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Implementing positivity constraints in 4-D resistivity time-lapse inversion

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Introduction

Over the last 25 years 2-D and 3-D resistivity surveys have been used for a wide range of engineering, environmental, hydrological and mineral exploration surveys (Loke et al. 2013). In some surveys, the purpose includes the monitoring of subsurface changes with time (Chambers et al. 2014). The 4-D smoothness-constrained inversion method (Loke et al. 2014) has proved to be a stable and robust method for the inversion of time-lapse data sets. This method inverts the data sets measured at different times simultaneously and it includes a temporal smoothness constraint to ensure that the resistivity changes in a smooth manner with time. In some surveys, such as infiltration experiments (Kuras et al. 2016), it is known that the subsurface resistivity should only decrease (or increase) with time. As the standard 4-D inversion method does not explicitly constrain the direction of the changes with time, this could result in artefacts where an increase in the resistivity is obtained in the inverse model while it is only expected to decrease (or vice versa). In this paper we describe a modification of the 4-D smoothness-constrained inversion method to remove such temporal artefacts.

Theory

The 4-D smoothness-constrained least-squares optimisation equation (Loke et al. 2014) is given by:

$$[\mathbf{J}_i^T \mathbf{R_d} \mathbf{J}_i + (\lambda_i \mathbf{W}^T \mathbf{R_m} \mathbf{W} + \alpha_i \mathbf{M}^T \mathbf{R_t} \mathbf{M})]\Delta \mathbf{r}_i = \mathbf{J}_i^T \mathbf{R_d} \, \mathbf{g}_i - (\lambda_i \mathbf{W}^T \mathbf{R_m} \mathbf{W} + \alpha_i \mathbf{M}^T \mathbf{R_t} \mathbf{M}) \, \mathbf{r}_{i-1} \qquad (1)$$

The Jacobian matrix $\mathbf{J}$ contains the partial derivatives of the (logarithm of the) apparent resistivity values with respect to the (logarithm of the) model resistivity values $\mathbf{r}$. λ is the damping factor vector. $\mathbf{g}$ is the data misfit vector. $\mathbf{W}$ is the roughness filter matrix. $\mathbf{R_d}$, $\mathbf{R_m}$ and $\mathbf{R_t}$ are weighting matrices used by the L1-norm inversion method (Farquharson and Oldenburg 1998). They are identical to the identity matrix if the L2-norm method is used. $\mathbf{M}$ is the difference matrix applied across the time models to minimize the change in the resistivity from one temporal model to the next. The model vector $\mathbf{r}$ has the following form if there are t time-series and each temporal model has m cells.

$$\mathbf{r}_i = (r_{11}..r_{1m} \, r_{21}..r_{2m} \ldots r_{t1}..r_{tm})^T = (\mathbf{r}_1 \, \mathbf{r}_2 \ldots \mathbf{r}_t)^T \qquad (2)$$

Equation (1) attempts to minimise the data misfit subject to the smoothness constraints. In some cases, particularly where there is a large resistivity variation across a boundary, it can produce a model where the resistivity increases with time where it is only expected to decrease (or vice versa). To ensure that the change of the resistivity with time only occurs in the expected direction, we make the following modifications to the inversion algorithm. To simplify the following discussion, we assume the resistivity is expected to decrease with time. We first carry out an inversion of the data using the standard 4-D inversion algorithm, as it generally provides a stable background model despite the possible temporal artefacts. We then compare the resistivity of each model cell in the first temporal model with the later temporal models. The mean value (p_j) of the jth model cell is calculated from the resistivity in the first temporal model (r_{1j}) and the maximum value achieved by the later time models (r_{kj}).

$$p_j = (r_{1j} + r_{kj})/2 \qquad (3)$$

If the jth model cell for the first temporal model is lower than lower than p_j, it is reset to this mean value. If the same cell for a later temporal model is higher than p_j, it is reset to the same mean value (corresponding to zero change with time). This is a rather crude procedure to reset the model values, and it usually causes an increase in the data misfit. Next a model refinement step is taken to reduce the data misfit subject to the constraint that the resistivity of any cell in the first temporal model is never lower than that of a later temporal model. This constraint is imposed by using the method of transformations. The cell resistivity (r_{1j}) for the first temporal model is replaced by a new variable, s_{1j}, using the following equation.

$$r_{1j} = p_j + s_{1j}^2 \qquad (4)$$

This transformation ensures that the cell resistivity is always higher than the mean value p_j. The following transformation is used for the cell resistivity (r_{ij}) for the later temporal models to ensure that it is always lower than the mean value p_j.

$$r_{ij} = p_j - s_{ij}^2 \qquad (5)$$

We then carry out two or three more iterations using equation (1) with the transformed variables. This usually reduces the data misfit to a similar level achieved without these constraints.

Results

We use the data from an infiltration experiment at the Hollin Hill (U.K.) landslide research site in North Yorkshire, UK. The landslide is situated on a south facing slope and is approximately 200 m long and 250 to 300 m wide with an average slope angle of 12°. It is a slow moving multiple earth slide-earth flow, with movement rates of up to 3.5 m/year (Merritt et al. 2014). An infiltration experiment was carried out on the landslide surface between 7th and 8th May 2014 to better understand the hydrological processes and the influence of any preferential flow paths. During the experiment, 0.973 m^3 of salt water (with a conductivity of 0.26 S/m) was sprinkled over a plot with an area of ~20 m^2. Figure 1a shows the resistivity inverse model from the initial data set before the infiltration. The infiltration started 1.3 hours later lasting for 6.7 hours. Eight time-lapse data sets were collected. Figure 1b shows the percentage resistivity model change between the initial data set and one collected after 7.3 hours. It shows a large region with negative changes in the top two layers. However, there is band to the right of the infiltration zone in the top layer (near the high resistivity zone in the top layer in Figure 1a) with increases of up to 30% that is an artefact. Figure1c shows the results where the zones with increased resistivity values are eliminated when the positivity constraint is applied. It also shows the infiltration pattern in the third layer more clearly.
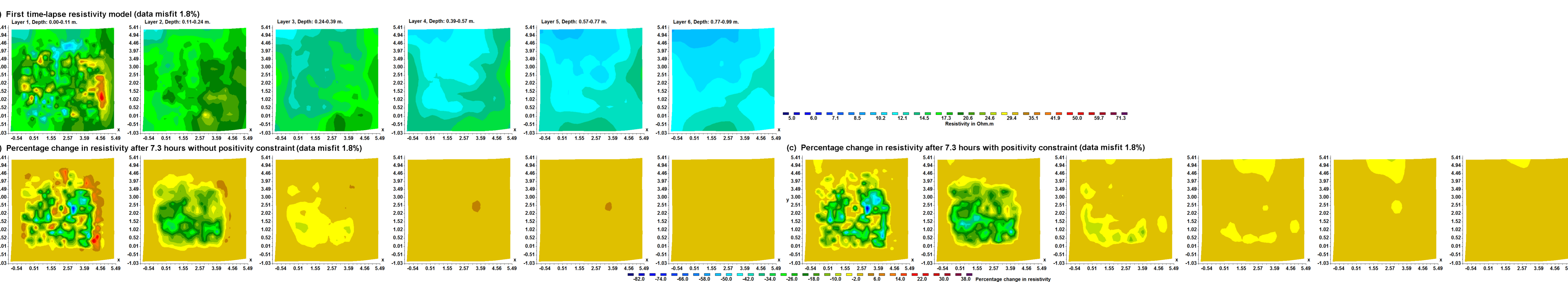

Figure 1. (a) Resistivity model for first data set shown as layers. Percentage resistivity change in the inverse model after 7.3 hours (b) without and (c) with positivity constraints.

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