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Network Capacity Enhancement in HetNets Using Incentivized Offloading Mechanism

BOKAMOSO BASUTLI[1], (Member, IEEE), JOSEPH M. CHUMA[1], (Member, IEEE), AND SANGARAPILLAI LAMBOTHARAN[2], (Senior Member, IEEE)
[1]Department of Electrical, Computer and Telecommunication Engineering, Botswana International University of Science and Technology, Private Bag 6, Palapye, Botswana
[2]Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, Loughborough LE11 3TU, U.K.
Corresponding author: Sangarapillai Lambotharan (s.lambotharan@lboro.ac.uk)

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ABSTRACT This paper investigates distributed algorithms for joint power allocation and user association in heterogeneous networks. We propose auction-based algorithms for offloading macrocell users (MUs) from the macrocell base station to privately owned small-cell access points (SCAs). We first propose a simultaneous multiple-round ascending auction (SMRA) for allocating MUs to SCAs. Taking into account the overheads incurred by SCAs during valuation in the SMRA, further improvements are proposed using techniques known as sub-optimal altered SMRA, the combinatorial auction with item bidding (CAIB), and its variations; the sequential CAIB and the repetitive CAIB. The proof for existence of the Walrasian equilibrium is demonstrated through establishing that the valuation function used by the SCAs is a gross substitute. Finally, we show that truthful bidding is individual rational for all of our proposed algorithms.

INDEX TERMS Auction, beamforming, HetNets, offloading, user admission, quality of service, Walrasian equilibrium, HetNets.

I. INTRODUCTION

As demand for data increases, the macrocell networks are becoming increasingly congested. Consequently, the macrocell base stations (MBSs) are unable to meet the quality of services requirement of all users, hence certain users will have to be dropped from service. Fortunately, there is an increase in the deployment of small cell access points (SCAs) which aim to ease the traffic congestion problem of macrocell networks. Since SCAs are low powered, they allow aggressive reuse of frequencies within the MBS coverage area. The network capacity and efficiency of the entire network can be improved by utilizing these SCAs to offload some of the MBS traffic.

A. RELATED WORKS

In traffic offloading, mobile users can connect to under utilized third party networks. Several works in the literature have investigated various mechanisms for benefits of offloading [1]–[7]. A system called Wiffler is proposed in [2] to augment mobile 3G capacity with WiFi. The work in [5] analyzed Erlang-like capacity in a setting with multiple macrocells deployed with picocells and femtocells. Their results showed that small cells can achieve higher network capacity with good energy efficiency. In [6] a small cell activation mechanism is proposed for offloading traffic from a macrocell to small cells while avoiding user QoS degradation. The main idea is to offload traffic to small cells in energy saving mode only when there is a significant energy saving gains. Hence this approach reduces the total energy consumption of the network. The work in [7] considered a centralized energy aware offloading scheme based on cloud-radio access network.

The SCAs are normally privately owned. There are three kinds of access modes, namely closed access, open access, and hybrid access. In closed access mode, only pre-registered users can be served. The open-access mode allows both pre-registered and unregistered users to access the network. In hybrid-access mode, access points allow unregistered users to gain access under some constraints. Jo *et al.* [8] and Quek *et al.* [9] recommended the hybrid-access mode for network capacity improvement. In order to motivate SCA owners to switch their SCAs from closed-access mode to hybrid-access mode, some incentives should be offered.

In order to address the issue of offloading, auction based algorithms have been proposed in [10]–[13]. Paris *et al.* [10]

and Paris *et al.* [13] formulated a combinatorial reverse auction problem wherein a set of mobile network operators (MNOs) acts as auctioneers and the wireless access points as bidders. The commodity in the auction is the under-utilized bandwidth of the access points. In their problem formulation, the access points submit bids to the MNOs who in turn select the access point of their interest. A reverse auction framework for fair and efficient access permission is proposed in [11]. In particular, the authors proposed a Vickery-Clarke-Groves (VCG) mechanism to maximize the social welfare of a network with one wireless service provider (WSP) and several femtocell owners. In their network model, the WSP is the buyer and the femtocells sell user access to serve WSP users. Chen *et al.* [11] addressed cell overlapping by partitioning the femtocell coverage area into small granularity of identical size (referred to as locations). This allows bids to be expressed as a function of access permission in each location. In order to tackle the complexity of VCG mechanism, Chen *et al.* [11] proposed a suboptimal algorithm with lower complexity. In [12], a network with multiple MBSs, third party owned femtocells and mobile users was considered. In order to allow the MBSs to offload some of their traffic to femtocells, the femtocells are required to submit bids to serve MBS. Considering that the sellers (femtocells) could incur significant overheads during valuation, the authors further proposed a system which allows imprecise valuations. Therefore the femtocells are allowed to estimate their valuations.

B. CONTRIBUTIONS

We explore a different class of auctioning algorithms that have been used in practice especially in spectrum auctioning. Our focus is on forward auctions. Incentives are offered to SCAs to participate in the auction which attracts more bidders hence increased level of competition among privately owned SCAs [14].

Hence the privately owned SCAs are willing to serve users from the MNOs so as to fully occupy their under-utilized spectral resources. On the other hand, it is the interest of MNOs to explore mechanisms that will increase their network capacity without deploying extra base stations (BSs) hence reducing the capital expenditure. We couple these standings of both parties to create a market place environment to develop auction-based algorithms.

Our focus is on a multi-unit auction settings wherein the bidders have budget constraints in terms of the maximum possible number of items they could bid. As we will show later, these budget constraints are private. In particular, we study simultaneous multiple-round ascending auction (SMRA) and combinatorial auction (CA) with item bidding. In the former, the items are sold simultaneously in an iterative manner. In CA with item bidding (CAIB), items are sold separately and independently in a one shot auction. Every bidder submits a single bid for each item, and each item is sold independently as in a single-item auction.

The contributions of our work are as follows:

- We propose and analyze auction based algorithms that jointly perform downlink beamformer design and user association. To the best of our knowledge, works that propose auction based mechanisms do not consider beamformer design in their mechanism design.
- We develop a novel valuation function that automatically monitors the resource budgets for bidders.
- We propose two forward SMRA based algorithms to facilitate the user offloading process. The first algorithm directly applies the classical SMRA which is used in spectrum auctioning. In order to reduce the valuation overheads incurred by the bidders, we propose a second algorithm, referred to here as the altered SMRA (ASMRA). These two algorithms are able to preserve the privacy of bidders' valuations.
- We further propose two forward CAIB algorithms; the sequential CAIB (SCAIB) and the repetitive CAIB (RCAIB). These algorithms use the second-price rule (i.e., VCG payment). In the RCAIB, standing highest bids are advertised to competitors and this could provide more information on the valuation and thus encourage retaliations. The SCAIB tries to avoid this problem.
- We show that truthful bidding leads to individual rationality and it is the best response for every bidder. We demonstrate that the truthful bidding leads to a Walrasian equilibrium (WE) where supply meets the demand.
- Thorough numerical analysis is conducted and validation of the proposed algorithms is carried out by comparing the proposed algorithms with the optimal solution for heterogeneous deployments.

This paper is structured as follows: Subsection I-C briefly annotates some basic terminologies found in auction theory. In section II, we present the system model, necessary assumptions and the problem formulation. Section III formulates the general combinatorial problem and analyze the existence of the WE. In section IV, we present the proposed algorithms. Bidding strategies are analyzed in section V. In section VI, we provide numerical results followed by concluding remarks in section VII.

Notations: In addition to the notations in Table 1, we use the following notations: We use the upper-case bold face and lower-case bold face letters for matrices and vectors, respectively. The notation $\|\cdot\|$ is the Euclidean norm. The operators $\Im(\cdot)$ and $\Re(\cdot)$ extract the imaginary part and the real part of their arguments, respectively. The regular and Hermitian transposes are denoted by $(\cdot)^{\mathsf{T}}$ and $(\cdot)^{\mathsf{H}}$, respectively. Finally, $\mathcal{CN}(\mu, \sigma^2)$ is the circular symmetric complex Gaussian distribution of a random variable with mean μ and variance σ^2.

C. BASIC TERMINOLOGY USED IN AUCTION THEORY

Here we present some basic terminologies commonly used in auction theory.

TABLE 1. Frequently used notations.

Notation	Definition
HU(s)	Host users, i.e., SCAs' pre-registered user(s).
GU(s)	Guest users, i.e., MBS user(s) requesting connection to the SCAs. A set of all GU(s) is denoted as $\mathcal{G}$.
MU(s)	Macrocell user(s) primarily served by the MBS. MUs being auctioned are called GUs.
$\mathcal{S}$	A set of all SCAs.
$\mathcal{M}_0$	A set of all MUs with cardinality $M_0 = \|\mathcal{M}_0\|$. $\mathcal{M}_0 = \{1, \ldots, M_0\}$.
$\mathcal{G}_s \subset \mathcal{G}$	A set of all GUs in the auction coverage area of the s-th SCA with cardinality $G_s = \|\mathcal{G}_s\|$.
$\mathcal{H}_s$	A set of all HU(s) and admitted GUs at the s-th SCA with cardinality $H_s = \|\mathcal{H}_s\|$.
$\mathcal{F}_s := \mathcal{H}_s \cup \mathcal{G}_s$	A set of HUs and GUs in an auction coverage area of the s-th SCA.
$\mathcal{A}_s$	The allocation/provisional set. A set of (provisionally) assigned GUs for SCA s.
$\mathcal{C}_g$	The competitors set. A set of SCAs competing for GU g.
$\mathcal{G}_s^i$	The conditional bidding set/conditional bid. A set of favourite GUs for SCA s.
$\mathcal{P}_s := \mathcal{G}_s$	The preference set. A set of GUs in the auction area of the SCA s arranged in the order of preference.
$\mathcal{R}_s \subset \mathcal{G}_s$	The remainder set. A set of GUs that are left over after determining the favourite set $\mathcal{G}_s^i$.
$\mathcal{L}_s \subset \mathcal{G}_s$	The loose set. A set of GUs that have been lost to other bidders.
$\mathcal{T}^i \subseteq \mathcal{S}$	A set of SCAs that have to answer queries from the MBS.

- *Commodity:* A commodity/good/item[1] is an object being traded. We treat the guest users (GUs) as the items.
- *Bidder:* A bidder/buyer is someone who wants to buy an item in the auction. In our problem, bidders are the SCAs.
- *Seller:* A seller is the owner of the items and is interested in selling his items. In our case the seller is the MNO.
- *Auctioneer:* An agent, usually appointed by the seller, who is responsible for conducting the auction proceedings. In our problem an auctioneer is the MBS.
- *Valuation:* It is the value of any item as perceived by the buyers.
- *Bid:* It is the proposal made by the bidder to the seller.
- *Price:* This is the value asked by the auctioneer/seller during an auction.
- *Utility:* The residual value or the benefit of participating in an auction.
- *Reverse or forward auction:* In a reverse auction the sellers compete for buyers while in a forward auction bidders compete for items.

In order to fit our system model into an auction environment, we introduce the following definitions:

- *Preference profile:* A set of all GUs that an SCA is willing to bid on, and sorted in the order of preference.
- *Valuation profile:* A set of all bids (i.e., valuations) corresponding to the preference profile.
- *Auction coverage area*: A prescribed area within which a bidder is allowed to bid.

[1] We choose to use the term item in the rest of the paper.

II. SYSTEM MODEL AND ASSUMPTIONS

Consider a single-cell multiple-input single-output (MISO) downlink network consisting of an MBS with *densely* deployed $\mathcal{S} = [1, \ldots, S]$ set of hybrid SCAs. We assume that the SCAs operate in a hybrid mode. We further assume that the MBS and the SCAs are operating in non-overlapping frequency bands. The latter assumption avoids intercell interference so that the valuation of an SCA will not depend on other SCAs actions. The SCAs can admit guest (GUs) with the provision that the performance of their host users (HUs) is not degraded. The MBS is equipped with M_{MBS} antennas and each SCA is equipped with M_{SCA} antennas where $M_{\text{MBS}} >> M_{\text{SCA}}$. There is a set $\mathcal{M}_0 = [1, \ldots, M_0]$ of macrocell users (MUs), that have to be served primarily by the MBS. It is assumed that at any given time, the MBS has commitment to serve M_0 MUs where $M_0 >> M_{\text{MBS}}$. The MBS and every SCA have a limit on the maximum transmission power, $p_0^{\max}$ and $p_s^{\max}$ respectively. We assume that all SCAs are connected to the MBS via wired backhaul links. All users are assumed to have a specific quality of service (QoS) requirement that needs to be met, otherwise they will be dropped.

A. PROBLEM FORMULATION

In conventional HetNets, MUs are served only by the MBS. This type of setting is shown to be very inefficient in terms of spectral usage and energy. It is likely that some of the resources of the SCAs may be under utilized by HUs. Network operators can take advantage of the availability of SCAs to serve some of their MUs, especially those at the boundary of the coverage area. To achieve this, SCAs should be incentivized to operate in hybrid access modes (i.e., to serve its own users and guest users). We achieve this using auction theory. In particular, we study the utilization of the forward ascending and the combinatorial auctions. If the number of GUs and SCAs is very high, it is highly probable that a GU may be in the auction coverage area of more than one SCA as shown in Figure 1. This stimulates a competitive market which is analyzed using an auction setup in which the MBS is the auctioneer, the SCAs are the bidders, and the MUs (or GUs from the perspective of SCAs) are the items. We must emphasize that the aim of the MBS is to offload as many MUs as possible.

B. GENERAL AUCTION ENVIRONMENT

The MBS intends to perform welfare-maximization for an economy with G heterogeneous items (i.e., GUs) via auctioning. It is assumed that all SCAs have private *marginal values* (see equation (1)) on the items and private budget constraints. In order to maximize their utilities, all bidders wish to admit their favorite GUs subject to the transmission

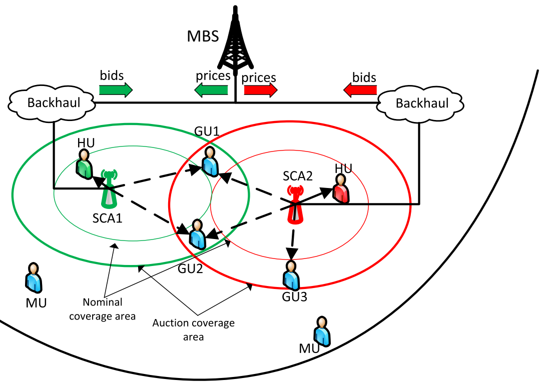

FIGURE 1. An auction market setting in a heterogeneous network. Guest users GU1 and GU2 are over-demanded items, and all MUs labeled users are under-demanded items.

power constraints and QoS requirements of their HUs.[2] Note that the budget constraints on bidders emanate from the constrained transmission powers. Later, we will show that these budgets constraints together with channel status and performance degradation of HUs, set the upper bound on the maximum number of GUs a bidder could accommodate.

Each bidder has private valuations $v_s(\mathcal{G}_s')$ for every possible bundle of GUs $\mathcal{G}_s' \subseteq \mathcal{G}_s$ in its auction coverage area. We immediately note that this will result in immense private parameters since each bidder has to compute the value of every possible bundle. A valuation function of G_s items is a function $v_s(\mathcal{G}_s) : 2^{G_s} \to \mathbb{R}$ such that $v_s(\emptyset) = 0$. We assume *free disposal*; hence, the monotonicity condition such that $v_s(\mathcal{G}_s') \leq v_s(\mathcal{G}_s^{\dagger})$ whenever $\mathcal{G}_s' \subseteq \mathcal{G}_s^{\dagger}$. Consider two disjoint sets $\mathcal{G}_s'$ and $\mathcal{G}_s^{\dagger\dagger}$ at the s-th SCA. The *marginal value* of $\mathcal{G}_s^{\dagger\dagger}$ with respect to the set $\mathcal{G}_s'$ can be defined as:

$$v_s(\mathcal{G}_s^{\dagger\dagger}|\mathcal{G}_s') = v_s(\mathcal{G}_s' \cup \mathcal{G}_s^{\dagger\dagger}) - v_s(\mathcal{G}_s^{\dagger\dagger}). \tag{1}$$

For a price profile $\mathbf{q} \in \mathbb{R}^G$, the utility of s-th bidder for acquiring G_s' GUs is a quasi-linear function defined as:

$$u_s(\mathcal{G}_s') = v_s(\mathcal{G}_s') - \sum_{g\in\mathcal{G}_s'} \mathbf{q}(g). \tag{2}$$

We assume that there are no externalities on the valuation functions of the bidders. Thus, the valuation of each bidder depends only on the set of items it acquires.

C. SYSTEM METRIC DESIGN

Let the set of HUs and GUs served by the s-th SCA be $\mathcal{H}_s$. Each user is denoted by index h. In the downlink, the transmitted signal for the h-th HU from the SCA s can be written as:

$$\mathbf{x}_h(t) = \mathbf{w}_h d_h(t), \tag{3}$$

[2]The term home user (**HU**) is used interchangeable to refer to preregistered users and **GUs** that are already admitted by an SCA. This is because once a **GU** is admitted, the SCA has the mandate to serve that **GU** as its preregistered user.

where $d_h(t) \in \mathbb{C}$ represents the information symbol at time t, and $\mathbf{w}_h \in \mathbb{C}^{M_{\text{SCA}}}$ is the unnormalized transmit beamforming vector for user h. Without loss of generality, assume that $\iota_h(t)$ is normalized such that $\mathbb{E}\{|d_h(t)|^2\} = 1$ as the power can be absorbed into $\mathbf{w}_h$, and that all data streams are independent such that $\mathbb{E}\{\iota_h(t)\iota_j(t)^*\} = 0$, if $h \neq j$. The received signal at the HU is given by:

$$y_h^s = \mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_h d_h(t) + \sum_{j\in\mathcal{H}_s\setminus h} \mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_j d_j(t) + \eta_h(t), \tag{4}$$

where $\mathbf{h}_{sh} \in \mathbb{C}^{M_{\text{SCA}}}$ is the random channel vector from the SCA s to the h-th HU, and $\eta_h(t) \in \mathcal{CN}(0, \sigma_h^2)$ is the circular symmetric zero mean complex Gaussian noise with variance σ_h^2. The notation $\mathcal{H}_s \setminus h$ means set $\mathcal{H}_s$ with element h removed. The instantaneous downlink SINR of the h-th HU served by the s-th SCA is given by

$$\mathsf{SINR}_h^s = \frac{|\mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_{sh}|^2}{\sum_{j\in\mathcal{H}_s\setminus h} |\mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_{sj}|^2 + \sigma_h^2}. \tag{5}$$

D. BIDDERS' VALUATION FUNCTIONS

Prior to bidding, each SCA has to determine its valuation of their favorite GUs. The objective of each bidder is to maximize its utility as defined in (2). Thus, given a preference set $\mathcal{P}_s \subseteq \mathcal{G}$ of GUs, a price profile $\mathbf{q}$, and a price increment δ, each bidder will have to solve (6), as shown at the bottom of the next page, at any instance of the auction. We note that solving (6) for every possible subset of $\mathcal{F}_s$ can be very costly. At this juncture, we make an essential assumption that all GUs require the same QoS in terms of SINR targets. This assumption ensures the monotonocity condition of the valuation function. The solution to (6) is attained by picking the most valuable items with the lowest prices. Whenever the price profile $\mathbf{q}$ contains identical values, e.g., at the start of an auction $\mathbf{q} = \mathbf{0}$, then the solution to (6) is achieved by admitting the largest possible set of users. This set will comprise of all users with the highest valuations.

Therefore, the cardinality of the very first favorite set gives the maximum number of users an SCA can admit. This first favorite set, denoted $\hat{\mathcal{G}}_s$, forms a budget constraint on the bidder. In an iterative auction, the cardinality of the favorite set can only decrease as the auction progresses. During the auction, the prices are bound to be different, hence, the bidder is obliged to determine the favorite set by exhaustively trying all possible combinations of the GUs available using (6). For a given favorite set $\mathcal{G}_s^*$, the valuation process will provide values that are downward-sloping such that $v_{s1} \geq v_{s2} \geq \cdots \geq v_{sG_s'}$. The total valuation of the favorite set is given by $v_s(\mathcal{G}_s^*) = \sum_{g\in\mathcal{G}_s^*} v_{sg}$.

The user maximization (UM) problem ($\mathscr{P}^{\text{UM}}$) and user admission (UA) problem ($\mathscr{P}^{\text{UA}}$) were discussed in [15]. The QoS targets of the HUs and GUs at the s-th SCA are defined as $\Xi^s = [\xi_1^s, \ldots, \xi_{H_s}^s, \xi_{H_s+1}^s, \ldots, \xi_{F_s}^s]$. Then $\mathscr{P}^{\text{UM}}$ at the s-th SCA is formulated as

$$\mathscr{P}_1^{\mathrm{UM}}: \ \texttt{maximize}\ \mathbf{card}(\mathcal{F}_s) \quad \texttt{subject to}\ \mathsf{SINR}_h^s \geq \xi_h^s,\ \ h \in \mathcal{F}_s, \quad \sum_{h\in\mathcal{F}_s} \|\mathbf{w}_{sh}\|_2^2 \leq p_s^{\max}. \tag{7}$$

where $\mathbf{card}(\cdot)$ is the cardinality. The problem in $\mathscr{P}_1^{\mathrm{UM}}$ is non-convex because the objective function and the QoS constraints are non-convex.

Let us define the matrix $\mathbf{W}_s = [\mathbf{w}_{sh}]_{h\in\mathcal{F}_s}$ by concatenating the column vectors $\mathbf{w}_{sh}$ at each SCA. We further introduce slack variables $\mathbf{a}^s = [a_1^s, \ldots, a_{H_s}^s, a_{H_s+1}^s, \ldots, a_{F_s}^s]$. By following the approach in [15], $\mathscr{P}_1^{\mathrm{UM}}$ is equivalently formulated as in (8), as shown at the bottom of this page.

Problem (8) can be solved using the convex programming package CVX [16]. To build up a preference set of GUs $\mathcal{P}_s \subseteq \mathcal{G}_s$, we sort the vector $\mathbf{a}^s$ in ascending order. The corresponding indices of the sorted $\mathbf{a}^s$ with the exclusion of the index of HUs give the preference profile $\mathbf{f}_s$. To build up an optimal favorite set $\mathcal{G}_s^* \subseteq \mathcal{P}_s$ and to determine the *marginal values* v_{sg} for each g-th user, the GUs are sequentially admitted beginning with the one corresponding to the smallest a_h^s. This is done by checking for feasibility at every admission by solving

$$\mathscr{P}^{\mathrm{UA}}: \ \underset{\{\mathbf{w}_{sh}\}}{\texttt{minimize}} \sum_{\forall h\in\mathcal{H}_s\cup g} \|\mathbf{w}_{sh}\|_2^2 \quad \texttt{subject to}\ \mathsf{SINR}_h^s \geq \xi_h^s,\ \ \forall h \in \mathcal{H}_s \cup g, \quad \sum_{\forall h\in\mathcal{H}_s\cup g} \|\mathbf{w}_{sh}\|_2^2 \leq p_s^{\max}, \tag{9}$$

using the CVX tool. When a newly admitted user makes the constraints in (9) infeasible, it is removed from the set $\mathcal{G}_s^*$.

At every admission stage, the bidders determine the marginal value of the newly admitted GU. Let the charge per unit of the data rate paid by every GU for connection, and the cost per unit power, be denoted as μ and κ, respectively. The marginal value of the admitted g-th GU is determined as

$$v_{sg} = \kappa \log_2(1 + \xi_g^s) - c_{sg}, \tag{10}$$

where the *marginal cost* c_{sg}, is given by

$$c_{sg} = \mu \left(\sum_{\forall k\in\mathcal{H}_s\cup g} \|\widehat{\mathbf{w}}_{sk}\|_2^2 - \sum_{\forall h\in\mathcal{H}_s} \|\mathbf{w}_{sh}\|_2^2 \right). \tag{11}$$

In (11), $\widehat{\mathbf{w}}_{sk}$ is the beamformer vector of the k-th HU given that GU g is admitted, and $\mathbf{w}_{sh}$ is the beamformer vector of the h-th HU before GU g is admitted. The first term within the parenthesis in (11) is the total power consumed after the connection of the g-th GU and the last term within the parenthesis is the total power consumption before g-th GU is admitted. Summing over all users before and after admission, the valuation in (10) is expressed as

$$v_s(g|\mathcal{H}_s) = v_s(\mathcal{H}_s \cup g) - v_s(g). \tag{12}$$

III. SURPLUS MAXIMIZATION PROBLEM

The objective of the MBS is to maximize its surplus, which is the number of users that are offloaded to the SCAs. We cast the general surplus maximization problem at the MBS as the following integer program (IP);

$$\begin{aligned}\mathscr{P}^{\mathrm{IP}}: \ \underset{x_{s\mathcal{A}_s}}{\texttt{maximize}}\ & \sum_{s\in\mathcal{S}}^{S} \sum_{\mathcal{A}_s\subseteq\mathcal{G}} v_{s\mathcal{A}_s} x_{s\mathcal{A}_s} \\ \texttt{subject to}\ & \sum_{j\in\mathcal{A}_s} \sum_{s} x_{s\mathcal{A}_s} \leq 1, \quad \forall j \in \mathcal{G}, \\ & \sum_{\mathcal{A}_s} x_{s\mathcal{A}_s} \leq 1, \quad \forall s \in \mathcal{S}, \\ & x_{s\mathcal{A}_s} \in \{0, 1\}, \quad \forall s \in \mathcal{S}, \mathcal{A}_s \subseteq \mathcal{G},\end{aligned} \tag{13}$$

where $\mathcal{A}_s$ is the possible allocation of set of GUs to the s-th bidder and $x_{s\mathcal{A}_s}$ is a binary decision variable, indicating

$$\underset{\mathcal{P}_s\subseteq\mathcal{G}\backslash\mathcal{A}_s}{\operatorname{argmax}} \left\{ v_s(\mathcal{H}_s \cup \mathcal{P}_s) - \left(\sum_{g\in\mathcal{A}_s} \mathbf{q}(g) + \sum_{g\in\mathcal{P}_s} (\mathbf{q}(g) + \delta) \right) \right\}, \tag{6}$$

$$\begin{aligned}\mathscr{P}_2^{\mathrm{UM}}: \ \underset{\{\mathbf{w}_h\},\{\mathbf{a}^s\}}{\texttt{minimize}}\ & \|\mathbf{a}^s\|_1 \\ \texttt{subject to}\ & \begin{bmatrix} \sqrt{1 + \frac{1}{\xi_h^s}}\mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_{sh} + a_h^s \\ \mathbf{h}_{sh}^{\mathsf{H}}\mathbf{W}_s \\ \sigma \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \quad h \in \mathcal{F}_s, \\ & \Im(\mathbf{h}_{sh}^{\mathsf{H}}\mathbf{w}_{sh}) = 0, \quad \forall h, \\ & \mathbf{a}^s = \mathbf{0}, \quad h = 1, \ldots, H_s, \\ & \mathbf{a}^s \geq \mathbf{0}, \quad h = H_s + 1, \ldots, F_s, \\ & \sum_{h\in\mathcal{F}_s} \|\mathbf{w}_{sh}\|_2^2 \leq p_s^{\max}, \quad \forall h.\end{aligned} \tag{8}$$

association of SCAs. $x_{s\mathcal{A}_s} = 1$ means SCA s is assigned to a set of GUs in the set $\mathcal{A}_s$ and otherwise $x_{s\mathcal{A}_s} = 0$. The first constraint ensures that every GU is matched with at most one SCA. The second constraint ensures that every SCA should get at most one bundle.

A. ALLOCATION AND PAYMENT RULES

Under the SMRA setting, the bidder with the highest bid wins and the payment is the price $\mathbf{q}$ minus the increment δ. For a CAIB auction, $\mathcal{A}_s(\mathbf{b})$ is used to denote the allocation for a bid profile $\mathbf{b}$. Let $\mathbf{b} = (\mathbf{b}_s, \mathbf{b}_{-s})$ denote the bid profile where SCA s bids $\mathbf{b}_s$ and all other SCAs bid $\mathbf{b}_{-s} = (\mathbf{b}_1, \ldots, \mathbf{b}_{s-1}, \mathbf{b}_{s+1}, \ldots, \mathbf{b}_S)$. In CAIB, the allocation and payment rules require the GU to be offered to the highest bidder at a price equal to the second highest bid. For a given allocation $\mathcal{A}_s(\mathbf{b}) \subseteq \mathcal{G}'_s$, the sum of the highest bids are denoted by

$$B^{\text{high}}(\mathcal{A}_s(\mathbf{b}), \mathbf{b}) = \sum_{g\in\mathcal{A}_s(\mathbf{b})} \max_t(\mathbf{b}_t(g)),$$
$$B^{\text{high}}_{-s}(\mathcal{A}_s(\mathbf{b}), \mathbf{b}_{-s}) = \sum_{g\in\mathcal{A}_s(\mathbf{b})} \max_{t\neq s}(\mathbf{b}_t(g)). \tag{14}$$

Using (2) and the second price rule, the utility of the s-th SCA is given by

$$u_s(\mathbf{b}) = v_s(\mathcal{A}_s(\mathbf{b})) - B^{\text{high}}_{-s}(\mathcal{A}_s(\mathbf{b}), \mathbf{b}_{-s}). \tag{15}$$

B. EXISTENCE OF THE WALRASIAN EQUILIBRIUM

In [17] and [18], it is argued that if the WE exists, any efficient allocation must solve the relaxed $\mathscr{P}^{\text{IP}}$. In order to address the existence of the WE in the SMRA and the CAIB, we require the following definitions.

Definition 1: Given a valuation function $v_s(\mathcal{G}_s)2^{G_s} \rightarrow \mathbb{R}$ and a vector of prices $\mathbf{q} \in \mathbb{R}$, the *demand* ($\mathcal{D}_s(v_s, \mathbf{q})$) of a bidder s at the price of $\mathbf{q}$ is given by

$$\mathcal{D}_s(v_s, \mathbf{q}) := \{\mathcal{G}'_s \subseteq \mathcal{G}_s : u_s(\mathcal{G}'_s) \geq u_s(\mathcal{G}^\dagger_s), \forall \mathcal{G}^\dagger_s \subset \mathcal{G}_s\} \tag{16}$$

Definition 2: An *allocation* is a partition of $\mathcal{G}$ into pairwise disjoint sets of items $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_S$.

Definition 3: Utilities of bidders are deemed to be *decreasing marginal utilities* if the marginal value of an item decreases as the number of already accumulated items increases. This is equivalently defined via the *submodular valuation* definition. A valuation function v_s is submodular if for a pair $\mathcal{G}'_s \subseteq \mathcal{G}^\dagger_s$ and a GU g, $v_s(g|\mathcal{G}'_s) \geq v_s(g|\mathcal{G}^\dagger_s)$.

Definition 4: A valuation function v_s is complementary free if for all sets of items $\mathcal{G}'_s$ and $\mathcal{G}^\dagger_s$, the following holds:

$$v_s(\mathcal{G}'_s) + v_s(\mathcal{G}^\dagger_s) \geq v_s(\mathcal{G}'_s \cup \mathcal{G}^\dagger_s). \tag{17}$$

C. SUBMODULARITY OF THE VALUATION FUNCTION

Theorem 1: The valuation function v_s in (12) is a submodular valuation function.

Proof: In [19] and [20] a valuation v_s is submodular if and only if *any* of the following conditions hold.

1) Decreasing marginal utilities: For any $g, g^\dagger \in \mathcal{G}$ and $\mathcal{G}'_s \subseteq \mathcal{G}$, then $v_s(g|\mathcal{G}'_s) \geq v_s(g|\mathcal{G}'_s \cup \{g^\dagger\})$.
2) Monotonicity: For any $\mathcal{G}'_s, \mathcal{G}^\dagger_s, \mathcal{G}^\ddagger_s \subseteq \mathcal{G}$, such that $\mathcal{G}' \subseteq \mathcal{G}^\dagger$, then $v_s(\mathcal{G}^\ddagger_s|\mathcal{G}'_s) \geq v(\mathcal{G}^\ddagger_s|\mathcal{G}^\dagger_s)$.
3) Complementary free: For any $\mathcal{G}'_s, \mathcal{G}^\dagger_s \subseteq \mathcal{G}$, then $v_s(\mathcal{G}'_s) + v(\mathcal{G}^\dagger_s) \geq v_s(\mathcal{G}'_s \cup \mathcal{G}^\dagger_s) + v(\mathcal{G}'_s \cap \mathcal{G}^\dagger_s)$.

It is sufficient to qualify for one of the conditions above. We choose the first one. From (5), let us write $\mathbf{w}_{sh} = \sqrt{\rho_h}\tilde{\mathbf{w}}_{sh}$, where ρ_h is the power and $\tilde{\mathbf{w}}_{sh}$ is the unit-norm beamforming direction for the h-th HU. We further denote power allocation vector $\boldsymbol{\rho}_s = [\rho_1, \ldots, \rho_{H_s}]$. The SINR in (5) can be expressed as

$$\mathsf{SINR}^s_h(\boldsymbol{\rho}_s) = \frac{\rho_h}{\mathcal{I}_h(\boldsymbol{\rho}_s)}, \tag{18}$$

where

$$\mathcal{I}_h(\boldsymbol{\rho}_s) = \min_{\|\tilde{\mathbf{w}}_{sh}\|=1} [\Psi_s(\tilde{\mathbf{w}}_{sh})\boldsymbol{\rho}_s]_h + \frac{\sigma_h}{|\mathbf{h}^{\mathsf{H}}_{sh}\tilde{\mathbf{w}}_{sh}|^2}. \tag{19}$$

The constant link gain matrix (i.e., a coupling matrix) Ψ_s for the s-th SCA is defined as

$$[\Psi_s]_{sh}(\tilde{\mathbf{w}}) = \begin{cases} \dfrac{|\mathbf{h}^{\mathsf{H}}_{sh}\tilde{\mathbf{w}}_{sj}|^2}{|\mathbf{h}^{\mathsf{H}}_{sh}\tilde{\mathbf{w}}_{sh}|^2}, & j \neq h, \\ 0, & j = h. \end{cases} \tag{20}$$

In [21] and [22], it was proven that $\mathcal{I}_k(\boldsymbol{\rho}_s)$ is a standard function. Now consider that the preference profiles of bidder s as $\mathcal{P}_s := \{g_1, g_2, \ldots, g_{u-1}, g_u, g_{u+1}, \ldots, g_{G_s}\}$. Let us assume that during sequential admission of GUs, the set of HUs is $\mathcal{H}_s := \{g_1, g_2, \ldots, g_{u-1}\}$ with the corresponding power allocation vector of $\boldsymbol{\rho}^{\mathcal{H}_s}_s$. Let us now assume that in the next admission, SCA s considers GU g_u, with the resulting power allocation vector $\boldsymbol{\rho}^{g_u|\mathcal{H}_s}_s$. Due to the monotonicity of $\mathcal{I}_k(\boldsymbol{\rho}_s)$ on $\boldsymbol{\rho}_s$ and using (18), we argue that $\mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_u|\mathcal{H}_s}_s \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{\mathcal{H}_s}_s$. Now suppose before admitting GU g_u, bidder s admits GU g_{u+1} first. Note that GU g_{u+1} has equal or lower preference to bidder s as compared to GU g_u. Let us denote the power allocation vector by $\boldsymbol{\rho}^{g_{u+1}|\mathcal{H}_s}_s$ when GU g_{u+1} is admitted first. With the same argument given earlier, the new power allocation vector will satisfy $\mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_{u+1}|\mathcal{H}_s}_s \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_u|\mathcal{H}_s}_s \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{\mathcal{H}_s}_s$. If GU g_u is admitted after GU g_{u+1} with the corresponding power allocation vector being $\boldsymbol{\rho}^{g_u|\mathcal{H}_s\cup\{g_{u+1}\}}_s$, it should be the case that $\mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_u|\mathcal{H}_s\cup\{g_{u+1}\}}_s \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_{u+1}|\mathcal{H}_s}_s \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_u|\mathcal{H}_s}_s \geq \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{\mathcal{H}_s}_s$.

Now by utilizing (12) and (11), we get

$$\begin{aligned} v_s(g_u|\mathcal{H}_s) &= \kappa \log_2(1+\xi^s_{g_u}) - \left(\mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_u|\mathcal{H}_s}_s - \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{\mathcal{H}_s}_s\right) \\ &\geq \kappa \log_2(1+\xi^s_{g_u}) - \left(\mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_u|\mathcal{H}_s\cup\{g_{u+1}\}}_s - \mathbf{1}^{\mathsf{T}}\boldsymbol{\rho}^{g_{u+1}|\mathcal{H}_s}_s\right) \\ &= v_s(g_u|\mathcal{H}_s \cup g_{u+1}) \end{aligned} \tag{21}$$

■

Lemma 1: The valuation function v_s is submodular for every subset $\mathcal{Q}_s$, and the marginal valuation function $v_s(\cdot|\mathcal{Q}_s)$ is complementary free.

Proof: Using theorem 1, we need to prove that for all $\mathcal{G}'_s, \mathcal{G}^\dagger_s \in \mathcal{G}_s$, it is such that $v_s(\mathcal{G}'_s) + v(\mathcal{G}^\dagger_s) \geq v_s(\mathcal{G}'_s \cup \mathcal{G}^\dagger_s) + v(\mathcal{G}'_s \cap \mathcal{G}^\dagger_s)$. Let $\mathcal{Q}_s := \mathcal{G}'_s \cap \mathcal{G}^\dagger_s$, $\bar{\mathcal{G}}'_s := \mathcal{G}'_s \setminus \mathcal{Q}_s$, and $\bar{\mathcal{G}}^\dagger_s := \mathcal{G}^\dagger_s \setminus \mathcal{Q}_s$. By using (1), we define the following marginal values:

$v_s(\mathcal{G}'_s) = v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}'_s \cup \mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}'_s \cup \bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s)$, and $v_s(\mathcal{G}'_s \cap \mathcal{G}^\dagger_s) = v_s(\mathcal{Q}_s)$. The third condition of theorem 1 can be equivalently written as

$$\begin{aligned} v_s(\mathcal{G}'_s) + v(\mathcal{G}^\dagger_s) &\geq v_s(\mathcal{G}'_s \cup \mathcal{G}^\dagger_s) + v(\mathcal{G}'_s \cap \mathcal{G}^\dagger_s) \\ &\Rightarrow v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s) \\ &\geq v_s(\bar{\mathcal{G}}'_s \cup \bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s) + v_s(\mathcal{Q}_s) \\ &\Rightarrow v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s \cup \bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s). \end{aligned} \tag{22}$$

This suggests that $v(\cdot|\mathcal{Q}_s)$ is complement free as per (17). With the properties of the interference function given in (19), and the conclusion in (21), we argue that $\mathbf{1}^\mathsf{T}\boldsymbol{\rho}_s^{\bar{\mathcal{G}}'_s|\mathcal{Q}_s} \leq \mathbf{1}^\mathsf{T}\boldsymbol{\rho}_s^{\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s}$, $\mathbf{1}^\mathsf{T}\boldsymbol{\rho}_s^{\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s} \leq \mathbf{1}^\mathsf{T}\boldsymbol{\rho}_s^{\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s}$, and $\mathbf{1}^\mathsf{T}\boldsymbol{\rho}_s^{\bar{\mathcal{G}}'_s|\mathcal{Q}_s} + \mathbf{1}^\mathsf{T}\boldsymbol{\rho}_s^{\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s} \leq \mathbf{1}^\mathsf{T}\boldsymbol{\rho}_s^{\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s}$. This confirms (22).

To conclude the proof, we need to prove that for all $\mathcal{Q}_s$ and $\bar{\mathcal{G}}'_s, \bar{\mathcal{G}}^\dagger_s \subseteq \mathcal{Q}^\mathsf{c}_s$, it is such that $v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s \cup \bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)$. Let us define $\mathcal{G}'_s = \bar{\mathcal{G}}'_s \cup \mathcal{Q}_s$ and $\mathcal{G}^\dagger_s = \bar{\mathcal{G}}^\dagger_s \cup \mathcal{Q}_s$. With these definitions, we get the same marginal valuations as before: $v_s(\mathcal{G}'_s) = v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, $v_s(\mathcal{G}'_s\cup\mathcal{G}^\dagger_s) = v_s(\bar{\mathcal{G}}'_s\cup\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)+v_s(\mathcal{Q}_s)$, and $v_s(\mathcal{G}'_s\cap\mathcal{G}^\dagger_s) = v_s(\mathcal{Q}_s)$. Due to v_s being a submodular function, the condition $v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s \cup \bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)$ is equivalently written as $v_s(\bar{\mathcal{G}}'_s|\mathcal{Q}_s) + v_s(\bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s) \geq v_s(\bar{\mathcal{G}}'_s \cup \bar{\mathcal{G}}^\dagger_s|\mathcal{Q}_s)$. Using the same arguments made above, the proof is established. ∎

D. GROSS-SUBSTITUTE OF THE VALUATION FUNCTION

A much stronger property of the valuation function is the gross-substitute condition.

Definition 5: In a market with m items, S agents, and valuations v_s, a *Walrasian equilibrium* (WE) is a price $\mathbf{q}^\star \in \mathbb{R}_+$ and a partition of goods in disjoint sets $\mathcal{G} := \cup_{s=1}^{S}\mathcal{A}_s$ such that $\mathcal{A}_s \in \mathcal{D}_s(v_s, \mathbf{q})$. The WE corresponds to the *market-clearing* prices where every bidder receives a bundle in his demand set [23]. At WE the following conditions must hold:

- Condition 1: Each bidder s is matched to its preferred item $g \in \operatorname{argmax}\{v_{sg} - q_g\}_{g\in G\cup\{\emptyset\}}$.
- Condition 2: An item $g \in \mathcal{G}$ is unsold only if $q_g = 0$.

Definition 6 (Gross Substitute Condition, Kelso and Crawford [24]): A valuation v_s over the items $\mathcal{G}_s$ satisfies the gross substitution (GS) condition if and only if for any price profile $\mathbf{q} \in \mathbb{R}$ and $\mathcal{G}'_s \in \mathcal{D}_s(v_s, \mathbf{q})$, if $\mathbf{q}'$ is a price profile such that $\mathbf{q}' \leq \mathbf{q}$, then there is a set $\mathcal{G}^\dagger_s \in \mathcal{D}_s(v_s, \mathbf{q}')$ such that $\mathcal{G}'_s \cap \{g : \mathbf{q}(g) = \mathbf{q}'(g)\} \subseteq \mathcal{G}^\dagger$.

In brief, definition 6 suggests that, if a bidder has GS valuation and demands a set $\mathcal{G}'_s$ items at the price profile $\mathbf{q}$, if the price of some of the items subsequently increases, the bidder still demands some of the items in $\mathcal{G}'_s$ whose price remained unchanged.

Proposition 1: The valuation function in (12) is a gross valuation function.

Proof: Consider a bidder s, v_s and $\mathbf{v_{-s}}$. Let the corresponding marginal values for the favourite set $\mathcal{G}'_s$ be denoted as $v_{s1}, v_{s2}, \ldots, v_{sG'_s}$. Suppose bidder s gets matched with all the **GUs** in its favourite set at price vector $\mathbf{q}$. We now introduce a new bidder t who has a favourite set $\mathcal{G}'_t$ such that $\mathcal{G}'_t \cap \mathcal{G}'_s := \{g_2\}$. Let us assume that $v_{sg} \geq v_{tg}, g \in \mathcal{G}'_s \setminus g_2$.[3] Assuming truthful bidding, bidder s will lose **GU** g_2 to bidder t as the price of **GU** g_2 increases. This change in allocation will result in a new power allocation vector $\boldsymbol{\rho}^\dagger_s$ such that $\boldsymbol{\rho}^\dagger_s \preceq \boldsymbol{\rho}_s$, with $\boldsymbol{\rho}^\dagger_s(g_2) = 0$. We invoke the monotonicity axiom for (19) from [21] and [22] and state that $\mathcal{I}_h(\boldsymbol{\rho}_s) \geq \mathcal{I}_h(\boldsymbol{\rho}^\dagger_s)$. With this being true, loss of **GU** g_2 will increase the marginal values of all other **GUs** in the favorite set $\mathcal{G}'_s \setminus g_2$ and thus making them more attractive to SCA s. ∎

E. COMPUTATION OF THE WE PRICES

The linear programming relaxation (LPR) of $\mathscr{P}^{\text{IP}}$ is:

$$\begin{aligned} \mathscr{P}^{\text{LPR}} : \underset{x_{pg}}{\texttt{maximize}} \ & \sum_{s=1} \sum_{\mathcal{A}_s\subseteq\mathcal{G}} v_{s\mathcal{A}_s} x_{s\mathcal{A}_s} \\ \texttt{subject to} \ & \text{Constraints 1 and 2 in (13)}, \\ & 0 \leq x_{s\mathcal{A}_s} \leq 1, \quad \mathcal{A}_s \in \mathcal{G},\ \forall s \in \mathcal{S} \end{aligned} \tag{23}$$

Even though $\mathscr{P}^{\text{LPR}}$ has $G + S$ variables, it has an exponential number of constraints. Works in [17] and [18] propose solving the dual of $\mathscr{P}^{\text{LPR}}$ by utilizing separation based linear programming algorithm. The dual linear programming relaxation (DLPR) is defined as:

$$\begin{aligned} &\mathscr{P}^{\text{DLPR}} : \\ &\underset{x_{pg}}{\texttt{minimize}} \sum_{s=1}^{S} u_s + \sum_{g\in G} \mathbf{p}(g), \\ &\texttt{subject to}\ u_s \geq v_s(\mathcal{A}_s) - \sum_{g\in\mathcal{A}_s} \mathbf{p}(g), \forall s \in \mathcal{S}, \mathcal{A}_s \in \mathcal{G} \\ &\qquad \mathbf{p}(g) \geq 0, u(s) \geq 0, \quad \forall s \in \mathcal{S}, g \in \mathcal{G}, \end{aligned} \tag{24}$$

where $\mathbf{p}$ and u_s are the Lagrange multipliers associated with the constraints in $\mathscr{P}^{\text{LPR}}$. For completeness, we state the following well known theorems:

Theorem 2 (First Welfare Theorem [23]): Suppose $(\mathbf{q}, \mathcal{A}_1, \ldots, \mathcal{A}_S)$ is a WE, then the allocation $(\mathcal{A}_1, \ldots, \mathcal{A}_S)$ corresponds to the allocation that maximizes the social welfare, i.e., the allocation maximizing $\sum_{s\in\mathcal{S}} v_s(\mathcal{A}_s)$.

Proof: Let $Q = \sum_{g\in\mathcal{G}} \mathbf{q}(g)$ be the sum of prices of all **GUs** and let the allocation $(\mathcal{A}^*_1, \ldots, \mathcal{A}^*_S)$ be any welfare maximizing allocation. Since $\mathcal{A}_s \in D(v_s, \mathbf{q})$, then by utilizing condition 1 of definition 5, it is the case that

$$v_s(\mathcal{A}_s) - q(\mathcal{A}_s) \geq v_s(\mathcal{A}^*_s) - q(\mathcal{A}^*_s). \tag{25}$$

Summing for all s, we get

$$\sum_{s\in\mathcal{S}} v_s(\mathcal{A}_s) - \sum_{s\in\mathcal{S}} q(\mathcal{A}_s) \geq \sum_{s\in\mathcal{S}} v_s(\mathcal{A}^*_s) - \sum_{s\in\mathcal{S}} q(\mathcal{A}^*_s). \tag{26}$$

[3] We assume all tie breaks are in favor of bidders s. In this proof we chose g_2 to be the only **GU** that bidder s got out bidden on. This is to simplify the proof.

Algorithm 1 SMRA Algorithm

Data: Initialization: $\delta > 0$, $\mathbf{q}(g) = 0, \forall g \in G$, $(S, G) \in \mathbb{Z}_+$, **GUs**' set $\mathcal{G} := \cup_{s=1}^{S} \mathcal{G}_s$, assignment set $\mathcal{A}_s = \emptyset, \forall s \in \mathcal{S}$, lost items set $\mathcal{L}_s = \emptyset, \forall s \in \mathcal{S}, \mathcal{G}_s^i = \emptyset, \forall s \in \mathcal{S}$, *auction round*: $i = 0$.
Result: Optimal Allocation set $\mathcal{A}_s^*, \forall s \in \mathcal{S}$.
1 **while** $\mathcal{T} \neq \emptyset || \cup_{s=1}^{S} \mathcal{G}_s^i \neq \emptyset$ **do**
2   $i \leftarrow i + 1$
3   Auctioneer asks each bidder for its conditional bidding set $\mathcal{G}_s^i$.
4   Bidders determine new preference profiles and marginal values for their favorite subset $\mathcal{G}_s^i$ (using (6)) of items not assigned to them, given the **GUs** they have admitted and the current prices $\mathbf{q}^i$.
5   Bidders submit their conditional bidding set.
6   Auctioneer set $\mathcal{T} = \emptyset$, $\mathcal{L}_s = \emptyset$
7   **for** $g \in \cup_{s=1}^{S} \mathcal{G}_s^i$ **do**
8   **if** $|\mathcal{C}_g^i| > 1$ **then**
9   pick an arbitrary bidder s: $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$
10       $\forall k \neq s, \mathcal{A}_k \leftarrow \mathcal{A}_k \setminus g, \mathcal{T} \leftarrow \mathcal{T} \cup \forall k \neq s$
11       $\mathcal{L}_k \leftarrow g, \forall k \in \mathcal{C}_g^i \setminus s$
12       $\mathbf{q}^{i+1}(g) \leftarrow \mathbf{q}^i(g) + \delta$
13
    else if $|\mathcal{C}_g^i| = 1$ **then**
14       $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g, \mathcal{G} \leftarrow \mathcal{G} \setminus g$
15   **else**
16   pick an arbitrary winner s from subset of the bidders in $\mathcal{T}$: $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$

Algorithm 2 ASMRA Algorithm

Data: See Algorithm 1.
Result: See Algorithm 1.
1 **while** $\mathcal{T} \neq \emptyset || \cup_{s=1}^{S} \mathcal{G}_s^i \neq \emptyset$ **do**
2   Algorithm 1 steps 2-3.
3   **if** $\mathcal{L}_s \neq \emptyset || i = 1$ **then**
4   Algorithm 1 step 4.
  else
5   **for** $g \in \mathcal{G}_s^{i-1}$ **do**
6   **if** $v_{sg} > \mathbf{q}(g)$ **then**
7         $\mathcal{G}_s^i \leftarrow g$
8   **else**
9         $\mathcal{G}_s^i \leftarrow \mathcal{G}_s^{i-1} \setminus g$
10   Algorithm 1 Steps 5 - 16.

Noting that $Q = \sum_{s \in \mathcal{S}} q(\mathcal{A}_s) = \sum_{s \in \mathcal{S}} q(\mathcal{A}_s^*)$ when we sum over all GUs that have non-zero price, we conclude that $\sum_{s \in \mathcal{S}} v_s(\mathcal{A}_s) \geq \sum_{s \in \mathcal{S}} v_s(\mathcal{A}_s^*)$. ■

Theorem 2 is complemented by the Second Welfare Theorem via the duality theorem in linear programming.

Theorem 3 (Second Welfare Theorem [23]): Suppose an integral optimal solution for $\mathscr{P}^{\text{LPR}}$ exists, then WE whose allocation is given also exists.

Proof: Let the optimal allocation to $\mathscr{P}^{\text{LPR}}$ be $(\mathcal{A}_1^*, \ldots, \mathcal{A}_S^*)$. Suppose the optimal solution to $\mathscr{P}^{\text{DLPR}}$ is given by $(\mathbf{p}^*, u_1^*, \ldots, u_S^*)$. We need to show that $(\mathbf{p}^*, \mathcal{A}_1^*, \ldots, \mathcal{A}_S^*)$ is a WE. Since Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for the optimality to $\mathscr{P}^{\text{LPR}}$ and $\mathscr{P}^{\text{DLPR}}$, then for each SCA for which $x_{s\mathcal{A}_s^*} > 0$, we have $x_{s\mathcal{A}_s^*} = 0$ in $\mathscr{P}^{\text{LPR}}$ and $u_s = v_s(\mathcal{A}_s^*) - \sum_{g \in \mathcal{A}_s^*} \mathbf{p}^*(g)$ in $\mathscr{P}^{\text{DLPR}}$ being true. Therefore, for any other bundle A_s we get

$$u_s = v_s(\mathcal{A}_s^*) - \sum_{g \in \mathcal{A}_s^*} \mathbf{p}^*(g) \geq v_s(\mathcal{A}_s) - \sum_{g \in \mathcal{A}_s} \mathbf{p}^*(g). \quad (27)$$

■

Theorem 3 means that, if $(\mathbf{p}, \mathcal{A}_1, \ldots, \mathcal{A}_S)$ is a WE and $(\mathcal{A}_1^*, \ldots, \mathcal{A}_S^*)$ maximizes the surplus $\sum_{s \in \mathcal{S}} v_s(\mathcal{A}_s^*)$, then $(\mathbf{p}, \mathcal{A}_1^*, \ldots, \mathcal{A}_S^*)$ is also a WE. Both theorems 2 and 3 suggest that the WE exists if there is strong duality between $\mathscr{P}^{\text{LPR}}$ and $\mathscr{P}^{\text{DLPR}}$. In order to solve $\mathscr{P}^{\text{DLPR}}$, we first propose two ascending auction algorithms (SMRA) and two CAIB algorithms based on the Walras' *tatônnement (i.e., trial and error) procedure* [25].

IV. THE SMRA AND CAIB ALGORITHMS

First we propose the iterative SMRA and ASMRA algorithms. These two algorithms preserve privacy of the valuations of SCAs. We further propose two versions of the CAIB. Initially, we propose a simultaneous CAIB (SCAIB) wherein the auctioneer runs different CAIBs in a sequential manner. In this setting, a bidder is allowed to submit a bid on a particular item only once. Finally, we propose a repetitive CAIB (RCAIB) wherein bidders are allowed to correct their bids by rebidding on items as long as they believe they constitute their favourite item set.

A. THE SIMULTANEOUS MULTIPLE-ROUND ASCENDING AUCTION MECHANISM

With reference to Algorithm 1, we describe how the WE is determined using an SMRA. First let us define a *conditional bidding set*[4] $\mathcal{G}_s^i$ at every iteration[5] i as the set that contains all the **GUs** which bidder s has to bids on given that it has admitted the *provisional set* $\mathcal{A}_s$. The assumption is that bidder s automatically bid on each **GU** $g \in \mathcal{A}_s$. In this regard, a bidder is not allowed to withdraw its bid on any **GU** that it has been matched with. For an SCAs to relinquish a **GU**, one of its competitors has to outbid it on that **GU**. The MBS predefines the set on which each SCA can bid on by setting

[4] Conditional bidding set is the favourite set. We sometimes refer to it as the conditional bid.

[5] We reserve the words iteration and round to describe the state of an iterative auction and a sequential auction respectively.

the auction coverage area for each bidder to $\alpha\varsigma_s$, where α is a scaling factor and ς_s is the SCA's nominal coverage area. The prices of all the GUs are initialized as $\mathbf{q}(g) = 0, \forall g \in \mathcal{G}$. The initial contact set $\mathcal{T}$ contains all SCAs with at least one auctioned GU in their auction coverage area. The set of GUs that are on auction $\mathcal{G}$ is initialized as all MUs that fall within the auction coverage areas of all the SCAs. For all SCAs, the provisional set $\mathcal{A}_s$, the conditional set $\mathcal{G}_s^i$, the loss set $\mathcal{L}_s$ are initialized as empty sets.

The Algorithm 1 iterates as follows: The MBS invites all bidders in the contact set $\mathcal{T}$ to indicate their conditional bidding sets. Each SCA submits its conditional bidding set $\mathcal{G}_s^i \subseteq G_s \setminus \mathcal{A}_s$ to the MBS, with the assumption that the price of each GU $g \in G_s \setminus \mathcal{A}_s$ has price $\mathbf{q}^i(g) = \mathbf{q}^{i-1}(g) + \delta$. The prices for all GU $g \in \mathcal{A}_s$ are assumed to be unchanged, i.e., $\mathbf{q}^i(g) = \mathbf{q}^{i-1}(g)$. In step 7 to step 16, the MBS updates the provisional sets by arbitrarily allocating a GU to any bidder that is interested in it. In the case where there is a tie, the winner is picked randomly. The MBS then updates the prices of all the GUs that are over demanded and updates the contact set.

Now suppose the current competitors set for GU g is empty. This implies that GU g does not appear in any of the conditional sets, $g \notin \cup_{s=1}^{S}\mathcal{G}_s^i$. The price for GU g is set to $\mathbf{q}^{i+1}(g) = \mathbf{q}^i(g)$ and the *provisional winner* remains unchanged, i.e., if $g \in \mathcal{A}_s$ during iteration i then $g \in \mathcal{A}_s$ in iteration $i+1$. If $|\mathcal{C}_g| = 1$, supply equals demand, then GU g is matched with the SCA $s \in \mathcal{C}_g$. Otherwise if $|\mathcal{C}_g| > 1$, then GU g is over demanded. Under this condition the GU g changes hands by arbitrarily being assigned to a bidder in the set $\mathcal{C}_g$ with the exception of its previous provisional winner. The same process is repeated until the contact set becomes empty or when the conditional bidding sets of all bidders become empty.

B. ALTERED SMRA ALGORITHM

Note that in Algorithm 1, for the SCAs to maximize their utilities, they are forced to exhaustively check for every possible bundle in $\mathcal{G}_s$ in every iteration. This is computationally costly for both the bidders and the auctioneer. In an attempt to reduce the overheads incurred by the SCAs during valuation, the MBS uses the activity rule as follows: once a bidder places a bid on a GU g, it must commit to bid on that GU g in every iteration. Otherwise if an SCA fails to submit a bid on GU g, it is erased from the competitors set $\mathcal{C}_g$ and it cannot re-enter later. Therefore a bidder is forced to commit bidding to its current favourite set until it loses at least one of the GUs. The SCAs are only allowed access to the prices of the GUs they are currently bidding on. Once an SCA registers a loss, the MBS reveals all the prices of the GUs in its remainder set to that particular SCA. The favourite set can now be augmented with new favourite GUs in the remainder set $\mathcal{R}_s := \mathcal{P}_s \setminus \hat{\mathcal{G}}_s$. For as long as the bidder does not exceed its budget, it is allowed to bid on the remainder set $\mathcal{R}_s$ whenever a loss occurs. The ASMRA is summarised in Algorithm 2.

Algorithm 3 SCAIB Algorithm

Data: Initialization: $\mathbf{q}(g) = 0, \forall g \in G, (S, G) \in \mathbb{Z}_+$, **GUs**' set $\mathcal{G} := \cup_s^S \mathcal{G}_s$, assignment set $\mathcal{A}_s = \emptyset, \forall s \in \mathcal{S}$, lost items set $\mathcal{L}_s = \emptyset, \forall s \in \mathcal{S}$, $\mathcal{G}_s^i = \emptyset, \forall s \in \mathcal{S}$, *auction number*: $r = 0$.

Result: Optimal Allocation set $\mathcal{A}_s^*, \forall s \in \mathcal{S}$.

while $\mathcal{T} \neq \emptyset || \cup_{s=1}^{S} \mathcal{G}_s^r \neq \emptyset$ **do**
- $r \leftarrow r + 1$
- MBS invites SCAs to submit bids, $\forall s \in \mathcal{T}$.
- **if** $\mathcal{L}_s \neq \emptyset || r = 1$ **then**
  - Bidders determines new preference profiles and valuations for their favourite subset $\mathcal{G}_s^r$ (using (6)) of items in the remainder set $\mathcal{R}_s$ given the **GUs** they have admitted.
- MBS collects bids from SCAs $\forall s \in \mathcal{T}$.
- Auctioneer set $\mathcal{T} = \emptyset, \mathcal{L}_s = \emptyset$
- **for** $g \in \cup_{s=1}^{S} \mathcal{G}_s^r$ **do**
  - **if** $|\mathcal{C}_g^r| > 1$ **then**
    - pick the current bidder s with the highest bid: $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g$
    - $\forall k \neq s, \mathcal{A}_k \leftarrow \mathcal{A}_k \setminus g$, contact only bidders who have submitted a bid $\mathcal{T} \leftarrow \mathcal{T} \cup \forall k \neq s$
    - $\mathcal{L}_k \leftarrow g, \forall k \in \mathcal{C}_g^r \setminus s$
  - **else**
    - $\mathcal{A}_s \leftarrow \mathcal{A}_s \cup g, \mathcal{G} \leftarrow \mathcal{G} \setminus g$

C. THE CAIB ALGORITHMS

We propose two different CAIB under the second price mechanism. Even though the second price mechanism has dominant strategies under single-item auction, it is unlikely to expect the same property to hold under CA [26]. Bhawalkar and Roughgarden [26] analyzed price of anarchy (PoA) in a non-truthful combinatorial auction when bidders have subadditive (i.e., submodular) valuations. In our proposed algorithms, we assume that the bidders are truthful. Firstly, we propose a sequential CAIB (SCAIB) where the auctioneer runs separate CAIB in a sequential manner. Secondly, we focus on a repeated CAIB (RCAIB) wherein the MBS runs a CAIB repetitively. Note that in SCAIB, once an SCA s has acquired GU g, it cannot be auctioned again unless it is a case such that $g \in \mathcal{R}_t, t \in \mathcal{C}_g \setminus s$. In contrast, in the RCAIB environment, the MBS publishes the winning bid to all potential bidders. Therefore, if any of the potential bidders is able to outbid the provisional winner, the GU is reassigned to the new provisional winner. This process is repeated until no new conditional bids are received from the SCAs.

D. SEQUENTIAL COMBINATORIAL AUCTION WITH ITEM BIDDING

Similar initialisations as in Algorithm 1 are carried forward in Algorithm 3. Unlike in the SMRA, here the MBS does

Algorithm 4 RCAIB Algorithm

Data: See Algorithm 1.
Result: See Algorithm 1.
1 **while** $\mathcal{T} \neq \emptyset || \cup_{s=1}^{S} \mathcal{G}_s^i \neq \emptyset$ **do**
2 $\quad i \leftarrow i + 1$
3 $\quad$ MBS invites SCAs to submit bids, $\forall s \in \mathcal{T}$.
4 $\quad$ Algorithm 2 step 3 - 9.
5 $\quad$ Algorithm 3 steps 6 - 7
6 $\quad$ **for** $g \in \cup_{s=1}^{S} \mathcal{G}_s^i$ **do**
7 $\quad\quad$ Algorithm 3 steps 9 - 12
8 $\quad\quad$ $\mathbf{q}^{i+1}(g) \leftarrow \text{argmax} \sum_{s \in \mathcal{C}_g^i} b_{sg}^r x_{sg}^r$

not post prices. Instead the MBS runs several single shot CAIB sequentially. In every CAIB, an SCA in the contact set submits bids on its conditional bidding set. An SCA can only bid on a GU at most once. In step 9, if a GU g has a competitors set such that $\mathcal{C}_g \neq \emptyset$, and it appears in at least one conditional bidding set $g \in \mathcal{G}_s, \forall s \in \mathcal{C}_g$, then it is provisionally assigned to the highest bidder at price equal the second highest bid. The provisional bidder will remain assigned to this GU if no competitor outbids it in the successive auction rounds. After losing some of its favourite GUs in the previous round, an SCA may advance some of GUs from its reminder set to form an entirely new conditional bidding set. Due to the budget constraint, it is clear that the conditional bidding sets will diminish as the number of CAIB are being run. Once no new conditional bids are submitted, the SCAIB halts.

E. REPETITIVE COMBINATORIAL AUCTION WITH ITEM BIDDING

The RCAIB is summarised in Algorithm 4. The prices of all the GUs is initialized at zero. In the very first iteration, the MBS collects bids on bidders favourite sets. The MBS allocates a GU to the current highest bidder. In the successive iterations, the MBS uses the current *standing highest bid* on a GU as the reserve price for that item. If an SCA loses some of its favourite GUs, the marginal values of the accumulated GUs in its provisional set increases due to reduced interference. This creates capacity for a new conditional bidding set. In step 4, the SCAs determine their new bidding sets which may contain the GUs that were previously lost. Therefore it is possible for an SCA to recoup a GU after it was lost to a competitor. This process is repeated until no new conditional set is available.

V. BIDDING STRATEGIES

In order for an auction to accurately discover the market prices, truthful bidding should at least be guaranteed. In all auctions unfaithfulness can manifest if any SCA has knowledge about the preference sets of its competitors. However, as it is difficult for any SCA to acquire preference profiles of other SCAs, we exclude this possibility in our mechanism design and analysis.

Unfortunately, the SMRA has two setbacks that may encourage bidders to deviate from their truthfully bidding strategies. In [27] it is claimed that if the valuation function of bidders satisfy gross-substitute condition, then truthful bidding becomes compatible with SMRA for any price trajectory. But due to the *demand reduction* and *snipping* issues in the ascending auctions, truthful bidding is unlikely to occur. Demand reduction is when a bidder requests for fewer items in order to lower competition, hence maximizing its utilities. Snipping is when a bidder observes the activities in the auction, without participation and then later makes an offer.

In the SMRA, the bidder uses the *local improvement method* to add, delete or replace items. This strategy has proven to find the optimal demand set when the valuation functions are gross-substitutes [28]. In SMRA an SCA's strategy can be influenced by what it can infer from the auction history. In iterative auctions like the SMRA, the action sets of bidders can be history-dependent and as a result, these sets get quite rich. The information learned by a bidder from its sequence of provisional sets of items, its sequence of conditional bids, and the price trajectory will have a great influence on its bidding strategy function. The strategy of a bidder is defined as the function that maps the bidder's valuation to any of other bidders' possible actions.

Definition 7: Let $\mathcal{V}_1, \ldots, \mathcal{V}_S$ be the possible private valuations of the bidders. A strategy profile $\mathbf{s} = [s_1 \cdots s_S]$ is an ex-post Nash equilibrium (EPNE) if, for every SCA s and valuation $v_s \in \mathcal{V}_s$, the action $s(v_s)$ is the best-response to every action profile $\mathbf{s}_{-s}(\mathbf{v}_{-s})$ where $\mathbf{v}_{-s} \in \mathcal{V}_{-s}$ [23].

Now, we intend to analyze if sincere bidding as a strategy profile can lead to an equilibrium. We assume that all bidders are *ex-post individual-rational*. That is, bidders play risk-free strategies in order to avoid getting negative utilities. Christodoulou *et al.* [29] argued that if bidders are ex-post individually-rational, and have submodular valuation functions, then every mixed Bayesian Nash equilibrium of a Bayesian auction (i.e., auctions in which valuations are private) provides a 2-approximation to the optimal social welfare.

A. INDIVIDUALLY RATIONAL BIDDING

With the assumption of ex-post individual rationality, an SCA will never take action that will result in negative utility. Let us define a bidding strategy to be *supportive* if it is individual rational.

Definition 8: Given the s-th SCA with provisional set $\mathcal{A}_s^i$ at iteration i, a conditional bid $\mathcal{G}_s^i$ is *secure* if for any given $\mathcal{A}_s' \subseteq \mathcal{A}_s^i \cup \mathcal{G}_s^i$, it is a case such that $v_s(\mathcal{A}_s') \geq q(\mathcal{A}_s')$.

Proposition 2: Denote $\mathcal{A}_s^i$, $\mathcal{G}_s^i$, and $\mathbf{q}^i$ as the provisional set, the conditional bidding set, and the price profile at iteration $i \in \mathbb{Z}_+$ with $i \leq \hat{i}$. If an SCA makes a non-secure bid during iteration $\hat{i}$, then there exist secure SCAs which can bid consistently with the history such that SCA s gets a negative utility in the final allocation.

Proof: Let us assume all other SCAs other than SCA s bid sincerely. Suppose that the s-th SCA bids inconsistently on a particular **GU** g and finally acquires it.[6] Let us define the maximum possible marginal value of **GU** $g \in \mathcal{G}_s \setminus \mathcal{A}_s^i$ as $\hat{v}_{sg} = v_s(\emptyset \cup g)$. Suppose **GU** g has the highest preference amongst all other **GUs** in the set $\mathcal{G}_s^{\hat{i}}$, then the marginal value of **GU** g during iteration $\hat{i}$ is $v_{sg}^{\hat{i}} = v_s(\mathcal{A}_s^i \cup g) - v_s(g)$. Then the **GU** g will contribute maximum utility of $u_{sg} = v_{sg}^{\hat{i}} - \mathbf{q}(g)$ to the total utility $u_s = v_s(\mathcal{A}_s) - \sum_{g \in \mathcal{G}_s} \mathbf{q}(g)$ earned by SCA s. If the bidder s bids inconsistently during iteration $\hat{i}$ with $\hat{v}_{sg} < \mathbf{q}(g)$, then there exists at least one SCA $t \in \mathcal{C}_g \setminus s$ who values **GU** g more. One of the outcomes in (28), as shown at the bottom of this page, are feasible at the end of the auction; where u_s^- means the utility is negative, u_s^+ means the utility is positive and u_s^{c} is the utility attained by bidding consistently and securely. The first case in (28a) follows immediately from the fact that $\hat{v}_{sg} < \mathbf{q}(g)$. The second case in (28b) suggests that if the absolute utility from winning **GU** g is greater than the absolute utility of other admitted **GUs** $\mathcal{G}_s$, then bidder s will get a negative utility. The third case in (28c) shows that bidder s might achieve a positive utility but the achievable utility cannot exceed that under consistent bidding. The proof shows that bidding securely is the best response for SCA s. ■

Proposition 2 carries forward to the ASMRA. Note that in ASMRA, if an SCA underbids on a particular **GU** so that it has access to the prices of the **GUs** in the remainder set, it is not allowed to rebid on that particular **GU** at a later stage. It will be a risky move for the SCA to underbid on any of the **GUs** in the current conditional bidding set as the prices of the **GUs** in the remainder set could possibly be very high.

Proposition 3: Truthful bidding is individual rational in the SCAIB and the RCAIB.

Proof: The proof below is for the SCAIB but it can easily be extended to RCAIB. Consider an SCA s during auction round r with conditional bidding set $\mathcal{G}_s^r$. On one hand, the SCA can have a truthful bidding function v_s^{c} that arranges the **GUs** in the set $\mathcal{G}_s^r$ according to its preference order and computes the truthful marginal values. On the other hand, the marginal values can be computed using another function v_s^{u}. For example, the function v_s^{u} could map the **GUs** to the values different from those they will have when v_s^{c} is used, simply by changing their order of preference. Both valuation functions will have the following cases: SCA s

1) bids truthfully and consistently according to the valuation function,
2) underbids on at least one of the **GUs**,
3) overbids on at least one of the **GUs**,
4) underbids and overbids on two different sets of **GUs**.

We now assume a set of competitors which have the conditional bidding set $\mathcal{G}_s^r$ exists as a subset of their conditional bidding sets. Let us fix a set $\mathcal{Y}_s^r \subseteq \mathcal{G}_s^r$ which contains the **GUs** that an SCA s underbids/overbids on.

Case 1: If an SCA bids truthfully, and consistently using the valuation function v_s^{c}, it gets a utility of $u_s^{\mathsf{c}}(\mathbf{b}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\mathbf{b}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\mathbf{b}_s), \mathbf{b}_{-s})$.

Case 2: If an SCA underbids with $\underline{\mathbf{b}}_s$, then there is a possibility that another SCA will outbid it on some of the **GUs** in $\mathcal{Y}_s^r$. Since the allocation is monotonic in $\mathbf{b}$, we get $\mathcal{A}_s(\underline{\mathbf{b}}_s) \subseteq \mathcal{A}_s(\mathbf{b}_s)$. Note that if an SCA loses some of the **GUs** $\mathcal{Y}_s^{r\mathsf{L}} \subset \mathcal{Y}_s^r$, then the marginal values of the remaining **GUs** in $\mathcal{G}_s^r \setminus \mathcal{A}_s(\underline{\mathbf{b}}_s)$ will increase. We denote the total increase of the marginal values by ϵ_s^r, and the utility contributed by the set $\mathcal{Y}_s^{r\mathsf{L}}$ as $u_s(\mathcal{Y}_s^{r\mathsf{L}})$. If $\epsilon_s^r > u_s(\mathcal{Y}_s^{r\mathsf{L}})$, the auction may suffer from *demand reduction*. Demand reduction is more prominent in iterative and sequential auctions wherein bidders use history reliant strategies for their next move. Fortunately, the SCAIB use one shot auctions in a sequential manner, therefore demand reduction could be very risky. Once more, if the SCAs use demand reduction under SCAIB to maximize their profit during auction r, they will exhibit a reduction on their budget, which was learned by the MBS in the first auction round, thereby reducing the number **GUs** they can bid on in the subsequent auction rounds. The utility attained for underbidding is $u_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\underline{\mathbf{b}}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\underline{\mathbf{b}}_s), \mathbf{b}_{-s})$.

Case 3: If an SCA overbids with $\bar{\mathbf{b}}_s$, then there is a possibility that it outbids its competitors on some of the **GUs** in $\mathcal{Y}_s^r$, and attains a set $\mathcal{Y}_s^{r\mathsf{W}} \subseteq \mathcal{Y}_s^r$ in the allocation set $\mathcal{A}_s(\bar{\mathbf{b}}_s)$ such that $\mathcal{A}_s(\underline{\mathbf{b}}_s) \subseteq \mathcal{A}_s(\mathbf{b}_s) \subseteq \mathcal{A}_s(\bar{\mathbf{b}}_s)$. The utility contributed by the set $\mathcal{Y}_s^{r\mathsf{W}}$ as $u_s(\mathcal{Y}_s^{r\mathsf{W}})$. By overbidding, the utility of SCA s is $u_s^{\mathsf{c}}(\bar{\mathbf{b}}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\bar{\mathbf{b}}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\bar{\mathbf{b}}_s), \mathbf{b}_{-s})$.

Case 4: Suppose an SCA submits a bid profile $\bar{\underline{\mathbf{b}}}_s$ with underbids and overbids. First let us denote the sets which an SCA s underbids and overbids on as $\underline{\mathcal{Y}}_s^r \subset \mathcal{Y}_s^r$ and $\bar{\mathcal{Y}}_s^r \subset \mathcal{Y}_s^r$, respectively. Further we denote the sets of **GUs** that an SCA s loses and wins for underbidding and overbidding as $\underline{\mathcal{Y}}_s^{r\mathsf{L}}$ and $\bar{\mathcal{Y}}_s^{r\mathsf{W}}$, respectively in the allocation set $\mathcal{A}_s(\bar{\underline{\mathbf{b}}}_s)$. The resulting utility is given by $u_s^{\mathsf{c}}(\bar{\underline{\mathbf{b}}}_s) = v_s^{\mathsf{c}}(\mathcal{A}_s(\bar{\underline{\mathbf{b}}}_s)) - B_{-s}^{\text{high}}(\mathcal{A}_s(\bar{\underline{\mathbf{b}}}_s), \mathbf{b}_{-s})$.

At the end of the auction round, the pay-off received by SCA s is given in (29), as shown at the bottom of next page.

[6] This proof can easily be extend to the case where the SCA bids inconsistently on a set of **GUs**. We chose one **GU** for simplicity.

$$u \begin{cases} {}_s^- = u_{sg}, & \text{if } \mathcal{A}_s^* := g, \quad (28a) \\ u_s^-, & \text{if } \mathcal{A}_s^* := \mathcal{G}_s^{\hat{i}} \cup g,\ |u_{sg}| > |u_{s\mathcal{G}_s^{\hat{i}}}|, \quad (28b) \\ u_s^+ \le u_s^{\mathbf{c}}, & \text{if } \mathcal{A}_s^* := \mathcal{G}_s^{\hat{i}} \cup g,\ |u_{sg}| \le |u_{s\mathcal{G}_s^{\hat{i}}}|, \quad (28c) \end{cases}$$

Underbidding can lead to *demand reduction* as shown in (29a). We note that an SCA cannot improve its utility by overbidding during a particular auction round. In (29e) and (29h), if $|u_s(\mathcal{Y}_s^{\mathsf{rW}})| > |u_s(\mathcal{A}_s(\bar{\mathbf{b}}_s)|$ or $|u_s(\bar{\mathcal{Y}}_s^{\mathsf{rW}})| > |u_s(\mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s)|$, then the utility of an SCA will be negative. Therefore the latter strategies are not *supportive*. Failure to maximize the utility during a particular auction round by unfaithful bidding will result in more utility loss in the forthcoming rounds because of the increased prices on the GUs and the decreasing valuations. When an SCA uses the bidding function v_s^{u}, then an SCA will have a combination of overbidding and underbidding. Following the same arguments stated above, there will be no improvement on the utility by unfaithful bidding. ∎

Since over-bidding is not individual rational, we assume *strong no-overbidding* [29], [30].

VI. NUMERICAL EXAMPLE

We consider a macrocell consisting of one MBS and 100 single antenna MUs that are uniformly distributed within the cell. The cell is assumed to have a nominal coverage of radius 500 m. The MBS is equipped with $M_{\mathrm{MBS}} = 50$ antennae. There are 25 privately owned SCAs within the coverage of the macrocell operating in a hybrid mode as shown in Figure 2. Each SCA serves one HU with a fixed QoS target of 2 bits/s/Hz. Each SCA is equipped with $M_{\mathrm{SCA}} = 8$ antennae and assumed to have a nominal coverage radius of 30 m. The transmission powers at the MBS and each SCA are $p_0^{\max} = 46$ dBm and $p_s^{\max} = 30$ dBm, respectively. The cost per unit power, and the cost per unit of data were set to $\mu = 0.0001$, and $\kappa = 0.1$, respectively. In order to reduce computational complexity and system overheads, the price increment in the SMRA and ASMRA were adapted using $\delta = 0.001 \times$ (target data rate)/(0.5 bits/s/Hz). The SCAs are allowed to bid for users within twice of their nominal coverage radius. This wider admissible coverage area ensures presence of considerable number of SCAs for competition. The MBS knows the locations of the SCAs and the MUs. The channels were modelled as of heterogeneous deployment in 3GPP LTE standard [32]. The model parameters are summarized in Table 2.

TABLE 2. Numerical parameters for numerical evaluation.

Description/Parameter	Value
Macrocell radius	500 m.
Smallcell radius	30 m.
MBS downlink transmit power $p_0^{\max}$	46 dBm.
SCA downlink transmit power $p_s^{\max}$	20 dBm.
MBS path and penetration loss at d (km)	$128.1 + 37.6\log_{10}(d)$ dB.
SCA path and penetration loss at d (km)	$127 + 30\log_{10}(d)$ dB.
Lognormal shadowing standard deviation	7 dB.
MBS-s minim distance constraint	35 m.
SCA-s minim distance constraint	3 m.
Noise variance σ^2	-127 dBm.
Wall attenuation	20 dB.
Number of MUs	100.
Number of HUs per SCA	1.
Number of MBS antennas M_{mbs}	50.
Number of SCA antennas M_{sca}	8.
Small scale fading distribution	$\mathbf{h}_{jk} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{jk})$.
Physical channel model	[31, Eq.(34)]

Figure 2 depicts the topographical overview of the network after running the SMRA, ASMRA, SCAIB and RCAIB algorithms. The target QoS of all the MUs was set to 8 bits/s/Hz. The green squares and red dots indicate the locations of the admitted MUs and the dropped MUs, respectively. The locations of HUs are indicated with the blue dots. The users served by the SCAs are indicated by connecting them using blue lines. All admitted users without connecting lines are served by the MBS. Figures 2(a) and 2(b) show the results of the SMRA and the ASMRA, respectively. Since ASMRA is a sub-optimal version of SMRA, we observe that the ASMRA occassionally fails to associate users to the closest SCAs. This is observed between SCA-1 and SCA-3, between SCA-22 and SCA-23, and between SCA-21 and SCA-24. The reason is the following. The SCA is confined to bid on a particular set of GUs until it experiences a loss. Once a loss

$$
u \begin{cases}
{}_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) > u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\mathbf{b}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \quad \epsilon_s^r > u_s(\mathcal{Y}_s^{\mathsf{rL}}), \quad (29a)\\
u_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) < u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\mathbf{b}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \quad \epsilon_s^r < u_s(\mathcal{Y}_s^{\mathsf{rL}}), \quad (29b)\\
u_s^{\mathsf{c}}(\underline{\mathbf{b}}_s) = u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\mathbf{b}}_s) := \mathcal{A}_s(\mathbf{b}_s), \quad \text{or if } \mathcal{A}_s(\underline{\mathbf{b}}_s) \subseteq \mathcal{A}_s(\mathbf{b}_s), \quad \epsilon_s^r = u_s(\mathcal{Y}_s^{\mathsf{rL}}), \quad (29c)\\
u_s^{\mathsf{c}}(\bar{\mathbf{b}}_s) = u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\bar{\mathbf{b}}_s) := \mathcal{A}_s(\mathbf{b}_s), \quad (29d)\\
u_s^{\mathsf{c}}(\bar{\mathbf{b}}_s) < u_s^{\mathsf{c}}(\mathbf{b}_s), & \quad \text{if } \mathcal{A}_s(\mathbf{b}_s) \subset \mathcal{A}_s(\bar{\mathbf{b}}_s), \quad u_s(\mathcal{Y}_s^{\mathsf{rW}}) < 0, \quad (29e)\\
u_s^{\mathsf{c}}(\underline{\bar{\mathbf{b}}}_s) < u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \quad \epsilon_s^r < u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \quad \bar{\mathcal{Y}}_s^{\mathsf{rW}} := \emptyset, \quad (29f)\\
 & \text{or if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r \le u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \quad u_s(\bar{\mathcal{Y}}_s^{\mathsf{rW}}) < 0, \quad (29g)\\
 & \text{or if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \quad u_s(\bar{\mathcal{Y}}_s^{\mathsf{rW}}) < 0, \underline{\mathcal{Y}}_s^{\mathsf{rL}}; = \emptyset, \quad (29h)\\
u_s^{\mathsf{c}}(\underline{\bar{\mathbf{b}}}_s) = u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) := \mathcal{A}_s(\mathbf{b}_s), \quad \text{or if } \epsilon_s^r = u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \quad \bar{\mathcal{Y}}_s^{\mathsf{rW}} := \emptyset, \quad (29i)\\
u_s^{\mathsf{c}}(\underline{\bar{\mathbf{b}}}_s) > u_s^{\mathsf{c}}(\mathbf{b}_s), & \text{if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r > u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \quad \bar{\mathcal{Y}}_s^{\mathsf{rW}} := \emptyset, \quad (29j)\\
 & \text{or if } \mathcal{A}_s(\underline{\bar{\mathbf{b}}}_s) \subset \mathcal{A}_s(\mathbf{b}_s), \epsilon_s^r > u_s(\underline{\mathcal{Y}}_s^{\mathsf{rL}}), \epsilon_s^r > |u_s(\bar{\mathcal{Y}}_s^{\mathsf{rW}})|. \quad (29k)
\end{cases}
$$

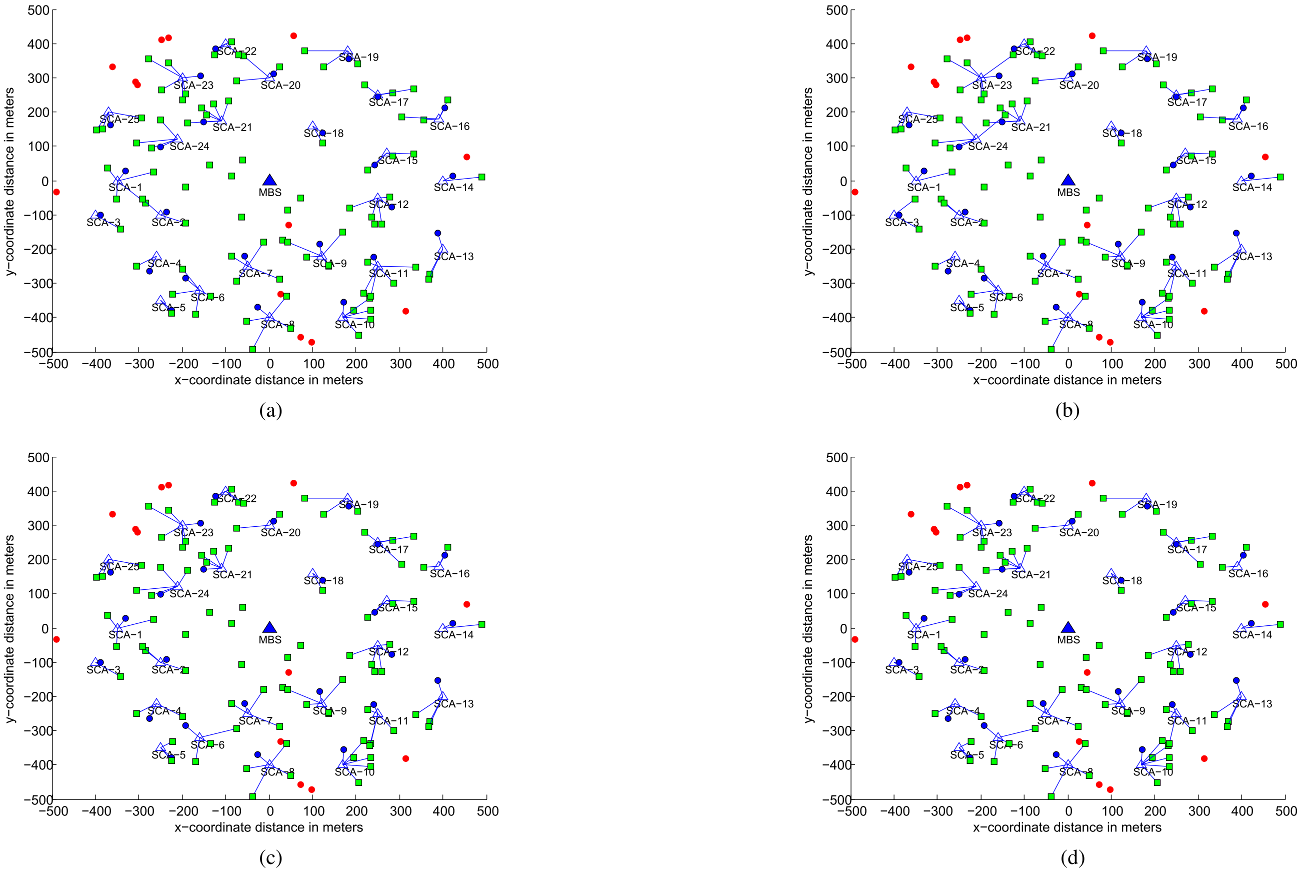

FIGURE 2. **Comparison of user association under different auctions. (a) User allocation under SMRA. (b) User allocation under ASMRA. (c) User allocation under SCAIB. (d) User allocation under RCAIB.**

is experienced, when the SCA attempts to bid for the GUs in the remaining set, the GUs that were cheaper and closer could have been sold to other SCAs.

The admitted users for the SCAIB and the RCAIB are shown in Figures 2(c) and 2(d). Even though these two alogrithms provide a very similar user association pattern, a difference is observed between SCA-21 and SCA-24. For this particular channel realization, all four algorithms admit the same users, but with a different association. These close performances are due to the sub-modularity and gross-substitute characteristics of the valuation functions.

A. GENERAL PERFORMANCE OF THE PROPOSED ALGORITHMS

The performances of the SMRA, ASMRA, SCAIB and RCAIB over 20 random channel realizations are provided in Figure 3. Figure 3(a) shows the average of the total admitted MUs/GUs jointly by the SCAs and MBS. The dotted line shows the average admitted MUs in the absence of auctioning, i.e., served only by the MBS. The solid lines depicts the performance of the four proposed algorithms. We observe that there is a huge improvement on user admission when SCAs are taking part in the auction. Even though the performances of the SMRA, the ASMRA, the SCAIB and the RCAIB are almost identical, we observe that the SCAIB provides a better user admission performance at lower target rates, while at higher target data rates, both versions of CAIB algorithms outperform SMRA and ASMRA algorithms. This is because for the CAIB algorithms, the MBS and the SCA are able to learn the market price of a particular GU in advance, hence the SCAs have the opportunity to explore other cheaper GUs quickly. The average performance of SMRA and ASMRA are identical. Figure 3(b) depicts the total number of dropped MUs. As expected and for the same reasons stated earlier, the CAIB algorithm maintains a lower dropped number of users as compared to SMRA and ASMRA.

In Figure 3(c), we illustrate the revenue generated by the MBS from the payments made by the MUs and the SCAs. Though very minimal, the differences between the revenues earned from MUs under the proposed algorithms suggest that the sets of MUs left behind after auctioning are different from one auction to the other. We notice that the SCAs make the lowest payments to the MBS under SMRA and highest payments under ASMRA. This is because the rule that demands the SCAs to commit to a bidding set until there is a loss, increases the competition and ultimately increases payments for the bidder under ASMRA. By conducting an auction, the MBS generates more revenue as compared to if it aims to serve MUs on its own.

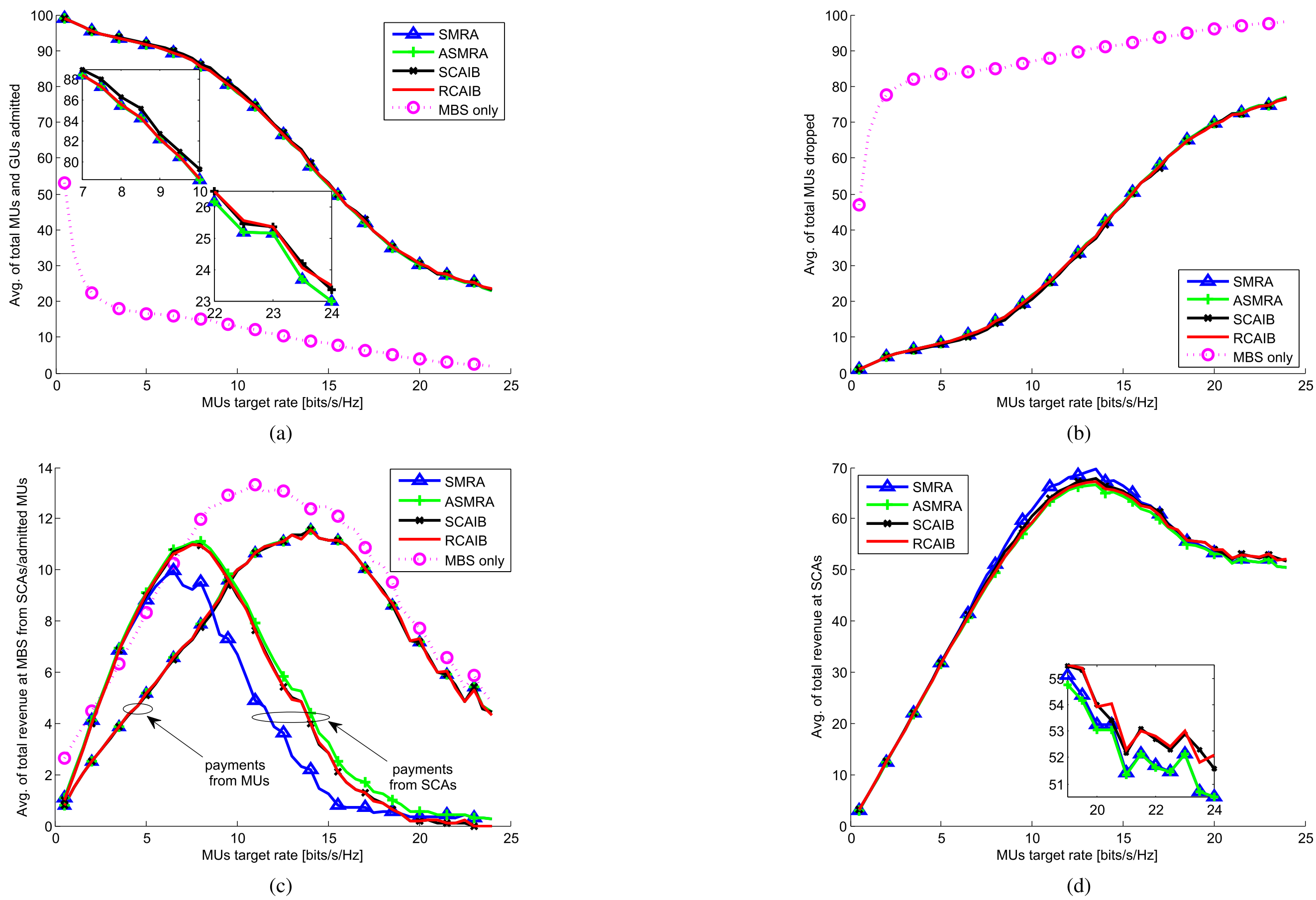

FIGURE 3. Average performance of the proposed BBWA and FBWA for 20 random channel realizations. There are 100 MUs and 25 SCAs. (a) Average of total MUs/GUs admitted by the SCAs and the MBS. (b) Average number of dropped MUs. (c) Average revenue generated by the MBS. (d) Average revenue generated by the SCAs.

The revenues generated by the SCAs using the SMRA, ASMRA, SCAIB and RCAIB algorithms are illustrated in Figure 3(d). At lower target data rates, the revenues earned by the SCAs under all the proposed algorithms are almost identical. The SCAs are able to generate the highest revenue when the data rate is in the range of 6 to 16 bits/s/Hz using SMRA. This is because of the absence of local improvement method in SCAIB. The effect of the local improvement method is weaker in RCAIB and much weaker in ASMRA. Hence the SCAs are deprived from maximizing their profit especially at moderate target rates. Nonetheless, the revenue generated under RCAIB and SCAIB is higher than that of SMRA and ASMRA at lower and higher target data rates when the competition is higher or lower, respectively. This is because of the swift price discovery in the CAIB algorithms, which allows SCAs to quickly discover GUs that have lower competition and lower prices. Ultimately the payments to the MBS are reduced. It is observed that the ASMRA always generates lowest revenue. This is because the SCAs are not allowed to explore other opportunities until they experience a loss on the set of GUs that they bid on.

Figures 4(a) and 4(b) show the average transmission powers at the MBS and SCAs. In Figure 4(a), we observe that both the SMRA and ASMRA consume identical transmission power. This is because both these algorithms perform equally in terms of the admitted and dropped GUs. The transmission power of the MBS under SMRA and ASMRA are comparable to that of the RCAIB at lower target data rates. At higher target data rates, the transmission power of SMRA and ASMRA are comparable to that of the SCAIB. When there is no auctioning, the MBS will have to use more transmission power with respect to power usage per user. In Figure 4(b), we note that the RCAIB required the least transmission power while SMRA required the most transmission power. This suggests that under the RCAIB, the SCAs chose those GUs that are closer to them while under the SMRA, the SCAs chose those GUs that are far from them. This reveals the effect of the local improvement method in the SMRA which allows the SCAs to identify cheaper GUs. Usually, GUs that are further away will be having lower bids (or prices). The performance of the ASMRA and the SCAIB in terms of the transmission power lies between that of the SMRA and the RCAIB.

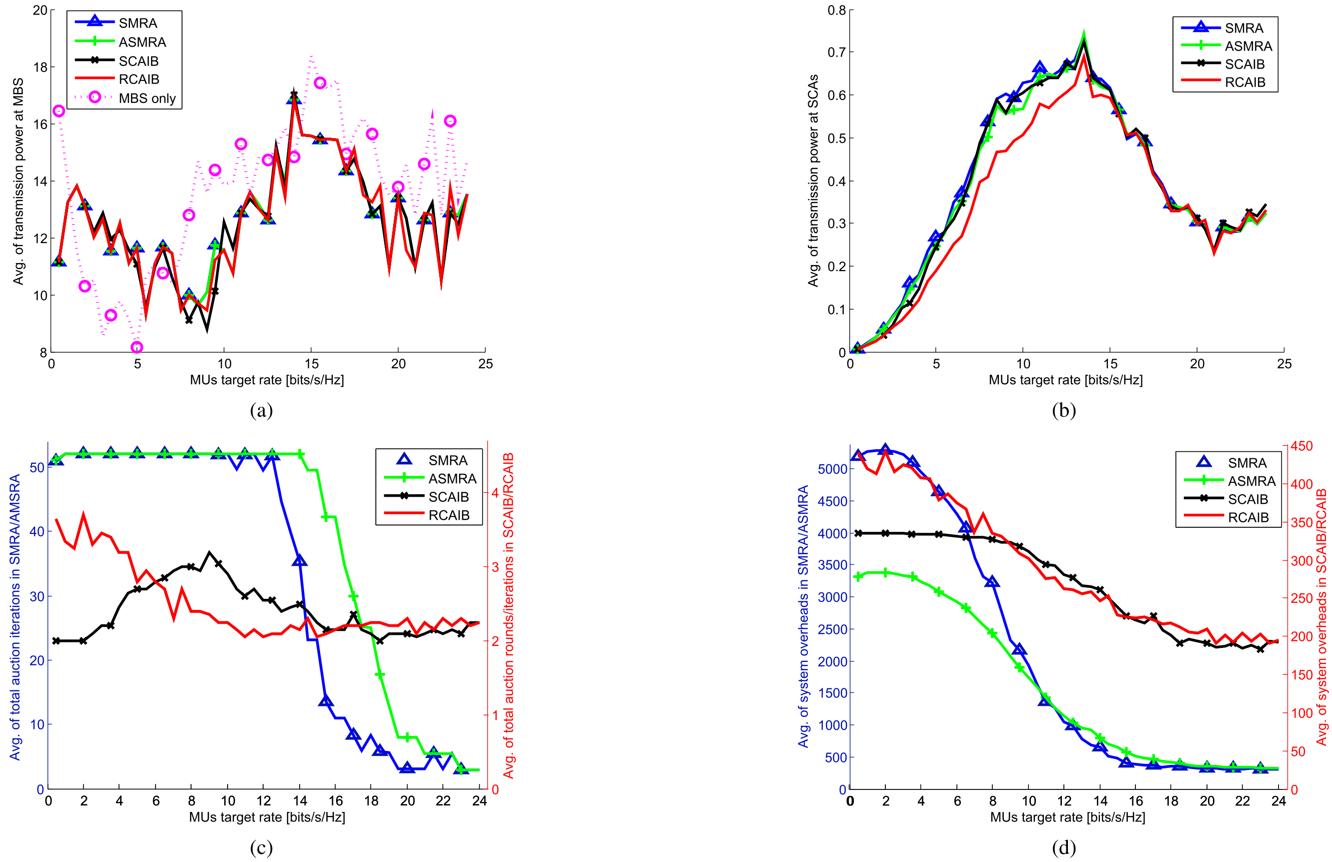

FIGURE 4. Comparison of SMRA, ASMRA, SCAIB, and RCAIB in terms of number of auction iterations/rounds and system overheads. (a) Average of total tranmission power at the MBS. (b) Average of total tranmission power at the SCAs. (c) Average auction rounds. (d) Average system overheads incurred.

Figure 4(c) and 4(d) show the number of auction rounds/iterations and the system overheads under each auction. The overheads are measured in terms of the number of invitations for bidding, number of bids submitted and the number of announcements made. The performance of the SMRA and ASMRA is indicated using the left y-axes while that of the the SCAIB and CAIB is indicated using the right y-axes. In Figure 4(c), we observe that the number of iterations/rounds reduces as the target data rate is increased. This is because at high target data rates, the GUs (mainly those far from the SCAs) become less attractive, which induce decoupled preference sets. Ultimately, the SCAs will be dropped out of the auction quickly, thereby increasing the convergence rate. Note that for the SMRA and ASMRA, a small increase in δ will increase the number of iterations significantly. This will be even worse when the values of the GUs are increased. As mentioned before, δ is adapted using $\delta = 0.001 \times$ (target data rate)/(0.5 bits/s/Hz). If the price increment is fixed to $\delta = 0.001$, the number of iterations required in the SMRA and the ASMRA can reach 10^4 for target rates between 5 bits/s/Hz and 10 bits/s/Hz. In Figure 4(c) the maximum number of iterations required is 52. The SCAIB and the RCAIB registered maximum of 3.5 auction rounds/iterations. This is due to the same reason as explained earlier for the CAIB algorithms, i.e., the rate of price discovery is very high and WE is quickly found in the auctions. The same behavior is observed in figure 4(d). The SMRA and ASMRA have large system overheads as compared to the SCAIB and the RCAIB. This is because for the SMRA and ASMRA, the price is increased by marginally small value at each iteration.

It is evident from the results that the auctioning mechanism is able to offload users form the macrocell to the SCAs, which results in enhancement of admitted users and revenue. The CAIB algorithms outperform the SMRA and the ASMRA in a wide range of perfomance metrics. Though the SMRA generates relatively more profit, it results in more system overheads and computational complexity. In terms of surplus maximization, the SCAIB is the most desirable. Since the SCAIB always generate the second highest revenues for the SCAs, the MBS will prefer the SCAIB.

B. OPTIMALITY OF THE PROPOSED ALGORITHMS

We compared the proposed algorithms to a centralized solution using an approach proposed in [33], which uses the branch-and-bound (BnB) method to solve user admission and association. In order to reduce the computational burden for the centralized system, we considered a network with 6 GUs and 2 SCAs only. The system parameters of the SCAs and the GUs remain unaltered. Figure 5 shows the user admission performance of the proposed auctions and the centralized solution. All our proposed auctioning methods perform very closer to the centralized (optimum) solution. especially at lower target rates.

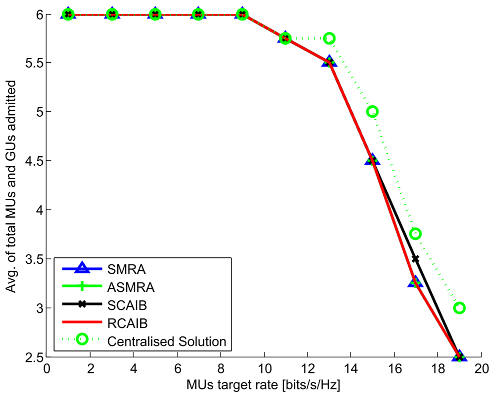

FIGURE 5. Average GUs admitted by the SCAs.

VII. CONCLUSION AND FUTURE DIRECTIONS

In this paper, we investigated a joint user offloading and downlink beamformer problem in heterogeneous networks. We formulated our problem as a combinatorial auction which readily provides a decentralized solution. The SCAs are able to design appropriate beamformers and admit users of the MBS using four different proposed auctioning mechanisms, namely SMRA, ASMRA, SCAIB, and RCAIB. Our analysis proved the existence of the Walrasian equilibrium for the proposed valuation functions. The SCAIB algorithm is the most preferred algorithm since it provides the highest admission rate and a competitive revenue for the SCAs. The proposed algorithms perform very closer to the centralised optimum solution. Considering all aspects including, user admission, power consumption and revenue, SCAIB should be preferred by both the MBS and SCAs.

Our future work will consider the case wherein the valuation function of a bidder is influenced by externalities. This may occur if the competing bidders are operating in the same frequency band. Last but not least, a problem wherein the items are compliments is also a future direction.

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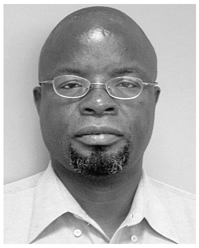
BOKAMOSO BASUTLI (M'11) received the Ph.D. degree in electronic, electrical and systems engineering from Loughborough University, U.K., in 2016. From 2008 to 2010, he was an Installation Engineer and a Lead Engineer at Singapore Technologies Electronics (Info-Software Systems). He was a Senior Telecommunications Engineer at the Civil Aviation Authority of Botswana from 2010 to 2012. He joined the Botswana International University of Science and Technology as a Founding Teaching Instructor in 2012, and he became a Lecturer with the Department of Electrical, Computer, and Telecommunications Engineering. His research interests are convex optimization, resource allocation, wireless communications, financial engineering, and game theory.



JOSEPH M. CHUMA (M'97) received the Ph.D. degree in electronic systems engineering from the University of Essex in 2001. He was with the University of Botswana as an Associate Professor and the Dean of the Faculty of Engineering and Technology. He joined as an Associate Professor in telecommunications engineering with the Department of Electrical, Computer and Telecommunication Engineering, Botswana International University of Science and Technology in 2014. He has over 24 years of experience in teaching and research, consultancy, and human resources development in telecommunication, computer, and electrical and electronics engineering in including CISCO computer networking. He has authored or co-authored three books, three book chapters, and many refereed published scholarly/scientific journal articles in the subject of telecommunications engineering. He holds three patents. He is a member of several professional bodies among which includes the IEEE, USA; IET, U.K.; and BIE, Botswana. He is a Professional Engineer registered with the Botswana Engineers Registration Board.

SANGARAPILLAI LAMBOTHARAN (SM'06) received the Ph.D. degree in signal processing from Imperial College London, London, U.K., in 1997. He was a Post-Doctoral Research Associate with Imperial College London until 1999. He was a Visiting Scientist with the Engineering and Theory Centre, Cornell University, USA, in 1996. From 1999 to 2002, he was with Motorola Applied Research Group, U.K. and investigated various projects, including physical link layer modeling and performance characterization of GPRS, EGPRS, and UTRAN. He was with Kings College London and Cardiff University as a Lecturer and a Senior Lecturer, respectively, from 2002 to 2007. He is currently a Professor of digital communications and the Head of the Signal Processing and Networks Research Group, Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, U.K. His current research interests include 5G networks, MIMO, radars, smart grids, machine learning, and network security. He has authored over 200 journal and conference articles in these areas.