Shared-Control for the Kinematic Model of a Mobile Robot

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***Abstract*— This paper presents a shared-control algorithm for the kinematic model of a mobile robot. The set of feasible position of the robot is defined by a group of linear inequalities. The shared-control strategy is based on a hysteresis switch and its properties are established by a Lyapunov-like analysis. Simulation results illustrate the effectiveness of the algorithm.**

I. INTRODUCTION

This paper deals with the shared-control problem for a simple mobile robot. The robot is ”driven” by a human operator which is ”supervised” by a feedback controller. The human operator provides velocity commands the robot except for ”emergency” situations, in which the feedback controller is active.

Mobile robots are widely used in probing [1], cleaning [2], military applications [3], industries [4], [5] and household tasks [6]. The present paper proposes a shared-control method to drive a wheeled mobile robot in the case in which the control authority is shared by a human operator and a feedback controller. This shared-control system is widely used in the modern world, for example in intelligent wheelchairs [7], teleoperations [8], transportations [9] and medicine [10]. The paper [11] has shown that there are four types of robot modes according to the level of system's autonomy and has demonstrated that collaborative control can increase performance and reduce error compared with manual operation, whereas [12] has introduced the idea of task-level adaptive shared-control to assembly systems and demonstrated that the resulting system is flexible and able to accomodate changes. The control authority shared between a human operator and a feedback controller is similar to that shared between two operators, which is commonly used in training systems [13]. For example, the paper [14] has quantified how much the interaction between two users and a slave robot, as well as the environment, occurs in a dual-user teleoperation system through a dominance factor which is chosen by means of experimental trials.

Shared control problems have been studied in [15], where a continuous shared control paradigm based on a constant sharing weight determined via simulations has been introduced for a brain-machine interface commanding a robot in reach and grasp tasks. In [16] the sharing weight is a variable calculated on-line. Finally in [17] and [18] the shared-control problem for fully actuated, linear, mechanical systems has been studied by utilizing ideas from [19] and [20].

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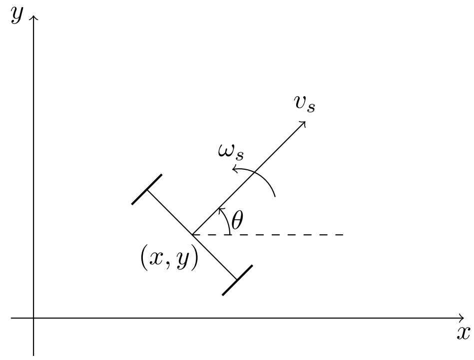

Fig. 1. Kinematic Model for a Unicycle-like Mobile Robot

In shared-control the aim is to let the human operator in charge of the system in ”normal” conditions, while the feedback control works only in ”dangerous” situations: this is similar to an obstacle avoidance problem. The obstacles are characterized by regions that cannot be visited by the robot depending on the states and the input. The potential field method is a commonly used way to solve the collision avoidance problem, but suffers from significant shortcomings discussed for example in [21], such as the existence of local minimizers and oscillatory trajectories when passing through a narrow corridor. These drawbacks can be partly alleviated with the use of Lyapunov-like methods [22]. However, this approach prevents the configuration of the system from reaching the boundary of the obstacle. This limitation can be overcome by the shared-control law presented in this paper, which allows the robot to move safely within a predefined, boundary-reachable, admissible set.

The rest of the paper is organized as follows. The problem is formulated in Section II together with some assumptions. The design of the shared-control is given in Section III where formal properties of the closed-loop system are presented. In Section IV two numerical examples to illustrate the performance of the shared-control are given. Finally, Section V gives some conclusions and suggestions for future work.

II. PROBLEM STATEMENT

In this section we formulate the shared-control problem for the kinematic model of a mobile robot. For simplicity we assume that the robot is a wheeled robot (see Figure 1), the dynamics of which are described by the equations

$$\begin{aligned} \dot{x} &= v_s \cos\theta, \\ \dot{y} &= v_s \sin\theta, \\ \dot{\theta} &= \omega_s, \end{aligned} \tag{1}$$

where (x, y) denotes the Cartesian coordinates of the center of the robot's mass, θ represents the angle between the (positive) heading direction and the x-axis, v_s and ω_s are the linear velocity and the angular velocity of the mobile robot, respectively. Note that v_s and ω_s are the inputs of the system.

Let

$$\begin{aligned} v_s(x,y,\theta,t) &= [1-k(x,y,\theta,v_h)]v_f(x,y,\theta,t) \\ &\quad +k(x,y,\theta,v_h)v_h(x,y,\theta,t), \\ \omega_s(x,y,\theta,t) &= [1-k(x,y,\theta,v_h)]\omega_f(x,y,\theta,t) \\ &\quad +k(x,y,\theta,v_h)\omega_h(x,y,\theta,t), \end{aligned} \tag{2}$$

where $u_h = [v_h, \omega_h]^T$, denoted as *h-control*, describes the human action, $u_f = [v_f, \omega_f]^T$, denoted as *f-control*, represents the feedback-control action and k, denoted as *sharing function*, quantifies how the control action is shared. In what follows we use the name *s-closed-loop* to denote the system described by equation (1) with input $u_s = [v_s, \omega_s]^T$ given by equation (2) and the name *h-closed-loop* to denote the system described by the equations

$$\begin{aligned} \dot{x} &= v_h \cos\theta, \\ \dot{y} &= v_h \sin\theta, \\ \dot{\theta} &= \omega_h. \end{aligned}$$

Note that the s-closed-loop system and the h-closed-loop system share the same state space $\mathcal{P} \times \mathcal{A} = \mathbb{R}^2 \times \mathbb{S}$, where $\mathcal{P}$ denotes the set of Cartesian positions in the plane and $\mathcal{A}$ denotes the set of heading angles. Suppose $\mathcal{P}_a \in \mathcal{P}$ is a given, closed and compact set that describes the admissible Cartesian configuration set for the system (1) and u_h is a given h-control. The shared-control problem can be posed as follows.

Given the system (1), an admissible configuration set $\mathcal{P}_a$ and an h-control u_h, find (if possible)

- an f-control u_f;
- a sharing function k;
- a safe set $\mathcal{R}_s(v_h) \triangleq \mathcal{P}_a \times \mathcal{A}_s \subset \mathcal{P}_a \times \mathcal{A} \triangleq \mathcal{R}(v_h)$;

such that the s-closed-loop system (1)-(2) has the following properties.

P1) The set $\mathcal{R}(v_h)$ is forward invariant.

P2) Let Ω_s and Ω_h be the Ω-limit set of the s-closed-loop and h-closed-loop, respectively. Then

$$\Omega_s = \begin{cases} \Omega_h & \text{if } \Omega_h \subset \mathcal{R}_s(v_h), \\ \Pi_{\mathcal{R}_s}(\Omega_h) & \text{if } \Omega_h \not\subset \mathcal{R}_s(v_h), \end{cases}$$

where $\Pi_{\mathcal{R}_s}(\Omega_h)$ is the projection of Ω_h into the set $\mathcal{R}_s(v_h)$, which will be defined in Section III-A.

P3) $u_s = u_h$ if $(x, y, \theta) \in \mathcal{R}_s(v_h)$.

Note that for any fixed v_h and any $p \in \mathcal{P}_a$, $\mathcal{A}_s$ is the set of all possible heading angles with which the robot is unable to hit the boundary of $\mathcal{P}_a$ within a short time.

III. DESIGN OF THE SHARED-CONTROL

In this section we give a solution to the shared-control problem presented in Section II in the case in which $\mathcal{P}_a$ is described by a group of linear inequalities, namely

$$\mathcal{P}_a = \{p \in \mathbb{R}^2 \ |Sp + T \leq 0\}, \tag{3}$$

where $p = [x, y]^T$, $S = [s_1^T, s_2^T, \ldots, s_m^T]^T \in \mathbb{R}^{m\times 2}$ and $T = [t_1, t_2, \ldots, t_m]^T \in \mathbb{R}^m$. Similarly to [17] and [18] we make the following assumption throughout the paper.

Assumption 1: If $m > 2$ then the matrices S and T are such that

$$\text{rank}(\begin{bmatrix} s_{r_1} \\ \vdots \\ s_{r_l} \end{bmatrix}) < \text{rank}(\begin{bmatrix} s_{r_1} & t_{r_1} \\ \vdots & \vdots \\ s_{r_l} & t_{r_l} \end{bmatrix}),$$

for all $l \in [3, m]$ and $r_1, r_2, \ldots, r_l \in \{1, 2, \ldots, m\}$.

A. Design of the f-control

In this section we design the f-control under the assumption $m = 2$. As discussed in [17] this is without loss of generality. In addition, as noted therein, N_c f-controls need to be designed and in general $N_c \leq \binom{m}{2}$. Consider now the i^{th} group of constraints

$$S^i p + T^i \leq 0, \tag{4}$$

where $S^i = [{s_1^i}^T, {s_2^i}^T]^T \in \mathbb{R}^{2\times 2}$ and $T = [t_1^i, t_2^i]^T \in \mathbb{R}^2$. Note that, as detailed in [17], S^i is invertible according to its construction. Define new variables q^i and $z^i = [z_1^i, z_2^i]^T$ as

$$q^i = S^i p + T^i,$$

and

$$z_j^i = \log \frac{q_j^i}{q_{r_j}^i}, \tag{5}$$

for all $j \in \{1, 2\}$, where $q_{r_j}^i$ (to be defined) describes the reference trajectory for the state q_j^i.

Define $q_r^i = [q_{r_1}^i, q_{r_2}^i]^T$ as

$$q_{r_j}^i = \begin{cases} q_d^i, & \text{if } q_{d_j}^i \leq (1-\frac{\sqrt{2}}{2})r - \epsilon, \\ -\epsilon, & \text{if } q_{d_j}^i \geq (\sqrt{2}-1)r - \epsilon, \\ h_j^i, & \text{otherwise}, \end{cases} \tag{6}$$

for $j \in \{1, 2\}$, where r is a positive constant, $h_j^i = -(r+\epsilon)+\sqrt{r^2 - [(\sqrt{2}-1)r - \epsilon - q_{d_j}^i]^2}$, $q_d^i = [q_{d_1}^i, q_{d_2}^i]^T = S^i p_d +$

T^i and p_d refers to the reference signal in the space $\mathcal{P}$. Note that, by definition, $q^i_{r_j}$ is a smooth and non-positive function. As a result, $\dot{q}_{r^i_j}$ and $\ddot{q}_{r^i_j}$ exist. Finally, define $(p^i_r, \alpha^i_r, v^i_r, \theta^i_r)$ as

$$\begin{aligned} p^i_r &= S^{i^{-1}}(q^i_r - T^i), \\ \alpha^i_r &= S^{i^{-1}}\dot{q}^i_r, \\ v^i_r &= \sqrt{{\alpha^i_{r_1}}^2 + {\alpha^i_{r_2}}^2}, \\ \theta^i_r &= \text{atan2}(\alpha^i_{r_2}, \alpha^i_{r_1}). \end{aligned} \tag{7}$$

Suppose (p_d, θ_d) is a point of the Ω-limit set of the h-closed-loop, *i.e.* $(p_d, \theta_d) \in \Omega_h$, and define the projection of (p_d, θ_d) into $\mathcal{R}_s$, relative to the i^{th} group of constraints, by

$$\Pi^i_{\mathcal{R}_s}(p_d, \theta_d) = (p^i_r, \theta^i_r),$$

where (p^i_r, θ^i_r) is defined by (7). Then the projection of Ω_h into the safe set $\mathcal{R}_s$ is defined by

$$\Pi^i_{\mathcal{R}_s}(\Omega_h) = \{s \in \mathcal{R}_s | s = \Pi^i_{\mathcal{R}_s}(p_d, \theta_d)\}, \forall (p_d, \theta_d) \in \Omega_h.$$

In addition, for any given α^i_r, the reference input signal u^i_r related to the i^{th} group of active constraints is defined as

$$u^i_r = \begin{bmatrix} v^i_r \\ \omega^i_r \end{bmatrix} = \begin{bmatrix} \sqrt{{\alpha^i_{r_1}}^2 + {\alpha^i_{r_2}}^2} \\ \frac{\text{d}}{\text{dt}}(\text{atan2}(\alpha^i_{r_1}, \alpha^i_{r_2})) \end{bmatrix}. \tag{8}$$

With the new variable z^i and the feedback controller u^i_f, the i^{th} group of constraints on p, *i.e.* $S^i p + T^i \leq 0$, can be removed and the system (1) can be rewritten as

$$\begin{aligned} \dot{z}^i_1 &= \frac{v^i_f \cos\theta^i}{e^{z^i_1} q^i_{r_1}} - \frac{v^i_r \cos\theta^i_r}{q^i_{r_1}}, \\ \dot{z}^i_2 &= \frac{v^i_f \cos\theta^i}{e^{z^i_2} q^i_{r_2}} - \frac{v^i_r \cos\theta^i_r}{q^i_{r_2}}, \\ \dot{\theta}^i &= \omega^i_f. \end{aligned} \tag{9}$$

Let

$$\theta^{i*} = \text{atan2}(e^{z^i_2}(\alpha^i_r \sin\theta^i_r - \gamma_2 z^i_2), e^{z^i_1}(\alpha^i_r \cos\theta^i_r - \gamma_1 z^i_1))$$

where $\gamma_1 > 0$ and $\gamma_2 > 0$.

Consider now the i^{th} Lyapunov function $L^i(z^i_1, z^i_2, \theta^i)$ given by the equation

$$L^i(z^i_1, z^i_2, \theta^i) = \frac{1}{2}[{z^i_1}^2 + {z^i_2}^2 + (\theta^i - \theta^{i*})^2], \tag{10}$$

and choose v^i_f and ω^i_f such that $\dot{L}^i < 0$ for all $(z^i_1, z^i_2) \neq (0, 0)$ and $\theta^i \neq \theta^{i*}$. One such a choice is given by

$$\begin{aligned} v^i_f &= \sqrt{\begin{array}{l} e^{2z^i_1}(v^i_r \cos\theta^i_r - \gamma_1 z^i_1)^2 \\ +e^{2z^i_2}(v^i_r \sin\theta^i_r - \gamma_2 z^i_2)^2 \end{array}}, \\ \omega^i_f &= \dot{\theta}^{i*} - \gamma_3(\theta^i - \theta^{i*}) - \frac{z^i_2 v^i_f \cos\frac{\theta^i + \theta^{i*}}{2} \text{sinc}\frac{\theta^i - \theta^{i*}}{2}}{q^i_2} \\ &\quad + \frac{z^i_1 v^i_f \sin\frac{\theta^i + \theta^{i*}}{2} \text{sinc}\frac{\theta^i - \theta^{i*}}{2}}{q^i_1}. \end{aligned}$$

This control can be pull back to the (q^i, θ) coordinates giving

$$\begin{aligned} v^i_f &= \sqrt{\begin{array}{l} (\frac{q^i_1}{q^i_{r_1}})^2(v^i_r \cos\theta^i_r - \gamma_1 \log\frac{q^i_1}{q^i_{r_1}})^2 \\ +(\frac{q^i_2}{q^i_{r_2}})^2(v^i_r \sin\theta^i_r - \gamma_2 \log\frac{q^i_2}{q^i_{r_2}})^2 \end{array}}, \\ \omega^i_f &= \frac{\log\frac{q^i_1}{q^i_{r_1}} v^i_f \sin\frac{\theta^i + \theta^{i*}}{2} \text{sinc}\frac{\theta^i - \theta^{i*}}{2}}{q^i_1} \\ &\quad - \frac{\log\frac{q^i_2}{q^i_{r_2}} v^i_f \cos\frac{\theta^i + \theta^{i*}}{2} \text{sinc}\frac{\theta^i - \theta^{i*}}{2}}{q^i_2} \\ &\quad + \dot{\theta}^{i*} - \gamma_3(\theta^i - \theta^{i*}), \end{aligned} \tag{11}$$

where

$$\theta^{i*} = \text{atan2}\left(\begin{array}{l} \frac{q^i_2}{q^i_{r_2}}(\alpha^i_{r_2} \sin\theta^i_r - \gamma_2 \log\frac{q^i_2}{q^i_{r_2}}), \\ \frac{q^i_1}{q^i_{r_1}}(\alpha^i_{r_1} \cos\theta^i_r - \gamma_1 \log\frac{q^i_1}{q^i_{r_1}}) \end{array} \right)$$

and $q^i_j = s^i_j p + t^i_j$, $q^i_{r_j} = s^i_j p_r + t^i_j$ for all $j \in \{1, 2\}$.

Lemma 1: Consider the f-closed-loop system (1)-(9) with $[v_s, \omega_s]^T = [v^i_f, \omega^i_f]^T$ given by (11), q^i_r given by (6), and v^i_r and θ^i_r given by (7). Assume $(x(0), y(0)) \in \mathcal{P}_a$. Then the system has the following properties.

P1) $(x(t), y(t)) \in \mathcal{P}_a$ for all $t \geq 0$;

P2) $\lim_{t\to\infty}(x(t) - p_{r_1}(t)) = \lim_{t\to\infty}(y(t) - p_{r_2}(t)) = 0$.

B. Shared Control Theorem

By Property (P3) in Section II we need to find the safe subset $\mathcal{R}_s$ before designing the sharing function k. Relative to the i^{th} group of constraints and a given h-control v_h, the safe, hysteresis and dangerous subsets $\tilde{\mathcal{R}}^i_s, \tilde{\mathcal{R}}^i_h$ and $\tilde{\mathcal{R}}^i_d$ are defined in equations (12) on the top of the next page, where[1] $\mathcal{Q}^i_a =$

[1] The notation $S\mathcal{P} + T$, with $S \in \mathbb{R}^{2\times 2}$, $T \in \mathbb{R}^2$, and $\mathcal{P} \in \mathbb{R}^2$ denotes the set defined as

$$\{x \in \mathbb{R}^2 \mid x = Sy + T,\ y \in \mathcal{P}\}.$$

$$\tilde{\mathcal{R}}_s^i(v_h) = \left\{ \; (q^i,\theta^i) \in \mathcal{Q}_a^i \times \mathbb{S} : (s_j^i[\cos\theta^i, \sin\theta^i]^T v_h) \leq \frac{1}{q_j^i + b_2} - \frac{1}{b_2} \text{ if } q_j^i \geq -b_2 \text{ for all } j \in \{1,2\} \; \right\}$$

$$\tilde{\mathcal{R}}_h^i(v_h) = \left\{ \begin{array}{ll} (q^i,\theta^i) \in \mathcal{Q}_a^i \times \mathbb{S} : & \exists j \in \{1,2\} \text{ such that } (s_j^i[\cos\theta^i, \sin\theta^i]^T v_h) > \frac{1}{q_j^i + b_2} - \frac{1}{b_2} \text{ and } q_j^i \geq -b_2 \\ & \text{and } (s_k^i[\cos\theta^i, \sin\theta^i]^T v_h) < \frac{1}{q_k^i + b_1} - \frac{1}{b_1} \text{ if } q_k^i \geq -b_1 \text{ for all } k \in \{1,2\} \end{array} \right\} \quad (12)$$

$$\tilde{\mathcal{R}}_d^i(v_h) = \left\{ \begin{array}{ll} (q^i,\theta^i) \in \mathcal{Q}_a^i \times \mathbb{S} : & \exists j \in \{1,2\} \text{ such that } (s_j^i[\cos\theta^i, \sin\theta^i]^T v_h) \geq \frac{1}{q_j^i + b_1} - \frac{1}{b_1} \text{ and } -b_1 \leq q_j^i < 0 \\ & \text{or } \exists j \in \{1,2\} \text{ such that } (s_j^i[\cos\theta^i, \sin\theta^i]^T v_h) > \frac{1}{q_j^i + b_1} - \frac{1}{b_1} \text{ and } q_j^i = 0 \\ & \text{or } \forall j \in \{1,2\} \text{ such that } q_j^i = (s_j^i[\cos\theta^i, \sin\theta^i]^T v_h) = 0 \end{array} \right\}$$

$S^i\mathcal{P}_a + T^i$, and $b_2 > b_1 > 0$. Note that $\tilde{\mathcal{R}}_s^i(v_h), \tilde{\mathcal{R}}_h^i(v_h)$ and $\tilde{\mathcal{R}}_d^i(v_h)$ are defined in the (q^i,θ^i) coordinates and can be pull back to the (p,θ) coordinates by the relations

$$\mathcal{R}_s^i(v_h) = \mathrm{diag}(\mathrm{S^i}^{-1}, 1)(\tilde{\mathcal{R}}_\mathrm{s}^\mathrm{i} - \mathrm{col}(\mathrm{T^i}, 0)),$$
$$\mathcal{R}_h^i(v_h) = \mathrm{diag}(\mathrm{S^i}^{-1}, 1)(\tilde{\mathcal{R}}_\mathrm{h}^\mathrm{i} - \mathrm{col}(\mathrm{T^i}, 0)),$$
$$\mathcal{R}_d^i(v_h) = \mathrm{diag}(\mathrm{S^i}^{-1}, 1)(\tilde{\mathcal{R}}_\mathrm{d}^\mathrm{i} - \mathrm{col}(\mathrm{T^i}, 0)),$$

where $\mathrm{col}(\mathrm{T^i}, 0)$ is a column vector obtained by stacking the number 0 under the vector T^i.

By construction, $\mathcal{R}_s^i(v_h)$, $\mathcal{R}_h^i(v_h)$ and $\mathcal{R}_d^i(v_h)$ have the following properties:

- $\mathcal{R}_s^i(v_h) \cup \mathcal{R}_h^i(v_h) \cup \mathcal{R}_d^i(v_h) = \mathcal{R}(v_h)$ for all $i \in \{1, 2, \ldots, N_c\}$;
- $\mathcal{R}_d^i(v_h) \cap \mathcal{R}_d^j(v_h) = \emptyset$ for all $i \neq j$ and $i, j \in \{1, 2, \ldots, N_c\}$;
- $\mathcal{R}_s^i(v_h) \cap \mathcal{R}_d^i(v_h) = \{(p, v_h) | S^i p + T^i = 0, v_h = 0\}$;
- $\mathcal{R}_d(v_h) = \mathcal{R}_d^1(v_h) \cup \cdots \cup \mathcal{R}_d^{N_c}(v_h)$,
  $\mathcal{R}_h(v_h) = \mathcal{R}_h^1(v_h) \cup \cdots \cup \mathcal{R}_h^{N_c}(v_h)$,
  $\mathcal{R}_s(v_h) = \mathcal{R}_s^1(v_h) \cap \cdots \cap \mathcal{R}_s^{N_c}(v_h)$.

For each group of constraints, the sharing function k^i can be defined as, see [23],

$$k^i(p,\theta,v_h) = \left\{ \begin{array}{ll} 1, & (p,\theta) \in \mathcal{R}_s^i(v_h) \setminus \mathcal{R}_d^i(v_h), \\ l^i(p,\theta,v_h), & (p,\theta) \in \mathcal{R}_h^i(v_h), \\ 0, & (p,\theta) \in \mathcal{R}_d^i(v_h), \end{array} \right. \quad (13)$$

where

$$l^i(p,\theta,v_h) = \left\{ \begin{array}{ll} 1, & \text{if } (p,\theta) \text{ enters } \mathcal{R}_h^i(v_h) \text{ from } \mathcal{R}_s^i(v_h), \\ 0, & \text{if } (p,\theta) \text{ enters } \mathcal{R}_h^i(v_h) \text{ from } \mathcal{R}_d^i(v_h). \end{array} \right.$$

Finally, the s-control is given by the equation

$$\begin{array}{rcl} u_s(p,\theta,v_h) & = & \sum_{i=1}^{N_c}[(1 - k^i(p,\theta,v_h))u_f^i(p,p_r,\theta_r,\alpha_r)] \\ & & + \min_{i=1}^{N_c} k^i(p,\theta,v_h) \; u_h. \end{array} \quad (14)$$

Lemma 2: Consider the system (1) with the shared-control input (11), (13), (14). Let $(p(t),\theta(t))$ be a trajectory of the system. Assume $(p(0),\theta(0)) \in \mathcal{R}_s(v_h(0))$. Suppose there exists $\bar{t} > 0$ such that $(p(\bar{t}),\theta(\bar{t})) \notin \mathcal{R}(v_h(\bar{t}))$. Then there exists a t_d such that $0 < t_d < \bar{t}$ and $(p(t_d),\theta(t_d)) \in \mathcal{R}_d(v_h(t_d))$.

Theorem 1: Consider the kinematic model (1) of a mobile robot with a given h-control u_h and the shared-control law given by (11)-(13)-(14). Assume the admissible configuration set $\mathcal{P}_a$ is defined by (3), $p(0) \in \mathcal{P}_a$ and the Ω-limit set Ω_h of the h-closed-loop is safe, *i.e.* $\Omega_h \subset \mathcal{R}_s$. Then there exist $\gamma_i > 0$, for all $i \in \{1,2,3\}$, and $b_2 > b_1 > 0$ such that the s-closed-loop system has the following properties.

(1) $p(t)$ stays in $\mathcal{P}_a$ for all $t \geq 0$.
(2) $\Omega_s = \Omega_h$.
(3) For all $t \geq 0$ such that $(p(t),\theta(t)) \in \mathcal{R}_s(v_h(t)) \setminus \mathcal{R}_d(v_h(t))$, $u_s(t) = u_h(t)$.

Remark 1: If $\Omega_h \not\subset \mathcal{R}_s$, then claim (2) in Theorem 1 should be modified as: $\Omega_s = \Pi_{\mathcal{R}_s}(\Omega_h)$.

IV. NUMERICAL EXAMPLES

In this section we discuss two case studies: in the first case $\mathcal{P}_a$ is convex and in the second case $\mathcal{P}_a$ is non-convex. Note that the state of the h-closed-loop system goes outside of $\mathcal{P}_a$ in both cases, *i.e.* $\Omega_h \not\subset \mathcal{R}_s$.

A. Convex $\mathcal{P}_a$

Consider the kinematic model (1) and the admissible region $\mathcal{P}_a$ defined by

$$\mathcal{P}_a = \{(x,y) | x \geq 2, \; y \geq 2\}. \quad (15)$$

Let the human operator generate a random (x,y) trajectory, the red, dashed-and-dotted, curve in Figure 2. The corresponding (x,y) trajectory of the s-closed-loop system is displayed by the green, dashed, curve in Figure 2. Figure 3 shows how the inputs and states of the h-closed-loop and s-closed-loop systems vary with time. $v_s(t) \neq v_h(t)$ and $\omega_s(t) \neq \omega_h(t)$ when $t \in (25, 28)$ and $t \in (34, 41)$ implies that the f-control is active and $k = 0$. With the shared-control, the robot moves along the boundary of $\mathcal{P}_a$ until the reference trajectory (*i.e.* the trajectory of the h-closed-loop) returns to the admissible set.

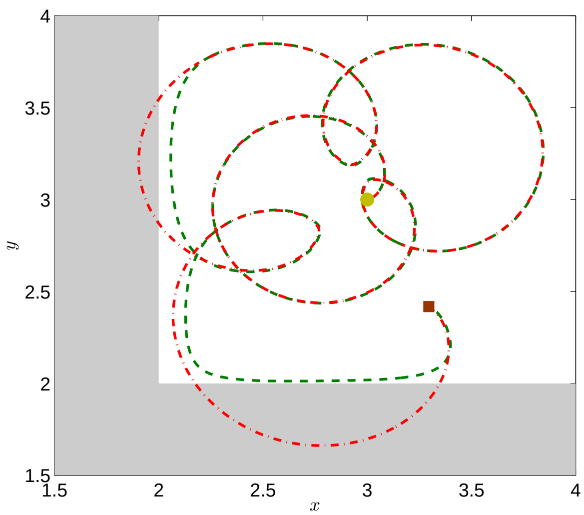

Fig. 2. (x, y) trajectories of the system (1) for the set $\mathcal{P}_a$ given in (15): h-closed-loop (red, dashed-and-dotted) and s-closed-loop (green, dashed). Round mark: the initial position of the robot. Square mark: the final position of the robot.

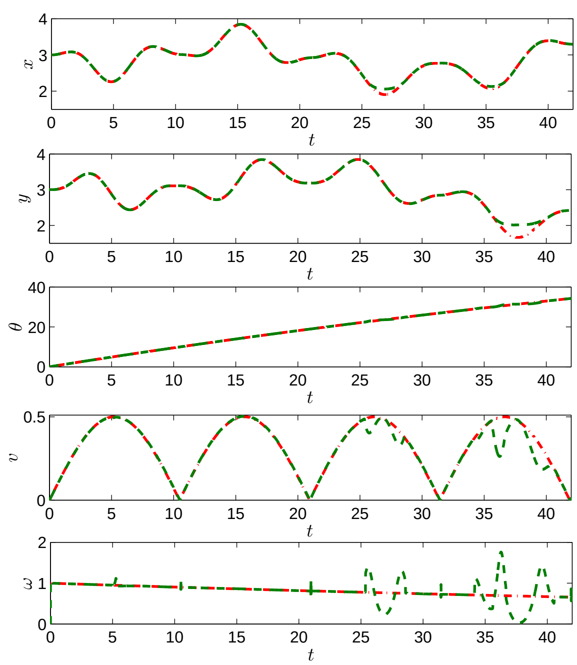

Fig. 3. Time histories of the variable x, y, θ, v and ω for the system (1) with the set $\mathcal{P}_a$ given by (15): h-closed-loop system (red, dashed-and-dotted) and s-closed-loop system (green, dashed).

B. Non-convex $\mathcal{P}_a$

As stated in [18], the shared-control algorithm can also be applied to non-convex admissible configuration sets defined by a group of linear inequalities complemented with logic conditions. To illustrate this scenario consider the system (1) and the non-convex region

$$\mathcal{P}_a = \left\{ (x,y) \;\middle|\; \begin{array}{c} 0 \leq x \leq 6,\ 0 \leq y \leq 3, \\ \text{and } y \in [0,1] \cup [2,3] \text{ if } x \in [2,4], \\ \text{and } x \in [0,2] \cup [4,6] \text{ if } y \in [1,2] \end{array} \right\}. \tag{16}$$

Assume the desired trajectory is a straight line described by

$$p_d(t) = [0.1t,\ 0.05t]^T.$$

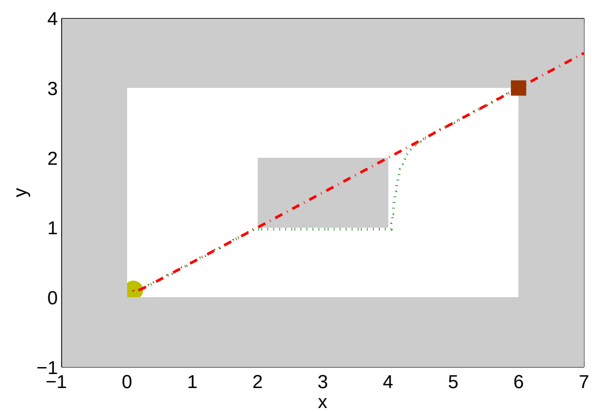

Fig. 4. (x, y) trajectories of the system (1) for the set $\mathcal{P}_a$ given in (16): h-closed-loop (red, dashed-and-dotted) and s-closed-loop (green, dashed). Round mark: the initial position of the robot. Square mark: the final position of the robot with the shared-control.

Figure 4 shows that the (x, y) trajectory of the system without shared-control goes through the non-admissible region (the grey shaded area in the center), while the (x, y) trajectory of the s-closed-loop system moves around it and goes along the boundary of $\mathcal{P}_a$ until the configuration of the h-closed-loop system enters $\mathcal{P}_a$ again. After 60s, the (x, y) trajectory of the h-closed-loop leaves $\mathcal{P}_a$ as the red, dashed-and-dotted, line in Figure 4 indicates, while the robot with the shared-control stops at the corner (the boundary) of the admissible region.

V. CONCLUSIONS

We have developed a solution to the shared-control problem for the kinematic model of a mobile robot. A hysteresis-based switch is used to unite the human input and the feedback control input based on the definitions of the sets $\mathcal{R}_s(v_h)$, $\mathcal{R}_h(v_h)$ and $\mathcal{R}_d(v_h)$. Even though the shared-control theory is designed for convex admissible configuration sets, it can also be applied to non-convex sets, as illustrated in Section IV-B. Two simple examples are given in Section IV to show the effectiveness of the shared-control.

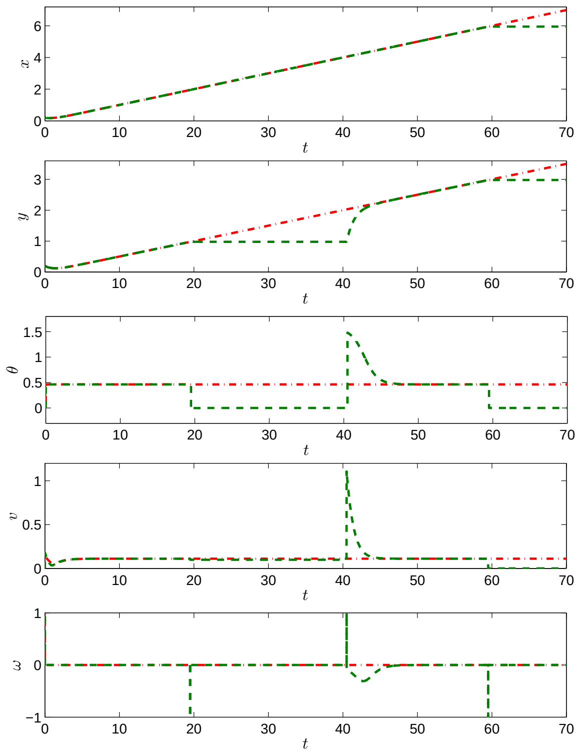

Fig. 5. Time histories of the variable x, y, θ, v and ω for the system (1) with the set $\mathcal{P}_a$ given by (16): h-closed-loop system (red, dashed-and-dotted) and s-closed-loop system (green, dashed).

Future research will focus on four-wheel car-like systems and trailer systems.

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