INTERACTION OF CRACKS WITH DISLOCATIONS
IN COUPLE-STRESS ELASTICITY

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Abstract. *In the present work we study the interaction of a finite-length crack with a climb dislocation within the framework of the generalized continuum theory of couple-stress elasticity. Our approach is based on the distributed dislocation technique. Due to the nature of the boundary conditions that arise in couple-stress elasticity, the crack is modeled by a continuous distribution of climb dislocations and constrained wedge disclinations. These distributions produce both standard stresses and couple stresses in the body. The final results are obtained by numerically solving a system of coupled singular integral equations with both Cauchy and logarithmic kernels. The results for the near-tip fields differ in several respects from the predictions of the classical fracture mechanics. In particular, the present results indicate that a cracked solid governed by couple-stress elasticity behaves in a more rigid way (having increased stiffness) as compared to a solid governed by classical elasticity. Also, the stress level at the crack tip region is appreciably higher, within a small zone adjacent to the tip, than the one predicted by classical elasticity while the crack-face displacements and rotations are significantly smaller that the respective ones in classical elasticity.*

1 INTRODUCTION

The interaction between a crack and a dislocation is a fundamental problem of fracture mechanics, since this interaction determines, in many cases, the macroscopic brittle or ductile material response. Extensive work on this problem is reported in the literature, within the framework of classical elasticity. Rice and Thomson[1] proposed an energy condition for dislocation emission from a of crack tip discussing the consequent way of fracture, brittle or ductile. Thomson[2] and Weertman[3] introduced the idea of a dislocation shielded crack and the concept of the dislocation free zone. The emitted dislocation is expected to glide away from the crack tip until the interaction force is balanced by the lattice friction force and the dislocation comes to rest. The distance between the crack tip and the point that the dislocation comes to rest is the dislocation free zone. Emitted dislocations are known to reduce the stress field in the vicinity of the crack tip and hence the local stress intensity factor. Later, Zhang and Li[4] employed the complex potential method to calculate the stress intensity factors at the crack tips and the image forces due to the presence of the dislocation. Markenscoff[5] provided a solution for the stress field ahead of the crack tip using integral equations. Additionally, there are numerous experimental observations of these phenomena and we may refer indicatively, to Kobayashi and Ohr[6], and Michot and George[7].

In the present work, the interaction of a finite-length crack with a climb dislocation is studied using a generalized continuum theory to account for the material microstructure. The theory employed is a particular case of the general approach of Mindlin[8]. The fundamental concepts of the couple-stress theory were first introduced by the Cosserat brothers[9], but the subject was generalized and reached maturity only in the 1960s through the works of Toupin[10], Mindlin and Tiersten[11], and Koiter[12]. Applications of the couple-stress elasticity theory, mainly on stress-concentration problems, met with some success providing solutions more adequate physically than classical-elasticity solutions. Work employing couple-stress theories on elasticity and

plasticity problems is also continued in recent years[13-17].

Our approach is based on the distributed dislocation technique (DDT) to construct the integral equations that describe the problem. Since Bilby's[18] pioneering work to model cracks by a distribution of dislocations, the DDT has been employed to analyze various crack problems in classical elasticity. The main advantage of the technique is that it provides detailed information about the crack problem demanding a relatively small computational cost compared to other analytical or numerical methods (for a detailed review we refer to Hills et al.[19]). Gourgiotis and Georgiadis[20,21] have recently applied the standard DDT to solve finite-length crack problems, under mode I, mode II and mode III conditions, within the framework of couple-stress elasticity.

Following the latter approach, the solution to the problem is obtained by the superposition of two auxiliary problems, an un-cracked medium subjected to the field of a climb dislocation and a cracked body loaded along the crack faces by equal and opposite tractions to those generated in the first auxiliary problem. The boundary conditions are met by distributing not only climb dislocations but also constant discontinuities of the rotation ω along the crack faces, named as *constrained* wedge disclinations[21]. The continuous distribution of these defects along the crack faces results in a coupled system of singular integral equations with Cauchy and logarithmic kernels.

2 BASIC EQUATIONS OF PLANE STRAIN IN COUPLE-STRESS ELASTICITY

The general idea in the so-called generalized continuum theories (one of which is the theory of couple stresses) is considering a continuum with material particles (macro-volumes), behaving like *deformable* bodies[22]. This behavior can easily be realized if such a material particle is viewed as a collection of sub-particles. It is further assumed that internal forces (called dipolar or double forces) are developed between the sub-particles. Although each pair of the dipolar forces has a zero resultant force, it gives generally a *non-zero* moment and therefore gives rise to stresses on a surface called couple-stresses. This means that a surface element may transmit, besides the usual force vector, a *couple* vector as well. One can interpret physically the couple-stresses as created by frictional couples resisting the relative rotation of the grains (sub-particles).

For a body that occupies a domain in the (x,y)-plane under plane strain conditions, the two-dimensional displacement field is described as

$$u_x \equiv u_x(x,y) \neq 0 , \quad u_y \equiv u_y(x,y) \neq 0 , \quad u_z \equiv 0 . \tag{1}$$

Assuming vanishing body forces and body couples the non-vanishing components of the *asymmetric* force-stress tensor and the couple-stress tensor are written as

$$\sigma_{xx} = (\lambda + 2\mu)\partial_x u_x + \lambda \partial_y u_y , \quad \sigma_{yy} = (\lambda + 2\mu)\partial_y u_y + \lambda \partial_x u_x , \tag{2}$$

$$\sigma_{yx} = \mu(\partial_y u_x + \partial_x u_y) + \mu\ell^2(\partial_x^3 u_y - \partial_x^2 \partial_y u_x + \partial_x \partial_y^2 u_y - \partial_y^3 u_x) ,$$

$$\sigma_{xy} = \mu(\partial_y u_x + \partial_x u_y) - \mu\ell^2(\partial_x^3 u_y - \partial_x^2 \partial_y u_x + \partial_x \partial_y^2 u_y - \partial_y^3 u_x) , \tag{3}$$

$$m_{xz} = 2\mu\ell^2(\partial_x^2 u_y - \partial_x \partial_y u_x) , \quad m_{yz} = 2\mu\ell^2(\partial_x \partial_y u_y - \partial_y^2 u_x) , \tag{4}$$

where ℓ is the characteristic material length accounting for couple-stress effects, expressed in dimensions of [length].

The equations of equilibrium written in terms of displacements are expressed as follows (ν is the Poisson's ratio)

$$(1-2\nu)^{-1}\partial_x\left[2(1-\nu)\partial_x u_x + \partial_y u_y\right] + \partial_y^2 u_x + \ell^2\left(\partial_x^3 \partial_y u_y - \partial_x^2 \partial_y^2 u_x + \partial_x \partial_y^3 u_y - \partial_y^4 u_x\right) = 0 ,$$

$$(1-2\nu)^{-1}\partial_y\left[2(1-\nu)\partial_y u_y + \partial_x u_x\right] + \partial_x^2 u_y + \ell^2\left(\partial_x^3 \partial_y u_x - \partial_x^2 \partial_y^2 u_y + \partial_x \partial_y^3 u_x - \partial_y^4 u_y\right) = 0 . \tag{5}$$

3 FORMULATION OF THE CRACK-DISLOCATION INTERACTION PROBLEM

Consider a straight finite crack of length 2α embedded in the infinite xy-plane (Fig. 1) and a discrete climb dislocation $\mathbf{b} = (0, b_y, 0)$ lying in the crack line at a distance d from the crack center. The crack faces are traction free and the body is considered to be in plane-strain conditions. The following mixed boundary conditions hold in the upper half-plane $(y \geq 0)$

$$\sigma_{yy}(x,0) = 0, \quad m_{yz}(x,0) = 0 \qquad \text{for } |x| < a , \tag{6}$$

$$\sigma_{yx}(x,0) = 0 \qquad \text{for } -\infty < x < \infty , \tag{7}$$

$$u_x(x,0) = 0, \quad \omega(x,0) = 0 \qquad \text{for } |x| > a . \tag{8}$$

The presence of a climb dislocation at a distance d from the origin induces the conditions

$$\sigma_{yy}(d,0) \to \sigma_{yy}^{disloc}, \quad \sigma_{yx}(d,0), \ \sigma_{xy}(d,0) \to 0, \quad m_{yz}(d,0) \to m_{yz}^{disloc}, \quad m_{yz}(d,0) \to 0 \quad \text{for } x = d , \tag{9}$$

where σ_{yy}^{disloc} and m_{yz}^{disloc} are the stresses and couple-stresses induced by a discrete climb dislocation in couple-stress elasticity[21].

The regularity conditions at infinity are given as

$$\sigma_{yy}^\infty, \ \sigma_{yx}^\infty, \ \sigma_{xy}^\infty, \ \sigma_{xx}^\infty \to 0, \quad m_{xz}^\infty, \ m_{yz}^\infty \to 0 \quad \text{as} \quad r \to \infty , \tag{10}$$

where $r \equiv (x^2 + y^2)^{1/2}$ is the distance from the origin. Eq. (10) denotes that there is no additional remotely applied loading to the problem.
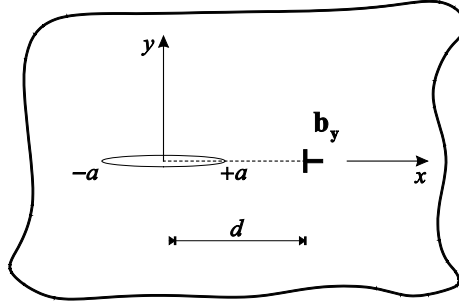

Figure 1. Cracked body interacting with a discrete climb dislocation

The solution to the original problem is obtained by the superposition of two auxiliary problems. Firstly, an un-cracked infinite medium subjected to the boundary conditions of eq. (9) is examined. It can be readily verified that along the line $y = 0$ both stresses and couple-stresses are generated. The corresponding stress field is obtained by application of the Fourier transform method in a half plane with the dislocation lying at its boundary, as it will be described in the next section.

In the second auxiliary problem (also referred as corrective solution), the original cracked medium is considered without the dislocation interaction. The only loading is now applied along the crack faces consisting of equal and opposite tractions to those generated in the un-cracked medium of the first auxiliary problem. The boundary conditions along the crack faces are written as

$$\sigma_{yy}(x,0) = -\sigma_{yy}^{disloc}(d-x,0), \quad m_{yz}(x,0) = -m_{yz}^{disloc}(d-x,0), \quad \sigma_{yx}(x,0) = 0 \qquad \text{for } |x| < a \tag{11}$$

Our intention is to solve the second auxiliary problem described by the boundary conditions of eqs. (7), (8) and (11). In classical elasticity, the normal stress of eq. (11) is produced by a continuous distribution of climb dislocations. In couple-stress elasticity this is not sufficient since a discrete climb dislocation produces both normal stresses σ_{yy} and couple-stresses m_{yz} along the line $y = 0$. On the other hand, the work conjugates of the reduced force traction $P_y = \sigma_{yy}n_y$ and the tangential couple traction $R_z = m_{yz}n_y$ are the normal displacement u_y and the rotation ω respectively. In light of the above, we are led to the conclusion that in order to generate the desired stress field (11) we need to distribute both discontinuities of the displacement u_y (climb dislocations) and the rotation ω. This was earlier pointed out by Gourgiotis and Georgiadis[21], who named the second type of defect *constrained* wedge disclinations.

4   GREEN'S FUNCTIONS

Our aim is to find the stress and couple-stress field, along the crack-line $y = 0$, induced by a discrete climb dislocation and a discrete *constrained* wedge disclination located at the origin of the (x,y)-plane. The above stress fields will serve as the Green's functions of our problem.

We impose at the origin of the (x,y)-plane a discrete climb dislocation with Burgers vector $\mathbf{b} = (0, b_y \equiv b, 0)$ and a discrete constrained wedge disclination with Frank vector $\mathbf{\Omega} = (0,0,\Omega)$. Considering the upper half-plane

$\left(-\infty < x < \infty,\ y \geq 0\right)$, we write the following boundary value problems

$$u_y\left(x,0^+\right) = -(b/2)H(x), \quad \omega\left(x,0^+\right) = 0, \quad \sigma_{yx}\left(x,0^+\right) = 0, \tag{12}$$

$$u_y\left(x,0^+\right) = 0, \quad \omega\left(x,0^+\right) = (\Omega/2)H(x), \quad \sigma_{yx}\left(x,0^+\right) = 0, \tag{13}$$

where $H(x)$ is the Heaviside step-function. It is clear by its description that the constrained wedge disclination departs from the concept of the classical wedge disclination since the discontinuity in the rotation does not induce any discontinuity in the displacement.

Both boundary value problems are attacked with the *Fourier* transform method using results from the distribution theory in the inversion procedure. By superposition of the solutions we obtain the following expressions for the normal stress σ_{yy} and the couple-stress m_{yz}

$$\sigma_{yy}\left(x, y = 0^+\right) = \frac{\mu b}{2\pi(1-\nu)x} + \frac{2\mu b}{\pi x}\left[\frac{2\ell^2}{x^2} - K_2\left(|x|/\ell\right)\right] - \frac{\mu\Omega}{\pi}\left[\frac{2\ell^2}{x^2} - K_2\left(|x|/\ell\right)\right] - \frac{\mu\Omega}{\pi}K_0\left(|x|/\ell\right), \tag{14}$$

$$m_{yz}\left(x, y = 0^+\right) = -\frac{\mu b}{\pi}\left[\frac{2\ell^2}{x^2} - K_2\left(|x|/\ell\right)\right] - \frac{\mu b}{\pi}K_0\left(|x|/\ell\right) + \frac{\mu\ell\Omega}{2\pi}\mathrm{sgn}(x)\cdot G_{1,3}^{2,1}\left(\left.\frac{x^2}{4\ell^2}\right| \begin{array}{c}1\\-1/2, 1/2, 0\end{array}\right), \tag{15}$$

where $K_i\left(|x|/\ell\right)$ is the ith order modified Bessel function of the second kind and $G_{c,d}^{a,b}()$ is the MeijerG function, which is a tabulated function. Eqs. (14) and (15) are the Green's functions of the problem. Concerning the nature of the above stress fields, we note the following: (i) Employing the asymptotic expansions of the modified Bessel and the MeijerG functions as $x \to 0$, we conclude that the normal stress σ_{yy} exhibits a Cauchy-type singularity due to the climb dislocation and a logarithmic singularity due to the constrained wedge disclination. On the contrary, the couple stress m_{yz} exhibits a Cauchy-type singularity due to the wedge disclination and a logarithmic singularity due to the climb dislocation. (ii) As $x \to \pm\infty$, it can be shown that $\sigma_{yy} \to 0$ while $m_{yz} \to \mp\mu\ell\Omega$, that is, the constrained wedge disclination does not induce normal stresses at infinity contrary to the classical wedge disclination which generates logarithmically unbounded normal stresses at infinity, both in classical and in couple-stress elasticity. (iii) When $\ell \to 0$ the couple-stress effects vanish and the expression for the normal stress σ_{yy} degenerates to the classical field for a discrete climb dislocation, $\sigma_{yy}\left(x, y = 0^+\right) = \mu b/2\pi(1-\nu)x$. We derive that the constrained wedge disclination induces stresses and couple-stresses only when couple-stress effects are taken into account. This is a convenient feature of the Green's functions of eqs. (14) and (15), since the respective Green's function of classical elasticity is recovered in the limit $\ell \to 0$.

5   REDUCTION OF THE PROBLEM TO A SYSTEM OF SINGULAR INTEGRAL EQUATIONS

The corrective stresses of eqs. (11) are generated by a continuous distribution of climb dislocations of magnitude b and constrained wedge disclinations of magnitude Ω along the crack faces $\left(|x| < a\right)$. The normal stress σ_{yy} and the couple-stress m_{yz} induced by the distribution of the two defects are derived by integrating the fields of eqs. (14) and (15) along the crack-faces. It is noted that the condition $\sigma_{yx}(x,0) = 0$ within the crack is automatically satisfied since no shear stress is produced by any of the defects. Satisfaction of the boundary conditions of eq. (11) yields the governing equations of the problem, a system of two coupled singular integral equations. Separating the singular and regular parts of the kernels we obtain

$$-\sigma_{yy}^{disloc}\left(d - x, 0\right) = \frac{\mu(3 - 2\nu)}{2\pi(1-\nu)}\int_{-a}^{a}\frac{B(\xi)}{x - \xi}d\xi + \frac{\mu}{\pi a}\int_{-a}^{a}W(\xi)\cdot\ln\frac{|x - \xi|}{\ell}d\xi + \frac{2\mu}{\pi a}\int_{-a}^{a}B(\xi)\cdot k_1(x,\xi)d\xi$$

$$-\frac{\mu}{\pi a}\int_{-a}^{a}W(\xi)\cdot k_2(x,\xi)d\xi, \qquad |x| < a, \tag{16}$$

$$-m_{yy}^{disloc}\left(d - x, 0\right) = -\frac{2\mu\ell^2}{\pi a}\int_{-a}^{a}\frac{W(\xi)}{x - \xi}d\xi + \frac{\mu}{\pi}\int_{-a}^{a}B(\xi)\cdot\ln\frac{|x - \xi|}{\ell}d\xi - \frac{\mu}{\pi}\int_{-a}^{a}B(\xi)\cdot k_2(x,\xi)d\xi$$

$$+\frac{\mu\ell}{2\pi a}\int_{-a}^{a}W(\xi)\cdot k_3(x,\xi)d\xi, \qquad |x| < a, \tag{17}$$

where $B(\xi)$ and $W(\xi)$ are, respectively, the dislocation and disclinations densities defined as

$$B(\xi)=\frac{db(\xi)}{d\xi}=-\frac{d\Delta u_y(\xi)}{d\xi}, \quad \Delta u_y(x)=-\int_{-a}^{x}B(\xi)d\xi, \tag{18}$$

$$W(\xi)=a\frac{d\Omega(\xi)}{d\xi}=a\frac{d\Delta\omega(\xi)}{d\xi}, \quad \Delta\omega(x)=\frac{1}{a}\int_{-a}^{x}W(\xi)d\xi, \tag{19}$$

and the kernels $k_i(x,\xi)$ for $i=1,\,2,\,3$ are written as

$$k_1(x,\xi)=\frac{a}{x-\xi}\left[\frac{2\ell^2}{(x-\xi)^2}-K_2\left(|x-\xi|/\ell\right)-\frac{1}{2}\right],$$

$$k_2(x,\xi)=\left[\frac{2\ell^2}{(x-\xi)^2}-K_2\left(|x-\xi|/\ell\right)\right]+K_0\left(|x-\xi|/\ell\right)+\ln\left(|x-\xi|/\ell\right), \tag{20}$$

$$k_3(x,\xi)=\operatorname{sgn}(x-\xi)\cdot G_{1,3}^{2,1}\left(\left.\frac{(x-\xi)^2}{4\ell^2}\right|_{-1/2,1/2,0}^{1}\right)+\frac{4\ell}{x-\xi}.$$

The left hand sides of eqs. (16) and (17) are derived from eqs. (14) and (15) for $x=d-x$ and $\Omega=0$. In eqs. (18) and (19), the quantity $\Delta u_y(x)$ represents the relative opening displacement and the quantity $\Delta\omega(\xi)$ the relative rotation between the two crack faces. Physically, the dislocation density can be interpreted as the negative of the slope whereas the disclination density as the curvature at any point between the crack faces. We note further that both densities are dimensionless. Asymptotic analysis near a mode I crack tip[23] showed that both the crack-face displacement u_y and the rotation ω behave as $r^{1/2}$ in the crack tip region. Consequently, both the dislocation and the disclinations densities have the same endpoint behaviors, i.e. $B(x)\sim x^{-1/2}$ and $W(x)\sim x^{-1/2}$. Thus, the unknown densities can be written as a product of a regular bounded function and a singular function characterizing the asymptotic behavior near the crack tips, under the following forms:

$$B(s)=\sum_{n=0}^{\infty}a_nT_n(s)(1-s)^{-1/2}, \quad W(s)=\sum_{n=0}^{\infty}b_nT_n(s)(1-s)^{-1/2}, \tag{21}$$

where $T_n(s)$ are the Chebyshev polynomials of the first kind.

Further, in order to ensure uniqueness of the values of the normal displacement and the rotation for a closed loop around the crack, the following normalized closure conditions must be satisfied

$$\int_{-1}^{1}B(t)dt=0, \quad \int_{-1}^{1}W(t)dt=0. \tag{22}$$

Substituting eq. (21) to (22), we immediately infer that $a_0=b_0=0$. For the numerical solution of the system of singular integral equations (16) and (17), the collocation method is employed. The singular integrals in these equations are computed in *closed form* as Cauchy principal value integrals whereas the regular integrals using the standard Gauss-Chebyshev quadrature. After appropriate normalization over the interval $[-1,\,1]$, the system takes the following discretized form:

$$-\frac{\mu b}{2\pi a(1-\nu)(d-t_k)}-\frac{2\mu b}{\pi a(d-t_k)}\left[\frac{2}{p^2(d-t_k)^2}-K_2\left(p|d-t_k|\right)\right]=-\frac{\mu(3-2\nu)}{2(1-\nu)}\sum_{n=1}^{N}a_nU_{n-1}(t_k)$$

$$-\mu\sum_{n=1}^{N}\frac{b_nT_n(t_k)}{n}+2\mu\sum_{n=1}^{N}a_n\frac{1}{m}\sum_{i=1}^{m}T_n(s_i)\cdot k_1(at_k,as_i)-\mu\sum_{n=1}^{N}b_n\frac{1}{m}\sum_{i=1}^{m}T_n(s_i)\cdot k_2(at_k,as_i), \tag{23}$$

$$\frac{\mu b}{\pi a}\left[\frac{2}{p^2(d-t_k)^2}-K_2\left(p|d-t_k|\right)\right]+\frac{\mu b}{\pi a}K_0\left(p|d-t_k|\right)=\frac{2\mu}{p^2}\sum_{n=1}^{N}b_nU_{n-1}(t_k)-\mu\sum_{n=1}^{N}\frac{a_nT_n(t_k)}{n}$$

$$-\mu\sum_{n=1}^{N}a_n\frac{1}{m}\sum_{i=1}^{m}T_n(s_i)\cdot k_2(at_k,as_i)+\frac{\mu}{2p}\sum_{n=1}^{N}b_n\frac{1}{m}\sum_{i=1}^{m}T_n(s_i)\cdot k_3(at_k,as_i), \tag{24}$$

where $p = a/\ell$, $t = x/a$ and $s = \xi/a$. The integration and collocation points are given, respectively, as $s_i = \cos\left[(2i-1)\pi/2m\right]$ for $i = 1,...,m$ and $t_k = \cos\left[k\pi/N\right]$ for $k = 1,...,N-1$. The $2N$ equations are solved to determine the unknown coefficients and the defect densities.

6 NUMERICAL RESULTS - DISCUSSION

In this section we present and briefly discuss some of the numerical results we have obtained.
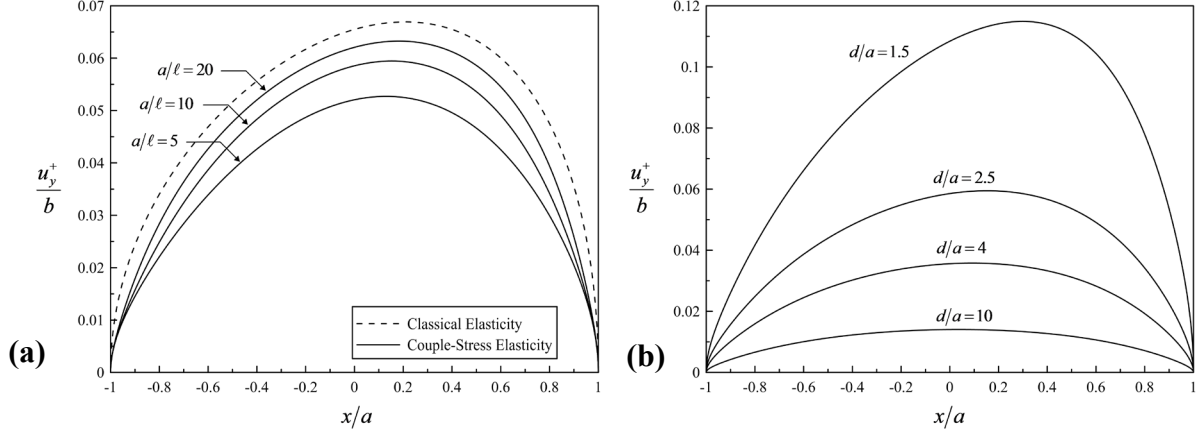

Figure 2. (a) Normalized upper-half crack displacement profile due to the interaction with a climb dislocation lying at $d/a = 2.5$. (b) Normalized upper-half crack displacement profile for various dislocation positions. The Poisson's ratio is $\nu = 0.3$.

In Fig. 2a, the dependence of the normal crack-face displacement to the ratio a/ℓ is displayed for a dislocation lying in a distance $d/a = 2.5$ in a medium with Poisson's ratio $\nu = 0.3$. It is observed that as the crack length becomes comparable to the characteristic length ℓ, the material exhibits a more *stiff* behavior, i.e. the crack-face displacements become smaller in magnitude compared to the respective ones in classical elasticity. Further, it is noted that the classical elasticity solution serves as an upper bound for couple-stress elasticity. In Fig. 2b, the influence of the dislocation distance to the crack-face displacement is examined. The material properties are $a/\ell = 10$ and $\nu = 0.3$. The resulting displacements vary significantly both in shape and magnitude.
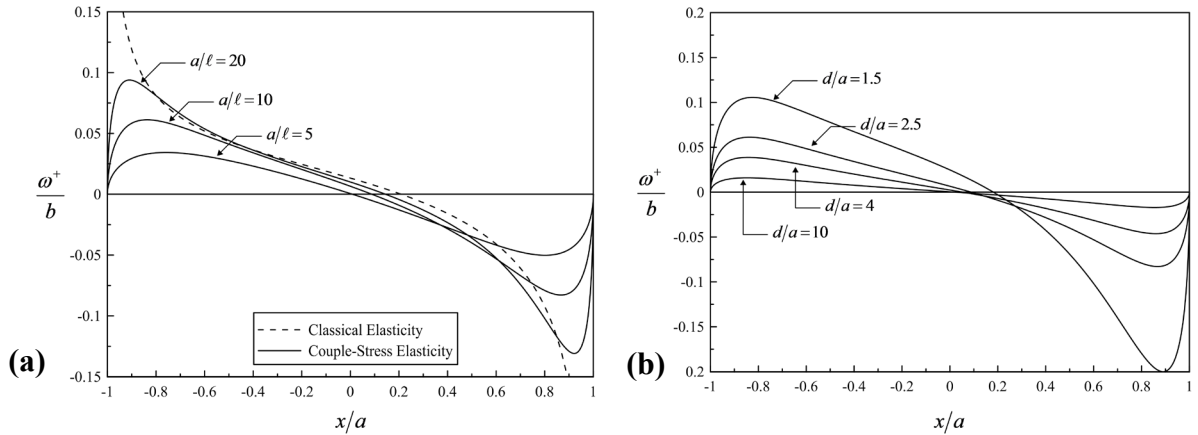

Figure 3. (a) Normalized upper-half crack rotation due to the interaction with a climb dislocation lying at $d/a = 2.5$. (b) Normalized upper-half rotation for various dislocation positions. The Poisson's ratio is $\nu = 0.3$.

The upper-half crack rotation for the same problems is depicted in Fig. 3. The produced fields are bounded contrary to the prediction of classical elasticity. In Fig. 3a, the variation of the crack-face rotation for several ratios a/ℓ is displayed. We note that as $\ell \to 0$, the rotation is pointwise convergent to the respective unbounded rotation in classical elasticity. In. Fig. 3b, we observe that as the dislocation approaches the crack, the rotation of the right crack tip increases significantly.

Next, we present the distribution of the normal stress σ_{yy} ahead or the right crack tip in Fig. 4a. For

convenience, a new variable $\bar{x} = x - a$ is introduced measuring the distance from the right crack tip. We notice that the couple-stress effects are dominant within a zone of length 3ℓ near the crack tip and 5ℓ near the dislocation core, whereas outside this zone the field gradually approaches the distribution given by classical elasticity. At $\bar{x}/\ell = 15$, the field becomes unbounded since this is where the climb dislocation lies. In Fig. 4b, the distribution of the couple-stress m_{yz} is depicted.
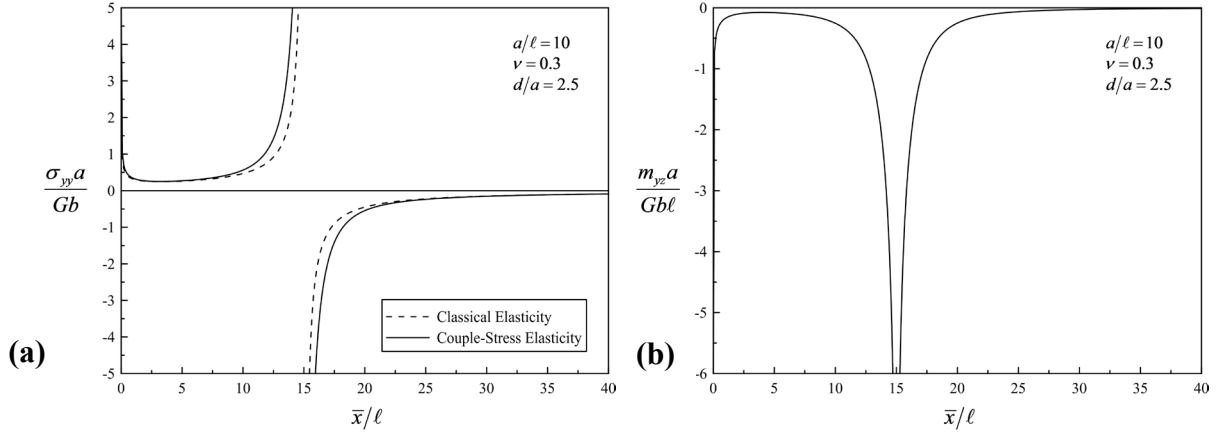

Figure 4. (a) Variation of the normal stress σ_{yy} ahead of the crack tip. (b) Variation of the couple-stress m_{yz} ahead of the crack tip.

In Fig. 5a, the variation of the ratio $K_I/K_I^{clas.}$ with the ratio ℓ/a for the right crack tip is presented. The stress intensity factor is defined as $K_I = \lim_{x \to a^+} \left[2\pi(x-a) \right]^{1/2} \sigma_{yy}(x,0)$. The dislocation is placed at $d/a = 2.5$ in all cases. We observe that the ratio of the stress intensity factors depends significantly on the Poisson's ratio and that there is a general increase in the stress intensity factor when couple-stress effects are considered. It should be noted that when $\ell/a = 0$, the above ratio should evidently approach unity. Therefore, the stress-ratio exhibits a finite jump discontinuity at the limit $\ell/a = 0$. This behavior has been reported in the past by other authors[20,21,24] who attributed it to the severe boundary layer effects of couple-stress elasticity in singular stress-concentration problems. It is also noted that the ratio, after an initial decrease for small values of ℓ/a, increases monotonically and tends to the value $(3 - 2\nu)$ as $\ell/a \to \infty$.
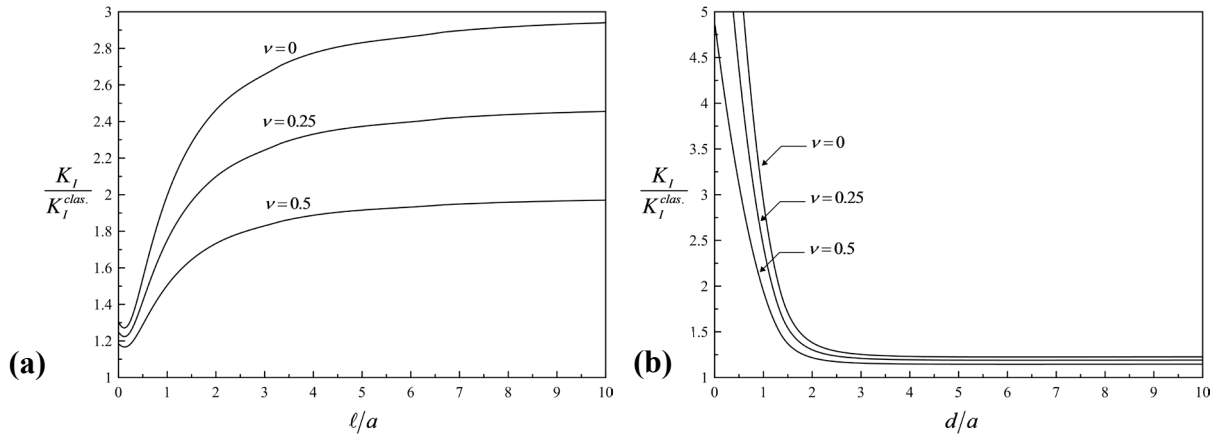

Figure 5. (a) Variation of the ratio of stress intensity factors in couple-stress elasticity and classical elasticity with ℓ/a. (b) Variation of the ratio of stress intensity factors in couple-stress elasticity and classical elasticity with d/a.

Finally, in Fig. 5b the variation of the ratio $K_I/K_I^{clas.}$ with the ratio d/a for the right crack tip, in a medium with $a/\ell = 5$. The ratio diminishes monotonically and quickly reaches a constant value at $d/a \sim 3$. Similar behavior is observed for the left crack tip as well.

7 CONCLUDING REMARKS

In this paper, the interaction of a discrete climb dislocation with a finite-length crack in the framework of couple-stress elasticity is studied. The distributed dislocation technique was employed for the analysis. Due to the nature of the boundary conditions that arise in couple-stress elasticity, the distribution of *constrained* wedge disclinations along the crack faces was also necessary. The continuous distribution of climb dislocations and constrained wedge disclinations along the crack faces results in a coupled system of singular integral equations with both Cauchy and logarithmic kernels which was solved numerically.

The present results indicated that the material microstructure of the couple-stress type has generally rigidity (smaller crack-face displacements and rotations) and aggravation (increase of stress ahead of the crack-tip) effects on the crack dislocation interaction problem. In particular, the crack-face displacements become significantly smaller than their counterparts in classical elasticity, when the length of the crack is comparable to the characteristic length ℓ of the material. Finally, it was observed that the stress intensity factor K_I is higher than the one predicted in classical elasticity in both crack tips.

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