Developing Use of Strategy in Childhood Mental Addition

By

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A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy

of Loughborough University

September 2001

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Abstract

The aim of this research was to look at the strategies used by children when doing mental addition problems of the varying levels of complexity. An authentic arithmetic task was designed for use in a school environment. The central aim was to study developing fluency in mental arithmetic as achieved through recruiting various strategies into solving more complex problems than those studied by existing research. The nature of mental addition strategies was inferred from children's solution times when doing sequences of sums. Three studies were carried out on 7-11 year old children from two local schools. The first study examined children's strategies at simple single-digit sums (of the type a+b), finding a dominance of counting methods and some emerging number-fact knowledge. This study also explored performance on more complex decade sums (of the type ab+c) and performance on three-digit serial addition sums (a+b+c). Both arrangements were followed up in more detail in the two subsequent studies. In the second study, the aim was to find out how children would use their existing strategic knowledge in conjunction with decomposition when required to do more complex decade sums. The results showed that children were capable of using decomposition-based strategies when required but only on certain types of sums. The third study looked at children's addend-reordering strategies when doing three-digit serial addition. Results suggested that although children were able to make strategic use of their existing number knowledge, they were reluctant to approach sums differently and relied primarily on counting methods. The overall pattern of results suggested slow progress towards versatile forms of mental addition and suggested there would be value in addressing this within curriculum design.

Keywords:
children's addition strategies; mental arithmetic; counting; decomposition; number-facts; solution times

Acknowledgements

I would like to express my deepest thanks to my supervisor, Dr Charles Crook, for his constant advice, support and guidance throughout the course of this work. I would also like to thank Charles for writing the software used in this thesis.

I would like to thank the head teachers and schoolteachers at the Outwoods Edge School and Wymeswold Primary School for their generous co-operation. A special thanks goes to all the children who took part in my studies.

I would like to thank Dr Thom Baguley and Nick Smith for their helpful advice and expertise on statistical matters.

I would also like to acknowledge the encouragement given to me by my Director of Research, Dr Dave Middleton.

I owe thanks to all my friends and my colleagues in my research group, as well as all the other individuals whose various contributions made this possible.

I would like to thank my brother for his highly significant contribution to this thesis.

Finally, my deepest thanks goes to my parents for their unfailing support, both financial and emotional, without which this work would not have been possible. It is to them that I dedicate this thesis.

Chapter 1

Introduction

How we acquire, understand and use mathematical concepts is crucial to our numerical competence, it relates to many aspects of everyday life. Research in this area is of both theoretical and practical significance. Numerical ability, as exemplified by the particular skill of mental arithmetic, has a very high cultural status in developed societies. But what does having this ability mean? How do we do arithmetic in our head? What computational strategies do we use? More particularly, how do we gain competence at mental arithmetic? What motivates learners to find new strategies?

The first three chapters of this thesis provide a review of existing mental arithmetic research. In the present chapter I consider the topical status of mental arithmetic in popular imagination. In Chapter 2 I then go on to look at the history of this area as a research problem, and the theoretical traditions within which it has been studied. Chapter 2 also looks at the emergence of mental/cognitive arithmetic as a disciplinary topic in its own right. In Chapter 3 I further focus on the theme of cognitive research in arithmetic processing. The studies reviewed include some research done with adults, as well as developmental research. Chapter 4 examines the methodology used in mental arithmetic research. In particular, I would like to highlight what I see as the limitations of the current methodology employed in developmental mental arithmetic research. In Chapters 5, 6, and 7 I discuss the results from my own research projects. Chapter 8 is the conclusion to this thesis.

1. The Current Status of Mental Arithmetic

In this chapter, my aim is to sketch the social and political background of research in the area of mental arithmetic.

1. 1 Popular Opinion on Mental Arithmetic

Mental arithmetic is an important everyday skill and a major aspect of an elementary education. Mental arithmetic is woven into our everyday lives. Arithmetic skills are required for such everyday tasks as handling money to buy lunch, doing grocery shopping, cooking etc. Adults regularly encounter mental arithmetic skills when doing building and construction tasks like DIY (do-it-yourself) or playing games like darts. Early on, children utilise arithmetic skills when taking part in "fun" activities such as playing board games like Monopoly, or even Pokemon. It provides children with the necessary skills for later mathematics and science education. Recently, there has been much emphasis on the mental arithmetic skills of children, or rather, there has been lament for their dismal lack of them. The national concern with mental arithmetic has preoccupied both the government and schools and this has been fuelled by the press, especially since children in the Far East have been shown to outperform those in the UK and the US (Geary, 1994; Stevenson, Chen and Lee; 1993). National concern for numeracy skills in general is evident in the formation of the National Numeracy Project, with its subsequent numeracy hour (National Numeracy Project, 1997, 1998).

Numeracy skills are essential if we are to fully participate in the modern world. But numeracy skills, especially complex numeracy skills, are also cognitively demanding and are therefore difficult to master. Most people live in cultures where (basic) numeracy is central to their everyday lives e. g. as a result of the evolution of currency and the need to deal with quantities for even the simplest trading transactions. In fact, arithmetic is so central to our everyday lives that, even where it is not taught, people (including children) will invent their own ways of dealing with numbers. For example, Carraher, Carraher and Schliemann (1985) looked at child street vendors in Brazil and found that they had developed their own mathematical practices for dealing with currency conversions, adding/ taking away large quantities of goods etc. If numeracy is so valuable a skill that we create ways of dealing with it even when it is not formally taught, then it is even more valuable for an increasingly technologically advanced society. Moving on from such street maths, as society becomes more technologically advanced, its members need evermore advanced numeracy skills. Thus any culture with a formal education system includes numeracy as part of its everyday curriculum e. g. as one of the three 'r's (reading, writing and arithmetic).

Compared to other subjects children learn in school, the area of numerical ability is unique. This is because the rules of arithmetic are generative, they allow any infinite number of sums to be obtained with knowledge of the rules and basic facts. It is an area of knowledge in which all items of knowledge can be retrieved from memory or computed due to their relationship with each other. All arithmetic facts have well-defined relationships to each other, something which does not hold true for most other disciplines. In geography, for example, we cannot "compute the capital of Guinea from first principles, nor derive it from the capital of Mali" (Butterworth, 1995). These very features also make arithmetic a cognitively intense activity. Yet, in spite of the *mental* nature of arithmetic activity and what it involves, it is not an exclusively private cognitive phenomenon. Arithmetic incorporates both mental and material aspects. Numerical ability is also about the artefacts and technologies we use when we interact with numbers. Because of the everyday nature of numerical tasks (e. g. buying and selling), doing arithmetic does not mean that the learner is isolated from the external world.

A significant consequence of this has been that numeracy standards have become a victim of intense scrutiny. This is not a new preoccupation, officials have deplored what they perceived as falling levels of numeracy since at least the middle of the last century (when they were supposedly high)! The topic of mental arithmetic has become a matter for fierce debate, an issue guaranteed to raise hackles and cause controversy. Although central to peoples' lives, this is an issue for debate.

Distinguished commentators in this area hold differing views on how children should be taught the important skill of mental arithmetic. In the following passage (taken from a newspaper report) Professor David Reynolds of the Numeracy Task Force highlights the government-backed programme for the teaching of mental arithmetic-seen as the solution to falling standards in numeracy. His status as a high profile policy adviser means that his opinions on this matter are important since they will undoubtedly influence policy.

> "A report by the task force...set up immediately after the general election, will call for the memorising of tables, a daily dose of 'mental maths' and an emphasis on whole-class teaching to ensure all children progress at a predetermined pace.
>
> In 40 per cent of the [National Numeracy] project's schools the results of 11-year-olds leapt by 15 per cent after a regime of mental arithmetic was introduced."
>
> (*The Observer*, 18th January 1998)

Journalists are aware of the significance of efficient numeracy skills and that research is needed in order to successfully execute high-profile programmes such as the Numeracy Project. Their views tend to reflect popular opinion. They are likely to influence public debate by alerting readers to the political motives for the necessity of such programmes since there is a competitive element behind such government-led drives i. e. in order to get ahead in a hi-tech world we need to be better than, for example, the Germans or the Japanese.

The setting up of the Numeracy Project shows that the government realises that there is a problem, and is keen on remedying the situation by doing something to raise numeracy standards. However, this raises issues regarding how this should be achieved and how it should be taught. For example, should it involve ritualistic methods i. e. “know by heart” which bring to mind rote-methods of teaching and the competitive elements underlying this need to “know by heart”.

> "The National Numeracy Project states, among other things, that numerate pupils, 'know by heart number facts such as number bonds, multiplication tables, doubles and halves.'

The task force has noted better results in countries such as Hungary and Switzerland which concentrate on mental arithmetic for the first few years of primary school, delaying written methods of calculation until pupils are eight or nine."

John Carvel, Education Editor

(*The Guardian,* 22nd January 1998)

But not everyone is in accord as to how these numeracy skills should be taught. In an interview for *The Observer*, Director of the Quality in Education Centre at the University of Strathclyde, John Macbeath, values the significance of being numerate but seems to be unsure about 'traditional' (such as those hinted at above) methods of teaching it.

"Personally I am all for numeracy and I am a whiz at mental arithmetic. I know my tables backwards and forwards because they were belted into me by an old sadist whose methods worked for a fast learner and devout coward like myself, but it terrified about a fifth of the class into lifelong number phobics."

John Macbeath, Director of the Quality in Education Centre at the University of Strathclyde

(*The Observer*, 22nd February 1998)

His comments suggest that he is also aware that the process of learning arithmetic, if not managed sensitively, can sometimes be a traumatic experience that can turn people off mathematics for life.

Of course, the lofty status accorded to mental arithmetic skill in our society (as well as the various methods implemented for its teaching) is itself a matter for discord. In addition, there is a possible emotionally affective element regarding the acquisition of arithmetic skills and the quality of that experience i. e. is it somehow "emotionally deadening" to learn these skills by rote, as pointed out above by John Macbeath. Or does the end result justify the means as Heinz Wolff claims. There seems to be discontinuity about the experience of learning arithmetic. Below Heinz Wolff scoffs at the notion that numeracy skills are unique and a sign of genius and suggests that anyone can master them with enough practice. He also suggests that the only way of learning such skills is through rote practice and, by making the analogy between arithmetic and other relatively difficult activities, claims that even if the actual method may not be enjoyable the end result will be worth the effort.

> "Getting the right answer to 7x8 instantly, a feat which recently got my picture in the Times, is not the measure of a mathematical genius. It is merely a tribute to the fact that I have learnt the result as a concept, just as I can recognise a horse without performing a zoological analysis. Detractors of rote learning forget the pleasure to be derived from doing something effortlessly, like skipping or roller-blading."
>
> (Heinz Wolff, in *Frontiers* 1998)

Raising another issue in mathematical cognition, Stanislas Dehaene points out the significant practical implications of calculators in our everyday lives, suggesting that certain arithmetic skills are likely to become obsolete in the future, but that this does not necessarily imply that arithmetic skills in general will no longer be essential. He goes on to argue that mental arithmetic is actually advantageous to our mathematical knowledge, so the very process of doing mental arithmetic is beneficial to us.

> "There is a general trend in mathematics towards inventing tools that discharge our brains from tedious work. Learning long division is doomed to disappear. The meaning behind calculations is more important than the mechanics...I am not saying that we should stop teaching arithmetic at school.

Mental manipulation of numbers is an excellent exercise that enhances mathematical intuition."

(Stanislas Dehaene, *The Guardian* 1998)

Dehaene's comments remind us of the dynamic nature of numeracy skills. By raising the increasing significance of calculators in our dealings with numeracy, he sums up the importance of being computationally numerate and that those skills may be more important than ever before. We need to be aware of what we are doing when using a calculator.

1. 2 The Preoccupation with Numeracy Standards

Section 1. 1 reminded us that, in the public arena, debate on numeracy standards is lively and brimming with differing personal opinions. Such public debate influences policy. It also shows that there are contradictory opinions on precisely how to improve these numeracy skills. Still, at the forefront of government led initiatives such as the National Numeracy Project is the view that children must "learn their arithmetic facts", master the simple number combinations so they can be recalled fluently and accurately. Regardless of the diversity of views in this area, the general consensus is that competent number fact knowledge is still highly valued and the governments in most Western countries are evermore concerned with raising what they perceive as falling standards.

Perhaps it is our popular belief in differential numerical ability that has renewed enthusiasm for somewhat mechanical teaching methods. This drive to raise falling standards has gained in intensity over the last few years (1997 onwards). Yet it is also likely to be the source of some tension due to the deep-seated belief, so thoroughly embedded in our culture, that mental arithmetic is "difficult" and that although some people will be good at arithmetic (hence the popular aphorism about "having a head for numbers"), most will not. This is the point that Heinz Wolff (see above) was stressing. It is as if we are aiming for an ideal perfect mastery of arithmetic facts that at the same time we know to be difficult to achieve. Possibly this may be why those who advocate rote methods of teaching arithmetic facts see it as such a popular remedy for dealing with this problem. Rote learning is essentially about committing facts (number facts in mental arithmetic) to memory so that they can be accurately retrieved at any time and so it is seen by some as the 'perfect' solution. This widespread belief that some people will be better at arithmetic than others and will always remain so reinforces the temptation to use drill-based methods. The view that mental arithmetic is inherently difficult is what may sometimes lead to educationalists and policy-advisers advocating blind rote methods in an attempt to mitigate this lack of confidence in our mental arithmetic skills.

Concern with this perceived lack of arithmetic ability is used to imply the "failure" of modern methods of schooling and to justify decisions to revert back to old-fashioned rote-methods that, in retrospect, are seen as having been more efficient somehow. It may be that such concerns are justified. After all number fact knowledge may only be a small component of more complex numerical skills, but it is undeniably a critical foundation for higher-level numerical problem-solving abilities. While this may be true, research suggests that extensive mastery of number facts (especially multiplication facts) is a rare occurrence. After years of classroom drill, university students still have relatively high error rates (near 30%) on certain number combinations such as 4x8 and 6x9 (Campbell and Graham, 1985). This suggests that even implementation of drill/rote/practice methods may fail to achieve 'perfect' mastery of number facts. However, even at younger ages errors are systematic rather than random thus suggesting that some problems are inherently more difficult than others and so harder to master. Even adults frequently use multiple strategies to solve simple number combinations instead of relying solely on retrieval from memory.

Therefore it is clear that competence involves going beyond number facts. It is not enough for children to have good mastery over the basic number facts themselves. Does improvement in arithmetic skills only involve rote learnt facts? Most adults probably use flexible strategies to manage and generate their number fact knowledge and apply it effectively and children should be encouraged to do the same. We should be looking at what good foundational skills are really about i. e. how children can use the foundational skills they do have effectively and strategically. Where do the weaknesses lie and how can this situation be made easier? For example, if confronted with a problem like 6+9+4 most adults could quickly come up with an answer by thinking strategically (6+4 = 10+9 = 19) but what about children? Children, even those who may otherwise be competent at number facts, may be slow at doing this. So it is not enough to have good number fact knowledge if it cannot be used strategically to make mental arithmetic easier.

1. 3 The Debate About Mechanical Teaching Methods

Recently, there seems to have been an increase in the trend to revert back to older (primitive and less optimal?) methods of teaching number facts although there are vehement denials by the popular press that this means a return to

> "A Victorian era when pupils spent their time chanting tables. There is no evidence that these old methods benefited slow learners. Professor Reynolds is recommending a daily maths lesson for every primary class, lasting 45-60 minutes and ideally in the morning." (*The Guardian*, January 1998).

This excerpt regarding Dr Reynolds' proposals also illustrates the ongoing tensions and debates in the recent press involving journalists' recognition of mental arithmetic as a topic of popular concern.

Professor Reynolds seems aware of the difficulty of successfully implementing such government initiatives. He knows that teachers and educationalists can be uneasy about such schemes (sometimes rightfully so because they are often held responsible for any failures) and tries to offer some reassurance. But there does seem to be a move to "make a definitive break with progressive ideas that said children should work at their own pace" (*The Observer*, January 1998). These progressive ideas (such as concepts first, number facts later) are based on the constructivist view of mathematics education that has influenced educational practice in the West. The constructivist view in education grew and developed mainly from the views of Piaget. The assumption behind this educational philosophy is that children are active learners and must construct mathematical knowledge for themselves. This will be discussed more thoroughly in a later chapter but the constructionist approach has come under some criticism recently for somehow 'failing to deliver' (Geary, 1995).

The most recent popular consensus seems to be that we tried drill and practice ('basics') to no avail, so we tried more concept-based approaches but that didn't work either. It seems to me that perhaps we now need to revert to more regimented methods like drill and practice (hence 'back to basics') but without all their negative connotations because at least they drummed in some knowledge of number facts even if it was only rote-learning of multiplication tables. The obvious conclusion to be derived from popular opinion seems to be that we should consider a return to older methods of drill-based rote learning but in a less draconian fashion (Isaacs and Carroll, 1999). So we need to retain something fairly traditional but make it richer, and more strategic.

Cross-cultural comparison has sometimes been used to support a return to drill-based rote learning but this needs careful evaluation. Effective arithmetic teaching varies from one culture to the next. For example, in Japan (where numeracy standards are high), although teaching practice is often viewed as being somewhat generally regimented, teachers hold a more exploratory view regarding teaching in some areas of mathematics (Stigler and Perry, 1990). A lesson is likely to consider only two or three problems, discussing them from many angles and exploring underlying principles and implications whereas an American lesson emphasises accuracy and speed, as opposed to understanding (Resnick, 1989). Thus, we need to exercise caution when making such comparisons. This is not an easily solved debate, we need to be careful when looking at the Far East because schooling, a cultural institution, varies across cultures and things are not always as they may seem on the surface.

Even in the West, traditional teaching methods are difficult to destabilise. There are conflicting beliefs about the nature of arithmetical expertise and the cultivation of basic number fact knowledge. Ginsburg, Klein and Starkey (1998) suggest psychologists contributing to mathematics education should take into account the forces that shape education. They show how traditional conceptions of the curriculum can survive in textbooks and with teachers, despite changes in the thinking of mathematics education researchers. They also show certain basic mathematical understandings have been neglected as a result of teachers' resilience to change. Traditional educational methods are still lurking beneath the gloss of modern constructivist methods.

In the face of all this concern and debate, the British government has been galvanised into taking critical action. It has recently agreed proposals for a National Numeracy Strategy that they claim is based on evidence about "what works". The Numeracy Task Force claim that their recommendations are based on research evidence. Brown, Askew, Baker, Denvir and Millett (1998) explore the extent to which this claim is supported. They conclude: 1) that the research findings are sometimes equivocal and allow differences of interpretation and admit that the complex nature of such findings suggests that "ministerial desires for simply telling 'what works' are unrealistic", but 2) that there are always many practical constraints on policy which are likely to override empirical evidence.

Above I have looked at some of the issues surrounding the area of early numeracy skills. In light of some of the concerns regarding the teaching of these skills, educators have attempted to deal with the situation through a variety of methods. Everyone agrees that there is a genuine need to bring numeracy knowledge to children in a way that is inherently interesting (Isaacs and Carroll, 1999). But can we retain the "regimented" drill and practice that children seem to find boring? One response has been to design computer games.

One example of doing this effectively is the maths education computer game Math Blaster (published by Davidson Knowledge Adventures). The program uses the sophisticated graphics and design of a conventional computer game to enliven what is essentially arithmetic skill practice at varying degrees of complexity. Math Blaster basically involves doing calculations but in a more interesting and enriched context (compared to traditional drill). It illustrates how the use of cleverly glossed drill-and-practice games means mental arithmetic practice need not be a deadening experience. One that, John Macbeath believes terrifies less competent children into "lifelong number phobics". So this deals with the motivational aspects of becoming numerate but it is still essentially about traditional arithmetic skill i. e. basic drill and practice under various cosmetic guises.

The intention of this thesis is to examine how children gain competence at mental arithmetic, more specifically mental addition. Considering the "knowing by heart" aspect of having knowledge of number facts suggests that some of it will be in the form of a "number fact dictionary" from which we pull out facts as and when we need them. But does it always have to be this way? We first need to look at what children do naturally (and creatively) when required to do sums and see just how foundational abilities can be taken forward to make their existing number knowledge work effectively for new problems. We also need to find out what age children recognise such possibilities, as well as finding out if this can be made to happen earlier.

The aim of this chapter has been to demonstrate why numerical ability matters so much, not only to educators and academics but also to politicians and popular commentators. There has been a growth in research on children's mathematical development in recent years. Numerical ability is a major focus of cognitive research, since it has a unique inherent structure that cannot always be applied to other forms of knowledge. All items of numerical knowledge can be retrieved from memory or they can be computed. So all arithmetic facts are, in some way, related to each other. The area of numerical ability is of vital theoretical and practical importance. The aim of developmental research in this area is to further investigate the developmental trend in the mastery of arithmetic knowledge, from an initial reliance on procedural methods e. g. counting to retrieval from a network representation of arithmetic facts.

To conclude, the extent of this public and political interest should encourage more psychological research on the processes underlying children's developing fluency with basic computations. Arguably, there is a particular urgency for more work on the topic of *mental* arithmetic. My own research will endeavour to make more visible the hidden processes that underlie such computations, indicating something of their developmental trajectory. First it is necessary to review existing psychological research that has been carried out in this spirit. I turn to this in the next chapter.

Chapter 2

2. Development of Research in Mental Arithmetic

Chapter 1 was about the topical nature of mental arithmetic research and how this has provoked popular media, thus highlighting the need for academic research. The current chapter will overview the development of research in mental arithmetic. It encapsulates some of the psychological concerns relevant to this topic. Section 2. 1 provides a brief historical perspective of arithmetic research in psychology. In Sections 2. 2, 2. 3, and 2. 4, I examine the research traditions within which arithmetic has been studied from its roots in Thorndike's behaviourist view on arithmetic to the development of the current cognitive tradition of mental arithmetic research. In 2. 5 I look at the tradition of cultural research in this area. In that section, I begin by differentiating between cultural psychological and traditional cross-cultural research and go on to look at the effect of language on numerical processing. I conclude the section by looking at other cultural factors that may influence numerical processing.

2. 1 Historical Perspective on Educational Practice

Historically, distinctions have been made between two different conceptions of arithmetical expertise. One emphasises the development of competence in calculation/ computation i. e. the manipulation of number fact relations; the other stresses the development of mathematical thinking that involves a deeper, and perhaps more abstract, understanding of numbers.

According to Cowan (1999), nineteenth century English elementary schools for working class children stressed the importance of the calculation aspect of arithmetic. This was because the children were not expected to continue their education beyond elementary school and the aim was to give them all the skills necessary for work. This emphasis on calculation was evident in the way it shaped educational policy at the time, e.g. the setting up of the payment-by-results scheme in 1862. The payment-by-results scheme meant that the amount of funding a school received depended on numbers of pupils and their performance in examinations. The examinations were based on standards set for each year and covered only arithmetic, reading and writing geared very much towards the demands of work and everyday life. Matthew Arnold, an inspector who was also a critic of the scheme, reported in 1869 that teachers only "taught to the test". This meant that for a reading test the same book would be read frequently. For an arithmetic test, children were taught mechanical rules for sums that they were then forced to practice repetitively. In addition to this, teachers were not concerned about teaching arithmetic principles or the science of arithmetic (Arnold, as cited in Gosden, 1969).

The system was slowly dismantled after 1890 but despite such influential critics as Arnold, reform of the curriculum was slow and the computational aspects of arithmetic continued to be emphasised. For example, in 1930 a primary school syllabus might suggest addition and subtraction up to 10 in the first year, up to 99 in the second year, up to 999 in the third year and to 10,000 in the fourth year (Schonell and Schonell, 1957). Inspectors would attribute children's deficiencies in either oral or written arithmetic to defects in teachers or children and not the method.

The Hadow Report (Great Britain Board of Education Consultative Committee, 1931) was a report on primary education that challenged these traditional views of the primary school curriculum. The report made new recommendations for a more child-centred curriculum with a wider scope than previous ones. Meanwhile, Jean Piaget was carrying out his constructivist research with its emphasis on the importance of children's conceptual mathematical knowledge. This was to have a far-reaching influence on how arithmetic came to be taught in schools.

Traditionalist attitudes changed as some educators began to believe that deficiencies in children's arithmetic were more likely to result from premature introduction to number facts and computation. Schonell and Schonell (1957) argued that such failure could be avoided if teachers grounded arithmetic in experience before progressing to calculations. They also claimed that practice in number facts without understanding had little value and that arithmetic problems should be rooted in familiar experience before written computation was required. However, the development of fast and accurate computational skill continued to be the goal.

As mentioned earlier there are two orientations towards arithmetic, the first is about the calculation aspect of arithmetic; the second view - the area more of interest to this thesis - is about a deeper more abstract understanding of mathematical principles. Despite internal debate on this matter, the computational aspect continued to be taught in primary schools in the early part of the century. The second view of arithmetical expertise came about as the result of the reform of the school curriculum in the 1950s prompted by the gap between university and school mathematics (Kilpatrick, 1992). It led to the secondary curriculum being revised in order to provide a better preparation for university mathematics. Another influence was the invention of calculators that rendered computational skill as being of less importance than knowledge and understanding of mathematical principles underlying calculations. In Britain, the aim of primary mathematics became to make children think for themselves, to give them knowledge and appreciation of mathematics as a creative subject, and to develop facility with number and quantity relationships (Schools Council, 1966). This new conception of arithmetical competence emphasised mathematical understanding. Educators relied on Piaget's research, for example, to justify the new emphasis on children's activity and experience and to provide insight into the difficulties children would have in understanding abstract mathematical ideas.

Recently, government bodies have stressed the importance of mental arithmetic and "back-to-basics". Cowan (1999) argues that this does not, however, involve retreating to the traditional conception of arithmetical expertise that exclusively emphasises calculations. This does raise questions on the precise nature of these "basics". Both advisors on the National Numeracy Project (1997, 1998) and Cowan (1998) would agree on the importance of encouraging fluency in mental arithmetic. However, there are unavoidable elements of "drill and practice" learning in mental arithmetic. For example, learning a number fact like $7+7 = 14$ cannot really be made into a more "constructive" process. Clearly, at an early level "$7+7 = 14$" must be learnt as a "fact" with little constructive processing behind it. Perhaps what matters most is the way in which such number fact knowledge (i. e. once it has become a "fact") can be manipulated swiftly and effectively to solve other types of related problems. Recognition of such deceptively simple strategies does make mental arithmetic learning into a constructive process. Such concerns are very much at the core of this thesis.

Even at the early stages of research there was recognition of two kinds of knowledge of arithmetic: 1) declarative knowledge (number facts/combinations) and 2) procedural knowledge ("knowledge about" or an understanding of arithmetic concepts and procedures). These two are intertwined.

1) Declarative knowledge (refers to stored knowledge of addition facts e. g. $2 + 3 = 5$ and depends on retrieval from memory)
2) Procedural knowledge (refers to stored knowledge about arithmetic e. g. $n \times 0 = 0$, $n + 0 = n$, or $n + 1 =$ one greater than the original number n and depends on rules such as commutativity, heuristics and computation)

It is because these two types of knowledge are interlinked that research should pay more attention to this relationship, rather than looking at one or the other in isolation.

From a brief history of how these two distinctions in mental arithmetic developed, in the following sections I will examine the research paradigms under which numeracy has been studied.

2. 2 Research in Mental Arithmetic

Mental arithmetic has been studied under four somewhat loosely defined research traditions. There is a considerable amount of overlap between these. Not all research has respected these distinctions, nor is it being claimed here that this should be the case. I have chosen to describe four theoretical frameworks in which computational mental arithmetic research has been carried out: 1) associative theory and stimulus-response (S-R) learning, 2) cognitive research based on response/reaction times, 3) research stressing the modular aspects of mental arithmetic, and 4) research looking at cultural resources for calculation. The first two are more closely linked than the latter. In the first two, mental calculation is considered an individual isolated process that is private and contained and where people act as calculators. The last two look at the biological and cultural influences that may situate such mental calculation.

2. 2. 1 Associative Theory and Stimulus-Response Learning

Thorndike (1922) carried out some of the earliest research in this area in his book entitled *The Psychology of Arithmetic*. This was rooted within the behaviourist tradition so dominant in the US at the time. Thorndike's theory was deeply entrenched in his theoretical concern for the "law of effect". Thorndike saw knowledge of the basic arithmetic facts, e. g. the 100 addition and 100 multiplication number facts, as the formation and subsequent strengthening of individual stimulus-response associations. Repeated presentations of the correct stimulus-response pairs meant that the arithmetic facts would be learned, i. e. simple facts learned with differing strengths

While Thorndike's theory itself may have been sound, his proposed method of teaching via carefully planned drill exercises came under heavy criticism as being a "mindless" rote method that ignored the principles of arithmetic and failed to teach "understanding". Even at the time, commentators (e. g. Brownell, 1928, 1935) argued against this approach and advocated meaningful instruction as opposed to rote practice as being crucial for better transfer to untaught number combinations. Brownell believed that drill methods distorted the goal of learning. He saw arithmetic skill as being about the ability to think quantitatively rather than being about the capacity for accuracy at a list of set arithmetic problems.

Thorndike's ideas did, however, set the stage for the more current cognitive arithmetic research that describes number fact representation as networks of associations that strengthen over time. This is because he was essentially looking at the formation of simple bonds and using a language for describing associations between the elements, associations that took the form of a stimulus-response. Currently, though, there seems to be a move away from this view and researchers are trying to distance themselves from the extreme versions of this somewhat simplistic theory. Cognitive theories are, nevertheless, continuous with S-R and associative theory in that both are concerned with hidden/ unseen processes i. e. the S-R bond is as invisible and abstract as a node in the mental modelling of cognitive theory. Both theories are reductionist, in that they reduce mental processing to elemental processes; because both S-R bonds in behaviourist theories and associative bonds in current cognitive research were about elemental associations. Although proponents of both might disagree, both were interested in behaviour. In stimulus-response theories, the focus was on the seemingly "visible" components of behaviour (responses), while cognitive psychology is about abstractions such as mental models. Where the continuity between S-R and cognitive theory lies is in the associations that are supported by both frameworks. But unlike earlier behaviourist theories, cognitive theorists want to model these as invisible relationships. Thus, cognitive theories differ from early S-R and associative theory in that they provide a richer mechanism for studying mental processes.

2. 2. 2 The Emergence of Cognitive Arithmetic and the Chronometric Method of Measurement

Until twenty or so years ago, mental arithmetic (in psychological experiments) was largely used as a distracter task in memory experiments (e. g. people counting backwards in 3's), a tool that would merit little research interest in its own right (Peterson and Peterson, 1959). There was very little interest in arithmetic knowledge itself. According to Ashcraft (1995), Groen and Parkman (1972) revived "genuinely cognitive" research into mental arithmetic. Since then (and especially within the last decade), however, mental or cognitive arithmetic has developed into an active area of interest and has generated a wealth of research. Ashcraft (1992 and 1995) believes that cognitive arithmetic is a rich enterprise because it provides links with other areas in cognitive psychology, and that the study of cognitive arithmetic can advance cognitive psychology as a whole. He believes work on mathematical cognition has applications outside traditional cognitive interests (e. g. studying working memory) in areas such as neurocognition, mathematics disabilities, cross-cultural differences and mathematics anxiety (Ashcraft, 1995).

For example, the underlying question concerning cognitive or mental arithmetic is simple: How do we do arithmetic without any overt/ tangible props? In more technical terms, this means asking: What do we do mentally when we perform simple arithmetic and what cognitive processes, memory representations and mental components are necessary for this skill?

Ashcraft (1995) attributes a surge in research on cognitive arithmetic to the cognitive revolution in psychology and believes that the "computer analogy" in particular was responsible for the revival of a certain type of method (namely, studying reaction time). Cognitive psychologists saw the potential of studying cognition via response times and mental arithmetic was an attractive candidate for their research. Although, cognitive psychologists had always been interested in using reaction/ response times as a measure of cognition, arithmetic provided them with a more optimal medium for doing this because it meant that mental processes could be manipulated directly. The topic of mental arithmetic had been pushed aside during the "heyday" of behaviourism, to re-emerge in the late 1960s and early 1970s. Between the 1930's and 1950's American experimental psychology involved studying a few narrow topics in human memory such as paired-associate learning. Behaviourist theorists were concerned only with S-R theory as applied to laboratory settings with strict experimental control in a somewhat introspectionist manner i. e they were not concerned with mental arithmetic as such. So they made little contribution to this area. Looking back, it seems that while behaviourist-inspired teaching techniques, e. g. for classroom management flourished, the area of education and educational psychology regarding mathematics and its teaching was left to languish.

According to Ashcraft (1995), the cognitive revolution and the developments that he collectively refers to as the "computer analogy" inspired the information-processing model and a preferred methodology in the use of RT as a primary dependent variable. Whatever the research interest, the emphasis was on stage-by-stage analysis of the components of performance and the flow of information through the processing system (Lachman, Lachman and Butterfield, 1979).

Reaction time emerged as a measure for practical purposes after World War II. Reaction time became the "gold standard" of dependent measures in such research, where the independent stages of processing were revealed by component RTs. Research involved analysing the "time" necessary for certain mental steps or stages to be completed. This research built on years of investigation on RT itself, i. e. the values of RT measurement under various conditions. Sternberg's (1969) research on short-term memory led to cognitive psychology's reliance on RT (or response latency) as an explanation of underlying mental processing. Posner's book *Chronometric Explorations of Mind* (1978) identified research that depends on RT as a measure as "mental chronometry" and uses RT evidence to define "chronometric" methods (Luce, 1986).

Reliance on RT became one of the fundamental tools of cognitive psychology and this had a significant impact on the area of "cognitive arithmetic". This influence is exemplified in the earliest major cognitive study of mental arithmetic (Groen and Parkman, 1972) and is still evident in current research. Chronometric methodology will be used as a measure in my own research.

Associative (S-R) theory and the cognitive approach both consider mental arithmetic to be a private activity that is contained within the individual. Both approaches are concerned with links between elemental processes i. e hidden mental processes because they look at the formation of bonds and learning associations all located mentally 'inside our heads'. But, in addition to traditional cognitive psychological explanations, mental arithmetic can also be influenced by external events beyond our private mental abilities. Thus, biology and culture also have a significant impact on the development of our numerical abilities. For example, research suggests that certain numerical abilities are innate and it is their further development that is culturally determined. Research has also found that cultural differences exist not only in the way mathematics is taught but also in our perception of what it is about. The next two sections look at how biology and culture contribute to our understanding of numeracy.

2. 2. 3 Modularity

There is currently a large amount of research looking into the "innate" nature of certain numerical skills or abilities and this has, to some extent, been influenced by the notion of mental modularity. According to Cole (1996) both modularity and cultural context affect our mental development He argued that the origins of the concept of modularity arose from the debate between Piaget and Chomsky (among others) on language development. Chomsky expanded from Piaget's claim that language construction depends on previously developed sensorimotor schemata. Chomsky argued for the existence of what is now referred to as "a language module".

In his book *The Modularity of Mind* (1983), Fodor applied the logic of Chomsky's theory of language to general cognitive development. Basically, Fodor claimed that:

1) Psychological processes are domain-specific. Information from the environment passes through special input systems or modules (special-purpose sensory transducers) that output data in a format processed by a "central processor".

2) The psychological principles that organise each domain are innate, in the sense that they have a fixed neural architecture, are fast and automatic and are "triggered" by the environment. That is, they are not constructed, as implied by Piagetian theory of stage development.

3) Each different domain is a separate mental module and they do not interact directly but through a "central processor".

4) The modules cannot be influenced by other parts of the mind.

Fodor proposed several modules in addition to language such as those implicated in the perception of colour, shape, three-dimensional relations and the recognition of voices and faces. Others have since suggested an even wider variety of possible modules: e.g., for mechanical causality, intentional movement, number, animacy and music (Hirschfeld and Gelman, 1994).

There are two versions of the modularity hypothesis, a weak version and a strong version. In the weak version of the modularity hypothesis, behaviour is richer and more complex than recognised by traditional theories of cognitive development. According to the strong version of the modularity hypothesis, the behavioural characteristics within domains do not really develop because they are innate and only need the right environment to develop. So culture affects the development of modules which are there from the outset.

Cole (1996) supports the weaker version of modularity as skeletal principles and starting points, because this can be effectively combined with cultural mediation. He argues that such a combination is a good way of describing the 'intertwining of "natural" and "cultural" lines of development as part of a single process'. Mathematics is a good concrete and representative example of how culture and modularity can be integrated, because there is enough evidence about phylogeny, ontogeny and the cultural organisation of thinking in this area.

Evidence suggests that some numerical abilities are innate because there are some numerical skills present in young babies (Gallistel and Gelman, 1992). Although there remains some debate about exactly how number is processed by preverbal infants. Research on infants has involved habituating them to visual displays (Starkey, Spelke, and Gelman, 1990; Wynn, 1992). Klein and Starkey (1988) favour the explanation that infants use a special perceptual process called subitizing. Subitizing is the ability to recognise very small number sets (sets of four or less) without counting. More complex counting procedures elaborate the primitive subitizing process. But according to Gallistel and Gelman (1992) infants actually have a preverbal enumeration system identical in its basic properties to the elementary numerical abilities found in non-human primates. Counting procedures are then imposed upon this initial set of constraints. Evidence of these early enumeration abilities is important because it provides the initial crude, module-like structure which can be supplemented by a more elaborate cultural system of mathematics (Cole, 1996).

Geary (1995) has moved this issue on from such studies on infant numeracy to propose that there are two kinds of numeracy skills: biologically primary and secondary. These are not exclusive to numeracy and various cognitive abilities (language as well as maths) fall into these two groups. Biologically primary cognitive abilities are found panculturally and across related species. Secondary cognitive abilities are found in some cultures but not found in others. Cultural practices influence cognitive abilities that are not completely related to evolutionary pressures. This is because the specialised neurocognitive systems that support biologically primary abilities can be used in a variety of ways apart from the original evolution-based function. The extent to which children in various cultures (and across generations) acquire secondary abilities varies directly with the extent to which formal cultural institutions (e. g. schools) emphasise them. For example, while language is found cross-culturally, the ability to read is not; so reading is therefore a biologically secondary cognitive domain. The following section looks at this distinction in more detail.

2. 2. 3. 1 Acquisition of biologically primary cognitive abilities

Just because certain abilities are biologically primary does not mean that experience is not necessary for their development. According to Gelman (1990) implicit knowledge or skeletal principles of the domain (skeletal principles of neurocognitive systems) provide only initial structure. For Trick (1992) the determination of numerosity or the quantity of small (less than 4) sets of visually presented objects involves preattentive processing in the visual system.

Arising from this, biologically primary mathematical abilities emerge early on in development. Evidence for the existence of this comes from various studies (Antell and Keating, 1983; Starkey, 1992; Starkey, Spelke, and Gelman, 1983, 1990) suggesting that human infants are sensitive to the numerosity of an array of up to three and sometimes four items, even as early on as their first week of life. They can also distinguish between homogeneous versus heterogeneous collections of objects and are sensitive to displays in motion. Intermodal studies (Starkey, Spelke and Gelman, 1983) are especially important because they suggest that an infant's sensitivity to numerosity is based on an abstract representation. That is, an infant's knowledge that a set of two items differs from a set of three items is not dependent on whether the items are seen or heard (as in a series of two or three drumbeats). Infants as young as five months are aware of the effects of addition and subtraction of one item on the quantity of a small set of items (Wynn, 1992). At eighteen months, human infants can recognise ordinal relationships, e. g. that 3 is more than 2 and 2 is more than 1. This general sensitivity to more than and less than is also evident in many animal species, such as cats, laboratory rats, African grey parrots and chimpanzees (Davis and Perusse, 1988). Well-controlled experimental studies have shown that non-human primates are able to make very precise ordinal judgements (Boysen, 1993; Boysen and Bernston, 1989).

So humans are born with the basic biological foundations of numeracy and our later experiences influence exactly how these will develop. They are, for example, "fleshed-out" through play. Eibl-Eibesfeldt (1989) described play as "self-activated practice" that allows children to rehearse and experiment with social roles and to start acquiring functional abilities. Saxe et al (1987) found children as young as 2 years frequently engaged in solitary or social play that involved numerical abilities such as counting toys. Piaget (1962) believed that not all but certain types of play activities were likely to be universal and therefore probably served similar functions across cultures.

Therefore, learning about numerical features of the environment is evident in children's activities across cultures and engaging in these activities fleshes out the skeletal principles associated with the biologically primary mathematical abilities. However, many abilities are not similarly advantaged by skeletal principles or contain a bias for engaging in activities that help in their acquisition. These abilities are classified as biologically secondary cognitive abilities.

2. 2. 3. 2 Acquisition of secondary cognitive abilities

Because development of these abilities does not have biological advantages, their acquisition is slow, effortful, and occurs only with sustained formal or informal instruction. To deal with this, complex societies have developed formal institutions (schools) so that children can acquire social and cognitive skills that otherwise would not emerge. Flynn (1987) noted that universal schooling is found only in technologically and socially complex societies. As the technological and social complexity of the society increases, so does the amount of formal schooling for children. Therefore, universal schooling even in complex societies is relatively recent. Geary (1994) points out that there is no reason that skills taught in schools are inherently interesting or enjoyable for children. Similarly, Ericsson, Krampe, and Tesch-Romer (1993) pointed out that deliberate practice that improves performance at a task is not inherently enjoyable even for experts in the area.

Thus, it is cultural institutions like schools that lead to difference in secondary mathematical abilities such as arithmetic. So while there are no differences between the biologically primary mathematical abilities of East Asian children and North American children, Geary (1994) argues there is a significant advantage for Asian children in secondary mathematical domains that rely on educational practice. The exception here is the possible influence of language structure (e. g. number words) on early development, an advantage that coincides with the start of formal schooling. This is an issue that will be discussed later on in Section 2. 2. 4. 3.

Biologically primary mathematical abilities include management of numerosity, ordinality, counting and very simple arithmetic (involving increases and decreases of sets of three or four items i. e. very simple addition and subtraction). Biologically secondary mathematical abilities include counting, number, and arithmetic. The former are found "pan-culturally and are evident in non-human primates and some other animal species" i. e. a sensitivity to numerosity has been found in cats, lab rats, the African grey parrot and chimpanzees, (Davis and Perusse, 1988; Boysen, 1993). The latter are culturally specific, and emerge with formal schooling so they must be taught.

Basic (biologically primary) numeracy skills are present in young babies and some animals, while complex (secondary) numeracy skills are taught in school and through formal/informal instruction. This distinction seems to have implications for encouraging the use of more drill and practice. So Geary (1995) recommends a "modified form of drill-and-practice" which, he argues, "is probably the only way to ensure the long-term retention of basic biologically secondary procedures". He argues that while the constructivist approach may be suitable for the acquisition of biologically primary mathematical abilities such as number and counting, it is not conductive to the development of biologically secondary skills. Geary (1995) is critical of the constructivist approach, because he believes that education researchers who have adopted this approach and put it into practice have ignored or dismissed a significant amount of relevant psychological research and theory. According to Geary (1995), constructivist philosophers and researchers fail to make the distinction between biologically primary and biologically secondary mathematical abilities and so treat all of mathematics as though it were a biologically primary domain. The selective use of psychological theory by some researchers, he believes, is highly problematic.

Compared to other cultures, such as the Asian culture, American culture is more liberal about the extent to which individuals are allowed to pursue their own self-interests or take part in inherently interesting activities. Geary argues that, apart from basic number and counting activities, many mathematics-related activities are not likely to be inherently interesting for most people. So it requires cultural values that reward mathematical development and strongly emphasise mathematics education in school, in order to develop complex secondary mathematics abilities. He concludes that constructivism is a reflection of current American cultural beliefs. As a result, this involves developing instructional methods that attempt to make learning complex mathematical skills an enjoyable activity, taken up due to interest and choice.

While these social-constructivist methods work well for the development of biologically primary skills, because constructivist activities are similar to contexts in which biologically primary activities naturally emerge (Eibl-Eibesfeldt, 1989), they are not sufficient for developing secondary skills. It is for secondary skills, many of which are not inherently enjoyable, that drill-and-practice techniques are useful. It is not that mathematics instruction should not be interesting or engaging, it should. It is just that an environment that supports the further development of biologically primary abilities is not sufficient for the acquisition of secondary abilities. Geary acknowledges that many constructivists hotly dispute the use of drill-and-practice and that formal drill-and-practice does not seem to be necessary for the acquisition and maintenance of many biologically primary cognitive abilities (such as language). Cultural values that support and foster students' interest in complex mathematics are essential because evolution has meant that children do not have a natural enjoyment of activities such as the drill-and-practice that is needed to master abilities in complex secondary domains. Practice, he argues, "provides an environment within which children can flesh out their understanding of the procedure and any associated conceptual knowledge".

Certainly, every attempt must be made to make drill-and-practice enjoyable and interesting for children. Even an associationist like Thorndike was concerned with the "meaningfulness" of arithmetic problems and recognised that such problems should be relevant to daily activities outside the classroom. Yet, drill-and-practice may be hard to avoid in developing arithmetical knowledge. Geary (1995) suggests that we cannot expect children to enjoy the process of acquisition of all secondary abilities. But in this case the ends justify the means, since the motivation to acquire these abilities comes from the needs of an increasingly complex society rather than the interests of the children.

If some numeracy skills are innate (biologically primary abilities) and biologically advantaged, while others (secondary abilities) depend mostly on formal educational practice, then the role of cultural practice is critical in determining what will be learnt. Culture has a significant impact on the development of numeracy since the teaching of mathematical abilities depends on the type of formal schooling a child receives and the cultural attitude about academic achievement. Cultural research has been carried out through various exciting and innovative methods. The next section will briefly examine some of the influential research done in a cultural framework.

2. 2. 4 Cultural Research

Like biology, culture also exemplifies an influence somewhat beyond our control in how it contributes to our development of numeracy. Maths is a cultural tool that has evolved in social time rather than just historically or biologically. It is crucial for us to understand the variety of conditions under which we acquire numerical skills and arithmetic knowledge. This section looks at how cultural variation affects the outcomes of learning. Cultural research can be realised from two perspectives. One is to take a cultural psychological perspective - in general: looking at how cognitive activities are affected by the cultural context in which they take place, e. g. to take control of cultural tools for doing number work. The other is more traditional comparative cross-cultural research. This often involves looking at how people in different cultures deal with the similar tasks, e. g. looking at the significance of number words found in different languages.

2. 2. 4. 1 Cultural psychological research

Numeracy is about manipulating number systems (including common tools like using fingers). This means invoking culture as a context for doing numeracy research. Models of number fact retrieval arising from a cognitive memory oriented approach (e. g. network retrieval and network interference models) have a tendency to bypass the fact that all this learning is going on in a social context. So they can seem more mechanical than active. The cognitive tradition does not seem to pay full attention to cultural research, because it is more concerned with the "internal" or "mental" aspect of computation rather than culture which is viewed as variables "out there" and "not in the head". Thus, the cognitive approach tends to neglect the fact that all this cognitive activity is going on within a cultural context.

Mathematics is, of course, a cultural product. Children's understanding of mathematics including mental arithmetic will ultimately depend on cultural practices especially resources encountered in education/school practice. This varies not only culturally but may even, to a lesser extent, vary from one school to another. Culture influences the way we do mathematics in both formal and informal settings. This has led to both cross-national research in children's of numerical ability as well as research looking at how the local cultural context affects numerical ability.

Some fascinating research has been carried out in this tradition. Researchers have carried out studies that have provided us with insight into how culture exerts a powerful influence on numerical and arithmetical ability. This is the reason behind studying cultural variation. It gives us insight into the competing sets of cultural tools within one culture e. g. "street" versus "school" mathematics (Nunes, Schlieman and Carraher, 1993 and Carraher, Carraher and Schlieman, 1985).

There are differences between formal and informal versions of mathematics. This was illustrated by Reed and Lave (1979) and Lave (1977) who looked at the way Vai tailors in Africa used Vai and English number systems in arithmetic problem solving. They looked at the how mathematical skills acquired by tailors in their everyday practices were used when they had to deal with unfamiliar problems. Lave also wanted to see how the tailors' formal educational experience affected their mathematical problem solving abilities. She presented the tailors with a set of tasks, including problems that required either school experience or tailoring experience in order to be solved. Lave (1977) found that when problems were modelled on school-type problems, the extent of schooling was a more significant predictor of performance than tailoring experience. For the problems that had been modelled on tailoring practice, the years of tailoring practice were a better predictor than years of schooling. Lave's research effectively illustrated how cultural practices directly influence our mathematical thinking. It turned out that there is a variety of methods people employ for dealing with problems involving maths. That is, mathematical ability varies because of the culture we grow up with.

Similarly, Carraher et al (1985) looked at the case of “street” versus “school” maths in practice. They looked at the social practices of mathematics among child street vendors in Brazil and found how arithmetical abilities varied depending on whether the task used was an "everyday" task or a "psychologist-imposed" task. The children engaged in different practices depending on the task. They generally found school-type arithmetic tasks harder to follow whereas similar tasks involving familiar “street” mathematics (e. g. currency conversions, adding/ taking away large quantities of goods) were simple.

This cultural variation in devising tools for dealing with numeracy was also studied by Saxe (1979) who found that the Oksapmin tribe of Papua New Guinea had a complex method of counting. In this method, they count by starting with the thumb of one hand and then pointing to twenty-seven places on the arms, head and body, ending with the little finger of the opposite hand. If they have to count further, they continue back up the wrist of the second hand and progress back around the body again. According to Saxe, this system of counting is used for everyday counting activities that do not require the use of calculational procedures e. g. such as counting pigs. It suffices because the Oksapmin usually do not need to engage in computations involving numbers. Saxe (1982) observed actual arithmetic calculations (similar to those studied by Klein and Starkey among US children) only among children who attended school and adults who became involved with the money economy of New Guinea. However, Saxe (1982) also observed Oksapmin children using the Oksapmin body-counting system to generate answers in school lessons. This study, in addition to those discussed above, all illustrate how culture influences how people have developed various methods for dealing with similar ideas.

Although the results are less immediately striking than those described above, more traditional cross-cultural research (i. e. giving the same task to school children in different cultures) has also shown cultural differences in performance on mental arithmetic tasks. Various explanations are cited for why these occur. Political interest in this area, especially in the West, has stemmed from research showing cultural differences between the mathematical competence of children in the US plus some Western European countries, and children from the Far East. These differences are more fine-grained and subtle than those that arise from other, more comparative cultural research interested in celebrating the cultural roots of numeracy rather than comparing systems (Saxe, 1982; Reed And Lave, 1979; Carraher et al, 1985). Yet these are seen as more fundamentally significant results because they may have far-reaching political implications in an international and hi-tech global society. Research has shown that children from the Far East seem to have superior mathematical and arithmetical skills than their Western counterparts and this is seen as a threat to economic competitiveness with the Far East (Geary, Bow-Thomas, Liu and Siegler, 1996; Reys and Yang, 1998).

2. 2. 4. 2 Cross-cultural research

Cultural differences are attributed to various factors such as language structure and schooling and these processes complement each other. Interestingly enough, language is an area that unites the cognitive and cultural approaches because this is where cultural differences feed into the information-processing cognitive approach. It is crucial that we understand how and why this occurs What follows is an example of how one methodology (cross-cultural research) affects the another (cognitive numerical processing).

2. 2. 4. 3 The effect of language on numerical processing

There is evidence that language structure influences digit span. The mean digit span for normal adults using different languages depends on the length of time it takes to pronounce the words for digits in those languages. For example, the digit span in Welsh is lower than digit-span in English because Welsh words take longer to say than their English equivalents (Ellis and Henelley, 1980). The converse is true for English and Chinese. This is often used as an explanation for the superior performance of Far Eastern children at arithmetical tasks.

Geary et al's (1996) findings support the view that the structure of Asian and English language number words influences the development of early numerical and arithmetical competency. Digit span is influenced by the speed with which number words can be spoken and is a language-related influence on children's strategy choices; especially on the use of finger counting as a back-up strategy. Chinese children were more likely to use verbal counting than finger counting as a back-up strategy. Exposure to mathematics in school is one reason for the national difference in mathematical achievement. Geary et al (1996) found that Chinese children's addition test performance (at all grades) improved significantly across the academic year compared to the American children.

The use of 10-based decomposition by Chinese children reflects both language and schooling effects. The structure of Chinese number words makes it easier to teach the base 10 system and base 10 problem-solving strategies. Therefore, Chinese teachers are more likely than American teachers to teach 10 based decomposition and Chinese students more likely to understand its usefulness.

Language may also affect Chinese children's speedy transition from back-up strategies to retrieval. According to Siegler (1998) and Siegler and Jenkins (1989), direct retrieval depends on the formation of associations between addition problems and their correct answers. These associations appear to develop with the use of back-up strategies. For the association to be formed, the problem's addends and its generated answer must be simultaneously active in working memory. The quantity of numbers that can be active in working memory is related to speed of counting.

While language influences seem to give Chinese children an early advantage in arithmetical competency, these should influence only the early stages of skill acquisition and not the ultimate level of skill that can be achieved in arithmetic. This is supported by the finding that American children who received their elementary school education in the 1930s developed the same level of competency (100% retrieval by 3rd grade) as the Chinese children in this study. So while language may influence early mathematical development, Geary et al (1996) argue it is an inadequate explanation for the advantages that East Asian children have over American children in almost all mathematical domains.

The early use of retrieval is not restricted to Chinese children. Adams and Hitch (1997) found that most British children used counting at mental addition tasks whereas German children were more likely to rely on retrieval or strategies that did not show overt signs of counting. They believe that this is because they receive more practice in mental arithmetic, they start school later and there is more emphasis on practising oral mental addition skills.

In conclusion, language is one of the factors that affects our arithmetic processing and one that can influence our early arithmetic development. However, it is an influence that may not have a lasting impact on our arithmetic skills because schooling seems to play an important role in evening out this early advantage.

2. 2. 4. 4 Other cultural factors that may influence numerical processing

In addition to cross-cultural factors, there also seem to be social factors that may influence children's mental arithmetic performance. I began by looking at almost crude cross-cultural comparisons to examine exactly where the differences lie. At one end we have the striking examples of how people manipulate numbers, followed by the more mundane differences between cultures that reveal interestingly coherent cultural practices. However, different social practices also affect mathematical development.

Saxe et al (1987) identified social processes in early number development such as age and social class differences in children's numerical understanding. They found that children are regularly engaged with social number activities involving number, though the nature of the numerical understandings and environments differ. Children's numerical environments are linked to their own understandings and to the socio-cultural context of their own development.

Uses of counting procedures to solve arithmetic problems are not unique to children raised in Western settings that place high value on early mathematical skills. Middle-class children are better at tasks involving cardinality, numerical reproduction and arithmetic. Social factors influence children's intellectual development through cognitive socialisation. Social organisation of young children's numerical activities can be understood as providing opportunities for children to use previously learnt skills on new functions and to elaborate and specialise existing strategies. This may have implications for children's later understanding of numbers as it suggests how some children from some families have different access to mathematical concepts in the classroom.

Children's socially organised experiences with numbers in everyday activities are emergent, they are negotiated in interactions and result from parent and child's adjusted efforts. Different social class groups are creating different environments in daily lives but for each group the process is the same. Numerical environments consist of active participation and negotiation.

Thus, Section 2. 2. 4 illustrates how cultural research encompasses a wide range of approaches, from a broad understanding of culture such as looking at different cultural practices in cross-cultural studies to cultural research at a micro-level such as studying interactions between a mother and child.

2. 3. Summary

The aim of this chapter was to look at the development of research in mental arithmetic as well as some of the broad research traditions within which mental arithmetic has been studied. Therefore, I have presented a general overview of the research traditions that have been concerned with psychological studies of human numerical computation. There are many strands from within this work can inform the practical concerns illustrated in Chapter 1. In my own work, I have chosen to focus on the tradition of cognitive psychology that has emphasised an understanding of the mental processes underlying simple computational activity. My particular interest will be in the *development* of such skill. Moreover, within that developmental trajectory, I shall concentrate on the emergence of skills that might be broadly termed "strategic". As a grounding for the empirical work that follows, I shall therefore review existing work in the cognitive tradition that seems to be relevant to these broad aims.

Chapter 3

3. Cognitive Research in Mental Arithmetic

Chapter 1 identified the topical nature of mental arithmetic research and sketched the social and political background of research in this area. Chapter 2 looked at the history of mental arithmetic as a research problem, the research traditions within which it was studied, and the gradual emergence of mental/cognitive arithmetic as a discipline in its own right. The present chapter takes up more specific issues of mental arithmetic research that were raised in Chapter 2 and looks at them in more detail. In this chapter, I will begin by looking at some of the early cognitive research in mental arithmetic. This will be followed by a discussion of current theories of simple arithmetic processing, such as the number-fact retrieval, and the evidence used to support them. I will then move on to look at issues of working memory and present schema-theory as an alternative to some of the current cognitive models on offer.

3. 1 Early Cognitive Research

Groen and Parkman (1972) and Parkman (1972) undertook the early explorations of cognitive research in this area. They carried out some of the original experiments looking at mental arithmetic and reaction times. They also pioneered the min model (the counting model that best fit their data). The min model set a precedent for future cognitive models and represented an important early achievement, since it was an observation of mental computation development over and above fact-retrieval. In the "min" model, subjects set an internal counting register to the larger of the two numbers (n, and m) being added (e. g. *m*) and then to increment this value by ones a total of *n* times i. e. until the smaller or minimum addend value was reached. At the end of this incrementing process, the counting register contained the value of the correct sum, and simple "read-out" (translation into an overt response) or comparison with the answer could occur.

They found that the size of the smaller number affects solution time in simple addition. The size of the smaller number was a good predictor of solution times on simple addition and solution times increase linearly with the size of the smaller number. But this was found to be the case only when this strategy (counting-on) was being used. The min model also failed to account for fast performance on tie problems and Groen and Parkman suggested this was due to these problems being stored directly in memory, and therefore retrieved by some direct access process. They also proposed that it was due to memory retrieval and that addition and multiplication "tables" were stored as hierarchical networks in LTM and RT was due to the location of the solution in the network. Although the min model was found to be insufficient early on in their research, later research extended and modified their findings. Theirs were among some of the earlier studies looking at the similar processes underlying simple mental addition and multiplication. Their research was essentially about looking into the counting-based strategies people used when doing mental arithmetic.

Another significant earlier study looking at the effects of practice, gender and individual differences on RT was carried out by Aiken and Williams (1973). Results showed that subjects used a variety of techniques in performing mental calculations and this depended on the type of arithmetic operation, amount of practice and individual differences in computational ability. They found that RT for addition increased linearly as a function of the smaller number. They claimed that certain well-learned sums were obtained by random access retrieval from memory. They also found subjects to be faster on addition problems having a sum of 10 and multiplication involving 5 and 1, a finding which has since been replicated in several studies. Even after practice, there was a significant linear relationship between number magnitude and mean RT. Their findings led them to conclude that several varieties of storage and retrieval mechanisms are in operation during the performance of mental arithmetic and this varies with the nature of the problem, amount of practice, level of motivation and individual differences in computational ability.

These early studies had implications for future cognitive research into mental arithmetic. They revived interest in research into mental arithmetic as a topic of interest in itself, instead of being a distracter task for memory experiments. The next section will look at the development of some of the current dominant theories of arithmetic processing.

3. 2 Current Theories of Simple Arithmetic Processing

There are three basic models of simple arithmetic and these have been revised and elaborated to account for new empirical evidence. Although referred to here as *current models* these are *developing models* in that they are under constant revision and change. The focus in these contemporary theories is on single-digit number bonds (this structural feature is common to all these cognitive models) and the way that these number facts are organised in human memory.

These models have in common several basic underlying assumptions. The first is that performance on simple number facts depends on retrieval from LTM. The second is that this memory representation is organised and structured according to the strength of individual connections and reflects amount of "relatedness" among the elements. The third assumption is that the strength of the associations with which the elements are stored, and therefore the probability or speed of retrieving information, depends on more than just the numerical characteristics i. e. what determines speed of response is the amount and type of exposure to those numbers.

Below is a brief summary of these models, which will then be discussed in greater depth. The models to be examined are summarised here as follows:

Ashcraft's network-retrieval model (Ashcraft, 1982; Stazyk, Ashcraft and Hamann, 1982). In the simplest network model the basic facts of addition and multiplication are stored in inter-related memory networks with number problems in the networks varying as a function of strength or accessibility. Each problem in memory is thus distinguished by the strength of its association in the network.

Campbell's network-interference model (Campbell and Graham, 1985; Campbell, 1987; Campbell and Clark, 1992; Campbell and Oliphant, 1992). This interprets retrieval from the organised network as a process heavily affected by interference. This seems to be the only current model to account for various interference and priming effects that are now well documented in the literature.

Siegler's distribution-of-associations model (Siegler and Shrager, 1984; Siegler and Jenkins, 1989; Shrager and Siegler, 1998). The basic representation of a problem in memory is accompanied not only by that problem's correct answer but also by the incorrect answers that the individual has generated or computed across experience.

Campbell, Siegler and Ashcraft are not only interested in correct/ accurate responses but are also interested in the specific errors observed in mental arithmetic. This is because these errors reveal aspects of the memory representations and processes that underlie retrieval i. e. they use errors as evidence for network interference because they find errors are more likely to be systematic (related to operands) rather than miscellaneous (unrelated to operands or correct answer). They identify types of errors and error rates are computed for each subject. The next three sections provide more in-depth information about each of these models.

3. 2. 1 Ashcraft's Network-Retrieval Model

Ashcraft (1982, 1987) proposed a network retrieval model of mental arithmetic performance. Basic addition and multiplication facts are stored in network representations with each learned fact represented as a node in a network structure. Simple addition and multiplication facts are represented in an associative network in memory, each with its own accessibility value. Ashcraft notes that is "absurd" to suggest that e. g. 7x4 is stored in some independent, isolated fashion from other knowledge of multiplication in the network. So what factors affect the influence of these strengths? The most recent version of the model (1987) accounts for the effect of early schooling on arithmetic, as well as other influences (e. g. frequency of occurrence, practice, small-fact bias), as having a direct effect on the strength of problem representation in long-term memory.

Ashcraft's experiments involved the use of reaction times as the dependent variable. Basically, adult performance on addition and multiplication facts can, to a great extent, be explained by memory retrieval. That is, in the simplest network model the basic facts of addition and multiplication are stored in interrelated memory networks in an organised interrelated network structure and that multiplication and addition are highly similar cognitive processes. The network contains associations to both correct and incorrect answers and these vary in strength.

But Ashcraft's theory neglects to make much mention of the procedural aspects of computation. Baroody (1983) attempted to provide an alternative to the network-interference explanation for chronometric (RT) trends in mental arithmetic. He argued that Ashcraft's model underestimates the role of procedural processes in the *efficient* production of number facts by adults. This is due to the questionable assumption that *all* procedural processes *are* or *remain* slow. According to Baroody (1983) procedural processes become more secure and interconnected and thus become automatic so that problem solving (even basic number facts) becomes more efficient. He concluded that there was a developmental shift from a reliance on slow-counting procedures to a reliance on automatic principled knowledge (analogous to syntactic language production). This seems to suggest that perhaps the strategic aspects of mental arithmetic have been somewhat neglected. It is possible that perhaps, for some sums at least, strategic processes become automatic and are thus carried out almost as swiftly as recalling number facts.

For example, the addition-subtraction inverse principles in stored procedural knowledge could be used to generate subtraction facts. This would be achieved through utilising automatic addition combinations, thus eliminating the need for another network in declarative memory. The developmental trend toward faster RT might be accounted for by the development of either declarative or procedural knowledge. This also accounts for difference in RT among various kinds of number-facts i.e. it arises from automaticity of procedures.

Barоody also claims that errors that reflect associative confusion (used as evidence for fact-retrieval by Ashcraft and other proponents of fact-retrieval) may actually be due to accessing the inappropriate rule in procedural knowledge. The "mastering" of number facts may be the development of procedural knowledge. It is, he claims, cognitively more economical to use procedural knowledge.

Although Ashcraft (1983) found Baroody's original idea intriguing and potentially important, he believed in his early criticism that its exposition was quite vague and speculative. According to Ashcraft (1983) Baroody's proposal only applied to special cases e.g. N+0 and N+1 and there were no concrete suggestions about the nature of the rules and heuristics especially for routine problems e.g. 4+3 and 8x5. Ashcraft (1983, 1985) argued that Baroody's alternative was an inadequate explanation of existing RT results. However, Baroody's (1994, 1999) proposals are now more sophisticated and do take into consideration existing RT results but I will return to this point later on.

In Ashcraft's model, young children compute simple sums by slow counting algorithms but older children and adults have stored it in memory and can retrieve it. However, Baroody (1983) claims such mental arithmetic performance (whole number facts) can be explained without resorting to the emphasis on declarative knowledge and memory retrieval, procedural knowledge is enough.

Ashcraft argues that the "confusion effect" in multiplication (e. g. recalling the answer to 4x6 (32) vs 4x8 (24), because both are answers to a “4x” problem) is support for the memory network approach, whereas Baroody provides no descriptions of rules other than N+0 and Nx0. Another issue, of the problem-size effect, shows that procedures are less automatic and more difficult to execute, and so this implies reliance on conscious rather than automatic processing, therefore a "sacrifice of efficiency and economy". If procedures were all that efficient there would be no problem-size effect. Procedural processing is thus inefficient and uneconomical. Fact storage in Ashcraft's model is comparable to storing the words of language for a language model, while according to Baroody, making sentences involves both rules and words.

According to Siegler's Distribution of Associations model (discussed further on in Section 3. 2. 3), the basic representation of a problem in memory is accompanied, in a rather mechanical fashion, by both correct and incorrect answers that the individual has generated or computed across experience. This is what may slow down responses for certain problems. Using this information, Ashcraft argues that while adult performance is interpreted as representing fact retrieval, other studies give evidence for strategy use even on simple addition and multiplication facts, perhaps because of a lack of confidence at recalling the correct answer.

In his own research project, Ashcraft (1995) looked at another less researched issue in mental arithmetic: the cognitive consequences of mathematics anxiety. Although he found that there was research available on mental arithmetic and mathematics anxiety, no one had considered looking at the possible cognitive consequences of mathematics anxiety, i. e. whether or not such anxiety makes any difference to how a person performs mental arithmetic. What was interesting was his finding that high maths anxious subjects achieve rapid RTs but at the cost of substantially higher errors (a speed-accuracy trade-off) in their desire to end the experiment and that there are physiological changes as a result of mathematics anxiety. This has significant implication for numeracy skills in general, as well as implications for educational practice especially for older children. This reminds us how useful it is to be confident at numeracy skills. Confidence in their general ability could affect children's competence at mental arithmetic although this would be more likely to become an issue for older children. Although it is doubtful that this would be the case with younger children, it may well be an issue that depends on early experiences with mental arithmetic.

3. 2. 2 Campbell's Network-Interference Model

In this further fact-retrieval model, the emphasis is on the interference from various number-facts in the network. In Campbell's network-interference model, retrieval from the organised network is viewed as a process heavily affected by interference. In this model, a presented problem activates the memory representations for a large number of related number facts. The strength of the activation of specific facts is determined by featural and magnitude similarity to the presented problem.

Campbell's (1995) network model simulates number-fact retrieval processes for single-digit multiplication facts up to 9 x 9 and addition facts up to 9 + 9. Campbell (1995) was a revision of Campbell and Oliphant's (1992) network-interference model of number fact retrieval. Although they first proposed an earlier model of their theory, I will be discussing the most recent one.

In the network-interference theory, whenever a problem is presented, memory codes that correspond to all the addition and multiplication facts in the network are activated to some degree (activation here is a metaphor for the changing strengths within the cognitive system). In this theory the dominant idea is that there are memory codes and processes that mediate retrieval of simple arithmetic facts such as 4 + 8 = 12 and 8 x 6 = 48. A problem activates memory representations for a large number of related number facts. When faced with a problem, memory codes corresponding to all the addition and multiplication facts in the network are activated to some degree. Adults and older children often rely on such retrieval strategies. Campbell (1995) claims that the network-interference model is a theory of number fact retrieval, not a general model of basic arithmetic skill. Within a general theory of basic arithmetic skill, he argues, the network-interference model provides a detailed theory of a component memory skill.

It is assumed that arithmetic memory involves both a magnitude code and physical codes. The magnitude code represents the approximate numerical size of the answer to a problem and primes the associated physical codes that represent exact answers. Physical codes for the problems are assumed to be visual or verbal associative units that consist of the operand pair, operation sign, and answer. Campbell refers to the physical-code representations of problems as nodes. Retrieval in this model involves a series of processing cycles, and each cycle represents a few tens of milliseconds of processing. On each cycle, each node receives excitatory input determined by both physical and magnitude code similarity to the presented problem. This excitatory input is modulated by inhibitory input that is proportional to the total activation associated with all other currently activated nodes. Across cycles, the strengths of the excitatory and counteracting inhibitory inputs gradually approach equilibrium, and a response is generated when one of the nodes in the network reaches a critical threshold level of activation. The excitatory input to the correct node is generally the same for all problems, so that differences in retrieval ability are mainly the result of differences in inhibition due to the activation of incorrect nodes.

One observation that provides support for this theory comes from one of the most frequently researched occurrences in cognitive arithmetic, i. e. the problem-size effect. The problem-size effect is the finding that the difficulty of simple arithmetic problems generally increases with numerical size. This is true for both children and adults, with correlations of about +0.6 to +0.8 observed between measures of problem-size and retrieval time (RT) on simple addition and multiplication problems (Ashcraft, 1987; Campbell and Graham, 1985; Geary, Widaman and Little, 1986; Miller, Perlmutter and Keating, 1984; Norem and Knight, 1930; Parkman and Groen, 1971). Various explanations have been offered for why the problem-size effect occurs.

Campbell and Graham (1985) proposed that the problem-size effect is due to larger number problems being learnt later on than smaller-number problems. This results in cumulative proactive interference that they claim might produce long-term effects in the impairment of retrieval for larger problems. They claim that although there is no direct evidence that such long-term cumulative interference effects exist, research has shown that children's performance on recently learned addition facts is disrupted when related multiplication facts are learnt (Miller and Paredes, 1990; Graham and Campbell, 1992).

Another reason for the problem-size effect may be that larger number problems are practised less frequently. Hamann and Ashcraft (1986) and Siegler (1988) found that larger problems appeared less often in elementary-school textbooks. Campbell (1995) also suggests it is likely that we come across smaller problems more frequently than larger ones in everyday context e. g. 2x3 or 2+3. However, McCloskey, Harley and Sokol (1991) believe that since all single-digit problems will probably have been encountered very often by adulthood, any differences arising from frequency should be minimal.

A further explanation for the problem-size effect is that numerical magnitude (the size of the number) has a direct effect on retrieval difficulty (Gallistel and Gelman, 1992). Support for this approach comes from magnitude effects found in arithmetic and numerical-comparison tasks. For example, most errors on simple arithmetic problems involve the correct answer to a "neighbour" problem (i. e. the answer to a problem if an operand was changed by +1/ -1 or +2/ -2; Campbell and Graham, 1985; Miller et al, 1984). In addition to this, in arithmetic verification tasks (e. g. 4+8 = 11: true or false?), false answers close to the true answers are more difficult to reject than false answers that are numerically further away.

Campbell (1995) concludes that the problem-size effect arises because larger-number problems are more similar in magnitude to their neighbours than smaller-number problems. Thus larger-number problems are more likely to activate neighbours, and so they encounter more interference due to inhibition from neighbours than smaller-number problems. This is due to the slowing of the rate of activation of the correct node for larger-number problems making them more susceptible to retrieval errors.

However, this account of fact-retrieval is not without its problems. There are other performance differences that cannot be explained by problem-size, although the latter is a good predictor of difficulty. Performance on problems with a repeated operand such as 3x3, or 8+8 (referred to as "ties") is better relative to "non-tie" problems of a similar magnitude (Campbell and Graham, 1985; Miller et al, 1984). Campbell (1995) believes that one explanation for this ties advantage may be a higher occurrence frequency relative to non-ties. Although Graham and Campbell (1992) found that, when practice frequency was controlled, learning and performance on tie "alphaplication" problems (these are arithmetic-like memory items consisting of letters rather than numbers) was still better than for non-ties.

Another reason, argues Campbell (1995), is that they may be "intrinsically" easier (Beem, Ippel and Markusses, 1987; Gallistel and Gelman, 1992). Graham and Campbell (1992) found evidence that tie and non-tie problems form categorically distinct sub-clusters of items because most errors on tie problems involved correct answers to other tie problems (52%). In contrast, tie answers were infrequent error responses to non-tie problems (4%). This propensity to confuse answers within but not between these subsets of problems leads Campbell to suggest that ties and non-ties form different categories of problems. Within the network interference model, this means that if activation is stronger within rather than across category boundaries, ties are easier because they cause relatively weak activation of the more numerous non-ties and so encounter less interference than non-ties. Campbell makes a similar observation for 5-times problems, and sum to 10 problems which also have faster and more accurate solution times and he attributes this as being due to these problems being categorically distinct (and so resulting in strong associative links).

In summary, Campbell (1995) proposed the network-interference model and a formula to test number fact retrieval in a computer simulation. He then tested it with both real and simulated subjects. He also found RTs to be longer for multiplication than addition in both actual and simulated data and that multiplication is more error-prone than addition. The model demonstrates that similarity based interference accounts for errors (where the incorrect answer is similar to the correct one or is the answer to another problem) and provides accurate prediction of variability in RTs of correct answers.

3. 2. 3 Siegler's Distribution of Associations Model

In Siegler's distribution-of-associations model (Siegler and Jenkins, 1989; Siegler and Shraeger, 1984), the model is distinguished by its attitude towards the representations of problems in memory which, in this theory, include both correct and incorrect answers that the individual has generated or computed over time and experience. So, due to its solution history, any particular problem may have a very strong, unique associative bond to its correct answer, or it may have several weaker associations, one to its correct answer and several to incorrect solutions. They used this distribution-of-associations model to describe the strategy choice process. Representation in memory involves associations of varying strengths between each problem and its correct and incorrect answers.

Siegler and Jenkins (1989) include another class of associations for each problem in which the most common or most favoured solution strategy for that problem is also represented. For example, counting as a solution process would also get attached to a problem on which it was likely to have been the most frequently invoked strategy. Therefore problems that have been solved with no or few errors across experience will be represented by a relatively strong association to its answer, as well as to the strategy responsible to obtaining that answer e. g. retrieval. However, a problem solved with an inconsistent accuracy, in addition to an erratic strategy, will mean multiple weak pathways of association for that problem accompanied by a slower, more error-prone performance.

This is a multiple procedure model because it accounts for the view that there are multiple routes to solution, i. e. individual children and adults often use multiple strategies to solve problems and this affects the strength and type of associations that form as a result (Siegler, 1988). Siegler (1988) argues that wise choices allow people to meet situational demands and overcome limited knowledge. He found that even young children can be fairly skilled at choosing strategies, e. g. between retrieval which may be fast and back-up strategies such as counting which are high in accuracy.

This is the most developmental of the three models discussed here, since it has been used to account for the performance of children and is the least concerned with processing effects in adults. Therefore, it is also the only model in which processing strategies other than retrieval strategies are prominent. This is because non-retrieval strategies are more likely to be used by child rather than adult samples. In this model a distribution of associations for each problem is built up through the cumulative experience of retrieval and other procedures. This includes a separate representation of each fact associated with a variety of possible answers (both correct and incorrect) and stored procedural knowledge about non-retrieval solutions. According to Siegler, in multiplication, there is a very strong continuity between the performance of young children learning to multiply and that of highly skilled adults.

Siegler and Shipley's (1995) revision of the distribution-of-associations model includes a computer simulation of strategic development called the adaptive strategy choice model (ASCM), designed to predict the way in which children learn strategies for mental arithmetic. Lemaire and Siegler (1995) found that improvements in speed and accuracy that generally accompany learning can reflect at least four types of specific strategy changes. Their findings supported several predictions of the adaptive strategy choice model. If retrieval is chosen, ASCM operates identically to its predecessor, the distribution-of-associations model. Acquisition of the min strategy, where the minimum addend is added to the larger one (Groen and Parkman, 1972), leads to an overall increase in speed and accuracy. Retrieval starts to be used more consistently on problems and is extended to larger size problems as well. An increase in frequency of use of the fastest and most accurate approaches can contribute to a general increase in speed and accuracy. Speed and accuracy will improve if children execute the strategies more efficiently. With learning, the relative frequencies of use of these strategies change, e. g. the more difficult the problem, the more frequent the use of back-up strategies.

In ASCM, the strategies operate on problems to give data about the particular answer that was generated, the speed with which the answer was generated, and whether the answer was correct. This information about particular answers, speeds and accuracy feeds back to the databases about the effectiveness of strategies and the difficulty of problems. The assumption is that stated answers become associated with the problem on which they are stated. Within ASCM, as the associative strength between a problem and its correct answer grows relative to associations between the problem and incorrect answers, the probability of retrieving the correct answer grows. This is followed by increasing use of more effective back-up strategies (strategies such as counting that we fall back on when unable to easily retrieve an answer) relative to less effective ones. According to Lemaire and Siegler (1995) a similar trend should be evident in multiplication. Children find it easier to add the larger multiplicand the number of times indicated by the smaller.

ASCM incorporates three levels of information: 1) global information (which is generic information averaged over all problems), 2) local information (which is information about individual problems), and 3) featural information (which is information about problems with particular characteristics). Problem-solving experience leads to children obtaining increasingly useful featural information, i. e. that there are patterns/ rules that can help solve certain problems.

By encoding featural information, ASCM can learn about the general characteristics of the answers that accompany problems as well as specific answers to specific problems. This includes the odd-even status i. e. when two odd numbers are multiplied, the answer is always odd and when two even numbers, or an odd and an even number are multiplied, the answer is always even. Although few adults and even fewer elementary school children know this rule explicitly, their performances seem to be affected by implicit knowledge of the odd-even pattern (Lemaire and Fayol, 1994). On verification tasks, both children and adults are faster at rejecting errors that have incorrect odd-even status for a pair of multiplicands (e. g. 5 x 4 = 21) than ones that have the correct status (e. g. 5 x 4 = 22). Siegler (1988) also found that children tend to produce errors that reflect odd-even status early on when learning to multiply. However, these errors are more likely to conform to the odd-even pattern of addition rather than multiplication, e. g. on problems with one add and one even operand (5 x 4) their errors tend to be odd numbers. An odd and an even number producing an odd number is the correct pattern for addition but not multiplication.

To conclude, Siegler's distribution-of-associations model and ASCM suggests that strategy choices are highly adaptive even early on in learning. The more accurate execution of the back-up strategies leads to stronger associations between each problem and its correct answer, and to weaker associations between the problem and incorrect answers. This in turn leads to more frequent use of retrieval. This model differs from the traditional view of children moving from Strategy A to Strategy B to Strategy C. In young children's arithmetic, both more and less advanced strategies coexist and compete with each other for a long time. Only gradually do more advanced strategies become prevalent.

So it is evident to Siegler that research shows that from an early stage becoming efficient at mental arithmetic is a dynamic process involving the introduction of new strategies, shifts toward a greater use of the more efficient existing strategies, improved execution of the strategies, and more adaptive choices among the strategies. According to Siegler and Jenkins (1989) it is not always the child with the most advanced knowledge who first discovers a new strategy. Innovators are likely to be those with the willingness to consider diverse strategies and to continue using them even when they are not working perfectly.

3. 3 Overview

At first glance it might be thought that these theories resemble behaviourist models but this is not the case. Cognitive psychology is concerned with mental models rather than the stimulus-response theory which was influential in the earlier part of this century. For example, Ashcraft, Campbell and Siegler talk about cognitive models and *associations*. Behaviourist models were considered theoretically limited and were not seen as being rich or powerful enough as explanations for behaviour. It is possible to claim that cognitive models are also rather mechanical in their general outlook. Ashcraft counters possible claims that network models are mechanical with the argument that cognitive psychology uses the notion of associations among facts, items of information, nodes in memory etc and is therefore a more dynamic, more adaptive process compared to the simple S-R theories. This does not suggest the formation of S-R associations in the Thorndike tradition. It is wrong, he claims, to equate associative network models with a S-R approach.

However, at a significant level network models *are* looking for mechanical associations but they are theoretically more sophisticated and therefore of richer and more productive capabilities. The earliest examples of cognitive research looked for more than just simple S-R associations. Cognitive models go beyond the behavioural level of association by conjuring up a "system of networks". For example, Campbell's (1995) network-interference model is meant to aid the construction of computer models that simulate children's thinking. Here the aim is to build a program that follows the rules of the model and behaves similarly to the way people behave when learning arithmetic. However, there is a tendency to bypass the fact that this learning is going on in a social context and so such theories can seem more passive than active.

Following on from this discussion of three prevalent theories of mental arithmetic processing, I will now look at some of the evidence used to provide support for network-retrieval models. Some of this comes from neuroscience research, more specifically from studies looking at selective impairment.

3. 3. 1 Neuroscience Research

One method of assessing cognitive (network/ fact-retrieval) models is by looking at how they are supported by neuropsychological research involving brain-damaged patients. It is important to take into consideration neurological research because mental arithmetic also involves neurological processes. As with all cognitive psychology, a lot can be learnt about processes (and certain functions associated with certain processes) from studying selective impairment, especially since new technology has allowed brain scans to become increasingly sophisticated. In mental arithmetic, selective associations between function and storage ability are crucial to arithmetic processing. The most convincing evidence for this comes from studies of people who have suffered from brain damage that has led to impairments in their arithmetical abilities. Selective impairment resulting from case studies of brain-damaged patients gives credibility to certain functional processes. If, for example, brain damage leaves a person unable to remember tie-sum number facts while still remembering other number-facts, then this is strong evidence for the view that this knowledge is somehow "distinctive" and is stored separately from other number-fact knowledge.

This has prompted neuropsychologists like Dehaene and Cohen (1995) to argue that while basic empirical findings in numerical processing are remarkably similar, there is disagreement about the models proposed to account for these data. While the arithmetic processing models themselves make similar predictions, it is difficult to discriminate between the models on the basis of peoples' performance which tends to be fairly consistent, e. g. the prevalence of the problem-size effect and the short RTs for tie-sums. The precise nature of the mental representation underlying simple number processing in adults remains a relatively controversial topic of debate.

Dehaene and Cohen (1995) proposed a model which accounts for the mental processes and neuroanatomical circuits involved in number processing and mental arithmetic, looking at the mental representation underlying simple number processing in adults. Their model elaborates on a previous "triple-code" model of arithmetic processing proposed by Dehaene (1992). It assumes that arabic and magnitude representations of numbers are available to both hemispheres but that the verbal representation that underlies arithmetic fact-retrieval is available only to the left hemisphere.

In the triple code model, arithmetic facts such as 2x3 = 6 cannot be retrieved unless the problem is coded into a verbal code "two times three" which then triggers the retrieval of the result "six" in the same verbal format. Support for the model comes from case studies of patients with various forms of neurological disorders that affect performance at numerical tasks. For multiplication, rote-memory retrieval is the main strategy. Various neurological conditions are shown to yield predictable impairments in the numerical domain. Dehaene and Cohen reject the phrenological notion of a "centre for calculation", or a single brain area where numerical knowledge would be centred.

Evidence from studies of neurological damage seems to support the network-retrieval model. Such evidence has played a significant role in testing models of mental representation of numbers (Campbell and Clark, 1988; Dehaene and Cohen, 1991). Dehaene and Cohen (1995) argue that since most studies have been carried out within the context of cognitive neuropsychology, they have looked at the behaviour of brain-lesioned patients at a purely functional level, without regard for brain localisation or lesion site. Therefore, models of number processing have also been studied within a functional level. Dehaene and Cohen (1995) are more interested in the networks of brain areas that underlie the functional architectures for number processing and how data from case-studies and functional brain-imaging techniques in normal subjects can be used to constrain models. Their intention is to provide an illustration of the form that a neuro-functional model of number processing could take. The assumption is that brain damage disrupts the network, perhaps by destroying or weakening associative links between nodes in the network. McCloskey (1992) found that three patients had considerable higher error rates for 8 x 8 than for 8 x 9 or 9 x 8. This supports low RT and error rates for tie problems that are seen as special cases (distinct facts) by proponents of the network model. It suggests that some distinct number-facts such as tie-sums may be stored separately from others, and this may be why they are so easy to recall i. e. generate faster RTs.

So, it seems that some of the most convincing evidence for network/ fact-retrieval models of arithmetic processing comes from such studies of selective impairment. Yet more evidence for network retrieval theories comes from studies looking at factors such as priming and errors that reveal much about our arithmetical processing.

3. 3. 2 Further Evidence Supporting Network Models

In addition, some of the evidence for the network theories comes from studies looking at people's performance at arithmetic tasks looking at priming and research on error patterns. LeFevre, Bisanze and Mrkonjic (1988) studied reaction times and found that activation of simple arithmetic facts is obligatory, in that activation of a sum occurs even when mental arithmetic is completely irrelevant to the task. They measured a

"Stroop effect" by presenting subjects with a combination such as 5 + 1 (the prime) and then replacing this display with a number (the target), the number is either one of the numbers presented previously (5 or 1), their sum (6) or some other number (e. g. 3). The subject was then required to indicate whether or not the number was presented previously in the combination display. They found evidence of an interfering Stroop effect. Subjects rejected sums more slowly than other numbers.

Their findings are compatible with the view that arithmetic knowledge is represented in an associative network and is accessed by means of spreading activation. They support network model theories that number facts are represented as nodes in a network of associative links and that arithmetic knowledge is a highly interconnected network of associations. They assume that the principles of semantic representation and processing that apply to network models of word knowledge also apply to arithmetic facts. Their findings also support the development of automaticity. This is the view that over time, some number facts may become automatised (Baroody, 1983, 1994). LeFevre et al (1988) concluded that although the task did not require the recall of combination sums, subjects did so automatically, and this slowed their response when evaluating a sum problem.

Evidence from error data also provides support for network theories of arithmetic processing, because they may reflect associative patterns that formed early on in learning and arithmetic development. While looking into this, Campbell and Graham (1985) claimed that despite research, no firm conclusions can be drawn about how basic arithmetic should be taught to children or about why the acquisition of simple number facts often presents a serious challenge to children. Some have argued that drill is necessary because the number facts need to be available effortlessly to avoid competition with higher level problem-solving processes. Campbell and Graham argue that the more thoroughly understood the basic operations are the more useful arithmetic tasks become as experimental tools. They believe that children initially use counting strategies to perform simple addition and that procedural models used by children are also used by adults. As a result, the problem-size effect in adults may reflect the presence of unconscious automatic counting procedures. The assumption in network-distance and procedural models of arithmetic memory is that the problem-size effect is a consequence of numerical magnitude. Since the problem-size effect is due to the search distance in the network, small number problems have fast response times (RT), because their operands are closely associated with the correct answer node in the memory structure, and larger number problems take longer because their answers occupy semantically remote regions of the network. Similarity among items to be memorised promotes interference, because competing associations are a source of confusion during learning and their impact is preserved in adult performance.

Analysing error patterns was useful because Campbell and Graham (1985) believe that errors reflect associative patterns. Activating a false association of a problem slows down correct retrieval and they found a positive correlation between problem error rates and RT. Children establish false associations during learning that weaken or interfere with the formation of correct associations. Looking at multiplication errors, they found that both adults and children are highly consistent in the errors they make and that children's errors start to resemble those of adults. They identified two kinds of multiplication errors: table errors (answers are correct for other simple problems e. g. 4 x 8 = 24 etc that are influenced by proximity) and miscellaneous errors (any other incorrect response e. g. 4 x 8 = 29). They found that errors were related to one or both of the operands in a problem. With increasing skill there is a tendency for errors to be numerically closer to the correct answer. So, errors reflect associative links between problems and false candidates. It is the frequency of occurrence which determines the strength of associations, i. e. the order in which arithmetic combinations are learned (small ones first, then larger ones). As a result, problems introduced later in the sequence must be learned in the context of many competing associations, whereas problems learned early are free from competition, and thus more free from errors. Ties (e. g. 4 x 4) are faster because associations form between operands and products and, since retrieval is slowed by false associations, ties which have only one number will activate fewer interfering false candidates. This would result in a strong association being formed between the tie-problem and it's correct response and lead to the fairly fast and accurate responses consistent with existing findings. Thus supporting the view that multiplication performance of both children and adults involves fact retrieval from an associative network.

Campbell and Graham (1985) also looked at how errors reflect associative patterns because errors are often systematic, i. e. errors are related to one or both of the operands in a problem. Thus multiplication errors reflect the activation of "false candidates" in an associative network structure in memory. They conclude that acquisition of simple multiplication skills is a process of associative bonding between problems and candidate answers. The problem-size effect is not due to distance searched through the network but is due to large problems being tested less often and occurring later in the learning sequence leading to weak correct associations and more/ stronger competing associations. There is, however, intrinsic variability among problems e. g. 4 x 8 (8 x 4) is one of the most difficult of problems whereas 8 x 9 (9 x 8) is *relatively* easy even though it is one of the last ones to be learnt. Network-interference explains multiplication performance but so does procedural (rule-based) knowledge. Any rule that constrains the candidate set for a problem should facilitate learning. The learning of multiplication facts can be facilitated by minimising the formation of false associates. They also believe that simple arithmetic can be used as a paradigm to define precise mechanisms of interference that have general implications for semantic memory and its development.

More support for the early versions of the network approach to mental arithmetic came from Stazyck, Ashcraft and Haman (1982). This earlier account of the network approach to mental multiplication looked at how multiplication facts are stored in an organised interrelated network structure. They had also argued that multiplication and addition have highly similar cognitive processes, in that there are no fundamental differences between fact-retrieval for addition and fact-retrieval for multiplication. Their results did not suggest adults usually reconstruct or calculate the simple multiplication facts. Instead of counting, adults use a nearly error-free fact retrieval process. However, infrequent extreme scores suggested that fluent retrieval was not absolutely error or difficulty free. Their results provided supporting evidence for the network approach.

The research discussed above provides some of the strongest evidence for the network based cognitive models of mental arithmetic. The following section will examine more closely some of the wider issues raised by adopting a network approach.

3. 3. 3 Overview of the Current Models of Simple Arithmetic Processing

To provide a brief summary, according to Ashcraft's network-retrieval model (Ashcraft, 1982; Stazyk, Ashcraft and Hamann, 1982), the simplest network model claims that the basic facts of addition and multiplication are stored in inter-related memory networks, with problems in the networks varying as a function of strength or accessibility. Meanwhile, Campbell's network-interference model (Campbell, 1987; Campbell and Clark, 1992; Campbell and Graham, 1985; Campbell, and Oliphant, 1992) interprets retrieval from the organised network as a process heavily affected by interference. This seems to be the only current model to account for various interference and priming effects that are now well documented in the literature. In Siegler's distribution-of-associations model (Siegler and Jenkins, 1989; Siegler and Shrager, 1984), the basic representation of a problem in memory is accompanied not only by that problem's correct answer but also by the incorrect answers that the individual has generated or computed across experience.

These are not isolated competing theories of number-fact processing. They are concerned with numerical processing and the formation of associations and/or networks of associations, and how these are accessed. They are rather more like variations on the same problems, i. e. how associations and associative links are formed in human memory because the three are quite closely linked but where each is a self-contained comprehensive theory of number fact processing in human memory. While each remains a self-contained comprehensive theory, this does not imply that they are in competition with each other. Nor are they static theories, since they are constantly being updated and developed to account for new findings. The cognitive theories mentioned above resource our understanding of how children learn mental arithmetic. Yet they are more provocative for certain areas e. g. how such research can be used to design computer simulations that will further our understanding of numerical processing by "learning" arithmetic in a way similar to that used by children learning arithmetic.

While it must be stressed that these models of simple arithmetic processing are not isolated competing theories (because the focus remains on single-digit numbers, instead of looking at sums which encourage the use of strategy and reordering digits), the domain of interest for all three theories seems relatively narrow. This is primarily because the three models of simple arithmetic processing outlined above are concerned with single number bonds. While this is appropriate for building models of number fact representations, it neglects the issue that real-life mental arithmetic is about more than just single-digit number bonds. It is about dealing with multi-digit problems i. e. adding up larger numbers or sequences of digits. Authentic addition problems usually involve two or more numbers with several digits.

3. 4 Implications for Education Practice

Since the domain of application of these theories (e. g. for teaching practice) is narrow, what these theories do not do enough of is put the results of their findings into practice for improving number fact processing. Campbell does consider this problem with the suggestion that perhaps number facts should be taught in a different order to the ones currently in practice, i. e. to minimise interference effects. Ashcraft looks in depth at how mathematics anxiety can hinder the process of learning number facts.

One of the proposals to arise from network models is the tentative suggestion that perhaps associative confusions could be minimised by not teaching multiplication problems in the context of times-tables. Maybe tradition has led educators into incorrectly believing that a well-ordered systematic introduction to number facts gives learners a helpful conceptual framework (Graham, 1987). On the other hand, there is no evidence as yet of whether any alternative orders would be better.

Graham (1987) suggests that computerised drills that deal with individualised needs are possible with computers in the classroom. Review programs could be designed so that all problems are periodically maintained while error-prone problems and responses are concentrated on. Graham also believes that computer drills would be much more practical than paper and pencil tests since more trials could be run and error histories automatically recorded. Furthermore, instant feedback at a computer screen would help identify the correct associations and minimise the impact of false retrievals. New technology allows more interactive, more adventurous forms of practice especially as computers become increasingly ubiquitous in classrooms. This allows for a potentially richer context for developing fluency at arithmetic skills.

Most research so far has focused on single-digit number facts and their retrieval from a network of stored number facts in human memory. However, this is changing and the focus of some research is shifting to how procedural rather than declarative knowledge is manipulated to solve arithmetic problems. While research focusing exclusively on single-digit number facts is useful for theorising about network models, real-life arithmetic is about larger numbers e. g. adding sequences of numbers such as 4+9+5+1+9. This type of serial addition quickly gets into larger double figures (such as 84 + 7), another form of neglected problem. Problems such as these can be solved more effectively by strategic use of existing number knowledge but younger children may be slow at doing this.

3. 5 The Role of Working Memory in Mental Arithmetic

3. 5. 1 Working Memory

Adding larger numbers involves calling up the network and this can be constrained by working memory. This raises the issue of what more complicated problems would demand in the way of working memory resources. Working memory is significant because mental arithmetic involves both simple networks of data and the complex constraints of working memory. Ashcraft (1995) suggests that adding numbers seems to rely mainly on the central executive component and the articulatory loop seems especially involved in counting, but that evidence for the role of the visuo-spatial sketchpad seems speculative and needs further research.

The long-term memory aspects of mental arithmetic lead researchers in this area to look at a more process-oriented explanation of arithmetic performance. This is done by examining performance by looking at the involvement of working memory. Ashcraft (1995) stresses the role of working memory in simple arithmetic as proposed by Baddeley and Hitch (1974). Their highly influential system of working memory comprised a number of components. Working memory refers to the temporary storage and manipulation of information. In this system, a central executive component is responsible for reasoning and language comprehension, decision making, and for holding retrieved and intermediate information for further processing. It is served by subsidiary specialised "slave" systems. Initially, two systems were proposed, the visuo-spatial sketch pad and the articulatory loop. The visuo-spatial sketchpad was considered to be responsible for the temporary storage and manipulation of visuo-spatial material and the articulatory loop performed a similar function for verbal material. Together these systems maintained and recycled phonological/ articulatory information in the auditory rehearsal system, maintaining visual and spatial information. This system has remained fairly intact (Baddeley, 1986, 1992) although the role of the components of working memory has become clearer and the articulatory loop is now referred to as the phonological loop.

The working memory model was able to account for a wide variety of data with relatively few assumptions (Logie, 1995). Although it was set in the context of dissociation with long-term memory, it argued that short-term memory could be usefully fractionated. It can also account for aspects of everyday cognition. The phonological loop seems to be involved in counting and in mental arithmetic (Logie and Baddeley, 1987; Logie, Gilhooly, and Wynn, 1994), in vocabulary acquisition by young children and by adults learning a second language (Gathercole and Baddeley, 1990). Working memory is also thought to be involved in problem-solving and in comprehension. Until recently, the role of the visuo-spatial working memory system has been less clear, although it was assumed to be separate from both the central executive and from the phonological loop.

Hitch (1978) did the earliest research on the involvement of working memory in arithmetic performance. His results demonstrated an important role for working memory in the computation of arithmetic answers for complex problems. He identified working memory as the storage system for initially presented operands and intermediate values computed during solution. But he did not account for possible working memory involvement in the simple fact retrieval process. Retrieval of basic facts, especially difficult ones, also uses working memory resources. This area has only recently been investigated.

Ashcraft, Donly, Halas and Vakali (1992) looked at the involvement of working memory resources in successful performance and the role of automaticity. They used the term "attention" to refer to mental resources or effort used in a cognitive task (a task that probes knowledge within the domain of simple arithmetic) with the assumption that attentional resources for cognition are limited. Handling numbers can be managed in two ways: the first is *conscious processing* which is the effortful, deliberate performance that relies on attentional resources. Whereas the second, *automatic* or *autonomous* processing is the very rapid, skilled performance accomplished with few, if any, demands on working memory resources.

A problem that has long intrigued researchers in this field is the problem size effect. Explanations of this effect in adults support memory representation of simple arithmetic fact knowledge in which the strength of the association or network connection between operands and answers affects the time taken for successful retrieval. For simple single-digit number facts in addition and multiplication adults' processing is mostly automatic. Even third graders show some automatic facilitation of retrieval on the smaller, less difficult multiplication problems. As mentioned earlier, this has been used to provide support for fact-retrieval theories of arithmetic processing.

Ashcraft et al (1992) looked at explicit manipulation of the working memory load as in the dual-task paradigm (where subjects were required to count the number of visually presented targets while performing various secondary tasks) used by Baddeley and Hitch (1974). They found that concurrent tasks influenced arithmetic performance. Important elements of simple fact retrieval in addition rely on working memory resources, especially for difficult problems. For two column problems, no-carry and carry represent the factor of problem difficulty. Even basic fact retrieval relies on working memory resources, at a subtle level for difficult ones and at a higher, more central level for complex addition with carrying. For adults, this is retrieval from organised memory (basic addition facts). Mental arithmetic should show clear evidence of automatic processing and there should be progression in childhood from conscious to automatic processing of basic facts.

Elementary school education stresses the need for "memorisation of the basic facts" to aid solution of more difficult problems, so there is a practical need for some degree of automaticity in performance on basic facts. The memory retrieval process in adults depends on an organised, interconnected memory representation of the facts. In semantic representation, the retrieval process involves spreading activation which activates both target and related information within the memory structure. Related information can therefore alter processing of an arithmetic stimulus. For example, any prime stimulus other than the correct answer has a disruptive effect and this is evidence that the prime is activating information in memory and has an influence on the retrieval process (Lemaire, Barret, Fayol and Abdi, 1994; Campbell, 1987).

There is evidence that small facts are "advantaged" (become automatised) early on at school level (Ashcraft and Christy, 1995). So it may be that automaticity will only develop quickly only for small addend problems. Multiplication knowledge, in particular, and possibly all arithmetic knowledge is prone to interference. Performance at less than automatic level implicates the working memory system. Strategic processing, especially on larger and more difficult facts is more common in adults than previously believed and competes with memory retrieval when the information is of low strength in memory. They conclude that there is lack of complete automaticity or autonomy in arithmetic performance.

3. 5. 2 Issues in Working Memory

The three structural components of working memory are relevant to mental arithmetic. Ashcraft (1995) found that arithmetic research in this area suggests that adding numbers seems to rely mainly on the central executive component and that the articulatory loop seems especially involved in counting. Evidence for the role of the visuo-spatial sketchpad seems speculative and needs further research. However, as yet, empirical evidence within these areas is scarce. The effects of the components of working memory, in particular the role of visuo-spatial working memory, in mental arithmetic require further research. Ashcraft (1995) identified how the three components of working memory are particularly relevant to mental arithmetic:

First, that adding numbers seems to rely especially on the central executive component. The central executive is presumably the component that retrieves and then manipulates number fact information. Retrieval of less accessible facts, i. e. those of lower strength in long-term memory, will presumably consume more of the central executive's resources. As a result, when such retrieval is accompanied by a secondary task that drains the central executive's resources, it should result in interference (slowing of the normal retrieval operation). This would occur whether performance was driven by relatively slow effortful retrieval or by more conscious application of a reconstructive strategy (non-retrieval processing). From our understanding of the working memory system in general, this would provide more compelling evidence than depending on verbal reports to demonstrate reliance on strategies (Geary and Wiley, 1991).

Secondly, the articulatory loop seems especially involved in counting, both the one-by-one incrementing process and genuine counting. Tasks or problems that rely on such counting mechanisms should therefore consume resources in the articulatory system. So keeping track of counted and to-be-counted items, and keeping track of one's progress in a counting sequence may also put demands on the central executive component. Any other reconstructive strategy that involves the counting mechanism should also consume resources from the articulatory loop.

Thirdly, evidence for a visuo-spatial sketchpad role in arithmetic performance is quite speculative.

Two further studies have looked at this issue. Logie et al (1994) and, more recently, Adams and Hitch (1997) have looked at the role of working memory in arithmetic problem solving. They argued that while there is extensive research on the acquisition of concepts and procedures for mental arithmetic, relatively little has been done on the role of working memory. The specific role of working memory in supporting mental calculation is still not well understood.

Logie et al's (1994) results led them to conclude that the central executive component of working memory is involved in performing the calculations required for mental addition and in producing approximately correct answers. Although visuo-spatial resources in working memory may also be involved in making approximations it is unlikely that mental arithmetic relies heavily on visualising. Their results suggest that the subvocal rehearsal component of working memory helps maintain accuracy in mental arithmetic, i.e. by keeping the sub-components active and this is supported by previous research on counting (Logie and Baddeley, 1987). So mental arithmetic may involve general purpose resources, verbal short-term storage and/or visual imagery as well as LTM. The phonological loop seems to be involved in counting and mental arithmetic.

Adams and Hitch (1997) looked at whether children's mental arithmetic is constrained by working memory. They found that the spans for mental addition were higher when the numbers to be added were visible throughout calculation than when they were not (working memory constraint). Their results support the assumption that working memory is a central general purpose resource supporting children's mental arithmetic. When working memory load was reduced by visual presentation, performance improved. They conclude that a natural task like mental addition which combines processing and storage as intrinsic components reflects working memory in a similar way to an artificial task. These studies isolate working memory as a constraint on arithmetical development.

Therefore, working memory presents us with the bigger picture in the area of mental arithmetic research. Whereas the network/ fact-retrieval models present us with the fairly simple view on arithmetic processing, working memory research highlights the complex underlying issues that can constrain arithmetic processing particularly in children. The next section is a brief look at the area of mathematical disabilities which, particularly in children, can also be due to short-comings in memory.

3. 7 Mathematical Disabilities

In light of what has just been discussed in the previous section, in cognitive development, constraints on mental arithmetic performance due to working memory are expected to be more severe in children. A significant number of arithmetical handicaps are due to deficiencies in short-term memory. There also seems to be evidence for working memory deficits in children with learning difficulties (Geary, 1990; Geary, Hamson and Hoard, 2000) since working memory deficits are exaggerated in children. Currently research is being carried out in the area of mathematical disabilities. Research has been carried out in which cognitive models are being applied to children who have difficulties in early educational settings. Geary's (1990, 1993) results suggest that a deficiency in working memory capacity is a component of children's developmental difficulties in mathematics.

Geary, Brown and Samaranayake (1991) tracked first grade (6-7 year old) children who had been identified as "mathematics disabled" across the nine-month school year along with a control group of normal first grade children. At the end of the year, a greater number of normal children showed evidence of a basic change in their arithmetic strategies for addition (from counting to retrieval-based performance). For problems that were still being solved by counting, the children showed faster performance. So normal children showed developmental progression towards more sophisticated strategies, as well as becoming more efficient at executing counting strategies when these were used. However, children who were initially identified as mathematics deficient failed to show greater reliance on retrieval at the end of the year. In addition to this, their counting-based performance remained slow and error-prone. They continued to rely on a less mature counting strategy, and their use of this strategy did not show improvement across the year. These findings suggest that working memory capacity constraints are exaggerated in children and this is especially so with subsets of children whose mathematical abilities are a problem.

In the above sections I provided a review of three current theories of arithmetic processing and the evidence that has been used to support them. I then went on to examine the role of working memory regarding mental arithmetic and also provided a brief view on the relevant themes from mathematical disabilities. In the following section I will consider schema-theory as an alternative approach to fact-retrieval/ network-based theories.

3. 8 Schema-theory

The network/ fact-retrieval based approach appears to offer a comprehensive method of studying and theorising about numerical development and the extensive evidence used to support its theories is seductive. However, the same facts and evidence used to provide such compelling evidence for network based approached can be deployed in another way to support alternative theories. We can take the same facts and the same evidence but we need not have the same theories to explain them. Schema-theory can be used to explain the results of several key studies used to support fact-retrieval models. Baroody (1994) compares fact-retrieval models with a schema-based view and argues that relational/ procedural knowledge is an important component in learning and representing basic number facts (e.g. 8x3 = 24) and offers recommendations for more clearly determining how the basic number facts are learned and represented.

3. 8. 1 An outline of schema-theory

Schema-theory refers to the building up of a representation from familiar subparts. The term refers to what is essential to category membership and connotes a plan or expectation that can be used to receive or organise incoming stimulation.

According to Fiske & Taylor (1984):

> "A schema is a cognitive structure that contains knowledge about the attributes of a concept and the relationships among those attributes. All types of schemata guide perception, memory, and schema-consistent information. Disconfirming or incongruent information requires more effort to process than congruent information; if that effort is made, it may be well-remembered."

Bartlett (1932) introduced the term "schema" into psychological literature, in the sense of an active organisation of past reactions, or of past experiences supposedly operating in any well-adapted response. He shifted emphasis onto cognitive aspects, as revealed to him by his memory experiments. Jean Piaget also used the term schema within a cognitive context. The Piagetian schema is the internal representation of some generalised class of situations that enable an organism to act in a co-ordinated fashion over a whole range of analogous situations.

Schema-based theory appears to be a viable alternative explanation for some of the findings discussed here, but it is one that is over-looked by proponents of fact-retrieval. The fundamental argument here is that there are schemas for procedures/ rules/ heuristics, as well as networks of number facts. There is a relative lack of RT research in this particular area that tests some of the underlying assumptions of this view especially since a considerable amount of existing research has been done on multiplication facts. The lack of research within the framework of schema-theory in an area such as mental addition, where it may have fascinating implications, needs to be addressed. Empirical research is needed to examine in detail some of the questions it raises.

There is agreement that children initially use a variety of strategies to reconstruct sums, differences and products of single-digit number combinations. They learn to respond to addition, subtraction and multiplication combinations efficiently (quickly and accurately) and, as experts, have some kind of network representation. However, theorists argue about the acquisition process and the nature of such network representation.

3. 8. 2 Evidence for schema-theory

Baroody (1994) looks at how evidence used to support fact retrieval models can be explained by the schema-based view. The schema-based view was originally developed to account for observations about children's early mental addition performance not predicted by fact-retrieval models at the time.

According to Baroody (1994), in the fact-retrieval theories described earlier in Section 3. 2 (Siegler's distribution-of-associations model, Campbell's network-interference model and Ashcraft's network-retrieval model errors) errors are due to retrieval strategy and reflect nature of the associative network. Practice is the key developmental mechanism and the retrieval network does not embody relational knowledge. However, Baroody argues that, contrary to Siegler's (1988) fact-retrieval model but consistent with the schema-based view, fact-related errors may increase on unpractised combinations i. e. as a result of searching for related answers.

One of the short-comings of the fact-retrieval models of mental arithmetic is their somewhat cavalier approach to alternative explanations for some of their findings. Baroody (1994) examines existing data used to support fact-retrieval theories and argues that their findings can also be used to support the schema-based view. For example, error patterns, the problem-size effect, and neurological data are used to support fact-retrieval theories but there is no reason why they should not be consistent with schema-theory.

For example, according to the distribution of associations model, every time an answer is stated or computed (correct or incorrect) a trace is laid down in LTM. As the number of traces build up in LTM, the bond or association between an answer and a problem is strengthened. So, the practice errors made in childhood shape the type and frequency of errors made as an adult.

In fact-retrieval models, the basic number combinations are stored as an independent system (the "arithmecon") and this is separate from conceptual or procedural knowledge (Campbell and Graham, 1985). Unlike other semantic systems, the associative network for the basic facts does not involve representations of relationships such as the commutativity principle.

Experts may use rules to generate the answers of combinations involving 0 or 1 but these rules are not considered an integral part of the network retrieval system. In the distribution of associations model, the representation consists of each single-digit combination, which has a unique associative strength with a correct answer. So even 8x2 and 2x8 (commuted combinations) are represented separately with their own degree of association with 16.

A schema-based view suggests that children and adults may both strategically employ related conceptual and factual knowledge to generate answers to "unrecallable" or unknown combinations. Proponents of fact-retrieval find that non-retrieval processes (back-up strategies) are basically slower than retrieval strategies. But according to the schema-based view, although some non-retrieval strategies such as (mental or finger) counting are slower, others such as reasoning or estimating may be almost as fast as retrieval. According to the schema-view, children or adults required to respond swiftly to unmastered combinations may draw on their existing knowledge to rapidly estimate an answer. They may even reason out the exact answer quite rapidly (i. e. in under a couple of seconds).

In the schema-based view, practice frequency alone cannot account for changes in mental arithmetic performance or the underlying mental representation of basic number facts. Insight or pattern recognition is important for the evolution of errors and mastering number facts. Relational knowledge may become embodied in the mental representation underlying the retrieval strategy. Children initially have few resources so their earliest estimation strategies for generating answers may be ineffective. People evolve procedures after a while as a result of practice and learning.

A good example of this is the way error patterns change because, with experience, children can bring more knowledge to bear and can devise more sophisticated estimation strategies. For example, they may restrict estimates of the product for 2x8 to 11 to 18. Even more sophisticated children may recognise all 2x are even and will only choose an even teen and if they know 2x7 = 14 the estimate for 2x8 may be further restricted to 16 or 18. In some cases they may master specific combinations with little or no practise e.g. 8+8 = 16 knowledge can be used to answer 8x2 = ? or knowing 8x2 = 16 and the commutative property of multiplication to answer 2x8 efficiently.

Patterns or relationships may become incorporated as an integral component in the processing of basic combinations by experts and shape the organisation of factual arithmetic knowledge. The basic combinations of number facts, procedures and representations may be represented in LTM as a structural framework consisting of a network of propositions and statements of relationships among facts. The associative network of number combinations and the semantic network of general arithmetic knowledge are not independent systems but are functionally dependent. Knowledge of commutativity permits related combinations like 8x2 and 2x8 to share the same data (answer) node in LTM.

Although over time schemas can become automatic, initially the schema underlying exact estimation strategies would be almost entirely under conscious control. With experience, elements of such processes may become automatic. As exact and inexact answer schemata become more automatic, they might serve as component parts for more complex schemata. Repeated reasoning out of an answer may lead to an association between the problem and the answer, *but* the schema-based view suggests that this is not necessary for efficient mental arithmetic. The advantage of automatized procedures over associative links is that the former is more cognitively economical (Baroody, 1983). For example, the schema for commutativity means that both 8x2 and 2x8 can be represented by a single-associative link rather than two separate links.

The schema-based view is more parsimonious compared to fact-retrieval models. Fact-retrieval models propose competition between the fact-retrieval network and procedural knowledge, whereas in the schema-based view the efficient rule-based answers to problems involving zero and 1 are not an exception or an oddity. In a fact-retrieval theory (e. g. Ashcraft's theory) these are oddities. The rules for these combinations are an integral aspect of a system of schemata that underlie the efficient production of all combinations including multi-digit combinations e. g. 79+1, 85-85, 104x0, 0/217.

There is a large amount of empirical evidence to support fact-retrieval models (especially distribution of associations). According to Baroody, however, the empirical basis for such models is not unequivocable. Some of the evidence supporting fact-retrieval models does not hold up under scrutiny (a lot of it may confound retrieved responses with fast non-retrieved responses). Current fact-retrieval models, if not inaccurate are at least incomplete because they do not account for estimation strategies and compiled procedures (such as derived-fact strategies). There is now growing evidence to support key aspects of the schema-based view.

Children become fluent at "surreptitiously and quickly" using back-up strategies. For example, on both retrieved-required and open-ended tasks, a child might reason that the product of 7x6 must be 6x6 plus 7 more but may be prone to mis-adding 36 and 7. Until recently, mental arithmetic error data of adults have been collected without regard to RT, so it had been difficult to get to the underlying processes. Evidence suggests some adults reason out some answers on timed tests and so it is entirely possible that factor-related and close-miss errors on mental arithmetic tasks are partly the result of reasoning strategies. An error of this sort suggests using a non-retrieval strategy.

The schema-based view also suggests that children and adults might use estimation strategies, which would also tend to produce factor-related and close-miss errors. Fact-retrieval theorists have often used evidence from research on errors to support their models (Campbell and Graham, 1985). However, Baroody argues that here is no direct evidence on a link between children's computational errors and subsequent mental arithmetic errors in adults. Baroody (1988) directly compared children's mental arithmetic errors to their frequency of computational errors and found no association between the two. In fact, existing evidence suggests that computational errors are not associated with error patterns or changes in these patterns. Error research could actually be used to support schema-theory.

So although incidental learning of erroneous answers may occur, it would be wrong to believe that *every* stated incorrect answer has an impact on LTM or that miscomputing answers is *necessary* for mental arithmetic errors to evolve as seems to be suggested by some researchers (e. g. Siegler). Not every stated incorrect answer becomes associated with a particular sum.

Most fact-retrieval models, including the distribution-of-associations model do not explicitly include a mechanism for pattern recognition or insight (Nesher, 1986). So they do not predict transfer (the mastery of unpractised combinations). However, recent research suggests that relational knowledge and transfer may play an important role in the learning of multiplication combinations involving larger numbers.

Baroody (1993) found 3rd graders apparently took advantage of their existing knowledge of addition doubles (tie-sums) such as 50+50 to master (respond correctly within 3 seconds) unpractised combinations like 50x2. Jerman (1970) found that with age children continued to use reasoning strategies with ever increasing efficiency. These results are consistent with the schema-based view that relationships are incorporated as schema and the use of the schema becomes increasingly automatic.

Various researchers have also noted that some adults use reasoning strategies (e.g. transforming 9+8 into the equivalent but easier combination of 10+7) to determine the sums of larger combinations. Though somewhat slower than that for retrieved answers the RTs for such reasoned answers may not be so much slower that they are considered as outliers and eliminated from a chronometric analysis. What needs to be done is to determine whether children are also capable of doing this. For example, primary school children may also be able to mobilise strategically their existing mental arithmetic knowledge (e.g. knowledge of ties) to solve problems such as 6+7, 7+8, and 8+9 as well as more complex multi-digit problems. This attempt at strategic behaviour on their part would be interesting to observe because it hints at strategy use that was probably not taught to them by anyone.

Evidence of how perceived responses become obligatory and affect responses is evident in the much investigated phenomena of the Stroop effect or obligatory activation LeFevre et al (1988). This has also been cited in support of fact-retrieval models. In theory, a fact-retrieval network is an autonomous or at least a semiautonomous system i.e. once set in motion by the display of a combination, the retrieval process should run its course (look up an answer) with no conscious effort or only some conscious control.

LeFevre et al (1988) measured a Stroop effect by presenting subjects with a combination such as 5+1 (the prime), and then replacing this display with a target number, the number is either one of the numbers presented previously (e. g. 5 or 1), their sum (6) or some other number (e.g. 3). The subject was then required to indicate whether or not the number was presented previously in the combination display. LeFevre et al (1988) found evidence of an interfering Stroop effect. Subjects were likely to reject sums more slowly than other numbers. They concluded that although the task did not require the recall of combination sums, subjects did so automatically, and this slowed their response when evaluating a sum problem. This interference effect was relatively short-lived and consistent with the spreading-activation-decay processes proposed fact-retrieval models.

These results are not, however, inconsistent with the schema-based view that a retrieval network consists of both facts and relationships, i. e. the schema-based view would predict a Stroop effect if subjects were presented a problem like 46-46 and then presented one of the digits (4 or 6) the difference (0), or some other number (e.g. 5). Half the problems in this experiment involved the addition of 1 which may involve (automatic) relational knowledge: the number-after rule i. e. n+1 = one greater than n. (Baroody, 1983 and 1985,).

Brain injury data is frequently used to support fact-retrieval theories. However, Baroody (1994) argues that the brain-injury data cited by Sokol, McCloskey, Cohen and Aliminosa (1991) are not entirely consistent with existing fact-retrieval models and provides some support for the schema-based view. Sokol et al (1991) found evidence for a zero-rule for multiplication. McCloskey, Harley and Sokol (1991) concluded that the pattern of impairments observed by Sokol et al (1990) was inconsistent with table-search models of fact-retrieval. For example, one subject had difficulty with 8x8 but no difficulty with 8x9 or 9x8, supporting Siegler's (1988) view that each fact is accessed independently according to its unique distribution of associations. Table-search models would predict that the search down the 8 column and across the 8 row would have been disrupted.

However, an analysis of the data presented by Sokol et al (1991) reveals an interesting pattern that contradicts this view. According to fact-retrieval models, in general, and the distribution-of-associations model, in particular, impairment in the recall of 2x6 should not necessarily imply the impairment of 2x6 because commuted combinations are stored independently in LTM. The schema-based view suggests that impairment with a combination should necessarily imply impairment of the commuted combination. According to Sokol et al (1991), the number of errors on problems are associated with that on commuted problems. The number of errors for a problem is a significant predictor of the number of errors for its commuted counterpart. Sokol et al (1991) concluded that though this pattern of results described above is consistent with the view that commuted problems access the same stored fact-representation, the close similarity of error-rates for commuted problems does imply a unified representation.

The evidence supporting fact-retrieval models and the distribution of associations model in particular is not as unequivocal as claimed. Research is needed to test the key assumptions of such models directly. But, as yet, there is no *direct* evidence that prior associations and computational errors shape the mental arithmetic errors of children or adults as suggested by proponents of fact-retrieval/network models. Some evidence actually appears inconsistent with such assumptions. Much evidence is ambiguous in that there are plausible alternative explanations e.g. confounding nonretrieved and retrieved responses may account for the problems size effect. Efficient proceduralisation may account for research used to resource fact-retrieval. Error priming or confusion effects may be due (at least in part) to nonautomatic responses. We need to be careful about drawing conclusions from existing data about how the basic combinations are internalised and then organised in LTM. Although there has been increased awareness about the effects of non-retrieval processes on mental arithmetic (Campbell, 1990; Koshmider and Ashcraft, 1991) possible confounding factors are still not given adequate attention.

To study the learning and representation of basic number combinations, researchers need to make every effort to eliminate the effects of non-retrieval processes and to distinguish between rule-based and fact-retrieval processes (Baroody, 1985). It is not merely enough to distinguish between fact-retrieval processes and counting based processes. There is more potentially going on than just this dichotomy. Whether or not the size effect is due to other non-retrieval processes such as automatic or relatively fast non-automatic reasoning is the real issue and what must be discounted, even though this is a much more difficult task. Discrediting purely counting-based explanations for adult mental arithmetic is like "knocking down a strawman".

The available evidence does not indicate whether fact-retrieval models are essentially correct but incomplete or fundamentally incorrect. Existing research seems consistent with the schema-based view and suggests that its basic principles are at least plausible. However, more research needs to be done before we can establish the validity of this view.

Further research is needed to study the role and development of estimation and reasoning strategies in mental arithmetic. Research is also needed to examine whether the representation of the basic number combinations involves both facts and relationships or facts only. Schema-theory would be a satisfactory explanation for why certain combinations of numbers may be faster than others. If tie problems have a shorter latency than other problems, schema theory would adequately explain why for example, for older children at least 9+8 would take less time than 9+7 if they were using relational knowledge to deduce that the answer would be one less than the answer for 9+9.

3. 9 Summary

Evidently, there are a variety of research perspectives relating to the development of the basic numerical skills in childhood. My own work will pursue the traditions established by the cognitive psychologists who have considered the concealed mental processes that mediate the solution of arithmetic problems that are unaided by external supports. In particular, I shall dwell on the case of mental addition.

At various points in the present chapter I have stressed that research has been focused on the particular case of two-digit sums - problems of the kind a+b in the case of addition. If we are to address the concerns of public commentators as they were identified in Chapter 1, it is important that we go beyond these theoretically interesting but narrow cases, to address forms of mental calculation that are more demanding - yet quite typical of everyday tasks. Thus, I shall converge here on an examination of cases of mental addition that have been neglected in the existing research literature. In particular, I shall consider situations where single-digits are added to two digit numbers, and where single digits are added in series.

However, the general aims of my research are presented in fuller detail in the following chapter. There, I shall also present the methodological basis of the empirical work to be reported.

Chapter 4

4. General Aims and an Appropriate Methodology

4. 1 Review

In the present chapter I will begin by setting out the aims and objectives of the current research project, including the methodology I have used. From the psychological aspects of doing mental arithmetic, I will then go on to evaluate the methodology used by others in this area, mentioning both its strengths and limitations.

In Chapter 1, I began by looking at the topical status of mental arithmetic and the social and political background of research in this area. I examined our national concern with numeracy skills and the significance of such skills in a technologically advanced society. I briefly looked at popular media opinion on numeracy including opinions on educational practice and the setting up of the Numeracy Task Force.

Chapter 2 was a bird's eye view of the development of research in mental arithmetic. Section 2 began by providing a historical perspective in this area. I did this by looking at the early historical significance of arithmetical ability and the evolution of arithmetical education and how it gradually led to an interest in arithmetical research. In Section 2. 1, I examined the research traditions within which arithmetic has been studied, from its roots in Thorndike's behaviourist perspective of arithmetic to the development of mental arithmetic as a research area. This involved looking at how arithmetic was viewed within associative theories, such as stimulus-response theories, where basic arithmetic facts were seen as stimulus-response associations. These associative theories became progressively cognitive in nature, going from simplistic stimulus-response theories to looking at associations within networks of number bonds. In Section 2. 2, I went on to look at the emergence of cognitive arithmetic and the chronometric method of measurement. This section looked at the evolution of mental arithmetic into a mature research area in its own right and how the growth in cognitive arithmetic research led to the emergence of reaction time as a widespread measure of cognitive abilities/ cognitive processing. In Section 2. 3, I moved on to consider another dimension of children's computation, i. e. the innate nature of certain numerical abilities and the existence of modules for mental arithmetic. This section also considered how two types of numeracy skills (biologically primary and secondary skills) might have implications for teaching practice. Section 2. 4 examined the tradition of cultural research in this area. I began by differentiating between cultural psychological and cross-cultural research and then looked at the effect of language on numerical processing. I concluded this section by looking at other cultural factors at work that may influence numerical processing.

Chapter 2 encapsulated a number of psychological concerns relevant to mental arithmetic. In Chapter 3 I chose to focus on one aspect of these concerns: cognitive research. Chapter 3 considered current models of simple arithmetic processing. It began by looking at early cognitive research in this area, and went on to look at three fact-retrieval based models of arithmetic processing, as well as providing supporting evidence for each one. Section 3. 2 looked at the role of working memory in mental arithmetic. In this chapter, I also provided a brief review of research (such as neurological research) which has been used as a lever to evaluate fact-retrieval theories. I made brief mention of the issue of mathematical disabilities. I concluded this chapter by examining schema-theory (Section 3. 3) as an alternative approach to some of the network models and looked at how evidence used to support network models can also be used to support schema-theory. I identified the areas in which further research would be beneficial for a better understanding of schema-theory.

4. 2 Generic Aims

My central aim has been to be more confident about the 'cognitive geography' of arithmetic development. In part, this means tracing this development to discover precisely where it is occurring across an important 3-4 year time span over which arithmetic skills are being vigorously cultivated. The aim of this project is to go beyond the analytic building block methods of the network models discussed earlier. Research on number fact knowledge has concentrated mainly on single-digit number facts because this feeds existing theories. Towards the end of Chapter 3, I suggested arithmetic research seemed to be concerned primarily with analytic number facts as building blocks of arithmetic knowledge. In the real world of computation, however, there may be issues that arise that these theories are not ready for. However, while there is a practical issue here, such findings may also be theoretically interesting.

As illustrated in Section 3. 8, schema-theory has been valuable in identifying gaps in existing research. Certain areas of numerical processing have been neglected by existing mental arithmetic research. For example, further research is needed to study the role and development of estimation and reasoning strategies in mental arithmetic. Research is also needed to examine whether the representation of the basic number combinations involves both facts and relationships, or facts only. Schema-theory would be a satisfactory explanation for why certain combinations of numbers may be faster than others, because sometimes children and adults may act more strategically. If tie problems have a shorter latency than other problems, schema theory would predict why, for example, 9+8 would take less time than 9+7. This could be explained if individuals were using relational knowledge to deduce that the answer would be one less than the answer for 9+9. The objective here is to look at how evolving knowledge for simple problems gets recruited into solving more difficult problems, e. g. using 9+9 to solve 9+8.

As discussed in an earlier section, there are two types of arithmetic knowledge. One is declarative knowledge; this refers to stored knowledge of addition facts e. g. $2+3 = 5$ and depends on retrieval from memory). The other is procedural knowledge; this refers to stored knowledge about arithmetic e. g. $nx0 = 0$, $n+0 = n$, or $n+1 = 1 > n$ (e. g. 6+1 will be *one* number more than the original number) and depends on rules, heuristics and computation.

A significant amount of cognitive arithmetic research so far has focused on the former (declarative knowledge), i. e. single-digit number facts and their retrieval from a network of stored number facts in human memory. However, this is changing. The focus of some research is shifting to how procedural as well as declarative knowledge is manipulated to solve arithmetic problems (Baroody, 1994 and 1999). While research focusing exclusively on single-digit number facts is useful for theorising network models, real-life arithmetic is often about more complex problems. This can include, for example, adding sequences of numbers such as 4+9+5+1+9: this type of addition (that will referred to in this thesis as serial addition) rapidly generates double figures (e. g. 84 + 7), i. e. the subtotals can become quite large. We may anticipate that problems such as these can be solved more effectively by strategic use of existing number knowledge, but younger children may be slow at doing this.

Furthermore, as will be identified later in this chapter, the methods used for carrying out mental arithmetic research on children are less than ideal and have their limitations. The methodology used for such research should ideally be relatively simple and easy to carry out in naturalistic conditions (for schoolchildren). These should allow me to carry out chronometric research, i. e. allow me to measure response times accurately and efficiently. Thus, a significant further aim of this project has been to refine a method. I accomplished this by designing an authentic classroom arithmetic task that would allow chronometric research to be carried out under naturalistic conditions. Therefore, my aim was to devise, test and implement a set of methods that were relatively fast to carry out with a large number of children, while looking at the emergence of addition strategies when children are doing mental addition. To this end I needed methods that were sensitive to this.

4. 2. 1 Generic Aims Summarised

My wider psychological aims are outlined below.

4. 2. 1. 1 Developmental Perspective

My research is about development. Because computation development normally accelerates at the primary/ junior school level, this is the period of development I shall be concerned with. In particular, I am interested in studying the development of fluency in mental arithmetic (particularly mental addition) through the emergence of new knowledge or strategic capability. For example, at what age do certain addition problems become number facts and when do these facts get recruited to help with the solution of the next more complex problems. Microgenetic research would be an optimal way of looking at change in this area. However, this is not going to be employed in this thesis, although methods will allow me to identify and follow up areas where microgenetic research would be beneficial. In short, my aim was to look at *developing* fluency in this area of calculation.

4. 2. 1. 2 Processes Underlying Simple Computation

My research is about basic cognitive processes. This is because my aim is to reveal in detail just what underpins the production of simple addition. For example, to what extent is counting being used as a solution process. From competency at computing simple single-digit addition sums to rather more complex (three-digit) serial addition sums, I want to examine how this changing fluency gets recruited into solving other more demanding problems. In addition, to study how these emerging core competencies are recruited into the service of solving more complex problems like exploiting ties (6+6) to reorder three digit problems or being faster at 9+8 than 9+7 because 9+8 is one less than the tie-sum 9+9. .

4. 2. 1. 3 Acquisition of Strategic Knowledge

My research is essentially about the acquisition of strategic knowledge, i. e. it asks what elements of existing knowledge get mobilised into solving complex addition problems. My aim is to infer the nature of mental strategies that are deployed for more demanding problems; for example, the use of sum to 10 number facts (e. g. 7+3) to solve more difficult problems. In addition, to look at what new problems arise, and what developments in strategy can be observed when children are asked to solve serial arithmetic tasks (as in computing the sum of a series of single-digits). I intend to look at how children process mental addition problems of the following two types: 1) decade problems, and 2) serial (three-digit) addition problems.

4. 3 Methodologies Currently Used to Study Mental Arithmetic

The mental arithmetic research described earlier has relied heavily on the use of response times (RT) as a measure of inferring mental processes. This involves the use of RT as a measure of hidden or unseen mental operations and processes that reflect separate stages or operations which occur between the presentation of a stimulus and a response (Ashcraft, 1982). Although chronometric research gives us valuable insight into the processes underlying mental arithmetic, it also raises some methodological concerns regarding self-report studies and reliance on RT as a dependent variable.

The models of processing described in Chapter 3 rely heavily on the use of chronometric analyses of mental arithmetic. Ashcraft's network-retrieval model, Campbell's network-interference model, and Siegler's distribution-of-associations and ASCM models rely extensively on the use of response times to infer cognitive processes. How do we deal with some of the problems that arise when we study these hidden, private processes? One way of doing this has been to rely on RT as a sole measure of processing. However, this approach of relying solely on RT as a dependent variable raises a number of methodological concerns, which will be examined in detail in the following sections. There will also be a brief discussion of the use of verbal reports (which involve people describing their solution processes immediately after solving a problem) in mental arithmetic research.

4. 3. 1 Response Times versus Verbal Reports

4. 3. 1. 1 Response Times

Siegler (1987, 1989) has made extensive use of chronometric methodology in his research and is aware of some of its limitations. He looks at the problems arising from conventional chronometric analyses being used as a sole index of cognitive activity (using mental subtraction and addition as an example), as this can lead to a distorted view of processes especially when people use diverse strategies such as counting and/or retrieval. He examines the view that either people cannot often accurately report their cognitive processes (Nisbett and Wilson, 1977) and that this may be an even greater problem for children, or that children's explanations do reflect their strategies (he provides evidence for the latter). He obtained both solution times and verbal reports on strategy use from each child on each trial. He found that multiple strategies were being used and that the RT data supported this. People can accurately describe their processing when they report immediately after the processing episode and when this episode was not too short.

There are advantages to using RT especially when solution times are classified (on the basis of the self-report) according to the strategy that generated them and then separately analysed. It is just that the very success of RTs as indirect indexes of cognitive activity may have led to an over-emphasis on their uses and to excessive scepticism about verbal reports as data. Siegler (1989) concludes that this model needs to explain how, with age, the speed and accuracy with which children execute different strategies increases. It also needs to account for their progressive movement toward more frequent use of the faster strategies such as retrieval and use of addition reference (i. e. using addition facts to solve subtraction problems) for their decreasing use of the slower strategies such as counting down and guessing.

4. 3. 1. 2 Verbal Reports

Cooney and Ladd (1992), however, questioned the validity of children's verbal reports about the cognitive processes underlying their mental arithmetic and found that immediately retrospective and concurrent verbal reports increased student's solution accuracy compared to a no verbal report condition. According to Cooney and Ladd, the two types of measure (studying both overt behaviour and use of immediately retrospective verbal reports) used in Siegler's (1989) study to infer that children used a variety of strategies are not consistent with results from previous studies of children's subtraction performance that used verbal report procedures (Carpenter and Moser, 1984; Fuson, 1984).

Verbal report data might seem to provide a more accurate picture of the mental operations underlying arithmetic but their use may be questionable on both theoretical and empirical grounds. Cooney and Ladd (1992) argue that accurate information about mental processes can be obtained by taking special precautions. Processes that have become automated through practice are unavailable to STM and so unavailable for verbal reports. Cognitive operations underlying mental multiplication may be automated in young children (3rd and 4th graders) and so their traces (in memory) are unavailable for reporting. Hence automated cognitive operations are not believed to be available for reporting and simple arithmetic is a task that may become automated even in young children.

However, rather than rejecting the use of verbal reports per se, they argue for the need to refine verbal protocol methodology. Any inaccuracies stem from incompleteness rather than fabrication through the reconstruction of events from memory. They encourage investigators to include a silent control group in research designs that utilise verbal reports.

To conclude, it seems that both response times and verbal reports provide a suitable measure for looking at strategic behaviour. However, in the ideal situation, both would be employed because when used together they would provide a richer, more detailed picture of events. Both qualitative and quantitative information is valuable for doing research in mental arithmetic.

4. 3. 2 Experimental Procedures Used In Current (RT) Research: Two Types of Mental Arithmetic Tasks

Above, I looked at some of the issues arising from the use of chronometric methods to study mental arithmetic, highlighting some of the problems associated with using this method. I will now look at some of the strengths and limitations associated with the specific types of procedures used to study mental arithmetic performance. Two types of procedure have been used to study mental arithmetic, production tasks and verification tasks. Each has its strengths and limitations.

4. 3. 2. 1 Production versus Verification

Production tasks require the participant to generate an answer (7x8 =_). Whereas in verification tasks the participant is presented with an answer and must state whether this is true or false (7x8 = 56, true/false). Little and Widaman (1995) argue that while some studies have used production tasks, these are few compared to verification task studies. In the production task, research looks at the time taken to produce an actual verbal answer without the presence of a stated answer. Simpler mental arithmetic processing is required because there is no comparison with a stated answer. It is, therefore, less susceptible to bias.

Ashcraft (1985) also considers the issue that where information processes are involved, verification requires at least an explicit decision stage that is presumably not present in the production task. He found that although there is a task effect in the performance of 1st grade children, there are similarities at later stages. He argues that task type (production vs. verification) is largely irrelevant for RT measures beyond 1st grade.

4. 3. 2. 2 Tools Used to Measure RT Within Verification and Production Tasks

Three methods seem to be common when measuring RT in mental arithmetic. These are voice-activated relay, manual timing and video timing.

Use of voice-activated relay seems to be the preferred method in most experiments when using a production task. Where the task is a verification task, i.e. true/false or yes/no, an internal (in-built) timer is used to record the time taken to press the right keys.

Campbell and Graham (1985) actually used manual timing (where an experimenter pressed a key) in research involving children instead of voice-activated relay, to avoid trials being spoiled by extraneous vocalisations (something which children are particularly prone to doing especially when doing arithmetic). However, this method is procedurally cumbersome as it is inherently rather awkward to use. It is also noisy, uncomfortable and disruptive of normal routine.

Siegler recorded RT through use of a digitiser that fed into the VCR and printed digital times across the bottom of the taped scene. This is because he is specifically interested in what strategies children are using and details of what they are doing at the time, especially because the children in his study were quite young (preschoolers).

In brief, the production methods used by others have involved recording RT and then asking children what methods they have used to come up with the answer. The strategies revealed are then classified into groups.

Siegler uses videotaped interviews and the children are filmed solving problems while the RT is recorded using a digitiser. The recording verifies their computational strategy even if they say they did not use it. The subsequent analysis of the RT usually supports the verbal reports of the strategy observed. Studies that relied on verbal reports found that RT data usually supported what was said.

Possibly, one of the reasons why verification tasks are so popular with researchers is because voice-activated relay/voice-activated recording software used with production tasks has been found to be rather "messy". It can be easily disrupted by unpredicted vocalisations/ change in responses (although this method is still frequently used). Some trials are lost because the response fails to trigger the relay, or because extraneous sounds stop the timer prematurely. It therefore relies on the continuous presence of the experimenter to record each response and/or record whether each response was correct or incorrect. So, fewer production trials than verification trials can be carried out in the same time period. Thus, the verification task is more convenient to use than the production task and it also allows false answer manipulations.

To conclude, neither of these methods is ideal. An aim of my project is achieve a balance by taking advantages of the strengths of both the production and the verification methods. The production task research looks at exact answers generated by the participant and therefore requires simpler mental processing, because there is no comparison with a stated answer. However, the verification task allows for a more accurate measure of response time due to the ease of using an in-built timer. My research method allows me to use a production task that will record both the answer and the response time accurately and easily and furthermore is easy to use with children in a classroom setting due to its mobility.

4. 4 Method

The aim of my research is to look at junior school children (7-11 year olds) as to whether they are slow in acquiring strategies that would ease their performance at mental addition. But previous mental arithmetic research has employed research methods that were relatively inflexible and awkward to use in settings other than in a laboratory or otherwise carefully constructed experimental setting. The method I will use is more flexible than the voice-activated relay previously used in production tasks. It is relatively easy to set up in a school environment since it requires only a classroom computer to run the software program.

Since my intention was to use a chronometric method with a production task to study the development of children's mental arithmetic skills, a school environment was an obvious setting. Children would be relaxed and at ease. Any program being used to study mental arithmetic in children should be designed to encourage the children's interest in the tasks they will be doing so there will be greater motivation on their part. This is an aspect of research that seems to have been neglected in some of the previous research in this area, although Siegler has touched on it through his use of video-recorders, i. e. the children involved were quite fascinated by the idea of being filmed and seeing themselves on video. The design of the program used here gave the child maximum control over the pace of the task. It also allowed the child to use the task quite independently of the researcher. The program should also give clear and rapid feedback. Therefore, in the current project, the time taken to reach the solution to a problem flashed across the screen and so provided the child with instant feedback on progress plus added incentive to do better. So, the program was designed to maximise the child's engagement with the task but in a non-threatening way, i. e. the only element of competition involved would be if the child were attempting to improve on his/her own solution time.

By doing so, I hope to gain some insight into the strategies children develop when doing mental arithmetic and how readily they make use of the most effective use of what they do know. My aim is to look at children's developing strategic methods for solving arithmetic problems and to help identify where teaching efforts need to be concentrated. In the next section, I will describe the general methods used in the current research project.

4. 4. 1 Contacting Schools

Arrangements were made to visit two local schools on a regular basis. Both schools used the Acorn A3000 series computers that were needed to run the program that would be used to obtain my data.

The following two schools from a medium-sized town were chosen to participate in this project:

Wymeswold Primary School (10-20 children in each year group). This school contained mostly middle-class rural children.

Outwoods Edge Primary School (20-30 children in each year group). This school contained middle/ working-class suburban children.

There was informed contact with the teachers and they were given generic information about the children's progress and an informal arrangement was made about the appropriate times to visit, in order to keep any disruptions to the lessons to a minimum. The computers on which the children were tested were usually kept in a corner of the classroom. The task required each child to come over to the computer and do an arithmetic task for about 15-20 minutes (more or less depending on the child's abilities).

4. 4. 2 Task Design

The study involved children in the junior school years solving arithmetic sums presented on their own classroom computers. So this ensured the setting was as natural as possible for children doing the mental arithmetic. The problems were appropriate for their age but challenging. The solutions were entered quite easily by clicking on screen numbers. A schematic picture of the screen layout can be found in the Appendix 1 (Diagram 1). The program provided feedback to the pupil and the teacher and more detailed information about which problems required longer thinking time to solve. This allowed the use of RT to make inferences about how children must be solving certain types of problems.

The initial task began with some introductory sessions at both schools to familiarise the children with the tasks that would be required of them later on. All children were familiar with using a mouse and graphical computer interface. However, they were all given an introductory practice session with the program. This got them used to the idea that they were to use the mouse, rather than the keyboard.

Although the program design was child-centred and allowed the child to complete the task without my being present, all the children did have sessions with the program in which I was available and seated nearby in order to advise and observe as necessary.

4. 4. 3 Tools and Techniques

Research into mental arithmetic has been carried out within two paradigms: production tasks and verification tasks. Production tasks require the participant to generate an answer (7x8 = _) whereas in verification tasks the participant is presented with an answer and must state whether this is true or false (7x8 = 56, true / false). The task used in this study was a production task since it required the participant to generate the appropriate answer.

Use of voice-activated relay seems to be the preferred method in most of the experiments when using a production task. Where the task is a verification task i. e. true / false or yes / no, an internal timer is used to record the time taken to press the right keys. Previous mental arithmetic research has employed methods that were inflexible and awkward to use in settings other than in a laboratory, or otherwise carefully constructed experimental setting. The method I used was more flexible than the voice-activated relay which has previously been used with production tasks. It was relatively easy to set up in a school environment, since it required only an Acorn computer to run the software. This would involve using a chronometric method with a production task to study the development of children's mental arithmetic skills.

The program was designed for Acorn A3000 series computers. It measured the time taken (to the nearest hundredth of a second) to solve various arithmetic problems. It was designed to be easy to use in a classroom with minimal interference from the researcher. The problems were prepared in a file and were presented in computer-generated random order. The names of the participants were entered into a further file along with details such as age, school year, gender, the types of problems that would be presented etc. The program recorded all the data from each individual in a data file to enable easy retrieval for future analysis. This meant that the data was stored and recorded in a self-contained form.

The participant clicked on his / her name and was presented with a set of problems that were generated in random order according to the constraints of the particular experimental design. The precise nature of the task varied depending on what issue was being studied. The tasks were all mouse and screen-keypad driven.

4. 4. 4 Participants

As pointed out in earlier chapters, computational development normally accelerates in the junior school age range (at 7+ years). The greatest change is during the early years of school, after which improvement in the basic adding skills is much less obvious. As a result, it is a significant curricular concern in junior school years. Relatively little is known about *early* competence beyond handling pairs of single-digit problems. The findings of this project should have implications for how we manage teaching priorities and could also have implications for the design of exercise material.

The participants were 7-11 year old primary school children and the arithmetic tasks were solved in the "computer corners" of the classrooms. This ensured that the task was as least intrusive as possible and kept disruption of regular classroom activities to a minimum level. This classroom setting was a naturalistic one for children, because it provided a familiar context for them, to do both arithmetic and use the computer in a non-threatening capacity. There was nothing threatening or unusual about the task the children were required to do i. e. there was nothing "scary" or "test-like" about it.

There were about 15-35 participants in each year group in both schools. The following year groups were tested:

Wymeswold	Years 3 & 4	(sharing one classroom)
	Years 4, 5, 6	(sharing one classroom)
Outwoods Edge	Year 3	
	Year 4	
	Year 5	
	Year 6	

Each child's age was recorded at the time they participated in the experiment. Their date-of-birth was also recorded.

4. 5 Ways forward: Competence and Development

In current mental arithmetic research, change is usually studied through longitudinal studies looking at the same children's performance at different ages and/or cross-sectional studies looking at the performance of different children. The research in this thesis falls into this latter group. While these methods yield valuable information about change they do not show exactly how change occurs. Longitudinal studies, for example, involve examining children at relatively wide-spaced intervals (1-3 years) and only rarely make more frequent observations. In mental arithmetic research, for example, it may involve testing children at the start and end of the school year and maybe once in between.

Generating a new strategy requires more mental resources than using a well-established one. Therefore there are fewer resources for monitoring what had been done and generating words to describe it. Understanding often comes only with use and use is especially important to children's understanding, i. e. to make accessible previously uncommon approaches. But children can be very slow to use new strategies. There almost seems to be a 'resistance to learning/ improvement'; a failure to learn despite repeated exposure to a problem. Children (and sometimes adults) often seem quite content to make use of adequate but inefficient strategies. They tend to stick with what has worked best in the past, where they did not use the wrong strategy but neither did they find the most optimal or most efficient strategy. It seems that once we have attained a certain level of competency at a task we are often resistant to improvement.

Siegler (1987) and Siegler and Jenkins (1989) have attempted to examine this issue by studying in detail how children discover new strategies. They are interested in the "strained and halting nature of the breakthroughs". In young children's arithmetic more and less advanced strategies coexist and compete for a long time. Only gradually do more advanced strategies become prevalent. But they looked only at young children's discovery of the min method of counting on in addition.

One way of studying mental arithmetic development is to use a microgenetic approach. Microgenetic methods are sensitive to a wider range of changes than alternative more frequently used methodologies. Greater use of this method will result in improvement in the quality of developmental theorising because it reveals crucial mediators of change and constrains forms of transition theories, i. e. where children gradually go through one stage then the next (Cauzinille-Marmeche and Julo, 1998). This is important where transition theories are central. Microgenetic methods offer a lot of promise once we have understood mechanisms for moving forward. For mental arithmetic development, the early primary school years (7+ years) are a crucial period of development. But what sort of progress is made during these junior school years? These areas need to be identified before we can move forward. We need to know what questions to ask before we can progress to studying change through microgenetic methodology.

Although the current project has not applied a microgenetic approach to mental arithmetic, the methodology has been designed in a way that can conveniently inform future research in this area. My research project uses one such mechanism for moving forward through the use of cross-sectional research. This will allow me to identify precisely the specific areas in mental arithmetic where development does or does not occur. By devising a methodology that would easily lend itself to a microgenetic approach, I can look at how various aspects of computational fluency i. e. use of certain strategies develop over time and to get beneath the change as it occurs.

Chapter 5

5. Children's strategies for single-digit, decade, and three-digit serial addition

In Chapter 1, I examined the social and political background of mental arithmetic research. In Chapter 2, I considered the development of research traditions in mental arithmetic and in Chapter 3 I reviewed the current theories of simple arithmetic processing. In Chapter 4, I discussed the general aims of this thesis and its underlying methodology. The current chapter will discuss the findings of my preliminary study. The aim of the present study was to look at children's performance at single-digit, decade, and three-digit (serial) addition - identifying any emerging strategies. This study was divided into two experiments. In Experiment 1, the children were presented with the 45 single-digit addition "number-facts" and a selection of "decade" sums (sums of the type ab+c) in order to identify their baseline performance on these sums. In Experiment 2, the children were presented with a selection of three-digit serial addition problems to investigate whether or not they could exploit their existing knowledge of numbers in a serial addition context. The aim of these two foundational studies was to discover any emergent patterns in the results, thus laying down the framework for the research in my following studies.

5. Introduction

There has been a strong tradition in mental arithmetic research of focusing on addition, subtraction and multiplication of the simplest kinds of sums as being the "building blocks" of mental arithmetic research. Groen and Parkman (1972) and Parkman (1972) carried out some of the earliest cognitive research in the area of mental arithmetic. Their research was chronometric, i. e. it looked at response times as indicators of mental processing. They found that the size of the smaller number affects solution time in simple addition. The size of the smaller number was a good predictor of solution times on simple addition. Solution times increased linearly with the size of the smaller number. However, this was found to be the case only when the min strategy was being used. The "min" strategy refers to the "minimum addend" counting on strategy where the individual sets an internal counting register to the larger of the two numbers being added, then increments this value by ones a total n number of times i. e. until the smaller or minimum addend value is reached. At the end of this incrementing process, the counting register contains the value of the correct sum. The "min" model as Groen and Parkman (1972) called it, also failed to account for fast performance on tie problems such as 7+7. Although the early min model was found to be insufficient, later research extended and modified its findings.

Another significant earlier study looking at mental arithmetic found that subjects used a variety of techniques in performing mental calculations and this depended on the type of arithmetic operation, amount of practice and individual differences in computational ability (Aiken and Williams, 1973). They found RT for addition was an increased linear function of the smaller number, and claimed certain well-learned sums were obtained by random access retrieval from memory. They also found subjects to be faster on addition problems having a sum of 10, a finding which has since been replicated in several studies.

However, these studies looked at adult performance. Both adults and children use a wide range of strategies to solve problems. Siegler (1988) found that individual children and adults often use multiple strategies. Wise choices allow people to meet situational demands and overcome limited knowledge. Even young children can be skilled at choosing strategies. Retrieval is fast (if number-facts have been mastered) and back-up strategies (such as counting) high in accuracy. According to Ashcraft (1985) children must "learn their arithmetic facts", i. e. master the simple number combinations so they can be recalled fluently and accurately. He found that from 7 years and onwards children's response times at number facts are similar to adults, shifting towards those indicating fact retrieval by middle grade. But children's ability to be strategic can be constrained by their willingness to actually do so in practice. Thus, children have the ability to make use of their number-fact knowledge but only if they are willing to exploit this.

Research has shown that children can initially use a variety of strategies to reconstruct sums, differences and products of single-digit number combinations, they learn to respond to addition, subtraction and multiplication problems efficiently (quickly and accurately) and experts (adults) have some kind of network representation. There is basic agreement on this but there is some debate surrounding the acquisition process and on the precise nature of the network representation, i. e. the extent to which declarative and/ or procedural strategies underlie the developmental changes in arithmetic skills. Certain number facts have been found to be more salient than others and this is for both children and adults. Tie-sums (problems with a repeated operand such as 3x3 and 8+8 have faster response times than other problems of a similar magnitude (Campbell and Graham, 1985; Miller, Perlmutter and Keating, 1984). Sum-to-10 problems also have also been found to have faster and more accurate solution times (as do 5-times problems).

Counting, however, is one of the ways children learn to be "strategic" about doing mental addition. Use of the min strategy is constructive to developmental research in mental arithmetic because the "min" strategy is one of the best and earliest examples of strategy use in children. This is because it requires children to identify the largest addend and then reorder them, placing the largest addend first, thus demonstrating a recognition of the addition principle of commutativity. Counting is one of the solution procedures children have available to them when doing mental arithmetic. Using number-fact knowledge is another. Some pairs of numbers such as tie-sums, and sum to 10s become "number-facts" earlier than others. If children can be better at some number facts can they then use this existing number fact knowledge strategically? The aim of this study is to identify the solution patterns that emerge as a result of doing single-digit mental addition, as well as a selection of decade sums. The single-digit sums would allow me to discover whether the performance of the children in my sample would be consistent with previous research. Thus, allowing me to echo the "building block" type of foundational research that has been carried out in this area. However, the area of decade addition is less well researched. So the aim here would be to discover any emergent strategies for this neglected context. In addition, I would be exploring a novel and authentic method for doing chronometric research in naturalistic classroom environments.

Previous research has focused on simple addition but, in real-life, addition problems are more complex and involve two or more numbers with several digits. There is a lack of mental arithmetic research looking at addition of sequences of numbers. Widaman and Little (1992) and Widaman et al (1989) theorised about how cognitive models account for three-digit additions. For example, the network-retrieval model would involve searching the memory network for the sum of the two largest addends and then either searching for the sum of the provisional sum and the smallest addend or increment the smallest addend onto the provisional sum. In the three-sum models, three-digit addition would mean incorporating the product of all three addends as a structural variable. In this, the product of the three addends is a direct function of the volume of a 3-D network that must be searched to arrive at the intersection of the three nodal values. The preceding three-sum model reflects simultaneous summing of all three addends rather than summing of two addends at a time. Subjects first obtained a provisional sum of the two largest addends, via the very fast and efficient memory network retrieval process and then incremented the smallest addend onto the provisional sum. However, this research was carried out on an adult sample, children's performance would necessarily fit this model.

Therefore, a fact-retrieval approach may not be adequate for describing what happens when children solve three-digit addition problems. As mentioned in 2.1 there are two kinds of arithmetic knowledge, declarative knowledge and procedural knowledge and these two are intertwined:

Declarative knowledge (about number facts and combinations) which refers to stored knowledge of addition facts e. g. $2 + 3 = 5$, and depends on retrieval from memory. Procedural knowledge ("knowledge about" or an understanding of arithmetic concepts and procedures) which refers to stored knowledge about arithmetic e. g. $n \times 0 = 0$, $n + 0 = n$, or $n + 1 =$ one greater than the original number, and depends on rules, heuristics, computation and the understanding of principles such as commutative and associative principles.

Research should, however, be paying greater attention to this relationship, rather than focusing exclusively on one or the other. Most chronometric research has focused on single-digit number facts and their retrieval from a stored network of number facts in human memory (Ashcraft, Fierman and Bartolotta, 1984). However, this is changing and the focus of some research is shifting to how procedural rather than declarative knowledge is manipulated to solve arithmetic problems (Sohn and Carlson, 1998). Baroody (1983, 1994, and 1999) portrayed strategy choice in mental arithmetic development as a move away from slow procedural processes such as the min strategy to faster and more principled procedural processes. As mentioned earlier, while research focusing exclusively on single-digit number facts is useful for theorising about network models, real-life arithmetic is about larger numbers, e. g. adding sequences of numbers such as 4+9+5+1+9 and this type of serial addition quickly gets into larger double figures (84+7). Problems such as these can be solved more effectively by strategic use of our existing number fact knowledge but younger children may be slow at doing this.

Experiment 1 was carried out in order to track changing fluency on the complete set of single-digit addition problems. It would also identify some performance benchmarks for decade sums of the type ab+c. Experiment 2 looked at children performance at the simplest sort of serial addition problems, those involving three-digits. Experiment 2 was carried out to find out whether this fluency (at single-digit sums) would be mobilised into three-digit serial addition problems.

The issue of individual difference is another matter in which researchers are sometimes interested. One aspect of individual variation that relates to arithmetic performance is gender (Geary, 1996). Maths is an area where this is an obvious issue. For various claims have been made about gender difference in children's mathematical ability. For example, in the area of mental arithmetic research, Carr and Jessup (1997) found evidence of gender differences in first-grade mathematics, in that girls were more likely than boys to use overt strategies and boys were more likely to use retrieval to solve addition and subtraction problems. However, boys and girls were equally able to solve basic maths problems but showed differences in the strategies they used for problem solving. Although looking at gender difference was not a central aim of this research, this study did consider any gender effects relating to general competency i. e. at the single-digit number-facts. A basic analysis of this found no significant differences between solution times for these sums. Therefore, this issue was not returned to for the rest of this study or the ones carried out in Chapter 6 and 7. Regarding the issue of other individual differences, the schools chosen for my research were those that would provide a good cross-section of the local area and would be thus be representative of the local population.

5. 1 Experiment 1

Experiment 1 looked at children's performance on the 45 single-digit addition facts from 1 + 1 to 9+9. Mixed in with these number facts were two-digit decade problems. This experiment was carried out to get preliminary information on children's performance at basic addition number facts. Response time measurements would indicate the relative difficulty of the various combinations in these sets.

5. 1. 2 Method

5. 1. 2. 1 Participants

A total of 167 participants were recruited from two local schools as described in Section 4.4:

Wymeswold Primary (10-20 children in each year group). This school's population consisted mostly of middle-class rural children.

Outwoods Edge Primary (20-30 children in each year group). This school's population consisted mostly of middle/ working class suburban children.

Table 5. 1

Year	male	female	Total number in each year
3	27	28	55
4	22	23	45
5	14	16	30
6	13	24	37
		Total number of participants	167

All the children were familiar with using a mouse and graphical computer interface. These are the numbers of children who took part in the study and completed all the sums. However, there were cases where a child's data for a particular sum or number combination was lost as a result of a computer error/ missing data and would not therefore be available for analysis. In these cases the child's data for the rest of the sums he/she completed would still be included in the analysis for those sums.

5. 1. 2. 2 Tools/ task

As described in Section 4.4, the programme was configured to deliver problems of the following type:

1) The 45 single-digit addition facts involving numbers 1-9. These 45 sums were presented in a random order generated by the program. Whether the smaller addend of the pair was written first or second was also randomly determined, e. g. for the 8+9 sum, half of the children would be given a 9+8, the other half would be given 8+9.

2) These problems were mixed with 9 decade (those involving 20s, 50s and 80s) problems of this type:

a) Problems in the twenties decade:

22+n (where the tens digit is repeated and sums to less than 10, e. g. 22+2)

23+n (where the tens digits differ and sum to less than 10, e. g. 23+4)

26+n (where the tens digits differ and sum to more than 10, e. g. 26+7)

b) Problems in the fifties decade:

52+n (where the tens digit is repeated and sums to less than 10, e. g. 52+2)

53+n (where the tens digits differ and sum to less than 10, e. g. 53+4)

56+n (where the tens digits differ and sum to more than 10, e. g. 56+7)

c) Problems in the eighties decade:

82+n (where the tens digit is repeated and sums to less than 10, e. g. 82+2)

83+n (where the tens digits differ and sum to less than 10, e. g. 83+4)

86+n (where the tens digits differ and sum to more than 10, e. g. 86+7)

Within each decade the problems would be within these constraints. The three types of decade problems were chosen to make sure the children received a representative selection of problems that would include both easy and difficult sums. The decade sums were mixed in with the single-digit sums so that they would not arouse any special attention.

This made up a total of 54 problems, which were divided into two sessions so that the task was not too demanding for the child. All problems were randomly selected within these constraints. For the decade problems, whether the smaller units digit was presented first (e. g. 23+4) or the larger (e. g. 24+3) was made to match the order for those two digits in their single-addition format for each participant. For example, when the 24 and 3 had been selected, if 4+3 was chosen for the single-digit case then 24+3 was presented. This allowed within subject comparisons (and its possible interactions) for this issue. The program recorded errors and presented that sum a second time at a random point within the remaining sequence. The time recorded for that case would be the sum of the two response times thus giving a longer time. Errors were thus incorporated in the analysis of the results. For this reason the logarithms of the RTs were calculated for each sum and these were analysed. Error rates for all sums can be found in Appendix 2: Tables 1 and 2.

5. 1. 2. 3 Procedure

The computers on which the children were tested were usually kept in a corner of the classroom. The task required each child to come over to the computer and do the arithmetic problems for about 15-20 minutes. Care was taken not to make the task too long or too tedious for the children. All participants were supervised while doing the task (as described in Section 4.4).

The participant would see his/ her name as a button on the screen and clicked on it. He/ she was then presented with the set of problems generated in a random order. The problem appeared on the screen along with a "got it" button. The participant was instructed to click on "got it" when he/ she had come up with the answer. The participant then saw a screen with a small number pad (numbered 0-9), an "OK" button and a "reset" button. The original problem disappeared from the screen but it would be brought back by clicking on "reset" if it was forgotten, "reset" could also be used if the child felt that he/ she had entered the wrong numeral when completing their answer. Each child had two trials at each sum. That is, if a child made an error on a sum, that sum would then re-appear later in the sequence at a point randomly allocated by the program. If the child made an error at that sum the second time then that would be classed as an error. Children received feedback about errors, getting an incorrect answer would bring up a message informing them "oops! Incorrect answer!".

Participants selected answers from the number pad by clicking on the number buttons and, when satisfied with the answer, clicked on "OK". The time taken to solve the problem would then flash on the screen, e. g. "correct answer in 4 seconds" or if the answer was incorrect the word "oops!". This was incorporated into the program to provide the child with instant feedback on progress and this instant feedback on solution times functioned as an incentive for the child to do better.

The response time used in the analysis was the time it took for a child to click on the first digit of the answer, not the time it took for him/ her to click on the "got it" or the "OK" button, although this latter time was the time that the child would see on the screen as feedback.

5. 1. 3 Results

The graphs below in Sections 5.1.3.1, 5.1.3.2, and 5.1.3.3 all plot absolute solution times, while the graphs in the results in Section 5.1.3.4 summarise the RT difference (solution time "overhead") between a single-digit sum and its decade realisation (e.g., RT (23+4) - RT (3+4)).

Results were analysed according to junior school year, since it has been found that this may be a more accurate measure of skill development in mental addition than age. For years of schooling reflects the amount of practice and formal instruction in addition (Widaman and Little, 1992). Widaman et al found that grade in school provided a better fit to the data than chronological age. School year would therefore be a more accurate guide to the developmental changes underlying efficient strategy use in both simple and complex addition. The summary statistic plotted in the following graphs is the median response time. The means and standard deviations can be found in Appendix 3. Where post hoc tests are described (in this and in other chapters), the results are based on the Tukey HSD post hoc analysis which was used throughout this thesis.

5. 1. 3. 1 Graphical results for single-digit sums

These are the findings from single-digit addition problems, starting from the 1+n to 9+n. The following graphs plot the median response times (RT) in centi-seconds. The results suggest that the smaller tie-sums from 1+1 to 5+5 (Figures 5.1 to 5.5) seem to be recognised as facts across all age groups. RT for number facts from 6+6 (Figures 5.6 to 5.9) onwards suggest counting is going on for all age groups, but this is less so for older children who do seem to be recognising the tie-sum facts. For these single-digit sums, half of the participants in each year groups would have received the large addend first (e. g. 5+4) and the other half the smaller addend first (e. g. (4+5). For the analysis these two were treated as the same number-fact i. e. 4+n or 5+n. The analysis for the results for the single-digit sums in this section was not carried out on the logarithms of the RTs.

Figure 5.1 illustrates the median solution times for the single-digit sums 1+n.

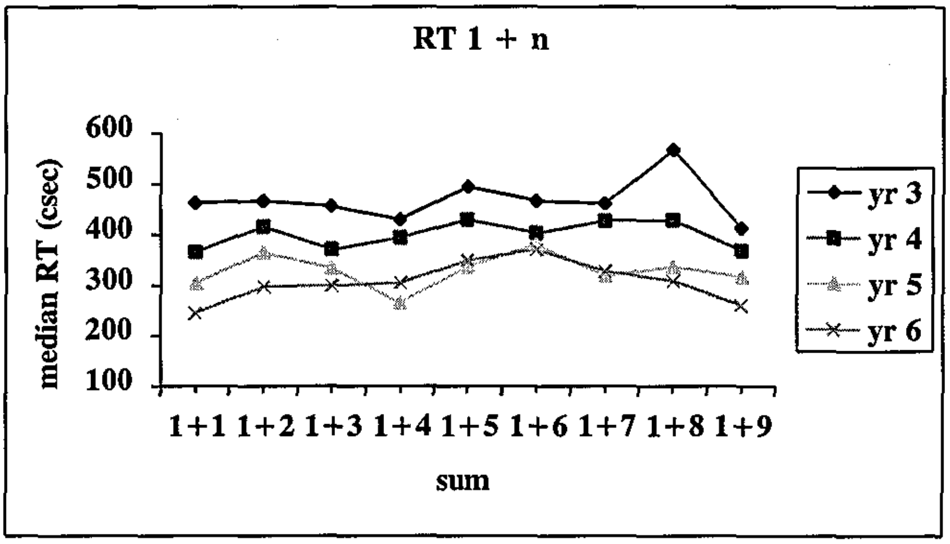

Figure 5.1 Solution times for 1+n.

A one-way ANOVA carried out on the RTs for the sum 1+2 and the tie-sum 1+1 for children in year 3 found that there was no significant difference between these solution times, $F(2, 113) = 0.59$. A one-way ANOVA carried out on the RTs for the sum 1+2 and the tie-sum 1+1 for children in year 4 found that there was a significant difference between these solution times, $F(2, 85) = 3.38$, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sum 1+2 and the tie-sum 1+1 for children in year 5 showed that there was no significant difference between these solution times, $F(2, 56) = 1.77$. A one-way ANOVA carried out on the RTs for the sum 1+2 and the tie-sum 1+1 for children in year 6 found that there was a significant difference between the solution times, $F(2, 71) = 3.9$, $p < 0.05$.

Figure 5.2 illustrates the median solution times for the single-digit sums 2+n.

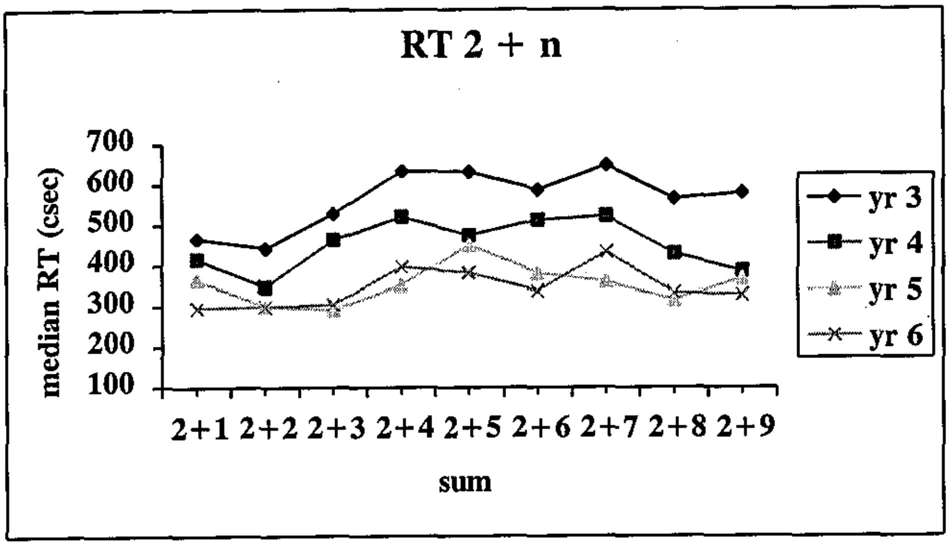

Figure 5.2 Solution times for 2+n.

A one-way ANOVA carried out on the RTs for the sums 2+1, 2+3 and the tie-sum 2+2 for children in year 3 showed that there was no significant difference between the solution times for these sums, $F(4, 168) = 1.54$. A one-way ANOVA carried out on the RTs for the sums 2+1, 2+3 and the tie-sum 2+2 for children in year 4 found that there was a significant difference between the solution times, $F(4, 127) = 2.98$, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sums 2+1, 2+3 and the tie-sum 2+2 for children in year 5 found that there was no significant difference between the solution times, $F(4, 85) = 1.24$. A one-way ANOVA carried out on the RTs for the sums 2+1, 2+3 and the tie-sum 2+2 for children in year 6 found that there was no significant difference between the solution times for these sums, $F(4, 105) = 0.45$.

Figure 5.3 illustrates the median solution times for the single-digit sums 3+n.

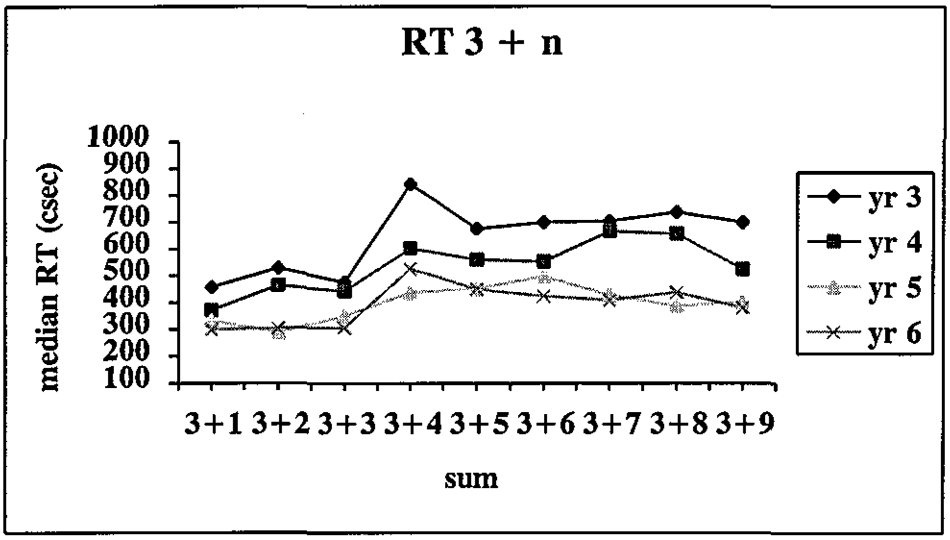

Figure 5.3 Solution times for 3+n.

A one-way ANOVA carried out on the RTs for the sums 3+2, 3+4 and the tie-sum 3+3 for children in year 3 showed that there was no significant difference between the solution times for these sums, $F(4, 166) = 0.44$. A one-way ANOVA carried out on the RTs for the sums 3+2, 3+4 and the tie-sum 3+3 for children in year 4 showed that there was no significant difference between the solution times for these sums, $F(4, 126) = 0.95$. A one-way ANOVA carried out on the RTs for the sums 3+2, 3+4 and the tie-sum 3+3 for children in year 5 showed that there was no significant difference between the solution times for these sums, $F(4, 85) = 2.05$. A one-way ANOVA carried out on the RTs for the sums 3+2, 3+4 and the tie-sum 3+3 for children in year 6 showed that there was a significant difference between the solution times for these sums, $F(4, 106) = 6.89$, $p < 0.05$.

The results above, for the single-digit sums 1+n to 3+n, suggest that small tie-sums seem to be salient number facts for children in all year groups. Counting seems to be the prevalent strategy for the youngest children, as well as some of the older children. The solution times for sums 1+n to 3+n seemed relatively flat. For the following sums one-way ANOVA were carried out on the four year groups to find out whether the solution time for the tie sum in each case would be significantly different from the sums adjacent to the tie sum.

Figure 5.4 illustrates the median solution times for the single-digit sums 4+n.

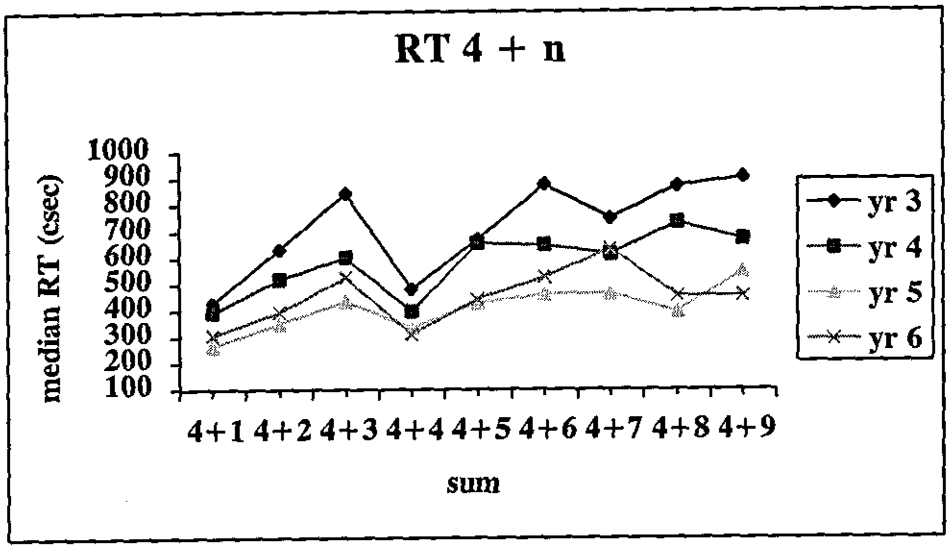

Figure 5.4 Solution times for 4+n.

A one-way ANOVA carried out on the RTs for the sums 4+3, 4+5 and the tie-sum 4+4 for children in year 3 found that there was no significant difference between the solution times, $F(4, 167) = 1.51$. A one-way ANOVA carried out on the RTs for the sums 4+3, 4+5 and the tie-sum 4+4 for children in year 4 found that there was no significant difference between the solution times, $F(4, 127) = 2.35$. A one-way ANOVA carried out on the RTs for the sums 4+3, 4+5 and the tie-sum 4+4 for children in year 5 found there was a significant difference between the solution times, $F(4, 85) = 2.68$, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sums 4+3, 4+5 and the tie-sum 4+4 for children in year 6 found that there was a significant difference between the solution times, $F(4, 107) = 3.35$, $p < 0.05$. This suggested that for the children in the older year groups 4+4 seems to be emerging as a distinct number fact.

Figure 5.5 illustrates the median solution times for the single-digit sums 5+n.

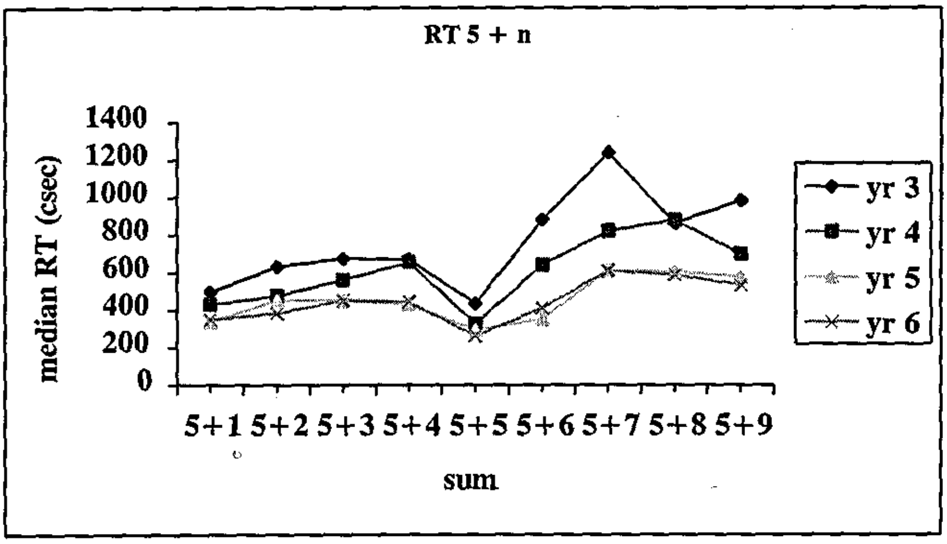

Figure 5.5 Solution times for 5+n.

A one-way ANOVA carried out on the RTs for the sums 5+4, 5+6 and the tie-sum 5+5 for children in year 3 found that there was no significant difference between solution times, F (4, 168) = 1.79. A one-way ANOVA carried out on the RTs for the sums 5+4, 5+6 and the tie-sum 5+5 for children in year 4 found that there was a significant difference between the solution times, F (4, 127) = 6.09, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sums 5+4, 5+6 and the tie-sum 5+5 for children in year 5 found that there was no significant difference between the solution times, F (4, 85) = 1.67. A one-way ANOVA carried out on the RTs for the sums 5+4, 5+6 and the tie-sum 5+5 for children in year 6 found that there was a significant difference between the solution times, F (4, 106) = 3.49, $p < 0.05$. This suggests that for children in year 4 and year 6 5+5 seems to be emerging as a distinct number fact.

Figure 5.6 illustrates the median solution times for the single-digit sums 6+n.

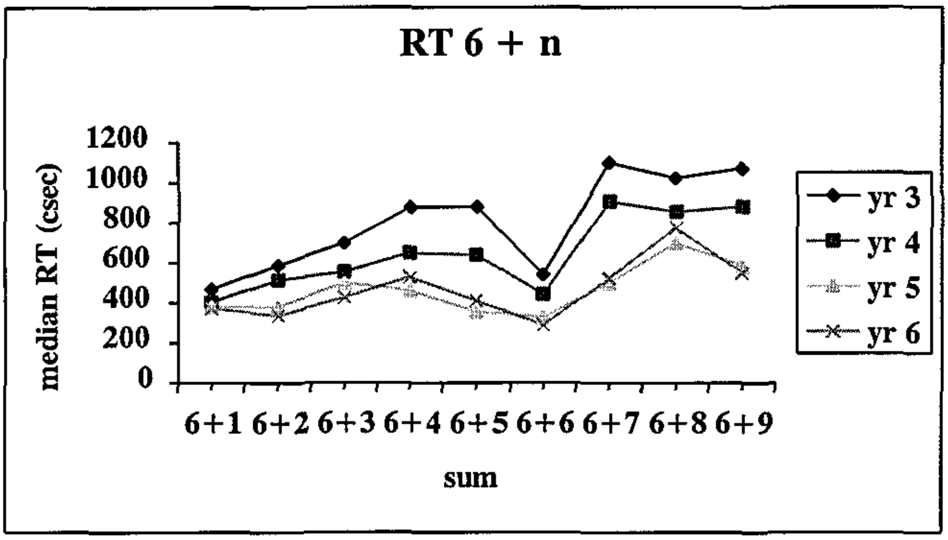

Figure 5.6 Solution times for 6+n.

A one-way ANOVA carried out on the RTs for the sums 6+4, 6+7 and the tie-sum 6+6 for children in year 3 found that there was no significant difference between the solution times, $F(4, 166) = 2.21$. A one-way ANOVA carried out on the RTs for the sums 6+4, 6+7 and the tie-sum 6+6 for children in year 4 found that there was a significant difference between the solution times, $F(4, 127) = 5.98$, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sums 6+4, 6+7 and the tie-sum 6+6 for children in year 5 found that there was a significant difference between the solution times, $F(4, 81) = 2.66$, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sums 6+4, 6+7 and the tie-sum 6+6 for children in year 6 found that there was a significant difference between the solution times for these sums, $F(4, 107) = 6.45$, $p < 0.05$. This suggests that for the children in the older year groups the 6+6 tie-sum does seem to be emerging as salient number-fact.

The graphical results for the single-digit sums 5+n (Figure 5. 5) and 6+n (Figure 5. 6) seem to suggest that 5+5 and 6+6 are becoming salient number facts for children in all years, although they are more likely to be salient number-facts for children in older year groups. However, for the oldest children 6+5 also seems to have a shorter solution time than 6+4, suggesting that perhaps this is being solved in a more strategic method i. e. that 6+5 is one less than 6+6.

Figure 5.7 illustrates the median solution times for the single-digit sums 7+n.

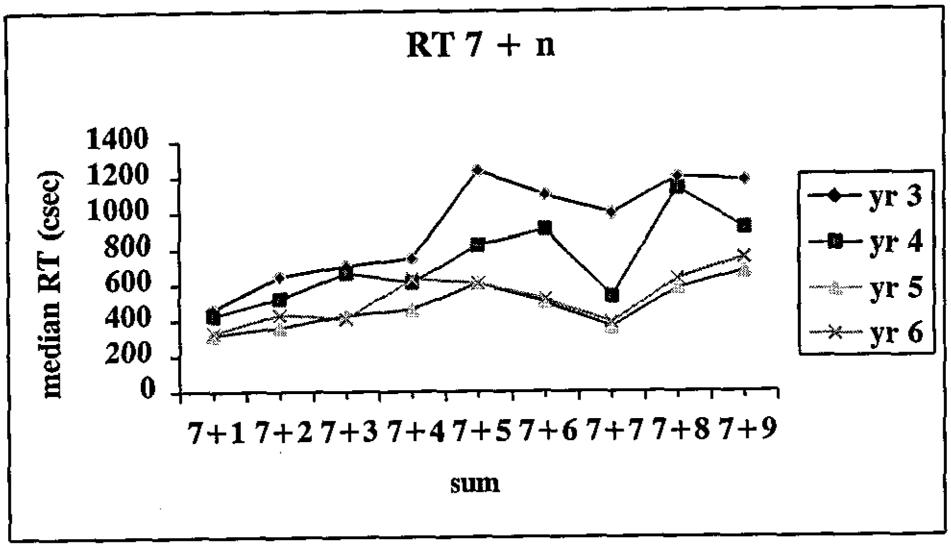

Figure 5.7 Solution times for 7+n.

A one-way ANOVA carried out on the RTs for the sums 7+6, 7+8 and the tie-sum 7+7 for children in year 3 found that there was no significant difference between the solution times, $F\ (4, 167) = 0.172$. A one-way ANOVA carried out on the RTs for the sums 7+6, 7+8 and the tie-sum 7+7 for children in year 4 found that there was no significant difference between the solution times, $F\ (4, 127) = 1.71$. A one-way ANOVA carried out on the RTs for the sums 7+6, 7+8 and the tie-sum 7+7 for children in year 5 found that there was a significant difference between the solution times, $F\ (4, 81) = 3.24$, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sums 7+6, 7+8 and the tie-sum 7+7 for children in year 6 found that there was no significant difference between the solution times, $F\ (4, 107) = 2.06$.

The results for the single-digit 7+n sums suggest that the tie-sum 7+7 is emerging as a number-fact for some older children and also some of the younger children (as illustrated by Figure 5.7). As with the results in Figure 5.6, the adjacent to tie-sum 7+6 also seems to have shorter times compared to 7+5.

Figure 5.8 illustrates the median solution times for the single-digit sums 8+n.

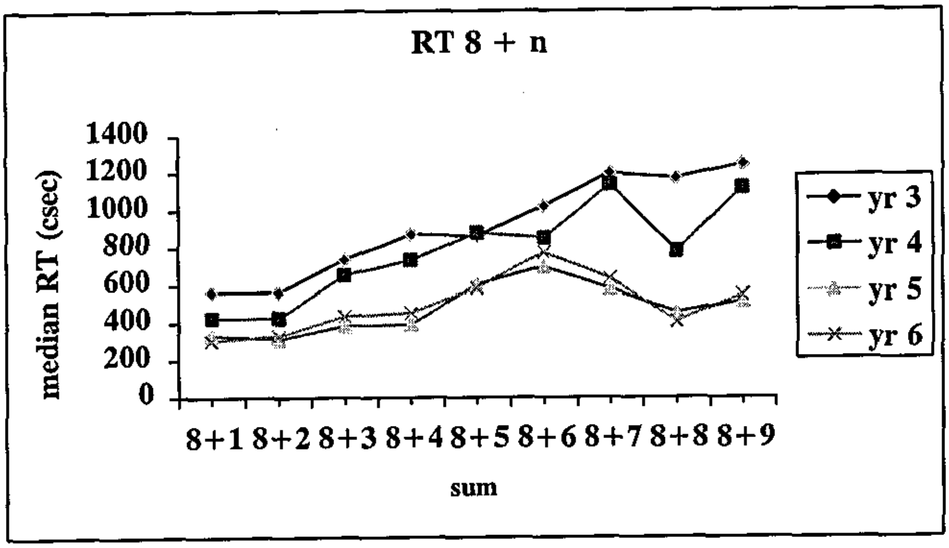

Figure 5.8 Solution times for 8+n.

A one-way ANOVA carried out on the RTs for the sums 8+7, 8+9 and the tie-sum 8+8 for children in year 3 found that there was no significant difference between the solution times for these sums, $F(4, 168) = 0.33$. A one-way ANOVA carried out on the RTs for the sums 8+7, 8+9 and the tie-sum 8+8 for children in year 4 found that there was no significant difference between the solution times, $F(4, 127) = 0.51$. A one-way ANOVA carried out on the RTs for the sums 8+7, 8+9 and the tie-sum 8+8 for children in year 5 found that there was a significant difference between these solution times, $F(4, 83) = 3.04$, $p < 0.05$. A one-way ANOVA carried out on the RTs for the sums 8+7, 8+9 and the tie-sum 8+8 for children in year 6 found that there was no significant difference between the solution times, $F(4, 105) = 0.28$.

The results for the single-digit 8+n sums suggest that counting seems to be the dominant solution strategy for children across all years. However, the patterns of results as illustrated by the graphs suggest that tie-sums do seem to be emerging as number facts for children in all years (i e. they do seem to be taking less time to solve than similarly large sums), while for the oldest children in years 5 and 6, the sums adjacent to tie-sums also seem to be emerging as number-facts.

Figure 5.9 illustrates the median solution times for the single-digit sums 9+n.

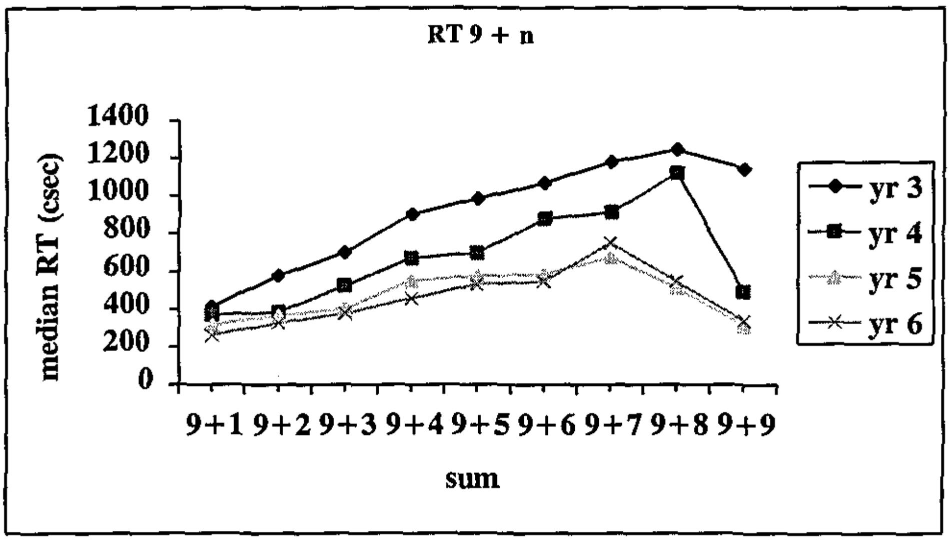

Figure 5.9 Solution times for 9+n.

A one-way ANOVA carried out on the RTs for the sums 9+8 and the tie-sum 9+9 for children in year 3 found that there was no significant difference between the solution times for these sums, F (2, 113) = 0.48. A one-way ANOVA carried out on the RTs for the sums 9+8 and the tie-sum 9+9 for children in year 4 found that there was no significant difference between the solution times, F (2, 85) = 0.54. A one-way ANOVA carried out on the RTs for the sums 9+8 and the tie-sum 9+9 for children in year 5 found that there was no significant difference between the solution times, F (2, 57) = 0.37. A one-way ANOVA carried out on the RTs for the sums 9+8 and the tie-sum 9+9 for children in year 6 found that there was no significant difference between these solution times, F (2, 70) = 1.58. This suggests that the tie-sum 9+9 was not yet a particularly salient number fact.

However, the trend in the results for the single-digit 9+n sums (as illustrated by Figure 5.9) does suggest, although this is not shown by the analysis, that the tie-sum 9+9 seems to be a salient number fact for children in the oldest year groups, and for some of the children in the youngest year groups. However, for children in years 5 and 6 the solution times for 9+8 seem to be dropping compared to solution times for 9+7, suggesting that perhaps the adjacent to tie-sums are being solved through a procedural "derived-fact" strategy compared to the counting strategies that seemed to be in use for the other sums.

The results illustrated in Figure 5.1 to 5.9 suggest that counting seems to be the prevalent strategy across all year groups, while tie-sums seem to be emerging as salient number-facts. For the younger children, this is limited to mostly the smaller tie-sums while for the older children this is for all single-digit sums. For the older children, the sums adjacent to tie-sums also seem to be emerging as number-facts.

5. 1. 3. 2 Graphical results for single-digit tie-sums

Figure 5.10 illustrates the median solution time for the single-digit tie-sums.

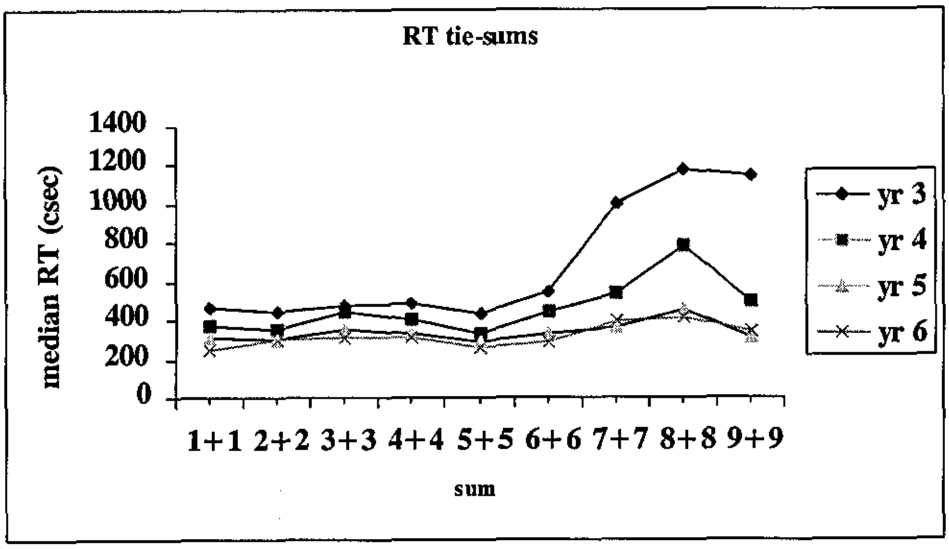

Figure 5.10 Solution times for single-digit tie-sums.

The results shown above suggest that tie-sums from 1+1 to 5+5 do seem to have become number-facts for chilren across all year groups. Tie-sums 6+6 to 9+9 seem to have bcome number-facts for the older children while for the youngest children they still seem to be resulting in long solution times.

5. 1. 3. 3 Graphical results for single-digit sum-to-10

Figure 5.11 illustrates the median solution times for the single-digit sum to10 sums.

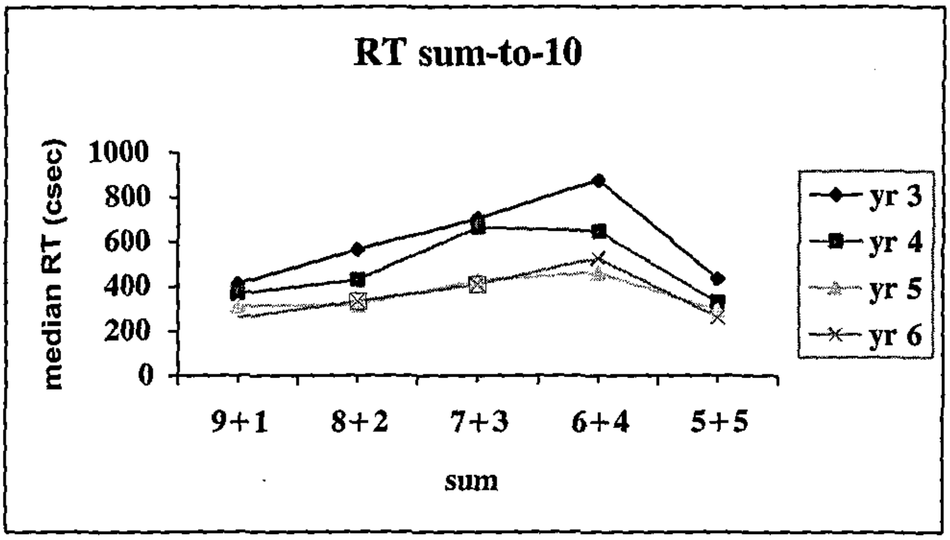

Figure 5.11 Solution times for single-digit sum to 10 sums.

The results for the single-digit sums involving a sum to 10 suggest that 9+1 and 5+5 seem to be salient number facts for children across all year groups.

5. 1. 3. 4 Results for decade sums

The graphs below illustrate the "extra time" or overhead for solving a decade sum compared to its single-digit counterpart i. e. RT (23+3) - RT (3+3). It was this time difference that was analysed in the following section. The graphs plot median RTs or RT overheads unless otherwise stated. The analysis was carried out on the logarithms of the actual RT difference.

Figure 5.12 illustrates the overall RT overhead for children across all years doing a decade sum.

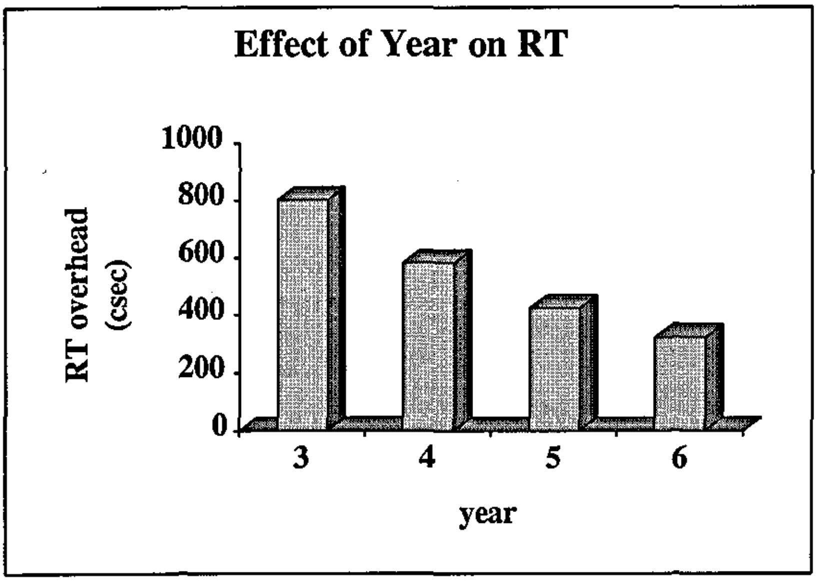

Figure 5.12 The effect of doing a decade sum on the overall RT overhead.

Figure 5.13 illustrates the effect of problem type on RT overhead for all three types of decade sums.

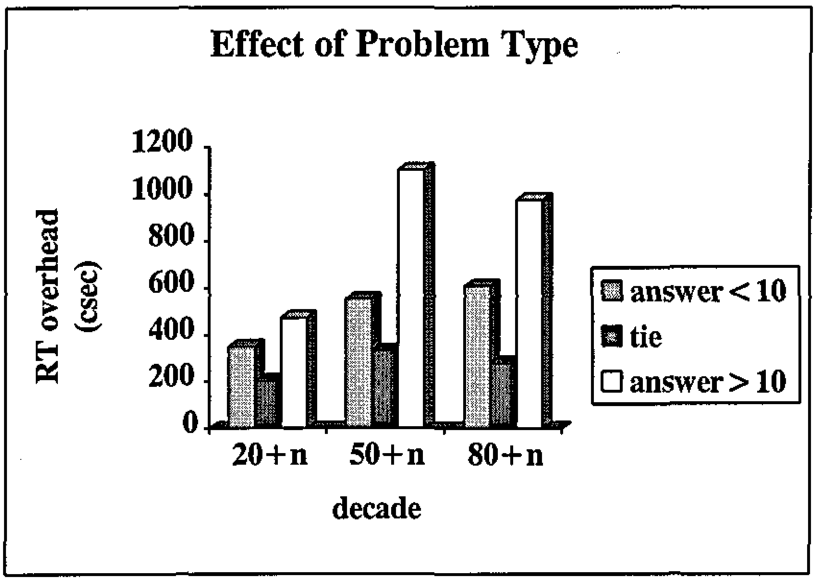

Figure 5.13 The effect of problem type on decade sums.

Figure 5.14 illustrates the RT overhead of the answer "crossing the decade boundary" on all decade sums.

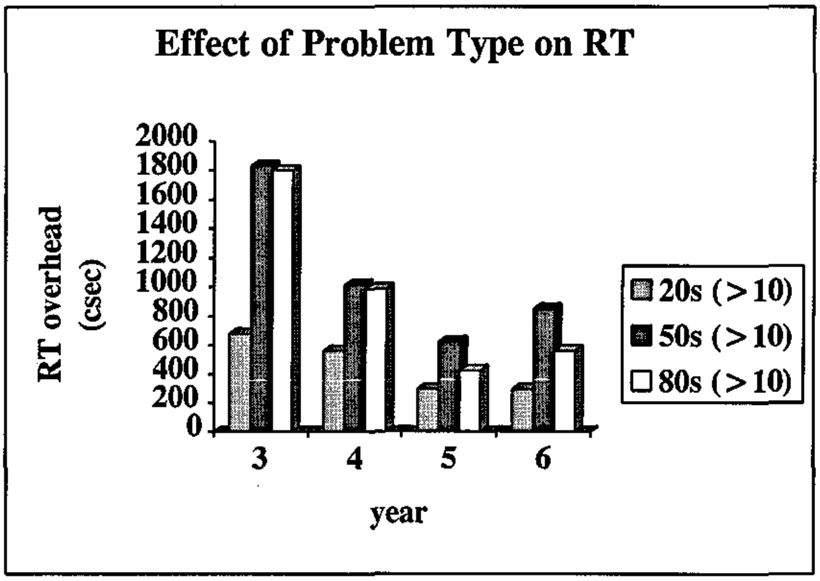

Figure 5.14 The effect of doing sums with answers that "cross the decade boundary".

Figure 5.15 illustrates the RT overhead for all decade sums.

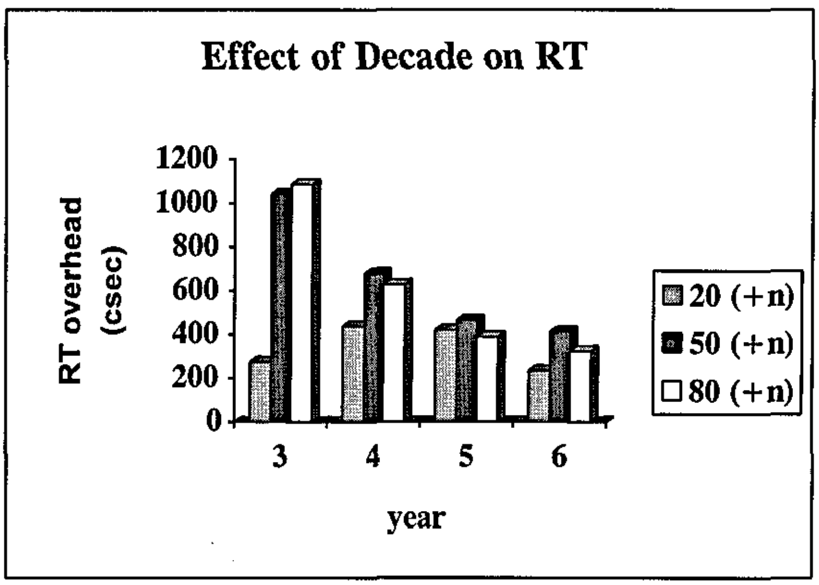

Figure 5.15 The effect of decade (20s, 50s, 80s sum) on solution overhead.

Figure 5.16 illustrates the effect of problem type on RT overhead.

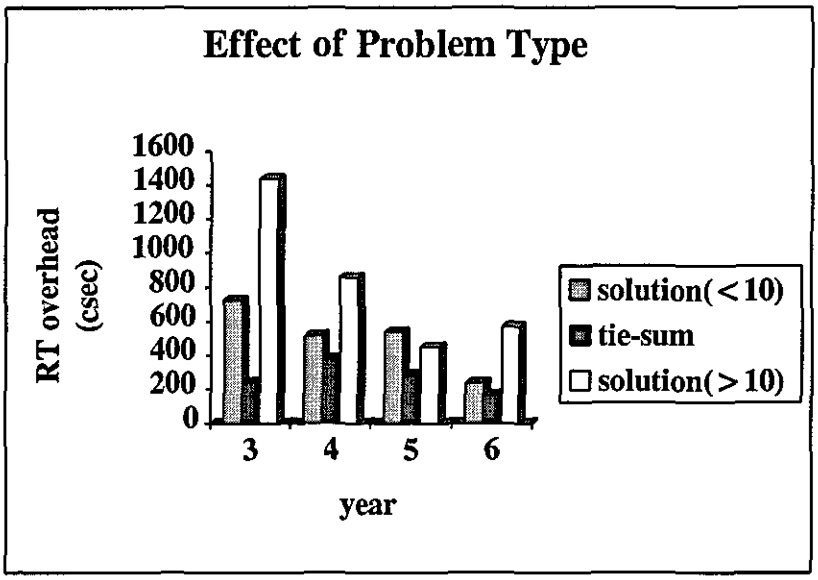

Figure 5.16 The effect of problem type (solution<10, solution>10, and tie-sum) on solution overheads.

5. 1. 3. 4. 1 Analysis for decade addition

An ANOVA carried out on the solution time differences (extra time) found that there was no significant overall effect of year, $F(3, 144) = 1.40$, suggesting that children across all year groups had similarly long solution times for decade sums. This is illustrated by Figure 5.12. There was a significant main effect of the decades (whether the sum was in the 20s decade, the 50s decade, or the 80s decade), $F(2, 143) = 9.99$, $p < 0.05$. This indicated that whether a problem involved a 20, 50 or an 80 did have an effect on the RT difference; children across all years had longer RTs for sums involving larger numbers such as 50 and 80 (as shown in Figure 5.15). There was a significant interaction between decade and year, $F(6, 288) = 2.59$, $p < 0.05$, suggesting that children in the youngest age groups had longer solution overheads than older children at sums that involved larger decade numbers (as illustrated by 5.15). There was a significant effect of problem type i. e. whether the answer of the problem remained within the same decade (answer <10, easy), involved a tie-sum, or crossed the decade boundary (was > 10, difficult), $F(2, 143) = 5.06$, $p < 0.05$. This meant that problems with answers that crossed the decade boundary were difficult across all sums (as shown in Figure 5.16). There was no significant interaction between problem type and year, F (6, 288) = 1, indicating that problems with answers that crossed the decade boundary were difficult for children across all year groups (as shown in Figure 5.14). There was a significant interaction between decade and problem type, $F (4, 141) = 2.53$, $p < 0.05$, indicating that sums with answers that cross the decade boundary (>10 problems) were the most difficult when they appeared in 50s or 80s problems as illustrated by Figure 5.13. There was no interaction between decades, problem type and year, F (12, 429) = 0.61.

Post hoc tests carried out on the results found that there was no significant difference between the mean RTs for the four year groups.

5. 1. 4 Discussion

The sums in this study were chosen to look into emerging patterns that would enable me to identify where the possible difficulties would be found. Results would help me decide which areas needed further investigation. Overall, the findings suggested that 1) children across all years seemed to be relying primarily on counting strategies, 2) that tie-sums are the earliest sums that emerge as number-facts, and 3) that children showed that they had the ability to use their existing knowledge (e. g. of ties) strategically. This seemed to be indicated by their recognising the tie-sum in the decade sums as illustrated by Figure 5.13 and Figure 5.14, as well as their recognition of adjacent to tie-sums in some of the single-digit sums.

The findings from the single-digit sums demonstrated that ties seemed to become increasingly salient for all age groups from 6+n onwards, i. e. there were short response times for ties so they seem to be becoming facts. However the results for 6+6 and onwards (Figure 5.10) suggest that counting still seems to be the preferred strategy. The graphs show a gradual increase in solution time which suggests counting is going on. Although they still appear to be counting, it is still taking them less time to do the tie-sums than the others. Figures 5.6 to 5.9 however show that RT for the sums adjacent to tie-sums also have lower RTs especially for the older children. This seems to suggest that next to ties are also being recognised as "facts" by older age groups. Findings for tie-sums support other findings (children encouraged to do "doubles" at school). However, findings for adjacent to tie-sums suggest these are taking less time and this seems to be evidence for strategy use. It seems that children using derived facts are being more strategic than those simply counting. Therefore, it would seem that children are being more procedural. It is possible that they are being more strategic by doing the tie-sum e. g. 9+9 because they either know it as a number fact and then address the sum next to the tie-sum e. g. 9+8 as they know it is one number less than the tie.

Figure 5.11 suggests that some sum-to-10 problems take less time than others. However, the overall pattern of results suggests that with the exception of 1+9 and 5+5 the sum-to-10 problems are not particularly salient for most children in this sample. Experiment 2 looked at this issue in more detail.

The results for the decade sums suggested that decade sums of the type ab+c were quite difficult for most children and seemed to result in consistently long solution times. The results suggested while children found sums in larger decade difficult, the sums that had the highest RT overhead were those with answers that crossed the decade boundary. These decade sums posed the biggest challenge for the children. Overheads for decade sums involving a tie suggested that children were recognising the tie-sum and were being strategic - using a transformational strategy like decomposition to solve these sums. Figures 5.13 and 5.16 suggest that tie-sums had fairly constant overheads even when they appeared in larger decades such as the 50s and the 80s. This issue would be investigated in greater detail in Chapter 6.

The consistently lower RTs for tie-sums support the findings of fact-retrieval research (Ashcraft, 1982; Campbell, 1987; Siegler and Jenkins, 1989). However, the lower RTs for the adjacent to tie-sums such as 6+7, 8+7, 9+8 do seem to support the schema theory proposed by Baroody (1994). The schema-based theory suggests that children and adults may both strategically employ related conceptual and factual knowledge to generate answers to "unrecallable" or unknown combinations. Proponents of fact-retrieval find that non-retrieval processes (referred to as back-up strategies) are basically slower than retrieval strategies. But according to the schema-based view although some non-retrieval strategies such as computing are slower, others such as reasoning or estimating may be almost as fast as retrieval. According to the schema theory children or adults required to respond swiftly to unmastered combinations may draw on their existing knowledge rapidly to estimate an answer. They may even reason out the answer in a relatively short time (Baroody, 1994).

In the schema-based view, practice frequency alone cannot account for changes in mental arithmetic performance or the underlying mental representation of basic number facts. Insight or pattern recognition is important for the evolution of errors and number fact knowledge. Relational knowledge may become embodied in the mental representation underlying the retrieval strategy. So, recognising the tie-sum as a number fact may encourage older children and some of the younger ones to use it strategically to solve another sum. Casual observation suggested that finger-counting was prevalent even among the oldest children. Even those children who had relatively fast RTs at the single-digit number facts would frequently rely on *fast* finger counting. However, it was also observed that that they would often begin by using their fingers, or mental counting (aloud) and then recognise that they were doing a tie-sum, or sum-to-10 and then say "I know that" or even something like "7+7, that's easy, I should know that".

Canobi, Reeve, and Pattison (1998) argued that children's use of retrieval strategies is related to their conceptual knowledge. They found that children who attend to and understand the relationships between problems are more able to store and retrieve addition combinations in memory. There is a link between conceptual understanding and storage and retrieval. Knowledge of addition principles may act as an organising framework for mental representations of the problem domain. They found that children who showed advanced conceptual understanding were relatively faster, more accurate and more flexible in their use of problem-solving compared to those who did not. This illustrated the relationship between conceptual knowledge and addition procedures. More research should look at the relationship between conceptual and procedural knowledge.

5. 2 Experiment 2

In Experiment 1, I measured performance on all possible single-digit addition pairs, and a selection of decade problems. The reason for investigating decade sums was to look at what happens when children solve sums of a greater complexity than simple single-digit sums, i. e. when adding sequences of numbers, the sum totals rapidly add up into larger double digits (decades). The aim of this second experiment was to look at just what makes those sequences of numbers (serial addition) so demanding for children. Would they be able to make use of their existing knowledge of number-facts such as tie-sum, sum to 10s or strategic reordering to use the min strategy to make these sums easier for to solve? My findings from Experiment 1 illustrated the difficulty children had when solving decade sums that had larger answers that crossed the decade boundary. The aim of this experiment was to find out where the difficult lay when it came to solving sequences of numbers, in this case three-digit serial addition.

Experiment 2 involved longer sequences of single-digit problems. Higher decade problems seemed to be persistently difficult. This led to interest in how performance might change as children progress through a long sequence of such problems that had the potential of generating large totals. Same digit (tie-sum) additions and some digits that sum to 10 seemed to have shorter RTs (Experiment 1). This could help in understanding children's performance on longer sequences of mental addition problems. What would children do when faced with longer sequences of addends that involved tie-sums and sum to 10 number facts? Would they be more likely to be strategic on sums on which these number-facts were made more visible, e. g. 4+4+3 as opposed to 4+3+4? Such sums could be solved more easily if children were strategic about reordering their addends, but only if they were willing to do so. Would they also be able to exploit their knowledge of the min reordering strategy in this case i.e. when faced with a sum such as a+b+9, would they reorder this into 9+a+b, thus making it easier to solve by counting?

Considerably less research has been done on addition problems involving more than two digits (Widaman et al, 1989). This study looked at serial addition problems of the simplest sort, three digit addition problems. Problems involving tie-sums and some sum-to-10 number-facts (such as 9+1 and 5+5) seemed easier for children and took less time to solve. Would children make strategic use of their addition number bond knowledge in solving these longer sequences? Would they be slow at strategically reordering sequences of digits e. g. 7+9+3 = 19 to 7+3 = 10+9 = 19 in order to optimise performance (which would result in both speed and accuracy)? With the exception of 9+1 and 5+5 sum to 10 number facts did not seem to emerge as particularly salient number facts. It is a cause for concern that after 6-7 years of primary school, children still do not seem to have picked up on the sum to 10 number facts given that they are potentially more powerful than tie-sums, for example. However, this experiment gave me the chance to look at this issue in greater detail because children might be more willing to recognise and exploit the strategic potential of sum to 10 number facts when the sum was generally more difficult to solve.

I looked at three digit (serial) addition problems where the sums were chosen to incorporate number facts such as tie-sums (Problem Type A) and sum-to-10 problems (Problem Type B). I also looked at problems where the largest addend was 9 to find out whether children's RTs would be affected by the position of the largest number i.e. would they have shorter times for problems which would encourage use of the min strategy (9+a+b)? The min strategy is usually studied in the context of single-digit sums but my aim was to look at this issue in the context of serial addition. The aim was to find out exactly how longer addition sequences were demanding for children and at what age children would start to exploit their existing knowledge strategically.

For example, a problem such as 7+5+3 might have a significantly longer RT than 7+3+5 because it was not being solved strategically i. e. was being solved in a linear way with the numbers being added up in the order in which they appeared. Similarly, a sum such as 4+n+4 might have a much longer solution time than 4+4+n because it was being solved in a fairly mechanical, linear method. A sum such as 4+4+n might have a much shorter solution time, so children could use their number-fact knowledge of 4+4 (because tie-sums have shorter RTs i. e. they are number-facts), and then add the third addend, unless they were using counting on the whole sums.

1) Problem Type A - three digit sums incorporating a tie-sum (4+4)
2) Problem Type B - three digit sums incorporating a sum-to-10 (7+3 and 9+1)
3) Problem Type C - three digit sums looking at reordering of the largest number (9+a+b, a+9+b, 9+a+b)
4) Problem Type D - three digit sums chosen as filler sums

5. 2. 2 Method

5. 2. 2. 1 Participants

A total of 175 participants were recruited from two local schools (for more detail see Section 4. 4. 1). The children in this experiment were the same children that took part in Experiment 1. However, as a result of new children in the classrooms, there were some more additions to the original sample. The new members of the classroom were given the sample lesson to familiarise them with the task. All the children were familiar with using a mouse and graphical computer interface.

Table 5. 2

Year	female	male	Total no. in each year
3	27	26	53
4	21	23	44
5	23	20	43
6	20	15	35
Total no. of participants	91	84	175

5. 2. 2. 2 Tools/ task

The following problems were carefully chosen so that they could be solved through strategic methods, simple counting or a combination of both. The problems were chosen in a way that would enable strategic reordering and also pick up the problems on which it was being used. The problems chosen fell into four categories. Problems in Problem Type D were used as filler sums only, and were not included in the analysis. All the problems appeared on the screen in a linear horizontal format i. e. a+b+c.

Problems in Problem Type A were chosen to discover whether or not children would exploit their knowledge of the tie-sum 4+4. The sums in Group 1 looked at the tie-sum 4+4 (+1) to find out whether or not the visible tie-sum would override the special "+1" case. Problems in Group 2 looked at the tie-sum 4+4 (+3) to find out whether or not 4+4 would be exploited.

Problems in Problem Type B were chosen to discover whether or not children would exploit their knowledge of sum to 10 number facts. The sums in Group 1 were chosen to look at knowledge of the sum to 10 7+3 and the sums in Group 2 were chosen to look at knowledge of the sum to 10 9+1 which also looks at the case of "+1".

Problems in Problem Type C were chosen to discover whether or not children would exploit the min reordering strategy and reorder the largest addend first. Similar solution times for these sums would suggest that they were doing so, whereas different solution times would suggest that they were not.

The 14 *three-digit addition problems were of the following type:*

Table 5. 3

Problem Type	**Sum**	**Problem**	**First addend**	**Second addend**	**Third addend**
A (Exploit tie-sum)					
Group 1	4+4	4+4 (+1)	4	4	1
			4	1	4
Group 2	4+4	4+4 (+3)	4	4	3
			4	3	4
			3	4	4
B (Exploit sum to 10)					
Group 1	7+3	7+3(+n)	7	3	(4,5,6)
			7	(4,5,6)	3
Group 2	9+1	9+1(+n)	9	1	(4,5,6)
			9	(4,5,6)	1
C (Reordering the largest addend)	9+a+b	9+a+b	9	(2,3,4,5)	(2,3,4,5)
		a+9+b	(2,3,4,5)	9	(2,3,4,5)
		a+b+9	(2,3,4,5)	(2,3,4,5)	9
D (Filler sums)	a+b+c	5+4+3	5	4	3
	b+a+c	4+5+3	4	5	3

Each child received 14 problems of the type shown above and for the sums in Problem Type B, the third addend (n) was randomly selected by the program from a combination of the digits 4, 5, 6, i.e. each participant would receive the sum to 10 (7+3) where n would be chosen randomly from this selection of 4, 5, or 6. For example, if a child received the sum-to-10 7+3+4 then he/ she could also receive the sum 7+4+3 but he/she could also receive 7+5+3. However, the difference between the size of this second or third addend would always be plus or minus one, i. e. the child could receive either 4 and 5, or 5 and 6 as the random addend but not 4 and 6. The aim of this was to prevent the child from actively noticing a similar pattern between the sums.

Similarly, in Problem Type C, in addition to the largest addend (9) the second and third addends a and b were randomly chosen from a selection of digits 2, 3, 4, or 5 so 9 would be constant but a and b were not. A tie-sum would only be presented in the context of Problem Type A, i. e. for Problem Type C a tie-sum such as 9+3+3 would not be presented.

5. 2. 2. 3 Procedure

All the children were familiar with the task because they had completed Experiment 1, which looked at their performance at basic single-digit number facts. The problems appeared on the computer screen in the same as those in Experiment 1, but this time they would see three-digit addition problems, instead of the single-digit addition problems mixed with the decade problems.

5. 2. 3 Results

Results were analysed according to junior school year. It has been found that this may be a more accurate measure of skill development in mental addition than age, because schooling reflects the amount of practice and formal instruction in addition (Widaman and Little, 1992). Widaman et al found that grade in school provided a better fit to the data than chronological age. School year would therefore be a more accurate guide to the developmental changes underlying efficient strategy use in both simple and complex addition. The summary statistic plotted in the following graphs is the median response time. Errors were dealt with as described earlier in Section 5.1.3. As mentioned earlier, the program recorded errors and presented that sum a second time at a random point within the remaining sequence. The time recorded for that case would be the sum of the two response times thus giving a longer time. Errors were thus incorporated in the analysis of the results. For this reason the logarithms of the RTs were calculated for each sum and these were analysed. All the analyses were carried out on the logarithms of the RTs. The error frequencies for these sums can be found in Appendix 2: Table 3. The means and standard deviations can be found in Appendix 3.

Results showed that three-digit addition problems generally took longer to solve than two-digit problems.

5. 2. 3. 1 Results for sums in Problem Type A (three-digit sums involving tie-sums)

The following results suggested that three digit serial addition problems including ties such as 4+4 (+1) and 4+4 (+3) take significantly less time than 4+1+4 and 4+3+4 and this is across all age groups. This is consistent with the findings in Section 5.1, which suggested that tie-sums do seem to emerge as salient number-facts.

Figure 5.17 illustrates the median solution times for three-digit sums involving the tie-sum 4+4 (+1).

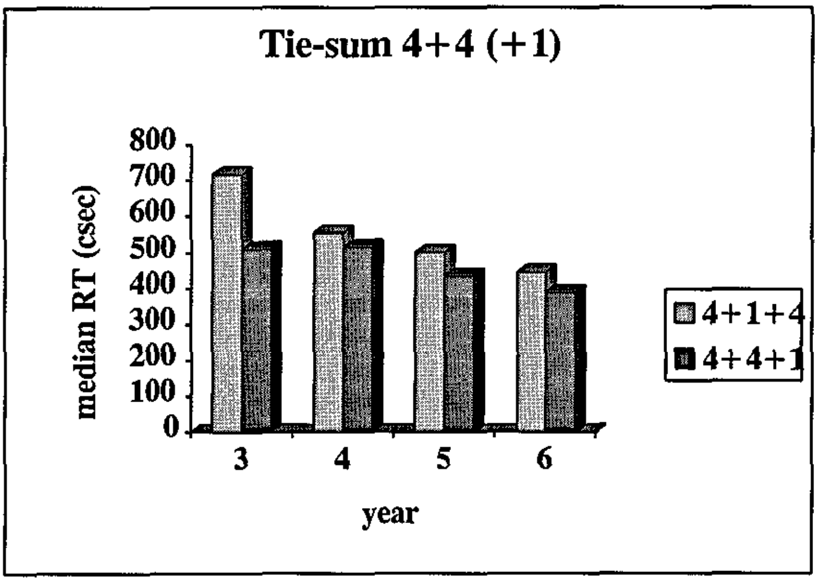

Figure 5.17 Solution times for 4+4 (+1)

An ANOVA on these results (illustrated in Figure 5.17) found a significant overall effect of year, $F (3, 168) = 6.19$, $p < 0.05$, suggesting that children in younger age groups had longer solution times for these sums. There was significant main effect of digit order, $F (1, 168) = 13.39$, $p < 0.05$. This indicates that children had faster solution times for sums such as 4+4+1 where the tie-sum was made more visible. It is possible that the "+1" is a potent distraction from forming the tie from the separated digits. There was no significant interaction between digit order and year, $F (3, 168) = 0.19$. This would suggest that children across all years had similarly long solution times for these two types of sums.

Post hoc tests on the results revealed that the RT for children in year 3 was not significantly different from the RT for children in year 4. However, the RT for year 5 was significantly different from the RT for year 3, $p < 0.05$, and the RT for year 6 was significantly different from the RT for year 3, $p < 0.05$. There was no significant difference between the RTs for year 4 and year 5. There was a significant difference between the RT for year 4 and year 6, $p < 0.05$. There was no significant difference between the RTs for year 5 and year 6.

Figure 5.18 illustrates the median solution times for three-digit sums involving the tie-sum 4+4 (+3).

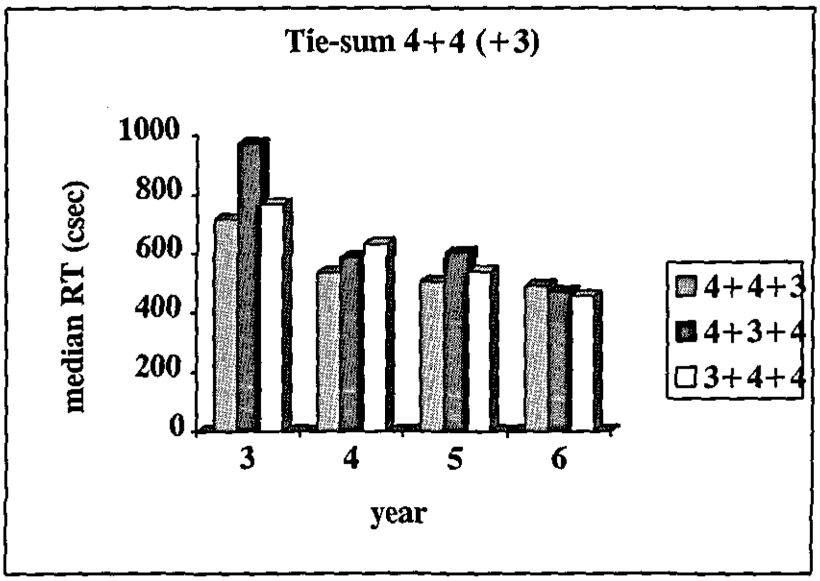

Figure 5.18 Solution times for 4+4 (+3)

An ANOVA on these results (illustrated as in Figure 5.18) found a significant overall effect of year, $F (3, 167) = 8.81$, $p < 0.05$. This suggests that children in younger age groups took generally longer to solve these sums. There was a significant main effect of digit order, $F (2, 166) = 7.61$, $p < 0.05$, suggesting that children has faster solution times for sums in which the tie-sum was made more visible such as 4+4 (+3) and (3+) 4+4 compared to sums in which the tie sum seemed less visible such as 4+3+4. There was no significant interaction between digit order and year, $F (6, 334) = 0.38$, suggesting that children across all years had similar solution times for these three types of problems.

Post hoc tests on the results showed that there was a significant difference between the RTs for children in year 3 and year 4, $p < 0.05$. There was also a significant difference between the RTs for year 3 and year 5, $p < 0.05$, and between the RTs for year 3 and year 6, $p < 0.05$. There was no significant difference between the RTs for year 4 and year 5, and there was no significant difference between the RTs for year 4 and year 6. There was no significant difference between the RTs for year 5 and year 6.

The results shown in Figures 5.18, demonstrate that tie-sum do seem to be emerging as salient number-facts, although this may be over-ridden by a "+1" distraction (Figure 5.17). Children across all years do seem to be willing to make use of their tie-sum number fact knowledge. The following section looks at results for three-digit sums that involved a sum to 10 number-fact.

5. 2. 3. 2 Results for sums in Problem Type B (three-digit sums involving a sum to 10)

The following results suggested that children did not seem to be making use of the sum to 10 number-fact 7+3, while they did seem to be taking advantage of the sum to 10 9+1. However, 9+1 also has the distinction of being a "plus 1" case. This would suggest that the 7+3 number-fact is not yet a salient number-fact for most children.

Figure 5.19 illustrates the median solution times for sums in Group 1, three-digit sums involving the sum to 10 7+3 (+n).

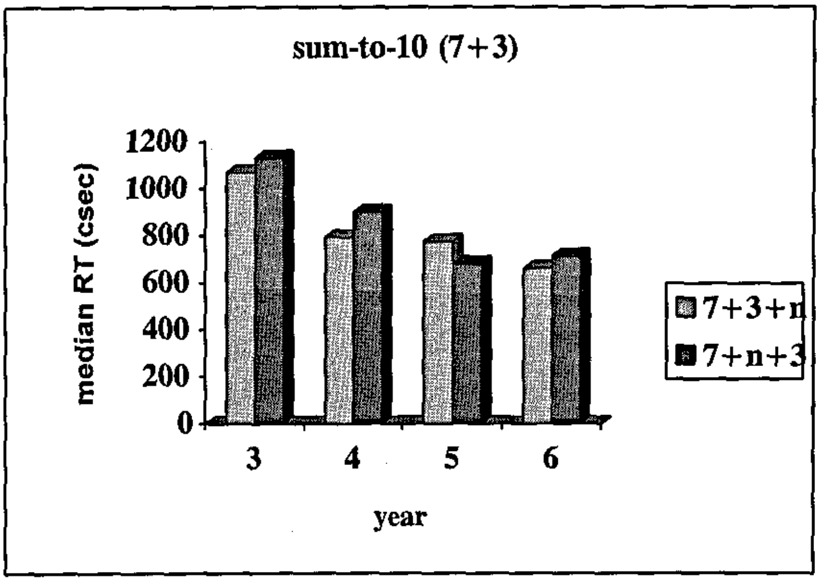

Figure 5.19 Solution times for sum to 10 (7+3+n)

An ANOVA on the results (illustrated in Figure 5.19) found that there was a significant overall effect of year, $F (3, 169) = 6.08$, $p < 0.05$, suggesting that younger children were taking longer to do these sums. There was no significant effect of digit order, $F (1, 169) = 2.13$. This indicates that children did not have significantly faster solution times for three-digit sums with the visible sum to 10 (7+3). It seemed that the sum to 10 7+3 was not being exploited as salient number-fact, but that the digits were probably being counted in turn. There was no significant interaction between digit order and year, $F (3, 169) = 0.15$.

Post hoc tests showed that there was no significant difference between the RTs for year 3 and year 4. There was, however, a significant difference between the RTs for year 3 and year 5, $p < 0.05$ and there was also a significant difference between the RT for year 3 and year 6, $p < 0.05$. There was no significant difference between the RTs for year 4 and year 5, and there was no difference between the RTs for year 4 and year 6. There was also no significant difference between the RTs for year 5 and year 6.

Figure 5.20 illustrates the median solution for times for sums in Group 2, three-digit sums involving the sum to 10 9+1 (+n).

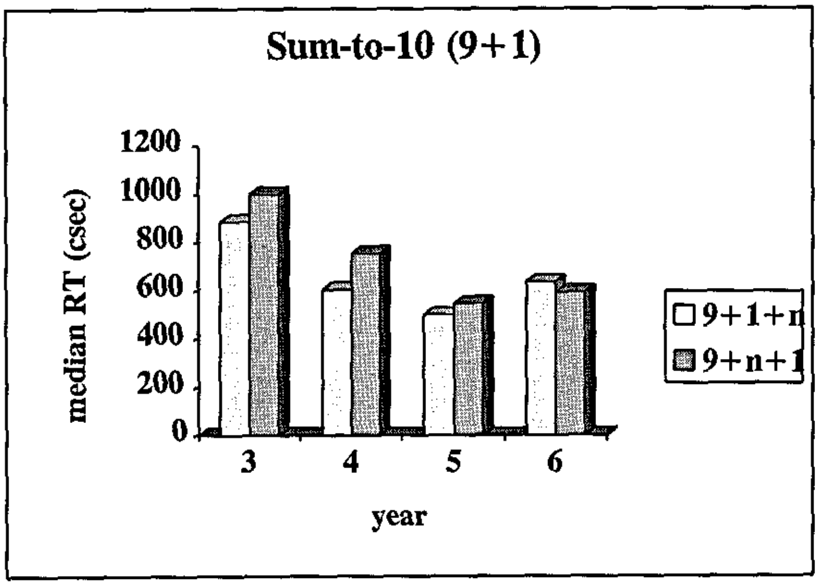

Figure 5.20 Solution times for sum to 10 (9+1+n).

An ANOVA on these results (illustrated in Figure 5.20) found that there was a significant overall effect of year, $F (3,167) = 8.65, p < 0.05$, suggesting that younger children had longer solution times for these sums. There was a significant main effect of digit order, $F (1,167) = 18.33, p < 0.05$. This suggested that children had significantly faster solution times for sums in which the sum to 10 (9+1) was made more visible (9+1+n) compared to sums in which it was not (9+n+1). There was no significant interaction between digit order and year, $F (3, 167) = 0.54$.

Post hoc tests on these results found that there was no significant difference between the RT for year 3 and year 4. However, the RT for year 3 varied significantly from the RT in year 5, $p < 0.05$ and the RT in year 6, $p < 0.05$. There was no significant difference between the RT for year 4 and year 5, and there was no significant difference between the RT for year 4 and 6. There was no significant difference between the RT for year 5 and year 6.

5. 2. 3. 3 Results for sums in Problem Type C (reordering of the largest addend 9)

The following set of results looked at the solution time for sums that reordered the largest addend 9 to find out whether or not the position of the largest addend would affect the solution times for these sums. That is, would RTs for these sums indicate whether or not the children were reordering the addends to select the largest number first, i. e. using the "min" strategy? The results suggested that children had long solution times for sums such as a+b+9 and that the youngest children had the longest times.

Figure 5.21 illustrates the solution times for three-digit sums involving a visibly large addend (9).

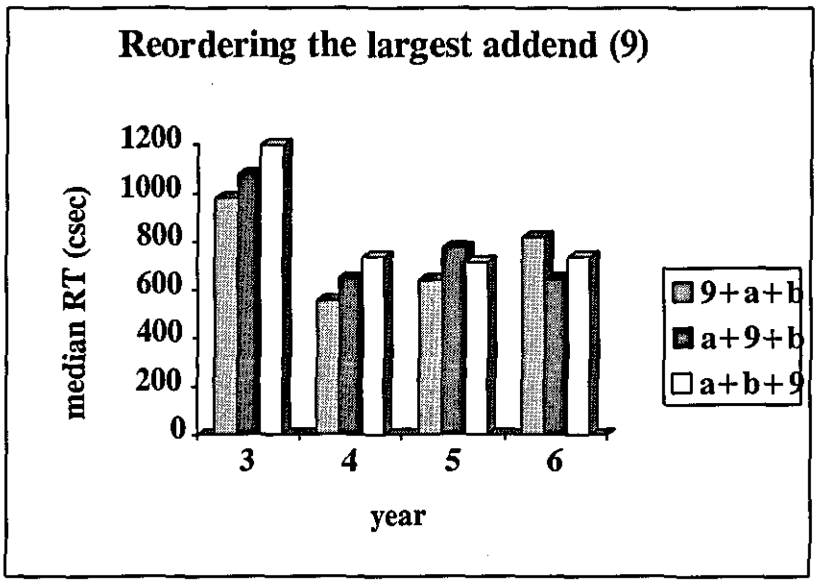

Figure 5.21 Solution times for sums with a visibly large addend.

An ANOVA on the results (illustrated in Figure 5.21) found that there was an overall significant effect of year, $F(3, 164) = 6.71$, $p < 0.05$, suggesting, as before, that younger children had longer solution times for these sums. There was no significant effect of digit order, $F(2, 163) = 0.69$, suggesting that the position of the largest addend did not affect the solution times for these sums. Children do seem to be reordering to select the largest addend first, i.e. they may be exploiting the min reordering strategy. The graphs do imply that the youngest children (in years 3 and might be counting, because it is taking them progressively longer to do sums in which the largest addend is not made as visible. However, there was no significant interaction between digit-order and year, $F(6, 328) = 1.57$.

Post hoc tests on these results found that there was no significant difference between the RTs for year 3 and year 4. However, there was a significant difference between the RTs for year 3 and year 5, $p < 0.05$, and there was also a significant difference between the RTs for year 3 and year 6, $p < 0.05$. There was no significant difference between the RTs for year 4 and year 5, and there was no significant difference between the RT for year 4 and year 6. There was no significant difference between the RTs for year 5 and year 6.

5. 2. 4 Discussion

The aim of this experiment was to look at children's solution times for three-digit serial addition with the intention to discover whether or not they were solving such sums strategically. The children were presented with three types of three-digit sums:

1) Problems designed to look at children's use of tie-sum number facts (Problem Type A, Group 1 and Group 2).
2) Problems designed to look at children's use of sum to 10 number facts (Problem Type B, Group 1 and Group 2).
3) Problems designed to look at children ability to use the min reordering strategy (Problem Type C).

The results from Experiment 2 showed that, overall, three-digit sums were difficult for most children because they took longer to solve compared to the two digit sums. One of the aims in looking at three-digit sums was to find out exactly what made longer addition sequences so demanding for children. The three-digit sums were particularly demanding for children in younger age groups and took them longer to solve. Doing three-digit mental addition can add a considerable memory load especially for younger children (Little and Widaman, 1992). The results also showed an expected developmental trend across all types of sums.

The results for the sums in Problem Type A suggested tie-sums such as 4+4 are salient number facts for children across all age groups and this supported earlier findings with single-digit sums (in Section 5.1) which showed the emergence of ties as early number facts. It took children significantly longer to solve the problem 4+3+4 compared to problems where the tie-sum appeared first. This seemed to suggest that children, at least for these smaller tie-sums, were using their existing number fact-knowledge of the tie-sum since the problems with the visible tie-sum (i. e. the tie appeared first) took less time. However, it also suggested that they are not doing this very efficiently, because it seems they are not using this knowledge when the tie-sum was paired with a +1 item. This may strongly prompt the 4+1 solution over the reordered 4+4 - thus leading to a longer overall solution time.

The results for the sums in Problem Type B found that 7+3 sum-to-10 problems do not seem to be emerging as salient number fact problems. This supported my findings in Experiment 1, which showed that the 9+1 (and 5+5) was the only sum-to-10 problem that seemed to emerge as a number fact. With the three-digit sums, whether the 7+3 appeared first did not have a significant effect on RT. However, there did seem to be a trend towards this as was illustrated by Figure 5.19.

The sums in Problem Type C (9+a+b) had been included to enable me to find out whether it would take children less time to solve problems where the largest number appeared first thus suggesting the children were using the min strategy. However, the problem type did not have an effect on their RT. This does suggest that reordering according to a min strategy is going on.

So, the children seemed to be making use of their existing number fact knowledge of ties. The findings from Experiment 2 showed that they were recognising ties and were attempting to make use of them.. These findings seemed to support the schema-theory view that relational knowledge may become embodied in the mental representation underlying the retrieval strategy.

Overall, the results suggested that children were, to an extent, being strategic by exploiting their existing number knowledge. But it seemed that children were not fully exploiting the potential of sum to 10 combinations. One general problem here might be that seeing the problems on the screen in a horizontal format inhibited them from being as strategic as they could be. That is, the order in which to add the digits had been more or less imposed upon them and they tended not to deviate from this. Therefore, one thing to consider was the possibility that seeing the problems in a linear format (9+1+n) may actually have prevented the children from being strategic and they were simply adding the numbers mechanically in the order in which they saw them. Another consideration was that it might not be enough to just look at RT for such problems, it may be more important to know the order in which the children were actually adding the numbers.

The next step in investigating serial addition would be to find out whether children would use an efficient reordering strategy if they were given control over the order in which they could add the numbers. In problems presented in a linear format the children might have thought the order in which to solve the problems had been imposed upon them and simply solved the sums as they saw them. So, when presented with serial addition sums in a linear format, they seemed to be fairly mechanical in adding them. Would this change if they were free to choose the order of the numbers they were required to add? This was studied in greater depth in Chapter 7.

5. 3 Conclusion

To conclude, the aim of the foundational research in this chapter was to set the framework for the following studies. I have looked at the patterns that would emerge as a result of children doing single-digit sums, a selection of decade sums and three-digit serial addition sums. This has helped me clarify the baseline performance and strategy-use for the children in my sample. Three types of solution procedures seemed to emerge from the findings in this chapter, counting, the use of number-facts and transformational strategies. Of these, counting seemed to be the most prevalent. However, children were also willing to use some number-facts such as tie-sums and, to a very limited extent, sum to 10 number facts. Transformational strategies such as the use of adjacent to tie-sums, and use of the min reordering strategy seemed to be exploited, but not to any great extent. These issues would be studied in further detail in the next two chapters.

Chapter 6

6. Importing single-digit solution procedures into double-digit decade sums

6. 1 Introduction

In 5.2, I began by looking at the 45 single-digit addition problems mixed with a selection of slightly more complex decade problems. The single-digit problems reflected the type ofchronometric research that is commonly carried out in this area. Results from these single-digit addition problems suggested that when children were doing single-digit problems their solution procedures (as inferred from the time taken to solve the problems) seemed dominated by a combination of three very basic solution methods. These solution methods involved counting procedures, number fact knowledge, and strategic procedures (such as using derived facts) which often seemed to be comprised of both counting and number fact knowledge. The latter are based on the idea that some forms of solution involve redefinition (transformation) of the problem in some way. This chapter will concern the continuity between my earlier findings on children's solution methods for single-digit sums and whether children will apply these solution methods to more complex double-digit situations.

The chapter will begin by summarising existing research on these solution methods used for mental addition. This will involve a discussion on the use of counting-based procedures and procedures involving number fact retrieval (e.g. for tie-sums), as well as more sophisticated *strategic* solution methods that go beyond simple number fact retrieval and counting (such as using derived-fact and decomposition). The introduction will conclude by describing the aims of the current study. In short, the research in this chapter will investigate the extent to which children export the solution methods they use to solve simple addition sums into complex problems to make them more manageable.

So, one vital question here concerns the extent to which children use these strategies when solving other, rather more complex, problems. Are strategies that emerge early in single-digit addition problems also evident in solutions to decade addition problems? The central aim of the present experiment was to investigate the extent to which the above mentioned strategies would be applied to decade problems. The decade problems children received in the study described in Section 5.2 had been designed to look at the general effects of doing complex decade sums. These problems had been chosen to look at how the absolute size of the numbers in a problem would affect the time taken to solve it and the effects on response times of crossing the boundaries between successive decades. However, the choice of actual sums that were suitable for clarifying this issue may not have been optimal for my current interest in revealing the strategies children might be using to make certain types of sums more manageable. The decade problems presented here were designed to reveal the more strategic solution methods that children could use.

The single-digit problems in 5.2 had been mixed in with slightly more complex decade problems. The children were originally presented with decade problems involving a single-digit being added to a double-digit decade problem. These fell into one of nine categories. In brief, the results showed that there was a significant overhead associated with simple addition in higher decades. The extent of this overhead was exaggerated for problems in which the single-digit answer was greater than 10 making the decade version of the problem cross into the next decade range, i.e. problems such as 24+8, where the answer (32) was in the thirties. These problems that crossed over into the next decade range were more difficult for children across all year groups. Sums that crossed this decade range did seem most difficult when they appeared in a larger decade problem (such as 50s or 80s problems). Yet, it was not so much a matter of appearing in a large (50s or 80s) decade that was the issue but *crossing the decade boundary* that made problems difficult.

These decade problems had been chosen as exploratory problems to allow generic understanding of the effect of decades on doing addition sums, i.e. within the constraints of certain categories the actual numbers being added were determined randomly by the program. This randomness of the actual sums that children received meant that it was not possible to target precisely what computations might be occurring for specific types of problems. It did not reveal enough about the underlying solution processes. So, while the earlier study was about looking at a narrow selection of sums, the present interest in looking at computational processes for decade sums demanded more specific comparisons. What would happen when children were faced with types of problems specifically chosen to make their computational methods more visible?

In the following section, I will review the strategies children have available to them when doing single-digit sums and how these might be recruited when children have to solve more complex sums.

6. 1. 1 Beyond single-digit computations

Developmental research in this area of mental arithmetic seems to have been strongly oriented towards the addition of single-digit number facts. In comparison, solving slightly more complex decade addition problems requires more than just fact-retrieval, especially for children. It also requires knowledge of procedures and principles of addition as well as how to use existing number fact knowledge strategically. My earlier results suggested that children could make use of their existing (derived) number fact knowledge (e.g. such as using their knowledge of the tie-sum 7+7 or 6+6 to solve 7+6) to solve other problems. Maybe children could be strategic when faced with rather more complex addition problems that could be solved faster and more efficiently with strategic use of number-fact knowledge. They could make decisions based on their existing knowledge or they could retreat to effective yet primitive counting strategies. What would happen when the problems were of a more complex nature? What, for example, would happen to tie-sum knowledge and the min strategy when children are faced with problems in a decade context?

The following sections look at some of the complex procedures children might utilise when dealing with mental addition beyond the single-digit case. Research on how children solve single-digit sums is useful for anticipating what they might do when confronted with sums involving larger numbers. Real-life everyday mental computation makes greater demands on children's arithmetic ability than do simple single-digit additions (such as a+b). To look at this in greater depth we need to study sums that go one step beyond such single-digit sums. This means looking at sums at the next level of complexity, such as adding a double-digit (decade) number to a single-digit sum (i.e. ab+c). Sums of this type (ab+c) were chosen because of their relevance to everyday mental arithmetic. For example, when adding up numbers in single columns, the totals rapidly start to add up to larger numbers and usually it is a case of single-digits being added to larger numbers e.g. when adding 5+9+7+6 we are sequentially adding 9 to 5, 7 to 14, 6 to 21: i.e. 5+9 =14, 14+7 = 21, 21+6 = 27 and so on. Where possible, and certainly when doing written addition, we tend to reduce larger sums into such single-digit columnar addition in order to make addition easier for us.

So, although it is unlikely that such problems (ab+c) would come up frequently in this isolated form, they are more likely to appear indirectly as a by-product of doing serial addition with single-digits (i.e. when adding strings or columns of single-digit numbers). When confronted with problems of the type ab+c, would children use counting solution procedures, or would they make use of more efficient and strategic procedures?

However, we need to start by reviewing some of the ways in which children can solve simple problems with particular reference to how these solutions might be made available for more complex tasks.

6. 1. 2 Taxonomy of methods for managing mental addition

The results reported in 5.2 suggested that when children solved single-digit sums they used more than one type of solution method. These can be best described as falling under the following three headings:

1) Counting (solution processes built upon incremental counting of successive adjacent numbers on the number line).
2) Retrieval or recall of known facts (solution processes which relied on the retrieval/ recall of number fact knowledge from memory).
3) Strategic deconstructive/ transformational methods (multi-step solution procedures involving breaking a sum down into simpler forms).

The next few sections will look at some of the options open to children when faced with cases of single-digit problems. Inevitably, counting remains a "basic" computational method that will still apply when moving on to complex decade sums. Direct recall of number-fact knowledge seems a somewhat less viable option in this case, i.e. if by this we mean recalling all the possible ab+c combinations that are logically possible. The challenge here is to look at what "strategic" transformations are going to be helpful. Reordering is one type of strategic behaviour. Another arithmetical principle, decomposition, also offers an opening for strategic thinking and thus a way of importing existing strategies into these more difficult problems.

6. 1. 2. 1 Addition with counting-based procedures

To begin with, we need to consider what types of counting methods children have available to them that they can use to solve mental addition problem. There are three aspects of the conduct of counting that invite analysis. First, a decision the child must make regarding the first term of the counting sequence - where to start. Second, a mechanism for incrementing numbers in a principled manner. Third, a decision that must be made about the last term in the counting sequence – where to stop.

The decision regarding the start number for a counting-based addition is the issue that has attracted most research. It has led to a four-way distinction. I will start here by looking at these four types of counting-start options, although the third of them will be revisited in a later section as an example of more strategic computational thinking.

Existing research has shown how children are strategic in managing counting-based solutions to addition problems with two addends (Carpenter and Moser, 1984; Baroody, 1987; Siegler, 1987). Their choices become increasingly efficient as they get older. Four basic counting-start procedures have been reported as available to children:

1) CA (counting all) - This requires counting out all i.e. both of the addends and is the least economical of the counting strategies.
2) CAL (counting all to the larger addend) - This requires counting to the larger addend and then counting on to the smaller addend. It also requires identifying the larger addend and ignoring addend order.
3) COF (counting on from the first addend) - This requires counting on from the first addend regardless of the size of the addends.
4) COL (counting on from the larger addend) - This is the most economical of the counting-based strategies available to children. It requires the child to first identify the larger addend, then count on from it the amount of the smaller addend while keeping track of the total counts. The COL procedure assumes that the order of the addends is irrelevant to their sum.

The most efficient of these is the COL strategy (also referred to as the "minimum addends" or "min" counting strategy) which implicitly assumes that the order of the addends is irrelevant to their sum. While the min procedure can be termed a counting-related "strategy", in a sense it is a "pre-counting" strategy because it requires *reordering* first and then counting. Children must reorder the digits, putting the larger number first and thus, it depends on children's willingness to reorder the digits before beginning to work on counting them. This involves utilising the addition principle of commutativity. For this reason, I shall return to COL (in Section 6. 1. 2. 3) as an example of an addition procedure that involves a "strategic" component.

How rigidly do children adhere to the counting strategies outlined above? In other words, how consistently do they adopt any particular counting strategy? This may not be a straightforward "stage-like" developmental process as they progress from one strategy to the next. Is it always a case of one counting strategy or the other i.e. to what extent do they discriminate between various counting strategies? According to Fuson (1992), children are rather careless about their starting addend for "marks" problems (what they referred to as written subtraction and addition situations including numerals, +, -, and =) as opposed to word-problems. This distinction is important, because different types of problems encourage different types of solution procedures, depending on how abstract or concrete the problem may be. For example, children may verbally count-all (i.e. use CA) beginning with the larger number but without necessarily understanding commutativity. Moreover, they may count all from the larger number on some problems but not others. Carpenter and Moser (1984) found little support for computer models that placed counting-on from first (COF) as a strategy preceding counting-on from larger. Their results suggested that these computer models did not adequately describe children's addition performance in practice. They found most children were using both of these procedures, with no strong order of which procedure was used first.

As far as the children in my own sample were concerned (Chapter 5), for a given (a+b) type, there were no apparent solution time differences between the larger number first case versus the smaller number first case. Results did not suggest that it took children longer to solve single-digit problems, where the larger number came first and vice versa. This reordering issue will be studied in this chapter in greater depth. It seemed that these children were at the stage where they were reordering (COL). This will be the assumption for single-digit problems in the following study. The fact that the "min" strategy was available to them when doing single-digit sums, suggested that it could be carried forward into a decade context – given the use of decomposition.

The second research issue relating to counting concerns execution of the counting sequence itself. That is, progressing through an appropriate sequence of numbers, incrementing from some start point (see discussion above) and progressing to some conclusion number (see discussion to follow). Evidently, doing this effectively depends upon children being confident with the numeral names on the number line.

Otherwise, the significant determinant of progress will be the pace at which children are able to execute that sequence. This, in turn, will be a matter of different times needed to vocalise the relevant words. In relation to the present interest in decade counting, it can be anticipated that counting words required to count sequences within the decades (e.g. forty-one, forty-two and so on) must take longer than simple unit counting (one, two etc). This matter of differential counting speed will be an issue developed later in this chapter.

The third issue relating to counting concerns how the child decides where to stop the sequence of number words. When counting for addition, children need to keep track of where they are at on the (mental) number-line as well as where to stop when they have finished counting. To do this successfully, they would normally rely on some visible marker of number-counted (most typically, their fingers). The point is that, compared to counting from zero, having to count on from any other number requires careful monitoring: since it requires keeping track of both the number to be added (determining the stop value) and the current number in an increasing sequence of counting. It is possible that, for small addends, children could use a kind of auditory subitized counting as "aids" or "props" (such as imposing rhythm on their counting process) to make it easier. While counting by subitizing is well explored for the visual domain (Gelman & Gallistel, 1978; Gallistel & Gelman, 1992; Starkey, Spelke, and Gelman, 1983; Starkey, 1992 & Wynn, 1992), it is possible that small numerosities are also directly perceived in the auditory modality. Cowan's (2001) review suggests this might apply to numbers around 4. In which case short count sequences might be executed such as to stop on a subitized perceptual cue, rather than from the visual monitoring of fingers (or similar external markers).

Evidently, the management of counting is not straightforward. Here, I am mainly concerned with how far this is the preferred procedure when the decade addition sums are tackled. Of the issues discussed above, the most relevant are the COL start procedure (to be returned to as a species of addition strategy) and the differential pace of counting associated with different number words (used here as a basis for determining whether decomposition is likely to have occurred).

6. 1. 2. 2 Addition based upon number-fact knowledge

Another basic computational resource available to children is memory, i.e. remembering specific addition number-bonds. Certain number-facts have been found to appear early in arithmetic development. To a great extent, the results for single-digit sums reported in Chapter 5 supported existing findings in this area. However, the results in 5.2 suggest that, contrary to existing findings (Aiken and Williams, 1973; Krueger and Hallford, 1984), sum-to-10 problems (with the exception of 5+5 and 9+1) did not seem to be particularly salient for these children. Yet the findings reported in 5.2 showed that some number-fact problems did take less time to solve than others, in particular, tie-sums (e.g. 6+6, 7+7, 8+8, and 9+9) took less time to solve than other similar sized sums.

But what would happen to this use of number-fact knowledge if the numbers being added became larger than single-digits? The results from the decade sums in 5.2 showed that when a tie-sum appeared in a decade context (e.g. 24+4) it also had a shorter RT regardless of whether it appeared in a larger decade such as 80+n or 50+n. This seemed to indicate that children were recognising tie number facts and making use of this knowledge in decade context. This also implied that they were making use of decomposition, because they were recognising that the problem could be broken down or decomposed into 20+4+4. The aim in 5.2 was to start by looking at children's existing number-fact knowledge, i.e. the 45 single-digit addition number facts as well as an exploratory selection of decade sums. In 5.2, the initial data had suggested that some number facts such as ties were more salient than others. But because the decade sums that involved tie-sums only looked at smaller tie-sums (those with an answer less than 10), it was uncertain whether this would still be the case if the decade involved a larger tie sum (with an answer greater than 10), e.g. 26+6. Would they still make use of their existing number fact knowledge and use a strategy that would also involve decomposition i.e. do 20+6+6? Or would they resort to an incremental COL strategy?

It must be noted that even when counting occurs, it can be taken over by other procedures such as recall of number facts because solution procedures are likely to vary with the particular numbers in a problem. Since smaller number facts are learnt before larger number facts, a recalled fact solution is more likely on problems containing small numbers (Fuson, 1992). So, a child who is likely to use counting procedures on most larger sums would still rely on number-fact knowledge on smaller sums, and then fall back to counting on other larger sums. In addition, some researchers have suggested that knowledge of a particular number triad (e.g. 7, 5, 12) might allow children to solve a more difficult problem type than they can solve without this knowledge of the number triad (Carpenter and Moser, 1984). Although this highlights the occasionally arbitrary nature of number fact knowledge, it also means that such knowledge can be a potentially powerful tool when doing larger problems. For example, a number triad (such as 7, 5, 12) can be embedded into other addition (and subtraction) problems. It would mean that whenever this factual knowledge (or other similar knowledge) appeared, it would be recognised and exploited. It can also appear embedded in more complex (decade) problems such as 37+5 if the child recognises that this also involves a 7+5 problem, and can therefore allow the child to do other sums derived from this factual knowledge. But this will only be beneficial if strategies such as decomposition are being used to solve a problem.

It seems unlikely that ax+n problems would benefit from number-fact knowledge directly. That is, it seems unlikely that such a large number of potential bonds would lead to many pairings that were directly recalled in this sense. If number-fact knowledge enters into such decade problems then it will surely be in relation to the single-digit pairings that might arise from an initial step of decomposing ax+n. That is, the concern of the empirical work reported below.

6. 1. 2. 3 Addition involving "strategic" solution procedures

Most smaller single-digit sums can be solved without too much difficulty by even somewhat inefficient counting strategies. Even those such as COF and CA. Perhaps having to solve more difficult sums forces children into being more strategic. When children have to do sums with larger numbers, the useful counting and derived-fact strategies described above will only come into effect if children are using decomposition. When doing decade sums, knowing that 7+7 = 14 is only helpful if you first recognise that 27+7 is the same as 20+7+7.

A more strategic use of this when solving unknown combinations of numbers is referred to as using "derived facts" which means using a known number fact to solve a problem via a transformation. It is a way of being "strategic" with existing number knowledge in order to do difficult sums on which we cannot reliably use retrieval. This can include a strategic combination of counting and number fact retrieval or a type of "smart counting". Use of derived-facts relies on solution processes that depend on retrieval from memory of a known number-fact. For example, using the known tie-sum 7+7 = 14 to solve 7+6. This works because it combines number-fact knowledge (7+7) with number-line knowledge (7+6 is one less than 14). This is an example of children being strategic in order to make sums more manageable. It illustrates what can occur between purely counting-based solutions and the use of number fact retrieval.

This is an area that seems to have been largely neglected by existing research. Fuson (1992) pointed out that at some stage children stop direct modelling (of the above mentioned counting strategies) and use "abbreviated counting strategies" flexibly. An example of abbreviated sequence procedures is that involving derived fact procedures in which the numbers in a problem are redistributed to become numbers whose sum or difference is already known.

Children also use derived fact strategies in which one addend and the sum are added to (or even subtracted from) in order to change those numbers into a known fact or sum, e.g. the sum 8+6 is 6+6 (+2) = 12+2 = 14 (using the known doubles fact 6+6 = 12). This is an example of children deploying their number knowledge in an elaborate way thus illustrating that even seemingly basic number-fact knowledge can be made richer and more creative. Relatively little research has been done on the conceptual structures required by different derived facts, or on the developmental relationships between sequence counting strategies and derived facts.

Counting and using number-fact knowledge are two of the procedures that are available to children when doing mental addition. Within these, children can be strategic as is demonstrated by reordering and using derived facts. But, in addition, there is another way of creatively exploiting both counting and retrieval that is more flexible than either counting or derived-facts alone, especially when solving more complex problems. Decomposition is a strategic procedure that affords both counting and retrieval but it is also one that requires greater knowledge and recognition of arithmetical principles. But before going on to look at decomposition in more detail, we need to consider what it means to be "strategic".

The process of drawing loose boundaries between "strategies" is not a straightforward one. How do we define what constitutes a "strategy" or "strategic behaviour"? Using derived-facts, or reordering numbers are more obvious examples of being strategic. For example, reordering 4+7 into 7+4 is a transformation that is strategic. Although this type of sum may eventually be solved by counting, the process of reordering the numbers to begin with the larger number first is an example of being strategic to make sums more manageable.

Therefore, single-digit mental addition problems can be solved by children by 1) counting 2) recalling number-facts, and 3) using transformation strategies. These transformations involve either exploiting number-fact knowledge and transforming a large addition problem into more manageable smaller ones, or on transformations based upon the principles of arithmetic (e.g. commutativity). In brief, this means that knowledge of number-bonds/ facts and knowledge of arithmetical principles can be strategically deployed to create transformations of difficult problems. These are "strategic" moves because they reduce difficult problems to short sequences of easy problems where "easy" entails the option of then applying counting and/ or number-fact knowledge directly.

So while counting and retrieval can be seen as fairly simple solutions, children could also have the ability to be strategic when dealing with difficult sums. The results in reported in Section 5.1 suggested that the transformational strategies that some children seemed to be using were a type of method which involved the use of both number-fact knowledge and counting i.e. a method involving active manipulation/ transformation of the sum. Of these solution procedures, the identified counting strategies (finger or mental) would be the most easily transferable to decade sums. But although the nature of single-digit addition meant that children doing such sums could rely on retrieval from memory as a solution procedure, this direct retrieval (of either number-facts or using the derived-facts strategy) is unlikely to be an option available for children when doing larger decade sums such as ab+c, unless they use decomposition. That is because in decade sums retrieval from memory only appears as a solution in relation to single-digit sums like 7+4 or 6+6 but only if the children were using more strategic procedures like decomposition, procedures which require breaking down complex problems into more simple ones. All of the strategic solution processes described can be used to help solve decade sums, but only when used in conjunction with decomposition.

Thus counting and even number-fact knowledge can be easy to transfer to decade sums, but to be truly effective i.e. "strategic" they require that the child be able to use decomposition. The sums chosen for the study below were designed to look at all three of the solution processes described above, i.e. to what extent would children be appropriating their single-digit solutions into solving complex decade problems?

Solution processes such as counting and derived-facts can be used together, while using the more flexible procedure of decomposition that is so inherent to using that existing knowledge. Decomposition allows more strategic manipulation of numbers and makes larger sums such as decade problems easier to solve than if only counting or retrieval-based procedures were used. But to what extent do children make use of this potentially powerful solution procedure? Children's use of decomposition is examined more thoroughly in the next section.

6. 1. 3 Development of decomposition strategies

Addition problems in a decade context can be solved more efficiently by using strategies that involve recognising properties of addition such as commutativity and decomposition. To what extent, if at all, do children make creative use of such addition principles? First, we need to consider the relevant properties of number and the principles of addition.

One principle of addition is additive composition. Addition of natural numbers has the property of closure because the sum of any set of natural numbers is a natural number i.e. any natural number (except for one) is the sum of other natural numbers (Cowan, 1999). According to Resnick (1983, 1986), children have intuitive knowledge of this additive composition. Decomposition strategies for solving addition problems require knowledge or awareness of additive decomposition e.g. 28+15=20+8+10+5. When solving a problem such as 37+5, if children do not decompose it into 30+7+5 then they will have to resort to counting, even though such a problem might be best solved through decomposition. Evidence suggests that children have knowledge of decomposition strategies. Putnam, deBettencourt, and Leinhardt (1990) found that children could give adequate explanations of derived-fact strategies for addition.

However, research suggests use of decomposition is uncommon. For example, Renton (1992) interviewed 6-10 year old children after they had done some sums to find out what strategies they could use and found that most children were able to show how to use counting strategies on the sums on which they said they used retrieval. Many also showed they could have used retrieval on problems on which they used counting strategies. However, very few of the children, even among the older ones, revealed that they could use decomposition to solve a problem.

According to Cowan (1999), this may not necessarily be due to ignorance of additive composition, because children are capable, in principle, of using and distinguishing a variety of strategies. Rather, this may be due to reluctance to make use of them, i.e. possessing knowledge about something is not a guarantee that it will be used.

Children can recognise the commutative property of addition fairly early on. Cowan and Renton (1996) demonstrated the relation between children's knowledge of commutativity (i.e. that order of addend does not affect the results of addition) and their use of counting strategies in addition. They found that children's methods of solving addition problems progressed towards optimal strategies (such as "min") that reverse the order of addends so that they have to do less counting. Most children between 6 and 10 years predicted that order of the addends would not change added quantity. They also tended to use strategies that reversed addend order. Renton and Cowan also found that 5 year olds were more likely to know commutativity than to actually use a strategy that reversed addend order to answer sums correctly. They concluded that children expect addition to be commutative before they actually start using strategies that disregard addend order, i.e. knowing commutativity precedes using strategies that presuppose it and does not derive from doing sums. Thus, while children can show awareness of principles of addition, putting this knowledge into practice may be a more difficult task.

Why should this be the case? For some children, whether or not the knowledge will get used might have something to do with attitude rather than just finding it difficult to put into practice. So even knowing about efficient strategies may not predispose some children to make use of them, because of their belief than this knowledge will not *guarantee* an accurate answer. That is, some children are unwilling to risk an inaccurate answer. Siegler (1988) found a group of children who showed good arithmetic knowledge but who resorted to counting rather than relying on more efficient strategies. Siegler referred to these children as "perfectionists" and suggested that they had to be extremely confident to rely on their strategic knowledge. While most children might know the principles of addition, they could be reluctant to actually use them. They were more likely to use reliable strategies that would give them accurate results than risk using newer ones that may lead to errors. This could be a possible explanation of children's cautious attitude in relation to decomposition strategies.

So, because children's use of decomposition cannot be taken for granted, we need more information on what situations prompt its use. In this study, I looked at how the strategies children used in a simple addition context would get used when they had to solve sums in more complex decade contexts. However, this could only be done effectively if the children were prepared to use decomposition-based strategies and it seemed uncertain that they would. The earlier sections of this chapter looked at children's development of basic procedures such as counting and number-fact knowledge and later sections moved towards children's use of increasingly advanced strategic solutions such decomposition. However, these sections were based upon children being strategic when solving single-digit sums. The next section will look at how the strategies discussed in these sections might get recruited into solving more complex decade sums.

6. 1. 4 Decade addition

Much of the research on children's mental addition has been concerned with simple single-digit problems. What occurs when children start adding decade sums? Investigating this would involve taking some of the solution possibilities described earlier and exploring them in a decade context. So, I used the following set of problems: Problem Type A in order to look at the relevance of *counting* following decomposition, Problem Type B to look at the relevance of *number-fact knowledge*, and Problem Type C to look at the issue of addend *reordering* (a strategic transformation) following decomposition. For every type of sum there might be two solution possibilities. These processes would involve either 1) counting or 2) a strategic solution process arising from decomposition. Problem Type A would be looking at decomposition and incremental counting, Problem Type B would be looking at decomposition and tie-sum number-facts and Problem C would be looking at decomposition and the addend reordering strategy. The basic counting-on strategy would be the other solution possibility on all the different types of sums.

In Sections 6.1.2. and 6.1.3 I looked at some of the basic solution procedures children have available to them when doing mental addition, from the use of counting-based procedures to more strategic transformational procedures such as the use of derived-fact knowledge. The challenge in this study was to discover whether children will exploit solution procedures via decomposition, in order to utilise their existing achievements. So, for example, when doing decade addition, they could use a combination of decomposition and strategic transformations e.g. 37+4 transformed from 34+7. First, I will review some of the existing research on decade addition.

6. 1. 4. 1 Existing research on decade addition problems

When research on mental addition problems has involved the addition of larger number problems, this has usually been in the context of working memory research (Hitch, 1978; Logie et al, 1994; Gathercole and Pickering, 2000). Such research has looked mainly at the role of the central executive and visuo-spatial components of working memory in performing the calculations required for mental addition. For example, Hitch (1978) used mental arithmetic to look at the relationship between operations and storage. He found that errors increased as the number of operands that had to be held in working memory increased (as opposed to having them written on paper). He also found that errors increased when subjects were required to write their answers in reverse order (hundreds, tens and units) because this would impose a delay on the reporting of some of the digits. In addition, response times and errors also increased as the number of embedded carry operations increased. These findings were used to indicate how working memory capacity was utilised: by having to hold more information in the working memory system, holding that information for a longer time and having to execute more steps or operations within working memory. Similarly Logie et al (1994) looked at the working memory components responsible for holding arithmetical operations. In their experiment, subjects had to do sums in which they were required to keep a running total of the sum in their mind while doing various distracter tasks. So, while researchers have used arithmetic processing to look more closely at working memory processes, they have not looked at the nature of the strategic processes spontaneously selected to execute such mental arithmetic.

Other chronometric research that is more relevant to our present interest and which looks at the mental addition of larger numbers has been carried out on adults by Widaman, Geary, Cormier and Little (1989) using a true/false verification paradigm. Widaman et al (1989) carried out a chronometric analysis of their general model of addition processing to validate the argument that processes involved in more complex forms of addition (such as the time taken to carry from the units to the tens column) were not adequately represented by models for simple arithmetic processing. They identified the need for models of mental processing that were richer than ones designed to account for the processing of simple single-digit number facts. They looked at four types of addition problems: sums involving two single-digit addends (e.g. 4+5 = 9), sums involving three-single-digit addends (e.g. 3+7+2 = 12), sums involving one single-digit and one double-digit addend (e.g. 34+7 = 41) and those involving two double-digit addends (e.g. 86+43 = 129). They found a substantial temporal overhead associated with doing problems involving a "carry" operation (referred to in the current research as "crossing the decade boundary") and they claimed that these findings revealed the extra demand added by each component of the problem.

Widaman et al (1992) studied children from the second, fourth and sixth grade as well as college students doing a selection of simple single-digit (e.g. 4+5 = 9) problems and a selection of complex double-digit problems (e.g. 86+43 = 129). They then fitted their componential models to the RT data from each individual and classified individuals according to whether they used a computational strategy or a retrieval strategy. Their results suggested that individuals seemed to be using the same strategies for the complex addition problems as they were for the simple addition problems. However, they do stress that this does not suggest that students at any grade were a homogenous group regarding strategy use. Because Widaman et al did not look at strategy use information for each problem, some individuals may have used retrieval as a solution to most problems and then used slower "back-up" counting processes for some problems when retrieval failed to produce the right answer.

Widaman's study is significant because it shows the straightforward relationship between arithmetical complexity (the "steps" or components" involved in larger sums) and the solution times. While Widaman et al concentrate on how longer sums take extra time because they have more "components" and therefore require some kind of "extra" processing, they do not consider just why having to do sums with more steps should take longer. So Widaman's study is useful because it shows that adding steps to a problem increases complexity; this, in turn, increases solution time which is probably due to the strategy being used.

6. 1. 4. 2 The mental demand of adding a "decade" component

The results reported in section 5.2 showed that, although doing decade addition sums was a manageable task for most children, doing such sums did take considerably longer. There was an overhead involved in doing decade sums; i.e. a sum like 87+5 becomes extremely difficult very rapidly. Solving such a problem without resorting to counting would require not only decomposing it but recomposing it as well, and that in turn would require a confidence in one's own numerical ability that may be beyond many children. Perhaps it is not surprising that some of them resort to counting, which in their experience is the most reliable method for producing an accurate answer. Strategies such as commutativity and reordering that require applying, as well as recognising addition principles, take up mental computational resources that children may be unwilling to invest. Adding extra steps to a problem can make it potentially easier to solve because it means breaking the problem down into simpler components that are quicker to deal with, with the results being brought together at the end. However, it may be that this very aspect of decomposition makes it harder for children to use. Perhaps because the more powerful solutions offered by decomposition are not taken up, such sums (e.g. 87+5) are seen as daunting and children fall back to reliable yet less efficient counting strategies. If children perceive a problem as being difficult enough because it involves a large number (for them more steps even if counting) then the prospect of decomposition (adding yet more steps) becomes even more unappealing for some children.

As my previous RT results suggested, adding a decade component may place considerable demand on children's mental arithmetic capabilities. The type of decade problems that will be discussed in this chapter will be those where a single digit is added to a double digit e.g. 24 + 5 as opposed to two double digit addends e.g. 24 + 23, since the latter are likely to require different types of processing and solution methods. Wolters, Beishuizen, Broers, and Knoppert (1990) presented 8-9 year olds children with (addition and subtraction) problems with sums > 20 and < 100. They studied children divided into groups depending on which of two solution procedures (as distinguished by Beishuizen, 1985) that they used most consistently. One was the "10-10" procedure (decomposition) and the "N-10" procedure. In the "10-10" procedure, both addends are spilt into tens and units which are added separately, and then combined again (e.g. 47+22: 40+20 = 60, 7+2 = 9, 60+9 = 69). In the "N-10" procedure, only the second addend is split up and the tens and units are added-on to the unsplit first addend (e.g. 47+22: 47+20 = 67, 67+2 = 69). Since the 10-10 method requires at least one more solution step than the N-10 method, use of this method should result in longer solution times. In addition, they also predicted that if the 10-10 method requires more procedures, and the problem difficulty increases (e.g. on problems such as 48+26 as compared to problems like 34+5), then the difference in the number of sub-problems required by the solution method also increases. Thus, they predicted that the increases in solution time with increasingly difficult problems will be smaller for children using the N-10 procedure than for those using the 10-10 procedure.

They argued that such difficult problems are calculated by using procedures in which the problem is broken down into subproblems for which solutions are retrieved from a declarative knowledge base. Their research was based on the view that most arithmetic procedures (e.g. for adding multiplying or dividing large numbers) consist of algorithms in which the problem is broken down into a series of subproblems at the declarative level with these subproblems being processed and solved serially.

Likewise mental calculation arithmetic procedures and problem variations requiring more solution steps will result in a larger memory load which will result in longer solution times. Therefore, increases in solution time will occur because each serially performed step takes time and because the temporary storage of data in working memory may lead to interference and confusion; correction of this also takes time. Thus, procedures requiring a greater number of subproblems will result in longer solution times, and procedures requiring less subproblems lead to faster solution times. Wolters et al's (1990) results were based on the assumption that performance in mental arithmetic is determined by characteristics of working memory (Baddeley and Hitch, 1974 and see section 4. 1). That is, because each operation takes time, arithmetic procedures and problems that require more sub-problems take longer to solve. In addition, with larger problems result in an increasing load on storage capacity and as a result there is a greater chance of interference and forgetting which lead to errors and additional time to correct them. They concluded that the increasing difference between solution procedures with increasing problem complexity is mainly due to the different number of subproblems and consequent differences in memory load.

Therefore, decomposing a problem into yet more steps may be something children, especially younger ones, find more daunting than simply counting, because it involves more solution steps. Where children are concerned, decomposition requires more intellectual effort than counting and so may be something to be avoided. If children are approaching these problems in a relatively mechanical way, then decomposition may not seem like an easier option. For many children, the idea of increasing the number of solution steps may seem too much effort, compared to counting. This is relevant to the research in the current study, because decomposition is the central issue of this chapter. A reluctance to use or attempt to use decomposition on decade sums would mean that children perceive decomposition as being more difficult or less efficient than counting. Perhaps because, for children, decomposition involves breaking the sum down into too many steps where counting does not.

To conclude, there is a lack of research that reveals precisely what computations may occur when children do addition involving larger numbers, especially addition of important ab+c type sums. Therefore, the focus in the following study will be on how the solution procedures children use for single-digit sums might get used for decade sums. For example a single-digit sum like 4+7 can be solved by retrieval or counting whereas a sum like 34+7 can solved by counting alone or, more efficiently, by decomposition plus reordering i.e. [(7+4)+30] and this would be reflected in the RT.

6. 1. 5 Concerns of the present study

6. 1. 5. 1 Central aim

The patterns that emerged from the results discussed in the previous chapter helped decide the problems that were chosen for the present study. In the study described in this chapter, children were presented with more specialised decade problems. These problems were designed to investigate when children start using their strategic knowledge of decomposition and decomposition-based counting/retrieval strategies to solve decade sums. Since my earlier results had shown a similarity between the performance of children from Years 5 and 6, I decided to limit my sample to children only from year 5 as the older age group.

In brief, earlier results from children doing decade sums showed that there was a significant overhead associated with simple addition in higher decades. The extent of this overhead was exaggerated for problems that crossed the decade boundaries. Problems that crossed the decade boundaries were more difficult for children across all year groups. Sums which crossed the decade boundary were most difficult when they appeared in a larger decade problem (such as 50s or 80s problems). However, one of the constraints of this study was that actual digits within each type of decade problem had been randomly selected and so the children may have been solving different problems e.g. for a sum whose answer crossed the decade boundary one child could have received 89+ 2 and another child could have received 89+8. Because the choice of problems in the earlier study was not systematic, it did not allow conclusions about actual computational procedures. However, these results had been useful in identifying the possible patterns and I now needed to look at more focused decade problems.

The problems in the research described in this chapter were chosen in order to enable a more confident claim to be made about children's strategies for decade problems. Problems would be of the type a+b (units or single-digit sums) or of the type xa+b (decade sum) and the analysis would always look at the RT "overhead", i.e. with any increase in RT being associated with "xa" as opposed to "a". This would reveal any step-like processes underlying the solution of decade problems, as inferred from the time taken to solve these problems. A longer time taken to solve a given problem would suggest that it is being solved through a less efficient (counting) strategy than a type of decomposition strategy or retrieval strategy which should have faster RTs. This meant presenting the children with a carefully chosen set of decade problems. As in 5.1, what would be analysed here was the extra load involved in doing certain decade problems because RTs on such problems should tease out any emerging computational patterns that would show whether or not children can use existing single-digit number knowledge to their advantage. This "cognitive load" or "temporal overhead" would be the extra time taken to do a decade sum as compared to its single-digit counterpart, i. e the RT for 37+4 minus the RT for 7+4. By doing this, I would be comparing the decade solution time with the baseline single-digit solution time, and the same would be done for tie-sums e.g. the RT for 26+6 minus the RT for 6+6.

The results would enable me to discover where the uptake of decomposition occurred, since the problems would be presented in a way that should facilitate the emergence of newer more efficient strategies. For example, would children solve a decade sum incorporating a tie number-fact through decomposition? If they were to use decomposition, then they could use their number-fact knowledge and this would be reflected in their RTs. Likewise, if a sum had smaller addends would they use decomposition or a counting-on strategy? If a sum was chosen so that it was more easily solved by using a decomposition-based strategy, would they use decomposition or would they resort to counting-on?

The aim in this study was to look at children importing strategic solution processes from single-digit sums into decade problems of the sort ab+c. This involved looking at how children would use the following:

1) Counting (incremental counting-on procedures) on problems with smaller addends.
2) Number fact knowledge (based on number-fact retrieval) such as knowledge of tie-sums whereby the only way that these solution processes can be imported successfully into a decade context is if they make use of decomposition.
3) Decomposition and addend reordering.

In light of this, three types of decade problems were chosen:

1) Problem type A designed to look at solution strategies on sums with smaller addends chosen to facilitate decomposition plus counting.
2) Problem type B designed to look at decomposition and the use of number-facts such as tie-sums.
3) Problem type C designed to look at the possible use of decomposition and reordering strategies.

Within each of the three sets of problems, children would receive a decade sum and its corresponding units/single-digit sum.

The particular problems (described in Table 6.2) were chosen after the results of the data in section 5.1 was analysed and revealed certain patterns. These problems would help pinpoint exactly where the difficulties lay; which combinations of numbers seemed to take the longest, and for which age groups. This allowed me to follow on and clarify the nature of the computational overhead associated with simple decade addition.

Type A problems were chosen to provide children with baseline sums that were simple enough to solve by counting or decomposition plus counting and would be helpful in discovering how children would solve such sums.

In problems of the sort in Type B, would children transfer their number fact knowledge (see 5.1) to decades as in recognising that the sums 26+6 or 23+3 involve tie-sums? Would this be seen so that this problem becomes relatively easy for them (faster to solve) or would they persist in counting up from 26 (longer solution time)? If both 23+3 and 26+6 are solved using decomposition then the RT overhead for both sums should be similar.

In Problem Type B, it meant looking at the specific issue of how crossing the decade boundary would affect the RT when the sums involved include a tie-sum (one which had been found to be a salient number fact), providing everything else remained constant. Given a sum like 26+6 and 6+6 and a sum like 23+3 and 3+3, the children would either be doing 1) some type of mechanical counting or 2) doing decomposition on the decade then adding on the single-digit sum (which may or may not have been counted).

If children were using a sophisticated decomposition strategy, e.g. transforming the decade problem 26+6 into [20+(6+6)] then in decade problems involving both a large and a small tie-sum (23 +3 and 26 +6) the computational overhead for the decomposition would be constant. That means that the RT for 26+6 would be greater than 6+6 and the RT for 23+3 would be greater than 3+3 but that the RT difference between the two types of sum should be constant. For example, if the RT for 3+3 was 3 seconds and the RT for 23+3 was 6 seconds and the RT for 6+6 was 4 seconds then the RT for 26+6 should be around 7 seconds. This would suggest that the two types of sums were being solved in a similar way, i.e. that the single-digit method (whatever it was) was being imported into the decade case. Would this be the case here? How far would these emerging strategies be used when children had to do decade addition problems that were specifically designed to encourage flexible strategy use?

When presented with another set of problems (Type C) that were complex enough to make counting possible but not necessarily as the optimal solution strategy, would children use decomposition to solve an addition problem such as 34+7 breaking it down into 30+(4+7) or even more optimally into 30+(7+4)? Or would they resort to mental or finger counting using the min or COL strategy? If they are using decomposition for a problem such as the one mentioned above, and the strategy of adding the larger number first is imported from the single-digit sums (7+4 instead of 4+7), then the problem 37+4 should not have a significantly longer overhead than 34+7. On the other hand, if they were relying primarily on counting then 34+7 would have a significantly longer overhead than 37+4 because the minimum addend counting model specifies that the RT increases in proportion to the size of the smaller addend.

6. 1. 5. 2 Role of presentation format

Presentation format, i.e. the visual format in which the participants received the actual sums (columns versus rows) was another issue that needed to be explored. Although this was not something their own research had investigated, Widaman and Little (1992) identified presentation format as a possible influence on mental arithmetic (in most of their own research they presented addition problems in a columnar form). According to Widaman and Little (1992), it would be interesting to discover whether there are detectable effects of presentation format on RT data for a given operation such as addition as well as finding out the effects of presentation format on other operations such as multiplication, subtraction and division.

The previous studies (in sections 5.1 and 5.2) had used a row format (e.g. 23 + 4 = 27). However, a columnar format could have had an influence on RTs since this is likely to be the one that children are most familiar with. A columnar presentation might encourage children to be more strategic than a row format. For example, seeing problems in a columnar format might invite a decomposition strategy more readily than seeing problems in a row, i.e. seeing the numbers visually lined up may encourage doing the single digit component first and then adding on the decade. For example, for 45+2 doing 5+2 first and then adding on 40. This may or may not involve some recognition of the principle of decomposition.

Therefore, presentation format was also manipulated in this study. Alternatively, it was possible that seeing larger decade problems in a horizontal/row format could lead to even the older children abandoning any emerging strategic behaviour and reverting to counting. This might be because such problems may be seen as problems in a more unfamiliar context and they would use a more reliable counting strategy.

In summary, the central aim of this study was to make strategic comparisons between a decade problem and its corresponding units problem and to investigate whether the computational overhead was or was not found to be constant. This would enable me to find out just what specific types of problems trigger decomposition.

6. 2 Method

6. 2. 1 Participants

A total of 89 participants were recruited from two schools (for more detail see Section 4. 4. 1 and 4. 4. 4).

Table 6. 1

Year	**Number**	**Male**	**Female**
3	34	17	17
4	21	10	11
5	34	14	20
Total no. of participants	89	41	48

All the children were familiar with using a mouse and graphical computer interface. These are the numbers of children who took part in the study and completed all the sums. However, there were cases where a child's data for a particular sum or number combination was lost as a result of a computer error and would not therefore be available for analysis. In these cases the child's data for the rest of the sums he/she completed would still be included in the analysis.

6. 2. 2 Tasks

The participants were presented with decade problems as well as the corresponding single-digit problems. The program (as described in 4. 4. 3) was configured to deliver the following sorts of problems:

Table 6. 2

Problem type	**Description**	**Decade Problem**	**Single-unit problem**
A	Designed to look at solution procedures that typically involve counting	45 + 1 45 + 2 45 + 3 45 + 4	5 + 1 5 + 2 5 + 3 5 + 4
B (tie-sum number facts)	Designed to look at decomposition and the use of number-fact knowledge (tie-sums)	23+3 26+6	3+3 6+6
C: 1 (answer crosses the decade boundary)	Designed to look at decomposition and the min strategy	34 + 7 37 + 4	4 + 7 7 + 4
C: 2 (answer does not cross decade boundary)	Designed to look at decomposition and the min strategy	33 + 6 36 + 3	3 + 6 6 + 3

It was planned that each participant receive a total of 22 problems. However, due to a computer error each participant received 21 problems because the single-unit problem (6+5) from a further Problem type C (derived number facts) was not presented although children still received the corresponding decade sum 26+5. These Type C (derived number-fact) problems had been chosen as an example of a problem which included an adjacent-to-tie sum but this was not included in the analysis because the corresponding single-digit sum was not included in the task.

The decade values themselves were varied because using the same decade continuously throughout the task would give an "unnatural" or "artificial" feel to the series. Children might recognise this and, as a result, might end up using the same strategy across all the problems. Using a variety of decades (e.g. 20s, 30s and 40s) would provide a more diverse and interesting range of sums than just using one decade alone.

For all types of problems, it was assumed that children would use one of two solution processes. One would be counting. The other would be decomposition plus "another process" which for Problems Type B would be decomposition plus use of tie-sum number-fact knowledge, and for Problem type C would be decomposition plus reordering.

Problems in Type A (45+n, and 5+n) were chosen to investigate the strategies children would use when counting would be expected, i.e. would they use incremental decade counting, or a combination of decomposition and unit counting. An incremental increase in RT as the smaller addend became larger would suggest that a counting strategy was being used. If the RT difference (overhead) was not constant, then decomposition plus unit counting was being used, i.e. the children were recognising 5+n and adding on 40. Indeed, for all the problems a constant overhead would suggest decomposition was being used, i.e. the comparison for all sums was about looking for a constant difference in RT.

Type B (tie-sum, number-fact) problems were chosen to explore whether children would use decomposition and knowledge of number-facts. The assumption here was that if the children were using decomposition then both 23+3 and 26+6 would have similar overheads if both the 6+6 and 3+3 tie-sums were recognised.

Type C problems were chosen to find out whether children would use a counting solution or whether they would import their knowledge of the min strategy into a decade context and use a decomposition strategy. Group C1 problems were chosen to look at the min strategy in sums which crossed the decade boundary (e.g. 34 + 7 = 41). Would they break 34+7 into 30+4+7 and 37+4 into 30+7+4 or 30+11, regardless of whether the smaller or larger number appears first. Or would they simply resort to a mental or finger counting strategy, in which case it should take them longer to do sums where the smaller unit (34/33) appears first? There should be no difference between overheads for 34+7 and 37+4 except for the constant overhead. If a difference was found, then this would be due to children not using decomposition but relying on counting. The problems in Group C2 were chosen to look at the min strategy in sums which did not cross the decade boundary (e.g. 33 + 6 = 39). There should be no significant difference between the overheads for 33+6 and 36+3 i. e the overhead should be constant unless counting was occurring. In each case, the digit order was reversed so that children received problems where the larger digit came first and problems where the larger digit came second. For both Group C1 and Group C2 problems, a similar overhead would suggest these decade sums were being solved in a similar way to the single-digit sums.

All of the problems were chosen not to make them too difficult or demanding for any child while making certain they were challenging enough to prevent children from getting bored. To minimise "unnecessary" demand on their abilities the problems chosen were in the low 30s and 40s range especially because the research in 5.1 had found that decade sums in the higher decades such as 80s were quite difficult for younger children. The results in 5.1. had also shown that there was not a big difference between 20s and 50s decades. The method for dealing with errors was as identified in 5.1.

Half of the participants in each year group received all the problems written horizontally in a row on the screen i.e. 45 + 1 and the other half received problems written in vertically in columns in the following format:

$$\begin{array}{r} 45 \\ + \quad 1 \end{array}$$

Whether each participant saw a horizontally (columnar) or a vertically (row) presented problem was determined randomly by the software. Fifty children received problems in a horizontal format and thirty-nine children received problems in a vertical format. In Year 3, 20 children received the horizontal format of the problems and 14 received the vertical format. In Year 4, 12 children received the horizontal format of the problems and 9 received the vertical format. In Year 5, 18 children received the horizontal format of the problems and 16 received the vertical format.

6. 2. 3 Procedure

The procedure was similar to that in 5.2 and 5.3 and followed the general pattern described in 4.2.

The computers on which the children were tested were usually kept in a corner of the classroom. The task required each child to come over to the computer and do the arithmetic problems for about 15-20 minutes. All participants were supervised while doing the task.

The participant would see his/her name as a button on the screen and clicked on it. He/she was then presented with the set of problems generated in a random order. The problem appeared on the screen along with a "got it" button. The participant was instructed to click on "got it" when he/she had come up with the answer. The participant then saw a screen with a small number pad (numbered 0-9), an "OK" button and a "reset" button. The original problem disappeared from the screen but it would be brought back by clicking on "reset" if it was forgotten, "reset" could also be used if the answer entered was incorrect.

The participant selected the answer from the number pad by clicking on the number buttons and, when satisfied with the answer, clicked on "OK". The time taken to solve the problem would then flash on the screen e.g. "correct answer in 4 seconds" or if the answer was incorrect the word "oops!". This was incorporated into the program to provide the child with instant feedback on progress plus added incentive to do better. Errors were dealt with as described in Section 5.1.2.3. That is, the program recorded errors and presented that sum a second time at a random point within the remaining sequence. The time recorded for that case would be the sum of the two response times thus giving a longer time. Errors were thus incorporated in the analysis of the results. For this reason i. e. to deal with the outlying RTs, the logarithms of the RTs were calculated for each sum and these were analysed. Error rates for all sums can be found in Appendix 2: Table 3.

6. 3 Results

The following results all involve graphs describing either absolute solution times, or the difference in RT between a decade problem and its corresponding single-digit problem. This latter will be referred to as the solution overhead. All graphs plot the median RT or median RT overhead. The issue of sum format was included as a variable in all the analyses, but it did not have a significant effect on response times for any sums in this study. The means and standard deviations can be found in Appendix 3. Where post hoc tests are described, the results are based on the Tukey HSD post hoc analysis.

6. 3. 1 Results for sums in Problem Type A (computing a small addend)

Figure 6.1 is a summary of the results for the single-digit sums 5+n.

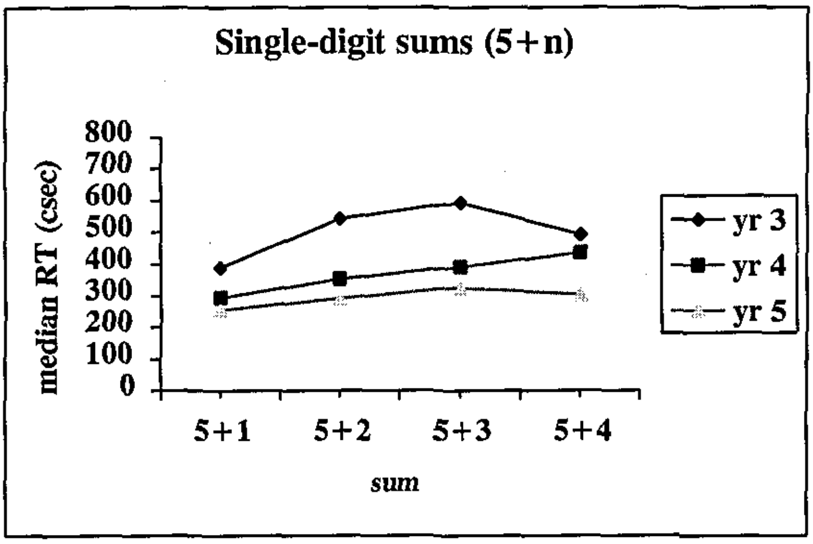

Figure 6.1 Solution times for the single-digit addition problems 5+n.

6. 3. 1. 1 Analysis (5+n)

An ANOVA on these results showed that there was a significant overall effect of year, $F(2, 83) = 19.72$, $p < 0.05$. There was no effect of format, $F(1, 83) = 0.09$, and there was no interaction between year and format, $F(2, 83) = 2.60$. As the graph illustrates, the younger children are taking significantly more time when doing even simple sums and this shows a developmental trend that would be expected. There was a significant main effect of addend size, $F(3,81) = 23.24$, $p < 0.05$. This confirms that as the addend is getting bigger the RT is increasing. This suggests that children are using some form of incremental counting process for these sums. There was no significant interaction between addend size and year, $F(6, 164) = 2.06$. The results for these single-digit sums suggest that the pace of counting increases with age. There was no interaction between addend size and format, $F(3, 81) = 0.17$. There was no significant interaction between addend size, year and format, $F(6, 164) = 0.85$.

Post hoc tests carried out on the results found that there was a significant difference between the RTs for year 3 and year 4, $p < 0.05$. There was a significant difference between the RTs for year 3 and year 5, $p < 0.05$. There was also a significant difference between the RTs for year 4 and year 5, $p < 0.05$.

Figure 6.2 is a summary of the results for the decade sums 45+n.

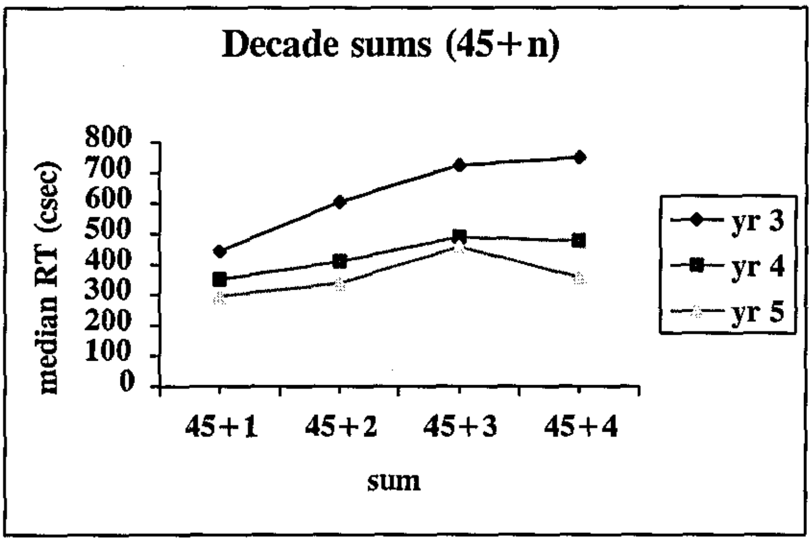

Figure 6.2 Solution times for the decade addition problems 45+n.

6. 3. 1. 2 Analysis (45+n)

An ANOVA on the results showed that there was a significant overall effect of year, $F(2, 83) = 13.34$, $p < 0.05$. There was no significant effect of format, $F(1, 83) = 0.18$, and there was no interaction between year and format, $F(2, 83) = 0.01$. It is taking the younger children significantly longer to solve these sums. The incremental increase in RT suggests that children are primarily using some form of counting (see Figure 6.2 above). There was a significant main effect of addend size, $F(3, 81) = 20.41$, $p < 0.05$. There was a significant interaction between addend size and year, $F(6, 164) = 2.38$, $p < 0.05$. Once again, this suggests that the mode of solution varies with age. There was no interaction between addend size and format, $F(3, 81) = 0.61$. There was no interaction between addend size, year and format, $F(6, 164) = 0.57$.

Post hoc tests carried out on the results found that there was a significant difference between the RTs for year 3 and year 4, $p < 0.05$ and between the RTs for year 3 and year 5, $p < 0.05$. There was no significant difference between the RTs for year 4 and year 5.

In order to judge whether decomposition plus counting is occurring, it is necessary to examine the overhead associated with the decade additions. If it is constant then decomposition might be assumed. This is better illustrated by the following set of results.

Figure 6.3 is a summary of results for the RT difference between 45+n and 5+n.

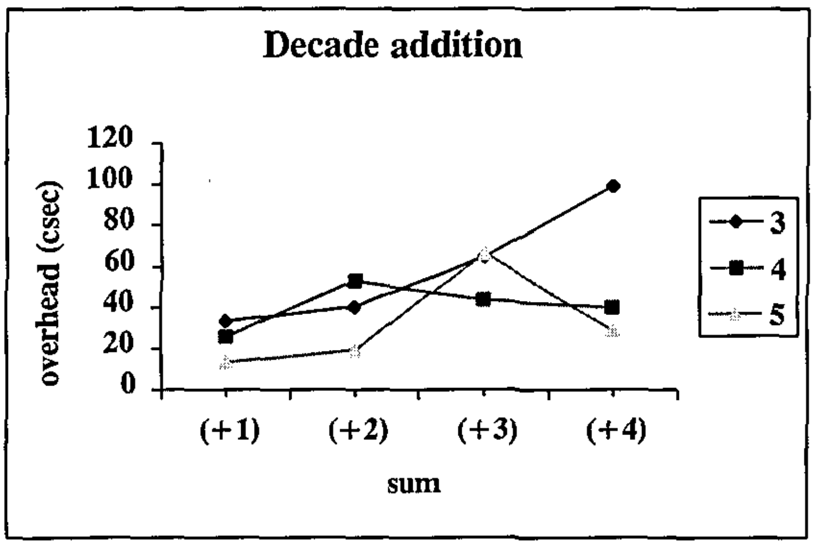

Figure 6.3 Solution overhead associated with the decade sums versus corresponding single-digit sums.

6. 3. 1. 3 Analysis (RT 45+n - RT 5+n)

For each participant, the results analysed were the differences in RT i.e. the RT for the single-digit sum in Figure 6.1 was subtracted from the RT for the decade sum in Figure 6.2 to give the "extra-time" or "decade overhead" as shown below in Figure

6.3. The ANOVA on the results found no significant overall effect of year, $F(2, 83) = 0.68$. There was no significant effect of presentation format, $F(1,83) = 0.05$, $p = 0.812$. There was a significant interaction between year and format, $F(2, 83) = 3.12$, $p < 0.05$. There was no significant effect of sum, $F(3, 81) = 0.64$ and there was no significant interaction between sum and year, $F(6, 164) = 0.70$. There was no significant interaction between sum and format, $F(3, 81) = 0.17$. There was no interaction between sum, year and format, $F(6, 164) = 0.44$.

These results show a slight cognitive load (about 30 csec) that is the effect of doing a decade problem and that this is constant (see Figure 6.3). The results suggest that decomposition and unit counting may be occurring. So, with simple decade sums the decade factor adds a constant computational overhead, i.e. it slows down RTs. Since this is small and constant the decade context does not contribute too much of an extra burden on mental computation.

Post hoc tests carried out on the results found that there was no significant difference between the mean RTs for the three year groups.

One of the aims of this study was to investigate what sorts of decade problems trigger decomposition and what sorts of decade problems trigger counting. The reason for not finding a constant overhead would be that counting on the 45+n sums must have a steeper RT function because of the slower counting pace in the decades. Figure 6.4 is a graph showing a prediction of what might be expected when the extra time to vocalise "40" is assumed to be a constant of one second, Figures 6.5, 6.6 and 6.7 show the actual results for the children in each year group.

Figure 6.4 illustrates a theoretical predicted comparison for the effect of adding a constant.

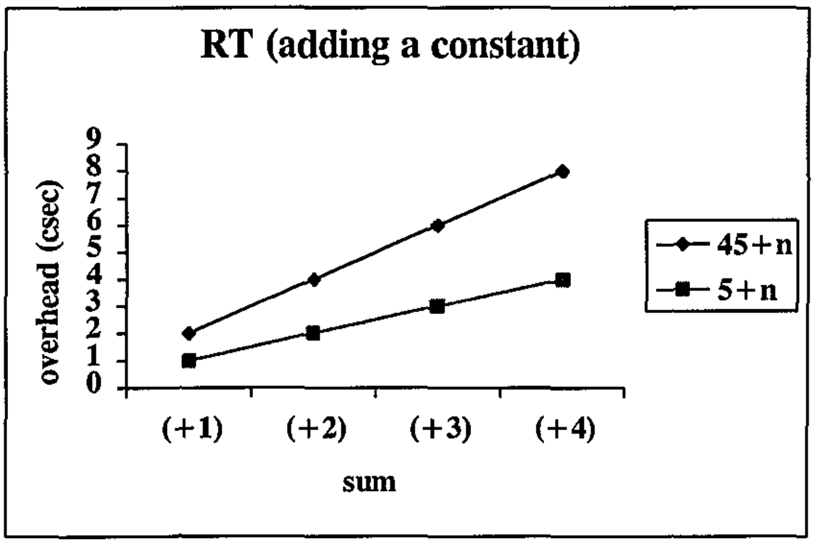

Figure 6.4 Predicted effect of adding a constant.

This is further illustrated by the graphs below showing the actual results. If children were using the counting-on strategy then the RT would be expected to increase as the sums became larger. The difference between the single-digit and the decade sum would be expected to become larger as the numbers being added became larger because of the time involved in each word being counted and this seems best illustrated by the actual data in Figure 6.5. The results shown in Figure 6.5 are the most similar to the theoretical comparison, suggesting that younger children, at least, may still be counting out the number words. However if children were using decomposition-based strategies then the RT for the decade sum and the single-digit sum should appear more constant as actually illustrated by Figure 6.6 and Figure 6.7. In the results from children in years 4 and 5, the RTs are fairly flat.

Figure 6.5 illustrates the effect of adding a constant for children in Year 3.

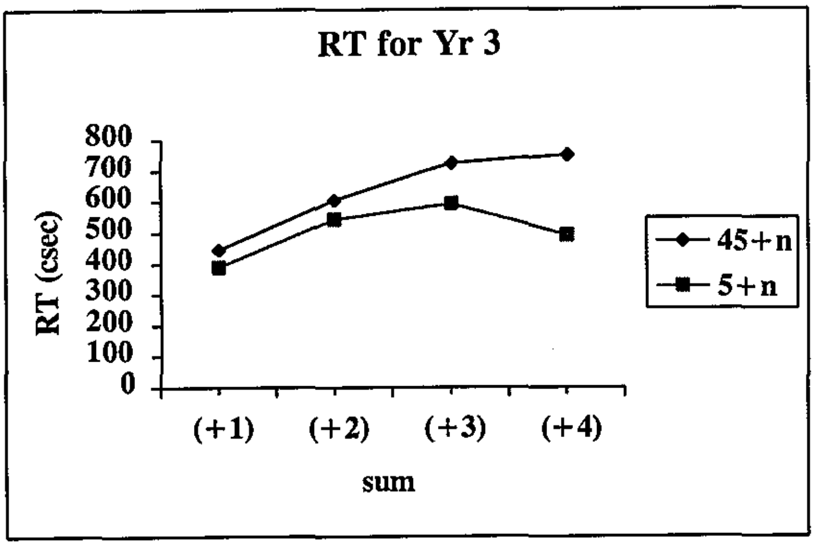

Figure 6.5 The effect of adding a constant for children in Year 3.

Figure 6.6 illustrates the effect of adding a constant for the children in Year 4.

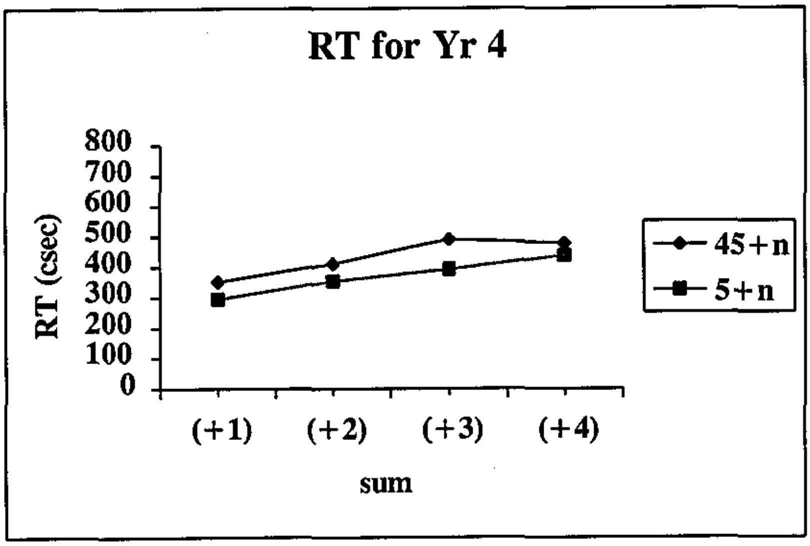

Figure 6.6 The effect of adding a constant for children in Year 4.

Figure 6.7 illustrates the effect of adding a constant for the children in Year 5.

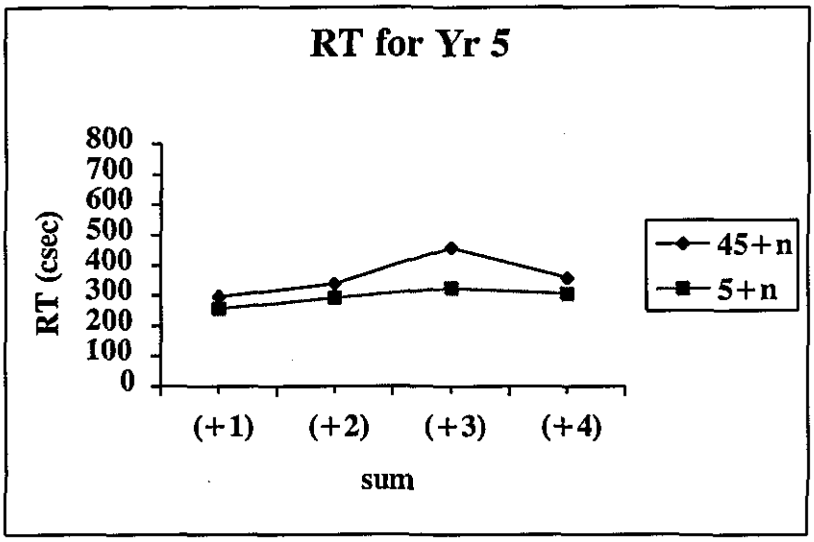

Figure 6.7 The effect of adding a constant for children in Year 5.

Figures 6.5, 6.6 and 6.7 illustrate that if children are using decomposition-based strategies then they are more likely to be children in the older age groups. Figures 6.5 and 6.6 show that the difference between the decade and the single-digit sum is relatively constant and this suggests that perhaps decomposition is only available to the older children. It also explains why there was no significant difference between the overhead for the decade sums.

These results suggest that it may be that decade sums with smaller addends are more likely to be solved through the use of decomposition (plus counting) as opposed to just decade counting. This could be because, with smaller addends, children across all year groups would be more confident of getting an accurate answer and, as pointed out earlier, accuracy is important to them. So, adding a small addend in a decade context might prompt decomposition whereas a large addend in a decade sum would make the problem seem too complex, in which case having to take the extra step required for decomposition would only make the problem more difficult. Thus, size of the addends might prompt decomposition. What other sums would also encourage decomposition? Would decade sums involving a tie-sum also encourage decomposition strategies? The following section looked at the results for decade sums (Problem Type B) that included a tie-sum number-fact.

6. 3. 2 Results for Problem Type B (decade sums incorporating a tie-sum)

Figure 6.8 is a summary of the results comparing RTs for decade sums incorporating a tie-sum with RTs for single-digit tie-sums.

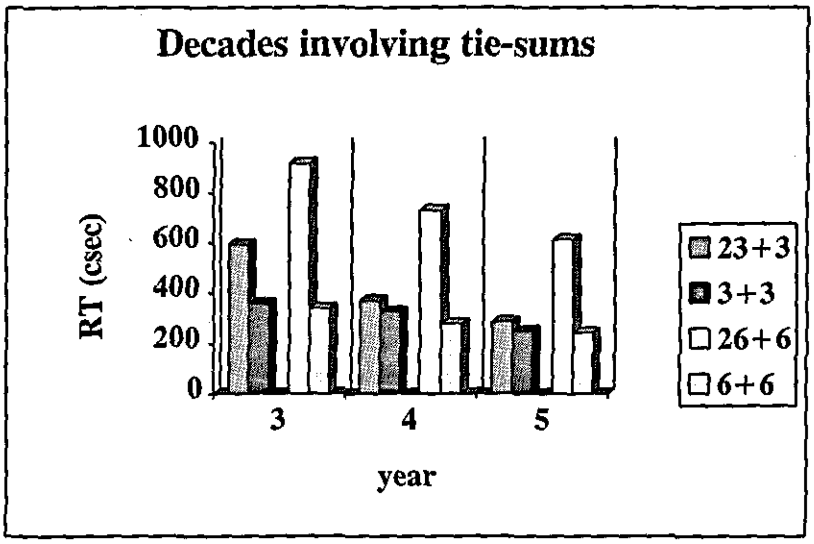

Figure 6.8 Solution times for decade problems with a tie-sum.

These results imply that children do seem to be decomposing and using tie-sum number-fact knowledge. The results illustrate that doing a decade sum that includes a tie-sum is fairly easy for most children when the answer does not cross the decade boundary. The difference between the decade sum and its single-digit counterpart is constant for all but the youngest children. Although the effect of doing the decade sum takes a little longer for the youngest children this difference is not great. However, when the decade sum has an answer that crosses the decade boundary this time difference increases and it takes children across all age groups significantly longer to solve. The results below show that there is a significantly larger overhead for decade tie-sums that cross the decade boundary.

Figure 6.9 is a summary of results for decade tie-sums.

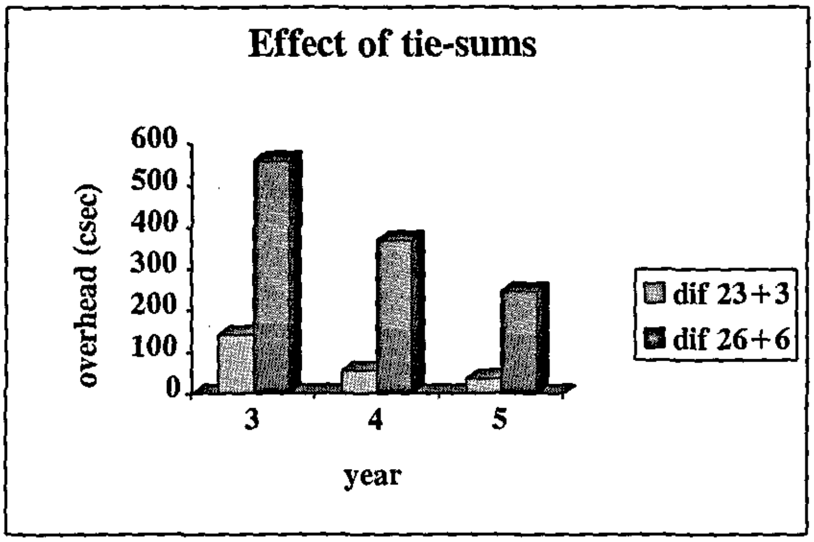

Figure 6.9 Solution overheads for tie-sum problems in a decade context.

These results suggested that there was a difference between the extra time taken to do a tie-sum that stays within the 20s decade boundary and one that crosses the decade boundary. Children are not therefore importing their single-digit tie-sum solutions wholesale into the decade context.

6. 3. 2. 1 Analysis (RT difference for decades with tie-sums)

An ANOVA carried out on the results in Figure 6.9 showed that there was no overall effect of year, $F(2,83) = 2.94$. There was no effect of format, $F(1,83) = 2.94$. There was no interaction between format and year, $F(2,83) = 1.18$. There was no interaction between problem type and format, $F(1, 83) = 0.002$, and there was no interaction between problem type, format and year, $F(2, 83) = 2.54$. There was a significant main effect of problem, $F(1,83) = 52.83$, $p < 0.05$. There was no interaction between problem type and year, $F(2,83) = 0.632$. The results show that, for all year groups, the overhead is significantly greater for 26+6 problems compared to 23+3 problems. These results suggest that when solving a tie-sum that crosses the decade boundary, children of all ages recognise the number fact 6+6 and attempt to use this tie-knowledge. Once they have done the tie-sum (6+6), they have problems when having to do the next stage of the problem (20+12) which leads them into the next decade (32). It is this recomposing aspect that they then find difficult as earlier results showed (5.2) crossing the decade boundary leads to significantly longer RTs for children of all age groups. If it was just crossing the boundary that was difficult then other sum that crossed the decade boundary (e.g. 34+7 and 37+4) should result in similarly long RTs but this was not the case (as illustrated in Figure 6.10 below).

Post hoc tests carried out on the results found that there was no significant difference between the mean RTs for the three year groups.

Figure 6.10 compares the RTs for decade tie-sum and sums that cross the decade boundary.

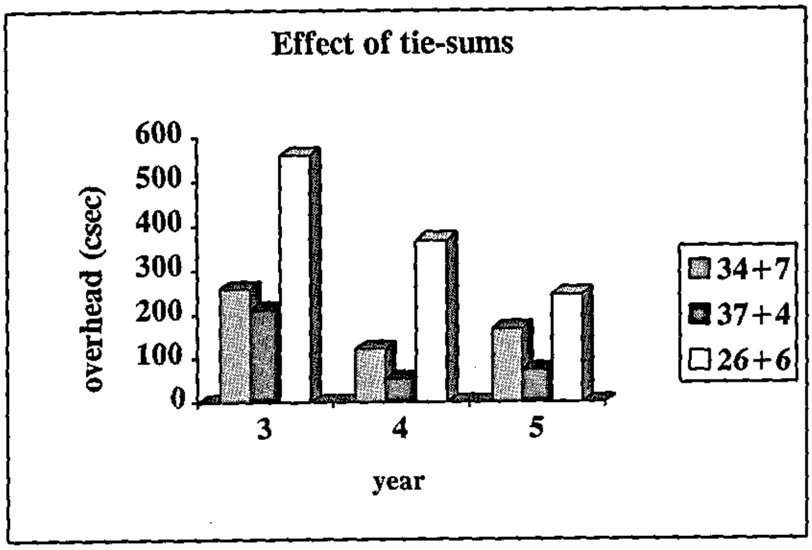

Figure 6.10 Solution times for decades involving tie-sums compared to decades that cross the decade boundary but do not involve a number-fact.

Figure 6.10 above suggests that 26+6 is being done differently to other sums that cross the decade boundary. One of the reasons that 26+6 had a significantly larger overhead than 23+3 could have been that it's answer crossing the decade boundary and thus was not being seen as a problem involving a tie-sum but as a +6 problem instead. However, the results shown above suggest that crossing the decade boundary was not the issue. The results illustrated above suggest that while children seem to be counting for the 34+7 and 37+4 problems, they are doing 26+6 differently.

This suggests that it is taking children across all year groups much longer to do 26+6 compared to 34+7 or 37+4, both of which cross the decade boundary and are therefore relatively difficult sums. There is no particular reason for this unless the children are using similar decomposition and/or counting strategy for 34+7 and 37+4 (as my other results suggest they are). However, although they are recognising the tie in 26+6 and trying to use it and are finding the next stage of recomposing the sum (adding 20) more difficult. For the younger children there seems to be more of a disadvantage in doing this since most of the children in the younger year groups may not yet know 6+6 as a fact. These results seem to suggest that although some of them may not yet actually know 6+6 they are still trying to use it. But it would seem that in decade sums involving a tie-sum that also cross the decade boundary i.e. involve a carry function, the cost of doing decomposition is having to do the recomposing and this is reflected in their RTs. Therefore, it would seem that decomposition, when it is first attempted, makes things more difficult for children i. e being strategic can have its price.

Figure 6.11 compares the RTs for decade sums with similar sums that cross the decade boundary.

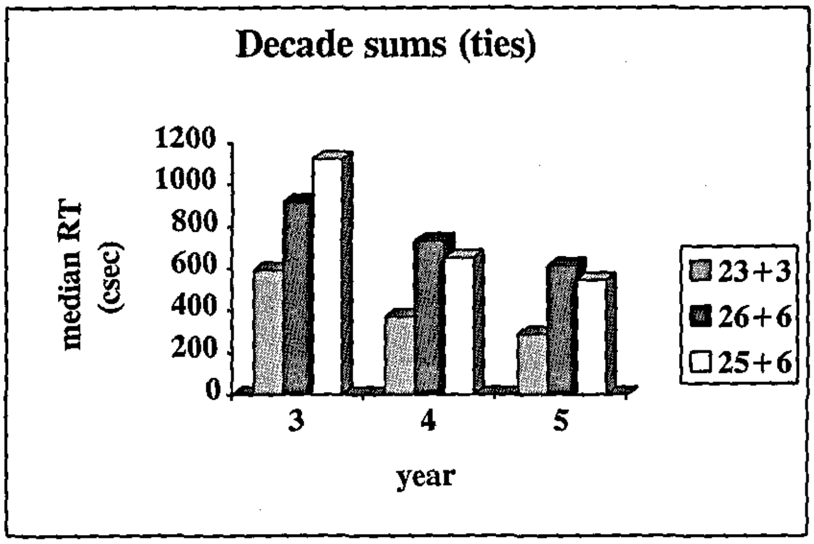

Figure 6.11 Solution times for decade sums involving a tie-sum compared with decade sums involving an adjacent to tie-sum.

Although the data for the single-digit problem 5+6 was not available in order to calculate the extra time, the RT for 25+6 compared to 26+6 suggests that for the older children 26+6 does seem to take less time than 25+6. This might suggest that children may be recognising 26+6 as incorporating a number fact, but then having to deal with the decade factor may make the problem more difficult compared to simply using counting for 25+6. For the younger children, though, the presence of a tie-sum may make this problem easier than 25+6 but then their overall RTs are higher for all sums suggesting that they are likely to be relying on counting for all sums.

In summary, the results in Figure 6.9 showed that the overhead difference was not constant for the decade tie-sums that crossed the decade boundary. If both 23+3 and 26+6 were being solved by decomposition then the RT overhead for both these sums would have been constant. But because this did not appear to be the case here, it could be suggested that children were not decomposing on these sums. However, the results in Figure 6.11 suggest that children did not seem to be counting either, so perhaps this reflects the transitional nature of the tie number-fact i.e. that it is being recognised as a potentially salient number fact but one that has not yet become an actual number-fact.

6. 3. 3 Results for Problem Type C (sums in Groups 1 & 2)

Figure 6.12 is a summary of results for the RTs for single-digit sums in Problem Type C.

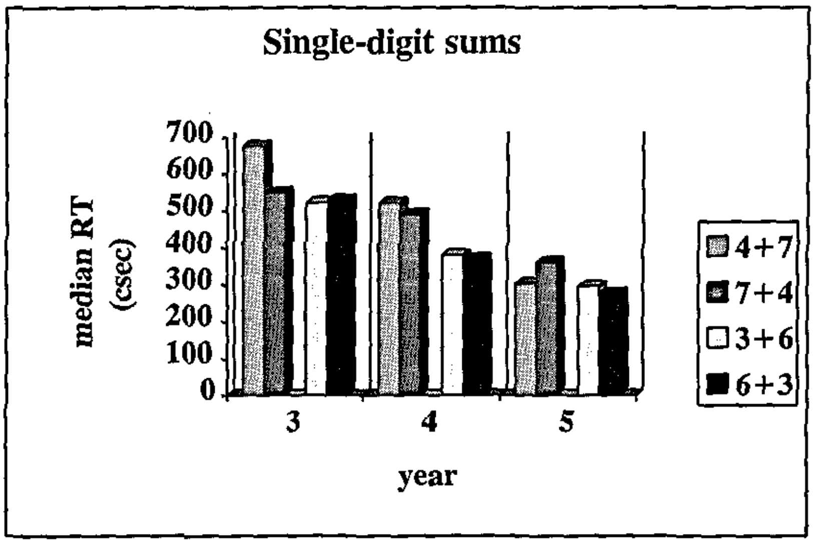

Figure 6.12 Solution times for the single-digit sums only for each year group.

6. 3. 3. 1 Analysis (single-digit sums in Problem Type C)

An ANOVA on the results for 7+4 and 4+7 found there was an overall effect of year on RT, $F(2, 83) = 7.64$, $p < 0.05$, illustrating that younger children take longer to do these single-digit sums. There was no effect of format, $F(1, 83) = 0.02$. There was no interaction between year and format, $F(2, 83) = 0.25$. There was no significant difference between the RT for 7+4 and 4+7, $F(1, 83) = 2.81$. There was no interaction between sum and year, $F(2, 83) = 0.25$, suggesting that year did not have a significant effect on RT for 7+4 and 4+7. There was no interaction between sum and format, F

(1, 83) = 0.29. There was no interaction between sum, year and format, F (2, 83) = 0.50. Post hoc tests on results found that there was no significant difference between the RTs for year 3 and year 4. There was a significant difference between solution times for year 3 and year 5, $p < 0.05$. There was no significant difference between RTs for year 4 and year 5.

An ANOVA on the results for 6+3 and 3+6 found that there was an overall effect of year on RT, F (2, 83) = 12.43, $p < 0.05$, illustrating that younger children take longer to do these single-digit sums. There was no effect of format, F (1, 83) = 0.27. There was no interaction between year and format, F (2, 83) = 0.44. There was no significant difference between the RT for 6+3 and 3+6, F (1, 83) = 3.46. There was no interaction between sum and year, F (2, 83) = 0.61, suggesting that year did not have a significant effect on RT for 6+3 and 3+6. There was no interaction between sum and format, F (1, 83) = 0.83. There was no interaction between sum, year and format, F (2, 83) = 0.81. Post hoc tests on the results found that there was a significant difference between RTs for year 3 and year 4, $p < 0.05$ and there was also a significant difference between the RTs for year 3 and year 5, $p < 0.05$. There was no significant difference between the RT for year 4 and year 5.

These results showed that there was no significant difference between the RT for the 7+4 and 4+7 and 6+3 and 3+6, suggesting that children across all year groups were recognising the commutative property of addition.

Figure 6.13 is a summary of results for RTs for decade sums in Problem Type C.

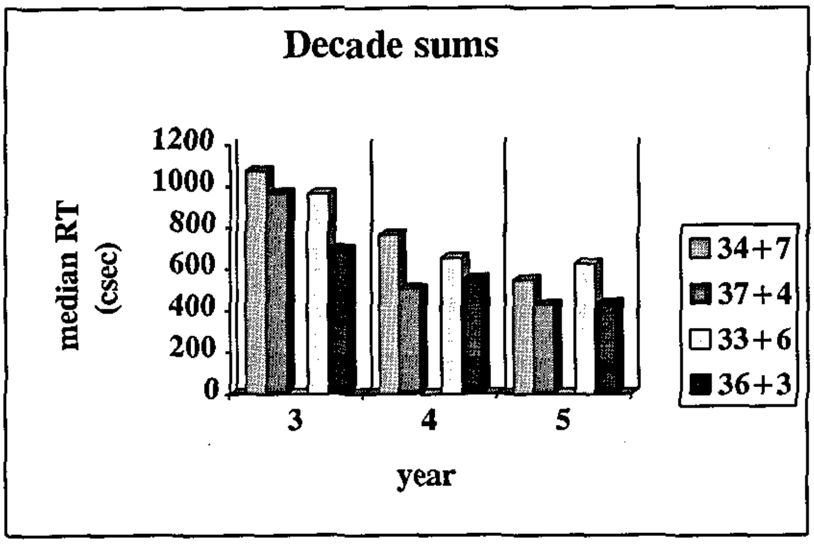

Figure 6.13 Solution times for the decade sums only for each year group.

6. 3. 3. 2 Analysis (decade sums in Problem Type C)

An ANOVA on the results for the sums 34+7 and 37+4 found that there was an overall effect of year, $F(2, 83) = 16.19$, $p < 0.05$, suggesting that younger children take longer to do these decade sums. There was no effect of format, $F(1, 83) = 1.25$). There was no interaction between year and format, $F(2, 83) = 0.52$. There was a significant difference between the RT for 34+7 and 37+4, $F(1, 83) = 20.87$, $p < 0.05$, suggesting that it is taking children across all years longer to do 34+7. There was no interaction between sum and year, $F(2, 83) = 0.47$. There was no interaction between sum and format, $F(1, 83) = 1.14$. There was no interaction between sum, year and format, $F(2, 83) = 0.60$. Post hoc tests on the results found that there was a significant difference between the RTs for year 3 and year 4, $p < 0.05$ and between the RTs for year 3 and year 5, $p < 0.05$. There was no significant difference between the RTs for year 4 and year 5.

An ANOVA on the results for the sums 33+6 and 36+3 found that there was an overall effect of year, $F(2, 83) = 8.5$, $p < 0.05$. There was no effect of format, $F(1, 83) = 0.001$. There was no interaction between year and format, $F(2, 83) = 0.68$ There was a significant difference between the RT for 33+6 and 36+3, $F(1, 83) = 20.29$, $p < 0.05$. This suggests that children across all years are taking longer to do 33+6. There was no interaction between sum and year, $F(2, 83) = 0.28$. There was no interaction between sum and format, $F(1, 83) = 1.7$. There was no interaction between sum, year and format, $F(2, 83) = 0.87$. Post hoc tests on the results found that there was a significant difference between the RTs for year 3 and year 4, $p < 0.05$ and between the RTs for year 3 and year 5, $p < 0.05$. There was no significant difference between the RTs for year 4 and year 5.

The results for the decade sums suggest that it is taking these children disproportionately longer to do decade sums where the smaller number comes first. This implies that when they see these sums in a decade context, the commutative property of addition is less visible to them and they resort to counting. The results for the 45+n problems illustrated in Figure 6.3 suggested that children might be using decomposition on decade sums involving a relatively small addend such as "+4". The results shown in Figure 6.4 do seem to suggest that adding the smaller number (3 or 4) takes less time than adding a larger number. Sums where the answer crosses the decade (34+7 and 37+4) seem to be more difficult for the younger children suggesting that they are most likely to be using fairly mechanical incremental counting for both types of decade sum since their RT increases according to the size of the addend. The reason for the increase in RT for the older children when doing the sums with the larger addend may be due to them still relying on counting for the single-digit units sum (3+6 and 4+7) and then adding on the decade.

The following results look at RT overhead, i. e. the difference between the decade sums and the corresponding single-digit sum.

Figure 6.14 is a summary of results looking at the effect of sums with answers crossing the decade boundary.

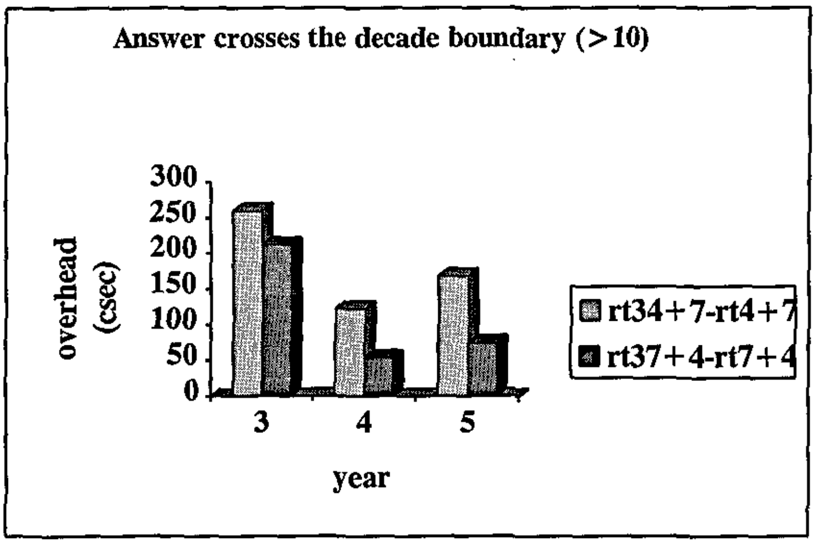

Figure 6.14 Solution overheads for the sums in Group1 where the answer crosses the decade boundary.

6. 3. 3. 3 Analysis (decade sums in Group 1)

An ANOVA on the results shown in Figure 6.14 found that there was no overall effect of year on the RT overhead, $F (2, 83) = 1.81$, suggesting that children across all year groups had similar RT overheads. There was no effect of format $F (1, 83) = 0.94$. There was no interaction between year and format, $F (2, 83) = 0.01$. There was no significant difference between the RT overheads for these two types of sums, $F (1, 83) = 2.61$, suggesting that the size of the second addend did not have a significant effect on the RT overhead. There was no interaction between sum and year, $F (2, 83) = 0.69$. There was no interaction between sum and format, $F (1, 83) = 1.23$. There was no interaction between sum, year and format, $F (2, 83) = 0.31$. Post hoc tests on the results found that there was no significant difference between the RTs for any of the three year groups.

The graph above (Figure 6.14) shows that it's taking longer for children (the overhead is greater) across all age groups to do sums where the smaller number appears first, when the addition problems cross the decade boundary. It could be that this is the case because children are counting incrementally for both types of sums in which case the size of the addend is what would affect RT. But it could also be that they are doing decomposition when the addend is smaller (+4) because that is just about manageable but for the larger addend (+7) it's seen as too complicated. So when doing +7 most children would just be counting on from 34 but when doing +4 they might attempt to use a more sophisticated strategy. It is taking the youngest year group much longer to do both types of sums suggesting that most of the younger children probably are using a counting strategy for both types of problems and even if they were attempting decomposition they could be struggling with it. But because both older age groups seem to have similar times for the sums with the smaller addend, it might mean that decomposition is a solution procedure that is only available to them when they are faced with a sum where it is not seen as somehow "adding" to the mental effort required to do the sum.

Figure 6.15 is a summary of results looking at the effect of sums with answers that do not cross the decade boundary.

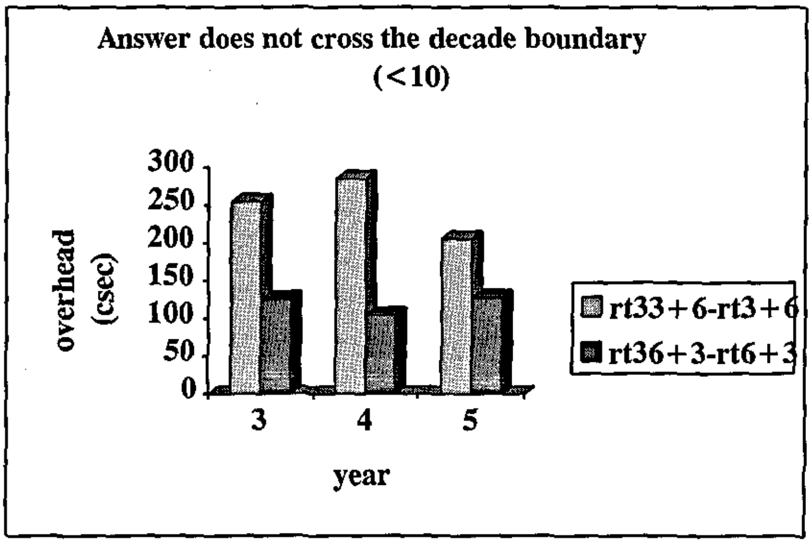

Figure 6.15 Solution overheads for the sums in Group 2 where the answer does not cross the decade boundary.

6. 3. 3. 4 Analysis (decade sums in Group 2)

An ANOVA on the results shown in Figure 6.15 found that there was no overall effect of year on the RT overhead, $F(2, 83) = 0.61$, suggesting that the overhead was similar for children from all year groups. There was no effect of format, $F(1, 83) = 0.61$. There was no interaction between year and format, $F(2, 83) = 0.42$. There was a significant difference between the RT overheads for these two types of sums, $F(1, 83) = 4.41$, $p < 0.05$, suggesting that the overhead was greater for sums with the larger addends (6) and that sums with a smaller addend (3) had a smaller overhead. There was no interaction between sum and year, $F(2, 83) = 0.53$. There was no interaction between sum and format, $F(1, 83) = 0.14$. There was no interaction between sum, year and format, $F(2, 83) = 0.14$. Post hoc tests on the results found that there was no significant difference between the RTs for the three year groups.

At first glance these results also suggest that children across all year groups are using counting strategies because it is taking them longer to do decade sums (where the answer does not cross the decade boundary) with the smaller addend. However, the RT difference is similar across all age groups for decade sums with a smaller addend (+3). It is taking children across all ages less extra time to do sums with a smaller addend compared to sums with a larger addend (+6). Thus suggesting that when doing 33+6 most children are likely to count on from 33 whereas when doing 36+3 they might use decomposition because it (6+3) involves adding a much smaller number (+3). Because the answer for these decade sums does not cross the decade boundary (unlike 34+7 and 37+4), this makes the sum easier for even the youngest children as illustrated by the smaller RT difference for 36+3 compared to the RT difference for 34+7.

This suggests that in both types of sums, those that cross the decade boundary (>10) and those that do no cross the decade boundary (<10) what seems to affect RT is the digit order or the size of the addend. One way of looking at this was to collapse the two types of sums to look at digit order (addend size) only. This would enable a comparison between sums with a large addend (+7 and +6) and sums with a smaller addend (+4 and +3).

Figure 6.16 illustrates the effect of sum-type on RT overhead.

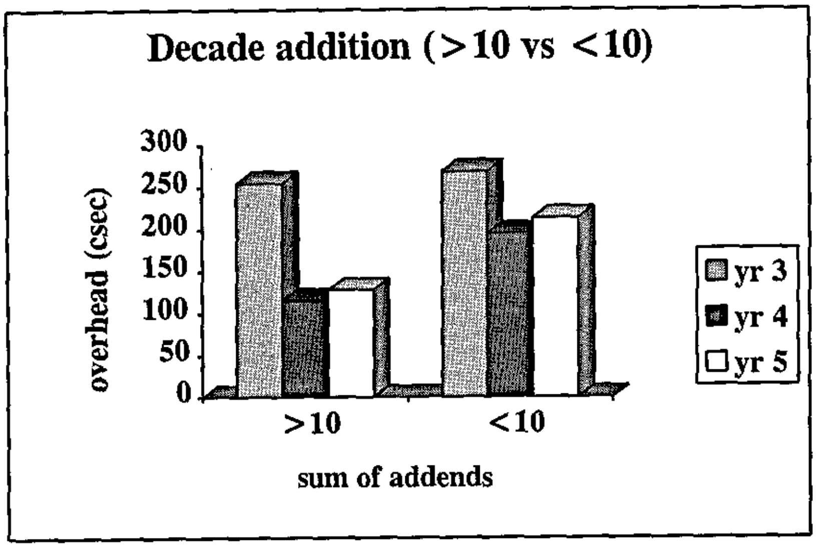

Figure 6.16 Solution overheads illustrating the effect of sum type.

Figure 6.17 illustrates the effect of addend size on RT overhead.

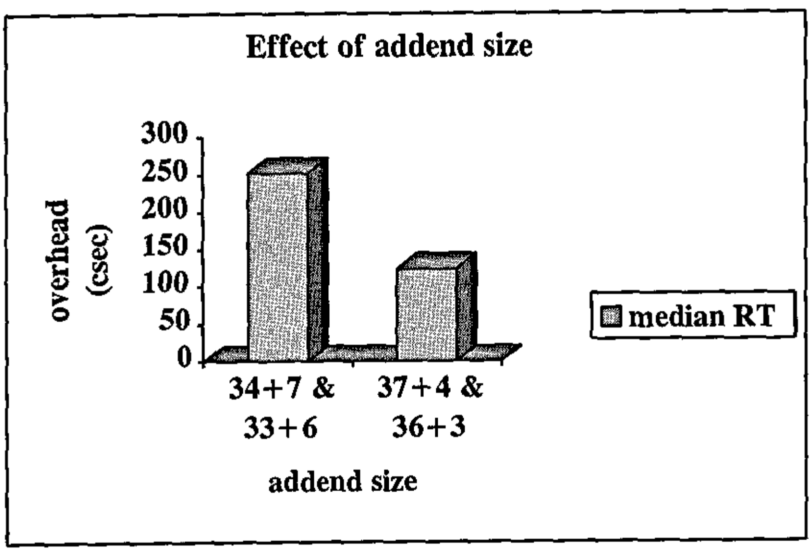

Figure 6.17 Solution overheads illustrating the effect of addend size (digit order).

6. 3. 3. 5 Analysis (decade sums in Groups 1 and 2)

An ANOVA on the results (illustrated in Figure 6.16) showed that there was no overall effect of year, $F (2, 83) = 1.13$. There was no effect of presentation format, $F (1, 83) = 0.03$. There was no interaction between year and format, $F (2, 83) = 0.12$. There was a significant main effect of addend size as illustrated by Figure 6.17, $F (1, 83) = 18.70$, $p < 0.05$. This suggests that the size of the addend did make a difference to the RT overhead. Decade sums with the smaller addend of 3 and 4 took significantly less extra time than decade sums with larger addends like 7 and 6 suggesting that the single-digit solution procedure (min) is not being imported wholesale to the decade case.

There was no interaction between addend size and year, $F (2, 83) = 0.00$. There was no interaction between addend size and format, $F (1, 83) = 0.00$. There was no interaction between addend size, year and format, $F (2, 83) = 0.00$.

These results, along with the findings from the results in the 45+n (Problem Type A) sums, suggest that perhaps it is smaller addends in decade sums are more likely to trigger decomposition. There was no significant effect of sum type as illustrated by Figure 6.16, $F (1, 83) = 0.14$. This would suggest that crossing the decade boundary did not have a significant effect on RT. However, there was a significant interaction between addend and sum type, $F (1, 83) = 8.11$, $p < 0.05$. This suggests that when doing decade sums, the RT will be affected whether or not the answer crosses the decade boundary depending on the size of the addends. This would make sense because if a child has to do a sum that crosses the decade boundary (has a carry function), then that is a difficult sum and addend size will have a important role in how that sum is solved, i.e. through the use of an efficient or inefficient strategy (see Figure 6.14). Furthermore, if a decade sum does not cross the decade boundary then it might be seen as an easy sum, but only if the addend is not too large (see Figure 6.15).

There was no interaction between sum type and format, $F (1, 83) = 0.44$. There was no interaction between sum type and year, $F (2, 83) = 0.16$. There was no interaction between sum type, year and format, $F(2, 83) = 0.24$. There was no interaction between addend size, sum type and year, $F(2, 83) = 0.62$. There was no interaction between addend size, sum type and format, $F(1, 83) = 1.54$. There was no interaction between addend size, sum type, year and format, $F(2, 83) = 0.37$.

Post hoc tests carried out on the results found that there was no significant difference between the mean RTs for the three year groups.

The findings so far seem to indicate that when doing decade sums, addition strategies can be prompted on sums with smaller addends and sums involving tie-sum number-facts.

To conclude this section, it would seem that when doing decade sums, sums involving number-facts such as tie-sums, along with smaller sums and sums with smaller addends are more likely to prompt decomposition. While decomposition-based strategy did not seem to be available to all the children, it did seem that it was perhaps being used, or at least attempted, for smaller sums and sums involving a visible number-fact such as a tie-sum.

6. 4 Discussion

The decade sums chosen in this study were designed to look at how children might be importing the strategies they used when doing simple single-digit sums (such as counting, and using derived number-fact retrieval) into doing more complex decade sums. The only way that such existing strategies might be optimally used when doing such decade sums would be if the children were to recognise and use decomposition based strategies. In order to do this I needed to consider what kinds of sums might encourage children to use more efficient strategies such as decomposition.

Three types of decade sums, along with their single-digit counterparts, were chosen, (see Table 6.2):

1) Problems designed to look at solution procedures for simpler decade problems with smaller addends such as 45+n (Problem type A).
2) Problems designed to look at decomposition and the use of number-fact knowledge such as tie-sums (Problem Type B).
3) Problems designed to look at decomposition and the min counting strategy (Problem type C) where Group 1 problems (34+7 and 37+4) had answers that crossed the decade boundary and Group 2 problems (33+6 and 36+3) had answers that remained in the same decade i.e. sums with both large and small addends.

The results illustrated by Figure 6.1, 6.2 and 6.3 showed that although the decade added a constant overhead, doing a decade sum itself was not problematic. These results showed a problem-size effect i.e. the RT increased as the size of the smaller digit increased and this was greater for the younger children. The results show an expected developmental effect for both single-digit and 45+n decade addition problems (Figure 6.1 and 6.2). This problem size effect suggested that most children across all year groups are using counting strategies for these sums. However, the RT overhead later suggested that perhaps for the sum 45+4 older children were using a decomposition strategy because the extra time taken for this sum was not as high as expected. It may have been that a sum like 45+4 is just about small enough for children to solve through decomposition, i.e. 5+4 = 9+40 = 49. Figure 6.3 shows that adding the decade shows a slight overall increase in the RT overhead which would be expected when children do larger decade sums. However, this is not significant, suggesting that simply doing a decade sum does not make decade sums much harder, i.e. doing smaller decade sums does not place too much demand on mental computational resources. It is when the problems become slightly more complex that this changes.

The findings from the sums in Problem type B showed that children were making an attempt at being strategic by trying to use their existing number-fact knowledge of tie-sums. Where the tie-sum involved was small (such as 3+3) the RT overhead was fairly short suggesting that, for even the youngest children, this number-fact knowledge was being used and they were using decomposition, i.e. recognising that

23+3 was 20+6. The larger tie-sum 6+6 which crossed the decade boundary posed more of a problem because it took significantly more extra time to solve compared to 23+3. Not only that, but it also took much longer to solve than the other sums that crossed the decade boundary (34+7 and 37+4). This suggested that perhaps for this sum the presence of the tie-sum did encourage decomposition but having done the tie-sum most children then had difficulty with adding 20 because this involved a carry function. A greater overhead for 26+6 compared to 34+7 and 37+4 suggests that this is being solved differently. They seemed to be recognising tie-sums such as 6+6 long before they actually become "facts", i.e. were retrieved swiftly from memory. Thus, it seemed that children were trying to take advantage of tie-sums but this was not always successful since this could lead to problems with recomposing the sum.

For the decade sums in Problem type C, the findings (as shown in Figure 6.12) demonstrated that the size of the addend does not have an effect on single-digit sums. It took children across all age groups about the same time to do 6+3 as 3+6 and 7+4 and 4+7. This suggested that these children were recognising the commutative property of addition. Although some children, especially those in Year 3, still seemed to be counting, these results suggested that these single-digit sums were not taking them too long to solve. From this it would follow that when they saw such sums in a decade context the addend order would not have a significant difference on RT, if children were using decomposition strategies. However, the results for the decade sums (Figure 6.13) showed that it was taking children longer to do those decade sums where the smaller digit appears first (or where the smaller addend was larger). For children in the youngest year group, this difference occurred for both types of decade problems where the answer crosses the decade boundary into the next decade (34+7 and 37+4) and where it stays in the same decade (33+6 and 36+3). When the results were analysed, the extra time taken to do sums where the addend involved a larger number (such as 6 or 7) was significantly different from extra time taken to do sums where addend was smaller (such as 4 or 3) suggesting that perhaps some children are making use of their knowledge of decomposition.

The significant effect of the addend size seemed to suggest that most children were using counting strategies. This was because a counting strategy would mean that it takes longer to count on 7 or 6 digits compared to 3 or 4 and this is what the results showed (Figure 6.13). However, it may also mean that some children were using decomposition on sums with smaller addends such as 3 or 4. This suggests that in both types of decade problem when a larger addend is involved most children resort to using counting strategies, rather than slightly more sophisticated decomposition strategies which would involve breaking down the problem and recognising concepts additive composition and commutativity and manipulating this knowledge efficiently to their advantage.

Therefore, the types of decade sums that would be most likely to prompt sophisticated decomposition strategies, or at least partial decomposition, would be less complex decade sums with smaller answers, decade sums with smaller addends and decade sums that included visible number-facts such as tie-sums. Further research would be needed to ascertain whether it is just the presence of a smaller addend that makes decomposition a viable strategy maybe because a smaller addend means more confidence in addition skill and more confidence in getting an accurate result. Certainly, children would have to be fairly confident in their ability to use decomposition before they would go to use it with larger sums where they would see themselves as more likely to make a mistake and thus resorting to reliable counting strategies which may not be efficient in the longer term. The problems in this study only considered two types of sums that involved a carry function where the answer crossed the decade boundary. Would crossing the decade boundary really be such an important factor (as it seemed to have been in this study) in determining whether or not decomposition would get used or at least attempted?

These results do seem to support findings of existing research that actual use of decomposition is uncommon (Renton, 1992, Cowan, 1999; and Cowan and Renton, 1996). It is likely that children may recognise concepts of additive composition, associativity and commutativity but not make effective use of them. When faced with difficult problems (in this case those involving decades), most children resort to counting strategies which they know from prior experience will give them an accurate answer. This would apply not only to those children who have good arithmetic knowledge and those in older age groups but also to those who are "perfectionists" and will rely on reliable counting strategies than novel ones that may lead to errors (Siegler, 1988). So, although children were relying on counting strategies, they may have been doing so because counting strategies were more likely than others to give them an accurate answer and casual observation suggested that this seemed quite important to many children including those with good mental arithmetic knowledge.

The results of this experiment show that the development in mental addition does depend on a blend of counting strategies and number fact knowledge, with most children relying primarily on counting strategies to solve addition problems that are slightly more complex than single-digit addition. For example, Figure 6.11 shows that doing a decade sum which involves a tie-sum (26+6) takes longer than sums such as 34+7 or 37+4 even though the latter are of a greater magnitude and appear to require slightly more complex processing. As Wolters et al (1990) suggested, arithmetic problems, and procedures that require more solution steps will result in a larger memory load which will result in longer solution times because each serially performed step takes time. So, although problems such as 37+4 and 34+7 are more complex, these results suggest that these are being solved through fairly mechanical counting strategies unless they involve smaller addends possibly because smaller addends are more easily solved by mental counting.

However, it is possible that when the children are faced with a problem such as 26+6 they recognise that this involves a tie-sum. Since results have shown that tie-sums have shorter RTs than other sums of a similar magnitude and are thus more salient as number facts they then attempt to use this knowledge. But once having done the tie-sum 6+6 they are then faced with having to add the decade 20 and this is when the sum crosses into the next decade 32 and becomes more difficult. Earlier results described in 5.2 showed that crossing the decade boundary made sums more difficult for children across all ages but particularly more so for the youngest age groups. For younger children, matters are complicated by the additional problem that, for many of them, 6+6, despite being recognised as something they "must" or "should" know "because it is easier", may not have yet become a number fact that is easily retrieved. So, attempting to be strategic may actually be slowing some of them down because in trying to be efficient they might actually end up increasing the number of steps in an addition sum and this puts too much demand on their mental computational resources.

As pointed out by Widaman and Little (1992), in the two proposed models of the development of strategy choices (Ashcraft, 1983; Baroody, 1983) choices are based upon computational or procedural strategies and memory retrieval or declarative strategies. In Ashcraft's model, strategy choice in mental addition development evolved from the use of slow, procedural strategies such as counting to a more efficient strategy such as memory network retrieval, which was the preferred and dominant strategy from the middle elementary school years onwards. In Baroody's (1983) model, however, strategy choice in mental addition development is a move from slower procedural processes such as "min" to much faster and more principled procedural processes. While Baroody admitted that some addition facts (e.g. ties) may be stored in long-term memory, he believed that individuals manipulate rules (such the identity element, $n+0 = n$), principles (such as commutativity, ties, and +1), and heuristics (such as reorganisation, recasting n+9 as $\{[n+10]-1\}$) in solving simple addition problems throughout development. According to Widaman and Little (1992), the underlying issue between both models is the degree to which procedural and/or declarative strategies underlie the changes occurring during the development of mental arithmetic skills. Whether mental addition development is characterised by a procedural or a declarative model at different ages, levels of schooling or transitional phases are an important empirical concern.

Although Widaman and Little (1992) had suggested that presentation format might have an influence on RT on mental arithmetic, format did not have an effect on RT on any of the addition problems in this study. However, this may not necessarily mean that format will not have any effect on RT on decade addition in other contexts.

6. 5 Conclusion

Overall, the results suggested that counting strategies were prevalent across all age groups when doing decade sums. It seemed that much performance at mental addition sums was based upon mental or finger counting across all age groups. However, the results also suggested that children might use decomposition strategies for certain types of decade problems, e.g. sums with smaller addends or problems that involved tie-sums. For simple decade problems, the decade problems were not much more difficult than the corresponding single-digit ones suggesting that these easier sums might be more likely to be solved through decomposition. However, when faced with more complex decade problems, children took longer to solve problems with larger addends. Therefore, children were not always strategically decomposing larger decade problems but were resorting to counting instead. Results also suggested that when faced with a decade problem involving a tie-sum that crossed the decade boundary, children of all ages seemed to recognise the tie number fact. However, this tended to slow them down. So, although children were trying to be strategic this seemed to be more of a hindrance than helpful. For some types of decade problems, children were being creative rather than mechanical in processing basic procedural knowledge. For some decade problems, it seemed that they were attempting to make use of decomposition-based strategies. For other types of decades, they were choosing to use strategies that they were familiar with, i.e. counting.

Chapter 7

7. Selection Strategies on Three-digit Serial Addition

7. 1 Introduction

Chapter 6 looked at how children would use their existing knowledge of decomposition dependent strategies to solve certain types of decade sums. These were sums that were more complex than the commonly studied simple single-digit (a+b) type integer addition. In particular, I considered sums that involved a decade in the ab+c format. Although children in this study seemed to be relying extensively on mental or finger counting, the results suggested that children were able to use decomposition strategies for certain kinds of decade problems. These included sums with smaller addends or problems that involved tie-sums. These results showed that while children may be generally reluctant to carry forward their existing knowledge, certain conditions do encourage them to be strategic. One of the reasons for looking at solution strategies for decade sums was the everyday authenticity of these problems. When adding strings or sequences of single-integer digits, the working totals soon get larger and more complex.

The aim in this chapter was to look at serial addition itself as a further variety of addition complexity. Although this would involve considering serial addition sums of a manageable length (involving 3 digits); that is, before they started adding up to higher numbers and while the sequence to navigate remained short. In this chapter, the introduction will begin by considering how patterns that emerged from my earlier research led to the questions raised in the present chapter, as well as resulting in the particular research design used for the current study. I will then go on to discuss the results from this study.

This chapter gives special consideration to the selection strategies children might use when they were required to do serial addition, and explores whether they would they still exploit their existing strategic knowledge of mental addition. That is, my previous research suggested that children had some capability for recruiting their existing knowledge of number-facts (such as tie-sums) and, to a much lesser extent, their knowledge of counting strategies (such as the min strategy) into a decade context. Would they be capable of using their number-fact knowledge and the min strategy in a serial addition context? There is a distinct lack of research in this area of serial addition (i. e. addition involving more than two pairs of numbers). Some research was done by Widaman et al (1989) in a preliminary attempt to theorise what occurs while solving three-digit sums. However, this research was carried out on an adult sample, and would not, therefore, be adequate for explaining children's strategies for such sums. The aim of this chapter is to look specifically at whether or not children would select strategically, in a certain order, combinations of addends when required to do three-digit serial addition. To achieve this would involve me utilising a serial addition task in which children were willing to reorder the addends to take advantage of this possibility. Would young children be prepared to do this?

Existing research has suggested that some number facts are easier (have faster solution times and more accurate responses) than other problems with numbers of a similar magnitude. These include number facts such as sum to 10 (Aiken and Williams, 1973; Ashcraft, 1982; Krueger and Hallford, 1984), although this was not replicated in my own findings - and problems with a repeated number i. e. tie-sums such as 3+3 or multiplication ties such as 8x8 (Campbell and Graham, 1985; Miller et al, 1984). Campbell and Oliphant (1992) reported that they had found 10 to be a relatively low-frequency error response in single-digit addition. The research reported earlier in 5. 1 supported the claim that tie-sums are more salient for becoming number-facts than other number combinations. This was further supported by the findings in Chapter 6 which suggested that tie-sums remain potent number facts even in a decade context and even if attempting to make use of the tie-sum slows down the solution process.

This raises the question of what would happen to children's existing number-fact knowledge when they were required to add more than two digits: could it be strategically mobilised? The results in my previous studies around decade problems had demonstrated that children were capable of being strategic, albeit to a rather more limited extent. What would they do if they were required to do sums that involved adding more than two single-digit numbers (i. e. the simplest type of serial addition involving three digits)? Would they transform these sums to take best advantage of their existing addition expertise?

Preliminary research in Section 5. 2 showed that three-digit mental addition added a considerable memory load and was quite challenging for children especially those in the younger age groups. This was reflected in much longer RTs on these types of sums. So, solving such problems strategically could make them much speedier to complete and this would be in the children's best interest. Three-digit mental addition would be an ideal format to examine this, because it was difficult enough to pose a challenge and simple enough to be solved mentally without being too stressful. To contextualise the present study, I will review these earlier findings in a little more detail.

7. 1. 1 Background Research

The research reported in Section 5. 2 was an early attempt to investigate children's strategic use of number-fact and reordering knowledge when doing three-digit mental addition. In that study children received three digit addition problems which incorporated number-facts such as tie-sums (e. g. 4+4) and sum to ten number-facts (e. g. 7+3). They also received sums that were designed to look at the position of the largest addend (9+a+b). The aim of that study was to find out whether or not there would be a shorter solution time for the sums in which the number-fact (or the position of the largest addend in the 9+a+b sums) was made more prominent. That is, a sum like 4+4+n should have a shorter RT than a sum like 4+n+4. Similarly a sum like 7+3+n and 9+1+n should also have shorter RTs than sums like 7+n+3 and 9+n+1. For the larger number reordering problems, sums where the larger addend appeared first such as 9+a+b should have similar solution times for a+b+9 and a+9+b. This would suggest that children were reordering the addends and were solving it strategically using min. If sums such as 9+a+b have significantly different solution times this would suggest children were rather slavishly counting on from the first addend without reordering to use the min counting strategy.

The results for the 9+a+b type sums (designed to look at the effect of reordering the largest addend first) showed that this type of sum generated comparable solution times, regardless of digit order. This suggested that children did not seem to be making use of their knowledge of the min counting strategy and reordering the numbers to put the largest number first. The min counting strategy implies that problems where the smaller addend is added to the larger addend should have lower RTs than problems where the larger addend is added to the smaller addend.

Results also demonstrated that the three digit sums that included a prominent tie-sum number-fact (4+4+n) did have significantly shorter RTs than the three digit sums in which the tie-sum was less prominent (4+n+4) and this was across all age groups (Figures 5.17 and 5.18). This reinforced the salience of the tie-sum number facts. Sums with a prominent 9+1 number fact (9+1+n) were also solved significantly faster than 9+n+1 (Figure 5.20). However, the problems in which the sum to 10 fact "7+3" was prominent did not have significantly shorter RTs than 7+n+3, although there did appear to be a trend towards shorter RTs for the 7+3+n sums (Figure 5.19). Unfortunately, this case is hard to interpret definitively, as the similar RTs might mean a reordering of the "3" or it might reflect a "passive" counting in which the 10+n offers no advantage of an immediate non-counted solution. However, my previous findings in Sections 5.1 and 5.1 suggest that reordering to use sum to 10 number facts would be unlikely.

This raises the question of why sum to 10 number-facts have failed to emerge as salient in my own research, especially since they are potentially more powerful than tie-sum number facts. Yet the latter seem to be more easily recognised and exploited. Tie-sums seem to emerge as number-facts in decade contexts as well as three-digit contexts and are also used by children to solve other related sums such as sums adjacent to tie-sums (Section 5. 2). Although the research reported in Section 5. 1 suggested that children did not seem to respect sum to 10 as a known number-fact, the three-digit serial addition used in Section 5. 2 was intended to have provided children with a suitably demanding context in which they would benefit from using this fact. If their reason for not taking advantage of the sum to 10 fact was due to a general reluctance to be strategic, then the serial addition scenario should have enticed them into doing so. Since research has shown that children are reluctant to make efficient use of their strategic knowledge unless they are confident enough to use it with ease, perhaps a serial addition context would have encouraged them to exploit their strategic knowledge. But children still seemed slow to use sum to 10 number-facts in a three-digit context. However, the RT based analysis is not straightforward - although suggestive. These preliminary observations needed to be pursued further in order to obtain a more confident picture of what was actually happening.

This was partly because of the possibility that the linear format in which the children were given the three digit sums was preventing them from being strategic. The pattern of results suggested that they were attempting to be strategic (i. e. they had slightly faster solution times for sums in which the "number-fact" was made obvious). Yet the findings from my previous study also suggested that it was difficult to look at strategic behaviour on three-digit mental addition without looking at the order in which the numbers were being added. Inferring strategy from RT may not be adequate - although it forms a useful part of any emerging picture about strategy development. Thus, the aim of this study was, first, to take into consideration the linear presentation of sums and the bias this may create towards not reordering and then to give children the freedom to impose their own order in selecting their addends. Secondly, my purpose was to find a way to explicitly observe their selection order, rather than simply infer it from the RT results.

7. 1. 2 Aim of Present Research

The study carried out in 5. 2 involved presenting children with problems written in a traditional horizontal format e. g. 7+3+n, and analysing the solution times. However, it was possible that seeing problems presented in this way might actually have discouraged some children from being strategic by exploiting the possibility of reordering moves. Instead of solving the problems in this way, some children could have added the numbers, slavishly respecting the order in which they appeared on the screen; particularly since existing research, as well as the research reported in Chapter 6, suggests that children can be rather slow at using their strategic knowledge. Even when presented with an opportunity to be strategic, most children are likely to rely on less efficient methods (such as mental or finger counting) that they know will work in the sense of providing them with an accurate answer. Seeing the problems in a linear format would further inhibit their strategic responses because the order of the addends was more or less being imposed upon them. What was needed was a methodology that would undermine traditional ordering of the addends and measure the effects of allowing children control over which addends to select first, thus allowing them the strong opportunity to exploit any strategic knowledge they might possess. In the following study, children were presented with numbers arranged randomly on the screen without any "+" or "=" symbols. They were then asked to click on the numbers in the order in which they were adding them. This would enable me to find out the order in which the numbers were being added.

Specifically, I would be able to find out which pairs of numbers they chose to add first i. e. which combinations of addends they perceived to be the easiest. Of course, they may make random choices or merely go for the closest addends to the mouse pointer. However, they could also be more strategic. They could choose the largest addend first, the largest pair of addends first, the addends that summed to 10 or tie-sum addends. If they were given control over which numbers to add, would they be systematic about the order in which they added those numbers? Would they be strategic about the numbers they would select or would they select the addends in no specific order? For older children, more competent younger children, and adults the most efficient method of dealing with a task like this might be to select the numbers according to their proximity to each other on the screen, because if you are fairly competent at arithmetic then $6+7+5$, for example, will not be sufficiently different from $7+7+4$ or even $7+3+5$ even when randomly positioned on the screen. So the order in which the numbers were added would not matter too much and would have only a minimal effect on RT and accuracy. But this would not be the case for younger children who may be competent at some number-facts and resort to mental or finger counting on others. In the case of the linear, left to right arithmetic format (as used in 5. 2) it would be done this way by children because they were being mechanically procedural while by adults because they were highly competent. So extrapolating from the physical proximity of the addends might be a factor affecting both very skilled (adult) and very unskilled (younger children) individuals.

In summary, the aim of this study was to discover the selection strategies used by the children when doing three digit mental addition. My previous research had shown that children were capable of using strategically their number fact knowledge (such as tie-sums) and their knowledge of the min reordering strategy. They were also able to make some use of their existing knowledge in a decade context. So, would they be able to make use of this knowledge in a serial addition context in which they could control which numbers to add first, and would this be evident from their selection of strategic number combinations?

As the results from my earlier study in Section 5. 2 suggested, three-digit sums were quite difficult for the children i. e. their average time on a three-digit sum was substantially higher, so any strategy that they can use must be helpful to them.

Three sorts of problems were chosen for the current study: problems inviting use of sum to 10 number facts such as 6+4 and 7+3 (Group 1), problems involving tie-sum number-facts such as 7+7 and 3+3 (Group 2), problems involving a visible large number (Group 3). The problems in Group 1 and Group 2 were designed to look at children's selection strategies and would reflect their use of sum to 10 and tie-sum number-fact knowledge and the problems in Group 3 were designed to look at their use of the min reordering strategy.

7. 2 Method

7. 2. 1 Participants

A total of 135 participants were recruited from two schools (for more detail see Section 4. 4. 1 and 4. 4. 4). All the children were familiar with using a mouse and graphical computer interface. These are the numbers of children who took part in the study and completed all the sums. However, there were cases where a child's data for a particular sum or number combination was lost as a result of a computer error and would not therefore be available for analysis. In these cases the child's data for the rest of the sums he/she completed would still be included in the analysis for sets of sums he/ she completed. For certain sums the participant would be excluded from the analysis as a result of missing data for that particular set of sums. This explains the occasional inconsistency between this table of participants and those in the results section (7.4.1).

Table 7.1

Year	Number	Male	Female
3	46	24	22
4	49	30	19
6	40	19	21
Total	135	73	62

7. 3. 2 Tasks

The participants received combinations of numbers that fell into three categories and within each there were subsets of sums (Table 7. 2). In Group 1, two types of sum to 10 (6+4, and 7+3) combinations were chosen because these were sum to 10 pairs in which the addends were relatively similar in magnitude (unlike 8+2) i. e. the small addend was large enough and was not a tie. This excluded 5+5 (tie-sum), 9+1 (adding "1") and 8+2. Children who selected the sum to 10 pairs first (e. g. 7+3+n or 3+7+n) would be seen as being strategic whereas those who chose the numbers in other orders (e. g. 7+n+3 or 3+n+7) would be seen as being less efficient with those who chose the smallest number first and the largest number last would be using the least efficient strategy (3+n+7). The "n" values were chosen to be both smaller than the smallest sum-to-10 addend, and larger than the largest addend.

In Group 2, two tie-sum combinations were chosen, one involving a large tie-sum (7+7) and one that involved a small tie sum (3+3). These sums were designed to investigate whether children would make use of their existing number-fact knowledge of tie-sums and sum to 10s. In these number combinations, children would be strategic if they chose the tie-sum first (e.g. 7+7+n), less so if they chose the tie-sum last (n+7+7) because this showed some recognition of the tie number-fact, and unstrategic if they were to select 7+n+7. Selecting the tie-sum first would suggest that they were recognising this as a number-fact and using this knowledge, while selecting the tie-sum addends last would suggest that although they were recognising the tie-sum the were not being particularly strategic.

The number combinations in Group 3 A involved adding an obviously large number (9) to two smaller ones, and were chosen to find out whether children would make use of their knowledge of the min strategy by reordering to choose the largest number first, followed by the two smaller ones. The number combinations in Group 3 B were chosen to look at children's selection strategies when there was a larger number (7) and "1" was being added, because this would shed light on the special "add one" case, i. e. would this "n+1" case encourage children to select this first. Children would be classified as being strategic in their selection of numbers if they chose the largest number first followed by "1".

The program was configured to deliver the following combination of numbers and the numbers within each problem were presented in a random order:

Table 7.2 Addend combinations as used in the program (sums in Group 1A).

Problem Type	**Descrip. of sum**	**Problem**	**Number combns.**	**Number of possible solutions**
Group 1 A	7+3+n	7+3+n (n<3)	7+3 (+2)	7+3+2
Sum to 10				3+7+2
				2+7+3
				2+3+7
				7+2+3
				3+2+7
		7+3+n (n>3)	7+3 (+4)	7+3+4
				3+7+4
				4+7+3
				4+3+7
				7+4+3
				3+4+7
		7+3+n (n>7)	7+3 (+8)	7+3+8
				3+7+8
				8+7+3
				8+3+7
				7+8+3
				3+8+7

Table 7.3 Addend combinations as used in the program (sums in Group 1B).

Problem Type	**Descrip. Of sum**	**Problem**	**Number combns**	**Number of possible solutions**
Group 1 B Sum to 10	6+4+n	6+4+n (n<4)	6+4 (+3)	6+4+3
				4+6+3
				3+6+4
				3+4+6
				6+3+4
				4+3+6
		6+4+n (n>4)	6+4 (+5)	6+4+5
				4+6+5
				5+6+4
				5+4+6
				6+5+4
				4+5+6
		6+4+n (n>6)	6+4 (+7)	6+4+7
				4+6+7
				7+6+4
				7+4+6
				6+7+4
				4+7+6

Table 7.4 Addend combinations as used in the program (sums in Group 2).

Problem Type	**Descrip. Of sum**	**Problem**	**Number combns.**	**Number of possible solutions**
Group 2 A Tie-sum (small number tie)	3+3+n	3+3+n (n<3)	3+3 (+2)	3+3+2 3+2+3 2+3+3
		3+3+n (n>3)	3+3 (+4)	3+3+4 3+4+3 4+3+3
Group 2 B Tie-sum (large number tie)	7+7+n	7+7+n (n<7)	7+7 (+6)	7+7+6 7+6+7 6+7+7
		7+7+n (n>7)	7+7 (+8)	7+7+8 7+8+7 8+7+7

Table 7.5 Addend combinations as used in the program (sums in Group 3).

Problem Type	**Descrip. Of sum**	**Problem**	**Number combns**	**Number of possible solutions**
Group 3 A Sum involving a larger number	x+y+z	9+x+y (x<9, y<9)	9+2+3	9+2+3 9+3+2 2+9+3 3+9+2 2+3+9 3+2+9
Group 3 B Sum involving a larger number +1	x+y+1	7+y+1	7+4+1	7+4+1 4+7+1 7+1+4 4+1+7 1+4+7 1+7+4

Within these three groups the participants received the following six types of sums.

1) three types of sum involving a sum to 10 (7+3)
2) three types of sum involving a sum to 10 (6+4)
3) two types of sum involving a large tie-sum (7+7)
4) two types of sum involving a small tie-sum (3+3)
5) sums which involved adding a 1 and a larger number
6) sums which involved adding a large number but did not involve adding "1"

The participants, therefore, received a total of 12 problems that could be solved in a variety of ways. For each number combination there were six possible solution choices e. g. in Group 1 A, for the number combination 7+3 (+2) the actual solution would be one out of 6 possible solutions. There were 3 categories of sums, and within each there were subsets of problems. Problems in Group1 A (6+4) and Group 1 B (7+3) were problems that included a sum to 10 that was not a tie (5+5) or a "+1" (9+1) and not a relatively easy sum to10 such as 8+2. Problems in Group 2 A involved a small-number tie-sum (3+3) and problems in Group 2 B involved a larger-number tie (7+7). Problems in Group 3 A were chosen to look at the use of the min-strategy in a serial addition context where all the numbers were different, there were no sum to 10s, and the problem included a large number. The problems in Group 3 B had been chosen to look at the effect of including a "+1" in a sum that involved a large number, where all the numbers were different, and there were no sum to 10s.

7. 3. 3 Procedure

The basic procedure followed the general pattern described in 4. 2.

The computers on which the children were tested were usually kept in a corner of the classroom. The task required each child to come over to the computer and do the arithmetic problems for about 15-20 minutes. All participants were supervised while doing the task.

The participant would see his/ her name as a button on the screen and clicked on it. He/ she was then presented with the set of problems generated in a random order. The problems would then appear on the screen in the form of randomly arranged numbers. The screen was ordered on an invisible 5x5 grid of number cells, providing the chance to see whether first choices were influenced by the distance from the starting mouse pointer position. No number had another number adjacent to it i. e. each number had a certain number of empty cells around it and it was this (i. e. the position of the number within the matrix) that was generated randomly. The position of the mouse would always be on a start/ finish bar at the bottom of the screen. There were no "+" or "=" signs on the screen. The participants were asked to add the numbers in any way they wanted using the method they found the easiest including finger counting. They were asked to click on each number as they added it, or (if they did this very fast) in the order in which they added it. The participant was also asked to say the answer out loud once he/ she had finished adding each number. Once they had done this they would click on the start/ finish bar at the bottom of the screen that would take them to the next sum. Each participant was given a demonstration (followed by a practice trial of 2+1+2 type sum) of the task that was required of them, to make certain that they knew what was expected of them.

The aim was to have the children click on each new number as soon as they were able to say the total it made when added e. g. if children were doing 7+4+3 they would say out "seven", "eleven", and "fourteen" as they successively clicked off the three numbers. The program recorded the order in which the children selected the numbers. For example, if a child clicked first on "7" then "3" then "4" then this would be the addend order that was recorded. The mouse returned to a random point on the baseline. The program also recorded the time taken to click on the numbers and the distance of the mouse pointer starting position from each target number. The program did not record the answers. Although the solution times were recorded, these would only provide a somewhat crude measure of performance only because the procedure did not give feedback of the RT to the children and their performance would not be well motivated for RT measuring. It must be stressed that although the solution times are represented in Figures 7.5, 7.6, 7.7, 7.12, 7.13 and 7.14, these are only presented here purely as an indication of the length of time (in this case total time spent on a sum from the time it appeared on screen until the final click on the answer bar) that was spent on a sum in which the less optimal combination of addends was selected. The RT data was not analysed and is used only to illustrate how long it may have taken to solve sums where certain combination of addends were selected.

7. 4 Results

For the ANOVA used in this section, the results were scored according to how strategically the addend combinations were selected. For example, if a child selected the addends 7+7+n then he/ she would be given a score of 3, if he/ she chose 7+n+7 then he/ she would be given a score of 1 and if he/ she selected n+7+7 then he/ she would be given a score of 2. Similarly participants who selected a sum-to-10 combination of 6+4+n or 7+3+n would be given a score of 6 and those who selected 4+n+6 or 3+n+7 would be given a score of 1.

7. 4. 1 Results for sums in Group 1 (sums involving a sum to 10 number-fact)

The results in this section looked at sums in Group 1 involving a sum to 10 number-fact.

For the analyses, sums in Group 1 were grouped into three types:

1) Type 1 grouped together sums in which the sum to 10 addends were selected first (such as 7+3+n and 3+7+n in Group 1 A, and 6+4+n and 4+6+n in Group 1 B).
2) Type 2 grouped together sums in which the sum to 10 addends were selected last (such as n+7+3 and n+3+7 in Group 1 A, and n+6+4 and n+4+6 in Group 1 B).
3) Type 3 grouped together sums in which the sum to 10 addends were not selected consecutively (such as 7+n+3 and 3+n+7 in Group 1 A, and 6+n+4 and 4+n+6 in Group 1 B).

7. 4. 1. 1 Results for sums in Group 1 A (7+3+n)

Figure 7.1 illustrates the results from sums in Group 1 A which involved the sum to 10 7+3(+n) and where n was less than 3.

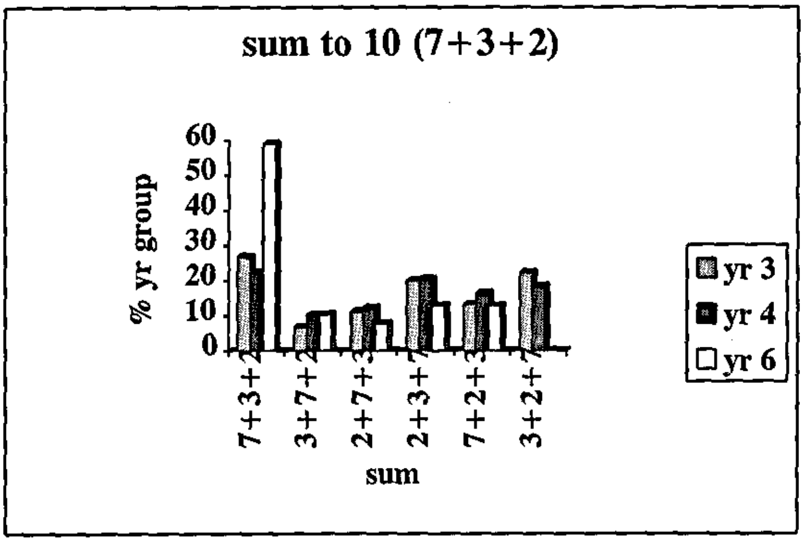

Figure 7.1 Sum to 10 (7+3+2)

The results shown in Figure 7.1 above suggest that most of the oldest children are likely to select the sum to 10 7+3+2 first when doing sums involving 7+3 and where the addend n was less than 3, and none of the oldest children selected 3+2+7 which would be the least efficient combination of addends. Younger children are almost as likely to select 3+2+7 suggesting that they are not being strategic about the order in which select the addends, perhaps because the addend (2) is not particularly large.

7. 4. 1. 1. 1 Analysis for sums in Group 1 A involving the sum to 10 [7+3 (+2)]

Table 7.6 Sum-type by year group cross tabulation for the sum 7+3 (+2).

Sum type * year group Crosstabulation

			year group			
			3	4	6	Total
Sum type	1	Count	15	16	27	58
		Expected Count	19.5	21.2	17.3	58.0
	2	Count	14	16	8	38
		Expected Count	12.8	13.9	11.3	38.0
	3	Count	16	17	5	38
		Expected Count	12.8	13.9	11.3	38.0
Total		Count	45	49	40	134
		Expected Count	45.0	49.0	40.0	134.0

A chi-square carried out on the results illustrated in Figure 7.1, chi-square (df = 4) = 14.215, p = 0.007, suggesting that children across all ages were more likely to select certain combinations of addends i. e most children were likely to select the larger addends first. However, the results in Figure 7.1 suggest that older children were more likely than the younger ones to select the sum to 10 or the larger digits first.

Figure 7.2 shows the pattern of results for sums in Group 1 A which involved the sum to 10 7+3(+n) and where n was greater than 3.

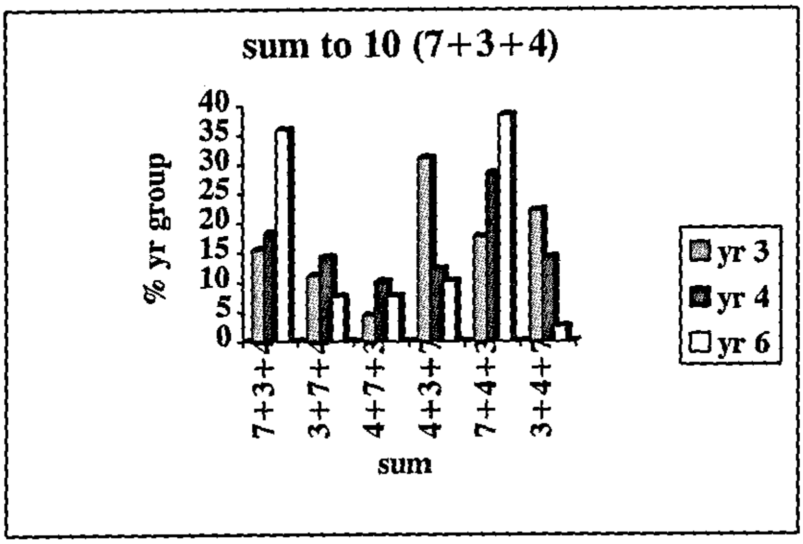

Figure 7.2 Sum to 10 (7+3+4)

The results pictured above in Figure 7.2 show that when doing a 7+3 sum to 10 where the addend n was greater than 3, most of the older children would select the sum to 10 first. However, they were also more likely to select the largest number 7 first suggesting that they could alternatively be taking advantage of the min strategy and reordering the addends optimally. They were least likely to select 3+4+7. However, the younger children do not appear to be selecting their addends in the most efficient order.

7. 4. 1. 1. 2 Analysis for sums in Group 1 A involving the sum to 10 [7+3 (+4)]

Table 7.7 Sum-type by year group cross tabulation for the sum 7+3 (+4).

sum-type * year group Crosstabulation

			year group			Total
			3	4	6	
sum-type	1	Count	12	16	17	45
		Expected Count	15.4	16.1	13.4	45.0
	2	Count	16	11	7	34
		Expected Count	11.7	12.2	10.1	34.0
	3	Count	18	21	16	55
		Expected Count	18.9	19.7	16.4	55.0
Total		Count	46	48	40	134
		Expected Count	46.0	48.0	40.0	134.0

A chi-square carried out on the results illustrated in Figure 7. 2, chi-square (df = 4) = 4.551, p = 0.336, found that children across all year groups did not seem to be strategic about their addend selection.

Figure 7.3 illustrates the pattern of results from sums in Group 1 A involving the sum to 10 7+3(+n) where n was greater than 7.

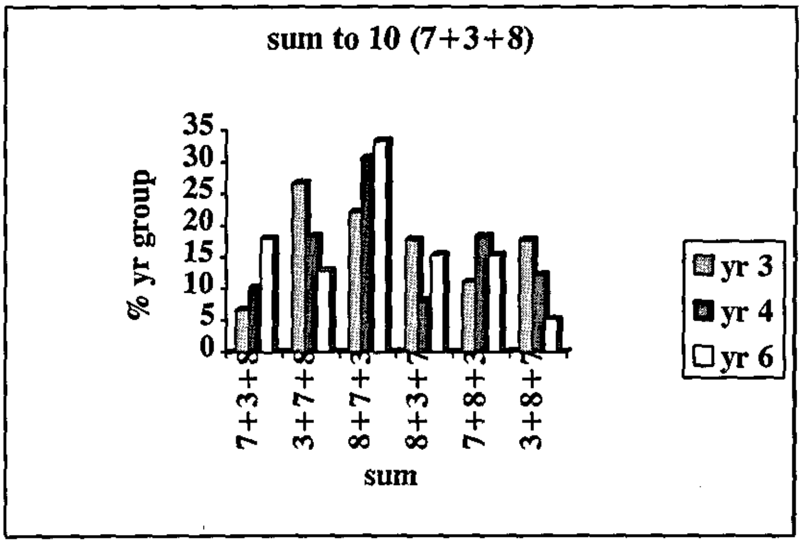

Figure 7.3 Sum to 10 (7+3+8)

The results in Figure 7.3 seemed to suggest that when doing a sum involving the sum to 10 7+3 where the addend n was greater than 7 most of the oldest children were more likely to select the largest addends first. This suggested that they were more likely to be relying on the min reordering strategy. The youngest children were as likely to select the largest addend last and were the least likely to select 7+3+8. The older children were more likely than the younger children to select both sum to 10 combinations (7+3+8 and 3+7+8) first.

7. 4. 1. 1. 3 Analysis for sums in Group 1 A involving the sum to 10 [7+3 (+8)]

Table 7.8 Sum-type by year group cross tabulation for the sum 7+3 (+8).

sum type * year group Crosstabulation

			year group			
			3	4	6	Total
sum type	1	Count	15	14	12	41
		Expected Count	14.2	14.8	12.0	41.0
	2	Count	18	19	19	56
		Expected Count	19.4	20.2	16.4	56.0
	3	Count	13	15	8	36
		Expected Count	12.5	13.0	10.6	36.0
Total		Count	46	48	39	133
		Expected Count	46.0	48.0	39.0	133.0

A chi-square carried out on the results illustrated in Figure 7.1, chi-square (df = 4) = 1.618, p = 0.806, suggested that children across all year groups were not being strategic about their addend selection.

Figure 7.4 shows a summary of the effect of year on sum order on sums involving the sum to 10 7+3 (+ n).

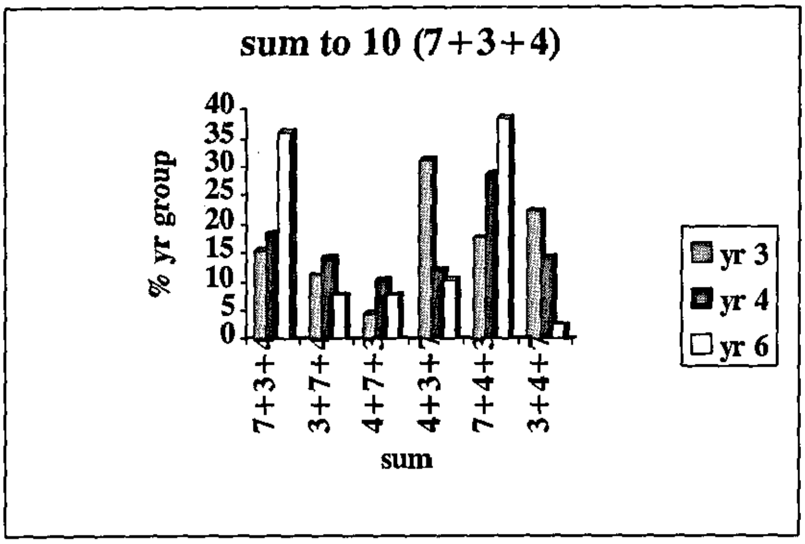

Figure 7.4 Sum to 10 (7+3+n)

The summary of results in Figure 7.4 show that when children are required to do sums involving the sum to 10 7+3+n, most children do not select the sum to 10 first. However, children in the oldest age groups are the most likely to select the sum to 10 first and they are the least likely to select the least efficient combination of addends (3+n+7) compared to younger children. Most of the children do not seem to be selecting the sum to 10 7+3 first suggesting that either 7+3 is not a salient number fact or that they may have this knowledge but are not taking advantage of it.

An ANOVA carried out on the results for all the sums in Group 1A found a significant overall effect of year, $F(2, 130) = 3.74$, $p < 0.05$. There was a significant main effect of the size of the third addend n, $F(1, 129) = 30.06$, $p < 0.05$, suggesting that size of the third addend did have an effect on the selection strategy. There was s significant interaction between the third addend size and year, $F(4, 260) = 15.75$, $p < 0.05$. This suggested that older children were significantly more likely to be influenced by the size of the third addend i. e. whether or not they selected the sum-to-10 first or the largest addend first depended on the size of the third addend n. There was a significant effect of sum order, $F(5, 126) = 24.31$, $p < 0.05$, suggesting that children across all ages were more likely to select certain combinations of addends i. e most children were likely to select the larger addends first. There was a significant interaction between sum order and year $F (10, 254) = 2.63$, $p < 0.05$. This meant that older children were more likely than the younger ones to select the sum-to10 or the largest digits first (as illustrated by Figure 7.4). There was a significant interaction between the third addend size and the sum order, $F (10, 121) = 14.53$, $p < 0.05$, suggesting that children's selection of addend order was influenced by the size of the third addend n i. e. children were more likely to select the larger addends first if the third addend n (8) was larger than 7. There was a significant interaction between third addend size, sum order and year, $F (20, 244) = 4.33$, $p < 0.05$. This suggested that older children's sum selection was more likely to be influenced by the size of the third addend compared to the younger children's.

It seemed that most of the children were not being particularly strategic about selecting their addends to make use of sum-to-10 number-facts.. Most of them did not seem to be making use of sum to 10 number-facts. It could be argued that perhaps most children were competent enough at mental addition that it did not matter what order they used for selecting their addends. Alternatively, because they were most likely using counting strategies, how they chose their addends did not make much difference. Would this be reflected in their overall solution times for these sums? That is, children who seemed to be making inefficient choices about addend selection might also have long solution times for these sums suggesting that their addend selection was actually holding them back and resulting in long solution times. Figures 7.5, 7.6 and 7.7 illustrate children's total solution times for the sum to 10 problems in Group 1 A (7+3).

Figure 7.5 is a summary of the solution times for the sum to 10 7+3 (+2).

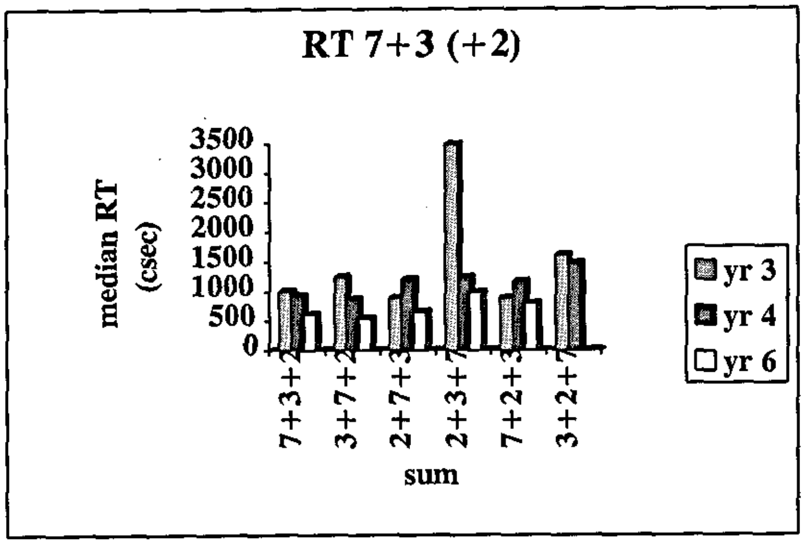

Figure 7.5 Solution times for sum to 10 7+3 (+2)

The solution times summarised in Figure 7.5 do suggest that children who were selecting the smallest addend first had the longest solution times, and the youngest children especially seem to be having the most difficulty. Children who were selecting the largest digit or the sum to 10 first had the shortest solution times.

Figure 7.6 is a summary of solution times for the sum to 10 7+3 (+4).

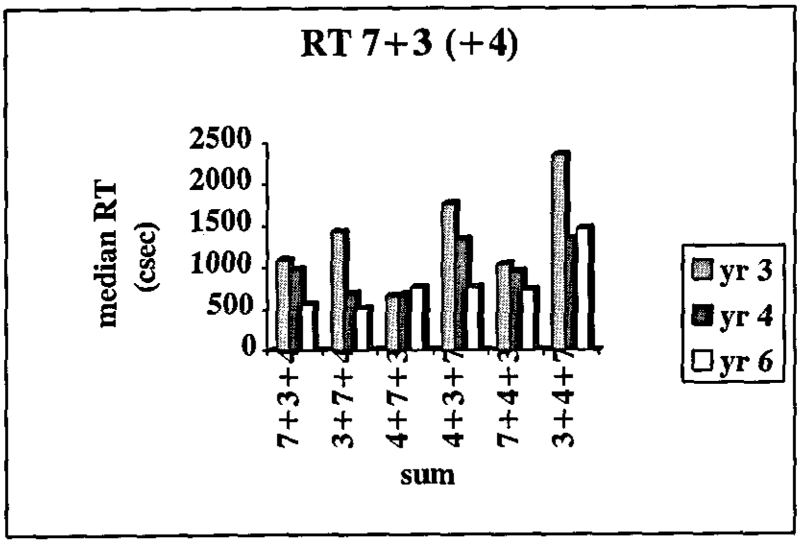

Figure 7.6 Solution times for sum to 10 7+3 (+4)

The solution times summarised in Figure 7.6 show that, as the addends become larger, the children's solution times also show an overall increase. Again, this has the greatest effect on the youngest children's solution times. Children who are selecting the smallest addends first have the longest solution times and the youngest children are the ones most likely to be doing so. Older children who select the smallest digits first are also the ones who have long RTs. Because the third addend "4" and "2" are not too large, it seems that some older children are not finding these sums too difficult. However, when the third addend (8) becomes much larger even the older children lose this advantage.

Figure 7.7 is a summary of solution times for the sum to 10 7+3 (+8).

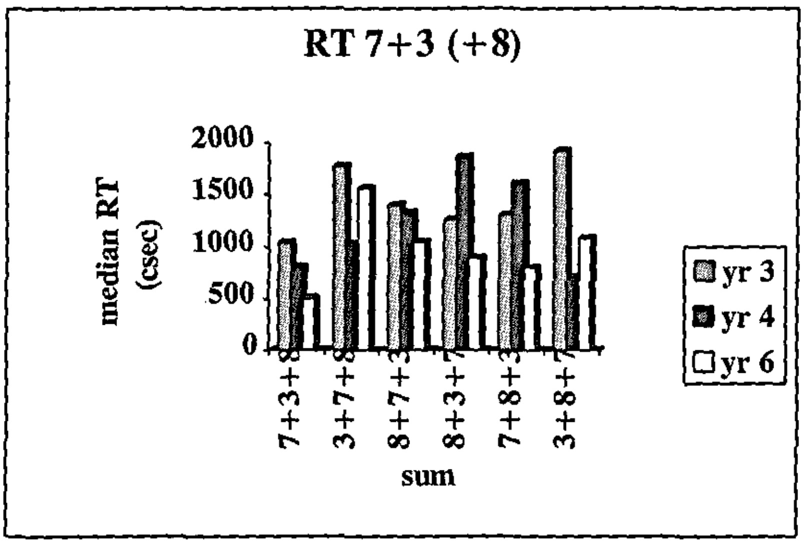

Figure 7.7 Solution times for sum to 10 7+3 (+8)

The solution times illustrated in Figure 7.7, show that when the size of the third addend increases solution times increase for children across all age groups. Interestingly, the children with the fastest solution times are those who select the sum to 10 (7+3) first, and this is the case for children across all ages. It seems that children who seem to be careless of addend order are also the ones paying the price of much longer solution times. Figures 7.5, 7.6 and 7.7 all indicate that there seems to be a pay off, accidental perhaps, for those children who are selecting the sum to 10 first even when the third addend n is the largest addend.

The sums in Group 1 A looked at children selection strategies when adding numbers that involved the sum to 10 7+3. The results showed that children were less likely to use sum to 10 knowledge and tended to rely on the min reordering strategy by selecting the largest addends first. The following section looks at their selection strategies when doing sums in Group 1 B involving the sum to 10 6+4.

7. 4. 1. 3 Results for sums in Group 1 B (6+4+n)

Figure 7.8 illustrates the pattern of results from sums in Group 1 B involving the sum to 10 6+4(+n) where n was less than 4.

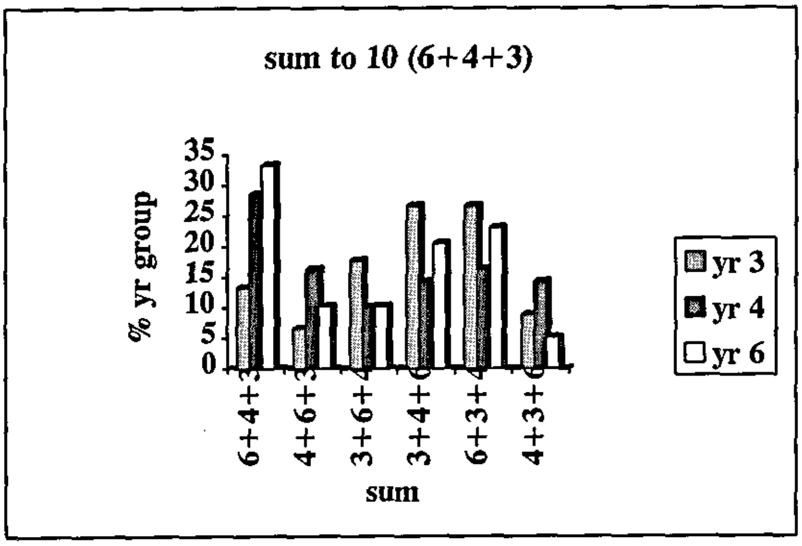

Figure 7.8 Sum to 10 (6+4+3)

The results in Figure 7.8 looked at children's selection strategies when doing sums involving the sum to 10 6+4 where the third addend n (3) was less than 4. These results suggest that most children do not seem to be taking advantage of the 6+4 number fact. Younger children were more likely to select the largest addend first suggesting that some of them seem to be reodering and using the min strategy.

However, just as many children seem to be selecting the addends in no particular order such as 3+4+6. The older children in years 4 and 6 did seem to be more likely to select 6+4 first suggesting that they may be adding the digits strategically. Alternatively they may still be selecting this combination because they are reordering and selecting the largest addend first.

7. 4. 1. 3. 1 Analysis for sums in Group 1 B involving the sum to 10 [6+4 (+3)]

Table 7.9 *Sum-type by year group cross tabulation for the sum* 6+4 (+3).

sum type * year group Crosstabulation

			year group			
			3	4	6	Total
sum type	1	Count	9	22	17	48
		Expected Count	16.1	17.6	14.3	48.0
	2	Count	20	12	12	44
		Expected Count	14.8	16.1	13.1	44.0
	3	Count	16	15	11	42
		Expected Count	14.1	15.4	12.5	42.0
Total		Count	45	49	40	134
		Expected Count	45.0	49.0	40.0	134.0

A chi-square test was carried out on the results illustrated in Figure 7. 8, chi-square (df = 4) = 8.205, p = 0.084. This suggests that the children across all year groups are not being strategic about selecting their addends.

Figure 7.9 illustrates the pattern of results from sums in Group 1 B involving the sum to 10 6+4(+n) where n was greater than 4.

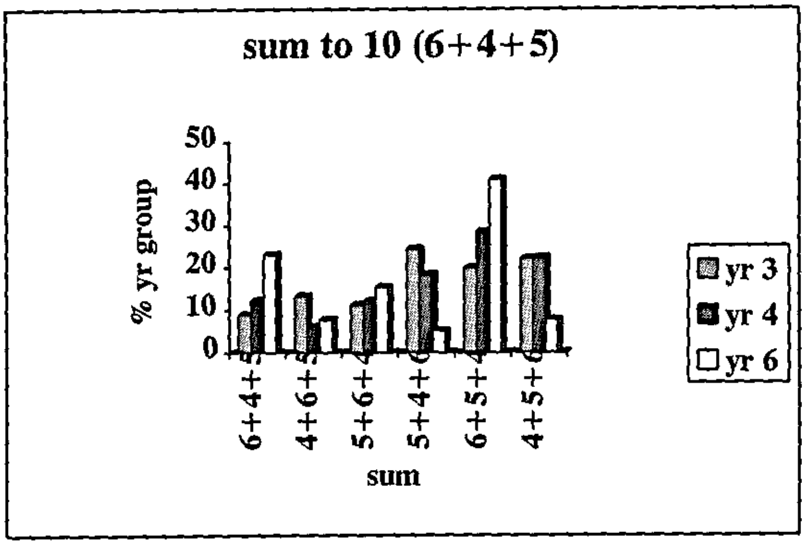

Figure 7.9 Sum to 10 (6+4+5)

The results shown above in Figure 7.9 suggest that when doing sums that involved a sum to 10 6+4 where the third addend n (5) was greater than 4, children in the oldest age group seemed to select the largest addend first. Most children did not seem to be taking advantage of the sum to 10 6+4. The oldest children were the most likely to select the sum to 10 first suggesting that they are being strategic and using their sum to 10 knowledge. However, they were more likely to select the largest addend first and this suggests that they wre more likely to be relying on the min strategy of reordering and adding the smaller addends to the larger ones.

7. 4. 1. 3. 2 Analysis for sums in Group 1 B involving the sum to 10 [6+4 (+5)]

Table 7. 10 Sum-type by year group cross tabulation for the sum 6+4 (+5).

sum type * year group Crosstabulation

			year group			Total
			3	4	6	
sum type	1	Count	10	9	12	31
		Expected Count	10.5	11.4	9.1	31.0
	2	Count	16	15	8	39
		Expected Count	13.2	14.4	11.4	39.0
	3	Count	19	25	19	63
		Expected Count	21.3	23.2	18.5	63.0
Total		Count	45	49	39	133
		Expected Count	45.0	49.0	39.0	133.0

A chi-square test was carried out on the results illustrated in Figure 7. 9, chi-square (df = 4) = 3.528, p = 0.474. This suggested that children across all year groups were not being strategic about their addend selection.

Figure 7.10 illustrates the pattern of results from sums in Group 1 B involving the sum to 10 6+4(+n) where n was greater than 6.

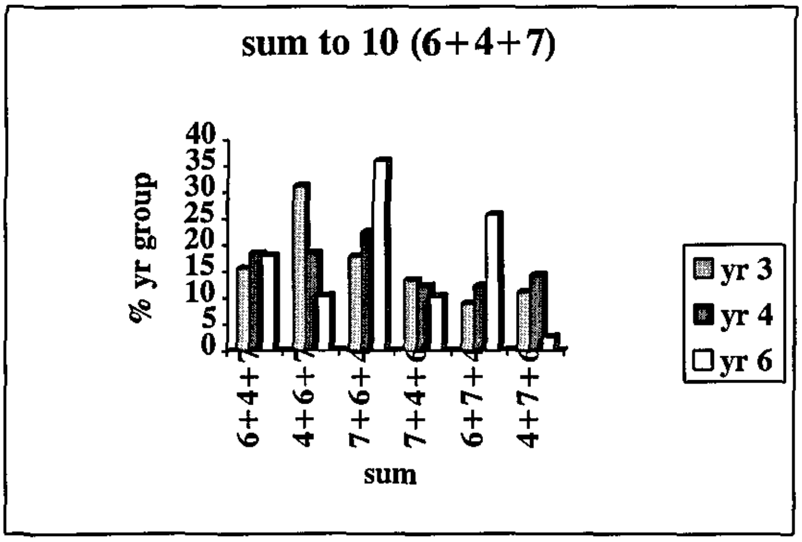

Figure 7.10 Sum to 10 (6+4+7)

The results shown above in Figure 7.10 also suggest that children in the oldest year group are more likely to be using the reodering strategy than use the sum to 10 number fact when doing sums where the third addend n (7) was greater than 6. The largest addend seems to be more visible to them than the sum to 10. The selection choices of the youngest children suggest that most of them do not seem to be selecting their addends in any particular order and this suggests that most of them are not being strategic about the order in which they are adding the digits.

7. 4. 1. 3. 3 Analysis for sums in Group 1 B involving the sum to 10 [6+4 (+7)]

Table 7.11 Sum-type by year group cross tabulation for the sum 6+4 (+7).

sum type * year group Crosstabulation

			year group			Total
			3	4	6	
sum type	1	Count	21	18	11	50
		Expected Count	16.7	18.2	15.2	50.0
	2	Count	14	17	18	49
		Expected Count	16.3	17.8	14.8	49.0
	3	Count	9	13	11	33
		Expected Count	11.0	12.0	10.0	33.0
Total		Count	44	48	40	132
		Expected Count	44.0	48.0	40.0	132.0

A chi-square test was carried out on the results illustrated in Figure 7. 10, chi-square (df = 4) = 3.853, p = 0.426. This suggests that children across all year groups are not being strategic about their addend selection.

Figure 7.11 shows a summary of the effect of year on sum order on sums involving the sum to 10 6+4 (+n).

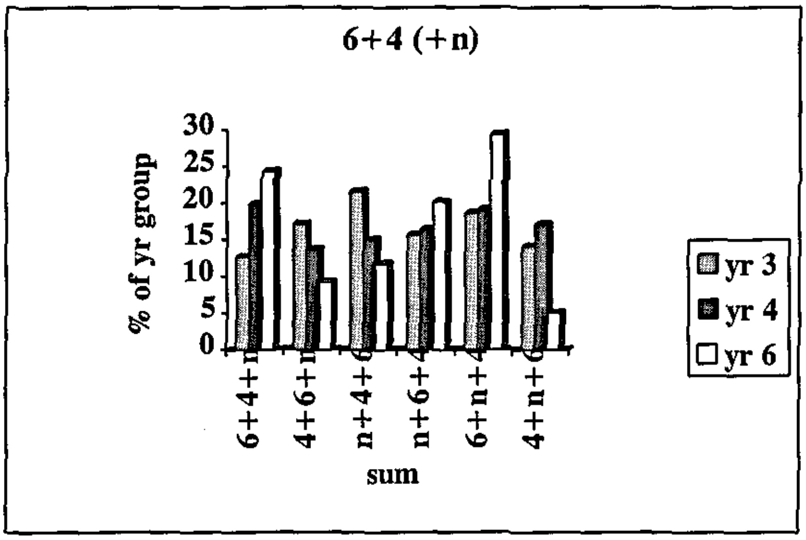

Figure 7.11 sum to 10 (6+4+n)

The summary of results in Figure 7.11 suggests that although older children are more likely than the younger ones to select the sum to 10 first, most of the younger children seem to be selecting their addends in no particular order.

An ANOVA on the results for all the sums in Group 1B found no overall effect of year, $F (2, 128) = 0.459$. There was a significant effect of addend size n, $F (2, 127) = 76.18$, $p < 0.05$, suggesting that the size of the third addend did affect solution strategy. There was a significant interaction between the third addend size and year, $F (4, 256) = 13.65$, $p < 0.05$. This suggested that older children were significantly more likely to be influenced by the size of the third addend i. e. whether or not they selected the sum-to-10 first or the largest addend first depended on the size of the third addend n. There was a significant effect of sum order, $F (5, 124) = 31.41$, $p < 0.05$. This suggested that children across all ages were more likely to select certain combinations of addends first. There was a significant interaction between the sum order and year, $F (10, 250) = 1.84$, $p < 0.05$, suggesting that older children were more likely than younger ones to select the sum-to-10 first or the largest addend first (as illustrated by Figure 7.8). There was a significant interaction between addend size and the sum order, $F(10, 119) = 17.52$, $p < 0.05$, suggesting that children's selection of addend order was influenced by the size of the third addend n. Children, therefore, were more likely to select the larger addends first if the addend n was larger than 6. There was a significant interaction between addend size, sum order and year, $F(20, 240) = 5.14$, $p < 0.05$. This suggested that older children's sum selection was more likely to be influenced by the size of the third addend compared to the younger children's.

The results for the sums in Group 1 suggested that, for the larger part, children were more likely to be using the min reordering strategy when selecting the order in which to add the digits and were selecting the largest addend first. Most children did not seem to be taking advantage of the sum to 10 number facts. It seemed that children were being strategic but their addend selection indicated that this was in favour of selecting addends that would enable them to count more efficiently as opposed to use their number-fact knowledge more efficiently.

As was the case with the 7+3 sum to 10 problems, children seemed reluctant to make use of the 6+4 sum to 10. When they did seem to be strategic, they favoured strategies that would make counting easier. Would this be reflected in their solution times for these problems? Would children who were using inefficient strategies also have longer solution times as shown by the results in Figures 7.5, 7.6 and 7.7? The following graphs are summaries for children total RTs for the sums in Group 1 B involving the sum to 10 (6+4).

Figure 7.12 is a summary of the solution times for the sum to 10 6+4 (+3).

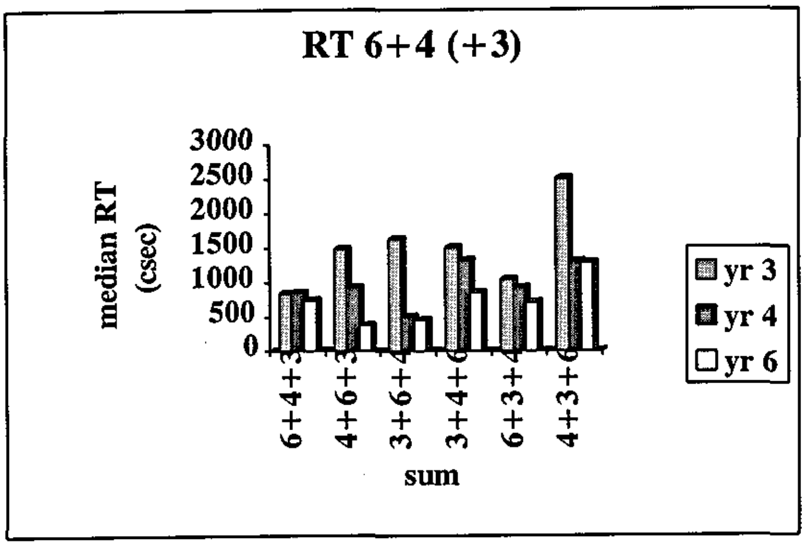

Figure 7.12 Solution times for sum to 10 6+4 (+3)

The results illustrated in Figure 7.12 suggest that selecting the smaller addends first results in longer solution times. This is more evident in the youngest children. It also suggests that children who do select the smaller addends first are probably most likely to be using an inefficient counting strategy.

Figure 7.13 is a summary of the solution times for the sum to 10 6+4 (+5).

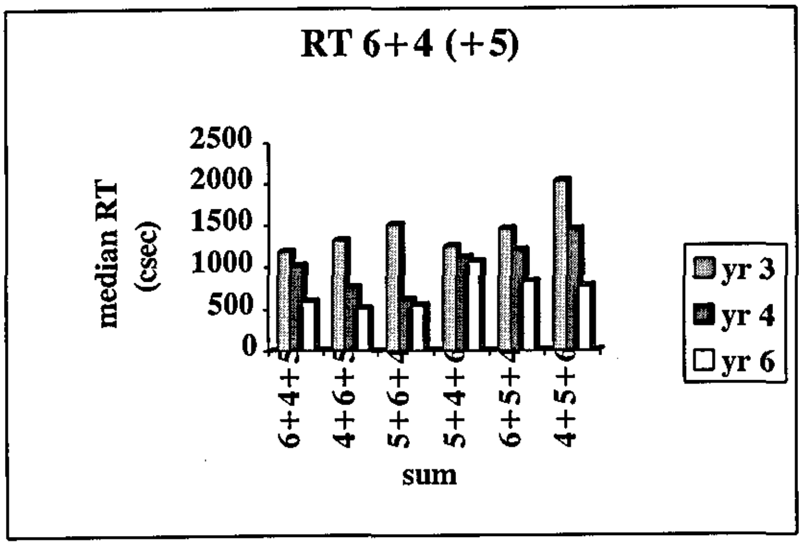

Figure 7.13 Solution times for sum to 10 6+4(+5)

As with the results shown in Figure 7.12, the solution times shown above suggest that children who select the smallest addends first seem to have long solution times. The younger children are more likely to do this. The solution times for the oldest children suggest that most of them are using similar strategies.

Figure 7.14 is a summary of the solution times for the sum to 10 6+4 (+7).

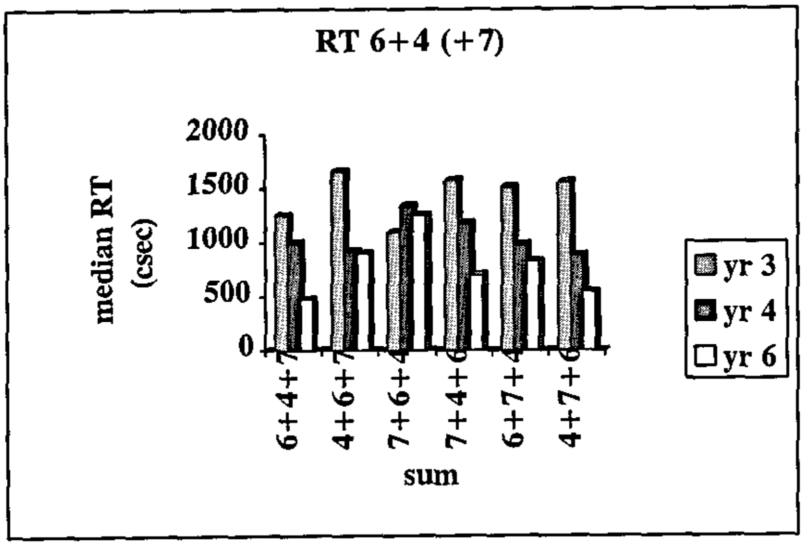

Figure 7.14 Solution times for sum to 10 6+4 (+7)

The solution times for the sums in Group 1 B all suggest that children across all ages find these sums quite difficult. Children who do not select the addends strategically seem to have the longest times and this is especially so for the youngest children. The solution times for the oldest children suggest that they are using the most consistent strategies. Older children who select the largest addend first have the longest times suggesting that they are resorting to counting. As with the solution time for the 7+3 sum to 10, there seems to be benefit in the form of shorter solution times for those children who do select the sum to 10 addends first.

The results in the next section looked at their selection strategies when doing sums in Group 2 which involved tie-sum number facts. The results from sums in Group 1 suggested that most children did not seem to rely on sum to 10 number-fact knowledge. The results in Group 2 looked at whether or not they would use their tie number fact knowledge.

7. 4. 2 Results for sums in Group 2 (sums involving a tie-sum number-fact)

The results in this section looked at addend selection for sums in Group 2 that involved tie-sum number facts.

7. 4. 2. 1 Results from sums in Group 2 A (small number tie-sum 3+3)

Figure 7.15 illustrates the pattern of results for sums in Group 2 A involving the tie-sum (3+3)+n where n was less than the tie-sum addends.

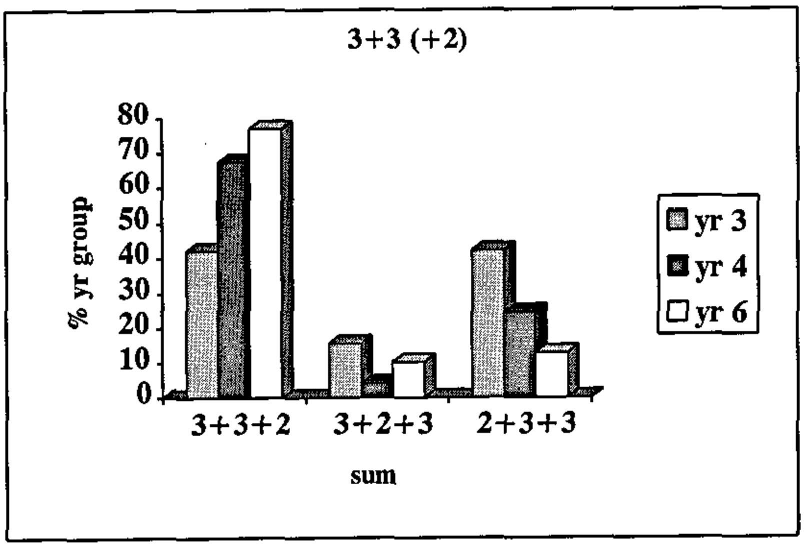

Figure 7.15 Tie-sum 3+3 (+2)

The results in Figure 7.15 suggested that when doing a sum that involved the tie-sum 3+3 (+n) where n (2) is less than the tie-sum addends most children will do the sum first. Children in the older age groups were most likely to select the tie-sum first. While the youngest chlidren were less likely than older children to do the tie-sum first, the results seemed to suggets that they are recognising the tie-sum because they are also more likely to select the tie-sum last ie. do 2+3+3. Children across all year groups were least likely to do 3+2+3 suggesting that they were making use of their tie-sum knowledge.

Figure 7.16 illustrates the pattern of results for sums in Group 2 A involving the tie-sum (3+3)+n where n was greater than the tie-sum addends.

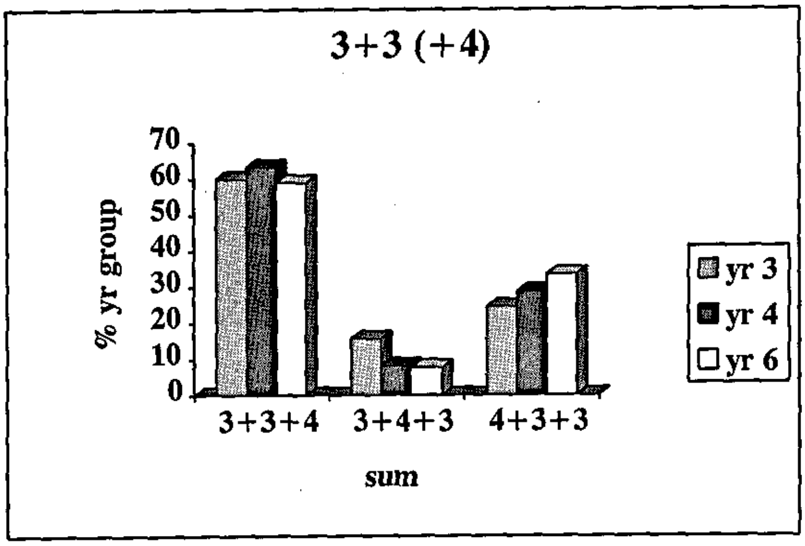

Figure 7.16 Tie-sum 3+3 (+4)

The results in Figure 7.16 suggested that when children had to do a sum involving the tie-sum 3+3 (+n) where n (4) was larger than the tie-sum addend children across all year groups were most likely to select the tie-sum first. Children in the youngest year group were more likely to do 3+4+3 than children in the older groups. Children were also more likely to do 4+3+3 than 3+4+3 suggesting that they are showing recognition of the tie-sum. For most of the children 3+3 has become a number fact and these results suggested they were taking advantage of this knowledge.

Figure 7.17 shows the summary of results for sums involving the tie sum 3+3(+n).

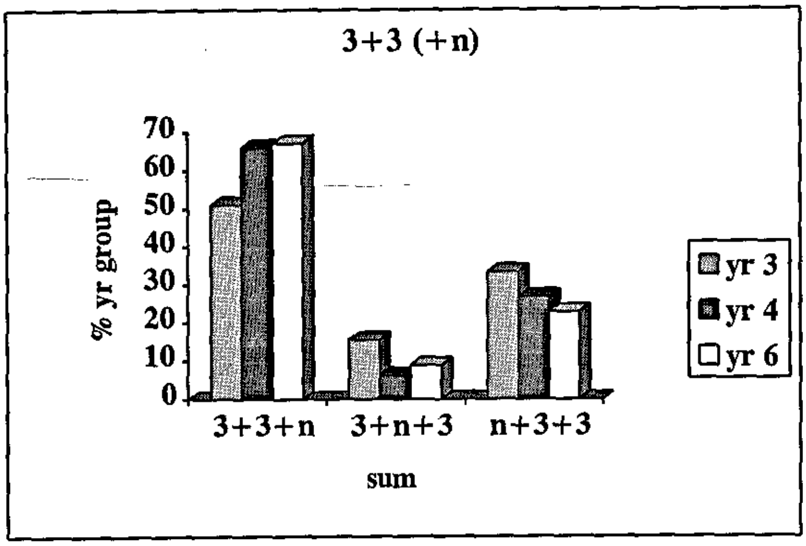

Figure 7.17 Tie-sum 3+3 (+n)

The results shown in Figure 7.17 suggested that children across all age groups were more likely to select the tie-sum first. This was more evident in older children where nearly 70% of the children in years 4 and 6 were likely to take advantage of their tie sum knowledge. Even children in the youngest age group were showing recognition of the tie-sum because they were selecting the tie-sum last.

Figure 7.18 shows a summary of the results for children across all years doing the tie-sum 3+3.

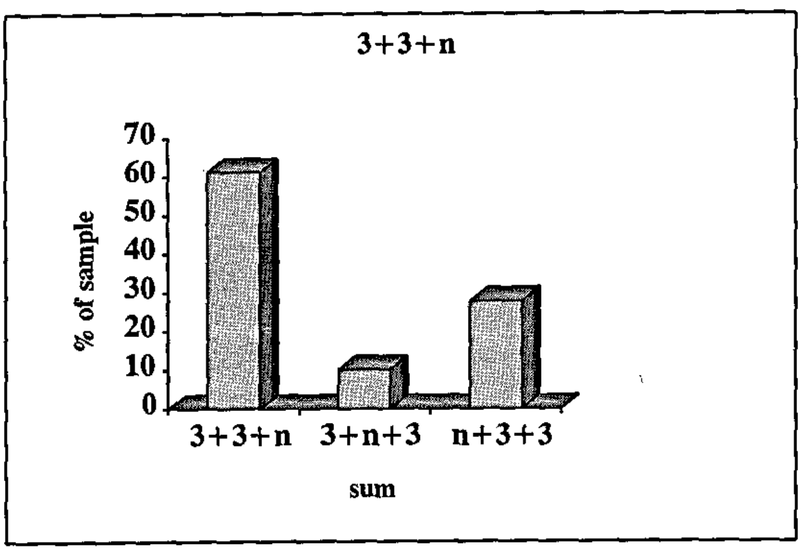

Figure 7.18 Tie-sum (3+3+n) across all year groups.

Figure 7.18 further illustrates that most children across all year groups were likely to select 3+3+n, followed by n+3+3 and were least likely to select 3+n+3.

7. 4. 2. 2 Analysis (sums in Group 2 A involving the small number tie-sum 3+3)

An ANOVA carried out on the results for sums involving the tie-sum 3+3 showed than there was no overall effect of year, $F(2, 128) = 2.83$. There was no effect of the size of the third addend (n), $F(1, 128) = 0.113$. This suggested that it did not make a significant difference whether the third addend n was greater than (n = 4) or less than (n = 2) the tie-sum addends and this is illustrated by the results shown in Figures 7.17 and 7.18. There was no interaction between the size of the third addend n and year, $F(2, 128) = 2.77$. There was a significant main effect of sum order, $F(2, 127) = 265.45$, $p < 0.05$. This suggested that significantly more children across all year groups were likely to select 3+3+n (as shown in Figure 7.17). There was no significant interaction between sum order and year group, $F(4, 256) = 1.52$. This suggested that children across all year groups were likely to select the tie-sum first (as shown by Figure 7.18). There was no interaction between the size of addend n and sum order, $F(2, 127) = 0.158$. This suggested that whether n was greater than or less than the tie-sum addends did not have a significant effect on the likelihood of children doing the tie-sum first. There was a significant interaction between the size of addend n, sum order and year group $F(4, 256) = 10.147$, $p < 0.05$. This suggested that the size of the addend n was more likely to affect the sum selection for children in younger year groups (as illustrated by Figure 7.17). Younger children were significantly more likely to do 3+3+n if the size of the addend n (4) was greater than the tie-sum addends.

These results suggested than the tie-sum 3+3 was a salient number-fact and most children across all age groups were willing to take advantage of their tie-sum number knowledge. However, the 3+3 tie-sum involves fairly small addends and the answers to the sums in Group 2 A were not greater than 10 and so could easily be solved through both number-fact knowledge or counting. Would the results for sums in Group 2 B involving the larger tie-sum 7+7 have similar patterns?

The results shown in the next section looked at the findings from sums in Group 2 B involving the tie-sum 7+7.

7. 4. 2. 3 Results from sums in Group 2 B (large number tie-sum 7+7)

Figure 7.19 illustrates the pattern of results from sums in Group 2 B involving the tie-sum 7+7(+n) where n was less than the tie-sum addends.

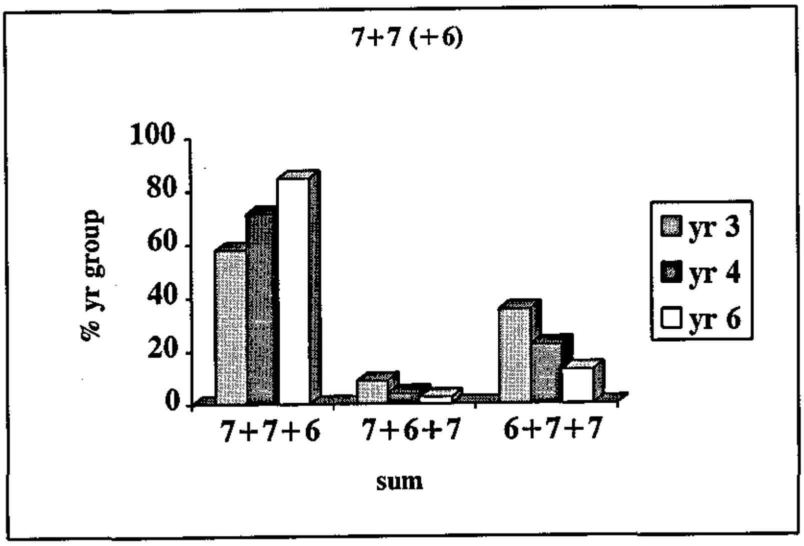

Figure 7.19 Tie-sum 7+7 (+6)

The results in Figure 7.19 suggested that when doing a sum that involved the tie-sum 7+7 (+n) where n (6) is less than the tie-sum addends most children will do the tie-sum first. Children in the older age groups were most likely to select the tie-sum first. Younger children seemed to be more likely to select the tie-sum last suggesting that although they seemed to be recognising the tie-sum 7+7 it was unlikely that this had become a number-fact. While the youngest chlidren were less likely than older children to do the tie-sum first, the results seemed to suggets that they are recognising the tie-sum because they are also more likely to select the tie-sum last ie. do 6+7+7. Children across all year groups were least likely to do 7+6+7, suggesting that they were being strategic and making use of their tie-sum knowledge.

Figure 7.20 illustrates the pattern of results from sums in Group 2 B involving the tie-sum 7+7(+n) where n was greater than the tie-sum addends.

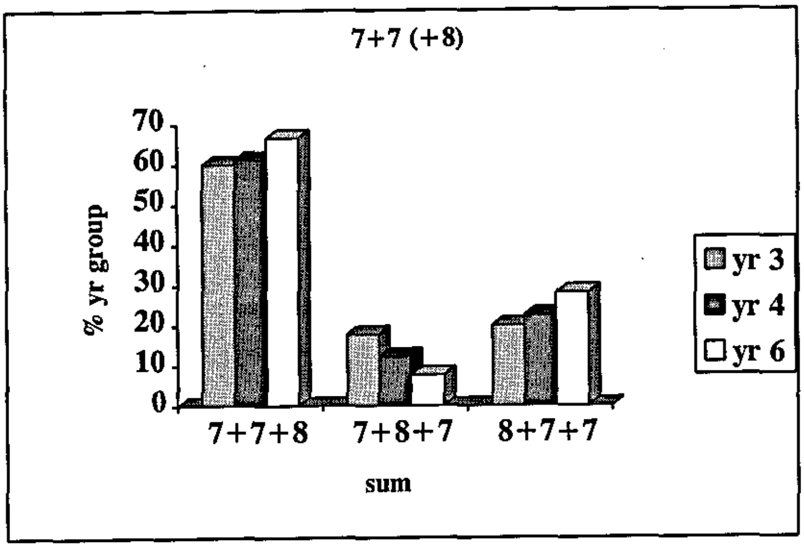

Figure 7.20 Tie-sum 7+7 (+8)

The results in Figure 7.20 suggested that when doing sums involving the tie-sum 7+7 (+n) where n was greater than the tie-sum addends, most children across all year groups were likely to do the tie-sum first. The oldest children were least likely to do 7+8+7. However, the older children were more likely to select the largest addend first suggesting that they are using the min reordering strategy instead of relying on tie-sum knowledge and this suggests that a large number of the older children as well as some of the younger ones are being quite strategic in their selection of addends.

Figure 7.21 shows a summary of the results from sums involving the tie-sum 7+7(+n).

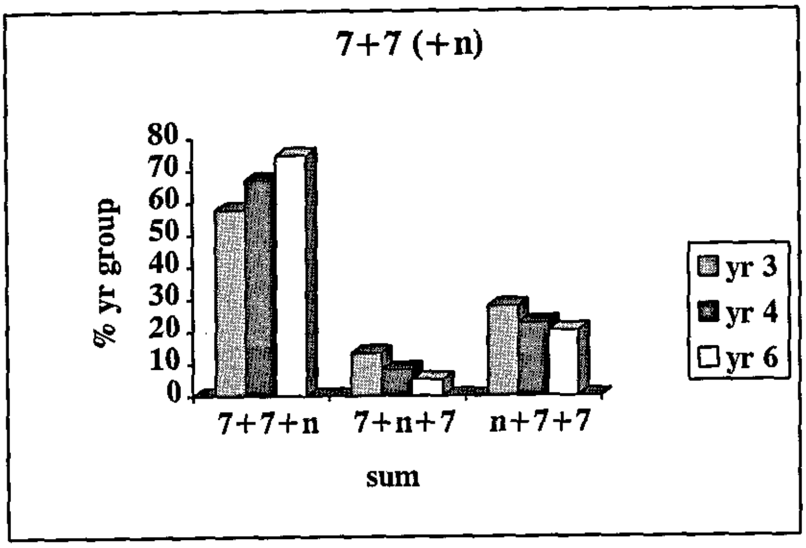

Figure 7.21 Tie-sum 7+7 (+n)

The summary of results in Figure 7.21 suggested that children across all age groups were more likely to select the tie-sum 7+7 first, followed by n+7+7 and 7+n+7. Most children seemed to be taking advantage of their tie-sum knowledge.

Figure 7.22 is a summary of the results for children across all years doing sums involving the tie-sum 7+7.

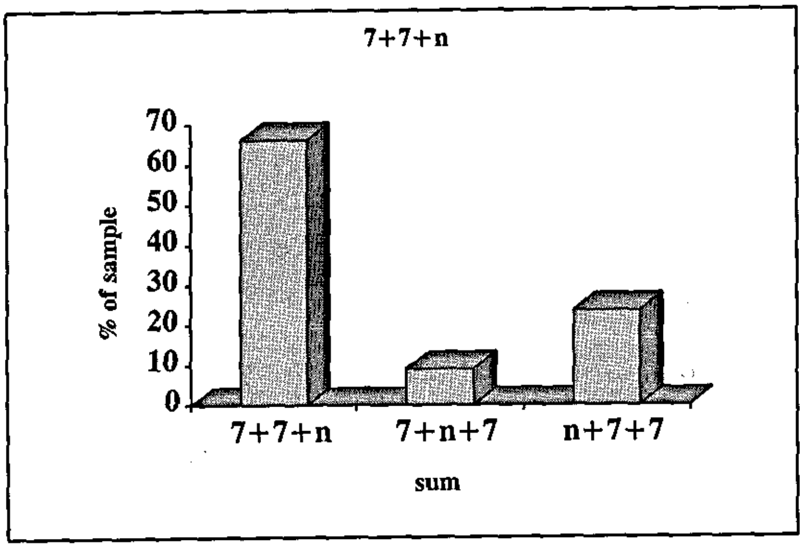

Figure 7.22 Tie-sum 7+7+n across all year groups.

Figure 7.22 further illustrates that most children across all year groups were likely to select 7+7+n, followed by n+7+7 and were least likely to select 7+n+7.

7. 4. 2. 4 Analysis (sums in Group 2 B involving the larger number tie 7+7)

An ANOVA carried out on the results for sums involving the tie-sum 7+7 showed that there was no overall effect of year, $F(2, 127) = 1.80$. The size of the third addend had a significant main effect on selection order, $F(1, 127) = 10.20$, $p < 0.05$. This suggested that whether n was greater than the tie-sum addends (n = 8) or was less than the tie-sum addends (n = 6) did have significant effect on the order in which the addends were selected i. e. as illustrated by Figure 7.19 more children across all age groups were likely to select the tie-sum (7+7) first when the third addend (8) was greater than the tie-sum addends. There was no significant interaction between addend size and year, $F(2, 127) = 0.905$. There was a significant effect of the sum order $F(2, 126) = 255.63$, $p < 0.05$. This suggested that significantly more children across all ages were likely to select the tie-sum first (as illustrated by Figure 7.22). There was no significant interaction between sum selection order and year, $F(4, 254) = 0.940$ (as illustrated by Figure 7.21). There was a significant interaction between the size of the third addend and the sum selection, $F(2, 126) = 5.70$, $p < 0.05$. This suggested that the size of the third addend did have a significant effect on the order in which the addends were selected. There was a significant interaction between the size of the third addend, the sum order and year group, $F(4, 254) = 4.75$, $p < 0.05$. This suggested that older children's sum selections were significantly more likely to be influenced by the size of the third addend.

These results suggested that the tie-sum 7+7 was a salient number fact for most children and that most children did seem to take advantage of their tie-sum number-fact knowledge. Children were willing to use their tie-sum knowledge for both smaller tie-sums such as 3+3 as well as larger tie-sums such as 7+7. However, it seemed that some of the older children were more likely to select the largest number first when the third addend n (8) was greater than the tie-sum addends.

The above sections looked at results from sums in Group 2, which investigated children's, selection strategies for sums involving tie-sums. The following section looks at the results for sums in Group 3 designed to investigate children's selection strategies when doing sums with a visibly large addend.

7. 4. 3 Results for sums in Group 3 (reordering of the largest addend)

The results in this section looked at addend selection for sums in Groups 3 and 4 designed to investigate reordering of the largest addend.

7. 4. 3. 1 Results for sums in Group 3 A (sums involving a larger number)

Figure 7.23 illustrates the pattern of results for sums in Group 3 A which looked at reordering of the addends in which the largest addend was 9.

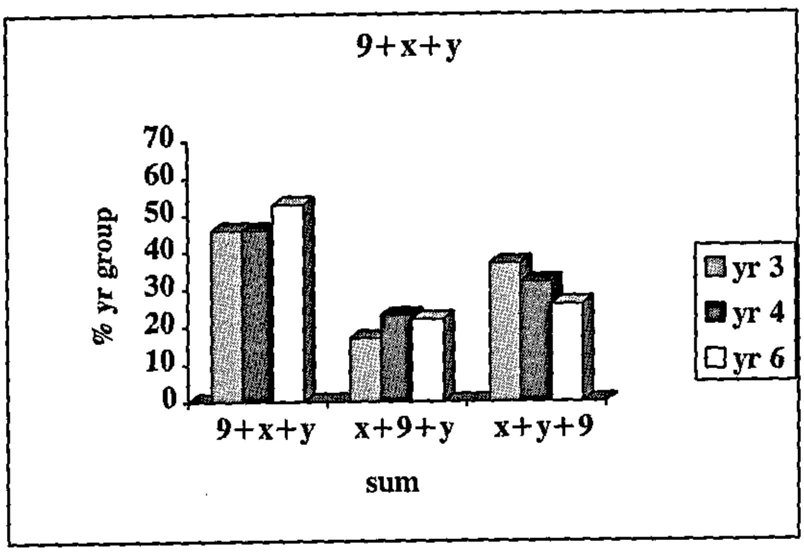

Figure 7.23 Large addend reordering (9+x+y)

The results in Figure 7.23 illustrate that when given a selection of numbers where the largest number was 9 and they could have done the sum in three ways: by selecting the largest addend first (9+x+y), the largest addend second (x+9+y) or the largest addend last (x+y+9), most children across all age groups seemed to select the largest addend first. However, quite a few of the children across all age groups were still not selecting the largest addend first. This is more pronounced for the children in the younger age groups. These results seemed to suggest that a large number of children may choose not to reliably take advantage of their knowledge of the min addend reordering strategy.

7. 4. 3. 2 Results for sums in Group 3 B (sums involving a larger number and "+1")

Figure 7.24 illustrates the pattern of results for sums in Group 4 which looked at reordering of the addends in which the largest addend was 7 and there was the possibly special case of "+1" involved.

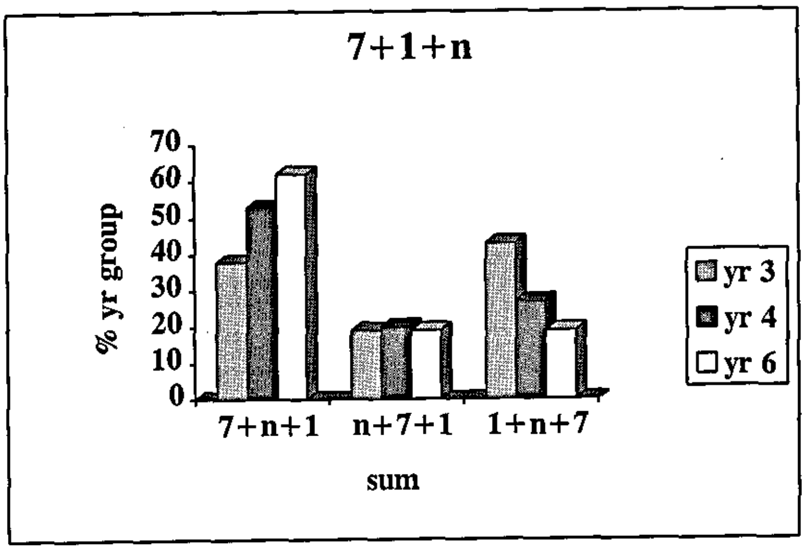

Figure 7.24 Large addend reordering (7+n+1)

These results in Figure 7.24 also seemed to suggest that most children in the older age groups will select the larger number first where the largest addend they see is 7 and a "+1" is involved. This was most evident with the older children because a larger percentage of the older children seemed to go for the largest addend first and this seemed to suggest that they are reordering and making use of the min strategy. However, most of the youngest children are not doing this, and are still selecting

1+n+7 i. e the least efficient combination of numbers. The results are very similar to the pattern reported in Figure 7.23 in which "n" was larger than one.

7. 4. 3. 3 Analysis (sums in Group 3)

An ANOVA carried out on the results for sums in Group 3 looking at reordering of the largest addend found no overall effect of year, $F(2, 130) = 1.39$. This is illustrated in Figures 7.23 and 7.24 suggesting that year did not make much difference to children's selection strategies for these sums. There was no effect of sum type, $F(1, 130) = 1.07$, and this suggested that it did not make a significant difference whether the sums were from Group 3 A (involved a large addend 9+x+y) or from Group 3 B (involved a large addend 7+n+1) as illustrated by Figures 7.23 and 7.24 the pattern of results for Groups 3 A and 3 B seem to be quite similar. There was no significant interaction between sum type and year, $F(2, 130) = 0.671$. There was a significant effect of addend order, $F(2, 129) = 33.90$, $p < 0.05$, suggesting (as shown in Figures 7.23 and 7. 24) that significantly more children across all age groups were likely to select the largest addend first for sums in both Group 3 A and 3 B. There was no interaction between addend order and year, $F(4, 260) = 1.72$, $p = 0.147$ and this suggested the age groups did not affect the order in which the addends were chosen. There was no interaction between sum type and addend order, $F(2, 187) = 0.187$, as illustrated above in Figure 7.23 and 7.24. Children across all year groups were not particularly affected by whether the sum was 9+x+y or was 7+n+1. There were no significant interactions between sum type, addend order and year, $F(4, 260) = 1.13$, $p = 0.343$. These results seemed to indicate that most children across ages were being strategic and using reordering to put the largest addend first i.e. were more likely to go for 9+x+y and 7+n+1.

It seemed that children were not always being strategic about the selection of their addends, unless the addends included a tie-sum. This suggested that perhaps another factor could be guiding their choices instead of the addends themselves and that was the proximity of the addends to the starting point or to each other. As pointed out earlier no addends were *directly adjacent* to each other. However, children who were not being strategic about their addend choices may have just been relying on this relative proximity to guide them.

7. 4. 4 Analysis (Spatial domain)

If the children were doing something other than making order selections based on the addends, then they might have been relying on the spatial domain i. e. the position of the numbers on the screen. Three sets of spatial measure were used to define the distance between the addends on the screen and this was on the basis of their position on a 5x5 grid. These were as follows: 1) the distance of each addend from the start/ finish bar (i. e position of the mouse at the bottom of the screen), 2) the position of the addends on the horizontal (x) axis of the grid (horizontal position), and 3) the position of the addends on the vertical axis on the grid (vertical position).

An ANOVA was carried out on the data from the first measure, distance of the addend from the start bar. The results found no overall effect of year, $F(2, 124) = 0.45$. The distance from the start bar for the first addend was significantly lower than the distance for the other two addends, $F(2, 123) = 13.47$, $p < 0.05$, suggesting that there is a non-numerical influence arising from selecting the "nearest digit" i. e. to some extent, the children are simply going for the nearest addend. The trend suggests that this increase is gradual, implying a gradual movement towards the last digit which seems to be the furthest from the start. There was no interaction between starting distance and year, $F(4, 248) = 1.04$.

An ANOVA on the data from the second measure, the position of the addends on the horizontal axis of the grid. The results found no overall effect of year, $F(2, 124) = 0.03$. There was a significant effect of the position of the addend on the horizontal axis, $F(2, 123) = 5.15$, $p < 0.05$. This seems to suggest that most children are biased to go from left to right, and some of them, if not selecting sums on the basis of the addends, are probably going to the leftmost part of the grid. There was no interaction between the horizontal position of the addend and year, $F(4, 248) = 0.75$.

An ANOVA on the data from the third measure, the position of the addends on the vertical axis of the grid. There results found no overall effect of year, $F(2, 124) = 0.17$. There was a significant effect of the position of the addend on the vertical axis (screen height), $F(2, 123) = 9.10$, suggesting that children were likely to be drawn towards the first addends up on the screen. There was no interaction between the vertical position of the addend and year, $F(4, 248) = 0.86$.

To conclude this section, it would seem that when doing three digit serial addition most children are unlikely to exploit the *sum to 10 number facts* 6+4 and 7+3. Instead they are more likely to take advantage of the min strategy and reorder the addends by selecting the larger ones first. Most children were, however, willing to make use of their tie-sum number knowledge of 3+3 and 7+7 and were likely to select the tie-sum addends first. When doing sums which involved a visibly large number they were also likely to reorder the addends in order to select the largest ones first, suggesting that they are making use of the min strategy in most cases. The results also implied that some children seemed to be relying on the spatial position of the addends suggesting that this can override their ability to be strategic.

7. 5 Discussion

The sum combinations chosen in this study were designed to look at children's selection strategies when doing three digit serial addition. The research reported in Section 5.2 and Chapter 6 had suggested that children were capable of being strategic with their knowledge of number fact knowledge of tie-sums and that they were capable of using their min reordering knowledge. They were also capable of applying this knowledge to solve complex decade sums and also, to a limited extent, to solve three-digit serial addition. But unlike my previous studies, the design of this study allowed children to select the order in they added the digits. This would enable a more confident claim to be made about whether children's use of their existing knowledge would be reflected in the order in which they selected their addend.

The children were presented with three types of number combinations which would appear randomly on a screen with no "+" or "=" signs (see Table 7. 2):

1) Problems designed to look at children's use of the sum to 10 number facts (Group 1 A and Group 1 B).
2) Problems designed to look at children's use of tie-sum number facts (Group 2 A and Group 2 B).
3) Problems designed to look at children's ability to use the min reordering strategy (Group 3 A and Group 3 B).

The findings for the sums in Group 1 suggested that when faced with the serial addition of three addends most children did not make use of the sum to 10 number fact. The sum to 10 number fact did not appear to be a salient number fact for most children, it was more likely to be used by older children and even then it was most likely to be selected if the larger sum to 10 addend was selected first. The results from both Group 1 A and 1 B (as illustrated by Figures 7.4 and 7.11) suggested that either sum to 10 had not yet become a "number-fact" for most children or that it was a fact but its potential was not being recognised. This was particularly evident when the third addend was a number largest than larger sum to 10 addend (e. g. 6+4+7 or 7+3+8) in which case most of the children, but especially the older ones, were likely to go for the largest addend first. Certainly if most of the children were relying primarily on counting strategies then the taking advantage of the sum to 10 knowledge would not be a priority i. e. if a child is using a relatively inefficient "counting-on" strategy then it would not matter which addends were selected first. For many of the younger children this did seem to be the case.

Alternatively, it may be that many of the older or more competent children were the most likely to select the addends in a fairly random order because they were competent enough at addition that the addend order did not make much difference to how they selected their addends. However, it seemed unlikely that this was the case. While some of the oldest children or the more able younger children could have been doing this, the older children were the most likely to be strategic about the order in which they selected their addends. They were more likely to select the largest addend first suggesting that they were using the min reordering strategy and this was more easily recognisable to them than the sum to 10 number fact. So they were being strategic in selecting the largest addend first in order, but perhaps not strategic enough. Selecting the largest addend first in order to use the min strategy does make counting more efficient but using the sum to 10 number-fact, even if it has not yet become a "fact", would make counting even more efficient especially in a three digit serial addition case where they would then be adding a digit onto 10. By selecting, for example, an addend combination like 8+7+3 or even 7+8+3 where the largest addends were selected first, the children would then have to count up to 15 and then add 3, which does make counting somewhat easier but is still a fairly difficult sum than adding the sum to 10 addends first.

The results from Group 1 supported the previous findings reported in Section 5. 2 which suggested that children were reluctant to use the sum to 10 number fact when doing serial addition. These results also supported the findings in section 5. 1 which suggested that sum to 10 number facts, with the exception of 9+1 and 5+5, were unlikely to be salient number facts for most children. Children across all ages seemed to be relying primarily on the min counting strategy. This was further illustrated by children's solution times for the problems in Group 1 which suggested that most children who seemed to be selecting less efficient combinations of addends also had the longest solution times. As the third addend became larger their solution times became longer, suggesting that most of the children were relying primarily on counting strategies. However, children who seemed to be selecting the sum to 10 combination first seemed to have the shortest solution times, and these were likely to be children in the oldest age group.

The findings for the sums in Group 2, however, suggested that while most children were unable or unwilling to make use of the sum to 10 number facts, they were willing to take advantage of their tie-sum knowledge. As illustrated in Figure 7.17 and Figure 7.21, most children were willing to exploit their tie-sum knowledge of 3+3 and 7+7. Children were more likely to select addend combinations such as n+7+7 and n+3+3 then combinations such as 7+n+7 or 3+n+3 suggesting that they were showing recognition of the tie-sums. This seems to support the findings in Chapter 6 that suggested than children showed recognition of the tie-sum and attempted to use it even if it had not quite become a "number-fact". Figures 7.18 and 7.22 suggested that children across all age groups were likely to be making use of their tie-sum knowledge although in both Group 2 A and 2 B the older children were most likely to be doing this. However, for sums in Group 2 B involving the tie-sum 7+7 and where the third addend (8) was larger than the tie-sum addends, it was the older children that were more likely than the younger ones who were likely to select the largest first.

These results from Group 2 support the previous findings in Sections 5. 1 and 5. 2 and Chapter 6 that tie-sums emerge as very salient number facts that prompt children to be strategic about their addend selection. Interesting enough, although tie-sums do emerge as highly salient number facts for children they are not particularly powerful number facts for children to use. They can make addition faster for the tie-sum but not much easier when adding more digits. That is, doing 7+7 might be quick and easy but then doing 14+8 is not a particularly easy sum, especially if relying on counting. However, at a higher, more advanced level recognising tie-sums makes addition easier i. e. recognising a string of identical digits means that they can be grouped together to either be added separately or even be multiplied.

The sums in Group 3 had been chosen specifically to investigate children's use of the min reordering strategy when doing sums that involved a visibly large addend (9) and a sums involving a large addend (7) "+1". The results from Group 3 A and 3 B did suggest that children from all age groups were more likely to select the largest addend first i. e. select 9+x+y. This indicated that children were being strategic about selecting their addends. Most of the older children did seem to be doing this. However, a large number of children did not seem to be doing this, and seemed nearly as likely to select 1+n+7 or x+y+9 which would be the least efficient combination of addends especially if counting was being used. As illustrated by Figure 7.23 and 7.24, quite a few children across all age groups did not seem to be selecting their addends in any particular order. The youngest children especially did not seem to be strategic about their selecting the most efficient combination of addends, suggesting that they were likely to be using the less efficient "counting-on" strategy as opposed to the min reordering strategy.

The results showed that children did seem to be strategic about their selection strategies when doing three-digit serial addition. Most children were willing to take advantage of their number-fact knowledge of tie-sum such as 3+3 and 7+7. By comparison, they seemed reluctant to take similar advantage of the sum to 10 number facts (6+4 and 7+3). This does not mean that they were not being strategic when doing sums that involved a sum to 10 number-fact because they were still being strategic about their addend selection. But instead of recognising and using sum to 10 knowledge they were more likely to use the min reordering strategy and select the largest addends first, and this was more likely to be the case with the older children. When the sums were designed to look specifically at the reordering of the largest addend, most of the older children were able to select the largest addend first. The younger children, however, were not always doing this. With the exception of the tie-sums, the youngest children did not seem to be strategic about their addend selection suggesting that they were likely to be relying on less efficient counting strategies. In addition, they seemed to be guided more by the position of the addends on the screen than by the addends themselves.

7. 6 Conclusion

Overall, the results indicated that while children are capable of recognising and exploiting some type of number-fact knowledge (such as tie-sums), they are unlikely to take advantage of potentially powerful sum to 10 number-facts. Most children seemed to be relying primarily on counting strategies as illustrated by their preference for selecting the largest addends first. The older children's selection strategies mostly reflected their use of the min reordering strategy, while the younger children seemed to be doing this but to a lesser extent. This supports my previous findings in Chapters 5 and 6 which suggested that while children can be strategic about using their existing knowledge effectively, they are likely to rely on methods that are most likely to provide them with a fast and accurate answer. Perhaps most children, especially the younger ones find it a more demanding task to try out new or unfamiliar strategies unless prompted by certain conditions. When given control over the choice of which digits to add first, most children chose the most familiar and reliable strategy that was available to them i.e. the min strategy, and were likely to use the number-fact knowledge (of tie-sums) that they felt most confident with.

Chapter 8

8. Conclusions

8. 1 Overview

The aim of this chapter is to bring together some of the issues arising from the research reported in this thesis. In Chapter 1, I began by examining the social and political background of research in the area of mental arithmetic: from popular opinion on mental arithmetic and preoccupation with numeracy standards, to the debate about teaching practice, how it is embedded in the educational policies and therefore likely to reflect the dominant attitudes of that particular historical period. In Chapter 2, I provided a review of the development of research in mental arithmetic and examined relevant psychological concerns. This included a brief historical perspective of arithmetic in psychology. I went on to look at the research traditions within which arithmetic research has been studied, from its roots in behaviourism to the current traditions of cognitive science as well as cultural research in this area. In Chapter 3, I concentrated on one of these research traditions, cognitive research in mental arithmetic. Chapter 3 was a deeper review of the existing research available in the area of mental arithmetic, in which I examined the current theories of simple arithmetic processing from evidence supporting the prevalent fact-retrieval theories to the schema-theory within which my own research is located. In Chapter 4, I went on to consider the methodology available for researching mental arithmetic that has been used in existing research, as well as outlining the methodology that would be developed and utilised in my own studies.

In Chapter 5, I reported the results from my two foundation studies designed to look at the strategies that would emerge from children doing single-digit sums, decade sums and a selection of three-digit serial addition sums. These studies formed the benchmark for my future research in Chapters 6 and 7. The findings from the study reported in Section 5.1 of Chapter 5 led to the problems I investigated in Chapter 6, looking at whether children would export the strategies they used on single-digit sums into decade sums. The findings from the study reported in Section 5.1 also led to the design of the study reported in Chapter 7. The research I reported in chapter 5, therefore, formed the platform for my later projects.

In Chapter 5, I investigated the three general types of mental addition problems that formed the basis of my later studies. The first of these problems was the single-digit case. While this has been a well-researched issue in this area, it clarified the baseline performance and solution processes on single-digit sums and helped identify some ground rules for the children in my sample. Decade sums and serial-addition sums are neglected areas in mental arithmetic research. The results from the decade and serial addition sums illustrated the complex nature of these problems. They were difficult for children, as indicated by their long solution times. Children took, on average, twice as long on these sums than on their single-digit counterparts. One of the reasons that decade sums might have been so demanding for children was the format in which they were presented to the children. That is, in my study the sums were presented in a linear (left to right) format which children are probably less familiar with compared to the columnar format more commonly used in school. This was one of the issues I rectified in the following chapter (6) that dealt with decade sums in greater depth.

The results from the serial addition sums in Chapter 5 suggested that children seemed to be exploiting certain number-facts such as tie-sums when doing three-digit problems, whereas they seemed to be reluctant to use other potential number-facts such as sum to 10. Yet it was not enough to consider only solution times when investigating such problems because the linear presentation format in which these sums were presented could have been preventing children from using their strategic knowledge. Thus, in Chapter 7, children received sums in the format of numbers on a screen for which they were given control over the order in which they selected their addends. This enabled me to identify more precisely whether or not they were selecting their addends efficiently.

The single-digit and the decade addition problems used in my first study (reported in Section 5.1 and 5.2) were chosen in order to find out which combinations of numbers children would find easiest and which combinations they would find difficult. The single-digit sums enabled me to discover which pairs of numbers were "number-facts" for my particular sample. In accordance with the existing research in the area, would tie-sum and sum-to-10 number facts be salient (be recalled from memory as opposed to counting) for these children? If these sums were becoming salient facts for the children, would their RTs for these sums suggest that they were being strategic with their knowledge? Similarly, the decade sums in this study were chosen to find out if any patterns emerged that would indicate whether children were relying on counting, use of number-facts or strategies such as decomposition. The findings from these studies would help in identifying what needed closer examination and whether the solution processes being used in these cases would be similar to processes used on sums involving a greater level of complexity. The three-digit serial addition sums chosen in Section 5. 2 were also chosen to see whether or not children were being strategic about their number knowledge.

Some clear patterns emerged from the findings in Chapter 5. Firstly, tie-sums emerged as salient number facts fairly early on. Perhaps this is because they are visually distinctive as well as being emphasised in school textbooks as "doubles". These emerged as number-facts that children were willing to exploit when solving more complex sums. Secondly, somewhat inconsistently with previous published findings, sum to 10 number facts did not emerge as salient number-facts, except for the special cases of 9+1 (adding "1") and 5+5 (tie-sum). This is a cause for concern, because sum to 10 is a potentially more powerful number-fact than ties. Thirdly, being strategic with number knowledge is essentially about transformations, from the early emergence of the "min" reordering strategy to make counting easier, to using existing number-fact knowledge such as derived facts. As indicated by my findings, for the older children at least, the sums "adjacent to tie-sums" (e. g. 9+8) also had faster RTs compared to similar sums such as 9+7. Using a strategy is about making such transformations, i. e. transforming one difficult sum into two easier ones and then bringing them back together. Children tend to rely primarily on counting procedures, while also using non-counting procedures and part counting procedures. The results in Chapter 5 confirmed the three types of solution processes that are employed by children and adults when doing mental addition. Some of these processes (such as the min strategy and transformation procedures) are strategic, while others (such as number-fact retrieval) are not.

The results from both studies in Chapter 5 suggested that these children did have the ability to be strategic, i. e. go beyond basic counting. My aim was then was to investigate this in greater detail. As found by existing research in this area, children were only likely to use or try to use newer or different strategies when the sums became too complex to solve through simple counting-on strategies. My aim was to find out whether or not certain complex sums could encourage children to be strategic and whether this could be inferred by their solution times for those sums. If children were able to approach some types of decade sums and serial addition sums strategically, then what other types of decade sums and three-digit sums would they be more likely to solve strategically? That is, could certain types of sums encourage children into being strategic?

Therefore, the decade sums I investigated in Chapter 6 were designed to look at some of the strategies identified in Section 5.1 and whether these would get recruited into solving decade sums where solution processes such as using number-fact knowledge and the min strategy could only be used effectively if the children were using decomposition. The results suggested that decomposition was being done some of the time on some of the sums, namely on decade sums with smaller addends and tie-sums. However, decomposition was not being exploited anything like as much as was possible.

Similarly, the three-digit serial addition sums chosen in Chapter 7 were designed to look at the selection strategies children would use when required to do such sums. The sums in Chapter 7 were chosen to investigate in greater depth some of the findings that emerged from Section 5.2 in Chapter 5. Certainly, there was some sign of strategic reordering but this was limited to addend combinations involving tie-sums, once again reinforcing the saliency of tie number-facts. Some strategic reordering seemed to be in favour of selecting the largest addends first, implying that most children seemed to be relying quite heavily on counting strategies. The sum to 10 number facts were not being exploited even though, bearing in mind how difficult they were for most children, they would have benefited from any strategy that would have aided them in doing these sums. Their solution times for these sums certainly suggested that not being strategic about their addend selection hinders their performance, i. e. results in quite long solution times, and children who *are* strategic take less time to solve them. So, while there were signs that some children were being strategic about their addend selection, most were not. There seemed to be a degree of inertia on the part of children, with the exception of tie-sum cases, most children were being relatively careless about addend selection and relying on spatial cues i. e. position of the addends on the screen to select their addends.

One of the aims of this thesis was to look at the relationship between procedural and number fact knowledge as suggested by schema-theory. My aim was to investigate whether or not I could use solution times to infer children's use of procedural knowledge. The results in Section 5.1 suggested this to be the case. My results showed that tie-sums had short RTs and this was consistent with existing research. However, the results for the adjacent to tie sums such as 6+7, 7+8, and 9+8 suggested that these were being solved through a combination of number-fact and procedural knowledge at least by some of the older children. The results in Chapter 6 also suggested that it was possible to infer children's strategic use of both number fact and procedural knowledge from their solution methods.

To conclude, the dominant finding of my research is that children are strikingly slow to adopt simple mental addition achievements into the context of more demanding tasks. Children in this research were chosen to represent a good sample of the junior school years. They came from two schools serving a wide catchment area and all the children from all relevant classes were included. There are thus grounds for confidence in the research outcomes. Yet, while these children were becoming fluent with tie-sums and although they did know about reordering possibilities (the min strategy), they were failing to internalise sum-to-10 number facts and they were by no means universal in their appropriation of simple number knowledge into complex cases. Decomposition (and the advantages that follow) was not comprehensively adopted, even among the older children. Reordering strategies in serial addition were present but far from universally so. I shall comment further on the implications of these observations, following a brief reflection on methodology.

8. 2 Reflections on methodology

One of the central aims of this thesis was to design and evaluate a method that would allow me to carry out chronometric research in a naturalistic classroom environment. Earlier (chronometric) research had often involved researchers using methods that were inflexible and awkward to use in natural settings because they involved setting up complex equipment that often relied on voice-activated timer relays (Campbell and Graham (1985) or setting up video-recording equipment to monitor children's strategies (Siegler and Jenkins, 1989). When inbuilt timers were used to record solution times these tended to be on verification (true/false) type tasks that do not reveal much information on just how long it takes individuals to solve sums as opposed to verify the answers. Such tasks would also be difficult to implement in developmental research.

My intention was to devise a production task (one that requires the participant to generate an answer) that were relatively simple to integrate into normal classroom activities, were easy for the children to use, and could provide a reliable measure of RT for a range of addition problems. My research showed that this method did allow me to carry out my research tasks in an efficient and flexible manner. It was relatively straightforward to set up in a busy school environment since it required only a computer that could run the relevant software. What the method did not allow me to do was to look at exactly how the children were doing mental addition the way verbal reports/ interviews would. Strategy, as always in chronometric research, was inferred from the patterns in children's solution times.

8. 2. 1 Ease of use for the researcher

The software made it very easy to take several measures of RTs as well as recording the addend selection for any combination of sums. It was simple to manipulate the task by entering the relevant program codes. The design of the classroom i. e. having the computer in a corner of the class meant that, after the first few visits, I was able to carry out my research with minimum disruption to the classroom activities. Doing the task became almost a routine experience for the children.

This method allowed me to carry out research in an unobtrusive manner that was comfortable for both the children, their teachers and myself. The software would be ideal for gathering large amounts of data for individual children e. g. for gathering longitudinal data looking at practice effects for the same sums. Although this was not something that was part of my research, this feature made it a good tool for doing microgenetic research into where the transitional processes lie i. e how and when new strategies would be recruited by individual children as inferred by their RTs/ selection strategies for various sums. Such research is sensitive to a wider range of changes and greater use of it results in improvement in quality of developmental theorising as it allows us to look at changes as they occur.

Development of mental arithmetic is about changing competency, from an initial reliance on laborious (finger) counting strategies to retrieval of number facts from memory and the use of conceptual (i. e. transformational) strategies such as decomposition and using derived facts. Mental arithmetic practice would benefit greatly from such research. As advocated by Kuhn (1995) and Siegler and Crowley (1991), the microgenetic method is a promising tool in the study of change. The goal of microgenetic research is to accelerate the change process by providing an individual frequent opportunities over a period of weeks or months to use the cognitive strategies being researched. Because mental arithmetic in children is developing continuously, it makes an ideal area for microgenetic analysis. Certainly, in mental arithmetic development the change process is not a simple transition from one mode of operation to another. Developing competence at mental arithmetic is all about multiple strategies being used and applied to a single-problem over the course of repeated presentations. Siegler (1985, 1989) pointed out the benefits of using microgenetic methods for studying the strategy construction process because they reveal characteristics of the processes that would be difficult to discover through the use of alternative methods. The microgenetic approach helps to reveal where the transitions occur and this important in areas such as mental arithmetic where transitional processes are central to development.

8. 2. 2 Ease of use for the participant

The easy-to-use interface meant that the children were quite capable of going through the task unsupervised although this did not occur. Even fairly young children would not have found it too demanding to use. This would make the task ideal for use in classroom arithmetic practice. The software was set up to record the name and age of each child and once they began using it would create a file for each individual. Right-clicking on each name would bring up this file and it would show the performance of that child i.e how many sums they answered correctly as well as how long it took them. Some of the teachers of the children in my study seemed quite interested in this function, because it allowed them to find out at a glance just how well their students were doing. They were interested in its potential as a classroom teaching tool, especially because the children were willing to use it.

The nature of the task also proved relatively popular with the children in my sample. In part, this was due to the RT for each sum flashing on the screen as soon as it was completed. This added a competitive element to the task that seemed to make it more interesting for the children. Most children treated it like a game and saw it as a welcome break from their normal classroom routine so it provided enough motivation for them to genuinely make an effort to do their best. I also found that accuracy seemed very important to most of the children. They were keen on making sure they had an accurate answer (whether or not they got the correct answer would also flash on the screen). Even if getting this correct answer meant getting a long RT as a result of using a laborious counting method, they were willing to make this effort. This observation would explain why some of the sums resulted in fairly long solution times. While most of the sums were quite difficult for children, children's desire for "getting the right answer" might have resulted in them being solved even more carefully, with some children counting out the answers. Such laborious (often finger-based) counting methods do, however, imply that this dependence on counting can have a detrimental affect on mental arithmetic especially in "timed test" case scenarios.

Another casual observation was that it did seem that the children who got the fastest times were not always the ones who relied on retrieval from memory, quite a few of them seemed to be relying on very fast counting. Interesting enough, some of the more competent/ older children also seemed to be the ones who tended to rely on counting strategies, because they were also the ones who were interested in getting accurate answers. Thus supporting Siegler's (1988) findings in which he identified "good students, not-so-good students and perfectionists". Perfectionists were students who, despite having greater speed and accuracy than other groups, were less likely to use retrieval of number-facts, relying instead on "back-up" strategies such as counting or derived facts.

8. 3 Implications

8. 3. 1 Practical Implications

The research in my thesis suggests that children do have the capacity to carry forward their simple strategies into more complex addition environments. This means that perhaps, in addition to teaching children about number-facts such as tie-sums (doubles), more attention needs to be given to more powerful number facts such as sum to 10, and to decomposition-based strategies. Nor is it enough just to have number fact knowledge, if this is to be stored in a passive sense. Knowledge of arithmetic facts alone does not help with more conceptual aspects of mathematical thinking and problem solving. Children need to know what to do with it, how to make use of it creatively and innovatively. For most children, counting is adequate enough when it comes to simple sums. From their point of view, why should they invest more effort? Perhaps schools should focus less on computational drill and more on getting children to understand why arithmetic procedures work for promoting long-term computational achievement.

According to Resnick (1989), children's invented arithmetic procedures show that they are able to construct basic principles of maths such as commutativity, complementarity of addition and subtraction, and associativity in intuitive forms well before such ideas are learnt in school. But school arithmetic does not build effectively on the base of children's informal knowledge. Resnick (1989) argues that children are taught to focus on the syntax of mathematics rather than its semantics where the sequence in which written numbers and calculations is the primary concern, in addition to emphasis on practice in memorising arithmetic combinations and rules for applying these in longer multidigit computations.

As pointed out by Siegler and Jenkins (1989), pretest knowledge of addition and related skills was not a good predictor of children's order of discovery of the min strategy. Children with the best knowledge of number-facts are not necessarily the first to discover new strategies. It is not always the child with the most advanced knowledge who first discovers a new strategy, but one who is willing to consider diverse strategies and to continue using them even when they do not always seem to be working. It is this willingness to try to use diverse strategies that must be encouraged. Some of the children in my sample seemed to be willing to do this. The results in Chapter 6, for example, suggested that they were willing to attempt to use a tie-sum when it appeared in a decade context even though doing so increased their solution times and made the sum seem harder.

This would suggest that while it is important for children to learn number-facts through practice, it is just as important for them to have conceptual understanding of principles such as decomposition for development of their addition skills, as pointed out by Isaacs and Carroll (1999). Conceptual understanding of one type of problem would enable children to solve other problems of a similar structure. The overall conclusion from my results was that while practice at number fact knowledge is crucial, the importance of conceptual knowledge must also be stressed. As Baroody (1994) argued, conceptual understanding both guides and constrains children's problem solving and as such must not be pushed aside in a bid to improve number-fact knowledge. With experience, children become faster and more accurate at solving addition problems and begin to use more sophisticated strategic solution processes such as commutitivity, decomposition and retrieval.

My own research leads me to concur with this view. Older children were the most likely to show recognition and use of more sophisticated strategic solution methods, and this seemed to be the case with most of the children in the oldest age groups. But even these older children were not being overly strategic. They still seemed to be relying mostly on counting, and were not always reordering addends optimally, considering that some of them were at the end of primary school i. e. have gone through six years of taught arithmetic. Perhaps strategies should be taught before drill and practice in number fact recall. However, evidence also suggests that the development of "invented" or creative strategies can be enhanced by practice.

The underlying reason for focus on number facts in school is the idea that it promotes the transition to "adult-like" patterns of retrieval. As Resnick (1989) notes, a dependence on counting directs children's attention away from the additive composition properties of numbers. My findings suggested that this does indeed seem to be the case. As illustrated by the results in chapter 6, children seem to rely on counting and counting-based strategies like min. When they are strategic, they seem to favour number facts such as tie-sums over sum to 10 number fact as demonstrated in Chapter 7. This is worrying because children who have difficulty learning mathematics are likely to rely on counting methods for a long time, and because counting methods do get the task done i. e. result in an accurate answer children's reliance on counting tends to go unnoticed.

However, this is not intended to paint a totally bleak landscape, but to point out just how slowly these children are making the steps into more advanced forms of addition computation. They do not seem to be using sum to 10 number-facts. They do not seem too eager to use decomposition, and they are not vigorous about reordering addends to optimise addition problems. They seem to manage well enough, but they are doing this through simple, rather than innovative and strategic methods. Addition strategies seem to remain at the primitive (but reliable) stage of mental or finger counting. This relative inertia regarding strategic solution methods is forgivable because such processes involve "hidden" processing. Classroom practice needs to take this into account and perhaps incorporate tasks that encourage children to be more strategic. Teachers can implement classroom tasks and activities that actively encourage children to use more flexible strategies based on conceptual knowledge of arithmetical strategies.

The methodology I used in my study can be used as a teaching practice aid to encourage children to use skills such as decomposition strategies and using number-fact knowledge in addition to tie-sum knowledge. Teachers were struck by the simplicity and design of the program I used in my task. Such a task could easily be modified and adapted to allow teachers to cultivate strategic moves that may initially seem like "long-cuts" but which may become short-cuts with even modest amounts of rehearsal. A task such as this one could allow the teacher to easily monitor their student's progress and to set up a range of tasks according to children's individual capabilities.

8. 3. 2 Research Implications

Due to time constraints, I was unable to investigate serial addition for sums with more than three-digits. This remains an area that would benefit from future research. What would happen to children's strategic knowledge when required to add sequences of perhaps four or more digits? Would they still be relying on counting strategies or would such sums provide enough of a challenge to encourage them to use more effective solutions processes?

The design of the task I used in my research would be ideal for looking at practice effects and individual differences in children's arithmetic performance. This is why the ease with which this task lends itself to microgenetic methods of studying change is so useful. According to Kuhn (1995) the purpose of the microgenetic method is to accelerate a natural change process by increasing the density of exercise above its normal level. This use of the microgenetic method has shown itself to be informative regarding the development of the skills examined and suggests the versatility of the method. This is precisely what developmental mental arithmetic research aims to look at. Development of competence at mental arithmetic is all about looking at change. Once the problem areas in arithmetic development are identified then something can be done to remedy them. More research of this sort means research that has significant ramifications for both arithmetic research in general and would also be extremely valuable in serving the interests of a future generation of children who, as a result of living in an increasingly hi-tech world, will have high basic numeracy requirements.

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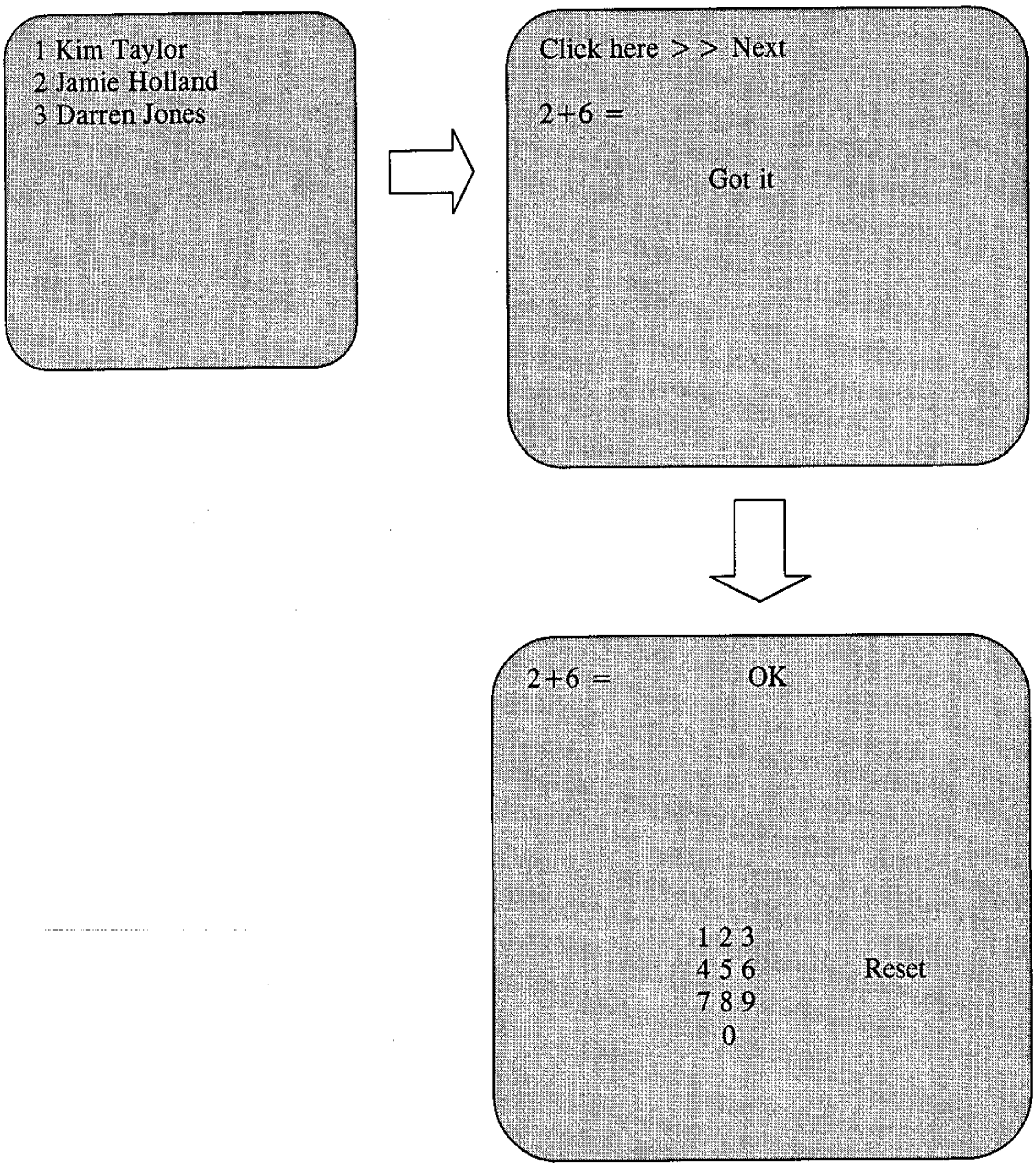
Appendix 1

Diagram 1: Schematic diagram of screen appearance



Appendix 2: Error Rates

All error frequencies include the number of times each participant selected an incorrect answer for a given sum. This means that they include the number of times each child would give an incorrect or a correct response. Because each child received two trials at a problem if they made a mistake the first time, this would mean that an error frequency of 3 for the sum 1+2, for example, could include the number of times an incorrect response was entered for that sum by the same child.

2. 1 Error rates for sums in Chapter 5

2. 1. 1 Sums in 5. 1 Experiment 1

2. 1. 1. 1 Error frequencies for single digit-sums

Table 1 Error frequencies for the 45 single-digit sums.

Sum	Response	Percent Year 3	Percent Year 4	Percent Year 5	Percent Year 6
1+1	wrong				
	right	100	100	100	100
1+2	wrong	3.3	2.2		
	right	96.7	97.8	100	100
1+3	wrong		8.5	3.3	5.1
	right	100	91.5	96.7	94.9
1+4	wrong		2.3	3.2	
	right	100	97.9	96.8	100
1+5	wrong	1.7	2.2		2.6
	right	98.3	97.8	100	97.4
1+6	wrong		2.2		2.6
	right	100	97.8	100	97.4
1+7	wrong	1.6			
	right	98.4	100	100	100
1+8	wrong	4.8	4.3		
	right	95.2	95.7	100	100
1+9	wrong	1.6	2.3		
	right	98.4	97.7	100	100
2+2	wrong				
	right	100	100	100	100
2+3	wrong	3.3		3.2	2.7
	right	96.7	100	96.8	97.3
2+4	wrong	1.7	4.3		
	right	98.3	95.7	100	100

Sum	Response	Percent Year 3	Percent Year 4	Percent Year 5	Percent Year 6
2+5	wrong	1.7	2.2		2.6
	right	98.3	97.8	100	97.4
2+6	wrong	1.6	4.3		
	right	98.4	95.7	100	100
2+7	wrong		4.3	3.3	
	right	100	95.7	96.7	100
2+8	wrong				
	right	100	100	100	100
2+9	wrong		2.3	6.3	2.6
	right	100	97.7	93.8	97.4
3+3	wrong	4.8		6.3	2.6
	right	95.2	100	93.8	97.4
3+4	wrong	12.5		11.8	2.6
	right	87.5	100	88.2	97.4
3+5	wrong		6.4	3.2	
	right	100	93.6	96.8	100
3+6	wrong	9.2	4.3	12.1	5.1
	right	90.8	95.7	87.9	94.9
3+7	wrong	4.8	2.2	3.2	5.1
	right	95.2	97.8	96.8	94.9
3+8	wrong	7.8	4.3		7.5
	right	92.2	95.7	100	92.5
3+9	wrong	5	2.2	9.1	
	right	95	97.8	90.9	100
4+4	wrong	1.7			
	right	98.3	100	100	100
4+5	wrong	6.2	4.3		2.6
	right	93.8	95.7	100	97.4
4+6	wrong	7.9	4.3	6.3	5.1
	right	92.1	95.7	93.8	94.9
4+7	wrong	3.3	6.4	6.7	7.5
	right	96.7	93.6	93.3	92.5
4+8	wrong	4.9	10.4		9.8
	right	95.1	89.6	100	90.2
4+9	wrong	7.8	10.2	11.8	5.1
	right	92.2	89.8	88.2	94.9
5+5	wrong			3.3	
	right	100	100	96.7	100
5+6	wrong	10.6	6.4	6.1	2.6
	right	89.4	93.6	93.9	97.4
5+7	wrong	10.8	12	6.3	14
	right	89.2	88	93.8	86
5+8	wrong	12.1		6.3	2.6
	right	87.9	100	93.8	97.4

Sum	Response	Percent Year 3	Percent Year 4	Percent Year 5	Percent Year 6
5+9	wrong right	6.3 93.8	4.4 95.6	9.7 90.3	14.3 85.7
6+6	wrong right	3.4 96.6	 100	3.6 96.4	 100
6+7	wrong right	11.8 88.2	12 88	3.3 96.7	7.1 92.9
6+8	wrong right	9.1 90.9	10.2 89.8	14.3 85.7	18.6 81.4
6+9	wrong right	9.2 90.8	10.2 89.8	9.1 90.9	 100
7+7	wrong right	11.8 88.2	10.4 89.6	6.3 93.8	 100
7+8	wrong right	12.3 87.7	15.4 84.6	12.5 87.5	15.9 84.1
7+9	wrong right	10.4 89.6	13.7 86.3	14.3 85.7	18.2 81.8
8+8	wrong right	6.3 93.7	8.3 91.7	3.3 96.7	14.3 85.7
8+9	wrong right	11.9 88.1	17 83	3.2 96.8	5.3 94.7
9+9	wrong right	13 87	17.3 82.7	6.3 93.8	5.1 94.9

2. 1. 1. 2 Error frequencies for decade sums

The sums for these decade problems correspond to the three types of decade sums as described in Section 5.1.2.2. The sum type given here is just an example of the type of sum the participants would have received as opposed to the exact sum which was randomised as described in Section 5.1.2.2.

Table 2 Error frequencies for decade sums.

Sum Type	**Response**	**Percent Year 3**	**Percent Year 4**	**Percent Year 5**	**Percent Year 6**
A) 23+n (<10)	wrong	11.5	9.3	6.2	5.1
	right	88.5	90.7	93.8	94.9
A) 26+n (>10)	wrong	15.9	25	19.4	9.8
	right	84.1	75	80.6	90.2
B) 53+n (<10)	wrong	14.9	15.7	7.7	1.3
	right	85.1	84.3	92.3	98.7
B) 56+n (>10)	wrong	24.7	26.3	16.7	29.4
	right	75.3	73.7	83.3	70.6
C) 83+n (<10)	wrong	19.4	2.2	3.2	3.9
	right	80.6	97.8	96.8	96.1
C) 86+n (>10)	wrong	26.3	23.2	11.8	20
	right	73.7	76.8	88.2	80

2. 1. 2 Sums in 5. 2 Experiment 2

2. 1. 2. 1 Error frequencies for three-digit sums

Table 3 Error frequencies for three-digit sums.

Sum	**Response**	**Percent Year 3**	**Percent Year 4**	**Percent Year 5**	**Percent Year 6**
4+4+1	wrong		4.3	2.3	
	right	100	95.7	97.7	100
4+1+4	wrong	7		2.3	2.7
	right	93	100	97.7	97.3
4+4+3	wrong	5.4	2.2	8.5	2.7
	right	94.6	97.8	91.5	97.3
4+3+4	wrong	3.6	2.3	4.4	5.3
	right	96.4	97.7	95.6	94.7

Sum	Response	Percent Year 3	Percent Year 4	Percent Year 5	Percent Year 6
3+4+4	wrong	21.2	2.2	8.5	2.7
	right	78.8	97.8	91.5	97.3
7+3+n	wrong	5.4	13.7	14.3	
	right	94.6	86.3	85.7	100
7+n+3	wrong	9.8	10.2	6.7	10
	right	90.2	89.8	93.3	90
9+1+n	wrong	6.9	6.4	2.3	
	right	93.1	93.6	97.7	100
9+n+1	wrong	16.9	8.2	10.4	16.3
	right	83.1	91.8	89.6	83.7
9+a+b	wrong	10.3	14	21.6	16.3
	right	89.7	86	78.4	83.7
a+9+b	wrong	11.7	13.7	12.5	10.3
	right	88.3	86.3	87.5	89.7
a+b+9	wrong	8.8	12.2	8.9	18.2
	right	91.2	87.8	91.1	81.8
5+4+3	wrong	12.9	4.3	2.3	12.2
	right	87.1	95.7	97.7	87.8
4+5+3	wrong	5.2	8.2	14.3	12.2
	right	94.8	91.8	85.7	87.8

2. 2 Error rates for sums in Chapter 6

2. 2. 1 Error frequencies for single-digit and decade sums

Table 4 Error frequencies for single-digit and decade sums.

Sum	Response	Percent Year 3	Percent Year 4	Percent Year 5
5+1	right wrong	100	100	100
5+2	right wrong	100	100	100
5+3	right wrong	97 3	100	97 3
5+4	right wrong	97 3	91 9	100
45+1	right wrong	100	91 9	94 6
45+2	right wrong	97 3	100	94 6
45+3	right wrong	100	91 9	92 8
45+4	right wrong	92 8	100	97 3
3+3	right wrong	100	100	100
23+3	right wrong	92 8	84 16	100
6+6	right wrong	90 10	100	97 3
26+6	right wrong	87 13	88 13	90 10
4+7	right wrong	97 3	100	97 3
7+4	right wrong	97 3	100	97 3
34+7	right wrong	83 17	96 4	90 10
37+4	right wrong	87 13	95 5	92 8
3+6	right wrong	97 3	100	100
6+3	right wrong	97 3	100	100

Sum	Response	Percent Year 3	Percent Year 4	Percent Year 5
33+6	right wrong	90 10	80 20	92 8
36+3	right wrong	100	91 8.7	92 8
25+6	right wrong	87 13	87 13	92 8

Appendix 3: Means and Standard Deviation

3. 1 Means and Standard Deviations for sums in Chapter 5

3. 1. 1 Sums in 5.1 Experiment 1

3. 1. 1. 1 Means and standard deviation for single-digit sums

Table 1 Means and standard deviation for single-digit sums

Sum	Year	Mean	Std Deviation
1+1	3	592.2414	684.6012
	4	410.2955	177.1728
	5	356.9310	176.9939
	6	280.1892	117.1525
1+2	3	692.2241	609.9116
	4	540.3636	413.2893
	5	410.7000	275.4538
	6	428.3243	316.4413
1+3	3	687.7458	881.0685
	4	528.8409	573.2186
	5	415.8000	269.7065
	6	343.8919	145.2429
1+4	3	574.8103	392.4266
	4	443.5581	286.4745
	5	369.1000	227.3778
	6	331.6216	129.9507
1+5	3	728.8571	628.7098
	4	544.2273	376.1820
	5	419.4333	277.0977
	6	460.6486	406.3771
1+6	3	812.3898	841.1910
	4	584.5227	550.3007
	5	405.2667	192.5556
	6	419.5676	210.6965
1+7	3	681.8644	510.9836
	4	449.6818	195.0721
	5	380.9355	184.4005
	6	405.1892	274.8387
1+8	3	750.6724	774.6498
	4	565.4773	366.5130
	5	417.3333	299.9761
	6	369.1111	181.5330

Sum	Year	Mean	Std Deviation
1+9	3	809.9825	922.3863
	4	473.3864	354.7879
	5	357.0333	168.9110
	6	303.8378	133.3297
2+2	3	556.3966	361.1634
	4	407.8409	221.4971
	5	343.9667	174.3464
	6	367.2703	280.1428
2+3	3	798.5088	673.5920
	4	674.5682	560.2176
	5	450.7000	586.6694
	6	370.5833	188.4803
2+4	3	997.3793	1057.8786
	4	739.9545	621.0831
	5	418.7667	255.9322
	6	424.4595	196.1813
2+5	3	983.0862	997.6494
	4	544.1136	206.2381
	5	481.2333	189.4596
	6	452.1316	195.2591
2+6	3	1254.2281	2475.9787
	4	621.2273	417.3275
	5	427.7000	182.3498
	6	398.1351	186.3723
2+7	3	888.4483	749.5560
	4	728.3409	528.4200
	5	534.0345	408.1940
	6	464.6216	278.6041
2+8	3	816.1071	945.7119
	4	690.0000	823.4540
	5	429.0333	312.9802
	6	435.5946	295.8627
2+9	3	837.4655	794.7153
	4	566.3256	483.6399
	5	506.9032	521.9203
	6	445.9730	554.4522
3+3	3	876.6034	1813.0162
	4	590.1163	483.1747
	5	484.1000	314.1290
	6	324.9189	103.4994
3+4	3	1070.3750	779.5840
	4	798.0909	618.2604
	5	671.4000	734.3298
	6	604.6579	382.8358

Sum	Year	Mean	Std Deviation
3+5	3	1026.1379	1289.4498
	4	784.3864	576.6824
	5	551.7667	337.8678
	6	524.7297	295.6233
3+6	3	1216.6140	1683.0742
	4	924.2500	1080.8200
	5	615.5333	598.6354
	6	494.8378	271.3865
3+7	3	1319.4237	1431.4877
	4	791.2727	647.3685
	5	495.9667	279.0350
	6	597.4865	541.6360
3+8	3	1516.4737	2291.8813
	4	870.2273	904.7848
	5	486.4828	338.7236
	6	577.2973	384.1062
3+9	3	1231.6842	1616.6746
	4	766.8182	597.9106
	5	661.5000	787.3411
	6	503.8378	352.1401
4+4	3	727.1034	664.1872
	4	556.9545	474.2614
	5	418.2000	332.0709
	6	348.8056	184.4499
4+5	3	1150.1207	1655.9828
	4	855.0682	646.2929
	5	482.0000	251.8137
	6	493.3784	422.8036
4+6	3	1491.1207	2065.7187
	4	952.1364	1074.2075
	5	728.6667	561.7257
	6	604.0270	460.3160
4+7	3	1147.0862	1004.0280
	4	879.1818	774.3612
	5	658.0345	595.5788
	6	638.5405	395.8269
4+8	3	1295.0847	1231.0344
	4	1035.2727	892.3349
	5	560.5484	401.3894
	6	589.4865	375.4473
4+9	3	1143.5789	937.2989
	4	1093.7273	1396.1696
	5	851.3333	742.6445
	6	567.6757	377.4724

Sum	Year	Mean	Std Deviation
5+5	3	659.5862	749.8480
	4	408.0000	270.5069
	5	428.9310	360.9453
	6	321.3784	194.6877
5+6	3	1297.8772	1664.7375
	4	867.5455	643.4213
	5	726.3548	761.7916
	6	489.1622	216.7971
5+7	3	1462.0508	934.4234
	4	1286.9773	1770.2728
	5	673.3000	375.2891
	6	867.6216	839.5008
5+8	3	1374.6491	1441.4553
	4	1112.0000	861.1699
	5	1036.9677	1192.9532
	6	707.9730	456.7837
5+9	3	1802.0678	2211.2677
	4	1028.2791	895.5417
	5	703.7931	616.5635
	6	665.4595	414.0096
6+6	3	883.3750	918.8196
	4	551.7955	367.7749
	5	469.8889	365.4987
	6	365.0541	230.5995
6+7	3	1618.1034	1367.9015
	4	1264.1364	1106.4875
	5	907.0714	761.9101
	6	721.4211	507.9416
6+8	3	1623.2105	1359.5563
	4	1617.7273	1664.4062
	5	1230.5000	2342.8736
	6	968.1351	725.1315
6+9	3	1586.2414	1465.2170
	4	1370.6364	1503.0501
	5	920.5000	1052.7570
	6	866.6842	999.5224
7+7	3	1492.5789	1396.8509
	4	955.8864	1054.1886
	5	599.0345	612.3386
	6	509.9459	344.7374
7+8	3	1550.6316	1198.4044
	4	1531.2045	1442.6559
	5	1150.9655	1548.4297
	6	870.8919	800.5472

Sum	Year	Mean	Std Deviation
7+9	3	1675.9138	1418.5412
	4	1232.8636	1052.7568
	5	1249.9333	1681.9455
	6	951.3243	786.1504
8+8	3	1858.5345	2503.3577
	4	1216.5909	1667.1507
	5	605.4483	539.1807
	6	847.2973	1138.8311
8+9	3	1805.9138	1449.3664
	4	1470.4318	1325.5224
	5	824.6667	690.4705
	6	770.0278	701.6555
9+9	3	1642.2931	1459.1441
	4	1257.1364	1802.9604
	5	707.2333	891.7002
	6	604.0541	549.4494

3. 1. 1. 2 Means and standard deviation for decade sums

The single-digit and decade sums here are examples of the types of decade sums used in the task instead of actual sums.

Table 2 Means and standard deviation for 20s single-digit and decade sums.

Means and standard deviation for 20s single-digit and decade sums

year		rt for 20s <10 sum (1+2)	rt for 20s <10 sum (21+2)	rt for 20s tie sum (2+2)	rt for 20s tie sum (22+2)	rt for 20s >10 sum (6+9)	rt for 20s >10 sum (26+9)
3.00	N	38	38	38	38	38	38
	Mean	983.2632	1203.1579	980.7105	905.5526	1502.1579	2182.5789
	Std. Dev	817.3971	901.0995	2151.8532	789.1600	1673.5637	2024.4576
4.00	N	44	44	44	44	44	44
	Mean	675.6364	1092.4318	451.5682	786.4545	1068.4091	1631.4318
	Std. Dev	433.1686	1833.3418	217.4094	818.1737	933.5884	1420.7706
5.00	N	30	30	30	30	29	30
	Mean	416.0333	1038.1667	390.2333	730.1333	949.6897	1248.7333
	Std. Dev	213.1010	1772.7341	220.3041	696.3391	933.5305	1292.0483
6.00	N	37	37	37	37	37	37
	Mean	436.0270	624.3784	323.5946	546.8649	835.4324	1129.2162
	Std. Dev	267.8175	554.8194	164.7015	275.5438	903.7425	959.4675
Total	N	149	149	149	149	148	149
	Mean	642.3221	993.5168	542.3893	745.9933	1098.2703	1570.2282
	Std. Dev	546.8912	1385.2290	1120.6703	693.2901	1179.0321	1526.1350

Table 3 Means and standard deviation for 50s single-digit and decade sums.

Means and standard deviation for 50s single-digit and decade sums

year		rt for 50s <10 sum (4+5)	rt for 50s <10 sum (54+5)	rt for 50s tie sum (4+4)	rt for 50s tie sum (54+4)	rt for 50s >10 sum (4+7)	rt for 50s >10 sum (54+7)
3.00	N	38	38	38	38	38	38
	Mean	921.7105	1762.3684	567.5526	1026.0789	1205.6053	3037.2368
	Std. Dev	1797.6957	1951.5521	403.2160	759.6762	1028.5202	4050.7619
4.00	N	44	44	44	44	44	44
	Mean	697.4091	1265.2727	477.2273	929.1364	921.8409	1936.4318
	Std. Dev	567.4074	1258.8020	254.1326	919.6593	703.8143	1474.1935
5.00	N	30	30	30	30	30	30
	Mean	413.0000	922.6667	427.3000	692.7000	747.4667	1369.8000
	Std. Dev	273.0518	1307.2359	246.3893	958.6223	613.9032	1239.2011
6.00	N	37	37	37	37	37	37
	Mean	400.6486	676.7568	366.2162	479.7838	783.9459	1630.7838
	Std. Dev	182.5888	610.2215	276.1258	459.9997	553.0761	1932.1393
Total	N	149	149	149	149	149	149
	Mean	623.6577	1176.9262	462.6443	794.6711	924.8591	2027.1879
	Std. Dev	984.9064	1415.5970	308.7367	817.0467	766.9555	2516.7091

Table 4 Means and standard deviation for 80s single-digit and decade sums.

Means and standard deviation for 80s single-digit and decade sums

year		rt for 80s <10 sum (2+3)	rt for 80s <10 sum (82+3)	rt for 80s tie sum (3+3)	rt for 80s tie sum (83+3)	rt for 80s >10 sum (5+6)	rt for 80s >10 sum (85+6)
3.00	N	38	38	38	38	38	38
	Mean	681.8684	1791.5789	926.0526	1272.3947	1462.9737	3262.5526
	Std. Dev	425.9102	1535.2885	2047.0641	1499.3531	1679.8702	3460.5958
4.00	N	44	44	44	44	44	44
	Mean	645.8636	1204.7955	520.6364	860.0455	1057.4091	2043.1364
	Std. Dev	401.7123	1717.8912	323.6255	581.5732	724.7016	1660.6331
5.00	N	30	30	30	30	30	30
	Mean	455.2667	943.9333	433.2667	686.2667	961.9667	1384.9000
	Std. Dev	228.0851	1218.9206	357.1248	660.8000	1029.3319	1111.5091
6.00	N	37	37	37	37	37	37
	Mean	484.5405	731.9459	346.6757	506.9189	707.2703	1267.6486
	Std. Dev	324.0089	638.1291	154.4839	413.9722	597.2514	1003.3734
Total	N	149	149	149	149	149	149
	Mean	576.6107	1184.5034	563.2416	842.5302	1054.6779	2029.0268
	Std. Dev	370.5933	1412.2919	1076.2571	931.7586	1107.7804	2214.0204

Table 5 Means and standard deviation for 20s decade overhead RT difference.

Means and standard deviation for 20s decade overheac difference

year		20s (answer < 10)	20s (tie-sum)	20s (answer > 10)
3.00	N	38	38	38
	Mean	219.895	-75.1579	680.421
	Std. Dev	1000.42	2139.32	1713.89
4.00	N	44	44	44
	Mean	416.795	334.886	563.023
	Std. Dev	1660.22	804.457	1350.14
5.00	N	30	30	29
	Mean	622.133	339.900	297.069
	Std. Dev	1757.39	584.855	1483.69
6.00	N	37	37	37
	Mean	188.351	223.270	293.784
	Std. Dev	531.152	320.050	661.787
Total	N	149	149	148
	Mean	351.195	203.604	473.743
	Std. Dev	1323.65	1205.43	1351.62

Table 6 Means and standard deviation for 50s decade overhead RT difference.

Means and standard deviation for 50s decade overheac
difference

year		50s (answer < 10)	50s (tie-sum)	50s (answer > 10)
3.00	N	38	38	38
	Mean	840.658	458.526	1831.63
	Std. Dev	1974.00	608.854	3739.33
4.00	N	44	44	44
	Mean	567.864	451.909	1014.59
	Std. Dev	1278.12	890.938	1393.81
5.00	N	30	30	30
	Mean	509.667	265.400	622.333
	Std. Dev	1106.74	841.468	1073.82
6.00	N	37	37	37
	Mean	276.108	113.568	846.838
	Std. Dev	537.812	465.518	1789.85
Total	N	149	149	149
	Mean	553.268	332.027	1102.33
	Std. Dev	1341.65	732.097	2295.02

Table 7 Means and standard deviation for 80s decade overhead RT difference.

Means and standard deviation for 80s decade overheac
difference

year		80s (answer < 10)	80s (tie-sum)	80s (answer > 10)
3.00	N	38	38	38
	Mean	1109.71	346.342	1799.58
	Std. Dev	1455.82	2082.19	2423.99
4.00	N	44	44	44
	Mean	558.932	339.409	985.727
	Std. Dev	1693.82	494.235	1423.70
5.00	N	30	30	30
	Mean	488.667	253.000	422.933
	Std. Dev	1155.57	630.982	808.640
6.00	N	37	37	37
	Mean	247.405	160.243	560.378
	Std. Dev	645.724	354.303	1091.79
Total	N	149	149	149
	Mean	607.893	279.289	974.349
	Std. Dev	1351.75	1126.63	1659.90

3. 1. 2 Sums in 5.2 Experiment 2

3. 1. 2. 1 Means and standard deviation for three-digit sums

Table 8 Means and standard deviation for Problem Type A

Means and standard deviation for sums in Problem Type A (tie sums)

year		4+1+4	4+4+1	4+4+3	4+3+4	3+4+4
3.00	Mean	1074.25	813.308	1328.29	1382.08	1250.64
	Std. Deviation	1274.43	674.797	1704.11	1298.24	1243.82
4.00	Mean	910.273	699.953	810.136	951.381	925.767
	Std. Deviation	944.375	556.745	666.170	762.773	861.554
5.00	Mean	647.465	552.860	699.744	775.698	853.535
	Std. Deviation	463.864	408.709	571.840	584.717	1197.47
6.00	Mean	534.200	456.200	526.543	622.714	574.886
	Std. Deviation	265.784	300.931	272.792	320.600	331.998
Total	Mean	818.684	648.150	880.661	970.791	936.293
	Std. Deviation	901.120	536.733	1074.90	912.407	1036.07

Table 9 Means and standard deviation for sums in Problem Type B

Means and standard deviation for sums in Problem Type B (sum-to-10)

year		7+3+n	7+n+3	9+1+n	9+n+1
3.00	Mean	1708.51	1618.47	1465.25	1539.02
	Std. Deviation	1751.04	1328.04	1489.73	1692.09
4.00	Mean	1537.65	1814.14	1190.02	1417.37
	Std. Deviation	1673.49	2649.83	2290.28	1719.43
5.00	Mean	1004.14	1100.24	643.286	971.762
	Std. Deviation	858.306	1054.17	476.319	1189.15
6.00	Mean	850.514	907.514	606.486	780.257
	Std. Deviation	624.445	646.784	286.303	499.358
Total	Mean	1319.63	1399.85	1020.98	1213.80
	Std. Deviation	1408.69	1658.34	1469.13	1434.86

Table 10 Means and standard deviation for sums in Problem Type C

eans and standard deviation for sums in Problem Tyı C (9+a+b)

year		9+b+c	a+9+c	a+b+9
3.00	Mean	2016.13	1925.92	1845.35
	Std. Deviation	2473.64	2049.92	1835.56
4.00	Mean	1401.30	1534.75	2058.28
	Std. Deviation	1677.64	1769.56	3648.54
5.00	Mean	1051.52	1038.72	935.000
	Std. Deviation	1529.84	1003.60	680.605
6.00	Mean	998.714	993.029	1001.11
	Std. Deviation	476.195	1199.54	751.437
Total	Mean	1419.85	1422.99	1502.50
	Std. Deviation	1816.67	1645.55	2179.25

Table 11 Means and standard deviation for sums in Problem Type D (filler sums)

Means and Standard Deviation for sums in Group D (filler sums)

year		5+4+3	4+5+3
3.00	Mean	1663.92	1374.68
	Std. Deviation	2072.00	1111.31
4.00	Mean	1296.50	1160.70
	Std. Deviation	1617.52	822.362
5.00	Mean	890.256	1071.28
	Std. Deviation	970.968	1118.56
6.00	Mean	748.257	796.057
	Std. Deviation	372.180	456.271
Total	Mean	1198.31	1130.61
	Std. Deviation	1522.14	959.510

3. 2 Means and standard deviation for sums in Chapter 6

3. 2. 1 Means and standard deviation for sums in Problem Type A

Table 12 Means and standard deviation for 5+n sums in Problem Type A.

eans and standard deviation for single-digit sums in Problem Type

year		5+1	5+2	5+3	5+4
3.00	Mean	417.294	740.471	781.676	648.471
	Std. Deviation	183.590	606.926	997.007	467.462
4.00	Mean	328.238	411.048	478.619	530.667
	Std. Deviation	111.085	155.814	240.433	291.094
5.00	Mean	287.882	342.559	372.029	394.118
	Std. Deviation	140.717	165.296	225.803	263.373
Total	Mean	346.843	510.730	553.674	523.506
	Std. Deviation	161.904	433.116	662.718	373.804

Table 13 Means and standard deviation for 45+n sums in Problem Type A.

Means and standard deviation for decade sums in Problem Type A

year		45+1	45+2	45+3	45+4
3.00	Mean	468.204	995.588	894.588	905.029
	Std. Deviation	200.516	910.525	570.086	846.426
4.00	Mean	389.333	468.143	509.429	513.857
	Std. Deviation	173.345	208.508	173.402	238.150
5.00	Mean	342.088	394.118	663.559	524.824
	Std. Deviation	183.178	209.706	643.004	607.998
Total	Mean	401.416	641.360	715.449	667.483
	Std. Deviation	193.928	645.388	554.365	674.888

Table 14 Means and standard deviation for RT 45+n - RT 5+n.

eans and standard deviation for RT difference for sums in Problem Type

year		rt45+1-rt5+1	rt45+2-rt5+2	rt45+3-rt5+3	rt45+4-rt5+4
3.00	Mean	50.9118	255.1176	112.9118	256.5588
	Std. Deviation	216.0086	1033.8699	1049.4034	814.6381
4.00	Mean	61.0952	57.0952	30.8095	-16.8095
	Std. Deviation	156.3777	236.4827	199.7480	316.8786
5.00	Mean	54.2059	51.5588	291.5294	130.7059
	Std. Deviation	192.2610	207.5829	602.7125	440.6371
Total	Mean	54.5730	130.6292	161.7753	143.9775
	Std. Deviation	192.1773	662.8689	754.8375	596.3463

3. 2. 2 Means and standard deviation for sums in Problem Type B

Table 15 Means and standard deviation for sums in Problem Type B.

eans and standard deviation for single-digit and decade sums in Problem Type

year		23+3	3+3	26+6	6+6	26+5
3.00	Mean	861.824	556.118	1245.65	492.824	1245.85
	Std. Deviation	963.814	583.606	887.215	346.327	683.151
4.00	Mean	722.714	411.048	1003.76	323.238	871.905
	Std. Deviation	852.434	290.096	687.199	140.319	799.203
5.00	Mean	371.382	296.618	694.471	324.853	601.294
	Std. Deviation	236.842	151.409	472.780	280.940	308.710
Total	Mean	641.640	422.753	978.011	388.640	911.382
	Std. Deviation	763.579	410.486	738.393	292.980	660.802

Table 16 Means and standard deviation for RT difference for sums in Problem Type B.

eans and standard deviation for RT difference fo
sums in Problem Type B

year		rt23+3-rt3 +3	rt26+6-rt6 +6
3.00	Mean	305.7059	752.8235
	Std. Deviation	755.9690	736.9436
4.00	Mean	311.6667	680.5238
	Std. Deviation	795.0351	701.1425
5.00	Mean	74.7647	369.6176
	Std. Deviation	208.0514	432.6877
Total	Mean	218.8876	589.3708
	Std. Deviation	622.2434	645.4107

3. 2. 3. Means and standard deviation for sums in Problem Type C

Table 17 Means and standard deviation for single-digit sums in Problem Type C.

eans and standard deviation for single-digit sums in Problem Type

year		4+7	7+4	3+6	6+3
3.00	Mean	970.235	776.294	871.971	763.029
	Std. Deviation	940.431	614.234	1438.80	729.918
4.00	Mean	612.667	539.857	481.143	389.238
	Std. Deviation	381.172	332.692	261.718	140.438
5.00	Mean	699.441	472.441	400.000	337.735
	Std. Deviation	1402.81	449.604	318.919	175.540
Total	Mean	782.416	604.427	599.449	512.360
	Std. Deviation	1061.04	511.448	936.707	505.477

Table 18 Means and standard deviation for decade sums in Problem Type C.

Means and standard deviation for decade sums in Problem Type C

year		34+7	37+4	33+6	36+3
3.00	Mean	1370.59	1155.41	1345.56	911.471
	Std. Deviation	864.250	929.822	1516.03	608.660
4.00	Mean	734.429	563.000	837.143	587.000
	Std. Deviation	336.786	265.624	597.920	337.271
5.00	Mean	762.529	503.088	707.618	549.176
	Std. Deviation	509.623	228.279	445.948	342.163
Total	Mean	988.191	766.427	981.888	696.506
	Std. Deviation	703.426	674.477	1050.18	487.630

Table 19 Means and standard deviation for RT difference for sums in Problem Type C.

eans and standard deviation for RT difference for sums in Problem Type

year		rt34+7-rt4 +7	rt37+4-rt7 +4	rt33+6-rt3 +6	rt36+3-rt6 +3
3.00	Mean	400.3529	379.1176	473.5882	148.4412
	Std. Deviation	1016.4478	812.4650	1786.4710	792.8394
4.00	Mean	121.7619	23.1429	356.0000	197.7619
	Std. Deviation	367.1783	366.5693	570.2159	314.6341
5.00	Mean	63.0882	30.6471	307.6176	211.4412
	Std. Deviation	1424.7104	421.6894	491.2128	276.0603
Total	Mean	205.7753	162.0000	382.4382	184.1461
	Std. Deviation	1097.0103	611.7519	1169.0701	536.3066