On the Economics of Consumer Stockpiling

by

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Abstract

Consumer stockpiling is a crucial retail phenomenon that has received wide academic attention. However, some related issues still remain unaddressed, with implications for many areas of economics policy. By focusing on consumer stockpiling, this thesis provides four theoretical essays to better understand these topics.

The first essay analyses the implications for demand elasticities. It proposes a general foundation to understand how empirical estimates of own- and cross-price elasticities of demand can be biased when the effects of consumer stockpiling are not fully considered. It suggests that both the own- and cross-price elasticity biases can be positive, negative, or zero depending upon intuitive theoretical conditions.

The second essay then places more structure on the above-mentioned general framework by developing a duopoly model of stockpiling with differentiated products. Within this model, the results show that the equilibrium measures of the own- and cross-price elasticity biases are both (weakly) positive. This essay then analyses when such biases matter most.

The third essay considers market entry. It introduces consumer stockpiling behaviour into an n-firm oligopoly with differentiated products. First, we show that for any finite number of firms, any symmetric equilibrium involves a positive level of consumer stockpiling. Second, by introducing free entry, we show that the excess entry theorem continues to hold under consumer stockpiling. Finally, we show how consumer stockpiling can result in biased empirical estimates of demand elasticities, and how this varies with the numbers of firm in the market.

The fourth essay introduces Behavioural-Based Price Discrimination (BBPD) into a storable product market. It shows that, in equilibrium, consumer stockpiling behaviour can be used as a device for the firm to perform BBPD. The results show that consumer stockpiling improves consumer surplus and profit despite the associated BBPD

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Chapter 1

Introduction

This thesis is broadly related to consumer stockpiling behaviour. It is often observed that consumers buy more for future consumption. Evidence of this phenomenon has been well documented for a host of retail products including cola, sugar, coffee, pasta, and laundry detergent, among many others (See Blattberg, Eppen and Lieberman, 1981; Neslin, Henderson and Quelch, 1985; Mela, Jedidi and Bowman, 1998; Pesendorfer, 2002).

This in turn generates dynamic effects that bring forward consumers' future demand. Due to this nature, consumer stockpiling has received wide academic attention. While some perspectives of it are well documented by practitioners and scholars, some are largely unaddressed, with implications for many areas of economic policy. In the spirit of this, this thesis aims to enhance our understanding of some selected topics regarding consumer stockpiling.

The core of this thesis is a collection of four theoretical essays on the economics of consumer stockpiling. While Chapter 2 presents a short note on a general framework, Chapters 3, 4, and 5 examine consumer stockpiling behavioural more extensively and comprehensively. In particular, Chapters 2 and 3 provide implications for demand estimation. Chapter 4 focuses on market entry, before Chapter

5 aims to explain how consumer stockpiling can be used to enable a special form of Behaviour-Based Price Discrimination (BBPD).

I now present a more detailed summary of this thesis. Chapter 2 and Chapter 3 mainly focus on the biased empirical estimates of own- and cross-price elasticities of demand that can result from consumer stockpiling. It has been well documented that most standard demand estimations employ a static methodology and so fail to disentangle consumers' true underlying demand from the dynamic effects of stockpiling. As a result, standard approaches produce biased estimates of own- and cross-price elasticities (Erdem *et al*, 2003; Hendel and Nevo, 2006a, 2006b, 2013; Perrone, 2017). Such biases give rise to a number of significant implications, including biased estimates of i) market power and the price effects of mergers, ii) the welfare effects of new product introductions, iii) the effects of tax changes on consumption, and iv) the profitability of sales promotions. Despite the obvious importance of this issue, the existing theoretical literature has offered little help. There are relatively few equilibrium models of consumer stockpiling, especially in markets with differentiated products, as most relevant for empirical work. In addition, such models have not been used to explicitly analyse the effects of stockpiling on demand elasticities. As a result, empirical researchers and policymakers have little guidance in knowing the potential nature of such biases, and when they matter most. This thesis therefore wishes to take some initial steps to fill in the gap. To this end, a general framework of biased elasticities is provided in Chapter 2, before a differentiated duopoly model is considered in Chapter 3.

Chapter 2 offers a general framework to consider such biases. Despite its simplicity, such an approach appears to have been overlooked within the literature. In more detail, this general framework supposes that a firm's observable demand in a given period is composed of two parts: i) a 'true' underlying demand that does not derive from any changes in consumers' inventories, and ii) an 'inventory' demand that derives solely from changes in consumers' inventories. If one measures the own-price or cross-price elasticity based on observed demand, rather than true demand, we show that the resulting biases can be positive, negative, or zero depending upon i) the difference between the elasticities of true demand and inventory demand, and ii) whether net stockpiling is positive or negative.

Chapter 3 places more structure on the framework of Chapter 2 by developing an example equilibrium model of stockpiling in an effort of further understanding the biases. To provide a meaningful analysis of the own- and cross-price elasticity biases, such a model necessarily requires a differentiated products oligopoly. Moreover, to fully understand the biases and how they may change with market parameters, we also require an equilibrium model with endogenous prices.

Despite the challenges of this task, we present a simple tractable model of a two-period differentiated products duopoly. We show that in any symmetric equilibrium, a (weak) subset of the consumers stockpile to avoid additional transaction costs. We find that the equilibrium measures of both the own- and cross-price elasticities biases are zero when product differentiation is low, but are strictly positive when product differentiation is high. To further consider when these biases matter most, we show that they are strictly increasing in the degree of product differentiation when net stockpiling is positive, but strictly decreasing in the degree of product differentiation in period 2, when net stockpiling is negative. This indicates to policymakers and econometricians when the exclusion of the dynamic effects of consumer stockpiling matters most.

Chapter 4 considers consumer stockpiling in a wider oligopoly setting with free entry. From the previous literature and Chapters 2 and 3 of this thesis, it is now commonly understood that ignoring dynamic effects of consumer stockpiling behaviour leads to biased empirical estimates of own- and cross-price elasticities, with significant implications for competition policy, tax policy, marketing and others. However, very little remains known about how the number of firms in a market affect consumer stockpiling behaviour and the consequent elasticity biases. By focusing on the role played by the number of firms in an oligopoly market, we aim to fill in this gap.

In particular, Chapter 4 considers consumer stockpiling behaviour, market entry, and the implied elasticities biases within a spatial n-firm oligopoly setting. As a primary contribution, this chapter firstly studies how the number of firms in market determines consumer stockpiling behaviour and the biases of own- and cross-price elasticities for a fixed level of market entry. The chapter then analyses how these conclusions change when free entry is allowed and the number of firms becomes endogenous. As a second contribution, because consumer stockpiling behaviour involves intra-period demand shifts that might potentially affect firms' entry decisions, this chapter studies the extent to which the excessive entry theorem still applies under consumer stockpiling.

The results show that in any symmetric equilibrium, a (weak) subset of consumers stockpiles to avoid additional transaction costs. Depending on the number of firms in the market, the biases of price elasticity can either be zero or positive. By treating the number of firms as endogenous, we show that excessive entry theorem still holds when consumers stockpile, but restrictions on entry do not necessarily increase social welfare. We finally consider how the biases of own- and cross-price elasticities vary with respect to transaction cost and product differentiation under free entry. Our findings suggest that the previous results of Chapter 2 are robust to free entry. The associated elasticities biases are strictly increasing in the degree of product differentiation when net stockpiling is positive, but strictly decreasing in the degree of product differentiation in period 2.

Chapter 5 considers a different aspect of consumer stockpiling by showing how it may enable a form of Behavioural-Based Price Discrimination (BBPD). Despite the literature of BBPD having received a wide attention in recent years, an analysis of such stockpiling-based price discrimination remains rare. Moreover, associated welfare analysis remains ambiguous in the literature. Addressing these omissions is important for the implications of consumer policy in relevant markets.

In more detail, we set up a two-period monopoly model with consumer stockpiling. In equilibrium, higher match value consumers stockpile in advance while consumers with lower match value do not. Hence, the firm can segment consumers according to their match value and perform BBPD.

Generally, the literature on BBPD is often associated with the recent technologies that allow firms to acquire information about consumers' previous purchase history. Meanwhile, this can also be done via observing consumers' actual stockpiling behaviour. The intertemporal demand substitution allows sellers to recognise those buyers that have not stockpiled and to price discriminate between consumers based on their past stockpiling behaviour.

We then examine the welfare effects of such BBPD. We show that being able to stockpile always increases aggregate consumer surplus and firm profits despite any potential BBPD. For the firm, this BBPD prompts it to optimally select lower prices in a way that increases its profits from the resulting increase in market demand. For the consumers, their surplus increases due to i) being able to stockpile and thereby reduce their expenditure on transaction costs, and ii) the reduced prices. Hence, policymakers should not be concerned by such a form of price discrimination.

Finally, it is important to note that while this thesis focuses on consumer stockpiling behaviour, its results can often be reinterpreted to apply to other contexts too, such as some forms of long-term contracts. These contracts allow consumers to pay a fixed-price in advance for the services and products that they receive in the future. Similar to consumer stockpiling, these contracts induce future demand to be shifted forward. It then follows that the monopolist can identify its previous consumers. Common examples include energy markets, mortgage markets, telecommunication markets, gym memberships, magazine subscriptions and bank services.

Chapter 2

Consumer Stockpiling and Demand Elasticity Biases: A General Framework

2.1 Introduction

Consumers often stockpile goods for future consumption. Evidence of this phenomenon has been well documented for a host of retail products including cola, sugar, coffee, pasta, and laundry detergent, among many others (See Blattberg, Eppen and Lieberman, 1981; Neslin, Henderson and Quelch, 1985; Mela, Jedidi and Bowman, 1998; Pesendorfer, 2002). However, most standard demand estimations employ a static methodology and so fail to disentangle consumers' true underlying demand from the dynamic effects of stockpiling. As a consequence, it is now commonly understood that standard approaches produce biased estimates of own- and cross-price elasticities.[1] Such biases give rise to a number of significant implications. For instance, such elasticities biases lead to i) underestimate of

[1]For instance, see Erdem *et al* (2003), Hendel and Nevo (2006a, 2006b, 2013), and Perrone (2016).

market power and the anti-competitive effects of mergers (e.g. Bresnahan, 1987; Hendel and Nevo, 2006a), ii) overestimates of the responsiveness of consumption following a tax change (e.g. Wang, 2015), and iii) exaggerations of the profitability of sales promotions[2]. In order to provide a clearer theoretical guidance about when these biases matter most, it is important to first know the characteristics. To this end, before later examining the implied biases with full equilibrium models of duopoly and oligopoly (in Chapters 3 and 4), this chapter firstly provides a general framework.

In more detail, our general framework supposes that a firm's observable demand in a given period is composed of two parts: i) a 'true' underlying demand that does not derive from any changes in consumers' inventories, and ii) an 'stockpiling' demand that derives solely from changes in consumers' inventories. If one measures the own-price or cross-price elasticity based on observed demand, rather than true demand, we show that the resulting biases can be positive, negative, or zero depending upon i) the difference between the elasticities of true demand and inventory demand, and ii) whether net stockpiling is positive or negative.

Despite its simplicity, such an approach appears to have been overlooked within the literature. It shows that any failure to take into account the role of stockpiling can lead to an over- or under-estimation of the true own- and cross-price demand elasticities, or even to no bias at all, depending on some intuitive conditions. This general framework provides a key foundation for Chapter 3 and 4 of this thesis, in which we place more structure on this framework by developing example equilibrium models of stockpiling within a differentiated products duopoly and oligopoly.

Empirical studies of stockpiling have a long history in marketing. Such studies typically seek to decompose estimated price elasticities into their different demand sources, such as stockpiling and brand switching, e.g. Gupta (1988), Bell

[2]More details about i)-iii) are provided in the appendix for the purposes of this thesis.

et al (1999), and the review by Gedenk *et al* (2010). An alternative empirical approach involves the development and testing of stockpiling predictions using reduced form analysis. Examples include Boizot *et al* (2001), Pesendorfer (2002), and notably Hendel and Nevo (2006a) who also provide an insightful discussion of the implications of stockpiling for demand elasticity estimation. The issue of elasticity estimation under stockpiling has been expanded in some recent studies that employ dynamic structural estimates (e.g. Erdem *et al*, 2003, Hendel and Nevo 2006b, 2013, and Perrone, 2016). For instance, Hendel and Nevo (2006b) use data on laundry detergent purchases to suggest that standard static methods overestimate own-price elasticities by 30% and underestimate cross-price elasticities by around 80%, while Perrone (2017) offers a quicker estimation method to suggest that own-price elasticities are overestimated by 20-100% using data on French food. In contrast, this chapter provides a simple, yet general theoretical framework to understand the factors involved in biased elasticity measurements.

2.2 A General Framework

This section provides a general theoretical foundation to study the biases that may result when own- and cross-price elasticities are calculated while ignoring the effects of consumer stockpiling.

Consider a market with $n \geq 1$ single product firms, $i \in \{1, ..., n\}$.[3] The product is storable in the sense that a purchased unit can be consumed either immediately or stockpiled for consumption in a later period; however it can only be consumed once. Denote $\widehat{Q}_{it} \geq 0$ as firm i's total 'observed' demand in period t. This is composed of two parts, $\widehat{Q}_{it} = Q_{it} + \Delta_{it}$. The first part, $Q_{it} \geq 0$, is firm i's 'true' underlying demand that does not derive from any changes in consumers' inventories. In contrast, the second part, $\Delta_{it} \geq -Q_{it}$, represents firm i's 'stockpiling' demand

[3] This can be easily extended to consider multi-product firms.

which is defined as the net demand that derives solely from changes in consumers' inventories. If stockpiling demand is positive, $\Delta_{it} > 0$, consumers in period t are stockpiling - adding to their inventories by more than they are reducing them. Here, the level of observed demand is larger than true demand, $\widehat{Q}_{it} > Q_{it}$. On the other hand, if stockpiling demand is negative, $\Delta_{it} \in [-Q_{it}, 0)$, consumers, on balance, are using their existing inventories to replace some or all of their true demand, Q_{it}. In this case, observed demand is less than true demand, $\widehat{Q}_{it} < Q_{it}$.

Now denote $Q_{it}(.)$ and $\Delta_{it}(.)$ as the demand functions for firm i in period t that relate to true demand and stockpiling demand, respectively. Beyond assuming that these demand functions are continuously differentiable, we can remain agnostic about how they are affected by changes in firm i's own price, p_{it}, or by changes in the price of some other firm $j \neq i$, p_{jt}. It then follows that the own- and cross-price elasticities of 'observed' demand for firm i in period t equal

$$\widehat{\rho}^{i}_{it}(.) = -\frac{\partial\left(Q_{it}(.) + \Delta_{it}(.)\right)}{\partial p_{it}} \cdot \frac{p_{it}}{Q_{it}(.) + \Delta_{it}(.)} \quad \text{and} \quad \widehat{\rho}^{j}_{it}(.) = \frac{\partial\left(Q_{it}(.) + \Delta_{it}(.)\right)}{\partial p_{jt}} \cdot \frac{p_{jt}}{Q_{it}(.) + \Delta_{it}(.)}.$$

Similarly, the own- and cross-price elasticities of 'true' demand for firm i in period t can be written respectively as

$$\rho^{i}_{it}(.) = -\frac{\partial Q_{it}(.)}{\partial p_{it}} \cdot \frac{p_{it}}{Q_{it}(.)} \qquad \text{and} \qquad \rho^{j}_{it}(.) = \frac{\partial Q_{it}(.)}{\partial p_{jt}} \cdot \frac{p_{jt}}{Q_{it}(.)},$$

and the own- and cross-price elasticities of stockpiling demand for firm i in period t are

$$\eta^{i}_{it}(.) = -\frac{\partial \Delta_{it}(.)}{\partial p_{it}} \cdot \frac{p_{it}}{\Delta_{it}(.)} \qquad \text{and} \qquad \eta^{j}_{it}(.) = \frac{\partial \Delta_{it}(.)}{\partial p_{jt}} \cdot \frac{p_{jt}}{\Delta_{it}(.)}.$$

Hence, the elasticities of observed demand may differ from the elasticities of true demand. Moreover, using observed demand to estimate the true demand elasticities may lead to biased results unless one takes into account the effects of stockpiling demand. The extent of any such bias can be measured by the difference between the observed and true elasticities, where the superscript $h = \{i, j\}$ allows us to refer to own-price or cross-price biases respectively:

$$\theta^h_{it}(.) = \widehat{\rho}^h_{it}(.) - \rho^h_{it}(.) \text{ for } h = \{i, j\}$$

After rewriting the expression for the (own- or cross-price) elasticity of observed demand as a weighted average of the elasticities of true demand and stockpiling demand,

$$\widehat{\rho}^h_{it}(.) = \rho^h_{it}(.) + \frac{\Delta_{it}(.)}{Q_{it}(.) + \Delta_{it}(.)} \left(\eta^h_{it}(.) - \rho^h_{it}(.)\right) \text{ for } h = \{i, j\}, \tag{2.1}$$

one can immediately state the following:

Proposition 2.1. *The bias between the observed and true (own- or cross-price) elasticity in period t equals:*

$$\theta^h_{it}(.) = \widehat{\rho}^h_{it}(.) - \rho^h_{it}(.) = \frac{\Delta_{it}(.)}{Q_{it}(.) + \Delta_{it}(.)} \left(\eta^h_{it}(.) - \rho^h_{it}(.)\right) \textit{ for } h = \{i, j\}. \tag{2.2}$$

i) The bias is zero if stockpiling demand is zero, $\Delta_{it}(.) = 0$, or the elasticities of stockpiling and true demand are equal, $\eta^h_{it}(.) = \rho^h_{it}(.)$.

ii) When stockpiling demand is positive, $\Delta_{it}(.) > 0$, the bias is positive (or negative) if the stockpiling demand elasticity, $\eta^h_{it}(.)$, is greater (or less) than the true demand elasticity, $\rho^h_{it}(.)$.

iii) When stockpiling demand is negative, $\Delta_{it}(.) < 0$, the bias is negative (or positive) if the stockpiling demand elasticity, $\eta^h_{it}(.)$, is greater (or less) than the true demand elasticity, $\rho^h_{it}(.)$.

Proposition 2.1 shows how both elasticity biases can, in principle, be negative, positive, or zero depending upon some intuitive conditions. This has two implications. First, the previous literature suggests that the own-price elasticity bias is always positive by focusing exclusively on the case where stockpiling demand is positive and where observed demand is more elastic than true demand. Proposition 2.1 shows that this corresponds to situations where the stockpiling demand elasticity is greater than the true demand elasticity, but stresses that is only one of several possible cases. Second, the literature has recognised that the cross-price elasticity bias may be either positive or negative, Hendel and Nevo (2006b, 2013). After providing a full theoretical foundation, Proposition 2.1 demonstrates how this ambiguity depends upon i) the relative magnitudes of the elasticities of stockpiling demand and true demand, $\eta_{it}^h - \rho_{it}^h$, and ii) whether the current value of consumers' (net) stockpiling demand, $\Delta_{it}(.)$, is positive or negative.

2.3 Conclusions

In this chapter, we have provided a theoretical analysis of the biases that can result when measuring own- and cross-price elasticities when the role of consumer stockpiling is ignored. We presented a general theoretical foundation to characterise the determinants of such biases. We showed that both the own- and cross-price elasticity biases can be positive, negative, or zero depending upon i) the difference between the elasticities of true demand and stockpiling demand, and ii) whether net stockpiling is positive or negative.

Appendix:

In this appendix, we provide some more detail to explain how ignoring the effects of consumer stockpiling behaviour can affect the estimation of market power, the impact of tax policy, and the profitability of firms' sales promotions. As further detailed in the main text of this chapter, denote $\widehat{Q}_{it}$ as firm's i's total 'observed' demand in period t, and let this be composed of two parts, $\hat{Q}_{it} = Q_{it} + \Delta_{it}$. The first part, Q_{it}, indicates firm's true demand, that is independent from consumer stockpiling. The second part Δ_{it} represents firm i's stockpiling demand. Figure 2.1 provides a simple demand and supply curve to help explain these.

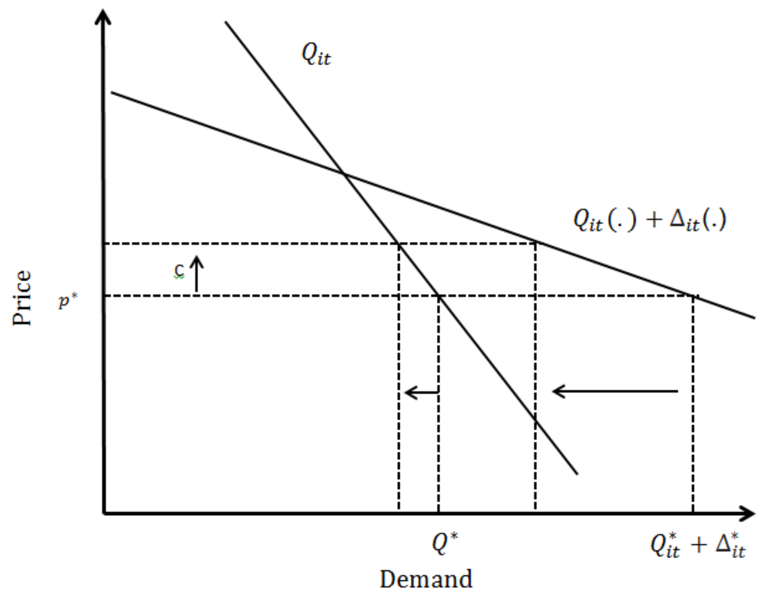

Figure 2.1: An Illustration of the Biases of Own-Price Elasticity

Market Power

The Lerner Index is a measure market power. It is defined by $m = \frac{p-mc}{p} = \frac{1}{\rho}$, where p is the price that maximises monopolist's profit, c is the marginal cost, and ρ is the price elasticity of demand. From Figure 2.1, it can be seen that the curve of observed demand, $\hat{Q}_{it} = Q_{it} + \Delta_{it}$, is flatter and elastic than that of true demand, Q_{it}. Hence if only observed demand is considered, then the Lerner index measured by observed demand is given by,

$$\hat{m} = \frac{1}{\hat{\rho}}$$

where $\hat{\rho}$ indicates price elasticity of observed demand, $\hat{\rho} = -\frac{\partial(Q_{it}+\Delta_{it})}{\partial p_{it}} \frac{p_{it}}{Q_{it}+\Delta_{it}}$.

Whereas the true (long-run) Lerner index, measured by true demand only is just

$$m = \frac{1}{\rho}$$

where ρ indicates price elasticity of true demand, $\rho = -\frac{\partial Q_{it}}{\partial p_{it}} \frac{p_{it}}{Q_{it}}$.

Hence, the bias of Lerner Index (market power) when a policymaker uses observed demand rather than true demand equals

$$\hat{m} - m = \frac{1}{\hat{\rho}} - \frac{1}{\rho}$$

which can be rewritten as,

$$\hat{m} - m = \frac{1}{\hat{\rho}} - \frac{1}{\rho} = \frac{\hat{\rho} - \rho}{\hat{\rho}\rho}$$

One can then find that the level of the bias of Lerner Index, $\hat{m} - m$, hinges on the difference between the elasticity of true demand and that of observed demand. The existing literature focusses on the case where $\hat{\rho} - \rho$ to suggest that the bias is positive such that ignoring the effects of stockpiling underestimates market power. However, in Section 2.2.1, we explain in detail that, depending on the Δ_{it}, the difference between elasticity of true demand and that of observed can be positive, negative, or zero upon some intuition conditions of stockpiling demand Δ_{it}. This clarifies the claims of the previous literature about the incorrect estimations of market power that can result from ignoring consumer stockpiling behaviour.

Tax Policy and Sales Promotion Profitability

We now show how the failure to account for consumer stockpiling can bias the estimated effects of tax policy and sales promotions. measurement of market power. This can be most easily explained graphically.

In Figure 2.1, the flatter demand curve indicates the observed demand, whereas the steeper curve indicates the true demand. Given this, if price increases by C, due to a tax increase, it can be observed that the expected reduction in observed demand is larger than the expected reduction in true demand. This coincides the findings of Wang (2015) that suggests that ignoring the effects of consumer stockpiling can lead to overestimates of the responsiveness of soda consumption following an increase in soda tax in the U.S.

Similarly, now consider a potential sales promotion where a firm considers a price cut. The increase in demand on the observed demand curve is larger than that on the true demand. This is consistent with some marketing literature suggesting that ignoring the effects of stockpiling can lead to an exaggerated estimates of the profitability of sales promotions (See Gupta 1988, Gednek *et al* 2010). However, the existing literature tends to solely focus on the intertemporal substitution of demand and ignore the effect of the biases of elasticity. The detailed illustration in Section 2.2.1 helps better understand to what extent exactly the profitability of sales promotions are exaggerated from ignoring consumer stockpiling behaviour.

Chapter 3

Equilibrium Consumer Stockpiling and Demand Elasticity Biases

3.1 Introduction

In the introduction of Chapter 2, the necessities of studying consumer stockpiling behaviour and the consequences of its failure have been summarised. Despite the obvious importance of this issue, the existing theoretical literature has offered little help. Indeed, as later detailed, there are relatively few equilibrium models of consumer stockpiling, especially in markets with differentiated products, as most relevant for empirical work. Moreover, such models have not been used to explicitly analyse the effects of stockpiling on demand elasticities. As a result, empirical researchers and policymakers have little guidance in knowing the potential nature of such biases, and when they may matter most.

Chapter 2 has already provided a general framework for analysing relevant biases of own- and cross-price elasticities. To illustrate some deeper results, this chapter places more structure on the framework by developing an example equilibrium model of stockpiling within a differentiated products duopoly. The results imply that static estimates of own- and cross-price elasticities can be *unbiased* if product differentiation is low, even though net consumer stockpiling is positive. Otherwise, static estimates are predicted to be positively biased, and either increasing or decreasing in the level of product differentiation depending on whether net consumer stockpiling is positive or negative, respectively.

In more detail, to better understand the biases suggested by Chapter 2, we place more structure on the framework by developing an example equilibrium model of stockpiling. To provide a meaningful analysis of the own- and cross-price elasticity biases, such a model necessarily requires a differentiated products oligopoly. Moreover, to fully understand the biases and how they may change with market parameters, we also require an equilibrium model with endogenous prices. Despite the challenges of this task, in this chapter we present a simple tractable model of a two-period differentiated products duopoly. In each period, consumers wish to consume exactly one unit, but may purchase a second unit in period 1 to store for consumption in period 2. As consistent with many retail markets, consumers incur positive transaction costs every time they make a purchase from a firm, but independent of the number of units bought. Such transaction costs can arise from the potential costs of visiting a firm, locating the product, ordering a delivery, or simply remembering to make a purchase.[1]

The model shows that any symmetric equilibrium involves positive consumer stockpiling. Intuitively, the transaction costs encourage consumers to stockpile in order to avoid such costs in the future. We show that the total level of stockpiling is increasing in the level of transaction cost, and decreasing in the level of product differentiation.

[1]Some common examples include the cost of visiting a store, locating an item in the store or delivering. Transaction cost helps explain why many consumers do their shopping weekly rather than daily. Later, we relax this condition by assuming that transaction cost is incurred only if the repeated trips to the store are made. For evidence of the importance of transaction costs for consumer decisions more generally, see Marshall and Pires (2017). In addition, Seiler (2013) provides evidence that transaction costs play large roles in explaining consumer's purchase decision on storable product market. The importance of transaction cost applies throughout the thesis.

By combining the insights of the general framework suggested by Chapter 2 with the equilibrium model, we then characterise the implied elasticity biases. The results depend on the extent of product differentiation. When product differentiation is relatively high, both the own- and cross-price elasticities biases are strictly positive. Furthermore, we find that the biases are increasing in the degree of product differentiation, and decreasing in the level of transaction costs. In contrast, if product differentiation is sufficiently low such that all consumers stockpile, then both elasticity biases can actually be negligible. Thus, to guide empirical researchers and policymakers, our results predict that elasticity biases matter most in markets with higher levels of product differentiation.

The chapter proceeds as follows. Section 3.2 reviews the related literature. To place more structure, Section 3.3 introduces the set-up of the duopoly stockpiling model, and Section 3.4 solves for equilibrium. The implications of the model for the associated biases are then presented and discussed in Section 3.5. Finally, Section 3.6 concludes. All proofs are listed in appendix.

3.2 Related Literature

This chapter aims to provide a better theoretical basis to analyse the effects of stockpiling on demand estimation. The empirical literature on stockpiling has already been reviewed in Chapter 2. For this chapter, we offer an equilibrium model of stockpiling to assess how product differentiation affects such biases. While some related models exist in the literature, they are surprisingly rare, and they have not been used to provide a detailed analysis of elasticity biases. From the existing models, some do not allow for endogenous prices and would therefore being unable to fully assess the biases and their comparative statics in a market equilibrium (e.g. the theoretical sections of Boizot *et al*, 2001; Hendel and Nevo 2006a, 2006b; Perrone, 2017). Others allow for endogenous prices, but assume that firms sell homogeneous products in a way that would limit any analysis of own- or cross-price elasticities (e.g. Salop and Stiglitz, 1982; Sobel, 1984; Pesendorfer, 2002; Hong *et al*, 2002; Bell *et al*, 2002; Anton and Das Varma 2005; Hosken and Reiffen, 2007).

This leaves only two papers, beyond our own, that consider stockpiling under differentiated products and endogenous prices. The first is Guo and Villas-Boas (2007), who present a two-period Hotelling model and show that consumers with relatively strong brand preferences are more likely to stockpile in period 1. Hence, as more consumers stockpile, price competition in period 2 becomes more intense because only the consumers with relatively weak brand preferences remain. This effect is sufficient to deter firms from lowering prices such that the equilibrium involves no consumer stockpiling. In their extensions, they briefly show how stockpiling can occur if i) consumers' preferences can change over time, or ii) consumers value the future sufficiently more than firms, but do not analyse the associated elasticity biases. Our equilibrium model of stockpiling builds on their analysis (albeit with a more flexible model of product differentiation) by demonstrating how transaction costs can provide an especially tractable and realistic source of positive stockpiling, before focusing on analysing the implications for elasticity biases.

The second paper is Hendel and Nevo (2013), who also develop an equilibrium model of stockpiling. However, rather than deriving any formal theoretical predictions about elasticity biases, they pursue a wider structural investigation into intertemporal price discrimination. Their model assumes an exogenous partition of consumers into storers and non-storers, and that firms commit to a price path over time. In contrast, the partition between storers and non-storers in our equilibrium model arises endogenously due to differences in consumers' brand preferences. We also allow for the more realistic scenario where firms have no price commitment. Similar to them, due to the challenges of fully demonstrating the existence of equilibria in dynamic models of stockpiling with differentiated products, we focus on fully characterising the unique local symmetric equilibrium. Any (global) symmetric equilibrium must necessarily have the properties of such a unique local symmetric equilibrium. For related reasons, Hendel and Nevo (2013) assume the concavity of their profit function. We return to further compare our results to the existing literature in Section 3.5.3.

3.3 A Model of Endogenous Stockpiling

In Sections 3.3 and 3.4, we present an equilibrium model of stockpiling. This model is then combined with the results of the previous section to study the implications for the elasticity biases in Section 3.5.

3.3.1 Assumptions

Consider a market where two firms, $i = \{A, B\}$, sell a single, horizontally differentiated, storable good with zero production costs over two periods, $t = 1, 2$. There is a unit mass of risk-neutral consumers with quasi-linear preferences who each want to consume one unit of the good per period. To model brand preferences, we use Perloff and Salop's (1985) random utility framework.[2] In particular, having bought product i in period t at price, p_{it}, let consumer m's net utility of consuming one unit of product i equal $u_{im}(p_{it}) = \varepsilon_{im} - p_{it}$, where consumer m's gross utility, ε_{im}, is a consumer-firm specific match value. Such match values are independently distributed across firms and consumers with $G(\varepsilon_{im}) = G(\varepsilon)\ \forall i, m$, and remain fixed throughout the game. For tractability, we assume that $G(\varepsilon)$ is a uniform distribution on $[a, b]$ with $a \geq 0$ and $\mu \equiv b - a > 0$ such that $G(\varepsilon) = \frac{\varepsilon - a}{\mu}$

[2]This framework is being used increasingly in a variety of applications, such as bundling (Zhou, 2017) and consumer search (Armstrong, 2016). With suitable restrictions, it flexibly encompasses the familiar Hotelling set-up as a special case. In our case, it is also useful in highlighting the different effects of consumers' preferences and consumers' transaction costs. In Chapter 4, we use Salop circular city framework, which is more related to Hotelling set-up, to consider some different issues in an n-firm oligopoly.

and $G'(\varepsilon) = g(\varepsilon) = \frac{1}{\mu}$ for $\varepsilon \in [a, b]$. The parameter, μ, can be interpreted as the degree of product differentiation.

Importantly, we assume that transactions are costly for consumers. Specifically, each time a consumer makes a purchase of one or more units from any given firm, the consumer incurs a transaction cost, $\kappa > 0$, as consistent with the cost of visiting the firm, locating the product, or ordering a delivery. As standard, we let each consumer's outside option be sufficiently unattractive such that they always *consume* one unit of the market good each period - although they need not *buy* each period due to the possibility of stockpiling. For simplicity, we also suppose that all agents have a discount factor close to one, as most appropriate for products that are purchased frequently (e.g. bottles of cola), and normalise any physical costs of stockpiling to zero.[3][4]

The timing of the game is then as follows. In period 1, each firm i simultaneously chooses its period 1 price, p_{i1}. Consumers then learn their match values for each firm and observe prices before making their purchase decisions. Each consumer must decide whether to buy either one unit for consumption in period 1 only, or an additional second unit to stockpile for consumption in period 2, and choose which firm(s) to buy from. In period 2, each firm then simultaneously chooses its period 2 price, p_{i2}. Consumers observe these prices, and any remaining 'active' consumers that did not stockpile in period 1 then choose which firm to purchase from. We consider (pure-strategy) symmetric equilibria where the firms set period 1 price, p_1^*, and period 2 price, p_2^*. In particular, as noted in Section 3.2, we focus on characterising on the unique local symmetric equilibrium. Hence, we need only consider local deviations around a potential symmetric equilibrium.

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[3] The model becomes less tractable for lower levels of the discount factor. However, one can show that an equilibrium with positive stockpiling will arise when the discount factor is sufficiently large.

[4] One can easily extend the model to allow consumers to have positive stockpiling costs, $s \geq 0$, as consistent with the costs of storing a product. The results then hinge on the level of net transaction costs, $(\kappa - s)$, rather than κ.

3.3.2 Benchmark Analysis

Let us briefly examine a benchmark where stockpiling is prohibited. In this case, the two periods are identical. In any period t, a given consumer m will purchase one unit from firm i rather than firm j if $u_{im}(p_{it}) - \kappa \geq u_{jm}(p_{jt}) - \kappa$. Hence, consumer m will prefer firm i if her relative brand preference for firm i, $\varepsilon_{im} - \varepsilon_{jm}$, exceeds the associated price difference, $p_{it} - p_{jt}$. As such, consumer m will buy one unit from firm i with probability $Pr(\varepsilon_{jm} \leq p_{it} - p_{jt} + \varepsilon_{im}) = G(p_{it} - p_{jt} + \varepsilon_{im})$, and firm i's demand in period t equals

$$Q_{it}(p_{it}, p_{jt}) = \int_a^b G(p_{it} - p_{jt} + \varepsilon)g(\varepsilon)d\varepsilon = \frac{1}{2} + \frac{p_{jt} - p_{it}}{\mu} \tag{3.1}$$

After applying the usual first order condition for a symmetric equilibrium, $p^*_{it} = -[Q_{it}(p^*_{it}, p^*_{it})/Q'_{it}(p^*_{it}, p^*_{it})]$, one obtains the standard equilibrium price and quantity. In each period, each firm sets $p^* = \frac{\mu}{2}$, sells to half of the consumers, $Q^* = \frac{1}{2}$, and earns (per-period) profits, $\pi^* = \frac{\mu}{4}$. Note that the equilibrium price and profits are increasing in the degree of product differentiation, μ.

3.4 Equilibrium Analysis

We now consider the equilibrium of the full game by permitting stockpiling. To begin, Section 3.4.1 analyses consumers' decisions and demand, before Section 3.4.2 then endogenises firms' behaviour.

3.4.1 Consumers' Decisions and Demand

We first characterise some features of consumers' stockpiling decisions and demand in period 1 for a given set of period 1 prices $\{p_{A1}, p_{B1}\}$, and expected period 2 prices, $\{p^e_{A2}, p^e_{B2}\}$. Then we consider period 2 demand given period 2 prices, $\{p_{A2}, p_{B2}\}$.

3.4.1.1 Period 1

Consider consumer m's options in period 1 given her match values $\{\varepsilon_{Am}, \varepsilon_{Bm}\}$. She could: i) stockpile by purchasing two units from some firm $i = \{A, B\}$ to gain

$$U_{im}^{S} = 2u_{im}(p_{i1}) - \kappa = 2(\varepsilon_{im} - p_{i1}) - \kappa, \tag{3.2}$$

ii) stockpile by purchasing one unit from each firm to gain

$$U_{m}^{S} = u_{Am}(p_{A1}) + u_{Bm}(p_{B1}) - 2\kappa = (\varepsilon_{Am} - p_{A1}) + (\varepsilon_{Bm} - p_{B1}) - 2\kappa, \tag{3.3}$$

or iii) not stockpile by purchasing one unit in each period to gain

$$U_{m}^{NS} = max\{u_{Am}(p_{A1}), u_{Bm}(p_{B1})\} + max\{u_{Am}(p_{A2}^{e}), u_{Bm}(p_{B2}^{e})\} - 2\kappa. \tag{3.4}$$

Then note the following. First, any stockpiling consumer m who stockpiles will always prefer to buy from a single firm under option i) rather than from two firms under option ii), because this avoids making two costly transactions, $max\left\{U_{Am}^{S}, U_{Bm}^{S}\right\} > U_{m}^{S}\ \forall m$. Hence, any consumer who buys their second unit from a firm in period 1 will also buy their first unit from the same firm. Second, consumer m will therefore stockpile from i if this yields a greater utility than: a) stockpiling at j, such that

$$\widehat{S}_{im} = U_{im}^{S} - U_{jm}^{S} = 2(\varepsilon_{im} - p_{i1}) - 2(\varepsilon_{jm} - p_{j1}) \geq 0, \tag{3.5}$$

and b) not stockpiling at all, such that

$$\widetilde{S}_{im} = U_{im}^{S} - U_{m}^{NS} \geq 0. \tag{3.6}$$

After defining $\psi_m \equiv \varepsilon_{Am} - \varepsilon_{Bm} \in [-\mu, \mu]$ as consumer m's relative brand preference for firm A, one can state the following. (Any proofs are listed in appendix.)

Lemma 3.1. *If consumer m finds it optimal to stockpile from firm A (or firm B) in period 1, then so will any other consumer k with $\psi_k \equiv \varepsilon_{Ak} - \varepsilon_{Bk} > \psi_m \equiv \varepsilon_{Am} - \varepsilon_{Bm}$ (or $\psi_k < \psi_m$).*

As in Guo and Villas-Boas (2007), this implies a positive relationship between relative brand preferences and the propensity to stockpile from a given firm. Consequently, we can derive the set of consumers that stockpile from A and B by identifying two marginal consumers. In particular, by using Lemma 1, one can define ψ_A^s as the lowest value of $\psi_m = \varepsilon_{Am} - \varepsilon_{Bm}$ at which consumer m prefers to stockpile from firm A, and ψ_B^s as the highest value of ψ_m at which consumer m prefers to stockpile from firm B. Consequently, as illustrated in Figure 3.1, those consumers with $\psi_m \in [\psi_A^s, \mu]$ constitute the set of consumers that stockpile from A, $X_A(\psi_A^s)$, and those consumers with $\psi_m \in [-\mu, \psi_B^s]$ constitute the set of consumers who stockpile from B, $X_B(\psi_B^s)$. The values of ψ_A^s and ψ_B^s will later be endogenised once we consider firms' pricing decisions.

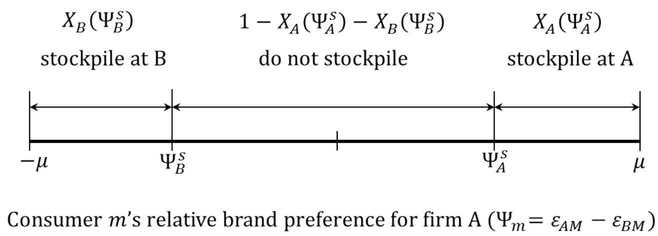


Figure 3.1: Consumers' Stockpiling Decisions

We know that any consumer who buys two units in period 1 will do so from the same firm. Hence, firm i's set of stockpiling consumers, $X_i(\psi_i^s)$, is always a weak subset of the total number of consumers who purchase from firm i in period 1. This leads to two possible cases. First, if $\psi_A^s = \psi_B^s$, then all consumers stockpile in period 1, $X_A(\psi_A^s) + X_B(\psi_B^s) = 1$. Second, if $\psi_A^s > \psi_B^s$, then a positive measure of consumers $1 - X_A(\psi_A^s) - X_B(\psi_B^s) \in (0, 1]$ do not stockpile and only buy one unit in period 1. By using the notation from Section 2.2, we can then state:

Lemma 3.2. *Firm i's total 'observed' demand in period 1, $\widehat{Q}_{i1}(.)$, is:*

$$\widehat{Q}_{i1}(.) = \begin{cases} 2Q_{i1}(p_{i1}, p_{j1}) = 1 + \frac{2}{\mu}(p_{j1} - p_{i1}) & \text{if } \psi_A^s = \psi_B^s \\ Q_{i1}(p_{i1}, p_{j1}) + X_i\left(\psi_i^s\right) = \frac{1}{2} + \frac{p_{j1} - p_{i1}}{\mu} + X_i\left(\psi_i^s\right) & \text{if } \psi_A^s > \psi_B^s \end{cases} \tag{3.7}$$

If $\psi_A^s = \psi_B^s$, all consumers stockpile. Hence, firm i's stockpiling demand is equal to its true period 1 demand, $X_i(.) = Q_{i1}(.)$, and so firm i's total period 1 demand equals $\widehat{Q}_{i1}(.) = 2Q_{i1}(.)$, where $Q_{i1}(p_{i1}, p_{j1})$ coincides with the demand in the benchmark, (3.1). If, instead, $\psi_A^s > \psi_B^s$, then only some consumers stockpile. Here, firm i's demand equals $\widehat{Q}_{i1}(.) = Q_{i1}(p_{i1}, p_{j1}) + X_i\left(\psi_i^s\right)$ because a total of $Q_{i1}(p_{i1}, p_{j1})$ consumers buy from firm i, of which $Q_{i1}(p_{i1}, p_{j1}) - X_i\left(\psi_i^s\right)$ buy one unit and $X_i\left(\psi_i^s\right)$ buy two units.

3.4.1.2 Period 2

For period 2 demand, one can then state:

Lemma 3.3. *Around any potential symmetric equilibrium, firm i's total 'observed' demand in period 2, $\widehat{Q}_{i2}(.)$, is:*

$$\widehat{Q}_{i2}(.) = \begin{cases} 0 & \text{if } \psi_A^s = \psi_B^s \\ Q_{i2}(p_{i2}, p_{j2}) - X_i\left(\psi_i^s\right) = \frac{1}{2} + \frac{p_{j2} - p_{i2}}{\mu} - X_i\left(\psi_i^s\right) > 0 & \text{if } \psi_A^s > \psi_B^s \end{cases} \tag{3.8}$$

If $\psi_A^s = \psi_B^s$, all consumers have stockpiled and so period 2 is inactive. However, if $\psi_A^s > \psi_B^s$, then consumers with $\psi_m \in (\psi_B^S, \psi_A^S)$ did not stockpile and so remain active. As in the benchmark, any such consumer will then buy one unit from firm i rather than j if $u_{im}(p_{i2}) - \kappa \geq u_{jm}(p_{j2}) - \kappa$. It then follows that firm $i's$ total period 2 demand equals $Q_{i2}(p_{i2}, p_{j2})$, from (3.1), minus those consumers that stockpiled from firm i in period 1, $X_i\left(\psi_i^s\right)$, from (3.8).

3.4.2 Firms' Decisions

Given consumer demand, we now analyse the firms' equilibrium decisions. First, we derive period 2 equilibrium prices for given levels of stockpiling. Second, we derive the equilibrium levels of stockpiling demand for given period 1 prices and expected period 2 prices, $X_i(p_{i1}, p_{j1}, p^e_{i2}, p^e_{j2})$, where consumers' expectations of period 2 prices are consistent with equilibrium, $p^e_{i2} = p^*_{i2}(X_i, X_j)$. Third, given the equilibrium levels of stockpiling demand, we then solve for period 1 equilibrium prices.

3.4.2.1 Period 2

Period 2 is active only if $\psi^s_A > \psi^s_B$ such that $X_A(\psi^s_A) + X_B(\psi^s_B) < 1$. If so, we can state:

Lemma 3.4. *Suppose $\psi^s_A > \psi^s_B$. Then, provided $3 - 4X_i(\psi^s_i) - 2X_j\left(\psi^s_j\right) > 0$ $\forall i,\ j \neq i \in \{A, B\}$, the unique period 2 equilibrium has*

$$p^*_{i2}\left(X_i(\psi^s_i), X_j\left(\psi^s_j\right)\right) = \frac{\mu}{6}\Big[3 - 4X_i(\psi^s_i) - 2X_j\left(\psi^s_j\right)\Big] > 0 \tag{3.9}$$

and $\widehat{Q}^*_{i2}(.) = Q_{i2}(p^*_{i2}, p^*_{j2}) - X_i(\psi^s_i) = \frac{1}{6}\Big[3 - 4X_i(\psi^s_i) - 2X_j\left(\psi^s_j\right)\Big] > 0.$

To characterise the properties of a potential symmetric equilibrium, we need only consider local deviations. Hence, provided that the firms' period 1 prices are sufficiently close such that their levels of stockpiling are not too dis-similar, with $3 - 4X_A(\psi^s_A) - 2X_B(\psi^s_B) > 0$ and $3 - 4X_B(\psi^s_B) - 2X_A(\psi^s_A) > 0$, Lemma 3.3 confirms that both firms will have positive period 2 equilibrium prices and demand. Moreover, as in Guo and Villas-Boas (2007), it suggests that i) period 2 prices are weakly lower than in the benchmark due to the potential absence of the stockpiling consumers who have relatively high brand preferences, and ii) the firm with the largest level of stockpiling sets a lower period 2 price because it has proportionately less consumers with a higher brand preferences.

3.4.2.2 Period 1

We now derive the equilibrium levels of stockpiling demand for given period 1 prices and expected period 2 prices. Following this, we solve for the equilibrium prices in period 1.

Equilibrium Stockpiling Demand

Denote $X_i(p_{i1}, p_{j1}, p^e_{i2}, p^e_{j2})$ as firm i's equilibrium level of stockpiling demand, where consumers expectations are correct if $p^e_{i2} = p^*_{i2}(X_i, X_j)$, for $i,\ j \neq i \in \{A, B\}$.

Proposition 3.1. *Around any symmetric equilibrium, the unique levels of stockpiling demand,* $\boldsymbol{X} = \{X_i(.), X_j(.)\}$, *equal:*

$$\boldsymbol{X} = \begin{cases} \{0, 0\} & \text{if } p_{i1} > \frac{\mu}{2} + \kappa,\ p_{j1} > \frac{\mu}{2} + \kappa \\ \left\{\frac{1}{2} - \left(\frac{2p_{i1} - p_{j1} - \kappa}{\mu}\right), \frac{1}{2} - \left(\frac{2p_{j1} - p_{i1} - \kappa}{\mu}\right)\right\} & \text{if } p_{i1} \in \left(\frac{\kappa + p_{j1}}{2}, \frac{\frac{\mu}{2} + \kappa + p_{j1}}{2}\right],\ p_{j1} \in \left(\frac{\kappa + p_{i1}}{2}, \frac{\frac{\mu}{2} + \kappa + p_{i1}}{2}\right] \\ \left\{\frac{1}{2}, \frac{1}{2}\right\} & \text{if } p_{i1} \leq \kappa,\ p_{j1} \leq \kappa \end{cases} \tag{3.10}$$

When making their decision of whether or not to stockpile, the proof verifies that an indifferent consumer at ψ^s_i optimally compares i) the cost of stockpiling in period 1 by buying a second unit from firm i, p_{i1}, versus ii) the cost of returning to buy a second unit in period 2 from firm i, rather than j, $p^*_{i2}(X_i, X_j) + \kappa$. Hence, if both firms' period 1 prices are sufficiently high, then no consumer finds it optimal to stockpile as $p_{i1} > p_{i2}^*(0,0)+\kappa$ for $i = \{A, B\}$. Similarly, if both firms' period 1 prices are sufficiently low, then all consumers find it optimal to stockpile as $p_{i1} \leq p_{i2}^*\left(\frac{1}{2}, \frac{1}{2}\right) + \kappa$ for $i = \{A, B\}$. This leaves the remaining case where both firms' period 1 prices are relatively moderate. Here, as in the middle line of (3.10), there exists a unique level of equilibrium stockpiling, such that $p_{i1} = p_{i2}^*(X_i, X_j)+\kappa$ for each firm. If firm i's period 1 price was below (above) this level for given levels of X_i and X_j, more (fewer) consumers would find it optimal to stockpile at the firm, which in turn would lower (raise) the firm's period 2 equilibrium price until this condition is satisfied.

Equilibrium Prices

We now complete the equilibrium by characterising period 1 prices. Firm i's associated profit function can be expressed as follows

$$\pi_i(.) = p_{i1}[Q_{i1}(p_{i1}, p_{j1}) + X_i(.)] + p_{i2}^*[Q_{i2}(p_{i2}^*, p_{j2}^*) - X_i(.)] \tag{3.11}$$

where firm i receives period 1 demand $\widehat{Q}_{i1} = Q_{i1}(p_{i1}, p_{j1}) + X_i(.)$ from (3.7) and (3.10), and (if active) sets a period 2 equilibrium price $p_{i2}^*(.)$, (3.9), and receives a period 2 equilibrium demand $\widehat{Q}_{i2}^* = Q_{i2}(p_{i2}^*, p_{j2}^*) - X_i(.)$ from (3.8). To begin, we can then note the following important result.

Proposition 3.2. *In any symmetric equilibrium with* $\kappa > 0$, *each firm receives a positive level of stockpiling demand.*

Proposition 3 contrasts with Guo and Villas-Boas's (2007) no-stockpiling result under zero transaction costs. Intuitively, if there is no stockpiling, we know from the benchmark that prices are equal across periods, $p_1^* = p_2^* = \frac{\mu}{2}$. However, when transaction costs are positive, these prices imply $p_1^* < p_2^* + \kappa$ such that consumers would optimally wish to stockpile to avoid incurring a second period transaction cost. Instead, any symmetric equilibrium with $\kappa > 0$ must therefore involve a positive level of stockpiling $X_i = X_j = X^* > 0$:

Proposition 3.3. *In any symmetric equilibrium:*

i) when product differentiation is low, $\mu \leq 3\kappa$, the unique level of stockpiling demand is $X^* = \frac{1}{2}$, where $p_1^* = min\{\frac{\mu}{2}, \kappa\}$, $\widehat{Q}_1^* = \frac{1}{2} + X^* = 1$ and $\widehat{Q}_2^* = 0$.

ii) when product differentiation is high, $\mu > 3\kappa$, the unique level of stockpiling demand is $X^* = \frac{3\kappa}{2\mu} \in \left(0, \frac{1}{2}\right)$, where $p_1^* = \frac{\mu-\kappa}{2}$, $p_2^* = \frac{\mu-3\kappa}{2}$, $\widehat{Q}_1^* = \frac{1}{2} + X^* < 1$ and $\widehat{Q}_2^* = \frac{1}{2} - X^* > 0$.

When product differentiation is low, $\mu \in (0, 3\kappa]$, competition is strong and prices are low relative to transactions costs, such that $p_1^* \leq \kappa$ holds in equilibrium. From Proposition 3.1, this implies that all consumers optimally stockpile, $X^* = \frac{1}{2}$. When product differentiation is high, $\mu > 3\kappa$, competition is weaker and prices are higher relative to transactions costs, such that $p_1^* > \kappa$. As such, Proposition 3.1 implies that $p_1^* = p_2^* + \kappa$ must hold in equilibrium and that this price relationship uniquely determines the equilibrium level of stockpiling demand, $X^* \in \left(0, \frac{1}{2}\right)$. To explore the determinants of stockpiling in more detail, we can now state:

Corollary 3.1. *In any symmetric equilibrium, the level of stockpiling, X^*, is (weakly) decreasing in the level of product differentiation, μ, and (weakly) increasing in the size of the transaction cost, κ.*

When product differentiation is low, $\mu \in (0, 3\kappa]$, all consumers stockpile and so the equilibrium level of stockpiling, $X^* = \frac{1}{2}$, is insensitive to small changes in product differentiation or transaction costs. However, this changes when product differentiation is high, $\mu > 3\kappa$, such that some consumers stockpile, $X^* \in \left(0, \frac{1}{2}\right)$.

To understand the intuition, first consider a marginal change in product differentiation, μ. Holding constant the level of stockpiling, X^*, both period prices strictly increase, but period 1 prices increase by more, such that $p_1^* > p_2^* + \kappa$. As a result, consumers are less inclined to stockpile, and X^* reduces until the point where $p_1^* = p_2^* + \kappa$ is restored. Now, consider a marginal change in the transaction cost, κ. Again, holding constant the level of stockpiling, X^*, the period 1 prices p_1^* strictly decrease, while $p_2^* + \kappa$ strictly increase, such that $p_1^* < p_2^* + \kappa$. As a result, consumers are more inclined to stockpile, and X^* increases until the point where $p_1^* = p_2^* + \kappa$ is restored.

3.5 Demand Elasticity Biases

Having completed our theoretical model of equilibrium stockpiling, we now combine its insights with the results from Section 2.2 to study the implied demand elasticity biases. Within the equilibrium model, period 1 net inventory demand is positive and equivalent to the equilibrium level of stockpiling, $\Delta_{i1}(.) = X_i(.) > 0$. Hence, from Proposition 2.1, the elasticity bias in period 1 equals

$$\theta_{i1}^h(.) = \widehat{\rho}_{i1}^h(.) - \rho_{i1}^h(.) = \frac{X_i(.)}{Q_{i1}(.) + X_i(.)} \left(\eta_{i1}^h(.) - \rho_{i1}^h(.)\right) \text{ for } h = \{i, j\}. \tag{3.12}$$

In contrast, period 2 net inventory demand is negative and equivalent to the equilibrium amount stockpiled in period 1, $\Delta_{i2}(.) = -X_i(.) < 0$, such that the elasticity bias in period 2 is

$$\theta_{i2}^h(.) = \widehat{\rho}_{i2}^h(.) - \rho_{i2}^h(.) = \frac{-X_i(.)}{Q_{i1}(.) - X_i(.)} \left(\eta_{i2}^h(.) - \rho_{i2}^h(.)\right) \text{ for } h = \{i, j\}. \tag{3.13}$$

In what follows, Section 3.5.1 evaluates the sign of each bias at the period's equilibrium price, p_t^*, before Section 3.5.2 analyses the associated comparative statics to understand when such biases matter most. Section 3.5.3 then discusses the results in the context of the previous literature. Henceforth, to ease exposition, we focus on firm A without loss of generality.

3.5.1 Signs of the Biases

First, consider the case of low product differentiation, $\mu \in (0, 3\kappa]$, where all consumers stockpile in period 1, $X^* = 0.5$, such that any period 2 analysis is redundant.

Proposition 3.4. *When product differentiation is low, $\mu \in (0, 3\kappa]$, the own- and cross-price elasticities of observed demand in period 1, $\widehat{\rho}^A_{A1}(p^*_1)$ and $\widehat{\rho}^B_{A1}(p^*_1)$, are unbiased, such that $\theta^A_{A1}(p^*_1) = \theta^B_{A1}(p^*_1) = 0$.*

This implies that demand elasticity estimates based only on observed demand need not be biased when there is positive stockpiling. For instance, when product differentiation is low, each consumer consumes one unit and stockpiles another, such that the level and slope of observed demand are doubled. Consequently, the own- and cross-price elasticities of observed demand remain equal to those for true demand, $\widehat{\rho}^h_{A1}(.) = -\frac{\partial(2Q_{A1}(.))}{\partial p_{A1}} \cdot \frac{p_{A1}}{2Q_{A1}(.)} = -\frac{\partial Q_{A1}(.)}{\partial p_{A1}} \cdot \frac{p_{A1}}{Q_{A1}(.)} = \rho^h_{A1}(.)$. An alternative intuition can be understood using (3.12). Here, given all consumers stockpile, the stockpiling demand equals the true demand, $X_A(.) = Q_{A1}(.)$, such that the associated own- and cross-price elasticities are the same, $\eta^h_{i1}(.) = \rho^h_{i1}(.)$.

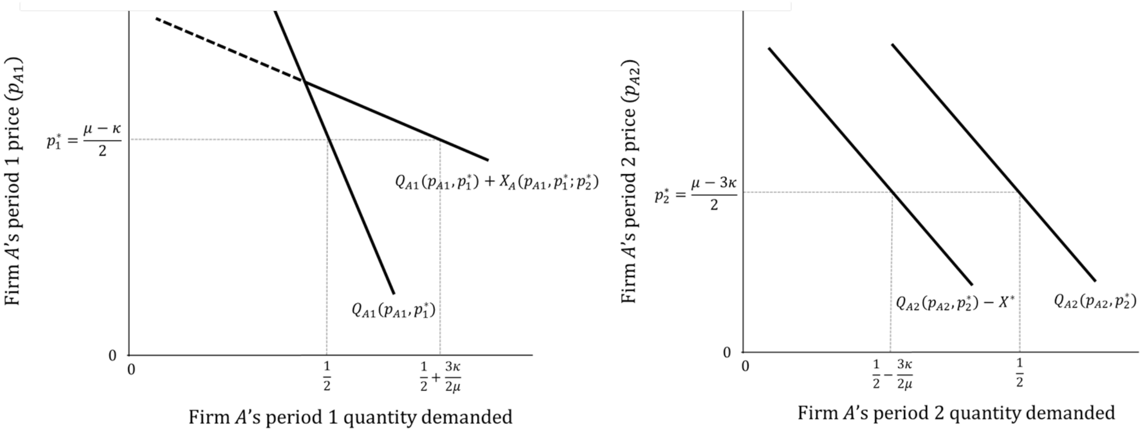
Now, consider the case of high product differentiation, $\mu > 3\kappa$, where some consumers remain active in period 2 with $X^* < 0.5$:

Proposition 3.5. *When product differentiation is high, $\mu > 3\kappa$:*

i) the own- and cross-price elasticities of observed demand in period 1, $\widehat{\rho}^A_{A1}(p^*_1)$ and $\widehat{\rho}^B_{A1}(p^*_1)$, are positively biased, such that $\theta^A_{A1}(p^*_1) = \frac{3(\mu-\kappa)}{\mu+3\kappa} - \left(\frac{\mu-\kappa}{\mu}\right) > 0$ and $\theta^B_{A1}(p^*_1) = \frac{2(\mu-\kappa)}{\mu+3\kappa} - \left(\frac{\mu-\kappa}{\mu}\right) > 0$;

ii) the own- and cross-price elasticities of observed demand in period 2, $\widehat{\rho}^A_{A2}(p^*_2)$ *and* $\widehat{\rho}^B_{A2}(p^*_2)$*, are positively biased, such that* $\theta^A_{A2}(p^*_2) = \theta^B_{A2}(p^*_2) = 1 - \left(\frac{\mu - 3\kappa}{\mu}\right) > 0.$

To gain an initial understanding of the intuition, consider the own-price elasticity biases for period 1 and 2 illustrated in Figure 3.2(a) and (b), respectively. Figure 3.2(a) shows that, in period 1, positive stockpiling demand ensures that i) the slope of the observed demand curve is flatter than the true demand curve, $\frac{\partial \widehat{Q}_{A1}(.)}{\partial p_{A1}} < \frac{\partial Q_{A1}(.)}{\partial p_{A1}} < 0$, and that ii) the observed quantity demanded is greater than the true quantity demanded, $\widehat{Q}_{i1} > Q_{i1}$. Both of these effects lead to an upward bias in the own-price elasticity of observed demand, $\theta^A_{A1}(p^*_1) = \widehat{\rho}^A_{A1}(p^*_1) - \rho^A_{A1}(p^*_1) > 0$. In contrast, Figure 3.2(b) shows that the observed demand in period 2 is parallel and to the left of the true demand, due to the fact that the equilibrium level of stockpiling is negative and independent of period 2 equilibrium prices. Intuitively, any price change in period 2 after consumers have stockpiled cannot affect the level of stockpiling demand in period 1. Therefore, the quantity demanded at a given price is lower for observed demand than the true demand, so the own-price elasticity of observed demand is greater than that of the true demand, $\theta^A_{A2}(p^*_2) = \widehat{\rho}^A_{A2}(p^*_2) - \rho^A_{A1}(p^*_2) > 0$.



(a) observed and true demand of period 1

(b) observed and true demand of period 2

Figure 3.2: Own-Price Demand Elasticity Biases

Now consider the explanation of each bias in terms of Proposition 2.1, starting with the period 1 own-price elasticity bias, $\theta^A_{A1}(p^*_1)$, in (3.12) with $i = h = A$. It follows that this bias is positive, $\theta^A_{A1}(p^*_1) > 0$, because the stockpiling demand is more elastic than the true demand, $\eta^A_{A1}(p^*_1) > \rho^A_{A1}(p^*_1)$. This is for two reasons: i) the slope of the stockpiling demand curve is flatter than the true demand curve, $\frac{\partial X_A(.)}{\partial p_{A1}} < \frac{\partial Q_{A1}(.)}{\partial p_{A1}} < 0$, and ii) the quantity demanded at p^*_1 is smaller for the stockpiling demand than the true demand, $X^* < Q^*_1$. A similar reason applies for why the period 1 cross-price elasticity of true demand is positively bias, $\theta^B_{A1}(p^*_1) > 0$. However, in this case, only the latter equivalent effect is present, because the marginal cross-price effects of the stockpiling demand and true demand are the same, $\frac{\partial X_A(.)}{\partial p_{B1}} = \frac{\partial Q_{A1}(.)}{\partial p_{B1}}$. Finally, consider the period 2 own- and cross-price elasticity biases in (3.13), which are positive, and happen to be equal in our model. Note that the own- and cross-price elasticities of inventory demand are zero, $\eta^h_{A2}(p^*_2) = 0$ for $h \in \{A, B\}$, as the equilibrium level of inventory demand in period 2 is unrelated to period 2 prices, $\frac{\partial X^*}{\partial p_{h2}} = 0$. Then, from (3.13), the period 2 own- and cross-price elasticity biases are positive, $\theta^h_{A2}(p^*_2) > 0$ for $h \in \{A, B\}$, because the own- and cross-price elasticities of true demand are positive, $\rho^k_{A1}(p^*_1) > 0$, and only a subset of consumers stockpile, $X^* \in (0, Q^*_1)$.

3.5.2 Comparative Statics

To understand when these biases matter most, we next consider how the biases vary with the level of product differentiation or transaction costs.

Proposition 3.6. *When product differentiation is high, $\mu > 3\kappa$:*

i) the period 1 own- and cross-price elasticity biases, $\theta^A_{A1}(p^*_1)$ and $\theta^B_{A1}(p^*_1)$, are strictly increasing in the level of product differentiation, μ, and strictly decreasing in the level of transaction costs, κ;

ii) the period 2 own- and cross-price elasticity biases, $\theta^A_{A2}(p^*_2)$ and $\theta^B_{A2}(p^*_2)$, are strictly decreasing in the level of product differentiation, μ, and strictly increasing in the level of transaction costs, κ.

Proposition 3.6 implies that a change in product differentiation or transaction costs have opposite effects on the elasticity biases depending on whether consumers are adding to their inventories (i.e. period 1) or consuming from their inventories (i.e. period 2).

To understand the underlying effects, first compare the impact of a increase in product differentiation, μ, on the period 1 and period 2 own-price elasticity biases, $\theta^A_{A1}(p^*_1)$ and $\theta^A_{A2}(p^*_2)$, expressed in (3.12) and (3.13) with $i = h = A$, respectively. In period 1, this bias gets larger as μ increases, because the difference between $\eta^A_{A1}(p^*_1)$ and $\rho^A_{A1}(p^*_1)$ becomes greater. The reason is that $\eta^A_{A1}(p^*_1)$ increases at a faster rate than $\rho^A_{A1}(p^*_1)$.[5] This effect is large enough to dominate a second offsetting effect that lowers the ratio $\frac{X^*}{Q^*_1+X^*}$, through a reduction in X^*. In contrast, the period 2 own-price elasticity bias reduces as μ increases, despite the fact that $\eta^A_{A2}(p^*_2) = 0$ and that $\rho^A_{A2}(p^*_2)$ strictly increases. The reason is that, in this case, the dominating force is a second offsetting effect that reduces the ratio $\frac{X^*}{Q^*_1-X^*}$.

Now consider an increase in transaction costs, κ. Here, the effects on the period 1 and 2 own-price elasticity biases, $\theta^A_{A1}(p^*_1)$ and $\theta^A_{A2}(p^*_2)$, are similar to that observed for a change in product differentiation. However, an increase in κ, has the opposite effects on the equivalent two forces just discussed for μ, but the same effect dominates in each case. For example, in period 1, the difference between $\eta^A_{A1}(p^*_1)$ and $\rho^A_{A1}(p^*_1)$ becomes smaller, because $\eta^A_{A1}(p^*_1)$ decreases at a faster rate than $\rho^A_{A1}(p^*_1)$. This effect dominates the second offsetting effect that raises the ratio $\frac{X^*}{Q^*_1+X^*}$, through an increase in X^*. Finally, comparable arguments also apply for the effects of μ and κ on the period 1 and 2 cross-price elasticity biases, $\theta^B_{A1}(p^*_1)$ and $\theta^B_{A2}(p^*_2)$.

[5] At first glance, it may seem strange that an increase in product differentiation makes both $\eta^A_{At}(p^*_1)$ and $\rho^A_{At}(p^*_1)$ more elastic. Commonly, an increase in product differentiation makes demand less elastic. However, here we are evaluating the elasticities at the equilibrium price, p^*_1, which also increases with μ. Consequently, there is a direct and indirect effect on these elasticities. The direct effect reduces $\eta^A_{A1}(p^*_1)$ and $\rho^A_{A1}(p^*_1)$ by making them less elastic, but the indirect effect raises them. In both cases, the indirect effects dominate such that $\rho^A_{A1}(p^*_1)$ and $\eta^A_{A1}(p^*_1)$ increase with μ.

3.5.3 Discussion and Relation to the Literature

The results of this section can be brought together in Figure 3.3, which illustrates the relationships between each bias and the degree of product differentiation, μ. It shows that there is no period 1 own- or cross-price elasticity bias when $\mu < 3\kappa$, and that the biases are positive and upward sloping thereafter. While the period 1 cross-price elasticity bias, $\theta^B_{A1}(p^*_1)$, is continuous with a kink at $\mu = 3\kappa$, period 1 own-price elasticity bias, $\theta^A_{A1}(p^*_1)$, is discontinuous at 3κ, jumping from 0 to $\frac{1}{3}$. In contrast, the period 2 own- and cross-price elasticity biases are only relevant when $\mu > 3\kappa$ in which case they are positively signed. They are strictly decreasing in μ over this range and tend to 1 as $\mu \to 3\kappa$.

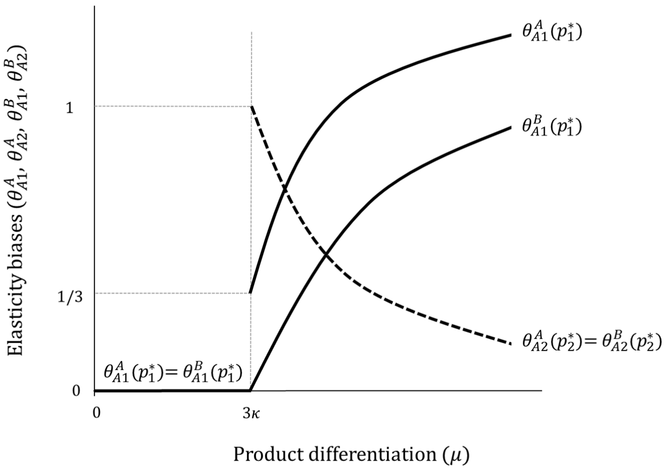


Figure 3.3: The Relationships between the Elasticity Biases and Product Differentiation

We now discuss the predicted signs of the biases in relation to the existing literature. By concentrating on situations where observed demand is more elastic than true demand, the previous literature has stressed that the own-price elasticity bias is always positive (e.g. Hendel and Nevo, 2006a, 2006b, 2013; Perrone, 2017). In contrast, the existing literature has recognised that the cross-price elasticity bias can be positive or negative, but has focussed on cases where it is negative (e.g. Hendel and Nevo, 2006b, 2013). For instance, Hendel and Nevo (2013) provide an intuition where a firm has undercut its rival and stimulated stockpiling in a previous period. Then if the firm raises its price, the true cross-price effect will be underestimated because consumers will be consuming from their inventories rather than switching to the rival.

Contrary to this literature, our general theoretical foundation has shown that the biases can be positive, negative, or zero depending upon some intuitive conditions. Moreover, within the example equilibrium model of this Section, our results demonstrate how both elasticity biases can be zero when product differentiation is sufficiently low, despite positive consumer stockpiling. For higher levels of product differentiation, we find that the own-price elasticity bias is positive as consistent with the existing literature, but suggest that the cross-price elasticity bias is also positive. This difference arises because of the following. In our model, the observed and true cross-price effects in period 2 are the same because period 1 stockpiling demand is insensitive to actual period 2 prices, such that $\frac{\partial \hat{Q}_{i2}}{\partial p_{j2}} = \frac{\partial Q_{i2}}{\partial p_{j2}} - \frac{\partial X_i}{\partial p_{j2}} = \frac{\partial Q_{i2}}{\partial p_{j2}}$. Thus, the elasticity bias is driven solely by the fact that the level of observed demand is less than the level of true demand, prompting it to be positive rather than negative, $\hat{\rho}_{i2} = \frac{\partial Q_{i2}}{\partial p_{j2}} \frac{p_{j2}}{\hat{Q}_{i2}} > \rho_{i2} \; \forall \hat{Q}_{i2} < Q_{i2}$.

Finally, we discuss when our predicted biases matter most. Figure 3.3 illustrates that the period 1 own- and cross-price elasticity biases are greater (less) than their period 2 counterparts when product differentiation is high (low). This implies that the own- and cross-price elasticity biases in period 1 (i.e. when consumers are added to their inventories) are of least concern in markets where product differentiation is low, but the elasticity biases in period 2 (i.e. when consumers are reducing their inventories) are of least concern when product differentiation is high. In addition, notice that an increase in transaction costs, κ, extends the range where there is no elasticity biases in period 1, but when the elasticity biases are positive, it shifts the period 1 elasticity biases down and the period 2 elasticity biases up. This implies that the period 1 own- and cross-price elasticity biases are of less concern in markets with high transaction costs, but the period 2 elasticity biases are of more concern.

3.6 Conclusions

In this chapter, we developed a full model of stockpiling in a differentiated products duopoly with endogenous prices over two periods. In any symmetric equilibrium, a (weak) subset of the consumers stockpile to avoid additional transaction costs. We found that the equilibrium measures of both the own- and cross elasticity biases are zero when product differentiation is low, but are strictly positive when product differentiation is high. To further consider when the biases matter most, we showed that they are strictly increasing in the degree of product differentiation when net stockpiling is positive, but strictly decreasing in the degree of product differentiation in period 2, when net stockpiling is negative.

Appendix:

Proof of Lemma 3.1. From (3.5) and (3.6), consumer m will stockpile if a) $\widehat{S}_{im} = U^S_{im} - U^S_{jm} \geq 0$ and b) $\widetilde{S}_{im} = U^S_{im} - U^{NS}_m \geq 0$. Suppose condition a) holds, with $U^S_{im} \geq U^S_{jm}$ which implies $(\varepsilon_{im} - p_{i1}) - (\varepsilon_{jm} - p_{j1}) \geq 0$. It then follows from (3.4) that $U^{NS}_m = u_{im}(p_{i1}) + max\{u_{Am}(p^e_{A2}), u_{Bm}(p^e_{B2})\} - 2\kappa$, such that condition b) can be rewritten as $\widetilde{S}_{im} = (\varepsilon_{im} - p_{i1}) - \max\{\varepsilon_{im} - p^e_{i2}, \varepsilon_{jm} - p^e_{j2}\} + \kappa \geq 0$. Therefore, $\widehat{S}_{im}$ (or $\widetilde{S}_{im}$) is then strictly (or weakly) increasing in consumer m's relative brand preference for firm i, $(\varepsilon_{im} - \varepsilon_{jm}) \in [-\mu, \mu]$. □

Proof of Lemma 3.2. If $\psi^s_A = \psi^s_B$, all consumers stockpile. Here, using (3.3), any given consumer will buy two units from i rather than j in period 1 if $2\,(\varepsilon_i - p_{i1}) - \kappa \geq 2\,(\varepsilon_j - p_{j1}) - \kappa$. This comparison reduces down to that in the benchmark, $\varepsilon_i - p_{i1} > \varepsilon_j - p_{j1}$. Hence, firm i's total period 1 demand equals $\widehat{Q}_{i1}(.) = 2Q_{i1}(.)$, where $Q_{i1}(p_{i1}, p_{j1})$ coincides with the benchmark demand, (3.1).

If, instead, $\psi^s_A > \psi^s_B$, some consumers only buy one unit in period 1. As in the benchmark, such consumers will buy one unit from firm i rather than j if $u_{im}(p_{i1}) - \kappa \geq u_{jm}(p_{j1}) - \kappa$ and one can define ψ_1 as the value of $\psi_m = p_{A1} - p_{B1}$ at which such a consumer would be indifferent. Hence, a total of $Q_{i1}(p_{i1}, p_{j1})$ consumers buy from firm i, of which $Q_{i1}(p_{i1}, p_{j1}) - X_i\,(\psi^s_i)$ consumers buy one unit and $X_i\,(\psi^s_i)$ consumers buy two units, such that total demand equals $\widehat{Q}_{i1}(.) = Q_{i1}(p_{i1}, p_{j1}) + X_i\,(\psi^s_i)$. □

Proof of Lemma 3.3. If $\psi^s_A = \psi^s_B$, all consumers stockpile and so $\widehat{Q}_{i2}(.) = 0$. If instead, $\psi^s_A > \psi^s_B$, then consumers with $\psi_m \in (\psi^S_B, \psi^S_A)$ did not stockpile and so remain active. As in the benchmark, any such consumer will then buy one unit from firm i rather than j if $u_{im}(p_{i2}) - \kappa \geq u_{jm}(p_{j2}) - \kappa$, and one can define ψ_2 as the value of $\psi_m = p_{A2} - p_{B2}$ at which such a consumer would be indifferent.

Around any symmetric equilibrium, $\psi_2 \in (\psi_B^s, \psi_A^s)$. Hence, there is a positive measure of consumers with $\psi_m \in (\psi_B^S, \psi_2)$ that strictly prefer to buy from firm B and a positive measure of consumers with $\psi_m \in (\psi_2, \psi_A^S,)$ that strictly prefer to buy from firm A. This implies that firm $i's$ total period 2 demand is equal to the benchmark demand, $Q_{i2}(p_{i2}, p_{j2})$ from (3.1), minus those consumers that stockpiled from firm i in period 1, $X_i\left(\psi_i^s\right)$ from (3.8). $\square$

Proof of Lemma 3.4. Suppose $\psi_A^s > \psi_B^s$. Then in any symmetric equilibrium, it must be the case that $\psi_2 \in (\psi_B^s, \psi_A^s)$ such that both firms have positive demand. Then one can use $\pi_{i2}(.) = p_{i2}\widehat{Q}_{i2}(.)$ with (3.8) to derive the firms' period 2 best responses for given stockpiling levels, $p_{i2}^*(p_{j2}) = \frac{p_{j2}}{2} + \frac{\mu}{2}\left(\frac{1}{2} - X_i(\psi_i^s)\right)$ for $i\ j \neq i \in \{A, B\}$. Solving simultaneously yields the unique period 2 equilibrium prices, $p_{i2}^* = \frac{\mu}{6}\Big[3 - 4X_i\left(\psi_i^s\right) - 2X_j\left(\psi_j^s\right)\Big]$, and substituting these back into (3.8) gives $\widehat{Q}_{i2}^*(.) = \frac{1}{6}\left[3 - 4X_i\left(\psi_i^s\right) - 2X_j\left(\psi_j^s\right)\right]$. All such prices and demands are positive if $4X_i\left(\psi_i^s\right) + 2X_j\left(\psi_j^s\right) < 3$ for all $i, j \neq i \in A, B$, which ensures $\psi_2 \in (\psi_B^s, \psi_A^s)$ as claimed. $\square$

Proof of Proposition 3.1. Having derived period 2 equilibrium prices, we first consider consumers' stockpiling decisions, before deriving the equilibrium levels of stockpiling demand as a function of period 1 prices in (3.10).

First, consider consumers' stockpiling decisions and initially suppose that each firm has positive period 2 demand with $\psi_2 \in (\psi_B^s, \psi_A^s)$. This implies that a consumer at ψ_i^s makes her stockpiling decision by comparing i) the net marginal benefits of stockpiling from firm i, with ii) the net marginal benefits of waiting to buy from firm i, rather than firm j, in period 2. From (3.6), this implies $\widetilde{S}_{im}(\psi_i^s) = (\varepsilon_{im} - p_{i1}) - (\varepsilon_{im} - p_{i2}^*(.)) + \kappa$. By construction, the consumer at ψ_i^s is indifferent between stockpiling, such that $\widetilde{S}_{im}(\psi_i^s) = 0$. Hence, this indifference requires $p_{i1} = p_{i2}^*\left(X_i, X_j\right) + \kappa$.

We are now in a position to derive the equilibrium levels of stockpiling demand as a function of period 1 prices. First, consider the top line of (3.10). Here, $p_{A1} > p^*_{A2}(0,0)+\kappa$ and $p_{B1} > p^*_{B2}(0,0)+\kappa$ such that no consumer finds it optimal to stockpile, $X_A = X_B = 0$. From (3.9), $p^*_{i2}(0,0) = \frac{\mu}{2}$ for both $i = \{A,B\}$, and so this case occurs when $p_{A1} > \frac{\mu}{2} + \kappa$ and $p_{B1} > \frac{\mu}{2} + \kappa$.

Second, consider the bottom line of (3.10). Here, $p_{A1} \leq p^*_{A2}\left(\frac{1}{2},\frac{1}{2}\right) + \kappa$ and $p_{B1} \leq p^*_{B2}\left(\frac{1}{2},\frac{1}{2}\right) + \kappa$ such that all consumers find it optimal to stockpile, $X_A = X_B = \frac{1}{2}$. Period 2 prices are unspecified as period 2 is inactive. However, if the marginal consumer at $\psi^s_A = \psi^s_B = 0$ were to deviate from stockpiling, we know from (3.9) that she should rationally expect zero period 2 prices, $\lim_{X_i \to 0.5} p^*_{i2}\left(X_i, \frac{1}{2}\right) = 0$. Hence, this case occurs when $p_{A1} \leq \kappa$ and $p_{B1} \leq \kappa$.

Third, consider the middle line of (3.10). Here, there exists a unique level of equilibrium stockpiling, $X_i(.) \in (0, \frac{1}{2})$ and $X_j(.) \in (0, \frac{1}{2})$, such that $p_{i1} = p^*_{i2}(X_i, X_j) + \kappa$ holds for each firm. To find such $X_i(.)$ and $X_j(.)$, one can insert p^*_{i2} from (3.9) to obtain

$$X_i(.) = \frac{3}{4} - \frac{X_j(.)}{2} - \frac{3(p_{i1} - \kappa)}{2\mu}. \tag{3.14}$$

After deriving a similar equation for $X_j(.)$ and solving simultaneously, one finds a unique level of $X_i(.) = \frac{1}{2} - \left(\frac{2p_{i1} - p_{j1} - \kappa}{\mu}\right)$ for $i\ j \neq i \in \{A, B\}$. For $X_i \in (0, \frac{1}{2})$, we require $p_{i1} \in \left(\frac{\kappa + p_{j1}}{2}, \frac{\frac{\mu}{2} + \kappa + p_{j1}}{2}\right]$ for each firm.

Finally, note that the levels of stockpiling and associated conditions in (3.10) are continuous as i) $\frac{1}{2} - \left(\frac{2p_{i1} - p_{j1} - \kappa}{\mu}\right) = \frac{1}{2}$ when $p_{i1} = p_{j1} = \kappa$, ii) $\frac{1}{2} - \left(\frac{2p_{i1} - p_{j1} - \kappa}{\mu}\right) = 0$ when $p_{i1} = p_{j1} = \frac{\mu}{2} + \kappa$, iii) $\frac{\kappa + p_{j1}}{2} = \kappa$ when $p_{i1} = \kappa$, and iv) $\frac{1}{2}\left[\frac{\mu}{2} + \kappa + p_{i1}\right] = \frac{\mu}{2} + \kappa$ when $p_{i1} = \frac{\mu}{2} + \kappa$. □

Proof of Proposition 3.2. From (3.10), $X_A = X_B = 0$ necessarily requires $p_{A1} > \frac{\mu}{2} + \kappa$ and $p_{B1} > \frac{\mu}{2} + \kappa$. However, we know from the benchmark analysis in Section 3.3.2 that $X_A = X_B = 0$ is consistent with $p_1^* = \frac{\mu}{2}$. This then leads to a contradiction as $p_1^* = \frac{\mu}{2} < \frac{\mu}{2} + \kappa$ for all $\kappa > 0$. $\square$

Proof of Proposition 3.3. First suppose that period 2 is active with $\psi_A^s > \psi_B^s$ such that $X_A(.) + X_B(.) < 1$. In any symmetric equilibrium each firm has positive period 2 demand with $\psi_2 \in (\psi_B^s, \psi_A^s)$. Using (3.8) and (3.9), firm i's profit function from (3.11) can then be rewritten as:

$$\pi_i(.) = p_{i1}[Q_{i1}(p_{i1}, p_{j1}) + X_i(.)] + \mu \left(\frac{3 - 4X_i(.) - 2X_j(.)}{6} \right)^2 \tag{3.15}$$

where $Q_{i1} = \frac{1}{2} + \frac{p_{j1} - p_{i1}}{\mu}$ from (3.7), and where $X_i(.)$ and $X_j(.)$ are given in (3.10). To maximise (3.15) with respect to p_{i1} note that $\frac{\partial \pi_{i1}}{\partial p_{i1}}$ equals

$$Q_{i1}(.) + X_i(.) + p_{i1} \left(\frac{\partial Q_{i1}(.)}{\partial p_{i1}} + \frac{\partial X_i(.)}{\partial p_{i1}} \right) + \frac{\mu}{3} (3 - 4X_i(.) - 2X_j(.)) \left(-\frac{2}{3} \frac{\partial X_i(.)}{\partial p_{i1}} - \frac{1}{3} \frac{\partial X_j(.)}{\partial p_{i1}} \right),$$

where $\frac{\partial Q_{i1}(.)}{\partial p_{i1}} = -\frac{1}{\mu}$, $\frac{\partial X_i(.)}{\partial p_{i1}} = -\frac{2}{\mu}$ and $\frac{\partial X_j(.)}{\partial p_{i1}} = \frac{\partial X_i(.)}{\partial p_{j1}} = \frac{1}{\mu}$. After expanding, enforcing symmetry with $p_{i1} = p_{j1} = p_1^*$, and setting equal to zero, one obtains a unique value for $p_1^* = \frac{\mu - \kappa}{2}$. There are no profitable local deviations as the associated second-order condition ensures local concavity, $\frac{\partial^2 \pi_i}{\partial p_{i1}^2} = -\frac{4}{\mu} < 0$. Then substituting p_1^* into (3.10), (3.9) and (3.1) provides the unique values for X^*, p_2^*, $\widehat{Q}_1^*$ and $\widehat{Q}_2^*$ as claimed. For period 2 to be active as assumed, we require $X^* = \frac{3\kappa}{2\mu} < 0.5$. This implies $\mu > 3\kappa$, which further ensures that the equilibrium is well-defined with non-negative prices.

Second suppose that period 2 is inactive with $\psi_A^s = \psi_B^s$ such that $X_A(.) + X_B(.) = 1$. Firm i's profit function then equals $\pi_i(.) = p_{i1}[\widehat{Q}_{i1}(p_{i1}, p_{j1})]$ which can be rewritten as follows using (3.7):

$$\pi_i(.) = p_{i1}[2Q_{i1}(p_{i1}, p_{j1})] = p_{i1} \left[\frac{1}{2} + \frac{p_{j1} - p_{i1}}{\mu} \right] \tag{3.16}$$

However, to ensure $X_A(.) + X_B(.) = 1$, we know from Proposition 3.1 that (3.16) must be maximised subject to $p_{i1} \leq \kappa$. After solving and enforcing symmetry, this leads to a unique local maximum with $p_1^* = \min\{\frac{\mu}{2}, \kappa\}$ and $X^* = \frac{1}{2}$. There are no profitable local deviations because the associated second-order condition ensures local concavity, $\frac{\partial^2 \pi_i}{\partial p_{i1}^2} = -\frac{2}{\mu} < 0$. When $\mu > 3\kappa$, we know from above that any symmetric equilibrium must have $X^* < 0.5$ which is inconsistent with this case. Hence, this case requires $\mu \in (0, 3\kappa]$.

□

Proof of Proposition 3.4. From (3.12), the bias for $h \in \{A, B\}$ is $\theta_{A1}^h\left(p_1^*\right) = \frac{X^*}{Q_1^*+X^*}\left(\eta_{A1}^h\left(p_1^*\right) - \rho_{A1}^h\left(p_1^*\right)\right)$, where $Q_1^* = \frac{1}{2}$ and $X^* = \frac{1}{2}$ from Proposition 3.3. From Lemma 3.2, $Q_{A1}\left(.\right) = X_A\left(.\right) = \frac{1}{2} + \frac{p_B - p_A}{\mu}$, such that $\eta_{A2}^h\left(p_1^*\right) = \rho_{A1}^h\left(p_1^*\right) = \frac{2p_1^*}{\mu}$. Thus, $\theta_{A1}^h\left(p_1^*\right) = 0$ for $h \in \{A, B\}$. □

Proof of Proposition 3.5. i) From (3.12), the bias for $h \in \{A, B\}$ is $\theta_{A1}^h\left(p_1^*\right) = \frac{X^*}{Q_1^*+X^*}\left(\eta_{A1}^h\left(p_1^*\right) - \rho_{A1}^h\left(p_1^*\right)\right)$, where $Q_1^* = \frac{1}{2}$, $X^* = \frac{3\kappa}{2\mu}$, and $p_1^* = \frac{\mu-\kappa}{2}$ from Proposition 3.3. Given $Q_{A1}\left(.\right) = \frac{1}{2} + \frac{p_B - p_A}{\mu}$ and $X_A\left(.\right) = \frac{1}{2} - \left(\frac{2p_{A1} - p_{B1} - \kappa}{\mu}\right)$ from Lemma 3.2 and Proposition 3.1, then $\rho_{A1}^h\left(p_1^*\right) = \frac{\mu-\kappa}{\mu}$ for $h \in \{A, B\}$, and $\eta_{A1}^A\left(p_1^*\right) = \frac{2(\mu-\kappa)}{3\kappa}$ and $\eta_{A1}^B\left(p_1^*\right) = \frac{(\mu-\kappa)}{3\kappa}$. Thus, $\theta_{A1}^A\left(p_1^*\right) = \frac{3(\mu-\kappa)}{\mu+3\kappa} - \left(\frac{\mu-\kappa}{\mu}\right) = \left(\frac{\mu-\kappa}{\mu}\right)\left(\frac{2\mu-3\kappa}{\mu+3\kappa}\right)$ and $\theta_{A1}^B\left(p_1^*\right) = \frac{2(\mu-\kappa)}{\mu+3\kappa} - \left(\frac{\mu-\kappa}{\mu}\right) = \left(\frac{\mu-\kappa}{\mu}\right)\left(\frac{\mu-3\kappa}{\mu+3\kappa}\right)$. Given $\mu > 3\kappa > 0$, these are both strictly positive.

ii) From (3.13), the bias in period 2 for $h \in \{A, B\}$ is $\theta_{A2}^h\left(p_2^*\right) = \frac{-X^*}{Q_2^*-X^*}\left(\eta_{A2}^h\left(p_2^*\right) - \rho_{A2}^h\left(p_2^*\right)\right)$, where $Q_2^* = \frac{1}{2}$, $X^* = \frac{3\kappa}{2\mu}$, and $p_2^* = \frac{\mu-3\kappa}{2}$ from Proposition 3.3 such that $\eta_{A2}^A(p_2^*) = \eta_{A2}^B(p_2^*) = 0$ and $\rho_{A2}^A(p_2^*) = \rho_{A2}^B(p_2^*) = \frac{2p_2^*}{\mu} = \frac{\mu-3\kappa}{\mu}$. Thus, $\theta_{A2}^h\left(p_2^*\right) = 1 - \left(\frac{\mu-3\kappa}{\mu}\right) = \frac{3\kappa}{\mu}$ for $h \in \{A, B\}$. Given $\mu > 3\kappa > 0$, these are both strictly positive. □

Proof of Proposition 3.6. Differentiating $\theta^A_{A1}(p^*_1) = \frac{3(\mu-\kappa)}{\mu+3\kappa} - \left(\frac{\mu-\kappa}{\mu}\right)$ with respect to μ and κ yields:

$$\frac{\partial\theta^A_{A1}(p^*_1)}{\partial\mu} = \frac{\kappa\left[11\mu^2-6\mu\kappa-9\kappa^2\right]}{\mu^2(\mu+3\kappa)^2} \qquad \text{and} \qquad \frac{\partial\theta^A_{A1}(p^*_1)}{\partial\kappa} = -\frac{\mu\left[11\mu^2-6\mu\kappa-9\kappa^2\right]}{\mu^2(\mu+3\kappa)^2},$$

respectively. Note that $[11\mu^2 - 6\mu\kappa - 9\kappa^2]$ can be rewritten as $[3\kappa+(1+2\sqrt{3})\mu][3\kappa+(1-2\sqrt{3})\mu]$ such that $\frac{\partial\theta^A_{A1}(p^*_1)}{\partial\mu} > 0$ and $\frac{\partial\theta^A_{A1}(p^*_1)}{\partial\kappa} < 0$ for all $\mu > 3\kappa$.

Furthermore, differentiating $\theta^B_{A1}(p^*_1) = \frac{2(\mu-\kappa)}{\mu+3\kappa} - \left(\frac{\mu-\kappa}{\mu}\right)$ with respect to μ and κ yields:

$$\frac{\partial\theta^B_{A1}(p^*_1)}{\partial\mu} = \frac{\kappa\left[7\mu^2-6\mu\kappa-9\kappa^2\right]}{\mu^2(\mu+3\kappa)^2} \qquad \text{and} \qquad \frac{\partial\theta^B_{A1}(p^*_1)}{\partial\kappa} = -\frac{\mu\left[7\mu^2-6\mu\kappa-9\kappa^2\right]}{\mu^2(\mu+3\kappa)^2},$$

respectively. Note that $[7\mu^2 - 6\mu\kappa - 9\kappa^2]$ can be rewritten as $[3\kappa+(1+2\sqrt{2})\mu][3\kappa+(1-2\sqrt{2})\mu]$ such that $\frac{\partial\theta^B_{A1}(p^*_1)}{\partial\mu} > 0$ and $\frac{\partial\theta^B_{A1}(p^*_1)}{\partial\kappa} < 0$ for all $\mu > 3\kappa$.

Finally, differentiating $\theta^h_{A2}(p^*_2) = \frac{3\kappa}{\mu}$ with respect to μ and κ yields $\frac{\partial\theta^h_{A2}\left(p^*_2\right)}{\partial\kappa} = \frac{3}{\mu} > 0$ and $\frac{\partial\theta^h_{A2}\left(p^*_2\right)}{\partial\mu} = -\frac{3\kappa}{\mu^2} < 0$ for $h \in \{A, B\}$, given $\mu > 0$ and $\kappa > 0$. $\square$

Chapter 4

Consumer Stockpiling and Market Entry

4.1 Introduction

Having studies how own- and cross-price elasticities are incorrectly biased from ignoring consumer stockpiling behaviour in a duopoly setting in Chapter 3, it will now be of interest to further investigate the role of number of firms, as most suitable stockpiling product market are oligopoly market. In the existing literature, very little remains known about how the number of firms affects consumer stockpiling behaviour and the consequent elasticity biases. Addressing this omission is imperative to further understand storable product markets and to help policymakers know when the exclusion of consumer stockpiling behaviour matters most.

In this chapter, we consider consumer stockpiling behaviour and the implied elasticity biases within an spatial n-firm oligopoly setting. As a primary contribution, this chapter firstly studies how the number of firms determines consumer stockpiling behaviour and the biases of own- and cross-price elasticities for a fixed level of market entry. This chapter then analyses how these conclusions change when free entry is allowed and the number of firms becomes endogenous. As a secondary contribution, since consumer stockpiling behaviour involves intra-period demand shifts that might potentially affect firms' entry decisions, this chapter studies the extent to which the excessive entry theorem still applies under consumer stockpiling.

In more detail, we introduce consumer stockpiling into a two-period differentiated products oligopoly based on Salop circular city model (1979). In each period, each consumer wishes to consume exactly one unit, but is allowed to purchase a second unit for future consumption. As consistent with many retail markets, consumers incur positive transaction costs every time they make a purchase from a firm. This transaction costs are independent of the number of units bought. Among other potential sources, such transaction costs can arise from the costs of locating a product in the supermarket, visiting a firm or ordering a delivery, or the costs of simply remembering to make a purchase.

We first treat the number of firms as exogenous, and show that for any positive transaction cost, any symmetric equilibrium must involve positive consumer stockpiling. This is because transaction cost encourages consumers to stockpile in advance so that any expected transaction cost in the future can be avoided. We then offer the result that the total level of stockpiling demand is weakly increasing in the number of firms. Intuitively, an increase in the number of firms ensures that the average distance between a consumer and their nearest firm reduces such that weakly more consumers find it optimal to stockpile.

Following Chapter 3, but with n-firm oligopoly rather than duopoly, we examine the signs of elasticity biases by comparing the elasticity of true demand, which is independent from consumer stockpiling behaviour, and that of observed demand, which derives from consumer stockpiling behaviour. Our findings show that with positive stockpiling equilibrium, biases of both own- and cross-price elasticities can be positive or zero, depending on the number of entrants in the market. When the number of firms is small, both own- and cross-price elasticities biases are strictly positive, and strictly increasing in the number of firms. On the contrary, in a market with large number of firms such that all consumers stockpile, then both own- and cross-price elasticities biases can be negligible. The implication of this result is that these biases of elasticity matter most in a market with small number of firms.

The latter part of this chapter then allows the number of firms to be endogenous. We first compare the equilibrium number of entrants with the socially efficient level. The entry of a new firm has several different effects i) it reduces firms' profits by reducing each firm's market share and the equilibrium prices in both periods, ii) it raises consumer surplus by reducing the prices in both periods, and by (weakly) increasing the level stockpiling such that consumers' expenditure on transaction costs also weakly fall. However, despite the extra inter-temporal effects on consumer stockpiling, we find that the negative effects on firms' profits always dominate such that entry is always excessive. Second, and most substantially, we then revisit the implied elasticity biases under free entry. We show that in a market with high fixed entry cost (equivalently small endogenous number of firms), the biases are increasing (decreasing) with product differentiation level (transaction cost). This confirms that the previous findings of Chapter 3 are robust to free entry and shows that the biases of own- and cross-price elasticities matter most in markets with a small number of firms.

The approach we use in this chapter is closely related to Guo and Villas-Boas (2007). We develop their duopoly model by a) showing how transaction costs provide a tractable and realistic source of positive stockpiling equilibrium, and b) by extending it to an oligopoly to consider firms' entry decisions. In addition, previous papers that empirically study the biases of price elasticity caused by ignoring consumer stockpiling behaviour are also relevant. (see Erdem *et al*, 2003. Hendel and Nevo, 2006b, and 2013, and Perrone, 2017). As opposed to these papers which argue cross-price elasticity is either negative or difficult to measure, we show that both-own and cross-price elasticity are either positive or zero depending on the sign and level of equilibrium stockpiling demand. This confirms the robustness of the results of Chapter 3, but also extends it by examining the role of the number of firms in the market under both fixed and free entry.

Previous theoretical models of consumer stockpiling in n-firm have been rare in the previous literature, and have focused on homogeneous product without free entry. (see Salop and Stiglitz, 1982, Sobel, 1984, and Pesendorfer 2002). These papers cover several topics, including price dispersion, price discrimination and price discount. In contrast, we study market entry problem and consumer stockpiling in a differentiated product n-firm market.

Finally, our study adds to the literature on excessive entry. In the seminal papers of oligopoly spatial model, Vickery (1964) and Salop (1979) shows that the equilibrium number of firms is larger than the socially optimal level. Some later literatures also confirm that excessive entry theorem applies in a non-spatial Cournot competition setting (see Mankiw and Whinston, 1986, Suzumura and Kiyono, 1987), and more papers have used the Salop circle model to analyse the extent to which excess entry applies in a variety of other settings (e.g. Gu and Wenzel 2009, 2012, 2015. Matsumura and Okamura 2006a, 2006b). The most recent paper is Chen and Zhang (2018) who study entry problem in a model of consumer search. In contrast, we revisit the market entry problem by considering it in a dynamic environment where consumer's demand varies with their stockpiling decisions and examine the relationship between the equilibrium and the socially efficient number of firms.

This chapter proceeds as follows. Section 4.2 introduces the model. Section 4.3 assumes a fixed number of firms. It presents the model and stockpiling equilibrium, before considering how the number of firms affects the implied biases of elasticities. Section 4.4 then allows the number of firms to be endogenous under free entry.

It compares the equilibrium number of firms to the socially optimal level, and then analyses the implications for the elasticity biases. Section 4.5 concludes. All proofs are in the appendix.

4.2 Model

Consider a market with $n \geq 2$ single product firms, $i \in \{2, ..., n\}$ sell a single, horizontally differentiated, storable product with zero cost over two periods, $t = 1, 2$. To capture firms' location, we use Salop's (1979) circular city framework. Specifically, there are $n \geq 2$ symmetric firms evenly located along the unit-long circle. Firms incur a fixed entry cost, denoted by $f \geq 0$, to enter the market. They compete in price to sell a single differentiated storable product which can either be consumed immediately or stockpiled for future consumption. But it can only be consumed once. Consumers of mass one are uniformly distributed along the circular city. Consumer's locations on the circle are fixed throughout the game and each consumer places a value of gross utility, λ, from consuming one unit of any product. We make an assumption that consumer's gross utility λ to be sufficiently large such that no consumer abstain from buying differentiated product. Define x_i as the distance between consumer's preferred location and its closest firm, say, firm i. When this consumer buying from firm i and paying price p_i, she derives utility as follows, where t can be interpreted as a parameter that measures product differentiation.

$$u_i(p_i) = \lambda - p_i - \mu x_i \quad x_i \in [0, \frac{1}{n}]$$

Similar to Chapter 3, we also assume that consumer's transactions are costly. Specifically, each time a consumer makes a purchase of one or more units from any given firm, she incurs a transaction cost, $\kappa > 0$, that is independent of the number of units bought. Typical examples include the costs of visiting a firm or ordering a delivery. Since the appropriate products that fit to this study are purchased and consumed frequently, any potential inventory cost is normalised to zero and intertemporal discounting factor is assumed to be zero.

What follows describes the timing of the game. The game consists of $t = 1, 2$ periods. Particularly, in period 1 each firm sets its period 1 price simultaneously. Consumers then realise their location and the distance between themselves and their closest firm, x, and observe the price of each firm before making decisions about where to buy and how many units to buy. If a consumer buys 1 unit for period 1 consumption and 1 unit for period 2 consumption, then she will no longer be active in period 2 market. In period 2, each firm sets period 2 price at the same time after observing its rivals' period 1 prices and consumers' period 1 actions (whether stockpiled or not). Remaining consumers then observe these prices and decide from which firm to buy. We look for pure-strategy symmetric equilibria where the firms set period 1 price, p_1^*, and period 2 price, p_2^*. Like Chapter 3, we focus on characterising on the unique local symmetric equilibrium. Hence, we need only consider local deviations around a potential symmetric equilibrium. Throughout, we will analyse pricing equilibria by supposing that firm i sets price p_{it} in period t while all other firms set the same rival price, p_{-it}.

4.3 Fixed Number of Firms

To begin, we treat the number of firms, n, as fixed and exogenous. In Section 4.3.1, we first examine a benchmark case where consumer stockpiling behaviour is not feasible. Section 4.3.2 then covers the equilibrium analysis, before examining the implied biases of own- and cross-price elasticities in Section 4.3.3.

4.3.1 Benchmark

This benchmark considers the case where consumer stockpiling is not feasible. Hence period 1 and period 2 markets become static and identical. As an assumption of the model, the n firms are symmetrically located along the unit-long circle such that the distance between each firm is $\frac{1}{n}$. Following Tirole (1988), firm i's rivals are the two firms that are located next to itself. Consider a marginal consumer, who is indifferent between buying from firm i and an adjacent firm, denoted by $-i$, is $\bar{x}$ away from firm i and $\frac{1}{n} - \bar{x}$ away from firm $-i$. If all firms other than firm i price at p_{-it}, then for this marginal consumer, it follows that,

$$u_{it}(p_{it}) = u_{-it}(p_{-it}) \Longleftrightarrow \lambda - p_{it} - \mu\bar{x} - \kappa = \lambda - p_{-it} - \mu(\frac{1}{n} - \bar{x}) - \kappa \quad x \in [0, \frac{1}{n}] \tag{4.1}$$

The equation above suggests that marginal consumer earns the same utility from purchasing in either firm i or firm $-i$. Isolating the expression of $\bar{x}$ in (4.1) yields,

$$\bar{x} = \frac{p_{-it} - p_{it}}{2t} + \frac{1}{2n} \tag{4.2}$$

$\bar{x}$ can be defined similarly as the number of consumers that buy from firm i on only one side. Adding those who are are located in between firm i and its rival on the other side firm i's total demand in period t equals $2\bar{x}$.

$$Q_{it}(p_{it}, p_{-it}) \equiv 2\bar{x} = \frac{p_{-it} - p_{it}}{\mu} + \frac{1}{n} \tag{4.3}$$

Applying the usual first order condition, $p^* = -\frac{Q_{it}(p_{it}, p_{-it})}{Q'_{it}(p_{it}, p_{-it})}$, and solving for the symmetric response functions gives rise to the symmetric equilibrium price of each firm $p_{it}^{NS*} = \frac{\mu}{n}$ for each period. As each firm has an equilibrium demand of $\frac{1}{n}$ per period, each firm then earns $\pi^{NS*} = \frac{2\mu}{n^2} - f$ in aggregate.

4.3.2 Equilibrium Analysis

Before proceeding to the main equilibrium analysis, we first cover some period 1 stockpiling preliminaries for a given set of period 1 prices $\{p_{i1},\, p_{-i1}\}$, and expected period 2 price $\{p^e_{i2},\, p^e_{-i2}\}$.

4.3.2.1 Period 1 Preliminaries

Consider a consumer that locates a distance x away from firm i and $(\frac{1}{n} - x)$ away from an adjacent firm $-i$. The consumer's period 1 available options involve three alternatives. Firstly, a consumer could choose to stockpile from firm i to gain[1],

$$U^S_i = 2u_i(p_{i1}) = 2(\lambda - p_{i1} - \mu x) - \kappa \tag{4.4}$$

This consumer could also build up inventory by buying one unit from firm i and stockpiling from another adjacent firm, $-i$. The utility gained from doing so is given by,

$$U^S_{-i} = u_i(p_{i1}) + u_{-i}(p_{-i1}) - 2\kappa = [\lambda - p_{i1} - \mu x - \kappa] + [\lambda - p_{-i1} - \mu(\frac{1}{n} - x) - \kappa] \tag{4.5}$$

Alternatively, the consumer could choose not to stockpile by purchasing one unit in each period to gain,

$$U^{NS} = (\lambda - p_{i1} - \mu x) - \kappa + Max\{u_i(p^e_{i2}),\, u_{-i}(p^e_{-i2})\} - \kappa \tag{4.6}$$

We first compare the options of stockpiling from the same store and stockpiling from different stores. Subtracting (4.5) from (4.4) gives $U^S_i - U^S_{-i} > 0$, which

[1] Quadratic disutility is used to fix the intractability problem of its linear counterpart, arising in the case of asymmetric locations with even number of firms. Otherwise, quadratic or linear disutility functions of location model are invariant. See Hoernig (2015).

suggests that if building up inventory is determined, consumer would always prefer to stockpile from the same store rather than from two different stores because stockpiling from the same firm avoids incurring transaction costs repetitively. This guarantees that consumer who stockpiles from any firm will always be a subset of consumer who buys from that firm in period 1. We then compare the options between stockpiling from a single firm and not stockpiling by buying a single unit in each period. Subtracting U^{NS} from U_i^S gives $-p_{i1} - \mu x + max\{tx + p_{i2}^e, \mu(\frac{1}{n} - x) + p_{-i2}^e\}$, which is weakly decreasing in x. This indicates that as x decreases, the relative desirability of stockpiling from firm i increases. We can now propose the following:

Lemma 4.1. *Consider a consumer located a distance $\bar{x}_i$ away from firm i. If this consumer finds it optimal to stockpile from firm i, then any other consumer located a distance $x \in [0, \bar{x}_i]$ away from firm i will also find it optimal to stockpile from firm i.*

Lemma 4.1 helps to characterise the location of marginal consumer. Specifically, consider an interval of the unit-long circle. This interval can be seen via Figure 4.1. It includes three firms, firm i and its two adjacent firms, denoted by $-i$. Define firm i's location as 0. Two adjacent firms will be at $1 - \frac{1}{n}$ and $\frac{1}{n}$, respectively. Since firms other than firm i are acting symmetrically, with out loss of generality we can just focus on half of Figure 4.1, either $[1 - \frac{1}{n}, 0]$ or $[0, \frac{1}{n}]$. We denote the (possibly empty) set of consumers who choose to stockpile from firm i in the first period as $[0, \bar{x}_i]$, the set of consumer who stockpile from firm i's adjacent firm as $[\frac{1}{n} - \bar{x}_{-i}, \frac{1}{n}]$. Thus, the remaining set $[x_i, \frac{1}{n} - \bar{x}_{-i}]$ gives consumers who will be active in period 2. Based on the different locations of $\bar{x}_i$ and $\bar{x}_{-i}$, we can state the following regarding firm i's observed demand in period 1.

Lemma 4.2. *Firm i's observed demand in period 1 is:*

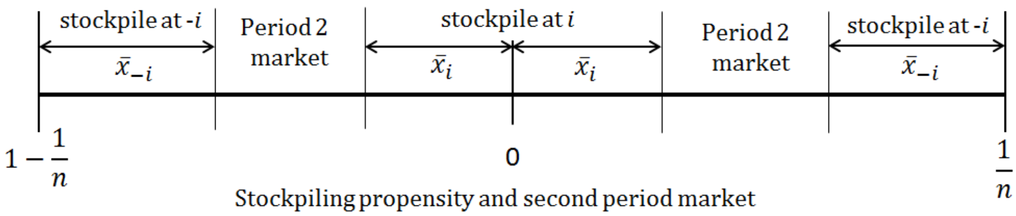


Figure 4.1: Consumer Stockpiling Decisions with Multifirms

$$\hat{Q}_{i1}(.) = \begin{cases} 2Q_{i1}(p_{i1},\, p_{-i1}) & \text{if } \bar{x}_i + \bar{x}_{-i} = \frac{1}{n} \\ Q_{i1}(p_{i1},\, p_{-i1}) + X_i(\bar{x}_i) & \text{if } \bar{x}_i + \bar{x}_{-i} < \frac{1}{n} \end{cases} \tag{4.7}$$

Lemma 4.2 suggests that firm i's observed demand in period 1 varies with locations of marginal consumers of both firms, $\overline{x}_i$ and $\overline{x}_{-i}$. Firstly, if the location of $\bar{x}_i$ and $\bar{x}_{-i}$ are overlapped such that the set $[\bar{x}_i,\, \frac{1}{n} - \bar{x}_{-i}]$ equals zero and therefore does not exist, all consumers stockpile by buying two units in period 1. In this way, firm i's period 1 observed demand is doubled and equals $\hat{Q}_{i1} = 2Q_{i1}(p_{i1},\, p_{-i1})$. In contrast, if $\bar{x}_i + \bar{x}_{-i} < \frac{1}{n}$ such that the set $[\bar{x}_i,\, \frac{1}{n} - \bar{x}_{-i}]$ is strictly positive, then firm $i's$ total demand equals $\hat{Q}_{i1} = Q_{i1}(p_{i1},\, p_{-i1}) + X_i(\bar{x}_i)$. A positive measure of consumers, $\frac{1}{n} - X_i(\overline{x}_i) - X_{-i}(\overline{x}_{-i})$, does not stockpile in period 1 and will therefore be active in period 2.

4.3.2.2 Period 2 Market

Given that consumer who stockpiles in period 1 is no longer active in period 2, from Lemma 4.2, we can state the period 2 market demand as follows

Lemma 4.3. *In any symmetric equilibrium, firm i's total period 2 observed demand, $\hat{Q}_{i2}$, is:*

$$\hat{Q}_{i2}(.) = \begin{cases} 0 & \textit{if } \bar{x}_i + \bar{x}_{-i} = \frac{1}{n} \\ Q_{i2}(p_{i2},\, p_{-i2}) - X_i(\bar{x}_i) & \textit{if } \bar{x}_i + \bar{x}_{-i} < \frac{1}{n} \end{cases} \tag{4.8}$$

If $\bar{x}_i$ and $\bar{x}_{-i}$ are overlapped such that $\bar{x}_i + \bar{x}_{-i} = \frac{1}{n}$, all consumers have stockpiled and so period 2 is inactive. However, if $\bar{x}_i + \bar{x}_{-i} < \frac{1}{n}$, then consumers with $x \in [\bar{x}_i,\, \frac{1}{n} - \bar{x}_{-i}]$ have not stockpiled and so remain active. As in the benchmark, any such consumer will then buy one unit from firm i rather than firm $-i$ if buying from firm i brings more utility. It then follows that firm i's period 2 observed demand equals $Q_{i2}(p_{i2},\, p_{-i2})$, from (4.3), minus those who have stockpiled from firm i in period 1, $X_i(\bar{x}_i)$, from (4.8).

4.3.2.3 Firm's Decisions: Period 2

In the case where period 2 market is active, period 2 market consists of those who didn't stockpile in period 1. From the previous Lemma, we know $\hat{Q}_{i2}(.) = Q_{i2}(p_{i2},\, p_{-i2}) - X_i(\bar{x}_i)$. Then from the benchmark we know $Q_{it}(p_{it},\, p_{-it}) \equiv 2\bar{x} = \frac{p_{-it} - p_{it}}{\mu} + \frac{1}{n}$. Now we can rewrite firm i's period 2 observed demand function as,

$$\hat{Q}_{i2} = [\frac{p_{-i2} - p_{i2}}{\mu} + \frac{1}{n} - X_i(\bar{x}_i)] \tag{4.9}$$

Accordingly, firm i's period 2 profit function can be written as

$$\pi_{i2} = p_{i2}\hat{Q}_{i2}(.) \tag{4.10}$$

Applying the first order condition of firm i's period 2 profit function yields the period 2 equilibrium price. We now state the following:

Lemma 4.4. *Suppose $\bar{x}_i + \bar{x}_{-i} < \frac{1}{n}$. Then, provided $\frac{1}{n} - \frac{(4X_i(\bar{x}_i)+2X_{-i}(\bar{x}_{-i}))}{3} > 0$, the unique period 2 equilibrium has*

$$p^*_{i2} = \mu[\frac{1}{n} - \frac{4X_i(\bar{x}_i) + 2X_{-i}(\bar{x}_{-i}))}{3}] > 0 \tag{4.11}$$

and $\widehat{Q}^*_{i2}(.) = Q_{i2}(p^*_{i2}, p^*_{-i2}) - X_i(\bar{x}_i) = \frac{2(X_i(\bar{x}_i)-X_{-i}(\bar{x}_{-i}))}{3} + \frac{1}{n} > 0$

To characterise the properties of a potential symmetric equilibrium, we need only consider local deviations. Hence, based on the assumption that all firms other than firm i set the same price and the period 1 prices of firm i and firm $-i$ are sufficiently close, their levels of stockpiling are similar, with $\frac{1}{n} - \frac{4X_i(\bar{x}_i)+2X_{-i}(\bar{x}_{-i}))}{3} > 0$ and $\frac{1}{n} - \frac{4X_{-i}(\bar{x}_{-i})+2X_i(\bar{x}_{-i}))}{3} > 0$. Given this, Lemma 4.4 confirms that all symmetric firms will have positive period 2 equilibrium prices and demand. Furthermore, it can be seen that the level of period 2 equilibrium prices are determined by the level of stockpiling demand. Intuitively, the more consumers stockpile in period 1, the less consumer are active in period 2. Facing less consumer, competing firms have more incentive to reduce its price. It can also be seen that both period 2 demand and period 2 equilibrium price are negatively related to the number of firms, n. This is due to the business-stealing effect caused by new entrants.

4.3.2.4 Firm's Decisions: Period 1

We now start examining the equilibrium levels of stockpiling demand for given period 1 prices and expected period 2 prices. After this, we solve for period 1 equilibrium.

Equilibrium Stockpiling Demand

Denote $X_i(p_{i1},\, p_{-i1},\, p^e_{i2},\, p^e_{j2})$ as firm i's equilibrium level of stockpiling demand, where consumers expectations are correct if $p^e_{i2} = p^*_{i2}(X_i,\, X_{-i})$.

Proposition 4.1. *Around any symmetric equilibrium, the unique levels of stockpiling demand,* $\mathbf{X} = \{X_i(.), X_{-i}(.)\}$, *equal:*

$$\mathbf{X} = \begin{cases} (0,0) \\ \left(\frac{3}{2} - \frac{3(p_{i1}-\kappa)}{\mu}, 0\right) \\ \left(\frac{1}{n} - \left(\frac{2p_{i1}-p_{-i1}-\kappa}{\mu}\right), \frac{1}{n} - \left(\frac{2p_{-i1}-p_{i1}-\kappa}{\mu}\right)\right) \end{cases} \tag{4.12}$$

Note that the first line of (4.12) requires $p_{i1} \geq \frac{\mu}{n} + \kappa$ and $p_{-i1} \geq \frac{\mu}{n} + \kappa$, the second line requires $p_{i1} < \frac{\mu}{n} + \kappa$ and $p_{-i1} \geq \frac{1}{2}\left[\frac{\mu}{n} + \kappa + p_{i1}\right]$, the third line requires $p_{i1} \leq \frac{1}{2}\left[\frac{\mu}{n} + \kappa + p_{-i1}\right]$ and $p_{-i1} \leq \frac{1}{2}\left[\frac{\mu}{n} + \kappa + p_{i1}\right]$.

In period 1, the proof verifies that marginal consumer with distance $\bar{x}_i$ away from firm i optimally compares the cost of stockpiling her second unit of product from firm i in period 1, and the cost of returning period 2 market to buy her second unit from firm i. The comparison can be mathematically written as,

$$U_i^S \lesseqqgtr U_i^{NS}$$

$$2(\lambda - \mu\bar{x}_i - p_{i1}) - \kappa \lesseqqgtr (\lambda - p_{i1} - \mu\bar{x}_i - \kappa) + (\lambda - p_{i2}^e - \mu\bar{x}_i - \kappa) \tag{4.13}$$

Simplifying yields,

$$p_{i1} \lesseqqgtr p_{i2}^e + \kappa \tag{4.14}$$

Depending on the level of transaction cost κ, (4.14) gives rise to three different scenarios. Firstly, if transaction cost κ is sufficiently high such that cost of buying in period 1 are low, then all consumers find it optimal to stockpile as $p_{i1} < p_{i2}^e + \kappa$. Similarly, if transaction cost κ is sufficiently small, then no consumer finds it optimal to stockpile as $p_{i1} > p_{i2}^e + \kappa$. If the transaction cost κ is on a moderate level. As consistent with the bottom line of equation (4.12), there exists a unique level of equilibrium stockpiling, such that $p_{i1} = p^e_{i2}(X_i,\, X_{-i}) + \kappa$. If firm i's period 1 price was below (above) this level for given levels of X_i and X_{-i}, more (fewer) consumers would find it optimal to stockpile at firm, which in turn would lower (raise) the firm's period 2 equilibrium price until this condition is satisfied.

Equilibrium Prices

Now we examine period 1 market from firms' side. In period 1 market, firms are aware of consumer's strategic intertemporal stockpiling behaviour. Thus, firm's profit maximisation problem can be described as,

$$\pi_i = p_{i1}[Q_{i1}(p_{i1},\, p_{-i1}) + X_i^*(.)] + p^*_{i2}[Q_{i2}(p^*_{i2},\, p^*_{-i2}) - X_i^*(.)] \tag{4.15}$$

where firm i receives period 1 demand $\hat{Q}_{i1} = Q_{i1}(p_{i1},\, p_{-i1}) + X_i(.)$ from (4.7) and (4.12), and (if active) sets a period 2 equilibrium price $p^*_{i2}(.)$ from (4.11), and receives a period 2 equilibrium demand $\hat{Q}_{i2} = Q_{i2}(p^*_{i2},\, p^*_{-i2}) - X_i(.)$ from (4.8). To begin, we can then note the following important result.

Proposition 4.2. *In any symmetric equilibrium with $\kappa > 0$, each firm receives a positive level of stockpiling demand.*

Proposition 4.2 is consistent with the result of Chapter 3, but is opposite to Guo and Villas-Boas (2007)'s equilibrium without consumer stockpiling. To get intuitions, prices in the benchmark case without consumer stockpiling are equal across both period 1 and 2, $p^*_1 = p^*_2 = \frac{\mu}{n}$. However, in our main framework, as the a positive transaction cost κ is introduced, it implies $p^*_1 < p^*_2 + \kappa$ such that there will be more consumers choose to stockpile in advance in an effort of avoiding a second period transaction cost. Thus, any symmetric equilibrium with any $\kappa > 0$ must involve with positive inventory demand of consumer, $X_i^* = X_{-i}^* = X^* > 0$.

Proposition 4.3. *In any symmetric equilibrium:*

i) when the number of firms is high, $n \geq \frac{2\mu}{3\kappa}$, the unique level of stockpiling demand is $X^* = \frac{1}{n}$, where $p_1^* = min\{\frac{\mu}{n}, \kappa\}$, $\hat{Q}_{i1}^* = \frac{1}{n} + X^* = \frac{2}{n}$, and $\hat{Q}_{i2}^* = 0$. The aggregate level of stockpiling demand is $nX^* = 1$.

ii) when the number of firms is low, $n \in [2, \frac{2\mu}{3\kappa})$, the unique level of stockpiling demand is $X^* = \frac{3k}{2\mu} \in (0, \frac{1}{n})$, where $p_1^* = \frac{\mu}{n} - \frac{k}{2}$, $p_2^* = \frac{\mu}{n} - \frac{3k}{2}$. The aggregate level of stockpiling demand is $nX^* = min\{\frac{3\kappa n}{2\mu}, 1\}$.

Proposition 4.3 suggests that whether the equilibrium is a corner solution or interior solution depends on the number of firms within the market. Intuitively, when the number of firms is large, competition is strong and prices are low relative to transaction cost because of the strong business-stealing effect, such that $p_1^* \leq \kappa$ holds in equilibrium. From Proposition 4.2, this implies all consumers optimally stockpile in advance, such that $X^* = \frac{2}{n}$. Instead, when the number of firms is small in the market, business-stealing effect is weak. As a result, competition is weaker such that price is relatively higher than the transaction cost. In equilibrium, $p_1^* = p_2^* + \kappa$ holds. Thus, there will be a strict positive proportion of, but not all, consumers stockpile in advance in period 1. To understand the role of the number of firms n in equilibrium stockpiling demand in more detail, we state the following:

Corollary 4.1. *In any symmetric equilibrium, the unique level of stockpiling demand of each symmetric firm X^* is weakly decreasing with n. The aggregate level of inventory demand $nX^* = min\{\frac{3\kappa n}{2\mu}, 1\}$, is weakly increasing in the number of firms, n.*

To understand Corollary 4.1, we first consider the case where n is low, $n \in [2, \frac{2\mu}{3\kappa})$, such that a proportion of consumers stockpile. From previous sections, we know the consumer's stockpiling decision depend upon (4.14). When n increases, p_{i1}^* and p_{i2}^* reduce by the same amount as $\frac{\partial p_{i1}^*}{n} = \frac{1}{n}$ and $\frac{\partial p_{i2}^*}{n} = \frac{1}{n}$. As a result, (4.14) does not change with n. The equilibrium stockpiling demand, X^*, for each firm is independent of the number of firms. In contrast, the increase in the number of firms does (weakly) increase the aggregate equilibrium stockpiling demand, nX^*. This is because, on average, consumers are now closer to their nearest firm. Now consider the case where n is larger, $n \geq \frac{2\mu}{3\kappa}$, such that all consumers stockpile. Now the storage demand for each firm is $\frac{1}{n}$, which is decreasing with the number of firms, and the aggregate equilibrium stockpiling demand, $nX^* = 1$, remains independent of n.

4.3.3 Elasticity Biases

In this section, we revisit the issue of the implied elasticities biases caused by ignoring consumer stockpiling behaviour. As opposed to Chapter 3, here we highlight the role of number of firms in determining the biases. Specifically, we first compare the elasticity of true demand, $Q_{it}(.)$, and the elasticity of observed demand that might have incorrectly included stockpiling demand, $\hat{Q}_{it}(.)$. Then we perform a comparative static analysis of n to study the role of it. The superscript $h = \{i, -i\}$ allows us to refer to own-price or cross-price biases respectively:

$$\hat{\rho}_{it}^h(.) = -\frac{\partial(Q_{it}(.) + X_i(.))}{\partial p_{it}} \frac{p_{it}}{(Q_{it}(.) + X_i(.))} \text{ and } \hat{\rho}_{it}^h(.) = \frac{\partial(Q_{it}(.) + X_i(.))}{\partial p_{-it}} \frac{p_{-it}}{(Q_{it}(.) + X_i(.))} \tag{4.16}$$

Similarly, own- and cross-price elasticity of true demand are given by

$$\rho_{it}^h(.) = -\frac{\partial Q_{it}(.)}{\partial p_{it}}\frac{p_{it}}{Q_{it}(.)} \qquad \text{and} \qquad \rho_{it}^h(.) = \frac{\partial Q_{it}(.)}{\partial p_{-it}}\frac{p_{-it}}{Q_{it}(.)} \tag{4.17}$$

Own- and cross-price elasticity of stockpiling demand are given by

$$\eta_{it}^h(.) = -\frac{\partial X_i(.)}{\partial p_{it}}\frac{p_i}{X_i(.)} \qquad \text{and} \qquad \eta_{it}^h(.) = \frac{\partial X_{it}(.)}{\partial p_{-it}}\frac{p_{-it}}{X_i(.)}$$

We compare the difference between the elasticity of observed demand and that of true demand to measure the bias that is calculated by incorrectly including consumers' stockpiling demand. Following the Proposition 2.1, the bias in period 1 can be expressed as,

$$\theta_{i1}^h(.) = \hat{\rho^h}_{i1}(.) - \rho_{i1}^h(.) = \frac{X_{i1}(.)}{Q_{i1} + X_{i1}(.)}(\eta_{i1}^h(.) - \rho_{i1}^h(.)) \; for \; h = \{i, -i\} \tag{4.18}$$

Stockpiling demand, X_i, in (4.18) is positive as consumers are adding inventories in period 1. Instead, the net stockpiling demand in period 2 is negative, and equivalent to the amount stockpiled in period 1. As such, the bias in period 2 is given by,

$$\theta_{i2}^h(.) = \hat{\rho}_{i2}^h(.) - \rho_{i2}^h(.) = \frac{-X_i(.)}{Q_{i2} - X_i(.)}(\eta_{i2}^h(.) - \rho_{i2}^h(.)) \; for \; h = \{i, -i\} \tag{4.19}$$

In what follows, (4.18) and (4.19) will mainly be used in these subsections. Section 4.3.3.1 examine the sign of each bias at equilibrium price, p_t^*, before Section 4.3.3.2 performs comparative static analysis to see the role of number of firms in determining such biases. Section 4.4 exploits the comparative static analysis with free entry number of entering firms to understand when such biases matter most. Hereafter, to ease exposition, we focus on firm i without loss of generality.

4.3.3.1 Signs of the Biases

We first consider the case of large number of firms, $n \geq \frac{3\mu}{2\kappa}$, such that the period 2 market is not active because all consumers chose to stockpile in period 1.

Proposition 4.4. *When the number of firms is large, $n \geq \frac{2\mu}{3k}$, the own- and cross-price elasticity in period 1 are unbiased, such that $\theta_{it}^h = \hat{\rho}_{it}(.) - \rho_{it}(.) = 0$*

Proposition 4.4 suggests that the elasticities based only on observed demand need not to be biased when there is positive stockpiling. This is because all consumers in period 1 stockpile such that $Q_{i1}(.) = X_i(.)$, leading the amount of own- and cross-price elasticities for observed demand equal to those for true demand, $\rho_{i1}^h(.) = -\frac{\partial(2Q_{i1}(.))}{\partial p_{i1}}\frac{p_{i1}}{2Q_{i1}(.)} = -\frac{\partial Q_{i1}(.)}{\partial p_{i1}}\frac{p_{i1}}{Q_{i1}(.)}$. Thus, the biases are zero. There is also an alternative way to understand Proposition 4 from (4.18). Given $Q_{i1}(.) = X_i(.)$, it follows that $\eta_{i1}^h(.) = \rho_{i1}^h(.)$. The elements within the bracket of (4.18) are therefore being zero.

Now we consider the case of small number of firms, where some consumers remain active in period 2 with a market level of stockpiling demand less than 1, $nX^* < 1$.

Proposition 4.5. *When the number of firms is small, $n \in [2, \frac{2\mu}{3k})$:*

i) the own- and cross- price elasticities of observed demand in period 1, $\hat{\rho}_{i1}^i(p_1^*)$, and $\hat{\rho}_{i1}^i(p_1^*)$ are positively biased, such that $\theta_{i1}^i(p_1^*) = \frac{3(2\mu-\kappa n)}{3\kappa n+2\mu} - \frac{2\mu-\kappa n}{2\mu} > 0$ and $\theta_{i1}^{-i}(p_1^*) = \frac{2(2\mu-\kappa n)}{2\mu+3\kappa n} - \frac{2\mu-\kappa n}{2\mu} > 0$;

ii) the own- and cross- price elasticities of observed demand in period 2, $\hat{\rho}_{i2}^i(p_2^*)$, and $\hat{\rho}_{i2}^{-i}(p_2^*)$ are positively biased, such that $\theta_{i2}^i(p_2^*) = \theta_{i2}^{-i}(p_2^*) = 1 - (\frac{2\mu-3\kappa n}{2\mu}) = \frac{3\kappa n}{2\mu} > 0$

The intuition of Proposition 4.5 is similar to Proposition 3.5. Particularly, in period 1, positive inventory demand ensures that the slope of the stockpiling demand curve is flatter than the true demand curve, $\frac{\partial X_i(.)}{\partial p_{i1}} < \frac{\partial Q_{i1}(.)}{\partial p_{i1}} < 0$. Moreover, at a given equilibrium price, p_1^*, the quantity demanded for true demand is larger than its counterpart for stockpiling demand, $Q_1^* > X^*$. Both these effects guarantees stockpiling demand is more elastic that true demand such that $\eta_{i1}^i(.) - \rho_{i1}^i(.) > 0$ in (4.18). For period 1 cross-price elasticity bias, even though the marginal responsiveness of cross-price of stockpiling demand and that of true demand are the same, $\frac{\partial X_i(.)}{\partial p_{-i1}} = \frac{\partial Q_{i1}(.)}{\partial p_{-i1}} > 0$, at a given equilibrium price, p_1^*, the true quantity demanded is again larger than the stockpiling quantity demanded, $Q_1^* > X^*$. Thus, cross-price elasticity of storage demand is again higher than that of true demand, $\eta_{i1}^{-i}(.) - \rho_{i1}^{-i}(.) > 0$.

Now consider the intuitions of positive biases of period 2 own- and cross-price elasticities. The elements with in the bracket in (4.19) is negative. This is because, firstly, stockpiling demand is independent of period 2 price, $\eta_{i2}^h(p_2^*) = 0$ for $h \in \{i, -i\}$. In addition, both own- and cross-price elasticities of true demand are positive, $\rho_{i2}^h(p_2^*) > 0$. Thus, the elements within the bracket in (4.19) is negative. Due to the negative stockpiling demand in period 2 and only a subset of consumers stockpile, the ratio of stockpiling demand and observed demand is negative, $\frac{-X_i^*(.)}{Q_{i2}^* - X_i^*(.)} < 0$.

4.3.3.2 Comparative Statics

To answer the key question of when the elasticities biases matter the most, we now perform a comparative static analysis on the number of firms, n. When n is large, there is no bias. When n is small, we can state the following:

Proposition 4.6. *When the number of firms is small, $n \in [2, \frac{2\mu}{3\kappa})$;*

i) the period 1 own- and cross-price elasticity biases, $\theta_{i1}^i(p_1^*)$, and $\theta_{i1}^{-i}(p_1^*)$, are strictly decreasing with the numbers of firms, n.

ii) the period 2 own- and cross-price elasticity biases, $\theta_{i1}^i(p_1^*)$, and $\theta_{i1}^{-i}(p_1^*)$, are strictly increasing with the numbers of firms, n.

Proposition 4.6 suggests that a unit change in n has opposite effects on the elasticity biases depending on whether consumers are building up their inventories or consuming their inventories. Here, we use equation (4.18) and (4.19) to understand this Proposition. As the number of firms increases, the ratio between stockpiling demand and observed demand, $\frac{X^*}{Q^*+X^*}$, increases (more positive) in period 1 and decreases (more negative) in period 2. This is because at a given equilibrium price, p_t^*, true demand is negatively related to n while stockpiling demand for each firm is independent of n. Next, consider how elements within the bracket of (4.18) and (4.19) vary with n. In period 1, both equilibrium prices and demand for each firm reduce as the number of firms increases, making storage demand and true demand more inelastic. However, elasticity of true demand reduces faster than that of storage demand, $\frac{\partial \eta_{i1}^h}{\partial n} < \frac{\partial \rho_{i1}^h}{\partial n} < 0$. Thus, $\eta_{i1}^h - \rho_{i1}^h$ becomes smaller. This effect is large enough to offset effect that increases $\frac{X^*}{Q+X^*}$. In period 2, the negative storage demand is independent of period 2 price such that $\eta_{i2}^h = 0$. Elasticity of true demand becomes more inelastic, $\frac{\partial \rho_{i2}^h}{\partial n} < 0$. Hence $\eta_{i2}^h - \rho_{i2}^h$ of (4.19) increases as n increases. By dominating the effect that reduces the ratio of $\frac{-X_i^*}{Q_{i2}^*-X_i^*}$, biases of price elasticity in period 2 increase as the number of firms increase.

4.4 Endogenous Number of Firms

In previous sections, the number of firms, n, was treated as exogenous. We now make n endogenous by considering free entry. Specifically, we follow standard assumptions (Tirole, 1988) such that each firm incurs fixed cost to enter the market. We first examine the equilibrium number of active firms when it is endogenously determined by the zero profit condition. We then consider the welfare implications by asking whether the optimal number of firms excessive or insufficient. To answer this, we need to calculate and compare the number of firms that would maximise the social welfare, and the endogenous number of firms that gives firms zero profit condition. Finally, to further understand when the biases of price elasticity matter most with an equilibrium number of firms, we revisit the comparative static analysis of price elasticity biases.

4.4.1 Benchmark

First, we consider free entry in the benchmark where stockpiling is prohibited. Under imperfect competition while free entry is allowed, the optimal number of firms is determined exogenously. Outsiders keep entering the market until the zero profit condition is fulfilled. Using the results of Section 4.3.1, this gives

$$\pi_i^* = \frac{2\mu}{n^2} - f = 0 \tag{4.20}$$

Solving (4.20) for n yields the equilibrium number of firms $n^{NS*} = \sqrt{\frac{2\mu}{f}}$. this value is as twice as the standard Salop equilibrium number of firms because we are considering a two-period market.

Now consider the case in which social planner maximises the welfare by controlling the level of entry. Define social welfare as the sum of consumer surplus and all firms' profits. Social welfare can be written as,

$$W_i(n) = CS(n) + n[\pi_i - f] \tag{4.21}$$

After expanding terms, this equals (4.22) below. Intuitively, the consumer welfare is composed of the gross utility for the purchased product, minus the price of purchased product, minus the transaction cost incurred, and minus the effect of product differentiation. This latter component of consumers' welfare can be explained as follows. Within each firm's demand, there are two sets of consumers located within the interval $[0, \frac{1}{2n}]$ away from the firm. Hence, the total effect of product differentiation per firm equals to $2\int_0^{\frac{1}{2n}} \mu x dx$, which when multiplied by the n firms gives $2n\mu \int_0^{\frac{1}{2n}} x dx$.

$$W_i(n) = \underbrace{2[\lambda - p_i^{*NS} - \kappa - 2n\int_0^{\frac{1}{2n}} \mu x dx]}_{\text{Consumers' welfare}} + \underbrace{n[\pi_i(p_i^{*NS}) - f]}_{\text{Firms' profits}} \tag{4.22}$$

Inserting benchmark equilibrium price and profit, $p_i^{*NS} = \frac{\mu}{n}$ and $\pi^{*NS} = \frac{2\mu}{n^2} - f$, and maximising (4.22) with respect to n yields $\frac{\mu}{2n^2} - f = 0$. Solving for n yields the socially optimal number of firms, $\hat{n}^{*NS}$.

$$\hat{n}^{NS*} = \sqrt{\frac{\mu}{2f}} < n^{NS*} = \sqrt{\frac{2\mu}{f}}$$

We can therefore infer that in the benchmark case, as expected and as consistent with standard Salop result, there is still excess entry.

4.4.2 Equilibrium Analysis

In this subsection we examine the equilibrium number of firms when stockpiling is permitted. Recall from previous sections that for a given number of firms, there may be a corner solution where all consumers stockpile or an interior solution in which a subset of consumers stockpile. In either case under free entry, the equilibrium number of firms will be exogenously determined by a zero profit condition. Using Proposition 3 and equation (4.15), the zero profit conditions for both firms can be written as,

$$\pi(n^*) - f = 0 \Longleftrightarrow \begin{cases} \frac{2\mu}{n^2} = f & \text{if } f < \frac{3\kappa^2}{\mu} \\ \frac{3\kappa^2 n^2 - 4\kappa n\mu + 4\mu^2}{2n^2\mu} = f & \text{if } f \in [\frac{3\kappa^2}{\mu}, \frac{3\kappa^2 - 2\kappa\mu + \mu^2}{2\mu}] \end{cases} \tag{4.23}$$

Solving (4.23) for n yields the equilibrium number of entrants. We now state the following

Lemma 4.5. *When consumer stockpiling is feasible, the equilibrium number of firms is i) given by (4.24), and ii) increasing with product differentiation, μ, and (weakly) decreasing with the level of the transaction cost, κ, and the fixed cost, f.*

$$n^* = \begin{cases} \sqrt{\frac{2\mu}{f}} & \text{if } f < \frac{3\kappa^2}{\mu} \\ \frac{2\mu(\sqrt{2f\mu-2\kappa^2}-\kappa)}{2f\mu-3\kappa^2} & \text{if } f \in [\frac{3\kappa^2}{\mu}, \frac{3\kappa^2-2\kappa\mu+\mu^2}{2\mu}] \end{cases} \tag{4.24}$$

First, consider the (upper) case in (4.24) where $f < \frac{3\kappa^2}{\mu}$ such that the transaction cost, κ, is relatively high and the fixed cost is relatively low. Here, all consumers are induced to stockpile in period 1, and the equilibrium number of entrants is identical to that of the benchmark case, $\sqrt{\frac{2\mu}{f}}$. This is because in this case, the period 1 market is doubled, $Q_{i1}^* = X_i^*$. Thus, the market size and equilibrium prices are identical, leading the equilibrium number of firms to also be identical. Now consider the other (lower) case in (4.24) where the transaction cost, κ, is now relatively lower and the fixed cost is relatively higher. Here, only a proportion of consumers stockpile in advance in equilibrium, and the subsequent prices and profits are lower than the previous case where all consumers stockpile, and so the equilibrium number of firms changes expression to $\frac{2\mu(\sqrt{2f\mu-2\kappa^2}-\kappa)}{2f\mu-3\kappa^2}$.

Consistent with conventional wisdom, Lemma 4.5 also suggests that in either case, the equilibrium number of firms is positively related to the level of product differentiation, μ, and negatively related to the level of the fixed cost, f. There is also a (weak) negative relationship with the level of the transaction cost, κ. Intuitively, when f is relatively small, the transaction cost has no impact on the number of entrants. However, when f becomes relatively larger, an increase of in the transaction cost discourages new entry. As κ increases, profit per firm reduces. This is because symmetric firms need to encourage consumers to buy by reducing its price to compensate increased transaction cost. Consequently, equilibrium number of firms has to decrease to maintain the free entry condition, $\pi(n) = f$.

4.4.3 Excess Entry

Before considering the impact of the elasticity biases, we are now in the position to consider whether the excess entry theorem applies under consumer stockpiling. Normally, there are two different benchmarks to compare the number of firms under free entry, a first-best benchmark in which the social planner chooses both the level of entry and the prices charged by firms, and a second-best benchmark in which the social planner can only control the level of entry, but not prices. Here, we consider a second-best benchmark. This is because in our framework, whether the prices can be regulated or not is trivial as the price changes have no impact on the total quantity purchased by consumers. i.e. demand is inelastic. We ask whether there is always excess entry into the market as it is the case in the standard Salop circular model.

Like Section 4.4.1, total social welfare is defined as the sum of consumer surplus and firm's profit. Any individual consumer's welfare is composed of the gross utility gained from consuming the product, minus the price paid, the transaction incurred from buying the product, and the effects of product differentiation, while any individual firm's welfare is subject to the revenue gained from selling the product, and entry cost incurred. Thus, the social total welfare maximisation problem when stockpiling is feasible can be written as the equation (4.25) shown below, while the second half is the sum of consumer surplus of period 1 demand, stockpiling demand, and period 2 demand.

$$W(n) = \begin{cases} \lambda - p_1^* - \kappa - 2n\int_0^{\frac{1}{2n}} \mu x dx + n(\pi_i^* - f) \\ [\lambda - p_1^* - \kappa - 2n\int_0^{\frac{1}{2n}} \mu x dx] + [nX^*(\lambda - p_1^*) + n\hat{Q}_2^*(\lambda - p_2^* - \kappa) - 2n\int_0^{\frac{1}{2n}} \mu x dx] + [n(\pi_i^* - f)] \end{cases} \tag{4.25}$$

Note that the first line of (4.25) indicates the case in the corner solution where all consumers stockpile, this case requires $f < \frac{3\kappa^2}{\mu}$. The second line of (4.25) indicates the interior solution where a proportion of consumers stockpile, this case requires

$f \in [\frac{3k^2}{\mu}, \frac{3k^2-2k\mu+\mu^2}{2\mu}]$.

Since the second line of (4.25) is very long, here I provide a note to specify. In particular, the first square bracket, $[\lambda - p_1^* - \kappa - 2n\int_0^{\frac{1}{2n}} \mu x dx]$, indicates the consumer surplus of the first unit that is purchased in period 1. It shows that each individual's utility is given by the match value, λ, minus price of the first unit, p_1^*, minus transaction cost, κ, and the effect of product differentiation, $2n\int_0^{\frac{1}{2n}} \mu x dx$. The second square bracket, $[nX^*(\lambda - p_1^*) + n\hat{Q}_2^*(\lambda - p_2^* - \kappa) - 2n\int_0^{\frac{Q_1^*}{2}} \mu x dx]$, indicates the consumer surplus of the second unit, in which some consumers buy in advance by stockpiling at p_1^* while some consumers buy in period 2 at p_2^*. The total number of consumers who stockpile the second unit in period 1 is nX^*. These consumers' utilities are subject to period 1 price, and not subject to transaction cost. Consumers who buy in period 2 is $n\hat{Q}_2^*$. These consumers' utilities are subject to period 2 price and the incurred transaction cost. Therefore, consumer surplus of stockpiling consumers, $nX^*(\lambda - p_1^*)$, plus consumer surplus of period 2 consumers, $n\hat{Q}_2^*(\lambda - p_2^* - \kappa)$, minus the effects of product differentiation, $2n\int_0^{\frac{1}{2n}} \mu x dx$, equals the total consumer surplus of the second unit. The third square bracket indicates the total industry profits.

The social planner would aim to select the number of firms that maximises total welfare. This is given by the solution of equation (4.26) and defined as the value of n that maximises $W(n)$.

$$W'(\hat{n}) - f = 0 \Longleftrightarrow \begin{cases} \frac{\mu}{2\hat{n}^2} = f & \text{if } f < \frac{3\kappa^2}{\mu} \\ \frac{3\kappa^2\hat{n}^2+\mu^2}{2\hat{n}^2\mu} = f & \text{if } f \in [\frac{3k^2}{\mu}, \frac{3k^2-2k\mu+\mu^2}{2\mu}] \end{cases} \tag{4.26}$$

The second order condition $W''(\hat{n}) < 0$ is satisfied. Thus, the efficient number of the firms, $\hat{n}$, is given by (4.26). If $f < \frac{3\kappa^2}{\mu}$, $\hat{n} = \sqrt{\frac{\mu}{2f}}$. If $f \in [\frac{3k^2}{\mu}, \frac{3k^2-2k\mu+\mu^2}{2\mu}]$, $\hat{n} = \frac{\mu}{\sqrt{2f\mu - 3\kappa^2}}$. Comparing $\hat{n}$ with the equilibrium number of firms n^* leads to the following Proposition,

Proposition 4.7. *There is always excessive entry. The equilibrium number of firms, n^*, always exceeds the socially optimal number of firms, $\hat{n}$.*

In line with the original excess entry theorems, (e.g. Vickery 1964, Salop 1979, Mankiw and Whinston 1986, Suzumura and Kiyono 1987), Proposition 4.7 suggests that the excess entry theorem always applies even in a two-period differentiated oligopoly where consumer can stockpile for future consumption. However, this result contrasts to some previous studies that develop the Salop circular city model to suggest a critical value of fixed cost that determines whether there is excessive or insufficient entry (e.g Matsumura and Okamura, 2006a, Gu and Wenzel, 2012 and 2015). In the general case, the entry of a new firm reduces each firm's market share and the equilibrium price under what is known as the business-stealing effect. On the other hand, from consumers' perspective, an additional entrant reduces the equilibrium price and enhances consumer surplus under the welfare-improving effect. Hence, whether entry is excessive or insufficient, depends on the relative sizes of these effects.

In our case, the entry of a new firm reduces firms' profits by reducing each firm's market share and the equilibrium prices in both periods. On the other hand, the additional entrant raises consumer surplus by reducing the prices in both periods, and also by (weakly) increasing the level stockpiling such that consumers' expenditure on transaction costs also weakly falls. However, Proposition 4.7 states that the firm-side business-stealing effect always dominates such that entry is always excessive, despite the extra inter-temporal effects on consumer stockpiling.[2] Hence, a policy implication of this suggests that restrictions on market entry may, but not necessarily, improve social welfare in an oligopoly storable product market.

[2]Finally, note that the additional positive effect on consumer surplus through the reduced transaction cost expenditure only arises when the equilibrium number of firms is small (with a relatively low transaction cost, κ, and a relatively high fixed cost, $f \in [\frac{3k^2}{\mu}, \frac{3k^2-2k\mu+\mu^2}{2\mu}]$). Hence, the excessive entry theorem is actually less acute when the equilibrium number of firms is smaller.

4.4.4 Elasticity Biases

From Chapter 3 and Section 4.3, we know how biases of own- and cross-price elasticities vary with respect to product differentiation or transaction cost when the number of firms n is exogenously given. In this section, we revisit this issue by inserting the equilibrium level of number of entrants, n^*, in an attempt to understand when these biases matter most with free entry.

Proposition 4.8. *When the number of firms is low,* $f \in [\frac{3k^2}{\mu}, \frac{3k^2-2k\mu+\mu^2}{2\mu}]$,

i) the period 1 own- and cross-price elasticity biases with equilibrium number of firms, $\theta_{i1}^{h*}(\kappa, \mu)$ *and* $\theta_{i1}^{h*}(\kappa, \mu)$, *are strictly increasing with product differentiation level, and decreasing with transaction cost.*

ii) the period 2 own- and cross-price elasticity biases with equilibrium number of firms, $\theta_{i1}^{h*}(\kappa, \mu)$ *and* $\theta_{i1}^{h*}(\kappa, \mu)$, *are strictly decreasing with product differentiation level and increasing with transaction cost.*

Proposition 4.8 shows the findings of Chapter 3 are robust to free entry, and indicates when the biases matter most with respect to the number of firms under free entry. To understand Proposition 4.8, first consider the direct effect. As μ increases, for equation (4.18) and (4.19), both $\eta_{i1}(p_1^*)$ and $\rho_{i1}(p_1^*)$ increase. But $\eta_{i1}(p_1^*)$ increases faster and leads the difference between $\eta_{i1}(p_1^*)$ and $\rho_{i1}(p_1^*)$ larger. Additionally, an increase of μ reduces the ratio $\frac{X^*}{Q_1^*+X^*}$. The former effect is large enough to offset the latter in period 1. Therefore, in period 1, keeping n^* constant, elasticity bias goes up as μ increases. Instead, in period 2, despite the fact that the elasticity of storage demand is independent of period 2 prices, $\eta_{i2}(p_2^*) = 0$, the period 2 observed demand increases, $\rho_{i2}(p_2^*) > 0$. The effect that reduces the ration $\frac{-X^*}{Q_2^*-X^*}$ dominates and therefore the bias in period 2 reduces as μ increases. This is very similar to what we have explained in Chapter 3. Additionally, there also exists an indirect effect through the equilibrium number of entrants, n^*. From Lemma 4.5, we know that as μ increases, the equilibrium numbers of entrants n^*

increases. From Proposition 4.6, we also know that biases of own- and cross- price elasticity are negatively related with n. Thus, an increase in μ reduces the bias of own- and cross-price elasticity indirectly. In aggregate, direct effect offsets the indirect effect. Therefore the biases of own- and cross-price elasticities in period 1 (2) are positively (negatively) related with product differentiation level, μ. The intuition of transaction cost, κ, is similar to the above discussion of μ. But the direction of the effect of κ is opposite to that of μ.

4.5 Conclusions

This study has extended a spatial model that incorporates consumer stockpiling behaviour (Guo and Villas-Boas 2007) into a n-firm oligopoly market. To this end, we derive a full model of stockpiling in a differentiated oligopoly with endogenous prices over two periods. The result show that in any symmetric equilibrium, a (weak) subset of consumer stockpile to avoid additional transaction costs. Depending on the number of firms in the market, the biases of price elasticity can either be zero or positive. By treating the number of firms as endogenous, we show that excessive entry theorem still holds when consumers stockpile, but that restrictions on entry do not necessarily increase social welfare. We finally consider how the biases of own- and cross-price elasticity vary with respect to transaction cost and product differentiation under free entry. Our findings suggest that the previous results of Chapter 3 are robust to free entry. The associated elasticity biases are strictly increasing in the degree of product differentiation when net stockpiling is positive, but strictly decreasing in the degree of product differentiation in period 2.

Appendix:

Proof of Lemma 4.1. From (4.4) and (4.6), any consumer will stockpile from i if a) $U_i^S - U_{-i}^S > 0$, and b) $U_i^S - U_i^{NS} > 0$. Suppose condition a) holds, which implies $(\lambda - p_{i1} - \mu x - \kappa) - [\lambda - p_{-ii} - \mu(\frac{1}{n} - x) - \kappa] > 0$. It then follows that $U_i^{NS} = U_i(p_{i1}) + Max\{u_i(p_{i2}^e) + u_{-i}(p_{-i2}^e)\} - 2\kappa$, such that condition b) can be rewritten as $U_i^S - U_i^{NS} = -\mu x - p_{i1} + max\{\mu x + p_{i2}^e, \mu(\frac{1}{n} - x) + p_{-i2}^e\}$. This is then strictly (weakly) increasing in x. □

Proof of Lemma 4.2. If $\bar{x}_i + \bar{x}_{-i} = \frac{1}{n}$, all consumers stockpile. Here, using (4.5), any given consumer will buy two units from i rather than $-i$ in period 1 if $2(\lambda - p_{i1} - \mu x) - \kappa \geq 2[\lambda - p_{-i1} - \mu(\frac{1}{n} - x)]$. This comparison reduces down to that in the benchmark, $\lambda - p_{i1} - \mu x > \lambda - p_{-i1} - \mu(\frac{1}{n} - x)$. Hence, firm i's total period 1 demand equals $\widehat{Q}_{i1}(.) = 2Q_{i1}(.)$, where $Q_{i1}(p_{i1}, p_{-i1})$ coincides with the benchmark demand, (4.2).

If, instead, $\bar{x}_i + \bar{x}_j < \frac{1}{n}$, some consumers only buy one unit in period 1. As in the benchmark, such consumers will buy one unit from firm i rather than $-i$ if $u_i(p_{i1}) - \kappa \geq u_{-i}(p_{i1}) - \kappa$ and one can define $\bar{x}$ as the value of $\bar{x}_{+i} = p_{i1} - p_{-i1}$ at which such a consumer would be indifferent. Hence, a total of $Q_{i1}(p_{i1}, p_{-i1})$ consumers buy from firm i, of which $Q_{i1}(p_{i1}, p_{-i1}) - X_i$ consumers buy one unit and $X_i(\bar{x}_i)$ consumers buy two units, such that total demand equals $\widehat{Q}_{i1}(.) = Q_{i1}(p_{i1}, p_{-i1}) + X_i(\bar{x}_i)$. □

Proof of Lemma 4.3. If $\bar{x}_i + \bar{x}_{-i} < \frac{1}{n}$, all consumers stockpile and so $\widehat{Q}_{i2}(.) = 0$. If instead, $\bar{x}_i + \bar{x}_{-i} < \frac{1}{n}$, then consumers with $x \in (\bar{x}_i, \bar{x}_{-i})$ did not stockpile and so remain active. As in the benchmark, any such consumer will then buy one unit from firm i rather than $-i$ if $u_i(p_{i2}) - \kappa \geq u_{-i}(p_{-i2}) - \kappa$, and one can define $\bar{x}_2$ as the value of $\bar{x}_2 = p_{i2} - p_{-i2}$ at which such a consumer would be indifferent. Around any symmetric equilibrium, $\bar{x}_2 \in (\bar{x}_i, \bar{x}_{-i})$. Hence, there is a positive measure of consumers with $x \in (\bar{x}_i, \bar{x}_j)$ that strictly prefer to buy from firm j and a positive measure of consumers with $x \in (\bar{x}_i, \bar{x}_2)$ that strictly prefer to buy from firm i, and a measure of consumer with $x \in (\bar{x}_2, \bar{x}_{-i})$ that strictly prefer to buy from firm $-i$. This implies that firm $i's$ total period 2 demand is equal to the benchmark demand, $Q_{i2}(p_{i2}, p_{-i2})$ from (4.2), minus those consumers that stockpiled from firm i in period 1, $X_i(\bar{x}_i)$ from (4.8). □

Proof of Lemma 4.4. Suppose $\bar{x}_i + \bar{x}_{-i} < \frac{1}{n}$. Then in any symmetric equilibrium, it must be the case that $\bar{x}_2 \in (\bar{x}_i, \bar{x}_{-i})$ such that both firms have positive demand, where $\bar{x}_2$ is defined as consumer who is indifferent between buying from firm i and $-i$. Then one can use $\pi_{i2}(.) = p_{i2}\widehat{Q}_{i2}(.)$ with (4.11) to derive the firms' period 2 best responses for given stockpiling levels, $p^*_{i2}(p_{-i2}) = \frac{p_{-i2}}{2n} + \frac{\mu}{2n}\left(\frac{1}{2} - X_i(\bar{x}_i)\right)$. Solving simultaneously yields the unique period 2 equilibrium prices, $p^*_{i2} = \mu[\frac{1}{n} - \frac{4X_i(\bar{x}_i)+2X_{-i}(\bar{x}_{-i})}{3}]$, and substituting these back into (4.9) gives $Q_{i2} = \frac{(X_i(\bar{x}_i)-X_{-i}(\bar{x}_{-i}))}{3} + \frac{1}{n}$. □

Proof of Proposition 4.1. Having derived period 2 equilibrium prices, we first consider consumers' stockpiling decisions, before deriving the equilibrium levels of stockpiling demand as a function of period 1 prices in (4.12).

First, consider consumers' stockpiling decisions and initially suppose that each firm has positive period 2 demand with $\bar{x}_2 \in (\bar{x}_i, \bar{x}_{-i})$. This implies that a consumer makes her stockpiling decision by comparing i) the net marginal benefits of stockpiling from firm i, with ii) the net marginal benefits of waiting to buy from firm i, rather than firm $-i$, in period 2. From Section 4.3.2.1, this implies $U_i^S = U_i^{NS}$. By construction, the consumer who is indifferent between stockpiling, such that $U_i^S - U_i^{NS} = 0$. Hence, this indifference requires $p_{i1} = p^*_{i2}\left(X_i, X_{-i}\right) + \kappa$.

We are now in a position to derive the equilibrium levels of stockpiling demand as a function of period 1 prices. First, consider the top line of (4.12). Here, $p_{i1} > p^*_{i2}(0,0) + \kappa$ and $p_{-i1} > p^*_{-i2}(0,0) + \kappa$ such that no consumer finds it optimal to stockpile, $X_i = X_{-i} = 0$. From (4.11), $p^*_{i2}(0,0) = \frac{\mu}{n}$ and so this case occurs when $p_{i1} > \frac{\mu}{n} + \kappa$ and $p_{-i1} > \frac{\mu}{n} + \kappa$.

Second, consider the bottom line of (4.12). Here, $p_{i1} \leq p^*_{i2}\left(\frac{1}{n}, \frac{1}{n}\right) + \kappa$ and $p_{-i1} \leq p^*_{-i2}\left(\frac{1}{n}, \frac{1}{n}\right) + \kappa$ such that all consumers find it optimal to stockpile, $X_i = X_{-i} = \frac{1}{n}$. Period 2 prices are unspecified as period 2 is inactive. However, if the marginal consumer at $\bar{x}_i = 0$ and $\bar{x}_{-i} = \frac{1}{n}$ were to deviate from stockpiling, we know from (4.11) that she should rationally expect zero period 2 prices, $\lim_{X_i \to \frac{1}{n}} p^*_{i2}\left(X_i, \frac{1}{n}\right) = 0$. Hence, this case occurs when $p_{i1} \leq \kappa$ and $p_{j1} \leq \kappa$.

Third, consider the middle line of (4.12). Here, there exists a unique level of equilibrium stockpiling, $X_i(.) \in (0, \frac{1}{n})$ and $X_{-i}(.) \in (0, \frac{1}{n})$, such that $p_{i1} = p^*_{i2}(X_i, X_{-i}) + \kappa$ holds for each firm. To find such $X_i(.)$ and $X_{-i}(.)$, one can insert p^*_{i2} from (4.11) to obtain

$$X_i(.) = \frac{3(\kappa - p_{i1})}{4\mu} + \frac{3}{4n} - \frac{X_{-i}}{2} \tag{4.27}$$

After deriving a similar equation for $X_{-i}(.)$ and solving simultaneously, one finds a unique level of $X_i(.) = \frac{1}{n} - \left(\frac{2p_{i1} - p_{j1} - \kappa}{\mu}\right)$. For $X_i \in (0, \frac{1}{n})$, we require $p_{i1} \in \left(\frac{\mu}{n} + \kappa, \frac{1}{2}\left[\frac{\mu}{n} + \kappa + p_{-i1}\right]\right]$ for each firm.

Finally, note that the levels of stockpiling and associated conditions in (4.12) are continuous as i) $\frac{1}{n} - \left(\frac{2p_{i1} - p_{-i1} - \kappa}{\mu}\right) = \frac{1}{n}$ when $p_{i1} = p_{-i1} = \kappa$, ii) $\frac{1}{n} - \left(\frac{2p_{i1} - p_{-i1} - \kappa}{\mu}\right) = 0$ when $p_{i1} = p_{-i1} = \frac{\mu}{n} + \kappa$. □

Proof of Proposition 4.2. From (4.12), $X_i = X_{-i} = 0$ necessarily requires $p_{i1} > \frac{\mu}{n} + \kappa$ and $p_{-i1} > \frac{\mu}{n} + \kappa$. However, we know from the benchmark analysis that $X_i = X_{-i} = 0$ is consistent with $p_1^* = \frac{\mu}{n}$. This then leads to a contradiction as $p_1^* = \frac{\mu}{n} < \frac{\mu}{n} + \kappa$ for all $\kappa > 0$. $\square$

Proof of Proposition 4.3. First suppose that period 2 is active with $\bar{x}_i + \bar{x}_{-i} < \frac{1}{n}$ such that $X_i(.) + X_{-i}(.) < \frac{1}{n}$. In any symmetric equilibrium each firm has positive period 2 demand with $\bar{x}_2 \in (\bar{x}_i, \bar{x}_{-i})$. Using (4.8) and (4.4), firm i's profit function from can then be rewritten as

$$\pi_i = p_{i1}[Q_{i1}(p_{i1}) + X_i^*(p_{i1})] + p_{i2}^*[Q_{i2} - X_i^*(p_{i1})]$$

Applying the first order condition with respect to p_{i1} and set it equals to zero, one obtains a unique value for $p_1^* = \frac{\mu}{n} - \frac{\kappa}{2}$. There are no profitable local deviations as the associated second-order condition ensures local concavity, $\frac{\partial^2 \pi_i}{\partial p_{i1}^2} = -\frac{2n}{\mu} < 0$. Then substituting p_1^* into (4.12), (4.4) and (4.2) provides the unique values for X^*, p_2^*, $\widehat{Q}_1^*$ and $\widehat{Q}_2^*$ as claimed. For period 2 to be active as assumed, we require $X^* = \frac{3\kappa}{2\mu}$. This implies $2 \leq n < \frac{2\mu}{3\kappa}$, which further ensures that the equilibrium is well-defined with non-negative prices.

Second suppose that period 2 is inactive with $\bar{x}_i + \bar{x}_{-i} = \frac{1}{n}$ such that $X_i(.) + X_{-i}(.) = 1$. Firm i's profit function then equals $\pi_i(.) = p_{i1}[\widehat{Q}_{i1}(p_{i1}, p_{-i1})]$ which can be rewritten as follows using

$$\pi_i = p_{i1}2[Q_{i1}(p_{i1}) + X_i(p_{i1})]$$

However, to ensure $X_i(.) + X_{-i}(.) = \frac{1}{n}$, we know from Proposition 4.1 that profit maximisation function must be maximised subject to $p_{i1} \leq \kappa$. After solving and enforcing symmetry, this leads to a unique local maximum with $p_1^* = \min\{\frac{\mu}{n}, \kappa\}$ and $X^* = \frac{2}{n}$. There are no profitable local deviations because the associated second-order condition ensures local concavity, $\frac{\partial^2 \pi_i}{\partial p_{i1}^2} = -\frac{n}{\mu} < 0$. When $2 \leq \frac{2\mu}{3\kappa} < n$, we know from above that any symmetric equilibrium must have $X^* < \frac{1}{n}$ which is inconsistent with this case. Hence, this case requires $n \geq \frac{2\mu}{3\kappa}$.

□

Proof of Proposition 4.4. The bias is $\theta_{i1}^h(p_1^*) = \frac{X^*}{Q_1^*+X^*}\left(\eta_{i1}^h(p_1^*) - \rho_{i1}^h(p_1^*)\right)$ for $h \in \{i, -i\}$, where $Q_1^* = \frac{1}{n}$ and $X^* = \frac{1}{n}$ from Proposition 4.3. From Lemma 4.2, $Q_{i1}(.) = X_i(.) = \frac{1}{2} + \frac{p_{-i} - p_i}{\mu}$, such that $\eta_{i2}^h(p_1^*) = \rho_{i1}^h(p_1^*) = \frac{2p_1^*}{\mu}$. Thus, $\theta_{i1}^h(p_1^*) = 0$ for $h \in \{i, -i\}$. □

Proof of Proposition 4.5. i) From (4.18), the bias for $h \in \{i, -i\}$ is $\theta_{i1}^h(p_1^*) = \frac{X^*}{Q_1^*+X^*}\left(\eta_{i1}^h(p_1^*) - \rho_{i1}^h(p_1^*)\right)$, where $Q_1^* = \frac{1}{2}$, $X^* = \frac{3\kappa}{2\mu}$, and $p_1^* = \frac{\mu}{n} - \frac{\kappa}{2}$ from Proposition 4.3. Given $Q_{i1}(.) = \frac{1}{n} + \frac{p_{-i}-p_i}{\mu}$ and $X_i(.) = \frac{1}{n} - \left(\frac{2p_{i1} - p_{-i1} - \kappa}{\mu}\right)$ from Lemma 4.2 and Proposition 4.1, then $\theta_{i1}^h(p_1^*) = \frac{3(2\mu-\kappa n)}{3\kappa n+2\mu} - \frac{2\mu-\kappa n}{2\mu} = \frac{(\kappa n-2\mu)(3\kappa n-4\mu)}{2\mu(3\kappa n+2\mu)}$ and $\theta_{i1}^h(p_1^*) = \frac{2(2\mu-\kappa n)}{2\mu+3\kappa n} - \frac{2\mu-\kappa n}{2\mu} = \frac{(\kappa n-2\mu)(3\kappa n-2\mu)}{2\mu(3\kappa n+2\mu)}$. Given $2 \leq n < \frac{2\mu}{3\kappa}$, these are both strictly positive.

ii) From (4.19), the bias in period 2 for $h \in \{i, -i\}$ is $\theta_{i2}^h(p_2^*) = \frac{-X^*}{Q_2^*-X^*}\left(\eta_{i2}^h(p_2^*) - \rho_{i2}^h(p_2^*)\right)$, where $Q_2^* = \frac{\mu}{n}$, $X^* = \frac{3\kappa}{2\mu}$, and $p_2^* = \frac{\mu}{n} - \frac{3\kappa}{2\mu}$ from Proposition 4.3 such that $\eta_{i2}^h(p_2^*) = \eta_{i2}^h(p_2^*) = 0$ and $\rho_{i2}^h(p_2^*) = \rho_{i2}^h(p_2^*) = \frac{2p_2^*}{\mu} = \frac{\mu-3\kappa n}{\mu}$. Thus, $\theta_{i2}^h(p_2^*) = 1 - \left(\frac{\mu-3\kappa n}{\mu}\right) = \frac{3\kappa n}{\mu}$ for $h \in \{i, -i\}$. Given $2 \leq n < \frac{2\mu}{3\kappa}$, these are both strictly positive. □

Proof of Proposition 4.6. This can be proved by taking first order derivatives of $\theta_{it}^h(p_t^*)$ with respect to n, for $h \in \{i, -i\}$,

$$\frac{\partial \theta_{i1}^i(p_1^*)}{\partial n} = \frac{\kappa}{2\mu(3\kappa n+2\mu)^2}[9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2]$$

Note that $9\kappa^2n^2+12\kappa nt-44\mu^2$ can be rewritten as $[3\kappa n+2\mu+4\sqrt{3}\mu][3\kappa n+2\mu-4\sqrt{3}\mu]$, which is negative, such that $\frac{\partial\theta_{i1}^i(p_1^*)}{\partial n}<0$ for all $n\in[2,\,\frac{2\mu}{3\kappa})$.

Furthermore, differentiating $\theta_{i1}^{-i}(p_1^*)=\frac{2(2\mu-\kappa n)}{2\mu+3\kappa n}-\frac{2\mu-\kappa n}{2\mu}=\frac{(\kappa n-2\mu)(3\kappa n-2\mu)}{2\mu(3\kappa n+2\mu)}$ with respect to n yields:

$$\frac{\partial\theta_{i1}^{-i}(p_1^*)}{\partial n}=\frac{\kappa}{2\mu(3\kappa n+2\mu)}[9\kappa^2n^2+12\kappa n\mu-28\mu^2]$$

Note that $9\kappa^2n^2+12\kappa n\mu-28\mu^2$ can be rewritten as $[3\kappa n+2\mu+4\sqrt{2}\mu][3\kappa n+2\mu-4\sqrt{2}\mu]$, which is negative, such that $\frac{\partial\theta_{i1}^{-i}(p_1^*)}{\partial n}<0$ for all $n\in[2,\,\frac{2\mu}{3n})$.

Finally, differentiating $\theta_{i2}^h(p_2^*)=\frac{3\kappa n}{\mu}$ with respect to n yields $\frac{\partial\theta_{i2}^h(p_2^*)}{\partial n}=\frac{3\kappa}{\mu}>0$ for all $n\in[2,\,\frac{2\mu}{3n})$. □

Proof of Lemma 4.5 . From Proposition 4.3, when $2\leq n<\frac{3\kappa}{2\mu}$, refer to the interior solution where $X^*<\frac{1}{n}$. For the interior solution, since firm's profit π_i^* is strictly deceasing with n, $\frac{\partial\pi_i^*}{\partial n}=\frac{2(\kappa n-2\mu)}{n^3}<0$ (since $\kappa<\frac{2\mu}{3n}$, $\kappa n-2\mu<0$ and $\frac{\partial\pi_i^*}{\partial n}<0$), the maximum relevant entry cost is given when $n=2$. Denote this cost by $\bar{f}$. Further, from another constraint condition $\kappa<\frac{2\mu}{3n}$, the minimum entry cost can be calculated as $n=\frac{2\mu}{3k}$. Denote this cost by $\underline{f}$. Now, the interior solution requires: $\underline{f}:=\frac{3k^2}{\mu}<f\leq\frac{3k^2-2k\mu+\mu^2}{2\mu}:=\bar{f}$. $\bar{f}\geq\underline{f}$ since $\bar{f}-\underline{f}=(\mu+\kappa)(\mu-3\kappa)\geq 0$ for all $n\in[2,\,\frac{2\mu}{3\kappa})$ and. When $f<\underline{f}$, refer to corner solution where $X^*=\frac{2}{n}$.

Solving the (4.23) for n yields $n^*=\sqrt{\frac{2\mu}{f}}$, when $f<\frac{3\kappa^2}{\mu}$ and $n^*=\frac{2\mu(\pm\kappa+\sqrt{2f\mu-2\kappa^2})}{2f\mu-3\kappa^2}$ when $f\in(\frac{3k^2}{\mu},\,\frac{3k^2-2k\mu+\mu^2}{2\mu}]$. The conditions of $f\in(\frac{3k^2}{\mu},\,\frac{3k^2-2k\mu+\mu^2}{2\mu}]$ guarantees there is only one positive solution of $n^*=\frac{2\mu(\pm\kappa+\sqrt{2f\mu-2\kappa^2})}{2f\mu-3\kappa^2}$.

(ii). Note that the optimal numbers of firm , n^*, must be calculated when$\pi(n^*,\,\kappa,\,\mu)=f$ holds. We first consider the impact of product differentiation level μ. Keep κ and f constant, as a unit of μ increases, the profit, $\pi(n^*,\,\kappa,\,\mu)$ has to increase, such that $\pi(n^*,\, \kappa,\, \mu) > f$. The zero profit condition can be maintained if optimal numbers of firm, n^*, increase since $\frac{\partial \pi^*(.)}{\partial n^*} < 0$. Therefore, an increase in t increases the optimal numbers of firm, $\frac{\partial n^*}{\partial \mu} > 0$. The intuition of the impact of κ is similar, but opposite to that of μ. For example, as κ increases, the equilibrium profit has to decrease such that, $\pi(n^*,\, \kappa,\, \mu) < f$. To re-achieve zero profit condition, n^*has to decrease. Thus, an increase in transaction cost reduces the optimal numbers of firm, $\frac{\partial n^*}{\partial \kappa} < 0$. The implication of entry cost is easier to obtain. From (4.23), we know $\pi(n^*,\, \kappa.\,, \mu) = f$. As f increases, the left hand side, $\pi(n^*,\, \kappa,\, \mu)$, has to increase. Keep κ and μ constant, this is only possible when n^*decreases. Hence, an increase of entry cost reduces the optimal numbers of firm, $\frac{\partial n^*}{\partial f} < 0$. □

Proof of Proposition 4.7 . The social planners aim to control the number of firm n to maximise the social welfare. The objective function is given by (4.25),

$$W(n) = \begin{cases} \lambda - p_1^* - \kappa - 2n \int_0^{\frac{1}{2n}} \mu x dx + n(\pi_i^* - f) \\ [\lambda - p_1^* - \kappa - 2n \int_0^{\frac{1}{2n}} \mu x dx] + [nX^*(\lambda - p_1^*) + n\hat{Q}_2^*(\lambda - p_2^* - \kappa) - 2n \int_0^{\frac{Q_1^*}{2}} \mu x dx] + [n(\pi_i^* - f)] \end{cases} \tag{4.28}$$

Note that the first line of (4.25) indicates the case in the corner solution where all consumers stockpile, this case requires $f < \frac{3\kappa^2}{\mu}$. In this case, $p_1^* = \frac{\mu}{n}$, $\pi_i^* = \frac{2\mu}{n^2}$. Applying the normal first order condition to $W(n)$ and reconstructing yield, $\frac{\mu}{2\hat{n}^2} - f = W'(\hat{n}) - f = 0$. Solving for $\hat{n}$ gives $\hat{n} = \sqrt{\frac{\mu}{2f}}$.

The second line of (4.25) indicates the interior solution where a proportion of consumers stockpile, this case requires $f \in [\frac{3k^2}{\mu}, \frac{3k^2 - 2k\mu + \mu^2}{2\mu}]$. In this case, $p_1^* = \frac{\mu}{n} - \frac{\kappa}{2}$, $p_2^* = \frac{\mu}{n} - \frac{3\kappa}{2}$. $X^* = \frac{3\kappa}{2\mu}$, $\hat{Q}_2^* = \frac{1}{n} - \frac{3\kappa}{2\mu}$, $\pi_i^* = \frac{3\kappa^2 n^2 - 4\kappa n\mu + 4\mu^2}{2n^2\mu}$. Applying the normal first order condition to $W(n)$ and reconstructing yield, $\frac{3\kappa^2 n^2 + \mu^2}{2n^2\mu} - f = W'(\hat{n}) - f = 0$. Solving for $\hat{n}$ gives $\hat{n} = \frac{\mu}{\sqrt{2f\mu - 3\kappa^2}}$.

To compare the n^* and $\hat{n}$, we just need to compare (4.26) and (4.23). Since $g(n)$ is decreasing, $\hat{n} > (=,<)n^*$ if $W'(\hat{n}) > (=,<)f$. Since $W'(\hat{n}) = f$, $W'(\hat{n}) > (=,<)f$ is equivalent to $W'(n^*) > (=,<)\pi(n^*)$. Since $W'(n) < \pi(n)^3$, we have $\hat{n} < n^*$. □

Proof of Proposition 4.8. i) From Proposition 4.5, we know the biases of own- and cross-price elasticities in period 1 are $\theta_{i1}^{i}(p_1^*) = \frac{3(2\mu-\kappa n)}{3\kappa n+2\mu} - \frac{2\mu-\kappa n}{2\mu} > 0$ and $\theta_{i1}^{-i}(p_1^*) = \frac{2(2\mu-\kappa n)}{2\mu+3\kappa n} - \frac{2\mu-\kappa n}{2\mu} > 0$. Substituting equation (4.23) into $\theta_{i1}^{i}(p_1^*)$ and $\theta_{i1}^{-i}(p_1^*)$ yields the biases of own- and cross-price elasticities with equilibrium numbers of entrants in period 1, $\theta_{i1}^{h}(n^*,\, \kappa,\, \mu)$ for $h \in \{i,\, -i\}$, where $n^*$is also a function of κ, and μ.

Take the first order derivatives of $\theta_{i1}^{i}(n^*,\, \kappa,\, \mu)$ w.r.t κ yields

$$\frac{\partial \theta_{i1}^{i}(n^*,\, \kappa,\, \mu)}{\partial \kappa} = \frac{\partial \theta_{i1}^{i}(n,\, \kappa,\, \mu)}{\partial \kappa} + \frac{\partial \theta_{i1}^{i}(n,\, \kappa,\, \mu)}{\partial n} + \frac{\partial n^*(\kappa)}{\partial \kappa}$$

$$\frac{\partial \theta_{i1}^{i}(n^*,\, \kappa,\, \mu)}{\partial \kappa} = \frac{n(9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2)}{2\mu(3\kappa n + 2\mu)^2} + \frac{\kappa(9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2)}{2\mu(3\kappa n + 2\mu)^2} \cdot \frac{\partial n^*(\kappa,\, \mu)}{\partial \kappa}$$

$$\frac{\partial \theta_{i1}^{i}(n^*,\, \kappa,\, \mu)}{\partial \kappa} = \frac{(9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2)}{2\mu(3\kappa n + 2\mu)^2} \cdot [n + \kappa \frac{\partial n^*(\kappa,\, \mu)}{\partial \kappa}]$$

$$\frac{\partial \theta_{i1}^{i}(n^*,\, \kappa,\, \mu)}{\partial \kappa} = \frac{(9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2)}{2\mu(3\kappa n + 2\mu)^2} \cdot \frac{[(-4\mu^2 f)[-(\kappa - \sqrt{2f\mu - 2\kappa^2})^2]}{\sqrt{2f\mu - 2\kappa^2}(2f\mu - 3\kappa^2)^2}$$

From the proof of Proposition 4.6, we know the first term on RHS is negative. Thus, on RHS, the first term is negative, the second term is positive, we can infer that $\frac{\partial \theta_{i1}^{i}(n^*,\kappa,\mu)}{\partial \kappa} < 0$.

[3] $g(n) - \pi(n) = \frac{4\kappa n - 3t}{2n^2}$. Provided that interior solution requires $n < \frac{2t}{3\kappa}$, $g(n) > \pi(n)$

Take the first order derivatives of $\theta_{i1}^{i}(n^{*},\, \kappa,\, \mu)$ w.r.t μ yields

$$\frac{\partial \theta_{i1}^{i}(n^{*},\, \mu)}{\partial \mu} = \frac{\partial \theta_{i1}^{i}(n,\, \mu)}{\partial \mu} + \frac{\partial \theta_{i1}^{i}(n,\, \mu)}{\partial \mu} + \frac{\partial n^{*}(\mu)}{\partial \mu}$$

$$\frac{\partial \theta_{i1}^{i}(n^{*},\, \mu)}{\partial \mu} = -\frac{\kappa n(9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2)}{2\mu^2(3\kappa n + 2\mu)^2} + \frac{\kappa\mu(9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2)}{2\mu^2(3\kappa n + 2\mu)^2} \cdot \frac{\partial n^{*}(\mu)}{\partial \mu}$$

$$\frac{\partial \theta_{i1}^{i}(n^{*},\, \mu)}{\partial \mu} = \frac{\kappa(9\kappa^2 n^2 + 12\kappa n\mu - 44\mu^2)}{2\mu^2(3\kappa n + 2\mu)^2} \cdot \frac{[(2\mu^2 f)[-(\sqrt{2f\mu - 2\kappa^2} - \kappa)^2]}{\sqrt{2f\mu - 2\kappa^2}(2f\mu - 3\kappa^2)^2}$$

From the proof of Proposition 4.6, we know the first term on RHS is negative. Thus, on RHS, the first term is negative, the second term is negative, we can infer that $\frac{\partial \theta_{ii1}(n^{*},\kappa,\mu)}{\partial \mu} > 0$.

Take the first order derivatives of $\theta_{i1}^{-i}(n^{*},\, \kappa,\, \mu)$ w.r.t κ yields

$$\frac{\partial \theta_{i1}^{-i}(n^{*},\, \kappa,\, \mu)}{\partial \kappa} = \frac{\partial \theta_{i1}^{-i}(n^{*},\, \kappa)}{\partial \kappa} + \frac{\partial \theta_{i1}^{-i}(n^{*},\, \kappa)}{\partial n} + \frac{\partial n^{*}(\kappa)}{\partial \kappa}$$

$$\frac{\partial \theta_{i1}^{-i}(n^{*},\, \kappa,\, \mu)}{\partial \kappa} = \frac{n(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2\mu(3\kappa n + 2\mu)^2} + \frac{\kappa(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2\mu(3\kappa n + 2\mu)^2} \cdot \frac{\partial n^{*}(\kappa)}{\partial \kappa}$$

$$\frac{\partial \theta_{i1}^{-i}(n^{*},\, \kappa,\, \mu)}{\partial \kappa} = \frac{(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2\mu(3\kappa n + 2\mu)^2} \cdot [n^{*} + \kappa\frac{\partial n^{*}(\kappa)}{\partial \kappa}]$$

$$\frac{\partial \theta_{i1}^{-i}(n^{*},\, \kappa,\, \mu)}{\partial \kappa} = \frac{(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2\mu(3\kappa n + 2\mu)^2} \cdot \frac{[(-4\mu^2 f)[-(\kappa - \sqrt{2f\mu - 2\kappa^2})^2]}{\sqrt{2f\mu - 2\kappa^2}(2f\mu - 3\kappa^2)^2}$$

From the proof of Proposition 4.6, we know the first term on RHS is negative. Thus, on RHS, the first term is negative, the second term is positive, we can infer that $\frac{\partial \theta_{i1}^{-i}(n^*, \kappa, t)}{\partial \kappa} < 0$.

Take the first order derivatives of $\theta_{i1}^{-i}(n^*,\, \kappa,\, \mu)$ w.r.t μ yields

$$\frac{\partial \theta_{i1}^{-i}(n^*,\, \kappa,\, \mu)}{\partial \mu} = \frac{\partial \theta_{i1}^{-i}(n^*,\, \mu)}{\partial \mu} + \frac{\partial \theta_{i1}^{-i}(n^*,\, \mu)}{\partial n} + \frac{\partial n^*(\mu)}{\partial \mu}$$

$$\frac{\partial \theta_{i1}^{-i}(n^*,\, \kappa,\, \mu)}{\partial \mu} = -\frac{\kappa n(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2\mu^2(3\kappa n + 2\mu)^2} + \frac{\kappa\mu(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2\mu^2(3\kappa n + 2\mu)^2} \cdot \frac{\partial n^*(\mu)}{\partial \mu}$$

$$\frac{\partial \theta_{i1}^{-i}(n^*,\, \kappa,\, \mu)}{\partial \mu} = \frac{\kappa(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2t^2(3\kappa n + 2\mu)^2} \cdot [\frac{\partial n^*(\mu)}{\partial \mu} \cdot \mu - n^*]]$$

$$\frac{\partial \theta_{i1}^{-i}(n^*,\, \kappa,\, \mu)}{\partial \mu} = \frac{\kappa(9\kappa^2 n^2 + 12\kappa n\mu - 28\mu^2)}{2\mu^2(3\kappa n + 2\mu)^2} \cdot \frac{[(2\mu^2 f)[-(\sqrt{2f\mu - 2\kappa^2} - \kappa)^2]}{\sqrt{2f\mu - 2\kappa^2}(2f\mu - 3\kappa^2)^2}$$

From the proof of Proposition 4.6, we know the first term on RHS is negative. Thus, on RHS, the first term is negative, the second term is negative, we can infer that $\frac{\partial \theta_{i1}^{-i}(n^*, \kappa, \mu)}{\partial \mu} > 0$.

ii) From Proposition 4.5, we know biases of own- and cross-price elasticities in period 2 are the same, $\theta_{i2}^{i}(p_2^*) = \theta_{i2}^{-i}(p_2^*) = \frac{3\kappa n}{2\mu}$. Inserting equation (4.23) yields biases of elasticity in period 2 with equilibrium level of entrants.

$$\theta_{i2}^{h*}(\kappa,\, \mu) = \frac{3\kappa(-\kappa + \sqrt{2f\mu - 2\kappa^2})}{2f\mu - 3\kappa^2} \qquad h \in \{i,\, -i\}$$

Take the first order derivatives of $\theta_{i2}^{h*}(\kappa,\, t)$ w.r.t κ and t yields,

$$\frac{\partial \theta_{i2}^{h*}(\kappa,\, \mu)}{\partial \kappa} = \frac{-6f\mu[-(\sqrt{2f\mu - 2\kappa^2} - \kappa)^2]}{\sqrt{2f\mu - 2\kappa^2}(2f\mu - 3\kappa^2)^2} > 0 \qquad h \in \{i,\, -i\}$$

$$\frac{\partial \theta_{i2}^{h*}(\kappa,\, \mu)}{\partial \mu} = \frac{3fk[-(\sqrt{2f\mu - 2\kappa^2} - \kappa)^2]}{\sqrt{2f\mu - 2\kappa^2}(2f\mu - 3\kappa^2)^2} < 0 \qquad h \in \{i,\, -i\}$$

□

Chapter 5

Consumer Stockpiling as a Form of Behavioural Based Price Discrimination

5.1 Introduction

In this chapter, we continue to investigate consumer stockpiling behaviour. One important and prevalent feature of this is that consumers who have stockpiled become inactive in the future. Hence, this allows seller to recognise those buyers that have not stockpiled and to price discriminate between consumers based on their past stockpiling behaviours.

Despite the literature of behavioural-based price discrimination (BBPD) having received a wide attention in recent years, analysis of such stockpiling-based BBPD remains rare. Moreover, associated welfare analysis remains ambiguous in the literature. Addressing these omissions is important for the implications of consumer policy in relevant market.

This chapter aims to help fill these gaps. It makes two main contributions. Firstly, it provides a dynamic model that considers consumer stockpiling behaviour. In equilibrium we show how consumer stockpiling can be used by firm as a special device to perform BBPD against consumers. Secondly and more substantially, we provide a welfare analysis of such effects of BBPD, and demonstrate that such price discrimination through stockpiling always improves both consumer surplus and total welfare. Hence, policymakers should not be concerned by such a form of price discrimination.

More specifically, we borrow the random utility choice model (Perloff and Salop 1985) to propose a two-period differentiated monopoly that incorporates consumer stockpiling behaviour. In each period, each consumer wishes to consume at most one unit, but is allowed to purchase the second unit to stockpile for future consumption. If consumers purchase in period 2, then a unit of transaction cost is incurred.

The model suggests that in any equilibrium, there is positive consumer stockpiling. Depending on the level of product differentiation, there are two cases in which all consumers stockpile and one case in which a proportion of consumers stockpile. In the corner solutions, all consumers stockpile in advance to save the expected expenditure on transaction cost in period 2. The firm endogenously sets a profit maximised price when the product differentiation level is sufficiently low relative to transaction cost. When the product differentiation level is moderate relative to transaction cost, this profit maximised price is constraint and thus the firm sets a lower price than the benchmark case. In the interior case where the level of product differentiation is higher relative to the transaction cost, consumers with higher match value stockpile in period 1, leaving those with low match value being active in period 2. Here, both period 1 and period 2 prices are lower than the benchmark case. This is because the firm needs to charge a lower price to low match value consumers in period 2. Then, in period 1 pricing, the firm needs to set its period 1 price higher than the period 2 price to sustain positive consumer stockpiling such that it can segment consumers to different groups according to their match value. This shows how the firm uses stockpiling as a device to identify consumers by their match values and charges discriminating prices against consumers.

We then examine the welfare effects. Particularly, we compare the welfare effects of BBPD with that of the normal pricing scheme. Despite some previous literature suggests that welfare effects are difficult to capture in a model of consumer stockpiling (Hendel and Nevo, 2013). Here, we provide a clear prediction relative to normal pricing benchmark that being able to stockpile always increases both firm's profits and consumers surplus. This is because with BBPD, i) consumers can buy at lower prices, and ii) consumers can save the transaction cost. It also increases the firm's profits. This is because even if performing BBPD means to charge lower prices than normal pricing scheme, profit loss can be compensated by the increased sale.

Finally, we provide a brief extension of an alternative transaction cost assumption. It was originally assumed consumers to incur transaction cost in period 2. Here, it is modified to be incurred for the second trip only. In this extension, we show how this alternative makes no difference to the welfare effects of BBPD if we assume market coverage.

As a bi-product, the findings of this chapter can also be used as an alternative explanations of price discrimination on some other industries that exist long-term contracts. These contracts allow consumers to pay a fixed-price in advance for the services and products that they receive in the future. Similar to consumer stockpiling, these contracts induce future demand to be shifted forward. It then follows that the monopolist can identify its previous consumers. Given this, the monopolist may choose to charge new consumers a different price. Common examples include telecommunication markets, gyms membership, magazine subscriptions and bank services.

In regards to the literature, this chapter firstly relates to the literature on consumer stockpiling. There are some marketing literature that focuses on the underestimation of increased demand from stockpiling in a price promotion period. (See Gupta, 1988; Bell *et al*, 1999; Gedenk *et al*, 2010; etc.). However, this does not allow for endogenous pricing and therefore is insufficient for an analysis of price discrimination. For some other studies that allow for endogenous pricing (e.g. Anton and Das Varma, 2005; Hosken and Reiffen, 2007; Guo and Villas-Boas, 2007), they either consider a quantity competition or considers equilibria in which consumers don't stockpile. This limits an analysis of welfare effects of stockpiling. Different from these above-mentioned paper, Hendel and Nevo (2003) study intertemporal price discrimination when consumers can store for future consumption. There are two types of consumers: price-sensitive consumer who stockpile for future, and less price-sensitive consumers who do not. Their result suggests that the welfare effect of BBPD is ambiguous, while we provide a clear and crisp result that how BBPD can strictly increase consumer surplus and total welfare.

More broadly, this Chapter is related to the wider literature on BBPD. Rossi *et al.* (1996) has pointed out that firms in many industries can price discriminate on the basis of purchase history of consumers. Since then, BBPD becomes a hot topic in the field of industrial organisation and quantitative marketing. In the literature, apart from some comprehensive surveys made by Armstrong (2006) and Fudenberg and Villas-Boas (2007). Most studies on BBPD focus on competitive price discrimination. Some of them focuses on consumer poaching where firm charges one price to its loyal consumer and a lower price to its rival's consumer. (see Fudenberg and Tirole, 2000; Hawswald and Marquez, 2006; Villas-Boas, 1999). Some other papers connect switching cost and BBPD (Chen, 1997; Shaffer and Zhang, 2000; and Taylor, 2003). They suggest that firm offers discounted price to compensate switching cost and thus gains less profits. Villas-Boas (2004) examines a monopolist selling to overlapping generations of heterogeneous consumers. The equilibrium involves cycles in price being charged to new consumers. For the welfare, he draws a result that the monopolist is worse off than if it could not perform price discrimination from recognising previous consumers. Jing (2011) considers a monopolist selling experienced product market where consumers' valuation can only be fully understood after purchase. Welfare effects is subject to the condition of the market. Our study differs from two perspectives. First, by solely focusing on stockpiling behaviour, we show how it can be used as devices towards BBPD, and secondly, we provide a welfare analysis of such effects of BBPD. Second, we demonstrate that that such price discrimination through stockpiling always improves both consumer surplus and total welfare.

This chapter proceeds as follows. Section 5.2 introduces the model. Section 5.3 and 5.4 present the main equilibrium analysis, before Section 5.5 provides the analysis of welfare effects and Section 5.6 shows an extension of alternative assumption of transaction cost. Finally, Section 5.7 concludes. All proofs are in the appendix.

5.2 Model

5.2.1 Assumptions

Consider a single product monopoly with zero production costs. The firm sells storable product over two periods, $t = 1, 2$. There is a unit mass of risk-neutral consumers with quasi-linear preferences, each of whom consumes at most one unit of the product per period. The market is not fully covered in a sense that consumers may choose not to buy at all in any given period. For a given price p_i, consumer m's net utility of consuming one unit is $u_m = \varepsilon_m - p_i$, where ε is a consumer specific match value. Each match value, ε, that remains fixed throughout the game and is independently distributed across consumers with $G(\varepsilon)$. We assume $G(\varepsilon)$ is continuous and twice differentiable on $[0, b]$ where $b > 0$. In particular, we focus on the uniform distribution with $G(\varepsilon) = \frac{\varepsilon}{b}$ and $g(\varepsilon) = \frac{1}{b}$. The parameter, b, is used and interpreted as the degree of product differentiation[1].

In our model, we assume that transactions are potentially costly for consumers. Such a transaction cost may be required in order to make a purchase and is independent of the number of units bought. Common examples includes the costs of visiting a firm or ordering a delivery. To ease exposition in the main model, we assume that transaction costs are zero in period 1, but equal to $\kappa \in (0, b)$ in period 2.[2] This captures the fact that repeat transactions are particularly costly for consumers and as we later show, it is the level of transaction costs in period 2, rather than period 1, that are important for consumers' stockpiling decisions. However, in Section 5.6, we show how this assumption can be relaxed to allow for positive transactions costs in both periods. For simplicity, we also suppose that all agents have a discount factor close to one, as most appropriate for products that are purchased frequently (e.g. bottles of cola).

We consider a one-shot game with two periods. In period 1, the firm chooses its period 1 price, p_1. The firm is unable to commit to its period 2 price. Consumers learn match values and observe the period 1 price before making period 1 purchase and stockpiling decisions - they can choose to not buy at all, to buy one unit, or to stockpile by buying two units. In period 2, the firm sets its period 2 price, p_2. Any remaining consumers that did not stockpile in period 1 then observe this price and choose whether to buy one or zero units. We focus then seek an equilibrium with equilibrium prices p_1^* and p_2^*.

5.2.2 Benchmark Analysis

We first briefly examine a benchmark case where consumer stockpiling is not feasible. In this case, the two periods are almost identical, apart from the transaction

[1] One may argue that this actually measures consumer heterogeneity. To keep the consistency throughout the thesis, I use it to index the level of product differentiation.

[2] To ensure that the whole market is active, κ cannot be too large, $\kappa < b$.

cost in period 2. In any period, a consumer will purchase one unit if his match value exceeds the cost of purchasing. As such, consumer will buy one unit in period 1 with probability $\Pr(\varepsilon - p_i \geq 0)$ and one unit in period 2 if $\Pr(\varepsilon - p_2 - \kappa \geq 0)$. Accordingly, firm's demand in period 1 and period 2 can be written as,

$$Q_1(p_1^{NS}) = 1 - G(p_1) = \frac{b - p_1}{b} \qquad Q_2(p_2^{NS} + \kappa) = 1 - G(p_2 + \kappa) = \frac{b - (p_2 + \kappa)}{b} \tag{5.1}$$

After applying the usual first order condition, one then obtain the non-storage equilibrium prices and quantity. In period 1, the firm sets $p_1^{*NS} = \frac{b}{2}$, and the equilibrium quantity is $Q_1^{NS} = \frac{1}{2}$. In period 2 the firm sets $p_2^{*NS} = \frac{b-\kappa}{2}$ and sells $Q_2^{NS} = \frac{b-\kappa}{2b}$. It can be seen that in period 2, the firm sets lower price. This is because the firm needs to offer a discounted price in period 2 to induce the consumers to incur the transaction cost. In aggregate, the firm earns $\pi^{*NS} = \frac{(b-\kappa)^2+b^2}{4b}$, whereas the equilibrium prices are increasing in the product differentiation b, and (weakly) decreasing in the transaction cost κ.

5.3 Equilibrium Analysis

We now start the main equilibrium analysis, where consumer stockpiling is feasible. Section 5.2.2 covers some important preliminary features of stockpiling decisions of consumers in period 1. Section 5.3.2 then endogenises the firm's behaviour.

5.3.1 Consumers' Decisions

We first characterise some features of consumer's stockpiling decisions and demand in period 1 for a given period 1 price, p_1, and expected period 2 price, p_2^e. Then we consider period 2 demand, for a given period 2 price, p_2.

5.3.1.1 Period 1

Consider any given consumer m's options with match value, ε_m, period 1 price, p_1 and expected period 2 price, p_2^e.

She could: i) choose to stockpile by buying two units in period 1 to gain

$$u_m^S = 2(\varepsilon_m - p_1)$$

ii) not to buy in period 1, but possibly to buy one unit in period 2 to gain

$$u_m^{'} = max\{\varepsilon_m - p_2^e - \kappa,\ 0\}$$

iii) buy one unit in period 1 and possibly to buy one unit in period 2 to gain

$$E(u_m^{NS}) = (\varepsilon_m - p_1) + max\{\varepsilon_m - p_2^e - \kappa,\ 0\}$$

Then note the following. First, if buying in period 1 gives consumer m negative payoffs, i.e. $\varepsilon_m - p_1 < 0$, then a) stockpiling in period 1 gives consumer m a negative payoff as well, and b) option iii) becomes dominated by option ii). Under this circumstance, consumer m never buys in period 1, but may buy in period 2 only, depending on the period 2 price. Secondly, if $\varepsilon_m - p_1 > 0$, consumer m will never choose option ii) to buy one unit only in period 2 because this is dominated by option i) or iii). Thus, the consumer must instead choose between i) and iii) and so will prefer option i) to stockpile if $S_m = u_m^S - E(u_m^{NS}) \geq 0$. It can then be shown that an increase in consumer m's match value weakly increases S_m. Now, we can state the following.

Lemma 5.1. *If consumer m with ε_m finds it optimal to stockpile in period 1, then so will any other consumer k with $\varepsilon_k > \varepsilon_m$. If consumer m with ε_m finds it optimal to not stockpile then so will any other consumer l with $\varepsilon_l < \varepsilon_m$.*

Lemma 5.1 supports the intuition that given the price of period 1 and expected period 2 price, whether consumer chooses to stockpile or not can be identified by their match value. Particularly, our model predicts that the consumers that are most likely to stockpile are those with relatively higher match values.

To proceed, it is useful to define $\bar{\varepsilon}$ as the match value of marginal consumer who is indifferent between stockpiling in period 1, and to define $X(\bar{\varepsilon})$ as the resulting level of stockpiling demand. As derived previously in (5.1), we also define $Q_1(.)$ as the level of consumer demand in period 1 absent the effects of stockpiling, and note that the firm's total level of demand (observed demand) in period 1 therefore equals $\hat{Q}_1(.) = Q_1(.) + X(.)$. We can then state the following:

Lemma 5.2. *Firm's observed demand in period 1, $\hat{Q}_1(.)$, is:*

$$\widehat{Q}_{i1}(.) = \begin{cases} 2Q_1(p_1) = \frac{2(b-p_1)}{b} & \text{if } \bar{\varepsilon} \leq p_1 \\ Q_1(p_1) + X(\bar{\varepsilon}) = \frac{b-p_1}{b} + X(\bar{\varepsilon}) & \text{if } \bar{\varepsilon} \in (p_1,\, b) \\ Q_1(p_1) = \frac{b-p_1}{b} & \text{if } \bar{\varepsilon} \geq b \end{cases} \tag{5.2}$$

This describes three cases. First, if $\bar{\varepsilon} \leq p_1$, all consumers stockpile. Hence, firm's stockpiling demand is equal to its true period 1 demand, $X(.) = Q_1(.)$, and so firm's total period 1 demand equals $\widehat{Q}_1(.) = 2Q_1(p_1)$, where $Q_1(p_1)$ coincides with the demand in the benchmark, (5.1). If, instead, $\bar{\varepsilon} \in (p_1,\, b)$, then only some consumers stockpile. Here, firm's observed demand equals $\widehat{Q}_1(.) = Q_1(p_1) + X(\bar{\varepsilon})$ since an aggregate of $Q_1(p_1)$ consumers buy of which $Q_1(p_1) - X(\bar{\varepsilon})$ buy one unit, and $X(\bar{\varepsilon})$ buys two units. Finally, if $\bar{\varepsilon} \geq b$, then no consumers stockpile, and so period 1 observed demand just corresponds to the benchmark case period 1 demand.

5.3.1.2 Period 2

We now move on to consider period 2 demand. Similar to period 1, we can define $Q_2(.)$ as the level of consumer demand in period 2 absent the effects of stockpiling, as derived previously in (5.1). We can then state the following.

Lemma 5.3. *The firm's observed demand in period 2, $\widehat{Q}_2$, is,*

$$\widehat{Q}_{i2}(.) = \begin{cases} 0 & \text{if } \bar{\varepsilon} \leq p_1 \\ Q_2(p_2+\kappa) - X(\bar{\varepsilon}) = \frac{b-(p_2+\kappa)}{b} - X(\bar{\varepsilon}) & \text{if } \bar{\varepsilon} \in (p_1,\, b] \\ Q_2(p_2+\kappa) = \frac{b-(p_2+\kappa)}{b} & \text{if } \bar{\varepsilon} \geq b \end{cases} \tag{5.3}$$

If $\bar{\varepsilon} \leq p_1$, all consumers have stockpiled and so period 2 is inactive. However, if $\bar{\varepsilon} \in (p_1,\, b)$, then consumers with $\varepsilon \in (p_1,\, \bar{\varepsilon}]$ did not stockpile and so remain active. As in the benchmark, any such consumer will then buy one unit in period 2. It then follows that firm's observed period 2 demand equals, $\hat{Q}_2 = Q_2(p_2+\kappa) - X(\bar{\varepsilon})$. If $\bar{\varepsilon} \geq b$, no consumers stockpiled and so period 2 observed demand just equals $\hat{Q}_2 = Q_2(p_2+\kappa)$.

5.3.2 Firm's Decisions

Using backwards induction, we now consider the firm's equilibrium decisions. We start from period 2 for a given level of consumer storage demand from period 1. We then derive the equilibrium levels of stockpiling demand for a given period 1 price and expected period 2 price, $X(p_1,\, p_2^e)$, where consumers' expectations of period 2 prices are consistent with the equilibrium , $p_1 = p_2^*(X)$. Finally, given the equilibrium levels of stockpiling demand, we then solve for period 1 equilibrium price.

5.3.2.1 Period 2

From (5.3), we know the period 2 market is active only if $\bar{\varepsilon} \in (p_1, b]$. From Section 5.3.1.1, given the period 2 price, period 2 observed demand comprises of those consumers that i) desire to buy in period 2 such that $\varepsilon - (p_2^e + \kappa) \geq 0$ and ii) did not stockpile in period 1. Suppose a proportion of consumers, indexed by $X(\bar{\varepsilon}) \in (0, 1]$, have stockpiled in period 1, such that $Q_1(.) - X(\bar{\varepsilon})$ consumers are potentially active within the market in period 2. Now we can state the following for the period 2 equilibrium,

Lemma 5.4. *Suppose $\bar{\varepsilon} \in (p_1, b]$ such that period 2 market is active. Then, if $b[1 - X(\bar{\varepsilon})] - \kappa \geq 0$, then the unique period 2 is achieved in equilibrium with,*

$$p_2^*(X(\bar{\varepsilon})) = \frac{1}{2}[b(1 - X(\bar{\varepsilon})) - \kappa] \geq 0 \tag{5.4}$$

and $\hat{Q}_2^*(.) = Q_2(p_2^*) - X(\bar{\varepsilon}) = \frac{1}{2b}[b(1 - X(\bar{\varepsilon})) - \kappa] \geq 0$

The optimal period 2 price, $p_2^*(X(\bar{\varepsilon}))$ is positive if $X(\bar{\varepsilon}) < \frac{b-\kappa}{b}$. Intuitively, those consumers that stockpiled in period 1 are the consumers with the highest match values. Hence the demand in period 2 consists of consumers with lower match values. It indicates that firm acquires consumer's match values and therefore sets period 2 discriminatory price by observing whether consumers have stockpiled and how many consumer have stockpiled. One can find that period 2 price has the following property. First, it is lower than period 2 equilibrium price in benchmark case $p_2^{*NS} = \frac{b-\kappa}{2}$. When storage demand reduces to 0, it collapses to benchmark case period 2 equilibrium price. Thus, it can be inferred that consumer stockpiling is the reason of price discrimination in period 2. When stockpiling demand exists, i.e, $X > 0$, different groups of consumers, identified by whether they stockpiled or not, have differences in the match value of the same product. In addition, period 2 price is also subject to the level of κ. This is because firm need to lower period

2 prices to attract and compensate consumers for the existence of transaction cost, which increases consumer's expenses to buy in period 2. Having observed differences of consumers' match value and the level of visit cost, firm can price discriminatingly in period 2 market.

5.3.3 Period 1

From the last section, we learned that period 2 price is determined by the level of stockpiling demand. In this section, we return to period 1 to examine the formation of stockpiling demand. We then solve for period 1 equilibrium price.

5.3.3.1 Equilibrium Stockpiling Demand

Denote $X(p_1, p_2^e)$ as firm's equilibrium level of stockpiling demand, where consumers expectations are correct if $p_1^e = p_2^e(X)$. As explained below, we can now state the following lemma.

Proposition 5.1. *The unique stockpiling demand in period 1 can be expressed as follows:*

$$X^*(.) = \begin{cases} 0 & \text{if } p_1 \geq \frac{b+\kappa}{2} \\ \frac{b+\kappa-2p_1}{b} & \text{if } \kappa < p_1 < \frac{b+\kappa}{2} \\ \frac{b-p_1}{b} & \text{if } p_1 \leq \kappa \end{cases} \tag{5.5}$$

When making the decisions of whether or not to stockpile, consumers optimally compare between the cost of stockpiling in period 1 p_1, and the cost returning to buy a second unit in period 2 $p_2 + \kappa$. This comparison is subject to the match value of each consumer, the level of period 1 price and expected period 2 price. Proposition 5.1 displays three different scenarios of stockpiling demand in period 1.

First, consider the first case where no stockpiling demand is facilitated. Here, all consumers who have purchased in period 1 find that stockpiling is less attractive than buying just one unit. From Section 5.3.1.1, it must be the case that $S^m = u_m^S - E(u_m^{NS}) < 0 \; \forall \varepsilon_m$. If $E(u_m^{NS}) = max\{\varepsilon - p_2^e - \kappa,\ 0\} = \varepsilon - p_2^e - \kappa$[3], rearranging yields the condition of operating zero storage demand, $p_1 > p_2 + \kappa$. Inserting equilibrium period 2 prices of equation (5.4) with $X = 0$, shows that this case requires $p_1 > \frac{b-\kappa}{2} + \kappa = \frac{b+\kappa}{2}$.

Second, consider the intermediate case where $X \in (0,\ Q(p_1))$. In this case, there exists a consumer who is indifferent between stockpiling and buying 1 unit in each period with $S^m = u_m^S - E(u_m^{NS}) = 0 \; \forall \varepsilon_m$, within those who have purchased in period 1. Rearranging yields $p_1 = p_2(.) + \kappa$. Substituting equation (5.4) and isolating the expression of X yields $X^*(.) = \frac{b+\kappa-2p_1}{b}$ and then get the condition $\kappa < p_1 < \frac{b+\kappa}{2}$.

Third, consider the third case where $X = Q_1(p_1)$ such that all consumers who buy one unit in period 1 also stockpile. Here, $S^m < 0 \; \forall \varepsilon_m$. Rearranging yields, $p_1 < p_2(X = Q(p_1) + \kappa$. Inserting equation (5.4) with $X = Q(p_1) = \frac{b-p_1}{b}$ shows that this case requires $p_1 < \kappa$.

5.3.3.2 Equilibrium Period 1 Price

We now move to derive the equilibrium by solving for the period 1 equilibrium prices. Given changes of stockpiling demand in period 1 and period 2, firm's profit maximisation problem can be written as,

$$\pi^S = p_1 \cdot [Q_1(p_1) + X(p_1^*)] + p_2^*(X^*) \cdot [Q_2(p_2) - X(p_1^*)] \tag{5.6}$$

where firm receives period 1 demand $\hat{Q}_1 = Q_1(p_1) + X(.)$ from (5.2) and (5.5), and (if active) sets a period 2 equilibrium price, p_2^*, (5.4), and receives a period 2

[3]If $E(u_m^{NS}) = 0$, then $S^m = 2(\varepsilon_m - p_1) < 0$. In this case consumer do not buy in period 1. The whole market is not active.

equilibrium demand $\hat{Q}_2 = Q_2(p_2) - X(.)$ from (5.3). To start with, we first cover the following important result.

Proposition 5.2. *In any symmetric equilibrium with $\kappa > 0$, there is a positive level of stockpiling demand.*

First, note that the benchmark case with no consumer stockpiling cannot qualify as an equilibrium when $\kappa > 0$. This is because zero stockpiling demand requires $p_1 > p_2 + \kappa$. But using Lemma 5.1 and Lemma 5.4, one can find that the benchmark $p_1^{*NS} = \frac{b}{2} < p_2^{*NS} + \kappa = \frac{b+\kappa}{2}$ implies the opposite. Hence, we now seek an equilibrium with $X^* > 0$. After defining Q_1^*, Q_2^* and X^* as the relevant quantities evaluated at equilibrium prices, we can then state the following:

Proposition 5.3. *There exists an unique equilibrium which is characterised as follows:*

i) If product differentiation is low, $b < 2\kappa$, then $X^* = Q_1^* = \frac{1}{2}$ and $\hat{Q}_2^* = Q_2^* - X^* = 0$, where $p_1^* = \frac{b}{2}$, $p_2^* = 0$.

ii) If product differentiation is moderate, $b \in [2\kappa, \frac{5\kappa}{2}]$, then $X^* = Q_1^* = \frac{b-\kappa}{b}$ and $\hat{Q}_2^* = Q_2^* - X^* = 0$, where $p_1^* = \kappa$, $p_2^* = 0$.

iii) If product differentiation is high, $b > \frac{5\kappa}{2}$, then $X^* = \frac{3\kappa}{2b} < Q_1^* = \frac{2b+\kappa}{4b}$ and $\hat{Q}_2^* = Q_2^* - X^* = \frac{2b+\kappa}{4\kappa} > 0$, where $p_1^* = \frac{b}{2} - \frac{\kappa}{4}$, $p_2^* = \frac{b}{2} - \frac{5\kappa}{4}$.

Proposition 5.3 establishes the unique equilibrium where consumers stockpile and where the firm sets prices endogenously. Depending on the level of product differentiation, there are three different cases. These can be understood as follows.

The first and the second cases suggest all consumers stockpile in equilibrium. i.e, $X^* = Q_1(p_1^*) = \frac{b-p_1^*}{b}$. In these two cases, consider the marginal consumer who is indifferent between stockpiling with $u^S \equiv 2(\varepsilon - p_1^*) = (\varepsilon - p_1^*) + (\varepsilon - p_2^* - \kappa) \equiv u^S$. If this consumer were to deviate from equilibrium by not stockpiling, she should rationally expect an equilibrium period 2 price $\lim_{X^* \to Q_1(p_1^*)} p_2^* = \frac{1}{2}(p_1^* - \kappa)$. Now, her benefits from not stockpiling can therefore be expressed as $u^{NS} = (\varepsilon - p_1^*) + (\varepsilon - (\frac{p_1^* - \kappa}{2}) - \kappa) = 2\varepsilon - \frac{3p_1^*}{2} - \frac{\kappa}{2}$. Hence, this consumer will stockpile as required in equilibrium only if $u^S > u^{NS} \iff p_1^* \leq \kappa$. Moreover, using a similar logic to Proposition 5.1, this condition is sufficient for all consumers to stockpile in equilibrium. Therefore, any equilibrium with $X^* = Q_1(p_1^*)$ requires $p_1^* \leq \kappa$. To derive p_1^*, provided that the period 1 price is less than κ, the firm selects p_1 to maximise (5.6), where $X = Q_1(p_1)$, such that (6) becomes $\pi = 2p_1 Q_1$ subject to $p_1 \leq \kappa$. After applying the normal first order condition, this leads to $p_1^* = \min\{\frac{b}{2}, \kappa\}$.

Hence, in the first case when the product differentiation level is sufficiently low relative to the level of transaction cost, $b < 2\kappa$, all consumers are inclined to stockpile in an effort of avoiding the relatively high transaction cost in period 2. This implies that the firm does not need to reduce its period 1 price to attract consumers to stockpile and firm's profit is unconstrained at $p_1^* = \frac{b}{2}$. In this case, the equilibrium level of stockpiling demand is $X^* = Q_1^* = \frac{1}{2}$

In contrast, in the second case when the product differentiation level is moderate relative to the level of transaction cost such that $b \in [2\kappa, \frac{5\kappa}{2}]$. In this case, the firm's profit maximisation price $\frac{b}{2}$ is bound by the $p_1^* \leq \kappa$ and therefore sets $p_1^* = \kappa < \frac{b}{2}$ to ensure the marginal consumer who is indifferent between stockpiling and not is just willing to stockpile with $u^S = u^{NS}$. In this case, the equilibrium level of stockpiling demand is $X^* = Q_1^* = \frac{b-\kappa}{b}$.

Finally, consider the third case with a higher level of product differentiation level relative to the level of transaction cost, $b > \frac{5\kappa}{2}$. Here, only a strict positive proportion of consumers with high match values stockpile in advance. This leaves the remaining consumers with low match values being active in period 2. Hence, in response to the consumers with relatively lower match values, the firm optimally sets a lower price in period 2, to maintain its market demand. Consequently, as the period 2 price goes down, the period 1 price also falls. Intuitively, from the discussion of Proposition 5.1, we know that $X \in (0, Q_1(.))$ requires $p_1^* = p_2^* + \kappa$. Thus, to ensure that a positive interior proportion of consumers are willing to stockpile, we require $p_1^* = p_2^* + \kappa$, and that this price relationship then uniquely pins down the proportion of consumers who stockpile. In particular, the appendix then shows that $p_1^* = \frac{b}{2} - \frac{\kappa}{4} = p_2^* + \kappa = \frac{b}{2} - \frac{5\kappa}{4} + \kappa$ and $X^* = \frac{3\kappa}{2b}$. Finally to ensure that equilibrium is well-defined with non-negative prices, it is necessary that $b > \frac{5\kappa}{2}$.

The last case in which only an interior proportion of consumers stockpile can also be understood from the perspective of a special form of BBPD. This is because being able to stockpile actually gives firm an opportunity to use consumers' stockpiling behaviour as a device to perform price discrimination for the unit of the product that will be consumed in period 2. Specifically, the firm sets period 1 price lower than the benchmark period 1 price to attract consumers with high match values to stockpile. By doing so, the firm is able to identify different groups of consumers with different match values from stockpiling behaviour. As a result, period 2 market only consists of those with lower match values. Having segmented the consumers, the firm then sets a even lower period 2 price to the remaining consumers.

5.4 Comparative Static Analysis

Before considering the welfare effects, we now analyse how the equilibrium level of stockpiling demand, X^*, varies with product differentiation and the transaction cost.

Corollary 5.1. *In equilibrium, the level of stockpiling demand X^* is weakly decreasing in the level of product differentiation, b, and increasing in the size of the transaction cost, κ.*

Corollary 5.1 illustrates how stockpiling demand varies with respect to exogenous market parameters.

In the first equilibrium case from Proposition 5.3 where the product differentiation is sufficiently low relative to the level of transaction cost, $b < 2\kappa$, equilibrium stockpiling demand is $X^* = \frac{1}{2}$. This is independent with any exogenous factors.

In the second equilibrium case where the product differentiation is moderate relative to transaction cost such that $b \in [2\kappa, \frac{5\kappa}{2}]$, the equilibrium stockpiling $X^* = \frac{b-\kappa}{b}$ is increasing in the product differentiation level and decreasing in the transaction cost. Intuitively, the demand function of period 1 demand and stockpiling demand are subject to the level of product differentiation, $Q_1(.) = X(.) = \frac{b-p_1}{b}$. As b increases, both stockpiling and period 1 demand increases. Similarly, the firm sets $p_1^* = \kappa$ in this case, as the level of transaction cost goes up, both period 1 demand and stockpiling demand go down.

In the last equilibrium case where the product differentiation is high relative to transaction cost, $b > \frac{5\kappa}{2}$, such that some consumers stockpile, $X^* \in (0, Q_1(.))$, the equilibrium level of stockpiling demand increases with product differentiation and decreases with transaction cost. To get the intuition, first, consider a unit change in product differentiation, b. Holding X^* constant, both period prices increase, but the period 1 price increases by more such that $p_1^* > p_2^* + \kappa$. As a result, consumers are less inclined to stockpile and X^* reduces until the point where $p_1^* = p_2^* + \kappa$ is restored. Next, consider a unit change in the transaction cost, κ. Compared to the period 1 price, the price 2 price is more responsive to the transaction cost. Holding X^* constant, p_1^* decreases, while $p_2^* + \kappa$ increases such that $p_1^* < p_2^* + \kappa$. As a result, consumers are more inclined to stockpile and X^* increases until the point where $p_1^* = p_2^* + \kappa$ is restored.

5.5 Welfare

Having characterised the equilibrium, we now consider the welfare effects. Here, we define define the total welfare as the sum of aggregate consumer surplus, CS, and firm profits, π.

$$W(.) = CS(.) + \pi(.) \tag{5.7}$$

5.5.1 Benchmark

Recall from the Section 5.2.2 that in the benchmark case where stockpiling is prohibited, the firm sets a period 1 equilibrium price, $p_1^{*NS} = \frac{b}{2}$, and a period 2 price $p_2^* = \frac{b-\kappa}{b}$, with demand $Q_1(.) = \frac{b-p_1}{b}$, and $Q_2(.) = \frac{b-(p_2+\kappa)}{b}$. Equilibrium firm profits then equal $\pi^{*NS} = p_1^* Q_1^* + p_2^* Q_2^* = \frac{b^2+(b-\kappa)^2}{4b}$. Given the levels of transaction costs, one can then also define consumer surplus in period 1 as $CS_1 = \int_{p_1^*}^{b} Q_1(p_1)dp_1$ and consumer surplus in period 2 as $CS_2 = \int_{p_2^*+\kappa}^{b} Q_2(p_2+\kappa)dp_2 \equiv \int_{p_2^*}^{b-\kappa} Q_2(p_2)dp_2$, such that total consumer surplus equals $CS^{*NS} = \frac{b^2+(b-\kappa)^2}{8b}$. After expanding (5.7), one then obtains $W^{*NS} = \frac{3[b^2+(b-\kappa)^2]}{8b}$

5.5.2 Main Model

By comparing these benchmark welfare values to the welfare values evaluated at equilibrium price p_1^*, we can now consider the welfare effects of stockpiling.

Proposition 5.4. *The possibility of consumer stockpiling always increases the firm's equilibrium profits, consumer surplus and total welfare.*

Proposition 5.4 summarises the welfare effects of stockpiling. This can be understood as follows.

First, consider the first equilibrium case where product differentiation is extremely low relative to transaction cost such that $b < 2\kappa$. The prices are the same as the benchmark case, so no extra demand is stimulated. However, being able to stockpile brings demand forward from period 2 to period 1. This allows consumer to buy their period 2 unit in period 1 without incurring the transaction cost. It therefore increases consumer surplus. Meanwhile, the firm also benefits from it because as suggested by benchmark case, the firm needs to offer a discounted price in period 2 to induce the consumers to incur the transaction cost. Thus, if all consumers stockpile in period 1, the firm can sell the period 2 demand at a higher period 1 equilibrium price. In aggregate, social welfare, which is given as the sum of consumer surplus and firm's profit as (5.7), increases in this case.

Second, consider the second equilibrium case where the product differentiation is moderate relative to transaction cost such that $b \in [2\kappa, \frac{5\kappa}{2}]$. Similar to the first case, all consumer stockpile. However, the firm now sets a lower period 1 price than in the benchmark. Thus, consumers benefit not only from bringing forward their consumption to avoid the transaction costs, but also from a lower price compared to the benchmark. It can be shown in the appendix that selling at this price still increases the firm's profit because this price attracts more consumers to buy and stockpile. From above, total welfare therefore increases in this case.

Finally. consider the third equilibrium case where the product differentiation is higher relative to transaction cost such that $b > \frac{5\kappa}{2}$. The period 2 market is active and only a proportion of consumers stockpile, while the firm sets both period 1 and period 2 prices lower than the benchmark case. The reduced prices have two effects. First, they create more demand in both periods. These increase consumer surplus. Second, the reduced price in period 1 attracts consumers to stockpile. Stockpiling consumers are better from saving the transaction cost expenditure in period 2. Thus, consumer surplus are better off. It can then be shown in the appendix that the firm is also better off because the increased sales compensate for the reduced prices. Hence, as both the consumers and the firm benefit, total welfare also rises in this case.

Proposition 5.4 can also be understood in terms of BBPD. The previous literature suggests that the effects of price discrimination and stockpiling model is typically complex and difficult to derive (Hendel and Nevo, 2013). But in the final stockpiling case of our analysis, we provide a crisp and clear prediction of it. When stockpiling is feasible, the firm sets a lower period 1 price to identify consumers with relatively high match values by attracting them to stockpile, and increase market demand. In period 2, after acknowledging that now only consumers with lower match values are active, the firm then sets its period 2 price lower than its period 1 price for these consumers in a way that benefits both the firm and the consumers.

In other words, the firm would like to enable stockpiling rather than not while policymakers would encourage consumers to stockpile in an effort of improving consumer surplus and total social welfare.

5.6 Alternative Transaction Cost Assumption

It was originally assumed that consumers incur a transaction cost if they make a purchase in period 2. We will now show how our results remain robust under a more realistic assumption where consumers incur the transaction cost only if they return to make a second transaction with the firm. However, in order to maintain tractability, this weaker transaction cost assumption requires us to make an additional assumption that the market is covered. In particular, this requires all consumers to consume (but not purchase) a unit in each period. Formally, this is consistent with consumers' match values that *are* distributed on the interval $[a, b]$ where $0 < a < b$ and where a is large enough to ensure that consumers always consume. Given this, option ii) in Section 5.3.1.1 becomes invalid. Any given consumer m only chooses between i) stockpiling in period 1 with utility: $u_m^s = 2(\varepsilon_m - p_1)$ and, iii) buying 1 unit in each period with expected utility:

$E(u_m^{NS}) = \varepsilon_m - p_1 + (\varepsilon_m - p_2 - t)$.[4], depending the one that maximises their utility. Now this extension with alternative transaction assumption coincides with the Lemma 1 of the main model. Using the same backward induction method as we did in the main model, it follows that the value of storage demand $X(p_1)$ the following pricing equilibrium and welfare results under this alternative visit cost assumption are now subject to the lower bound of the distribution of consumer's match value. Accordingly, our result of the welfare effects remains robust.

5.7 Conclusions

It is often observed that consumers stockpile for future consumption. What we focus in this chapter is that how the stockpiling can be used as a device towards BBPD and its welfare implications. Based on a storable product, we set up a two-period monopoly. In the unique equilibrium, due to the existence of transaction costs, consumers stockpile. Depending on the level of product differentiation, there are two cases where all consumers stockpile and one case where a proportion of consumers stockpile. In the latter case, where product differentiation level is high relative to transaction cost, higher match value consumers stockpile in advance while consumers with lower match value do not. Hence, the firm can segment consumers according to their match value and perform BBPD.

In regards to welfare effects, We show that being able to stockpile always increases aggregate consumer surplus and firm profits despite any potential BBPD. For the firm, this BBPD prompts it to optimally select lower prices in a way that increases its profits from the resulting increase in market demand. For the consumers, their surplus increases due to i) being able to stockpile and thereby reduce their expenditure on transaction costs, and ii) the reduced prices. Hence, policymakers should not be concerned by such a form of price discrimination.

[4]It requires that a is sufficient large such that $a - p_1 < t$.

We hope that future research can build on our work in at least three ways. First, further work should generalise, expand, and test out findings to develop the implications of competing market where more than one firms are in the market. Second, future work would be useful if more consumer factors, such as uncertainty and risk aversion, are taken into account. Finally, and more generally, we hope that future research can build on our framework to analyse further storable product related questions.

Appendix

Proof of Lemma 5.1. : Period 1 consumer's choice can be summarised as follows: i), $u_m^s = 2(\varepsilon_m - p_1)$, ii), $u_m^{'} = Max\{\varepsilon_m - p_2^e - t,\ 0\}$,and iii), $E(u_m^{NS}) = (\varepsilon_m - p_1) + Max\{\varepsilon_m - p_2^e - t,\ 0\}$. First consider if $(\varepsilon_m - p_1) < 0$, then i) will be a dominated strategy since $u_m^s < 0$. Furthermore, ii) will be superior than iii). Thus, if purchasing in period 1 gives consumer a negative payoff such that $(\varepsilon_m - p_1) < 0$, option ii), which is irrelevant to storage demand, is the dominating strategy. On the other hand, if $(\varepsilon_m - p_1) > 0$, ii) will be the dominated strategy. Under such a case consumer will choose to stockpile if $S^m = u_m^s - E(u_m^{NS}) > 0$. One can verify that S^m is weakly increasing in ε_m by taking first order derivatives of S^m to ε_m, which equals to zero. This completes the proof.

$\square$

Proof of Lemma 5.4. : Suppose the match value of marginal consumer $\bar{\varepsilon} \in [0,\ b)$, such that firm have positive storage demand. Then one can use $\pi_2 = p_2\hat{Q}_2(.)$ with (5.3) to derive firm's period 2 price for a given level of stockpiling demand. Applying normal first order condition yields, $p_2^*(.) = \frac{1}{2}[b(1 - X(\bar{\varepsilon})) - \kappa] \geq 0$. $\square$

Proof of Proposition 5.1. : Once we have derived firm's period 2 equilibrium price, we can use it to derive period 1 equilibrium level stockpiling demand. Firstly, consider consumers' stockpiling decisions. From Section 5.3.1, we know consumer optimally compares the cost of stockpiling and the cost of waiting until period 2. If $S^m = u_m^S - E(u_m^{NS}) = 0$, consumers are indifferent between stockpiling. By construction, this indifference requires $p_1 = p_2(X) + \kappa$.

In the case of no stockpiling demand that is suggested by the top line of (5.5). In this case, $X = 0$, such that no consumer finds it optimal to stockpile. It must then follow that $p_1 > p_2(0) + \kappa$. From (5.4), $p_2^* = \frac{b-\kappa}{2}$ when $X = 0$. Therefore, this case requires, $p_1 > \frac{b-\kappa}{2} + \kappa$, rearranging yields $p_1 > \frac{b+\kappa}{2}$.

Second, consider another 'corner case' in which all consumer stockpiles. In this case, all consumer who has bought in period 1 finds it optimal to stockpile, such that $Q_1(.) = X(.) = \frac{b-p_1}{b}$. It must then follow that $p_1 < p_2(\frac{b-p_1}{b}) + \kappa$. By construction, it requires $p_1 < \kappa$.

Lastly, consider the intermediate case in which some consumer stockpiles. In this case, there exists an unique level of equilibrium stockpiling, $X \in (0, Q_1(.))$, such that $p_1 = p_2(X) + \kappa$ holds for firm. To obtain such X, one can insert p_2^* from (5.4) and isolate the expression of X to yield $X = \frac{b+\kappa-2p_1}{b}$.

Finally, note that the levels of stockpiling and associated conditions in (5.5) are continuous when $p_1 > 0$ □

Proof of Proposition 5.3. : a) If $b < \frac{5\kappa}{2}$ such that all consumer stockpiles, firm's profit maximisation function can be written as,

$$\pi = p_1[Q_1(.) + X(.)]$$

where $Q_1(.) = X(.) = \frac{b-p_1}{b}$. Applying the normal first order condition yields, $p_1 = \frac{b}{2}$. Note that, for all consumer stockpiling to be facilitated, it also requires $p_1 < \kappa$. Therefore, if $b < 2\kappa$, firm charges period 1 equilibrium price, $p_1^* = \frac{b}{2}$. If $2\kappa < b < \frac{5\kappa}{2}$, firm charges $p_1^* = \kappa$.

b). If $b > \frac{5\kappa}{2}$, firm's profit maximisation problem can be written as,

$$\pi^S = p_1 \cdot [Q_1(p_1) + X(p_1^*)] + p_2^*(X^*) \cdot [Q_2(p_2) - X(p_1^*)] \tag{5.8}$$

where $Q_1(p_1) = \frac{b-p_1}{b}$ and $Q_2(p_2) = \frac{b-(p_2+\kappa)}{b}$, and where $p_2^*(X^*)$ and $X(p_1^*)$ are given by (5.4) and (5.5).

After solving the first order condition of (6) with respect to p_1[5], one obtains

$$p_1^* = \frac{b}{2} - \frac{\kappa}{4}$$

Together with $p_2^* = \frac{b}{2} - \frac{5\kappa}{4}$, $X^* = \frac{3\kappa}{2b}$, $\pi^* = \frac{(2b-\kappa)^2}{8b} + \frac{\kappa^2}{b}$. This case requires $p_2^* \geq 0$ or $b \geq (5\kappa/2)$. $\square$

Proof of Corollary 5.1. : The proof can be straightforwardly done by taking first order derivatives of X^* w.r.t $z \in \{b, \kappa\}$ respectively.

$\square$

Proof of Proposition 5.4. : From Section 5.2.2, we know equilibrium price of benchmark case is $p_1^{*NS} = \frac{b}{2}$, and $p_2^{*NS} = \frac{b-\kappa}{2}$. In this case, firm's profit function is $\pi^{*NS} = p_1^{*NS}Q_1(.) + p_2^{*NS}Q_2(.) == \frac{(b-\kappa)^2+b^2}{4b}$. From Section 5.1, we know the consumer surplus of benchmark case is $CS^{*NS} = \frac{b^2+(b-\kappa)^2}{8b}$, and total welfare of the benchmark case is $\frac{3[b^2+(b-\kappa)^2]}{8b}$.

If stockpiling is feasible. From Section 5.3.3.2, there are three different cases,

a). If $b < 2\kappa$, all consumer stockpiles while firm charges $p_1^* = \frac{b}{2}$. In this case the profit function is $\pi^* = 2p_1^*[Q_1(.)] = \frac{b}{2}$. It can then be inferred that being able to stockpile gives firm more profits from $\pi^* - \pi^{*NS} = \frac{\kappa(2b-\kappa)}{4b} \geq 0$. Consumer's welfare is given by $CS^* = 2\int_{p_1^*}^{b} Q_1(.) = \frac{b}{4}$. It can then be inferred that in this case being able to stockpile gives consumer more surplus from $CS^* - CS^{*NS} = \frac{\kappa(2b-\kappa)}{8b} > 0$. From above, it follows that social welfare also increases in this case.

b). If $2\kappa \leq b \leq \frac{5\kappa}{2}$, all consumer stockpiles while firm charges $p_1^* = \kappa$. In this case, the profit function is $\pi^* = 2p_1^*[Q_1(.)] = \frac{2\kappa(b-\kappa)}{b}$. It can then be inferred that being

[5] The second order condition is given by, $\frac{\partial \pi^2}{\partial p_1^2} = -\frac{4}{b} < 0$

able to stockpile gives firm more profits from $\pi^* - \pi^{*NS} = \frac{[(b-\frac{5\kappa}{2})+\frac{\sqrt{7}}{2}\kappa][(b-\frac{5\kappa}{2})-\frac{\sqrt{7}}{2}\kappa]}{-2b} > 0$ if $2\kappa \leq$ b $\leq \frac{5\kappa}{2}$. Consumer's welfare is given by $CS^* = 2\int_{\kappa}^{b} Q_1(.) = \frac{(b-\kappa)^2}{b}$. It can then be inferred that when $2\kappa \leq b \leq \frac{5\kappa}{2}$, being able to stockpile gives consumer more surplus from $CS^* - CS^{*NS} = \frac{3[(b-\frac{7\kappa}{6})+\frac{\sqrt{7}}{6}\kappa][(b-\frac{7\kappa}{6})-\frac{\sqrt{7}}{6}\kappa]}{4b} > 0$. From above, it follows that social welfare also increases in this case.

c). If $b > \frac{5\kappa}{2}$, some consumer stockpiles while firm charges period 1 price $p_1^* = \frac{b}{2} - \frac{\kappa}{4}$, and period 2 price $p_2^* = \frac{b}{2} - \frac{5\kappa}{4}$. Firm's profit maximisation function is now given by (5.6) and equals $\pi^* = \frac{(2b-\kappa)}{8b} + \frac{\kappa^2}{b}$. It is straightforward to find that $\pi^* - \pi^{*NS} = \frac{7\kappa^2}{8b} > 0$.

For consumer surplus, If $b > \frac{5\kappa}{2}$, in equilibrium where there is a strict positive proposition of consumer stockpile with $X^* = \frac{3\kappa}{2b} \in [0,\, Q_1(p_1)]$, we have $p_1^* = p_2^* + \kappa$,

$$CS = \int_{p_1^*}^{b} [Q_1(p_1) + X^*]dp_1 + \int_{p_2^*+\kappa}^{b} [Q_2(p_2+\kappa) - X^*]dp_2$$

$$= \int_{p_1^*}^{b} Q_1(p_1)dp_1 + \int_{p_2^*+\kappa}^{b} Q_2(p_2)dp_2$$

$$= \frac{(\kappa+2b)^2}{16b}$$

as $p_1^* = p_2^* + \kappa$, and where $p_1^* = \frac{b}{2} - \frac{\kappa}{4}$, $p_2^* = \frac{b}{2} - \frac{5\kappa}{4}$, $X^* = \frac{3\kappa}{2b}$, $Q_1(p_1) = \frac{b-p_1}{b}$, $Q_2(p_2) = \frac{b-(p_2+\kappa)}{b}$.

Given the total consumer surplus of non-stockpiling benchmark case is $CS^{*NS} = \frac{b^2+(b-\kappa)^2}{b}$

$$CS - CS^{*NS} = \frac{\kappa(8b-\kappa)}{16b}$$

This is strictly positive because $b > \frac{5\kappa}{2}$.

It then follows that both profits and consumer surplus increase, and therefore welfare, in this case. $\square$

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