{s_n}D & 0 & M_nD \\
& & & & & & & & & M_nD & M_n\,{}_n & R_{r_n}+L_{r_n}D & L_{r_n}\dot{\theta}_n \\
& & & & & & & & & -M_n\dot{\theta}_n & M_nD & -L_{r_n}\dot{\theta}_n & R_{r_n}+L_{r_n}D
\end{bmatrix}
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ \vdots \\ i_{sd_n} \\ i_{sq_n} \\ i_{rd_n} \\ i_{rq_n} \end{bmatrix}
\quad \ldots 2.7
$$

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \\ \\ \\ \\ \\ \\ \\ \vdots \\ \\ \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\left[\begin{array}{cccccccccccccccc}
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 & 0 & & & & & & & & \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 & & & & & & & & \\
M_1D & M_1\theta_1 & R_{r_1}+L_{r_1}D & L_{r_1}\theta_1 & 0 & 0 & 0 & 0 & & & & & & & & \\
-M_1\theta_1 & M_1D & -L_{r_1}\theta_1 & R_{r_1}+L_{r_1}D & 0 & 0 & 0 & 0 & & & & & & & & \\
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 & & & & & & & & \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & & & & & & & & \\
0 & 0 & 0 & 0 & M_2D & M_2\theta_2 & R_{r_2}+L_{r_2}D & L_{r_2}\theta_2 & & 0 & & & & & & \\
 & & & & -M_2\theta_2 & M_2D & -L_{r_2}\theta_2 & R_{r_2}+L_{r_2}D & & & & & & & & \\
 & & 0 & & & & & & & & & & & & & \\
 & & & & & & 0 & & & & & & & & & \\
 & & & & & & & & \ddots & & & & & & & \\
 & & & & & & & & & R_{s_{n-1}}+L_{s_{n-1}}D & 0 & M_{n-1}D & 0 & -(R_{s_n}+L_{s_n}D) & 0 & -M_nD & 0 \\
 & & & & & & & & & 0 & R_{s_{n-1}}+L_{s_{n-1}}D & 0 & M_{n-1}D & 0 & -(R_{s_n}+L_{s_n}D) & 0 & -M_nD \\
 & & & & & & & & & & & & & M_nD & M_n\theta_n & R_{r_n}+L_{r_n}D & L_{r_n}\theta_n \\
 & & & & & & & & & & & & & -M_n\theta_n & M_nD & -L_{r_n}\theta_n & R_{r_n}+L_{r_n}D
\end{array}\right]
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ i_{sd_3} \\ i_{sq_3} \\ i_{rd_3} \\ i_{rq_3} \\ \vdots \\ i_{sd_n} \\ i_{sq_n} \\ i_{rd_n} \\ i_{rq_n} \end{bmatrix}
\quad \ldots .2.8
$$

but it has only one pair of voltage terms on the left-hand side. However, it will be noticed that various additional submatrices are introduced into the impedance matrix.

Equations 2.7 and 2.8 both provide a full electrical description for the group of motors. Since the quantities in the impedance matrix are all real, a solution on a digital computer is possible for either set of equations. For this purpose both Equation 2.7 and Equation 2.8 may be rewritten,

$$[v] = [R][i] + [L][Di] + \sum_{k=1}^{n} \dot{\theta}_k \, [G_k][i] \qquad \ldots\ldots \quad 2.9$$

where the matrices [R], [L] and $[G_k]$ are formed respectively from the resistance terms, the coefficients of D, and the coefficients of $\dot{\theta}_k$ in the overall impedance matrix.

2.5 Electromagnetic Torque

When the electrical differential equations of a motor have been obtained, as in Equation 2.3, the electromagnetic torque developed by the motor is given by[17]

$$T_e = p\ [i_t][G][i]$$

$$= p\ M(i_{rd}\ i_{sq} - i_{rq}\ i_{sd})$$

where p is the number of pairs of poles, [i] is the current matrix of Equation 2.3, $[i_t]$ is the transpose of [i] and [G] is the matrix formed from the coefficients of $\dot{\theta}$ in the impedance matrix. Thus, the electromagnetic torque developed by the m th motor of the n motor group is

$$T_{em} = p_m \, [i_t][G_m][i]$$

where $[G_m]$ is formed only from the coefficients of $\dot{\theta}_m$. Since many of the currents in [i] and $[i_t]$ do not flow in the m th motor they obviously do not contribute to the torque developed by this motor, which can therefore be written in the much simpler form

$$T_{em} = p_m \, M_m \, (i_{rdm} \, i_{sqm} - i_{sdm} \, i_{rqm}) \qquad \ldots\ldots \; 2.10$$

2.6 General Mechanical Equations

Underlying Equation 2.10 is the assumption that the rotor core may be represented as a rigid body with no flexibility. In such a case all points on a given radius always retain the same relative angular position, and there is no twist between the two ends of the core.

Each motor of the group will normally be coupled to a mechanical load inertia through a shaft, which may be flexible. The electromagnetic torque developed by the m th motor of the group is equal to the sum of the opposing torques due to acceleration of the motor inertia and to windage and friction losses, together with any torsional torque arising from the distortion of the flexible shaft. For the m th motor the torque balance can be stated mathematically as,

$$T_{em} = J_m D^2 \theta_m + T_{fm} + k_m (\theta_m - \theta_{\ell m}) \qquad \ldots.. \ 2.11$$

where J_m is the rotor inertia of this motor, T_{fm} is the combined windage and friction torque as a function of speed, and θ_m and $\theta_{\ell m}$ are, respectively, the angular position of the rotor and the load.

The torsional torque transmitted through the shaft is equal to the sum of the accelerating torque of the load, $J_{\ell m} D^2\theta_m$, the friction and windage torques, $T_{f\ell m}$, and the torque applied at the load $T_{\ell m}$. That is

$$k_m (\theta_m - \theta_{\ell m}) = T_{f\ell m} + J_{\ell m} D^2 \theta_{\ell m} + T_{\ell m} \qquad \ldots.. \ 2.12$$

where $J_{\ell m}$ is the moment of inertia of the coupled load.
On combining Equations 2.11 and 2.12 we obtain

$$D^2(\theta_m - \theta_{\ell m}) = \frac{T_{em}}{J_m} - \frac{T_{fm}}{J_m} - \frac{k_m}{J_m}(\theta_m - \theta_{\ell m}) - \frac{k_m}{J_{\ell m}}(\theta_m - \theta_{\ell m})$$

$$+ \frac{T_{f\ell m}}{J_{\ell m}} + \frac{T_{\ell m}}{J_{\ell m}} \qquad \ldots.. \ 2.13$$

Since the torque transmitted through the shaft is given by

$$T_{tm} = k_m \, (\theta_m - \theta_{m\ell}) \qquad \ldots\ldots \; 2.14$$

it follows that

$$D^2(\theta_m - \theta_{\ell m}) = D^2 \, \frac{T_{tm}}{k_m} \qquad \ldots\ldots \; 2.15$$

and therefore that

$$\frac{1}{k_m} \, D^2 \, T_{tm} = \frac{T_{em}}{J_m} - \frac{T_{fm}}{J_m} + \frac{T_{f\ell m}}{J_{\ell m}} + \frac{T_{\ell m}}{J_{\ell m}} - \frac{T_{tm}}{J_m} - \frac{T_{tm}}{J_{\ell m}}$$

If there is no load torque applied at the coupled inertia, $T_{\ell m}$ is zero, and we obtain

$$T_{em} = \frac{J_m}{k_m} \, D^2 \, T_{tm} + T_{fm} - T_{f\ell m} \, \frac{J_m}{J_{\ell m}} + \frac{J_m + J_{\ell m}}{J_{\ell m}} \, T_{tm} \qquad .. \; 2.16$$

If there is no coupled inertia, the corresponding reduced form of Equation 2.16 is obtained by putting $(\theta_m - \theta_{\ell m})$ in Equation 2.11 to zero, so that

$$T_{em} = J_m \, D^2 \, \theta_m + T_{fm}$$

To obtain the variation in the motor speed during transient condition, it is necessary to express the acceleration in the form

$$D^2\,\theta_m = \frac{T_{em} - T_{fm} - T_{tm}}{J_m} \qquad \ldots\ldots\ 2.17$$

and then to integrate this equation. Equation 2.17 is obtained from Equations 2.11 and 2.14, and in the case of no coupled inertia the acceleration becomes

$$D^2\,\theta_m = \frac{T_{em} - T_{fm}}{J_m} \qquad \ldots\ldots\ 2.18$$

2.7 Equations During Disconnection

When a group of induction motors working in the steady state condition is disconnected from the supply, the currents and fluxes in the machines do not fall instantaneously to zero, but rather decay at a rate depending on the parameters of all the connected machines. The interaction between the machines during the time of disconnection is an extremely important factor in determining the behaviour of the machines when they are subsequently reconnected to either the same or a different supply.

During disconnection there is no voltage applied to the machine terminals. The summation of the stator currents of the group is therefore zero, or using Figure 2.2,

$$i_{sd_1} + i_{sd_2} + i_{sd_3} + \ldots\ldots + i_{sd_n} = 0 \qquad \ldots\ldots\ 2.19$$

and

$$i_{sq_1} + i_{sq_2} + i_{sq_3} + \ldots\ldots + i_{sq_n} = 0 \qquad \ldots\ldots 2.20$$

On replacing the first two rows of Equation 2.8 by Equations 2.19 and 2.20, we obtain Equation 2.21 from which the behaviour of the group during disconnection can be determined. Although Equation 2.21 has 4n equations in 4n unknowns, the constraints of Equations 2.19 and 2.20 enable it to be reduced to (4n - 2) equations in (4n - 2) unknowns. This is, however, a very tedious process to perform in the general case, although it will be carried out later when groups of two and three machines are investigated.

Equation 2.21 provides a full electrical description for the group of motors during disconnection. Since the quantities in the impedance matrix are all real, a solution on a digital computer is possible. For this purpose Equation 2.21 may be rewritten,

$$[0] = [R]_{dis}\ [i] + [L]_{dis}\ [Di] + \sum_{k=1}^{n} \theta_k\ [G_k][i] \ \ldots\ 2.22$$

where the matrices $[R]_{dis}$, $[L]_{dis}$ and $[G_k]_{dis}$ are formed respectively from the resistance terms, the coefficients of D, and the coefficients of $\dot{\theta}_k$ in the overall impedance matrix.

$$
\begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\ \vdots \\0\\0\\0\\0 \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & & & & & & & & \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & & & & & & & & \\
M_1 D & M_1 \dot\theta_1 & R_{r_1}+L_{r_1} D & L_{r_1}\dot\theta_1 & 0 & 0 & 0 & 0 & & & & & & & & \\
-M_1 \dot\theta_1 & M_1 D & -L_{r_1}\dot\theta_1 & R_{r_1}+L_{r_1} D & 0 & 0 & 0 & 0 & & & & & & & & \\
R_{s_1}+L_{s_1} D & 0 & M_1 D & 0 & -(R_{s_2}+L_{s_2} D) & 0 & -M_2 D & 0 & & & & & & & & \\
0 & R_{s_1}+L_{s_1} D & 0 & M_1 D & 0 & -(R_{s_2}+L_{s_2} D) & 0 & -M_2 D & & & & & & & & \\
 & & & & M_2 D & M_2\dot\theta_2 & R_{r_2}+L_{r_2} D & L_{r_2}\dot\theta_2 & & & & & & & & \\
 & & 0 & & -M_2\dot\theta_2 & M_2 D & -L_{r_2}\dot\theta_2 & R_{r_2}+L_{r_2} D & & & & & & & & \\
 & & & & & & & 0 & & & & 0 & & & & \\
 & & & & & & & & \ddots & & & & & & & \\
 & & & & & & & & & R_{s_{n-1}}+L_{s_{n-1}} D & 0 & M_{n-1} D & 0 & -(R_{s_n}+L_{s_n} D) & 0 & -M_n D & 0 \\
 & & & & & & & & & 0 & R_{s_{n-1}}+L_{s_{n-1}} D & 0 & M_{n-1} D & 0 & -(R_{s_n}+L_{s_n} D) & 0 & -M_n D \\
 & & & & & & & & & 0 & 0 & 0 & 0 & M_n D & M_n\dot\theta_n & R_{r_n}+L_{r_n} D & L_{r_n}\dot\theta_n \\
 & & & & & & & & & 0 & 0 & 0 & 0 & -M_n\dot\theta_n & M_n D & -L_{r_n}\dot\theta_n & R_{r_n}+L_{r_n} D
\end{bmatrix}
\begin{bmatrix} i_{sd_1}\\ i_{sq_1}\\ i_{rd_1}\\ i_{rq_1}\\ i_{sd_2}\\ i_{sq_2}\\ i_{rd_2}\\ i_{rq_2}\\ i_{sd_3}\\ i_{sq_3}\\ i_{rd_3}\\ i_{rq_3}\\ \vdots \\ i_{rq_{n-1}}\\ i_{sd_n}\\ i_{sq_n}\\ i_{rd_n}\\ i_{rq_n} \end{bmatrix}
\qquad \ldots 2.2
$$

2.7.1 Electromagnetic torques

During the time in which the currents of the machines are decaying, each stator and rotor will produce an mmf and there will be a resultant flux in the air-gap of each machine. The interaction between the currents of each machine will result in the production of an electromagnetic torque, which for the m th motor of the group is

$$T_{em} = p_m \quad [i_t][G_m][i] \qquad \ldots\ldots \; 2.23$$

where $[i_t]$ is the transpose of the current matrix [i] of the machines during disconnection.

2.7.2 Analytical solution of the current equations

The components of the motor currents expressed in d-q form in Equation 2.21 can be transformed into their corresponding positive and negative sequence instantaneous symmetrical components. By following this approach, the number of equations required to define the conditions of the group during disconnection is therefore reduced to one half the number required when the equations are in d-q form, i.e. to (2n - 1) equations. When the d,q equations are so transformed, there is no change in the number of equations which are produced. However, since the two sequence components are merely complex conjugates, the negative-sequence components add no extra information to that provided by the positive-sequence component, and only the positive-sequence set of equations need be considered. However, an analytical solution of these equations still remains possible only if we assume that the speeds of the motors during disconnection remain constant, when the non-linear differential equations of the system become linear.

The relationships between the instantaneous positive-sequence component of a voltage or a current, and the corresponding d-q components, are[52]

$$v_{sp} = \frac{v_{sd} + j\, v_{sq}}{\sqrt{2}} \qquad \ldots\ldots\ 2.24$$

$$v_{rp}\, e^{j\theta} = \frac{v_{rd} + j\, v_{rq}}{\sqrt{2}} \qquad \ldots\ldots\ 2.25$$

$$i_{sp} = \frac{i_{sd} + j\, i_{sq}}{\sqrt{2}} \qquad \ldots\ldots\ 2.26$$

$$i_{rp}\, e^{j\theta} = \frac{i_{rd} + j\, i_{rq}}{\sqrt{2}} \qquad \ldots\ldots\ 2.27$$

Since the actual values of the voltage and current are given by $2/\sqrt{3}$ times the real part of their positive-sequence components, it follows from the above equations that the method of instantaneous symmetrical components combines into one operation the transformation from 3-phase instantaneous values to 2-phase rotating axis values, with the transformation from 2-phase rotating axis values to 2-phase stationary axis values.

Substitution of Equations 2.23 to 2.26 into Equation 2.21, which represents the general equations of the group of motors during disconnection, provides the positive-sequence component of the system equations. It is obvious that since all the stator windings are connected in parallel, that

$$v_{sp_1} = v_{sp_2} = v_{sp_3} = \dots\dots\dots\dots v_{sp_n}$$

and that

$$i_{sp_1} + i_{sp_2} + i_{sp_3} + \dots\dots\dots\dots i_{sp_n} = 0 \qquad \dots\dots 2.28$$

Lyon[2] has shown that the positive-sequence equation for the stator of an induction motor is

$$v_{sp} = (R_s + L_s\ D)\ i_{sp} + MD\ (i_{rp}\ e^{j\theta})$$

and for the rotor

$$0 = M\ (D - j\ \dot{\theta})\ i_{sp} + [R_r + L_r\ (D - j\ \dot{\theta}\ )\ ]\ (i_{rp}\ e^{j\theta})$$

It therefore follows that the differential equations for the loops formed by the first and second stators, the second and third stators, down to the (n-1)th and n th stators are given by

$$[(R_{s_1}+L_{s_1}D)i_{sp_1}+M_1D(i_{rp_1}e^{j\theta_1})]-[(R_{s_2}+L_{s_2}D)i_{sp_2}+M_2D(i_{rp_2}e^{j\theta_2})]=0$$

$$[(R_{s_2}+L_{s_2}D)i_{sp_2}+M_2D(i_{rp_2}e^{j\theta_2})]-[(R_{s_3}+L_{s_3}D)i_{sp_3}+M_3D(i_{rp_3}e^{j\theta_3})]=0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$[(R_{s_{n-1}}+L_{s_{n-1}}D)\,i_{sp_{n-1}}+M_{n-1}\,D\,(i_{rp_{n-1}}e^{j\theta_{n-1}})]\ -$$

$$[(R_{s_n}+L_{s_n}D)\,i_{sp_n}+M_n\,D\,(i_{rp_n}e^{j\theta_n})]=0$$

Similarly, the differential equations for the rotor circuits of the machines of the group are given by:

$$M_1(D-j\dot{\theta}_1)\,i_{sp_1}+[R_{r_1}+L_{r_1}(D-j\dot{\theta}_1)]\,(i_{rp_1}e^{j\theta_1})=0$$

$$M_2(D-j\dot{\theta}_2)\,i_{sp_2}+[R_{r_2}+L_{r_2}(D-j\dot{\theta}_2)]\,(i_{rp_2}e^{j\theta_2})=0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$M_n(D-j\dot{\theta}_n)\,i_{sp_n}+[R_{r_n}+L_{r_n}(D-j\dot{\theta}_n)]\,(i_{rp_n}e^{j\theta_n})=0$$

After taking Laplace transforms of the stator and rotor differential equations, together with Laplace transform of Equation 2.27, and performing a great deal of algebraic manipulation, we obtain the Laplace transform of the stator current of the m th motor of the group during disconnection in the form

$$L(i_{spm}) = \frac{k_{0m} S^{2n-2} + k_{1m} S^{2n-3} + k_{2m} S^{2n-4} + \ldots + k(2n-2)m}{b_{0m} S^{2n-1} + b_{1m} S^{2n-2} + b_{2m} S^{2n-3} + \ldots + b(2n-1)m} \quad \ldots\ 2.29$$

where k_{0m}, k_{1m}, ... and b_{0m}, b_{1m} etc. are constants determined by the speeds and the inductance and resistance parameters of the machines. The total set of equations defining the group of motors contains 2n-1 independent currents, and the denominator of Equation 2.29 is therefore a polynomial in S of order 2n-1. In the solution of a set of linear equations of this form, the order of the numerator is one less that of the denominator, that is 2n-2 in this case.

When all the machines' parameters and speeds, and therefore all the coefficients of the denominator of Equation 2.28 are known, this can be broken into (2n-1) factors, using a subroutine normally available in a computer library. The computer available at Loughborough is an ICL (1904 A), with a subroutine able to solve a polynomial of the fortieth order.

If the roots of the denominator are λ_1, λ_2, λ_3, ...λ_{2n-1}, Equation 2.29 can be rewritten as

$$L(i_{spm}) = \frac{a_{0m} S^{2n-2} + a_{1m} S^{2n-3} + a_{2m} S^{2n-4} + \ldots + a_{(2n-2)^m}}{(S - \lambda_1)(S - \lambda_2)(S - \lambda_3)\ldots\ldots(S - \lambda_{2n-1})} \quad \ldots. \ 2.30$$

where $a_{0m} = \dfrac{k_{0m}}{b_{0m}}$, $a_{1m} = \dfrac{k_{1m}}{b_{1m}}$, etc.

Using the inverse-integral method, the inverse Laplace transform of a set of equations similar to Equation 2.30 will give the corresponding time domain expressions for i_{sp_1}, i_{sp_2} etc. Since the negative-sequence component is the conjugate of the positive-sequence component, the actual stator phase currents then follow from $2/\sqrt{3}$ the real part of these expressions as

$$i_{s_1} = \frac{2}{\sqrt{3}} \text{ Re } [A_{11} e^{\lambda_1 t} + A_{12} e^{\lambda_2 t} + \ldots\ldots + A_{1(2n-1)} e^{\lambda_{2n-1} t}]$$

$$i_{s_2} = \frac{2}{\sqrt{3}} \text{ Re } [A_{21} e^{\lambda_1 t} + A_{22} e^{\lambda_2 t} + \ldots\ldots + A_{2(2n-1)} e^{\lambda_{2n-1} t}]$$

....

....

$$i_{s_n} = \frac{2}{\sqrt{3}} \text{ Re } [A_{n1} e^{\lambda_1 t} + A_{n2} e^{\lambda_2 t} + \ldots\ldots + A_{n(2n-1)} e^{\lambda_{2n-1)} t}]$$

and for the rotors

$$i_{r_1} = \frac{2}{\sqrt{3}} \text{Re} \left[B_{11} e^{(\lambda_1-\dot{\theta}_1)t} + B_{12} e^{(\lambda_2-\dot{\theta}_1)t} + \ldots\ldots + B_{1(2n-1)} e^{(\lambda_{2n-1}-\dot{\theta}_1)t}\right]$$

$$i_{r_2} = \frac{2}{\sqrt{3}} \text{Re} \left[B_{21} e^{(\lambda_1-\dot{\theta}_2)t} + B_{22} e^{(\lambda_2-\dot{\theta}_2)t} + \ldots\ldots + B_{2(2n-1)} e^{(\lambda_{2n-1}-\dot{\theta}_2)t}\right]$$

....

....

$$i_{r_n} = \frac{2}{\sqrt{3}} \text{Re} \left[B_{n1} e^{(\lambda_1-\dot{\theta}_n)t} + B_{n2} e^{(\lambda_2-\dot{\theta}_n)t} + \ldots\ldots + B_{n(2n-1)} e^{(\lambda_{2n-1}-\dot{\theta}_n)t}\right]$$

where A_{11}, A_{12}, ... and B_{11}, B_{12} etc. are functions of the machine speeds and parameters.

2.7.3 Analytical derivation of electromagnetic torque

In instantaneous symmetrical component form, the electromagnetic torque developed by the m th motor is

$$T_{em} = 6\, p_m\, M_m\, I_m\ \left[i_{spm}\left(i_{rpm}\, e^{j\theta_m}\right)^*\right] \qquad \ldots\ldots\ 2.31$$

where the asterisk denotes the conjugate of the terms within the brackets. The torque developed is a motoring or a generating torque, depending on whether the sign of

$I_m\,[i_{spm}\,(i_{rpm}\,e^{j\theta_m})^*]$ is respectively positive or negative.

With the results of the stator and rotor currents known in analytical form from the previous section, Equation 2.31 enables an analytical result for the electromagnetic torque to be developed.

2.7.4 Derivation of the terminal voltage

The positive-sequence component of the common voltage across the terminals of the stators during disconnection is

$$v_{sp} = (R_{s_1} + L_{s_1} D)\, i_{sp_1} + M_1 D\,(i_{rp_1} e^{j\theta_1}) \quad \ldots\ldots\ 2.32$$

and the first phase of the stator voltage is obtained from

$$v_A = \frac{2}{\sqrt{3}} \operatorname{Re}(v_{sp})$$

so that with the expressions of the stator and rotor currents known, an expression for v_A can be developed.

The analysis outlined in the previous sections is obviously not amenable to a general solution, and more detailed considerations are derived later, when the two machine situation is investigated.

2.8 Reswitching Condition

The general electrical and mechanical equations for the group of motors on reswitching are precisely the same as those given for switching conditions (Sections 2.4 and 2.6), except that the left-hand side of the electrical equations (i.e. the voltage matrix) is determined according to the type of the supply to which the machines are re-switched.

CHAPTER 3

MODELLING OF A GROUP OF INDUCTION MOTORS WITH WEAK SUPPLY

In Chapter 2, a model of a group of induction motors was developed when the source was considered as infinite, so that the voltage applied to the motor terminals remained constant under all conditions. If, however, the source has a significant internal impedance, or if the machines are started by a reduced voltage technique in which additional impedance is added to the source, the previous analysis of the performance of the motors is clearly no longer directly valid.

The starting current of an induction motor is typically, from 5 to 8 times its full-load current, and the starting of a group of large or even medium-size motors may draw from the source a current which is not only excessive but is also at a very low power factor. This current will reduce the supply voltage by an amount which depends on the short-circuit capacity of the network feeding the motors, while this capacity depends, in turn, on the impedance of the circuit elements forming the network. Normally, induction motors are operated from power supplies with a rated voltage which is less than the distribution voltage, and which is often referred to as the consumer voltage. To reduce the distribution voltage to the rated voltage of the motors step-down transformers are required. The impedance of these and the connecting cables, together with the starting currents drawn by the motors, will determine the magnitude of the depression in the consumer voltage. The reduction in the supply voltage will not only affect the motors to be started, but will also affect other motors running on the same supply. Since the torque of a motor is proportional to the square of the terminal voltage, the torque developed by a running motor will be reduced during the period when other motors are started, and the running motor will slow down and demand more current from the supply. In an extreme case, the motor torque may be reduced by such an amount that it is no longer capable of driving the load, when both the motor and its load will be brought rapidly to rest.

To overcome the undesirable reduction in the source voltage which may occur when a group of induction motors is started, the starting current is reduced by means of a reduced-voltage starting arrangement, e.g. series resistors or reactors, a star-delta switch or an auto transformer. In this situation, the supply to the machines will still be the previous non-stiff supply, with the starting arrangement introducing an additional series impedance. Thus, although the terminal voltage of the motors will still be very substantially reduced at starting, that fed to other consumers from the same supply will not be so much affected.

There is, however, a limitation on the use of reduced voltage starters, since the torque developed by the motor will correspondingly be decreased. If the load torque exceeds the motor torque at any point below full-load speed, the motors will fail to accelerate beyond this speed, and under this condition the motor will rapidly overheat.

3.1 General Electrical Equations

The mathematical model which represents a group of induction motors connected to a common weak supply can be formulated by two different approaches. The first of these is through the use of the d,q machine equations coupled with the electrical equations of the supply in phase quantities rather than in d,q values. By regarding the equations of the machines and the supply as separate but linked through the voltage at the motor terminals, the total number of equations required is reduced by three.

The second approach is to consider the machines and the supply as a complete system, and from a study of the system to obtain a single set of d,q equations. These two approaches are explained and compared in the following sections, although a selection of the most appropriate method cannot be made until after their computer solution has been discussed.

3.1.1 General electrical equation with supply equations in phase values

In Equations 2.7 and 2.8 of Chapter 2, the general electrical equations were presented for a group of induction motors fed from a stiff supply. When the supply impedance is taken into consideration then, in addition to Equation 2.7, we require the electrical differential equations of the supply.

The instantaneous values of the 3-phase voltages of the supply can be represented by:

$$v_a = \hat{V} \operatorname{Sin} (\omega t + \delta) \qquad 3.1$$

$$v_b = \hat{V} \operatorname{Sin} (\omega t - \frac{2\pi}{3} + \delta) \qquad 3.2$$

$$v_c = \hat{V} \operatorname{Sin} (\omega t - \frac{4\pi}{3} + \delta) \qquad 3.3$$

where $\hat{V}$ is the peak voltage and δ is the instant on the voltage wave at which the supply is switched to the machines. Further, if the supply resistance and inductance are R' and L' respectively, we obtain for the instantaneous values of the voltages fed to the motors:

$$v_{ga} = v_a - (R' + L'\, D)\, i_a \qquad 3.4$$

$$v_{gb} = v_b - (R' + L'\, D)\, i_b \qquad 3.5$$

$$v_{gc} = v_c - (R' + L'\, D)\, i_c \qquad 3.6$$

where v_{ga}, v_{gb}. v_{gc} are the instantaneous values of the voltages applied on the machines, and

$$i_a = i_{a1} + i_{a2} + \ldots\ldots. + i_{an}$$

$$i_b = i_{b1} + i_{b2} + \ldots\ldots. + i_{bn}$$

$$i_c = i_{c1} + i_{c2} + \ldots\ldots. + i_{cn}$$

where i_{a1}, i_{b1}, i_{c1} are the instantaneous currents of the first motor, i_{a2}, i_{b2}, i_{c2} those of the second motor, and so on. The d-q components of the voltages at the terminals of the motors are,

$$v_{sdg} = \sqrt{\frac{2}{3}}\ (v_{ga} - 0.5\ v_{gb} - 0.5\ v_{gc}) \qquad 3.7$$

$$v_{sqg} = \sqrt{\frac{2}{3}}\ \left(\frac{\sqrt{3}}{2}\, v_{gb} - \frac{\sqrt{3}}{2}\ v_{gc}\right) \qquad 3.8$$

and on replacing v_{sd} and v_{sq} in Equation 2.7 by v_{sdg} and v_{sqg} from these equations, the result obtained is Equation 3.9, which together with Equations 3.1 to 3.6 then fully models the group with the supply impedance taken into account.

3.1.2 General electrical equations of the machines and supply in d-q values

Since the machines are all referred to a stationary frame of axes (d-q), and the supply impedance is clearly stationary with respect to that frame, the d,q components of the supply voltages are

$$v_{sd} = (R' + L'\ D)\ i_{sd} + (R_{s_1} + L_{s_1} D)\ i_{sd_1} + M_1 D\ (i_{rd_1}) \qquad 3.10$$

$$v_{sq} = (R' + L'\ D)\ i_{sq} + (R_{s_1} + L_{s_1} D)\ i_{sq_1} + M_1 D\ (i_{rs_1}) \qquad 3.11$$

$$
\begin{bmatrix}
v_{sdg}\\ v_{sqg}\\ 0\\ 0\\ v_{sdg}\\ v_{sdg}\\ 0\\ 0\\ \vdots\\ v_{sdg}\\ v_{scg}\\ 0\\ 0\\ v_{sdg}\\ v_{scg}\\ 0\\ 0
\end{bmatrix}
=
\left[\begin{array}{cccccccccccccc}
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & & & & & & & & & \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & & & & & & & & & \\
M_1D & M_1\theta_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot{\theta}_1 & & & & & & & & & \\
-M_1\dot{\theta}_1 & M_1D & -L_{r_1}\dot{\theta}_1 & R_{r_1}+L_{r_1}D & & & & & & & & & \\
& & & & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & & & & & \\
& & & & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & & & & & \\
& & & & M_2D & M_2\theta_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot{\theta}_2 & & & & & \\
& & & & -M_2\dot{\theta}_2 & M_2D & -L_{r_2}\dot{\theta}_2 & R_{r_2}+L_{r_2}D & & & & & \\
& & & & & & & & \ddots & & & & \\
& & & & & & & & & & & & \\
& & & & & & & & & & & & \\
& & & & & & & & & & & & \\
& & & & & & & & & & & & \\
& & & & & & & & & R_{sn}+L_{sn}D & 0 & M_nD & 0\\
& & & & & & & & & 0 & R_{sn}+L_{sn}D & 0 & M_nD\\
& & & & & & & & & M_nD & M_n\dot{\theta}_n & R_{rn}+L_{rn}D & L_{rn}\dot{\theta}_n\\
& & & & & & & & & -M_n\dot{\theta}_n & M_nD & -L_{rn}\dot{\theta}_n & R_{rn}+L_{rn}D
\end{array}\right]
\begin{bmatrix}
i_{sd_1}\\ i_{sq_1}\\ i_{rd_1}\\ i_{rq_1}\\ i_{sd_2}\\ i_{sq_2}\\ i_{rd_2}\\ i_{rq_2}\\ \vdots\\ i_{sdn-1}\\ i_{sqn-1}\\ i_{rdn-1}\\ i_{rqn-1}\\ i_{sdn}\\ i_{sqn}\\ i_{rdn}\\ i_{rqn}
\end{bmatrix}
\quad \ldots 3.
$$

where

$$i_{sd} = i_{sd_1} + i_{sd_2} + i_{sd_3} + \ldots\ldots + i_{sd_n} \qquad 3.12$$

$$i_{sq} = i_{sq_1} + i_{sq_2} + i_{sq_3} + \ldots\ldots + i_{sq_n} \qquad 3.13$$

On using Equations 3.10 to 3.13 in Equation 2.8 we obtain Equation 3.14, which are the general electrical equations for the machines together with the supply in a compatible d-q form.

3.2 Comparison Between the Two Approaches

As can be seen from a comparison of Equations 3.9 and 3.14, the two approaches by which the general electrical equations were derived have different advantages and disadvantages from an analytical point of view. We find that although they have the same general form, irrespective of the elements contained in the two impedance matrices, only Equation 3.14 is sufficient to provide a full solution. Equation 3.9 is not sufficient by itself, and when it is used Equations 3.4, 3.5 and 3.6 are also needed. Despite this apparent advantage of Equation 3.14, the choice of which of the approaches to adopt will depend on the relative time required for the computer solution of the equations. This point will be discussed when the cases of two and three motor groups are dealt with in Chapters 7 and 8.

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\left[\begin{array}{ccccccccccccccccc}
(R+R_{s_1})+(L+L_{s_1})D & 0 & M_1 D & 0 & R+LD & 0 & 0 & 0 & & & & & & & & & \\
0 & (R+R_{s_1})+(L+L_{s_1})D & 0 & M_1 D & 0 & R+LD & 0 & 0 & & & & & & & & & \\
M_1 D & M_1 \dot{\theta}_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot{\theta}_1 & 0 & 0 & 0 & 0 & & & & & & & & & \\
-M_1 \dot{\theta}_1 & M_1 D & -L_{r_1}\dot{\theta}_1 & R_{r_1}+L_{r_1}D & 0 & 0 & 0 & 0 & & & & & & & & & \\
R_{s_1}+L_{s_1}D & 0 & M_1 D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2 D & 0 & & & & & & & & & \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1 D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2 D & & & & & & & & & \\
 & & & & M_2 D & M_2 \dot{\theta}_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot{\theta}_2 & & & & & & & & & \\
 & 0 & & & -M_2 \dot{\theta}_2 & M_2 D & -L_{r_2}\dot{\theta}_2 & R_{r_2}+L_{r_2}D & & & & & & & & & \\
 & & & & & & & & 0 & & & & & & & & \\
 & & & & & & & & & R_{s_{n-1}}+L_{s_{n-1}}D & 0 & M_{n-1}D & 0 & -(R_{s_n}+L_{s_n}D) & 0 & -M_n D & 0 \\
 & & & & & & & & & 0 & R_{s_{n-1}}+L_{s_{n-1}}D & 0 & M_{n-1}D & 0 & -(R_{s_n}+L_{s_n}D) & 0 & -M_n D \\
 & & & & & & & & & 0 & 0 & 0 & 0 & M_n D & M_n \dot{\theta}_n & R_{r_n}+L_{r_n}D & L_{r_n}\dot{\theta}_n \\
 & & & & & & & & & 0 & 0 & 0 & 0 & -M_n \dot{\theta}_n & M_n D & -L_{r_n}\dot{\theta}_n & R_{r_n}+L_{r_n}D
\end{array}\right]
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ \vdots \\ i_{sd_n} \\ i_{sq_n} \\ i_{rd_n} \\ i_{rq_n} \end{bmatrix}
\quad \ldots 3.1
$$

CHAPTER 4

COMPUTER PROGRAM ORGANIZATION

Figure 4.1 shows a flow chart of the computer program developed for the numerical solution of the electrical and mechanical equations of a group of induction motors, connected either to a stiff or to a weak supply. The program can, in principle, be generalized for any number of induction motors, although the computing time required will naturally increase as more motors are connected, and may eventually become excessive.

The numerical integration of the electrical and mechanical differential equations of the system is based on a 4th-order Runge-Kutta method (this and other methods of numerical integration are discussed in Appendix A). Runge-Kutta methods have been extensively used in previous investigations of the dynamic behaviour of both induction and synchronous machines, and their validity and accuracy have been established in several different studies, provided that the step length is properly chosen. The main advantages of using the Runge-Kutta method are that it is inherently self-starting, which aids considerably the handling of any discontinuities, (such as disconnection and reswitching), and that any adjustment of the step-length which may be required is readily achieved.

The computer program commences by reading in the input data which are

i) matrices [R], [L], and [G], as defined in Equation 2.9 and obtained from the impedance matrix of Equation 2.7 for the case of a stiff-supply, or from either Equation 3.9 or Equation 3.14, when the supply is weak. In addition, matrices $[R]_{dis}$, $[L]_{dis}$ and $[G]_{dis}$, defined by Equation 2.22, and obtained from the impedance matrix of Equation 2.21, are also required.

ii) the initial values of the currents and speeds.

iii) the mechanical parameters.

Generally speaking, the program structure has three main parts - starting, disconnection and reswitching. Each part is dealt with in detail as follows.

4.1 Starting Condition

If the motors are started while electrically inert the initial supply and rotor currents are all zero, whether the motors are started from a stiff or from a weak supply.

If the motors are connected to a stiff supply, or if the supply impedance is included in the impedance matrix, then the initial values of the motor voltage is equal to the supply voltage. If, however, the motors are connected to a weak supply, then it is necessary to determine the initial value of the motor voltages.

4.1.1 Initial values of motor voltages

Figure 4.2 shows the single-phase equivalent circuit of the group of motors at rest, with the stator and rotor resistances and leakage-reactances of each machine of the group are assumed equal and the magnetizing currents are neglected. From this figure we obtain

$$v = v_g + (R' + L' D)i \qquad \ldots\ldots \qquad 4.1$$

where v and v_g are the instantaneous values of the supply and motor voltages respectively. Also,

$$v_g = (R_{s_1} + R_{r_1})i_1 + (L_{s_1} + L_{r_1} - 2M_1)Di_1 \qquad 4.2$$

$$= (R_{s_2} + R_{r_2})i_2 + (L_{s_2} + L_{r_2} - 2M_2)Di_2 \qquad 4.3$$

$$= (R_{s_3} + R_{r_3})i_3 + (L_{s_3} + L_{r_3} - 2M_3)Di_3 \qquad 4.4$$

....

....

$$= (R_{s_n} + R_{r_n})i_n + (L_{s_n} + L_{r_n} - 2M_n)Di_n \qquad 4.5$$

Since at $t = 0$, $i_1 = i_2 = i_3 = \ldots\ldots = i_n = 0$, it follows that

$$(Di_1)_0 = \frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_1} + L_{r_1} - 2\,M_1}\;(Di_1)_0 \qquad \ldots\ldots \qquad 4.6$$

$$(Di_2)_0 = \frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_2} + L_{r_2} - 2\,M_2}\;(Di_1)_0 \qquad \ldots\ldots \qquad 4.7$$

$$(Di_3)_0 = \frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_3} + L_{r_3} - 2\,M_3}\;(Di_1)_0 \qquad \ldots\ldots \qquad 4.8$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$(Di_n)_0 = \frac{L_{s_1} + L_{r_1} - 2\,M}{L_{s_n} + L_{r_n} - 2\,M_n}\;(Di_1)_0 \qquad \ldots\ldots \qquad 4.9$$

On adding Equations 4.5 to 4.9 we obtain,

$$(Di)_0 = \left[\frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_1} + L_{r_1} - 2\,M_1} + \frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_2} + L_{r_2} - 2\,M_2} + \ldots\ldots + \frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_n} + L_{r_n} - 2\,M_n}\right](Di_1)_0 \qquad \ldots\ldots \qquad 4.10$$

where Di is the rate-of-change of the supply current, and the suffix ($_0$) indicates values at the instant of switching.

From Equation 4.10 we obtain,

$$(Di_1)_0 = \frac{1}{K_G}(Di)_0 \qquad \ldots\ldots \qquad 4.11$$

where

$$K_G = 1 + \frac{L_{s_1} + L_{r_1} - 2M_1}{L_{s_1} + L_{r_1} - 2M_1} + \frac{L_{s_1} + L_{r_1} - 2M_1}{L_{s_2} + L_{r_2} - 2M_2} + \ldots\ldots +$$

$$+ \frac{L_{s_1} + L_{r_1} - 2M_1}{L_{s_n} + L_{r_n} - 2M_n}$$

On substituting Equation 4.11 in Equation 4.2, we obtain the voltage across the motors as:

$$v_g = \frac{L_{s_1} + L_{r_1} - 2M_1}{K_G}(Di)_0 \qquad \ldots\ldots \qquad 4.12$$

and from Equation 4.1 we obtain the supply voltage as,

$$v = L'(Di)_0 + \frac{L_{s_1} + L_{r_1} - 2M_1}{K_G}(Di)_0$$

so that

$$(Di)_0 = \frac{v}{L' + (L_{s_1} + L_{r_1} - 2M_1)/K_G} \qquad \ldots\ldots \qquad 4.13$$

Thus if the value of the supply voltage at the instant of switching is known, the voltage across the motors at this instant is

$$v_g = v\left[1 - \frac{L^{\prime}}{\left[L^{\prime} + \frac{(L_{s_1} + L_{r_1} - 2M_1)}{K_G}\right]}\right] \qquad 4.14$$

4.1.2 Program description of starting condition

After obtaining the initial conditions, the procedure followed by the program is as follows:

i) a subroutine is used to invert the inductance matrix [L].

ii) the applied voltages are transformed to their d-q components.

iii) a subroutine is used to compute the multiplication of [R][i], [G][i] and $[i_t][G][i]$.

iv) the rates-of-change of the currents are computed from

$$[D_i] = [L]^{-1} \{[v] - [R][i] - \sum_{k=1}^{n} \dot{\theta}_k [G_k][i]\}$$

v) the rates-of-change of the currents are numerically integrated to obtain the currents.

vi) the electromagnetic torque for the mth motor is computed from,

$$T_{em} = p_m M_m (i_{rd_m} i_{sq_m} - i_{sd_m} i_{rq_m}) \quad \ldots\ldots \quad 4.15$$

vii) the variation of the speed of any motor is taken into account using the mechanical equation of the machine, and the values of the speeds are obtained by numerically integrating the acceleration, given by

$$D^2 \theta_m = \frac{T_{em} - T_{fm} - T_{tm}}{J_m} \quad \ldots\ldots \quad 4.16$$

where T_{tm} is put to zero if the motor is unloaded.

viii) if the 3-phase rotor currents are required, the d,q currents have to be transformed to rotating axes. For this transformation the angular position of the rotor, θ, is required. To obtain this, the speed is numerically integrated from

$$D \theta_m = \dot{\theta}_m \quad \ldots\ldots \quad 4.17$$

ix) the computed values of the d-q currents, speeds and the angular position of the rotors at the end of the first step are taken as the initial values for the next integration step. This process is repeated until the next switching operation (i.e. disconnection) occurs.

4.2 Disconnection Condition

i) The initial values of the speeds and the angular-position of the rotors are taken as those immediately before disconnection.

ii) The initial values of the currents are obtained by applying the constant-flux-linkage theorem to each machine individually (see next section).

iii) Matrices $[R]_{dis}$, $[L]_{dis}$ and $[G]_{dis}$ are then called.

iv) The inductance matrix $[L]_{dis}$ is inverted and the rates-of-change of the currents are obtained from,

$$[Di] = [L]^{-1}_{dis} \left\{ 0 - [R]_{dis} [i] - \sum_{k=1}^{n} \dot{\theta}_k [G_k]_{dis} [i] \right\}$$

v) The rates-of-change of the currents are numerically integrated to obtain the currents.

vi) The torque during disconnection is computed for each motor from Equation 4.15.

vii) The speeds of the motors are obtained by numerically integrating Equation 4.16 for each motor.

viii) The angular position of the rotors are obtained by integrating Equation 4.17 for each motor.

4.2.1 Application of constant-flux-linkage theorem

On considering that the flux linkages of all the stator and rotor circuits remains the same, immediately before and after disconnection, we obtain for the n rotor circuits of the n machines

$$L_{r_1}(i_{rd_1})_0 + M_1(i_{sd_1})_0 = L_{r_1}(i_{rd_1})_1 + M_1(i_{sd_1})_1$$
$$\dots \quad 4.18$$
$$L_{r_1}(i_{rq_1})_0 + M_1(i_{sq_1})_0 = L_{r_1}(i_{rq_1})_1 + M_1(i_{sq_1})_1$$

$$L_{r_2}(i_{rd_2})_0 + M_2(i_{sd_2})_0 = L_{r_2}(i_{rd_2})_1 + M_2(i_{sd_2})_1$$
$$\dots \quad 4.19$$
$$L_{r_2}(i_{rq_2})_0 + M_2(i_{sq_2})_0 = L_{r_2}(i_{rq_2})_1 + M_2(i_{sq_2})_1$$

....

....

$$L_{r_n}(i_{rd_n})_0 + M_n(i_{sd_n})_0 = L_{r_n}(i_{rd_n})_1 + M_n(i_{sd_n})_1$$
$$4.20$$
$$L_{r_n}(i_{rq_n})_0 + M_n(i_{sq_n})_0 = L_{r_n}(i_{rq_n})_1 + M_n(i_{sq_n})_1$$

Equations 4.18, 4.19 and 4.20 apply to the first, the second and the nth motor respectively, and the suffices $_0$ and $_1$ denote the values immediately before and after disconnection, giving a total of 2n equations. Because of the relationships

$$i_{sd_1} + i_{sd_2} + i_{sd_3} + \ldots\ldots\ldots + i_{sd_n} = 0$$

$$i_{sq_1} + i_{sq_2} + i_{sq_3} + \ldots\ldots\ldots + i_{sq_n} = 0$$

imposed on the stator currents, the 4n unknowns of Equations 4.18 to 4.20 are reduced to 4n-2 unknowns overall. Since we have 2n equations for the rotor circuits, a further (2n-2) equations are required for the stator circuits for a solution to be possible. These equations are obtained by applying the constant-flux-linkage theorem to the stator circuits of the first (n-1) machine. In d-q form this gives

$$L_{s_1} (i_{sd_1})_0 + M_1 (i_{rd_1})_0 = L_{s_1} (i_{sd_1})_1 + M_1 (i_{rd_1})_1$$

$$L_{s_1} (i_{sq_1})_0 + M_1 (i_{rq_1})_0 = L_{s_1} (i_{sq_1})_1 + M_1 (i_{rq_1})_1 \qquad \ldots \quad 4.21$$

$$L_{s_2} (i_{sd_2})_0 + M_2 (i_{rd_2})_0 = L_{s_2} (i_{sd_2})_1 + M_2 (i_{rd_2})_1$$

$$L_{s_2} (i_{sq_2})_0 + M_2 (i_{rq_2})_0 = L_{s_2} (i_{sq_2})_1 + M_2 (i_{rq_2})_1 \qquad \ldots \quad 4.22$$

....

....

$$L_{s_{n-1}} (i_{sd_{n-1}})_0 + M_{n-1} (i_{rd_{n-1}})_0 = L_{s_{n-1}} (i_{sd_{n-1}})_1 + M_{n-1} (i_{rd_{n-1}})_1$$

$$L_{s_{n-1}} (i_{sq_{n-1}})_0 + M_{n-1} (i_{rq_{n-1}})_0 = L_{s_{n-1}} (i_{sq_{n-1}})_1 + M_{n-1} (i_{rq_{n-1}})$$

$$\ldots \quad 4.23$$

where the suffices $_0$ and $_1$ again denote the values immediately before and after disconnection. Equations 4.21, 4.22 and 4.23 are derived for the stators of the first, the second and the (n-1)th motor respectively, and by this means we obtain the additional 2(n-1) equations required for a complete definition of the machines. The left-hand side of Equations 4.18 to 4.23 are all known from the values of the currents computed at the end of the starting period and the unknowns all contained in the right-hand side.

The equations for the rotors can be rewritten in the alternative form,

$$\psi_{rd_1} = L_{r_1} (i_{rd_1})_1 + M_1 (i_{sd_1})_1 \quad \ldots\ldots \quad 4.24$$

$$\psi_{rq_1} = L_{r_1} (i_{rq_1})_1 + M_1 (i_{sq_1})_1 \quad \ldots\ldots \quad 4.25$$

$$\psi_{rd_2} = L_{r_2} (i_{rd_2})_1 + M_2 (i_{sd_2})_1 \quad \ldots\ldots \quad 4.26$$

$$\psi_{rq_2} = L_{r_2} (i_{rq_2})_1 + M_2 (i_{sq_2})_1 \quad \ldots\ldots \quad 4.27$$

....

....

$$\psi_{rd_n} = L_{r_n} (i_{rd_n})_1 + M_n (i_{sd_n})_1 \quad \ldots\ldots \quad 4.28$$

$$\psi_{rq_n} = L_{r_n} (i_{rq_n})_1 + M_n (i_{sq_n})_1 \quad \ldots\ldots \quad 4.29$$

and for the stators in the form

$$\psi_{sd_1} = L_{s_1} (i_{sd_1})_1 + M_1 (i_{rd_1})_1 \quad \ldots\ldots \quad 4.30$$

$$\psi_{sq_1} = L_{s_1} (i_{sq_1})_1 + M_1 (i_{rq_1})_1 \quad \ldots\ldots \quad 4.31$$

$$\psi_{sd_2} = L_{s_2} (i_{sd_2})_1 + M_2 (i_{rd_2})_1 \quad \ldots\ldots \quad 4.32$$

$$\psi_{sq_2} = L_{s_2} (i_{sq_2})_1 + M_2 (i_{rq_2})_1 \quad \ldots\ldots \quad 4.33$$

....

....

$$\psi_{sd_{n-1}} = L_{s_{n-1}} (i_{sd_{n-1}})_1 + M_{n-1} (i_{rd_{n-1}})_1 \quad \ldots\ldots \quad 4.34$$

$$\psi_{sq_{n-1}} = L_{s_{n-1}} (i_{sq_{n-1}}) + M_{n-1} (i_{rq_{n-1}})_1 \quad \ldots\ldots \quad 4.35$$

where ψ is the flux linkage for the specified winding immediately before disconnection. Equations 4.24 to 4.35 can be combined and expressed in matrix form as

$$[\psi] = [L_{CF}]\ [i] \quad \ldots\ldots \quad 4.36$$

where

$$[\psi] = \begin{bmatrix} \psi_{rd_1} \\ \psi_{rq_1} \\ \psi_{rd_2} \\ \psi_{rq_2} \\ \vdots \\ \psi_{rd_n} \\ \psi_{rq_n} \\ \psi_{sd_1} \\ \psi_{sq_1} \\ \psi_{sd_2} \\ \psi_{sq_2} \\ \vdots \\ \psi_{sd_{n-1}} \\ \psi_{sq_{n-1}} \end{bmatrix} \quad \text{and} \quad [i] = \begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ \vdots \\ i_{sd_{n-1}} \\ i_{sq_{n-1}} \\ i_{rd_{n-1}} \\ i_{rq_{n-1}} \\ i_{rd_n} \\ i_{rq_n} \end{bmatrix}$$

and the matrix $[L_{CF}]$ is formed from the coefficients of i_{sd_1}, i_{sq_1}, i_{rd_1}, i_{rq_1}, etc. Equation 4.36 can be rewritten in the form

$$[i] = [L_{CF}]^{-1} \quad [\psi] \qquad \ldots\ldots \qquad 4.37$$

so that on computing the inverse of the matrix $[L_{CF}]$ and premultiplying $[\psi]$ by $[L_{CF}]^{-1}$ we obtain the values of the initial currents at the beginning of the disconnection interval.

4.5 Reswitching Condition

After reswitching, the matrices used at starting are recalled, and the initial values of the currents, speeds and angular positions of the rotors are those immediately before the instant of reswitching. The voltage matrix is determined by transforming the supply voltage to which the group is reconnected into d,q components. If the group is restored to the same supply the 3-phase voltage of the supply is transformed into d-q components. If the machines are plugged the 3-phase voltage with reversed phase sequence is transformed to d,q system. In star-connected machines this merely requires a reversal of the lagging phase of the 2-phase voltage so that this becomes the leading phase; but in delta connected machines a phase shift in both voltages of $\frac{\pi}{3}$ is required in addition to the reversal of their phase-sequence (Figure 4.3). For star-delta starting a phase shift of $\pi/6$ is required in addition to a reduction in the magnitude of the applied voltage of $1/\sqrt{3}$ when the machines are star-connected. The numerical solution then continues as described in Section 4.1.2.

CHAPTER 5

EXPERIMENTAL PROCEDURE

Experimental investigations to validate the theoretical study and computer program of the previous chapters were performed using the motors detailed in Table 1. The electrical parameters of the machines were obtained from the results of open-circuit and short-circuit tests, with the usual assumption being made for each machine that the short-circuit impedances of the rotor and stator are equal. The moments of inertia of the rotors of the machines were estimated by measuring their dimensions, and assuming a specific gravity of 7.8 in the well-known formula for a cylindrical mass.[31] Standard deceleration tests were used to determine the windage and friction torques of the machines, the experimental results being recorded in Figure 5.1. The curve obtained for each motor was then used to determine the friction and windage torques, by calculating the deceleration at two points from the tangents of the curve at these points and then using the torque balance equation

$$-J D \dot{\theta} = K_1 + K_2 \, (\dot{\theta}/314.2)^2$$

in which K_1 and K_2 are the coefficients of the friction and windage torques respectively and $\dot{\theta}$ is the speed. Since J is already known, this procedure provides two equations in the two unknowns K_1 and K_2.

To record the transient currents, a calibrated current shunt was included in series with one line of each motor. The starting transients were obtained by switching the motors at rest to the supply. When the supply is to be assumed weak an 11.0 ohm resistance was connected in series with each line of the supply.

The electrical and mechanical parameters of the machines referred to, are shown in detail in Table I.

5.1 Measurement of the Torque

The measurement of the torque of an induction motor can be directly accomplished either by measuring the acceleration of the rotor, or the reaction of this torque on the stator. With losses assumed negligible, the acceleration is proportional to the electromagnetic torque of the motor and several methods are available by which this can be accomplished.

If the motor is loaded and its acceleration is measured then it is necessary to know the equation of motion of the rotating system in order to determine the torque transmitted to the coupled load and the acceleration of this load. Alternatively, the torque transmitted to the coupled load can be measured, although information of the mechanical system dynamics is again needed. In either the unloaded or loaded situation these measurements are sufficient to assess the validity of the model proposed for the motor and its load.

5.1.1 <u>Measuring the acceleration by a drag-cup generator</u>

A drag-cup induction generator[32,33,34] is a brushless device, which gives an output voltage directly dependent on either the velocity or the acceleration of the motor. No electrical noise due to slip rings and brushgear arises, and the output signal is sufficiently large to be fed directly to a recording device. The only problem which arises is that of calibrating the output in terms of torque, and this is often obtained by measuring the acceleration of the shaft caused by a known weight falling freely under the action of gravity.

The drag-cup generator is normally used to measure angular velocity. However, when an alternating excitation at a fixed frequency is applied to one of the windings, the second winding gives an output of the same frequency, but with an amplitude proportional to the speed of its rotor. When it is required to measure acceleration, excitation is produced by a constant magnitude direct voltage. Rotation of the rotor then produces induced rotor emfs proportional to the speed, and because of the negligible rotor inductance the resultant flux is perpendicular to the excitation mmf and links only the output winding. Since this flux varies directly with the speed, the emf induced in the output winding is directly proportional to the acceleration.

Other methods of measuring torque are discussed in Appendix B.

TABLE I

DATA FOR MACHINES INVESTIGATED

Machine Number	I	II	III	IV
Rated power, kW	0.75	1.5	2.25	5.6
Full-load speed, rpm	960	2760	1380	955
Full-load line current, A	3.1	3.9	9.0	11.9
Stator connection	Delta	Star	Delta	Delta
Number of poles	6	2	4	6
Frequency, Hz	50	50	50	50
Line voltage, V	230	230	230	400
Full-load torque, Nm	7.4	5	15.5	56
R_s, R_r, Ω	6.25	4.0	3.24	2.45
L_s, L_r, H	0.6	0.7216	0.36	0.437
M, H	0.57	0.7	0.33	0.42
J, kg-m^2	0.055	0.0027	0.0195	0.102
T_f, Nm	$1+0.8\eta^2$	$0.08+0.509\eta^2$	$0.5+0.4\eta^2$	$0.06+1.2\eta^2$

CHAPTER 6

TRANSIENT BEHAVIOUR OF A SINGLE MACHINE

6.1 Starting Condition

As was shown in Equation 2.3 of Section 2.3, the electrical differential equations of an induction motor, referred to stationary d,q axes, are

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + L_s D & 0 & MD & 0 \\ 0 & R_s + L_s D & 0 & MD \\ MD & M\dot{\theta} & R_r + L_r D & L_r\dot{\theta} \\ -M\dot{\theta} & MD & -L_r\dot{\theta} & R_r + L_r D \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} \qquad \ldots\ldots \quad 6.1$$

or, briefly

$$[v] = [R][i] + [L][Di] + \dot{\theta}\,[G][i] \qquad \ldots\ldots \quad 6.2$$

where

$$[R] = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{bmatrix}, \quad [L] = \begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix},$$

$$[G] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & M & 0 & L_r \\ -M & 0 & -L_r & 0 \end{bmatrix}, \quad [v] = \begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix} \quad \text{and}$$

$$[i] = \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}.$$

Equation 6.1 can be rewritten in 2-phase instantaneous symmetrical component form, by using the relationships between the d,q components and the corresponding positive-sequence components given in Equations 2.24 - 2.27. The results thereby obtained are

$$\begin{bmatrix} v_{sp} \\ v_{rp}\, e^{j\theta} \end{bmatrix} = \begin{bmatrix} R_s + L_s D & MD \\ M(D - j\dot{\theta}) & R_r + L_r(D - j\dot{\theta}) \end{bmatrix} \begin{bmatrix} i_{sp} \\ i_{rp}\, e^{j\theta} \end{bmatrix} \qquad \ldots\ldots \quad 6.3$$

where ($i_{rp}\ e^{j\theta}$) and ($v_{rp}\ e^{j\theta}$) are respectively the positive-sequence components of the rotor current and voltage referred to the stator.

Since the supply voltage is defined and $v_{rp}\ e^{j\theta}$ is equal to zero, Equation 6.3 contains only two unknowns, compared with the four unknowns of Equation 6.1. An analytical solution for the machine currents therefore becomes more readily available, although it can still only be obtained if the speed $\dot{\theta}$ is considered to remain constant. Expressions for i_{sp} and $i_{rp}\ e^{j\theta}$ can be obtained, as shown in Appendix C, and expressions for i_{sd}, i_{sq}, i_{rd} and i_{rq} subsequently derived by using Equations 2.24-2.27. Either directly or by using these expressions, the current in the stator windings is obtained in the form

$$i = A \sin(\omega t + \phi_1) + B\, e^{-\alpha_1 t} \sin(\beta_1 t + \phi_2) + C\, e^{-\alpha_2 t} \sin(\beta_2 t + \phi_3) \qquad \ldots\ldots \quad 6.4$$

where A, B, C, ϕ_1, ϕ_2 and ϕ_3 are all functions of the machine parameters and the switching conditions of the supply, while β_1, β_2, α_1 and α_2 are functions of the machine parameters and the starting speed.

The transient stator current is thus composed of a single sinusoidal component, which is the steady-state current, together with two exponentially decaying and oscillatory components of different frequencies. It can be seen from Appendix C that

$$\beta_1 + \beta_2 = \dot{\theta}$$

i.e. $\beta_1 < \dot{\theta}$ and $\beta_2 < \dot{\theta}$

so that the rotor speed is always greater than the synchronous speeds corresponding to β_1 and β_2. The stationary axis rotor currents will have the same form as the currents in the stator.

An expression for the electromagnetic torque of the machine may conveniently be obtained by substituting for the d,q current, in

$$T = pM\ (i_{rd}\, i_{sq} - i_{rq}\, i_{sd}) \qquad \ldots\ldots \qquad 6.5$$

from which various torque components will arise as a result of interaction between the different components of the stator and rotor currents. Examination of the current expressions of Appendix C, shows that the corresponding components of the currents in the direct and quadrature axis windings of the stator and the rotor, are $\pi/2$ out of phase. This results in three rotating magnetic mmf's being produced by the stator and three similar rotating mmf's by the rotor currents. One of each of these sets of mmf's is the steady-state component, rotating with an angular velocity of ω rad/s with respect to the stator. The other two mmf's of each set rotate with angular velocities of β_1 and β_2 rad/s respectively, with respect to the stator, and they also decay exponentially. The torque components produced from the interaction of the stator and rotor currents may be written as, (see the torque expression of Appendix C)

i) $E\, e^{-2\alpha_1 t}$

ii) $F\, e^{-2\alpha_2 t}$

iii) $G\, e^{-(\alpha_1 + \alpha_2)t} \sin[(\beta_1 - \beta_2)t + \phi_4]$

iv) $H\, e^{-\alpha_1 t} \sin[(\omega - \beta_1)t + \phi_5]$

v) $N\, e^{-\alpha_2 t} \sin[(\omega - \beta_2)t + \phi_6]$

vi) The steady-state component

where E, F, G, H, N, ϕ_4, ϕ_5 and ϕ_6 are all functions of the machine parameters, the speed at connection and the magnitude of the supply voltage, although not of the instant at which the supply is connected. Three of the torque components (i), (ii) and (vi), are unidirectional, resulting from the interaction of stator and rotor mmf's of the same angular velocity. One of the components, (vi), is the steady-state torque resulting from the interaction between stator and rotor mmf's rotating with ω rad/s.

The other two unidirectional components decay exponentially, and result from the interaction of the stator and rotor mmf's with synchronous speeds corresponding to β_1 and β_2 respectively. The other three components of the torque (iii), (iv) and (v) are alternating and exponentially decaying. These result from interaction between stator and rotor mmf's of different angular frequencies, and their frequencies are determined by the differences in the angular velocities of the two fields involved. However, although there is a difference in the number of components in the transient currents and torque, the longest time constants are the same and the resultant transient current will decay after the same length of time as the transient torque.

We may notice that, unlike the expressions for the transient currents, the components of the expression for the transient torque developed are independent of the switching angle δ. This is clearly to be expected, since the rotor and stator windings are balanced and the torque developed depends on the amplitude of the resultant flux produced by the combined stator and rotor mmf's with the net rotor current.

6.2 Disconnection Condition

When an induction motor operates in a steady-state condition, the mmf produced by the three phases of the rotor rotates at synchronous speed in space, and it lags the synchronously rotating stator mmf by a fixed angle. If the motor is disconnected from the supply, and the stator currents are then assumed to fall instantaneously to zero, the rotor mmf will now rotate with the rotor and decay with the time constant of the rotor circuit. The machine will thus act as a synchronous generator, running below the synchronous speed corresponding to the supply frquency, and with the decaying flux of the rotor taking the place of the excitation. A 3-phase voltage of varying frequency and amplitude will thus be generated in the stator, with a magnitude decaying with the time constant of the rotor and a frequency corresponding to the speed.

Since the stator currents are zero, Equation 6.1 reduces during disconnection to

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} R_r + L_r D & L_r\dot{\theta} \\ -L_r\dot{\theta} & R_r + L_r D \end{bmatrix} \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} \quad \ldots\ldots \quad 6.6$$

or, briefly

$$[0] = [R]_{dis}\,[i] + [L]_{dis}\,[Di] + \dot{\theta}\,[G]_{dis}[i] \quad \ldots \quad 6.7$$

where

$$[R]_{dis} = \begin{bmatrix} R_r & 0 \\ 0 & R_r \end{bmatrix}, \quad [L]_{dis} = \begin{bmatrix} L_r & 0 \\ 0 & L_r \end{bmatrix}$$

$$[G]_{dis} = \begin{bmatrix} 0 & L_r \\ -L_r & 0 \end{bmatrix}, \quad [i] = \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix}$$

and the stator voltages are

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = \begin{bmatrix} MD & 0 \\ 0 & MD \end{bmatrix} \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} \quad \ldots\ldots \quad 6.8$$

If the speed $\dot{\theta}$ remains constant, Equations 6.6 become linear differential equations and they may be solved analytically for i_{rd} and i_{rq}. It is shown in Appendix C how this may be used to provide equations for the rotor currents and for the stator voltage.

6.3 Reswitching Condition

During disconnection from the supply, the decaying rotor mmf of the motor will lag at an increasing rate behind the normal steady-state position of the stator mmf. If the motor is re-switched to the original supply, or to a different supply, a transient torque will be developed. Since this torque will be dependent on the magnitude of the rotor currents and the position of the rotor at the time of reswitching, it clearly also depends on the magnitude and phase of the stator voltage at this time. For a given magnitude, the transient torque will be a maximum when the supply voltage is about 180^0 out of phase with the stator voltage.

6.3.1 Electrical and mechanical equations

Equations 6.1 for the starting condition apply also to the reswitching condition, although the initial values of the rotor currents are, of course, no longer zero, but are as provided by the conditions at the end of the disconnection period. For the evaluation of the torque, Equation 6.5 is again used.

The mechanical system equations of the machine given in Section 2.4 obviously apply also in the present condition.

6.4 Numerical Solution and Computer Program

The procedure followed in Chapter 4 for the numerical solution of the electrical and mechanical equations of the group, may also be followed in the solution of the electrical and mechanical differential equations of a single machine. The definition of matrices [R], [L] and [G] during starting and reswitching are given in Section 6.1. The definition of matrices $[R]_{dis}$, $[L]_{dis}$ and $[G]_{dis}$ during disconnection are given in Section 6.2. For computation of the electromagnetic torque Equation 6.5 is used. If the motor is loaded, computation of the speed is performed using Equation 2.17, together with the equation for the transmitted torque (Equation 2.16), and if the motor is unloaded Equation 2.18 is used.

If the actual rotor currents are required the angular position of the rotor is determined using Equation 4.17, and the currents subsequently obtained from

$$\begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 0 \\ \cos(\theta - \frac{4\pi}{3}) & \sin(\theta - \frac{4\pi}{3}) & 0 \end{bmatrix} \begin{bmatrix} i_{rd} \\ i_{rq} \\ 0 \end{bmatrix}$$

at a rotor position defined by the angle θ.

The computer program again consists of three parts - starting, disconnection and reswitching, and the same procedure discussed in detail in Chapter 4 is applied. The flow chart of the computer program for a group of motors shown in Figure 4.1 applies also for a single machine, and the listings of the computer program for the single machine are given in Appendix D.

6.5 Transient Conditions in a Single Induction Motor

6.5.1 Connection condition

Using the data for each of the machines given in Table 1, the transient starting currents were computed, and comparisons with the recorded values of the same currents are shown in Figures 6.1(a) - (d), with the instant of switching to the red-to-yellow line voltage being indicated. From Figures 6.1 we see that the degree of agreement is generally good for all the machines investigated. The most notable feature of these curves is that for the first few cycles the experimental result is somewhat greater than the predicted result. The major reason for this is that the high currents drawn from the supply following connection of the machines are sufficient to cause saturation in the leakage flux paths, particularly in the teeth, and so to reduce the leakage reactances of both the stator and the rotor.

Experimental results for the transient torque of machine IV, as recorded by a calibrated drag-cup tachogenerator, following connection at standstill and at 0.95 p.u. speed are shown in Figure 6.2, together with computed results for the same conditions. Comparisons between the two sets of results in these figures show that the agreement between theoretical and experimental values is again as good as that achieved by other authors. We may notice that the experimental result is now less than the computed result, although this is again due to the differences of the machine inductances arising from saturation.

6.5.1.1 Transient current

Computed values of transient currents were obtained to investigate how the magnitude of the peak current depends on the instant in the applied voltage cycle at which connection is made. Figures 6.3(a) - (d) were obtained for machines I, II, III and IV respectively. These show how the values of the first to the fourth peaks are dependent on the instant of switching, when the machine is connected at standstill. Corresponding results for connecting the same machines at 0.45 p.u. and 0.9 p.u. speed and with no initial rotor currents flowing are shown in Figures 6.3. From these figures the first current peak is seen to be little affected by the initial speed but greatly affected by the switching angle, whereas the subsequent peaks are all greatly affected by both the speed and the angle of switching. If the switching occurs in the early part of a cycle the first peak current is more dependent upon speed than when the switching occurs later in the cycle.

A physical interpretation of the effect of the speed on the current peaks can be based on the fact that sudden application of the stator voltage creates both the normal and the transient rotating mmf's, together with a transient decaying mmf. This later mmf is stationary in space, and since the resultant flux in the machine is zero at the instant of switching, it has a spatial direction opposed to that of the initial value of the rotating mmf. At the same time, the transient currents depend on the rotor speed, because of the associated rotational emf's, and these emf's also depend on the presence of the transient mmf. The rotational emf's will have an appreciable effect only after the first half cycle, by which time the transient magnetizing current has had sufficient time to become established. We may notice in Figure 6.3 that the first peak of the current when the switching angle is zero is equal but opposite to the second peak when the switching angle is π. Similarly, the second peak when the switching angle is zero is equal but opposite to the third peak when the switching angle is π, and this is repeated again for the third and fourth peaks. This result arises since, as the switching angle approaches π, the first current peak will inevitably be very small, and the switching conditions will result in a current transient almost the negative of that which follows switching at an angle of zero.

6.5.1.2 Transient torque

Figures 6.4(a) - 6.7(a) show computed torque patterns for machines I, II, III and IV respectively, following connection at an initial speed of 0.4 p.u., and each of the first three maxima and minima of these patterns are used to establish one point on Figures 6.4(b) - 6.7(b). These latter diagrams show the variations of the maximum and minimum values of the initial torque peaks with the speed on connection, and indicate that the most severe torque in the normal operating speed range is in the forward direction and occurs on starting from rest. As the speed on connection increases the first peak of the torque decreases, until at speeds in excess of 0.5 p.u. it becomes negative and opposed to the direction of rotation. After becoming negative, the value of the torque peak increases steadily as the speed on connection increases, and the maximum negative torque occurs following connection at synchronous speed.

6.5.2 Disconnection condition

Figures 6.8(a) - (c) show comparisons between the recorded and computed voltages across the motor terminals, following disconnection from the supply of machines I, II and III, and good agreement between the experimental and theoretical curves is again evident.

The voltages shown are of varying and decreasing amplitude and frequency, with the voltage decay governed by the time constant of the open-circuited motor and the frequency decay by the time constant of the mechanical system.

6.5.3 Reswitching condition

Figures 6.9(a) - (d) show comparisons between the computed and experimental results of the transient currents, following reconnection to the same supply of machines I, II, III and IV respectively, after a disconnection beginning at the instants shown. A comparison between predicted and experimental results for the transient torque following disconnection and reconnection of machine IV is shown in Figure 6.10. Similar comparisons for plugging and for star-delta starting of the same machine are given in Figure 6.11. Both in these figures and in Figure 6.9 the agreement between the two sets of results is generally good, with any differences being explainable on the same basis as for the starting condition.

6.5.3.1 Reswitching to the same supply

When a motor is disconnected from the supply a changing phase shift arises between the generated emf across the stator terminals and the supply voltage. The degree of this change is obviously dependent upon the loading on the motor.

To investigate the effect of the duration of interruption of the supply on the first peaks of the transient torque when the motor is subsequently reconnected, this torque was computed for the unloaded machine, following reconnection after increasing periods of interruption. Figures 6.12(a) to 6.15(a) show the patterns of the torque curves following reconnection after an interruption of 10 ms, for machines I, II, III and IV respectively. From a series of such curves Figures 6.12(b) to 6.15(b) were developed to illustrate the variations of the first two peaks of torque with increasing interruption. Comparing Figures 6.12(b) to 6.15(b) with Figures 6.4(b) to 6.7(b), we see that whereas while the first peak following connection of an electrically inert machine is negative at a speed greater than 0.5 p.u, that when the machine is reconnected at the same speed but with currents in the rotor is positive and less severe. This is mainly due to the motoring action of the current carrying rotor in the net transient mmf produced on connection being superimposed on and exceeding the braking action of the electrically inert machine in the net transient mmf.

Figures 6.12(b) to 6.15(b) show that the magnitude of the peak torque initially increases with the time of the supply interruption. Following a maximum value of about 1.9 p.u. for machine I, 2.4 p.u. for machine II, 0.9 p.u. for machine III, and 2.1 p.u. for machine IV, the value of the first peak torque thereafter decreases. Although the peak emf generated in the stator by the rotor currents decreases as the time of interruption increases, the phase shift increases such that the resultant of the generated stator emf and the supply voltage increases. The maximum value of the peak torque occurs when the phase shift is about 180^0, and after this the phase shift and the peak torque both begin to decrease.

To investigate the effect of changing the instant at which the motor is disconnected from the supply, Figures 6.16(b) to 6.19(b) were developed. These are drawn for constant length of interruption and show that under this condition the instant at which the motor is disconnected is of no importance and the transient torque pattern remains unchanged. The results for the current peaks are shown in Figures 6.20(b) to 6.23(b) and show that the peak current is changing between positive and negative values with a rapid transition happening at intervals of 10 ms.

6.5.3.2. Plugging

When the phase-sequence of the supply to a motor is quickly reversed, while currents are still flowing in the rotor circuit, a very rapid braking effect is produced. The torque and the currents during the brief braking period while the motor slows to standstill are extremely large.

To investigate how the peak currents and torques depend upon the time taken in changing the phase-sequence, computed results for each of the machines were obtained. Figures 6.24(a) to 6.27(a) show the torque patterns produced by each machine when a delay of 70 ms occurs before the application of the reversed phase-sequence voltage. The largest of the torque peaks of many such patterns are plotted against the delay time, to provide Figures 6.24(b) to 6.27(b). The torque peak variations are seen to vary between maximum and minimum values, with a greatest torque peak occurring when the supply and the stator voltages are in antiphase and the smallest peak when they are cophasal.

Figures 6.28(a) to 6.31(a) show patterns for the stator currents following plugging of each machine, after supply interruptions of 10 ms. From a series of such curves Figures 6.28(b) to 6.31(b) were produced, showing the variations of the maximum current peak with the delay. From these figures we see that for each motor the peak current is alternately positive and negative, with a rapid transition occurring at intervals of about 10 ms.

6.5.3.3 Star-delta starting

When the phase voltage of an induction motor is reduced, the line current and the developed torque are each also reduced. Although reduced voltage starting may cause the current to be less, the presence of the decaying rotor current at the instant when full voltage is applied in star-delta starting may lead to transient currents, which though of brief duration, may be more severe than those during direct connection at the same speed but without any currents flowing in the rotor.

A series of star-delta starting transients for the unloaded machines were computed, for increasing periods of delay between star connected operation and the application of full voltage when reconnected in delta. Figures 6.32(b) to 6.35(b) show that the peak torque is never very severe for the four machines (I, II, III and IV). The corresponding current variations are shown in Figures 6.36(b) to 6.39(b). Although, the transient condition is affected by the angular position of the decaying rotor mmf axis at the instant of applying full voltage, and because the motors are unloaded, no high currents and torques are produced.

The current results, similar to the results following reswitching to the same supply and plugging, are again alternating between positive and negative limits, with a rapid transition occurring at intervals of about 10 ms. This transition arises as a result of the maximum peak in the current transient not always being the first peak.

6.6 Conclusions

This chapter has been devoted mainly to establishing the validity of the digital-computer program simulating the single 3-phase induction motor. The idealized model of the motor was used in preparing a digital-computer program, and subsequently used for the evaluation of the overall performance of several machines. The comparisons of the measured and computed results generally showed very good agreement, for the transient currents following connection and reconnection to the same supply of each of four machines used. The stator voltages induced by the decaying rotor currents following disconnection were also accurately predicted.

The predicted results for the transient torques developed following connection, reconnection to the same supply, star-delta starting and plugging all showed reasonably good agreement with the experimental results, except in the case of plugging. The quite considerable deviations there may be attributed partly to the very considerable effects of local saturation on leakage inductances of the machine, and partly to the assumption that the rotor of the machine is a rigid body. Although this latter assumption appears reasonable, it may not be valid under the very high torques developed following plugging.

However, it is considered that the validity of the induction motor model and the digital-computer program developed for the solution of the machine equations have been fully established for the majority of situations likely to be encountered in practice.

CHAPTER 7

TRANSIENT BEHAVIOUR OF TWO MOTORS

In Chapter 2, the general theory modelling a group of n induction motor was presented. However, to clarify the theory and to illustrate its application, a group of a definite number of machines needs to be investigated. In this Chapter the theory of a group of two motors is considered in detail.

7.1 Connection Condition

If the two machines are electrically inert and connected directly to the supply, the transient behaviour of one of the machines will affect the transient behaviour of the other, since any voltage drop in the supply impedance will cause a corresponding dip in the common terminal voltage applied to the two machines. The amount of this reduction will depend both on the size of the machines and on the short-circuit capacity of the network. Thus the behaviour of the two motors is interactive, and the performance of each machine may be much different from what it would be if used alone. However, if the supply is stiff, and no voltage dip is caused by the current drawn, the behaviour of the two motors in parallel is exactly the same as their behaviour when each machine is connected alone. This condition will be considered first, to show clearly the approach which is adopted.

7.1.1 Stiff-supply

The electrical differential equations of any group of n-motors as given in Equation 2.8, are reduced in the case of two motors to Equation 7.1, or briefly,

$$[v] = [R][i] + [L][Di] + \dot{\theta}_1 [G_1][i] + \dot{\theta}_2 [G_2][i] \qquad 7.2$$

where

$$[v] = \begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, [R] = \begin{bmatrix} R_{s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{s_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{r_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{r_1} & 0 & 0 & 0 & 0 \\ R_{s_1} & 0 & 0 & 0 & -R_{s_2} & 0 & 0 & 0 \\ 0 & R_{s_1} & 0 & 0 & 0 & -R_{s_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_{r_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{r_2} \end{bmatrix}$$

$$[L] = \begin{bmatrix} L_{s_1} & 0 & M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{s_1} & 0 & M_1 & 0 & 0 & 0 & 0 \\ M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 \\ L_{s_1} & 0 & M_1 & 0 & -L_{s_2} & 0 & -M_2 & 0 \\ 0 & L_{s_1} & 0 & M_1 & 0 & -L_{s_2} & 0 & -M_2 \\ 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} \end{bmatrix}$$

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 & 0 \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 \\
M_1D & M_1\dot{\theta}_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot{\theta}_1 & 0 & 0 & 0 & 0 \\
-M_1\dot{\theta}_1 & M_1D & -L_{r_1}\dot{\theta}_1 & R_{r_1}+L_{r_1}D & 0 & 0 & 0 & 0 \\
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D \\
0 & 0 & 0 & 0 & M_2D & M_2\dot{\theta}_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot{\theta}_2 \\
0 & 0 & 0 & 0 & -M_2\dot{\theta}_2 & M_2D & -L_{r_2}\dot{\theta}_2 & R_{r_2}+L_{r_2}D
\end{bmatrix}
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \end{bmatrix}
\qquad \ldots\ldots \quad 7.1
$$

$$[G_1] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 \\ -M_1 & 0 & -L_{r_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_2] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} \\ 0 & 0 & 0 & 0 & -M_2 & 0 & -L_{r_2} & 0 \end{bmatrix}$$

and

$$[i] = \begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \end{bmatrix}$$

If Equation 7.1 is compared with the equivalent equation obtained by simply writing down the equations for the separate machines (see Equation 2.7), several noticeable differences appear. Thus, in the impedance matrix which results, terms only arise in the upper left and lower right quadrants, and the column matrix of the system voltages contains two further entries in the fifth and sixth rows. Although these differences are not significant at this stage, they become important and offer advantages in favour of Equation 7.1 in the considerations given to the non-stiff supply situation in the next section.

7.1.2 Non-stiff Supply

As was shown in Chapter 3, there are two approaches to the inclusion of the supply impedance in the system model. In the first of these, the electrical equations of the supply in 3-phase form are required together with the equations of the machines in d,q form given in Equation 7.1. The equations of the supply are

$$v_{ga} = v_a - (R' + L'D)\, i_a \qquad \ldots\ldots \quad 7.3$$

$$v_{gb} = v_b - (R' + L'D)\, i_b \qquad \ldots\ldots \quad 7.4$$

$$v_{gc} = v_c - (R' + L'D)\, i_c \qquad \ldots\ldots \quad 7.5$$

where v_{ga}, v_{gb} and v_{gc} are the instantaneous voltages applied to the two machines, and

$$i_a = i_{a_1} + i_{a_2}$$

$$i_b = i_{b_1} + i_{b_2}$$

$$i_c = i_{c_1} + i_{c_2}$$

where i_{a_1}, i_{b_1} and i_{c_1} are the instantaneous phase currents of the first motor and i_{a_2}, i_{b_2} and i_{c_2} are those of the second motor. We may notice that when this approach is followed, transformations from instantaneous phase quantities to d,q quantities and vice versa are required throughout the numerical solution of the overall system equations. For example, the d,q components of the common terminal voltage are found at every stage of the solution, from

$$v_{sdg} = \sqrt{\frac{2}{3}} \left(v_{ga} - \frac{1}{2} v_{gb} - \frac{1}{2} v_{gc}\right) \quad \ldots\ldots \quad 7.6$$

$$v_{sqg} = \sqrt{\frac{2}{3}} \left(\frac{\sqrt{3}}{2} v_{gb} - \frac{\sqrt{3}}{2} v_{gc}\right) \quad \ldots\ldots \quad 7.7$$

which replace v_{sd} and v_{sq} in Equation 7.1.

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
(R'+R_{s_1})+(L'+L_{s_1})D & 0 & M_1 D & 0 & R'+L'D & 0 & 0 & 0 \\
0 & (R'+R_{s_1})+(L'+L_{s_1})D & 0 & M_1 D & 0 & R'+L'D & 0 & 0 \\
M_1 D & M_1 \dot{\theta}_1 & R_{r_1}+L_{r_1} D & L_{r_1} \dot{\theta}_1 & 0 & 0 & 0 & 0 \\
-M_1 \dot{\theta}_1 & M_1 D & -L_{r_1} \dot{\theta}_1 & R_{r_1}+L_{r_1} D & 0 & 0 & 0 & 0 \\
R_{s_1}+L_{s_1} D & 0 & M_1 D & 0 & -(R_{s_2}+L_{s_2} D) & 0 & -M_2 D & 0 \\
0 & R_{s_1}+L_{s_1} D & 0 & M_1 D & 0 & -(R_{s_2}+L_{s_2} D) & 0 & -M_2 D \\
0 & 0 & 0 & 0 & M_2 D & M_2 \dot{\theta}_2 & R_{r_2}+L_{r_2} D & L_{r_2} \dot{\theta}_2 \\
0 & 0 & 0 & 0 & -M_2 \dot{\theta}_2 & M_2 D & -L_{r_2} \dot{\theta}_2 & R_{r_2}+L_{r_2} D
\end{bmatrix}
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \end{bmatrix}
\qquad \ldots\ldots \quad 7.8
$$

In the second approach, the impedance of the supply is included in an overall set of d,q equations, leading to Equation 7.8 in which R' and L' are the resistance and inductance of the supply, and v_{sd} and v_{sq} are the direct and quadrature components of the stiff supply voltage behind the supply impedance, obtained by transforming the phase voltages of the supply into their corresponding d,q components.

In either approach, the electromagnetic torque developed by the first machine is given by

$$T_{e_1} = p_1 M_1 (i_{sq_1} i_{rd_1} - i_{sd_1} i_{rq_1}) \qquad \ldots\ldots \qquad 7.9$$

and that produced by the second machine by

$$T_{e_2} = p_2 M_2 (i_{sq_2} i_{rd_2} - i_{sq_2} i_{rq_2}) \qquad \ldots\ldots \qquad 7.10$$

7.2 Disconnection Condition

When the two machines working in a steady-state condition are disconnected from the common supply, both their stator and rotor currents and their fluxes decay at a rate determined by the loads and the parameters of the two machines, from initial values determined by the load conditions and the time of disconnection. During disconnection there will be a transfer of power between the two machines, and although at first one machine will act alternately as a motor and as a generator, this will eventually settle down to a situation where the machine with the greater moment of inertia feeds power to the other until the fluxes in both machines have decayed to zero. While flux still exists, the speeds of the two machines will consequently decay at a closely similar rate, although in both machines this may be at a considerably different rate from that when used alone. This interaction between the two motors during supply interruption is an important factor, affecting considerably the transient behaviour when the two machines are subsequently reswitched to the same or to a different supply.

The electrical equations during disconnection of any number of machines (as given in Equation 2.21) reduce for two motors to

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (R_{s_1}+R_{s_2})+(L_{s_1}+L_{s_2})D & 0 & M_1 D & 0 & -M_2 D & 0 \\ 0 & (R_{s_1}+R_{s_2})+(L_{s_1}+L_{s_2})D & 0 & M_1 D & 0 & -M_2 D \\ M_1 D & M_1 \theta_1 & R_{r_1}+L_{r_1} D & L_{r_1}\theta_1 & 0 & 0 \\ -M_1\theta_1 & M_1 D & -L_{r_1}\theta_1 & R_{r_1}+L_{r_1} D & 0 & 0 \\ -M_2 D & -M_2\theta_2 & 0 & 0 & R_{r_2}+L_{r_2} D & L_{r_2}\theta_2 \\ M_2\theta_2 & -M_2 D & 0 & 0 & -L_{r_2}\theta_2 & R_{r_2}+L_{r_2} D \end{bmatrix} \begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{rd_2} \\ i_{rq_2} \end{bmatrix} \qquad \ldots\ldots \quad 7.11$$

where during disconnection

$$i_{sd_2} = - i_{sd_1}$$

$$i_{sq_2} = - i_{sq_1}$$

Equation 7.11 can be rewritten briefly as

$$[0] = [R]_{dis}[i] + [L]_{dis}[Di] + \dot{\theta}_1 [G_1]_{dis}[i] + \dot{\theta}_2 [G_2]_{dis}[i] \qquad \ldots\ldots \quad 7.12$$

in which

$$[R]_{dis} = \begin{bmatrix} R_{s_1} + R_{s_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{s_1} + R_{s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{r_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{r_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{r_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{r_2} \end{bmatrix},$$

$$[L]_{dis} = \begin{bmatrix} L_{s_1} + L_{s_2} & 0 & M_1 & 0 & -M_2 & 0 \\ 0 & L_{s_1} + L_{s_2} & 0 & M_1 & 0 & -M_2 \\ M_1 & 0 & L_{r_1} & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 \\ -M_2 & 0 & 0 & 0 & L_{r_2} & 0 \\ 0 & -M_2 & 0 & 0 & 0 & L_{r_2} \end{bmatrix}$$

$$[G_1]_{dis} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 \\ -M_1 & 0 & -L_{r_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_2]_{dis} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -M_2 & 0 & 0 & 0 & L_{r_2} \\ M_2 & 0 & 0 & 0 & -L_{r_2} & 0 \end{bmatrix}$$

7.2.1 Electromagnetic Torques

Although the machines are disconnected from the supply, the alternating currents which continue to flow in the stator circuits of the two machines produce a rotating mmf in the air-gap of each machine. The interaction between decaying currents in the stator and the rotor circuits will result in the production of an electromagnetic torque, which for the two machines is

$$T_{e_1} = p_1 \, [i_t][G_1]_{dis} \, [i] \qquad \ldots\ldots \qquad 7.13$$

and

$$T_{e_2} = p_2 \, [i_t][G_2]_{dis} \, [i] \qquad \ldots\ldots \qquad 7.14$$

respectively.

7.3 Analytical Solution During Disconnection

When the two motors are disconnected from the common supply, three electrical circuits are obtained from the two separate rotor circuits and the circuit formed by the stators of the two machines. Since the currents flowing in the stator circuits are the same in magnitude, but opposite in sign, the instantaneous symmetrical component form of the electrical equations are

$$M_1 (D - j\dot{\theta}_1) i_{sp} + [R_{r_1} + L_{r_1} (D - j\dot{\theta}_1)] (i_{rp_1} e^{j\theta_1}) = 0$$

$$-M_2 (D - j\dot{\theta}_1) i_{sp} + [R_{r_2} + L_{r_2} (D - j\dot{\theta}_2)] (i_{rp_2} e^{j\theta_2}) = 0$$

$$[(R_{s_1} + R_{s_2}) + (L_{s_1} + L_{s_2})D] i_{sp} + M_1 D (i_{rp_1} e^{j\theta_1}) - M_2 D (i_{rp_2} e^{j\theta_2}) = 0$$

Taking Laplace transform we obtain

$$M_1 (S - j\theta_1) I_1 + [R_{r_1} + L_{r_1} (S - j\theta_1)] I_2 = M_1 i_{sp_0} + L_{r_1} i_{rp_{10}} \qquad \ldots\ldots \quad 7.15$$

$$-M_2 (S - j\theta_2) I_1 + [R_{r_2} + L_{r_2} (S - j\theta_2)] I_3 = -M_2 i_{sp_0} + L_{r_2} i_{rp_{20}} \qquad \ldots\ldots \quad 7.16$$

$$[(R_{s_1} + R_{s_2}) + (L_{s_1} + L_{s_2})s]\, I_1 + M_1\, s\, I_2 - M_2\, s\, I_3 = (L_{s_1} + L_{s_2}) i_{sp_0} +$$

$$+ M_1\, i_{rp_{10}} - M_2\, i_{rp_{20}}$$

..... 7.17

where I_1, I_2 and I_3 are respectively, the Laplace transform of i_{sp}, $i_{rp_1} e^{j\theta_1}$ and $i_{rp_2} e^{j\theta_2}$, and i_{sp_0}, $i_{rp_{10}}$ and $i_{rp_{20}}$ are the initial values of these currents. As shown in Appendix E, Equations 7.15, 7.16 and 7.17 may be solved for I_1, I_2 and I_3. Inverse Laplace transform may then be taken to give the positive-sequence component of the stator current as

$$i_{sp} = \frac{e^{\lambda_1 t}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} [\lambda_1^2\, i_{sp_0} + \lambda_1 (a - jb) + (c - jd)]$$

$$+ \frac{e^{\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} [\lambda_2^2\, i_{sp_0} + \lambda_2 (a - jb) + (c - jd)]$$

$$+ \frac{e^{\lambda_3 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} [\lambda_3^2\, i_{sp_0} + \lambda_3 (a - jb) + (c - jd)]$$

..... 7.18

the positive-sequence component of the rotor current of the first machine as

$$i_{rp_1} e^{j\theta_1} = [\lambda_1^2 \, i_{rp_{10}} + (a_1 - j\, b_1)\, \lambda_1 + (c_1 - j\, d_1)] \frac{e^{\lambda_1 t}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}$$

$$+ [\lambda_2^2 \, i_{rp_0} + (a_1 - j\, b_1)\, \lambda_2 + (c_1 - j\, d_1)] \frac{e^{\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)}$$

$$+ [\lambda_3^2 \, i_{rp_{10}} + (a_1 - j\, b_1)\, \lambda_3 + (c_1 - j\, d_1)] \frac{e^{\lambda_3 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}$$

and the positive-sequence component of the rotor current of the second machine as

$$i_{rp_2} e^{j\theta_2} = \frac{e^{\lambda_1 t}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \; [\lambda_1^2 \, i_{rp_{20}} + (a_2 - j\, b_2)\lambda_1 + (c_2 - j\, d_2)]$$

$$+ \frac{e^{\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} [\lambda_2^2 i_{rp_{20}} + (a_2 - j\, b_2)\, \lambda_2 + (c_2 - j\, d_2)]$$

$$+ \frac{e^{\lambda_3 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} [\lambda_3^2 i_{rp_{20}} + (a_2 - j\, b_2)\, \lambda_3 + (c_2 - j\, d_2)]$$

The positive-sequence component of the common stator voltage is given in Appendix E as

$$v_{sp} = \{\lambda_1^3 (i_{sp_0} L_{s_1} + M_1 i_{20}) + \lambda_1^2 [i_{sp_0} R_{s_1} + (a - j b) L_{s_1} +$$

$$M_1 (a_1 - j b_1)] + \lambda_1 [(a - j b) R_{s_1} + L_{s_1} (c - j d) +$$

$$M_1 (c_1 - j d_1)] + R_{s_1} (c - j d)\} \frac{e^{\lambda_1 t}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}$$

$$+ \{\lambda_2^3 (i_{sp_0} L_{s_1} + M_1 i_{rp_{10}}) + \lambda_2^2 [R_{s_1} i_{sp_0} + L_{s_1} (a - j b) +$$

$$M_1 (a_1 - j b_1)] + \lambda_2 [(a - j b) R_{s_1} + L_{s_1} (c - j d) +$$

$$M_1 (c_1 - j d_1)] + R_{s_1} (c - j d)\} \frac{e^{\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)}$$

$$+ \{\lambda_3^3 (i_{sp_0} L_{s_1} + M_1 i_{rp_{10}}) + \lambda_3^2 [i_{sp_0} R_{s_1} + L_{s_1} (a - j b)$$

$$+ M_1 (a_1 - j b_1)] + \lambda_3 [(a - j b) R_{s_1} + L_{s_1} (c - j d)$$

$$+ M_1 (c_1 - j d_1)] + R_{s_1} (c - j d)\} \frac{e^{\lambda_3 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}$$

On applying the previous analytical solution to the group consisting of the 0.75 kW and 1.5 kW motors the roots of the characteristic equation (see Appendix E) which have the form

$$\lambda = \alpha + j\,\beta$$

and when the machine parameters are substituted in this equation the time constants and frequencies of the components are obtained as

$$\lambda_1 = -\,8.0857 + j\;313.4569$$

$$\lambda_2 = -\,122.58 + j\;273.2098$$

$$\lambda_3 = -\,98.343 + j\;40.11$$

from which we notice that the component with the greatest time constant of 123.6 ms has the greatest frequency of 49.98 Hz while the second component has a time constant of 8.16 ms and a frequency of 43.48 Hz. The third component has a time constant of 10.17 ms and a frequency of 6.383 Hz.

From these results, the currents and the voltages following disconnection are seen to be composed of a lightly damped alternating component at a frequency nearly equal the frequency of the supply, a highly damped alternating component with a frequency of 43.5 Hz and a low frequency component at 6.383 Hz.

7.4 Reswitching Condition

When, after a certain time, the two motors are reconnected to a common supply, they may be treated in precisely the same way as following the original connection. The only difference is that the initial currents are now not zero, but have values given from the considerations of the disconnection period given in Section 7.2.

Equations 7.1 and 7.2 are used again to represent the two machines when reconnected. However, the voltage matrix on the left hand side of both of these equations needs to be determined according to the reconnected supply voltage.

7.5 Mechanical System

The mechanical system equations for the two machines can be derived directly from the general mechanical equations given in Section 2.4 of Chapter 2, by replacing m by 1 and 2 respectively.

7.6 Numerical Solution and Computer Program

The procedure discussed previously in Chapter 4 for the n-machine group is also followed in the numerical solution of the electrical and mechanical equations of the 2-machine group. The program is divided into three sections; connection, disconnection and reconnection.

7.6.1 Connection Condition

To perform the numerical integration of the electrical differential equation, Equation 7.2 may be rewritten as

$$[Di] = [L]^{-1}\{[v] - [R][i] - \dot{\theta}_1 [G_1][i] - \dot{\theta}_2 [G_2][i]\} \quad .. \qquad 7.19$$

where the matrices [R], [L], $[G_1]$ and $[G_2]$ are given in Section 7.1.1 for the case of a stiff supply, and when the supply is non-stiff they can be obtained from Equation 7.8. The initial conditions for the currents are zero for electrically inert machines, and the initial values of the speeds are also zero if the machines are started from rest. The voltage matrix [v] contains only zeros and terms representing the supply voltage. However, if the second approach for the case of the non-stiff supply is used, the initial values of the currents and speeds remain the same, but the initial values of the common voltages across the machines need to be known. It was shown in Equation 4.14 of Section 4.1.1 that the common voltage applied to a group of n-motors is

$$v_g = \left[1 - \left[\frac{L^`}{L^` + \dfrac{(L_{s_1} + L_{r_1} - 2\,M_1)}{K_G}} \right] \right] v \; .$$

where in the case of two motors K_G is given by

$$K_G = 1 + \frac{L_{s_1} + L_{r_1} - 2M_1}{L_{s_2} + L_{r_2} - 2M_2} \qquad \ldots\ldots \qquad 7.20$$

Computation of the electromagnetic torque developed by the motors can be performed using Equations 7.9 and 7.10. The variation of the speed of each motor is taken into account by numerically integrating the mechanical equations of both machines; thus for the first machine

$$D^2\,\theta_1 = \frac{T_{e_1} - T_{f_1} - T_{t_1}}{J_1} \qquad \ldots\ldots \qquad 7.21$$

if it is loaded and

$$D^2\,\theta_1 = \frac{T_{e_1} - T_{f_1}}{J_1} \qquad \ldots\ldots \qquad 7.22$$

if it is unloaded. Correspondingly, for the second machine

$$D^2\,\theta_2 = \frac{T_{e_2} - T_{f_2} - T_{t_2}}{J_2} \qquad \ldots\ldots \qquad 7.23$$

and

$$D^2\,\theta_2 = \frac{T_{e_2} - T_{f_2}}{J_2} \qquad \ldots\ldots \qquad 7.24$$

If the 3-phase rotor currents are required, the d,q currents need to be transformed to rotating axes, and for this transformation the angular position of the rotor θ is required.

To obtain this, the speed is numerically integrated using the differential equations

$$D\,\theta_1 = \dot{\theta}_1 \qquad \ldots\ldots \quad 7.25$$

for the first machine, and

$$D\,\theta_2 = \dot{\theta}_2 \qquad \ldots\ldots \quad 7.26$$

for the second machine.

The values of the currents, speeds and angular positions obtained at the end of each step-length are taken as initial values for the next step-length, and the process is repeated until the next switching operation, i.e. disconnection of the group from the supply.

7.6.2 Disconnection Condition

Following interruption of the supply the matrices $[R]_{dis}$, $[L]_{dis}$, $[G_1]_{dis}$ and $[G_2]_{dis}$ are as given in Section 7.2, and the initial conditions of the speeds are taken as those immediately before the interruption. To perform the numerical integration of the electrical equations during disconnection, Equation 7.12 is rewritten as

$$[Di] = [L]_{dis}^{-1}\,\{[0] - [R]_{dis}[i] - \dot{\theta}_1[G\,]_{dis}[i] - \dot{\theta}_2[G\,]_{dis}[i]\} \qquad \ldots\ldots \quad 7.27$$

which may be numerically integrated. The electromagnetic torque during disconnection is computed from Equations 7.13 and 7.14. The speeds of the two machines are obtained by numerically integrating their accelerations given by Equations 7.22 and 7.24 if the motors are loaded or Equations 7.23 and 7.25 if they are unloaded. The angular-positions of the rotors are obtained by integrating Equations 7.25 and 7.26. However, before the numerical integration can begin, it is necessary to determine the initial values of the currents on disconnection, and these are determined using the constant-flux-linkage theorem applied to the rotor and stator circuits of the two machines as described in the following section.

7.6.2.1 Application of constant-flux-linkage theorem

Applying constant flux linkage considerations to the d- and q-axes of the stator and rotor circuits of the two machines leads to

$$L_{r_1} (i_{rd_1})_0 + M_1 (i_{sd_1})_0 = L_{r_1} (i_{rd_1})_1 + M_1 (i_{sd_1})_1$$

$$L_{r_1} (i_{rq_1})_0 + M_1 (i_{sq_1})_0 = L_{r_1} (i_{rq_1})_1 + M_1 (i_{sq_1})_1$$

$$L_{r_2} (i_{rd_2})_0 + M_2 (i_{sd_2})_0 = L_{r_2} (i_{rd_2})_1 + M_2 (i_{sd_2})_1$$

$$L_{r_2}(i_{rq_2})_0 + M_2(i_{sq_2})_0 = L_{r_2}(i_{rq_2})_1 + M_2(i_{sq_2})_1$$

$$L_{s_1}(i_{sd_1})_0 + M_1(i_{rd_1})_0 = L_{s_1}(i_{sd_1})_1 + M_1(i_{rd_1})_1$$

$$L_{s_1}(i_{sq_1})_0 + M_1(i_{rq_1})_0 = L_{s_1}(i_{sq_1})_1 + M_1(i_{rq_1})_1$$

where the suffices $_{0,1}$ denote the conditions immediately before and immediately after disconnection. In matrix form, these equations are

$$\begin{bmatrix} \psi_{rd_1} \\ \psi_{rq_1} \\ \psi_{rd_2} \\ \psi_{rq_2} \\ \psi_{sd_1} \\ \psi_{sq_1} \end{bmatrix} = \begin{bmatrix} M_1 & 0 & L_{r_1} & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 \\ -M_2 & 0 & 0 & 0 & L_{r_2} & 0 \\ 0 & -M_2 & 0 & 0 & 0 & L_{r_2} \\ L_{s_1} & 0 & M_1 & 0 & 0 & 0 \\ 0 & L_{s_1} & 0 & M_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} (i_{sd_1})_1 \\ (i_{sq_1})_1 \\ (i_{rd_1})_1 \\ (i_{rq_1})_1 \\ (i_{rd_2})_1 \\ (i_{rq_2})_1 \end{bmatrix}$$

where

$$\begin{bmatrix} \psi_{rd_1} \\ \psi_{rq_1} \\ \psi_{rd_2} \\ \psi_{rq_2} \\ \psi_{sd_1} \\ \psi_{sq_1} \end{bmatrix} = \begin{bmatrix} M_1 & 0 & L_{r_1} & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 \\ -M_2 & 0 & 0 & 0 & L_{r_2} & 0 \\ 0 & -M_2 & 0 & 0 & 0 & L_{r_2} \\ L_{s_1} & 0 & M_1 & 0 & 0 & 0 \\ 0 & L_{s_1} & 0 & M_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} (i_{sd_1})_0 \\ (i_{sq_1})_0 \\ (i_{rd_1})_0 \\ (i_{rq_1})_0 \\ (i_{rd_2})_0 \\ (i_{rq_2})_0 \end{bmatrix}$$

and once the initial values have been determined, the variation of the stator and rotor currents during disconnection can be calculated using Equation 7.27.

7.6.3 Reswitching Condition

Following reswitching, the original [R], [L], $[G_1]$ and $[G_2]$ matrices used during connection condition are recalled, and the initial values of the currents, speeds and angular positions of the rotors are those immediately before the instant of the re-switching. The numerical solution then continues, using a voltage matrix determined for the supply to which the machines have been reconnected.

The flowchart of the computer program for the two motors situation is virtually that of the n-motor situation shown in Figure 4.1. The listings of the computer program simulating the two machines are given in Appendix F.

The selection of which of the two approaches described provides the most useful method of solution with a non-stiff supply depends on the time taken on the computer. The minimum length of the step which may be used in the numerical integration process is an important factor in determining the time which will be consumed.

Granborg[35] has shown, by forming an explicit relationship between the computational time increment and the coefficients of the differential equation system, that the limit of numerical stability in a Runge-Kutta method is reached with a step-length of 2.8 times the system smallest time constant. However, during the present integration, it was found that using a step-length equal to the smallest time constant of the system gave good stability of the numerical solution.

In Appendix A the method is given by which the time constants of a group of motors are obtained. For a 2-machine system formed from the 0.75 kW and 1.5 kW motors described in Table I, it was found that when using the first approach described for a weak supply, a step length of 1 ms is sufficiently small to enable a stable solution to be obtained, while the second approach was followed it was necessary to decrease this to 0.5 ms, due to the drastic change in the impedance matrix brought about by the inclusion of the supply impedance. Although additional computation needs to be performed in the first approach, less time is consumed than in the second approach with its much reduced step-length. For this reason the first approach is preferred.

7.7 Transient Conditions in 2-Motor Group

A group of two motors was formed from the 0.75 kW and the 1.5 kW motors detailed in Table I. The computer program described in Section 7.6 was used to predict the transient currents, torques and speeds following connection of the group to a stiff-supply and to a non-stiff supply simulated by inclusion of an 11.0Ω resistor in each supply line.

The computer program was used also to predict the transient conditions following disconnection of the group of motors from the supply, as well as following reconnection, plugging and star-delta starting.

7.7.1 Connection Condition

Figures 7.1(a) and 7.1(b) show comparisons between the predicted and measured currents in the b-line of the 0.75 kW motors and the r-line of the 1.5 kW motor, when connected to a stiff supply at the instant indicated by the voltage waveform.

From these results it is clear that the 1.5 kW motor takes a much shorter time to reach its steady-state condition than does the 0.75 kW, due mainly to the relatively high moment of inertia of the smaller machine. The differences in the behaviour of the two motors are clearly sufficient to ensure that an adequately searching investigation may be made of the effect of switching to a non-stiff supply.

From the correlations in Figure 7.1, it can be seen that the predicted and measured currents are in uniformly good agreement. As would be expected, the computed currents given here coincide with those computed when each of the motors is connected separately, but at the same instant on the supply cycle, since there is no interaction between the two machines. The transient torques and speeds of the two machines are shown in Figures 7.2(a) and 7.2(b). Again, the electromagnetic torque patterns obtained are precisely the same as those when each machine is connected separately to a stiff supply. Also from these figures, it is now clear that the 1.5 kW motor achieves its steady-state condition after about 8.5 cycles of the supply while the 0.75 kW motor takes about 13.5 cycles to reach this condition. As further confirmation of the accuracy with which the computer program predicts the experimental behaviour, Figure 7.3 shows the close correlation which exists between the theoretical and measured results for the torque developed in a group formed by the 5.6 kW and the 2.25 kW motors detailed in Table I.

With the 11.0Ω resistors included in the circuit, the starting currents of the two motors were obtained experimentally and theoretically, as shown in Figure 7.4. The common terminal voltage of the two machines was also measured and predicted, with the results shown in Figure 7.5. Because of the long time involved, drawing the instantaneous values of the currents and voltages will not provide more information than is given by simply recording the positive peak values as in the figures. The negative peak values are simply the mirror image in the time axis of the positive peak values.

From Figures 7.4 and 7.5 it is clear that the predicted and measured results are again in reasonably good agreement. In view of the now lengthy run up time of the motor, the differences which are evident in the time at which the voltage across the second machine rises to its final value may be accounted for by the difficulties in obtaining a precise measure of the friction and windage torques of the two machines. When the machines are loaded, the shape of both the current and the common voltage will depend significantly on the characteristics of the loads on the machines, which will then control the value of the accelerating torques for more than will the motor parameters. The computed torque patterns and the speeds of the 0.75 kW motor and the 1.5 kW motor are shown in Figures 7.6(a) and 7.6(b) respectively.

Consideration of Figures 7.4 and 7.5 shows that the current of the 0.75 kW motor (Figure 7.4(a)) begins with a constant amplitude of 1.093 p.u., and that of the 1.5 kW motor (Figure 7.4(b)) with an amplitude of 1.278 p.u. The currents of both machines remain constant until about 50 cycles of the supply frequency have passed, when the 1.5 kW motor begins to accelerate, and the current falls to a constant amplitude of 0.2778 p.u. As a consequence of the reduced voltage drop in the supply impedance, the common voltage rises from its previous value of 0.294 p.u. to a new value of 0.496 p.u. This quite considerable increase causes the current drawn by the 0.75 kW motor to rise to 1.79 p.u., and it remains at about this level until the motor begins to accelerate after about 150 cycles of the supply frequency. The current then falls to 0.39 p.u., which is the final steady-state no-load value, and the common voltage rises to a corresponding value of 0.9 p.u., causing an increase in the current drawn by the 1.5 kW motor to a final steady-state of 0.3056 p.u. It is clear therefore that the acceleration times of the two machines are not mutually independent, and that the moment of inertia and the load characteristic of each will affect the acceleration time of the other. From Figures 7.6(a) and 7.6(b), we see that the torque developed by each motor is initially oscillatory, in the same way as when the machines were considered in isolation. The torque developed by the 0.75 kW motor soon attains a constant value, although the speed has not changed significantly from zero. After the initial oscillations have ceased, the torque developed by the 1.5 kW motor continues to increase, and the speed of this motor rises almost uniformly. When the 1.5 kW motor reaches the speed corresponding to the common voltage, its current and torque fall rapidly and the resultant increase in the common voltage causes an increase in both the torque and the speed of the other machine. This situation continues until full speed is reached, when the current and torque of the 0.75 kW motor fall rapidly and a voltage almost equal to the supply voltage is applied to the motors. However this does not change very much either the torque developed by the machines or their speeds.

7.7.2 Disconnection Condition

The common voltage across the two motors after disconnection from the supply was measured, together with the current flowing in the common stator circuit. A prediction was made of both the stator voltage and the current using the computer program described in Section 7.6, and Figures 7.7(a) and 7.7(b) show the close correlation obtained between the experimental and computed results. Computed results for all the 3-phase rotor currents of both the 0.75 kW and the 1.5 kW motor are shown in Figure 7.8. From these figures it is clear that the rotor currents are no longer simply unidirectional and exponentially decaying, as in the case of a single machine disconnected from the supply, but that there are some oscillations preceeding a unidirectional decay, i.e. the rotor currents now contain a highly damped alternating component and a much less damped direct component, with the initial values of the rotor currents being dependent on the instant of disconnecting the machines from the supply. If the period of disconnection is short, the oscillatory component will clearly have a major effect on the characteristics following any reconnection.

The results shown in Figure 7.9 reveal that the stator currents during disconnection are mainly oscillatory and lightly damped, with an additional but much more heavily damped unidirectional component. As with the rotor currents, the initial values of both stator and rotor currents depend on the instant at which the machines are disconnected. The heavily damped d.c. component in the stator results in the heavily damped a.c. component in the rotor, and the less damped d.c. component in the rotor results in the less damped a.c. component in the stator.

The air-gap powers of the two machines are shown in Figure 7.10. From this figure it is evident that there is a transfer of power between the two machines, and although at first one machine acts alternately as a motor and as a generator, the situation rapidly settles down to one where the machine having the greater moment of inertia feeds power to the other machine, until the fluxes of both machines have decayed to zero. As shown in Figure 7.11 the speeds of the two machines decay at a closely similar rate, although during the first part of the curve there is an oscillation due to the changing mode of operation of the machines. The speeds of the machines, when compared with their speeds when disconnected separately from the supply, show that the only significance difference is during the initial part of the curve, and that after the speed oscillation have ceased they become very close to one another.

Figure 7.12 shows a comparison between the common terminal voltage of the group and each motor in isolation. From this figure it is clear that, as would be expected, the rate at which the voltage across the group decays lies between the rates at which the voltages of the two motors decay when each is in isolation. Also, since the 0.75 kW motor has the smaller rotor time constant, its voltage decays more rapidly than both that of the 1.5 kW motor alone and that of the group, whether the machines are connected in star or in delta.

7.7.3 Reconnection Condition

The stator currents following reconnection of the 0.75 kW/1.5 kW motor group to the original stiff supply, after disconnection beginning at the instants shown, are recorded in Figure 7.13. The computed and measured values of the currents show about the same measure of agreement as was evident in the previous switching and disconnection conditions. This justifies again both the accuracy of the model developed for the system and its digital simulation, and enables full investigations to be made on a computational basis alone.

7.7.3.1 Reswitching to the same supply

To investigate the effect of the duration of the supply interruption on the transient torque, these torques were computed following reconnection after several different lengths of interruption. Figures 7.14(a) and 7.14(b) show the torque patterns following reconnection of the two machines after an interruption of 100 ms, and from a series of such curves Figures 7.14(a) and 7.14(b) were developed to show the variation of torque peaks with an increasing length of interruption. These figures show that the magnitude of the positive torque peak of the 0.75 kW motor increases initially to a maximum of 1.253 p.u. after a 10 ms decay, decreases to a minimum of 1.1824 p.u. after a 20 ms delay, increases again to a maximum of 1.689 p.u. before finally decreasing continuously for greater lengths. The positive torque peak of the 1.5 kW motor decreases initially to a minimum of 0.54 p.u., after a 10 ms delay, before increasing to a maximum of 1.49 p.u. and afterwards decreasing.

As Figure 7.10 makes clear, for any supply interruption of up to 10 ms duration the 0.75 kW machine is acting as a generator and the 1.5 kW as a motor immediately before reconnection, and as a consequence of this the power drawn by the former machine tends to increase and that drawn by the latter machine tends to decrease. The torque developed by the machines naturally follows a similar pattern. After an interruption lasting between 10 and 20 ms the modes of operation on reconnection have reversed, and the patterns and the variations of the torque of the two machines become in a reversed sense.

Figure 7.15(a) shows a comparison between the torque peaks following reswitching of the 0.75 kW motor alone and in parallel with the 1.5 kW motor, and Figure 7.15(b) shows a similar comparison for the 1.5 kW motor. From these figures it is clear that the positive torque peak of the 0.75 kW motor in isolation is greater than when in parallel with the 1.5 kW motor, and that after a delay disconnection of 75 ms the positive peak of the machine in isolation is less than when connected in parallel with the 1.5 kW motor. However, the negative torque peaks in isolation are always greater than when the machine is reswitched in parallel with the 1.5 kW motor. Referring to the comparison in Figure 7.12 between the terminal voltage of the machine when disconnection from the supply alone and in the group, we see that the decaying voltage of the 0.75 kW motor in isolation is less than when it is in the group. Further, the reswitching torque depends mainly on the differences in magnitudes and the phase shift between the supply voltage and the terminal voltage of the machine at the instant of reswitching; consequently, since during disconnection the voltage of the machine in isolation is less than when in the group, the torque peak of the machine in isolation should be greater than when in the group. This is true, as is evident from Figure 7.15(a), for both the second torque peak and the first part of the first peak. The difference between the peaks after a disconnection delay of 75 ms is attributed to the difference in the speed transients of the machine following re-switching, which will affect the values of the transient currents and consequently make the torque peak of the machine in the group greater than when in isolation, although the terminal voltage of the group is less than the terminal voltage of the machine in isolation before the reswitching operation.

Figure 7.15(b) shows a similar comparison for the 1.5 kW motor. Since the terminal voltage of the machine in isolation is greater than the terminal voltage of the group, the arguments for the 0.75 kW motor are the reverse of those for the 1.5 kW motor. The positive torque peak of the machine in the group is greater than when in isolation, until the disconnection delay is about 40 ms, when the situation becomes reversed. Whatever the disconnection delay, the negative torque peak of the machine in the group is always greater than the negative torque peaks of the machine in isolation.

Figures 7.16(a) and 7.16(b) show the variation of the torque peaks following reconnection of the group after a constant length of interruption but a variable instant of disconnection, and as is expected these peaks are all the same. This is clearly due to the amplitude and the phase shift of the voltages generated within the machines remaining constant with respect to the supply, so that the transient speeds are always the same, and identical torque patterns are produced.

7.7.3.2 Plugging

When the phase-sequence of the stiff supply to the group of motors is reversed, a braking effect is produced. If currents are still flowing in the stator and rotor circuits, the torques and the currents immediately following this operation will obviously be different from those when the machines are electrically inert.

To investigate how the currents and torques of the two motors depend upon the time taken in changing the phase-sequence of the supply, computed results for both machines were obtained. Figures 7.17(a) and 7.18(a) show the torque patterns produced by the two machines, when a delay of 10 ms occurs before the application of the reversed phase-sequence voltage. The greatest of the torque peaks of such patterns are plotted against the delay time to provide Figures 7.17(b) and 7.18(b). The resulting torque peak variations are seen here to oscillate between a maximum and a minimum value with a greatest torque peak corresponding to the condition when the supply and the stator voltages are in antiphase and a minimum torque peak to the condition when they are cophased, depending upon the orientation of the rotor and the instant of application of the reversed phase sequence voltage. Figure 7.19(a) shows a comparison between the torque peaks of the 0.75 kW motor when plugged in isolation and when plugged in parallel with the 1.5 kW motor, and Figure 7.19(b) shows a similar comparison for the 1.5 kW motor alone and in parallel with the 0.75 kW machine, both the greatest and the smallest torque peaks are less when the machine is plugged in isolation than when it is in the group, whereas for the 1.5 kW machine, both the greatest and the smallest torque peaks are greater than when the machine is in the group. This would be expected from the comparisons between the decaying voltages of the machines in isolation and in the group, shown in Figure 7.12, from which it can be seen that the 0.75 kW motor has a greater terminal voltage. Since the plugging voltage has a reverse phase-sequence, the torque produced by the 0.75 kW motor in isolation is less than when in the group while that of the 1.5 kW motor is greater than when in the group. This conclusion is the opposite of that reached when the machines are simply reconnected to the same supply. However, the torque peaks decrease as the delay of interruption increases whether the machine is in isolation or in a group.

Figures 7.20(a) and 7.21(a) show patterns of the stator currents following plugging of the two machines after supply interruption of 10 ms. From a series of such curves Figures 7.20(b) and 7.21(b) were produced, showing the variations of the maximum current peak with delay. From these curves we see that the peak current changes alternately between positive and negative with a rapid change every 10 ms.

7.7.3.3 Star-delta starting

A series of star-delta starting transients were computed for the 0.75 kW/1.5 kW motor group, for increasing delays between star-connected operation and application of the full voltage, with the machines connected in delta. Figures 7.22(a) and 7.23(a) show the patterns followed by the torques of the 0.75 kW and 1.5 kW motors respectively, following a delay of 5 ms. From a series of such curves Figures 7.22(b) and 7.23(b) were developed to show the variations in the torque peaks of the two motors following increasing periods of interruption. It is clear from these figures that the torque peaks are not very high when compared with the direct on-line starting torque peaks recorded in Figures 6.32(b) and 6.33(b). During the initial stages of these torque variations the torque peaks of the 0.75 kW motor and the 1.5 kW motor exhibit a slight increase and decrease, this behaviour again depending on the mode of the machine at the instant of reconnection.

The computed reconnection currents corresponding to Figures 7.22 and 7.23 are shown in Figures 7.24 and 7.25, for the 0.75 kW and 1.5 kW motor respectively. Again, the maximum current peaks change alternately between positive and negative with a rapid change about every 10 ms. This is similar to the plugging condition and it is mainly because the maximum peak is not always the first peak.

Figures 7.26(a) and 7.26(b) show comparisons between the torque peaks of the 0.75 kW and the 1.5 kW motors respectively, when started alone and in the group. By referring to Figure 7.12(b), we see that the voltage of the 0.75 kW motor in isolation is less than in the group, while that of the 1.5 kW motor is greater than when in the group. The torque developed following connection of the 0.75 kW machine in delta is consequently greater when the machine is in isolation than when in the group, except for the initial part of the negative torque peak curve which is affected by the speed transients. On the other hand the torque of the 1.5 kW motor in the group is greater than the torque in isolation.

7.8 Conclusions

An idealized model of a group of two motors was used in developing a digital-computer program for the evaluation of the overall performance of the group. Comparisons of measured and computed results yielded very good agreement, and confirmed the reliability of the results obtained on a computational basis only, whether the group is connected to a stiff or to a weak supply, or disconnected and reconnected to the same or a different stiff supply.

When the two motors are connected simultaneously to a stiff-supply, their transient behaviour is the same as if each motor was connected separately. However, when the group of motors is connected to a weak supply, the transient behaviour is interactive, and the performance of one motor will affect that of the other. The electrical as well as the mechanical parameters of one motor affect the behaviour of the other motor and vice versa. The time taken by a motor to achieve its steady-state situation will depend on the parameters of the machines, as well as the weakness of the supply, i.e. its short-circuit capacity.

When the motors are disconnected from the supply, currents continue to flow in the common stator circuit and also in the rotor circuits. The rotor currents are not unidirectional, as in the case of a single machine, but have heavily damped alternating components and a lightly damped unidirectional component. The characteristics of the stator currents are opposite to those of the rotor currents, i.e. the stator current has heavily damped direct componentsand lightly damped alternating components. Both the stator and the rotor currents are composed of three exponentially decaying components, all with different time constants and frequencies. There are also relatively small alternating electromagnetic torques developed, which result in the machine with the greatest moment of inertia acting as a generator and the other machine as a motor, until the air-gap power and the speeds of both machines have decayed to zero. The speeds of the two machines oscillate initially, before settling down to decrease at closely similar rates. The joint terminal voltage is less in magnitude than that of the machine with the largest time constant in isolation, and greater than that of the machine with the smallest time constant i.e. the time constant of the group lies between the valuesof the time constants of the two machines individually.

Following reswitching to the same stiff supply, the negative torque peaks of the machine having the smallest time constant are greater than if the machine is reswitched in the group. The negative torque peak of the machine with the largest time constant is less than if the machine is reswitched in the group.

Following plugging, the machine with the smallest time constant has peak torques smaller in isolation than when in the group, whereas the machine with the largest time constant has peak torques larger in isolation than if it is plugged in the group.

For star-delta starting, the machine with the smallest time constant has torque greater in isolation than when in the group, and the machine with the greatest time constant has torque peaks smaller in isolation than when in the group.

CHAPTER 8

TRANSIENT BEHAVIOUR OF A GROUP OF THREE MOTORS

An investigation of the transient performance of a 3-motor group is a logical extension of the investigation of the 2-motor group recorded in Chapter 7. Again, if a group of three motors is simultaneously connected, then disconnected and finally reconnected to a weak supply, the transient performance of each motor is affected by the transient performance of the other motors of the group. On the other hand, if the group of motors is connected to a stiff-supply the performance of each motor of the group will be precisely the same as when the motor is connected alone, although this will not be the case for a reconnection situation.

8.1 Connection Condition

8.1.1 Stiff-supply

The electrical differential equations of a group of three motors are obtained by reducing Equation 2.7, which models a group of n-motor, to the equation for three motors only. This leads directly to Equation 8.1. If the alternative formulation of the equations is used, Equation 2.8 is reduced to Equation 8.2. Both Equations 8.1 and 8.2 can be rewritten briefly as

$$[v] = [R][i] + [L][Di] + \dot{\theta}_1 [G_1][i] + \dot{\theta}_2 [G_2][i] + \dot{\theta}_3 [G_3][i] \qquad 8.3$$

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
M_1D & M_1\dot{\theta}_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot{\theta}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-M_1\dot{\theta}_1 & M_1D & -L_{r_1}\dot{\theta}_1 & R_{r_1}+L_{r_1}D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & M_2D & M_2\dot{\theta}_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot{\theta}_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -M_2\dot{\theta}_2 & M_2D & -L_{r_2}\dot{\theta}_2 & R_{r_2}+L_{r_2}D & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{s_3}+L_{s_3}D & 0 & M_3D & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{s_3}+L_{s_3}D & 0 & M_3D \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_3D & M_3\dot{\theta}_3 & R_{r_3}+L_{r_3}D & L_{r_3}\dot{\theta}_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -M_3\dot{\theta}_3 & M_3D & -L_{r_3}\dot{\theta}_3 & R_{r_3}+L_{r_3}D
\end{bmatrix}
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ i_{sd_3} \\ i_{sq_3} \\ i_{rd_3} \\ i_{rq_3} \end{bmatrix}
\qquad 8.3
$$

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
M_1D & M_1\dot{\theta}_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot{\theta}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-M_1\dot{\theta}_1 & M_1D & -L_{r_1}\dot{\theta}_1 & R_{r_1}+L_{r_1}D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 & 0 & 0 & 0 & 0 \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & M_2D & M_2\dot{\theta}_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot{\theta}_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -M_2\dot{\theta}_2 & M_2D & -L_{r_2}\dot{\theta}_2 & R_{r_2}+L_{r_2}D & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & -(R_{s_3}+L_{s_3}D) & 0 & -M_3D & 0 \\
0 & 0 & 0 & 0 & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & -(R_{s_3}+L_{s_3}D) & 0 & -M_3D \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_3D & M_3\dot{\theta}_3 & R_{r_3}+L_{r_3}D & L_{r_3}\dot{\theta}_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -M_3\dot{\theta}_3 & M_3D & L_{r_3}\dot{\theta}_3 & R_{r_3}+L_{r_3}D
\end{bmatrix}
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ i_{sd_3} \\ i_{sq_3} \\ i_{rd_3} \\ i_{rq_3} \end{bmatrix}
\tag{8.2}
$$

where

$$[R_3] = \begin{bmatrix}
R_{s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R_{s_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & R_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & R_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
R_{s_1} & 0 & 0 & 0 & -R_{s_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R_{s_1} & 0 & 0 & 0 & -R_{s_2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & R_{r_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{r_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & R_{s_2} & 0 & 0 & 0 & -R_{s_3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & R_{s_2} & 0 & 0 & 0 & -R_{s_3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{r_3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{r_3}
\end{bmatrix}$$

$$[L_3] = \begin{bmatrix}
L_{s_1} & 0 & M_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_{s_1} & 0 & M_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
L_{s_1} & 0 & M_1 & 0 & -L_{s_2} & 0 & -M_2 & 0 & 0 & 0 & 0 & 0 \\
0 & L_{s_1} & 0 & M_1 & 0 & -L_{s_2} & 0 & -M_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_{s_2} & 0 & M_2 & 0 & -L_{s_3} & 0 & -M_3 & 0 \\
0 & 0 & 0 & 0 & 0 & L_{s_2} & 0 & M_2 & 0 & -L_{s_3} & 0 & -M_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_3 & 0 & L_{r_3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_3 & 0 & L_{r_3}
\end{bmatrix}$$

$$[G_1] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -M_1 & 0 & -L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_2] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -M_2 & 0 & -L_{r_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_3] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_3 & 0 & L_{r_3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -M_3 & 0 & -L_{r_3} & 0 \end{bmatrix}$$

$$[v_3] = \begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad [i_3] = \begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ i_{sd_3} \\ i_{sq_3} \\ i_{rd_3} \\ i_{rq_3} \end{bmatrix}$$

The electromagnetic torques developed by the three motors are, respectively:

$$T_{e1} = p_1 M_1 (i_{sq_1} i_{rd_1} - i_{sd_1} i_{rq_1}) \quad \ldots\ldots \quad 8.4$$

$$T_{e2} = p_2 M_2 (i_{sq_2} i_{rd_2} - i_{sd_2} i_{rq_2}) \quad \ldots\ldots \quad 8.5$$

$$T_{e3} = p_3 M_3 (i_{sq_3} i_{rd_3} - i_{sd_3} i_{rq_3}) \quad \ldots\ldots \quad 8.6$$

8.1.2 Non-stiff supply

When the group is connected to a weak supply two approaches can be used to model the system, just as when the cases of the two-motor and the n-motor groups were investigated. In the first approach, the supply voltages are related to the common voltages at the machine terminals by the equations:

$$v_{ga} = v_a - (R` + L`D)\ i_a \quad \ldots\ldots \quad 8.7$$

$$v_{gb} = v_b - (R` + L`D)\ i_b \quad \ldots\ldots \quad 8.8$$

$$v_{gc} = v_c - (R` + L`D)\ i_c \quad \ldots\ldots \quad 8.9$$

where

$$i_a = i_{a_1} + i_{a_2} + i_{a_3}$$

$$i_b = i_{b_1} + i_{b_2} + i_{b_3}$$

$$i_c = i_{c_1} + i_{c_2} + i_{c_3}$$

in which i_{a_1}, i_{b_1}, i_{c_1} and i_{a_2}, i_{b_2}, i_{c_2} and i_{a_3}, i_{b_3}, i_{c_3} are the instantaneous currents of the three motors. In addition to Equations 8.4 to 8.6, which represent the interlinking between the stiff supply voltage and the common machine voltage, Equation 8.1 is also used in calculating the performance of the motors. However, v_{sd} and v_{sq} are no longer the d and q axis components of the stiff-supply, but need to be replaced by v_{sdg} and v_{sqg} where

$$v_{sdq} = \sqrt{\frac{2}{3}} \left(v_{ga} - \frac{1}{2} v_{gb} - \frac{1}{2} v_{gc}\right) \qquad \ldots\ldots \qquad 8.10$$

$$v_{sqg} = \sqrt{\frac{2}{3}} \left(\frac{3}{2} v_{ga} - \frac{3}{2} v_{gc}\right) \qquad \ldots\ldots \qquad 8.11$$

If the second approach is followed, the supply impedance is included in the impedance matrix of the group and this leads to Equation 8.12.

$$
\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
(R'+R_{s_1})+(L'+L_{s_1}D) & 0 & M_1D & 0 & R'+L'D & 0 & 0 & 0 & R'+L'D & 0 & 0 & 0 \\
0 & (R'+R_{s_1})+(L'+L_{s_1}D & 0 & M_1D & 0 & R'+L'D & 0 & 0 & 0 & R'+L'D & 0 & 0 \\
M_1D & M_1\theta_1 & R_{r_1}+L_{r_1}D & L_{r_1}\theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-M_1\theta_1 & M_1D & -L_{r_1}\theta_1 & R_{r_1}+L_{r_1}D & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 & 0 & 0 & 0 & 0 \\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & M_2D & M_2\theta_2 & R_{r_2}+L_{r_2}D & L_{r_2}\theta_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -M_2\theta_2 & M_2D & -L_{r_2}\theta_2 & R_{r_2}+L_{r_2}D & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & -(R_{s_3}+L_{s_3}D) & 0 & -M_3D & 0 \\
0 & 0 & 0 & 0 & 0 & R_{s_2}+L_{s_2}D & 0 & M_2D & 0 & -(R_{s_3}+L_{s_3}D & 0 & -M_3D \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_3D & M_3\theta_3 & R_{r_3}+L_{r_3}D & L_{r_3}\theta_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -M_3\theta_3 & M_3D & -L_{r_3}\theta_3 & R_{r_3}+L_{r_3}D
\end{bmatrix}
\begin{bmatrix} i_{sd_1} \\ i_{sq_1} \\ i_{rd_1} \\ i_{rq_1} \\ i_{sd_2} \\ i_{sq_2} \\ i_{rd_2} \\ i_{rq_2} \\ i_{sd_3} \\ i_{sq_3} \\ i_{rd_3} \\ i_{rq_3} \end{bmatrix}
\qquad 8.12
$$

8.2 Disconnection Condition

When the motors are disconnected from the supply their stators remain interconnected, so that the subsequent performance is again not independent. Equation 2.21, which represents a group of n-motor during disconnection, can be reduced to represent a group of three motors by the following equation, Equation 8.13, which can be rewritten briefly as

$$[0] = [R]_{dis}\,[i] + [L]_{dis}[Di] + \{\dot{\theta}_1[G]_{dis} + \dot{\theta}_2\,[G]_{dis} + \dot{\theta}_3\,[G]_{dis}\}\,[i] \qquad \ldots\ldots \quad 8.14$$

where

$$[R]_{dis} = \begin{bmatrix} R_{s_1} & 0 & 0 & 0 & -R_{s_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{s_1} & 0 & 0 & 0 & -R_{s_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ R_{s_3} & 0 & 0 & 0 & R_{s_2}+R_{s_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{s_3} & 0 & 0 & 0 & R_{s_2}+R_{s_3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_{r_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{r_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{r_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_{r_3} \end{bmatrix}$$

$$
\begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}
=
\begin{bmatrix}
R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 & 0 & 0\\
0 & R_{s_1}+L_{s_1}D & 0 & M_1D & 0 & -(R_{s_2}+L_{s_2}D) & 0 & -M_2D & 0 & 0\\
M_1D & M_1\dot\theta_1 & R_{r_1}+L_{r_1}D & L_{r_1}\dot\theta_1 & 0 & 0 & 0 & 0 & 0 & 0\\
-M_1\dot\theta_1 & M_1D & -L_{r_1}\dot\theta_1 & R_{r_1}+L_{r_1}D & 0 & 0 & 0 & 0 & 0 & 0\\
R_{s_3}+L_{s_3}D & 0 & 0 & 0 & \begin{matrix}(R_{s_2}+R_{s_3}D)+\\(L_{s_2}+L_{s_3}D)\end{matrix} & 0 & M_2D & 0 & 0 & 0\\
0 & R_{s_3}+L_{s_3}D & 0 & 0 & 0 & \begin{matrix}(R_{s_2}+R_{s_3})+\\(L_{s_2}+L_{s_3}D)\end{matrix} & 0 & M_2D & 0 & 0\\
0 & 0 & 0 & 0 & M_2D & M_2\dot\theta_2 & R_{r_2}+L_{r_2}D & L_{r_2}\dot\theta_2 & 0 & 0\\
0 & 0 & 0 & 0 & -M_2\dot\theta_2 & M_2D & -L_{r_2}\theta_2 & R_{r_2}+L_{r_2}D & 0 & 0\\
-M_3D & -M_3\dot\theta_3 & 0 & 0 & -M_3D & -M_3\dot\theta_3 & 0 & 0 & R_{r_3}+L_{r_3}D & L_{r_3}\dot\theta_3\\
M_3\dot\theta_3 & -M_3D & 0 & 0 & M_3\dot\theta_3 & -M_3D & 0 & 0 & -L_{r_3}\dot\theta_3 & R_{r_3}+L_{r_3}D
\end{bmatrix}
\begin{bmatrix} i_{sd_1}\\ i_{sq_1}\\ i_{rd_1}\\ i_{rq_1}\\ i_{sd_2}\\ i_{sq_2}\\ i_{rd_2}\\ i_{rq_2}\\ i_{rd_3}\\ i_{rq_3} \end{bmatrix}
\qquad \ldots.\ 8.13
$$

$$[L]_{dis} = \begin{bmatrix} L_{s_1} & 0 & M_1 & 0 & -L_{s_2} & 0 & -M_2 & 0 & 0 & 0 \\ 0 & L_{s_1} & 0 & M_1 & 0 & -L_{s_2} & 0 & -M_2 & 0 & 0 \\ M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ L_{s_3} & 0 & 0 & 0 & L_{s_2}+L_{s_3} & 0 & M_2 & 0 & 0 & 0 \\ 0 & L_{s_3} & 0 & 0 & 0 & L_{s_2}+L_{s_3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 \\ -M_3 & 0 & 0 & 0 & -M_3 & 0 & 0 & 0 & L_{r_3} & 0 \\ 0 & -M_3 & 0 & 0 & 0 & -M_3 & 0 & 0 & 0 & L_{r_3} \end{bmatrix}$$

$$[G_1]_{dis} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ -M_2 & 0 & -L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_2]_{dis} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -M_2 & 0 & -L_{r_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[G_3]_{dis} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -M_3 & 0 & 0 & 0 & -M_3 & 0 & 0 & 0 & L_{r_3} \\ M_3 & 0 & 0 & 0 & M_3 & 0 & 0 & 0 & -L_{r_3} & 0 \end{bmatrix}$$

Since the three motors are interconnected, the stator currents of the third motor of the group is given in terms of the currents of the other motors by

$$i_{sd_3} = -(i_{sd_1} + i_{sd_2})$$

$$i_{sq_3} = -(i_{sq_1} + i_{sq_2})$$

8.2.1 Electromagnetic Torques

Following disconnection, the stator and rotor currents of the three machines do not fall instantaneously to zero but decay with a duration determined by the time constants of the system. The interaction between the decaying currents in the stators and the rotors will result in the production of electromagnetic torques, which for the three machines are given by Equations 8.4 to 8.6 respectively.

8.3 Reswitching Condition

When the three machines are reconnected to the same or to a different supply before the complete decay of the currents, Equation 8.1 is used if the supply is stiff. If the group reconnection is to a weak supply, Equations 8.7 to 8.9 are needed in addition to Equation 8.1 if the first approach is followed, or Equation 8.12 if the second approach is used. We may notice that the only difference between the representation of the group when connected and when reconnected lies in the initial conditions of the currents, speeds and voltages.

The voltage matrix is, again, determined by the type of the supply to which the group is reconnected.

8.4 Mechanical System

The mechanical system equations of the three machines are derived directly from the general mechanical equations of the n-motor group, by replacing m by 1,2 and 3 in the equations of Section 2.4 for the first, second and third motor respectively.

8.5 Numerical Solution and Computer Program

The same procedure followed in Chapter 4, for a group of n-motors, is applied here for a group of three motors. As before, the computer program is divided into three parts; connection, disconnection and reconnection.

8.5.1 Connection condition

When the supply is stiff, either Equation 8.1 or Equation 8.2 may be rewritten as

$$[Di] = [L]^{-1}\{[v] - [R][i] - \dot{\theta}_1[G_1][i] - \dot{\theta}_2[G_2][i] - \dot{\theta}_3[G_3][i]\} \qquad \ldots\ldots \quad 8.15$$

to enable it to be numerically integrated. Matrices [R], [L], $[G_1]$, $[G_2]$ and $[G_3]$ of Equation 8.15 are formed by the resistance terms, the coefficients of D, the coefficients of $\dot{\theta}_1$, the coefficients of $\dot{\theta}_2$ and the coefficients of $\dot{\theta}_3$ respectively. The initial conditions of the currents and speeds are taken as zero, since the group is initially both electrically and mechanically inert.

If the supply is weak, and the second approach is used, the initial value of the common voltage across the group is required, in addition to the initial conditions of the currents and speeds. Following Chapter 4, the required voltage is given by

$$vg = \left[1 - \left[\frac{L'}{L' + \dfrac{(L_{s_1} + L_{r_1} - 2\,M_1)}{K_G}} \right] \right] v$$

where, for a group of three motors,

$$K_G = 1 + \frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_2} + L_{r_2} - 2\,M_2} + \frac{L_{s_1} + L_{r_1} - 2\,M_1}{L_{s_3} + L_{r_3} - 2\,M_3}$$

The electromagnetic torque developed by each machine of the group is computed by using Equations 8.4 to 8.6. The variation of the speed of each motor is computed by numerically integrating the acceleration, which for the three motors is

$$D^2\theta_1 = \frac{T_{e_1} - T_{f_1} - T_{t_1}}{J_1} \quad \ldots\ldots \quad 8.16$$

$$D^2\theta_2 = \frac{T_{e_2} - T_{f_2} - T_{t_2}}{J} \quad \ldots\ldots \quad 8.17$$

$$D^2\theta_3 = \frac{T_{e_3} - T_{f_3} - T_{t_3}}{J_3} \quad \ldots\ldots \quad 8.18$$

when the machines are loaded. If the machines are unloaded T_{t_1}, T_{t_2} and T_{t_3} are all zero, and the above equations reduce to

$$D^2\theta_1 = \frac{T_{e_1} - T_{f_1}}{J_1} \quad \ldots\ldots \quad 8.19$$

$$D^2\theta_2 = \frac{T_{e_2} - T_{f_2}}{J_2} \quad \ldots\ldots \quad 8.20$$

$$D^2\theta_3 = \frac{T_{e_3} - T_{f_3}}{J_3} \qquad \ldots\ldots \qquad 8.21$$

If the 3-phase rotor currents of each motor are required, the d,q currents have to be transformed to their rotating axes equivalents. For this transformation the angular position of the rotor is required, and this can be obtained by numerically integrating the speed of each motor, thus

$$D\theta_1 = \dot{\theta}_1 \qquad \ldots\ldots \qquad 8.22$$

$$D\theta_2 = \dot{\theta}_2 \qquad \ldots\ldots \qquad 8.23$$

$$D\theta_3 = \dot{\theta}_3 \qquad \ldots\ldots \qquad 8.24$$

The computed values of the d,q currents, speeds and angular positions at the end of the first step-length are taken as the initial values for the next integration step. The process is repeated until the next switching operation (i.e. disconnection) occurs.

8.5.2 Disconnection condition

Following disconnection Equation 8.13 is used, which can be rewritten as

$$[Di] = [L]^{-1}_{dis} \{[0] - [R]_{dis}\,[i] - \dot{\theta}_1[G_1]_{dis}[i] - \dot{\theta}_2[G_2]_{dis}[i] - \dot{\theta}_3[G_3]_{dis}[i]$$

$$\ldots\ldots \qquad 8.25$$

where the matrices $[R]_{dis}$, $[L]_{dis}$, $[G_1]_{dis}$, $[G_2]_{dis}$ and $[G_3]_{dis}$ are as given in Section 8.2.

The initial values of the speeds and angular positions of the rotors, required when Equation 8.25 is numerically integrated, are taken as those immediately before disconnection. The initial values of the motor currents are obtained by applying constant-flux-linkage considerations to the stator and rotor circuits of each motor of the group.

The electromagnetic torque developed by each machine is computed using Equations 8.4 to 8.6. The variations in the speeds are obtained by numerically integrating Equations 8.16 to 8.18 if the machines are loaded, or Equations 8.19 to 8.21 when they are unloaded. The angular positions of the rotors are obtained by numerically integrating Equations 8.22 to 8.24.

8.5.2.1 Initial values of currents

On considering the flux linkages in each of the stator and rotor circuits to remain constant immediately before and after disconnection we obtain

$$L_{r_1}(i_{rd_1})_0 + M_1(i_{sd_1})_0 = L_{r_1}(i_{rd_1})_1 + M_1(i_{sd_1})_1$$

$$L_{r_1}(i_{rq_1})_0 + M_1(i_{sq_1})_0 = L_{r_1}(i_{rq_1})_1 + M_1(i_{sq_1})_1$$

$$L_{r_2}(i_{rd_2})_0 + M_2(i_{sd_2})_0 = L_{r_2}(i_{rd_2})_1 + M_2(i_{sd_2})_1$$

$$L_{r_2}(i_{rq_2})_0 + M_2(i_{sq_2})_0 = L_{r_2}(i_{rq_2})_1 + M_2(i_{sq_2})_1$$

$$L_{r_3}(i_{rd_3})_0 + M_3(i_{sd_3})_0 = L_{r_3}(i_{rd_3})_1 + M_3(i_{sd_3})_1$$

$$L_{r_3}(i_{rq_3})_0 + M_3(i_{sq_3})_0 = L_{r_3}(i_{rq_3})_1 + M_3(i_{sq_3})_1$$

$$L_{s_1}(i_{sd_1})_0 + M_1(i_{rd_1})_0 = L_{s_1}(i_{sd_1})_1 + M_1(i_{rd_1})_1$$

$$L_{s_1}(i_{sq_1})_0 + M_1(i_{rq_1})_0 = L_{s_1}(i_{sq_1})_1 + M_1(i_{rq_1})_1$$

$$L_{s_2}(i_{sd_2})_0 + M_2(i_{rd_2})_0 = L_{s_2}(i_{sd_2})_1 + M_2(i_{rd_2})_1$$

$$L_{s_2}(i_{sq_2})_0 + M_2(i_{rq_2})_0 = L_{s_2}(i_{sq_2})_1 + M_2(i_{rq_2})_1$$

where the suffices $_0$, $_1$ denote conditions immediately before and after disconnection. If the left-hand sides of the above equations are replaced by ψ, then in matrix form these equations become

$$
\begin{bmatrix} \psi_{rd_1} \\ \psi_{rq_1} \\ \psi_{rd_2} \\ \psi_{rq_2} \\ \psi_{rd_3} \\ \psi_{rq_3} \\ \psi_{sd_1} \\ \psi_{sq_1} \\ \psi_{sd_2} \\ \psi_{sq_2} \end{bmatrix}
=
\begin{bmatrix}
M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & M_1 & 0 & L_{r_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M_2 & 0 & L_{r_2} & 0 & 0 \\
-M_3 & 0 & 0 & 0 & -M_3 & 0 & 0 & 0 & L_{r_3} & 0 \\
0 & -M_3 & 0 & 0 & 0 & -M_3 & 0 & 0 & 0 & L_{r_3} \\
L_{s_1} & 0 & M_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & L_{s_1} & 0 & M_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & L_{s_2} & 0 & M_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & L_{s_2} & 0 & M_2 & 0 & 0
\end{bmatrix}
\begin{bmatrix} (i_{sd_1})_1 \\ (i_{sq_1})_1 \\ (i_{rd_1})_1 \\ (i_{rq_1})_1 \\ (i_{sd_2})_1 \\ (i_{sq_2})_1 \\ (i_{rd_2})_1 \\ (i_{rq_2})_1 \\ (i_{rd_3})_1 \\ (i_{rq_3})_1 \end{bmatrix}
\qquad \ldots\ldots \quad 8.26
$$

Since the left-hand side of Equation 8.26 is known from the values of the currents immediately before disconnection, the values of the currents immediately after disconnection are obtained.

8.5.3 Reswitching Condition

The equations used for the connection condition are used also for the reconnection condition, but the initial values of the currents and speeds are no longer zero and are taken as those immediately before reconnection. The initial conditions of the voltage applied are determined by the type of the supply to which the group is reconnected.

The choice of the step-length is, again, dependent on the smallest time constant of the system. As is shown in Appendix A, this is determined by the reciprocal of the largest eigenvalue, obtained after writing the system equation in state-variable form. By this method, a step-length of 0.5 ms was found to give numerical stability of the Runge-Kutta integration method used when the first approach for the weak supply is followed and a step-length of 0.25 ms when the second approach is used.

The flow-chart of the computer program is shown in Figure 4.1 and the listings of the program are given in Appendix G.

8.7 Transient Condition in 3-motor Group

A group of three motors was formed from the 0.75 kW, 1.5 kW and 2.25 kW motor detailed in Table I. The computer program described in the previous section was used to predict the currents, torques and speeds following connection, disconnection and reconnection of each group to a stiff supply, and also following connection to a supply with an 11.0 ohm resistance in each supply line.

The program was used also to predict the currents and torques following plugging and star-delta starting to the stiff supply.

8.7.1 Connection condition

Figures 8.1(a) - 8.1(c) show a comparison between the predicted and measured stator currents in the r-line of the 0.75 kW and 1.5 kW motors and the b-line of the 2.25 kW motor, when this group was connected to a stiff supply. From these results it is clear that the 1.5 kW motor has the shortest acceleration time, the 0.75 kW motor the longest acceleration time and the 2.25 kW motor a time in between those of the other two motors. The differences in the behaviour of the three machines are sufficiently large to ensure that an accurate investigation may be made of the effect of switching to a non-stiff supply.

Figure 8.2 shows a correlation between the experimental and computed results of the current and torque developed by the 5.6 kW motor when the 0.75 kW/2.25 kW/5.6 kW group was connected to the supply.

From Figures 8.1 and 8.2 it is clear that the predicted and measured currents and torques are in good agreement. The computed results in these figures are, of course, precisely the same as the computed results when each of the machines is connected separately at the same instant on the voltage wave. This confirms the accuracy of the more complicated program of the group, since its results agree with the results obtained from the program of one machine.

Figures 8.3(a) - 8.3(c) show the torques and the speeds of the 0.75 kW, 1.5 kW and 2.25 kW motors, following connection of the group to the stiff supply. The electromagnetic torque patterns obtained are again precisely the same as those when each machine is connected separately to a stiff supply, since there will be no interaction between the different machines. It can be seen from the figures that the 1.5 kW motor achieves its steady-state after about 8.5 cycles of the supply, the 2.25 kW motor after 9.5 cycles and the 0.75 kW motor after about 13.5 cycles, confirming observations that can be made from the current results of Figure 8.1.

With the 11.0 ohm resistances included in the circuit, the starting currents of the three motors were obtained experimentally and theoretically, and a comparison between these is shown in Figures 8.4(a) - 8.4(c). The common terminal voltage was also measured and predicted, with the results shown in Figure 8.5. Again, due to the long time involved, peak rather than instantaneous values are indicated in the curves. From Figures 8.4 and 8.5 it is clear that the all predicted and measured comparisons show reasonably good agreement, but not as good as that for results obtained on a shorter time period. Because of the lengthy run-up time of the machines, the differences which do arise may be accounted for mainly by inaccuracies in the values obtained for the friction and windage torques of the machines, which have a much greater influence than when the supply is stiff. When the machines are loaded, the shape of both the current and the common voltage curves will depend significantly on the characteristics of the loads on the machines, which will then control the value of the accelerating torque far more than will the motor parameters. Figures 8.6(a) - 8.6(c) show the computed torque and speeds of the 0.75 kW, 1.5 kW and 2.25 kW motors respectively, corresponding to Figures 8.4 and 8.5.

In investigating Figures 8.4 and 8.5 we see that the current of the 0.75 kW motor begins with a constant amplitude of 0.76 p.u., that of the 2.25 kW motor with an amplitude of 0.67 p.u. and that of the 1.5 kW motor with an amplitude of 1 p.u. The currents of the three machines remain practically constant until after about 200 cycles of the supply frequency, when the 1.5 kW motor begins to accelerate, and its current falls rapidly to an almost constant amplitude of 0.11 p.u. At the same time, the common voltage rises from its previous value of 0.1 p.u. to 0.12 p.u. This increase in the common voltage causes an increase in the currents of the 0.75 kW and the 2.25 kW machines, and this condition persists until the 2.25 kW motor begins to accelerate after about 350 cycles of the supply. After acceleration, the motor draws a reduced current of 0.16 p.u., and the common voltage will correspondingly rise to a value of 0.24 p.u., causing the currents drawn by the 0.75 kW and the 1.5 kW motors to rise to 1.4 p.u. and 0.3 p.u. respectively. This new condition continues until the 0.75 kW motor, which has the greatest moment of inertia, accelerates after about 475 cycles of the supply, when its current is reduced to a value of 0.6 p.u., which is its final steady-state no load value. The common voltage then rises to the final and steady-state value. The increase in the motor voltage causes the current drawn by the 1.5 kW and 2.25 kW motors also to increase to their steady-state no-load values of 0.33 p.u. and 0.31 p.u. respectively. It is clear from this discussion that the acceleration times of the three machines are not mutually independent, and that the moment of inertia and the load characteristics of each machine will affect the acceleration times of the other machines of the group.

The electromagnetic torque developed by the 0.75 kW, 1.5 kW and the 2.25 kW is shown in Figures 8.6(a) - 8.6(c). From these figures we see that the torque developed by each motor is initially oscillatory. That developed by the 0.75 kW motor soon attains an almost constant value, although the speed has not changed significantly from zero. After the initial oscillations the torque of the 1.5 kW motor continues to increase, and the speed of this motor rises almost uniformly. After about 200 cycles of the supply have elapsed this motor, which has the smallest moment of inertia, achieves the speed corresponding to the common voltage, and as its current and torque fall rapidly the resultant increase in the common voltage causes an increase in both the torques and the currents of the other machines. This continues until full speed of the 2.25 kW motor is reached, when the current and torque of this motor fall rapidly and the further increase of the common voltage causes an increase in both the torque and the speed of the 0.75 kW motor. This continues until full speed of this motor is reached, when its current and torque fall rapidly and a voltage almost equal to the supply voltage is applied to the three machines. However this final change does not alter very much either the torque developed by the machines or their speeds.

8.7.2 Disconnection condition

The common voltage across the three motors during disconnection was measured, together with the currents flowing in the stator circuits of the individual machines. The common voltage and the stator currents were also predicted, using the computer program described in Section 8.6. Figure 8.7 shows the close correlation obtained. Computed results for the rotor phase currents of the three machines are also shown in Figure 8.8, from which it is clear that, as in Section 7.7.2, the rotor currents are at all times oscillatory and remain so for a considerable time, although their amplitude naturally decays as the time of disconnection increases. The alternating nature of the rotor currents has an effect on the characteristics following any subsequent reconnection, depending on the instant the machines are reconnected.

The air-gap powers of the three machines are shown in Figure 8.9, from which it can be seen that there is an interchange of power between the three machines. Although each machine initially acts alternately as a motor and as a generator, the situation eventually settles down to one where the machine of the greatest moment of inertia is feeding power to the other two.

As Figure 8.10 shows, the speeds of the three machines decay at a closely similar rate, and during the first part of the speed curve, there are small oscillations due to the changing mode of operation of the machines. After the oscillations have ceased the speeds become close to the speeds when the machines are considered in isolation.

Figure 8.11 shows a comparison between the terminal voltage following disconnection of the group from the supply and each machine disconnected in isolation. From this figure it is seen that the time constant of the decaying voltage is greater than the time constants of both the 0.75 kW and 2.25 kW machines in isolation but less than that of the 1.5 kW machine.

8.7.3 Reconnection condition

The currents following reconnection of the 3-motor group to the original stiff supply, after a disconnection beginning at the instant indicated are shown in Figure 8.12. The computed and measured values of the currents show good agreement. This further justifies the accuracy of the model adopted for the system and its digital simulation, and enables a full investigation to be made on a computational basis alone.

8.7.3.1 Reswitching to the same supply

To investigate the effect of the duration of the supply interruption on the transient torque, these torques were computed following reconnection after several different lengths of interruption from the same instant of disconnection. In Figures 8.13(a) - 8.15(a), torque patterns following reconnection after an interruption of 80 ms of the supply to the 0.75 kW, 1.5 kW and 2.25 kW motors are shown, and from a series of such curves Figures 8.13(b) - 8.15(b) were produced to show the variation of the torque peaks with increasing length of supply interruption. From these figures it can be seen that the positive torque peak of the 0.75 kW motor increases initially to a maximum of 1.49 p.u. after a 10 ms delay, decreases to a minimum of 1.42 p.u., after a delay of 20 ms and increases again to a maximum of 3.19 p.u., before decreasing continually as the length of interruption increases. Meanwhile, the positive torque peak of the 1.5 kW motor decreases initially to a minimum of 0.21 p.u. after a delay of 10 ms, increases to a maximum of 1.28 p.u. after a delay of 20 ms and decreases to a minimum of 1.54 p.u. after the delay reaching 30 ms, before increasing to a maximum of 1.18 p.u. and afterwards decreasing. The positive torque peak of the 2.25 kW motor initially increases to a maximum of 0.71 p.u. after a delay of 10 ms and decreases to a minimum of 0.56 p.u. after a delay of 20 ms, before increasing to a maximum of 0.87 p.u., and finally decreasing as the delay of interruption increases.

Reference to Figure 8.9, which shows the variation of the air-gap power of the three machines following disconnection from the supply, shows that after a 5 ms delay the 0.75 kW and the 2.25 kW machines are generating and the 1.5 kW machine is motoring. When the delay is increased to 8 ms, the machines are still in the same mode of operation, and when reconnected to the supply the machines which are generating draw more power to enable them to transfer to a motoring mode, while the machine which is motoring draws less power since it continues in the same mode. It is for that reason that the torque peak of the 0.75 kW and 2.25 kW machines tend to increase following reconnection, whereas that of the 1.5 kW machine tends to decrease.

Figures 8.16(b) - 8.18(b) show the variation of the torque peak following reconnection of the group, after an interruption of constant length but at a variable starting instant in the supply cycle. As was the case in Figure 7.16, these variations are again all the same, i.e. independent of the instant of interruption, as long as the period of this interruption is constant.

Figure 8.19(a) shows a comparison between the torque peaks following reswitching of the 0.75 kW motor alone and in the 3-motor group. Figures 8.19 (b) and 8.19(c) show similar comparisons for the 1.5 kW and 2.25 kW motors respectively. From these figures the positive torque peak of the 0.75 kW motor in isolation is seen to be initially greater than when in the group, but after a delay disconnection of 60 ms the positive peak in isolation is less than when in the group. On the other hand, the negative torque peak of the machine in isolation is always greater than when in the group. Referring to the comparison in Figure 8.11(a) between the terminal voltage of the machine when disconnected from the supply alone and in the group, we see that the decaying voltage of the 0.75 kW motor in isolation is less than when in the group. Since the reswitching torque depends on the differences in magnitudes and the phase-shift between the supply voltage and the terminal voltage of the machine at the instant of reswitching, the torque peak of the machine in isolation should be greater than when in the group. The difference between the positive peaks is attributable to the difference in the speed transients of the machine.

The comparison of the 1.5 kW motor in Figure 8.19(b) shows that the positive torque peak of the machine when in the group is greater than when in isolation. After a delay of 65 ms the positive torque peak of the machine is isolation is greater than when in the group, while the negative torque peak is always greater when the machine in the group than when in isolation. This can be explained by referring to Figure 8.11(b), which shows that during disconnection the terminal voltage of the machine in isolation is greater than when in the group; hence the torque developed following reconnection of the machine in the group is greater than when in isolation, except for that part of the positive torque peak which is due to the differences in the transient speeds.

Figure 8.19(c) shows that the torque peaks of the 2.25 kW motor when reswitched to the supply in isolation is greater than when in the group, which is, again, due to the decaying voltage of the machine in isolation during disconnection being less than when in the group.

8.7.3.2 Plugging

An investigation was made to establish how the currents and torques of the three machines change with the time taken for plugging to the stiff supply. Figures 8.20(a) - 8.22(a) show computed torque patterns for a period of 10 ms between disconnection and application of the reversed phase-sequence supply to the group of the 0.75 kW, 1.5 kW and 2.25 kW motors respectively. From a series of such curves Figures 8.20(b) - 8.22(b) were developed to show the variation of the largest torque peak with the period of supply interruption. Depending upon the orientation of the rotor and the instant of application of the reversed phase sequence voltage, the torque peak variations are seen to oscillate between a maximum and a minimum value, with a greatest torque peak corresponding to a condition when the supply and the stator voltages are in antiphase and a minimum torque peak to the condition when they are cophasal.

Figure 8.23(a) shows a comparison between the torque peaks of the 0.75 kW motor when plugged in isolation and when plugged in the group; Figure 8.23(b) shows a similar comparison for the 1.5 kW motor, and Figure 8.23(c) for the 2.25 kW motor. From these figures we see that, in the case of the 0.75 kW machine, both the greatest and the smallest torque peaks are less when the machine is plugged in isolation than when it is in the group. For the 1.5 kW machine, both the greatest and the smallest torque peaks are greater when the machine is in the group, and for the 2.25 kW machine both the greatest and the smallest torque peaks are less than when the machine is in isolation. This is expected from the comparisons between the decaying voltage of the machines in isolation and in the group, shown in Figure 8.11, from which it can be seen that both the 0.75 kW and the 2.25 kW motors have smaller terminal voltages in isolation than when in the group, whereas that of the 1.5 kW machine is greater. Since the plugging voltage has a reverse phase sequence, the torque produced by the 0.75 kW and 2.25 kW machines in isolation is less than when in the group, while the torque developed by the 1.5 kW machine is greater in isolation. This conclusion is the opposite of that reached when the machines are reconnected to the same supply. The torque peaks decrease as the delay of interruption increases whether the machine is in isolation or in a group.

Figure 8.24(a) - 8.26(a) show stator current patterns following plugging of the three machines after a supply interruption of 80 ms, and from a series of such patterns Figures 8.24(b) - 8.26(b) were produced, showing the variations of the maximum current peak with delay. The maximum current peaks are seen to be alternately positive and negative, with a rapid change from positive to negative about every 10 ms.

8.7.3.3 Star-delta starting

Starting transients were computed for the 0.75 kW/ 1.5 kW/2.25 kW motor group, for an increasing period of interruption between the star-connected starting condition and the delta connected running condition. Figures 8.27(a) - 8.29(a) show the torque patterns of the 3-machines following a supply interruption of 10 ms, and from a series of such curves Figures 8.27(b) - 8.29(b) were produced to show the variation of the torque peak as the length of the supply interruption is increased. From these figures it can be seen that the torque peaks are never high for any of the three machines.

The computed current results for a delay of 10 ms are shown in Figures 8.30(a) - 8.32(a). From these patterns Figures 8.30(b) - 8.32(b) are developed. From these figures the maximum current peaks are seen to alternate between positive and negative values.

Figures 8.33(a) - 8.33(c) show comparisons between the torque peaks of the 0.75 kW, 1.5 kW and 2.25 kW motors respectively, when started alone and in the group. By referring to Figure 8.11(b), the voltages of the 0.75 kW motor and the 2.25 kW motor in isolation are less than when in the group, while that of the 1.5 kW motor is greater. The torque developed following connection of the 0.75 kW motor or the 2.25 kW motor in delta is consequently greater when the machine is in isolation than when in the group. On the other hand the torque of the 1.5 kW motor in the group is greater than when in isolation.

8.8 Conclusions

The idealized model of a group of three motors was used in developing a digital-computer program, for the evaluation of the overall performance of the group. Results obtained from this program showed good agreement with experimental results, when the group was connected to either a stiff or to a non-stiff supply, disconnected and reconnected to the same or to a different supply.

When the 3-motor group is connected to a stiff-supply the transient currents and torques developed are precisely the same as when each motor is connected seperately, since there can be no interaction between the machines. When the group is connected to a weak supply, the impedance of the supply causes interaction between the machines, and the transient performance of each motor is affected by the transient performance of the others. As currents are drawn from the supply there is a reduction in the voltage common to the three motors, and both the electromagnetic and the accelerating torques are therefore reduced. As a consequence of this, the run-up time of the machines may be very long, depending on the short-circuit capacity of the supply and the characteristics of the loads coupled to the motors. Even at no load, a quite small source impedance will produce an appreciable increase in the run-up time, and the less the moment of inertia and the opposing friction and windage, the quicker will be the motor run-up. The electromagnetic torque produced by each motor was found to be initially oscillatory. Although the motor which ran-up to its steady-state speed most rapidly was only slightly affected by the run-up of the other two machines, the run-up time of each of these was significantly affected by the presence of the other two. The motor which ran-up in the second shortest time was affected principally by the run-up of the motor with the shortest run-up time but only slightly affected by the other motor, whereas the motor with the longest run-up time was affected by the acceleration of both the other two machines.

Following disconnection of the three machines from the supply, both the stator and the rotor circuits of each carry decaying unidirectional and alternating currents, and one of the machines acts as a motor and the other two as generator, or vice versa, depending on the parameters of the motors and the stored mechanical energy at the instant of disconnection.

The torque developed during disconnection is oscillatory, and the mode of operation of the motors changes before a unidirectional operation is achieved which continues until the air-gap fluxes have decayed to zero.

When the group of motors is disconnected and reconnected to the same stiff supply, the behaviour of the machines is affected by their interactive nature during disconnection. During reswitching the machine with the smallest open-circuit time constant has torque peaks greater in isolation when when in the group, while the machines of the greater time constant has smallest torque peaks. Furthermore, as long as the period of the supply interruption is constant, variation of the instant on the cycle at which the machines are disconnected will not affect the torque pattern developed.

When the machines are plugged, the machine with the smallest open-circuit time constant will have a greater torque peak in the group than when in isolation, whereas those of the machines with the greatest open-circuit time constant will be smallest in the group than when in isolation.

When the group is star-delta started, the torque peaks developed by the machine with the smaller time constant in isolation is greater than when the machine is in the group and vice versa.

REFERENCES

1. Stanley, H.C. "An analysis of the induction motor", Trans. A. I.E.E., 1938, 57, p.751.

2. Lyon, W.V. "Transient analysis of alternating current machinery", (Wiley, 1954).

3. Takeuchi, T.J. "Transient phenomena at instant of changing power source of induction motor while it is running", E.T.J., Japan, 1960, 6, p.3.

4. Chidambara, H.R., and Ganapathy, S. "Transient torques in three-phase induction motors during switching operations", Trans. A.I.E.E., 1962, 81, p.47

5. Gilfillan, F.S., and Kaplan, E.L. "Transient torques in squirrel-cage induction motors with special reference to plugging", Trans. A.I.E.E., 1941, 60, p.1200.

6. Maginniss, L.J., and Schultz, N.R. "Transient performance of induction motors", Trans. A.I.E.E., 1946, 63, p.641.

7. Hughes, F.M., and Aldred, A.S. "Transient characteristics and simulation of induction motors", Proc. I.E.E., 1964, 111, p.2041.

8. Rudenbürg, R. "Transient performance of electric power systems", (McGraw Hill, New York, 1950).

9. Enslin, N.C., Kaplan, W.M. and Davies, J.L. "Influence of transient switching currents and fluxes on the torque developed by a squirrel-cage induction motor", Proc. I.E.E., 1966, 113, p.1035.

10. Wood, W.S., Flynn, F., and Shanmugasundaram, A., "Transient torques induction motors, due to switching of the supply", Proc. I.E.E., 1966, 113, p.819.

11. Slater, R.D., Wood, W.S., Flynn, F.P. and Simpson, R., "Digital computation of induction motor transient torque pattern", Proc. I.E.E., 1966, 113, p. 819.

12. Smith, I.R., and Sriharan, S. "Transient performance of the induction motor", Proc. I.E.E., 1966, 113, p.1173.

13. Smith, I.R., and Sriharan, S. "Induction motor reswitching transients", Proc. I.E.E., 1967, 114, p.503.

14. Smith, I.R., and Sriharan, S. "Transients in induction machines with terminal capacitors", Proc. I.E.E., 1968, 115, p.519.

15. Ramsden, V.S., Zorbas, N., and Booth, R.R. "Prediction of induction motors dynamic performance in power systems", Proc. I.E.E., 1968, 115, p.511.

16. Park, R.H., "Two-reaction theory of synchronous machines", Trans. A.I.E.E., 1929, 48, p.716.

17. Smith, I.R. and Hamill, B. "Effect of parameter variations on induction-motor transients", Proc. I.E.E., 1973, 120, p.1489.

18. Kalsi, S.S. "Switching transients in large deep-bar squirrel-cage induction motor", Trans. I.E.E.E., 1973, PAS-92, p.1260.

19. Snider, L.A., and Smith, I.R. "Prediction of performance of induction motor/synchronous-generator combination", Proc. I.E.E., 1973, 120, p.79.

20. Stephen, D.D. "Effect of system voltage depressions on large a.c. motors", Proc. I.E.E., 1966, 113, p.500.

21. Lewis, D.G. and Marsh, W.D. "Transfer of steam-electric generating-station auxiliary busses", Trans. A.I.E.E. 1955, 74, p.322.

22. Humpage, W.D., Durrani, K.E. and Carvalho, V.F. "Dynamic-response analysis of interconnected synchronous-asynchronous-machine group", Proc. I.E.E., 1969, 116, p.2015.

23. Kron, G. "Classification of the reference frames of synchronous machines", Trans. A.I.E.E., 1950, Pt.II, 69, p.720.

24. Kron, G. "Tensor analysis of networks", (Wiley 1939).

25. Lynn, J.W. "Tensors in electrical engineering", (Edward Arnold, 1963).

26. Kalsi, S.S. and Adkins, B. "Transient stability of power systems containing both synchronous and induction machines", Proc. I.E.E., 1971, 118, p.1467.

27. Kalsi, S.S., Stephen, D.D. and Adkins, B. "Calculation of system-fault currents due to induction motors", Proc. I.E.E., 1971, 118, p.201.

28. Abdel-Hakim, M.M. and Berg, G.J. "Dynamic single-unit representation of induction motor groups", Trans. I.E.E.E., 1976, PAS-95, p.155.

29. Sriharan, S. "Digital simulation of a group of induction motors", Proc. I.E.E., 1975, 122, p.1399.

30. Lathrop, C.M. and Schleckser, C.E. "Problems confronting the designer in starting large motors", Trans. A.I.E.E., 1951, 70, p.1351.

31. Lawden, D.F. "A course in applied mathematics", (The English Universities Press, 1970).

32. Burridge, R. and Umamaheswaran, V.S. "Accelerometer characteristics of a drag-cup induction machine", Trans. I.E.E.E., 1968, PAS-87, p.1603.

33. Wood, W.S., and McNaull, J. "A simple method of recording angular acceleration", Engineer, 1962, 2-3, p.569.

34. Law, J. and Novonty, D.W. "The two-phase accelerometer" Trans. I.E.E.E., 1964, PAS-83, p.614.

35. Granborg, B.S.H. "Computational stability in system analysis", Proceeding of the 7th Annual Allerton conference on circuit and system theory (I.E.E.E.) New York, 1969, p.476.

36. Gill, S. "A process for the step-by-step integration of differential equations in an automatic digital computing machine", Proc. Cambridge, Phil.Soc. 1951, 47, p.96.

37. Ralston,A. "Runge-Kutta methods with minimum error bounds", M.A.T.C., 1962, 16, p.431.

38. Milne, W.E. "Numerical integration of ordinary differential equations", Amer. Math. Monthly, 1931, 38, p.455.

39. Hamming, R. "Stable predictor-corrector methods for ordinary differential equations", J. Assoc. Computing Machinery, 1959, 6, p.37.

40. Ralston, A. and Wilf, H.S. "Mathematical methods for digital computers", (Wiley, 1967).

41. Williams, S. and Smith, I.R. "Fast digital computation of 3-phase thyristor bridge circuits", Proc. I.E.E., 1973, 120, p.791.

42. Sayce, L.A. "The preparation and industrial application of diffraction grating", Trans. Soc. Instr., 139, 1957.

43. Dunworth, A. "A digital instrumentation for angular velocity and acceleration", Trans. Instr. Measur., 18, No.2.

44. Hajiroussou, M. "An investigation into the transient torque characteristics of a small induction motor", PhD Thesis, 1972, University of Birmingham.

45. Carter, B.C., Shannon, J.F. and Forshaw, J.R. "Measurement of displacement and strain by capacity methods", Proc. I.M.E., 1945, 154, p.215.

46. Mills, C.H.G. "A capacitance type torquemeter", J.of Sci. Instruments, 1948, 25, p.151.

47. Sudan, R.N., Manohar, V.N. and Adkins, B. "The measurement of transient torque and load angle in model synchronous machines", Proc. I.E.E., 1960, 107A, p.51.

48. Sriharan, S. "Induction motor transients", PhD Thesis, 1967, University of Birmingham.

49. Ford, H., and Douglas, A. "The measurement of torque in shafts", Engineering, 1949, p.481.

50. Enslin, N.C.V. "A method of using microwave for measuring small displacement, and a torque-meter using this principle", Proc. I.E.E., 1954, p.522.

51. Montgomery, C.G. "Technique of microwave measurements", (McGraw Hill, New York, 1947).

52. Hancock, N.N. "Matrix analysis of electrical machinery", (Pergamon Press, 1964).

APPENDIX A

NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

Numerical methods for the solution of ordinary differential equations may be put in two categories - numerical integration methods (mainly predictor-corrector) and solution by successive substitutions (mainly Runge-Kutta methods). The advantages of Runge-Kutta methods are that they are self-starting and easy to program for digital computers. Although predictor-corrector methods are of greater accuracy and error-estimating ability, Runge-Kutta methods still find application in starting the numerical solution and in changing the interval of integration.

A.1 Runge-Kutta methods

Runge-Kutta methods[36,37] are used for solving differential equations by means of successive substitutions, without requiring the explicit definition or evaluation of any derivatives beyond the first. For the solution of the equation $y' = f(x,y)$ with given initial condition $y_0 = y(x_0)$, the general Runge-Kutta method of order m at a sequence of points $x_1, x_2, \ldots\ldots$ is

$$y_{n+1} - y_n = k = \sum_{i=1}^{m} w_i\, k_i \qquad \text{A.1}$$

where $y_r = y(x_r)$, the w_i's are constants and

$$k_i = hf\left(x_n + \alpha_i h,\ y_n + \sum_{j=1}^{i-1} \beta_{ij} k_j\right)$$

where $h = x_{n+1} - x_n$ and $\alpha_1 = 0$. By choosing the α_i's and β_{ij}'s properly, the expansion of the right-hand side of Equation A.1 can be made identical with Taylor series expansion of k about x_n through the term in h^m.

To determine the suitable values for the α_i's and β_{ij}'s, the k_i's are expanded by the use of Taylor series. For fourth order Runge-Kutta methods i.e. when m=4, this will result in three equations for k_2, k_3 and k_4 in terms of the function and its high derivatives. By inserting these three equations into Equation A.1 an explicit expression for k will result, containing eight terms of orders up to and including h^4. If each term is compared with the corresponding term of the Taylor series expansion for k, a two-parameter system of equations (with α_2 and α_3 as parameters) will result, and two further consistent relations between the parameters may be arbitrarily imposed in order to determine the solution. It is the relationship between the parameters which determines the relative properties of the various types of Runge-Kutta solution. With the fourth order Runge-Kutta solution α_2 and α_3 are both equal to $\frac{1}{2}$, and consequently $\beta_{21} = \beta_{32} = \frac{1}{2}$, $\beta_{43} = \alpha_4 = 1$, all other $\beta_{ij} = 0$, $\omega_1 = \frac{1}{2}$, $\omega_2 = \frac{1}{3}$, $\omega_3 = \frac{1}{3}$, and $\omega_4 = \frac{1}{6}$. The resulting solution is then

$$y_{n+1} - y_n = k = \frac{1}{6}\left(k_1 + 2\,k_2 + 2\,k_3 + k_4\right)$$

in which

$$k_1 = h\,f\,(x_n,\ y_n)$$

$$k_2 = h\,f\,(x_n + \frac{1}{2}\,h,\ y_n + \frac{1}{2}\,k_1)$$

$$k_3 = h\,f\,(x_n + \frac{1}{2}\,h,\ y_n + \frac{1}{2}\,k_2)$$

$$k_4 = h\,f\,(x_n + h,\ y_n + k_3)$$

A.2 Predictor-corrector methods

While the Runge-Kutta method starts each integration step independent of the other steps, and uses only rates-of-change of the variables integrated within each step, the predictor-corrector methods[38,39,40] make use of values of the variables and of their first derivatives obtained in previous steps. From these, a prediction is made at the beginning of each step of the values of the integrated variables at the end of it, and many formulae may be derived that approximate to these values from a knowledge of the previous ones.

The accuracy of prediction is then dependent upon the number of previous values of the variables and their derivatives to which recourse is made, and on the relative significance attached to each by their weighting in the formula for prediction. Errors may be checked by comparing the prediction formula with the exact infinite-series expansion of the integral over a small step, from x_0 to x_{0+h}. By appropriate choice of the coefficients of the previous values used, an exact correspondence is sought between the prediction formula and the first few terms of the expansion. When correspondence is achieved in the more important early terms, the remaining terms in the infinite series represent the error incurred in prediction. This error is referred to as being of order p when the predictor formula is accurate up to, and including, the (p-1)th power of the step interval h. The shorter the step interval the smaller is this error of truncation, and to this must be added the round-off errors and also those inherited from previous steps, in order to give the total error incurred at the end of any one step of integration.

Having in this way derived an estimate of moderate accuracy, it is now necessary to reduce the error by the application of a corrector formula. The latter, using the first prediction, and other previous values as in the predictor formula, yields a final value for the integral, which is of higher accuracy, at the end of the step. In many cases, a single application of the corrector is sufficient, but it may be applied several times iteratively, until no change in the values of the variables takes place on its successive application. To promote an efficient computation, the corrector formula should have as low a truncation error as that of the predictor.

The general form of the commonly used group of predictor-corrector formulae may be written as

$$y_{n+1} = a_n y_n + a_{n-1} y_{n-1} + \ldots + a_{n-p} y_{n-p} +$$

$$h(b_{n+1} y'_{n+1} + b_n y'_n + \ldots + b_{n-p} y'_{n-p}) + C_n \qquad \text{A.2}$$

in which C_n is the error incurred during the (n+1)th step, and where the coefficients in the predictor formulae are, as a rule, different from those in the corrector formulae. In particular, the coefficient b_{n+1} will be zero in all predictor formulae.

Predictor-corrector methods may be summarized as follows.

A.2.1 Midordinate-trapezoidal method

Using formula A.2, with the midordinate rule used to predict the value $\overline{y}_{n+1}$ of the integral at the (n+1)th step, a solution is obtained in terms of one previous value and one derivative

$$\overline{y}_{n+1} = y_{n-1} + 2h\, y'_n$$

with the error being of the order h^3. The prediction is corrected by the trapezoidal rule, and the final value of the integral at the step considered is

$$y_{n+1} = y_n + \frac{h}{2} (y'_n + \bar{y}'_{n+1})$$

which is also of third-order accuracy.

A.2.2 Adams-Bashforth method

By increasing the number of previous values used in prediction and subsequent correction, the accuracy of integration may be improved at the expense of more complicated formulae, and therefore of an increased amount of computation and storage space.

The fifth order Adams-Bashforth method uses three previous derivative values in the predictor formula, the latter being

$$\bar{y}_{n+1} = y_n + \frac{h}{24} (55\, y'_n - 59\, y'_{n-1} + 37\, y'_{n-2} - q\, y'_{n-3})$$

and two previous derivative values in the corrector formula which is

$$y_{n+1} = y_n + \frac{h}{24} (g\, \bar{y}'_{n+1} + 19\, y'_n - 5\, y'_{n+1} + y'_{n-2})$$

A.2.3 Methods of Milne, Hamming and Ralston

These methods are shown in Table A.1.

A.3 Step-length choice in Runge-Kutta methods

To choose the appropriate length of the step of integration required in a Runge-Kutta solution, so as to ensure both accuracy and numerical stability, the system equations are arranged in the standard state-variable form as,

$$[\dot{x}] = [A][x] + [B][u]$$

The time constants of the system are found by obtaining the reciprocal of the eigenvalues of A . Granborg[35] has shown that, for a 4th-order Runge-Kutta procedure, the limit of numerical stability is reached with a step-length 2.8 times the system smallest time constant. However, Williams and Smith[41] adopted a figure of 1.0 times the smallest time constant of the system.

Method	Predictor	Intermediate Step	Corrector	Final step
Milne	$\bar{y}_{n+1}=y_{n-3}+\frac{4h}{3}(2\ y'_n-y'_{n-1}+2\ y'_{n-2})$	-	$y_{n+1}=y_{n-1}+\frac{h}{3}(y'_{n+1}+4y'_n+y'_{n-1})$	-
Hamming	$y_{n+1}=y_{n-3}+\frac{4h}{3}(2y'_n-y'_{n-1}+2y'_{n-2})$	$\tilde{y}_{n+1}=\bar{y}_{n+1}-\frac{112}{121}(\bar{y}_{n}-y_n)$	$y_{n+} = \frac{1}{8}[g\ y_n-y_{n-2}+$ $3h(\tilde{y}'_{n+1}+2\ y'_n-y'_{n-1})$	$\hat{y}_{n+1}=y_{n+1}+$ $\frac{9}{121}(\bar{y}_{n+1}-y_{n+1})$
Ralston 1	$y'_{n+1}=7\ y'_n-21\ y'_{n-1}+35\ y'_{n-2}$ $-35\ y'_{n-3}+21\ y'_{n-4}-7\ y'_{n-5}$ $+\ y'_{n-6}$	-	$y_{n+1}=\frac{1}{3}(2\ y_n+2\ y_{n-1}$ $-\ y_{n-2})+\frac{1}{27}(y_{n-3}-y_n)$ $+\frac{h}{3}(y'_{n+2}+\frac{10}{3}y'_n-y'_{n-1}$	-
Ralston 2	$y'_{n+1}=\frac{1}{9}(49\ y'_n-98\ y'_{n-1}+70y'_{n-2}$ $+\ 28\ y'_{n-3}-56\ y'_{n-4}-14\ y'_{n-5}$ $+\ 58\ y'_{n-6}-35\ y'_{n-7}$ $+\ 7\ y'_{n-8})$	-	$y_{n+1}=\frac{1}{3}(2\ y_n+2\ y_{n-1}$ $-\ y_{n-}+\frac{1}{27}(y_{n-3}-y_n)$ $+\frac{h}{3}(y'_{n+1}+\frac{10}{3}y'_n-$ $y'_{n-1})$	-

TABLE A.1

APPENDIX B

METHODS OF MEASURING TORQUE

B.1 Unloaded motor

B.1.1 Tachometer using diffraction grating

The application of diffraction gratings made by the Metron-N.P.L[42] process have proved very suitable for linear measurement. They consist of a strip of plate glass, bearing a thin layer of synthetic resin, the surface being moulded with a large number of transverse grooves, usually 1000-5000 per inch, of very even 'saw-tooth' cross section.

Circular diffraction gratings are readily used for the continuous measurement of angular displacement in a corresponding way to linear diffraction grating. For the purposes of measuring angular velocity and acceleration[43,44], the tachometer consists of two diffraction gratings, the main and the index gratings. The main grating is a circular glass surface covered with radial lines and attached to the shaft of the motor. The index grating is a much smaller glass disc covered with lines of the same spacing as the main grating. The index grating is mounted within a sensing head, inside which there is also a photodiode and an optical system, consisting of an exciter lamp at the focal length of a collimating lens. The sensing head is attached to an aluminium plate, rigidly supported on the motor bed-plate. By adjusting the optical system, parallel light is made to fall on the two gratings, and the index grating is turned about its axis until the optimum pattern of Moire fringes is obtained.

Rotation of the main grating relative to the fixed index grating results in a changing pattern of Moire fringes and these are detected by the photo-diode in the form of a sinusoidal signal, each cycle of which corresponds to the traverse by the main grating of one grating space. The sinusoidal output from the photo-diode is fed to a high-gain amplifier with a high cut-off frequency, so that a square wave output with a very short rise time is obtained at all motor speeds, the pulse width being inversely proportional to the speed. During a transient the amplifier output pulses may not be of uniform width. A monostable multivibrator is used to reduce them to a uniform width and the spacing between consecutive pulses may be used as a measure of the rotational speed.

If the number of pulses occurring in successive equal time intervals are counted, it is a simple matter to determine both the velocity and the acceleration of the motor. The necessary data storage and processing can be performed either by electronic circuitry specially developed for the purpose[43] or by the use of an available digital computer.[44]

B.1.2 The use of perforated disc

If a series of holes are drilled near the circumference of a disc fixed to the shaft of the motor, and one side of the disc is irradiated by a light source, a series of light pulses are produced on the other side when the disc rotates. The light pulses are arranged to fall on a photodiode, where they are converted to electrical form. Electronic counting of the number of pulses arriving in fixed time intervals, and subsequent data processing circuitry, then enables the acceleration and velocity to be determined in a way substantially the same as when the pulses are produced by a diffraction grating.

B.2 Loaded motor

When the motor is loaded, measurement of the transmitted torque can be performed by one of several methods, which differ only in how the strain in the rotating shaft affects some electrical or magnetic quantity. The methods generally adopted can be classified as follows.

B.2.1 Electromagnetic methods

The principle of these methods is that when a shaft of magnetic material is strained, its permeability will change. If the shaft completes a magnetic circuit in which a flux is established, the reluctance of the magnetic circuit will change with the strain in the shaft, and this will cause a change in the flux which can be used to provide an output signal proportional to the torque causing the initial change in reluctance. An example of a torque measuring device based on this principle is the torductor.

B.2.1.1 Torductor

The form of torductor particularly useful for detecting changes in the permeability of a rotating shaft is the ring torductor, which consists of three sets of pale rings arranged side by side around the shaft as shown in Figure B.1. These rings constitute a central excitation ring, and two pick-up rings, with mutual spacings of one pole pitch. The excitation ring is excited from a 100 Hz, 220V supply and the outer rings are connected in series and phase opposition to each other. In this way the torductor measures the torsion in the shaft as two mutually orthogonal components, one is tension and the other is compression and there is no output signal from the rings when no torque is transmitted by the shaft. The output which results when the shaft is strained is rectified and fed to a recording device.

B.2.2 Strain gauge methods

The principle of these methods is that the strain in the shaft will cause a change in a capacitance in a capacitance strain gauge, or a resistance in a resistance strain gauge. The change in this electrical quantity provides an output which is proportional to the transmitted torque.

B.2.2.1 Capacitance strain gauge

A capacitance strain gauge[45,46] consists of four parallel plate condensers connected in parallel. One plate of each capacitor is fitted to each of two circular ebonite discs, as in Figure B.2(a). The discs are mounted on the motor shaft, with the two sets of plates parallel and separated by a small air gap; and the four capacitances so formed are connected in parallel, with one side earthed to the motor shaft and the other brought out through a slip ring. The two discs are mounted so that the length of shaft by which they are separated is strained, and the capacitance is changed, Figure B.2(b). The change in capacitance is a measure of the torque and means for its accurate determination must be provided. To accomplish this, the capacitance strain gauge is made part of the tuned circuit of a coupled r.f. oscillator. The gauge is energised from an alternating source at about 5 kHz.

Due to the variations in the contact resistance at the slip ring, amplitude-modulated components will occur at the output of the oscillator, and due to the strain in the rotating shaft frequency-modulated component will appear. The undesirable amplitude-modulated components are eliminated by use of a limiter stage, and the required frequency-modulated signal is converted to an amplitude modulated signal in a discriminator stage. The output of the discriminator is rectified to obtain a unidirectional signal which is then amplified and fed to a recording device.

B.2.2.2 Resistance strain gauge

A resistance strain gauge[47,48,49] measures the surface elongation or compression of the shaft. The principal axes of strain in a circular shaft subjected to torsion occur at 45^0 to the axis of the shaft, the strains being at right angles and equal, but opposite in sign, i.e. one is tensile and the other is compressive. Thus, if a resistance strain gauge is mounted on the surface of a shaft, along a 45^0 helix, it will be subjected to the maximum strain arriving from the torque. A single gauge could be used in this way, but it would be responsive not only to the torsional strain, but also to bending strain. Furthermore, any change in the brush-contact resistance at the sliprings would swamp the small changes at the gauge. In practice therefore four gauges, on helices at right angles and on opposite sides of the shaft, are connected in a Wheatstone bridge arrangement and the output signal from the bridge is amplified before the slip rings. It is, therefore, necessary to have slip rings associated with the strain gauge bridge, two to connect the input to the bridge and two for the output signal. The most important feature in the choice of the slip-ring and the brushes is to keep the contact resistance small and constant. Small diameter slip rings should be used to reduce the running velocity to a minimum while to minimise the noise the brushes are made from silver-carbon with a fairly high pressure on stainless steel or silver slip rings. The output signal of the bridge is amplified and fed to a recording device (Figure B.3).

B.3 Measurement of the reaction on the stator

Measurement of the torque developed by the rotor of an induction motor can be obtained from measurement of its reaction on the stator. This can be accomplished by using, for example, a microwave cavity[50] or a load cell as the necessary transducer.

B.3.1 Use of a microwave cavity

A microwave cavity[51] is an enclosed chamber in which a dielectric medium, often air, is surrounded by a conducting material. The cavity may be a rectangular box, a cylinder, a sphere, etc. The cavity can obtain an electromagnetic field, which varies periodically when and only when the frequency of the field has certain definite values. Inside the cavity oscillations of energy occur between the electric and the magnetic fields, as energy is transferred through the cavity at a definite frequency which depends on the size and shape of the conducting surface.

In making connections to external circuits for exciting electromagnetic waves in the cavity, and for absorbing energy from them, small coupling probes are inserted through holes cut in the metal enclosure, and since energy will be radiated through these holes it is desirable to keep them as small as possible.

To use the microwave cavity as a torque-meter an oscillator is used to feed the cavity with a signal at a fixed frequency through the input coupling probe. The output from the output coupling probe is fed directly to a recording device if the torque is pulsating. If the torque is constant, the output is connected to a detector which converts the variations in transmitted power to variations in direct current, which are then amplified and fed to a recording device.

For the case of an induction motor, the stator core with its winding is suspended by rigid cantilevers. The resonance of the mechanical system excited by the slotting of the motor and bearing vibrations is suppressed by an attenuating network. A rigid link transforms the rotational movement of the stator, due to the reaction of the torque developed by the rotor, to a linear displacement of the diaphragm of a microwave cavity. The cavity is calibrated by an attachment enabling a fixed torque produced (say) by a lever arm and weight to be applied to the stator.

B.4 The use of a load cell

The reaction of the torque developed by the induction motor on the stator can be detected by means other than the microwave cavity. One simple way in which this may be accomplished is by mounting the feed of the motor or short pillars, and recording the deformation of the pillars by resistance or semiconductor strain gauges. The pillars must be sufficiently sturdy for any stator movement to be extremely small. The strain gauges are connected in a bridge formation, and the output is amplified and fed to a recording device.

APPENDIX C

C.1 Analytical solution for stator and rotor currents for connection of a single machine

Equations 6.1 are non-linear differential equations. By assuming that the speed may be considered as constant for the first few cycles following connection to the supply the equations become linear.

The solution of Equation 6.1 is facilitated by the introduction of 2-phase instantaneous symmetrical components, thereby reducing the number of the equations involved to two. These equations are

$$\begin{bmatrix} v_{sp} \\ 0 \end{bmatrix} = \begin{bmatrix} R_s + L_s D & MD \\ M(D - j\dot{\theta}) & R_r + L_r(D - j\dot{\theta}) \end{bmatrix} \begin{bmatrix} i_{s_p} \\ i_{r_p} \; {}_je^{\theta} \end{bmatrix} \qquad \text{C.1}$$

The positive-sequence component of a balanced set of voltages is obtained from

$$\begin{bmatrix} v_{sp} \\ v_{sn} \\ v_{sz} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \text{C.2}$$

where $a = j_e^{\frac{2\pi}{3}}$

so that the positive-sequence component v_{sp} is

$$v_{sp} = \frac{\sqrt{3}\ Vm}{\sqrt{2}}\ e^{j(\omega t+\delta)}$$

Applying Laplace transform and solving Equations C.1 for I_{s_p} and I_{r_p}, we obtain

$$I_{s_p} = \frac{V}{\sigma L_s}\ \frac{k_r + (S - j\,\dot{\theta})}{(S - j\,\omega)(S + \alpha_1 - j\,\beta_1)(S + \alpha_2 - j\,\beta_2)} +$$

$$\frac{i_{sp_0}\ \sigma L_r L_s\ S + (R_r - j\,\dot{\theta}\,L_r)(L_s\, i_{sp_0} + M\, i_{rp_0})}{\sigma L_r L_s\ (S + \alpha_1 - j\,\beta_1)(S + \alpha_2 - j\,\beta_2)}$$

C.3

and

$$I_{r_p} = \frac{VM}{\sigma L_s L_r}\ \frac{(j\,\dot{\theta} - S)}{(S - j\,\omega)(S + \alpha_1 - j\,\beta_1)(S = \alpha_2 - j\,\beta_2)} +$$

$$\frac{i_{rp_0}\ \sigma L_s L_r\ S + R_s\ (M\, i_{sp_0} + L_r\, i_{rp_0}) + j\,\dot{\theta}\,M\,(i_{sp_0}\, L_s + M\, i_{rp_0})}{\sigma L_s L_r\ (S + \alpha_1 - j\,\beta_1)(S + \alpha_2 - j\,\beta_2)}$$

C.4

where I_{sp} and I_{rp} are the Laplace transform of i_{s_p} and $i_{r_p} e^{j\theta}$, respectively, and i_{sp_0} and i_{rp_0} are their initial values. In the above equations the following simplifcation substitutions are introduced

$$\sigma = 1 - \frac{M^2}{L_s L_r}$$

$$\alpha_1 = \frac{k_r + k_s}{2\sigma} + \mathrm{Re}\,\sqrt{Z}$$

$$\alpha_2 = \frac{k_r + k_s}{2\sigma} - \mathrm{Re}\,\sqrt{Z}$$

$$\beta_1 = \frac{\dot{\theta}}{2} - \mathrm{Im}\,\sqrt{Z}$$

$$\beta_2 = \frac{\dot{\theta}}{2} + \mathrm{Im}\,\sqrt{Z}$$

where

$$Z = \left[\left(\frac{k_r + k_s}{2\sigma}\right)^2 - \left(\frac{\dot{\theta}}{2}\right)^2 - \frac{k_r k_s}{\sigma}\right] + j\,\frac{\dot{\theta}}{2}\,\frac{(k_s - k_r)}{\sigma}$$

$$k_s = \frac{R_s}{L_s}$$

$$k_r = \frac{R_r}{L_r}$$

Taking inverse transforms of Equations C.3 and C.4 we obtain

$$i_{sp} = \frac{V}{\sigma L_s} \frac{\{k_r + [-\alpha_1 + j(\beta_1 - \dot{\theta})]\}[\cos\beta_1 t + j\sin\beta_1 t]}{[-\alpha_1 + j(\beta_1 - \omega)][\alpha_2 - \alpha_1 + j(\beta_1 - \beta_2)]} e^{-\alpha_1 t}$$

$$+ \frac{V}{\sigma L_s} \frac{\{k_r + [-\alpha_2 + j(\beta_2 - \dot{\theta})]\}[\cos\beta_2 t + j\sin\beta_2 t]}{[-\alpha_2 + j(\beta_2 - \omega)][(\alpha_1 - \alpha_2) + j(\beta_2 - \beta_1)]} e^{-\alpha_2 t}$$

$$+ \frac{V}{\sigma L_s} \frac{[k_r + j(\omega - \dot{\theta})][\cos\omega t + j\sin\omega t]}{[\alpha_1 + j(\omega - \beta_1)][\alpha_2 + j(\omega - \beta_2)]}$$

$$+ e^{-\alpha_1 t} \frac{[(-\alpha_1 + j\beta_1)\sigma L_r L_s i_{sp_0} + (R_r - j\dot{\theta} L_r)(i_{sp_0} L_s + M i_{rp_0})](\cos\beta_1 t + j\sin\beta_1 t}{\sigma L_r L_s [\alpha_2 - \alpha_1 + j(\beta_1 - \beta_2)]}$$

$$+ \frac{[(-\alpha_2 + j\beta_2)\sigma L_r L_s i_{sp_0} + (R_r - j\theta L_r)(i_{sp_0} L_s + M i_{rp_0})][\cos\beta_2 t + j\sin\beta_2 t]e^{-\alpha_2}}{\sigma L_r L_s [\alpha_1 - \alpha_2 + j(\beta_2 - \beta_1)]}$$

C.5

and

$$i_{r_p} \; e^{j\theta} = \frac{VM}{\sigma L_r L_s} \; \frac{j(\dot{\theta} - \omega)(\cos \omega t + j \sin \omega t)}{[\alpha_1 + j(\omega - \beta_1)]\;[\alpha_2 + j(\omega - \beta_2)]} +$$

$$\frac{VM}{\sigma L_r L_s} \; \frac{[j(\dot{\theta} - \beta_1) + \alpha_1][\cos \beta_1 t + j \sin \beta_1 t]\; e^{-\alpha_1 t}}{[-\alpha_1 + j(\beta_1 - \omega)]\;[\alpha_2 - \alpha_1 + j(\beta_1 - \beta_2)]} +$$

$$\frac{VM}{\sigma L_r L_s} \; \frac{[j(\theta - \beta_2) + \alpha_2]\;[\cos \beta_2 t + j \sin \beta_2 t]\; e^{-\alpha_2 t}}{[-\alpha_2 + j(\beta_1 - \omega)]\;[(\alpha_1 - \alpha_2) + j(\beta_2 - \beta_1)]} +$$

$$\frac{e^{-\alpha_1 t}\;[(\cos\beta_1 t + j\sin\beta_2 t)\;(-\alpha_1 + j\beta_1)(i_{rp_0}\sigma L_s L_r) + R_s(M\, i_{sp_0} + L_r\, i_{rp_0}) + j\dot{\theta} M(i_{sp_0} L_s + M\, i_{rp_0})]}{\sigma L_s L_r\;[(\alpha_2 - \alpha_1) + j(\beta_1 - \beta_2)]}$$

$$+ \frac{e^{-\alpha_2 t}\;(\cos\beta_2 t + j\sin\beta_2 t)[(-\alpha_2 + j\beta_2)(i_{rp_0}\sigma L_s L_r) + R_s(M i_{rp_0} + L_r i_{rp_0}) + j\dot{\theta} M(i_{sp_0} L_s + M i_{rp_0})]}{\sigma L_s L_r\;[\alpha_1 - \alpha_2 + j(\beta_2 - \beta_1)]}$$

C.6

Since the relationships between the instantaneous symmetrical components and the d,q components are

$$i_{s_p} = \frac{i_{sd} + j\, i_{sq}}{\sqrt{2}} \qquad \text{C.7}$$

$$i_{r_p}\, e^{j} = \frac{i_{rd} + j\, i_{rq}}{\sqrt{2}} \qquad \text{C.8}$$

applying Equations C.5 and C.6 in Equations C.7 and C.8 we obtain

$$i_{sd} = \frac{V}{\sigma L_s \sqrt{b_1}\, c_1} [(\omega - \dot{\theta}) \cos (\omega t + \gamma - \phi_1 - \phi_2) + k_r \sin (\omega t + \gamma - \phi_1 - \phi_3)]$$

$$+ e^{-\alpha_1 t} \left[\frac{V}{\sigma L_s \sqrt{a_1}\, b_1} \left[- \beta_2 \cos (\beta_1 t + \gamma - \phi_1 - \phi_3) + (k_r - \alpha_1) \sin \right.\right.$$

$$\left. \beta_1 t + \gamma - \phi_1 - \phi_3) \right] + \frac{\sqrt{2}\, (B_{22} (\alpha_2 - \alpha_1) - A_{22} (\beta_1 - \beta_2))}{\sigma L_r L_s ((\alpha_2 - \alpha_1)^2 + (\beta_1 - \beta_2)^2)} \Bigg]$$

$$+ e^{-\alpha_2 t} \left[\frac{V}{\sigma L_s \sqrt{a_1}\, c_1} [\beta_1 \cos (\beta_2 t + \gamma - \phi_2 - \phi_3) - (k_r - \alpha_3) \sin \right.$$

$$\left. \beta_2 t + \gamma - \phi_2 - \phi_3)] + \frac{\sqrt{2}\, (B_{44} (\alpha_1 - \alpha_2) - A_{44} (\beta_2 - \beta_1))}{\sigma L_r L_s ((\alpha_1 - \alpha_2)^2 + (\beta_2 - \beta_1)^2)} \right]$$

C.9

$$i_{sq} = \frac{V}{\sigma L_s \sqrt{b_1 c_1}} \left[k_r \cos(\omega t + \gamma - \phi_1 - \phi_2) - (\omega - \theta) \sin(\omega t + \gamma - \phi_1 - \phi_2) \right]$$

$$+ e^{-\alpha_1 t} \left[\frac{V}{\sigma L_s \sqrt{b_1 a_1}} \left[(k_r - \alpha_1) \cos(\beta_1 t + \gamma - \phi_1 - \phi_3) + \beta_2 \sin(\beta_1 t + \gamma - \phi_1 - \phi_3) \right. \right.$$

$$\left. + \frac{\sqrt{2}\,(A_{22} (\alpha_2 - \alpha_1) + B_{22} (\beta_1 - \beta_2))}{\sigma L_r L_s ((\alpha_2 - \alpha_1)^2 + (\beta_1 - \beta_2)^2)} \right]$$

$$+ e^{-\alpha_2 t} \left[\frac{V}{\sigma L_s \sqrt{a c}} \left[(k_r - \alpha_2) \cos(\beta_2 t + \gamma - \phi_2 - \phi_3) + \beta_1 \sin(\beta_2 t + \gamma - \phi_2 - \phi_3) \right. \right.$$

$$\left. + \frac{\sqrt{2}(A_{44} (\alpha_1 - \alpha_2) + B_{44} (\beta_2 - \beta_1))}{\sigma L_r L_s ((\alpha_1 - \alpha_2)^2 + (\beta_2 - \beta_1)^2)} \right]$$

C.10

$$i_{rd} = \frac{V\,k_4\,(\omega - \theta)}{\sigma\,L_s\,\sqrt{b_1\,c_1}}\ \cos\,(\omega t + \gamma - \phi_1 - \phi_2) +$$

$$e^{-\alpha_1 t}\left[\frac{V\,k_4}{\sigma\,L_s\,\sqrt{a_1\,b_1}}\ [\beta_2\ \cos\,(\beta_1 t + \gamma - \phi_1 - \phi_3) + \alpha_1\ \sin\,(\beta_1 t + \gamma - \phi_1 - \phi_3)]\right.$$

$$\left. + \frac{[A_{88}\,(\alpha_2 - \alpha_1) - A_{77}\,(\beta_1 - \beta_2)]\ \sqrt{2}}{\sigma\,L_r\,L_s\,(\,(\alpha_2 - \alpha_1)^2 + (\beta_1 - \beta_2)^2)}\right]$$

$$e^{-\alpha_2 t}\left[\frac{-V\,k_4}{\sigma\,L_s}\ [\beta_1\ \cos\,(\beta_2 t + \gamma - \phi_2 - \phi_3) + \alpha_2\ \sin\,(\beta_2 t + \gamma - \phi_2 - \phi_3)]\right.$$

$$\left. + \sqrt{2}\ \frac{[A_{111}\,(\alpha_1 - \alpha_2) - A_{112}\,(\beta_2 - \beta_1)]}{\sigma\,L_s\,L_r\,(\,(\alpha_1 - \alpha_2)^2 + (\beta_2 - \beta_1)^2)}\right]$$

C.11

$$i_{rq} = \frac{V\,k_4\,(\omega - \dot{\theta})}{\sigma\, L_s\, \sqrt{b_1\, c_1}}\ \sin\,(\omega t + \gamma - \phi_1 - \phi_2) +$$

$$e^{-\alpha_1 t}\left[\frac{V\,k_4}{\sigma\, L_s\, \sqrt{a_1\, b_1}}[\alpha_1\, \cos\,(\beta_1 t + \gamma - \phi_1 - \phi_3) - \beta_2\, \sin\,(\beta_1 t + \gamma - \phi_1 - \phi_3)]\right.$$

$$\left. + \frac{[A_{77}\,(\alpha_2 - \alpha_1) + A_{88}\,(\beta_1 - \beta_2)]\ \sqrt{2}}{\sigma\, L_r\, L_s\,(\,(\alpha_2 - \alpha_1)^2 + (\beta_1 - \beta_2)^2)}\right]$$

$$e^{-\alpha_2 t}\left[\frac{-V\,k_4}{\sigma\, L_s\, \sqrt{a_1\, c_1}}\ [\alpha_2 \cos\,(\beta_2 t + \gamma - \phi_2 - \phi_3) - \beta_1 \sin\,(\beta_2 t + \gamma - \phi_2 - \phi_3)]\right.$$

$$\left. + \sqrt{2}\ \frac{[A_{111}\,(\alpha_1 - \alpha_2) + A_{112}\,(\beta_2 - \beta_1)]}{\sigma\, L_r\, L_s\,(\,(\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2)}\right]$$

C.12

where

$$a_1 = (\alpha_1 - \alpha_2)^2 + (\beta_1 - \beta_2)^2$$

$$b_1 = \alpha_1^{\,2} + (\omega - \beta_1)^2$$

$$c_1 = \alpha_2^{\,2} + (\omega - \beta_2)^2$$

$$\phi_1 = \tan^{-1} \frac{\omega - \beta_1}{\alpha_1}$$

$$\phi_2 = \tan^{-1} \frac{\beta_2 - \beta_1}{\alpha_1 - \alpha_2}$$

If the expressions for the currents given by Equations C.9 to C.12 are substituted in Equation 6.9 for the torque, the components of the resulting expression are found as

i) $A_1 e^{-2\alpha_1 t}$

ii) $A_2 e^{-2\alpha_2 t}$

iii) $A_3 e^{-(\alpha_1 - \alpha_2)t} \sin [(\beta_1 - \beta_2)t + \psi_1]$

iv) $A_4 e^{-\alpha_1 t} \sin [(\omega - \beta_1)t + \psi_2]$

v) $A_5 e^{-\alpha_2 t} \sin [(\omega - \beta_2)t + \psi_3]$

vi) Steady-state component

C.2 Analytical solution for current and voltage following disconnection

The differential equations of the currents during disconnection are

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_r + L_r D & L_r \dot{\theta} \\ -L_r \dot{\theta} & R_r + L_r D \end{bmatrix} \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} \qquad \text{C.13}$$

The Laplace transform of Equation C.13 yields,

$$0 = (R_r + S\,L_r)\, I_{r_d} + \dot{\theta}\, L_r\, I_{r_q} - i_{r_d}(0)\, L_r \qquad \text{C.14}$$

$$0 = -L_r\, \dot{\theta}\, I_{r_d} + (R_r + S\,L_r)\, I_{r_q} - i_{r_q}(0)\, L_r \qquad \text{C.15}$$

Equations C.14 and C.15 can be rewritten as,

$$\begin{bmatrix} i_{r_d}(0)\, L_r \\ i_{r_q}(0)\, L_r \end{bmatrix} = \begin{bmatrix} R_r + S\,L_r & \dot{\theta}\, L_r \\ -L_r \dot{\theta} & R_r + S\,L_r \end{bmatrix} \begin{bmatrix} I_{r_d} \\ I_{r_q} \end{bmatrix}$$

and consequently

$$I_{r_d} = \frac{(R_r + S\,L_r)\,L_r\,i_{r_d}(0) - \dot{\theta}\,L_r^{2}\,i_{r_q}(0)}{[(R_r + S\,L_r)^2 + (\dot{\theta}\,L_r)^2]}$$

and

$$I_{r_q} = \frac{\dot{\theta}\,L_r^{2} i_{r_d}(0) + (R_r + S\,L_r)\,L_r\,i_{r_q}(0)}{[(R_r + S\,L_r)^2 + (\dot{\theta}\,L_r)^2]}$$

Therefore,

$$I_{r_d} = \frac{[R_r\,L_r\,i_{r_d}(0) - \theta\,L_r^{2}\,i_{r_q}(0)] + S\,L_r^{2}\,i_{r_d}(0)}{L_r^{2}\quad (\frac{R_r}{L_r} + S)^2 + \dot{\theta}^2 -}$$

i.e.

$$I_{r_d} = \frac{R_r\,i_{r_d}(0) - \dot{\theta}\,L_r\,i_{r_q}(0) + S\,L_r\,i_{r_d}(0)}{L_r\,[(\frac{R_r}{L_r} + S)^2 + \dot{\theta}^2\,L_r]} \qquad \text{C.16}$$

and

$$I_{r_q} = \frac{R_r\,i_{r_q}(0) + \dot{\theta}\,L_r\,i_{r_d}(0) + S\,L_r\,i_{r_q}(0)}{L_r\,[(\frac{R_r}{L_r} + S)^2 + \dot{\theta}^2]} \qquad \text{C.17}$$

By taking the inverse transforms of Equations C.16 and C.17 we obtain

$$i_{r_d} = \hat{i} \sin (\theta t + \psi) e^{-t/\tau_0} \qquad \text{C.18}$$

and

$$i_{r_q} = \hat{i} \cos (\theta t + \psi) e^{-t/\tau_0} \qquad \text{C.19}$$

where

$$\hat{i} = \sqrt{i_{r_d}^{2}(0) + i_{r_q}^{2}(0)}$$

$$\psi = \tan^{-1} \frac{i_{r_q}(0)}{i_{r_d}(0)}$$

Substituting for i_{r_d} and i_{r_q} from Equations C.18 and C.19 into the equations of v_{s_d} and v_{s_q} we obtain

$$v_{s_d} = M D i_{r_d}$$

$$= [M \dot{\theta} \hat{i} \cos (\dot{\theta} t + \psi) - \frac{M \hat{i}}{\tau_0} \sin (\dot{\theta} t + \psi)] e^{-t/\tau_0}$$

and

$$v_{s_q} = M D i_{r_q}$$

$$= [- M \dot{\theta} \hat{i} \sin (\dot{\theta} t + \psi) - \frac{M \hat{i}}{\tau_0} \cos (\dot{\theta} t + \psi)] e^{-t/\tau_0}$$

or in alternative form as

$$v_{s_d} = \hat{v} \sin(\dot{\theta} t + \phi) e^{-t/\tau_0}$$

$$v_{s_q} = \hat{v} \cos(\dot{\theta} t + \phi) e^{-t/\tau_0}$$

where

$$\hat{v} = \sqrt{(M \dot{\theta} \hat{i})^2 + (\frac{M \hat{i}}{\tau_0})^2}$$

and $\phi = \tan^{-1} (\frac{-\dot{\theta} L_r}{R_r})$

APPENDIX D

COMPUTER PROGRAM LISTINGS FOR A SINGLE MACHINE

```
      MASTER TOTAL TRANSIENT IN SINGLE MOTOR
      DIMENSION AL(4,4),G(4,4),R(4,4),TRM(4,4)
      DIMENSION TC(4,4),GI(4,1),RI(4,1),CI(4,1)
      DIMENSION DI(4,1),V(4,1),VER(4,4),DIF(4,1)
      DIMENSION D(4,4),AK1(4),AK2(4),AK3(4),AK4(4)
      DIMENSION RI1(2,1)
      DIMENSION DIF1(2,1),R22(2,2),GI1(2,1)
      DIMENSION AL12(2,2),V1(2,1),CI2(2,1),VER1(2,2)
      DIMENSION G22(2,2),AL22(2,2),DI1(2,1),D1(2,2)
      DIMENSION AKI1(2),AKI2(2),AKI3(2),AKI4(2)
C     *** READS IN PARAMETERS ***
  904 READ(5,1)((R(I,J),I=1,4),J=1,4)
      READ(5,1)((AL(I,J),I=1,4),J=1,4)
      READ(5,1)((G(I,J),I=1,4),J=1,4)
    1 FORMAT(4F0.0)
      READ(5,10)((AL12(I,J),I=1,2),J=1,2)
      READ(5,10)((R22(I,J),I=1,2),J=1,2)
      READ(5,10)((G22(I,J),I=1,2),J=1,2)
      READ(5,10)((AL22(I,J),I=1,2),J=1,2)
   10 FORMAT(2F0.0)
      READ(5,905)AJ1,PP,A11,A22,A33
      READ(5,905)DEL,H,DUR,VM,VM1
  905 FORMAT(5F0.0)
      READ(5,902)NCC,NCC1,NCC2
  902 FORMAT(3I0)
      READ(5,907)EPS1,EPS2,EPS3
  907 FORMAT(3F0.0)
      PI=4.0*ATAN(1.0)
      TOP=2.0*PI
      TOPI=2.0*PI/3.0
      FOPI=2.0*TOPI
      R1=A11+A22*(DTHETA/314.2)**2
      NC=0
      DO 11 I=1,4
   11 CI(I,1)=0.0
C     *** INVERSION OF MATRIX L ***
      CALL MB02A(AL,VER,DETER,4,3,4)
   12 CONTINUE
      L=0
      VA=VM*SIN(100.0*PI*DUR+DEL)
      VB=VM*SIN(100.0*PI*DUR-TOPI+DEL)
      VC=VM*SIN(100.0*PI*DUR-FOPI+DEL)
      DUR=DUR+H
C     *** TRANSFORMATION FROM THREE PHASE TO D-Q AXES ***
   48 VD=SQRT(2.0/3.0)*(VA-0.5*VB-0.5*VC)
      VQ=SQRT(2.0/3.0)*((SQRT(3.0)/2.0)*VB-(SQRT(3.0)/2.0)*VC)
```

```
C      *** INITIAL CONDITIONS OF VOLTAGES ***
       V(1,1)=VD
       V(2,1)=VQ
       V(3,1)=0.0
       V(4,1)=0.0
C      *** GENERATE MATRICES FOR R*I,G*I AND T ***
    13 CALL MZ01B(RI,R,CI,4,4,1,4,4,4)
       CALL MZ01B(GI,G,CI,4,4,1,4,4,4)
       CALL MZ02B(TRM,CI,GI,1,4,1,1,4,4)
       T=PP*TRM(1,1)
       DO 14 K=1,4
    14 DIF(K,1)=V(K,1)-RI(K,1)-DTHETA*GI(K,1)
C      *** EVALUATE THE RATES OF CHANGE OF CURRENTS ***
       CALL MZ01B(DI,VEP,DIF,4,4,1,4,4,4)
C      *** COMMENCE A RUNGE-KUTTA CYCLE OF NUMERICAL INTEGRATION ***
       L=L+1
       GO TO (15,25,35,45),L
    15 DO 16 I=1,4
    16 AK1(I)=DI(I,1)*H
C      *** TRANSFORMATION FROM D-Q TO THREE PHASE CURRENTS ***
       CIRA=SQRT(2.0/3.0)*(CI(3,1)*COS(THETA)+CI(4,1)*SIN(THETA))
       CIRB=SQRT(2.0/3.0)*(CI(3,1)*COS(THETA+TOPI)+CI(4,1)*SIN(THETA+
      1TOPI))
       CIRC=SQRT(2.0/3.0)*(CI(3,1)*COS(THETA+FOPI)+CI(4,1)*SIN(THETA+
      1FOPI))
       CISA=SQRT(2.0/3.0)*(CI(1,1))
       CISB=SQRT(2.0/3.0)*(-0.5*CI(1,1)+SQRT(3.0)*CI(2,1)/2.0)
       CISC=SQRT(2.0/3.0)*(-0.5*CI(1,1)-SQRT(3.0)*CI(2,1)/2.0)
       PI=VA*CISA+VB*CISB+VC*CISC
       WRITE(6,888)VA,VB,VC,CISA,CISB,CISC
   888 FORMAT(1X,5X,3HVA=,F7.2,3X,3HVB=,F7.2,3X,3HVC=,F7.2,3X,3HIA=,F7.2,
      13X,3HIB=,F7.2,3X,3HIC=,F7.2)
       WRITE(6,777)DUR,T,DTHETA,THETA
   777 FORMAT(1X,5X,5HTIME=,F8.6,3X,2HT=,F7.2,3X,2HN=,F7.2,3X,6HANGLE=,F
      16.3)
       DTHET1=(T-R1)*PP*H/AJ1
       THET1=DTHETA*H
       DO 18 I=1,4
    18 CI(I,1)=CI(I,1)+0.5*AK1(I)
       DTHETA=DTHETA+0.5*DTHET1
       THETA=THETA+0.5*THET1
       GO TO 13
    25 DO 19 I=1,4
    19 AK2(I)=DI(I,1)*H
       DTHET2=(T-R1)*PP*H/AJ1
       THET2=DTHETA*H
       DO 20 I=1,4
```

```
   20 CI(I,1)=CI(I,1)-0.5*(AK1(I)-AK2(I))
      DTHETA=DTHETA-0.5*(DTHET1-DTHET2)
      THETA=THETA-0.5*(THET1-THET2)
      GO TO 13
   35 DO 21 I=1,4
   21 AK3(I)=DI(I,1)*H
      DTHET3=(T-R1)*PP*H/AJ1
      THET3=DTHETA*H
      DO 22 I=1,4
   22 CI(I,1)=CI(I,1)+AK3(I)-0.5*AK2(I)
      DTHETA=DTHETA-0.5*DTHET2+DTHET3
      THETA=THETA-0.5*THET2+THET3
      GO TO 13
   45 DO 23 I=1,4
   23 AK4(I)=DI(I,1)*H
      DTHET4=(T-R1)*PP*H/AJ1
      THET4=DTHETA*H
      DO 24 I=1,4
   24 CI(I,1)=CI(I,1)-AK3(I)+(AK1(I)+2.0*(AK2(I)+AK3(I))+AK4(I))/6.0
      DTHETA=DTHETA+(DTHET1+2.0*(DTHET2+DTHET3)+DTHET4)/6.0-DTHET3
      R1=A11+A22*(DTHETA/314.2)**2
      THETA=THETA+(THET1+2.0*(THET2+THET3)+THET4)/6.0-THET3
      NC=NC+1
      IF(THETA.LT.TOP) GO TO 26
      THETA=THETA-TOP
   26 CONTINUE
C     *** HAVE ENOUGH ITERATION BEEN PERFORMED? ***
      IF(NC.LT.NCC)GO TO 12
      IF(NC.LT.(NCC+NCC1)) GO TO 100
      IF(NC.LT.(NCC+NCC1+NCC2)) GO TO     49
  100 WRITE(6,27)
   27 FORMAT(//52H D   I   S   C   O   N . N   E   C   T   I   O   N//1
      CI2(1,1)=CI(3,1)+A33*CI(1,1)
      CI2(2,1)=CI(4,1)+A33*CI(2,1)
C     *** INVERSION OF MATRIX L22 ***
      CALL MB02A(AL22,VER1,DETER1,2,3,2)
   28 CONTINUE
      L=0
C     *** GENERATE MATRICES FOR R*I AND G*I ***
   29 CALL MZ01B(RI1,R22,CI2,2,2,1,2,2,2)
      CALL MZ01B(GI1,G22,CI2,2,2,1,2,2,2)
      DO 30 K=1,2
   30 DIF1(K,1)=0.0-RI1(K,1)-DTHETA*GI1(K,1)
C     *** EVALUATE THE RATES OF CHANGE OF CURRENTS ***
      CALL MZ01B(DI1,VER1,DIF1,2,2,1,2,2,2)
C     ***COMMENCE A RUNGE-KUTTA CYCLE OF NUMERICAL INTEGRATION *****
      L=L+1
      GO TO (31,41,51,61),L
   31 DO 32 I=1,2
   32 AK1(I)=DI1(I,1)*H
```

```
      CRA=SQRT(2.0/3.0)*(CI2(1,1)*COS(THETA)+CI2(2,1)*SIN(THETA))
      CRB=SQRT(2.0/3.0)*(CI2(1,1)*COS(THETA+TOPI)+CI2(2,1)*SIN(THETA+
     1TOPI))
      CRC=SQRT(2.0/3.0)*(CI2(1,1)*COS(THETA+FOPI)+CI2(2,1)*SIN(THETA+
     1FOPI))
      WRITE(6,1243)CRA,CRB,CRC,DTHETA,THETA
 1243 FORMAT(5F20.3)
      DTHET1=(-R1)*PP*H/AJ1
      THET1=DTHETA*H
      DO 33 I=1,2
   33 CI2(I,1)=CI2(I,1)+0.5*AKI1(I)
      DTHETA=DTHETA+0.5*DTHET1
      THETA=THETA+0.5*THET1
      GO TO 29
   41 DO 34 I=1,2
   34 AKI2(I)=DI1(I,1)*H
      DTHET2=(-R1)*PP*H/AJ1
      THET2=DTHETA*H
      DO 36 I=1,2
   36 CI2(I,1)=CI2(I,1)-0.5*(AKI1(I)-AKI2(I))
      DTHETA=DTHETA-0.5*(DTHET1-DTHET2)
      THETA=THETA-0.5*(THET1-THET2)
      GO TO 29
   51 DO 37 I=1,2
   37 AKI3(I)=DI1(I,1)*H
      DTHET3=(-R1)*PP*H/AJ1
      THET3=DTHETA*H
      DO 38 I=1,2
   38 CI2(I,1)=CI2(I,1)+AKI3(I)-0.5*AKI2(I)
      DTHETA=DTHETA-0.5*DTHET2+DTHET3
      THETA=THETA-0.5*THET2+THET3
      GO TO 29
   61 DO 39 I=1,2
   39 AKI4(I)=DI1(I,1)*H
      DTHET4=(-R1)*PP*H/AJ1
      THET4=DTHETA*H
      DO 40 I=1,2
   40 CI2(I,1)=CI2(I,1)-AKI3(I)+(AKI1(I)+2.0*(AKI2(I)+AKI3(I))+
     1AKI4(I))/6.0
      DTHETA=DTHETA+(DTHET1+2.0*(DTHET2+DTHET3)+DTHET4)/6.0-DTHET3
      R1=A11+A22*(DTHETA/314.2)**2
      THETA=THETA+(THET1+2.0*(THET2+THET3)+THET4)/6.0-THET3
C     *** DETERMINATION OF STATOR VOLTAGES ***
      CALL MZ01B(V1,A,I2,DI1,2,2,1,2,2,2)
      VSA=SQRT(2.0/3.0)*(V1(1,1))
      VSB=SQRT(2.0/3.0)*(-0.5*V1(1,1)+SQRT(3.0)*V1(2,1)/2.0)
      VSC=SQRT(2.0/3.0)*(-0.5*V1(1,1)-SQRT(3.0)*V1(2,1)/2.0)
      WRITE(6,1523)VSA,VSB,VSC
 1523 FORMAT(3F20.2)
      R1=0.206+0.382*(DTHETA/314.2)**2
```

```
      IF(THETA.LT.TOP) GO TO 2626
      THETA=THETA-TOP
 2626 CONTINUE
      DUR=DUR+H
      NC=NC+1
C     *** HAVE ENOUGH ITERATION BEEN PERFORMED?****
      IF(NC.LT.(NCC+NCC1)) GO TO 28
      WRITE(6,47)
   47 FORMAT(//42H R    E    S    W    I    T    C    H    I    N    G)
      CI(1,1)=0.0
      CI(2,1)=0.0
      CI(3,1)=CI2(1,1)
      CI(4,1)=CI2(2,1)
   49 CONTINUE
      L=0
      VA=VM1*SIN(100.0*PI*DUR+EPS1)
      VB=VM1*SIN(100.0*PI*DUR+EPS2)
      VC=VM1*SIN(100.0*PI*DUR+EPS3)
      DUR=DUR+H
      IF(NC.LT.(NCC+NCC1+NCC2)) GO TO 48
      STOP
      END
      FINISH
      ****
```

APPENDIX E

ANALYTICAL SOLUTION FOLLOWING DISCONNECTION OF A GROUP OF TWO MOTORS

The electrical equations of the two machines following disconnection from the common supply are,

$$M\,(D - j\,\dot{\theta}_1)\; i_{sp} + [R_{r_1} + L_{r_1}\,(D - j\,\dot{\theta}_1)]\;(i_{rp_1}\, e^{j\theta_1}) = 0$$

$$-M_2(D - j\,\dot{\theta}_2)\; i_{sp} + [R_{r_2} + L_{r_2}\;(D - j\,\dot{\theta}_2)]\;(i_{rp_2}\; e^{j\theta_2}) = 0$$

$$[(R_{s_1} + R_{s_2}) + (L_{s_1} + L_{s_2})D]\; i_{sp} + M_1\;\; D\,(i_{rp_1}\; e^{j\theta_1}) - M_2\; D\,(i_{rp_2} e^{j\theta_2}) = 0$$

Taking Laplace transform we obtain

$$\begin{bmatrix} M_1\;(S - j\,\dot{\theta}_1) & R_{r_1} + L_{r_1}(S - j\,\dot{\theta}_1) & 0 \\ -M_2\;(S - j\,\dot{\theta}_2) & 0 & R_{r_2} + L_{r_2}(S - j\,\dot{\theta}_2) \\ (R_{s_1} + R_{s_2}) + (L_{s_1} + L_{s_2})S & M_1 S & -M_2\; S \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$= \begin{bmatrix} M_1 i_{sp_0} + L_{r_1} i_{r_{10}} \\ \\ -M_2 i_{sp_0} + L_{r_2} i_{r_{20}} \\ \\ (L_{s_1} + L_{s_2}) i_{sp_0} + M_1 i_{r_{10}} - M_2 i_{r_{20}} \end{bmatrix} \qquad \text{E.1}$$

where I_1, I_2 and I_3 are the Laplace transforms of i_{sp}, $i_{rp_1} e^{j\theta_1}$ and $i_{rp_2} e^{j\theta_2}$ respectively, and i_{sp_0}, $i_{r_{10}}$ and $i_{r_{20}}$ are the initial values of the stator current, first rotor current and the second rotor current.

The inverse Laplace transform of Equation E.1 yields,

$$\begin{bmatrix} I_1 \\ \\ I_2 \\ \\ I_3 \end{bmatrix} = \frac{1}{|Z|} \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ \\ Z_{21} & Z_{22} & Z_{21} \\ \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} M_1 i_{rp_{10}} + L_{r_1} i_{r_{10}} \\ \\ -M_2 i_{rp_{10}} + L_{r_2} i_{r_{20}} \\ \\ (L_{s_1} + L_{s_2}) i_{10} + M_1 i_{r_{10}} - M_2 i_{r_{20}} \end{bmatrix}$$

E.2

where

$$Z_{11} = S^2\,(-M_1 L_{r_2}) - S M_1 (R_{r_2} - j L_{r_2} \dot{\theta}_2)$$

$$Z_{12} = S^2\,(M_2 L_{r_1}) + S M_2 (R_{r_1} - j L_{r_1} \dot{\theta}_1)$$

$$Z_{13} = S^2\,(L_{r_1} L_{r_2}) + S L_{r_2} (R_{r_1} - j L_{r_1} \dot{\theta}_1) + L_{r_1} (R_{r_2} - j L_{r_2} \dot{\theta}_2)$$

$$+ (R_{r_1} - j L_{r_1} \dot{\theta}_1)(R_{r_2} - j L_{r_2} \dot{\theta}_2)$$

$$Z_{21} = S^2[L_{r_2} (L_{s_1} + L_{s_2}) - M_2{}^2] + S[(L_{s_1} + L_{s_2})(R_{r_2} - j L_{r_2} \dot{\theta}_2)$$

$$+ L_{r_2} (R_{s_1} + R_{s_2}) + j M_2{}^2 \dot{\theta}_2] + (R_{r_2} - j L_{r_2} \dot{\theta}_2)(R_{s_1} + R_{s_2})$$

$$Z_{22} = S^2\,(-M_1 M_2) + S\,(j M_1 M_2 \dot{\theta}_1)$$

$$Z_{23} = S^2\,(-M_1 L_{r_2}) + S\,[-M_1 (R_{r_2} - j L_{r_2} \dot{\theta}_2) + j M_1 \dot{\theta}_1 L_{r_2}]$$

$$+ j M_1 \dot{\theta}_1 (R_{r_2} - j L_{r_2} \dot{\theta}_2)$$

$$Z_{31} = S^2\,(L_{r_1} L_{r_2}) + S\,[L_{r_2} (R_{r_1} - j L_{r_1} \dot{\theta}_1) + L_{r_1} (R_{r_2} - j L_{r_2} \dot{\theta}_2)]$$

$$+ (R_{r_1} - j L_{r_1} \dot{\theta}_1)(R_{r_2} - j L_{r_2} \dot{\theta}_2)$$

$$Z_{32} = S^2\,(-M_1 L_{r_2}) + S\,[j L_{r_2} M_1 \dot{\theta}_1 - M_1 (R_{r_2} - j L_{r_2} \dot{\theta}_2)]$$

$$+ j M_1 \dot{\theta}_1 [R_{r_2} - j L_{r_2} \dot{\theta}_2]$$

$$Z_{33} = S^2 (M_2 L_{r_1}) + S[M_2 (R_{r_1} - j L_{r_1} \dot{\theta}_1) - j M_2 \dot{\theta}_2 L_{r_1}]$$

$$+ [R_{r_1} - j L_{r_1} \dot{\theta}_1] [- j M_2 \dot{\theta}_2]$$

and $|Z| = (S - \lambda_1)(S - \lambda_2)(S - \lambda_3)$.

In the equation for $|Z|$, λ_1, λ_2 and λ_3 are the roots of the characteristic equation

$$\lambda^3 + (\alpha - j \beta) \lambda^2 + (\gamma - j \eta) \lambda + (\xi_1 - j \xi_2) = 0$$

in which

$$\alpha = \frac{R_{r_2} L_{r_2} (\sigma_1 L_{s_1} + L_{s_2}) + R_{r_1} L_{r_2} (L_{s_1} + \sigma_2 L_{s_2}) + L_{r_1} L_{r_2} (R_{s_1} + R_{s_2})}{L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})}$$

$$\beta = \dot{\theta}_1 + \dot{\theta}_2$$

$$\gamma = \frac{R_{r_1} R_{r_2} (L_{s_1} + L_{s_2}) + (R_{s_1} + R_{s_2})(L_{r_1} R_{r_2} + L_{r_2} R_{r_1})}{L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})} - \dot{\theta}_1 \dot{\theta}_2$$

$$\eta = \frac{L_{r_1} R_{r_2} \dot{\theta}_1 (\sigma_1 L_{s_1} + L_{s_2}) + R_{r_1} L_{r_2} \dot{\theta}_2 (L_{s_1} + \sigma_2 L_{s_2}) + (\dot{\theta}_1 + \dot{\theta}_2) L_{r_1} L_{r_2} (R_{s_1} + R_{s_2})}{L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})}$$

$$\xi_1 = \frac{(R_{s_1} + R_{s_2})(R_{r_1} R_{r_2} - L_{r_1} L_{r_2} \dot{\theta}_1 \dot{\theta}_2)}{L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})}$$

$$\xi_2 = \frac{(R_{s_1} + R_{s_2})(\dot{\theta}_1 L_{r_1} R_{r_2} + \dot{\theta}_2 L_{r_2} R_{r_1})}{L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})}$$

$$\sigma_1 = 1 - M_1^{\,2}/L_{r_1} L_{s_1}$$

$$\sigma_2 = 1 - M_2^{\,2}/L_{r_2} L_{s_2}$$

From equation E.2

$$I_1 = \{Z_{11} (M_1 i_{sp_0} + L_{r_1} i_{r_{10}}) + Z_{12} (- M_2 i_{sp_0} + L_{r_2} i_{r_{20}})$$

$$+ Z_{13} [(L_{s_1} + L_{s_2}) i_{sp_0} + M_1 i_{r_{10}} - M_2 i_{r_{20}}] \}/[(S - \lambda_1)(S - \lambda_2)(S - \lambda_3)]$$

E.3

$$I_2 = Z_{21} (M_1 i_{sp_0} + L_{r_1} i_{r_{10}}) + Z_{22} (- M_2 i_{sp_0} + L_{r_2} i_{r_{20}}) +$$

$$+ Z_{23} [(L_{s_1} + L_{s_2}) i_{sp_0} + M_1 i_{r_{10}} - M_2 i_{r_{20}}] \} / (S - \lambda_1)(S - \lambda_2)(S - \lambda_3)$$

E.4

$$I_3 = \{Z_{31} (M_1 i_{sp_0} + L_{r_1} i_{r_{10}}) + Z_{32} (- M_2 i_{sp_0} + L_{r_2} i_{r_{20}})$$

$$+ Z_{33} [(L_{s_1} + L_{s_2}) i_{sp_0} + M_1 i_{r_{10}} - M_2 i_{r_{20}}]\} / [(S - \lambda_1)(S - \lambda_2)(S - \lambda_3)]$$

E.5

The inverse Laplace transforms of E.3, E.4 and E.5 give

$$i_{sp} = \frac{e^{\lambda_2 t}}{(\lambda_1 - \lambda_2)(\lambda_2 - \lambda_3)} \quad [\lambda_2^2 i_{10} + \lambda_2 (a - jb) + (c - jd)]$$

$$+ \frac{e^{\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \quad [\lambda_2^2 i_{10} + \lambda_2 (a - jb) + (c - jd)]$$

$$+ \frac{e^{\lambda_2 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \quad [\lambda_3^2 i_{10} + \lambda_3 (a - jb) + (c - jd)]$$

and

$$i_{rp} = \frac{\lambda_1^2 \, i_{20} + (a_1 - j\,b_1)\,\lambda_1 + (c_1 - j\,d_1)}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_2)} \, e^{(\lambda_1 - \dot{\theta}_1)t}$$

$$+ \frac{e^{(\lambda_2 - \dot{\theta}_1)t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \left[\lambda_2^2 \, i_{20} + (a_1 - j\,b_1)\,\lambda_2 + (a_1 - j\,d_1)\right]$$

$$+ \frac{e^{(\lambda_3 - \dot{\theta}_1)t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \left[\lambda_3^2 \, i_{20} + (a_1 - j\,b_1)\,\lambda_3 + (c_1 - j\,d_1)\right]$$

and

$$i_{rp} = \frac{e^{\lambda_1 t}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \left[i_{r_{30}} \, \lambda_1^2 + (a_2 - j\,b_2)\,\lambda_1 + (c_2 - j\,d_2)\right]$$

$$+ \frac{e^{\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \left[i_{r_{30}} \, \lambda_2^2 + (a_2 - j\,b_2)\,\lambda_2 + (c_2 - j\,d_2)\right]$$

$$+ \frac{e^{\lambda_3 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \left[i_{r_{30}} \, \lambda_3^2 + (a_2 - j\,b_2)\,\lambda_3 + (c_2 - j\,d_2)\right]$$

where

$$a=\frac{[L_{r_1}R_{r_2}(\sigma_1 L_{s_1}+L_{s_2})+L_{r_2}R_{r_1}(L_{s_1}+\sigma_2 L_{s_2})]\,i_{10}+M_1 L_{r_2}R_{r_1}i_{20}-M_2 L_{r_1}R_{r_2}i_{30}}{L_{r_1}L_{r_2}(\sigma_1 L_{s_1}+\sigma_2 L_{s_2})}$$

$$b=\frac{[\dot\theta_1(L_{s_1}+\sigma_2 L_{s_2})+\dot\theta_2(L_{s_2}+\sigma_1 L_{s_1})]\,i_{10}+M_1\dot\theta_1 i_{20}-M_2\dot\theta_2 i_{30}}{\sigma_1 L_{s_1}+\sigma_2 L_{s_2}}$$

$$c=\frac{(R_{r_1}R_{r_2}-L_{r_1}L_{r_2}\dot\theta_1\dot\theta_2)\,[(L_{s_1}+L_{s_2})\,i_{10}+M_1 i_{20}-M_2 i_{30}]}{L_{r_1}L_{r_2}(\sigma_1 L_{s_1}+\sigma_2 L_{s_2})}$$

$$d=\frac{(R_{r_2}L_{r_1}\dot\theta_1+R_{r_1}L_{r_2}\dot\theta_2)\,[(L_{s_1}+L_{s_2})\,i_{10}+M_1 i_{20}-M_2 i_{30}]}{L_{r_1}L_{r_2}(\sigma_1 L_{s_1}+\sigma_2 L_{s_2})}$$

and

$$a_1=[(M_1 L_{r_2}R_{s_1}+M_1 L_{r_2}R_{s_2})\,i_{10}+(L_{r_1}L_{s_1}R_{r_2}+L_{s_2}R_{r_2}L_{r_1}+L_{r_1}L_{r_2}R_{s_1}$$

$$+L_{r_1}L_{r_2}R_{s_2}-M_1^2 R_{r_2})\,i_{20}+M_1 M_2 R_{r_2}i_{30}]\,/[L_{r_1}L_{r_2}(\sigma_1 L_{s_1}+\sigma_2 L_{s_2})]$$

$$b_1 = (M_1 M_2^2 \dot{\theta}_1 - M_1 M_2^2 \dot{\theta}_2 - M_1 L_{r_2} L_{s_1} \dot{\theta}_1 - M_1 L_{r_2} L_{s_2} \dot{\theta}_1)\, i_{10}$$

$$+ (L_{r_1} L_{r_2} L_{s_2} \dot{\theta}_2 + L_{r_1} L_{r_2} L_{s_1} \dot{\theta}_2 - M_2^2 L_{r_1} \dot{\theta}_2 - M_1^2 \dot{\theta}_1 L_{r_2})\, i_{20}$$

$$+ (M_1 M_2 L_{r_2} \theta_2)\, i_{30}] / [L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})]$$

$$c_1 = \{[R_{r_2} M_1 (R_{s_1} + R_{s_2}) + M_1 L_{r_2} \dot{\theta}_1 \dot{\theta}_2 (L_{s_1} + L_{s_2})]\, i_{10}$$

$$+ [R_{r_2} (R_{s_1} + R_{s_2}) L_{r_1} + M_1^2 L_{r_2} \dot{\theta}_1 \dot{\theta}_2]\, i_{20} - M_1 M_2 L_{r_2} \dot{\theta}_1 \dot{\theta}_2\, i_{30}\} /$$

$$[L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})]$$

$$d_1 = \{[L_{r_2} \dot{\theta}_2 M_1 (R_{s_1} + R_{s_2}) - M_1 \dot{\theta}_1 R_{r_2} (L_{s_1} + L_{s_2})]\, i_{10}$$

$$+ [L_{r_1} L_{r_2} (R_{s_1} + R_{s_2}) \dot{\theta}_2 - M_1^2 \dot{\theta}_1 R_{r_2}]\, i_{20}$$

$$+ M_1 M_2 R_{r_2} \dot{\theta}_1\, i_{30}\} / [L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})]$$

and

$$a_2 = \{[R_{r_1} L_{r_2} M_1 + L_{r_1} R_{r_2} M_1 + M_1 M_2 R_{r_2} + M_2 R_{r_1} (L_{s_1} + L_{s_2})] i_{10}$$

$$+ [L_{r_1} (R_{r_1} L_{r_2} + L_{r_1} R_{r_2}) + M_2 M_1 R_{r_1}] i_{20}$$

$$+ - M_1 R_{r_2} L_{r_2} - M_2^2 R_{r_1} \quad i_{30}\} / [L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})]$$

$$b_2 = \{[M_1 L_{r_1} L_{r_2} (\dot\theta_1 + \dot\theta_2) + M_1 M_2 L_{r_2} (\dot\theta_1 + \dot\theta_2) + M_2 L_{r_1} (\dot\theta_1 + \dot\theta_2)(L_{s_1} + L_{s_2})] i_{1}$$

$$+ [L_{r_1}^2 L_{r_2} (\dot\theta_1 + \dot\theta_2) + M_1 M_2 L_{r_1} (\dot\theta_1 + \dot\theta_2)] i_{20} + [M_2^2 L_{r_1} - L_{r_2}^2 M_1](\dot\theta_1 + \dot\theta_2) i_{30}\} /$$

$$[L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})]$$

$$c_2 = \{[M_1 (R_{r_1} R_{r_2} - L_{r_1} L_{r_2} \dot\theta_1 \dot\theta_2) - M_1 M_2 L_{r_2} \dot\theta_1 \dot\theta_2 - M_2 L_{r_1} \dot\theta_1 \dot\theta_2 (L_{s_1} + L_{s_2}] i_{10}$$

$$+ [L_{r_1} (R_{r_1} R_{r_2} - L_{r_1} L_{r_2} \dot\theta_1 \dot\theta_2) - M_1 M_2 L_{r_1} \dot\theta_1 \dot\theta_2] i_{20}$$

$$+ i_{30} [M_1 L_{r_2}^2 \dot\theta_1 \dot\theta_2 + M_2^2 L_{r_1} \dot\theta_1 \dot\theta_2]\} / [L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})]$$

$$d_2 = \{[M_1 L_{r_1} R_{r_2} (\dot{\theta}_1 + \dot{\theta}_2) + M_1 M_2 R_{r_2} \dot{\theta}_1 + M_2 R_{r_1} \dot{\theta}_2 (L_{s_1} + L_{s_2})] i_{10}$$

$$+ [L_{r_1}^2 R_{r_2} (\dot{\theta}_1 + \dot{\theta}_2) + M_1 M_2 R_{r_1} \dot{\theta}_2] i_{20}$$

$$+ [- L_{r_2} M_1 R_{r_2} \dot{\theta}_1 - M_2^2 R_{r_1} \dot{\theta}_2] \} / [L_{r_1} L_{r_2} (\sigma_1 L_{s_1} + \sigma_2 L_{s_2})]$$

The positive-sequence component of the common stator voltage is obtained from

$$v_p = R_{s_1} i_{sp} + L_{s_1} \frac{d\, i_{sp}}{dt} + M_1 \frac{d\,(i_{rp_1} e^{j\theta_1})}{dt}$$

or

$$v_p = \frac{e^{\lambda_1 t}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \{ \lambda_1^3 (i_{10} L_{s_1} + i_{20} M_1) + \ldots$$

$$+ \lambda_1^2 [i_{10} R_{s_1} + L_{s_1} (a - jb) + M_1 (a - jb_1)]$$

$$+ \lambda_1 [(a - jb) R_{s_1} + L_{s_1} (c - jd) + M_1 (c_1 - jd_1)] + [(c - jd) R_{s_1}]\} +$$

$$\frac{e^{\lambda_2 t}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \{ \lambda_2^3 (i_{10} L_{s_1} + i_{20} M_1) + \lambda_2^2 [R_{s_1} i_{10} + L_{s_1} (a-jb) +$$

$$+ M_1 (a - jb_1)]$$

$$+ \lambda_1 [(a - jb) R_{s_1} + L_{s_1} (c - jd) + M_1 (c_1 - j d_1)] + (c - jd) R_{s_1} \}$$

$$+ \frac{e^{\lambda_3 t}}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \{\lambda_3^3 i_{10} L_{s_1} + M_1 i_{20}) + \lambda_3^2 [R_{s_1} i_{10} + L_{s_1} (a-jb) + M_1 (a_1 - j b_1)]$$

$$+ \lambda_3 [(a - jb) R_{s_1} + (c - jd) L_{s_1} + M_1 (c_1 - j d_1)] + [(c - jd) R_{s_1}]\}$$

APPENDIX F

COMPUTER PROGRAM LISTINGS FOR A GROUP OF TWO MACHINES

```
      MASTER TOTAL TRANSIENT IN TWO MOTORS
      DIMENSION AL(8,8),R(8,8),G1(8,8),TRM1(8,8),G2(8,8)
      DIMENSION TC(1,8),GI1(8,1),RI(8,1),CI(8,1),GI2(8,1)
      DIMENSION DI(8,1),V(8,1),VER(8,8),DIF(8,1)
      DIMENSION D(8,8),AK1(8),AK2(8),AK3(8),AK4(8),TRM2(8,8)
      DIMENSION DI2(6,1)
      DIMENSION D1(6,6),DIS(6,6),AM(6,1),VERD(6,6)
      DIMENSION GI12(6,1),G12(6,6),DIF1(6,1),AKI1(6),AKI2(6),AKI3(6),AKI4(
     14(6)
      DIMENSION GI22(6,1),G22(6,6),TRM11(6,6),TRM22(6,6)
      DIMENSION AL22(6,6),VER1(6,6),RI13(6,1),R22(6,6),CI2(6,1)
      DIMENSION RI1(4,1),GI11(4,1)
      DIMENSION V1(4,1),G11(4,4),R11(4,4),AL1(4,4),CI1(4,1),DI1(4,1)
      DIMENSION ALDI(4,1)
C     *** READS IN PARAMETERS ***
      READ(5,1)((R(I,J),I=1,8),J=1,8)
      READ(5,1)((AL(I,J),I=1,8),J=1,8)
      READ(5,1)((G1(I,J),I=1,8),J=1,8)
      READ(5,1)((G2(I,J),I=1,8),J=1,8)
      READ(5,98)((R22(I,J),I=1,6),J=1,6)
      READ(5,98)((AL22(I,J),I=1,6),J=1,6)
      READ(5,98)((G12(I,J),I=1,6),J=1,6)
      READ(5,98)((G22(I,J),I=1,6),J=1,6)
      READ(5,98)((DIS(I,J),I=1,6),J=1,6)
      READ(5,555)((R11(I,J),I=1,4),J=1,4)
      READ(5,555)((AL1(I,J),I=1,4),J=1,4)
      READ(5,555)((G11(I,J),I=1,4),J=1,4)
      READ(5,610)A11,A22,B11,B22,RSP,AI1,BIT,CIT
      READ(5,616)VM,VM1,PP,PP2,AJ1,AJ2,DTHETA,DTHETA2
      READ(5,616)NCC,NCC1,NCC2,DEL,AA1,AA2,AB1,AB2
      READ(5,610)AC1,AC2,AD1,AD2,AE1,AE2,AF1,AF2
      READ(5,626)DUR,EPS1,EPS2,EPS3
  626 FORMAT(4F0.0)
  616 FORMAT(8F0.0)
  555 FORMAT(4F0.0)
   98 FORMAT(6F0.0)
    1 FORMAT(8F0.0)
      PI=4.0*ATAN(1.0)
      TOP=2.0*PI
      FOPI=2.0*TOPI
      R1=A11+A22*(DTHETA/314.2)**2
      R2=B11+B22*(DTHETA2/314.2)**2
      NC=0
      DO 11 I=1,8
   11 CI(I,1)=0.0
C     *** INVERSION OF MATRIX L ***
```

```
2222 CALL MB02A(AL,VER,DETER,8,3,8)
  12 CONTINUE
     L=0
     VA=VM*SIN(100.0*PI*DUR+DEL)
     VB=VM*SIN(100.0*PI*DUR-TOPI+DEL)
     VC=VM*SIN(100.0*PI*DUR-FOPI+DEL)
     VAM=VA-RSP*AIT
     VBM=VB-RSP*BIT
     VCM=VC-RSP*CIT
     DUR=DUR+H
C    *** TRANSFORMATION FROM THREE PHASE TO D-Q AXES ***
  48 VDM=SQRT(2.0/3.0)*(VAM-0.5*VBM-0.5*VCM)
     VQM=SQRT(2.0/3.0)*((SQRT(3.0)/2.0)*VBM-(SQRT(3.0)/2.0)*VCM)
C    *** INITIAL CONDITIONS OF VOLTAGES ***
     V(1,1)=VDM
     V(2,1)=VQM
     V(3,1)=0.0
     V(4,1)=0.0
     V(5,1)=VDM
     V(6,1)=VQM
     V(7,1)=0.0
     V(8,1)=0.0
C    *** GENERATE MATRICES FOR R*I,G*I AND T ***
  13 CALL MZ01B(RI,R,CI,8,8,1,8,8,8)
     CALL MZ01B(GI1,G1,CI,8,8,1,8,8,8)
     CALL MZ01B(GI2,G2,CI,8,8,1,8,8,8)
     CALL MZ02B(TRM1,CI,GI1,1,8,1,1,8,8)
     CALL MZ02B(TRM2,CI,GI2,1,8,1,1,8,8)
     T=PP*TRM1(1,1)
     T2=PP2*TRM2(1,1)
     DO 14 K=1,8
  14 DIF(K,1)=V(K,1)-RI(K,1)-DTHETA*GI1(K,1)-DTHETA2*GI2(K,1)
C    *** EVALUATE THE RATES OF CHANGE OF CURRENTS ***
     CALL MZ01B(DI,VER,DIF,8,8,1,8,8,8)
C    *** COMMENCE A RUNGE-KUTTA CYCLE OF NUMERICAL INTEGRATION ***
     L=L+1
     GO TO (15,25,35,45),L
  15 DO 16 I=1,8
  16 AK1(I)=DI(I,1)*H
C    *** TRANSFORMATION FROM D-Q TO THREE PHASE CURRENTS ***
     CISA=SQRT(2.0/3.0)*(CI(1,1))
     CISB=SQRT(2.0/3.0)*(-0.5*CI(1,1)+SQRT(3.0)*CI(2,1)/2.0)
     CISC=SQRT(2.0/3.0)*(-0.5*CI(1,1)-SQRT(3.0)*CI(2,1)/2.0)
     CISA2=SQRT(2.0/3.0)*(CI(5,1))
     CISB2=SQRT(2.0/3.0)*(-0.5*CI(5,1)+SQRT(3.0)*CI(6,1)/2.0)
     CISC2=SQRT(2.0/3.0)*(-0.5*CI(5,1)-SQRT(3.0)*CI(6,1)/2.0)
     CIRC1=SQRT(2.0/3.0)*(CI(7,1)*COS(THETA2+FOPI)+CI(8,1)*
    1SIN(THETA2+FOPI))
     CIRB1=SQRT(2.0/3.0)*(CI(7,1)*COS(THETA2+TOPI)+CI(8,1)*SIN(THETA2
    1+TOPI))
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```
      CIRA1=SQRT(2.0/3.0)*(CI(7,1)*COS(THETA2)+CI(8,1)*SIN(THETA2))
      CIRA=SQRT(2.0/3.0)*(CI(3,1)*COS(THETA)+CI(4,1)*SIN(THETA))
      CIRB=SQRT(2.0/3.0)*(CI(3,1)*COS(THETA+TOPI)+CI(4,1)*SIN(THETA
     1+TOPI))
      CIRC=SQRT(2.0/3.0)*(CI(3,1)*COS(THETA+FOPI)+CI(4,1)*SIN(THETA
     1+FOPI))
      P2=VA*CISA2+VB*CISB2+VC*CISC2
      P1=VA*CISA+VB*CISB+VC*CISC
      WRITE(6,977)P1,P2
  977 FORMAT(10X,F15.3,20X,F15.3)
      WRITE(6,17)DUR,T,T2,DTHETA,DTHETA2
   17 FORMAT(10X,5HTIME=,F6.5,3X,3HT1=,F6.2,3X,3HT2=,F6.2,3X,3HN1=,F6.2,
     13X,3HN2=,F6.2)
      WRITE(6,8000)THETA,THETA2
 8000 FORMAT(2F15.5)
      WRITE(6,3025)VA,VB,VC,VAM,VBM,VCM
 3025 FORMAT(10X,3HVA=,F7.2,3X,3HVB=,F7.2,3X,3HVC=,F7.2,3X,
     14HVAM=,F7.2,3X,4HVBM=,F7.2,3X,4HVCM=,F7.2)
      WRITE(6,888)CISA,CISB,CISC,CISA2,CISB2,CISC2
  888 FORMAT(10X,4HIA1=,F7.2,3X,4HIB1=,F7.2,3X,4HIC1=,F7.2,3X,4HIA2=,F
     17.2,3X,4HIB2=,F7.2,3X,4HIC2=,F7.2)
      WRITE(6,1001)CIRA,CIRB,CIRC,CIRA1,CIRB1,CIRC1
 1001 FORMAT(10X,5HIRA1=,F7.2,3X,5HIRB1=,F7.2,5X,5HIRB2=,F7.2,3X,5HIRC2=,
     1,F7.2)
 9090 CONTINUE
      DTHET1=(T-R1)*PP*H/AJ1
      DTHE1=(T2-R2)*PP2*H/AJ2
      THET1=DTHETA*H
      THE1=DTHETA2*H
      DO 18 I=1,8
   18 CI(I,1)=CI(I,1)+0.5*AK1(I)
      DTHETA=DTHETA+0.5*DTHET1
      DTHETA2=DTHETA2+0.5*DTHE1
      THETA=THETA+0.5*THET1
      THETA2=THETA2+0.5*THE1
      GO TO 13
   25 DO 19 I=1,8
   19 AK2(I)=DI(I,1)*H
      DTHET2=(T-R1)*PP*H/AJ1
      DTHE2=(T2-R2)*PP2*H/AJ2
      THET2=DTHETA*H
      THE2=DTHETA2*H
      DO 20 I=1,8
   20 CI(I,1)=CI(I,1)-0.5*(AK1(I)-AK2(I))
      DTHETA=DTHETA-0.5*(DTHET1-DTHET2)
      DTHETA2=DTHETA2-0.5*(DTHE1-DTHE2)
      THETA2=THETA2-0.5*(THE1-THE2)
      THETA=THETA-0.5*(THET1-THET2)
      GO TO 13
   35 DO 21 I=1,8
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21 AK3(I)=DI(I,1)*H
   DTHET3=(T-R1)*PP*H/AJ1
   DTHE3=(T2-R2)*PP2*H/AJ2
   THET3=DTHETA*H
   THE3=DTHETA2*H
   DO 22 I=1,8
22 CI(I,1)=CI(I,1)+AK3(I)-0.5*AK2(I)
   DTHETA=DTHETA-0.5*DTHET2+DTHET3
   DTHETA2=DTHETA2-0.5*DTHE2+DTHE3
   THETA=THETA-0.5*THET2+THET3
   THETA2=THETA2-0.5*THE2+THE3
   GO TO 13
45 DO 23 I=1,8
23 AK4(I)=DI(I,1)*H
   DTHET4=(T-R1)*PP*H/AJ1
   DTHE4=(T2-R2)*PP2*H/AJ2
   THET4=DTHETA*H
   THE4=DTHETA2*H
   DO 24 I=1,8
24 CI(I,1)=CI(I,1)-AK3(I)+(AK1(I)+2.0*(AK2(I)+AK3(I))+AK4(I))/6.0
   DTHETA=DTHETA+(DTHET1+2.0*(DTHET2+DTHET3)+DTHET4)/6.0-DTHET3
   DTHETA2=DTHETA2+(DTHE1+2.0*(DTHE2+DTHE3)+DTHE4)/6.0-DTHE3
   R1=A11+A22*(DTHETA/314.2)**2
   R2=B11+B22*(DTHETA2/314.2)**2
   THETA=THETA+(THET1+2.0*(THET2+THET3)+THET4)/6.0-THET3
   THETA2=THETA2+(THE1+2.0*(THE2+THE3)+THE4)/6.0-THE3
   CID=CI(1,1)+CI(5,1)
   CIO=CI(2,1)+CI(6,1)
   NC=NC+1
C     *** HAVE ENOUGH ITERATION BEEN PERFORMED? ***
   IF(NC.LT.NCC)GO TO 12
   IF(NC.LT.(NCC+NCC1)) GO TO 100
   IF(NC.LT.(NCC+NCC1+NCC2)) GO TO 49
100 WRITE(6,27)
27 FORMAT(50H D    I    S    C    O    N    N    E    C    T    I    O    N)
   AA=AA1*CI(3,1)+AA2*CI(1,1)
   AB=AB1*CI(4,1)+AB2*CI(2,1)
   AC=AC1*CI(7,1)+AC2*CI(5,1)
   AD=AD1*CI(8,1)+AD2*CI(6,1)
   AE=AE1*CI(1,1)+AE2*CI(3,1)
   AF=AF1*CI(2,1)+AF2*CI(4,1)
   AM(1,1)=AA
   AM(2,1)=AB
   AM(3,1)=AC
   AM(4,1)=AD
   AM(5,1)=AE
   AM(6,1)=AF
   CALL MB02A(DIS,VERD,DETER1,6,3,6)
   CALL MZ01B(CI2,VERD,AM,6,6,1,6,6,6)
C     *** INVERSION OF MATRIX AL22 ***
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      CALL MB02A(AL22,VER1,DETER2,6,3,6)
   28 CONTINUE
      L=0
C      *** GENERATE MATRICES FOR R*I AND G*I    ***
   29 CALL MZ01B(RI13,R22,CI2,6,6,1,6,6,6)
      CALL MZ01B(GI12,G12,CI2,6,6,1,6,6,6)
      CALL MZ01B(GI22,G22,CI2,6,6,1,6,6,6)
      CALL MZ02B(TRM11,CI2,GI12,1,6,1,1,6,6)
      CALL MZ02B(TRM22,CI2,GI22,1,6,1,1,6,6)
      T=PP*TRM11(1,1)
      T2=PP2*TRM22(1,1)
      DO 30 K=1,6
   30 DIF1(K,1)=0.0-RI13(K,1)-DTHETA*GI12(K,1)-DTHETA2*GI22(K,1)
      CALL MZ01B(DI2,VER1,DIF1,6,6,1,6,6,6)
C     *** COMMENCE A RUNGE-KUTTA CYCLE OF NUMERICAL INTEGRATION ***
      L=L+1
      GO TO (31,41,51,61),L
   31 DO 32 I=1,6
   32 AKI1(I)=DI2(I,1)*H
      CIRA1=SQRT(2.0/3.0)*(CI2(3,1)*COS(THETA)+CI2(4,1)*SIN(THETA))
      CIRA2=SQRT(2.0/3.0)*(CI2(5,1)*COS(THETA2)+CI2(6,1)*SIN(THETA2))
      CIRB1=SQRT(2.0/3.0)*(CI2(3,1)*COS(THETA+TOPI)+CI2(4,1)
     1*SIN(THETA+TOPI))
      CIRB2=SQRT(2.0/3.0)*(CI2(5,1)*COS(THETA2+TOPI)+
     1CI2(6,1)*SIN(THETA2+TOPI))
      CIRC1=SQRT(2.0/3.0)*(CI2(3,1)*COS(THETA+FOPI)+CI2(4,1)*SIN(THETA+
     1FOPI))
      CIRC2=SQRT(2.0/3.0)*(CI2(5,1)*COS(THETA2+FOPI)+CI2(6,1)*
     1SIN(THETA2+FOPI))
      CISA1=SQRT(2.0/3.0)*(CI2(1,1))
      CISB1=SQRT(2.0/3.0)*(-0.5*CI2(1,1)+SQRT(3.0)*CI2(2,1)/2.0)
      CISC1=SQRT(2.0/3.0)*(-0.5*CI2(1,1)-SQRT(3.0)*CI2(2,1)/2.0)
      THET1=DTHETA*H
      THE1=DTHETA2*H
      DO 33 I=1,6
   33 CI2(I,1)=CI2(I,1)+0.5*AKI1(I)
      THETA=THETA+0.5*THET1
      THETA2=THETA2+0.5*THE1
      GO TO 29
   41 DO 34 I=1,6
   34 AKI2(I)=DI2(I,1)*H
      THET2=DTHETA*H
      THE2=DTHETA2*H
      DO 36 I=1,6
   36 CI2(I,1)=CI2(I,1)-0.5*(AKI1(I)-AKI2(I))
      THETA=THETA-0.5*(THET1-THET2)
      THETA2=THETA2-0.5*(THE1-THE2)
      GO TO 29
   51 DO 37 I=1,6
   37 AKI3(I)=DI2(I,1)*H
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      THET3=DTHETA*H
      THE3=DTHETA2*H
      DO 38 I=1,6
   38 CI2(I,1)=CI2(I,1)+AKI3(I)-0.5*AKI2(I)
      THETA=THETA-0.5*THET2+THET3
      THETA2=THETA2-0.5*THE2+THE3
      GO TO 29
   61 DO 39 I=1,6
   39 AKI4(I)=DI2(I,1)*H
      THET4=DTHETA*H
      THE4=DTHETA2*H
      DO 40 I=1,6
   40 CI2(I,1)=CI2(I,1)-AKI3(I)+(AKI1(I)+2.0*(AKI2(I)+AKI3(I))+
     1AKI4(I))/6.0
      THETA=THETA+(THET1+2.0*(THET2+THET3)+THET4)/6.0-THET3
      THETA2=THETA2+(THE1+2.0*(THE2+THE3)+THE4)/6.0-THE3
      IF(THETA.LT.TOP) GO TO 2121
      THETA=THETA-TOP
 2121 IF(THETA2.LT.TOP) GO TO 3131
      THETA2=THETA2-TOP
 3131 CONTINUE
      CI1(1,1)=CI2(1,1)
      CI1(2,1)=CI2(2,1)
      CI1(3,1)=CI2(3,1)
        CI1(4,1)=CI2(4,1)
      DI1(1,1)=DI2(1,1)
      DI1(2,1)=DI2(2,1)
        DI1(3,1)=DI2(3,1)
      DI1(4,1)=DI2(4,1)
      CALL MZ01B(RI1,R11,CI1,4,4,1,4,4,4)
      CALL MZ01B(ALDI,AL1,DI1,4,4,1,4,4,4)
      CALL MZ01B(GI11,G11,CI1,4,4,1,4,4,4)
      DO 666 K=1,4
  666 V1(K,1)=RI1(K,1)+ALDI(K,1)+DTHETA*GI11(K,1)
      VSA1=SQRT(2.0/3.0)*(V1(1,1))
      VSB1=SQRT(2.0/3.0)*(-0.5*V1(1,1)+SQRT(3.0)*V1(2,1)/2.0)
      VSC1=SQRT(2.0/3.0)*(-0.5*V1(1,1)-SQRT(3.0)*V1(2,1)/2.0)
      WRITE(6,8484)VSA1,VSB1,VSC1
 8484 FORMAT(3F10.5)
      DUR=DUR+H
      NC=NC+1
      R1=A11+A22*(DTHETA/314.2)**2
      R2=B11+B22*(DTHETA2/314.2)**2
      IF(NC.LT.(NCC+NCC1)) GO TO 28
      WRITE(6,47)
   47 FORMAT(44H R    E    S    W    I    T    C    H    I    N    G )
      CI(1,1)=CI2(1,1)
      CI(2,1)=CI2(2,1)
      CI(3,1)=CI2(3,1)
      CI(4,1)=CI2(4,1)
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```
      CI(5,1)=-CI2(1,1)
      CI(6,1)=-CI2(2,1)
      CI(7,1)=CI2(5,1)
      CI(8,1)=CI2(6,1)
   49 CONTINUE
      L=0
      VA=VM1*SIN(100.0*PI*DUR+EPS1)
      VB=VM1*SIN(100.0*PI*DUR+EPS2)
      VC=VM1*SIN(100.0*PI*DUR+EPS3)
      VAM=VA-RSP*AIT
      VBM=VB-RSP*BIT
      VCM=VC-RSP*CIT
      DUR=DUR+H
      IF(NC.LT.(NCC+NCC1+NCC2)) GO TO 48
      STOP
      END
      FINISH
      ****
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APPENDIX G

COMPUTER PROGRAMLISTINGS FOR A GROUP OF THREE MACHINES

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      MASTER TOTAL TRANSIENT THREE MOTORS
      DIMENSION AL(12,12),R(12,12),G1(12,12),TRM1(12,12),G2(12,12)
      DIMENSION TC(1,12),GI1(12,1),RI(12,1),CI(12,1),GI2(12,1),DI2(10,1)
      DIMENSION DI(12,1),V(12,1),VER(12,12),DIF(12,1),TRM3(12,12)
      DIMENSION D(12,12),AK1(12),AK2(12),AK3(12),AK4(12)
      DIMENSION TRM2(12,12),D1(10,10),DIS(10,10),AM(10,1),G3(12,12) ,
     1AL22(10,10),VER1(10,10),RI13(10,1),R22(10,10),G22(10,10),
     1TRM11(10,10),TRM22(10,10),TRM33(10,10),AKI4(10),GI22(10,1)
      DIMENSION G33(10,10),GI33(10,1)
      DIMENSION VERD(10,10),GI12(10,1)
      DIMENSION G12(10,10),DIF1(10,1),AKI1(10),AKI2(10),AKI3(10)
      DIMENSION CI2(10,1),RI1(4,1),GI11(4,1),V1(4,1),G11(4,4),R11(4,4)
      DIMENSION AL1(4,4),CI1(4,1),DI1(4,1),ALDI(4,1),GI3(12,1)
C     $$$ READS IN PARAMETERS $$$
      READ(5,1)((R(I,J),I=1,12),J=1,12)
      READ(5,1)((AL(I,J),I=1,12),J=1,12)
      READ(5,1)((G1(I,J),I=1,12),J=1,12)
      READ(5,1)((G2(I,J),I=1,12),J=1,12)
      READ(5,1)((G3(I,J),I=1,12),J=1,12)
      READ(5,555)((DIS(I,J),I=1,10),J=1,10)
      READ(5,555)((R22(I,J),I=1,10),J=1,10)
      READ(5,555)((AL22(I,J),I=1,10),J=1,10)
      READ(5,555)((G12(I,J),I=1,10),J=1,10)
      READ(5,555)((G22(I,J),I=1,10),J=1,10)
      READ(5,555)((G33(I,J),I=1,10),J=1,10)
      READ(5,98)((R11(I,J),I=1,4),J=1,4)
      READ(5,98)((AL1(I,J),I=1,4),J=1,4)
      READ(5,98)((G11(I,J),I=1,4),J=1,4)
      READ(5,616)A11,A22,B11,B22,RSP,AIT,BIT,CIT
      READ(5,636)C11,C22,DTHETA3,PP3,AJ3
      READ(5,616)VM,VM1,PP,PP2,AJ1,AJ2,DTHETA,DTHETA2
      READ(5,616)NCC,NCC1,NCC2,DEL,AA1,AA2,AB1,AB2
      READ(5,616)AC1,AC2,AD1,AD2,AE1,AE2,AF1,AF2
      READ(5,626)DUR,EPS1,EPS2,EPS3
      READ(5,646)AG1,AG2,AH1,AH2,AI1,AI2,AJ1,AJ2
  646 FORMAT(8F0.0)
  636 FORMAT(5F0.0)
    1 FORMAT(12F0.0)
  626 FORMAT(4F0.0)
  616 FORMAT(8F0.0)
  555 FORMAT(10F0.0)
   98 FORMAT(4F0.0)
      PI=4.0*ATAN(1.0)
      R1=A11+A22*(DTHETA/314.2)**2
      R2=B11+B22*(DTHETA2/314.2)**2
      R3=C11+C22*(DTHETA3/314.2)**2
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      TOP=2.0*PI
      TOPI=2.0*PI/3.0
      FOPI=2.0*TOPI
      NC=0
      DO 11 I=1,8
   11 CI(I,1)=0.0
C     *** INVERSION OF MATRIX    L ***
      CALL MB02A(AL,VER,DETER,12,3,12)
   12 CONTINUE
      L=0
      VA=VM*SIN(100.0*PI*DUR+DEL)
      VB=VM*SIN(100.0*PI*DUR-TOPI+DEL)
      VC=VM*SIN(100.0*PI*DUR-FOPI+DEL)
      DUR=DUR+H
   38 CONTINUE
      VAM=VA-RSP*AIT
      VBM=VB-RSP*BIT
      VCM=VC-RSP*CIT
C     ***  TRANSFORMATION FROM THREE PHASE TO D-Q AXES ***
   48 CONTINUE
      VDM=SQRT(2.0/3.0)*(VAM-0.5*VBM-0.5*VCM)
      VQM=SQRT(2.0/3.0)*((SQRT(3.0)/2.0)*VBM-(SQRT(3.0)/2.0)*VCM)
C     ***  INITIAL CONDITIONS OF VOLTAGES ***
      V(1,1)=VDM
      V(2,1)=VQM
      V(3,1)=0.0
      V(4,1)=0.0
      V(5,1)=VDM
      V(6,1)=VQM
      V(7,1)=0.0
      V(8,1)=0.0
      V(9,1)=VDM
      V(10,1)=VQM
      V(11,1)=0.0
      V(12,1)=0.0
C     *** GENERATE MATRICES FOR R*I,G*I AND T        ***
   13 CALL MZ01B(RI,R,CI,12,12,1,12,12,12)
      CALL MZ01B(GI1,G1,CI,12,12,1,12,12,12)
      CALL MZ01B(GI2,G2,CI,12,12,1,12,12,12)
      CALL MZ01B(GI3,G3,CI,12,12,1,12,12,12)
      CALL MZ02B(TRM1,CI,GI1,1,12,1,1,12,12)
      CALL MZ02B(TRM2,CI,GI2,1,12,1,1,12,12)
      CALL MZ02B(TRM3,CI,GI3,1,12,1,1,12,12)
      T=PP*TRM1(1,1)
      T2=PP2*TRM2(1,1)
      T3=PP3*TRM3(1,1)
      DO 14 K=1,12
   14 DIF(K,1)=V(K,1)-RI(K,1)-DTHETA*GI1(K,1)-DTHETA2*GI2(K,1)-
     1DTHETA3*GI3(K,1)
C     ***  EVALUATE THE RATES OF CHANGE OF CURRENTS ***
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      CALL MZ01B(DI,VER,DIF,12,12,1,12,12,12)
C     *** COMMENCE OF RUNGE KUTTA CYCLE OF NUMERICAL INTEGRATION ***
      L=L+1
      GO TO (15,25,35,45),L
   15 DO 16 I=1,12
   16 AK1(I)=DI(I,1)*H
C     *** TRANSFORMATION FROM D-Q TO THREE PHASE CURRENTS ***
      CISA=SQRT(2.0/3.0)*(CI(1,1))
      CISB=SQRT(2.0/3.0)*(-0.5*CI(1,1)+SQRT(3.0)*CI(2,1)/2.0)
      CISC=SQRT(2.0/3.0)*(-0.5*CI(1,1)-SQRT(3.0)*CI(2,1)/2.0)
      CISA2=SQRT(2.0/3.0)*(CI(5,1))
      CISB2=SQRT(2.0/3.0)*(-0.5*CI(5,1)+SQRT(3.0)*CI(6,1)/2.0)
      CISC2=SQRT(2.0/3.0)*(-0.5*CI(5,1)-SQRT(3.0)*CI(6,1)/2.0)
      CISA3=SQRT(2.0/3.0)*(CI(9,1))
      CISB3=SQRT(2.0/3.0)*(-0.5*CI(9,1)+SQRT(3.0)*CI(10,1)/2.0)
      CISC3=SQRT(2.0/3.0)*(-0.5*CI(9,1)-SQRT(3.0)*CI(10,1)/2.0)
      WRITE(6,8989)DUR,T,T2,T3,DTHETA,DTHETA2,DTHETA3
 8989 FORMAT(10X,5HTIME=,F6.5,2X,2HE=,F6.3,2X,3HT2=,F6.3,2X,3HT3=,F6.3,2
     1X,3HN1=,F6.1,2X,3HN2=,F6.1,2X,3HN3=,F6.1)
      WRITE(6,3010)VAH,VBH,VCH
      WRITE(6,3010)VA,VB,VC
 3010 FORMAT(10X,3HVA=,F7.3,3X,3HVB=,F7.3,3X,3HVC=,F7.3)
      WRITE(6,9)CISA,CISB,CISC,CISA2,CISB2,CISC2,CISA3,CISB3,CISC3
    9 FORMAT(10X,4HIA1=,F4.1,2X,4HIB1=,F4.1,2X,4HIC1=,F4.1,2X,4HIA2=,
     1F4.1,2X,4HIB2=,F4.1,2X,4HIC2=,F4.1,2X,4HIA3=,F4.2,2X,4HIB3=,F4.2,2
     1X,4HIC3=,F4.2)
      WRITE(6,3011)AIT,BIT,CIT
 3011 FORMAT(3F30.5)
 9090 CONTINUE
      DTHET1=(T-R1)*PP*H/AJ1
      DTHE1=(T2-R2)*PP2*H/AJ2
      DTH1=(T3-R3)*PP3*H/AJ3
      THET1=DTHETA*H
      THE1=DTHETA2*H
      TH1=DTHETA3*H
      DO 18 I=1,12
   18 CI(I,1)=CI(I,1)+0.5*AK1(I)
      DTHETA=DTHETA+0.5*DTHET1
      DTHETA2=DTHETA2+0.5*DTHE1
      DTHETA3=DTHETA3+0.5*DTH1
      THETA=THETA+0.5*THET1
      THETA2=THETA2+0.5*THE1
      THETA3=THETA3+0.5*TH1
      GO TO 13
   25 DO 19 I=1,12
   19 AK2(I)=DI(I,1)*H
      DTHET2=(T-R1)*PP*H/AJ1
      DTHE2=(T2-R2)*PP2*H/AJ2
      DTH2=(T3-R3)*PP3*H/AJ3
      THET2=DTHETA*H
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      THE2=DTHETA2*H
      TH2=DTHETA3*H
      DO 20 I=1,12
20    CI(I,1)=CI(I,1)-0.5*(AK1(I)-AK2(I))
      DTHETA=DTHETA-0.5*(DTHET1-DTHET2)
      DTHETA2=DTHETA2-0.5*(DTHE1-DTHE2)
      DTHETA3=DTHETA3-0.5*(DTH1-DTH2)
      THETA=THETA-0.5*(THET1-THET2)
      THETA2=THETA2-0.5*(THE1-THE2)
      THETA3=THETA3-0.5*(TH1-TH2)
      GO TO 13
35    DO 21 I=1,12
21    AK3(I)=DI(I,1)*H
      DTHET3=(T-R1)*PP*H/AJ1
      DTHE3=(T2-R2)*PP2*H/AJ2
      DTH3=(T3-R3)*PP3*H/AJ3
      THET3=DTHETA*H
      THE3=DTHETA2*H
      TH3=DTHETA3*H
      DO 22 I=1,12
22    CI(I,1)=CI(I,1)+AK3(I)-0.5*AK2(I)
      DTHETA=DTHETA-0.5*DTHET2+DTHET3
      DTHETA2=DTHETA2-0.5*DTHE2+DTHE3
      DTHETA3=DTHETA3-0.5*DTH2+DTH3
      THETA=THETA-0.5*THET2+THET3
      THETA2=THETA2-0.5*THE2+THE3
      THETA3=THETA3-0.5*TH2+TH3
      GO TO 13
45    DO 23 I=1,12
23    AK4(I)=DI(I,1)*H
      DTHET4=(T-R1)*PP*H/AJ1
      DTHE4=(T2-R2)*PP2*H/AJ2
      DTH4=(T3-R3)*PP3*H/AJ3
      THET4=DTHETA*H
      THE4=DTHETA2*H
      TH4=DTHETA3*H
      DO 24 I=1,12
24    CI(I,1)=CI(I,1)-AK3(I)+(AK1(I)+2.0*(AK2(I)+AK3(I))+AK4(I))/6.0
      DTHETA=DTHETA+(DTHET1+2.0*(DTHET2+DTHET3)+DTHET4)/6.0-DTHET3
      DTHETA2=DTHETA2+(DTHE1+2.0*(DTHE2+DTHE3)+DTHE4)/6.0-DTHE3
      DTHETA3=DTHETA3+(DTH1+2.0*(DTH2+DTH3)+DTH4)/6.0-DTH3
      R1=A11+A22*(DTHETA/314.2)**2
      R2=B11+B22*(DTHETA2/314.2)**2
      R3=C11+C22*(DTHETA3/314.2)**2
      THETA=THETA+(THET1+2.0*(THET2+THET3)+THET4)/6.0-THET3
      THETA2=THETA2+(THE1+2.0*(THE2+THE3)+THE4)/6.0-THE3
      THETA3=THETA3+(TH1+2.0*(TH2+TH3)+TH4)/6.0-TH3
      CID=CI(1,1)+CI(5,1)+CI(9,1)
      CIQ=CI(2,1)+CI(6,1)+CI(10,1)
      AIT=SQRT(2.0/3.0)*CID
```

```
      BIT=SQRT(2.0/3.0)*(-0.5*CID+SQRT(3.0)*CIQ/2.0)
      CIT=SQRT(2.0/3.0)*(-0.5*CID-SQRT(3.0)*CIQ/2.0)
      NC=NC+1
      IF(THETA.LT.TOP) GO TO 26
      THETA=THETA-TOP
   26 IF(THETA2.LT.TOP) GO TO 1111
      THETA2=THETA2-TOP
 1111 IF(THETA3.LT.TOP) GO TO 1112
      THETA3=THETA3-TOP
 1112 CONTINUE
C     **** HAVE ENOUGH ITERATION BEEN PERFORMED **** ?
 9888 IF(NC.LT.(NCC)) GO TO 12
      IF(NC.LT.(NCC+NCC1)) GO TO 100
      IF(NC.LT.(NCC+NCC1+NCC2)) GO TO 49
  100 WRITE(6,27)
   27 FORMAT(50H D    I    S    C    O    N    N    E    C    T    I    O    N)
      AA=AA1*CI(3,1)+AA2*CI(1,1)
      AB=AB1*CI(4,1)+AB2*CI(2,1)
      AC=AC1*CI(7,1)+AC2*CI(5,1)
      AD=AD1*CI(8,1)+AD2*CI(6,1)
      AE=AE1*CI(11,1)+AE2*CI(9,1)
      AF=AF1*CI(12,1)+AF2*CI(10,1)
      AG=AG1*CI(1,1)+AG2*CI(3,1)
      AH=AH1*CI(2,1)+AH2*CI(4,1)
      AI=AI1*CI(5,1)+AI2*CI(7,1)
      AJ=AJ1*CI(6,1)+AJ2*CI(8,1)
      AM(1,1)=AA
      AM(2,1)=AB
      AM(3,1)=AC
      AM(4,1)=AD
      AM(5,1)=AE
      AM(6,1)=AF
      AM(7,1)=AG
      AM(8,1)=AH
      AM(9,1)=AI
      AM(10,1)=AJ
      CALL MB02A(DIS,VERD,DETER1,10,3,10)
      CALL MZ01B(CI2,VERD,AM,10,10,1,10,10,10)
C     *** INVERSION OF MATRIX AL22 ****
      CALL MB02A(AL22,VER1,DETER2,10,3,10)
   28 CONTINUE
      L=0
C     **** GENERATE MATRICES FOR R*I ,GI ,AND T ****
   29 CALLMZ01B(RI13,R22,CI2,10,10,1,10,10,10)
      CALL MZ01B(GI12,G12,CI2,10,10,1,10,10,10)
      CALL MZ01B(GI22,G22,CI2,10,10,1,10,10,10)
      CALL MZ01B(GI33,G33,CI2,10,10,1,10,10,10)
      CALL MZ02B(TRM11,CI2,GI12,1,10,1,1,10,10)
      CALL MZ02B(TRM22,CI2,GI22,1,10,1,1,10,10)
      CALL MZ02B(TRM33,CI2,GI33,1,10,1,1,10,10)
```

```
      T=PP*TRM11(1,1)
      T2=PP2*TRM22(1,1)
      T3=PP3*TRM33(1,1)
      DO 30 K=1,10
   30 DIF1(K,1)=0.0-RI13(K,1)-DTHETA*GI12(K,1)-DTHETA2*GI22(K,1)-DTHETA3
     1*GI33(K,1)
      CALL M201B(DI2,VER1,DIF1,10,10,1,10,10,10)
C     **** COMMENCE A RUNGE-KUTTA CYCLE OF NUMERICAL INTEGRATION ****
      L=L+1
      GO TO (31,41,51,61),L
   31 DO 32 I=1,10
   32 AK11(I)=DI2(I,1)*H
      SA1=SQRT(2.0/3.0)*(CI2(1,1))
      SB1=SQRT(2.0/3.0)*(-0.5*CI2(1,1)+SQRT(3.0)*CI2(2,1)/2.0)
      SC1=SQRT(2.0/3.0)*(-0.5*CI2(1,1)-SQRT(3.0)*CI2(2,1)/2.0)
      SA2=SQRT(2.0/3.0)*(CI2(5,1))
      SB2=SQRT(2.0/3.0)*(-0.5*CI2(5,1)+SQRT(3.0)*CI2(6,1)/2.0)
      SC2=SQRT(2.0/3.0)*(-0.5*CI2(5,1)-SQRT(3.0)*CI2(6,1)/2.0)
      RA1=SQRT(2.0/3.0)*(CI2(3,1)*COS(THETA)+CI2(4,1)*SIN(THETA))
      RA2=SQRT(2.0/3.0)*(CI2(7,1)*COS(THETA2)+CI2(8,1)*SIN(THETA2))
      RA3=SQRT(2.0/3.0)*(CI2(9,1)*COS(THETA3)+CI2(10,1)*SIN(THETA3))
      RB1=SQRT(2.0/3.0)*(CI2(3,1)*COS(THETA+TOPI)+CI2(4,1)*SIN(THETA+TOPI)
     1))
      RB2=SQRT(2.0/3.0)*(CI2(7,1)*COS(THETA2+TOPI)+CI2(8,1)*SIN(THETA2+TOP
     1PI))
      RB3=SQRT(2.0/3.0)*(CI2(9,1)*COS(THETA3+TOPI)+CI2(10,1)*SIN(THETA3+TO
     1TOPI))
      RC1=SQRT(2.0/3.0)*(CI2(3,1)*COS(THETA+FOPI)+CI2(4,1)*SIN(THETA+FOPI)
     1))
      RC2=SQRT(2.0/3.0)*(CI2(7,1)*COS(THETA2+FOPI)+CI2(8,1)*SIN(THETA2+FOP
     1PI))
      RC3=SQRT(2.0/3.0)*(CI2(9,1)*COS(THETA3+FOPI)+CI2(10,1)*SIN(THETA3+FO
     1FOPI))
      WRITE(6,9393)SA1,SB1,SC1,SA2,SB2,SC2
 9393 FORMAT(5X,4HIA1=,F7.2,3X,4HIB1=,F5.2,3X,4HIC1=,F5.2,3X,
     14HIA2=,F5.2,3X,4HIB2=,F5.2,3X,4HIC2=,F5.2)
      WRITE(6,9595)RA1,RB1,RC1,RA2,RB2,RC2,RA3,RB3,RC3
 9595 FORMAT(10X,4HIA1=,F4.1,2X,4HIB1=,F4.1,2X,4HIC1=,F4.1,2X,4HIA2=,
     1F4.1,2X,4HIB2=,F4.1,2X,4HIC2=,F4.1,2X,4HIA3=,F4.2,2X,4HIB3=,F4.2,2X,
     1X,4HIC3=,F4.2)
      WRITE(6,9696)DUR,T,T2,T3,DTHETA,DTHETA2,DTHETA3
 9696 FORMAT(10X,5HTIME=,F7.6,3X,2HT=,F7.2,3X,3HT2=,F7.2,3X,
     13HT3=,F7.2,3X,3HN1=,F6.2,3HN2=,F6.2,3X,3HN3=,F6.2)
      DTH1=(T3-R3)*PP3*H/AJ3
      DTHET1=(T-R1)*PP*H/AJ1
      DTHE1=(T2-R2)*PP2*H/AJ2
      DTH1=(T3-R3)*PP3*H/AJ3
      THET1=DTHETA*H
      THE1=DTHETA2*H
      TH1=DTHETA3*H
```

```
      DO 33 I=1,10
   33 CI2(I,1)=CI2(I,1)+0.5*AKI1(I)
      DTHETA=DTHETA+0.5*DTHET1
      DTHETA2=DTHETA2+0.5*DTHE1
      DTHETA3=DTHETA3+0.5*DTH1
      GO TO 29
   41 DO 34 I=1,10
   34 AKI2(I)=DI2(I,1)*H
      DTHET2=(T-R1)*PP*H/AJ1
      DTHE2=(T2-R2)*PP2*H/AJ2
      DTH2=(T3-R3)*PP3*H/AJ3
      THET2=DTHETA*H
      THE2=DTHETA2*H
      TH2=DTHETA3*H
      DO 36 I=1,10
   36 CI2(I,1)=CI2(I,1)-0.5*(AKI1(I)-AKI2(I))
      DTHETA=DTHETA-0.5*(DTHET1-DTHET2)
      DTHETA2=DTHETA2-0.5*(DTHE1-DTHE2)
      DTHETA3=DTHETA3-0.5*(DTH1-DTH2)
      THETA=THETA-0.5*(THET1-THET2)
      THETA2=THETA2-0.5*(THE1-THE2)
      THETA3=THETA3-0.5*(TH1-TH2)
      GO TO 29
   51 DO 37 I=1,10
   37 AKI3(I)=DI2(I,1)*H
      DTHET3=(T-R1)*PP*H/AJ1
      DTHE3=(T2-R2)*PP2*H/AJ2
      DTH3=(T3-R3)*PP3*H/AJ3
      THET3=DTHETA*H
      THE3=DTHETA2*H
      TH3=DTHETA3*H
      DO 38 I=1,10
   38 CI2(I,1)=CI2(I,1)+AKI3(I)-0.5*AKI2(I)
      DTHETA=DTHETA-0.5*DTHET2+DTHET3
      DTHETA2=DTHETA2-0.5*DTHE2+DTHE3
      DTHETA3=DTHETA3-0.5*DTH2+DTH3
      THETA=THETA-0.5*THET2+THET3
      THETA2=THETA2-0.5*THE2+THE3
      THETA3=THETA3-0.5*TH2+TH3
      GO TO 29
   61 DO 39 I=1,10
   39 AKI4(I)=DI2(I,1)*H
      DTHET4=(T-R1)*PP*H/AJ1
      DTHE4=(T2-R2)*PP2*H/AJ2
      DTH4=(T3-R3)*PP*H/AJ3
      THET4=DTHETA*H
      THE4=DTHETA2*H
      TH4=DTHETA3*H
      DO 40 I=1,10
   40 CI2(I81)=CI2(I,1)-AKI3(I)+(AKI1(I)+2.0*(AKI2(I)+AKI3(I))
```

```
     1+AKI4(1))/6.0
      DTHETA=DTHETA+(DTHET1+2.0*(DTHET2+DTHET3)+DTHET4)/6.0-DTHET3
      DTHETA2=DTHETA2+(DTHE1+2.0*(DTHE2+DTHE3)+DTHE4)/6.0-DTHE3
      DTHETA3=DTHETA3+(DTH1+2.0*(DTH2+DTH3)+DTH4)/6.0-DTH3
      R1=A11+A22*(DTHETA/314.2)**2
      R2=B11+B22*(DTHETA2/314.2)**2
      R3=C11+C22*(DTHETA3/314.2)**2
      THETA=THETA+(THET1+2.0*(THET2+THET3)+THET4)/6.0-THET3
      THETA2=THETA2+(THE1+2.0*(THE2+THE3)+THE4)/6.0-THE3
      THETA3=THETA3+(TH1+2.0*(TH2+TH3)+TH4)/6.0-TH3
      CI1(1,1)=CI2(1,1)
      CI1(2,1)=CI2(2,1)
      CI1(3,1)=CI2(3,1)
      CI1(4,1)=CI2(4,1)
      DI1(1,1)=DI2(1,1)
      DI1(2,1)=DI2(2,1)
      DI1(3,1)=DI2(3,1)
      DI1(4,1)=DI2(4,1)
      CALL MZ01B(RI1,R11,CI1,4,4,1,4,4,4)
      CALL MZ01B(ALDI,AL1,DI1,4,4,1,4,4,4)
      CALL MZ01B(GI11,G11,CI1,4,4,1,4,4,4)
      DO 666 K=1,4
  666 V1(K,1)=RI1(K,1)+ALDI(K,1)+DTHETA*GI11(K,1)
      VSA1=SQRT(2.0/3.0)*(V1(1,1))
      VSB1=SQRT(2.0/3.0)*(-0.5*V1(1,1)+SQRT(3.0)*V1(2,1)/2.0)
      VSC1=SQRT(2.0/3.0)*(-0.5*V1(1,1)-SQRT(3.0)*V1(2,1)/2.0)
      DUR=DUR+H
      NC=NC+1
      IF(NC.LT.(NCC+NCC1)) GO TO 28
      WRITE(6,47)
   47 FORMAT(//42H R    E    S    W    I    T    C    H    I    N    G)
      CI(1,1)=CI2(1,1)
      CI(2,1)=CI2(2,1)
      CI(3,1)=CI2(3,1)
      CI(4,1)=CI2(4,1)
      CI(5,1)=CI2(5,1)
      CI(6,1)=CI2(6,1)
      CI(7,1)=CI2(7,1)
      CI(8,1)=CI2(8,1)
      CI(9,1)=-CI2(1,1)-CI2(5,1)
      CI(10,1)=-CI2(2,1)-CI2(6,1)
      CI(11,1)=CI2(9,1)
      CI(12,1)=CI2(10,1)
   49 CONTINUE
      L=0
      VA=VM1*SIN(100.0*PI*DUR+EPS1)
      VB=VM1*SIN(100.0*PI*DUR+EPS2)
      VC=VM1*SIN(100.0*PI*DUR+EPS3)
      VAN=VA-RSP*AIT
      VBN=VB-RSP*BIT
```

```
VCH=VC-RSP*CIT
DUR=DUR+H
IF(NC.LT.(NCC+NCC1+NCC2)) GO TO 48
STOP
END
FINISH
****
```

APPENDIX H

SUBROUTINES USED THROUGHOUT THE PROGRAMS

```
C       *** MB02A INVERTS A MATRIX ***
C       M=DIMENSION OF  MATRIX
C       IA=FIRST DIMENSION OF (A) AND (B)
C       DET=VALUE OF DETERMINANT
C       IP=1 DET=DETERMINANT ,ADJOINT NOT FOUND
C       IP=2 DET=DETERMINANT,(A) ADJOINT OF (B)
C       IP=3 DET=DETERMINANT,(A) IS INVERSE OF (B)
        SUBROUTINE MB02A(B,A,DET,M,IP,IA)
        DIMENSION B(IA,IA),A(IA,IA),C(20,20),D(20),IND(20),JND(20)
        AMAX=0.0
        DO 2 I=1,M
        IND(I)=I
        JND(I)=I
        DO 2 J=1,M
        A(I,J)=B(I,J)
        IF(ABS(A(I,J))-AMAX)2,2,0
      3 AMAX=ABS(A(I,J))
        I4=I
        J4=J
      2 CONTINUE
        D(1)=1.0
        MM=M-1
        DO 11 J=1,MM
        IF(I4-J)6,6,0
      4 D(1)=-D(1)
        ISTO=IND(J)
        IND(J)=IND(I4)
        IND(I4)=ISTO
        DO 5 K=1,M
        STO=A(I4,K)
        A(I4,K)=A(J,K)
        A(J,K)=STO
      5 CONTINUE
      6 IF(J4-J)8,8,0
      9 D(1)=-D(1)
```

```
      ISTO=JND(J)
      JND(J)=JND(J4)
      JND(J4)=ISTO
      DO 12 K=1,M
      STO=A(K,J4)
      A(K,J4)=A(K,J)
      A(K,J)=STO
   12 CONTINUE
    8 AMAX=0.0
      J1=J+1
      DO 11 I=J1,M
      STO=-A(I,J)/A(J,J)
      DO 10 K=1,M
      A(I,K)=A(I,K)+STO*A(J,K)
      IF(K-J)10,10,15
   15 IF(ABS(A(I,K))-AMAX)10,10,17
   17 AMAX=ABS(A(I,K))
      I4=I
      J4=K
   10 CONTINUE
      A(I,J)=STO
   11 CONTINUE
      DO 18 I=1,MM
      D(I+1)=D(I)*A(I,I)
   18 CONTINUE
      DET=D(M)*A(M,M)
      PROD1=1.0
      IF(IP-2)99,19,0
   16 PROD1=1.0/DET
   19 DO 20 J=1,M
      DO 21 K=1,J
      C(K,J)=0.0
   21 CONTINUE
      DO 22 K=J,M
      C(K,J)=A(K,J)
   22 CONTINUE
      C(J,J)=1.0
      PROD=PROD1
      DO 30 I=1,MM
      I2=M-I
      I1=I2+1
      STO1=C(I1,J)
      C(I1,J)=D(I1)*STO1*PROD
      IF(ABS(STO1)-ABS(A(I1,I1)))25,25,26
   25 STO=STO1/A(I1,I1)
      DO 27 K=1,I2
      C(K,J)=C(K,J)-STO*A(K,I1)
   27 CONTINUE
      PROD=PROD*A(I1,I1)
      GO TO 30
```

```
26 STO=A(I1,I1)/STO1
   DO 28 K=1,I2
   C(K,J)=A(K,I1)-STO*C(K,J)
28 CONTINUE
   PROD=-PROD*STO1
30 CONTINUE
   C(1,J)=D(1)*C(1,J)*PROD
20 CONTINUE
   DO 40 I=1,M
   K=IND(I)
   DO 40 J=1,M
   L=JND(J)
   A(L,K)=C(J,I)
40 CONTINUE
99 RETURN
   END


C     MZ01B MULTIPLIES TWO MATRICES****
      SUBROUTINE MZ01B(C,A,B,I,J,K,IC,IA,IB)
      DIMENSION C(IC,K),A(IA,J),B(IB,K)
      DO 1 II=1,I
      DO 1 KK=1,K
      C(II,KK)=0.0
      DO 1 JJ=1,J
      C(II,KK)=C(II,KK)+A(II,JJ)*B(JJ,KK)
    1 CONTINUE
      RETURN
      END


C     ***MZ02B MULTIPLIES A TRANSPOSE OF ONE MATRIX BY A MATRIX
      SUBROUTINE MZ02B(C,A,B,I,J,K,IC,IA,IB)
      DIMENSION A(IA,I),B(IB,K),C(IC,K)
      DO 1 II=1,I
      DO 1 KK=1,K
      C(II,KK)=0.0
      DO 1 JJ=1,J
      C(II,KK)=C(II,KK)+A(JJ,II)*B(JJ,KK)
    1 CONTINUE
      RETURN
      END
```

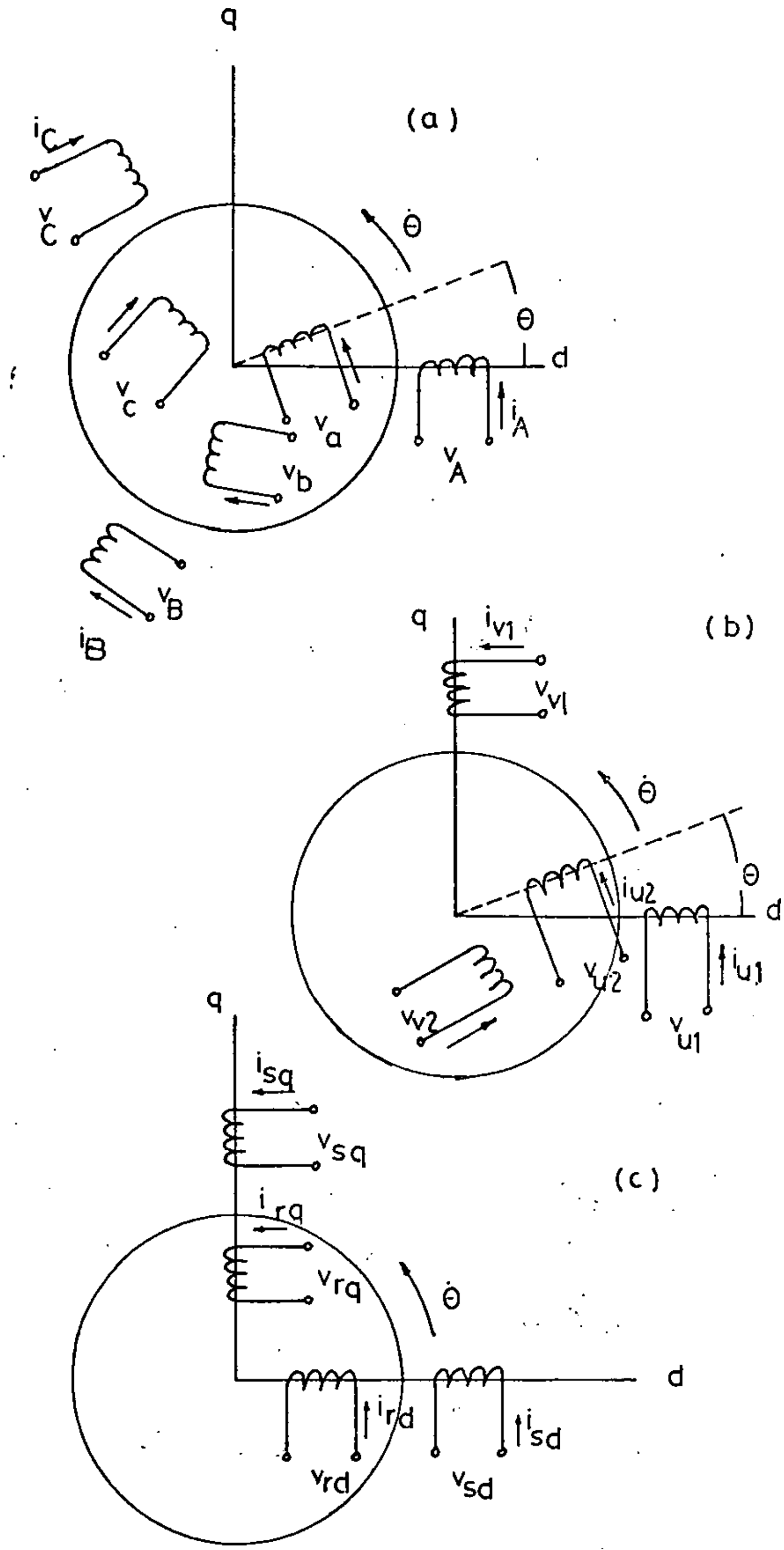


Figure 2·1 (a) Balanced 3-phase induction motor

(b) Balanced 2-phase indution motor

(c) Stationary axis equivalent

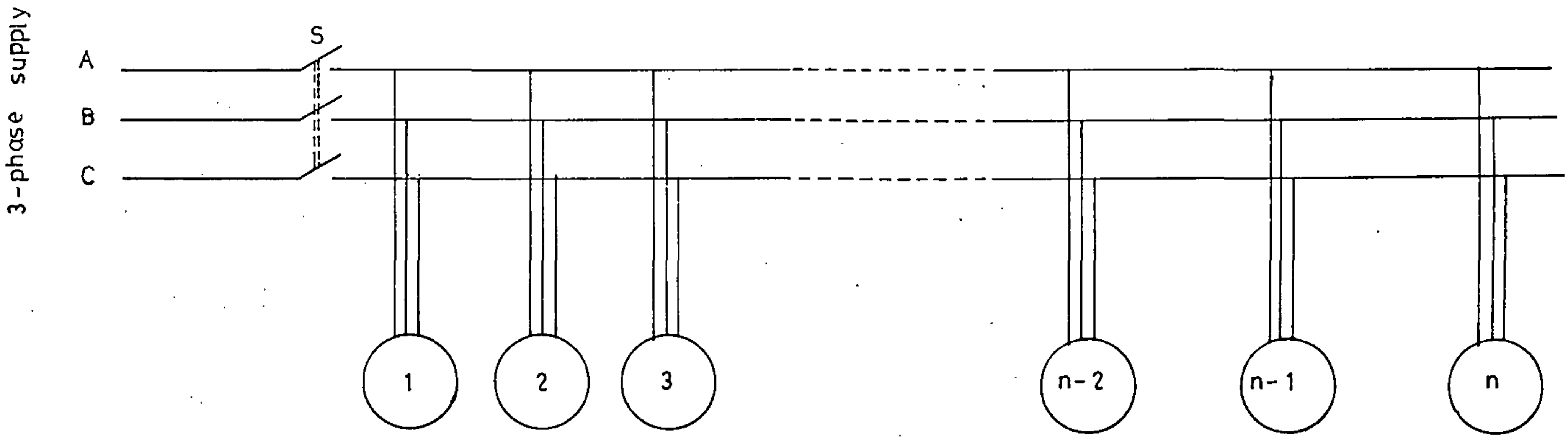


Figure 2.2 Parallel connection of n-indution motor

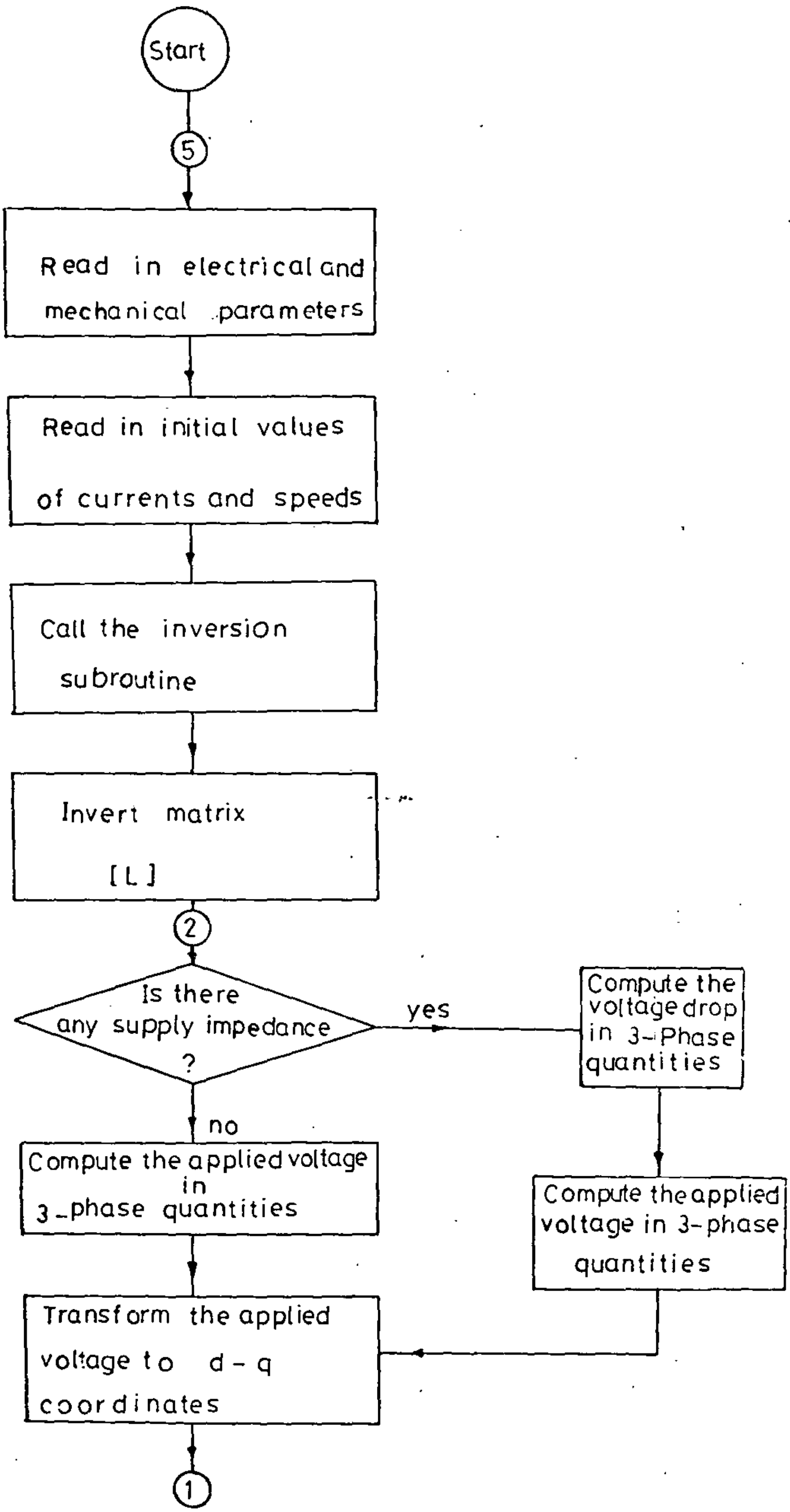


Figure 4·1 Flow chart (Sheet 1)

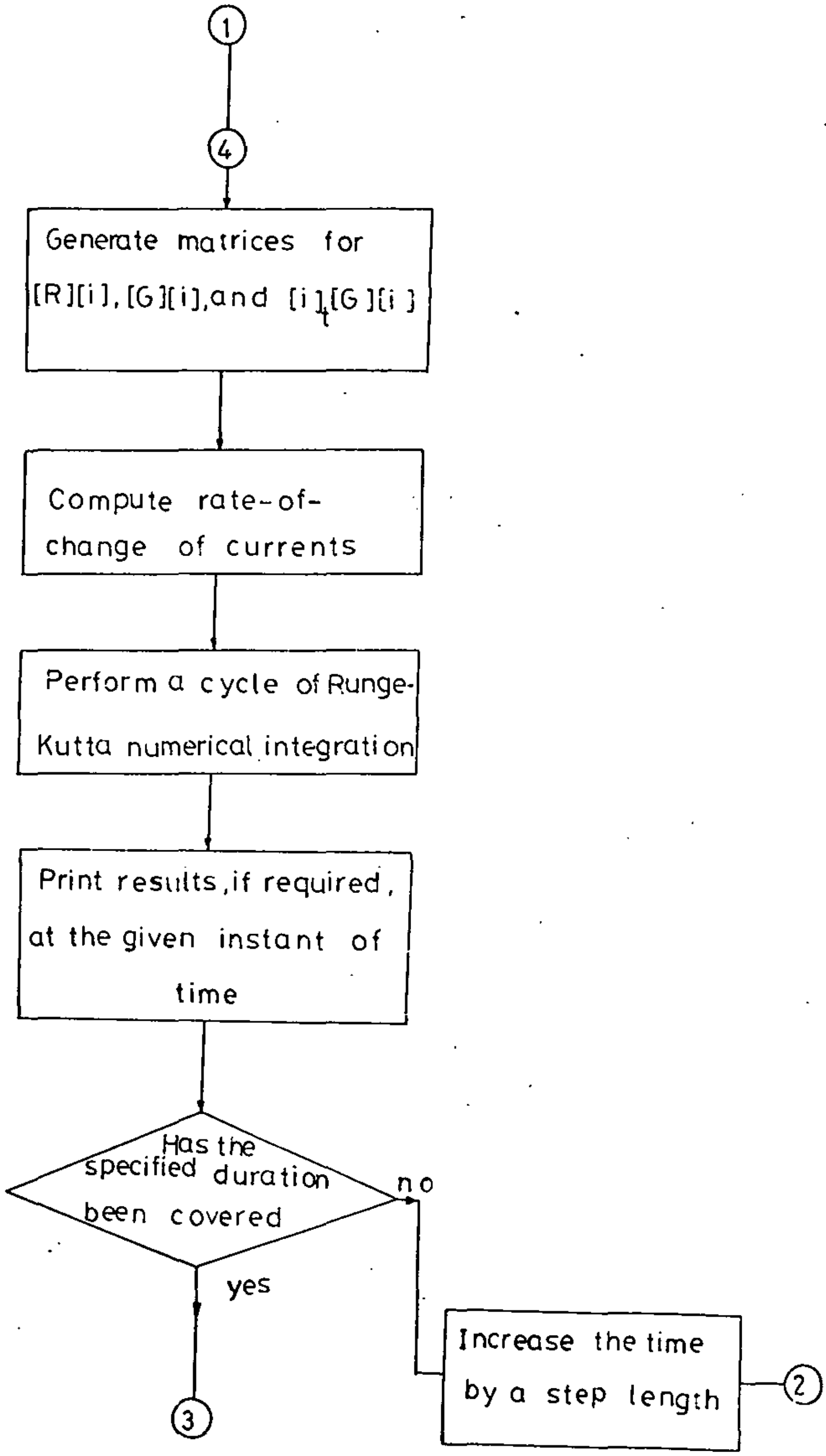


Figure 4·1 Flow chart (Sheet 2)

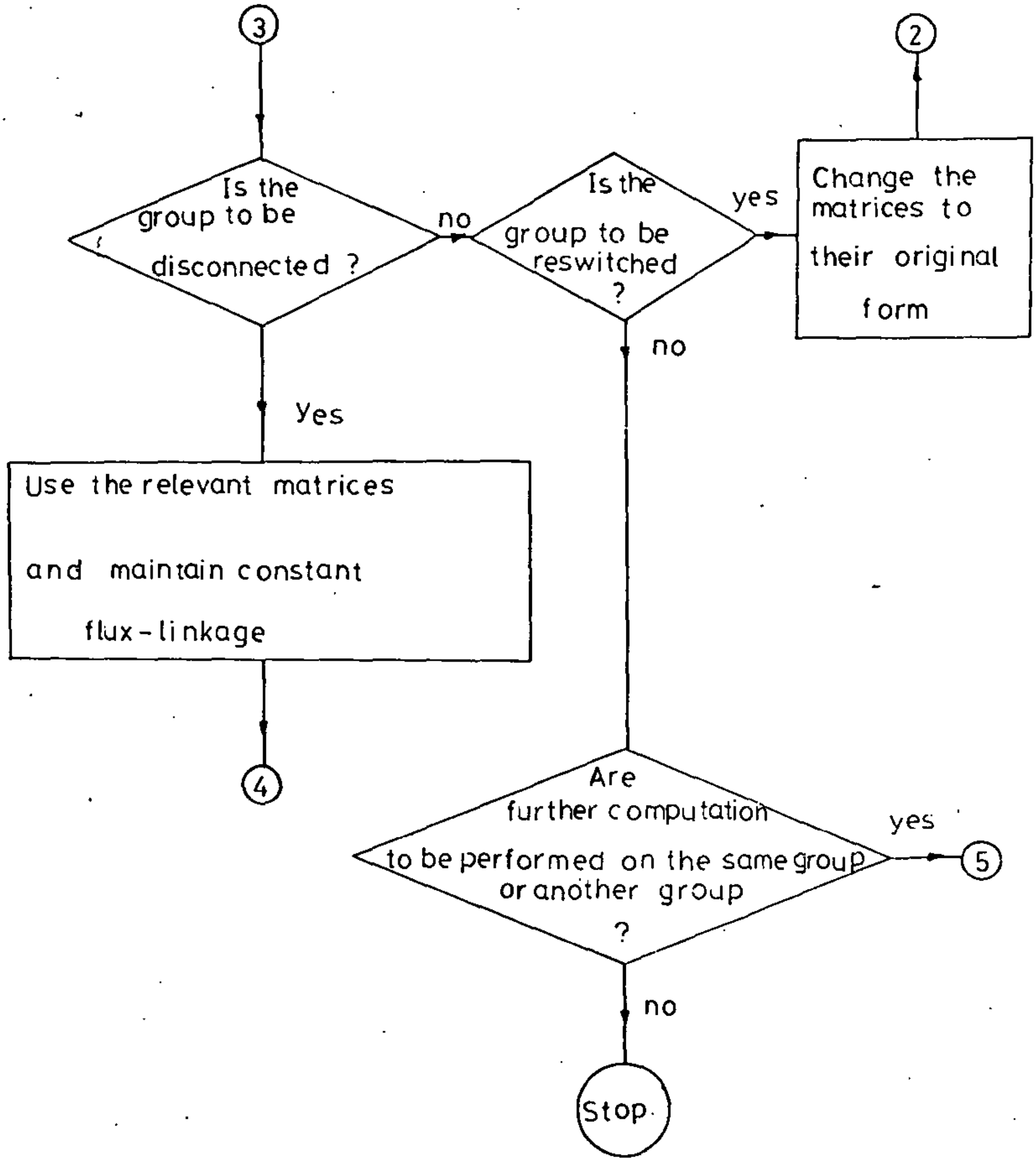


Figure 4·1 Flow chart (Sheet 3)

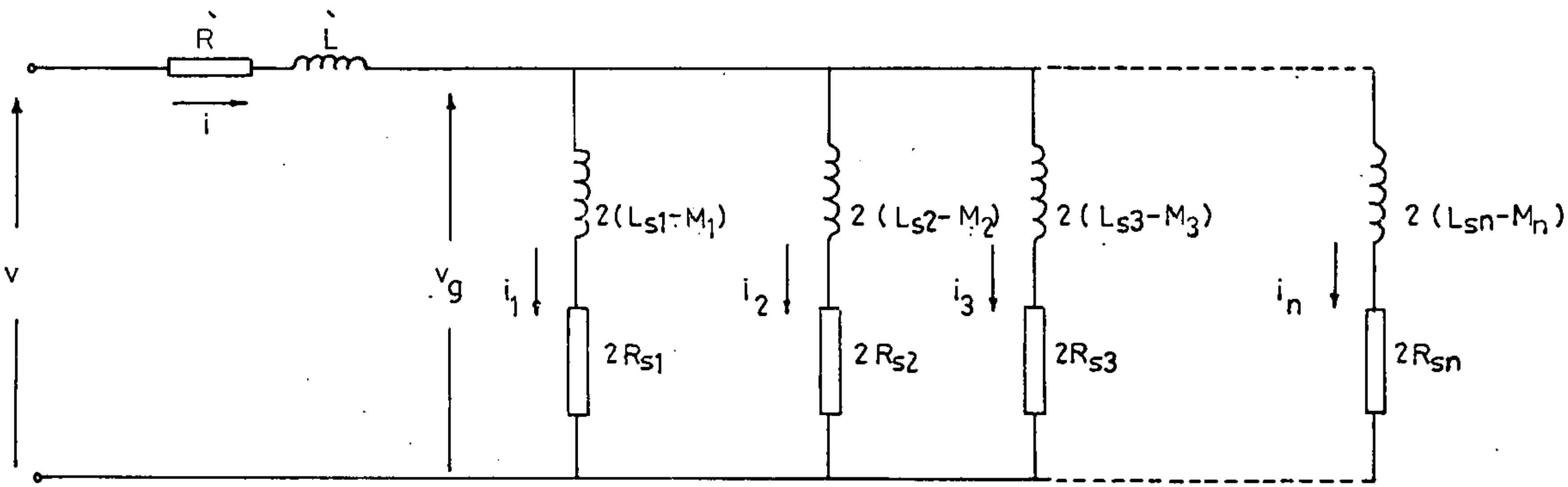


Figure 4·2 Single-phase equivalent circuit at starting.

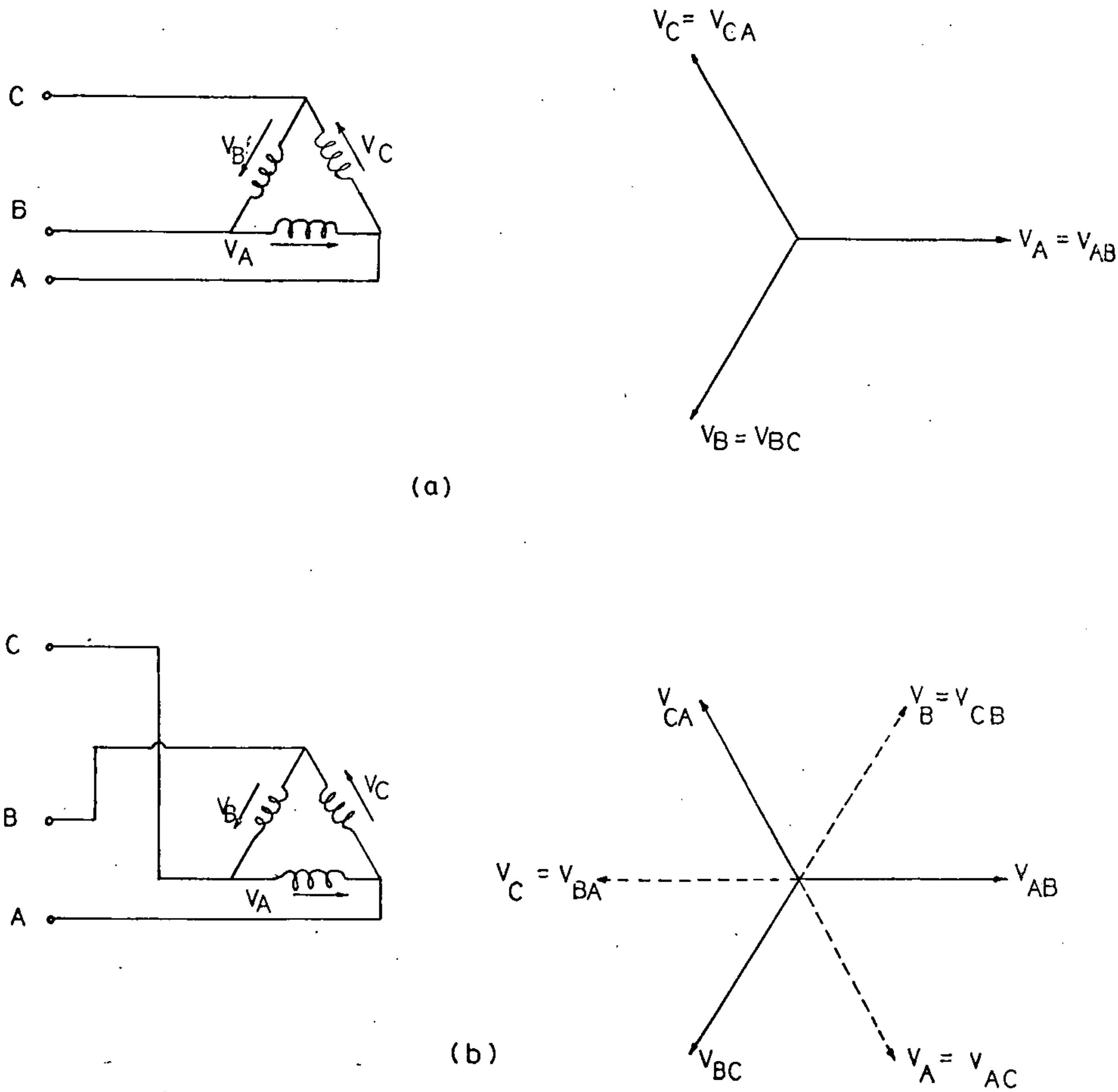


Figure 4·3 Induction motor phase voltages.

(a) Phase sequence ABC.

(b) Phase sequence ACB.

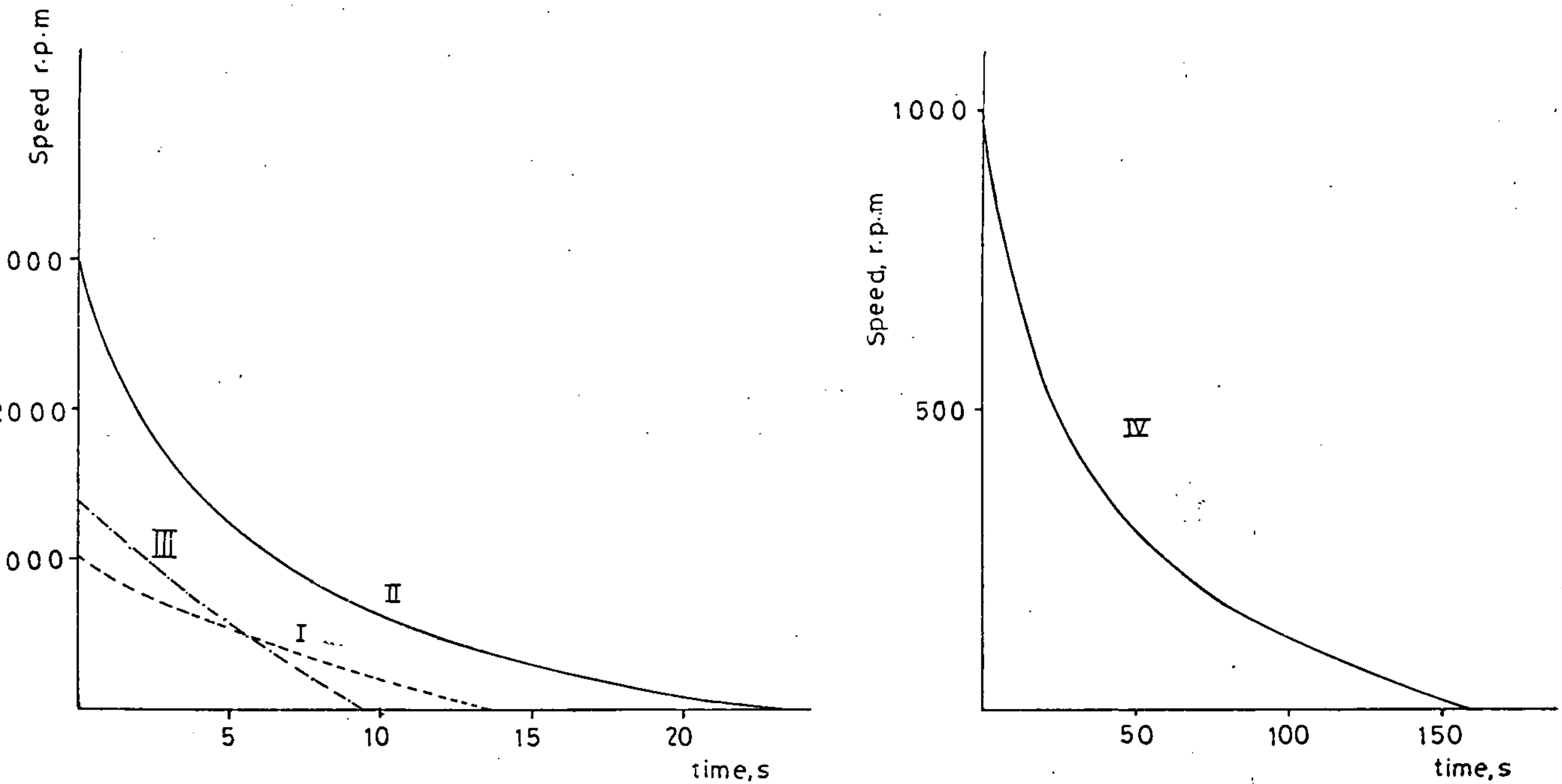


Figure 5·1 Deceleration tests

motor I 0.75 kW, 6-pole

motor II 1·5 kW, 2-pole

motor III 2·25 kW, 4-pole

motor IV 5·6 kW, 6-pole

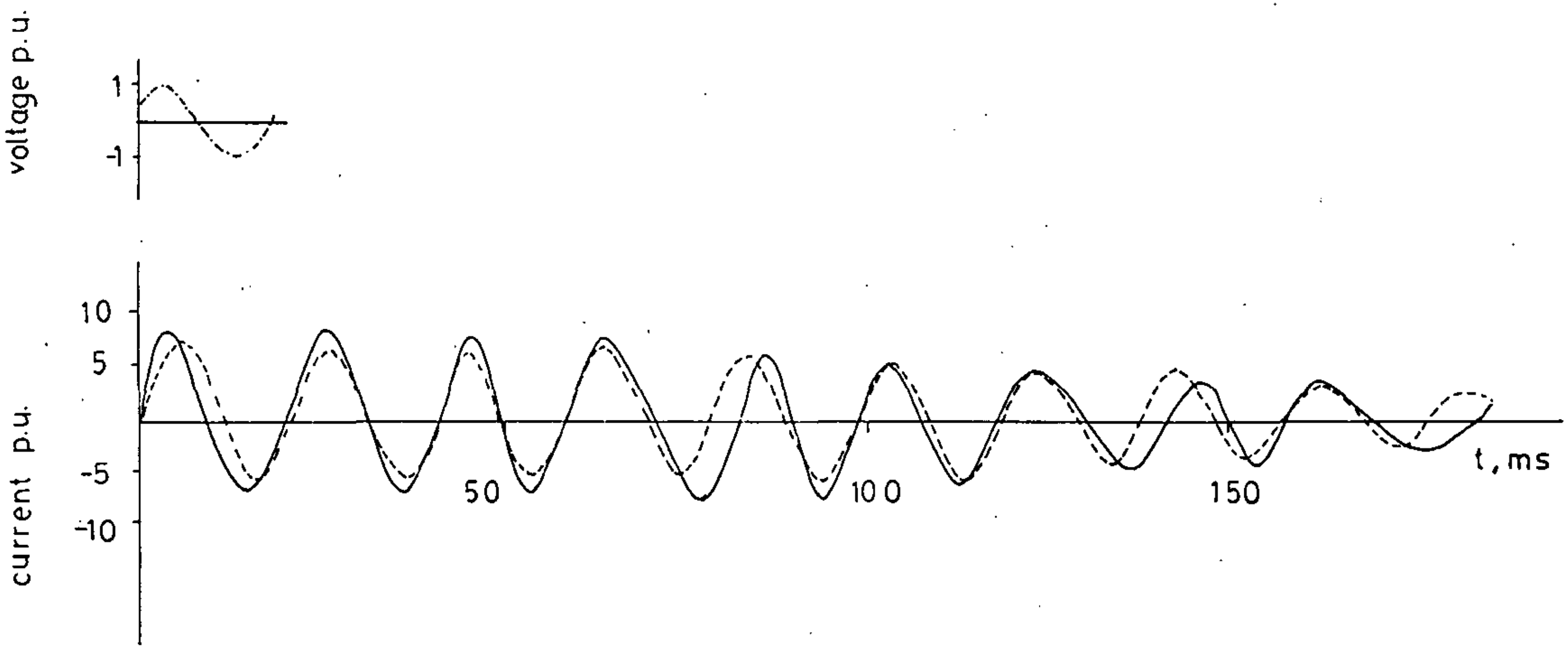


Figure 6·1(a) Transient current in r-line following direct-to-line starting of 0·75 kW motor.

Computed results ------- , Experimental results ———

r-y line voltage -·-·-·-·-

1 p.u. voltage = 345 V , 1 p.u. current = 4·4 A

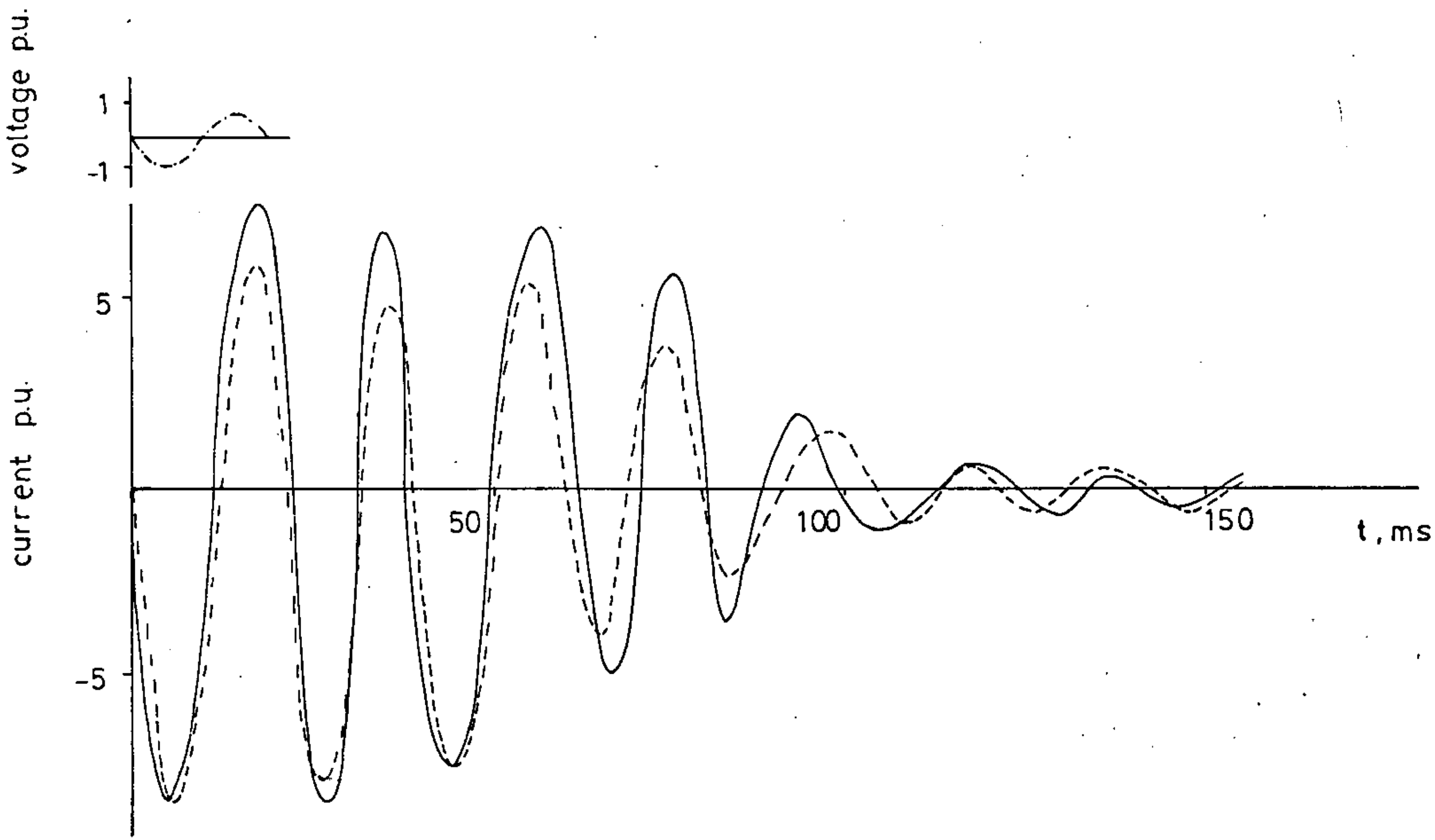


Figure 6·1(b) Transient current in r-line following direct-to-line starting of 1·5 kW motor.

Computed results - - - - , Experimental results ——— , r-y line voltage —·—·—

1 p.u. voltage = 345 V , 1 p.u. current = 5·5 A

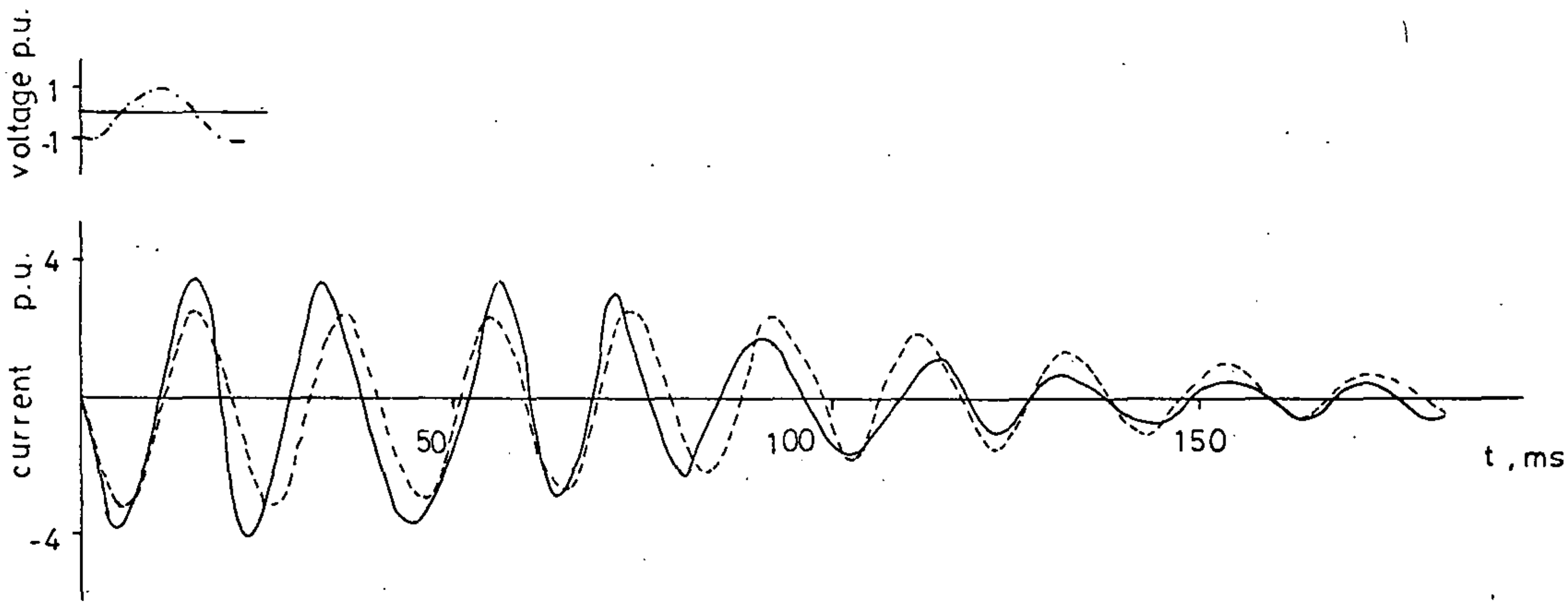


Figure 6·1(c) Transient current in r-line following direct-to-line starting of 2·25 kW motor.

Computed results------ , Experimental results —— , r-y line voltage -·-·-

1 p.u. voltage = 345 V , 1 p.u. current = 12·7 A

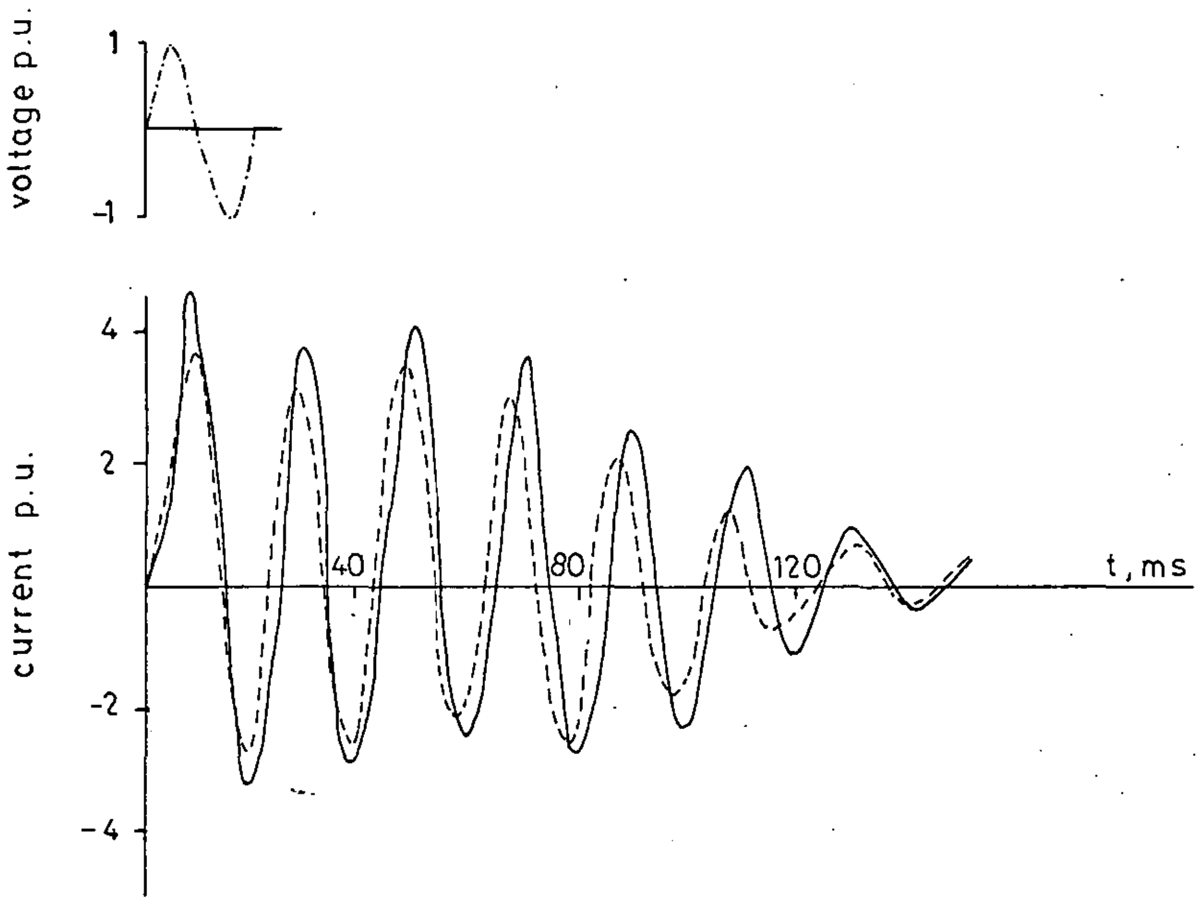


Figure 6.1(d) Transient current in r-phase following direct-to-line starting of 5.6 kW motor.
Computed results ----- , Experimental ——— , r-y line voltage —·—·—
1 p.u. voltage = 565.7 V , 1 p.u. current = 16.8 A

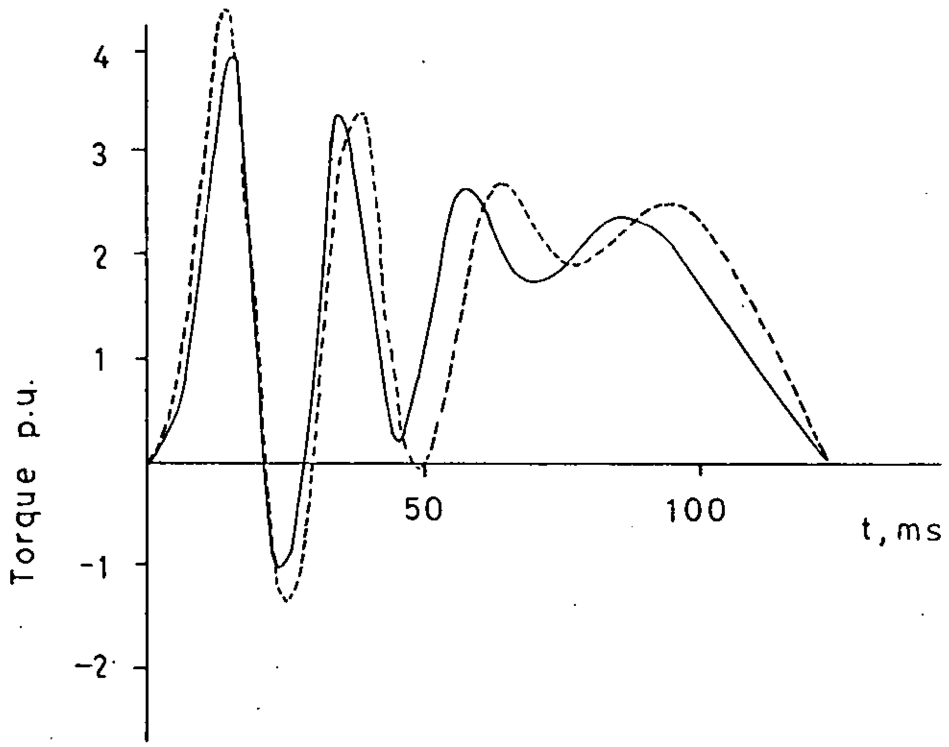


Figure 6·2 (a) Transient torque following connection of 5·6 kW motor at rest.

Computed results ------

Experimental results ———

1 p.u. torque = 56 Nm

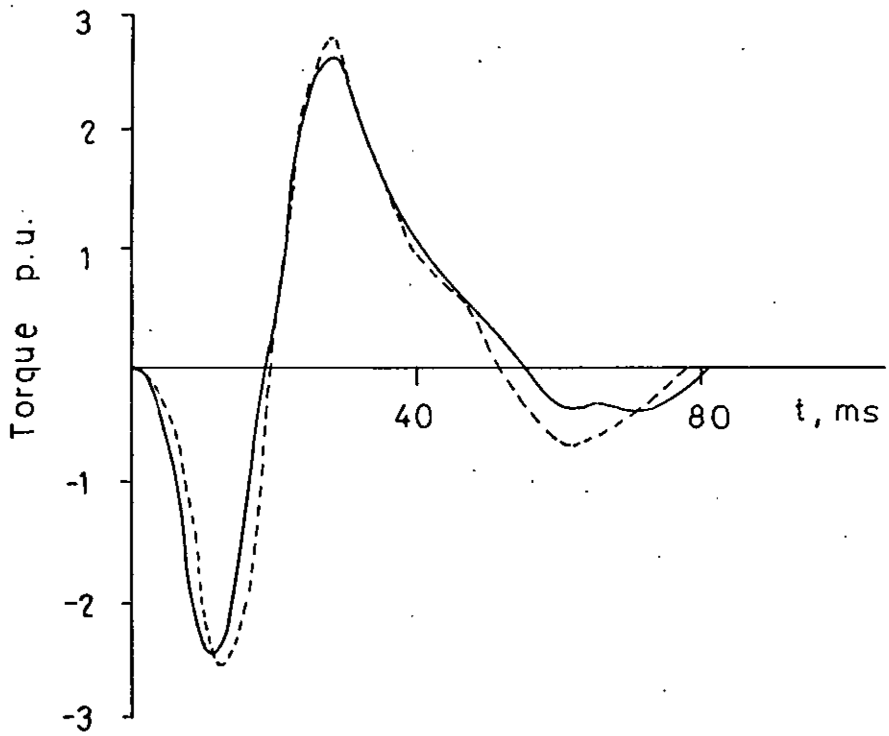


Figure 6·2(b) Transient torque following connection of 5·6 kW motor at 0·95 p.u. speed.

Experimental results ———

Computed results - - - - -

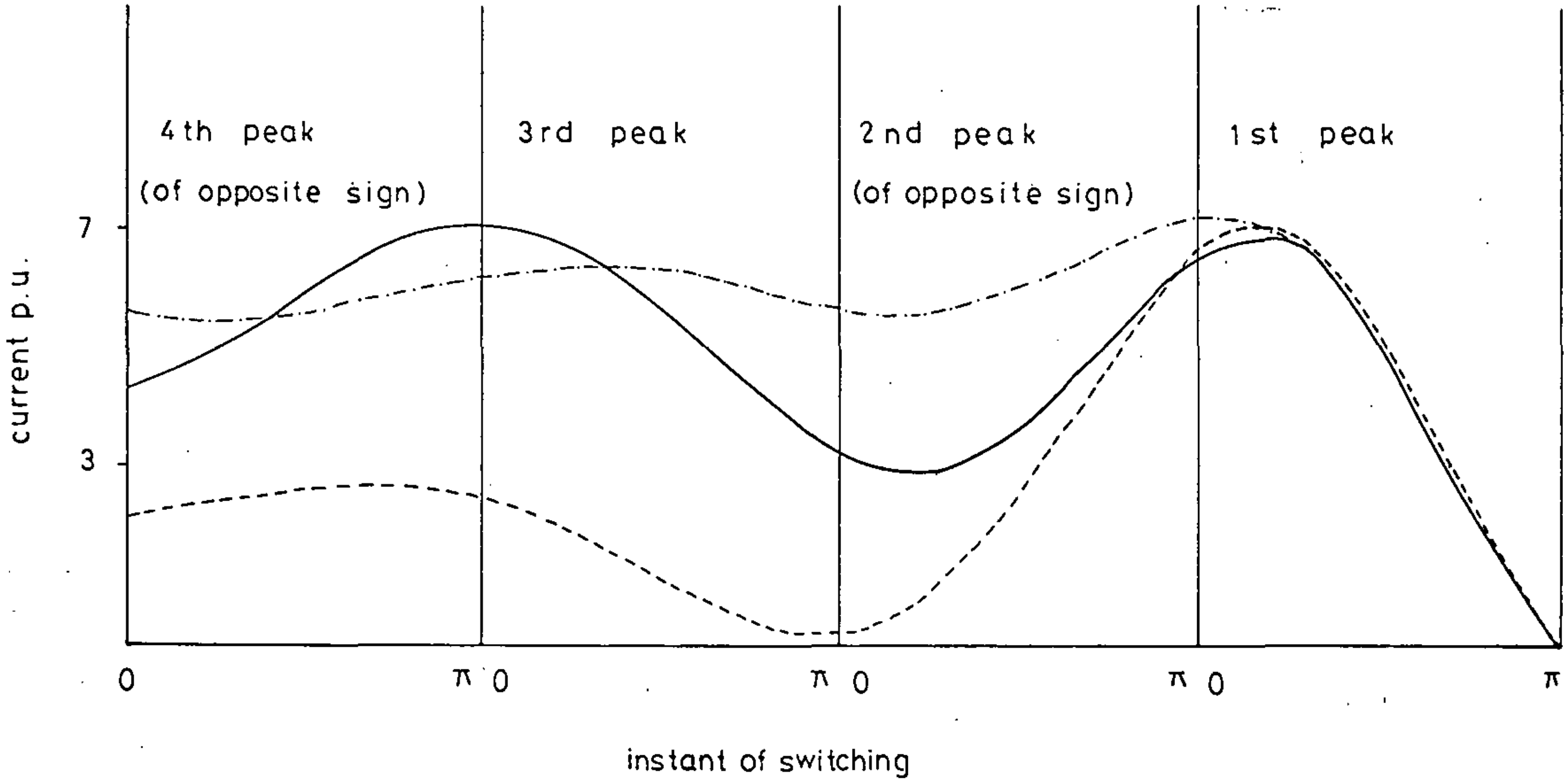


Figure 6·3(a) Variation of peak current with instant of switching of 0·75 kW motor.
standstill —·—·—·— , 0·45 p.u. speed ———— , 0·9 p.u. speed - - - - - -
1 p.u. current = 4·4 A

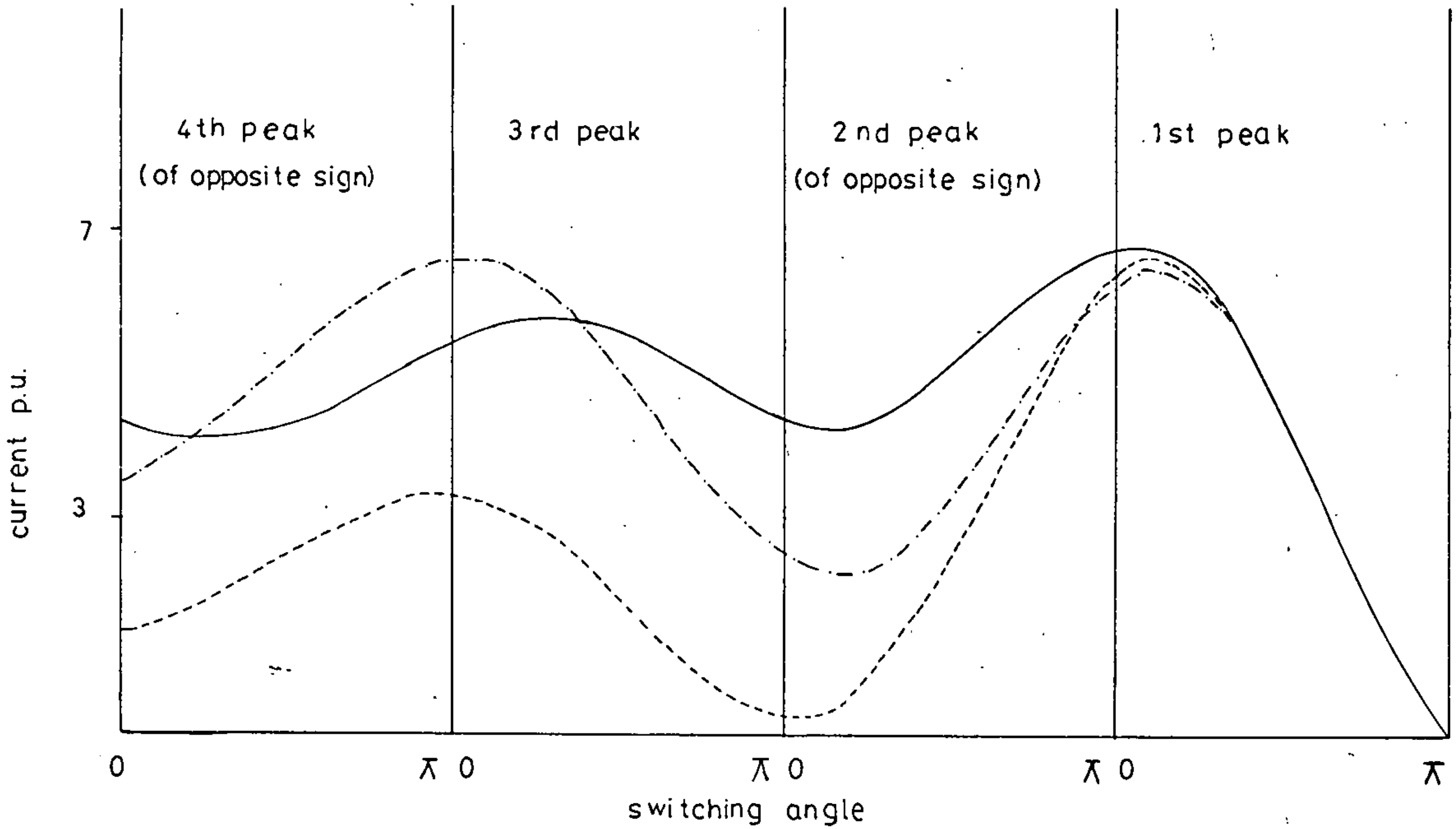


Figure 6·3(b) Variation of peak current with instant of switching of 1·5 kW motor.

standstill ———— ,0·45 p.u. speed —·—·—·— ,0·9 p.u. speed - - - - - -

1 p.u. current = 5·5 A

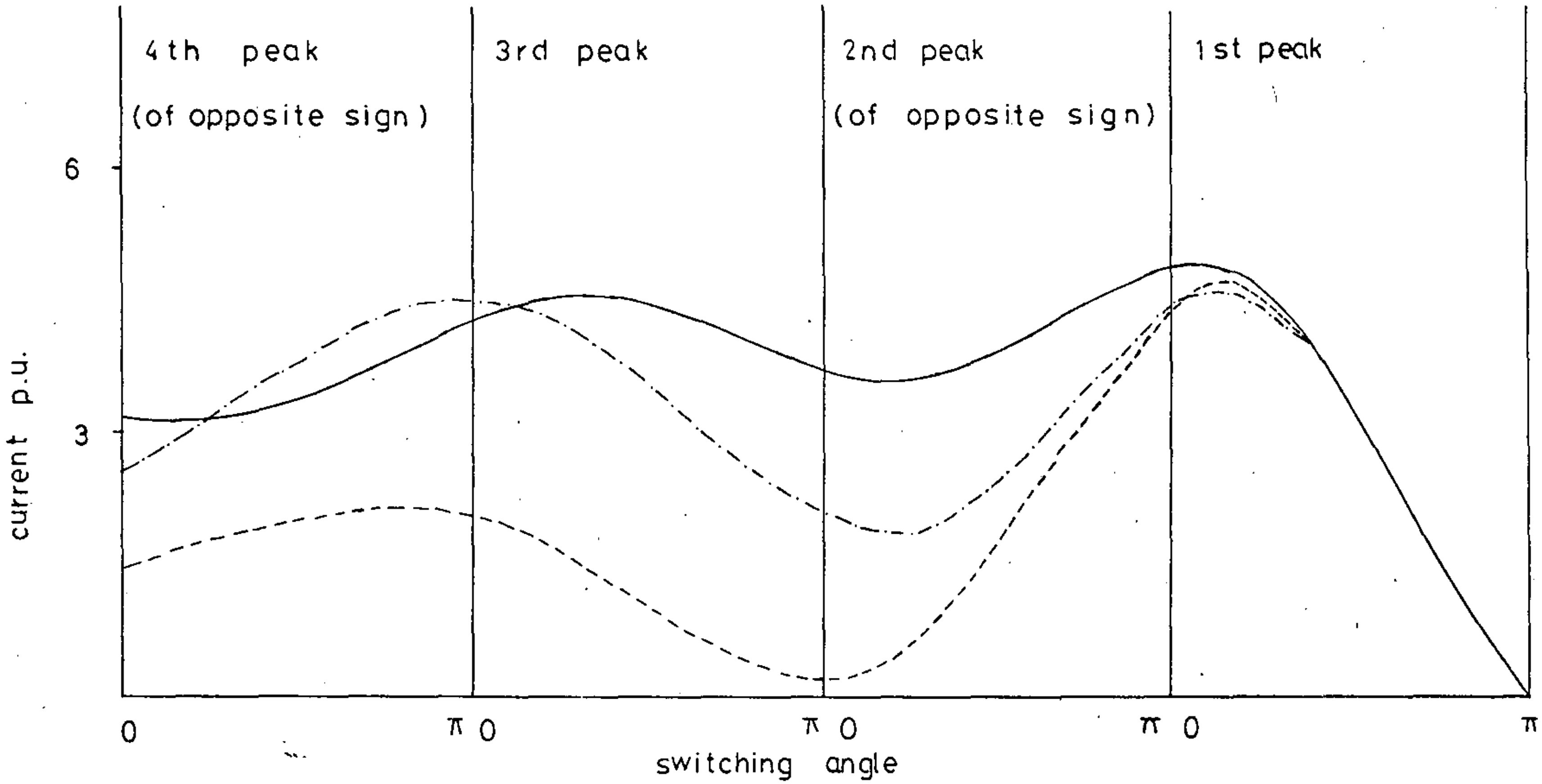


Figure 6·3 (c) Variation of peak current with instant of switching of 2·25 kW motor.
standstill ———, 0·45 p.u. speed –·–·–·, 0·9 p.u. speed - - - - -

1 p.u. current = 12·7 A

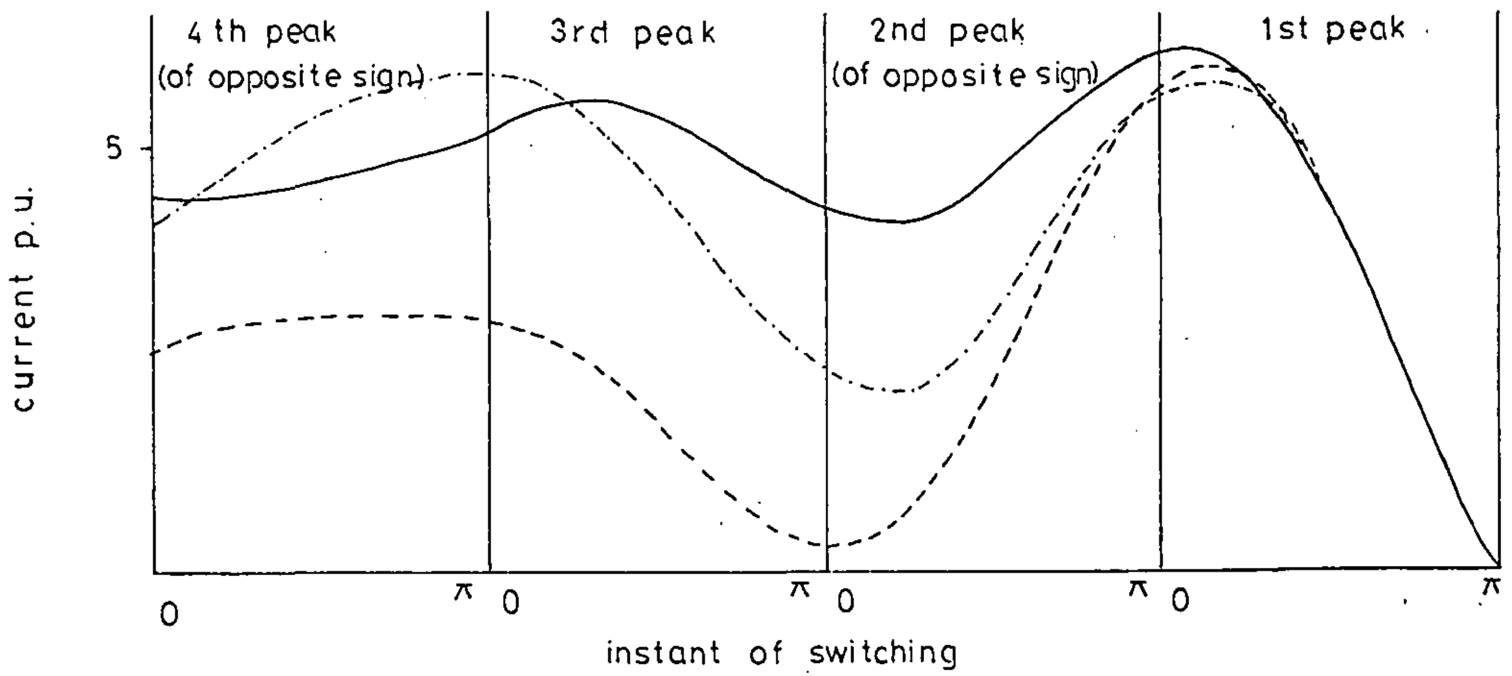


Figure 6·3 (d) Variation of peak current with instant of switching of 5·6 kW motor.
standstill ———, 0·45 p.u. speed —·—·—, 0·9 p.u. speed - - - - -

1 p.u. current = 16·8 A

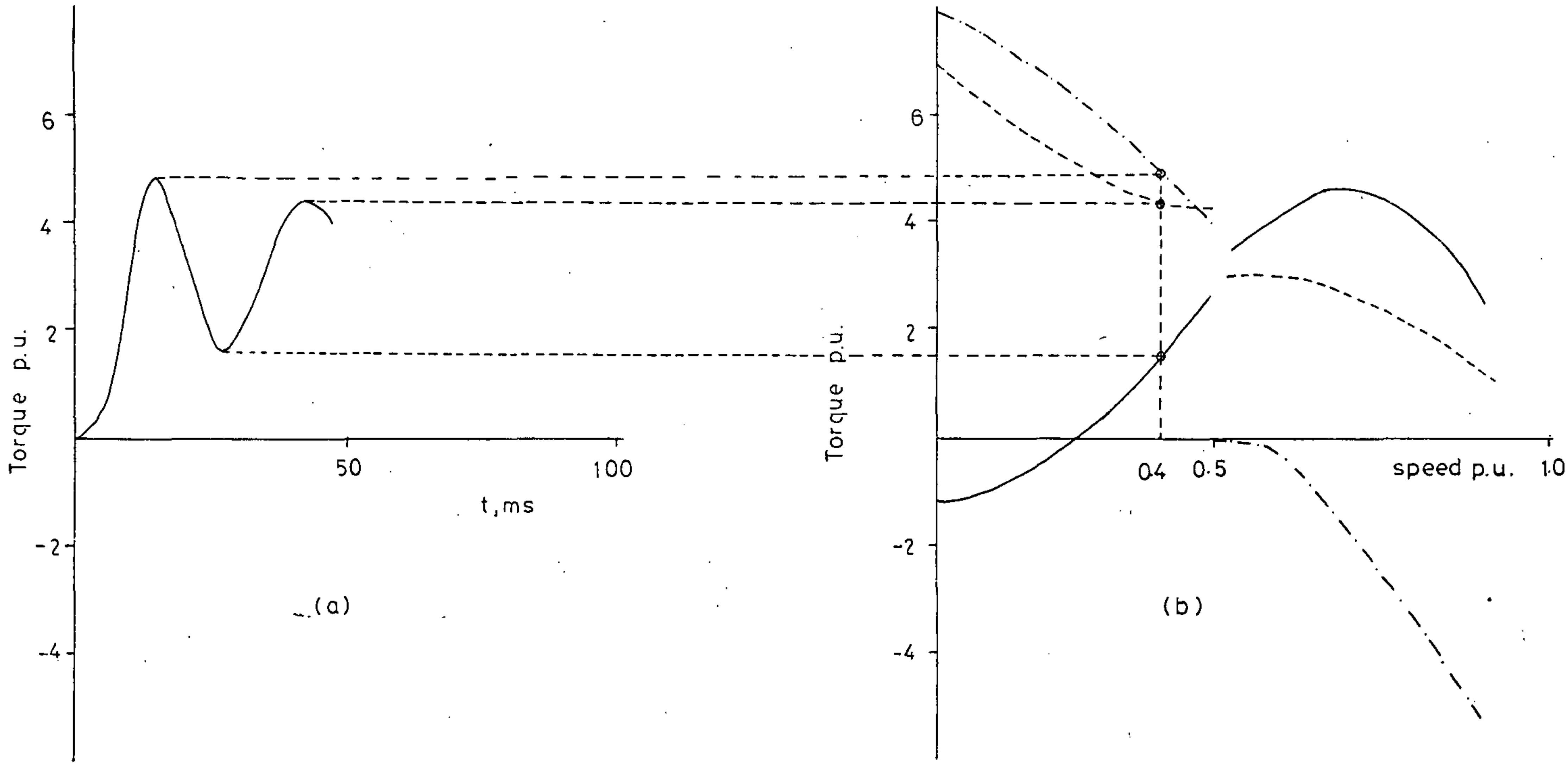


Figure 6.4 Effect of speed on connection on maximum and minimum values of transient torque of 0.75 kW motor.

(a) Computed torque pattern following connection at 0.4 pu speed.
(b) Variation of maximum and minimum torque values with speed.
1st -·-·-·, 2nd ———, 3rd - - - - -, 1 p.u. torque = 7.4 Nm.

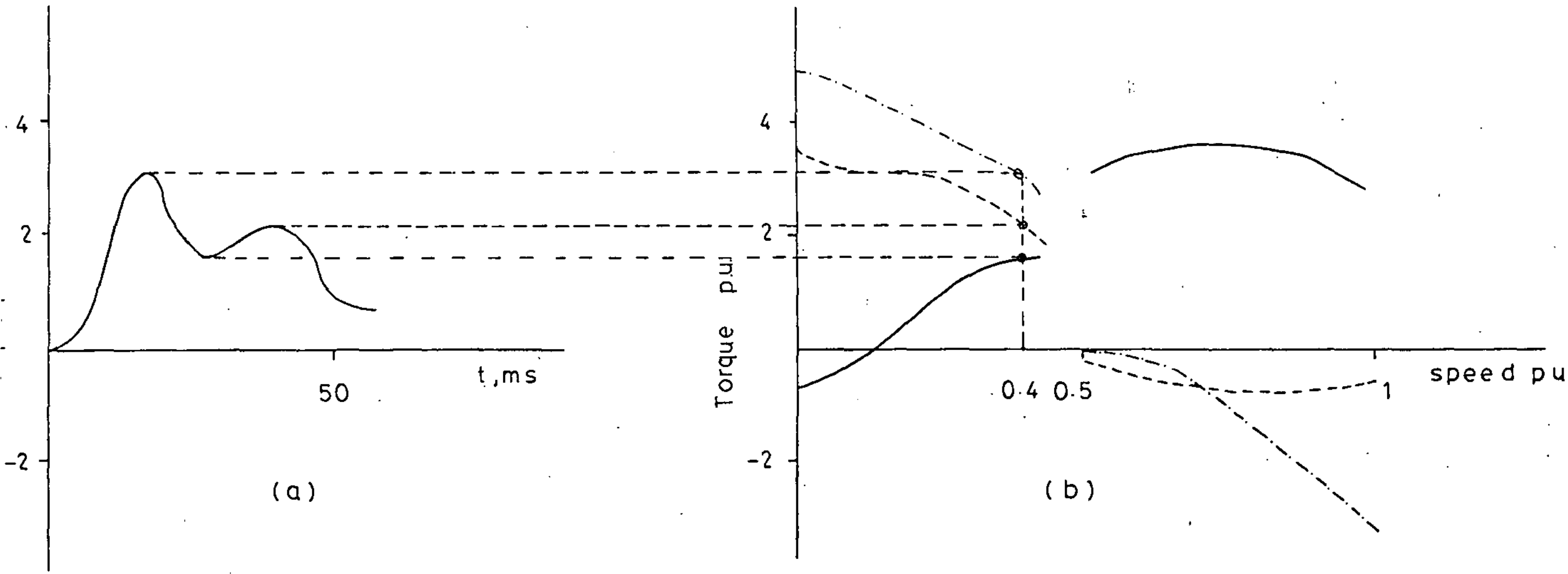


Figure 6.5 Effect of speed on connection on maximum and minimum values of transient torque of 1.5 kW motor.

(a) Computed torque pattern following connection at 0.4 p.u. speed.

(b) Variation of maximum and minimum torque values with speed.

1st —·—·—·—, 2nd ———— ,3rd - - - - - - - , 1 p.u. torque = 4.7 Nm.

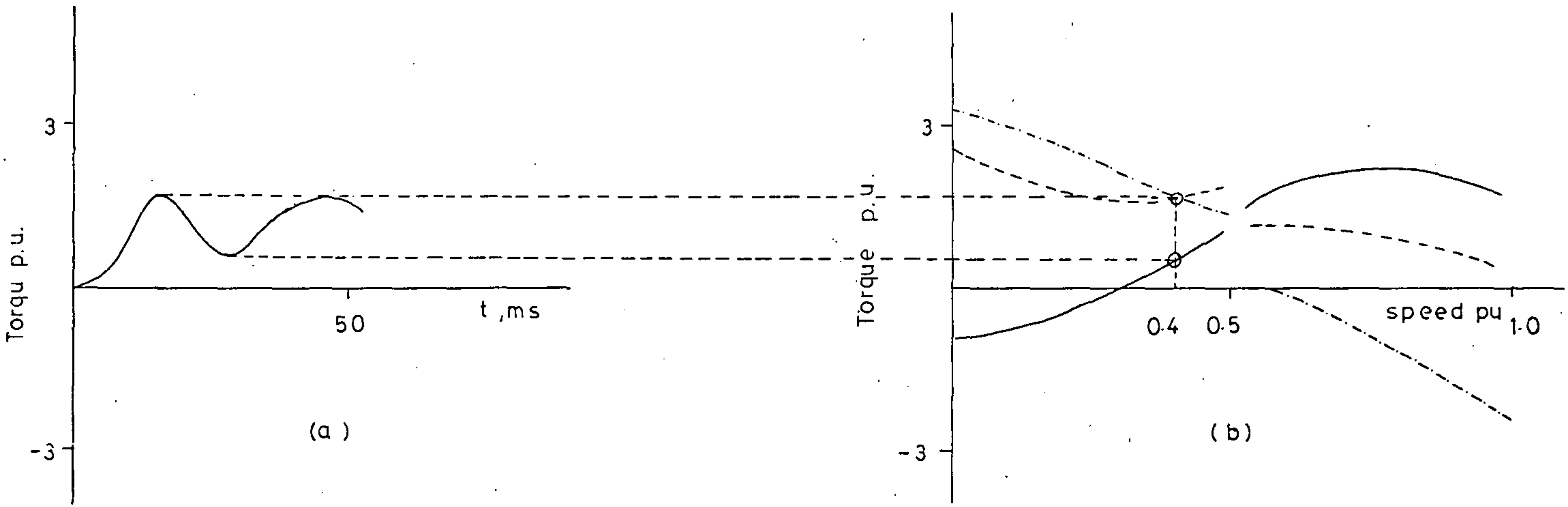


Figure 6.6 · Effect of speed on connection on maximum and minimum values of transient torque of 2.25 kW motor.

(a) Computed torque pattern following connection at 0.4 pu speed.

1st —·—·—, 2nd ———, 3rd - - - - - -, 1 p.u. torque = 15.5 Nm.

(b) Variation of maximum and minimum torque values with speed.

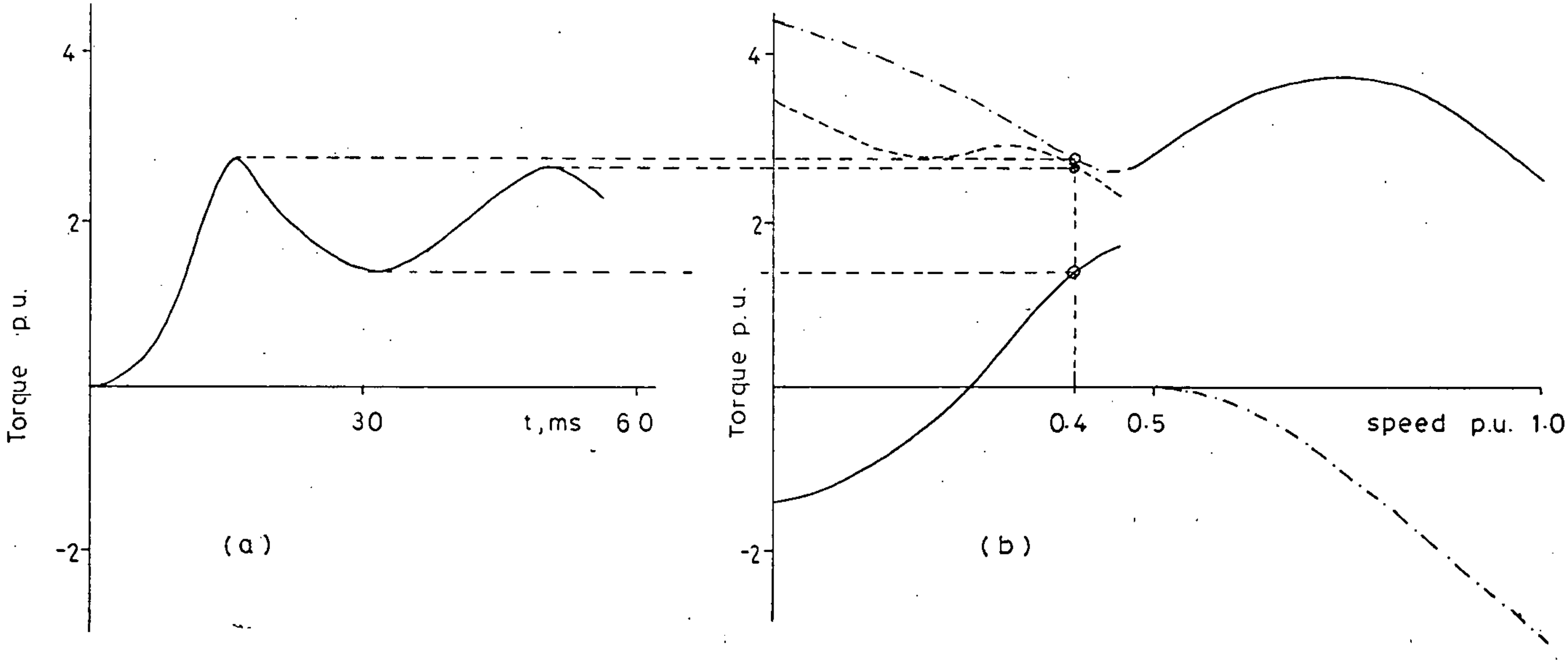


Figure 6.7 Effect of speed on connection on maximum and minimum values of transient torque of 5.6 kW motor.
(a) Computed torque pattern following connection at 0.4 p.u. speed.
(b) Variation of maximum and minimum torque values with speed.
1st –·–·–·– , 2nd ———— , 3rd - - - - - - - , 1 p.u. torque = 56 Nm.

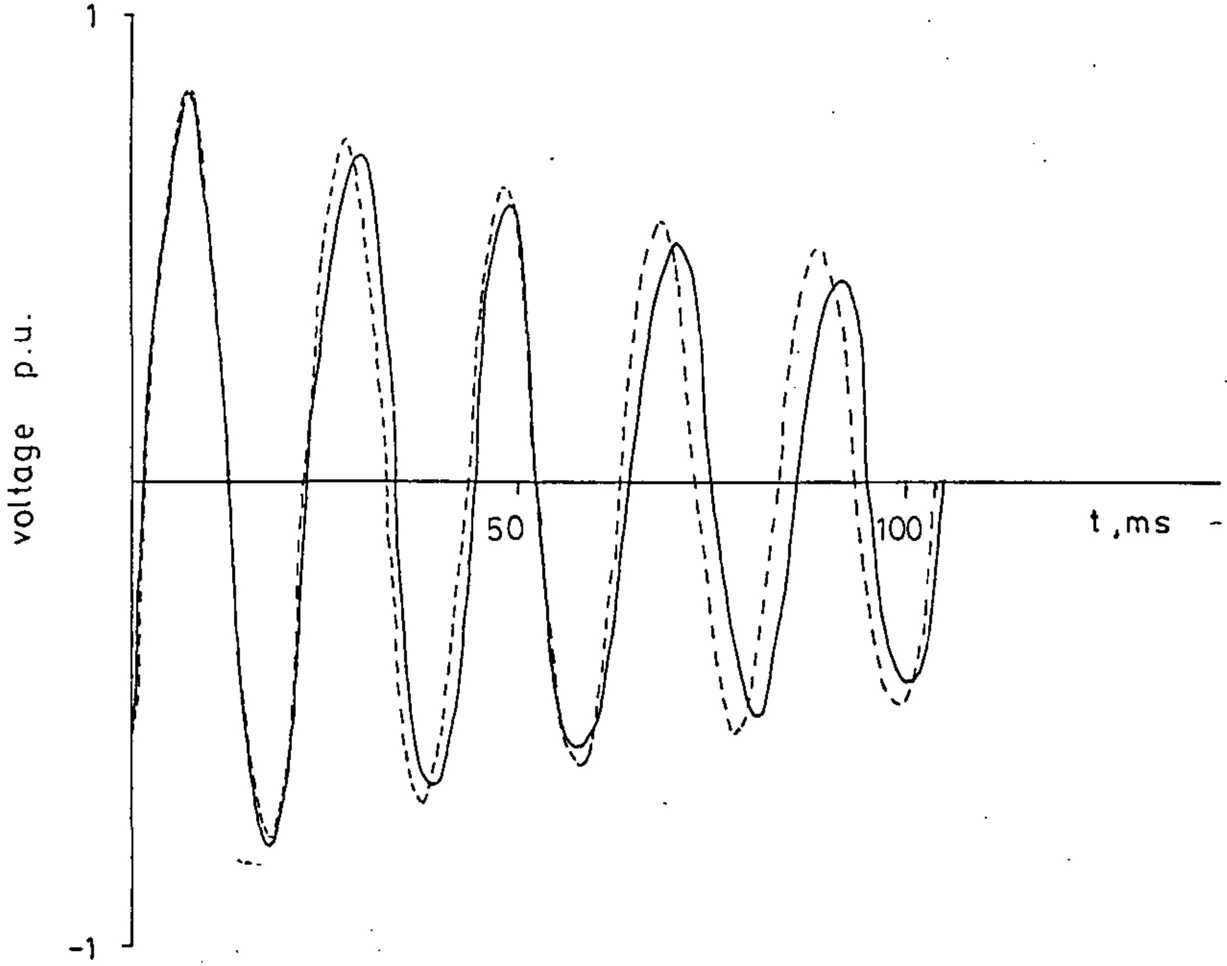


Figure 6.8 (a) Terminal voltage following disconnection of 0.75 kW motor.

Computed results --------, Experimental results ———

1 p.u. voltage = 345 V.

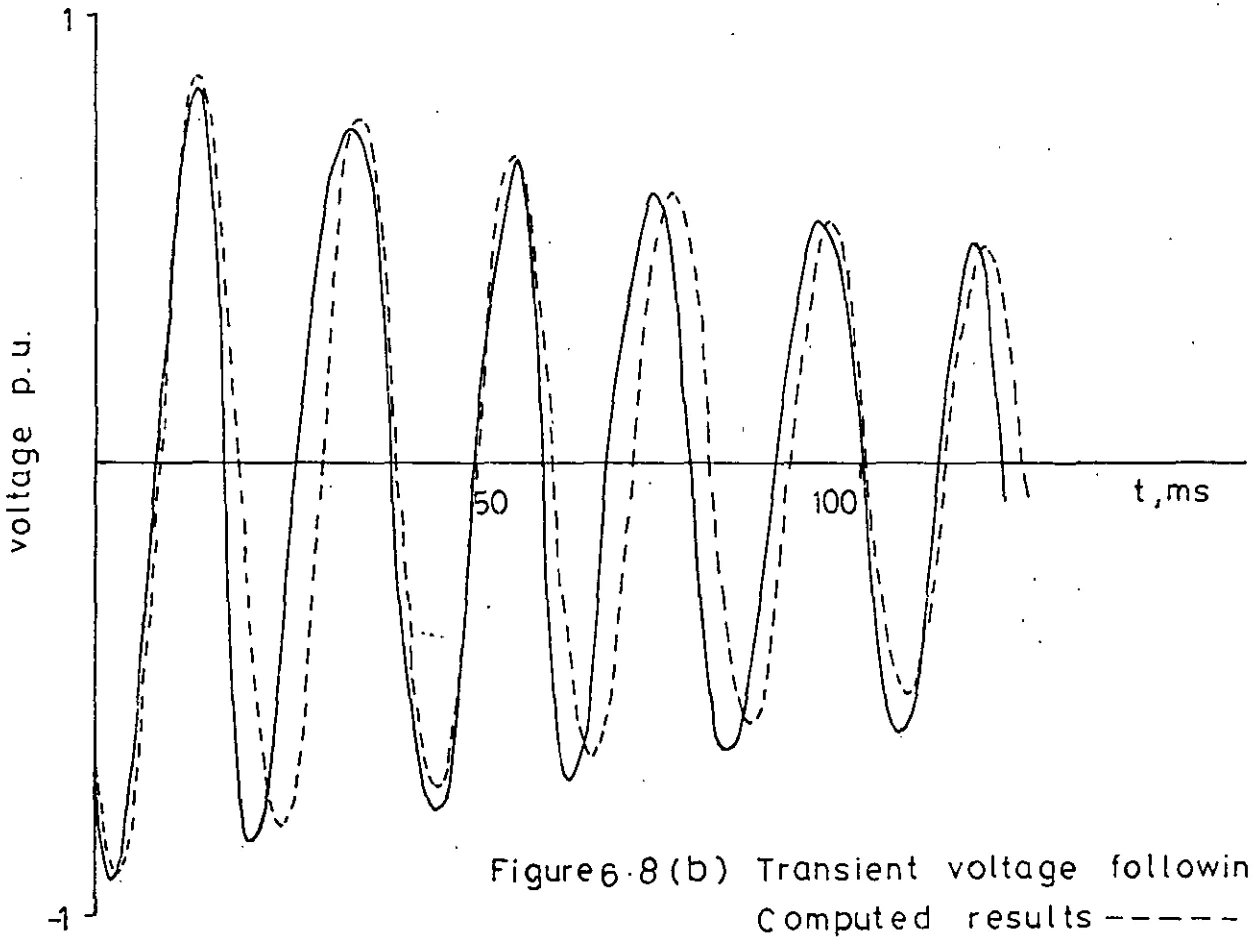


Figure 6·8 (b) Transient voltage following disconnection of 1·5 kW motor.
Computed results - - - - - , Experimental results ———

1 p.u. voltage = 345 V.

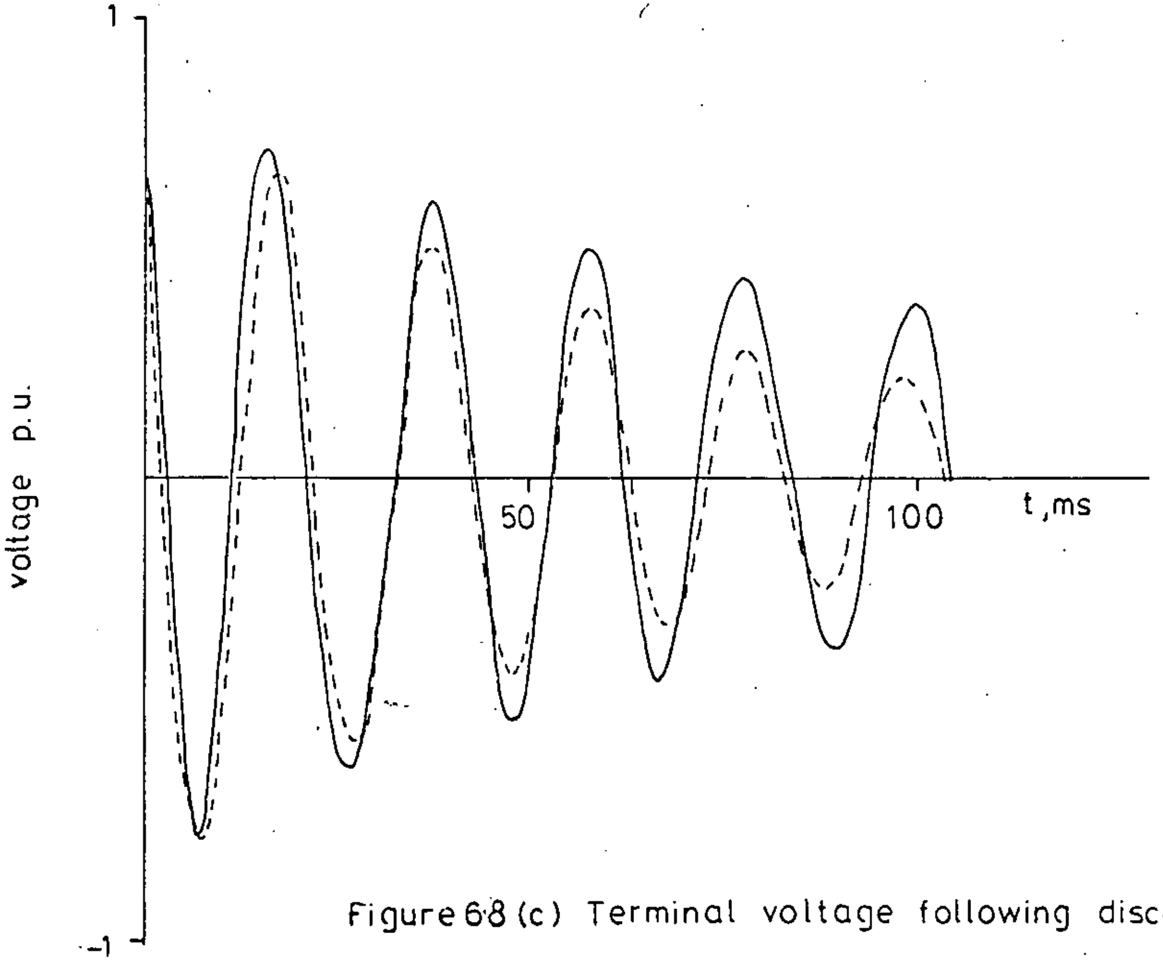


Figure 6·8 (c) Terminal voltage following disconnection of 2·25 kW motor.

Computed results --------, Experimental results ————

1 p.u. voltage = 345 V.

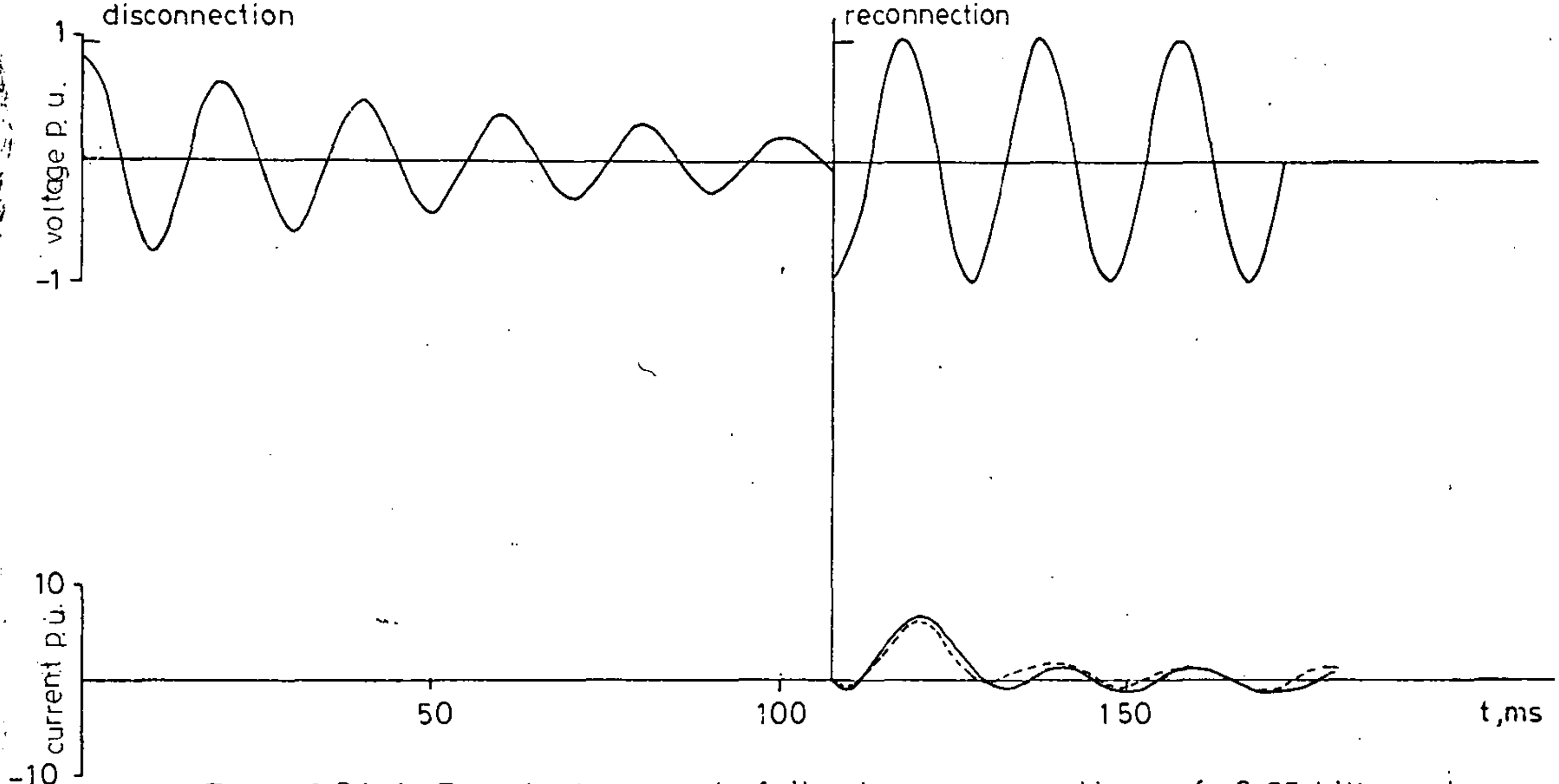


Figure 6·9(a) Transient current following reconnection of 0·75 kW motor.
Computed results ------ , Experimental results ——

1 p.u. current = 4·4 A , 1 p.u. voltage = 345 V

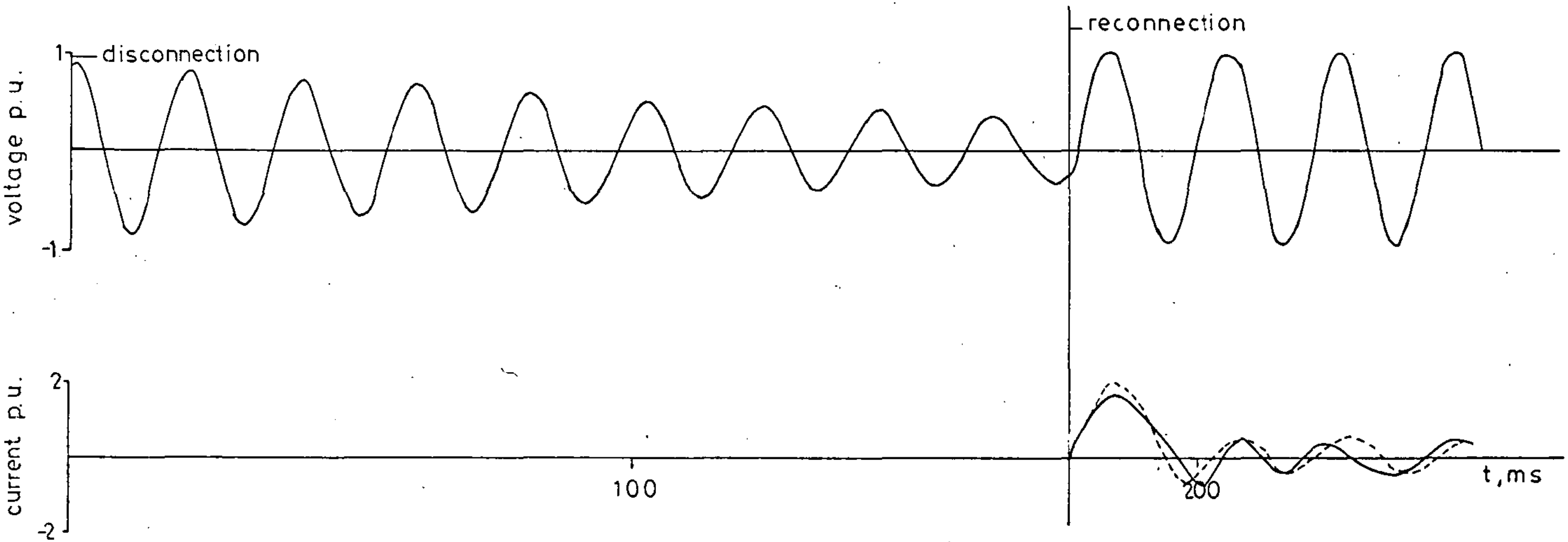


Figure 6·9(b) Transient current following reconnection of 1·5 kW motor.
Computed results --------, Experimental results ————
1 p.u. current = 5·5 A, 1 p.u. voltage = 345 V

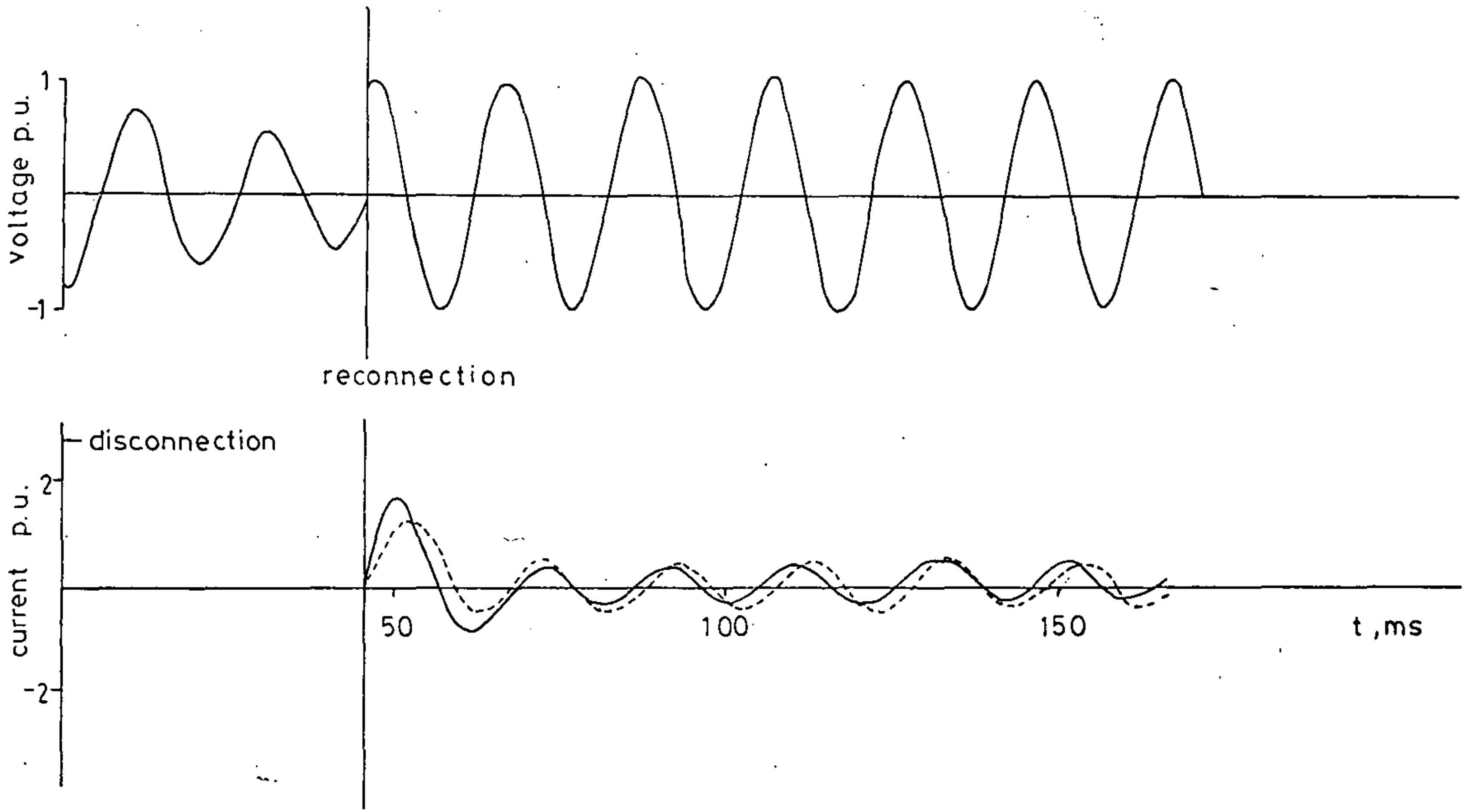


Figure 6·9 (c) Transient current following reconnection of 2·25 kW motor.

Computed results ---------, Experimental results ———

1 p.u. current = 12·7 A , 1 p.u. voltage = 345 V.

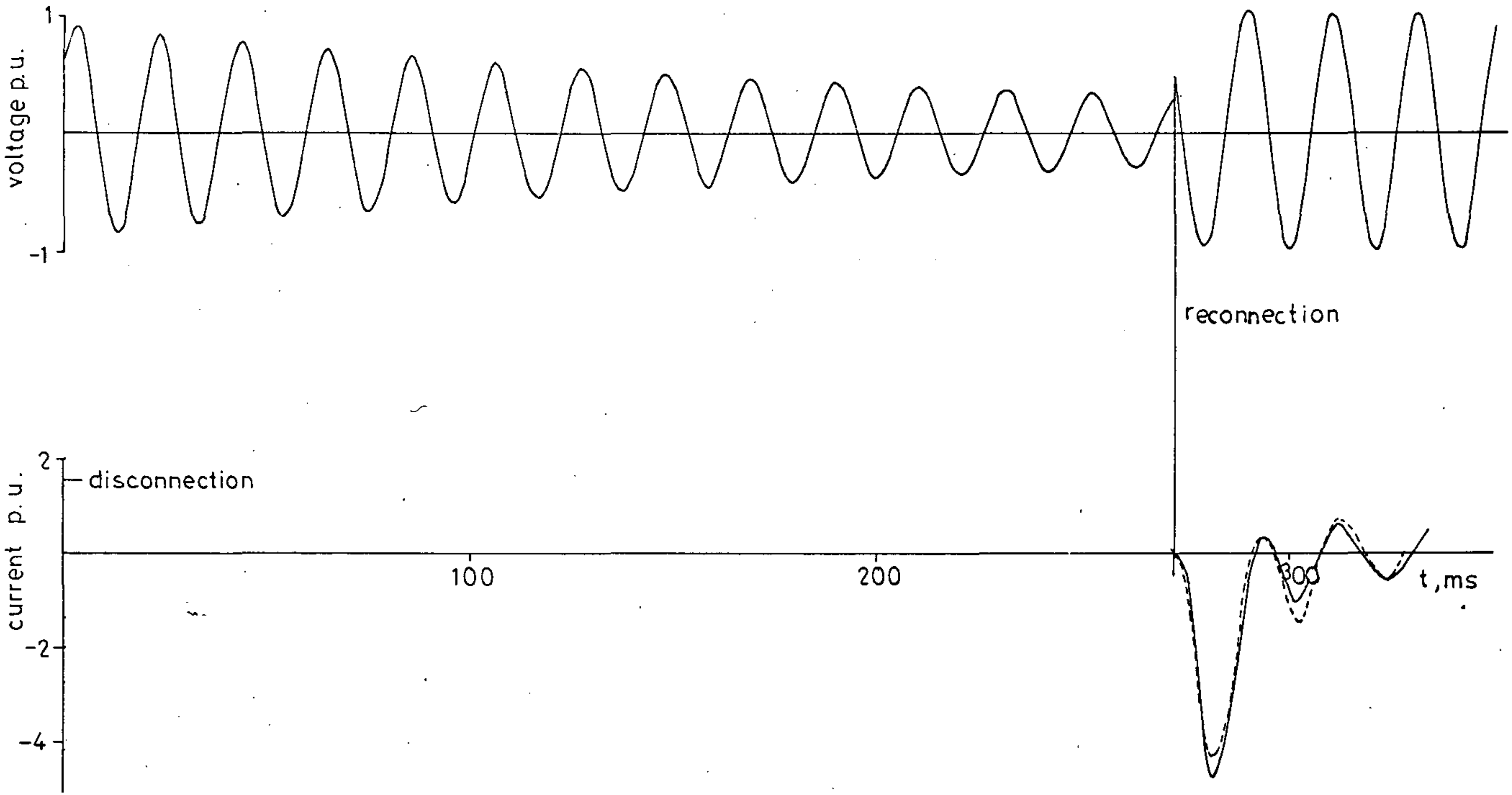


Figure 6.9(d) Transient current following reconnection of 5.6 kW motor.
Computed results ------, Experimental results ———
1 p.u. current = 16.8 A , 1 p.u. voltage = 565.7 V.

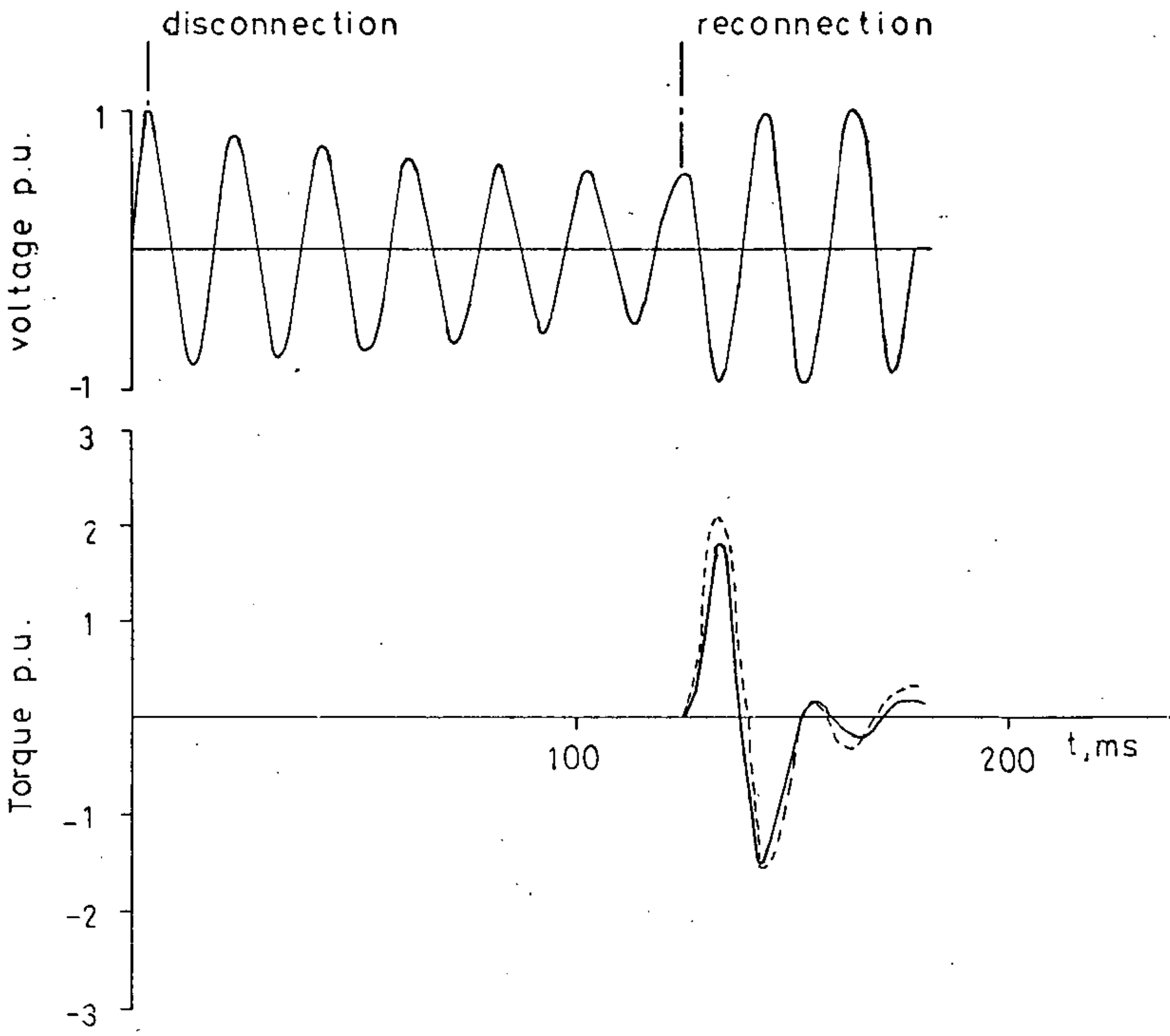


Figure 6·10 Transient torque following reconnection of 5·6 kW motor.

Computed results - - - - - -

Experimental results ————

1 p.u. torque = 56 Nm.

1 p.u. voltage = 565·7 V

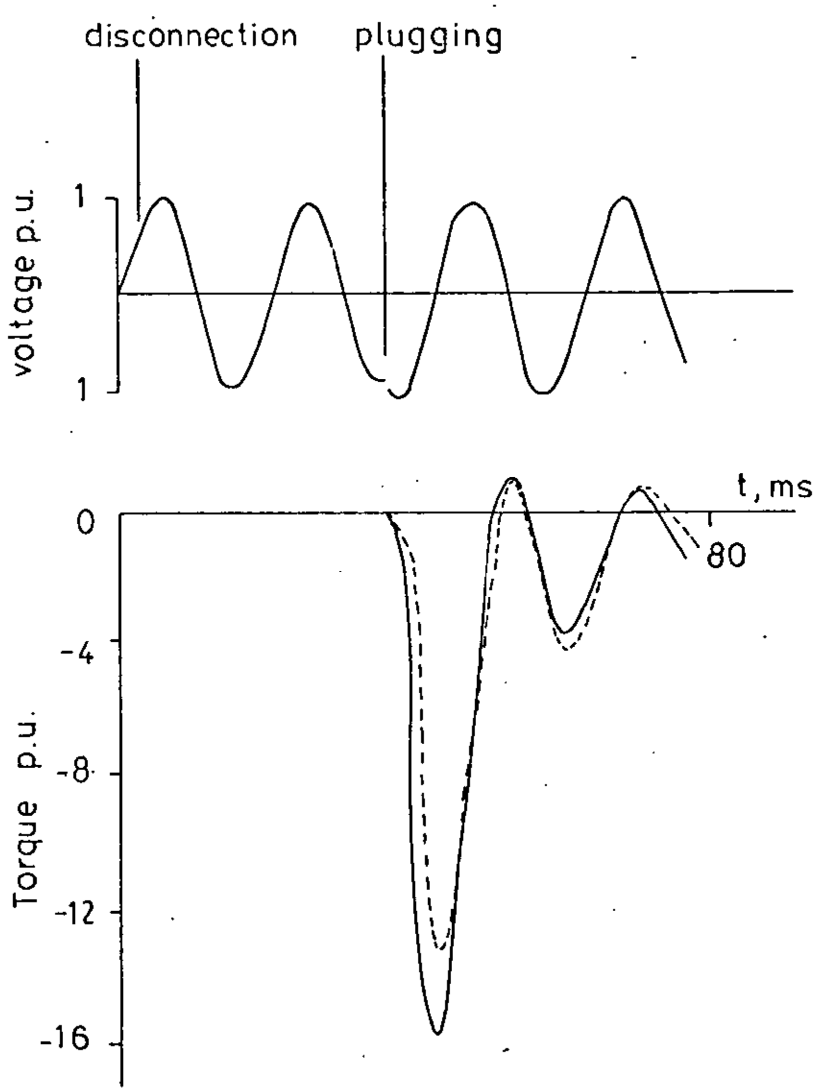


Figure 6.11(a) Transient torque following plugging of 5.6 kW motor.
Computed results ------
Experimental results ———
1 p.u. torque = 56 Nm, 1 p.u. voltage = 565.7 V

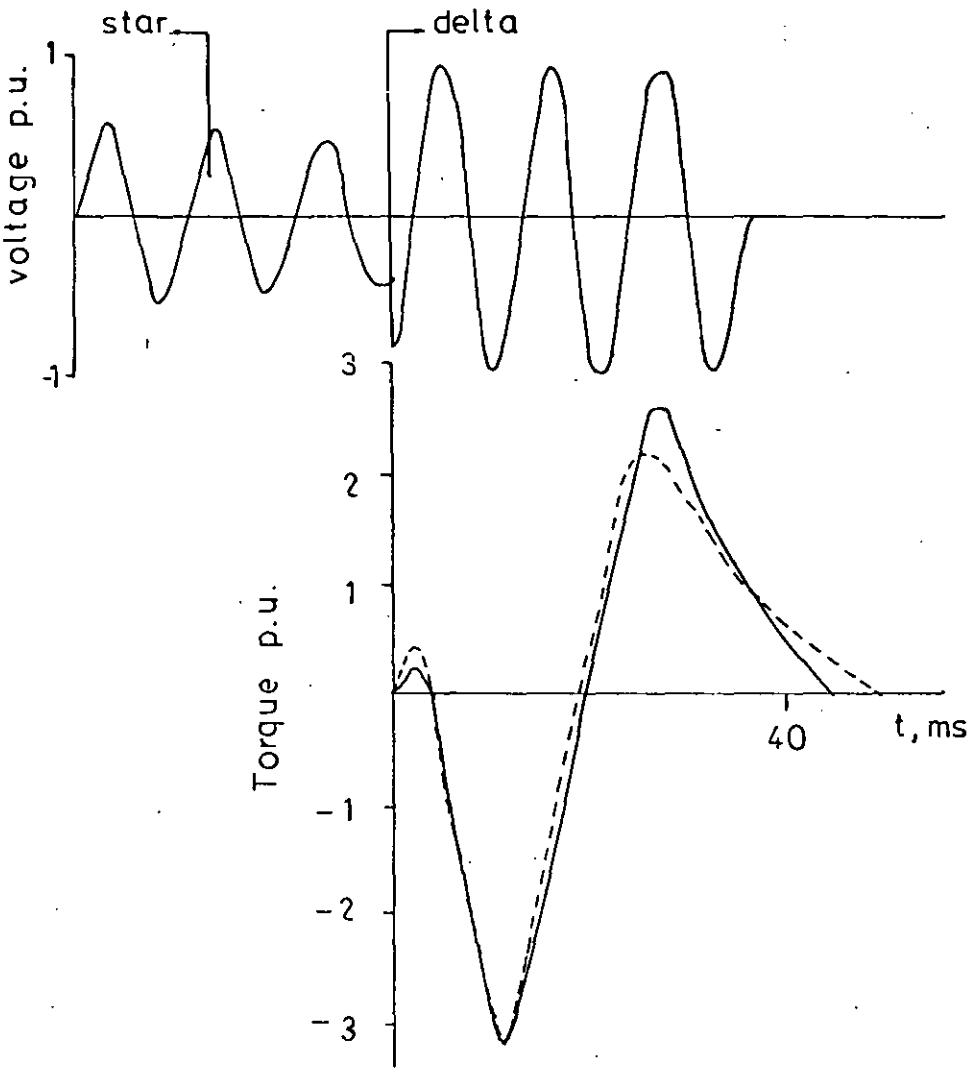


Figure 6.11(b) Transient torque following star-delta starting of 5.6 kW motor.
Computed results - - - - - -
Experimental results ———

1 p.u. torque = 56 Nm , 1 p.u. voltage = 565.7 V

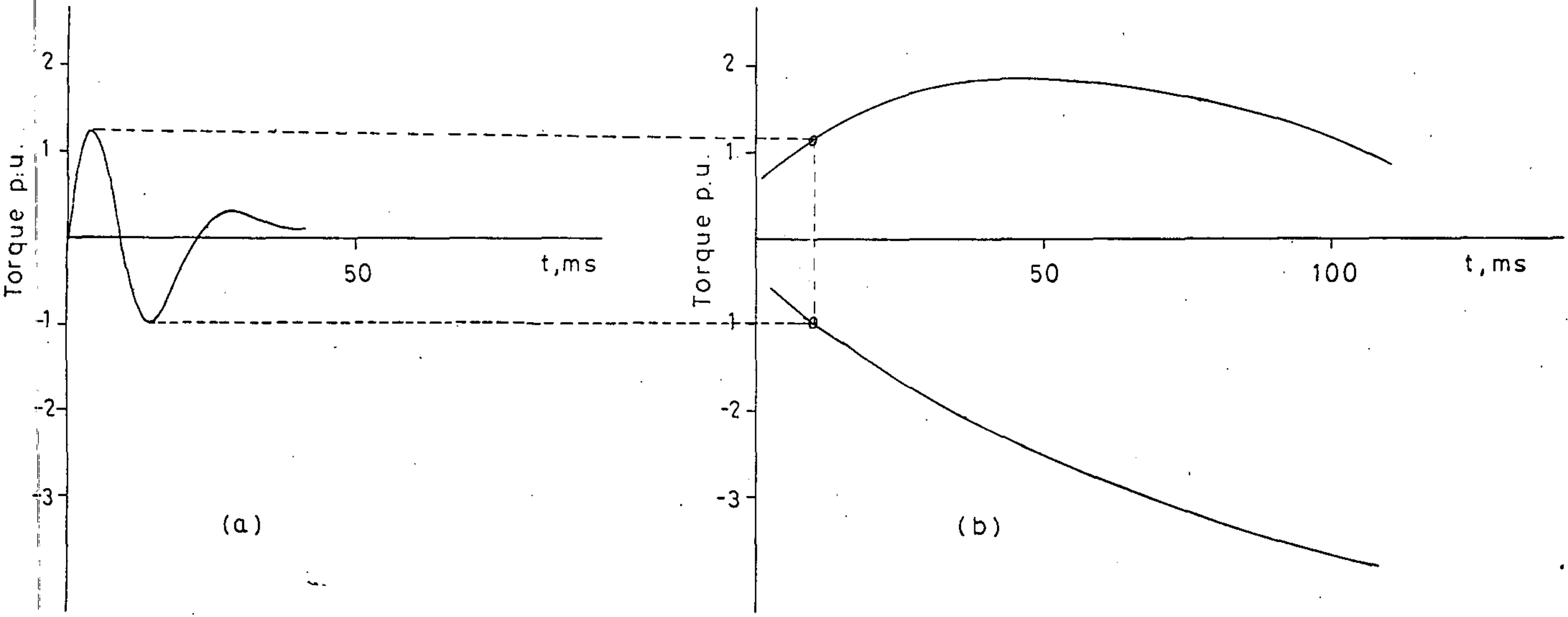


Figure 6.12 Torque following reconnection of 0.75 kW motor.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.
1 p.u. torque = 7.5 Nm.

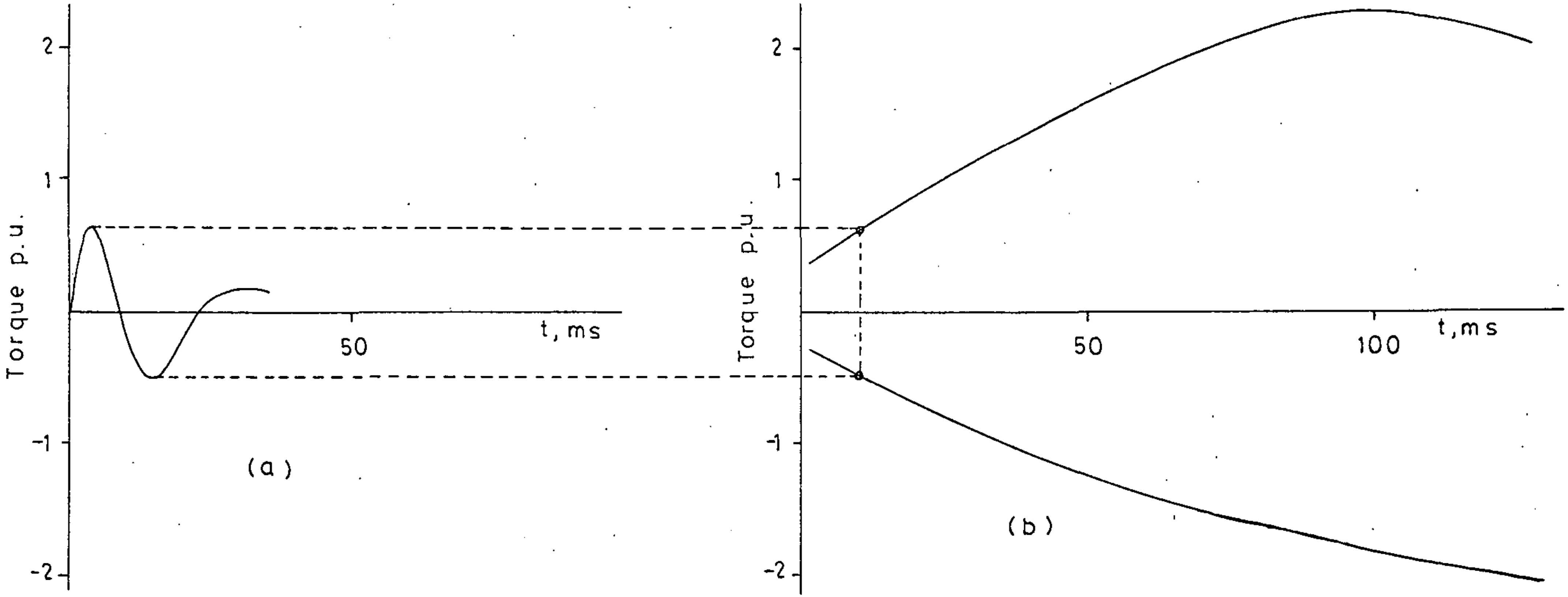


Figure 6.13 Torque following reconnection of 1.5 kW motor.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.

1 p.u. torque = 4.7 Nm.

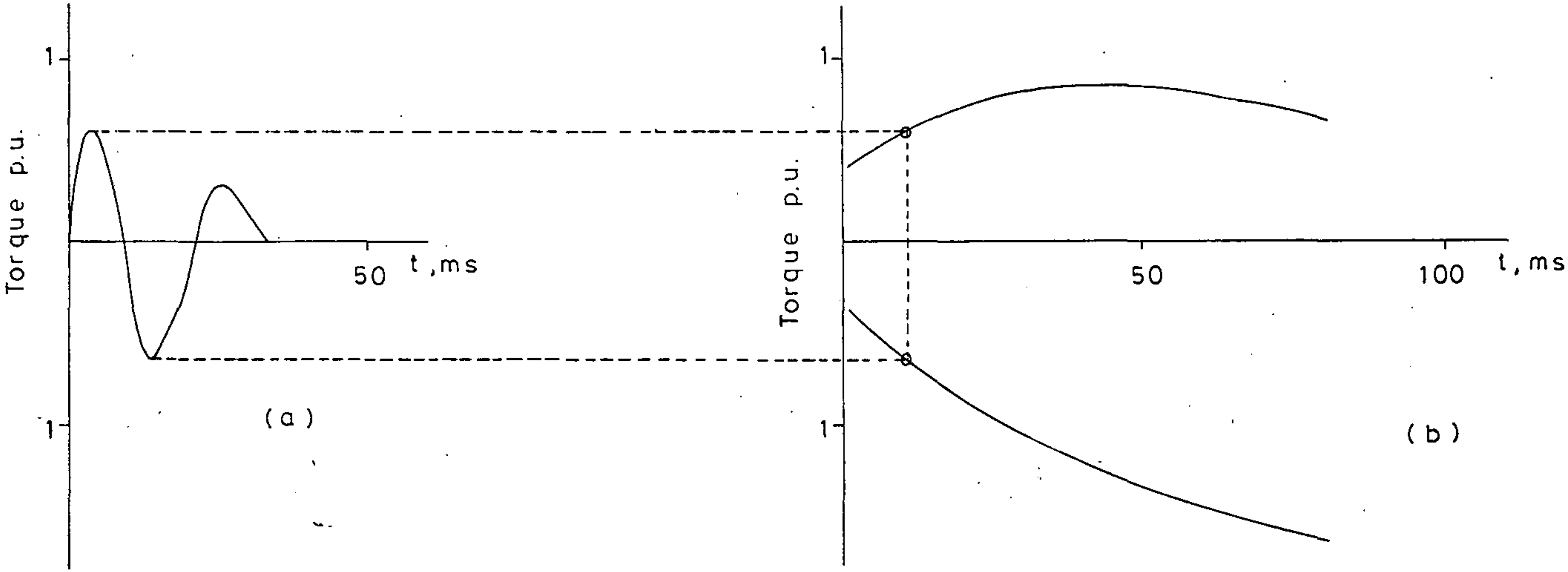


Figure 6.14 Torque following reconnection of 2.25 kW motor.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 15.5 Nm.

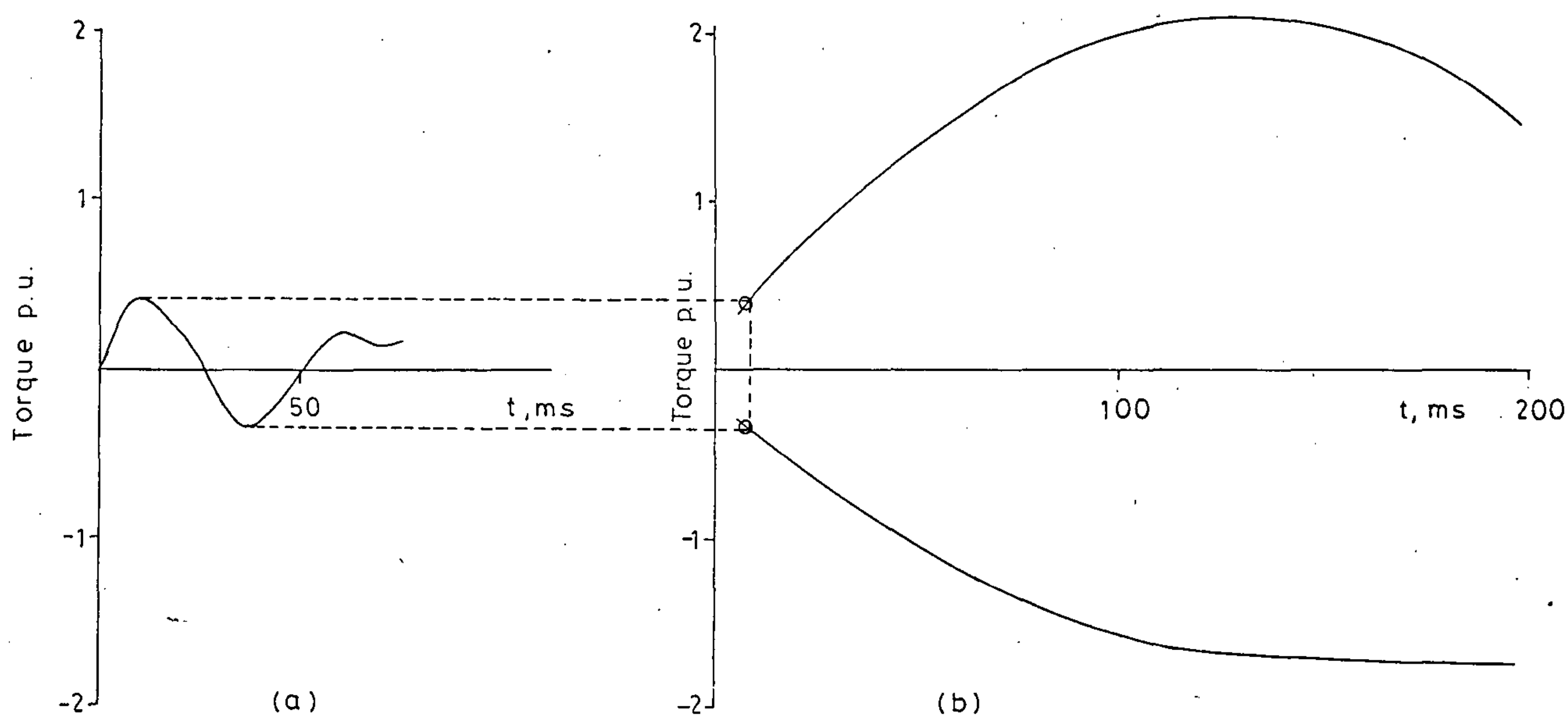


Figure 6·15 Torque following reconnection of 5·6 kW motor.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.
1 p.u. torque = 56 Nm.

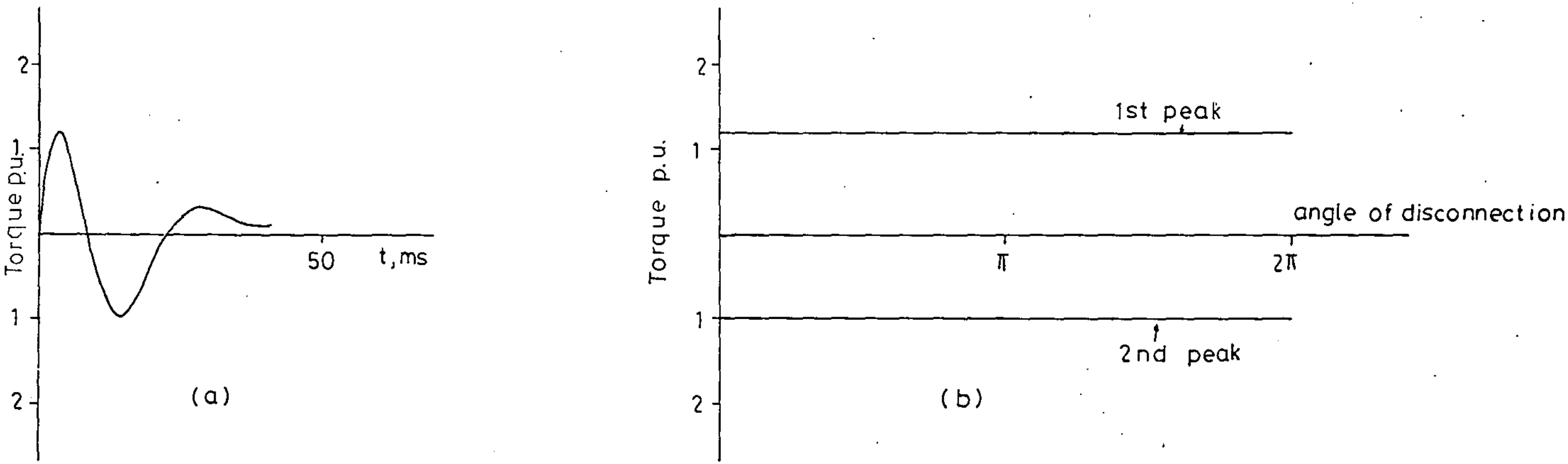


Figure 6.16 Torque following reconnection at constant supply interruption and different instant of connection of 0.75 kW motor.
(a) Pattern curve.
(b) Variation of peak torque with instant of disconnection (constant supply interruption of 10 ms).
1 p.u. torque = 7.5 Nm.

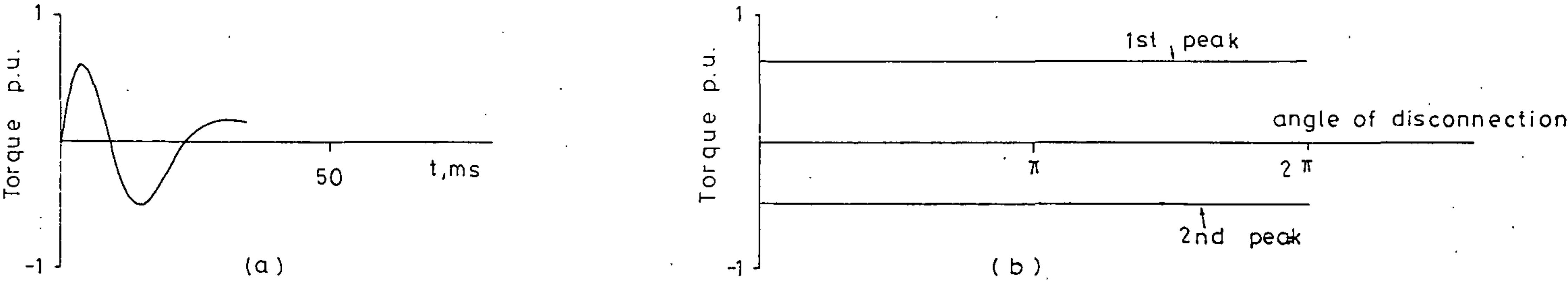


Figure 6.17 Torque following reconnection at constant supply interruption and different instant of connection of 1.5 kW motor.

(a) Pattern curve.

(b) Variation of peak torque with instant of disconnection (constant supply interruption of 10 ms)

1 p.u. torque = 4.7 Nm.

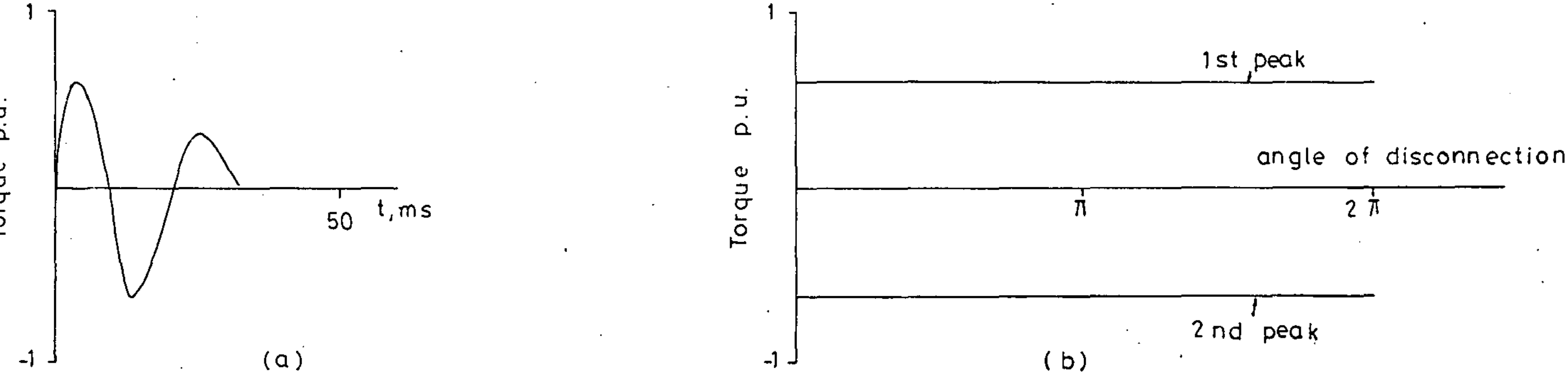


Figure 6.18 Torque following reconnection at constant supply interruption and different instant of connection of 2.25 kW motor.

(a) Pattern curve.

(b) Variation of peak torque with instant of disconnection (constant supply interruption of 10 ms).

1 p.u. torque = 15.5 Nm.

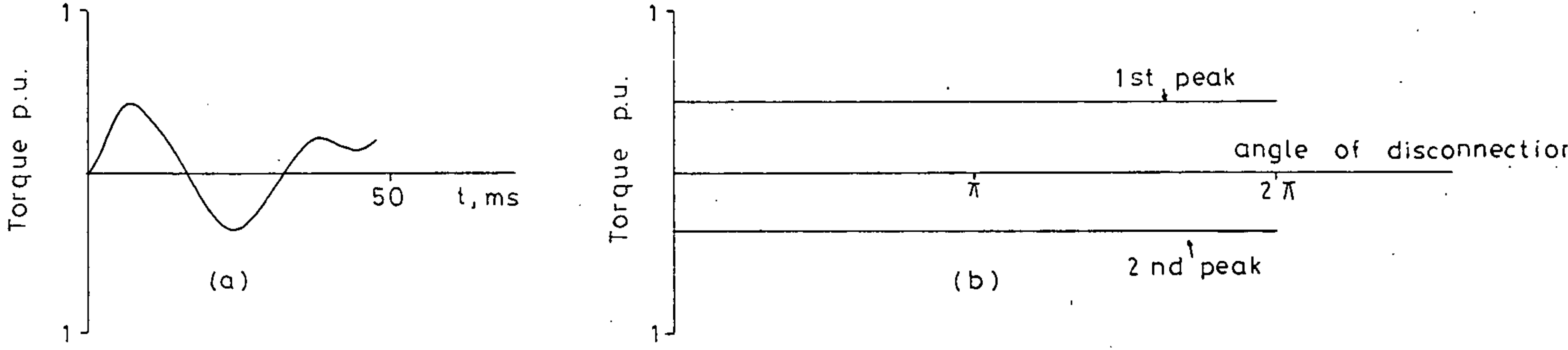


Figure 6·19 Torque following reconnection at constant supply interruption and different instant of connection of 5·6 kW motor.

(a) Pattern curve.

(b) Variation of peak torque with instant of disconnection (constant supply interruption of 10 ms)

1 p.u. torque = 56 Nm.

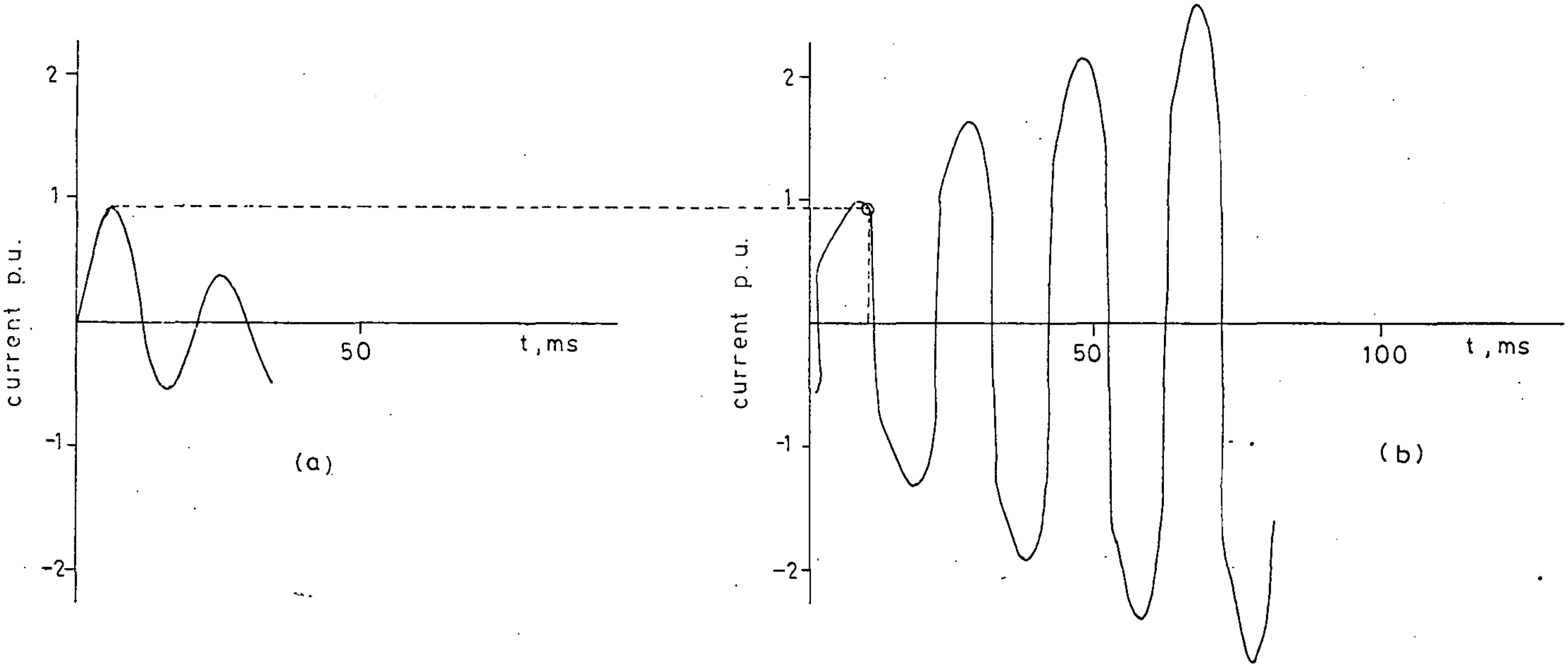


Figure 6·20 Current following reconnection of 0.75 kW motor.
(a) Computed transient current pattern.
(b) Variation of peak current with delay.
1 p.u. current = 4·4 A

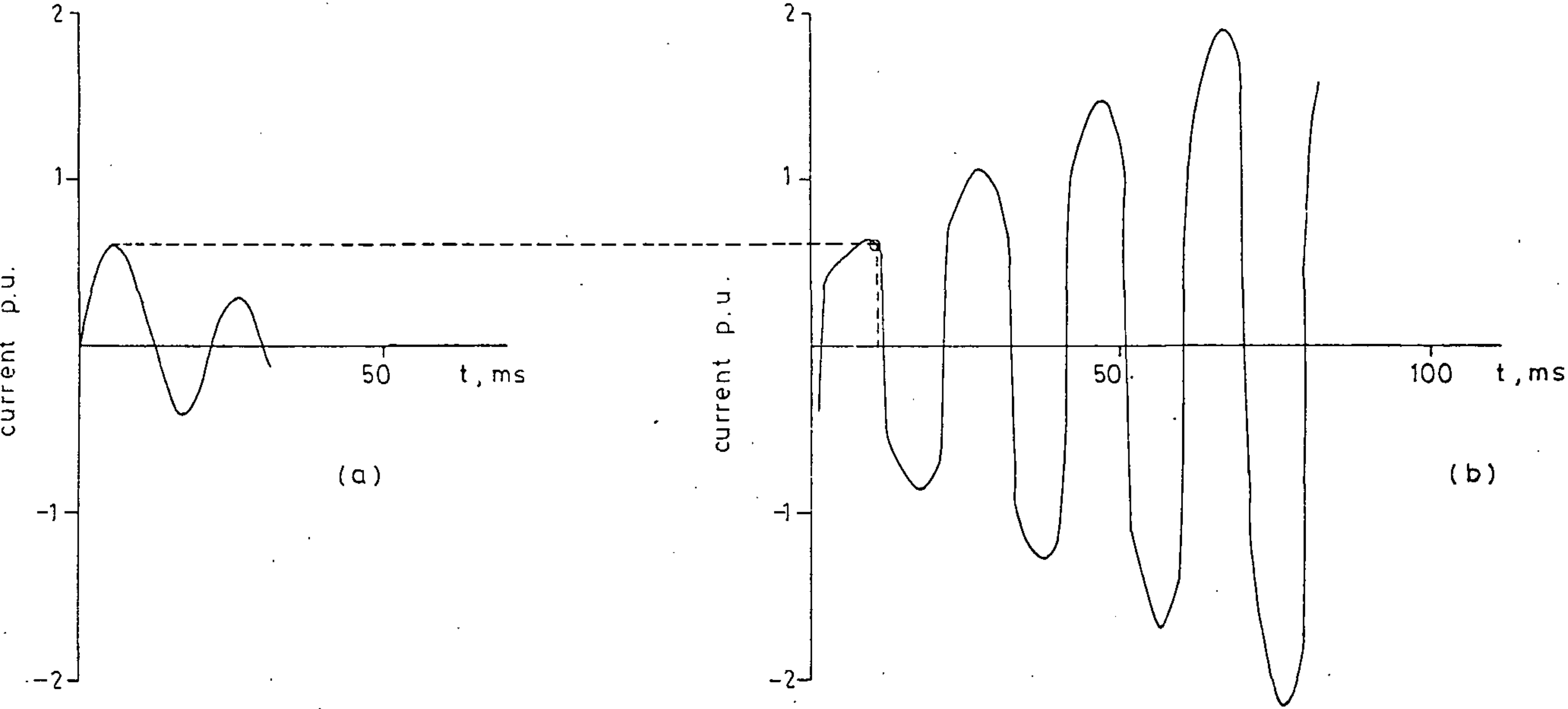


Figure 6·21 Current following reconnection of 1·5 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 5·5 A

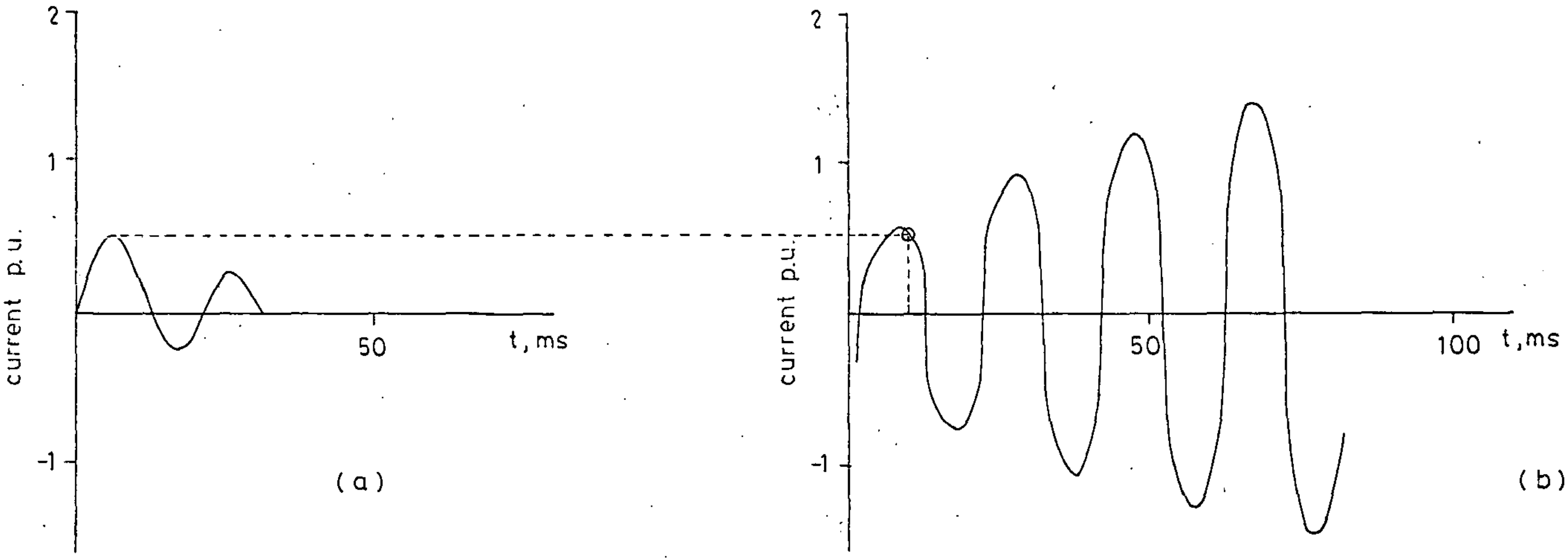


Figure 6·22 Current following reconnection of 2·25 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 12·7 A

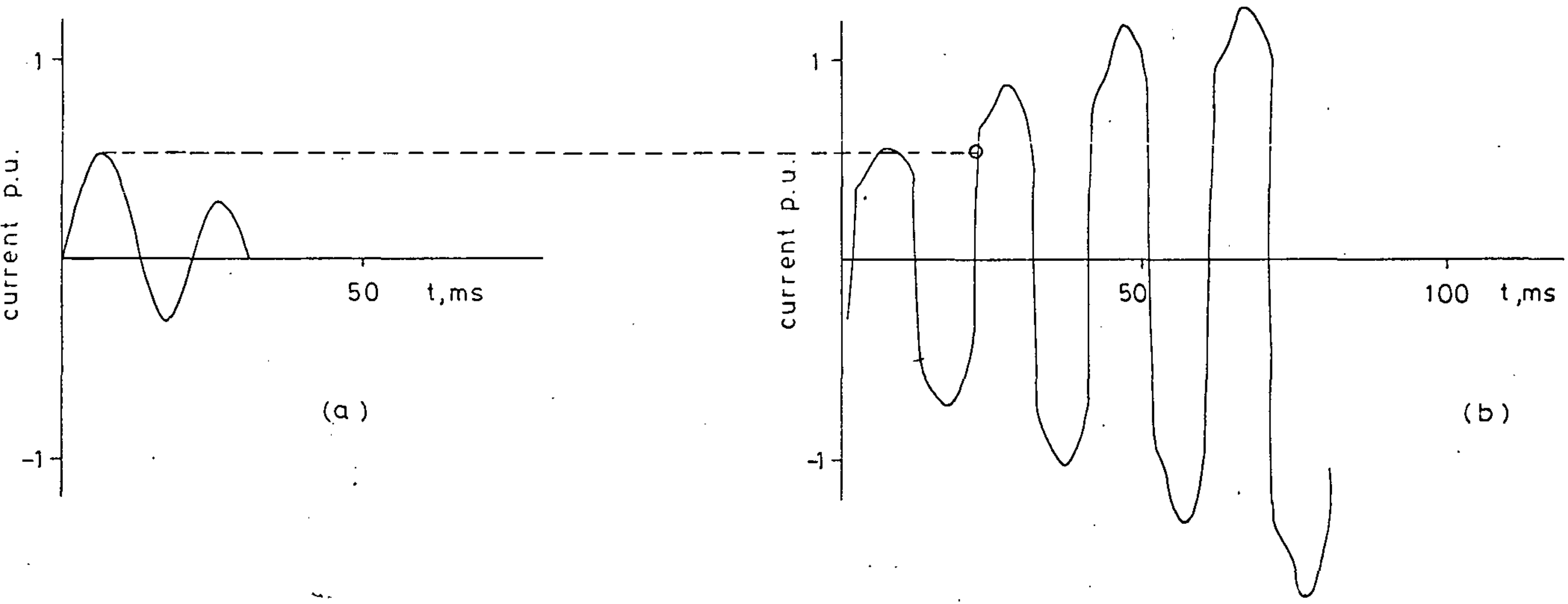


Figure 6·23 Current following reconnection of 5·6 kW motor.
(a) Computed transient current pattern.
(b) Variation of peak current with delay.
1 p.u. current = 16·8 A.

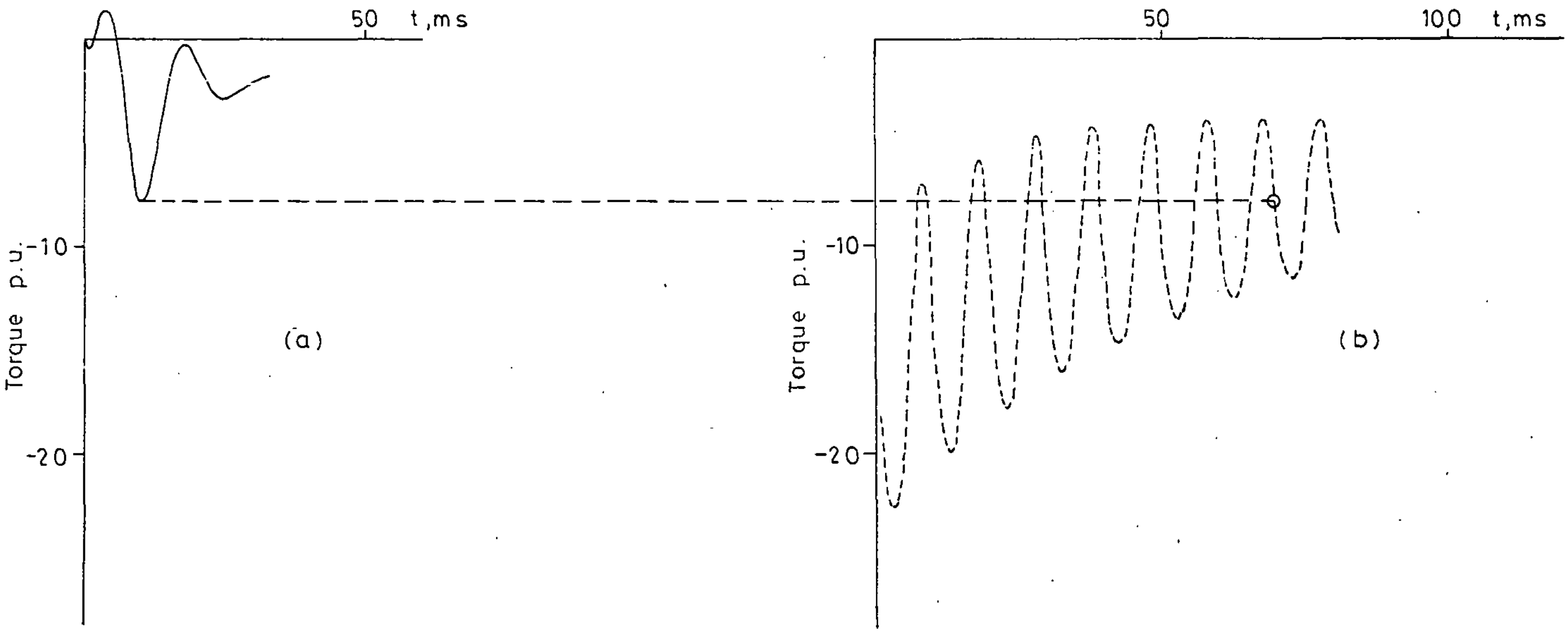


Figure 6.24 Torque following plugging of 0.75 kW motor.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.
1 p.u. torque = 7.4 Nm.

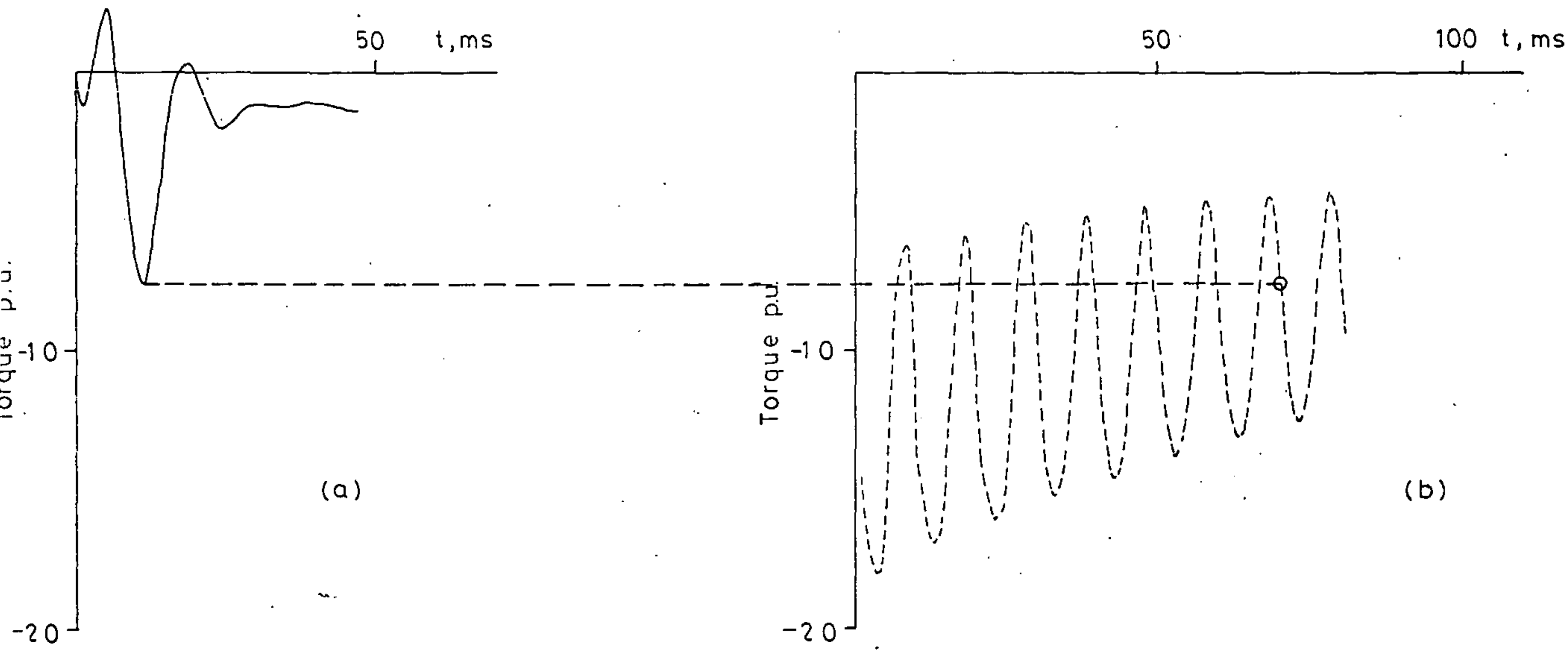


Figure 6·25 Torque following plugging of 1·5 kW motor.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 4·7 Nm .

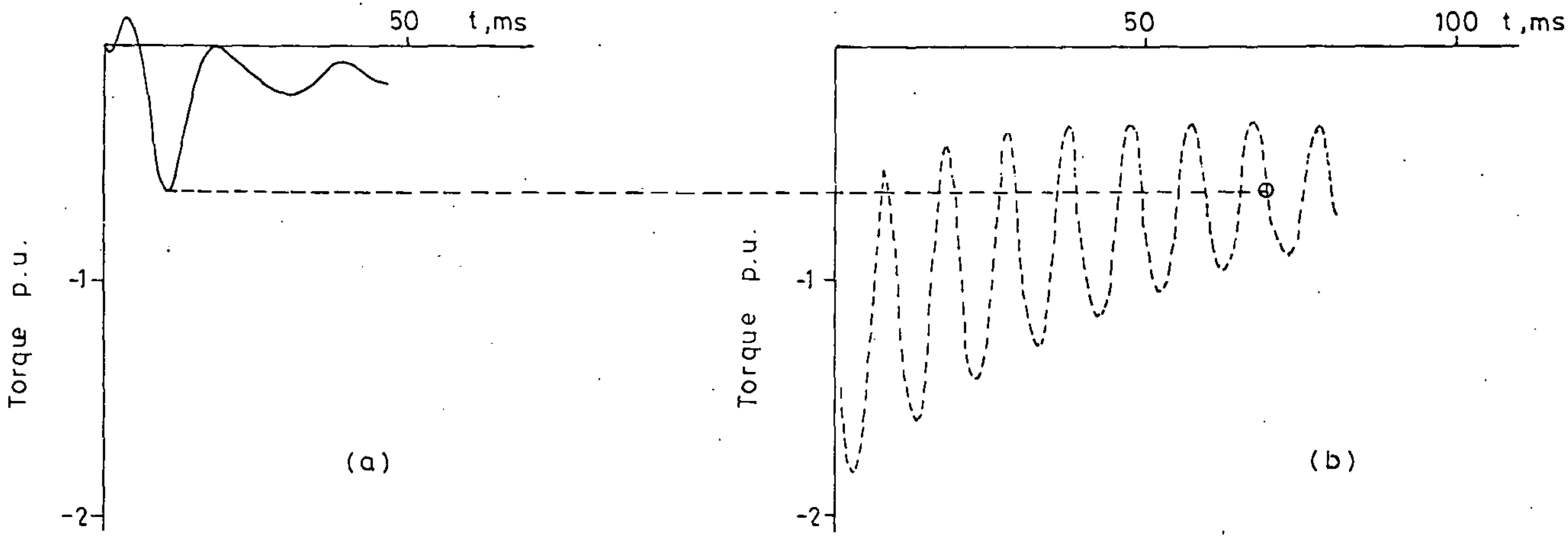


Figure 6·26 Torque following plugging of 2·25 kW motor.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 15·5 Nm.

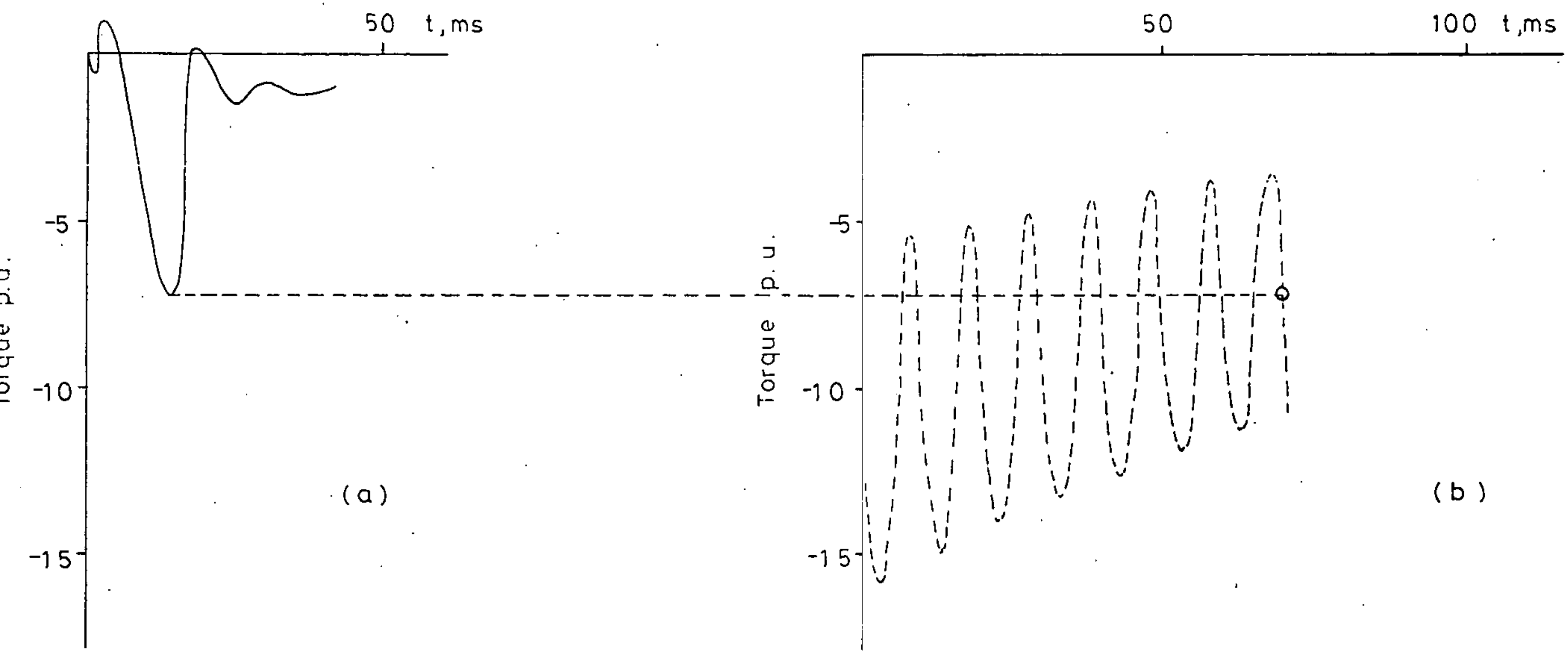


Figure 6.27 Torque following plugging of 5.6 kW motor.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.
1 p.u. torque = 56 Nm.

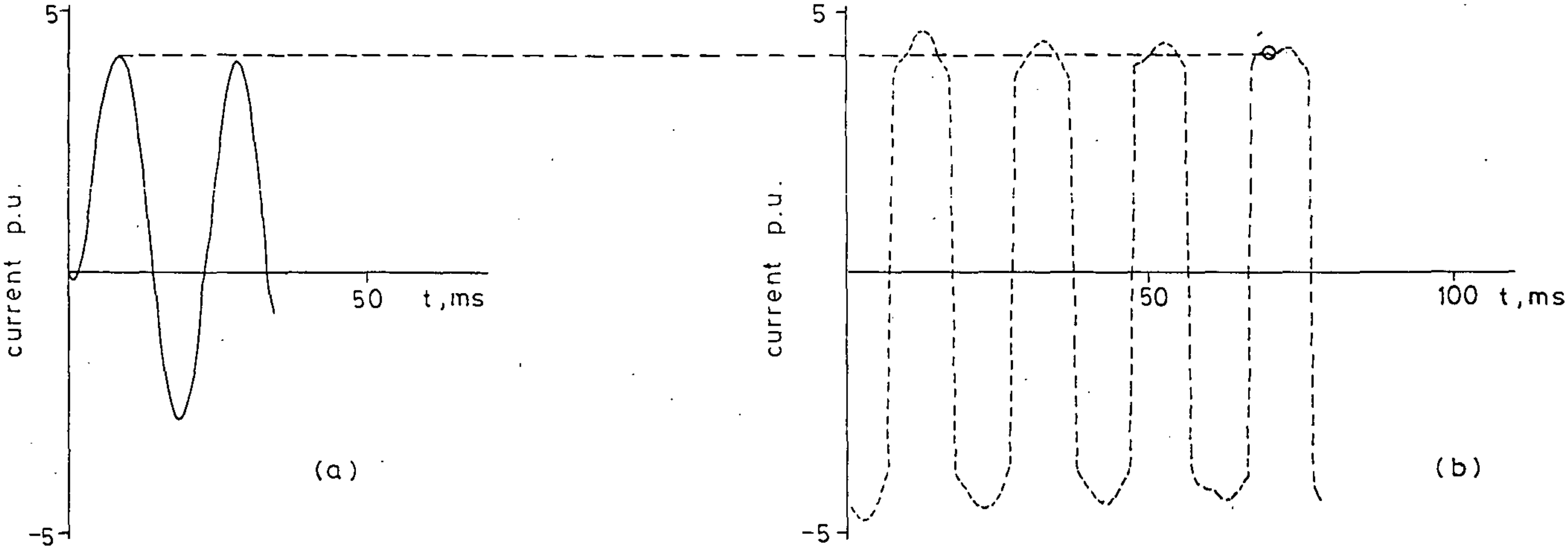


Figure 6·28 Current following plugging of 0·75 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 4·4 A

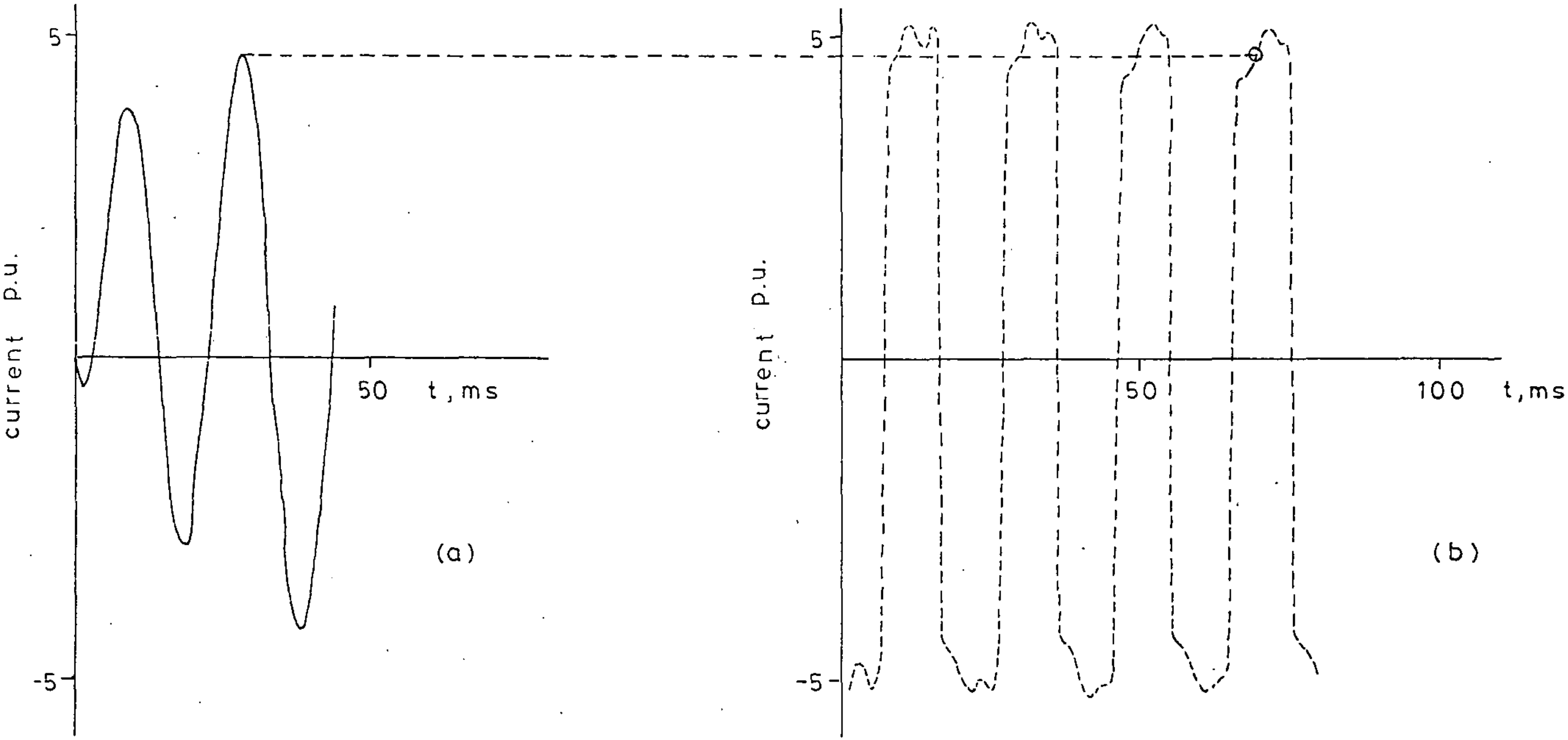


Figure 6·29 Current following plugging of 1·5 kW motor.
(a) Computed transient current pattern.
(b) Variation of peak current with delay.
1 p.u. current = 5·5 A

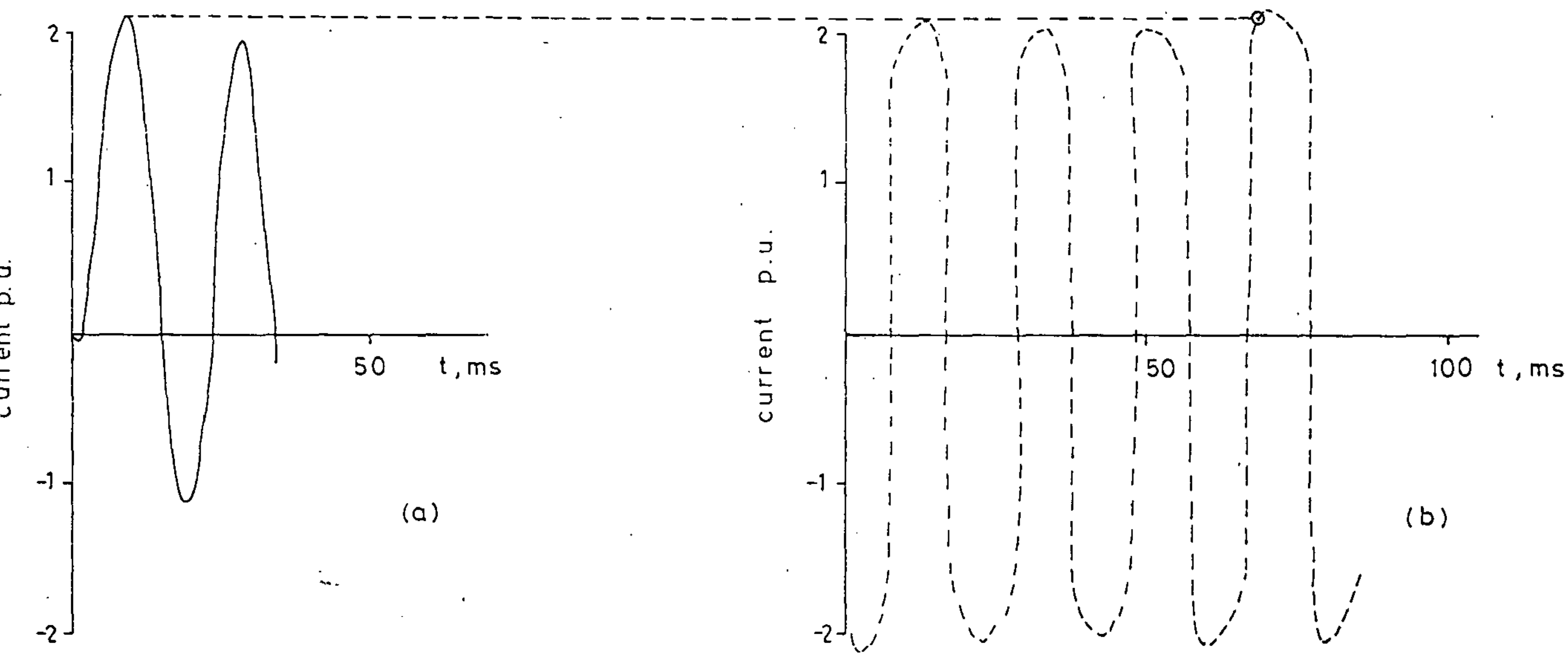


Figure 6.30 Current following plugging of 2.25 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 12.7 A

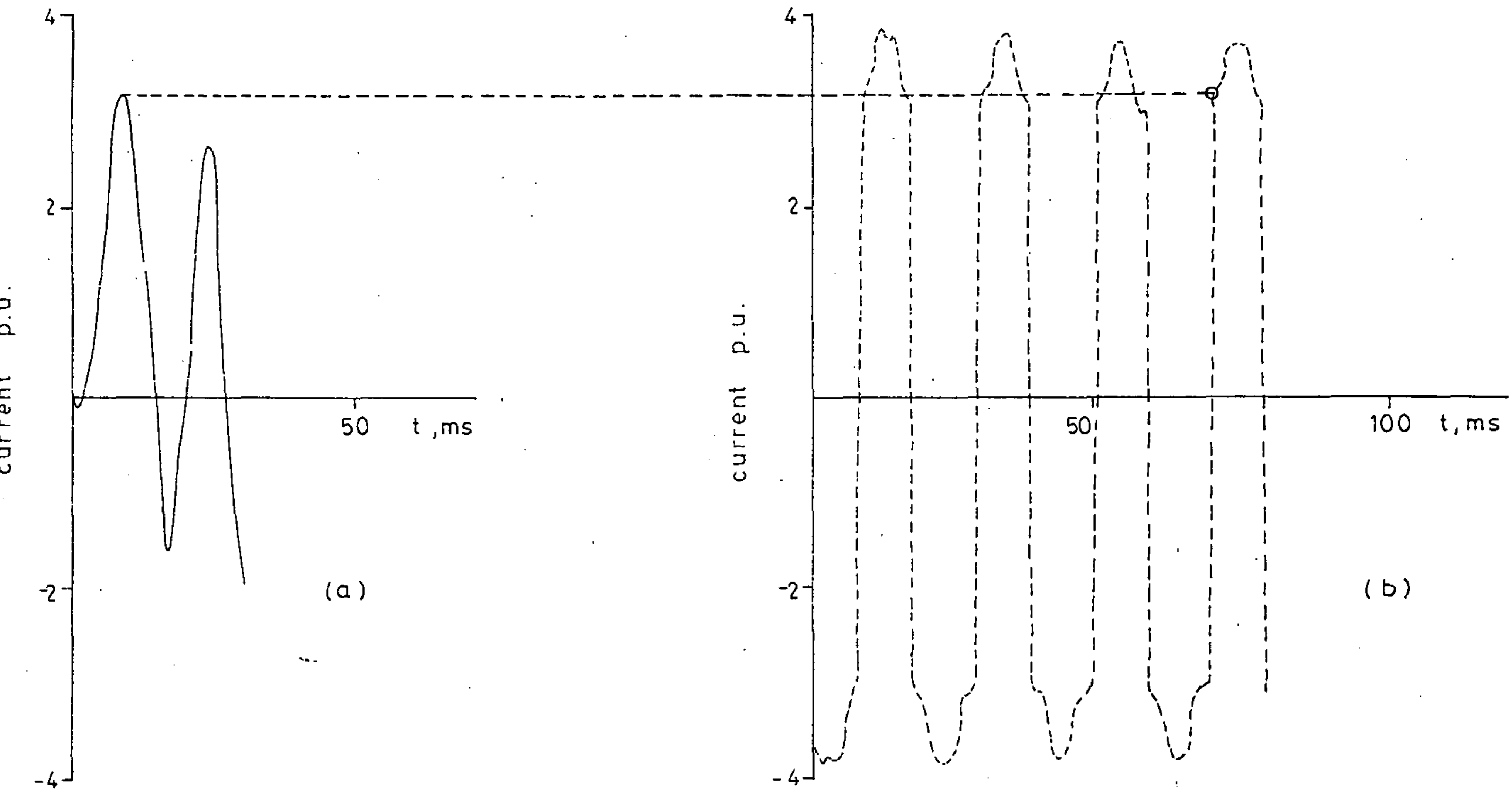


Figure 6.31 Current following plugging of 5.6 kW motor.
(a) Computed transient current pattern.
(b) Variation of peak current with delay.
1 p.u. current = 16.8 A

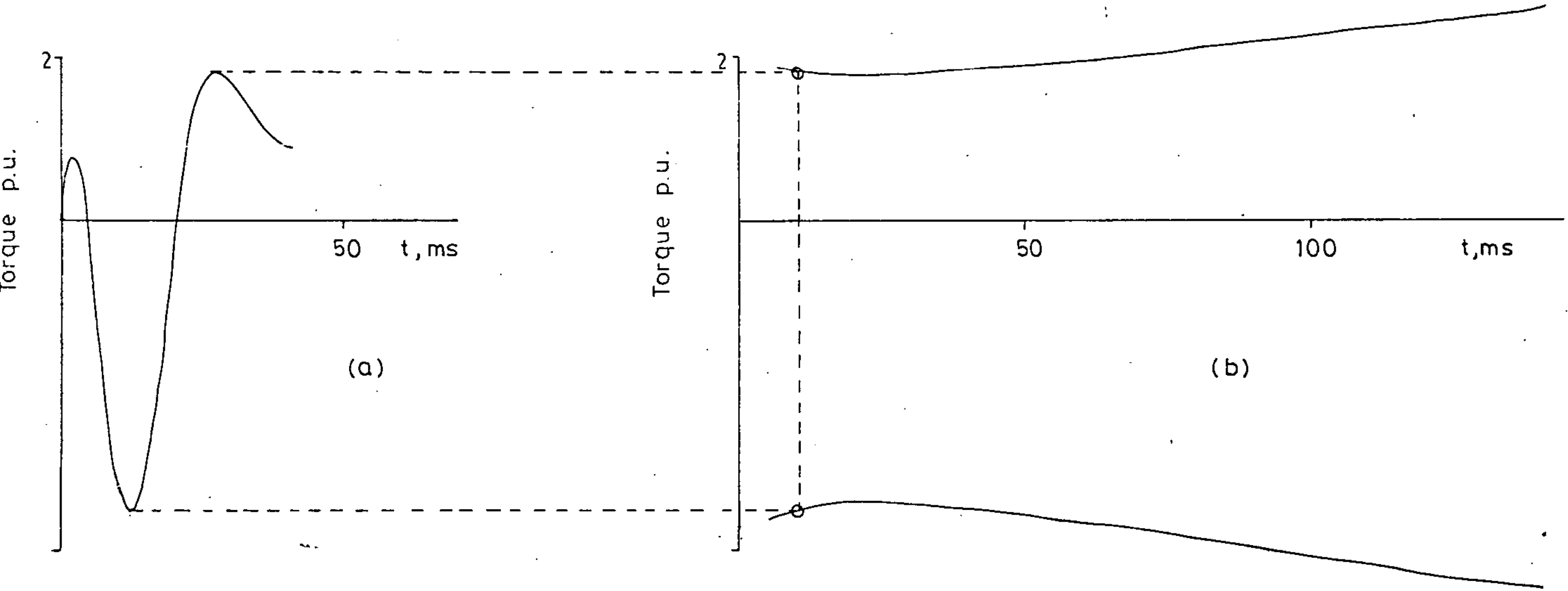


Figure 6.32 Torque following star-delta starting of 0.75 kW motor.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 7.4 Nm.

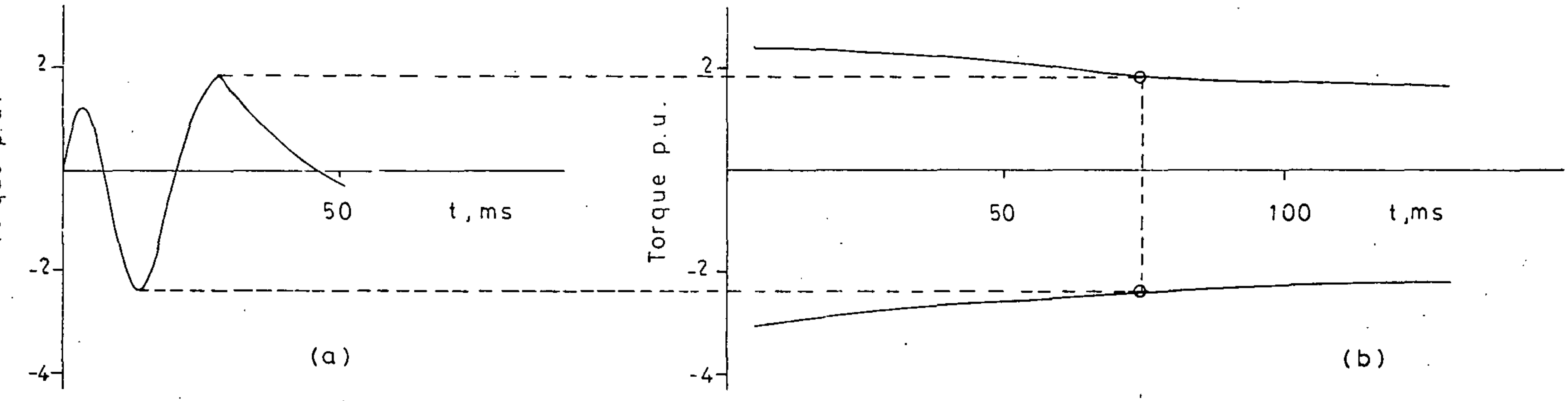


Figure 6.33 Torque following star-delta starting of 1.5 kW motor.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 4.7 Nm.

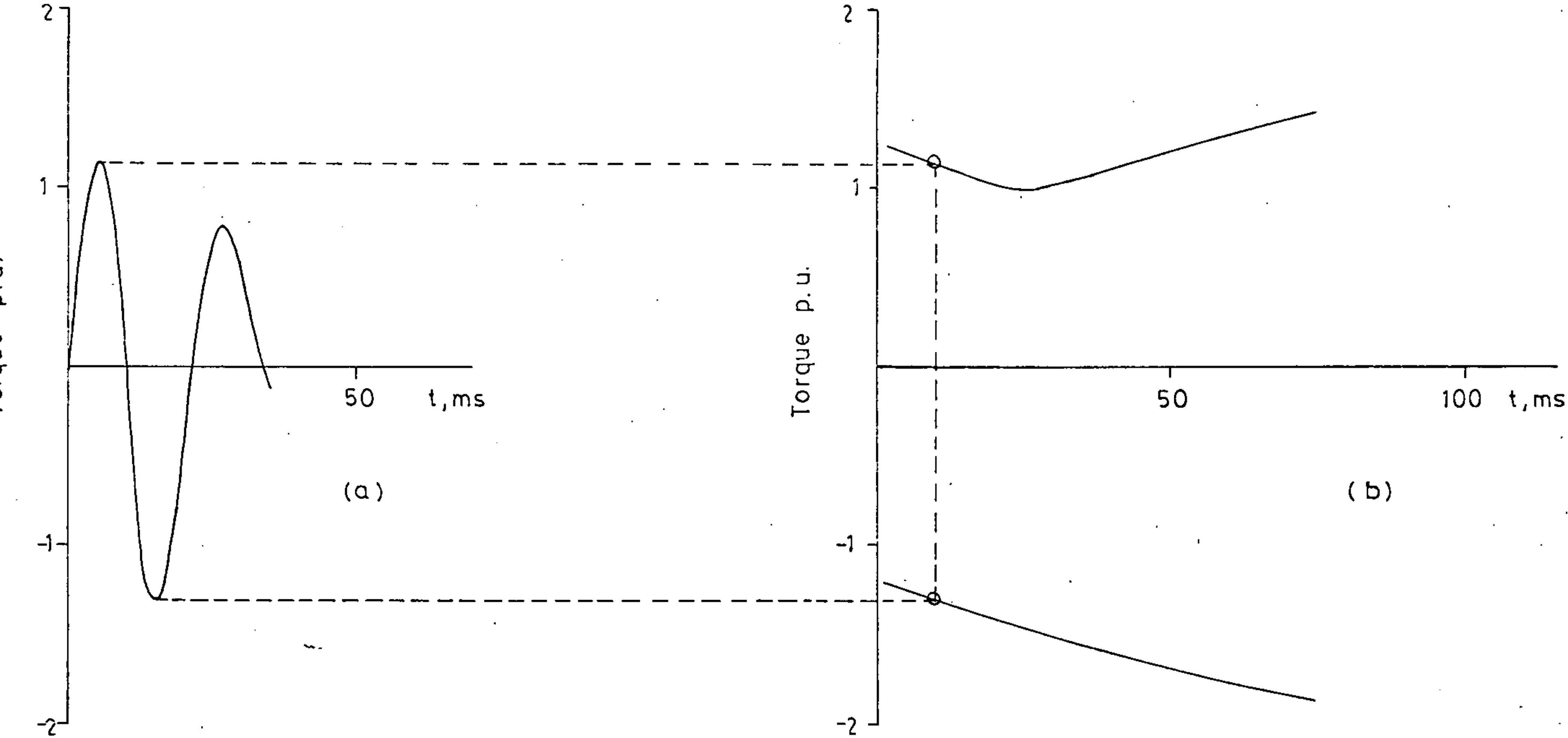


Figure 6.34 Torque following star-delta starting of 2.25 kW motor.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.
1 p.u. torque = 15.5 Nm.

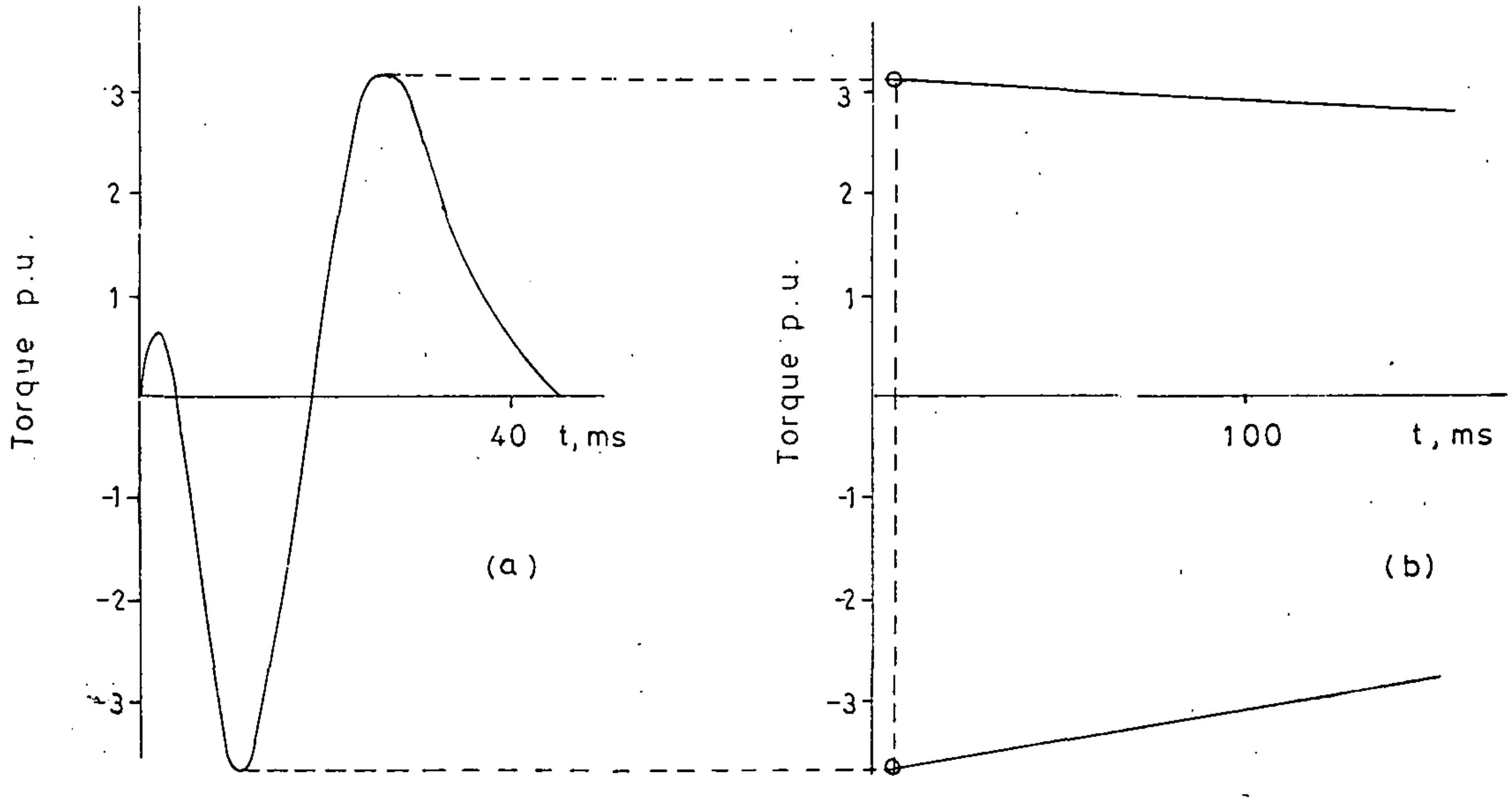


Figure 6·35 Torque following star-delta starting of 5·6 kW motor.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.
1 p.u. torque = 56 Nm.

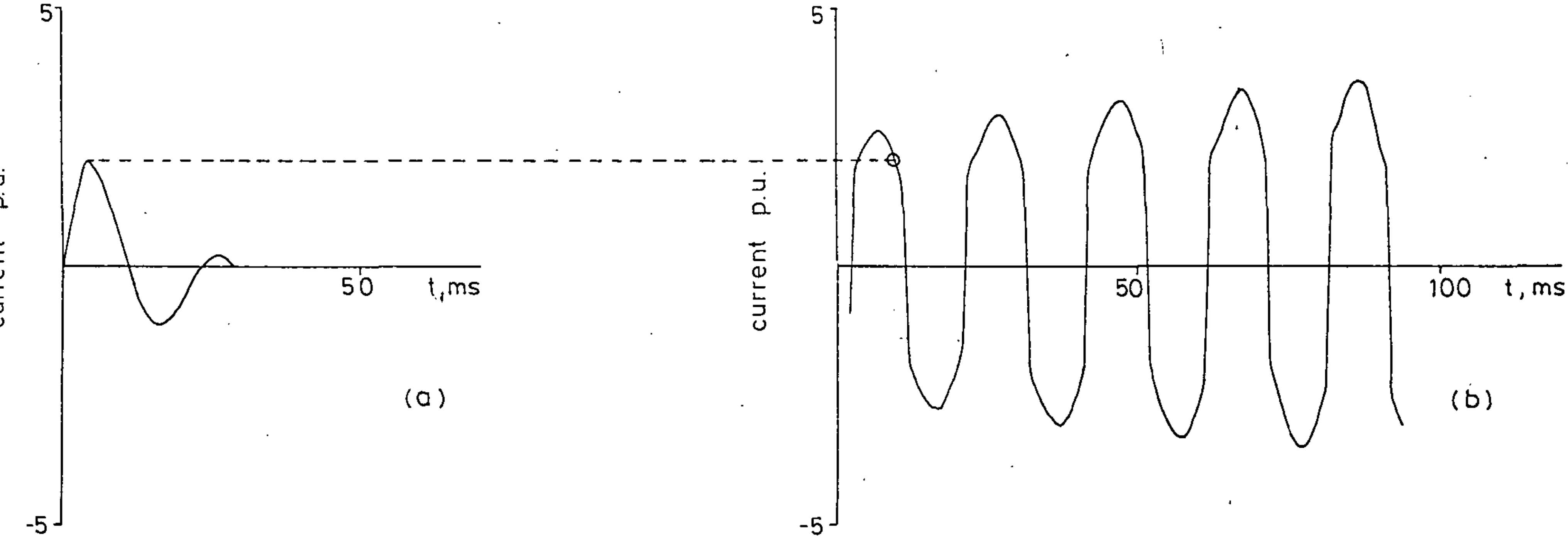


Figure 6·36 Current following star-delta starting of 0·75 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 4·4 A.

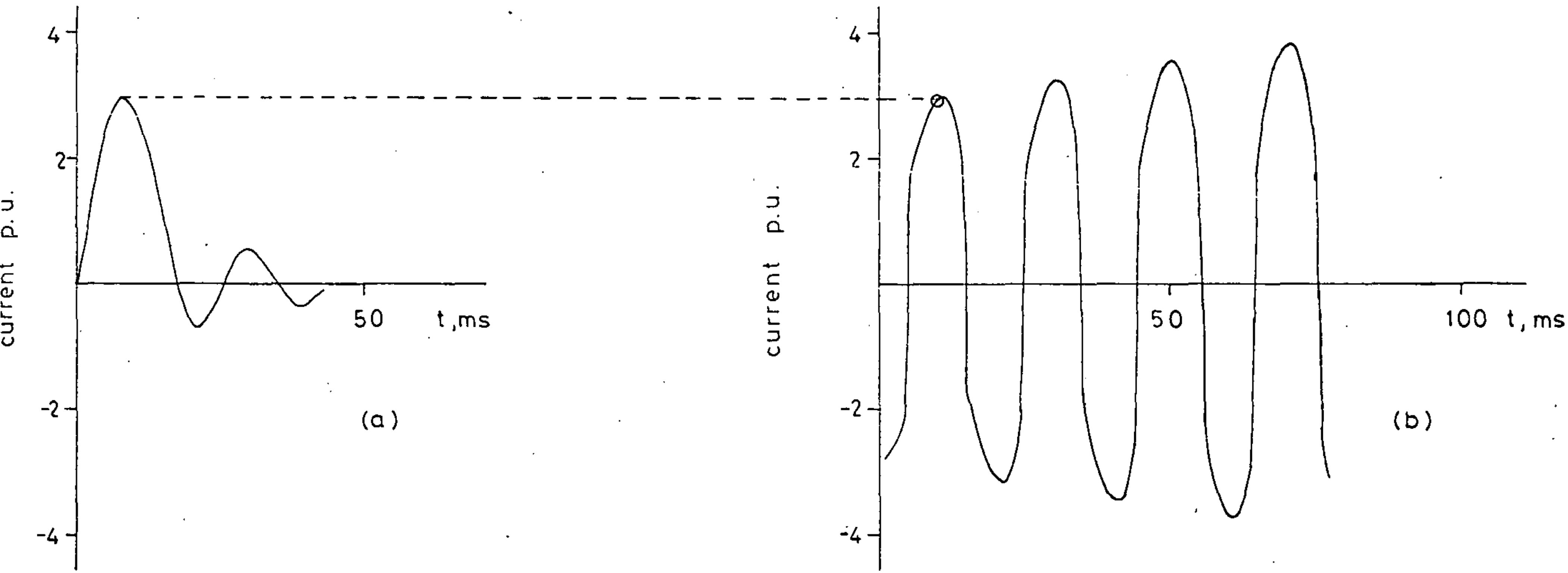


Figure 6.37 Current following star-delta starting of 1.5 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 5.5 A.

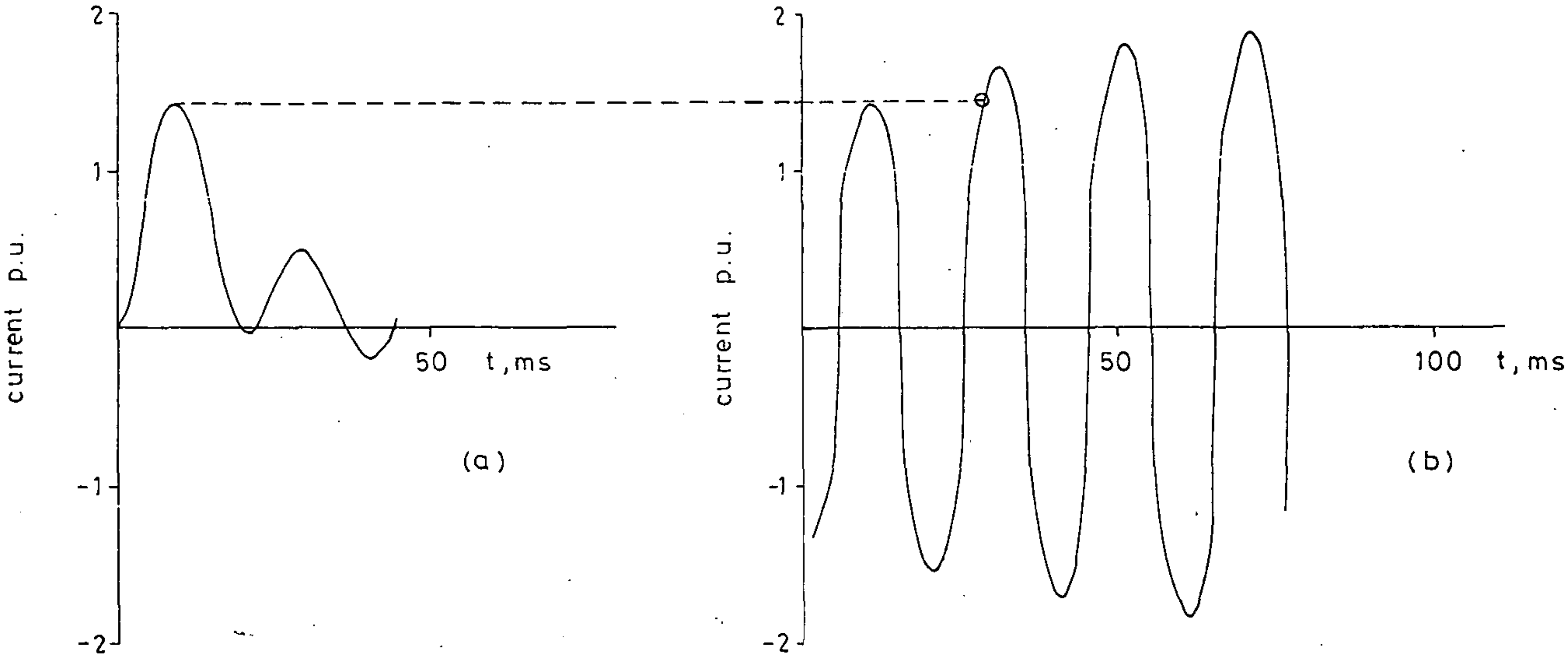


Figure 6.38 Current following star-delta starting of 2.25 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 12.7 A.

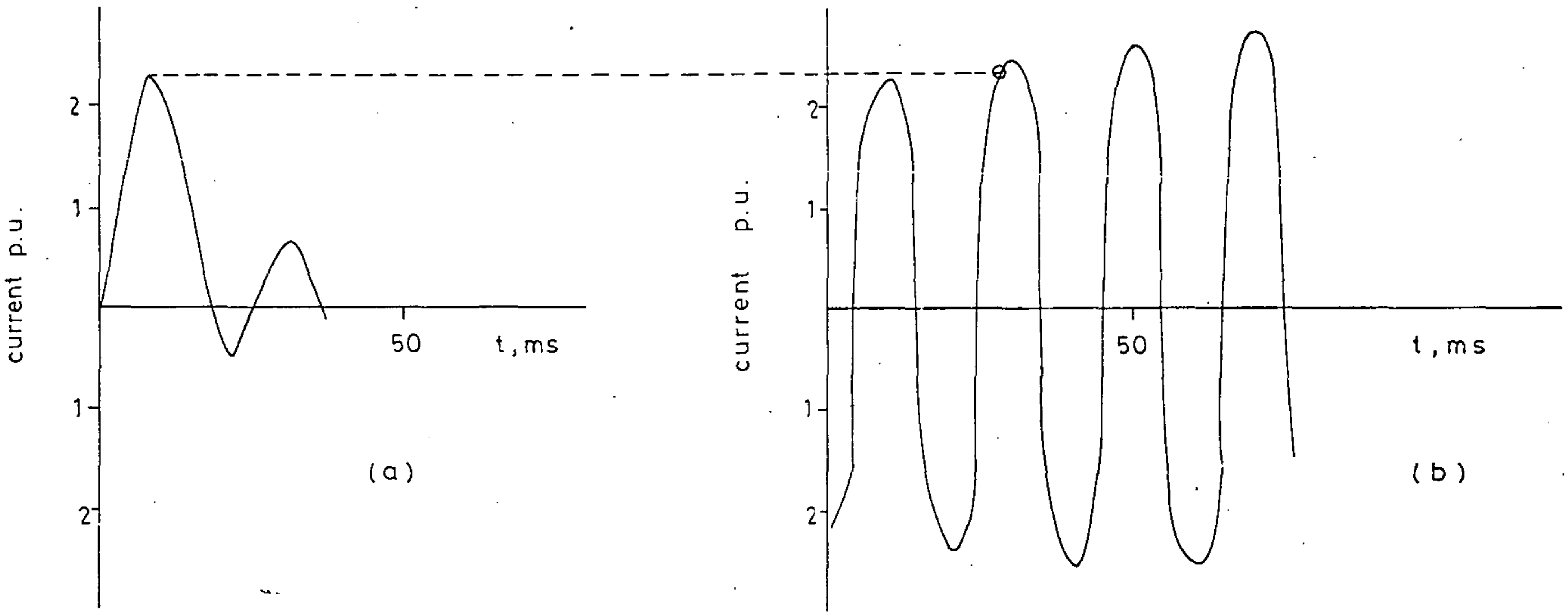


Figure 6.39 Current following star-delta starting of 5.6 kW motor.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 16.8 A.

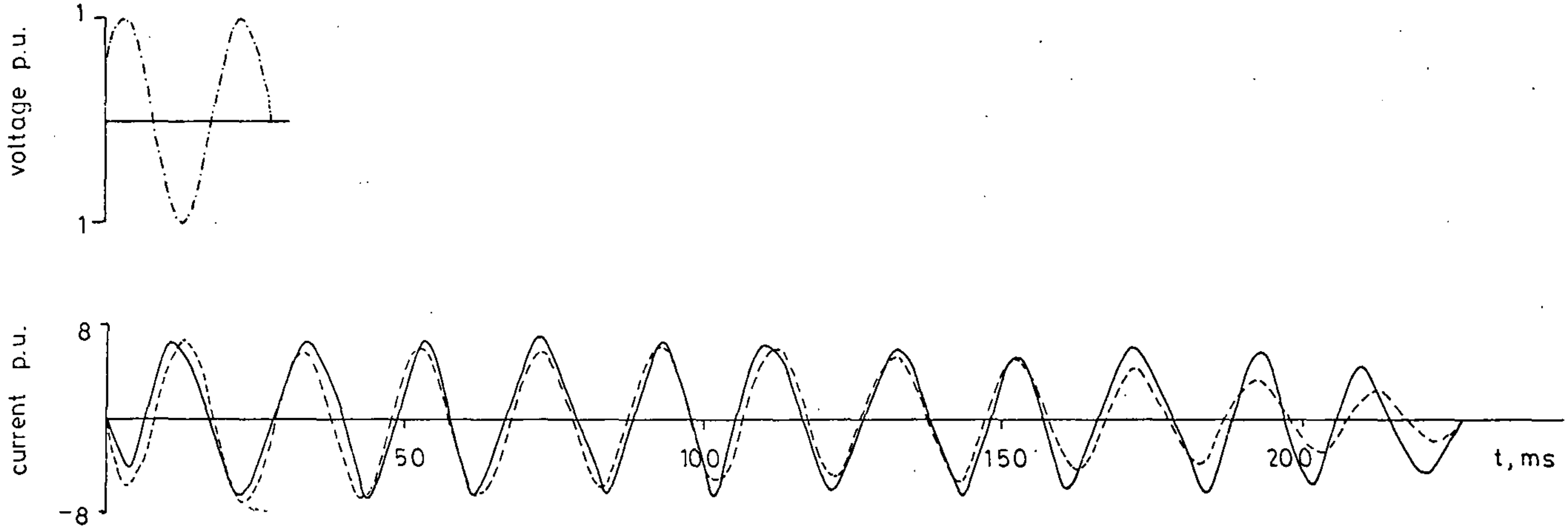


Figure 7.1(a) Transient current in b-line of 0.75 kW motor following direct-to-line starting of 0.75 kW/1.5 kW motor group.

Computed results ------, Experimental results —— , r-y line voltage –·–·–

1 p.u. voltage = 345 V , 1 p.u. current = 4.4 A .

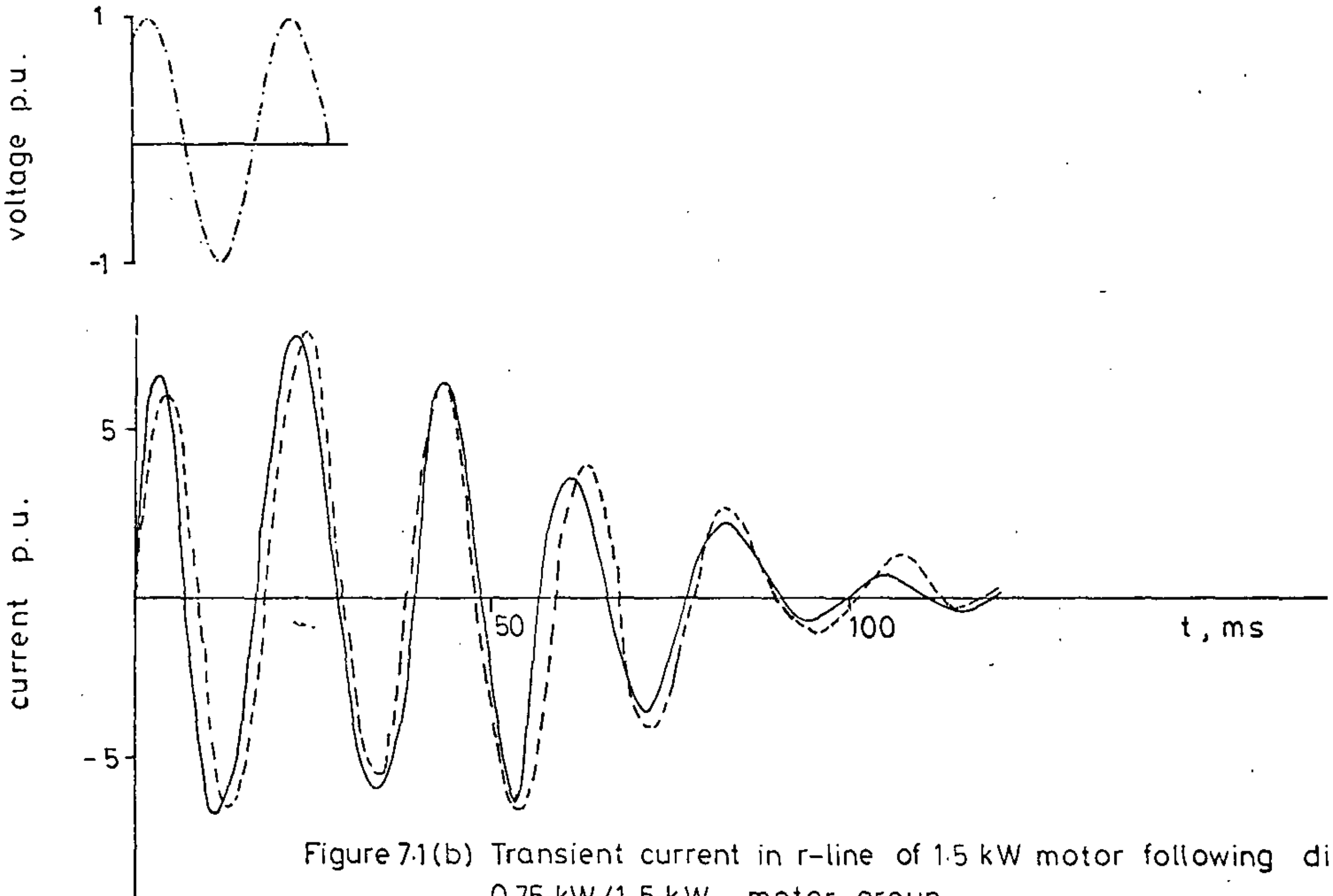


Figure 7.1(b) Transient current in r-line of 1.5 kW motor following direct-to-line starting of 0.75 kW/1.5 kW motor group.

Computed results----- ,Experimental results —— , r-y line voltage -·-·-·-

1 p.u. voltage = 345 V , 1 p.u. current = 5.5 A

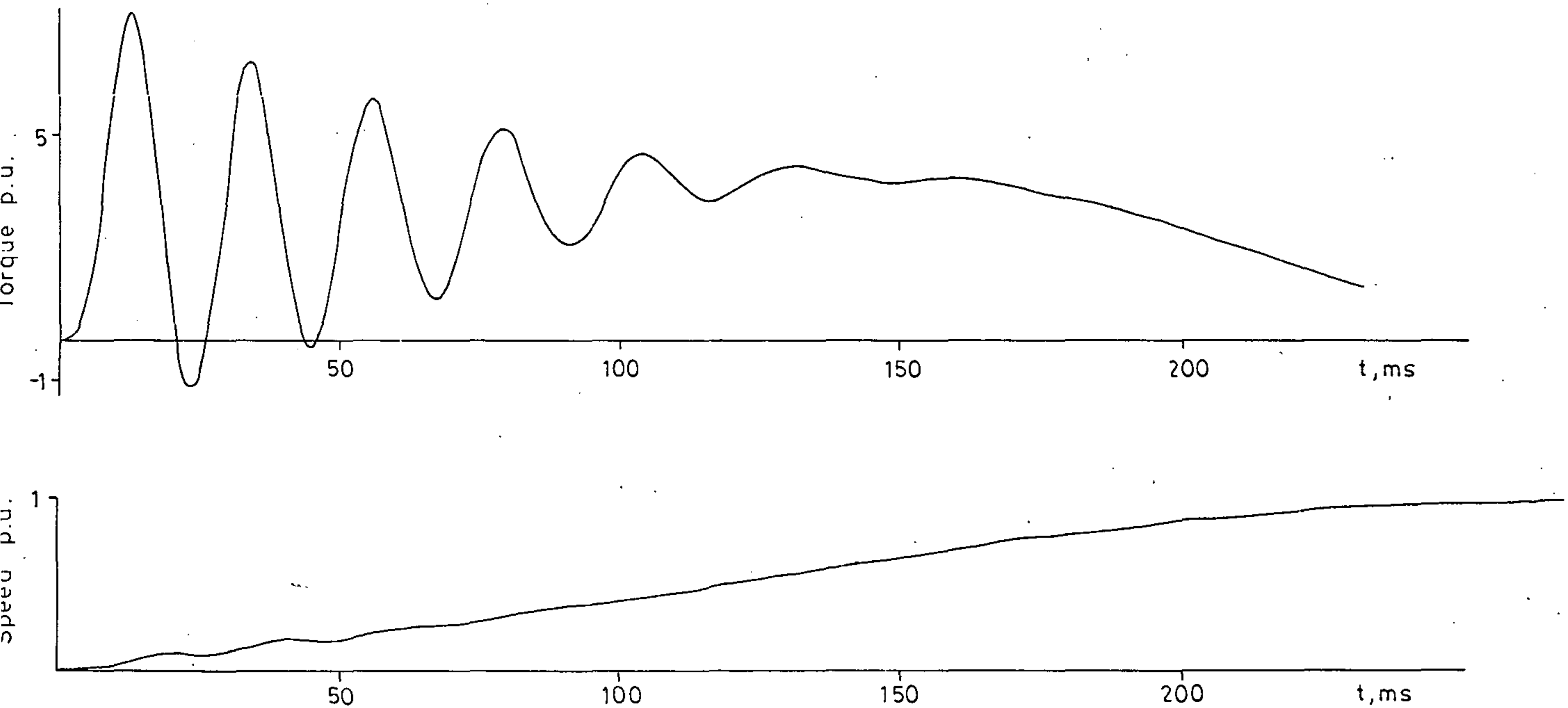


Figure 7·2 (a) Computed torque and speed of 0·75 kW motor following direct-to-line starting of 0·75 kW/1·5 kW motor group.

1 p.u. torque = 7·4 Nm , 1 p.u. speed = 314·2 elec. rad./sec.

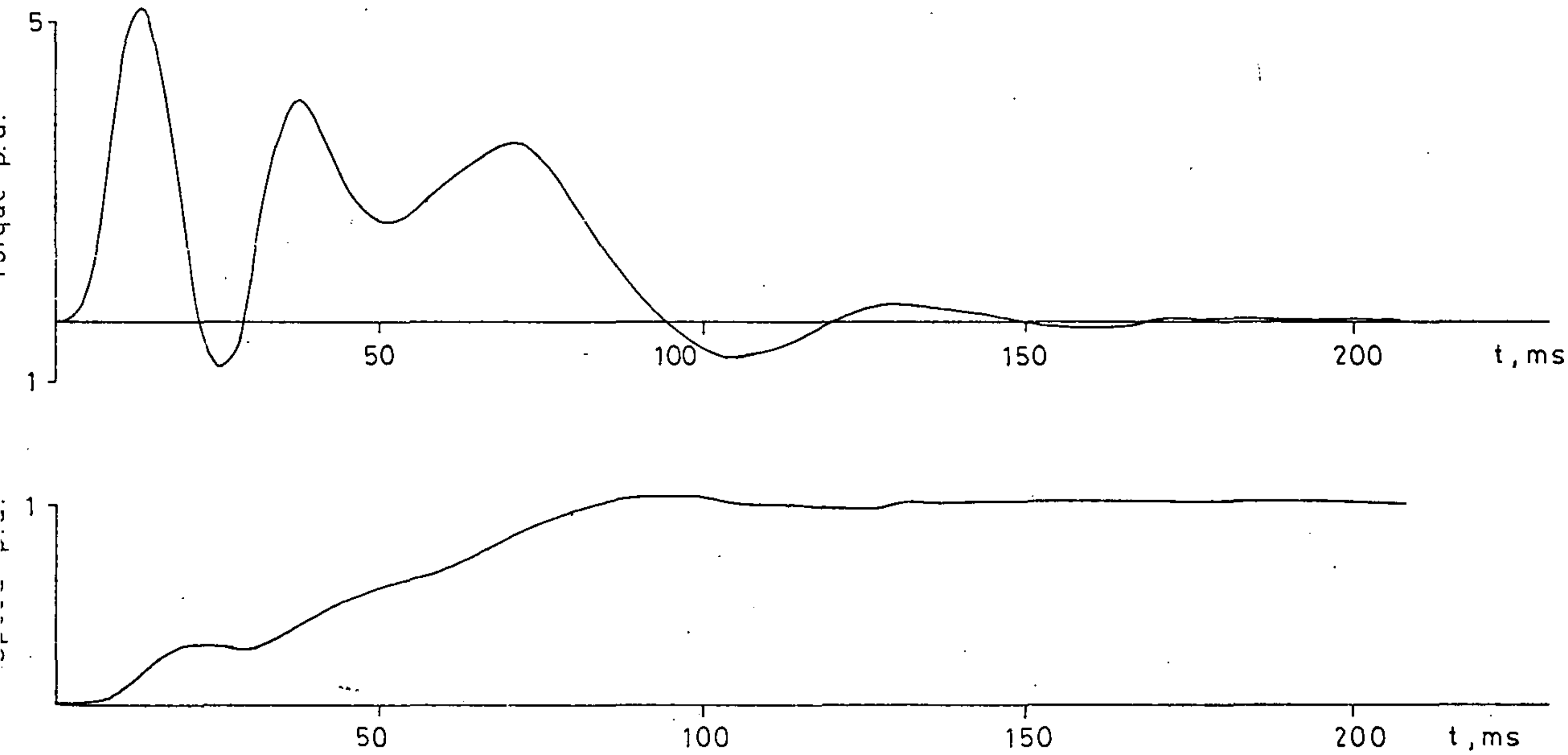


Figure 7·2(b) Computed torque and speed of 1·5 kW motor following direct-to-line starting of 0·75 kW/1·5 kW motor group.
1 p.u. torque = 4·7 Nm , 1 p.u. speed = 314·2 elec. rad./sec.

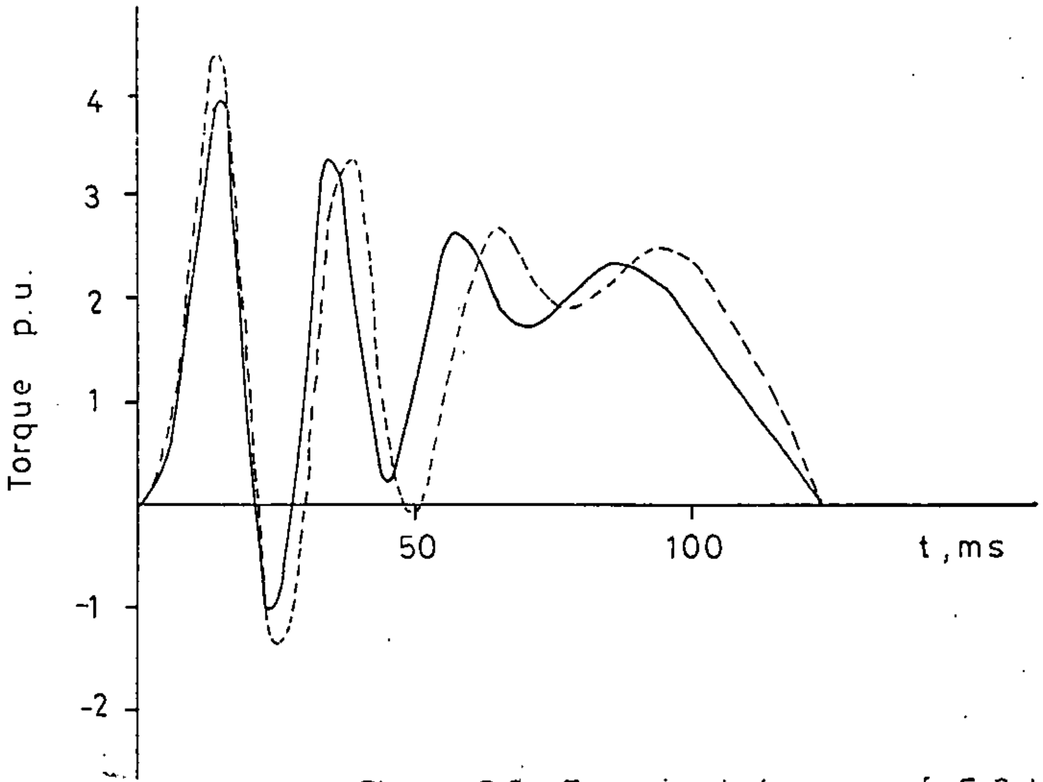


Figure 7·3 Transient torque of 5·6 kW motor following direct-to-line starting of 5·6 kW/2·25 kW motor group.
Computed results - - - - - , Experimental results ———
1 p.u. torque = 56 Nm.

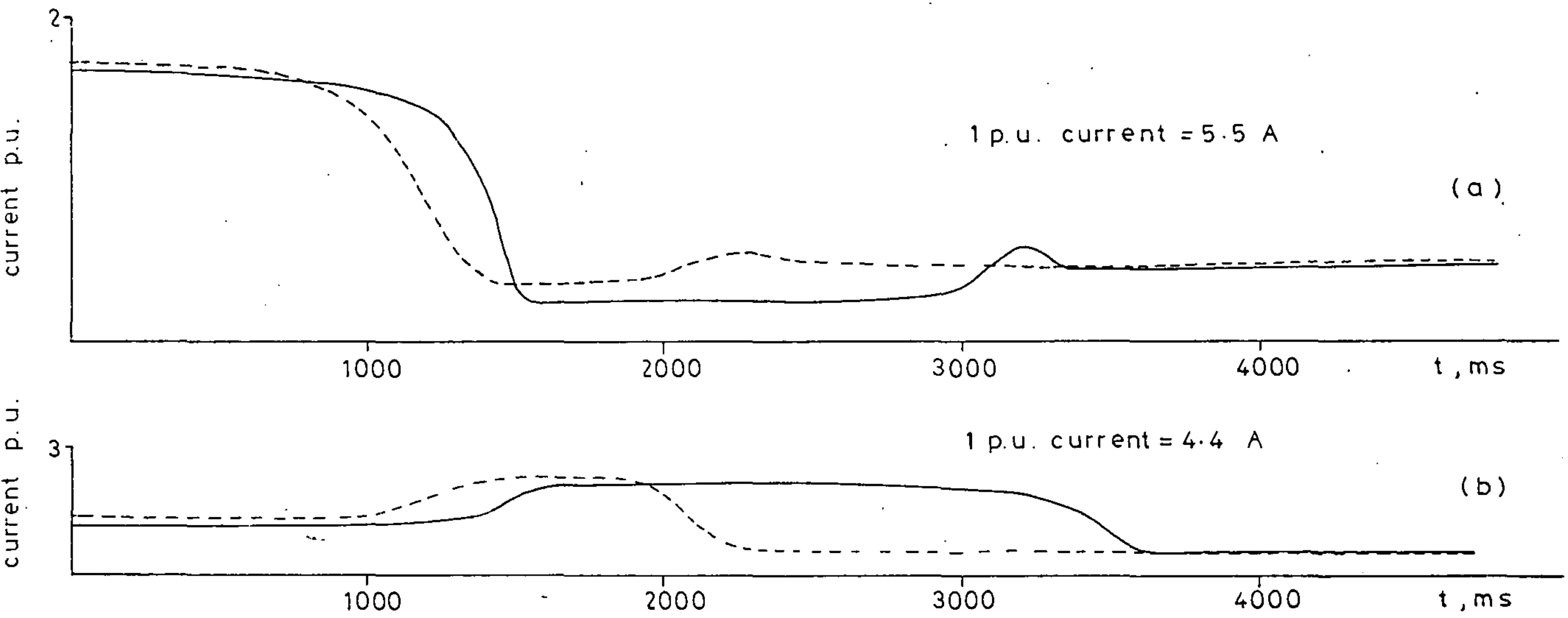


Figure 7.4 Transient current following connection of 0·75 kW/1·5 kW motor group to a non-stiff supply. (supply resistance = 11·0 ohm)
(a) Transient current of 1·5 kW motor. (b) Transient current of 0·75 kW motor.
Computed results --------, Experimental results ———

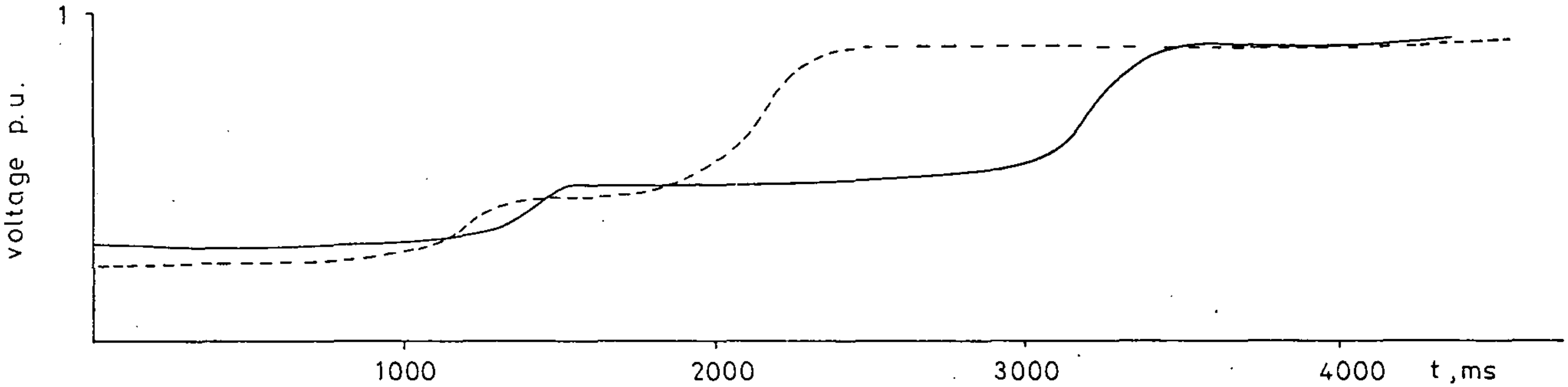


Figure 7·5 Terminal voltage following connection of 0·75 kW/1·5 kW motor group to a non-stiff supply. (supply resistance = 11·0 ohm)

Computed results------ ,Experimental results ———

1 p.u. voltage = 345 V·

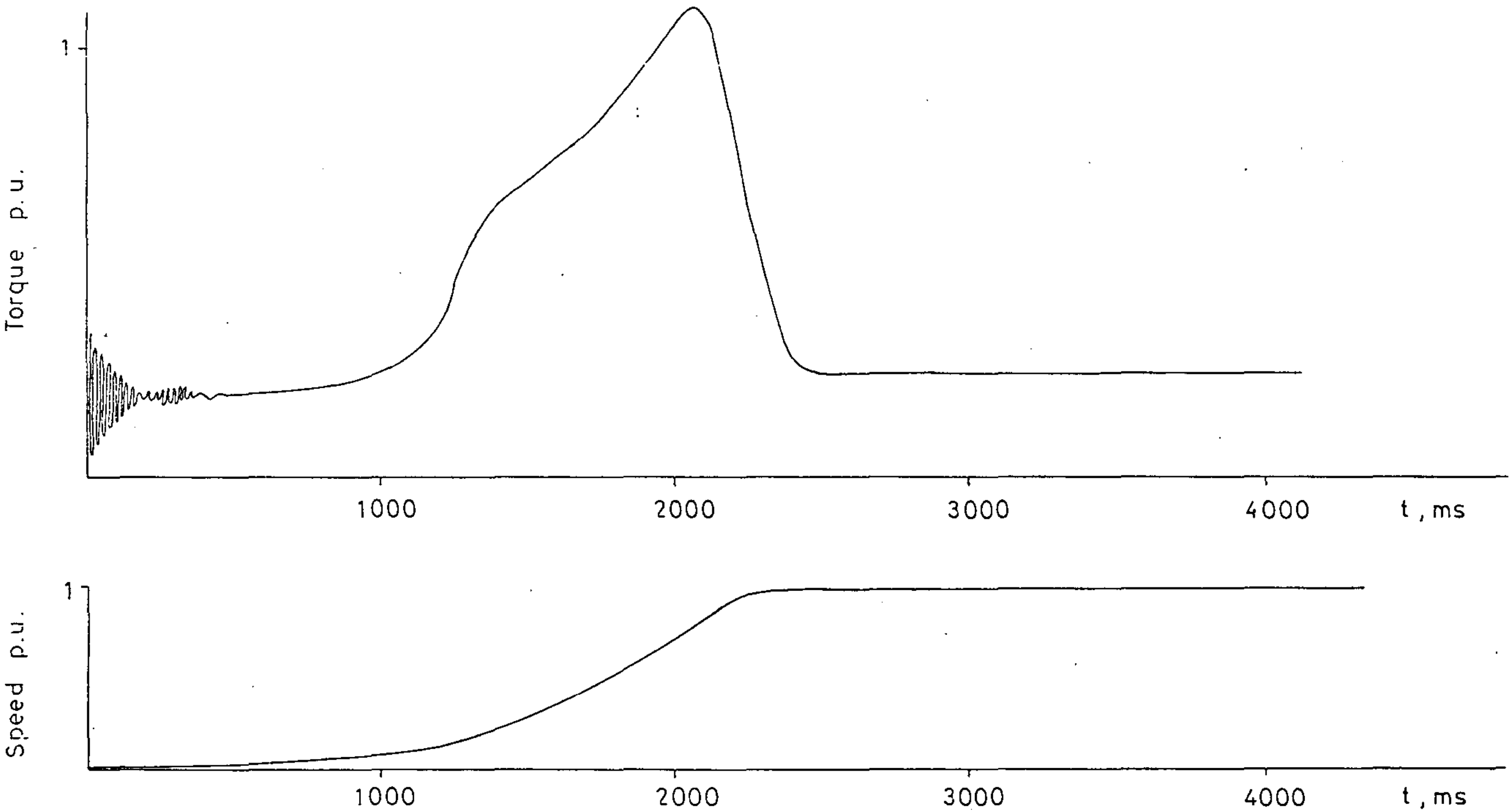


Figure 7·6 (a) Transient torque and speed of 0·75 kW motor following connection of 0·75 kW/1·5 kW motor group to non-stiff supply.

(supply resistance = 11·0 ohm)· 1 p.u. torque = 7·4 Nm, 1 p.u. speed = 314·2 elec.rad/sec.

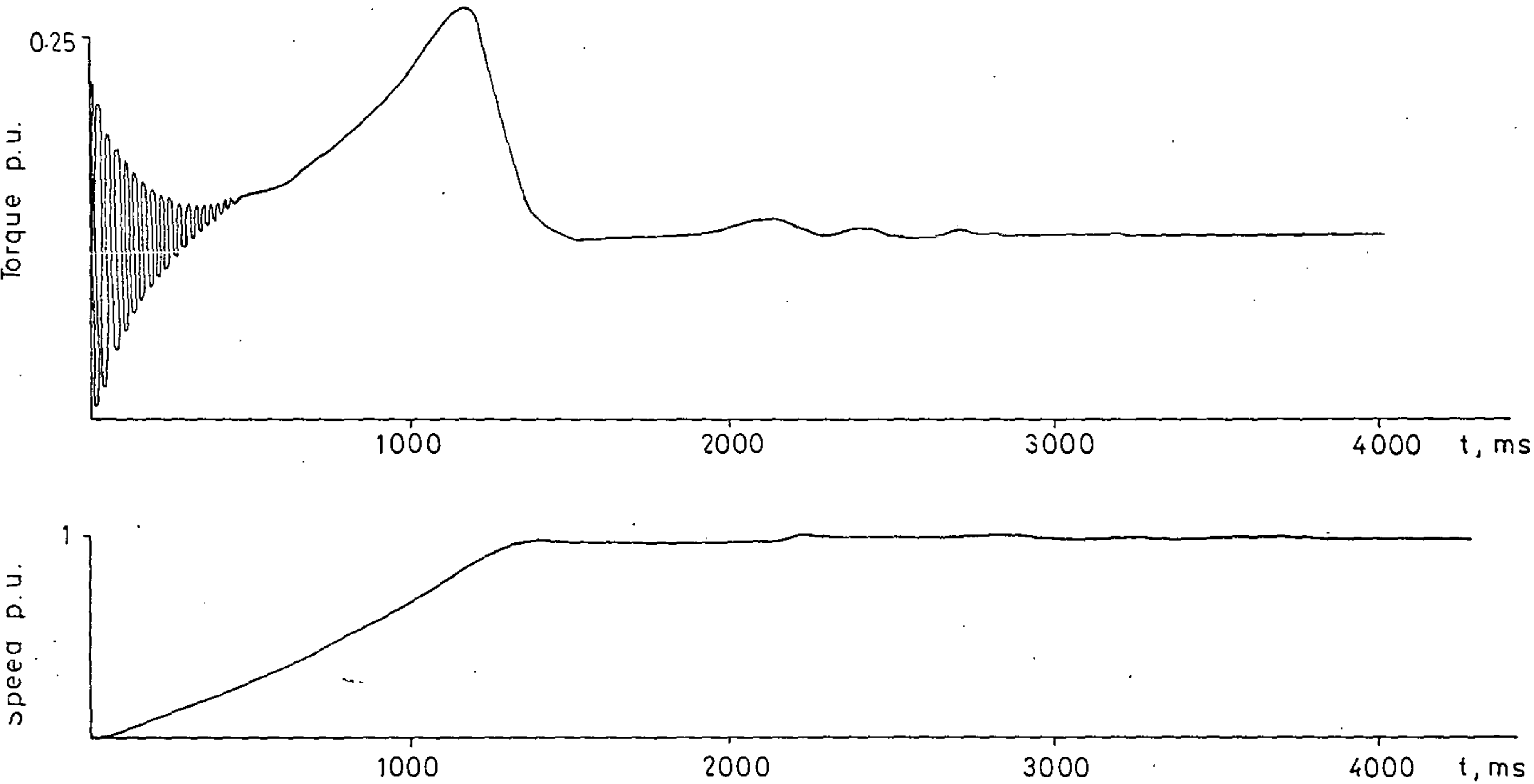


Figure 7·6(b) Transient torque and speed of 1·5 kW motor following connection of 0·75 kW/1·5 kW motor group to non-stiff supply. (supply resistance = 11·0 ohm)
1 p.u. torque = 4·7 Nm. , 1 p.u. speed = 314·2 elec. rad/sec.

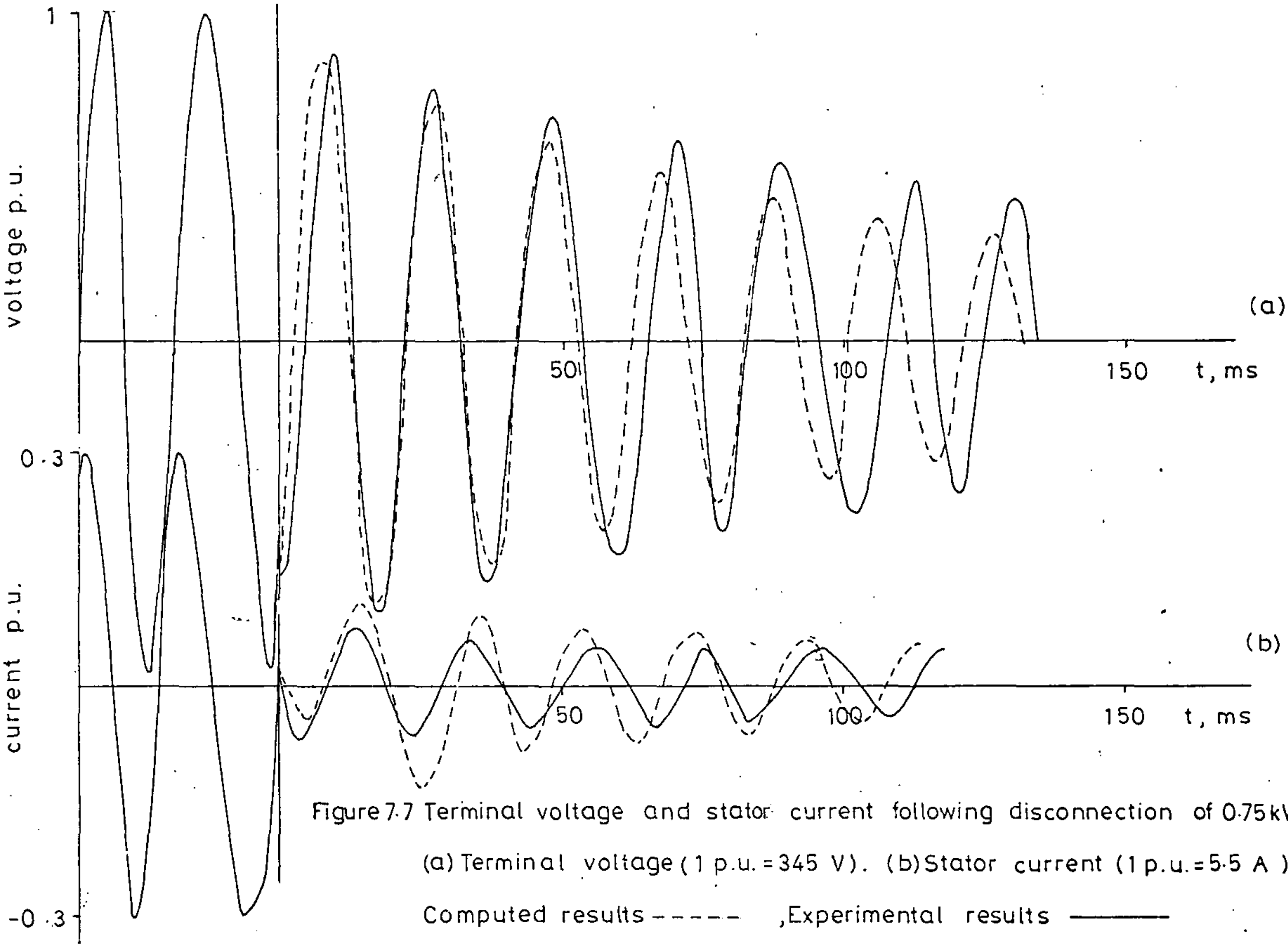


Figure 7·7 Terminal voltage and stator current following disconnection of 0·75 kW/1·5 kW motor group

(a) Terminal voltage (1 p.u. = 345 V). (b) Stator current (1 p.u. = 5·5 A).

Computed results - - - - - , Experimental results ———

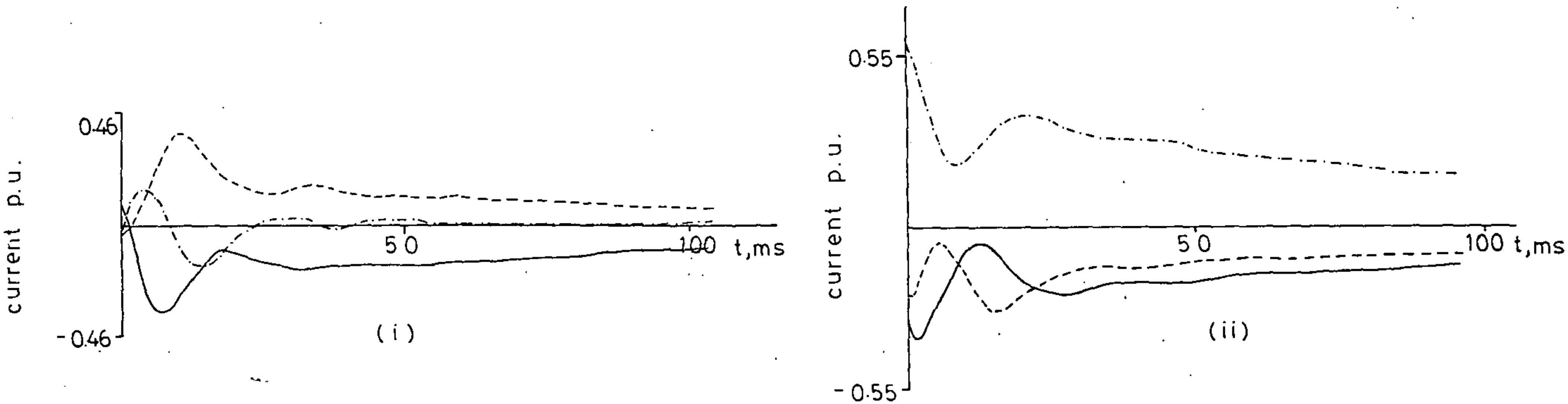


Figure 7.8 Computed rotor currents following disconnection of 0.75 kW/1.5 kW motor group.

(i) 0.75 kW motor (1 p.u. current = 4.4 A).

(ii) 1.5 kW motor (1 p.u. current = 5.5 A).

a-phase ——— , b-phase ----- , c-phase -·-·-

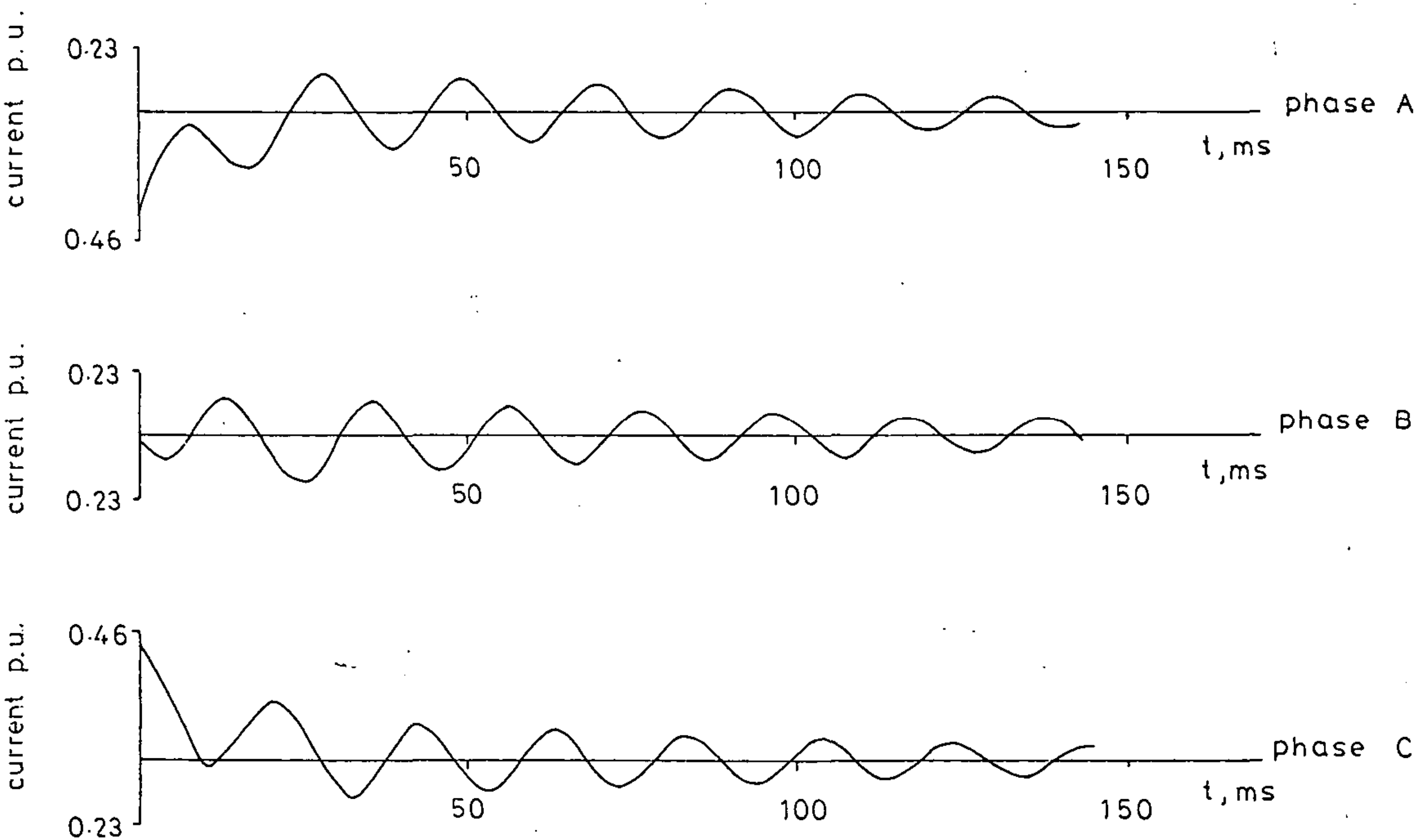


Figure 7·9 Stator current following disconnection of 0·75 kW / 1·5 kW motor group.

1 p.u. current = 4·4 A

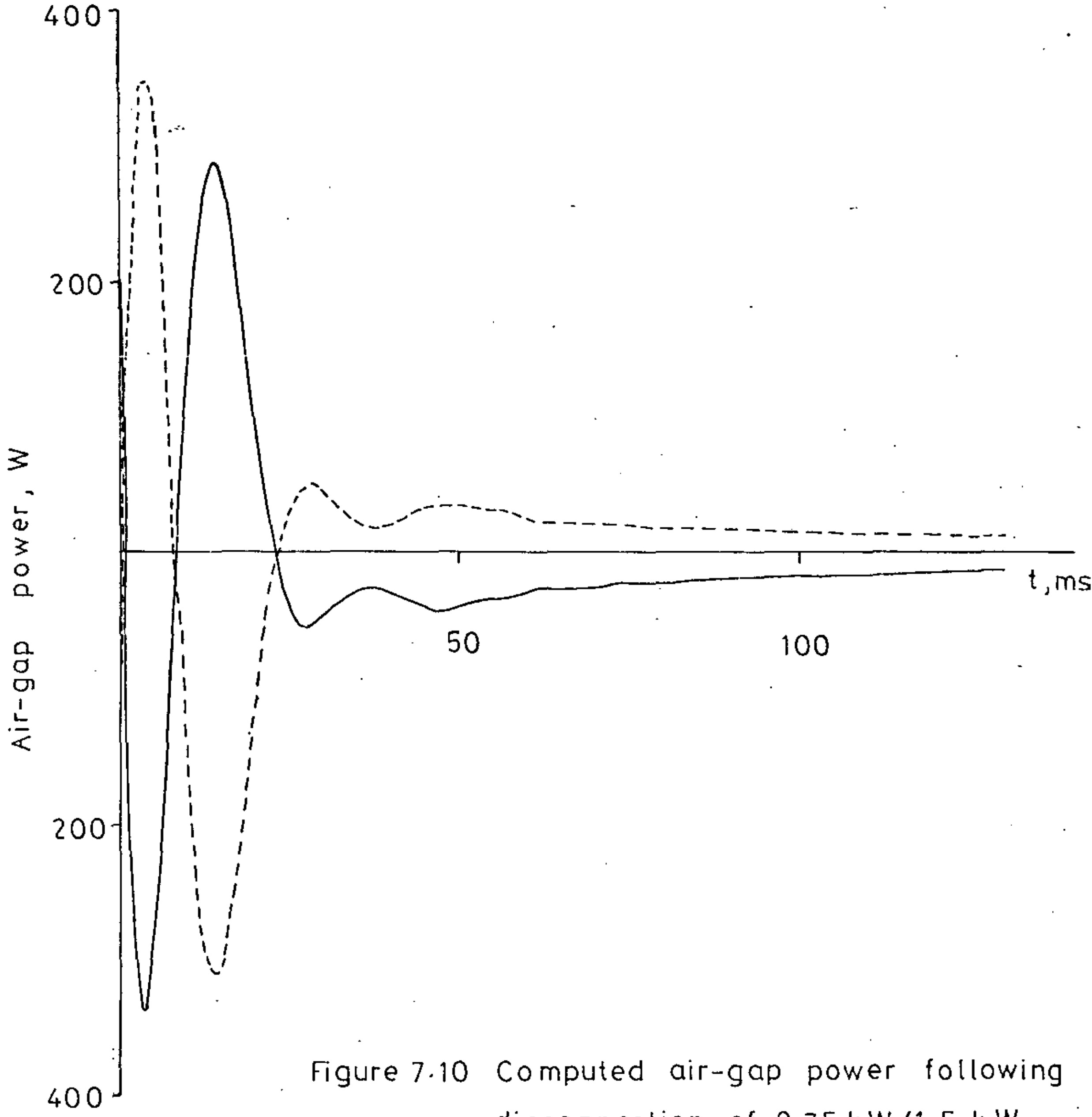


Figure 7·10 Computed air-gap power following disconnection of 0·75 kW/1·5 kW motor group.

——— 0·75 kW motor.

- - - - - 1·5 kW motor.

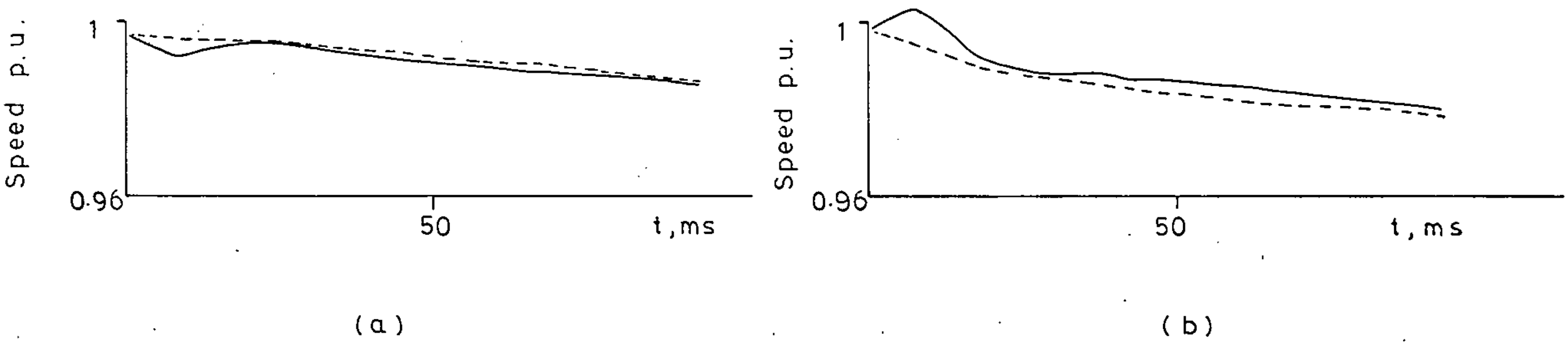


Figure 7.11 Computed speeds following disconnection of 0.75kW/1.5kW motor group.

(a) 0.75 kW motor. (b) 1.5 kW motor.

results in 0.75 kW/1.5 kW motor group ————

results in isolation - - - - - - -

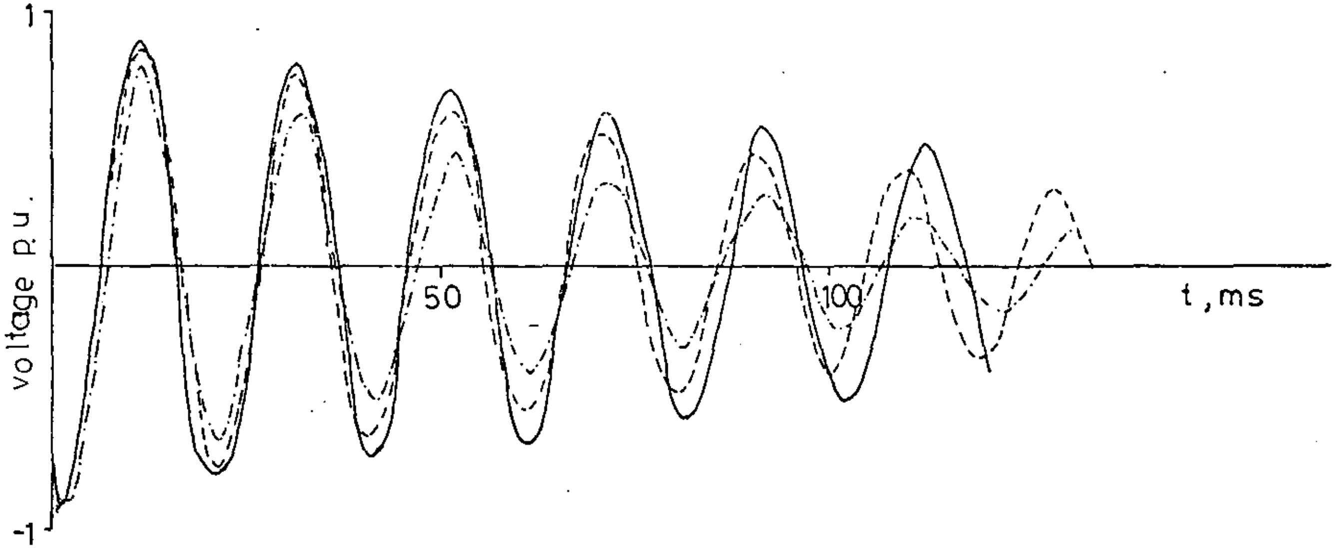


Figure 7·12 Comparison between terminal voltages of 0·75 kW/1·5 kW motor group and 0·75 kW and 1·5 kW motors in isolation following disconnection.

- - - - - - - 0·75 kW/1·5 kW motor group.

-·-·-·-·- 0·75 kW motor.

——— 1·5 kW motor.

1 p.u. voltage = 345 V.

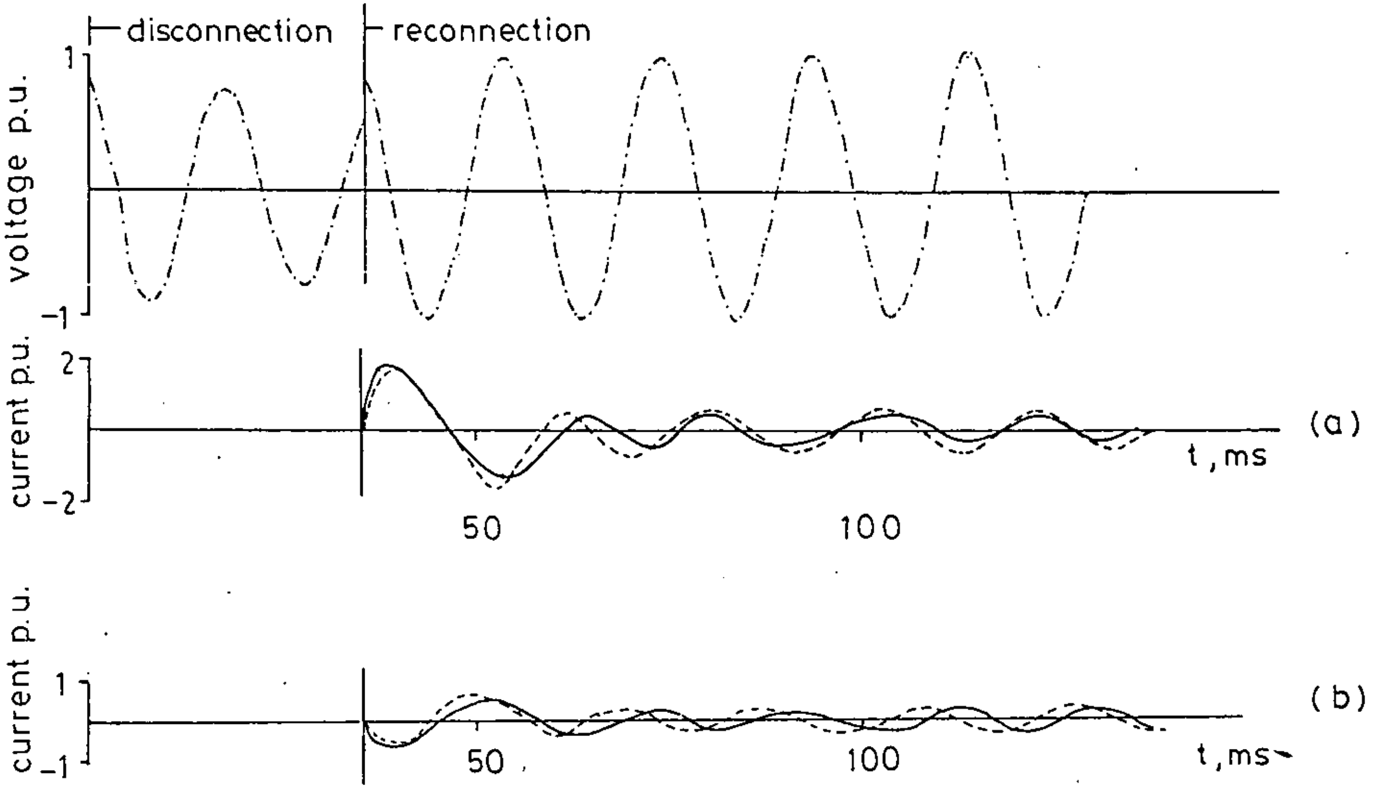


Figure 7.13 Transient current following reconnection of 0.75 kW/1.5 kW motor group.

(a) r-line current of 1.5 kW motor (1 p.u. current = 5.5 A)

(b) b-line current of 0.75 kW motor (1 p.u. current = 4.4 A)

Computed results -------- , Experimental results ————

r-y line voltage -·-·-·- , 1 p.u. voltage = 345 V.

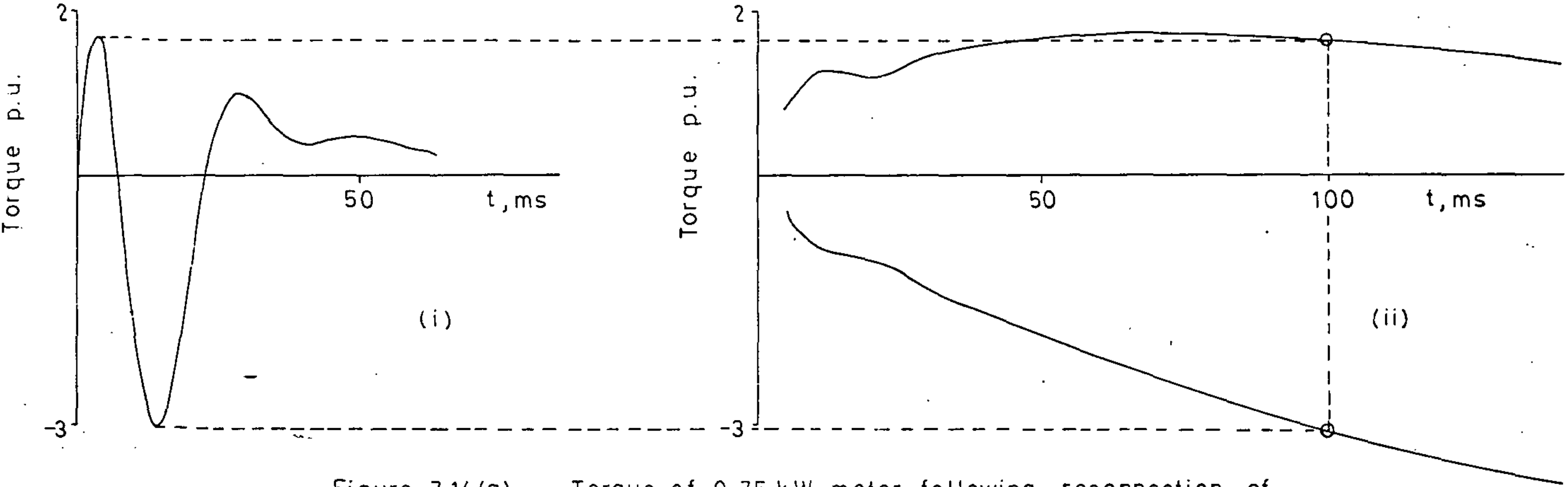


Figure 7.14(a) Torque of 0·75 kW motor following reconnection of 0·75 kW/1·5 kW motor group.

(i) Computed transient torque pattern.

(ii) Variation of peak torque with delay.

1 p.u. torque = 7·4 Nm.

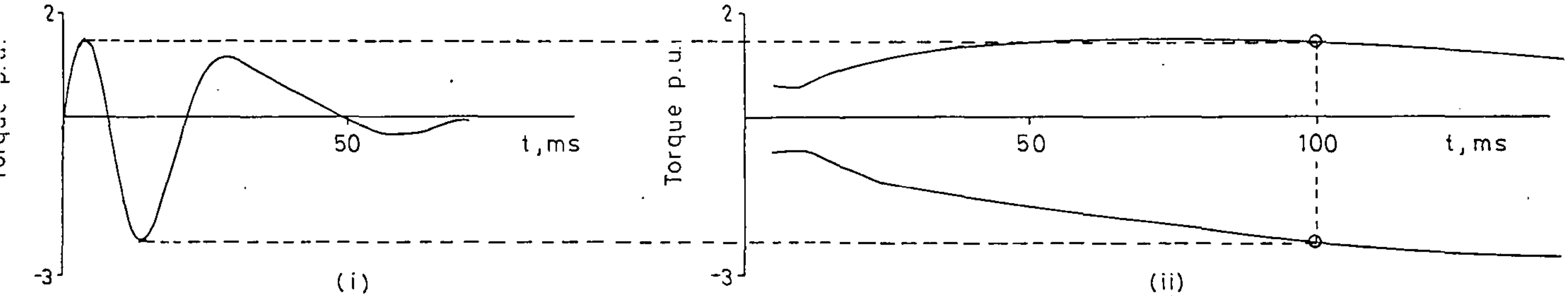


Figure 7·14 (b) Torque of 1·5 kW motor following reconnection of 0·75 kW/1·5 kW motor group.

(i) Computed transient torque pattern.

(ii) Variation of peak torque with delay.

1 p.u. torque = 4·7 Nm.

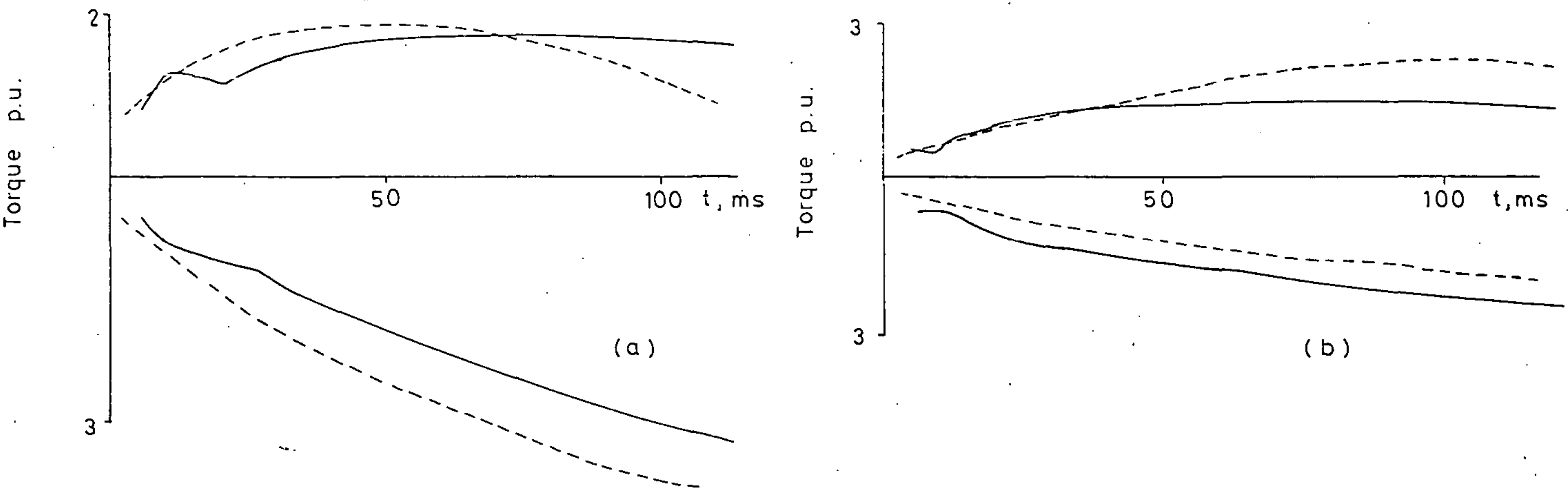
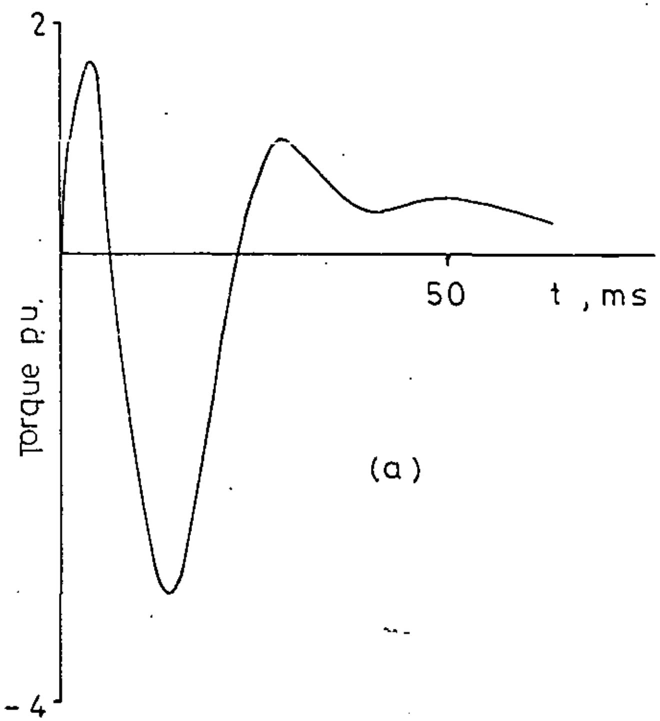


Figure 7.15 Comparison between torque peaks following reconnection of 0.75 kW and 1.5 kW motors in isolation and in 0.75 kW/1.5 kW motor group.

(a) Variation of peak torque with delay of 0.75 kW motor (1 p.u. torque = 7.4 Nm).

(b) Variation of peak torque with delay of 1.5 kW motor (1 p.u torque = 4.7 Nm).

motor in isolation ------ , motor in group ———



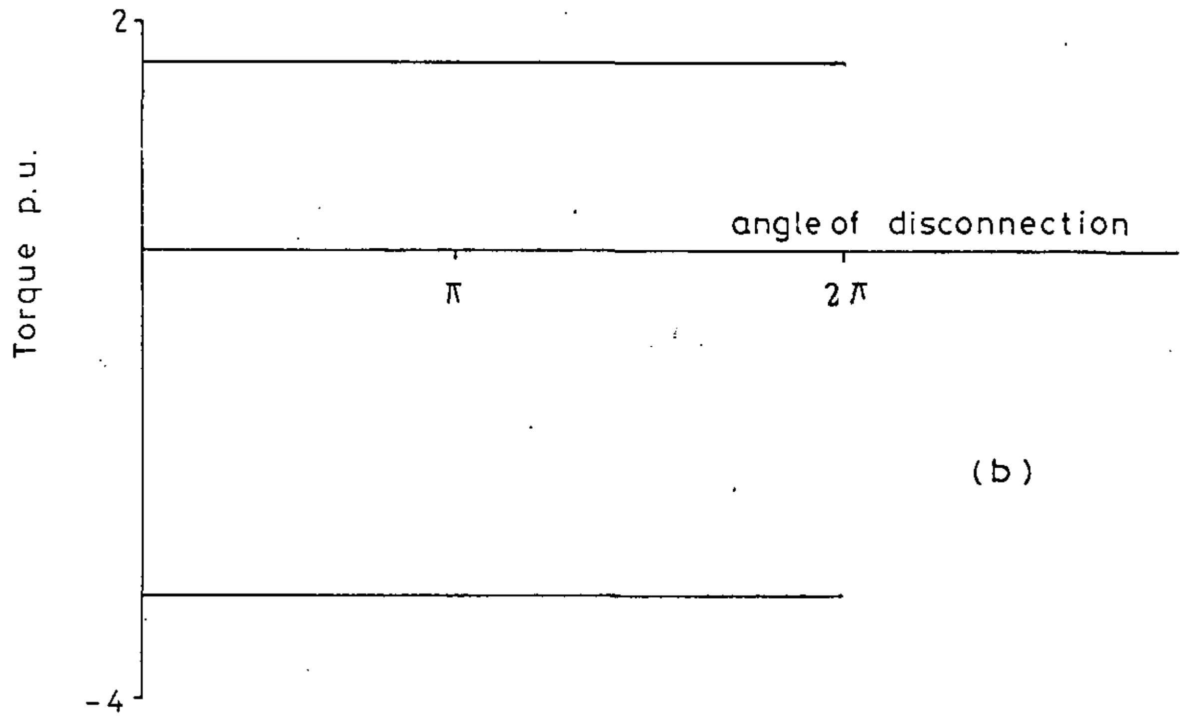


Figure 7·16 (a) Torque of 0·75 kW motor following reconnection of 0·75 kW/1·5 kW motor group at constant supply interruption and different instant of connection.

(a) Transient torque pattern.

(b) Variation of peak torque with angle of disconnection.

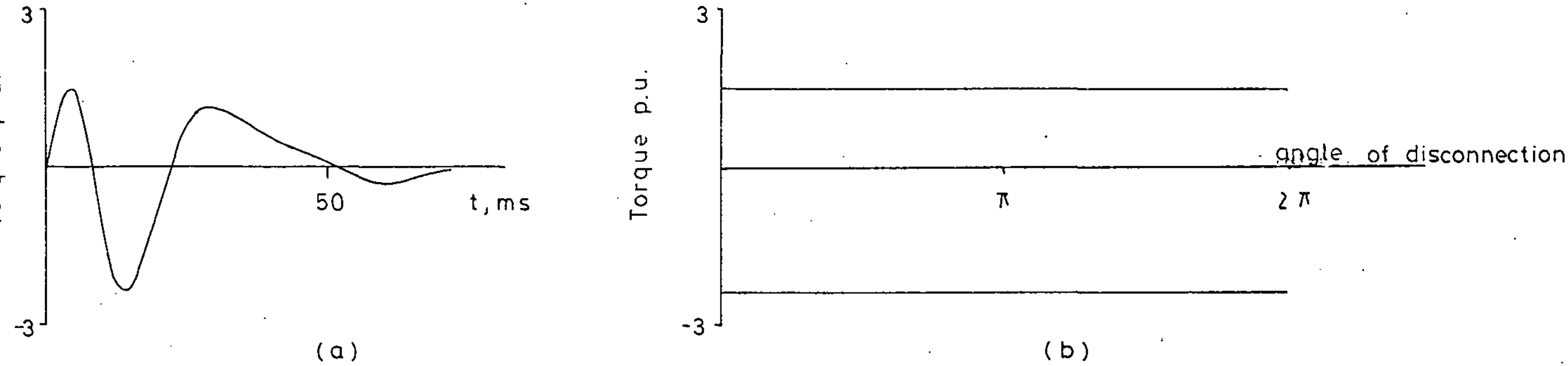


Figure 7.16 (b) Torque of 1.5 kW motor following reconnection of 0.75 kW/1.5 kW motor group at constant supply interruption and different instant of connection.

(a) Transient torque pattern.

(b) Variation of peak torque with angle of disconnection.

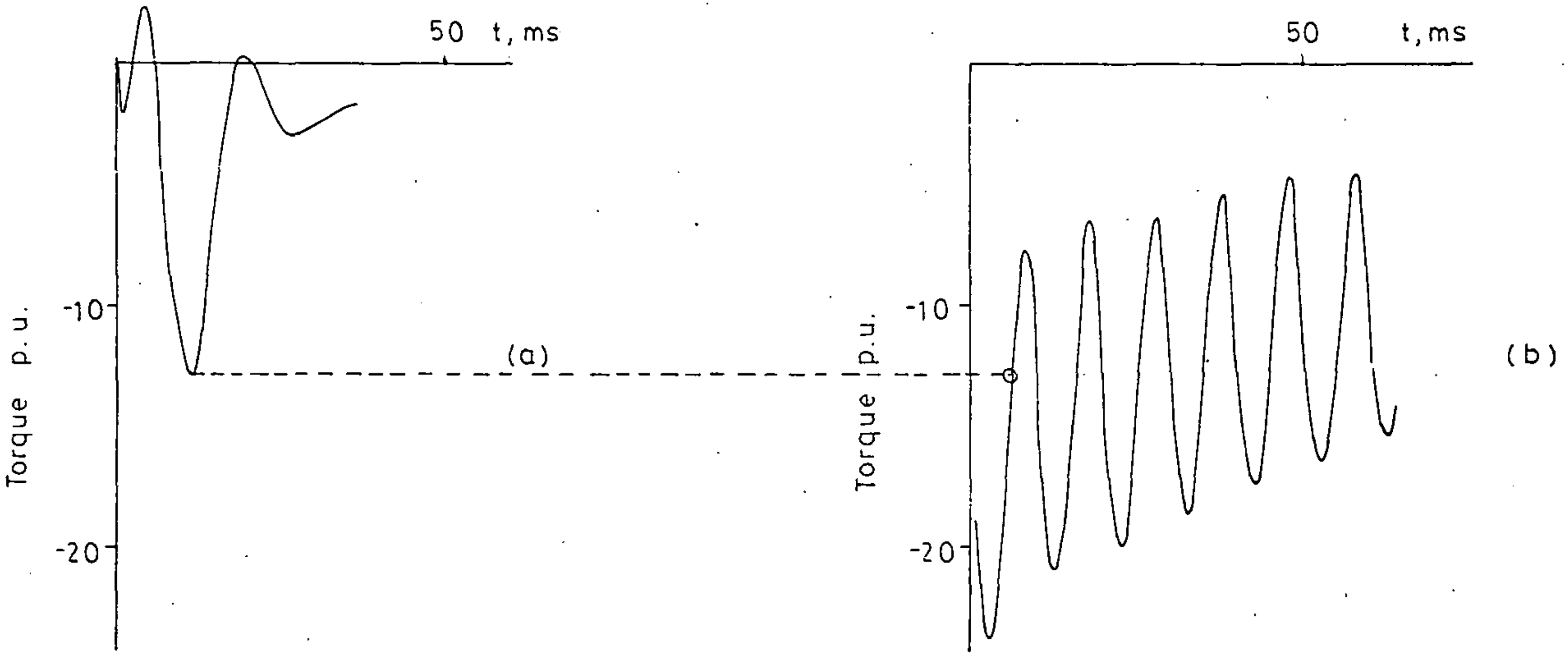


Figure 7·17 Torque of 0·75 kW motor following plugging of 0·75 kW/1·5 kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 7·4 Nm.

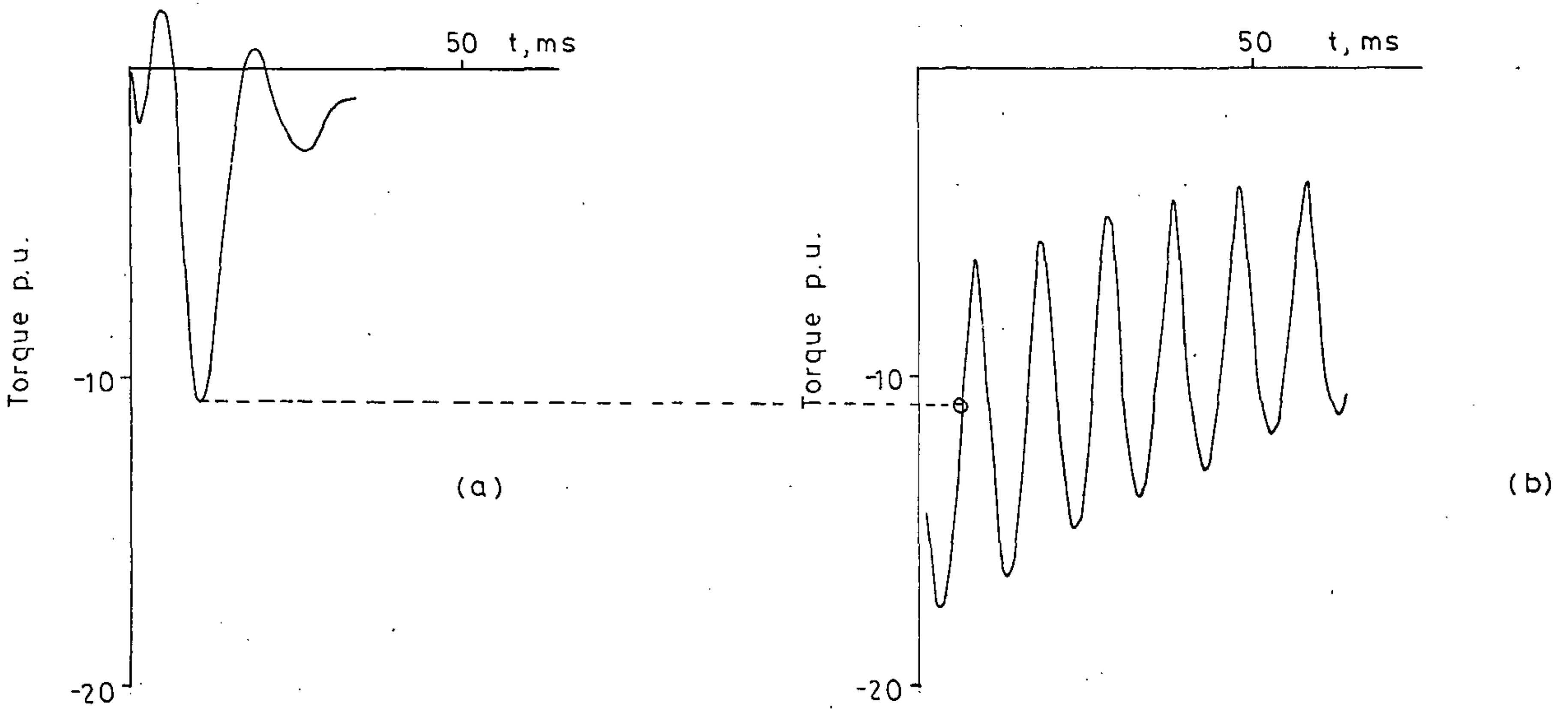


Figure 7·18 Torque of 1·5 kW motor following plugging of 0·75 kW/1·5 kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 4·7 Nm.

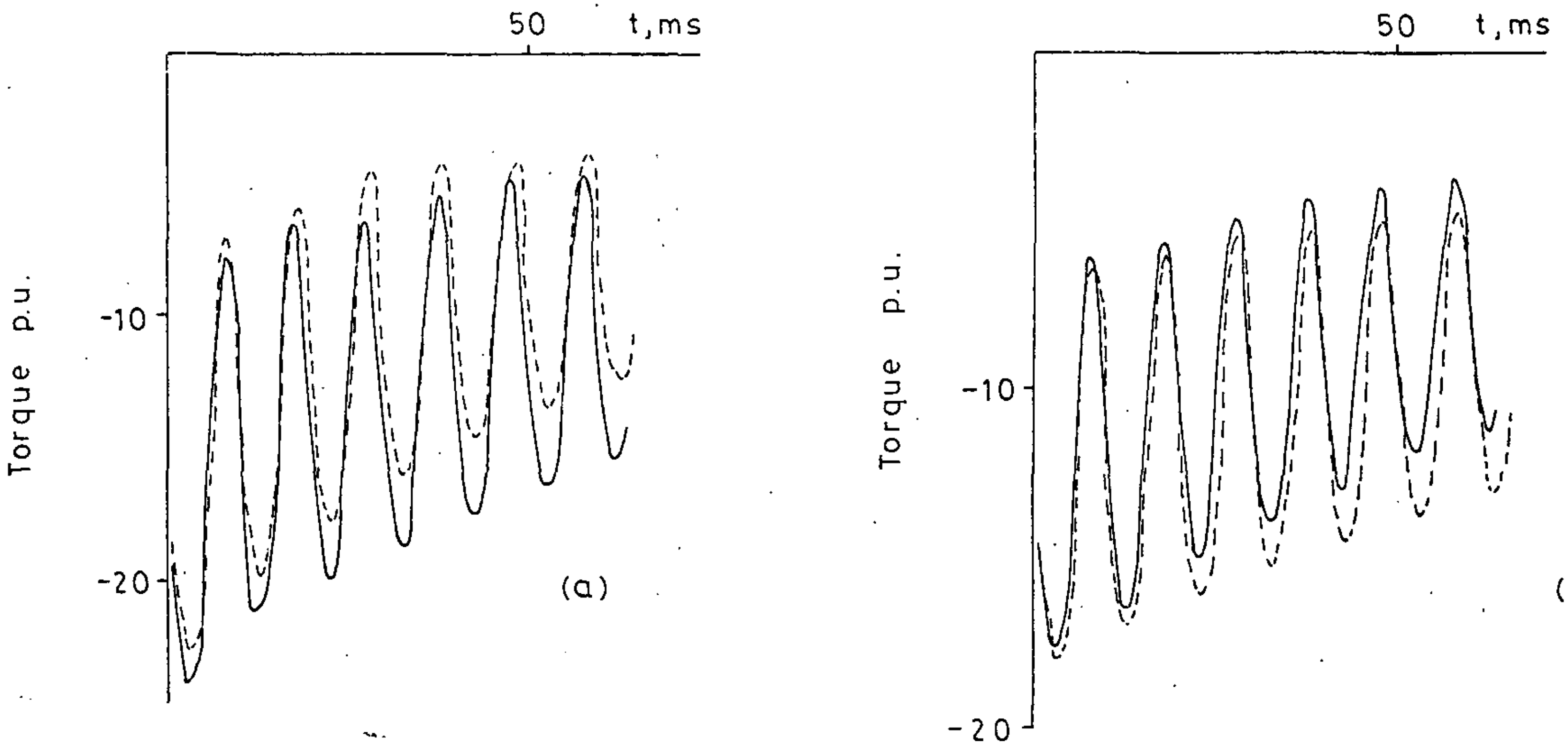


Figure 7·19 Comparison between torque peaks following plugging of 0·75 kW/1·5 kW motor group and 0·75 kW and 1·5 kW motors in isolation.

(a) Variation of peak torque with delay of 0·75 kW motor (1 p.u. torque = 7·4 Nm).

(b) Variation of peak torque with delay of 1·5 kW motor (1 p.u. torque = 4·7 Nm).

motor in isolation --------, motor in group ————

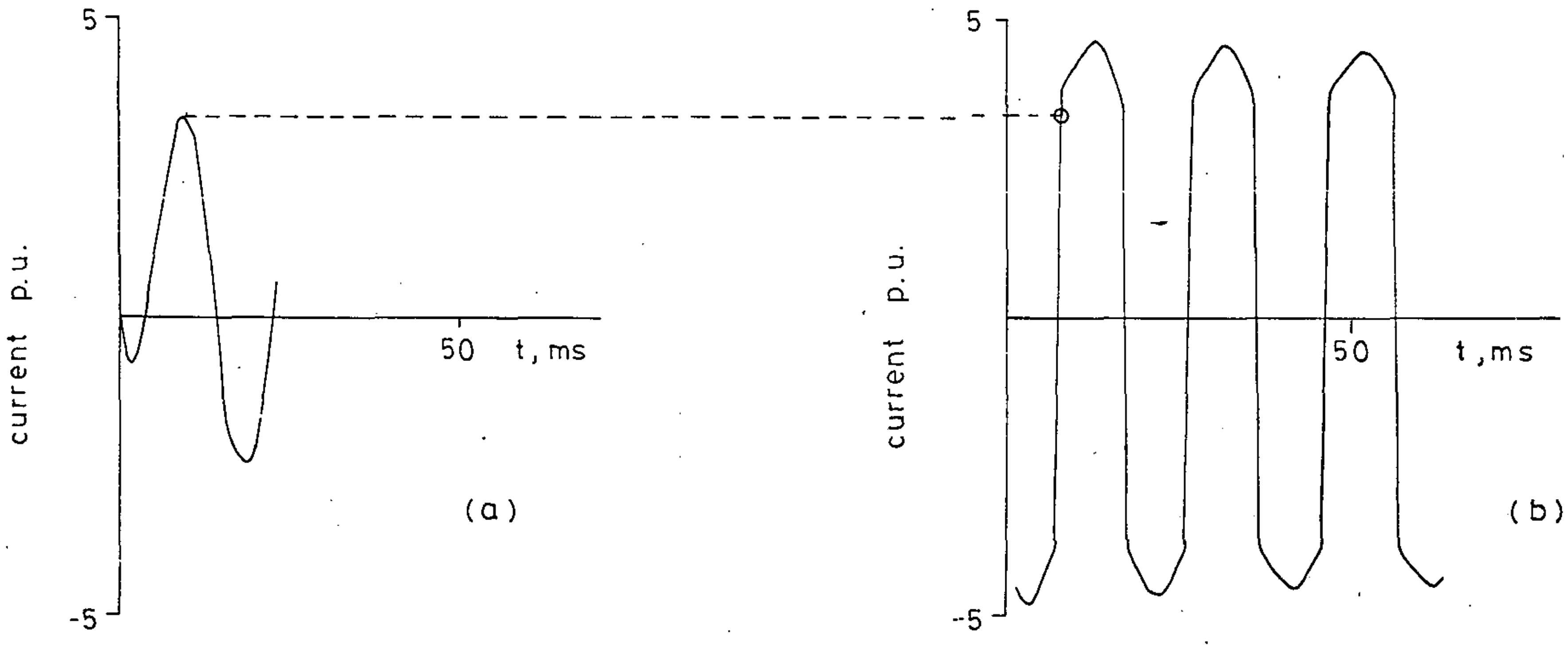


Figure 7·20 Current of 0·75 kW motor following plugging of 0·75 kW/1·5 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 4·4 A

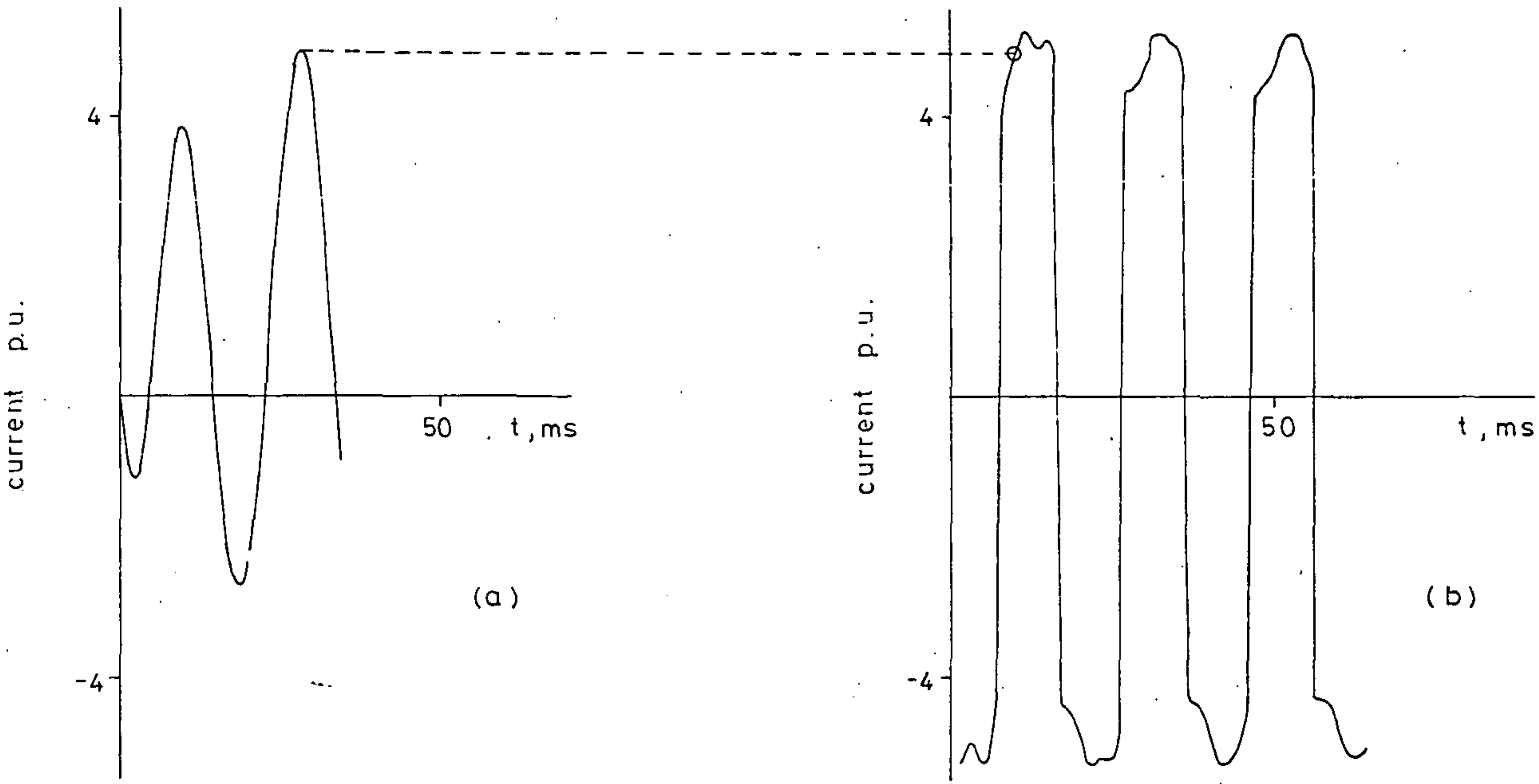


Figure 7·21 Current of 1·5 kW motor following plugging of 0·75 kW/1·5 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 5·5 A.

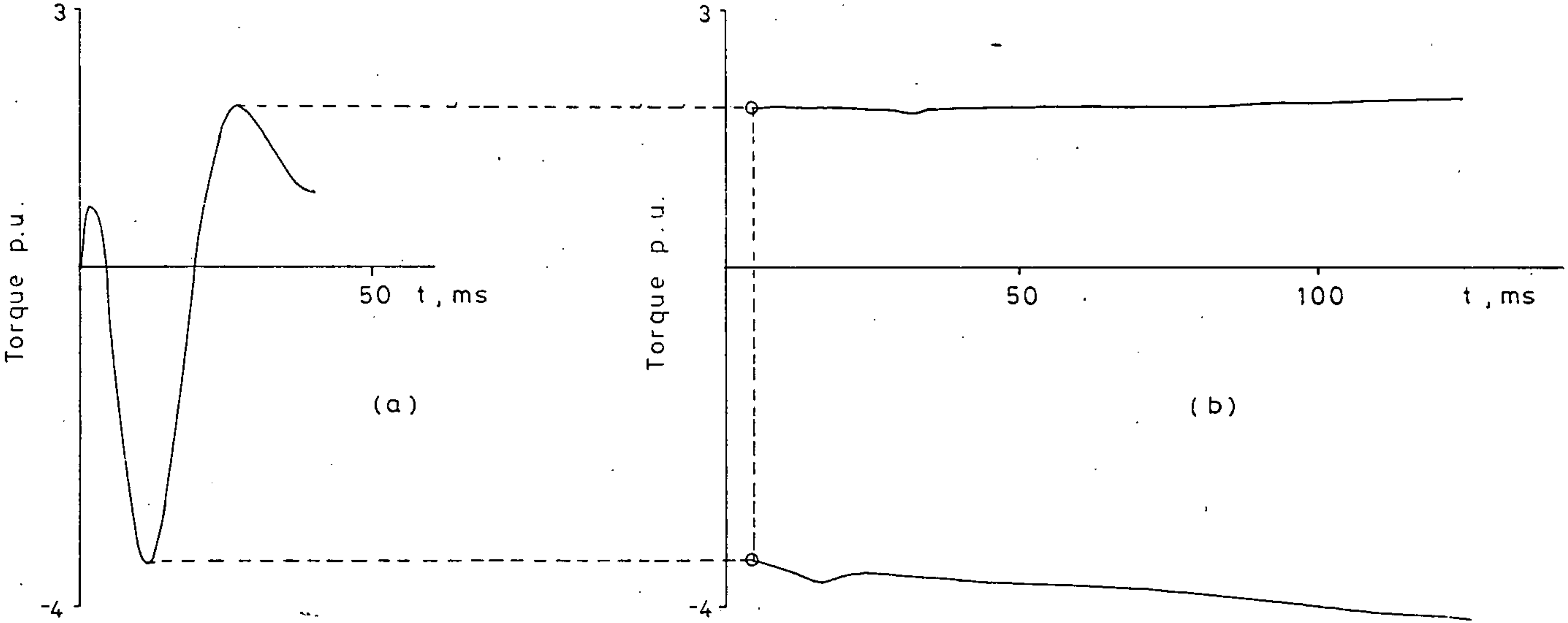


Figure 7·22 Torque of 0·75 kW motor following star-delta starting of 0·75 kW/1·5 kW motor group.
(a) Computed transient torque pattern.
(b) Variation of peak torque with delay.
1 p.u. torque = 7·4 Nm.

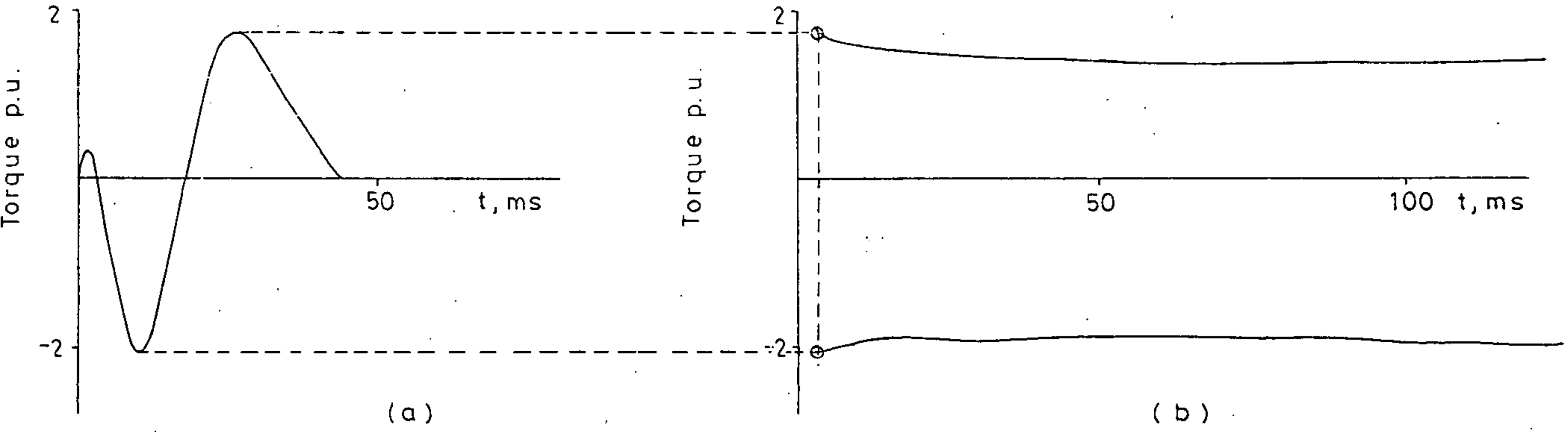


Figure 7·23 Torque of 1·5 kW motor following star-delta starting of 0·75 kW/1·5 kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 4·7 Nm.

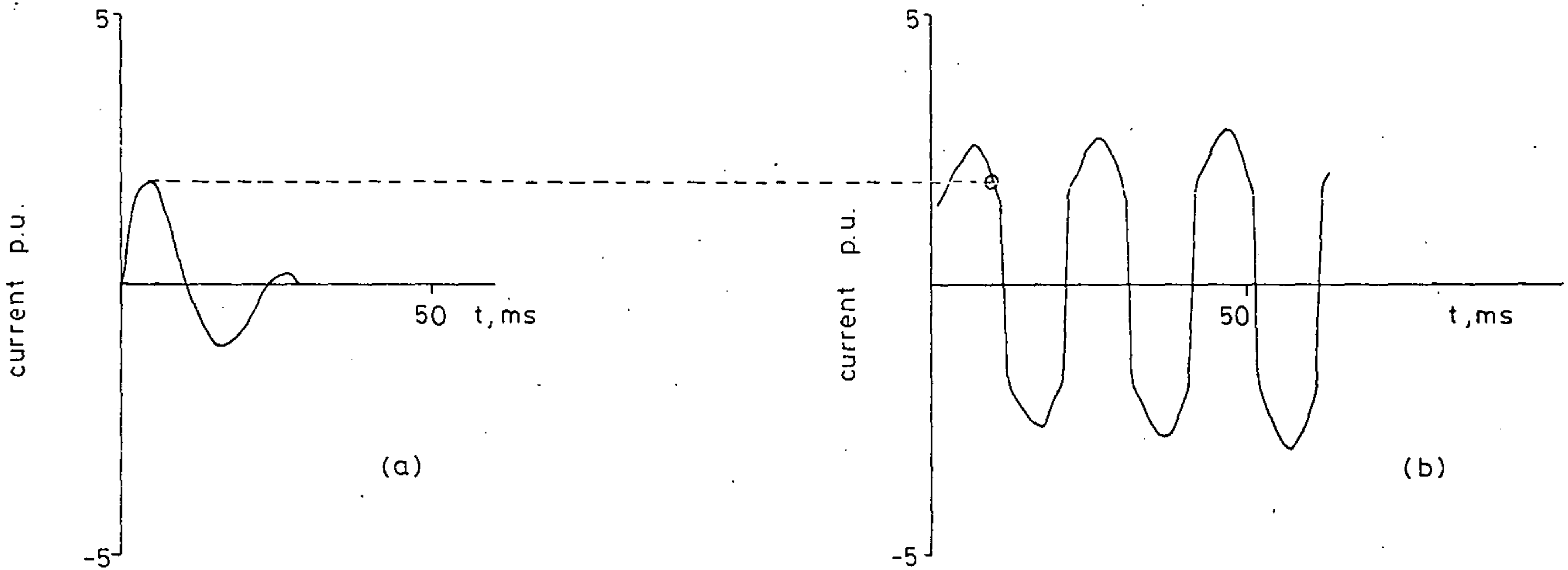


Figure 7·24 Current of 0·75 kW motor following star-delta starting of 0·75 kW/1·5 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 4·4 A.

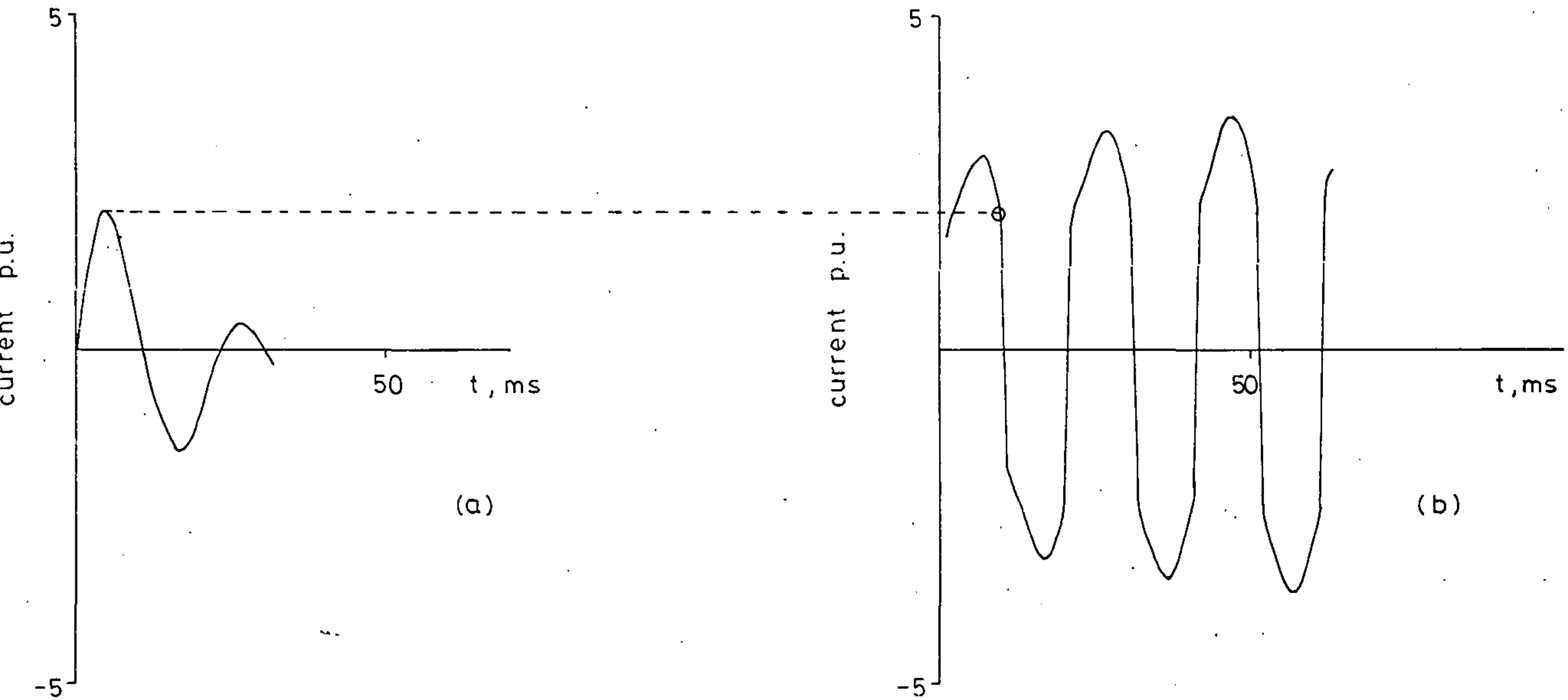


Figure 7.25 Current of 1.5 kW motor following star-delta starting of 0.75 kW/1.5 kW motor group.
(a) Computed transient current pattern.
(b) Variation of peak current with delay.
1 p.u. current = 5.5 A.

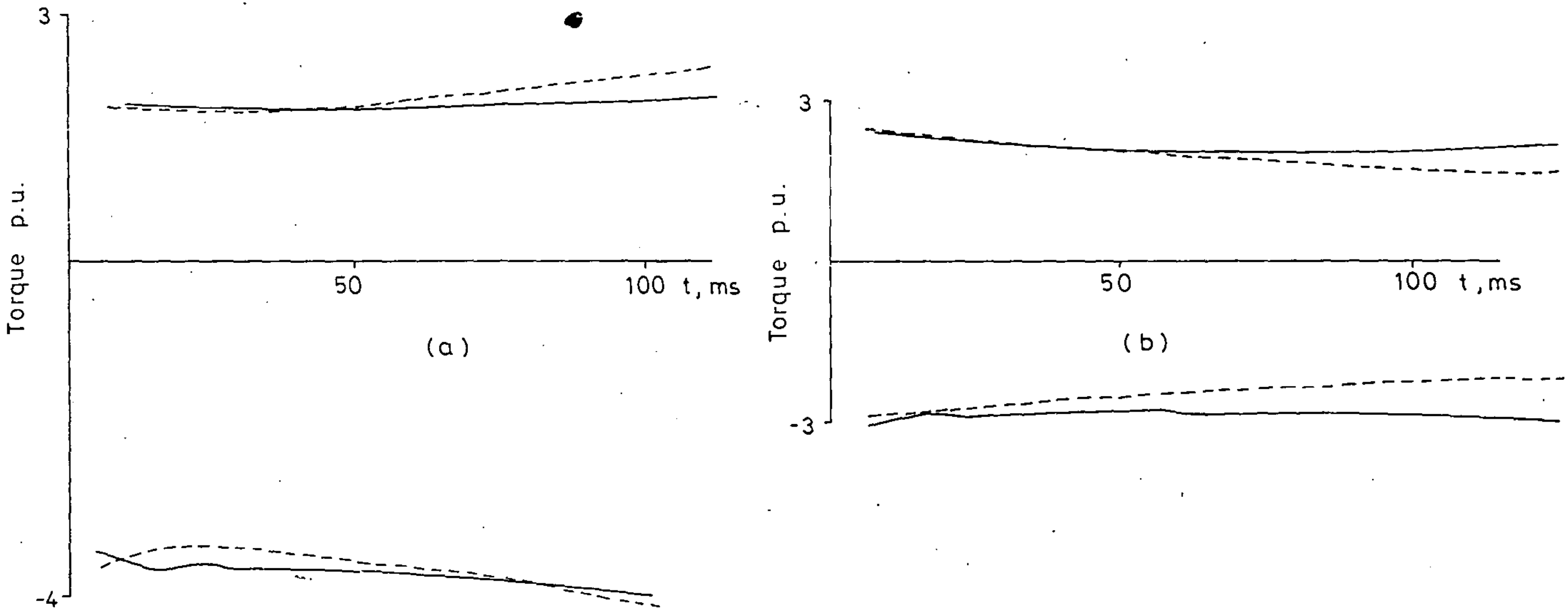


Figure 7.26 Comparison between torque peaks following star-delta starting of 0.75 kW and 1.5 kW motors in isolation and in 0.75 kW/1.5 kW motor group.

(a) Variation of peak torque with delay of 0.75 kW motor (1 p.u. torque = 7.4 Nm).

(b) Variation of peak torque with delay of 1.5 kW motor (1 p.u. torque = 4.7 Nm).

motor in isolation - - - - - - , motor in group ————

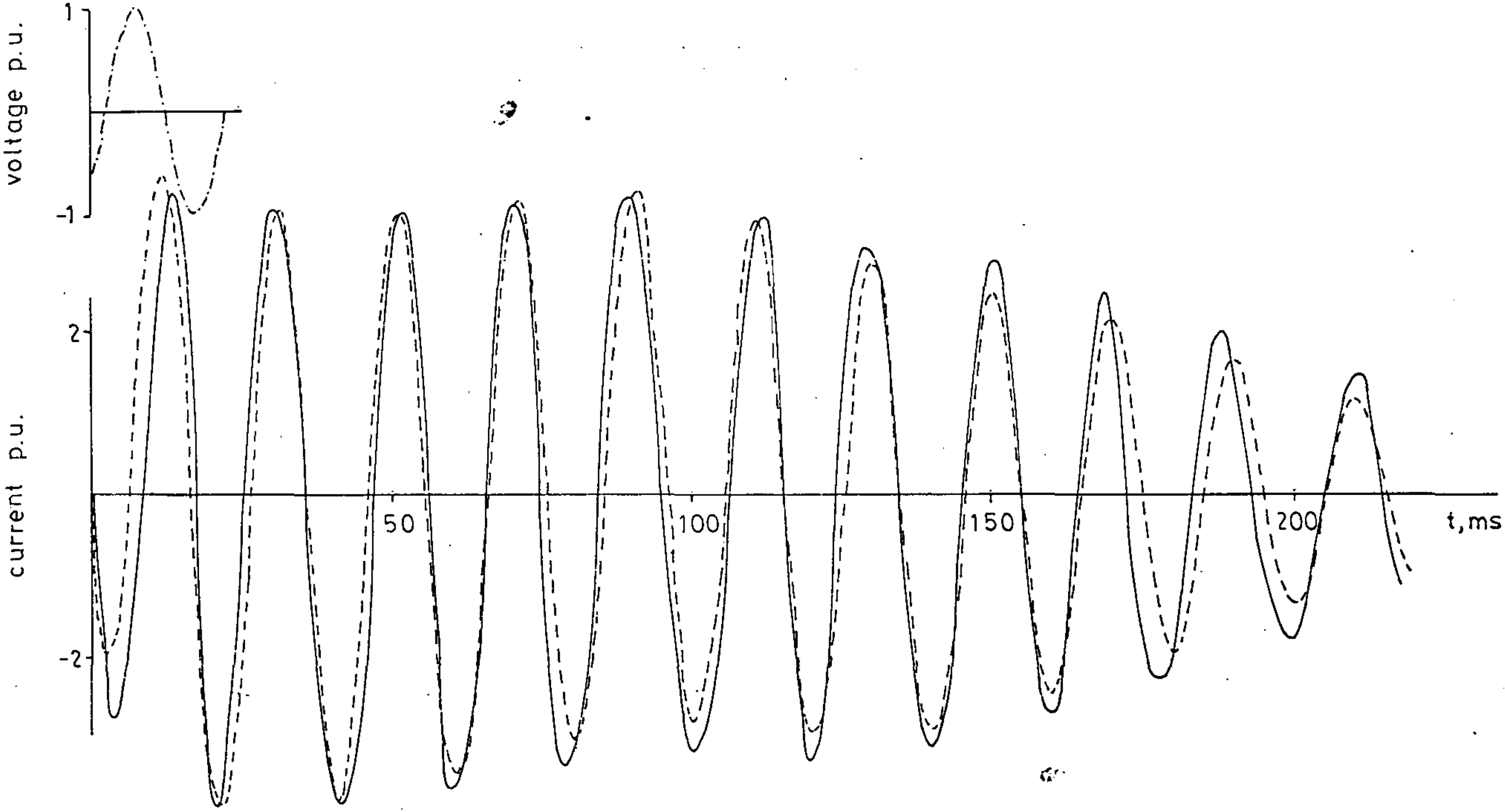


Figure 8.1(a) Transient current in r-line of 0.75 kW motor following direct-to-line starting of 0.75 kW/1.5 kW/2.25 kW motor group.

Computed results----- ,Experimental results ——— ,r-y line voltage —·—·—·—

1 p.u. voltage = 345 V , 1 p.u. current = 4.4 A.

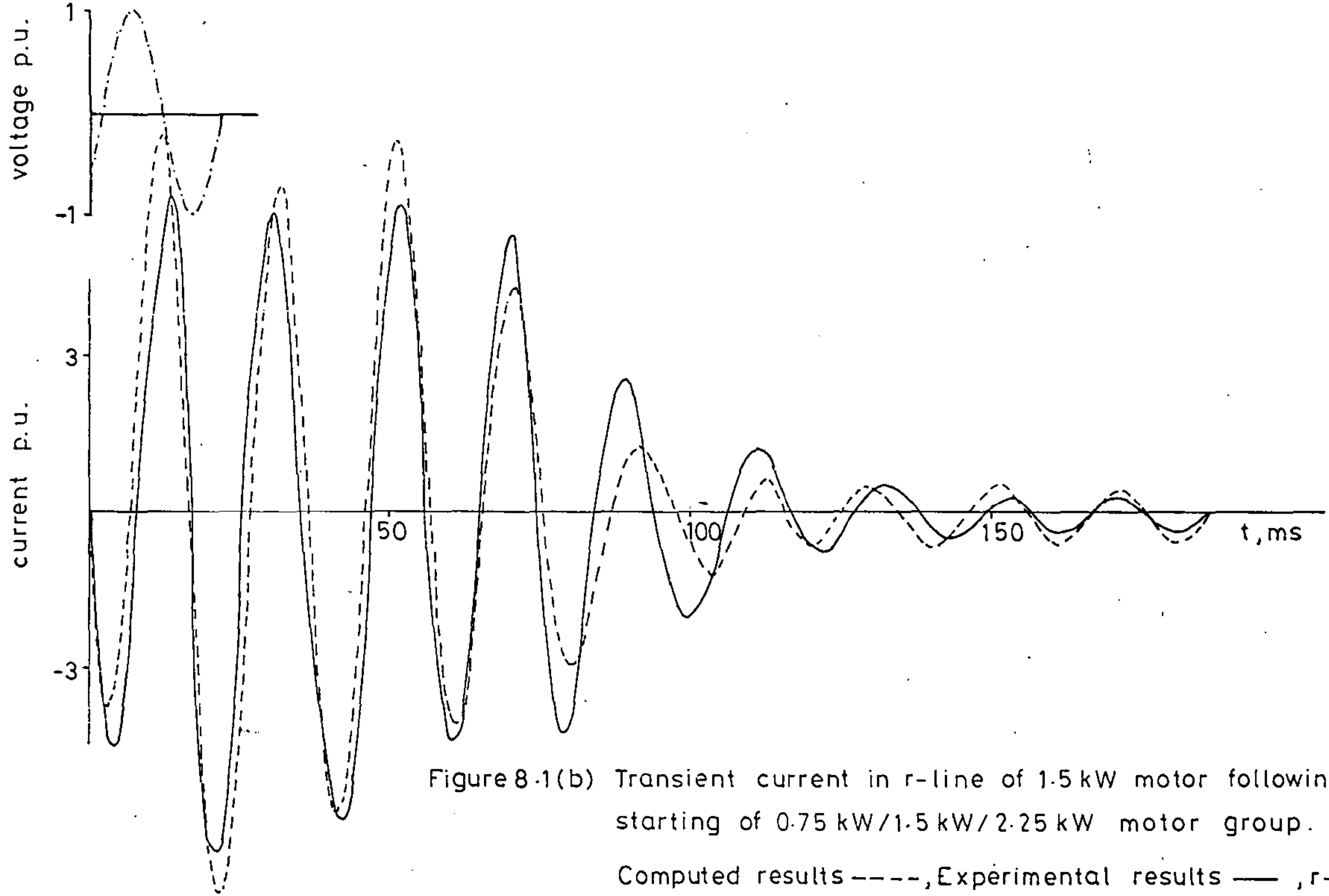


Figure 8·1(b) Transient current in r-line of 1·5 kW motor following direct-to-line starting of 0·75 kW/1·5 kW/2·25 kW motor group.

Computed results ----, Experimental results —— , r-y line voltage -·-·-·-

1 p.u. voltage = 345 V , 1 p.u. current = 5·5 A .

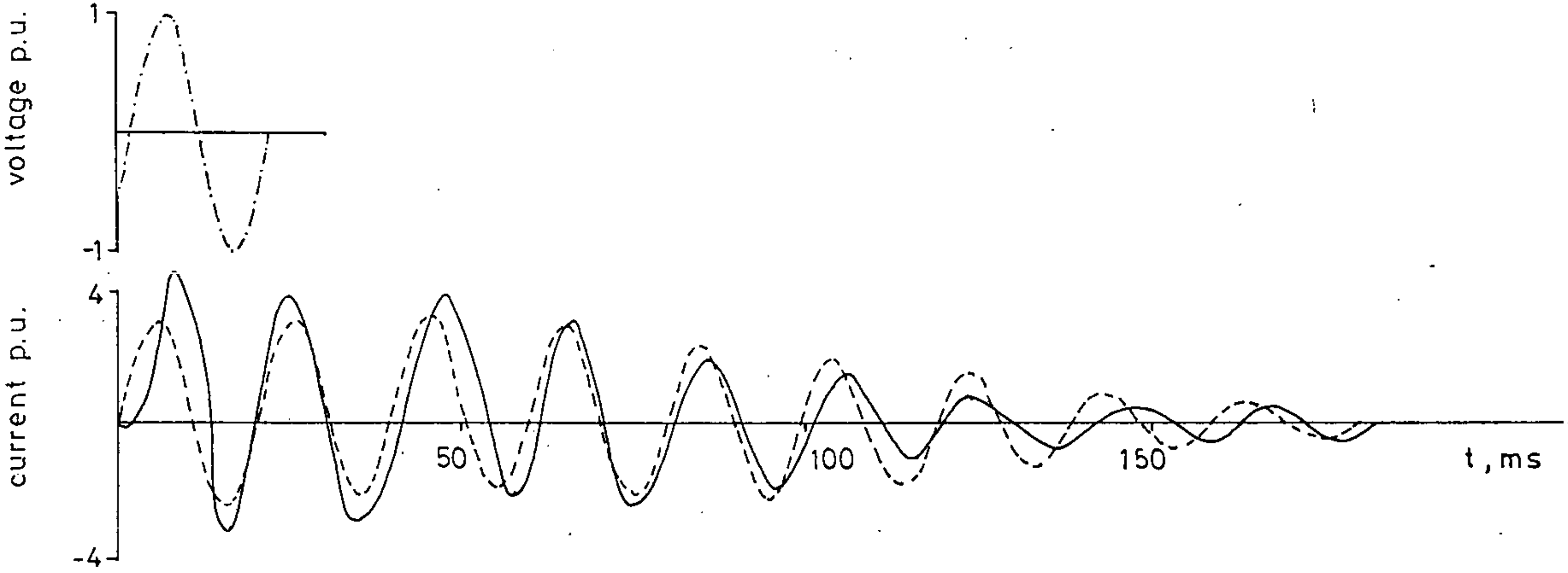


Figure 8.1(c) Transient current in b-line of 2.25 kW motor following direct-to-line starting of 0.75 kW/1.5 kW/2.25 kW motor group.

Computed results ------ , Experimental results ——— , r-y line voltage -·-·-·-

1 p.u. voltage = 345 V , 1 p.u. current = 12.7 A .

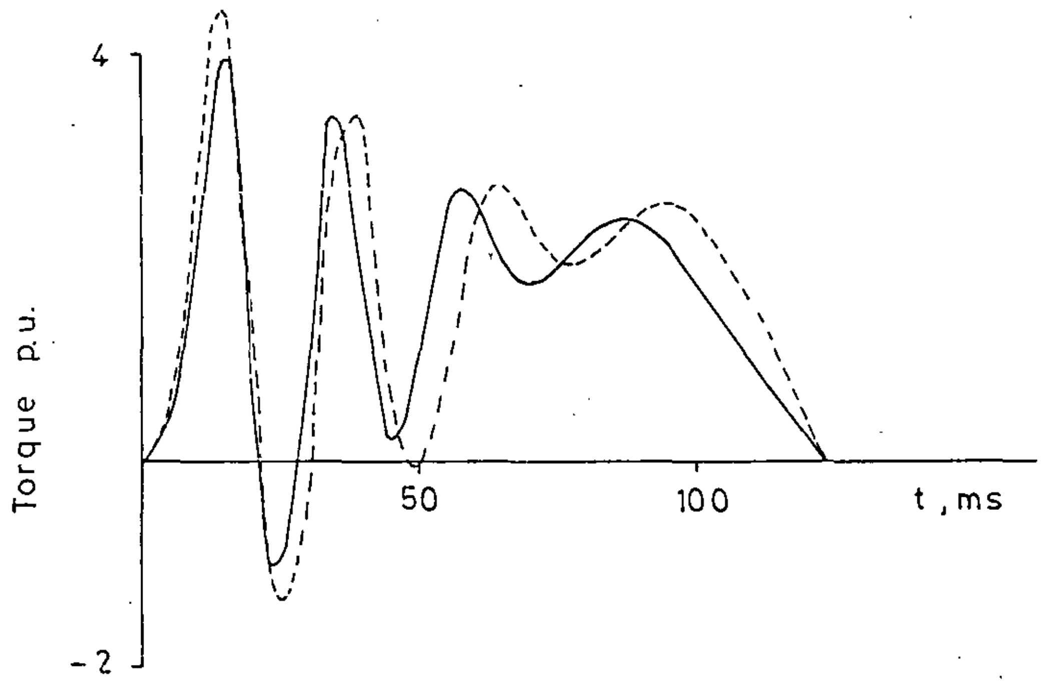


Figure 8·2 Transient torque of 5·6 kW motor following direct-to-line starting of 0·75 kW/ 2·25 kW/ 5·6 kW motor group.

Computed results - - - - - - , Experimental results ———

1 p.u. torque = 56 Nm.

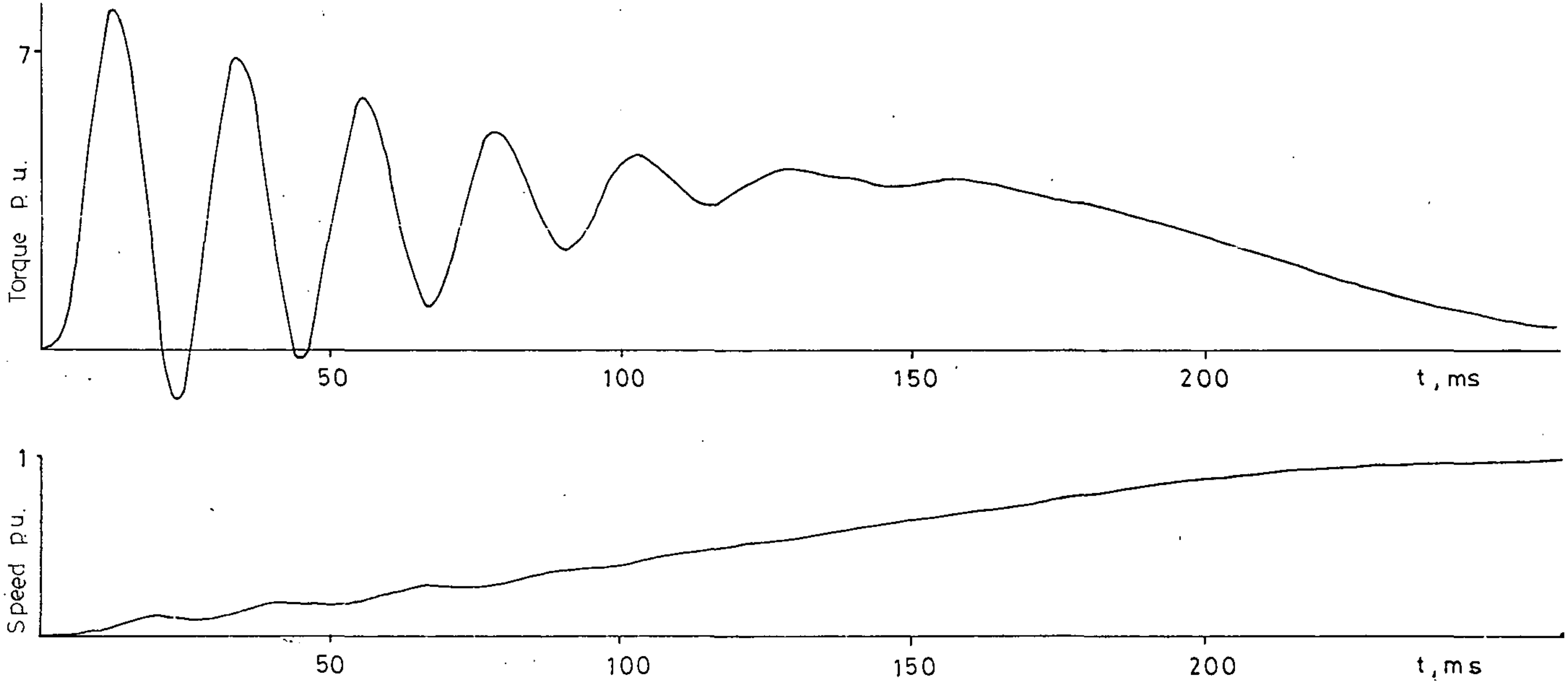


Figure 8.3(a) Computed torque and speed of 0.75 kW motor following direct-to-line starting of 0.75 kW/1.5 kW/2.25 kW motor group.

1 p.u. torque = 7.4 Nm , 1 p.u. speed = 314.2 elec. rad./sec.

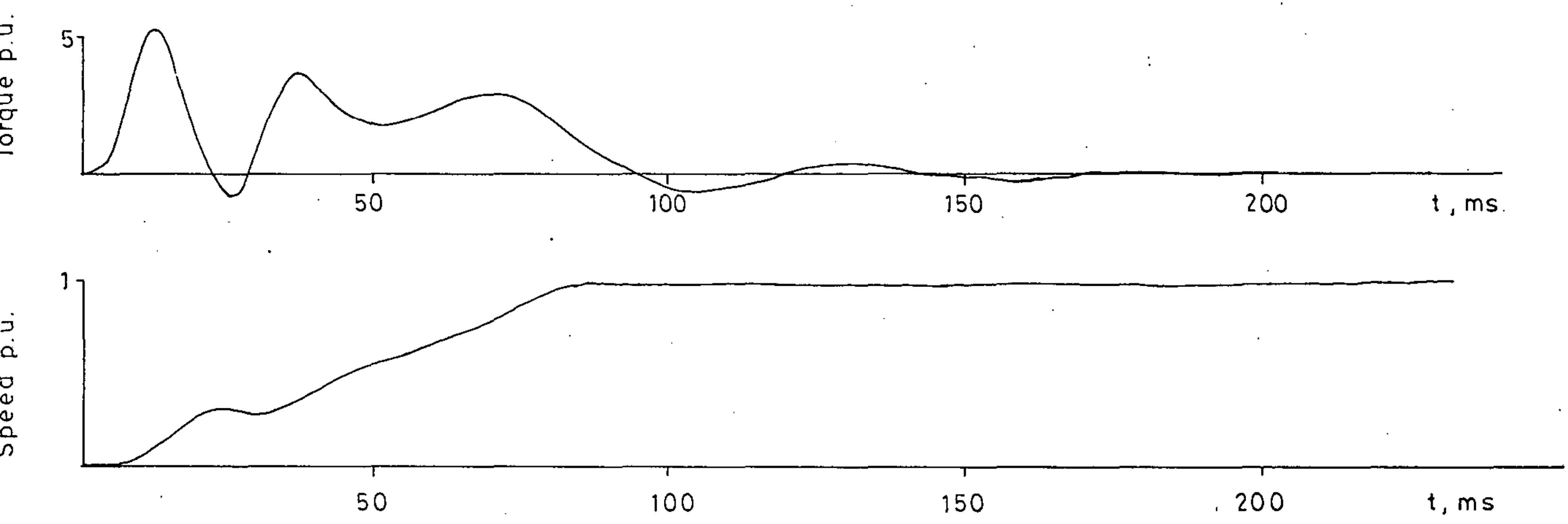


Figure 8.3 (b) Computed torque and speed of 1.5 kW motor following direct-to-line starting of 0.75 kW/ 1.5 kW / 2.25 kW motor group.

1 p.u. torque = 4.7 Nm , 1 p.u. speed = 314.2 elec. rad./sec.

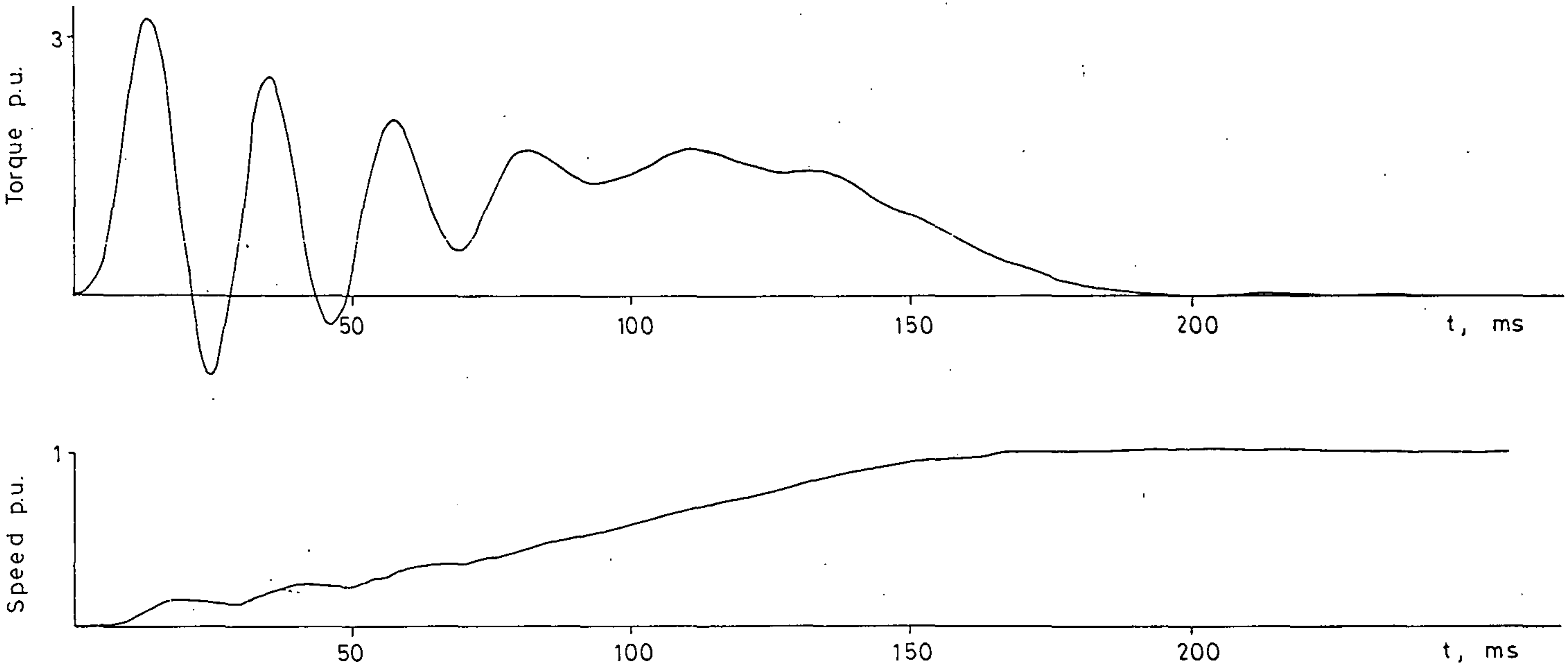


Figure 8·3(c) Computed torque and speed of 2·25 kW motor following direct-to-line starting of 0·75 kW/1·5 kW / 2·25 kW motor group.

1 p.u. torque = 15·5 Nm , 1 p.u. speed = 314·2 elec. rad./sec.

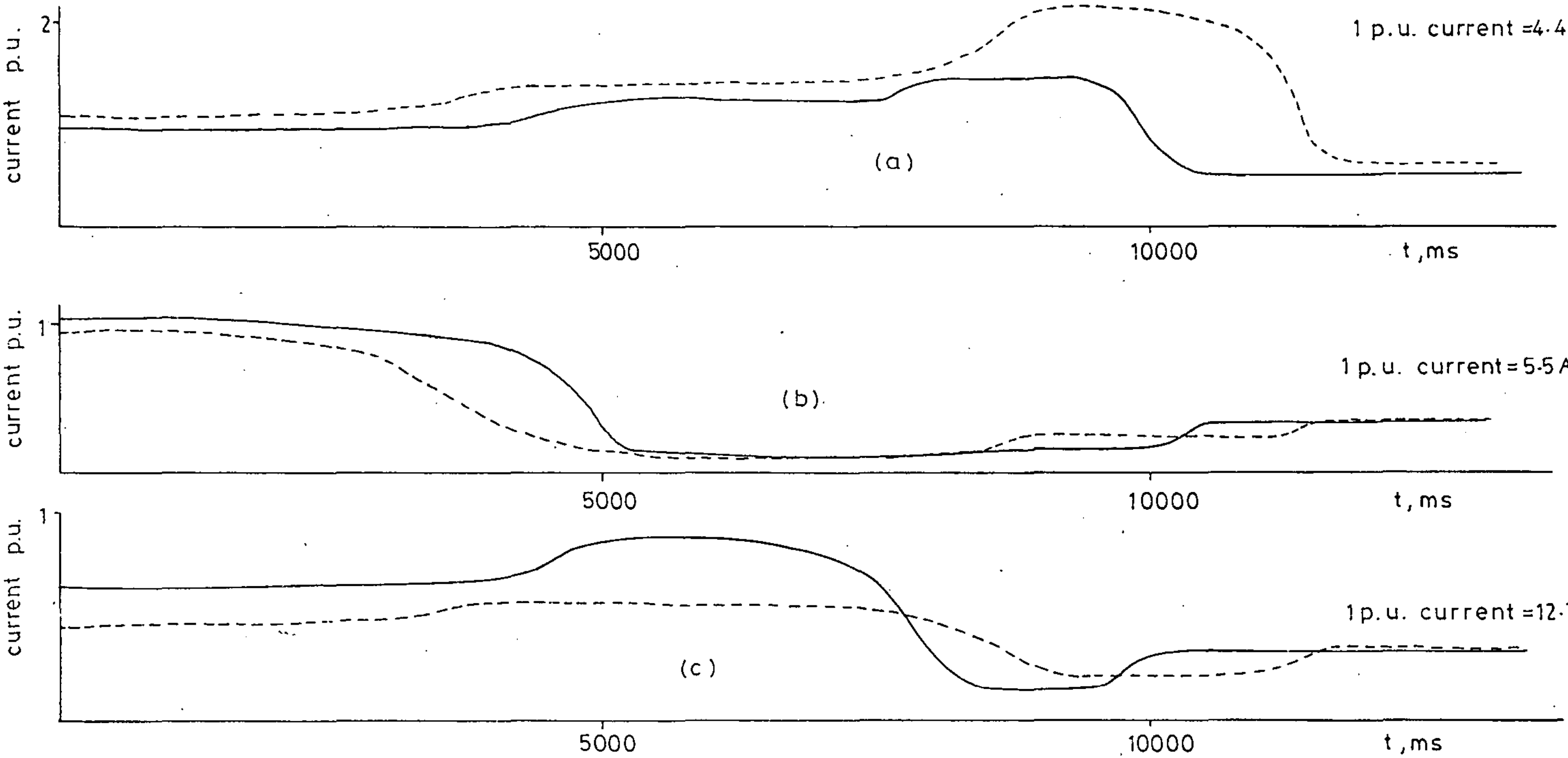


Figure 8.4 Transient current following connection of 0.75 kW/1.5 kW/2.25 kW motor group to a non-stiff supply (supply resistance = 11.0 ohm).

(a) Transient current of 0.75 kW motor. (b) Transient current of 1.5 kW motor. (c) Transient current of 2.25 kW motor. Computed results------ , Experimental results ———

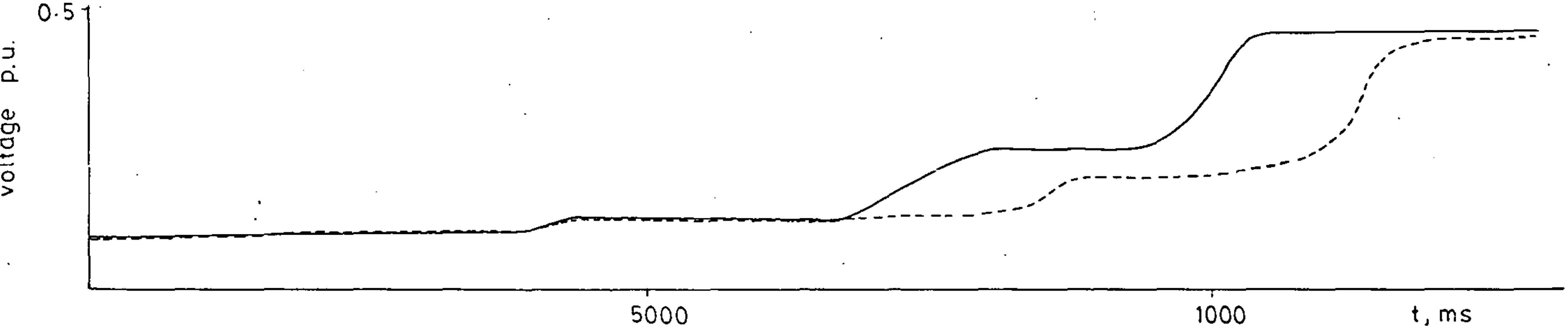


Figure 8·5 Terminal voltage following connection of 0·75 kW/1·5 kW/2·25 kW motor group to a non-stiff supply (supply resistance = 11·0 ohm).

Computed results ------ , Experimental results ———

1 p.u. voltage = 345 V.

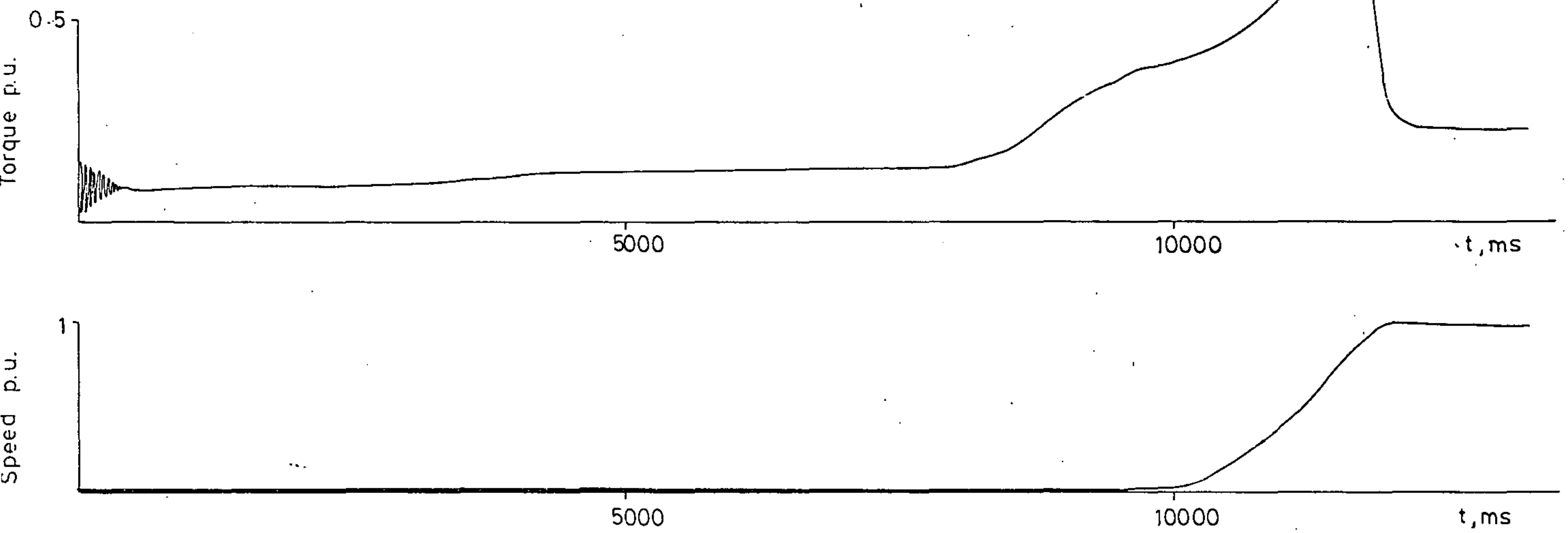


Figure 8.6(a) Transient torque and speed of 0.75 kW motor following connection of 0.75 kW/1.5 kW/2.25 kW motor group to a non-stiff supply (supply resistance = 11.0 ohm).

1 p.u. torque = 7.4 Nm , 1 p.u. speed = 314.2 elec. rad./sec.

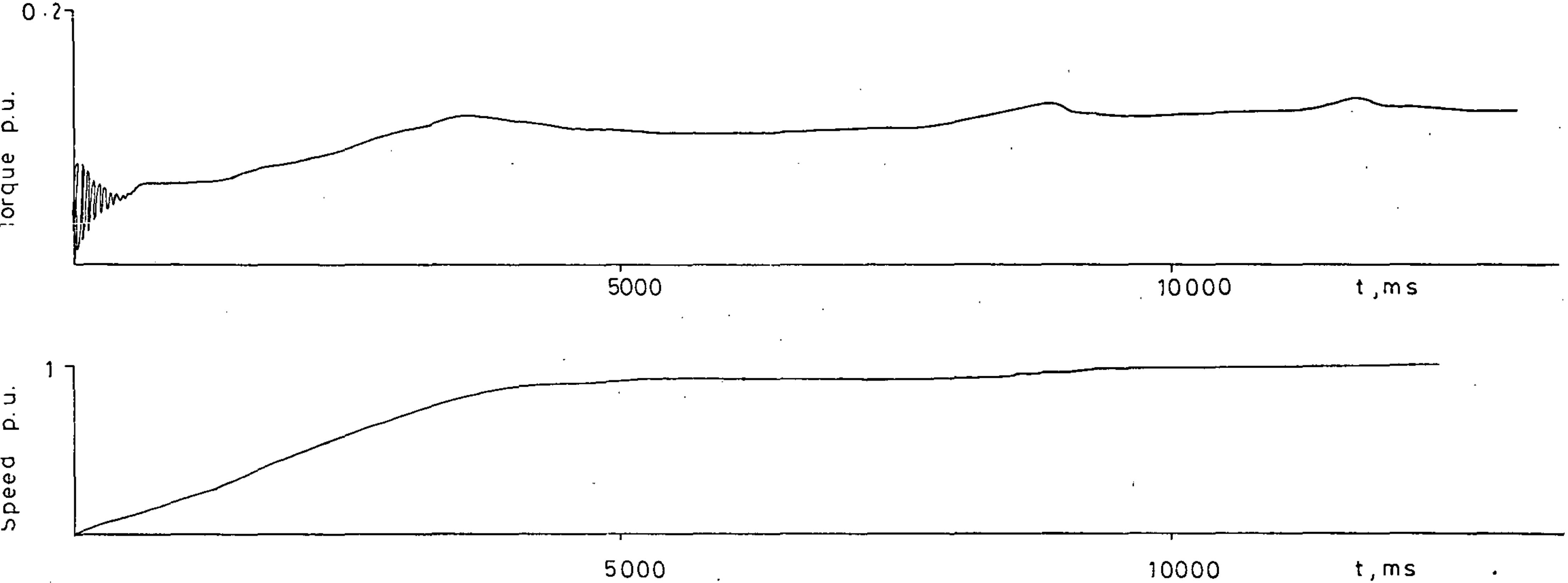


Figure 8·6(b) Transient torque and speed of 1·5 kW motor following connection of 0·75 kW/1·5 kW/2·25 kW motor group to a non-stiff supply(supply resistance=11·0 ohm)

1 p.u. torque = 4·7 Nm , 1 p.u speed = 314·2 elec. rad./sec.

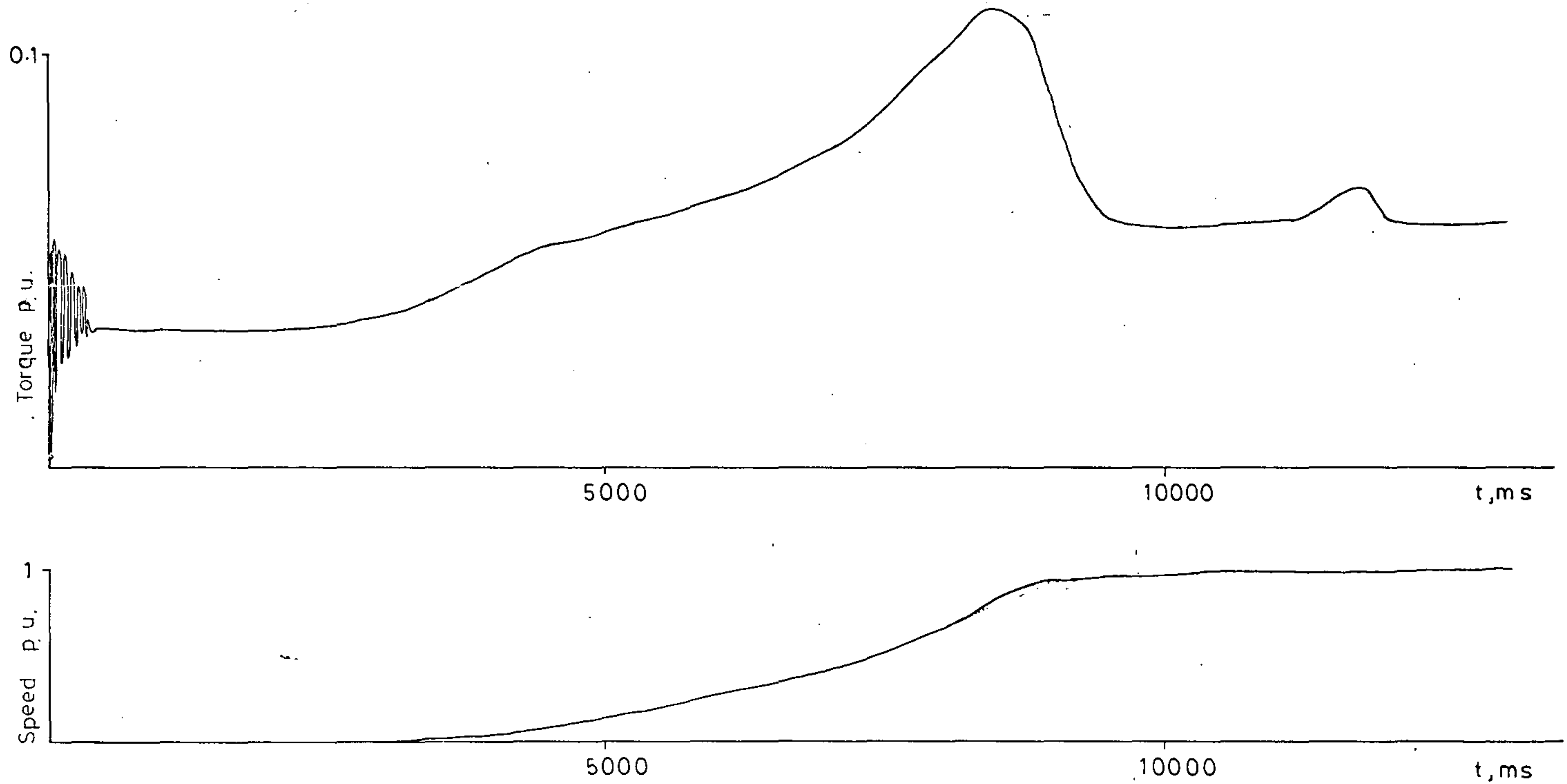


Figure 8·6 (c) Transient torque and speed of 2·25 kW motor following connection of 0·75 kW/1·5 kW/2·25 kW motor group to a non-stiff supply (supply resistance = 11·0 ohm).

1 p.u. torque = 15·5 Nm. , 1 p.u. speed = 314·2 elec. rad./sec.

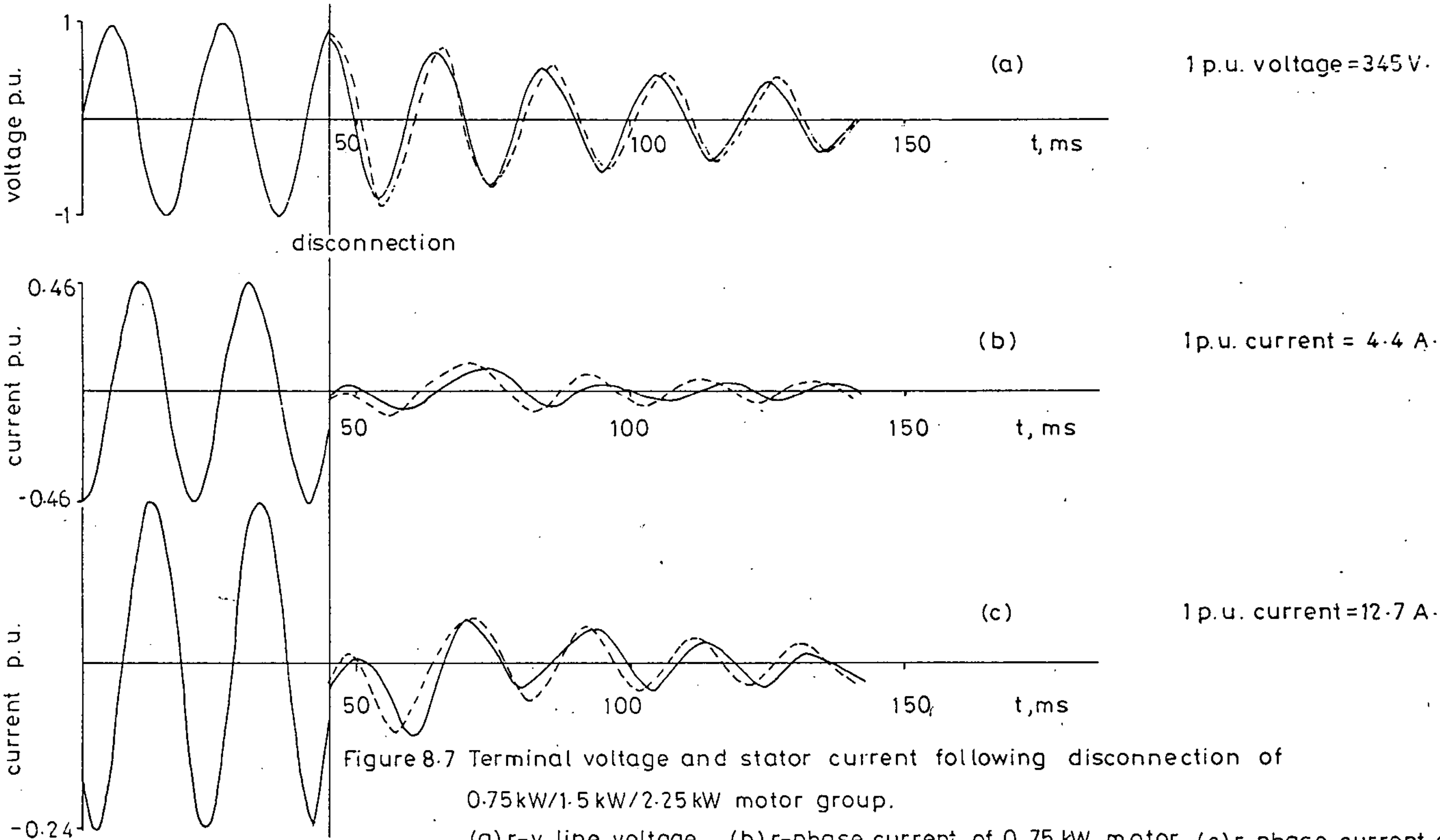


Figure 8.7 Terminal voltage and stator current following disconnection of 0.75 kW/1.5 kW/2.25 kW motor group. (a) r-y line voltage. (b) r-phase current of 0.75 kW motor. (c) r-phase current of 2.25 kW motor. Computed results - - - - -, Experimental results ———

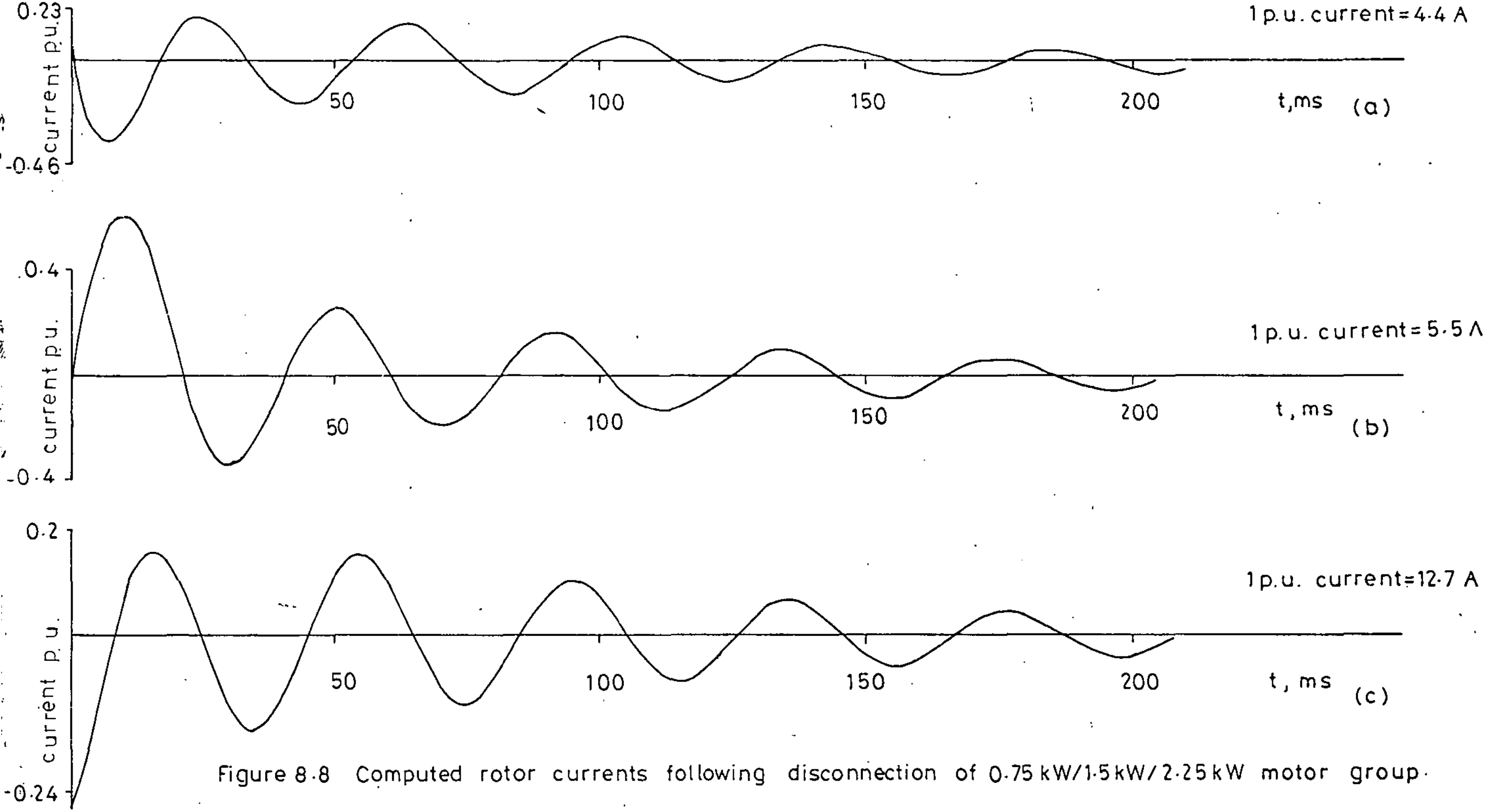


Figure 8.8 Computed rotor currents following disconnection of 0.75 kW/1.5 kW/2.25 kW motor group.

(a) r-phase current of 0.75 kW motor. (b) r-phase current of 1.5 kW motor.

(c) r-phase current of 2.25 kW motor.

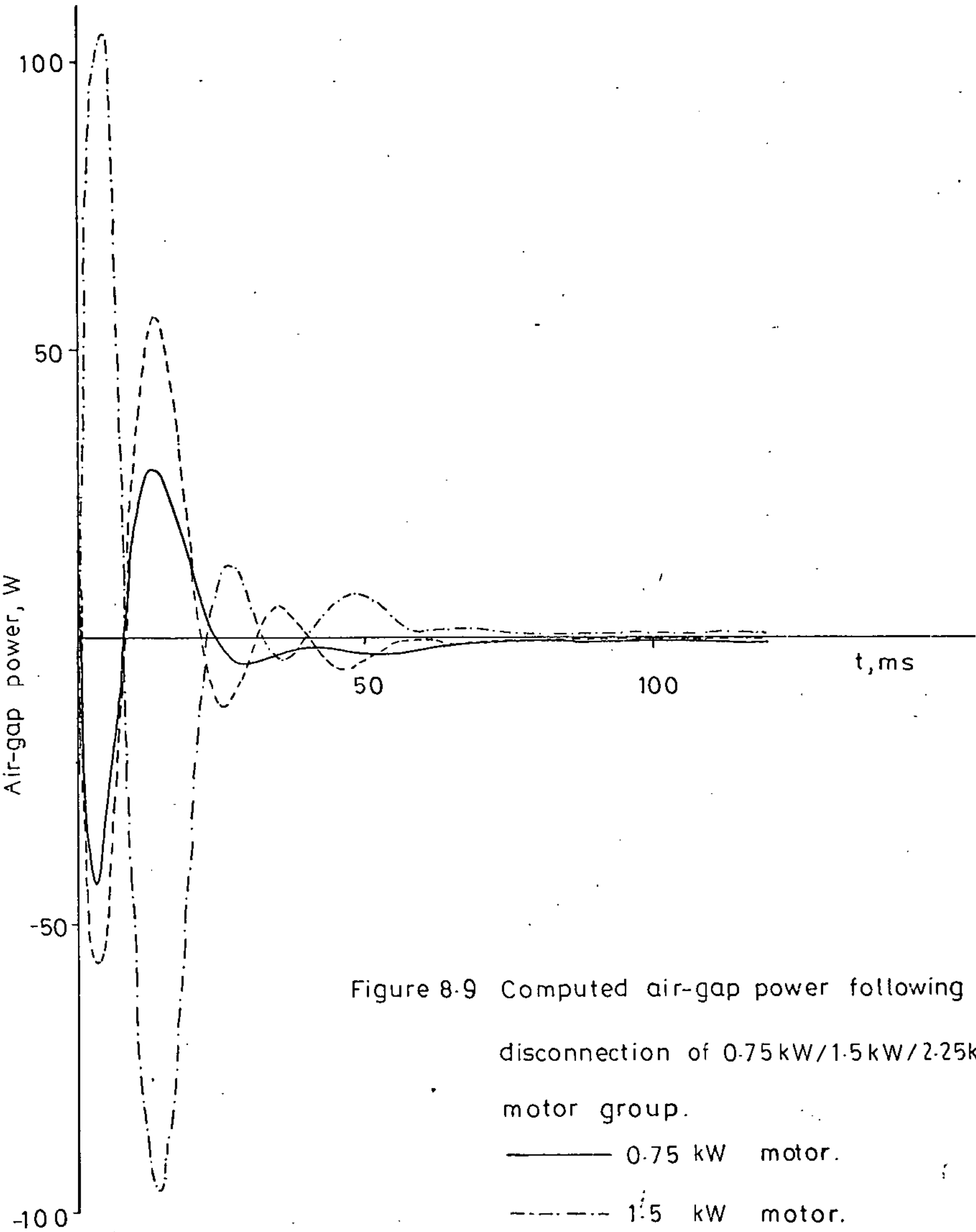


Figure 8.9 Computed air-gap power following disconnection of 0.75 kW/1.5 kW/2.25 kW motor group.

——— 0.75 kW motor.

-·-·-· 1.5 kW motor.

- - - - - 2.25 kW motor.

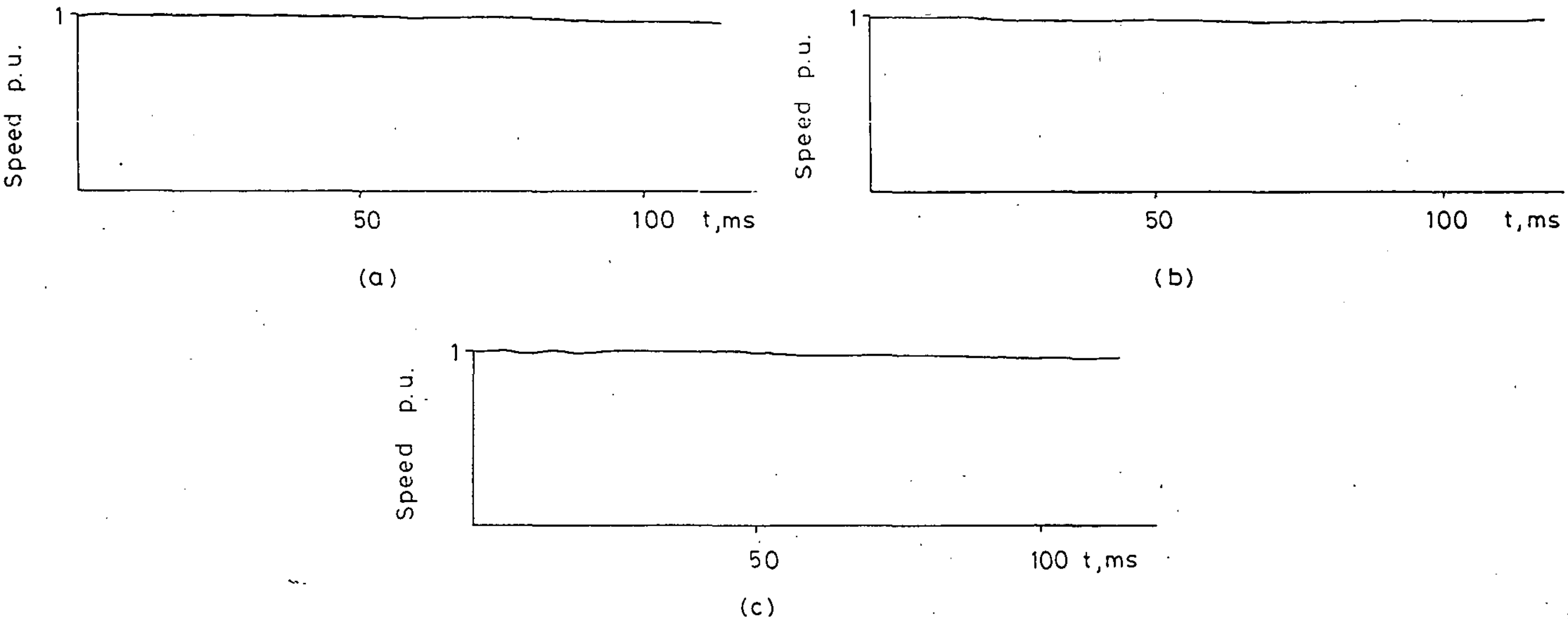


Figure 8·10 Computed speeds following disconnection of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) 0·75 kW motor. (b) 1·5 kW motor. (c) 2·25 kW motor.

1 p.u. speed = 314·2 elec. rad./sec.

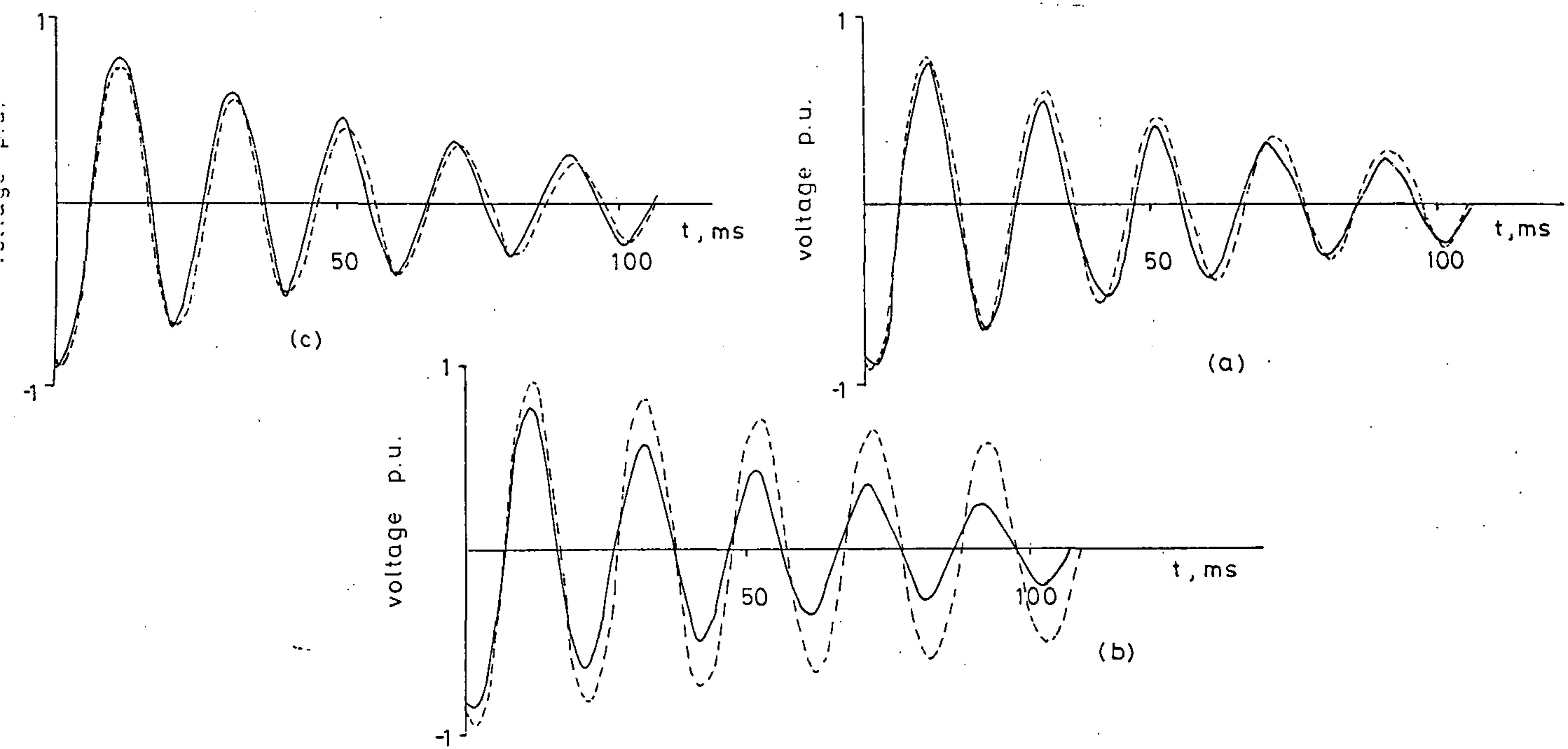


Figure 8.11 Comparison between terminal voltages of 0.75 kW/1.5 kW/2.25 kW motor group and 0.75 kW, 1.5 kW, 2.25 kW motors in isolation following disconnection. (1 p.u. voltage = 345 V)

(a) 0.75 kW motor. (b) 1.5 kW motor. (c) 2.25 kW motor.

motor in isolation ------- , motor in group ————

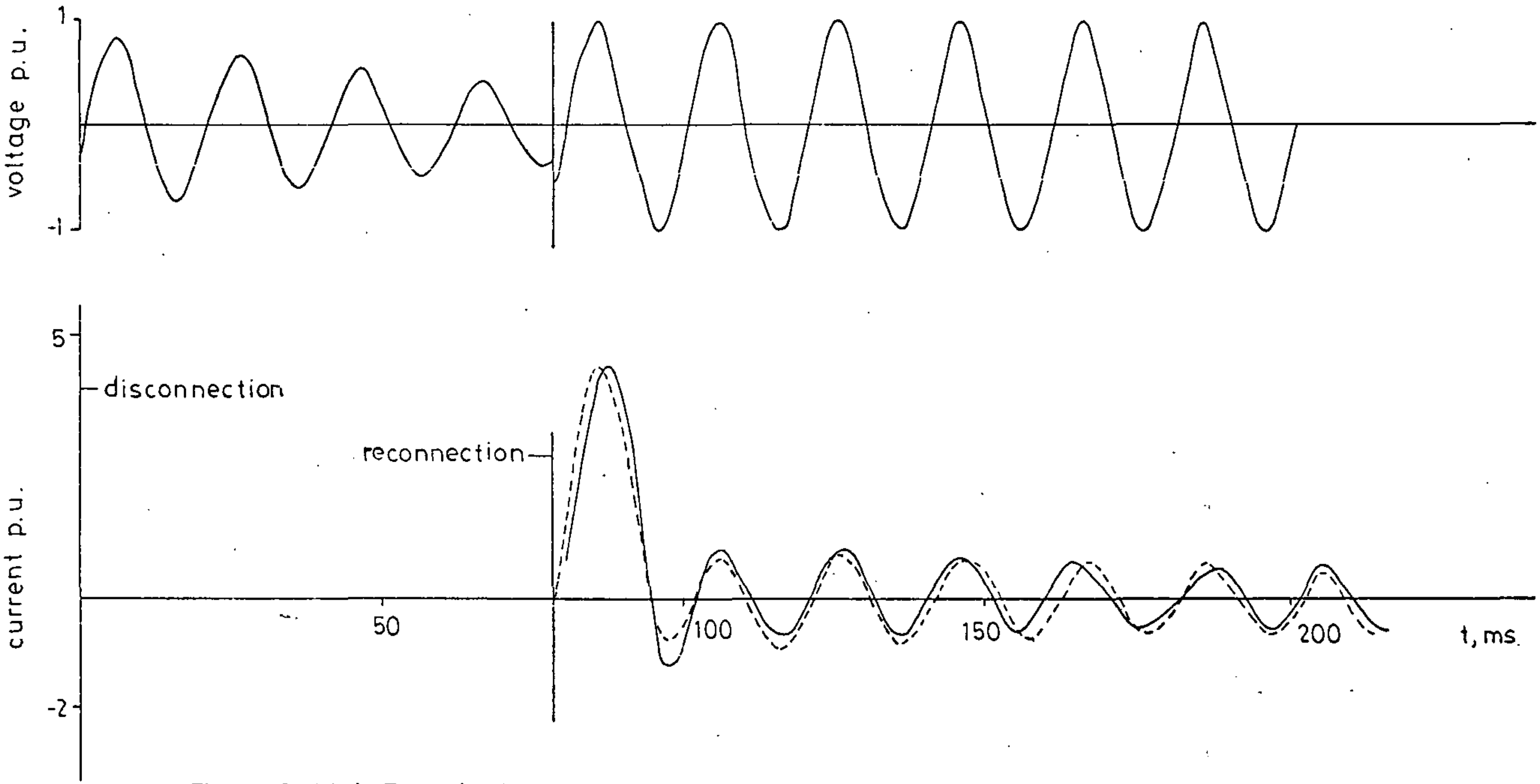


Figure 8.12(a) Transient current of 0.75 kW motor following reconnection of 0.75 kW/1.5 kW/2.25 kW motor group.

1 p.u. r-line current = 4.4 A , 1 p.u. r-y line voltage = 345 V .

Computed results ------ -- , Experimental results ————————

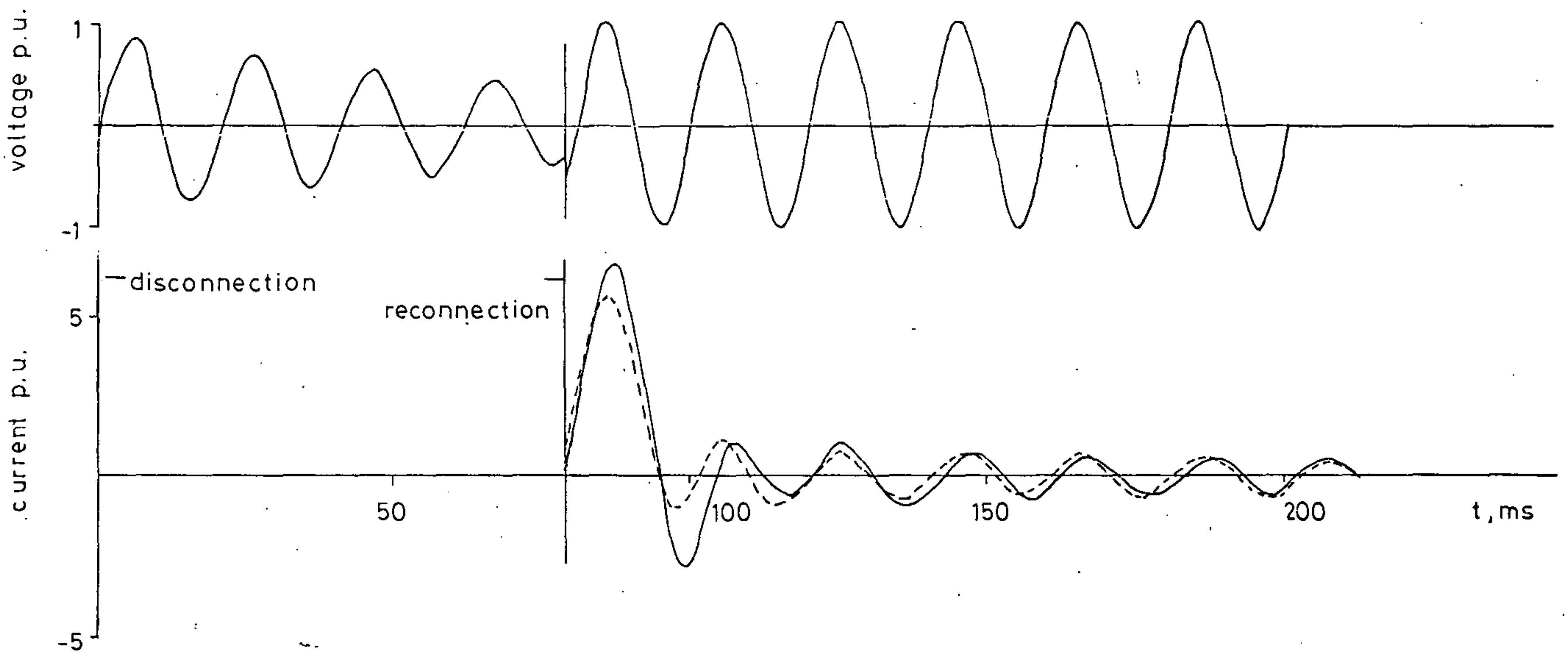


Figure 8.12(b) Transient current in r-line of 1·5 kW motor following reconnection of 0·75 kW/1·5 kW/ 2·25 kW motor group.

1 p.u. current = 5·5 A , 1 p.u. r–y line voltage = 345 V.

Computed results -------- , Experimental results ————————

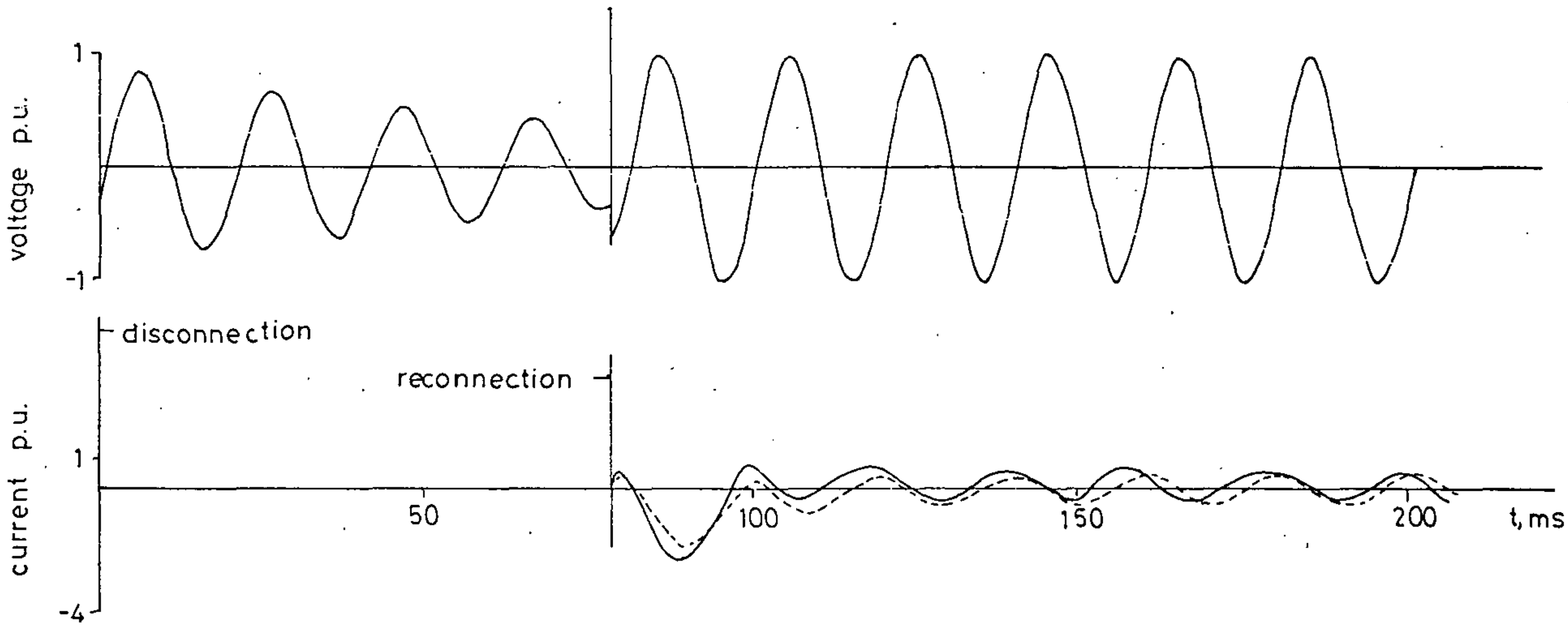


Figure 8.12(c) Transient current in b-line of 2.25 kW motor following reconnection of 0.75 kW/1.5 kW/2.25 kW motor group.

1 p.u. current = 12.7 A , 1 p.u. r-y line voltage = 345 V.

Computed results --------- , Experimental results ———

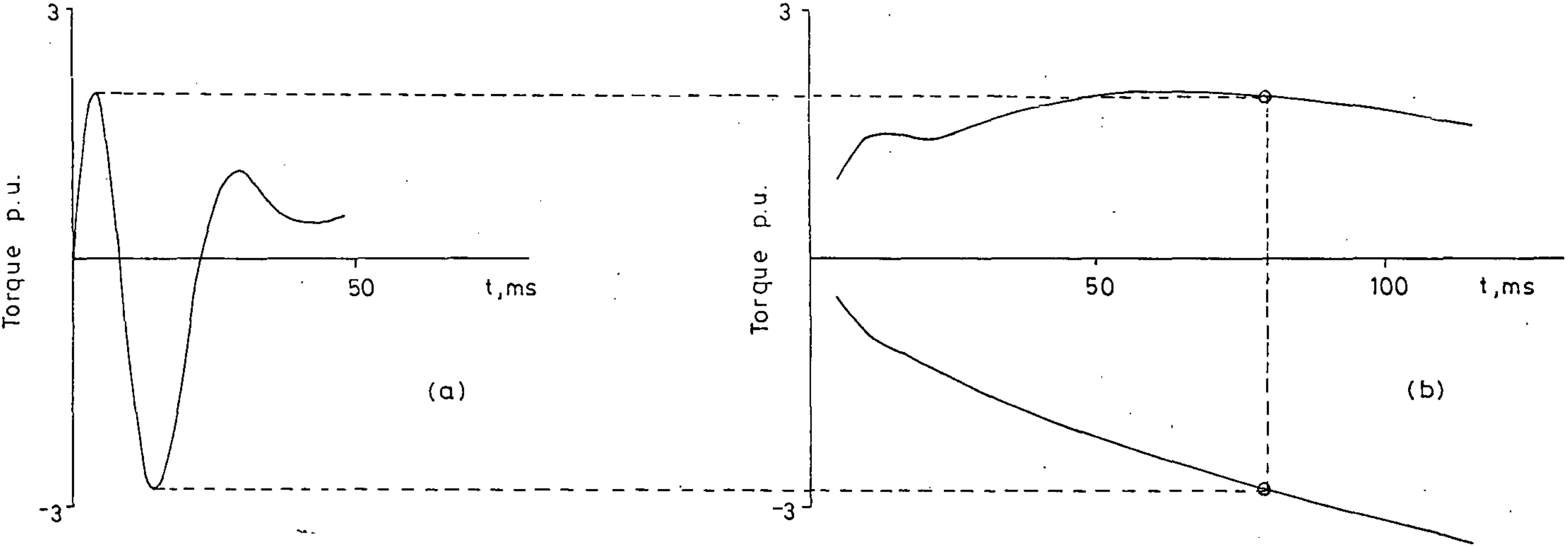


Figure 8.13 Torque of 0.75 kW motor following reconnection of 0.75 kW/1.5 kW/2.25 kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 7.4 Nm.

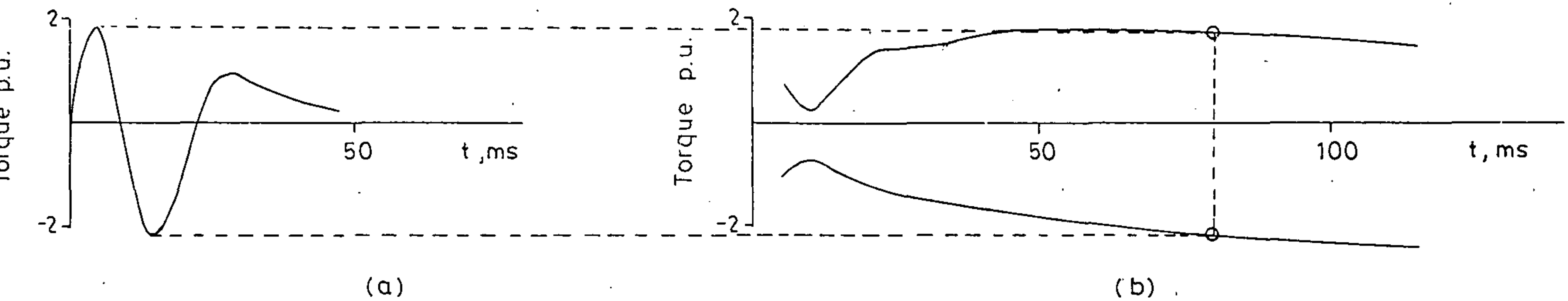


Figure 8.14 Torque of 1.5 kW motor following reconnection of 0.75 kW/1.5 kW/2.25 kW motor group.

(a) Transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 4.7 Nm.

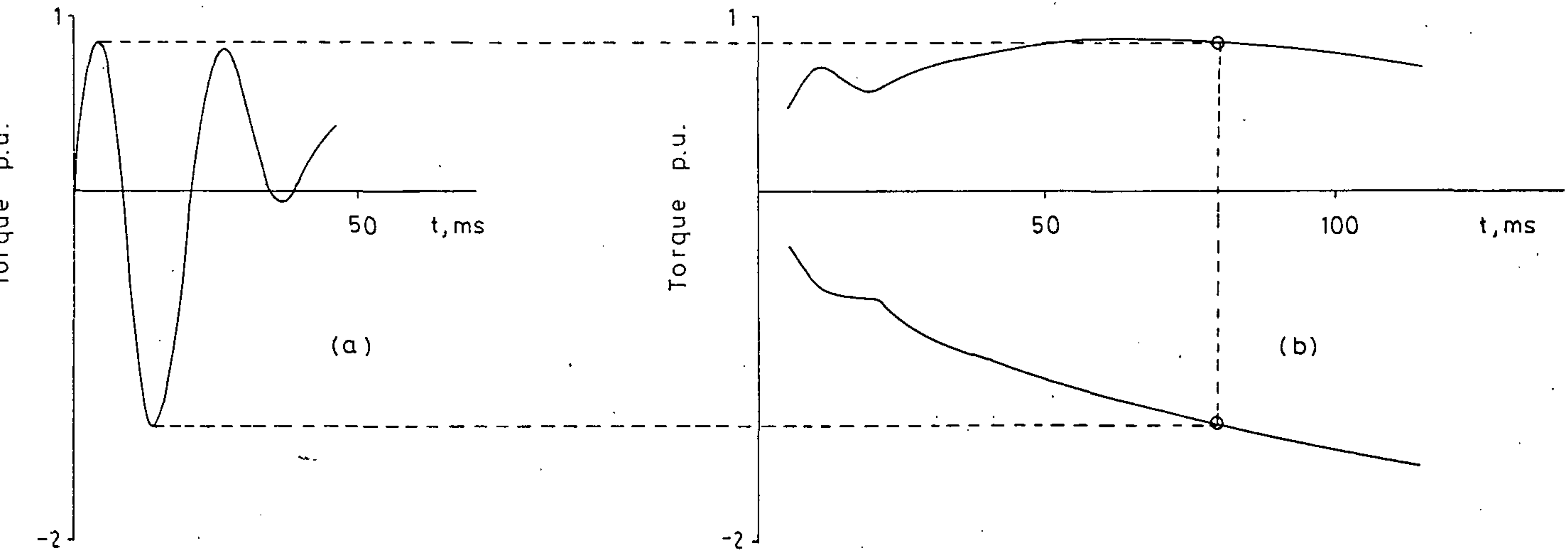


Figure 8·15 Torque of 2·25 kW following reconnection of 0·75 kW/1·5 kW/2·25 kW motor.

(a) Transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 15·5 Nm.

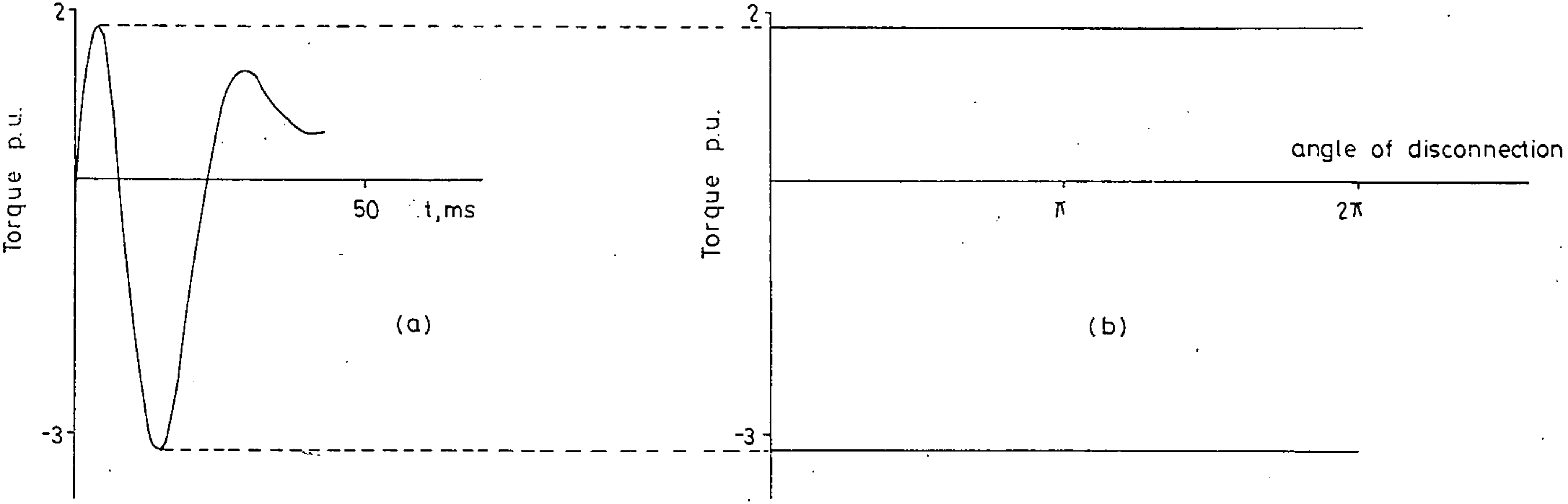


Figure 8·16 Torque of 0·75 kW motor following reconnection of 0·75 kW/1·5 kW/2·25 kW motor group at constant supply interruption and different instant of disconnection.

(a) Computed transient torque pattern.

(b) Variation of peak torque with angle of disconnection.

1 p.u. torque = 7·4 Nm.

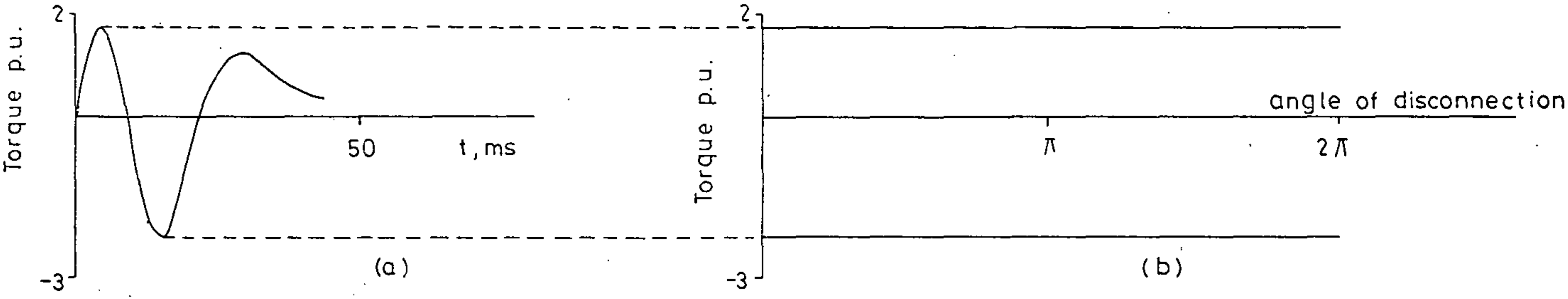


Figure 8.17 Torque of 1.5 kW motor following reconnection of 0.75 kW/1.5 kW/2.25 kW motor group at constant supply interruption and different instant of disconnection.

(a) Computed transient torque pattern.

(b) Variation of peak torque with angle of disconnection.

1 p.u. torque = 4.7 Nm.

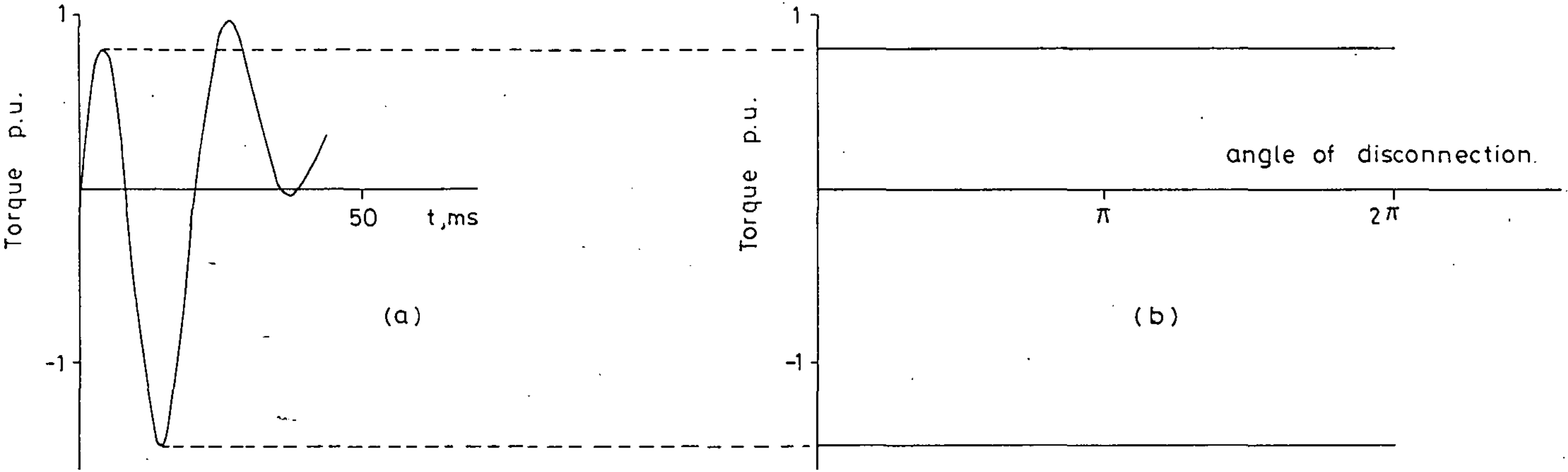


Figure 8·18 Torque of 2·25 kW motor following reconnection of 0·75 kW/1·5 kW/2·25 kW motor group at constant supply interruption and different instant of disconnection.

(a) Computed transient torque pattern.

(b) Variation of peak torque with angle of disconnection.

1 p.u. torque = 15·5 Nm.

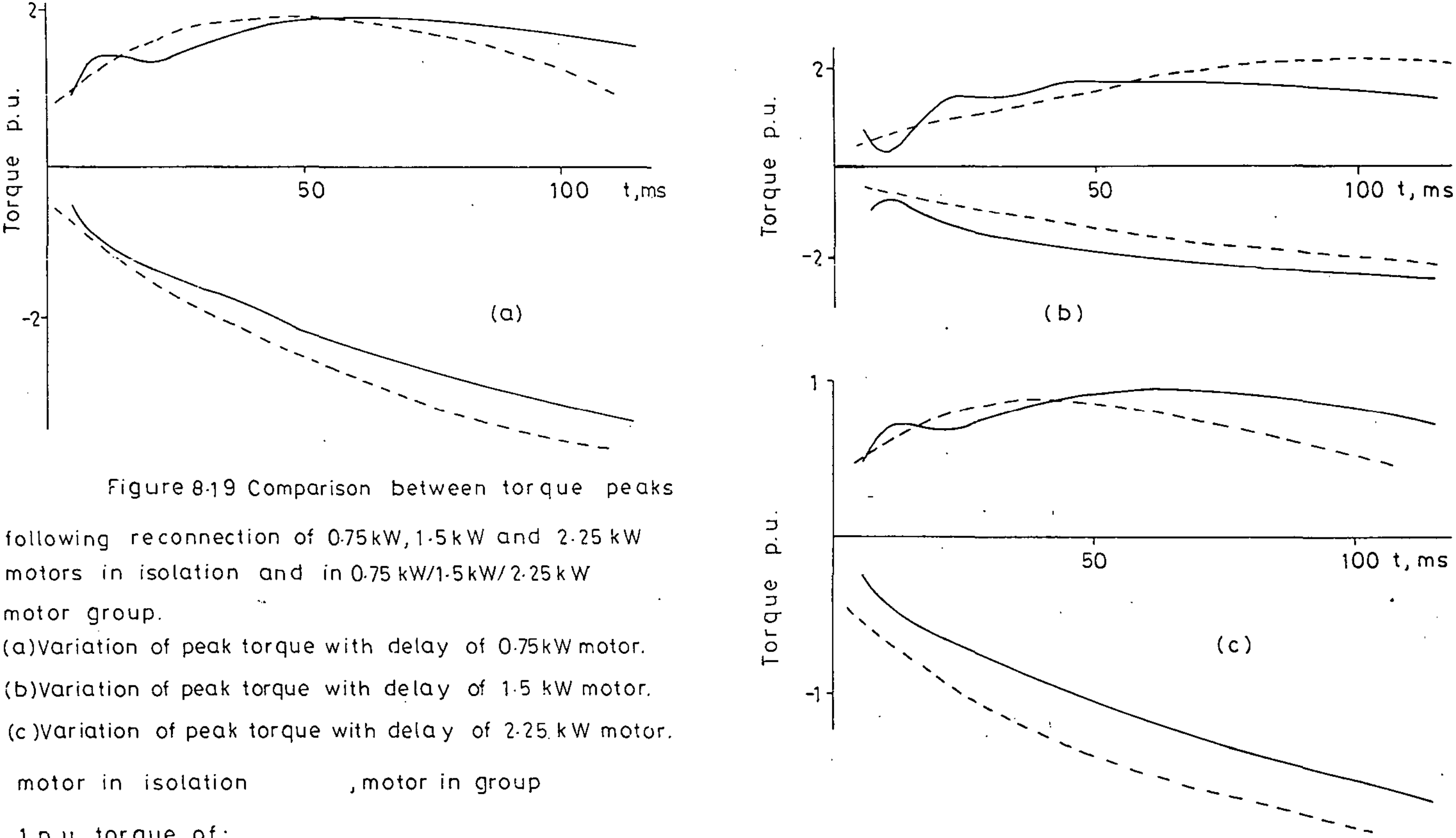


Figure 8·19 Comparison between torque peaks following reconnection of 0·75kW, 1·5kW and 2·25kW motors in isolation and in 0·75kW/1·5kW/2·25kW motor group.

(a) Variation of peak torque with delay of 0·75kW motor.

(b) Variation of peak torque with delay of 1·5 kW motor.

(c) Variation of peak torque with delay of 2·25 kW motor.

motor in isolation , motor in group

1 p.u. torque of:

0·75 kW motor = 7·4 Nm. , 1·5 kW motor = 4·7 kW , 2·25 kW motor = 15·5 Nm.

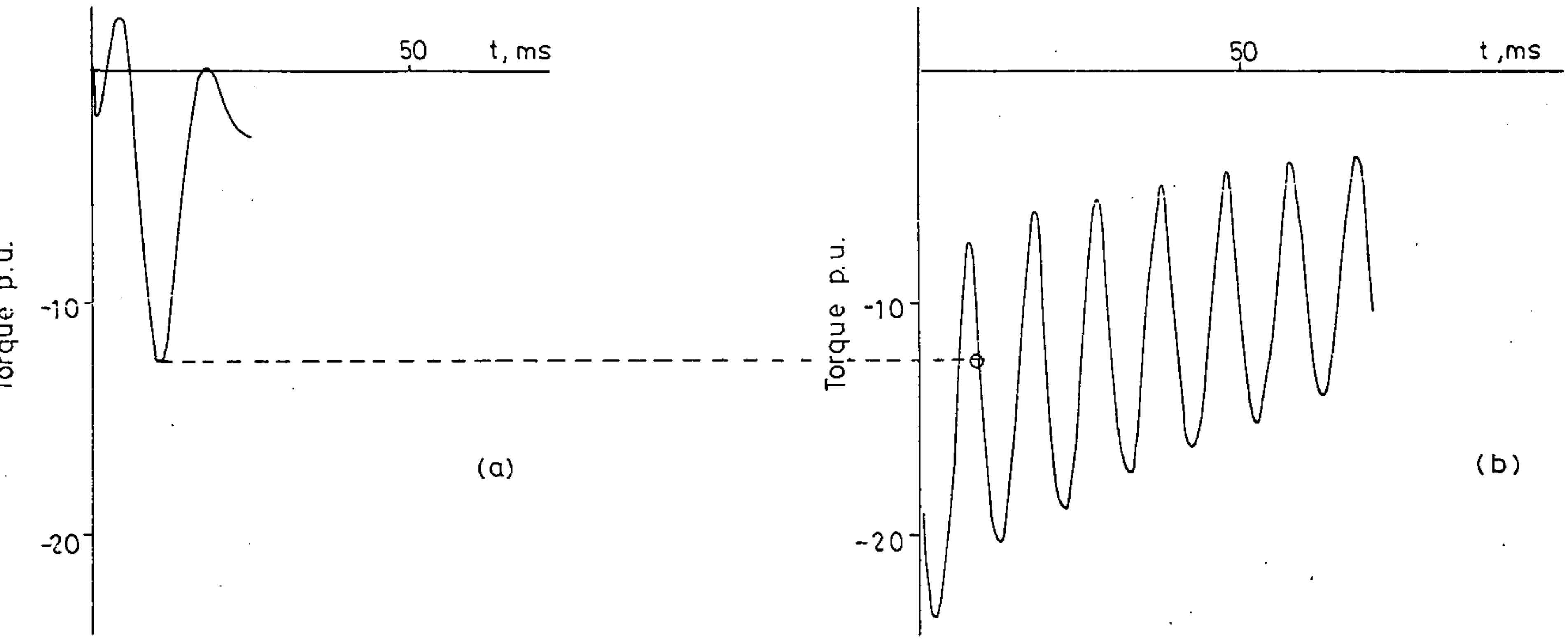


Figure 8·20 Torque of 0·75 kW motor following plugging of 0·75 kW/1·5kW/2·25 kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 7·4 Nm.

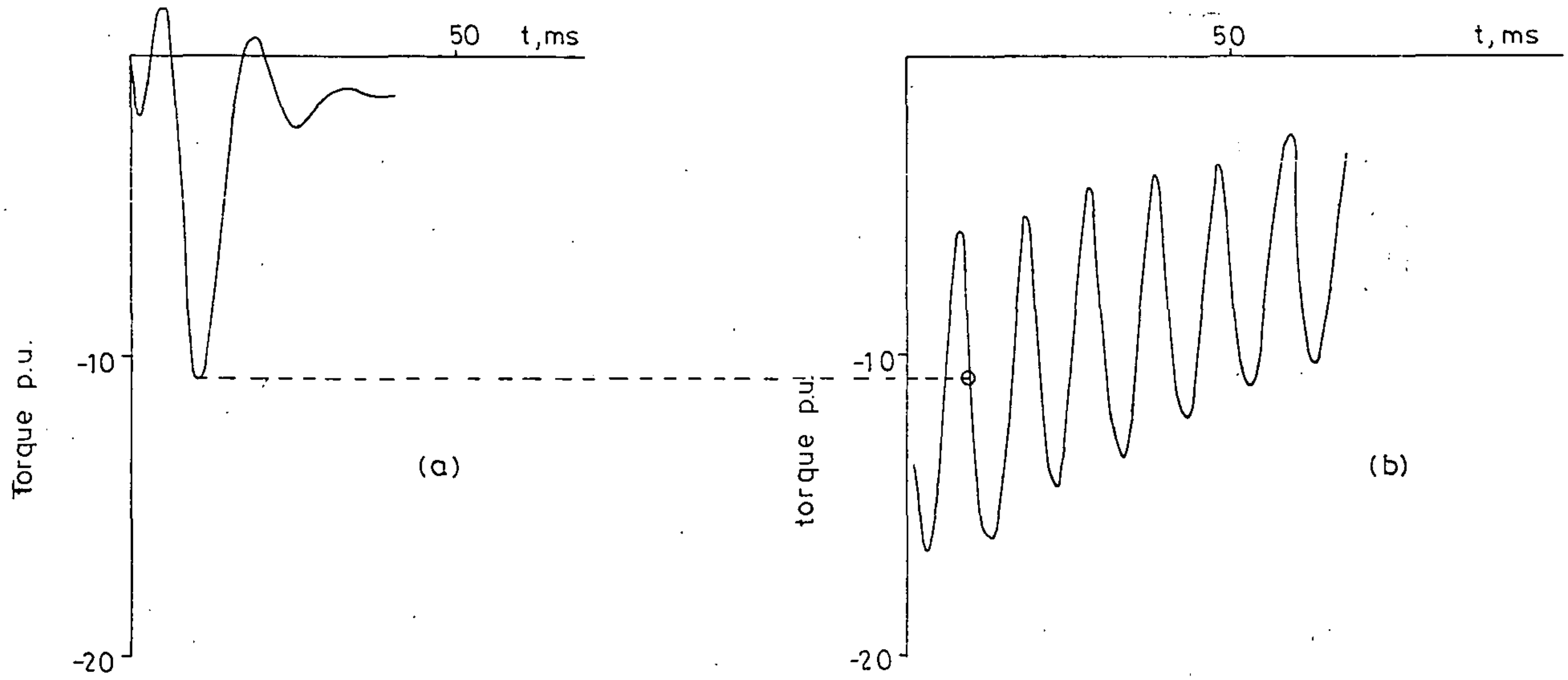


Figure 8.21 Torque of 1·5 kW motor following plugging of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) Computed transient torque peak.

(b) Variation of peak torque with delay.

1 p.u. torque = 4·7 Nm.

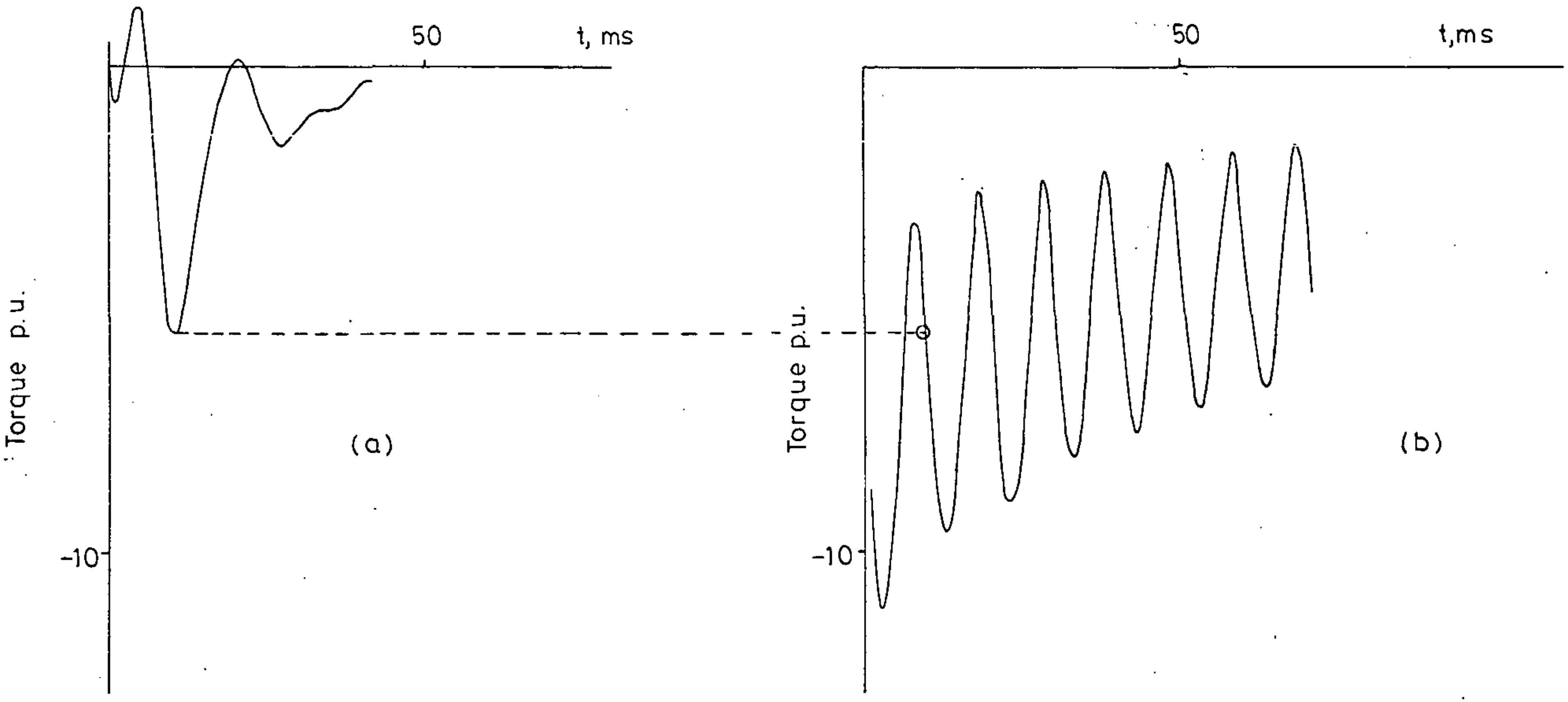
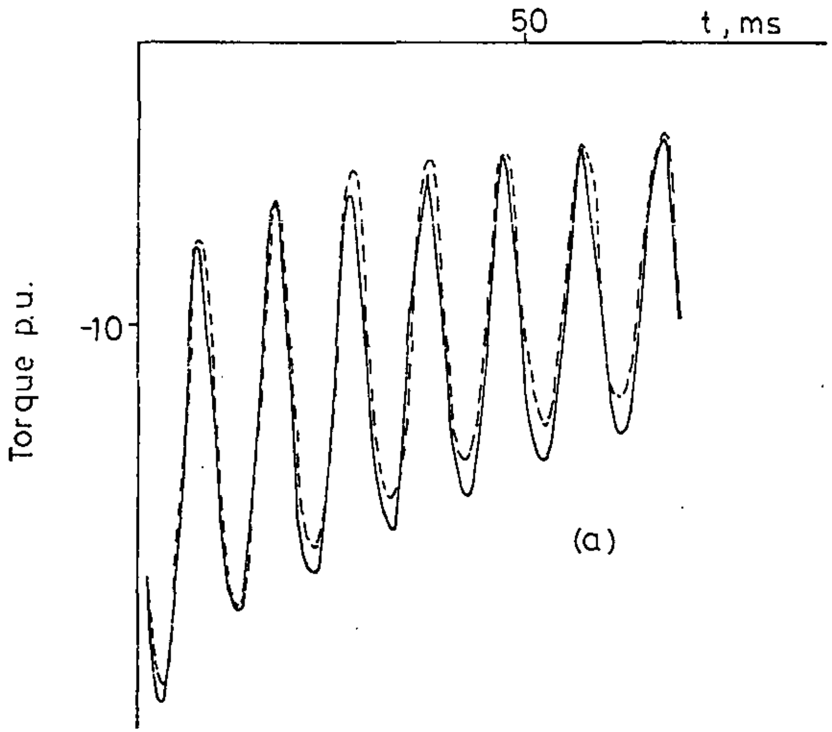


Figure 8·22 Torque of 2·25 kW motor following plugging of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 15·5 Nm.



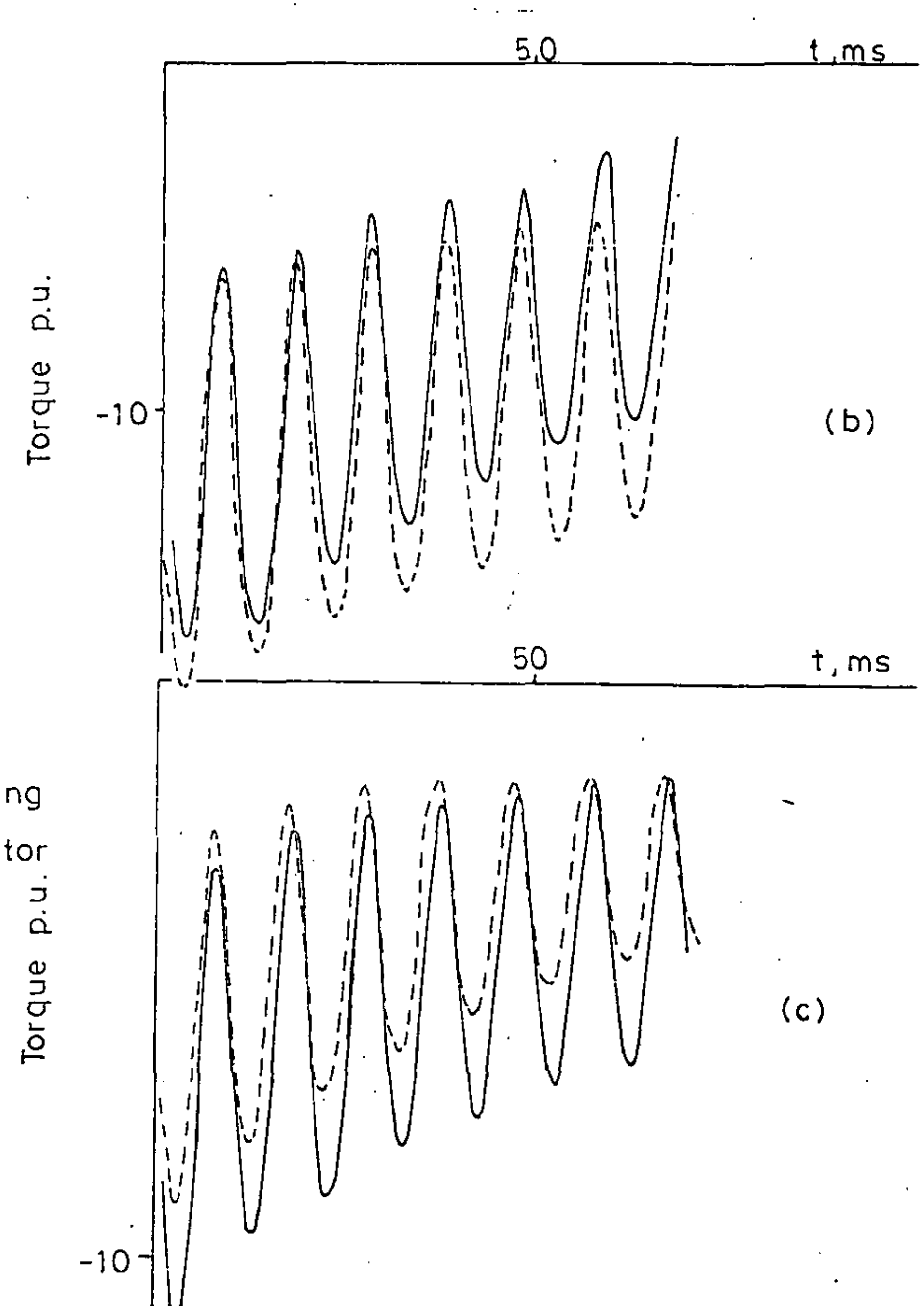


Figure 8·23 Comparison between torque peaks following plugging of 0·75 kW/1·5 kW/2·25 kW motor group and 0·75 kW, 1·5 kW and 2·25 kW motors in isolation.

(a) 0·75 kW motor. (b) 1·5 kW motor.

(c) 2·25 kW motor.

motor in isolation------ , motor in group ———

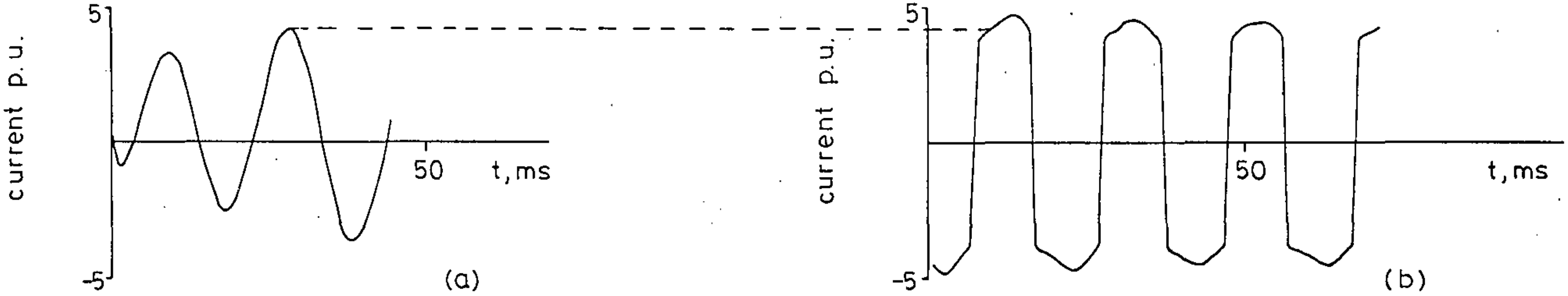


Figure 8·24 Current of 0·75 kW motor following plugging of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 4·4 A.

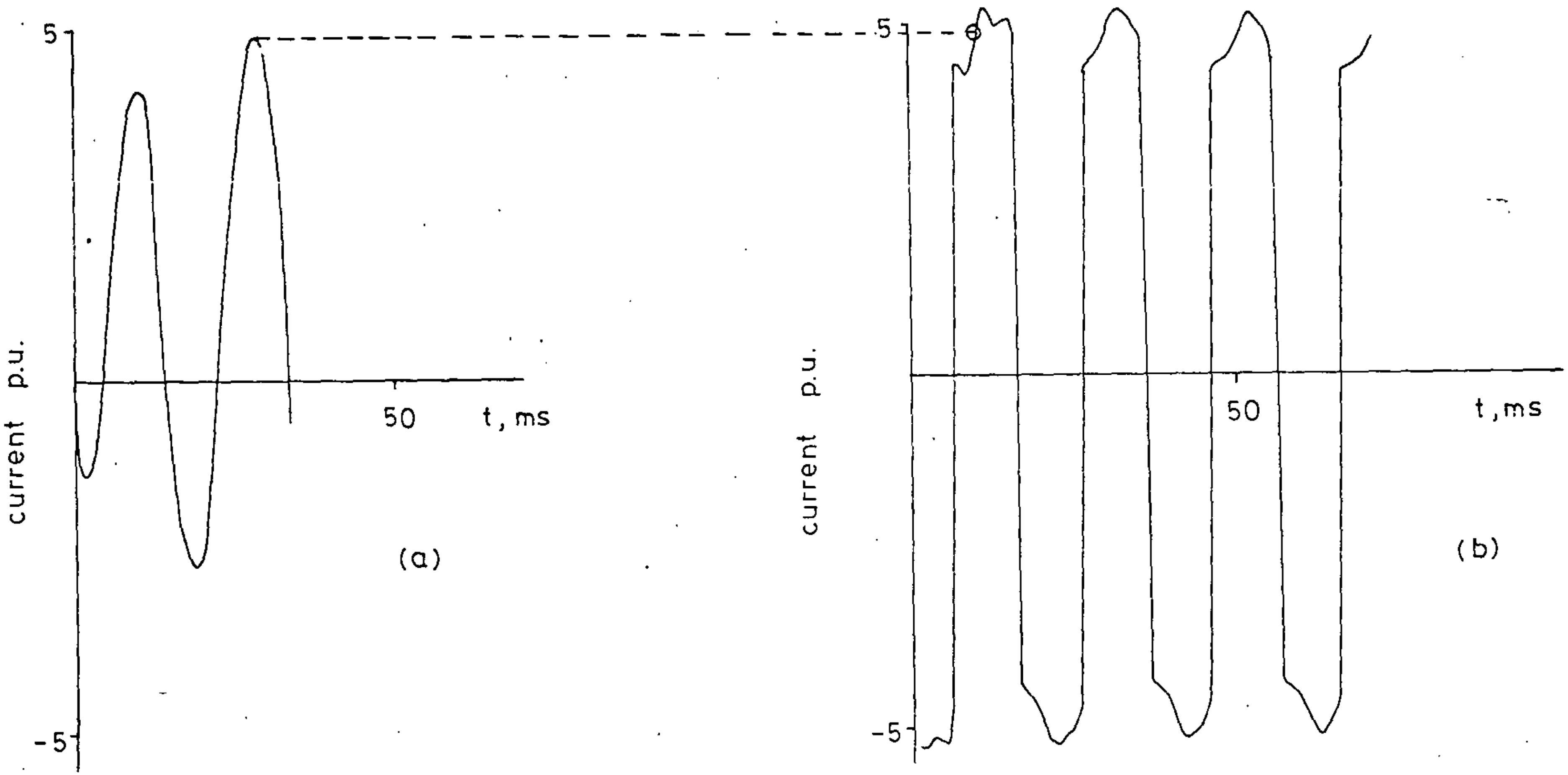


Figure 8·25 Current of 1·5 kW motor following plugging of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 5·5 A.

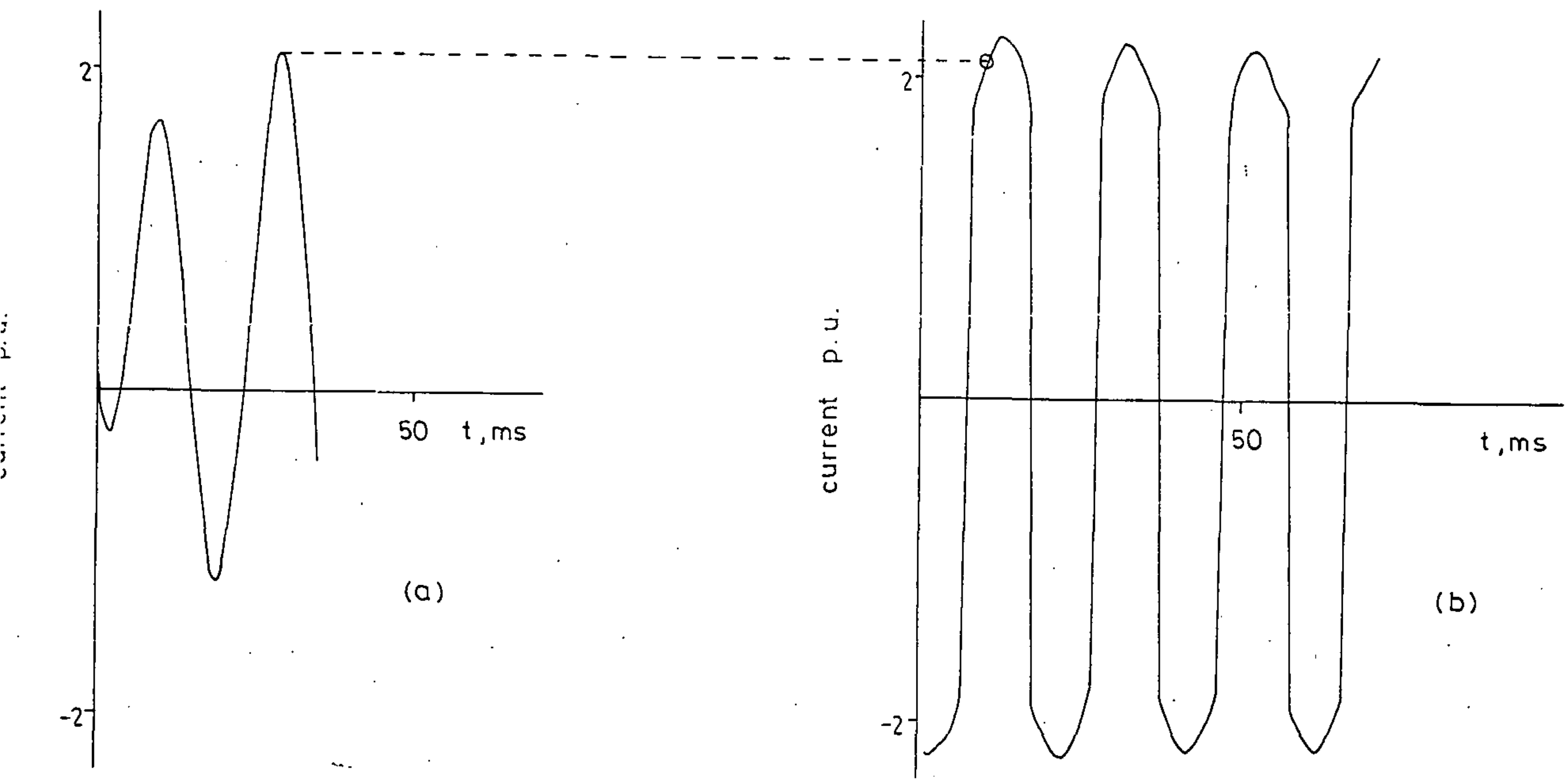


Figure 8.26 Current of 2·25 kW motor following plugging of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 12·7 A.

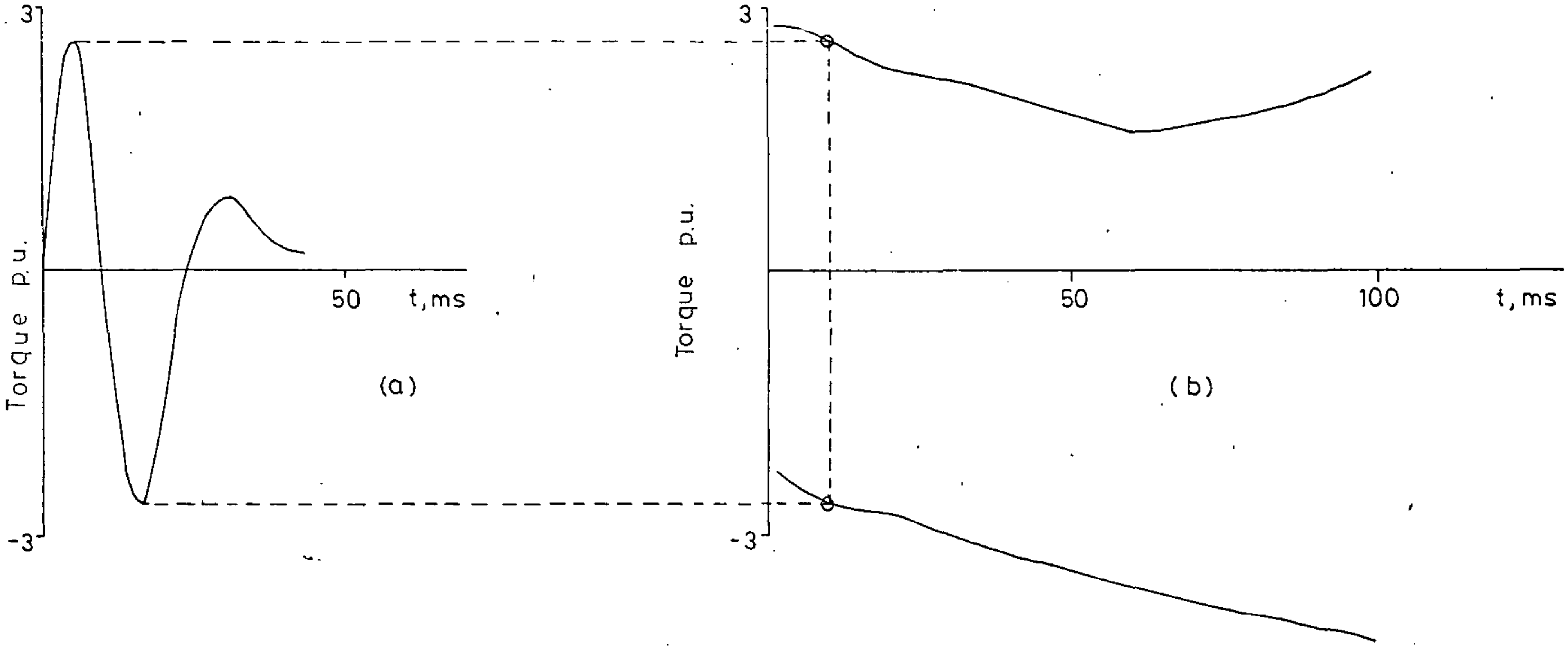


Figure 8·27 Torque of 0·75kW motor following star-delta starting of 0·75kW/1·5kW/2·25kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 7·4 Nm.

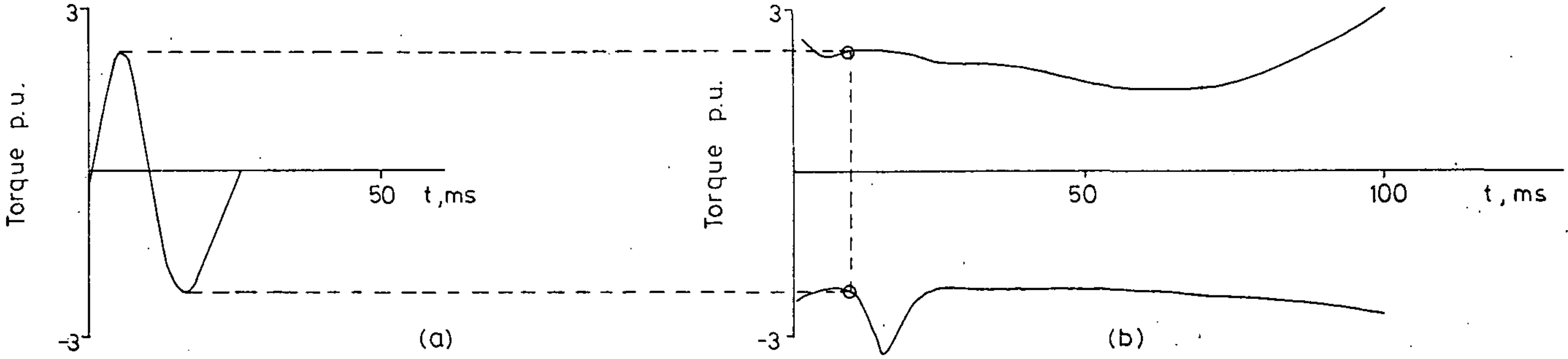


Figure 8·28 Torque of 1·5kW motor following star-delta starting of 0·75kW/1·5kW/2·25kW motor group.

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 4·7 Nm.

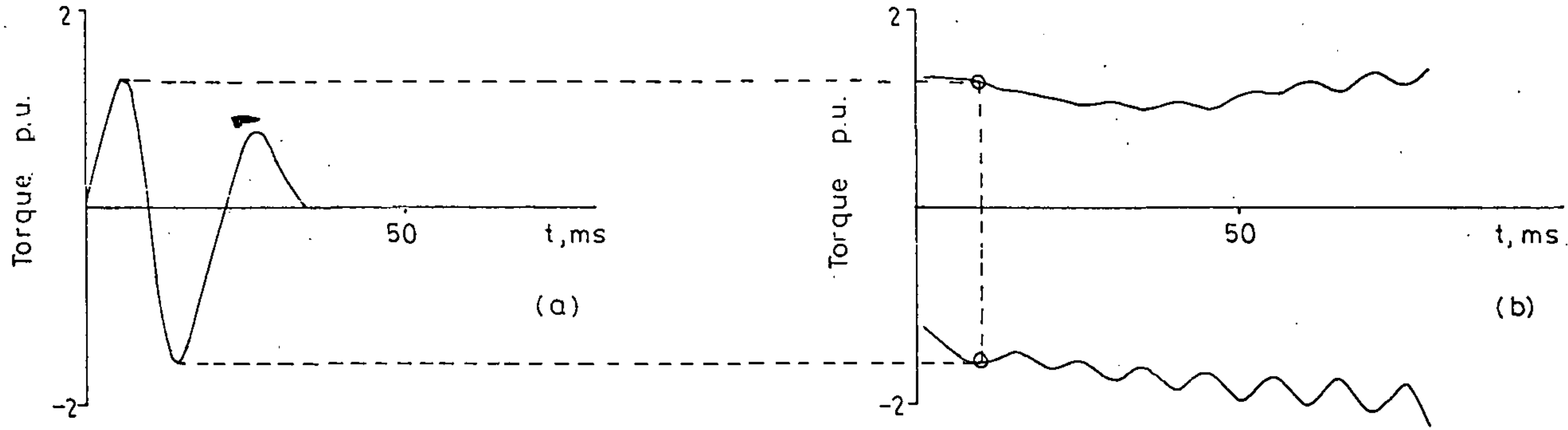


Figure 8·29 Torque of 2·25 kW motor following star-delta starting of 0·75 kW/1·5 kW/2·25 kW motor grou

(a) Computed transient torque pattern.

(b) Variation of peak torque with delay.

1 p.u. torque = 15·5 Nm.

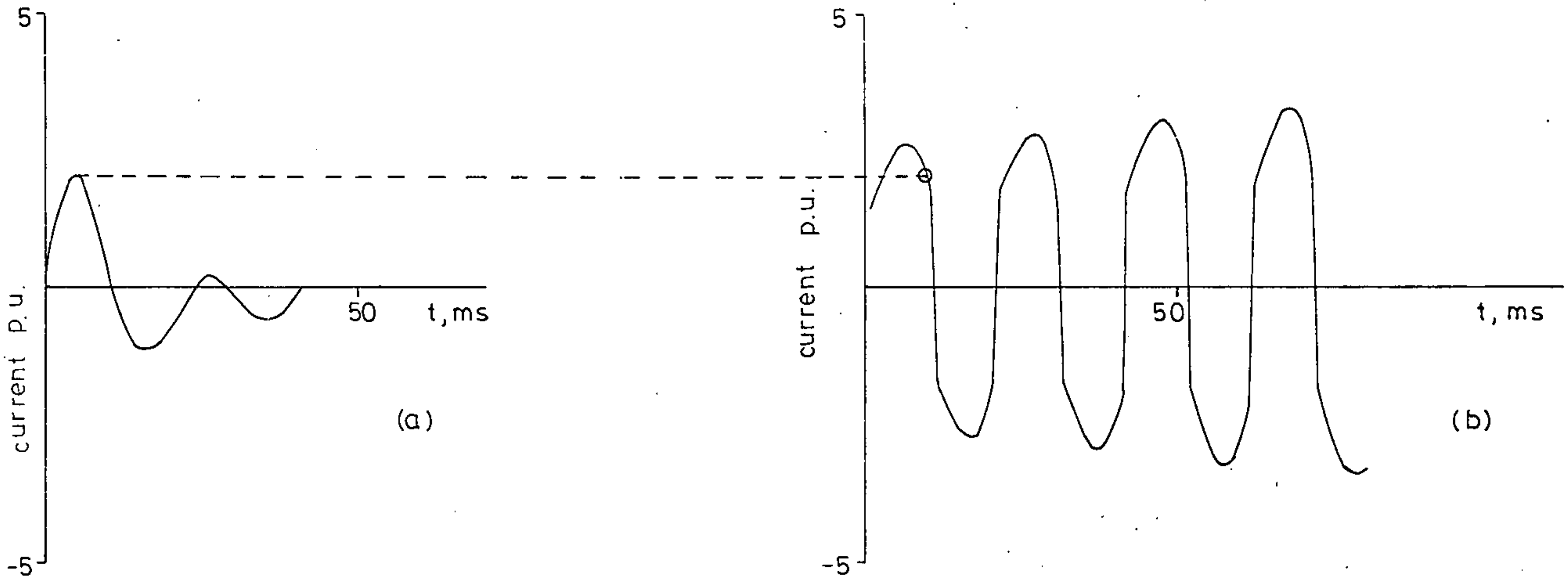


Figure 8.30 Current of 0.75 kW motor following star-delta starting of 0.75 kW/1.5 kW/2.25 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 4.4 A.

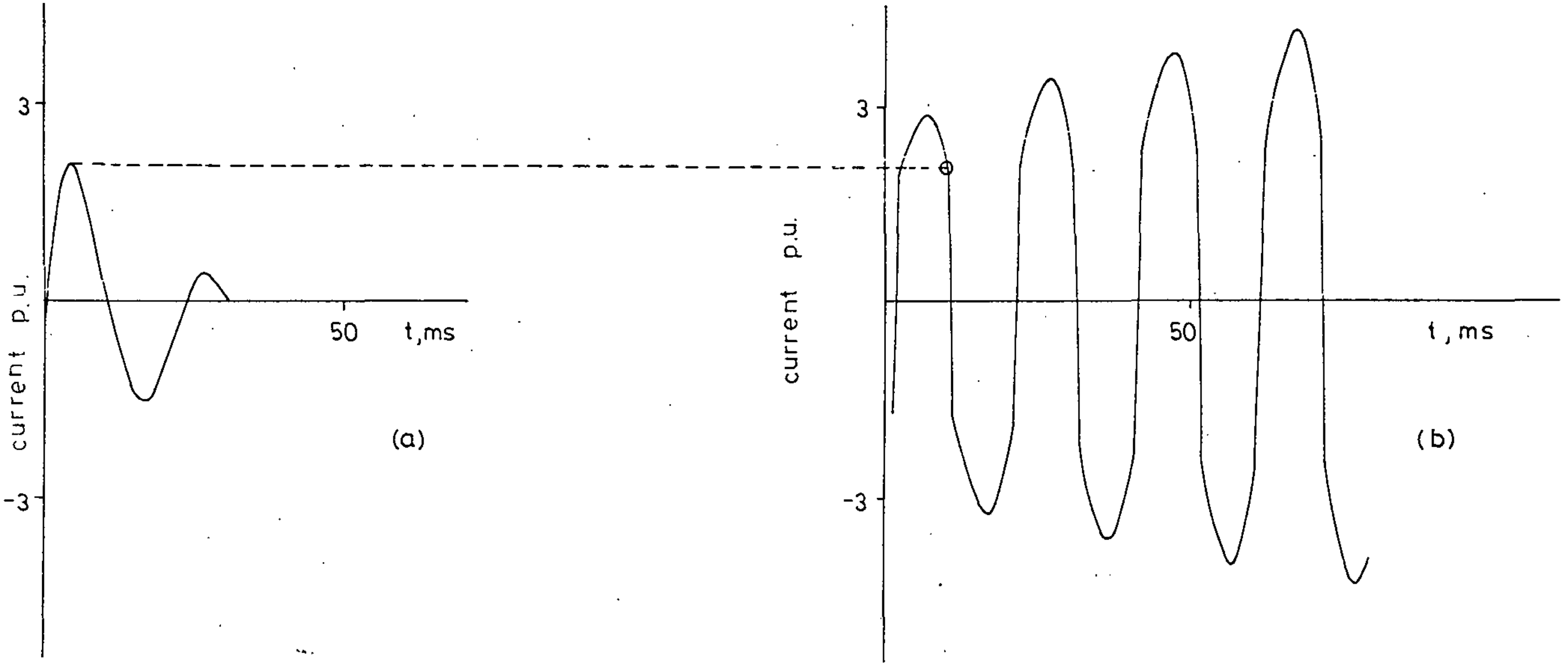


Figure 8·31 Current of 1·5 kW motor following star-delta starting of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 5·5 A.

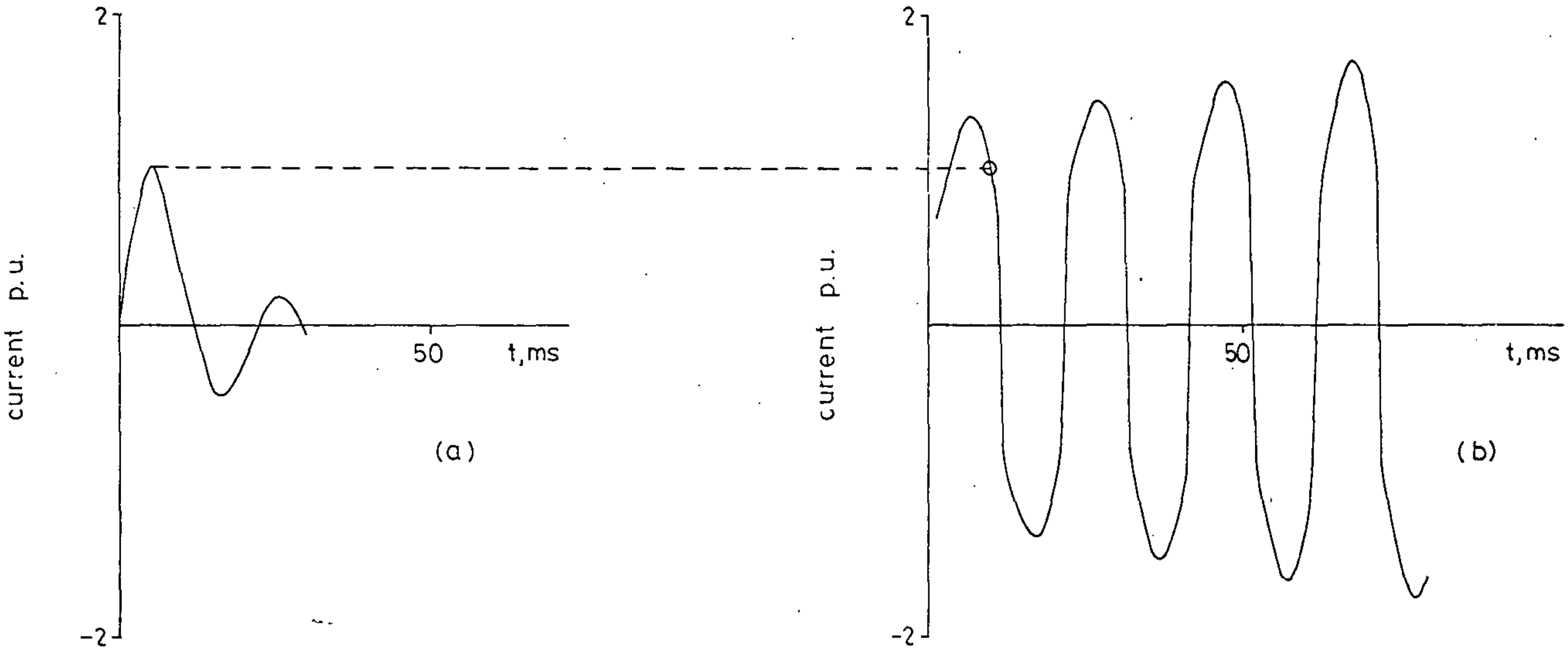
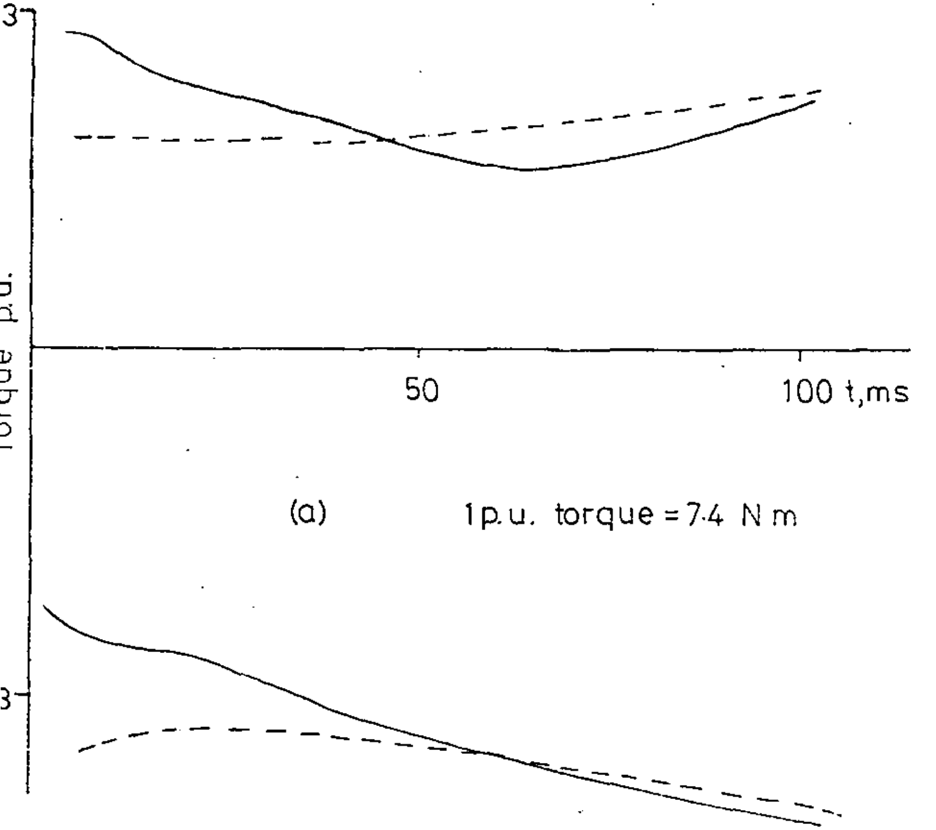


Figure 8.32 Current of 2·25 kW motor following star-delta starting of 0·75 kW/1·5 kW/2·25 kW motor group.

(a) Computed transient current pattern.

(b) Variation of peak current with delay.

1 p.u. current = 12·7 A.



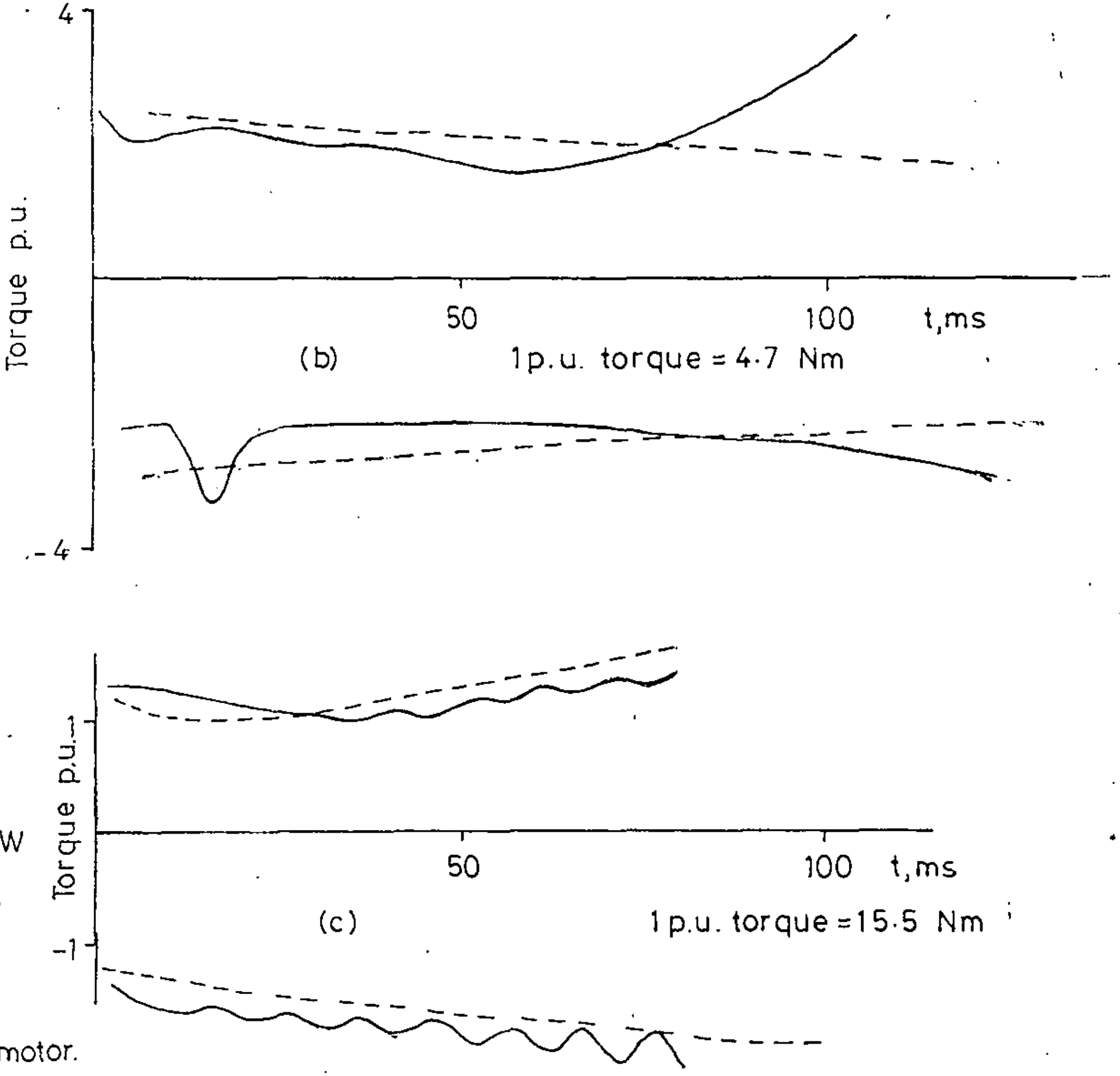


Figure 8.33 Comparison between torque peaks following star-delta starting of 0.75 kW, 1.5 kW, 2.25 kW motors in isolation and in 0.75 kW/1.5 kW/2.25 kW motor group.

(a) Variation of peak torque with delay of 0.75 kW motor.

(b) 1.5 kW motor. (c) 2.25 kW motor.

motor in isolation - - - - - - , motor in group ————

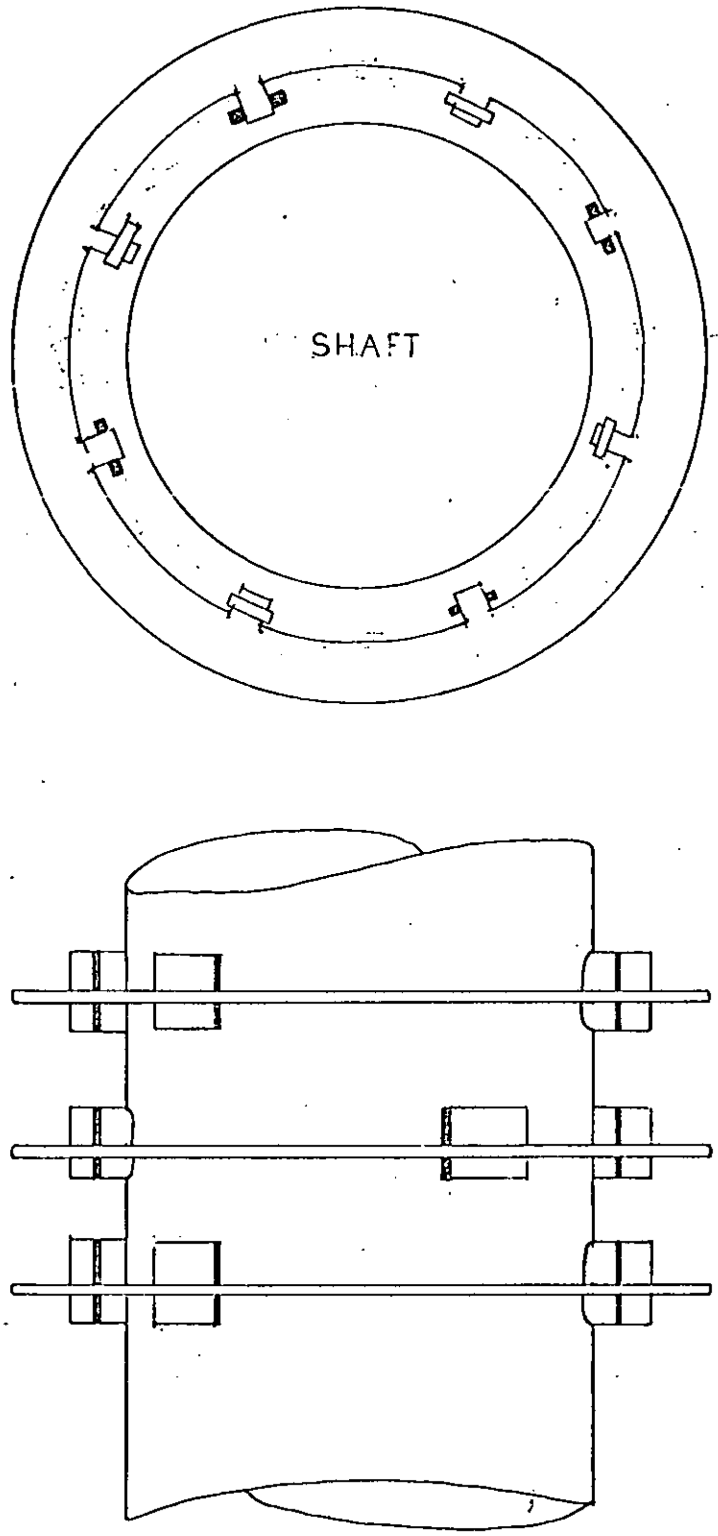


Figure B.1 The torductor.

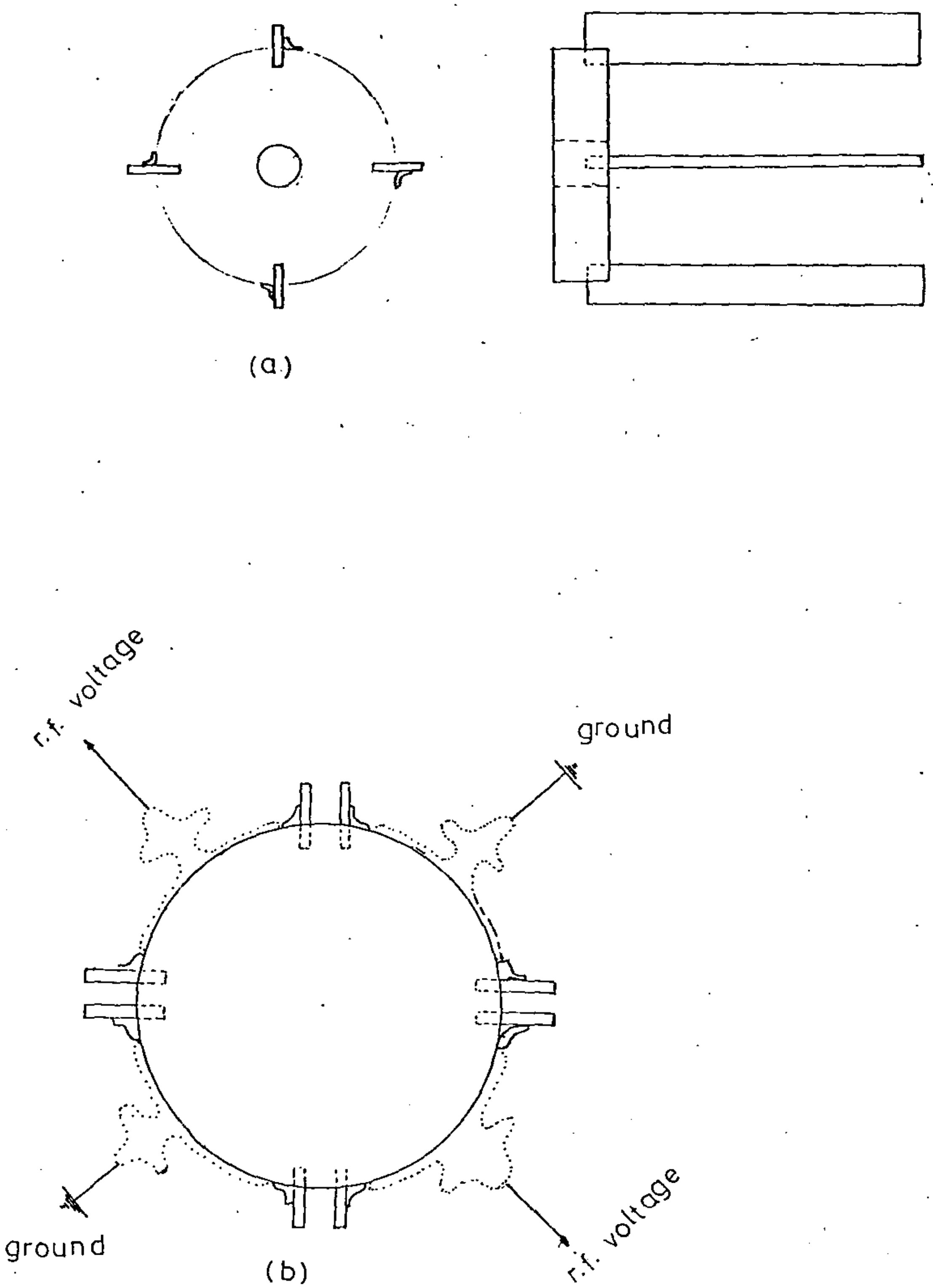


Figure B.2 Capacitance strain-gauge

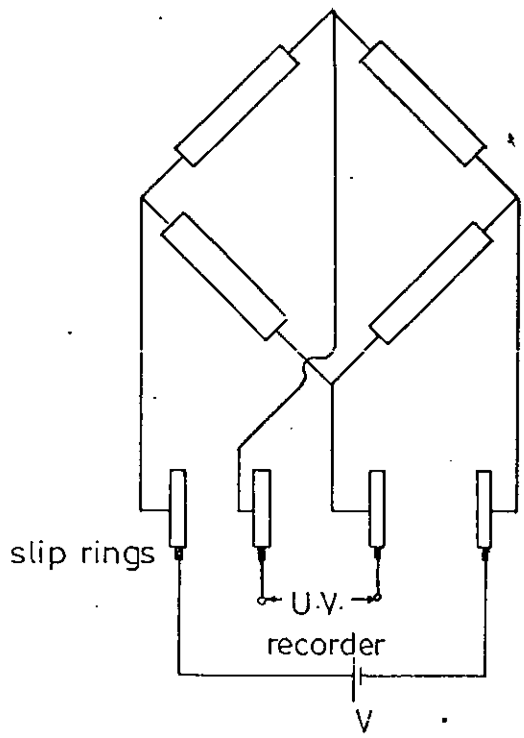


Figure B.3 Resistance strain-gauge.