Student Partners in Task Design in a computer medium to promote Foundation students' learning of mathematics

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A team consisting of three mathematics education teacher-researchers, four former Foundation students (called Student Partners, SPs), and two analytic assistants worked together to produce mathematical tasks in a computer medium for the mathematical learning of current Foundation students (FSs). We have explored the collaboration between the SPs and researchers, the processes and outcomes of task design, and the contribution of the collaboration to tutorial teaching of FSs. We seek insight into the learning of all concerned of mathematics, mathematics teaching, task design and personal-professional development. The project is ongoing. Here we introduce the project and present early findings – specifically related to task design and the contribution of SPs.

Keywords: Teachers' and students' practices at university level; Novel approaches to teaching; The role of digital and other resources in university mathematics education; Students as partners in task design.

INTRODUCTION TO THE CATALYST PROJECT

We report from an exciting new development project (2016-18: The Catalyst Project[1]) in which mathematics teacher-educators collaborate with former Foundation students as partners in designing computer-based mathematical tasks for current Foundation students. Our research explores the collaboration of participants, the design of tasks, teaching of students in tutorials, use of computer software for teaching and learning mathematics and the learning of all concerned.

The mathematics learning of students in our university Foundation Studies programme (henceforth Foundation Studies students – FSs) is the focus of this developmental project. These are students who need a higher-level qualification than they hold currently in order to be able to enter the first year of their desired undergraduate programme (e.g., programmes in engineering or science). For such programmes, mathematics is an essential component. All FSs are required to pass their year-long module in mathematics. It has been observed that teaching and learning in this module in the past has been rather procedurally based: students have been introduced to and expected to learn the application of procedures to mathematical problems and have been examined on their procedural competency. A current aim is to make the module more conceptually based, and to explore the use of computer-based tasks for this purpose. Our previous research has shown that:

a) The involvement of peer learning-teaching in a mathematics module has resulted in participating students gaining higher marks, with associated experiences reported as positive to their understanding of concepts; (SYMBoL, e.g., Duah, Croft & Inglis, 2013; Solomon, Croft, Duah, & Lawson, 2014);

b) The cultural differences between teacher-researchers designing an innovation in mathematics teaching-learning and students engaged in learning mathematics through the innovation have contributed positively to outcomes/higher marks. (ESUM, e.g., Jaworski, Robinson, Matthews, & Croft, 2012).[ii]

Beyond this activity, we have found very little other research involving students' engagement through partnerships in mathematics teaching and learning. There is relevant work in Higher Education more generally involving *Partnership Learning Communities* (Healey, Flint and Harrington, 2014), but this does not include mathematics specifically. A recent special issue of NoMAD, the Nordic Journal of research in mathematics education, included several papers in which teachers were involved in exploring their own practice. The reports address three themes, one being 'innovative approaches to teaching and learning, with emphasis on student participation in the educational process' (Goodchild & Jaworski, 2017). One paper, in particular, reported positively on students' activity in oral presentations as a tool for promoting metacognitive regulation in *Real Analysis* (Naalsund & Skogholt, 2014). Student engagement and understanding were seen to improve through their participative activity. Searches to date have revealed no other relevant work.

Building on our experiences in SYMBoL and ESUM, we sought to design mathematical tasks which would challenge the FSs in new ways, engage them visually and actively and promote the beginnings of a new learning culture. Recognising the value of *student design of resources* and *peer support* as demonstrated in the SYMBoL Project we built both of these aspects into our Catalyst project. One of the Catalyst aims was: To promote collaboration between staff and students that results in higher degrees of confidence, motivation and learning in mathematics and a new culture in the teaching-learning of mathematics (e.g., HEA, 2014). We hoped to learn from the various elements of this project in ways that would have relevance beyond mathematics, particularly in the inclusion of former FSs as Student Partners (SPs) in course design and teaching. Our own learning from working with students in these ways was also a central aim, with the intention to bring staff and student cultures into focus through our joint activity. The reflections of the students on their activity, both SPs and FSs, were seen as an important outcome of the project. Thus, our innovation in the Catalyst Project has two main areas of inquiry:

- The design of computer-based tasks in *matrices* and *complex numbers* for the FSs using *Autograph* software.
- The involvement of former FSs (the SPs) in the design of teaching and in the tutorial teaching of the current FSs.

According to its specification, the Foundation Studies programme provides "*An opportunity to make it onto a degree course at Loughborough University*". Within the programme there is a wide range of student experience in mathematics from GCSE grade C to A level grade A. We focus on a mathematics module called 'Applicable Mathematics' which prepares students to take up degree programmes in Science or Engineering. The two semesters focus on the following topics: *Semester 1:* Algebra, Logarithms, Inequalities, Functions, Trigonometry, Vectors, Differentiation, Integration, Sequences; *Semester 2*: Polynomials, Partial Fractions, Further Calculus, Conic Sections, Vectors**,** Matrices, Complex Numbers.

The project has focused on the teaching of *Matrices* and *Complex Numbers* in Semester 2 in 2017. The three project leaders (PLs) have worked with four SPs to design tasks using the computer software *Autograph* in the two topic areas. Tasks are for use in tutorials with Foundation Students (FSs). SPs are former FSs: in the previous year group they were successful in having achieved grades at the levels required for transition to programmes in Mechanical Engineering, Chemical Engineering, Physics and Chemistry. At the time of their recruitment, they were first year students in their current programmes. In addition, two doctoral students in Mathematics Education were recruited as "Analytical Assistants" to support data collection and analysis. Thus, nine participants have been involved in the project, with differing roles.

THEORETICAL BACKGROUND

We are concerned with *learning* at a number of levels.

- FSs learning of mathematics;
- SPs learning of mathematics, task design and participation with staff in preparing for undergraduate learning;
- Mathematics teachers and researchers learning about the design of teaching in partnership with students.

This learning is influenced by a wide range of factors which include the curriculum, and institutional settings within the broader sociocultural setting. Some of these factors we can seek to influence; others are less amenable to innovation. We take a fundamentally Vygotskian (e.g., Vygotsky, 1978; Wertsch, 1991) perspective recognising particularly mediation by people and tools that support learning; goal-directed activity and action related to learning and teaching; scientific concepts that require pedagogic mediation; and the zone of proximal development in which mediation fosters learning and development. We engage particularly with digital tools, their design and use, and the ways in which they mediate the learning process through both support and challenge for making sense of mathematical concepts.

An important theoretical concept is that of “partnership” between staff and SPs (e.g., Healey, Flint and Harrington, 2014). Relationships within the partnership have resulted in the design of mathematical tasks and their use with the FSs. The nature of this partnership is central to project outcomes, in terms of the designed tasks and their use. We see ourselves as having formed a ‘Learning Community’ in which *co-learning* is an important concept (Wagner, 1997), and which demonstrates tenets of a *Community of Practice,* such as *mutual engagement* and *joint enterprise* (Wenger 1998) and a *Community of Inquiry,* such as *critical alignment* (Jaworski 2006).

METHODOLOGY

We take a developmental research approach, consistent with our Vygotskian theory, which both studies project development and learning within the project and contributes to development and learning (Goodchild, 2008; Jaworski, 2003). Mediation and tool use, for example, can be seen in an interactive stance of reflection and negotiation in which participants engage together in activity and action with growth of mutual understanding and co-learning (Wagner, 1997). Analysis begins in questioning of what is done and achieved and is formalised through scientific inquiry addressing a range of data through recognised methods.

Research questions and data

Our *Research Questions* relevant to this paper are as follows:

- How have SPs engaged with task design and what has been the outcomes and issues arising?
- How have FSs worked with the designed tasks?
- What have we learned about the FSs’ learning of mathematics with the designed tasks? What issues arise?

Data, which are being analysed to address these questions include:

- The involvement of SPs and staff in the design process as shown through recordings of project meetings, SP reflections/reports, documents (collected at the design meetings and from the SPs’ own work).
- The tasks, and their use as seen through observation, screen capture and discussion. The teacher’s narratives from her reviewing of tutorial data.

Analytical approach to date

Reflection and negotiation have taken place through meetings, discussion, sharing and review of designed tasks leading to increased awareness of issues in design and communication. This collaborative co-learning has involved a bringing to collective consciousness of key elements of the design process and issues to be resolved.

Analysis of collected data according to research questions has been qualitative, focusing on data from the design meetings, and from tutorials with FSs in which designed tasks have been used. A process of data reduction has summarised and coded recorded data, allowing initial identification of key elements and issues relating to research questions. The process is cyclic with developing depth of inquiry and insight to significant issues expressed through analytical memos. The tasks themselves have also been a focus of scrutiny which is ongoing. These analyses are as yet in their very early stages, so what we report below is tentative and indicative. Here, we discuss some emergent findings in task design and use of tasks with FSs.

EMERGENT FINDINGS

Task Design

The teacher/lecturer of the Foundation course provided course notes on the two topics, Complex Numbers (CN) and Matrices. An expert in Autograph gave the group an induction into its use and potential for mathematical representation and exploration. SPs were asked to review the notes and think about possible tasks using Autograph. Task design, in 2 SP pairs (one to each topic) took place over 6 weeks and across 4 meetings – finding times for these meetings, from timetables of 9 partners, over a short time period was challenging. At the meetings, SPs' presented their ideas to the whole team, hesitantly in the beginning but with growing confidence in response to expressed appreciation and suggestions from the team. The pair working on tasks in *complex numbers* were quick to provide examples and to modify them according to suggestions in meetings. The pair who worked on matrices found it harder to get going. Tasks in complex numbers appeared to be more readily achievable than in *matrices* where concepts seemed less amenable to digital representation/questioning – it became necessary both to identify the barriers and to find some resolution. Collaboration between the SP pair and the PLs, focusing on the mathematics of matrices and the learner difficulties suggested by SPs, resulted in a set of tasks on matrices. One of the PLs also designed a set of tasks in GeoGebra focusing on matrix arithmetic. The emerging 'raw' tasks, consisting of an Autograph (or GeoGebra) file with brief associated notes, were then 'prepared' by the FS lecturer to make them ready for FS use. We see two examples of the prepared tasks, one for each topic, in Figure 1. Certain characteristics, incorporated by the SPs into these tasks can be seen in the examples; FSs have to:

- use several display features of Autograph to “see” mathematical objects and relationships;
- undertake some associated calculation by hand;
- relate movements on their screen to the handwork and theory involved;
- reflect on specific results to develop a more general awareness of concepts.

Some of these demands turned out to be a challenge for many FSs as we see below.

> Question 2: Open the Autograph File Task 2
>
> There are three complex numbers labelled z_1, z_2 and z. z_1 is fixed while z_2 and z can be moved. Select z_2 and move it until z reaches the position 3 + i.
>
> (a) What complex number is z_2?
>
> (b) Right click and “Unhide All" to check your answer.
>
> (c) What is the relationship between z_1, z_2 and z?
>
> (d) Explore subtraction of complex numbers in Autograph.
>
> (e) Now calculate by hand:
>
> With $z_1 = -1 - 3i$ and $z = 3 + i$, find z_2 such that $z_2 - z_1 = z$.
>
> (f) Draw (by hand) all three complex numbers on an Argand diagram.
> Give a geometric interpretation of the relationship between z and z_1 and z_2.

> Question 7: Open the Autograph File Matrices 5
>
> On this page you see two straight lines. Their equations are $4x - y = 14$ and $7x + 4y = a$
>
> (a) By hand, using matrices, calculate the value of a so that the solution to the simultaneous equations is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
>
> (b) When you have a solution, use the “constant controller” to vary the value of a until the point (3, 2) is clearly displayed.
>
> (c) Select both lines by holding down the “Shift" key. Both lines should have changed colour.
>
> Go to “Object" in the menu bar and choose “Solve f(x) = g(x)".
>
> A point is displayed. To see what the co-ordinates of this solution are, go to “View" in the menu bar and choose “Results Box". Does your solution make Autograph show the intersection of the lines to be x = 3 and y = 2?

Figure 1: Examples of prepared tasks in complex numbers and matrices

Use of tasks with Foundation students

The tasks were used in timetabled tutorials with the FSs, who were asked also to comment on the tasks for the research. Those agreeing to participate were audio-recorded in conversation with the teacher and one researcher. From analysis to date we are gaining some insight into FS participation with the tasks. We have recorded instances of FS requesting help from the teacher, asking questions, explaining their solutions, revealing mathematical insight, surprise, or lack of understanding. In interactions with the FS, the teacher responds to students, asks questions, explains, and provides technical information. The dialogue in Figure 2 shows a teacher-student exchange, relating to Question 2 above, that reveals an issue in the student's engagement with the task that is typical of several such exchanges:

We see that the student had engaged with the task, moving z_2 as instructed. However, he did not know what is meant by "the relationship between z1, z2 and z" although he had written that "z_2-z_1 gives you z". It seems a case of not understanding the meaning of the word "relationship", although he had written the relationship. We are learning here about language issues in working between the visual mode of Autograph and the symbolic mode of expressing complex relationships.

T: What are you doing in Question 2? [She looks at what he has written] 3+i... Haa! What did you find out? What relationship?

FS: I don't know what it means.

T: You don't know what that means? Well they are connecting aren't they? You must be doing something with them. If they are moving together [z1, z2 and z are moving together] we do something to them and then you get the third.

FS: I wrote that z_2-z_1 gives you z.

T: So you did write it down. For [part] c, that is the relationship. Yes, that's what we meant.

FS: OK

T: So the relationship is subtraction. You are subtracting two complex numbers. How did you know that though? Did you know that from the picture? Or did you do something else?

FS: I worked it out.

T: Ah ok. So you actually did the calculation. T=Teacher FS= Foundation Student

result. Whether this is as well as discerning it from the movements in Autograph or instead of this, is not clear. Thus, it might be that the student had used Autograph effectively and made links with the symbolic forms. Or it may be that he had sidestepped interpretation of the visual and instead had worked out the result symbolically (the latter perhaps being a more familiar task).

The teacher's reflective narrative relating to the recordings from the second tutorial on CN reveals the following example [Teacher's written words are italicised]:

> *[Student] found the additive relationship by adding separately the real parts and then the imaginary parts. He says "is it bisecting the angle?" I reply "it's something to do with it".* [She asks about her related lecture presentation and mentions a comparison with *vectors*.] *The student correctly relates vector addition to "adding head to tail" and that this forms a "triangle".* [She indicates that he does not seem to understand what is meant by a "geometric interpretation".]

The teacher's reflection suggests a student engaging correctly with several concepts including complex addition and vector representation. Yet, again, we see a problem with language – the term "geometric interpretation" is unfamiliar to the student. One student, asked about his work on the Autograph tasks, replied that he found the questions “hard to read” and could not “understand the way they are worded”.

The two examples quoted are typical of recorded exchanges. We see from these

a) Students not understanding the words or expression of ideas in the written questions (e.g., “relationship”, “geometric interpretation”)

b) Students not articulating explicit conclusions from what they see on screen – rather using the familiar forms of calculation to answer questions.

These observations lead us to question both the Autograph tasks and the wording of the tasks. How might we have worded the tasks differently so that students would engage with what they could see on screen and discern the mathematical relationships that the task was designed to reveal? How might we wish to modify the task itself so that students engage visually rather than depending on calculation? The challenge for the team here is twofold: to design a teaching approach that introduces the language we want students to use and enables students to become familiar with its use; and to design tasks that are revealing of concepts in and of themselves, so that students can see visually what they familiarly work out in calculation. These seem to be important elements of the learning culture we are trying to foster.

We are aware that many FSs come to their university course from school or college where their mathematical enculturation towards success in national final examinations may have encouraged a procedural perspective on learning mathematics (Minards, 2013) and that we see the results of this to some extent in their response to the designed tasks. As well as looking critically therefore at the design of the tasks, we have to consider the wider mathematical culture, the nature of teaching that seeks to interact with this culture and the ways in which the tasks can be incorporated within the teaching-learning interface.

CONCLUSION AND FURTHER RESEARCH

The developmental nature of the project can be seen through the development of mathematical tasks for Foundation students by SPs and PLs in partnership, the subsequent use of the tasks in tutorials with the FSs, and issues arising from task design and use revealed both in practice and in analysis of data from the various events. An aim of the project was to foster conceptual understanding of mathematics by the FSs. We see above some issues arising from the nature of the tasks and the ways in which they are written, and from the ways in which FSs’ mathematical experience influences their engagement with the tasks.

Because analysis is in its very early stages, we are not yet in a position to report on many of the aspects of learning in the project (such as aspects of the learning of the SPs). However, already we can start to see indications of important learning and the feedback element of the developmental research. Co-learning has been demonstrated between the FSs and the teacher-researchers – FSs' learning of mathematics has raised issues for the intention and preparation of tasks which offer challenges to the researchers and for future work with the SPs. The tasks and their design have been mediational tools not only for the mathematical learning of the students but also for the awareness of the researchers about teaching-learning issues, not least the issue of language in which tasks are expressed. The project is ongoing, both in terms of teaching-learning development and of analysis of the data collected so far.

A major issue for the project has been the timescale as dictated by the funding body and university organisation of teaching. We had barely half a semester to recruit SPs, initiate the design process, hold 4 spaced meetings, prepare the tasks for FSs and hold the tutorials. The project end coincides with the time for the next cohort to reach the teaching of matrices and complex numbers, so we could not build this into the project. We expect to use the same tasks again with the new cohort and collect further data, informed by our experiences a year ago. Since our data is extensive, in depth analysis is ongoing from which more in-depth reporting should be possible. We expect to be reporting further on the many aspects of this project.

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[i] Supported by HEFCE (Higher Education Funding Council for England) Catalyst Fund: Innovations in learning and teaching, and addressing barriers to student success **A:** Small-scale, 'experimental' innovation in learning and teaching.

[ii] SYMBoL – Second Year Mathematics Beyond Lectures – was a project designed to support teaching in two second year mathematics modules, Vector Calculus and Complex Analysis. Students who had experienced these modules were employed over a summer period to design resources in collaboration with mathematics staff. The resources were used in subsequent delivery of the modules and a peer support system was initiated in which third year undergraduates supported their second year counterparts.

ESUM – Engineering Students Understanding Mathematics – was a developmental research project involving an innovation in mathematics teaching seeking to engage students more conceptually with mathematics through inquiry-based activity, a computer-based learning environment, small group tasks and an assessed small group project.

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