A D.C. MOTOR MODEL INCLUDING ARMATURE REACTION

by

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CONTENTS

SUMMARY

In a d.c. machine the armature reaction reduces the total flux per pole because of saturation effects at heavy armature currents. In this project the separately excited d.c. motor is modelled by considering the total flux per pole as a function of armature and field currents. The machine model is linearized so that it is valid only for small changes in the neighbourhood of steady state conditions.

The machine model when represented in the matrix form is a set of first order differential equations in which changes in the armature voltage, field voltage and load torque are the inputs, whereas changes in the armature current, field current and motor speed represent the response of the machine.

The parameters of the machine measured are for an 800 watt Westinghouse generalised machine connected as a separately excited d.c. motor. A mechanically coupled d.c. machine was used as a d.c. generator to load the test machine. Experiments were done for the transient and steady state responses of change in armature current and speed.

This machine model is then simulated on a digital computer to calculate the time response of change in armature current and speed by applying a change in field voltage and armature voltage.

The calculated response of the machine model is compared with the experimentally obtained responses of the machine.

The two sets of results are found to be similar.

LIST OF MAJOR SYMBOLS

VA	Supply voltage
VF	Field voltage
ΔVA	Change in armature voltage
ΔVF	Change in field voltage
IA	Steady state armature current
IF	Steady state field current
ΔIA	Change in armature current
ΔIF	Change in field current
RA	Armature resistance
RA1	Brush contact resistance
RF	Field resistance
Te	Electrical torque developed by the machine
Tℓ	Load torque
T_{loss}	Lost torque
ΔTℓ	Change in load torque
ΔTe	Change in electrical torque
N	Speed of motor in revs/min
ΔN	Change in speed of motor in rev/min
J	Moment of inertia
D1	Constant of friction
Φ	Flux per pole
ψ_A	Total flux linkage with armature winding
ψ_F	Total flux linkage with field winding
P	Number of poles
Z	Total number of conductors
A	Number of parallel paths

E	Back emf
Ke	Back emf constant
Kt	Torque constant
Kd	Distribution factor
Kp	Pitch factor
KΦ	Leakage factor
KW	Kd.Kp winding factor
La	Self inductance of the armature winding
Lf	Self inductance of the field winding
D(ΔψA)	Derivative of the change in armature flux linkage
D(ΔψF)	Derivative of the change in field flux linkage

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CHAPTER I

INTRODUCTION

The d.c. motor has high capital and maintenance costs because it has a copper commutator. On the other hand the induction motor is cheap, robust and requires little maintenance.[1] In addition the frame size of an induction motor is smaller than that of a d.c. machine for a given rating. Therefore even though the d.c. motor has superior speed torque characteristics, the induction motor replaced d.c. motors especially in the situations where sparking may cause explosion.[3,5]

With the advent of thyristors the need for a separate rotating machine power unit and, consequently, the need for large space is reduced and overall economy of machine running is increased because the power unit for a.c./d.c. conversion becomes much smaller in size, with reduced cost of maintenance.[5] Also the reliability is increased if electronic components such as diodes and transistors are used in the control circuit because they occupy less space and they are less costly compared to motor generator sets.[7,8] Also time and the cost of maintenance is reduced. These economies are further enhanced by integrated circuits which are rapidly and increasingly replacing the conventional electronic components.[9,10]

Researches in commutator-less d.c. motors using thyristors for commutation instead of the commutator and brush set arrangement, is leading to still reduced cost, size of the d.c. motor, easy maintenance and increased reliability.

By using digital methods for the control of speed of a d.c. motor, the accuracy between desired speed and obtained speed could be achieved up to 0.1% at the lower end of the range and 0.02% for the majority of the range. Accuracy up to .002% is claimed. Also the digital method of control of speed eliminates the drawbacks of the analogue control system, like ageing of components and change in characteristics due to temperature variation. The signal can also be transmitted over a long distance without distortion, if remote control is required.[4,12,11]

A thyristor converter, when combined with a separately excited d.c. motor, gives a drive system which is superior in many ways. For example speed could be controlled by an armature voltage control and field voltage control, and better speed control provides high production rates, better production quality and increased process automation.[9] Furthermore, for a separately excited d.c. motor, the speed torque characteristic could either be constant speed varying torque or varying speed constant torque, or both could vary independently.[9] When field weakening is used with constant armature voltage, constant horsepower output is obtained with decreasing torque and increasing speed. A separately excited d.c. motor, when supplied by a thyristor rectifier, is capable of running in all the four quadrants and therefore regenerative braking is possible.[8,5]

The weakening of the field near the full load condition increases the speed such that it becomes uncontrollable and the motor becomes unstable. It could be controlled by using a suitable control system. For developing a control system it is necessary to predict the steady state and transient responses of the machine accurately and therefore modelling of the machine is a fundamental step towards designing a control system.[2]

By modelling any machine mathematically its response could be predicted for any input condition like field or armature voltage. This prediction of machine response is useful for the control engineer.[13] It is therefore necessary that the model should be as near a representation of the actual machine as possible. The closeness of the machine model with the actual machine can be judged by comparing their responses under the same input conditions. Also the usefulness of any model over another may be judged according to ease with which its parameters could be measured.

Modelling may be done in a number of ways, for example, by obtaining the transfer function[14,21] of the machine or writing the machine equations for armature and field voltages[15,21] and load torque. It may be modelled by changing the input to the machine around the steady state conditions.[16] Their time response could be found either by Laplace transform or solving the simultaneous differential equations or using a numerical method. But all these models ignore the effect of armature reaction. The effect of armature reaction is the weakening of the total flux per pole due to the saturation of iron of the stator.[17] Therefore they do not take into account the changing of parameters due to change in armature current. Sinha[18] has considered the armature reaction as a voltage added or subtracted from the back e.m.f. and developed ways of calculating it. The armature reaction term needs to be calculated by experiment[2].

The object of the thesis is to model a separately excited d.c. motor taking armature reaction into account. This is done by taking flux per pole as a function of armature and field current. The machine model is linearized so that it is valid only for small changes in the neighbourhood of steady state conditions. The gradients or partial derivatives, that is change in flux per pole with change in armature current and change in flux pole with change in field current, are the main parameters of the machine. These new parameters are termed G1 and G2 respectively.

The parameters of the machine are determined on a Westinghouse generalised machine connected as a separately excited d.c. motor. Another d.c. machine coupled to the generalised machine was used as a separately excited d.c. generator. The experiments for parameter measurement are described in Chapter 4.

A computer program is developed for the transient response of the machine model around steady state conditions. This program uses a subprogram already developed by my supervisor to integrate a set of three first-order differential equations using a numerical method.

The Gino library subroutines are used to plot the transient response. Tests are then performed to obtain the time response of the machine and recorded on a u.v. recorder. Results obtained from computer simulation and from the machine are then compared.

CHAPTER 2

ARMATURE REACTION

2.1 General

In this chapter, the effect of small changes in either the field voltage or the armature voltage on armature reaction and the total change in the flux linkage with field and armature windings will be discussed. Initially, the m.m.f's produced by the armature winding and the field winding are considered separately. Then their combined effect is considered. It is shown that the resultant m.m.f. varies with a change in either of the inputs. Then assuming flux per pole proportional to the m.m.f., the change in flux per pole with change in input is considered. Finally the effects of saturation are discussed. In this case the flux produced is not proportional to the m.m.f. and the effect of a change in an input on the change in flux per pole in the air gap is explained.

2.2 M.M.F. Due to Armature Current Alone

If the field of the machine is not energised and a current is passed through the armature, the ampere turns of the armature will produce an m.m.f. wave with a shape, as shown in Fig. 2.1[19,20] The steps in the waveform are caused by the armature slots. However it will be assumed in the remainder of the thesis that the effect of the slotting can be neglected and in this case the m.m.f. waveform becomes triangular.

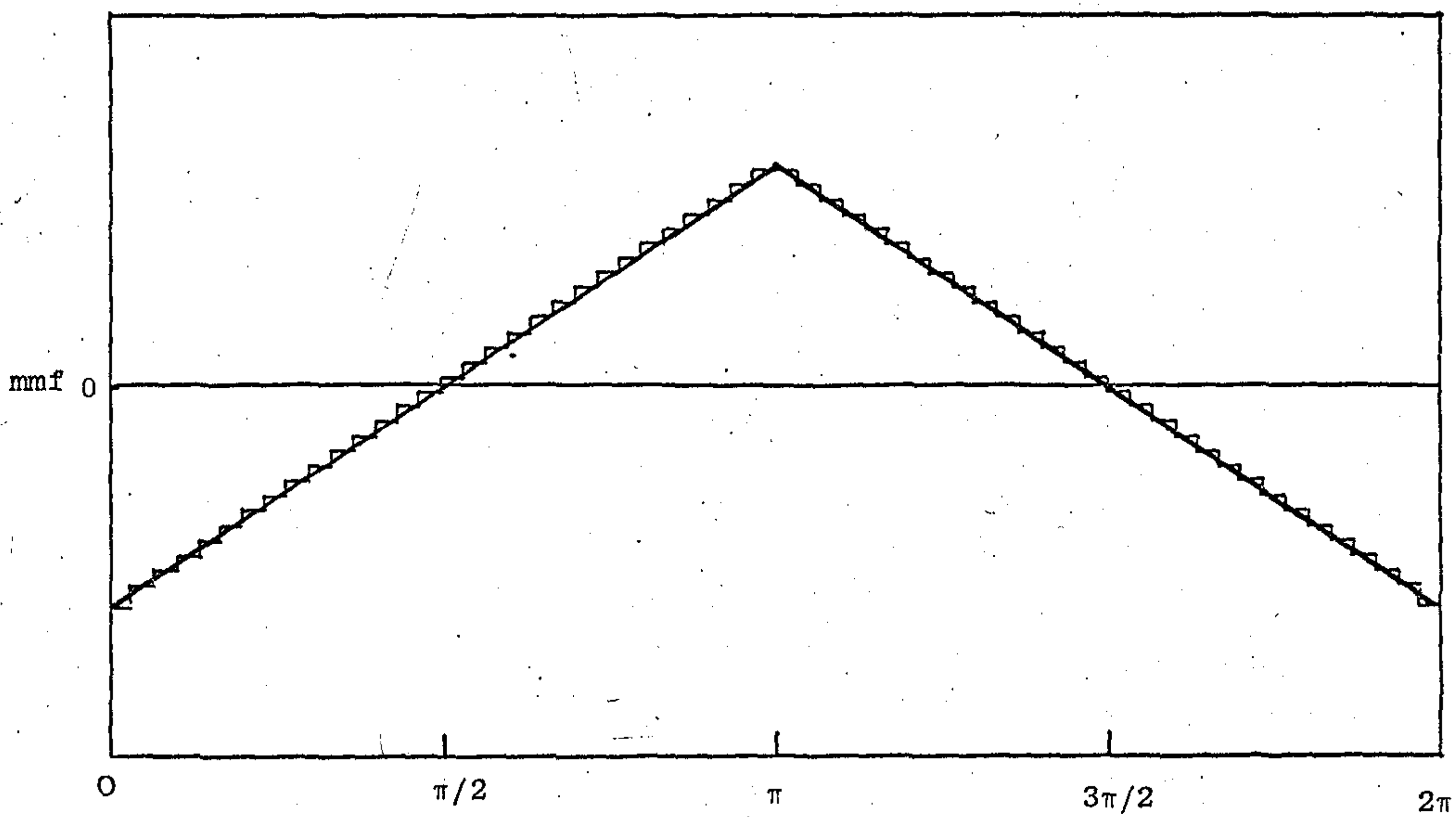


FIG. 2.1 Armature mmf alone

This is the approximation of the actual m.m.f. shape. The armature m.m.f. is zero at the centre of the pole and a maximum at the magnetic neutral axis. On either side of the centre of the pole or magnetic axis, the shape of the m.m.f. curve is the same, but its sign is reversed.[20,25] The magnitude of this m.m.f. at every point is proportional to the current passing through the coil. The net area of the armature m.m.f. curve is zero.

2.3 M.M.F. Due to Field Current Alone

Usually the d.c. machine has one or more pairs of salient poles. The winding is placed on the pole pieces. The shape of the m.m.f. produced depends upon the shape of the pole face and the magnitude of the m.m.f. depends upon coil current.

The machine on which the tests are performed does not have salient poles. The stator field winding is embedded in slots in the cylindrical surface. The inner periphery of the stator is divided into four quadrants, each quadrant having a similar winding layout. The m.m.f. shape is again stepped one, and shown in Fig. 2.2. The shape of the field m.m.f. is the same as shown in Fig. 2.2 but the magnitude is dependent upon the amount of current passing through the field winding.

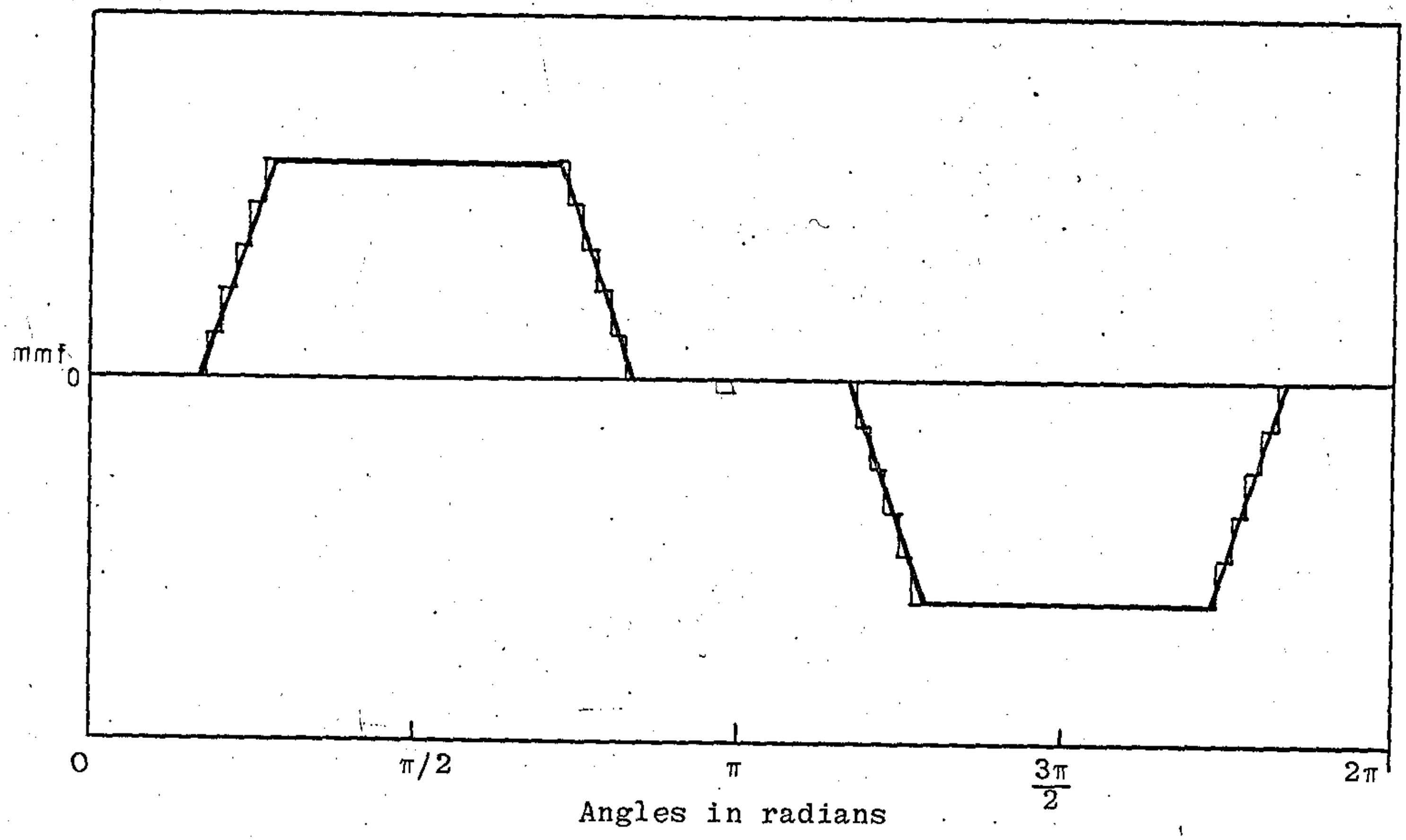


FIG. 2.2 Field mmf alone

2.4 Resultant M.M.F. Shape Due to Field Current and Armature Current

In the running condition both the armature and the field m.m.f. are acting simultaneously. If the resultant of the two m.m.f.'s is drawn then on on side of the magnetic axis, the m.m.f. will be additive, while on the other side of the axis it will be subtractive. The final result is that the m.m.f. is strengthened on one side and weakened on the other sideof the magnetic axis. Thus the resultant m.m.f. is distorted in shape due to the presence of armature m.m.f. and also the magnetic neutral axis is shifted from the original no load neutral axis. The resultant m.m.f. wave is shown in Fig. 2.3.[20,24,25]

2.5 Effect of a Change in Input on the Resultant M.M.F.

Suppose a change is made in the armature voltage. This change will cause a change in armature current and the armature m.m.f. will change. After this change, m.m.f. is added on one side of the magnetic axis and subtracted on the other side, and there will be a change in the resultant m.m.f. curve. The shape of the resultant will distort but the total area of the m.m.f. will be zero. Similarly the shape of the resultant m.m.f. will be affected if the change is made in the field voltage, which will cause a change in field current and field m.m.f.

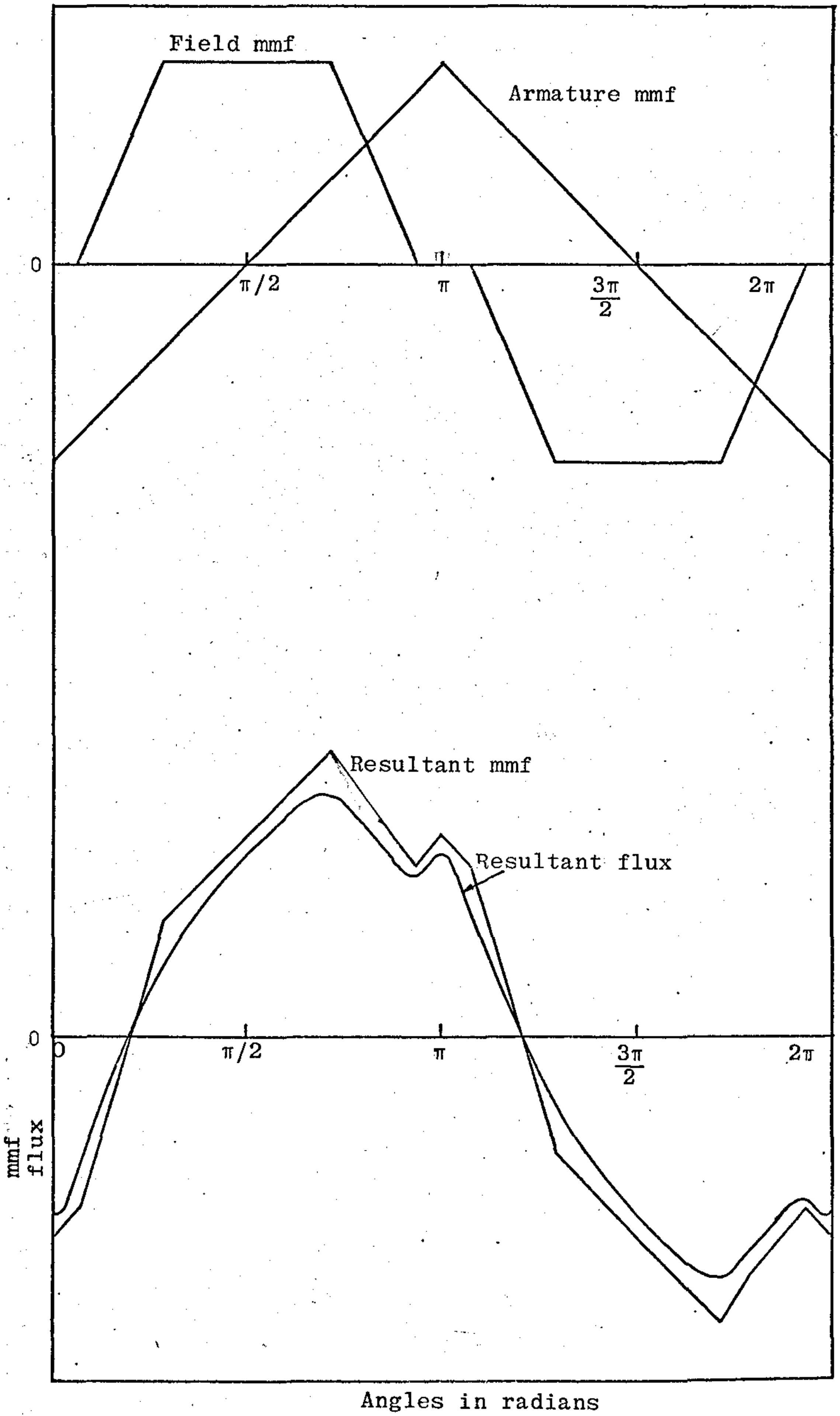


FIG. 2.3 Resultant mmf and flux

2.6 The effect of Change of Input on the Total Flux per Pole

The flux per pole is required rather than the m.m.f. It would have been simpler if this m.m.f. was directly proportional to the flux produced by it, but after a certain point (in BH curve) the flux density and hence the flux produced becomes a non-linear function of m.m.f.[23,25,27]

Firstly it is assumed that flux per pole is proportional to the resultant m.m.f., i.e. the iron is unsaturated for all m.m.f. values.

So long as the iron is unsaturated, the m.m.f. required to overcome the reluctance of the iron parts of the magnetic circuit is small compared with the air gap m.m.f. But in the case of saturation of iron, the m.m.f. drop can no longer be neglected in the iron parts of the magnetic circuit. The reluctance of iron parts increases and becomes comparable with the reluctance of the air gap. In the case of saturation the flux produced by the same m.m.f. is less than the flux produced when there is no saturation.[17,20,23,25]

In the case of armature current near the full load condition, the resultant m.m.f. has a very high value at one side of the magnetic axis. This high value of the m.m.f. saturates the iron parts which results in more m.m.f. drop in the iron parts.[17,20] This drop is comparable to the drop in the air gap. The flux produced can be given by the relation:

$$\Phi = \frac{\text{m.m.f.}}{(\text{Reluctance of the air gap} + \text{Reluctance of the iron})} \tag{2.6.1}$$

Due to saturation the reluctance of the iron parts becomes comparable to the reluctance of the air gap, and as the degree of saturation is increased, the reluctance of the iron increases. Equation 2.6.1 represents the situation when the iron is saturated and, due to saturation, the properties of iron are changed. The reluctance of the iron, which was negligible compared to the air gap reluctance when there was no saturation, now becomes comparable to the air gap reluctance. The effect of saturation is such as to reduce the flux produced.[17,19,22,23,25]

If a change is made in the armature input, there will be a change in the armature m.m.f. Suppose the change in the armature m.m.f. is increasing, this change will be additive to the field m.m.f. on one side of the magnetic axis. This increase in m.m.f. will change the properties of iron so as to increase the reluctance of the iron. Although there will be an increase in the flux, the increase in flux will be less than it would have been in the absence of saturation.

On the other side of the magnetic axis, this increase in armature m.m.f. is subtracted from the field m.m.f. and therefore the resultant m.m.f. will be less than what it was before the change in the armature input, and this reduction in armature m.m.f. will take the iron out of saturation. The overall effect of an increase in the armature input is that there is a net reduction of flux per pole.

Along the same line of argument, the effect of a change in the field m.m.f. on the total flux per pole can be discussed.

CHAPTER 3

MACHINE MODEL

3.1 General

In representing a separately excited d.c. motor by a mathematical model, there can be three independent input variables.

i) Field voltage, which is applied to energize the field winding.

ii) Armature voltage which is applied to energize the armature winding.

iii) Torque input.

In this model of a separately excited d.c. motor the starting point is the steady state equation of these input variables. Then these inputs are changed by small amounts and the expressions for changes are derived. This model for change in the input variable is then arranged as a set of first-order differential equations.

3.2 Steady State Equations for Separately Excited d.c. Motor

The steady state equations for separately excited d.c. motors may be written as:

$$VA = (RA + RA1).IA + E$$

$$VF = RF\ .IF$$

$$Te = T_{\ell} + T_{Loss}$$

$$Te = K_t\ \Phi\ IA$$

3.3 Equations for Small Changes

When inputs are changed by small amounts from their steady state values, they will produce changes in field flux linkage and armature flux linkage, which will initiate a transient response. These changes of the input will result in changes of the outputs which are armature current, field current and speed.

The total flux per pole can be represented as:

$$\Phi = \Phi\ (IA, IF)$$

and the change in total flux per pole is given by:

$$\Delta\Phi = \left(\frac{\partial\Phi}{\partial IA}\right)_{IF} . \Delta IA + \left(\frac{\partial\Phi}{\partial IF}\right)_{IA} . \Delta IF \qquad (3.3.1)$$

$\left(\frac{\partial\Phi}{\partial IA}\right)_{IF}$ is the change in flux per pole with change in armature current, when IF is a constant.

$\left(\frac{\partial\Phi}{\partial IF}\right)_{IA}$ is the change in flux per pole with change in field current when IA is a constant.

If ψ is the total flux linkage:

$$\Delta\Psi F = \Delta\psi F\ (IA, IF) \qquad (3.3.2)$$

$$\Delta\psi A = \Delta\psi A\ (IA, IF) \qquad (3.3.3)$$

where:

$\Delta\psi F$ = change in total field flux linkage

$\Delta\psi A$ = change in total armature flux linkage.

Let d/dt = D.

Taking the partial derivatives of 3.3.2 and 3.3.3 with respect to time:

$$D(\Delta\psi F) = \left(\frac{\partial\psi F}{\partial IA}\right)_{IF} \cdot D(\Delta IA) + \left(\frac{\partial\psi F}{\partial IF}\right)_{IA} \cdot D(\Delta IF) \quad (3.3.4)$$

$$D(\Delta\psi A) = \left(\frac{\partial\psi A}{\partial IA}\right)_{IF} \cdot D(\Delta IA) + \left(\frac{\partial\psi A}{\partial IF}\right)_{IA} \cdot D(\Delta IF) \quad (3.3.5)$$

where:

$\left(\frac{\partial\psi F}{\partial IF}\right)_{IA}$ = change in total field flux linkage with change in field current, keeping armature current constant.

$\left(\frac{\partial\psi F}{\partial IA}\right)_{IF}$ = change in total field flux linkage with change in armature current, keeping field current constant.

$\left(\frac{\partial\psi A}{\partial IF}\right)_{IA}$ = change in total armature flux linkage with change in field current, keeping armature current constant.

$\left(\frac{\partial\psi A}{\partial IA}\right)_{IF}$ = change in total armature flux linkage with change in armature current, keeping field current constant.

Equation for change in armature input

For a steady state condition, the armature circuit equation may be written as:

$$VA = (RA + RA1)\ IA + E \quad (3.3.6)$$

If the input voltage is changed by a small amount ΔVA the result is:

$$VA + \Delta VA = (RA + RA1)(IA + \Delta IA) + D(\Delta\psi A) + E + \Delta E \tag{3.3.7}$$

$D(\Delta\psi A)$ is due to the total inductance of the armature winding

Subtracting (3.3.6) from (3.3.7)

$$\Delta VA = (RA + RA1)\ \Delta IA + D(\Delta\psi A) + \Delta E \tag{3.3.8}$$

The back e.m.f. is represented by:

$$E = K_e\ \Phi N$$

where K_e = ZP/60.A is the back e.m.f. constant

$$\Delta E = K_e . N\Delta\Phi + K_e .\ \Phi\Delta N$$

$$= Ke.N.(\frac{\partial\Phi}{\partial IA}).\Delta IA + Ke.N(\frac{\partial\Phi}{\partial IF}).\Delta IF + Ke.\Phi.\Delta N \tag{3.3.9}$$

Let RT = RA + RA1

Substituting equations (3.3.9) and (3.3.5) into (3.3.8):

$$\Delta VA = RT.\Delta IA + (\frac{\partial\psi A}{\partial IA})_{IF}.D(\Delta IA) + (\frac{\partial\psi A}{\partial IF})_{IA}.D(\Delta IF) + K_e N(\frac{\partial\Phi}{\partial IA})\Delta IA$$

$$+ K_e.N.(\frac{\partial\Phi}{\partial IF}).\Delta IF + K_e.\Phi.\Delta N$$

Rearranging

$$\Delta VA = [RT+K_e.N(\frac{\partial\Phi}{\partial IA})]\Delta IA+K_e.N(\frac{\partial\Phi}{\partial IF})_{IA}.\Delta IF+K_e.\Phi\Delta N+(\frac{\partial\psi A}{\partial IA})_{IF}.D(\Delta IA)$$

$$+(\frac{\partial\psi A}{\partial IF})_{IA}.D(\Delta IF) \qquad (3.3.10)$$

The steady state equation relating the field voltage to the field current is:

$$VF = RF.IF \qquad (3.3.11)$$

If a change is made in the field voltage, the result is:

$$VF + \Delta VF = RF(\Delta IF + IF) + D(\Delta\psi F) \qquad (3.3.12)$$

$D(\Delta\psi F)$ is due to the total inductance of the field winding

Subtracting (3.3.11) from (3.3.12) and substituting (3.3.4):

$$\Delta VF = RF.\Delta IF + (\frac{\partial\psi F}{\partial IA})_{IF} D(\Delta IA) + (\frac{\partial\psi F}{\partial IF})_{IA} D(\Delta IF) \qquad (3.3.13)$$

<u>Torque equation</u>

The torque developed by a motor is given by:

$$T_e = K_t \Phi IA$$

where $K_t = ZP/2\pi A$ is torque constant

If the torque is changed by small amounts in the neighbourhood of a steady state:

$$T_e + \Delta T_e = K_t\ (\Phi + \Delta\Phi)(IA + \Delta IA)$$

Neglecting the second order terms:

$$\Delta T_e = K_t.\Phi.\Delta IA + K_t\ \Delta\Phi.IA \qquad (3.3.14)$$

Steady state
electrical torque can also be represented by:

$$T_e = \text{load torque} + \text{viscous torque}$$

$$= T_\ell + 2\pi ND1$$

If the torque is changed by a small amount in the neighbourhood of the steady state value:

$$T_e + \Delta T_e = T_\ell + \Delta T_\ell + (2\pi J/60).D\Delta N + (2\pi D1/60).(N + \Delta N)$$

$$\Delta T_e = \Delta T_\ell + (2\pi J/60).D\Delta N + (2\pi D1/60).\Delta N \qquad (3.3.15)$$

Equating (3.3.14) and (3.3.15) and rearranging

$$\Delta T_\ell = K_t.\Phi\Delta IA + K_t.\Delta\Phi.IA - (2\pi J/60).D\Delta N - (2\pi D1/60).\Delta N$$

Substituting for $\Delta\Phi$ from equation (3.3.1)

$$\Delta T_\ell = K_t\ (\Phi + \frac{\partial\Phi}{\partial IA}.IA)\Delta IA + K_t\ (\frac{\partial\Phi}{\partial IF}).IA.\Delta IF - (2\pi J/60).D\Delta N$$

$$- (2\pi D1/60).\Delta N \qquad (3.3.16)$$

Combining equations (3.3.10), (3.3.13) and (3.3.16) together a set of first order linear differential equations is obtained, representing the separately excited d.c. motor:

$$\Delta VA= [RT+K_e.N(\frac{\partial\Phi}{\partial IA})]\ \Delta IA+[K_e N.\ \frac{\partial\Phi}{\partial IF}]\Delta IF+K_e\Phi\Delta N+(\frac{\partial\psi A}{\partial IA})_{IF}.D(\Delta IA)$$

$$+\ (\frac{\partial\psi A}{\partial IF})_{IA}.\ D(\Delta IF)$$

$$\Delta VF=RF.\Delta IF+(\frac{\partial\psi F}{\partial IA})_{IF}.D\Delta IA+(\frac{\partial\psi F}{\partial IF})_{IA}.D\Delta IF$$

$$\Delta T_\ell\ = K_t[\ \Phi\ +(\frac{\partial\Phi}{\partial IA}).IA]\ \Delta IA\ +\ K_t.IA(\frac{\partial\Phi}{\partial IF}).\Delta IF-(2\pi D/60).\Delta N$$

$$-(2\pi J/60).\ D\Delta N$$

For simplicity let

$$\frac{\partial\Phi}{\partial IA} = G1,\quad \frac{\partial\Phi}{\partial IF} = G2,\ (\frac{\partial\psi A}{\partial IA})_{IF} = F1,\quad (\frac{\partial\psi A}{\partial IF})_{IA} = F2,$$

$$\frac{\partial\psi F}{\partial IA})_{IF} = H1,\quad (\frac{\partial\psi F}{\partial IF})_{IA} = H2.$$

$R\ \ \ = RT + K_e.N.G1$

$RP\ = K_e.N.G2$

$RTA = K_t\ (G1\ IA + \Phi)$

$RTF = K_t.IA.G2$

The simplified model in terms of new constants is:

$$\begin{bmatrix} \Delta VA \\ \Delta VF \\ \Delta T_\ell \end{bmatrix} = \begin{bmatrix} R & RP & K_e\Phi \\ O & RF & O \\ RTA & RTF & -\frac{2\pi D1}{60} \end{bmatrix} \begin{bmatrix} \Delta IA \\ \Delta IF \\ \Delta N \end{bmatrix} + \begin{bmatrix} F1 & F2 & O \\ H1 & H2 & O \\ O & O & -\frac{2\pi J}{60} \end{bmatrix} \begin{bmatrix} D\Delta IA \\ D\Delta IF \\ D\Delta N \end{bmatrix} \quad (3.3.17)$$

Let:

$$Y = \begin{bmatrix} \Delta VA \\ \Delta VF \\ \Delta T_\ell \end{bmatrix}, \quad X = \begin{bmatrix} \Delta IA \\ \Delta IF \\ \Delta N \end{bmatrix}, \quad A = \begin{bmatrix} R & RP & K_e\Phi \\ O & RF & O \\ RTA & RTF & -\frac{2\pi D1}{60} \end{bmatrix}$$

and $C = \begin{bmatrix} F1 & F2 & O \\ H1 & H2 & O \\ O & O & -\frac{2\pi J}{60} \end{bmatrix}$

Equation (3.3.17) can now be represented by:

$$Y = AX + C\,\frac{dX}{dt} \quad (3.3.18)$$

multiplying equation (3.3.18) by C^{-1} and rearranging

$$\frac{dX}{dt} = -\,C^{-1}.A.X + C^{-1}Y \quad (3.3.19)$$

where: X is the state vector of the system

Y is the control vector of the system.

$$|C| = \frac{2\pi J}{60} \; [F2.H1 - H2.F1]$$

$$= \frac{-2\pi J}{60} \; EL$$

where: EL = H2.F1 - F2.H1

$$C^{-1} = \begin{bmatrix} H2/EL & -F2/EL & 0 \\ -H1/EL & F1/EL & 0 \\ 0 & 0 & \frac{-60}{2\pi J} \end{bmatrix}$$

Substituting C^{-1} in equation (3.3.19)

$$\begin{bmatrix} D\Delta IA \\ D\Delta IF \\ D\Delta N \end{bmatrix} = -\begin{bmatrix} H2/EL & -F2/EL & 0 \\ -H1/EL & F1/EL & 0 \\ 0 & 0 & \frac{-60}{2\pi J} \end{bmatrix} \begin{bmatrix} R & RP & Ke\Phi \\ 0 & RF & 0 \\ RTA & RTF & \frac{-2\pi Df}{60} \end{bmatrix} \begin{bmatrix} \Delta IA \\ \Delta IF \\ \Delta N \end{bmatrix}$$

$$+ \begin{bmatrix} H2/EL & -F2/EL & 0 \\ -H1/EL & F1/EL & 0 \\ 0 & 0 & \frac{-60}{2\pi J} \end{bmatrix} \begin{bmatrix} \Delta VA \\ \Delta VF \\ \Delta T_{\ell} \end{bmatrix}$$

Simplifying:

$$\begin{bmatrix} D\Delta IA \\ D\Delta IF \\ D\Delta N \end{bmatrix} = \begin{bmatrix} -R.H2/EL & -RP.H2/EL+RF\ .F2/EL & -K_e\Phi H2/EL \\ R.H1/EL & -RF\ .F1/EL+RP.H1/EL & K_e\Phi H1/EL \\ \frac{RTA}{2\pi J}.60 & \frac{RTF}{2\pi J}.60 & -D/J \end{bmatrix} \begin{bmatrix} \Delta IA \\ \Delta IF \\ \Delta N \end{bmatrix} + \begin{bmatrix} H2/EL & -F2/EL & 0 \\ -H1/EL & F1/EL & 0 \\ 0 & 0 & \frac{-60}{2\pi J} \end{bmatrix} \begin{bmatrix} \Delta VA \\ \Delta VF \\ \Delta T_\ell \end{bmatrix}$$

(3.3.20)

Equation (3.3.20)[28] is a set of first-order differential equations representing a separately excited d.c. motor. Integration of this set of equations will give the time response of the machine.

3.4 Modification in the Machine Model

Change in armature input and field input is made by varying the series resistance in the armature circuit and field circuit. Equations in terms of input variable are written taking series resistance into account. This series resistance is then changed by connecting another resistance in parallel and another equation is then written representing the changed condition. The remainder of the analysis is then carried out along the same lines as done previously.

Armature Circuit

If R2 is a series resistance:

$$VA = (RA + RA1)\ IA + IA.R2 + E \qquad (3.4.1)$$

Another resistance R1 is switched in parallel with R2:

$$VA = (RA+RA1)(IA+\Delta IA) + (\frac{R2.R1}{R1+R2})(IA+\Delta IA) + E+\Delta E + D(\Delta\psi A) \tag{3.4.2}$$

Subtracting (3.4.1) from (3.4.2):

$$O = (RA+RA1)\Delta IA+\Delta E+D(\Delta\psi A) + (\frac{R1.R2}{R1+R2})\Delta IA+IA(\frac{R1.R2}{R1+R2} - R2)$$

$$\frac{R2^2}{(R1+R2)} IA = (RA+RA1+ \frac{R1.R2}{R1+R2})\Delta IA+\Delta E + D(\Delta\psi A) \tag{3.4.3}$$

Rest of the analysis is carried out as in Section 3.2.

<u>Field circuit:</u>

$$VF = IF.RF + IF.R3 \tag{3.4.4}$$

If another resistance R4 is switched in parallel with R3:

$$VF = (IF + \Delta IF)RF + \frac{R3.R4}{(R3+R4)} (IF + \Delta IF) + D(\Delta\psi F) \tag{3.4.5}$$

Subtracting equation (3.4.4) from (3.4.5)

$$O = RF.\Delta IF + R3.R4/(R3+R4).\Delta IF+((R3.R4/(R3+R4))-R3)IF + D(\Delta\psi F)$$

$$(R3^2/(R3+R4)).IF=(RF + R3.R4/(R3+R4))\Delta IF + D(\Delta\psi F)$$

The remainder of the analysis is carried out as in Section 3.3. No change in load torque is made and the equation for change in load torque remains the same as in Section 3.3.

Let:

$$R = RA+RA1 + R1.R2/(R1+R2)+K_e.N.G1$$

$$RFF = RF + \frac{R3.R4}{R3+R4}$$

The machine model expressed in terms of first order differential equations and arranged in matrix form is given below:

$$\begin{bmatrix} D\Delta IA \\ D\Delta IF \\ D\Delta N \end{bmatrix} = \begin{bmatrix} -R.H2/EL & -RP.H2/EL+RFF.F2/EL & -K_e\Phi H2/EL \\ RH1/EL & -RFF.F1/EL+RP.H1/EL & K_e\Phi H1/EL \\ \frac{RTA}{2\pi J}.60 & \frac{RTF}{2\pi J}.60 & -D/J \end{bmatrix} \begin{bmatrix} \Delta IA \\ \Delta IF \\ \Delta N \end{bmatrix}$$

$$+\begin{bmatrix} H2/EL & -F2/EL & 0 \\ -H1/EL & F1/EL & 0 \\ 0 & 0 & \frac{-60}{2\pi J} \end{bmatrix} \begin{bmatrix} (R2^2/R2+R1)IA \\ R3^2/(R3+R4).IF \\ \Delta T_\ell \end{bmatrix}$$

3.5 Modification Considering the Dynamic Loading

The load on the test motor is a separately excited mechanically coupled d.c. generator. The flux per pole of the generator is constant and hence the torque on the shaft of the generator is proportional to the armature current. Any change in the armature input voltage or the field input voltage of the motor brings about a change in the speed and hence changes the generator emf and current passing through a resistance connected as a load on the generator. It follows therefore that the load torque T on the motor changes.

If:

$\Phi 1$ is the flux per pole of the generator

Ke1 is the emf constant of the generator

Kt1 is the torque constant of the generator

RL is the load resistance

N is the speed in rev/min

RT is the brush contact resistance and the armature resistance

T_g is the generated electrical torque

E1 is the generated emf

KA is constant

then the emf E1 generated is given by:

$$E1 = (RT + RL)\, IA1$$

$$T_g = Kt1.\Phi 1.IA1$$

$$T_g = \frac{Kt1.\Phi 1.Ke1.\Phi 1.N/60}{(RT + RL)}$$

$$T_g = KA.N/60 \qquad (3.5.1)$$

$$\Delta T_g = KA.\Delta N/60 \qquad (3.5.2)$$

If: T_{gf} is the friction and windage torque of the generator

T_{mf} is the friction and windage torque of the motor

T_{gi} is the torque lost in the iron losses of the generator

T_{mi} is the torque lost in the iron losses of the motor.

the electrical torque developed by the motor is given by:

$$T_e = T_g + T_{gf} + T_{mf} + T_{gi} + T_{mi}$$

$$T_{mf} + T_{gf} = 2\pi ND1$$

$$T_g = T_e - \frac{2\pi N.D1}{60} - T_{gi} - T_{mi}$$

Let the power post in the iron of the generator be C1 watts and the power lost in the iron of the motor be C2.

$$T_g = Kt.\Phi.IA - \frac{2\pi N.D1}{60} - \frac{C1 + C2}{2\pi\frac{N}{60}} \qquad (3.5.3)$$

Substitute T_g in equation 3.5.1 in equation 3.5.3:

$$KA.\frac{N}{60} = Kt.\Phi.IA - \frac{2\pi N.D1}{60} - \frac{C1 + C2}{2\pi \frac{N}{60}}$$

$$KA = \frac{Kt.\Phi.IA}{N/60} - 2\pi D1 - \frac{(C1 + C2)}{2\pi(\frac{N}{60})^2} \qquad (3.5.4)$$

The load torque on the motor

$$T_\ell = T_g + T_{gi}$$

If the variation in the speed is small enough to make T_{gi} constant:

$$\Delta T_\ell = \Delta T_g$$

Substituting ΔT_ℓ for ΔT_g in equation 3.5.2

$$\Delta T_\ell = KA.\Delta N/60$$

Substituting ΔT_ℓ in the set of equations 3.4.6

the equations overleaf are obtained.

$$\begin{bmatrix} D\Delta IA \\ \\ D\Delta IF \\ \\ D\Delta N \end{bmatrix} = \begin{bmatrix} R.HZ/EL & -RP.HZ/EL+RFF.F2/EL-Ke.\Phi.HZ/EL \\ \\ R.H1/EL & -RFF.F1/EL+RP.H1/EL \;\; Ke.\Phi.H1/EL \\ \\ \frac{RTA}{2\pi J}.60 & \frac{RTF}{2\pi J}.60 \qquad -\frac{D1}{J} - \frac{KA}{2\pi J} \end{bmatrix} \begin{bmatrix} \Delta IA \\ \\ \Delta IF \\ \\ \Delta N \end{bmatrix}$$

$$+ \begin{bmatrix} H2/EL & -F2/EL & 0 \\ \\ -H1/EL & F1/EL & 0 \\ \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R2^2/(R2+R1).IA \\ \\ R3^2/(R3+R4).IF \\ \\ 0 \end{bmatrix}$$

(3.5.5)

CHAPTER 4

PARAMETER MEASUREMENTS

4.1 General

Generally parameters such as the resistances of the armature and field windings, self and mutual inductances, moment of inertia and constant of friction are supplied by the manufacturer of the machine and taken as such if measurements are not possible. But generally they are fixed quantities and not dependent either on field current or armature current. The brush drop of the machine is assumed to be constant at 2V and the brush contact resistance is not measured for different armature currents, but calculated by dividing the brush contact drop with armature current. The parameters of the motor model, which has been derived in the previous chapter, are functions of field and armature current, flux per pole and speed. In this chapter expressions are derived for two main parameters G1 and G2. Experiments are performed and characteristics are drawn to calculate G1 and G2.

The resistance of the armature and field winding increases with temperature. It is difficult to estimate the temperature of field and armature winding separately. Approximate temperatures can be found if it is assumed that the flow of heat is taking place from rotor to stator, and temperature of the field winding and armature winding is the same. The constant of friction is derived as a function of the square of the speed. An expression for power loss in friction and windage is derived. Tests are performed to obtain the characteristics relating power lost in friction and windage to the speed.

4.2 Expression for G1[28]

The expression for G1, that is change in flux per pole with change in armature current can be derived by differentiating flux per pole with respect to armature current IA.

The speed of the motor can be expressed as:

$$N = (NO + DN.IA/IFL) \qquad (4.2.1)$$

where NO is no load speed of the motor and DN is the difference of speed between no load and full load. Speed increases linearly from no load to full load, when field current and armature voltage are held constant.

$$VAA = VA - IA.RA1$$

where VAA is the voltage across armature winding.

Back e.m.f. E = VAA - IA.RA

$$\Phi = \frac{VAA - IA.RA}{K_e N.} \qquad (4.2.2)$$

$$\Phi = \frac{VAA - IA.RA}{K_e\ (NO + DN.IA/IFL)} \qquad (4.2.3)$$

Flux per pole Φ induces a back e.m.f. E in the armature winding whose resistance is RA. The reason for taking only IA.RA in the expression is that RA is the armature coil resistance, in which back e.m.f. is induced and RA1, the brush resistance, is considered as an external resistance. To calculate the change in flux per pole with change in armature current, differentiate equation (4.2.3) with respect to IA and simplifying

$$\left(\frac{\partial \Phi}{\partial IA}\right)_{IF} = G1 = \frac{-(NO + DN.IA/IFL).RA - (VAA-IA.RA)DN/IFL}{K_e\ (NO+DN.IA/IFL)^2} \tag{4.2.4}$$

4.3 Expression for G2[28]

Change in flux per pole with change in field current is calculated by utilising the fact that the speed of the motor is a function of armature and field currents.The speed of the motor N can therefore be differentiated with respect to field current.

$$N = NO + (IA/IFL).DN \tag{4.3.1}$$

Differentiating (4.3.1) with respect to IF

$$\left(\frac{\partial N}{\partial IF}\right)_{IA} = \left(\frac{\partial NO}{\partial IF}\right)_{IA} + \left(\frac{\partial DN}{\partial IF}\right)_{IA}.IA/IFL \tag{4.3.2}$$

Total flux per pole is given by:

$$\Phi = \frac{VAA - IA.RA}{K_e.N} \quad (4.3.3)$$

Differentiating (4.3.3) with respect to IF:

$$\left(\frac{\partial\Phi}{\partial IF}\right)_{IA} = G2 = \frac{(VAA - RA.IA)}{K_e}.\left(\frac{1}{-N^2}\left(\frac{\partial N}{\partial IF}\right)_{IA}\right) \quad (4.3.4)$$

Substituting (4.3.1) and (4.3.2) into (4.3.4):

$$G2 = \frac{-(VAA - RA.IA)}{K_e(NO+DN.\frac{IA}{IFL})^2}\left[\left(\frac{\partial NO}{\partial IF}\right)_{IA} + \frac{IA}{IFL}\left(\frac{\partial DN}{\partial IF}\right)_{IA}\right] \quad (4.3.5)$$

4.4 Estimation of G1 and G2[28]

For estimating G1 and G2 at constant field current the following constants are needed:

i) No load speed.

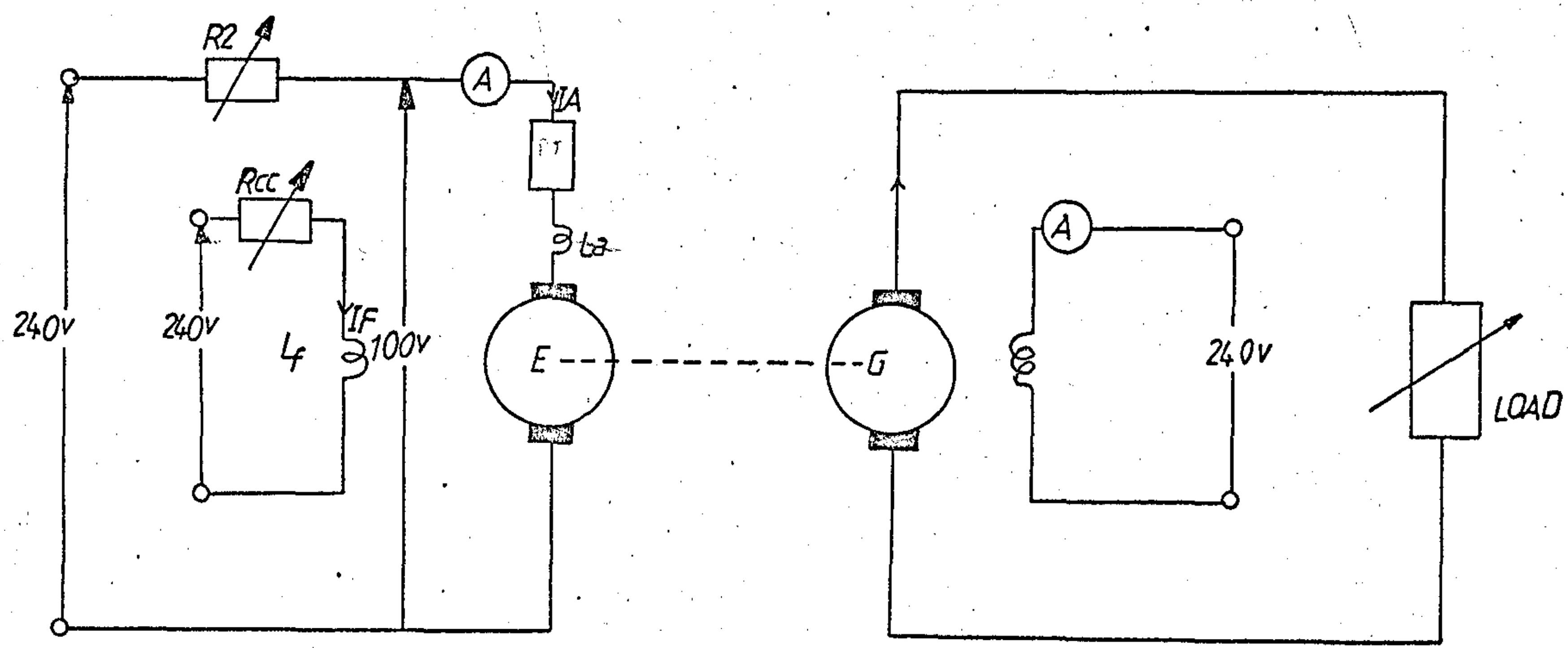
ii) Change in speed between no load and full load.

iii) $(\partial NO/\partial IF)_{IA}$

iv) $(\partial DN/\partial IF)_{IA}$

v) Armature resistance at a particular load current.

Described below is the simple experimental set-up which determines the first four parameters required for the determination of G1 and G2. Fig. 4.1 shows the circuit diagram.



Generalised machine

Rotor drive motor as loading generator

FIG. 4.1 Circuit diagram for determining the characteristics of a separately excited d.c. motor.

The generalised machine is connected as a separately excited d.c. motor. The voltage across the armature is kept constant at 100V throughout the experiment by adjusting R2.

The generalised machine is mechanically coupled to another d.c. machine. This machine is excited at its rated field current. This machine is used to load the test machine. The test machine field is energized and the field current is kept constant throughout for each set. Each set consists of loading the machine from no load to full load. Full load current for the generalised machine is 8A. For every load current sufficient time is allowed so that the armature and field winding temperature becomes constant. The speed of the motor is noted for that armature current and plotted. A rising straight line is obtained. The point of intersection of this line at zero current and full load current are the no load and full load speed. These N Vs. IA characteristics are drawn for different field currents. These characteristics are shown in Fig. 4.2.

The difference between the no load and the full load speed DN is obtained and then plotted against the corresponding field current and is shown in Fig.4.3. The tangent on any point on this curve gives the slope $(\partial DN/\partial IF)_{IA}$. A curve showing the relationship between the no load speed NO and the field current is shown in Fig.4.4. The tangent at any point on this curve gives the slope $(\partial NO/\partial IF)$.

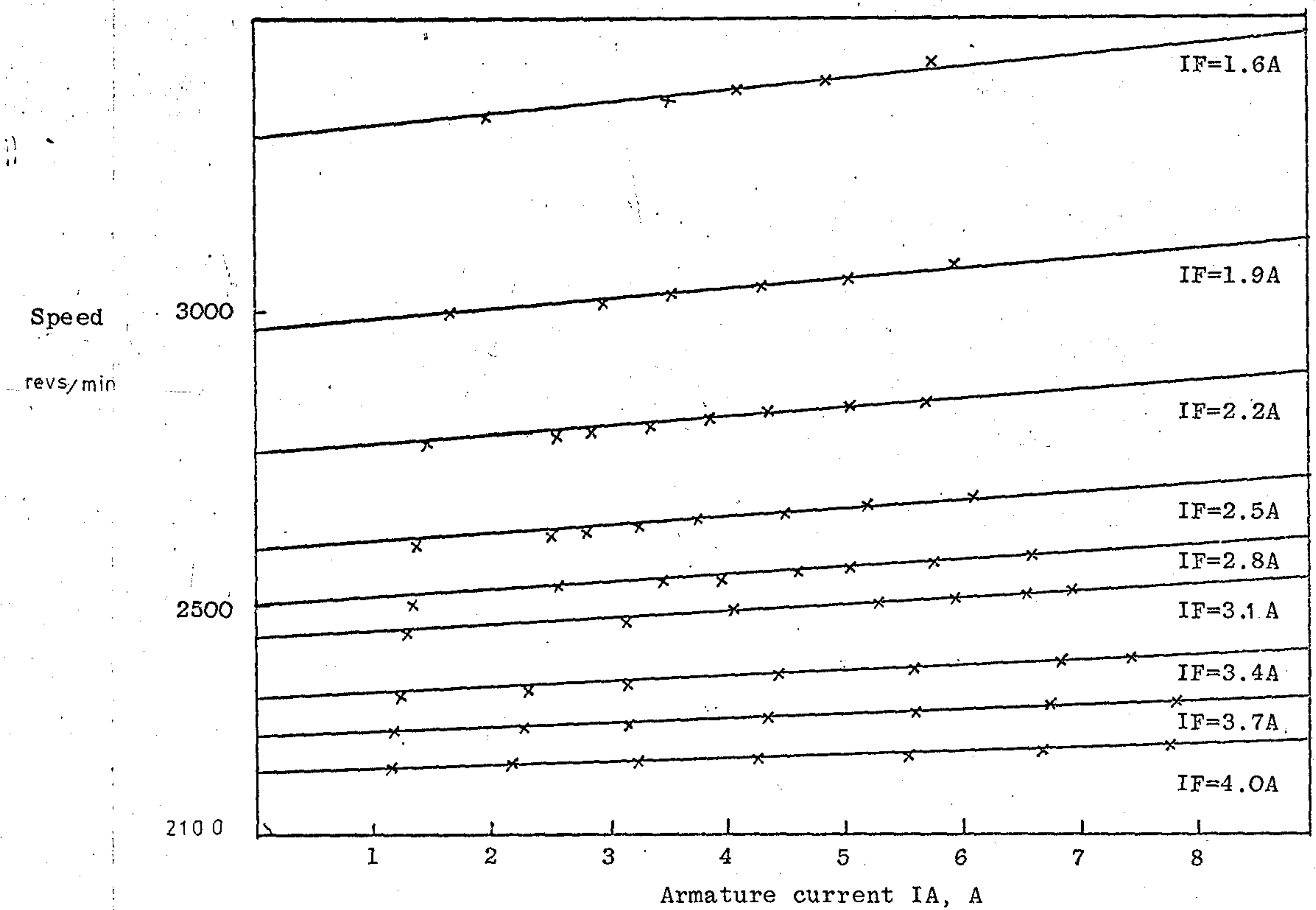


FIG. 4.2 Relation between motor speed and armature current. Each characteristic is for constant field current and constant armature voltage

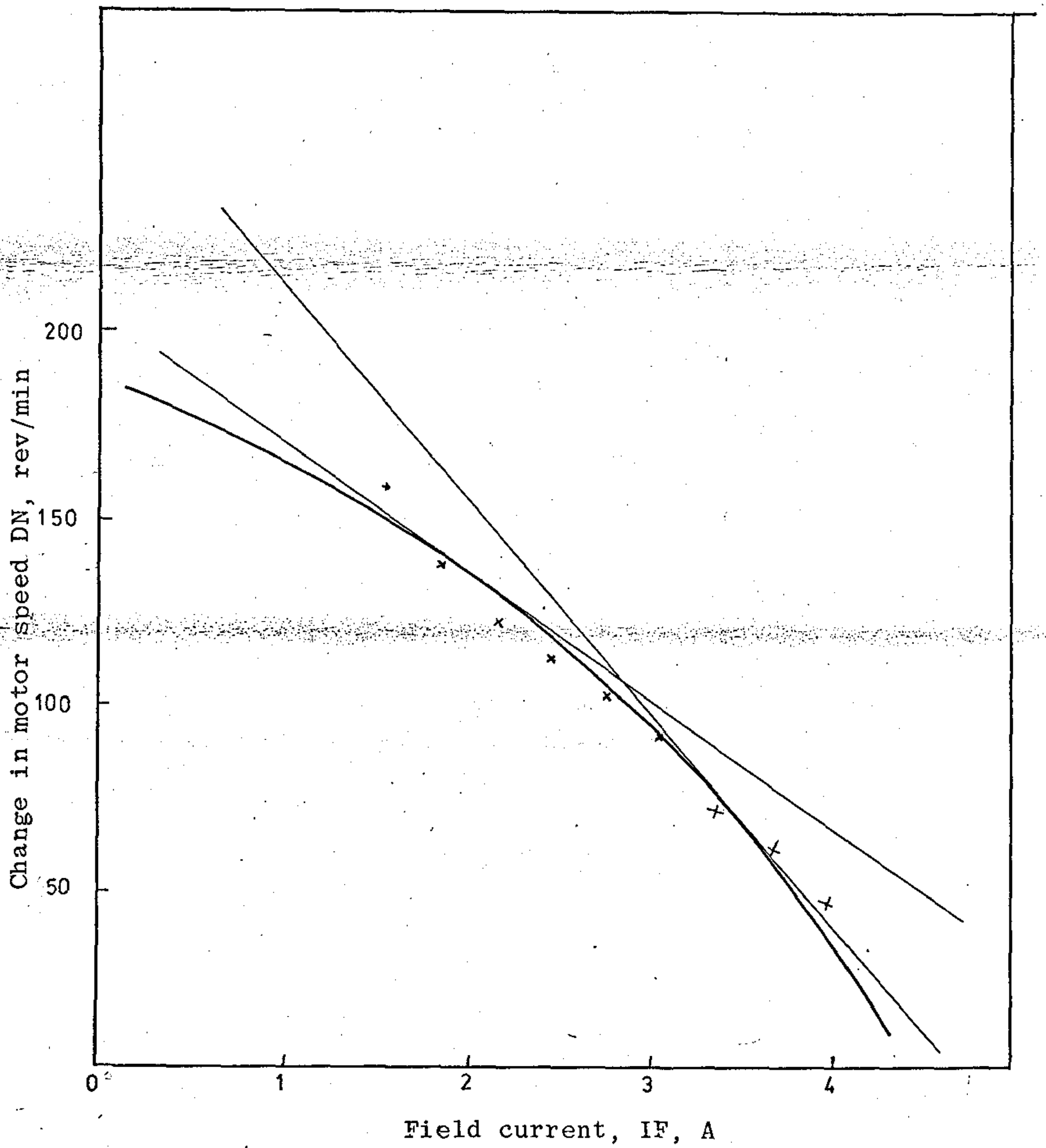


FIG. 4.3 Relationship between field current and change in motor speed

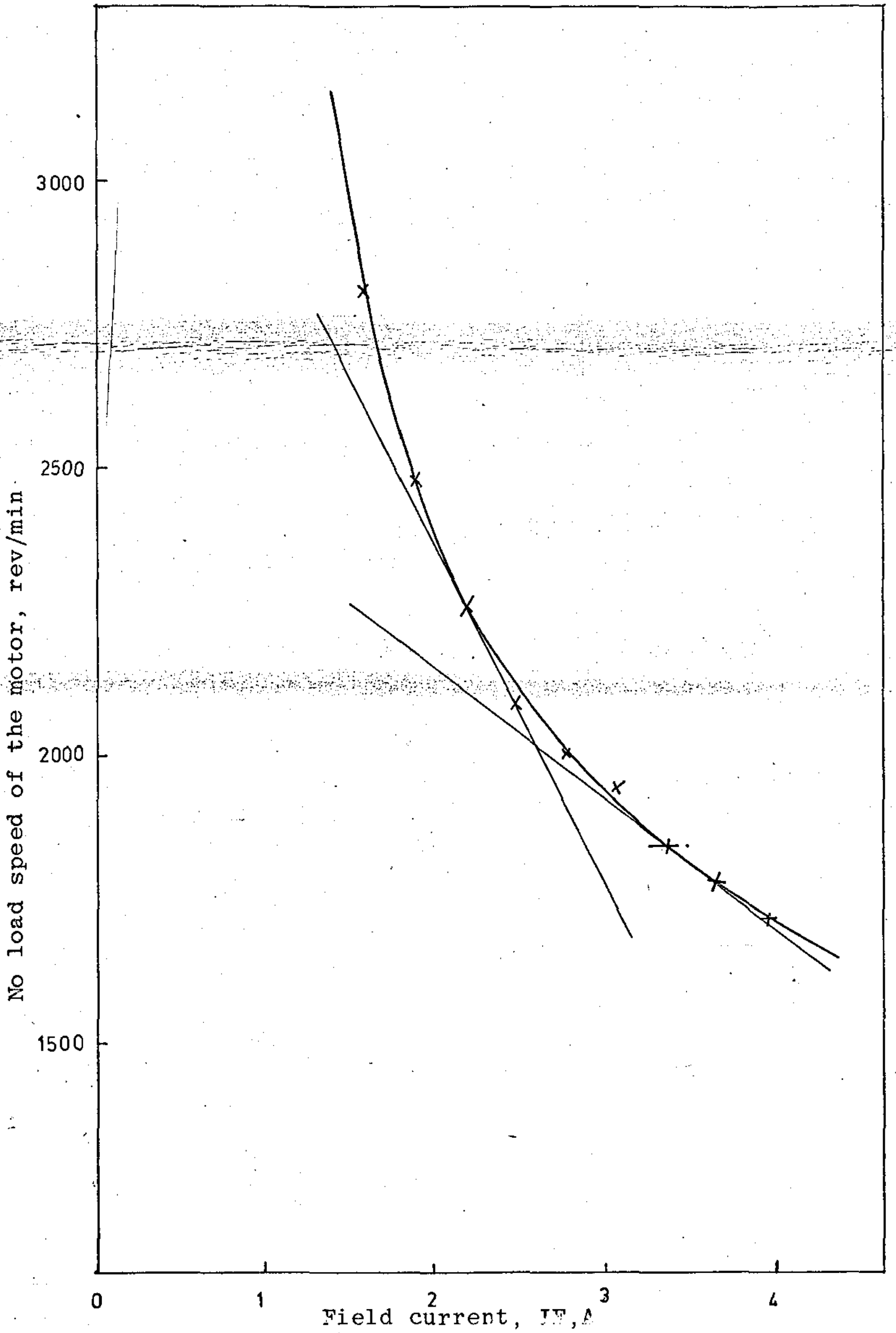


FIG. 4.4 Relationship between field current and the no load speed of the motor

Table I shows the values of no load speed, full load speed and change in speed, for different values of field current.

TABLE I

No load speed, full load speed and change in speed at different field currents

Field Current Amps.	No load Speed rev/min	Full load Speed rev/min	Change in Speed rev/min
1.6	3295	3450	155
1.9	2965	3100	135
2.3	2755	2875	120
2.5	2590	2700	110
2.8	2495	2595	100
3.1	2440	2530	90
3.4	2340	2410	70
3.7	2270	2330	60
4.0	2210	2255	45

4.5 Measurement of Armature Resistance at Different Load Currents

The temperature of the machine increases with the increase in armature current. Heat is generated due to armature copper loss, eddy current and hysteresis losses, friction and windage loss and field winding copper loss. Iron losses are mainly in the armature[17] and friction loss is in the shaft and bearing and between brush and commutator, are all on the armature side of the machine. Therefore heat transfer is taking place from armature winding to field winding. At constant field current the rise in temperature of the machine is dependent upon the armature current. For steady state armature current the rise in temperature can be assumed the same for both field and armature windings.

There is a portion of field winding in the generalised machine accessible to the tip of the thermocouple. The tip of the thermocouple is placed on the portion of the field winding which is facing the armature winding. The rise in temperature of the field winding is assumed to be the same as that of the armature in the steady state condition. A graph is then drawn between the rise in temperature and the armature current and is shown in Fig. 4.5. Table 2 shows the rise in temperature at different load currents. The resistance of the armature winding and the field winding is then calculated for any temperature rise by:

$$R_h = R_c \{1 + \alpha(T_h - T_c)\} \tag{4.5.1}$$

where:

R_h = resistance of hot winding

R_c = resistance of cold winding

α = temperature coefficient of resistance

= 0.04

$(T_h - T_c)$ = rise in the temperature of the winding in ^{o}C.

T_c = ambient temperature

TABLE 2

Rise in temperature of the machine at different armature currents

Armature Current Amps	Ambient Temperature °C	Rise in Temperature °C
1.2	21	14
2.4		19
3.5		23
4.6		27
5.6		31
6.6		35
7.6		38
8.6		42

Resistance of armature winding at 21°C = 0.46 ohm

By using formula 4.5.1 and the rise in temperature at different load currents from the graph, rise in temperature Vs. IA, the resistance of the armature winding at different load currents can be calculated. Fig. 4.6 shows the RA Vs. IA graphically and Table 3 shows the armature resistance at different load currents.

TABLE 3

Armature resistance at different armature current

Armature Current Amps	Armature Resistance Ohms
1	0.485
2	0.495
3	0.502
4	0.509
5	0.517
6	0.524
7	0.530
8	0.537

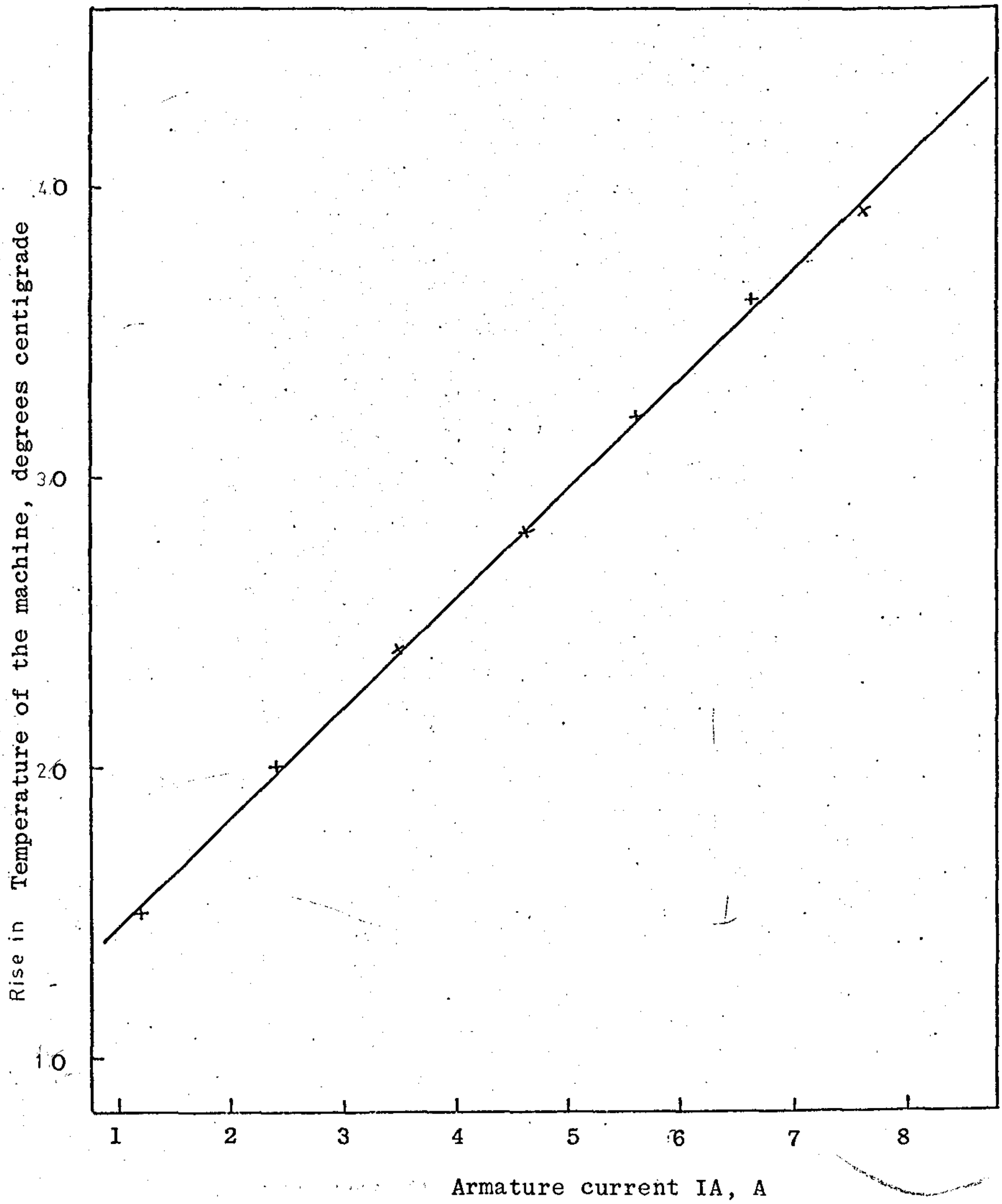


FIG. 4.5 Relationship between armature current and rise in temperature of the machine .

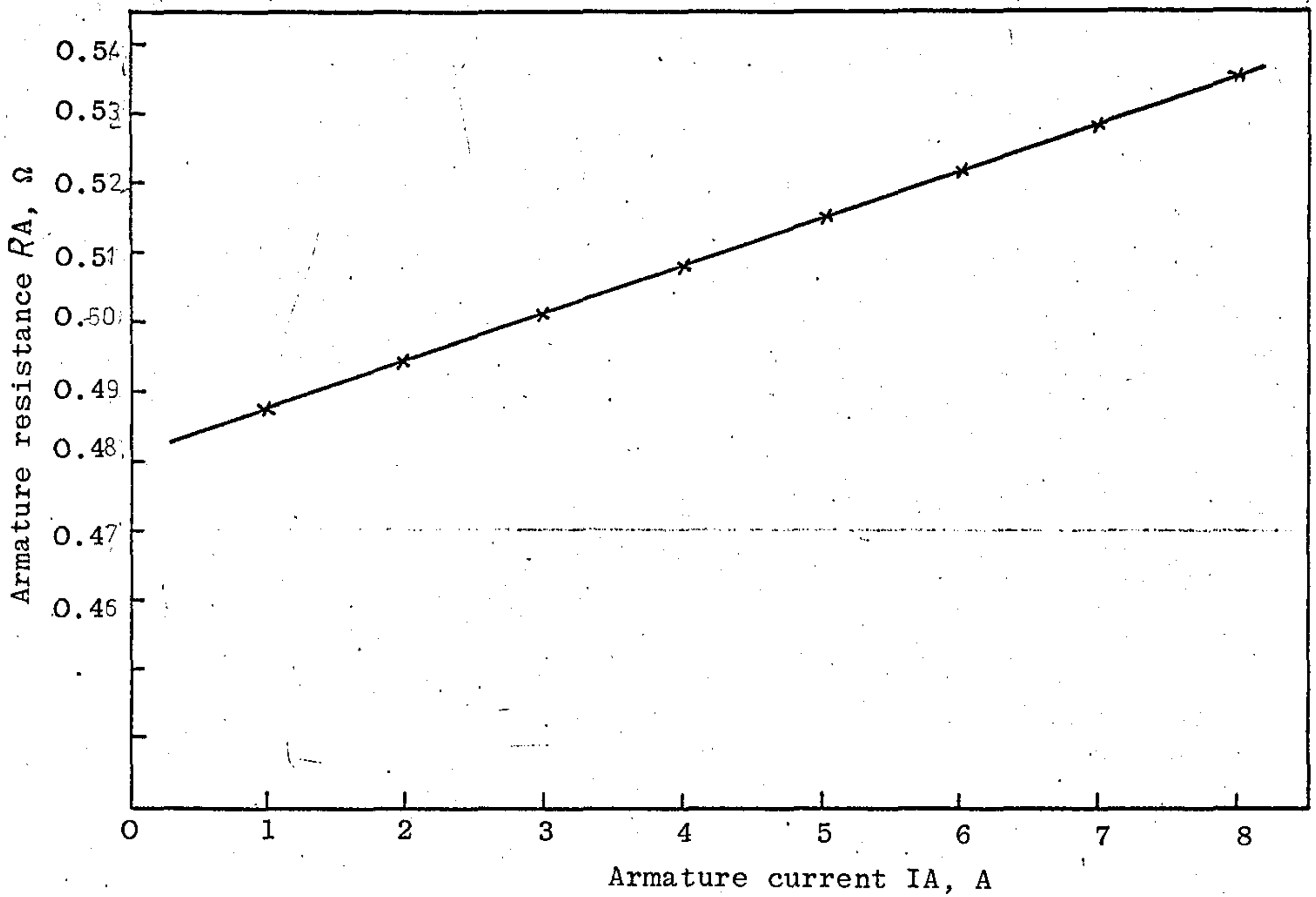


FIG. 4.6 Relationship between armature current and armature resistance when the ambient temperature is 21°C

4.6 Measurement of Brush Contact Resistance

Brush resistance is measured indirectly, by applying voltage at armature terminals after locking the armature and measuring the current passing through the winding. Armature voltage divided by the current gives the brush contact resistance and armature winding resistance at room temperature. From this armature winding resistance, at room temperature, is subtracted. This gives brush contact resistance. A curve is drawn for brush contact resistance Vs. armature current. Fig. 4.7 shows the relationship between brush contact resistance and armature current and the Table 4 shows the brush contact resistance at different load currents.

TABLE 4

Brush contact resistance at different armature currents

Current through armature Amps	Voltage applied Volts	Total resistance at 21 °C RT Ohms	Brush contact resistance RT-0.46 Ohms
1	1.85	1.85	1.39
2	3.05	1.52	1.06
3	3.80	1.27	0.81
4	4.60	1.15	0.69
5	5.2	1.04	0.58
6	6.10	1.01	0.55
7	6.80	0.97	0.51
8	7.40	0.925	0.465

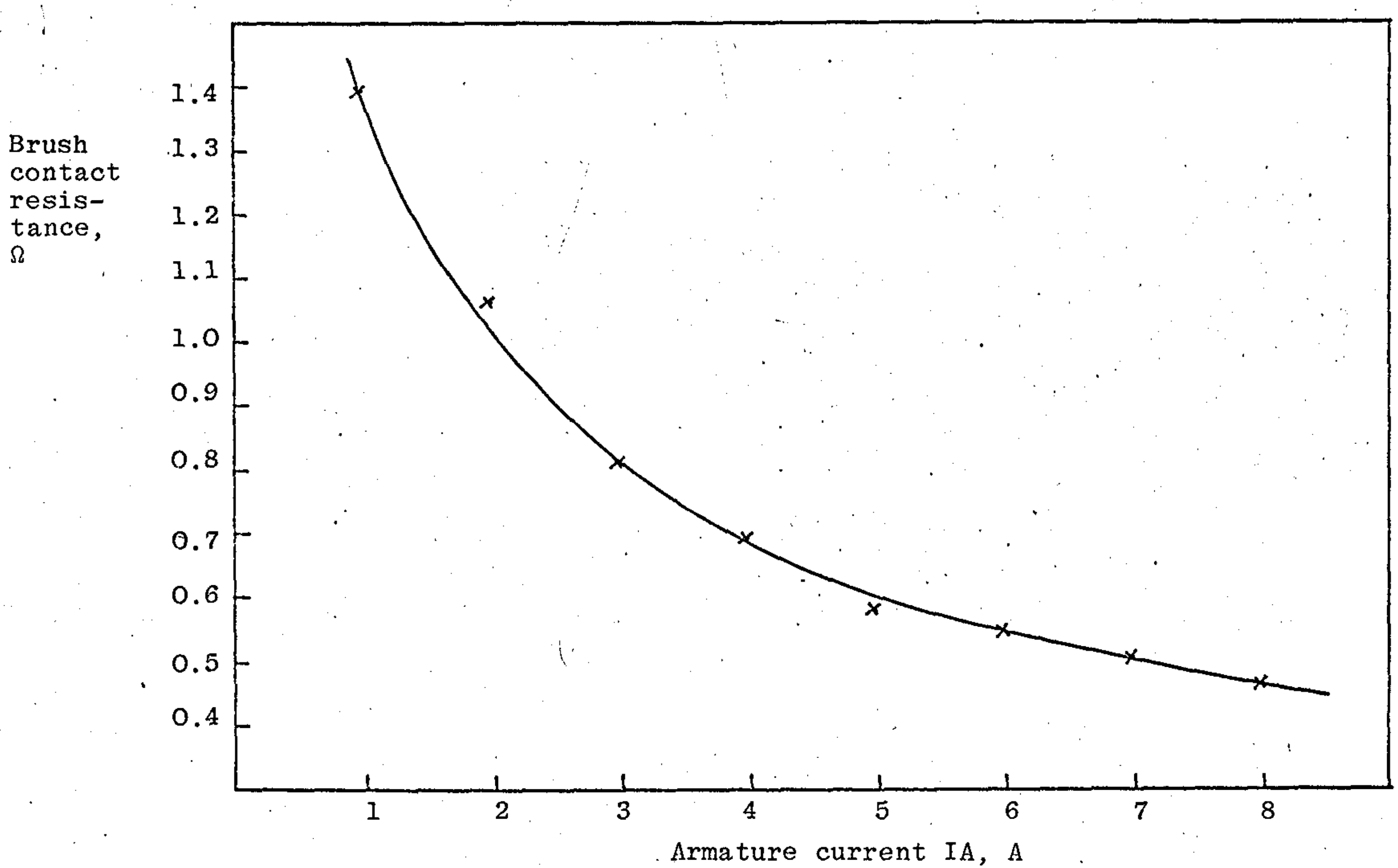


FIG. 4.7 Relationship between armature current and brush contact resistance

4.7 Measurement of Field Resistance

The field winding is accessible for measurement. The resistance of the field winding at room temperature is measured by applying voltage to it and measuring the current. It is assumed previously that the heat flow takes place from the armature to the stator and at a steady state condition the temperatures of the armature and the field winding are the same. Therefore at a steady state load current the temperature of the field winding is the same as the temperature of the armature winding. Also because the calculation for transient response is carried out around a steady state armature current and a constant-field current, the field resistance is calculated for those temperatures which correspond to that particular armature current. Figure 4.8 shows the relation between field resistance and armature current and Table 5 shows the field resistance at different load currents.

Measured resistance of field winding = 2.44Ω at 21°C.

TABLE 5

Field resistance including temperature effect

ambient temperature 21°C

Armature current Amps	Rise in the temperature of the machine °C	Field resistance taking machine temperature into account, Ohms
1	13.6	2.57
2	17.4	2.61
3	21.0	2.65
4	24.8	2.68
5	29.6	2.73
6	32.4	2.76
7	36.4	2.80
8	41.9	2.85

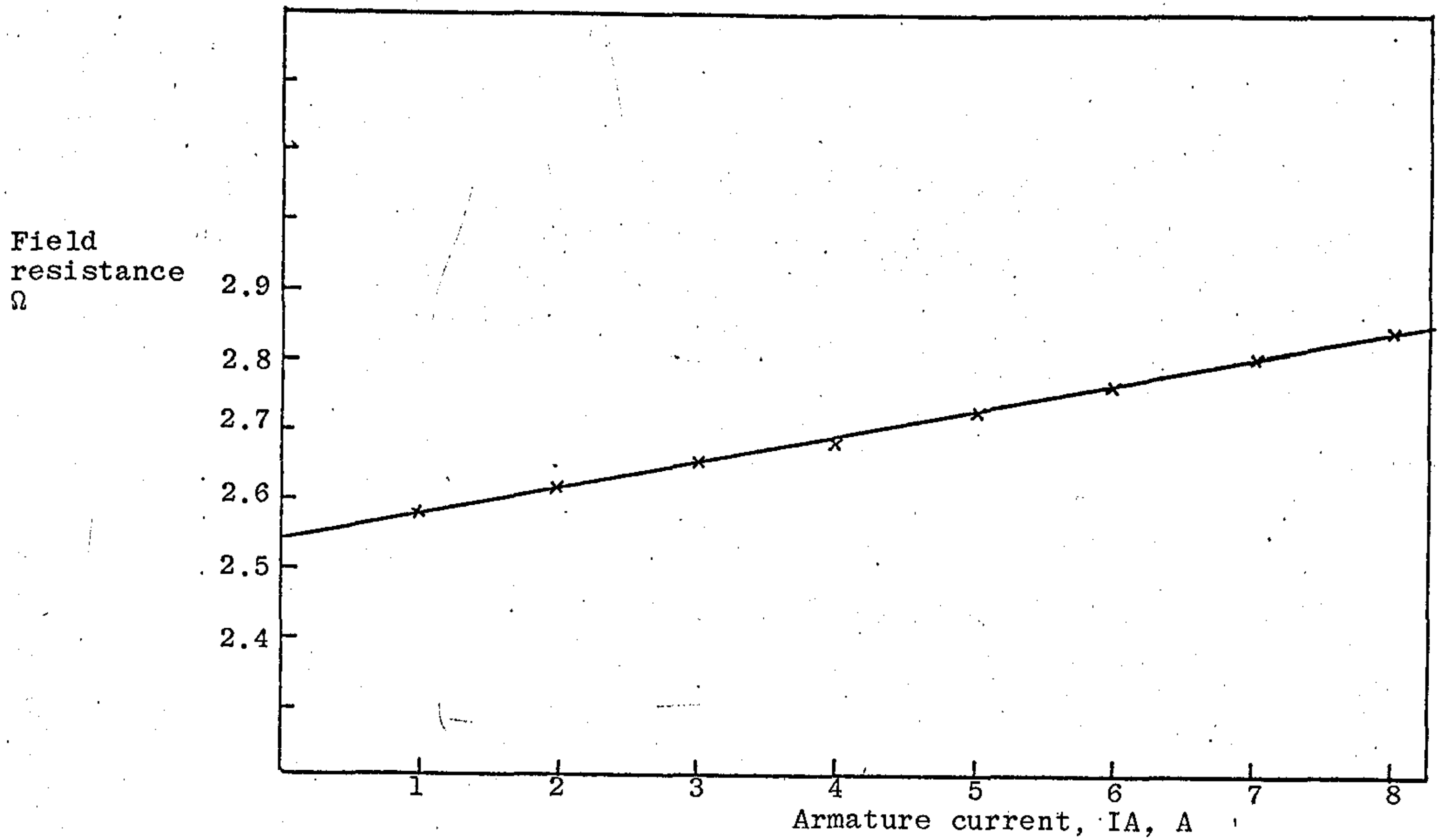


FIG. 4.8 Relationship between armature current and field resistance

ambient temperature 21°C

4.8 Measurement of the Constant of Viscous Friction

The electrical torque generated by the motor is given by:

$$T_e = K_t . \Phi . IA$$

A fraction of this generated torque is absorbed in bearing and brush friction, windage, hysteresis and eddy current. Hysteresis and eddy current losses could be put together as iron loss or core loss, as they are eliminated from the final equation for friction and windage loss.

Friction and windage loss is a function of the square of the speed and the torque loss in it is represented by:

$$T_{FW} = 2\pi ND1/60 \qquad (4.8.1)$$

where D is the constant of friction. If the power lost in friction and windage is P_L then

$$P_L = \frac{2\pi N . T_{FW}}{60} \qquad (4.8.2)$$

substituting 4.8.1 in 4.8.2

$$P_L = \frac{2\pi N}{60} . \frac{2\pi ND1}{60}$$

$$D1 = \frac{P_L}{(2\pi N/60)^2} \qquad (4.8.3)$$

The constant of friction can be calculated if the power loss in friction and windage could be estimated at different speeds.

For the Westinghouse generalised machine, the power lost in friction and windage is the sum of the power loss in friction and windage of the generalised machine and another machine which is coupled to it and represented by P_L.

There are three unknowns for the two machines, one is the core loss of the test or generalised machine, and second is the core loss of the loading machine. Third, is the power loss in friction and windage which is common for both the machines. Three equations will be needed to eliminate core loss, and the final equation could be obtained in the form of applied armature voltages to the generalised machine and the loading machine and copper losses in them. The core loss, though, depends upon the flux density and which, in turn, depends upon both the field and armature current. For the purposes of our calculations of power loss in friction and windage, it will be assumed that the core loss depends only on the field current and not upon the armature current.

The two machines are run in three ways. Firstly, the loading generator or rotor drive motor is run at full excitation leaving the test machine or generalised machine unexcited. Secondly, the test machine, or generalised machine, is run at full excitation, leaving the rotor drive motor unexcited. Thirdly, the test machine or generalised machine is run at full excitation and the rotor drive motor field is excited to its rated value. The speed is kept constant for all the three runs.

The brush drop of the rotor drive motor is taken as 2V from the manual of the machine.

When the R.D.M. is running as the motor and G.M. unexcited:

$$V_{RD}.I_{RD} = (R_{ARD}I_{RD}+2)I_{RD}+ \text{Core loss of R.D.M.} + P_L \qquad (4.8.4)$$

When the G.M. is running as the motor and R.D.M. unexcited:

$$V_{G1}.I_{G1} = R_{G1}.I^2_{G1} + \text{Core loss of G.M.}+P_L \qquad (4.8.5)$$

When the G.M. is running as the motor and R.D.M. excited to rated field:

$$V_{G2}.I_{G2} = R_{G2}.I^2_{G2} + \text{Core loss of G.M.} + \text{Core loss of R.D.M.} + P_L \qquad (4.8.6)$$

where: R.D.M. is abbreviation of the rotor drive motor.

G.M. is abbreviation of the generalised machine.

V_{RD} = voltage applied to R.D. motor when G.M. field unexcited

I_{RD} = current passing through R.D.M.

V_{G1} = voltage applied to G.M. when R.D.M. field is unexcited

I_{G1} = current passing through G.M.

V_{G2} = voltage applied to the G.M. when R.D.M. field fully excited

I_{G2} = current passing through G.M. when R.D.M. field fully excited

R_{ARD} = armature resistance of R.D. motor

R_{G1} = armature and brush contact resistance of generalised machine corresponding to current I_{G1}

R_{G2} = armature and brush contact resistance of G.M. corresponding to current I_{G2}

Subtract (4.8.5) from (4.8.6):

$$V_{G2}.I_{G2}-V_{G1}.I_{G1} = (R_{G2}.I_{G2}^2-R_{G1}.I_{G1}^2) + \text{Core loss of R.D.M.}$$

$$\text{Core loss of R.D.M.} = (V_{G2}.I_{G2}-V_{G1}.I_{G1})-(R_{G2}.I_{G2}^2 - R_{G1}.I_{G1}^2)$$

$$= C1$$

Substitute core loss of the rotor drive motor in equation (5.8.4):

$$V_{RD} \cdot I_{RD} = (R_{ARD} \cdot I_{RD} + 2) I_{RD} + C + P_L$$

$$P_L = V_{RD} \cdot I_{RD} - (R_{ARD} \cdot I_{RD}^2 + 2 I_{RD}) - C$$

P_L is determined for different speeds and a curve is drawn for P_L versus speed and shown by Figure 4.12. The coefficient of friction and windage can then be calculated for any speed.

Figures 4.9 , 4.10 and 4.11 show the circuit diagrams for the experiments carried out, whose results are given in Tables 6, 7 and 8 respectively.

Core loss of R.D.M and GM can be calculated by equations 4.8.4 and 4.8.5 respectively. Figure 4.13 shows the relation between speed and power loss in the iron of G.M and R.D.M

TABLE 6

Rotor drive motor as motor and generalised machine unexcited.

Speed N rpm	V_{RD} Volts	I_{RD} Amps	$V_{RD}.I_{RD}$ Watts	R_{ARD} Ohms	$I_{RD}(I_{RD}.R_{ARD}+2)$ Watts
500	40.5	0.55	22.48	0.72	1.33
1000	79.0	0.67	52.93		1.66
1500	117.0	0.77	90.24		1.96
2000	115.5	0.86	133.73		2.25
2500	194.8	0.96	187.00		2.84
3000	233.3	1.10	256.30		3.07

TABLE 7

Generalised machine as motor and rotor drive motor unexcited

Speed N rpm	V_{G1} Volts	I_{G1} Amps	$V_{G1}.I_{G1}$ Watts	R_{G1} Ohms	$I_{G1}^2 . R_{G1}$ Watts
500	23.0	0.76	17.59	2.05	1.20
1000	43.8	0.89	39.20	1.95	1.56
1500	65.2	1.05	68.46	1.90	2.10
2000	86.1	1.15	99.01	1.82	2.41
2500	107.8	1.30	140.00	1.74	2.94
3000	128.8	1.40	180.32	1.71	3.35

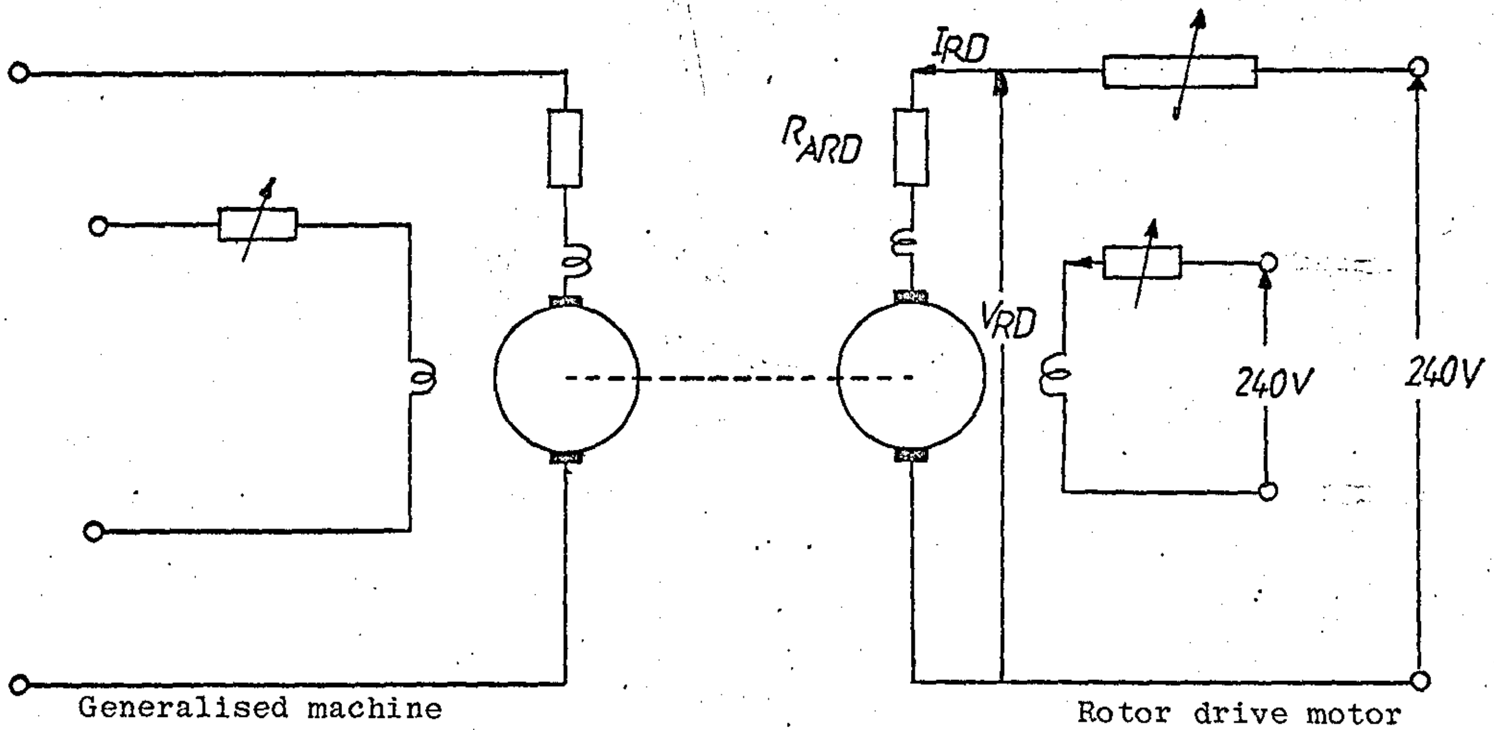
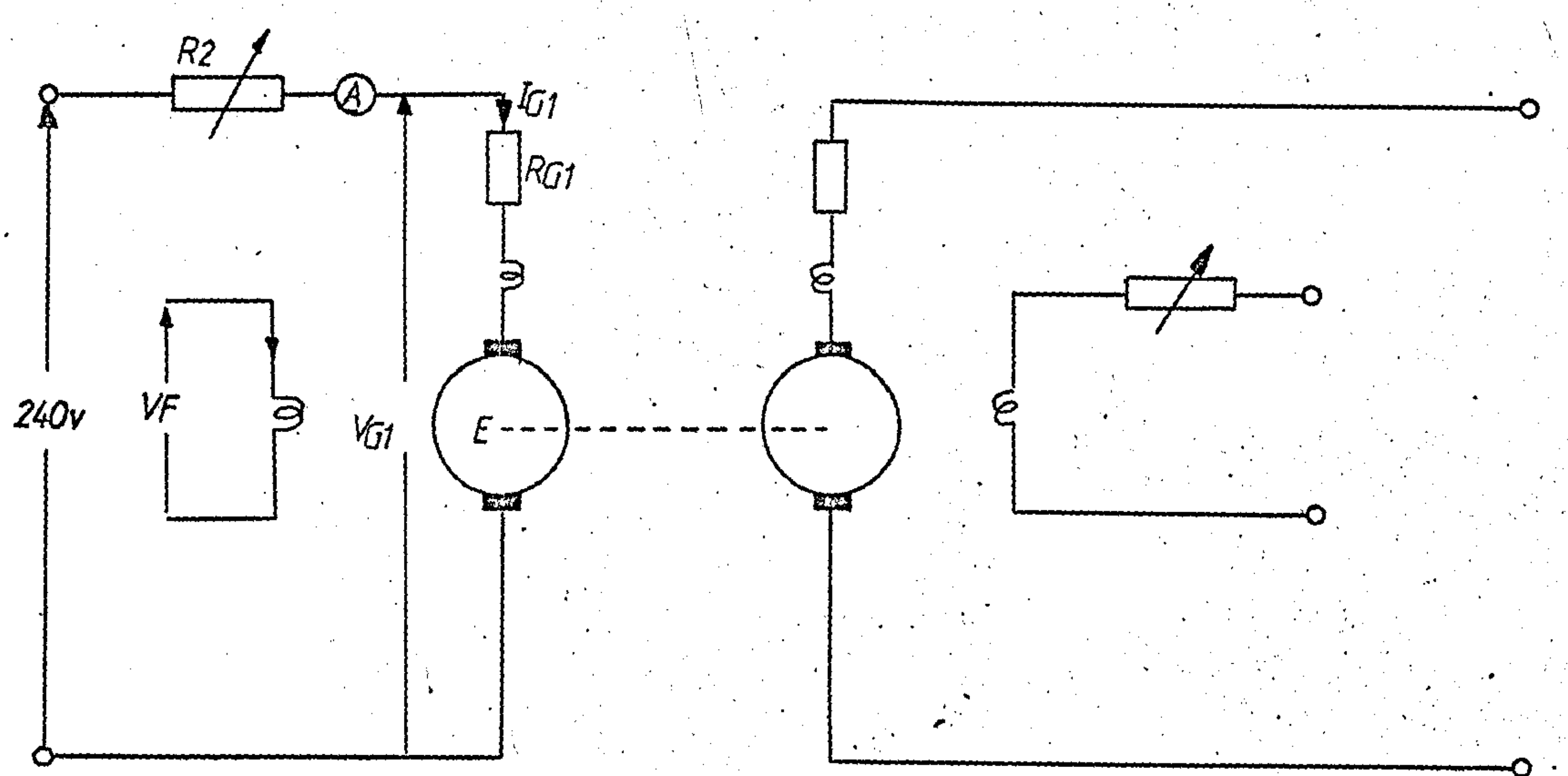


FIG. 4.9 R.D.M. as motor at rated field and variable V R D for different speed. Field of G.M. unexcited.



Generalised Machine

Rotor Drive Motor

FIG. 4.10 G.M. as motor at rated field and variable VG 1 for different speed. Field of R.D.M. unexcited.

TABLE 8

Generalised machine as motor and rotor drive motor fully excited

Speed N rpm	V_{G2} Volts	I_{G2} Amps	$V_{G2}I_{G2}$ Watts	R_{G2} Ohms	$I_{G2}^2 \cdot R_{G2}$ Watts
500	23.4	1.15	26.96	1.82	2.41
1000	45.0	1.40	63.00	1.71	2.35
1500	65.6	1.60	104.96	1.63	4.17
2000	87.1	1.80	156.78	1.56	5.06
2500	108.7	2.00	217.78	1.52	6.08
3000	130.0	2.20	286.00	1.47	7.12

TABLE 9

Calculations for power lost in friction and windage P_L

Speed N rpm	$A = V_{G2} \cdot I_{G2} - V_{G1} \cdot I_{G1}$ Watts	$B = R_{G2} I_{G2}^2 - R_{G1} I_{G1}^2$ Watts	$C = A - B$ Watts	$P_L = V_{RD} \cdot I_{RD} - (R_{ARD} \cdot I_{RD}^2 + 2 \cdot I_{RD}) - C$ Watts
500	9.36	1.21	8.15	13.00
1000	23.80	1.79	22.01	29.26
1500	36.50	2.07	34.42	54.86
2000	57.77	2.65	55.12	76.36
2500	77.40	3.14	74.26	109.90
3000	105.68	4.05	101.63	151.60

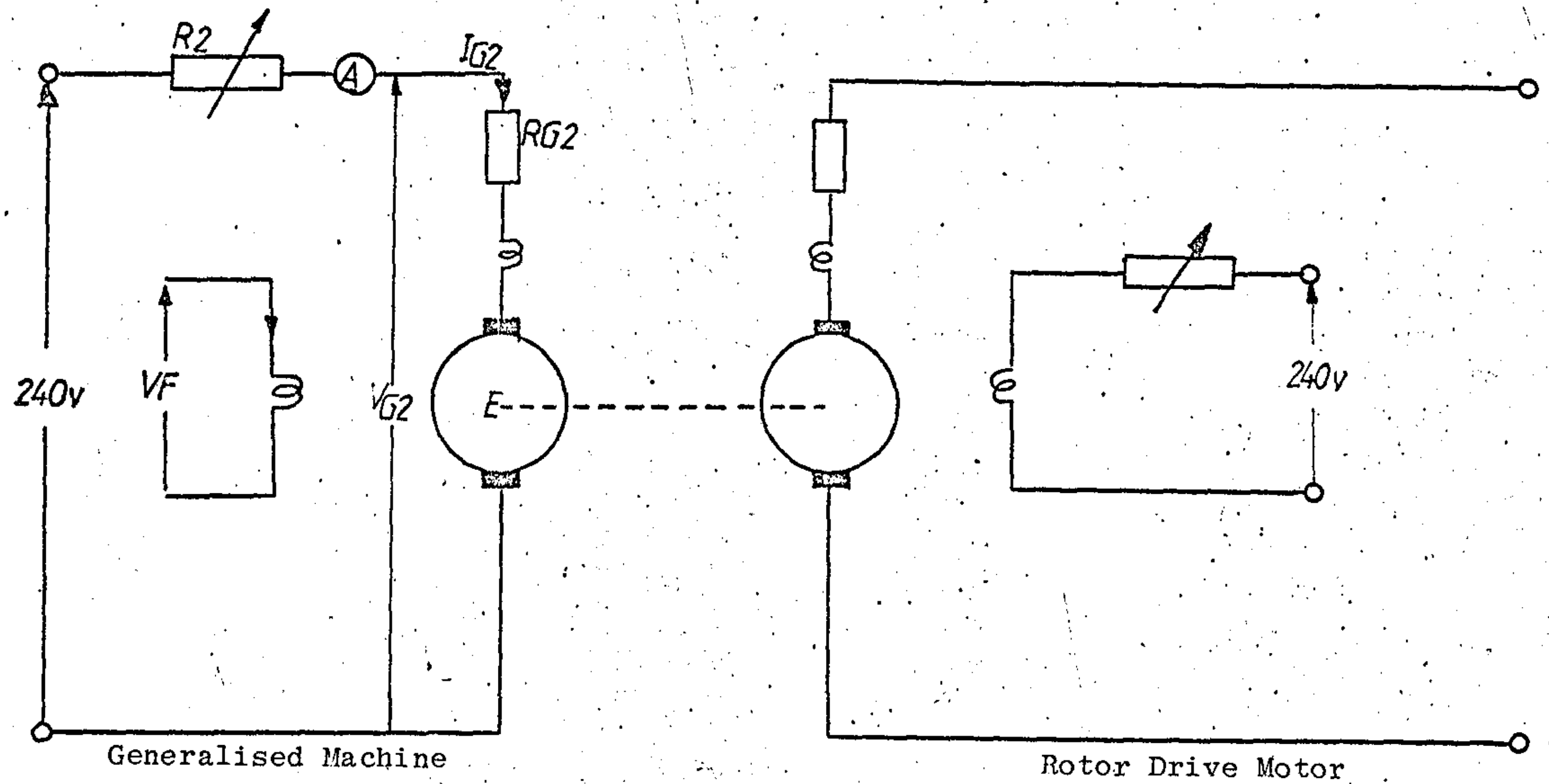


FIG. 4.11 G.M. as motor at rated field and variable VG 2 for different speed. Field of R.D.M., fully excited.

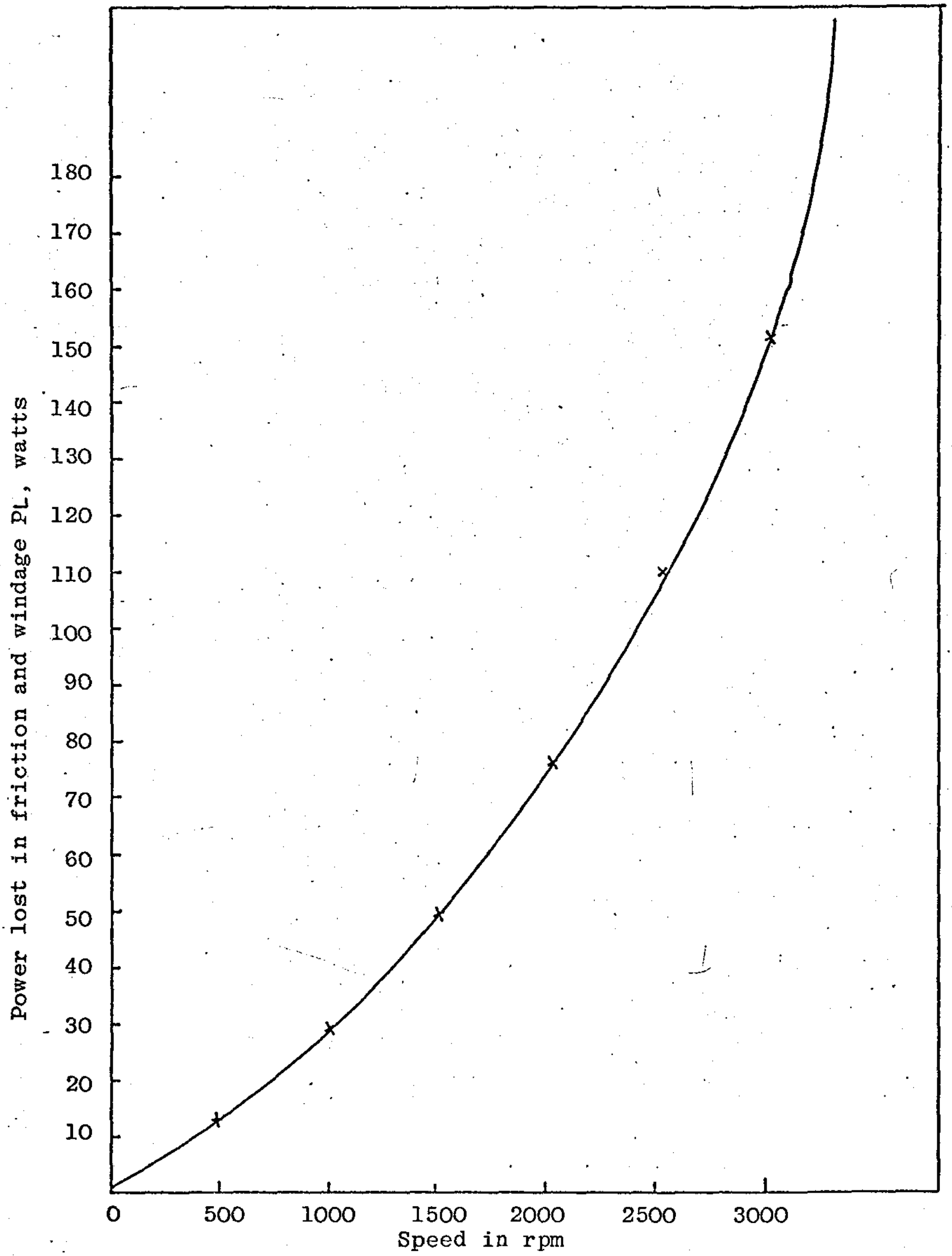


FIG. 4.12 Relationship between speed and power lost in friction and windage

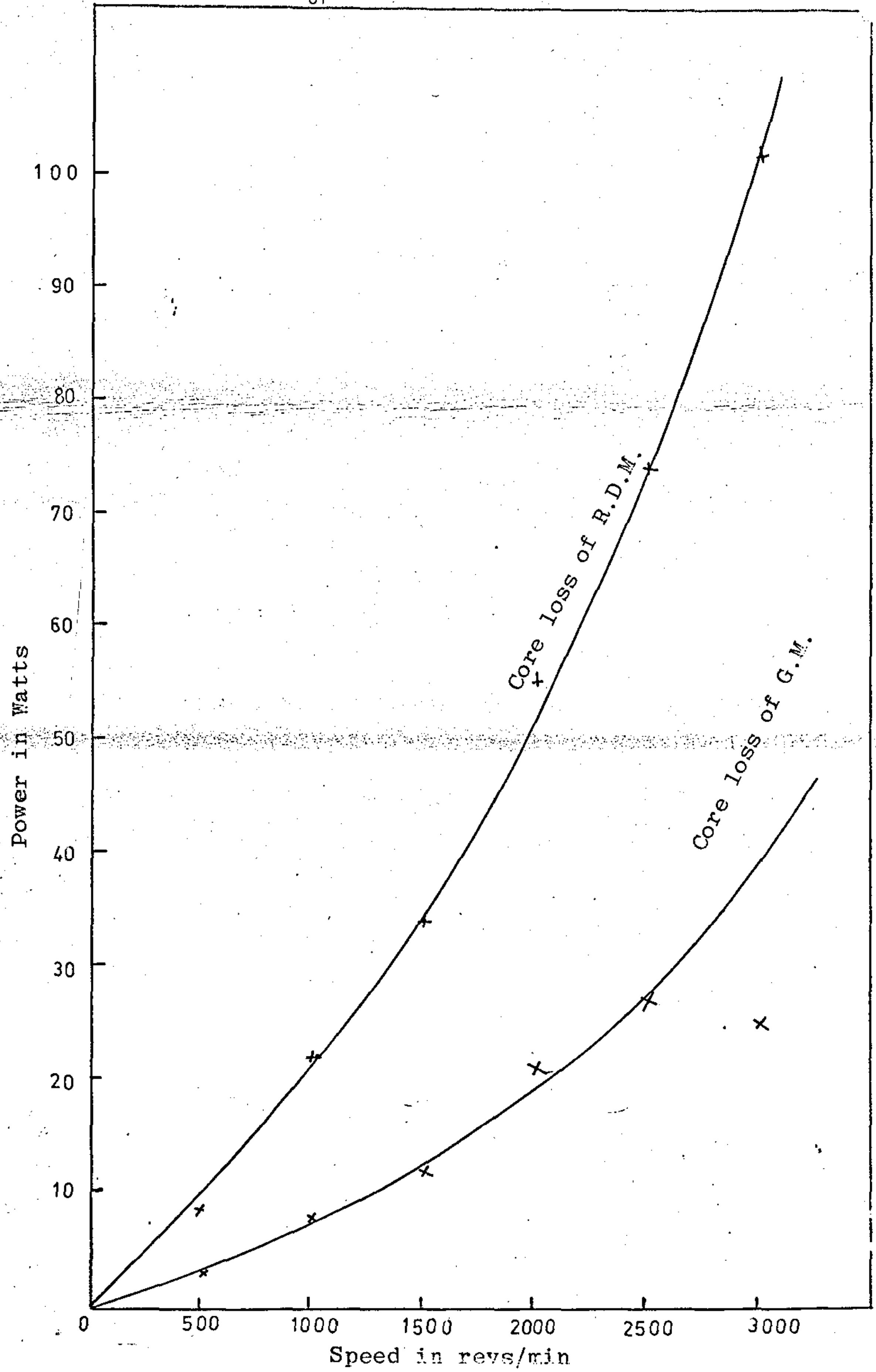


FIG. 4.13 Relationship between speed and power lost in the iron of the G.M. and R.D.M.

4.9 Estimation of F1, F2, H1, H2

F1, or the change in the armature flux linkage with change in armature current when the field current is kept constant, is the self inductance of armature. The self inductance is obtained from the manual of the machine[26]

$$F1 = 0.05 \text{ Henry}$$

F2, or the change in armature flux linkage with change in field current is zero if the armature current is constant, because the field and armature windings are effectively perpendicular to each other and there is no flux linkage between them. Any change in flux linkage due to change in magnetic properties because of saturation of the rotor iron are considered to be small and therefore neglected.

H1, or the change in field flux linkage with change in armature current when the field current is kept constant will have a value, because the change in armature current will change the magnitude of the resultant mmf, which in turn will change the magnetic properties of the stator iron. H1 can be calculated as follows:

Number of stator slots	36
Winding groups used	2
Number of slots per winding group	9
Coils per group	9
Number of turns per coil	18

Total number of turns	HF	18 x 9 x 2
	HF	324
	Kp	0.819
	Kd	0.956
	KW	0.783
	KΦ	1.10

Change in the field flux linkage when armature current is changed and the field current is kept constant:

$$(\partial\psi F/\partial IA)_{IF} = HF.G1.K\Phi.KW \text{ Henry}$$

H2, or change in flux linkage with the field winding when the field current is changed and the armature current is kept constant is:

$$(\partial\psi F/\partial IF)_{IA} = HF.G2.K\Phi.KW \text{ Henry}$$

KΦ is the leakage factor for the field winding defined as the ratio of the total flux linking the field winding divided by the mutual flux linking the field and armature windings

CHAPTER 5

COMPUTER PROGRAMMING

5.1 General

The solution of the set of three first order differential equations, which represent the machine, is carried out using a numerical method and a program written in Fortran language is used for digital computer simulation. The parameters of the machine are calculated for different armature currents.

The numerical method of solution of first order differential equations divides the time of integration into small and equal intervals. Then using this interval with derivatives of the state and the value of the state at any instant of time, the value of state for the next interval of time is predicted. Using the new state as the initial value, derivatives of the new state are calculated, which is used for calculating the value of the new state. Proceeding in this way the transient response of the mathematical model is determined.

It can be observed from Figures 5.1.1a and 5.1.1b that the integration by the numerical method is more accurate if the interval of time between the calculation of successive states is small. But for a given time of integration, smaller values of interval mean more numbers of calculations. It is therefore necessary to calculate the transient response of the machine model for different intervals of time to ensure the accuracy of the result.

The program used is given in Appendix 1 and a flow chart is shown by Figures 5.1.2a and 5.1.2b.

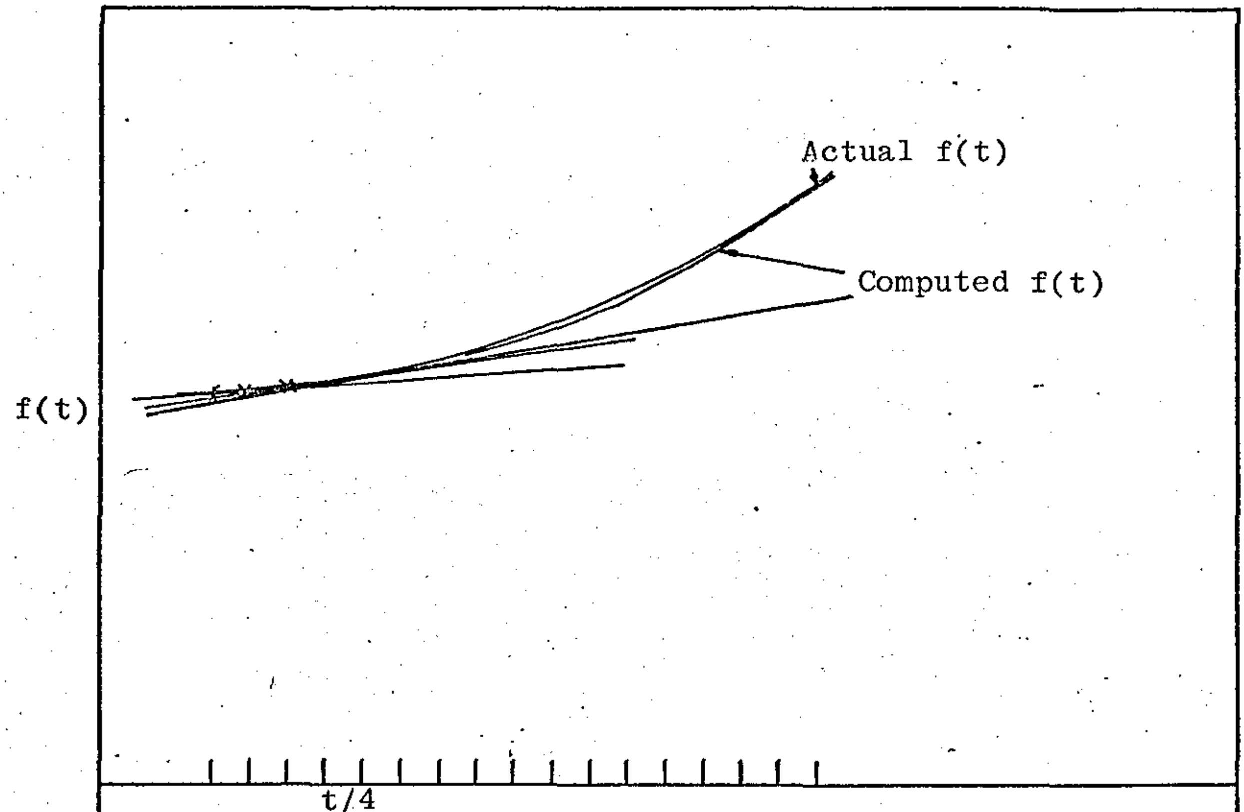


FIG. 5.1.1.b

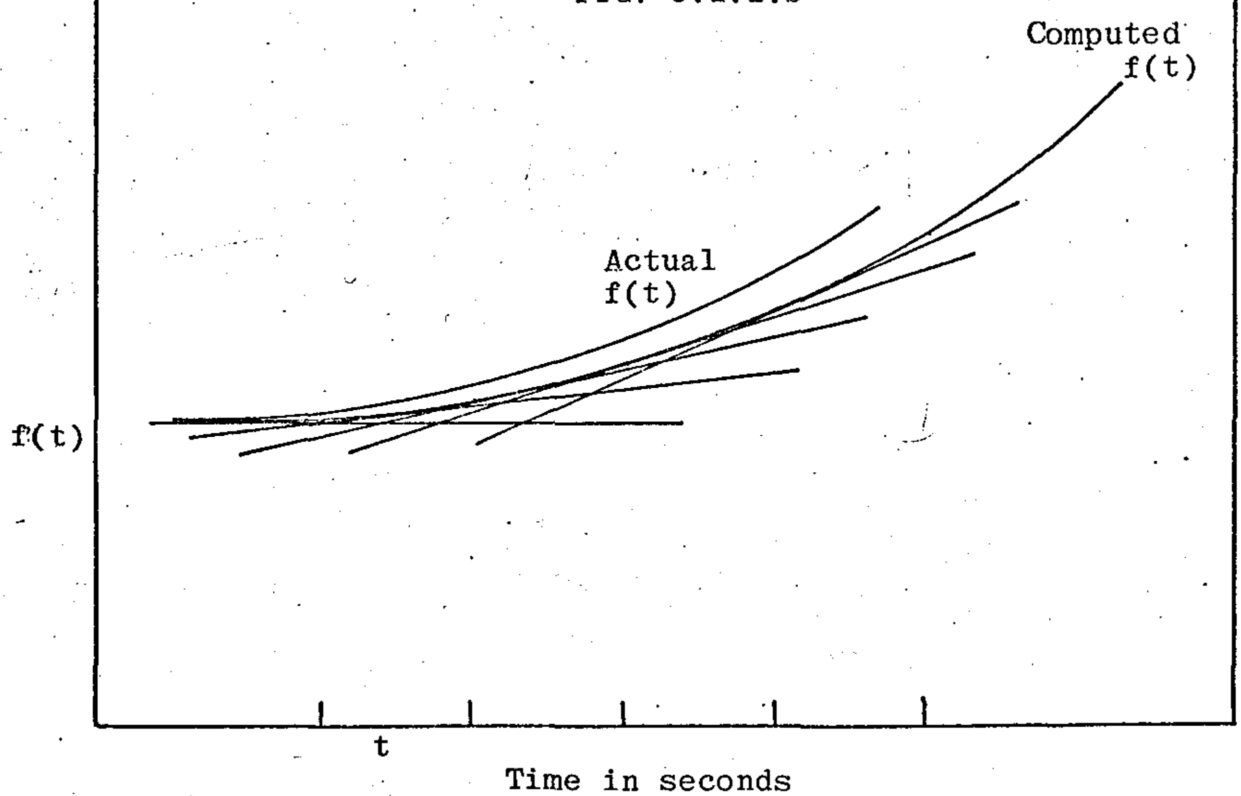


FIG. 5.1.1a

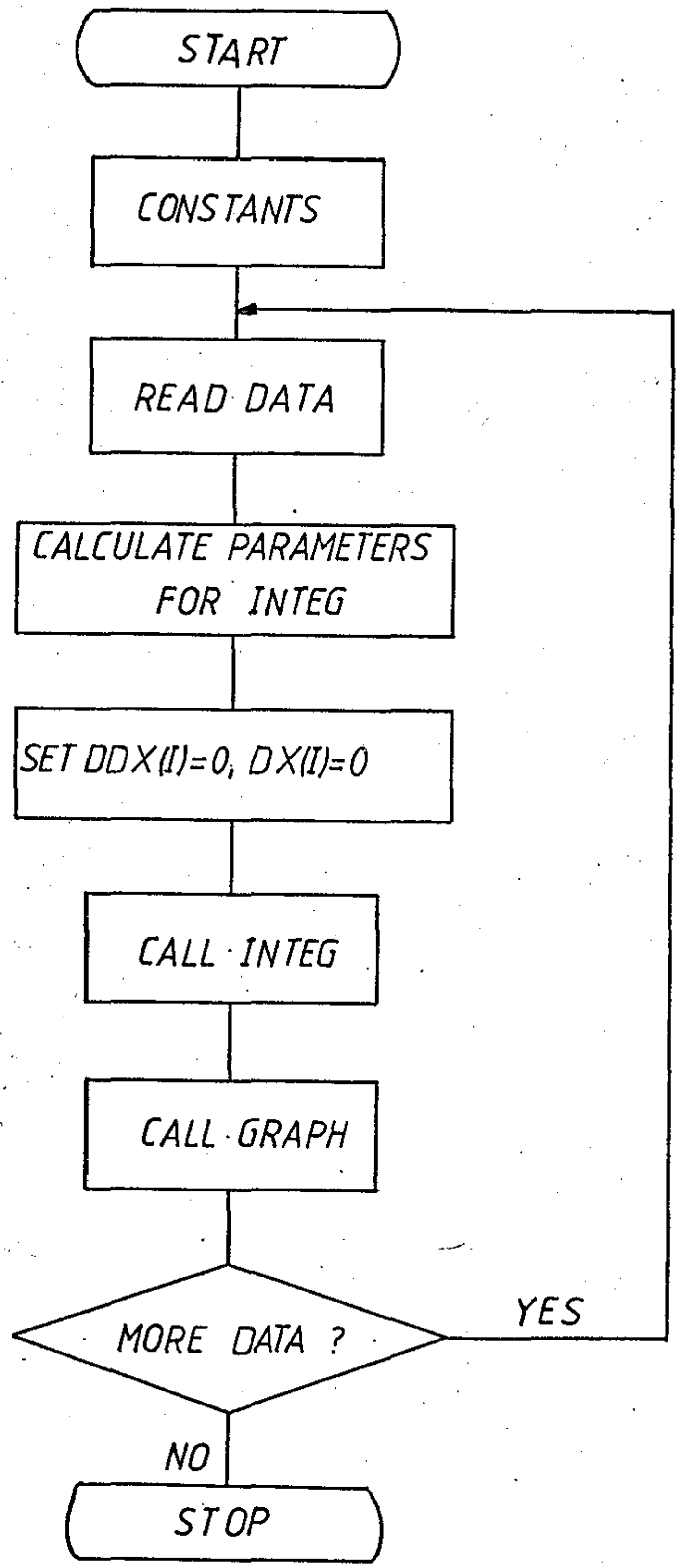


FIG. 5.1.2a FLOW CHART, MAIN PROGRAM

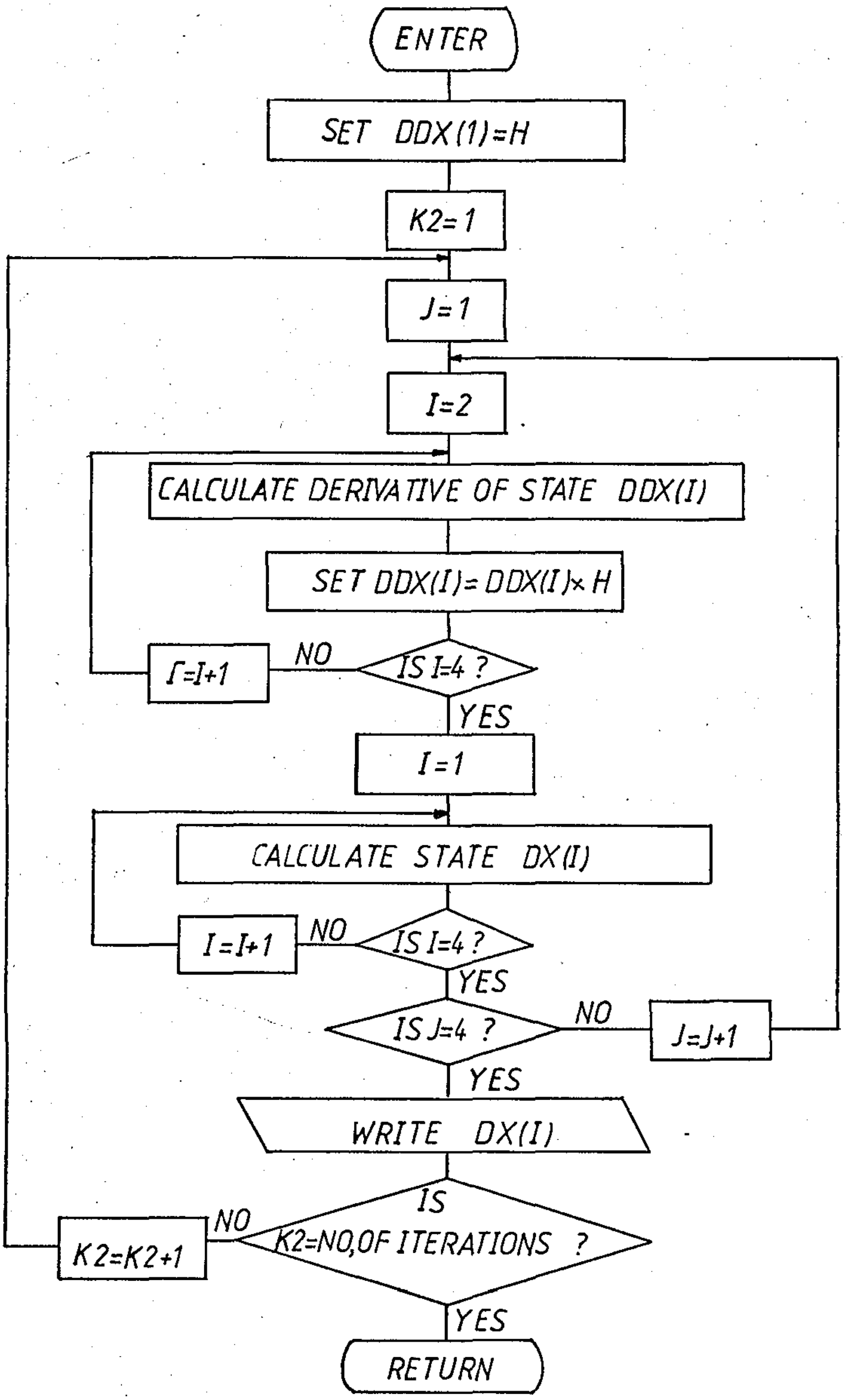


FIG.5.1.2b FLOW CHART, SUBROUTINE

CHAPTER 6

MACHINE RESPONSE

6.1 General

To verify the validity of results obtained by computer simulation of the separately excited d.c. motor model, it is necessary to compare the results, with the results obtained from the machine run under the same conditions and with the same changes as in the program. The change in the output is then recorded. The comparison of the two results gives the validity of the machine model.

The steady state output quantities involving the armature current, field current and speed of the motor are compared to the change in these quantities. To record the changes only the output signal or part of it is balanced with another independent source and zero signal in voltage form is fed to the recorder. The recorder then plots the change in output against time whenever a change in input is made.

6.2 Method of Change in Armature Input

The supply voltage is fixed at 240 volts. A variable series resistance is used to keep the voltage at 100 volts at the armature terminals. If this resistance is changed, then the drop across it will change as well, depending upon the change in resistance, there will be a change in armature current and change in back e.m.f. As shown in Chapter 3, the change in the series resistance can be achieved by switching in a resistance in parallel with the series resistance, and the value of the resistance can be calculated by assuming the value of change in the input voltage at the armature terminals. This parallel resistance is different for different steady state armature currents. This change in input will bring about a change in the output, i.e. change in armature current and the speed of the motors. The scheme for the change in armature input is shown in Figure 6.3.1.

6.3 Method of Change in the Field Voltage

The field input is supplied by a variable voltage power supply. For making a change in the field circuit input, again a resistance combination is used. A standard resistance of 0.1 ohms is used in series with the field winding. To make the change, another standard resistance of 0.1 ohms is connected across the first-standard resistance or one resistance is taken out of parallel combination, depending upon the effect of field strengthening or field weakening has to be considered. This will change the over all resistance of the circuit and set about a change in field current.

The circuit diagram for making the change in the field input is shown in Figure 6.3.2

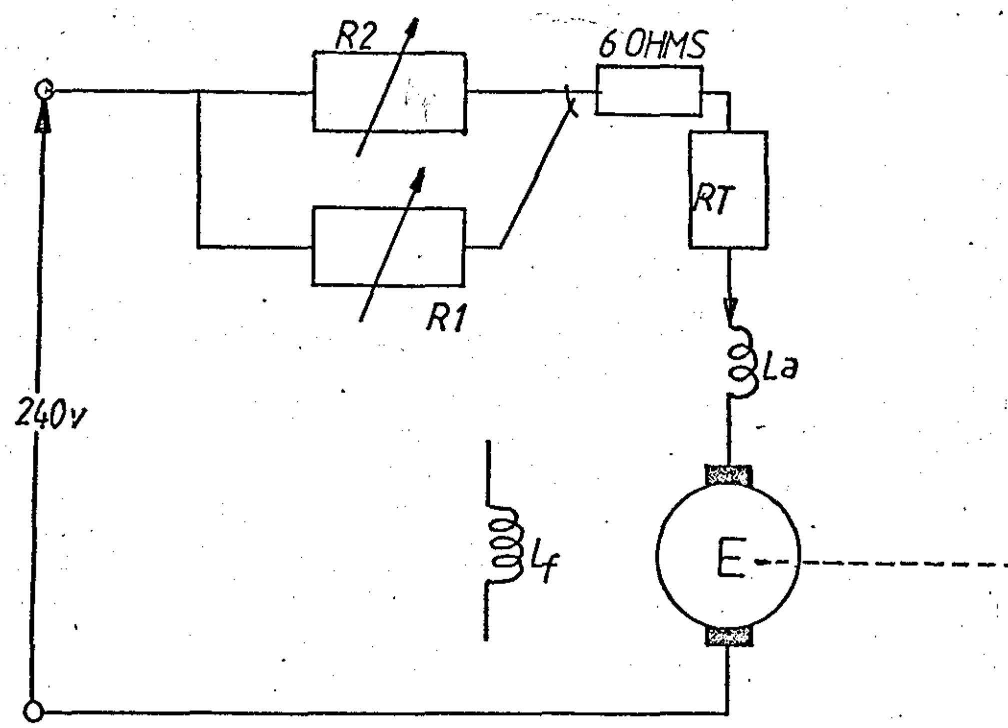


FIG. 6.3.1 Method of change in armature input voltage

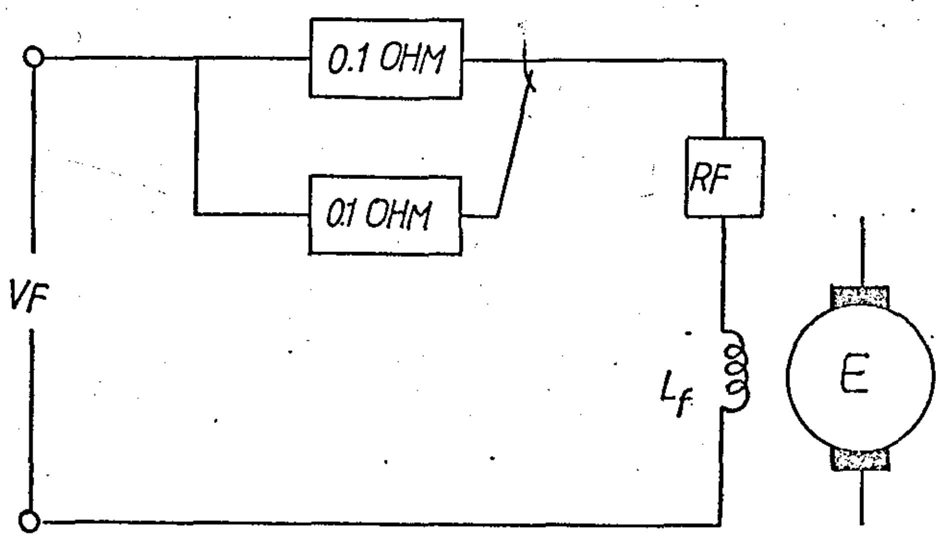


FIG. 6.3.2 Method of change in field input voltage

6.4 Measurement of Change in the Output

The steady quantities are large compared to the change in these quantities. The recording instrument has a range of sensitivity from 0.1V/cm to 50V/cm. If the changes are recorded as superimposed on the steady state values, and low sensitivities, then they may hardly be distinguishable. On the other hand, if the record is made in higher sensitivities of the recorder, to make changes in the output distinguishable, the amplitude of the output will be more than the width of the recording paper.

Figures 6.4.1 and 6.4.2 show schemes which represent the method of recording the change in the output like change in speed and change in armature current. Variable voltage source B is so adjusted that voltage across AA' is zero. A change is then made in the field voltage or armature voltage to produce a change in the output, this change in output is recorded by the U.V. recorder.

The output from the tachogenerator, and the armature current has ripples superimposed on d.c. The magnitude of the ripple is comparable to the change, when only the change is recorded. The presence of ripple makes the recognition of the amplitude of the response difficult. A low pass filter is used to suppress these ripples. The output needed is a zero frequency voltage, therefore the d.c. level will be unaffected while high frequency will filter out, making recorded output more smooth.

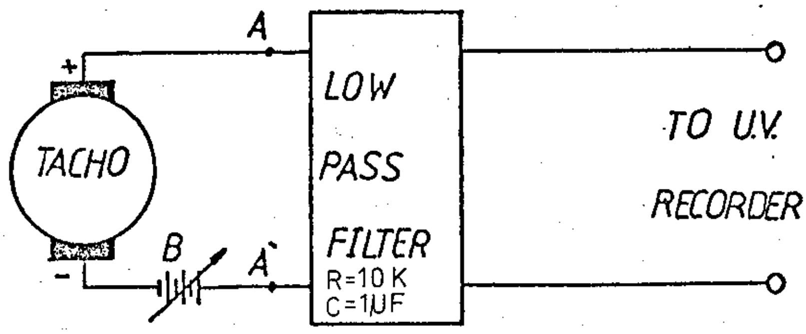


FIG. 6.4.1 Measurement of change in speed

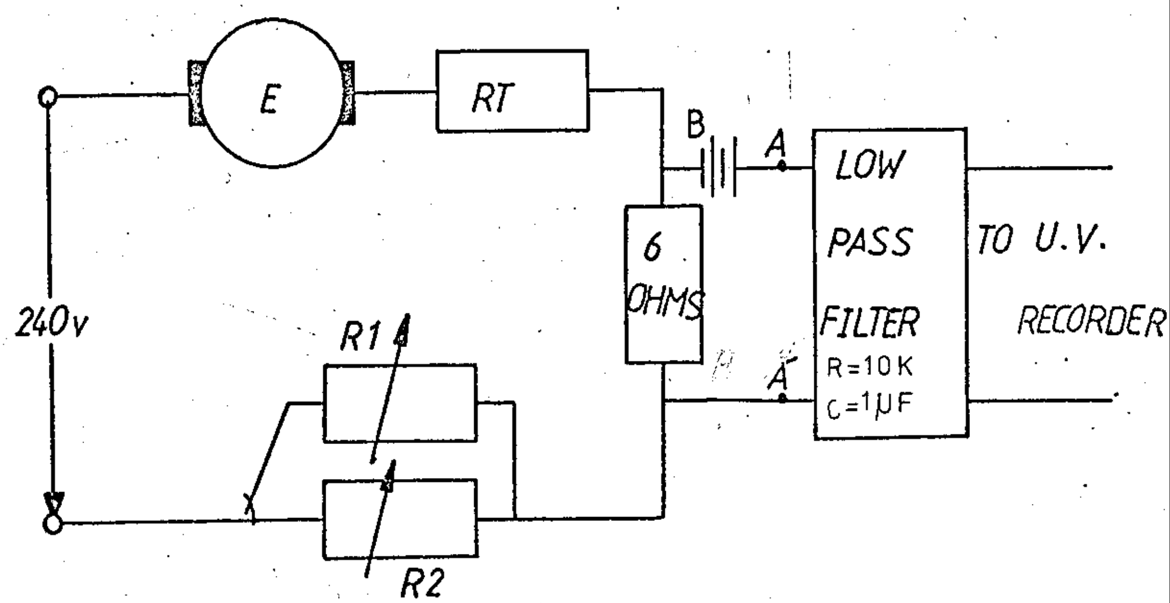


FIG. 6.4.2 Measurement of change in armature current when change in armature input voltage is made

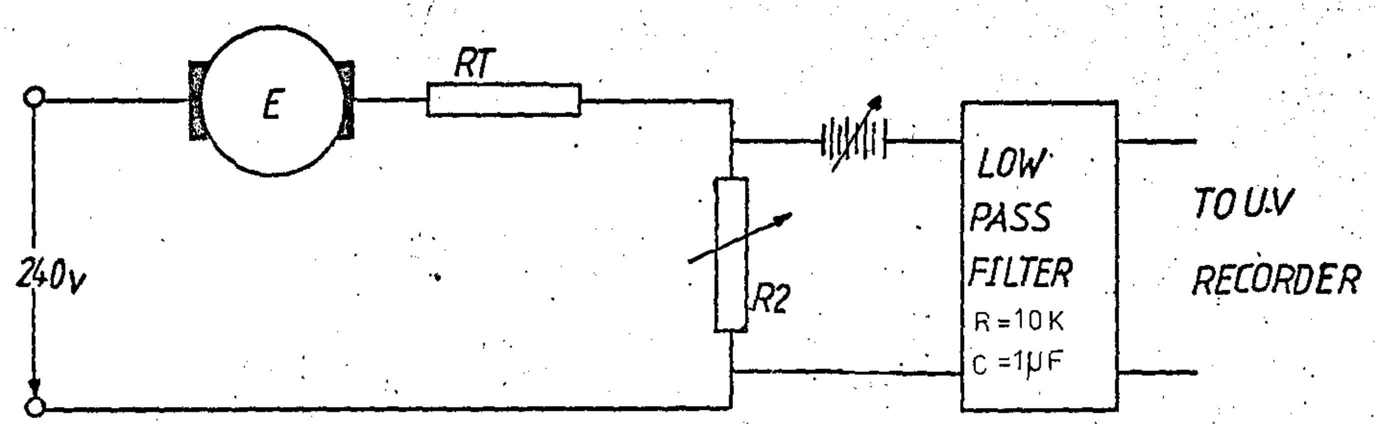


FIG. 6.4.2a. Measurement of change in armature current when change in field input voltage is made

CHAPTER 7

RESULTS AND DISCUSSION

7.1 Introduction

In this chapter the responses of the machine model, in terms of change in speed and change in armature current are compared with the same responses of the actual machine, when the change in either field input or armature input is made. Two steady state values of field current are taken. One near the rated field condition at 3.4A and another for weakened field condition at 2.2A.

Three steady state values of armature current are taken around which a change is made, i.e. 4A, 5A, 6A. The reason for doing so is that the excessive sparking made it difficult to operate the machine beyond that armature current and especially at weakened field current.

The results are tabulated for each of the two steady state values of field current, indicating which input is changed and what output is compared. Amplitude of the machine response is compared with the model response at different instants for transient and steady state values, for each steady state value of armature current. Actual and computed responses for the same instant of time are then used to calculate the percentage error between the machine response and the model response.

The graph plotted for the response of the machine model, using Gino facilities, and the graph obtained from the U.V. recorder for each steady state condition of field current, armature current and for each of the input conditions, are given soon after each table for computed and machine responses

7.2 Discussion

As can be seen from the results tabulated, percentage error between the machine and the model response is normally within $\pm$ 15%. The comparison of transient and steady state response between the model and the machine would have been closer but for two main reasons.

The reason for not having a closer comparison is that the machine has non-salient poles. As can be seen from Figure 2.3, that due to a small air gap, the flux density, which is normally very small in the interpolar region due to large air gap in the salient pole d.c. machine, at the neutral or brush axis is quite high and the emf induced in the short circuited turns is also high, thus increasing the sparking during commutation. Because of this sparking there is a voltage variation of about $\pm$1 volt, around steady state value of 100 volts applied at the armature terminal. Also if the load torque increases with speed then the motor speed may hunt below and above an average value.[20] This variation in speed and voltage makes it difficult to determine the average value precisely.

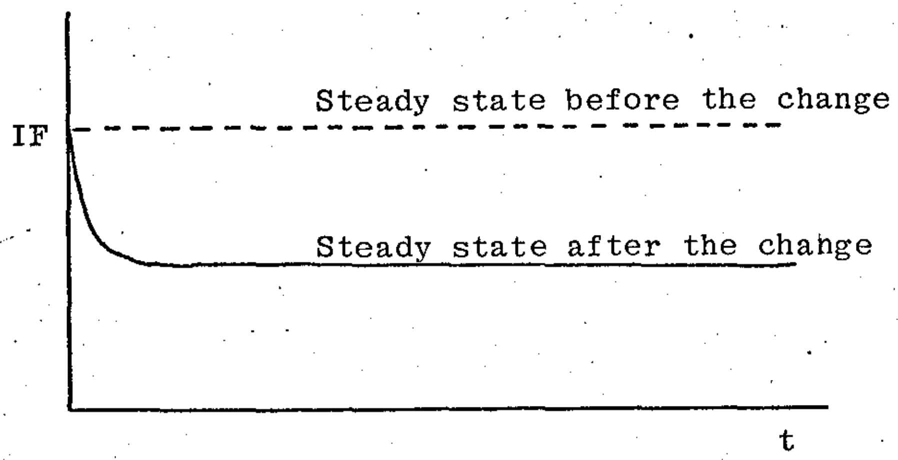


Fig. 7.2.1a

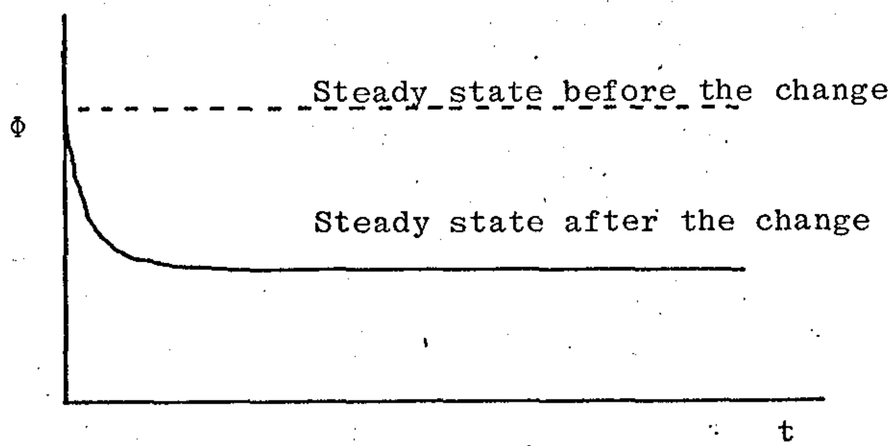


Fig. 7.2.1b

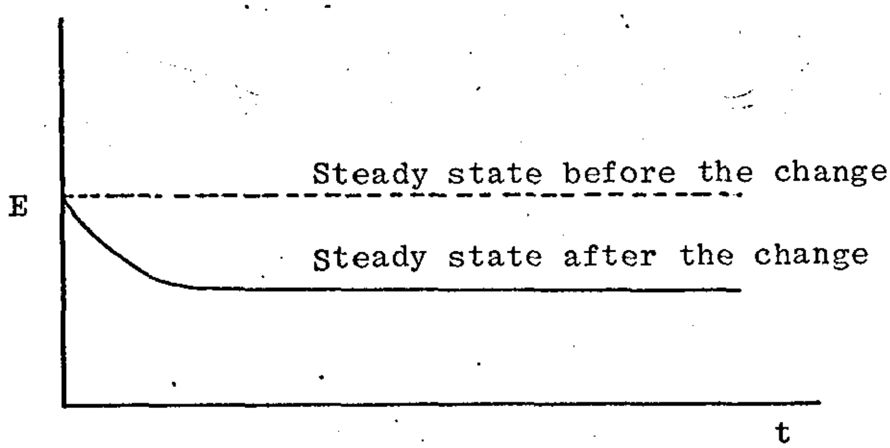


Fig. 7.2.1c

Change in field current, flux per pole and back emf when field input is changed.

TABLE 10

Steady state field current 3.4A
Change in series field resistance from 0.05 to 0.1 ohm
Comparison of change in armature current.

Time in Secs	Steady State Armature Current A	Theoretical Change in Armature Current, A	Practical Change in Armature Current, A	Percentage Difference Between Practice and Theory
0.1	4A	0.0155	0.014	-10.71
0.2		0.0177	0.016	-10.62
0.3		0.0182	0.018	- 0.55
0.5		0.0184	0.020	8.00
1.0		0.0196	0.021	6.66
5.0		0.0213	0.023	7.39
0.1	5	0.0195	0.018	- 8.33
0.2		0.0224	0.021	- 7.14
0.3		0.0230	0.023	0.0
0.5		0.0235	0.024	2.08
1.0		0.0244	0.025	2.40
5.0		0.0273	0.028	2.50
0.1	6	0.0235	0.022	- 6.81
0.5		0.0285	0.030	5.00
1.0		0.0300	0.034	11.70
5.0		0.0332	0.035	5.14
0.1	7	0.028	0.027	- 3.70
0.2		0.033	0.034	2.80
0.5		0.034	0.036	5.55
1.0		0.036	0.039	7.69
5.0		0.044	0.044	0.0

$$\text{Percentage difference} = \frac{\text{Practical value} - \text{Theoretical value}}{\text{Practical value}} \times 100\ \%$$

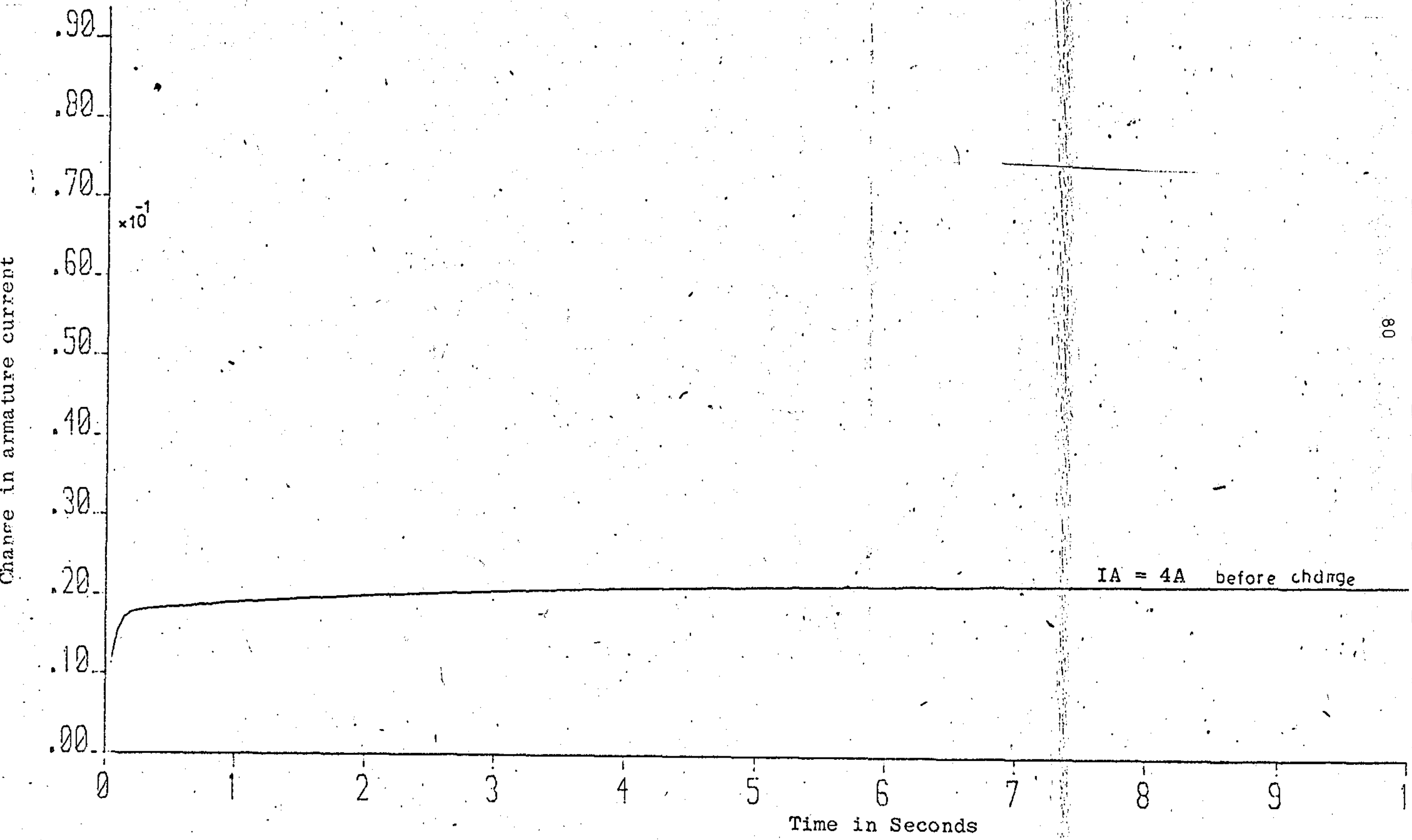
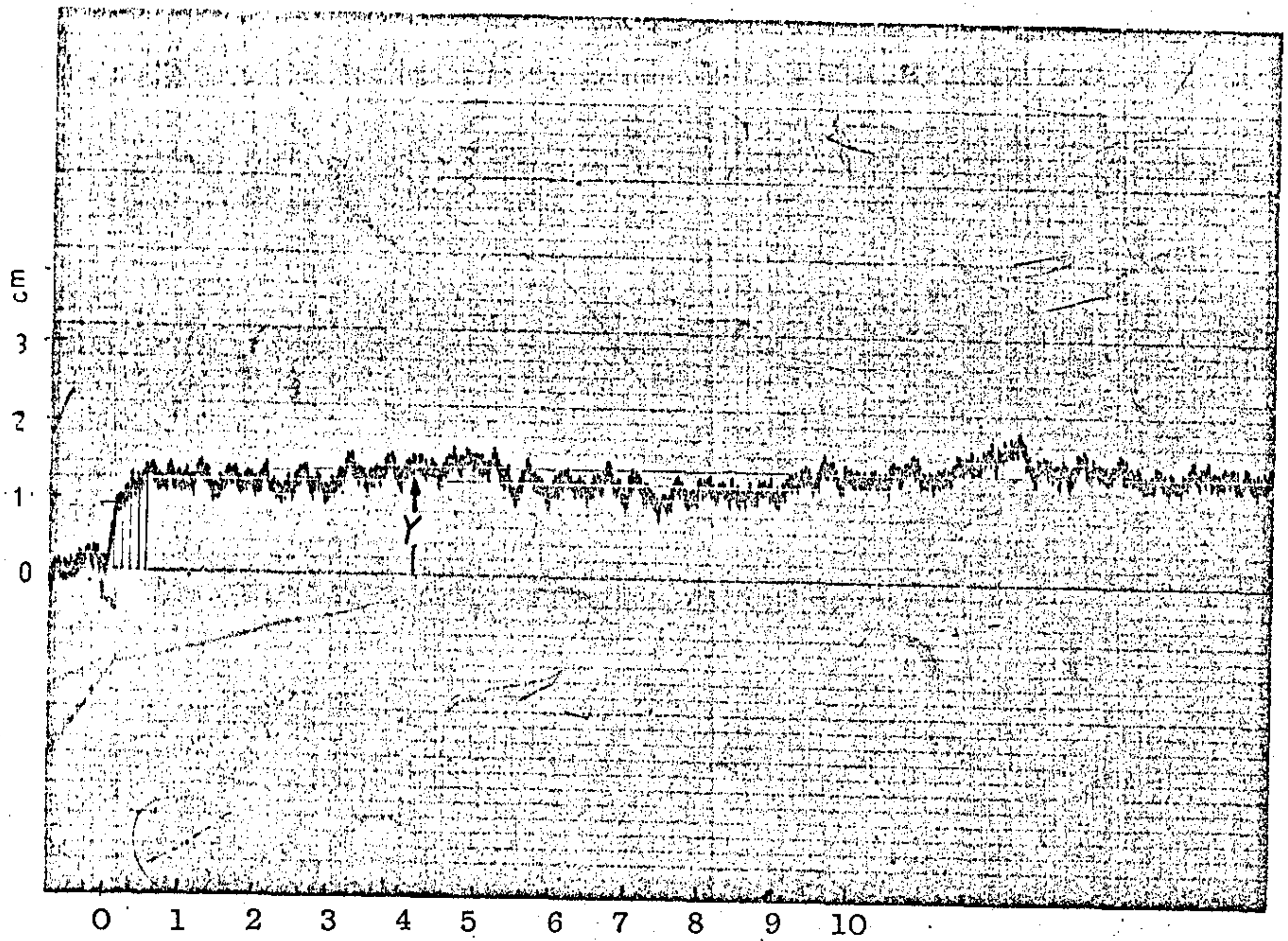


FIG. 7.2.2 Machine model response after a change in the series field resistance

Change in voltage across 35Ω resistance

Change in armature current $\frac{Y.\text{sensitivity}}{35}$

Sensitivity 0.5V/cm



Time in seconds

IA = 4A

FIG. 7.2.3 Machine response

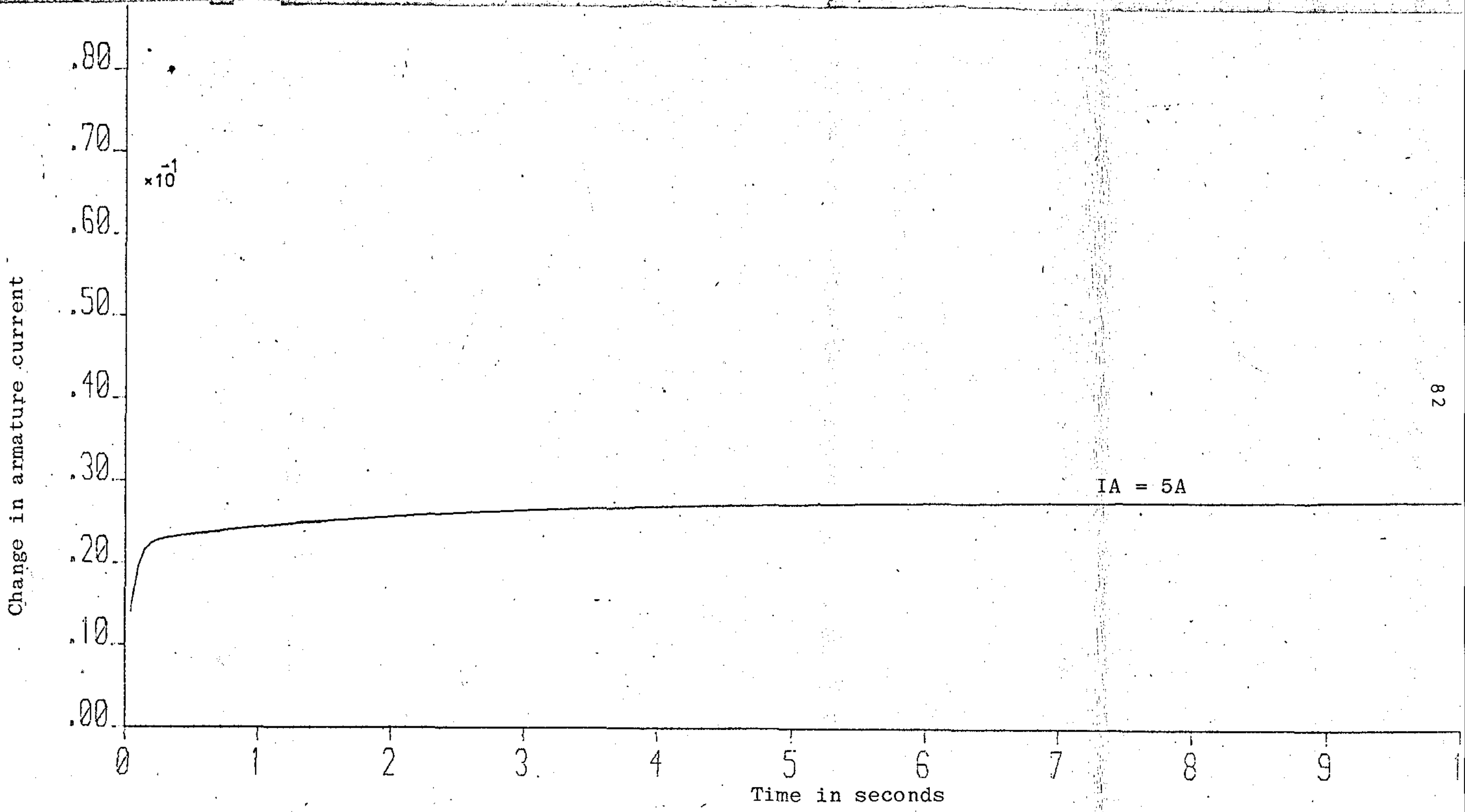


FIG. 7.2.4. Machine model response

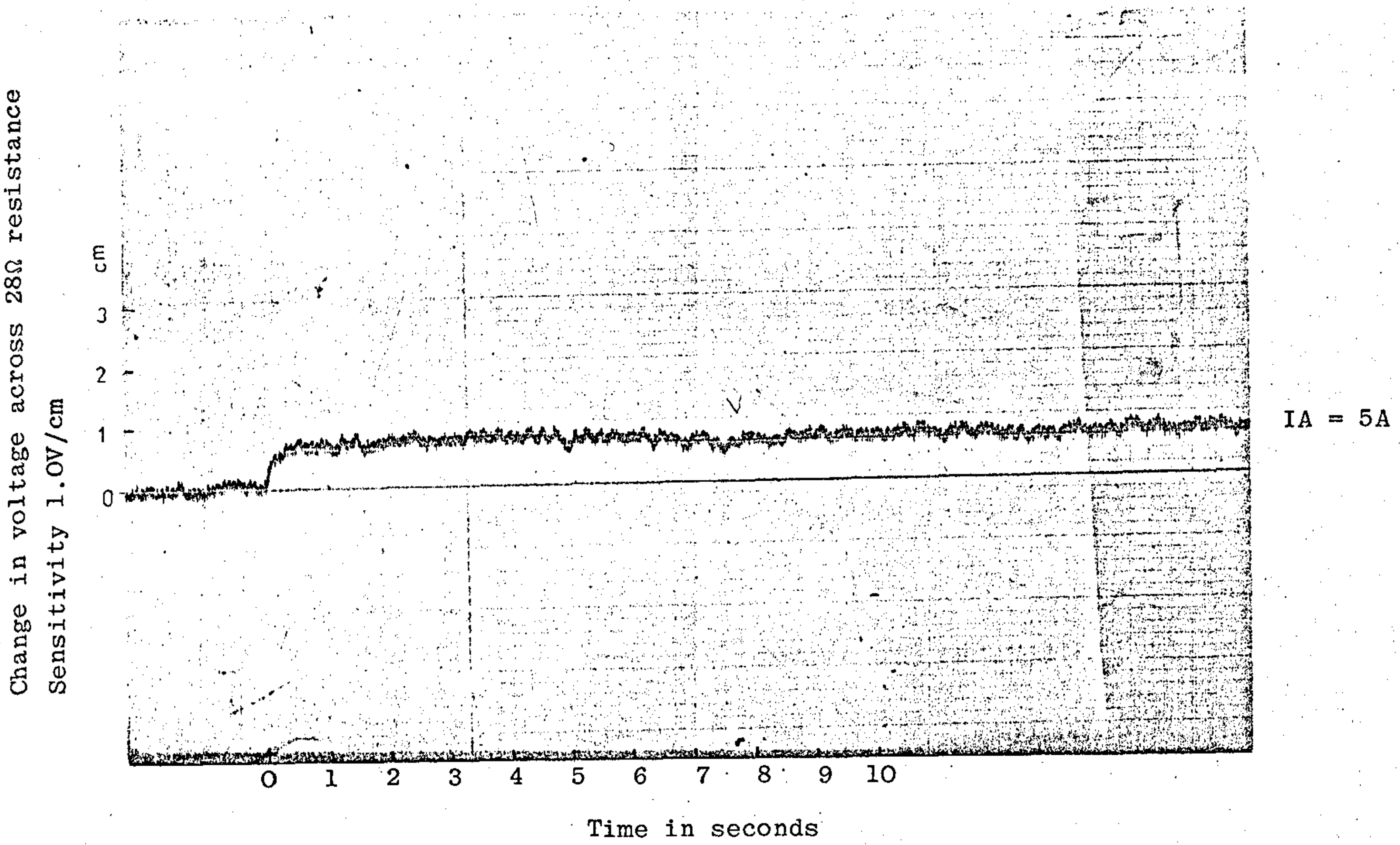


FIG. 7.2.5 Machine Response

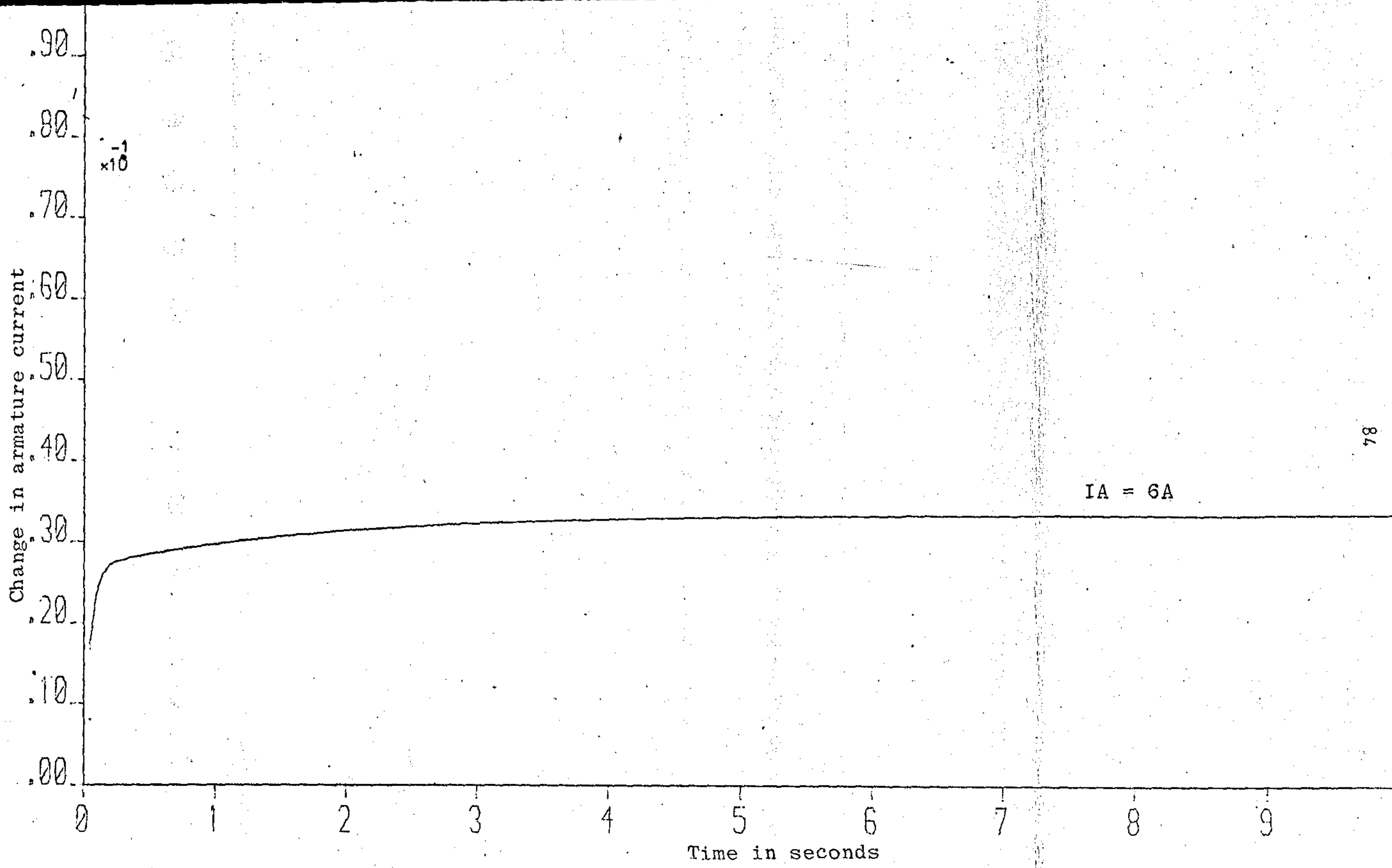


FIG. 7.2.6. Machine model response

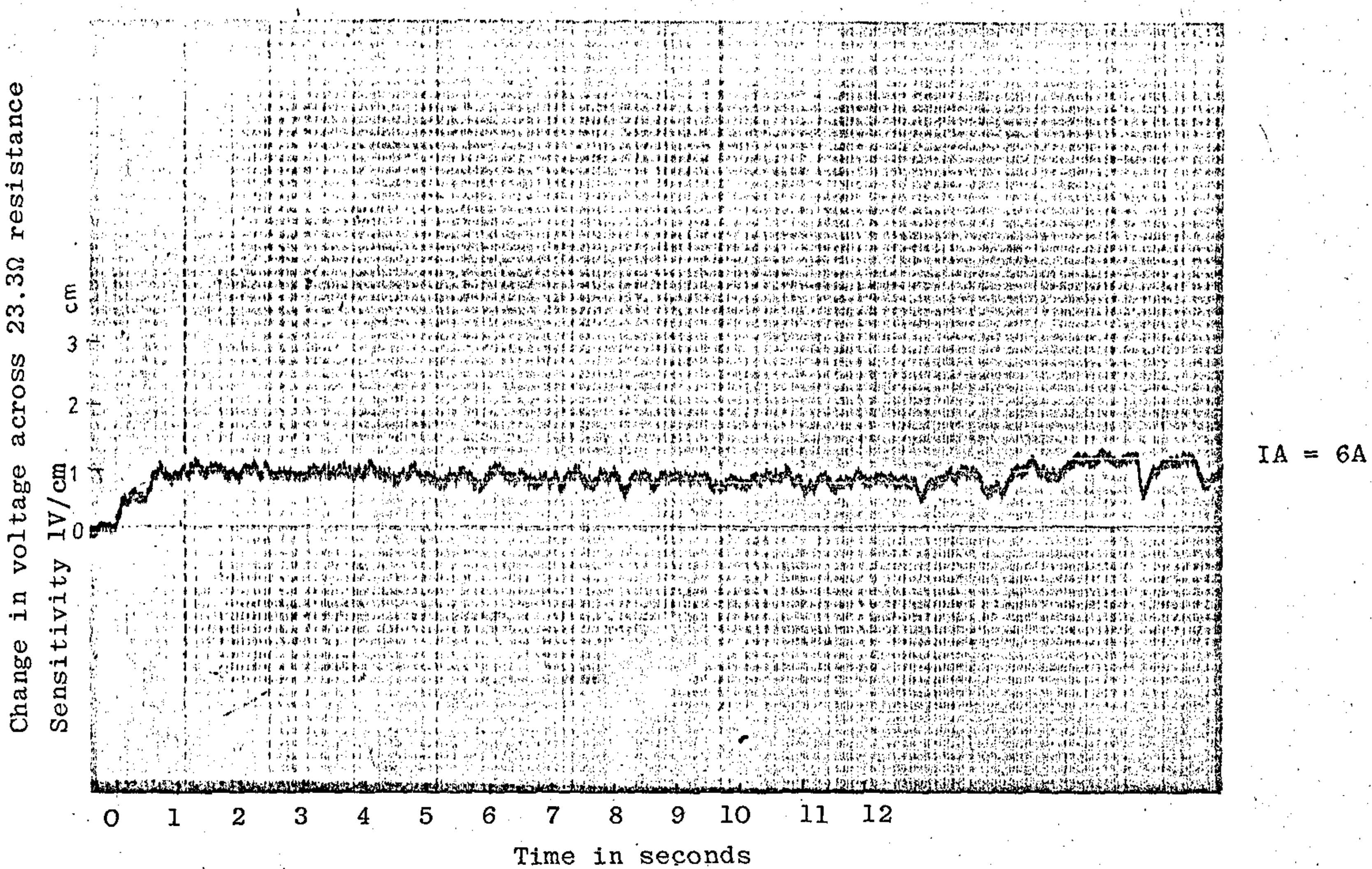


FIG. 7.2.7 Machine response

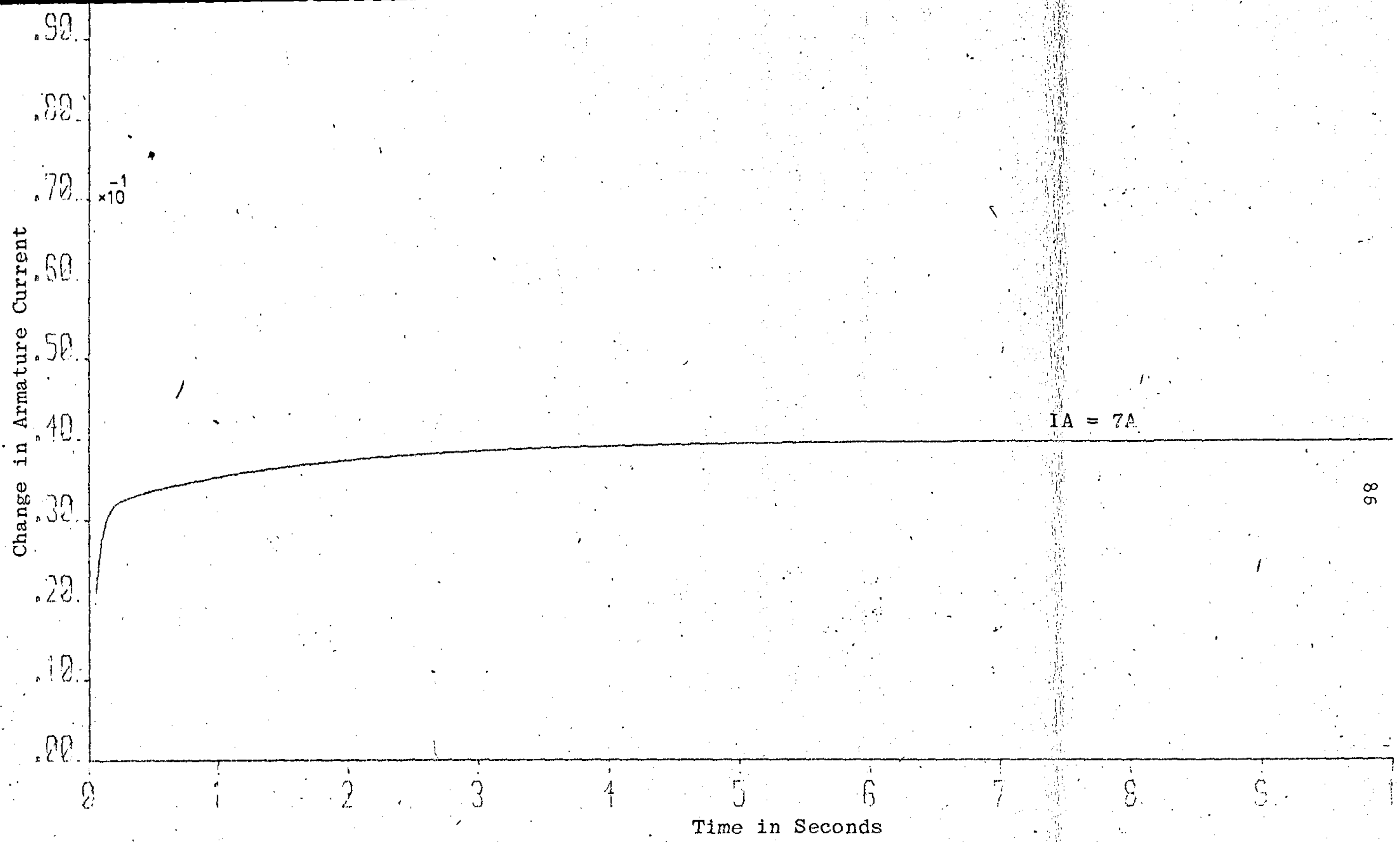


FIG. 7.2.8 Machine Model Response

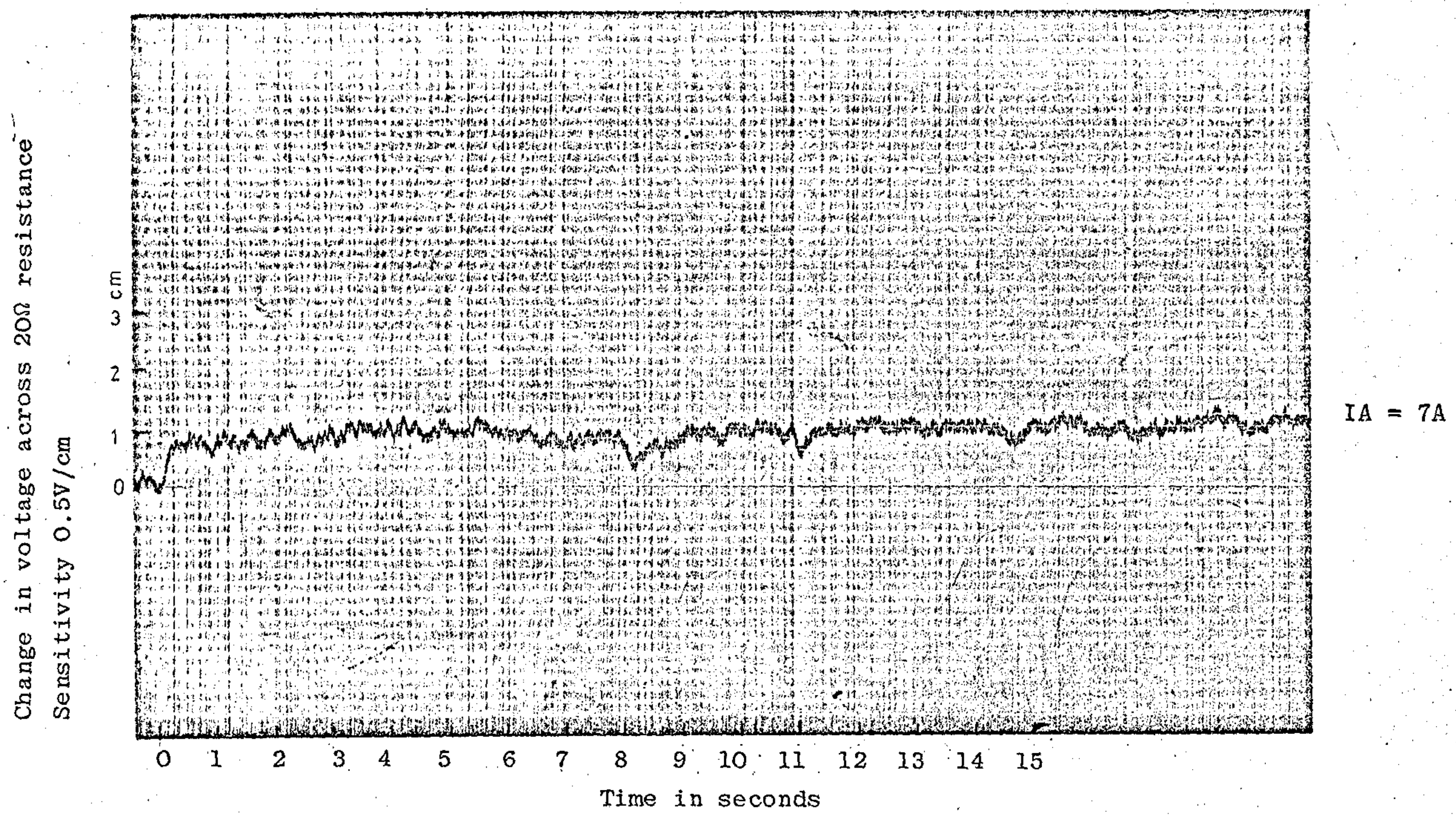


FIG. 7.2.9 Machine response

TABLE 11

Steady state field current 3.4A
Change in armature series resistance
Comparison of change in armature current.

Time in Secs.	Steady State Armature Current A	Theoretical Change in Armature Current, A	Practical Change in Armature Current, A	Percentage Difference Between Practice and Theory
0.1	4A	0.0280	0.025	-12.00
0.2	4900 Ohms resistance connected in parallel with 37 ohms resist.	0.0276	0.030	8.00
0.3		0.0271	0.029	6.55
0.5		0.0263	0.028	2.59
1.0		0.0245	0.027	5.78
2.5		0.0207	0.022	5.91
5.0		0.0178	0.018	1.10
0.1	5A	0.0348	0.032	- 8.75
0.2	3920 Ohms resistance connected in parallel with 28 ohms resist.	0.0343	0.033	- 3.94
0.3		0.0340	0.037	8.11
0.5		0.0324	0.032	- 1.25
1.0		0.0300	0.030	0.0
2.5		0.0250	0.027	7.41
5.0		0.0218	0.023	6.57
0.1	6A	0.0416	0.042	0.95
0.2	3266 Ohms resistance connected in parallel with 23.33 ohms. resist	0.0410	0.045	8.88
0.3		0.040	0.046	13.04
0.5		0.0384	0.043	10.69
1.0		0.0350	0.038	7.89
2.5		0.0292	0.028	- 4.28
5.0		0.026	0.025	4.00

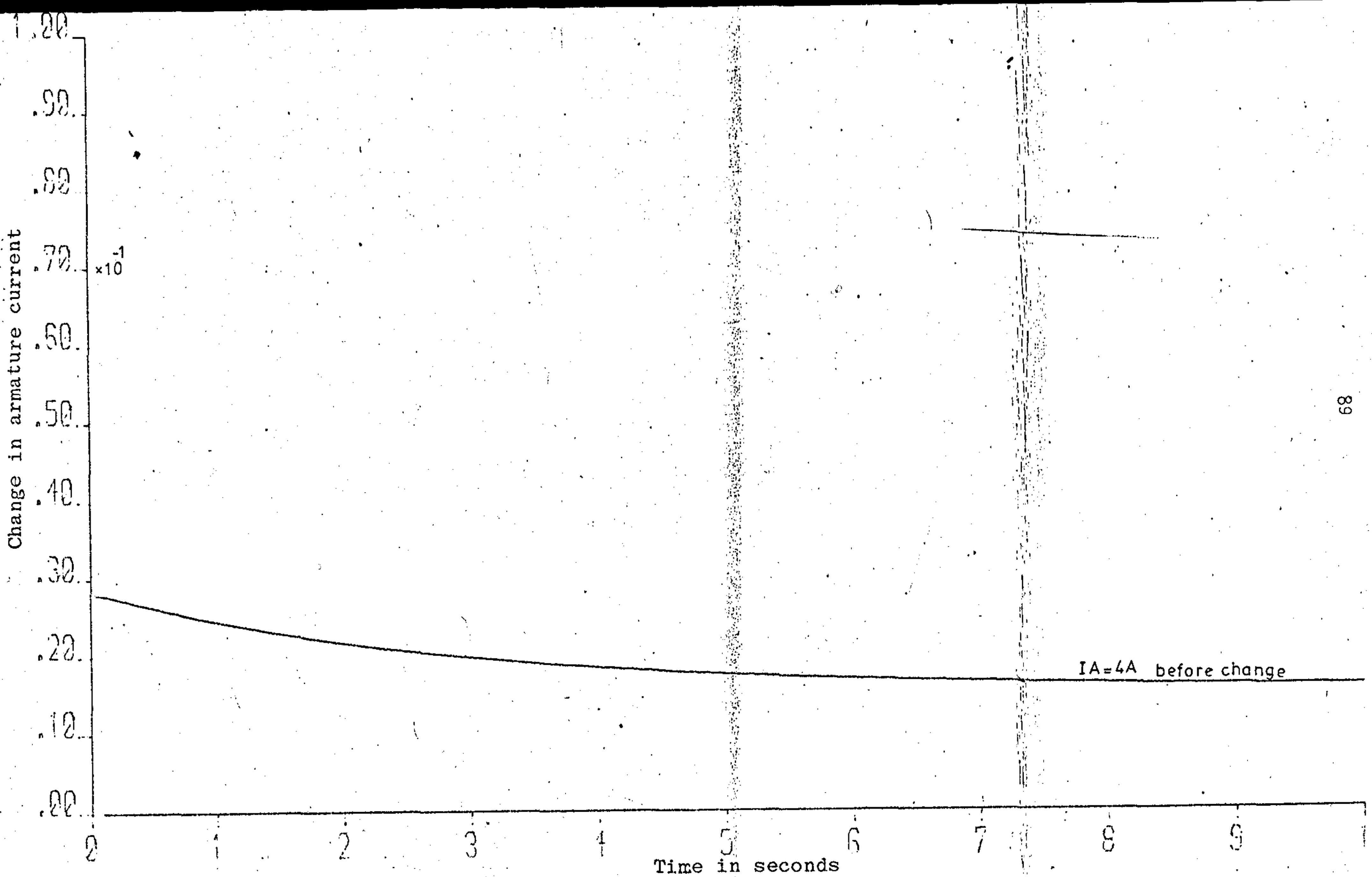


FIG. 7.2.10 Machine Model Response
after a change in the series armature resistance

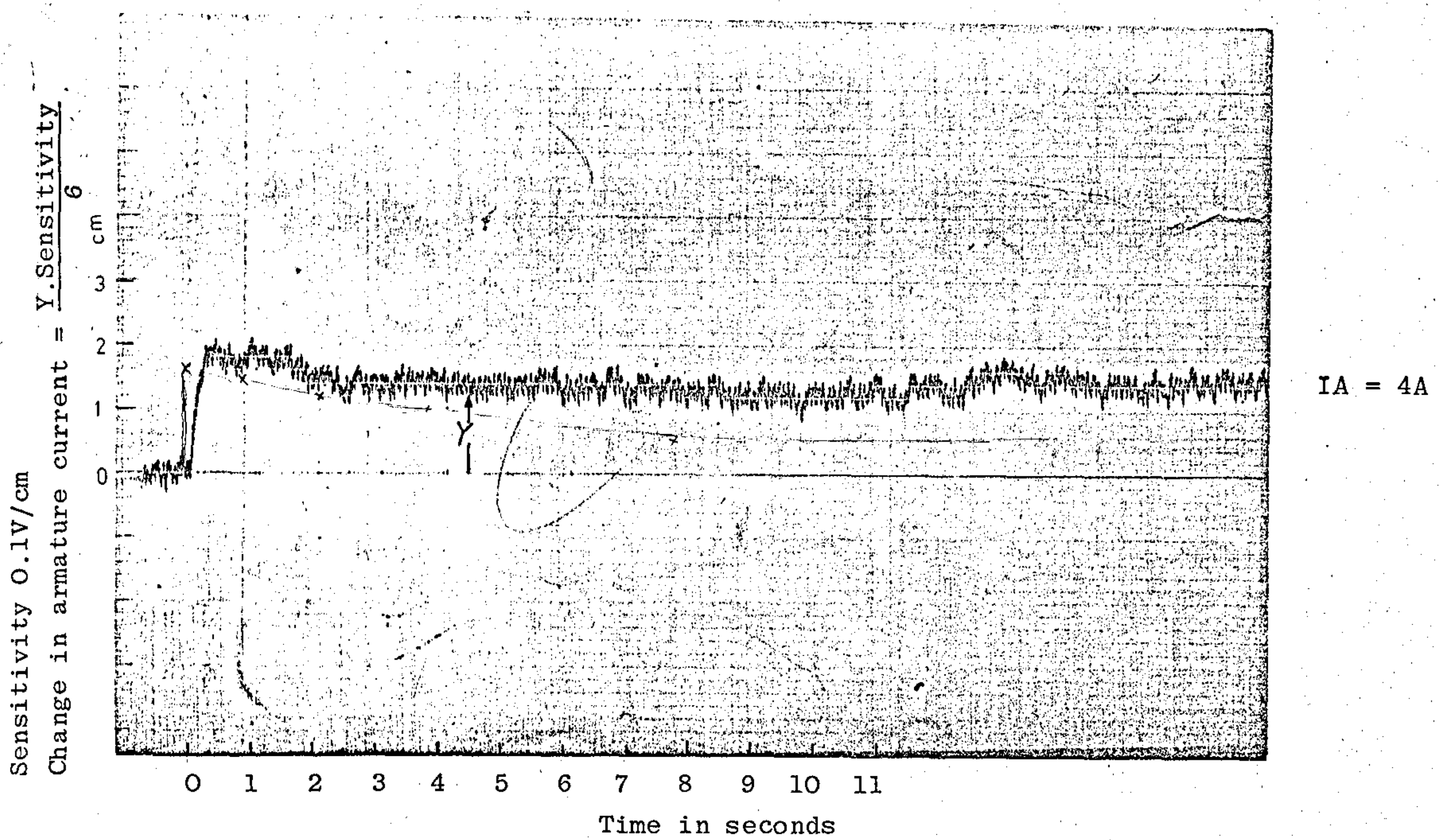


FIG. 7.2.11 Machine response

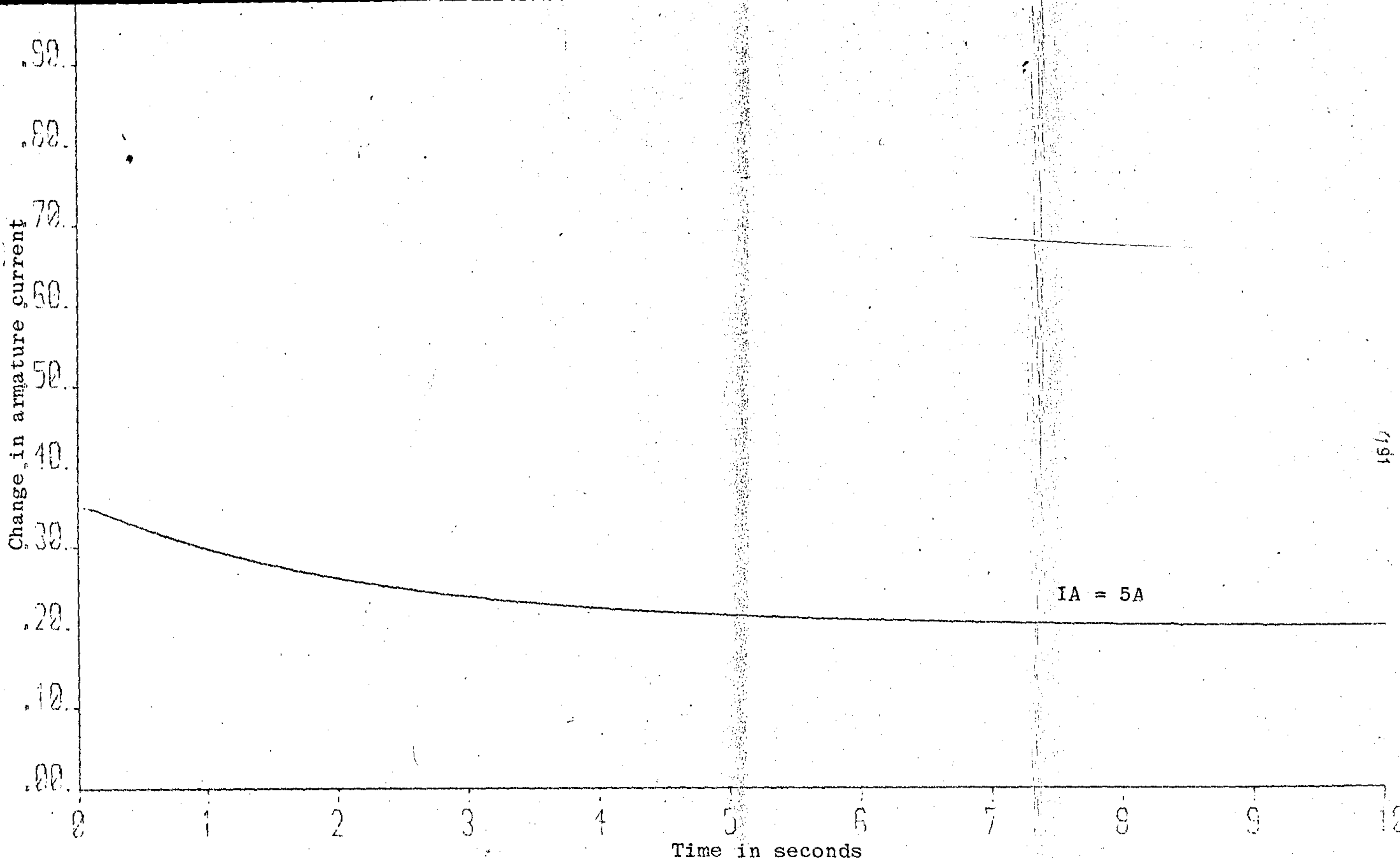


FIG. 7.2.12 Machine Model response

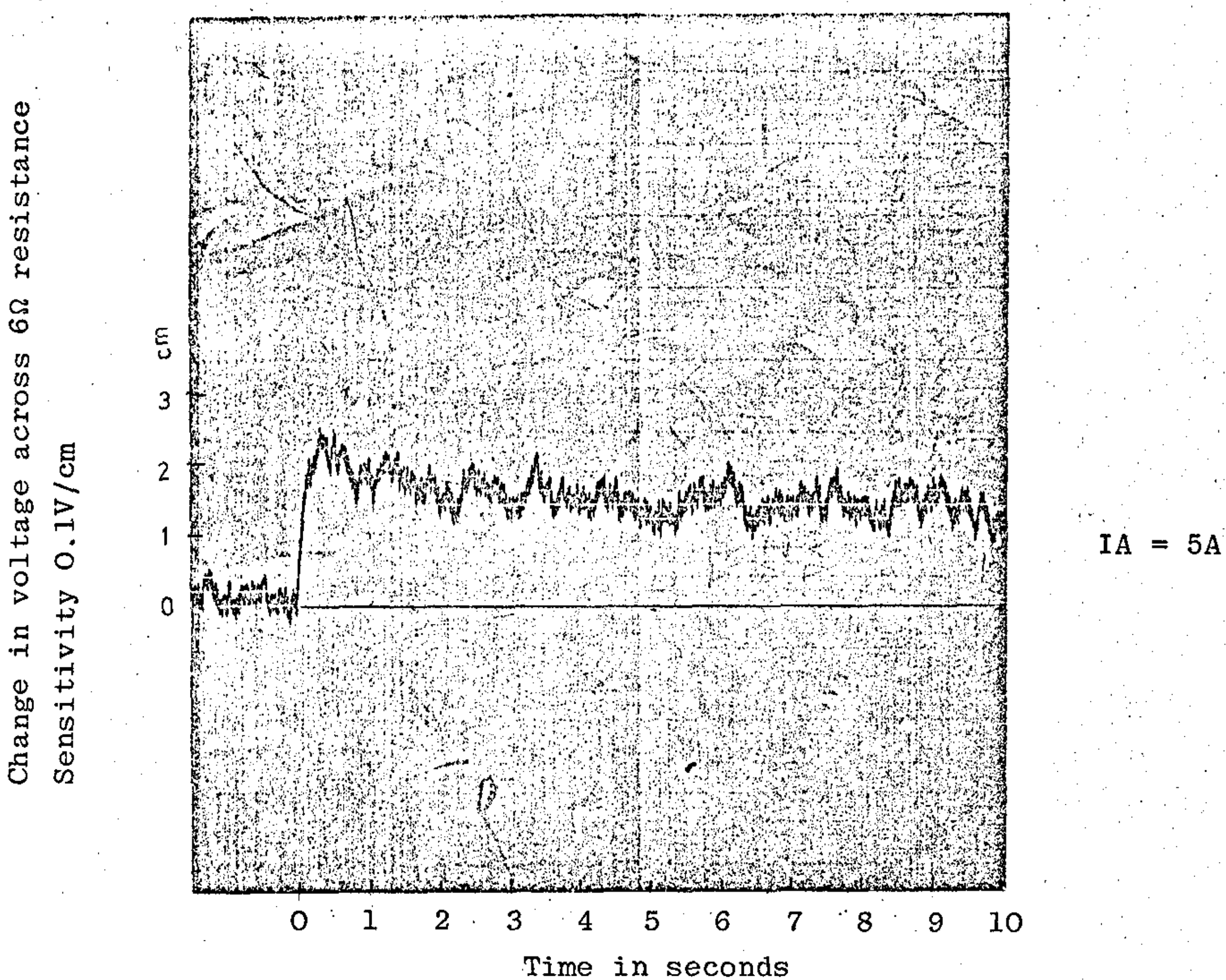


FIG. 7.2.13 Machine response

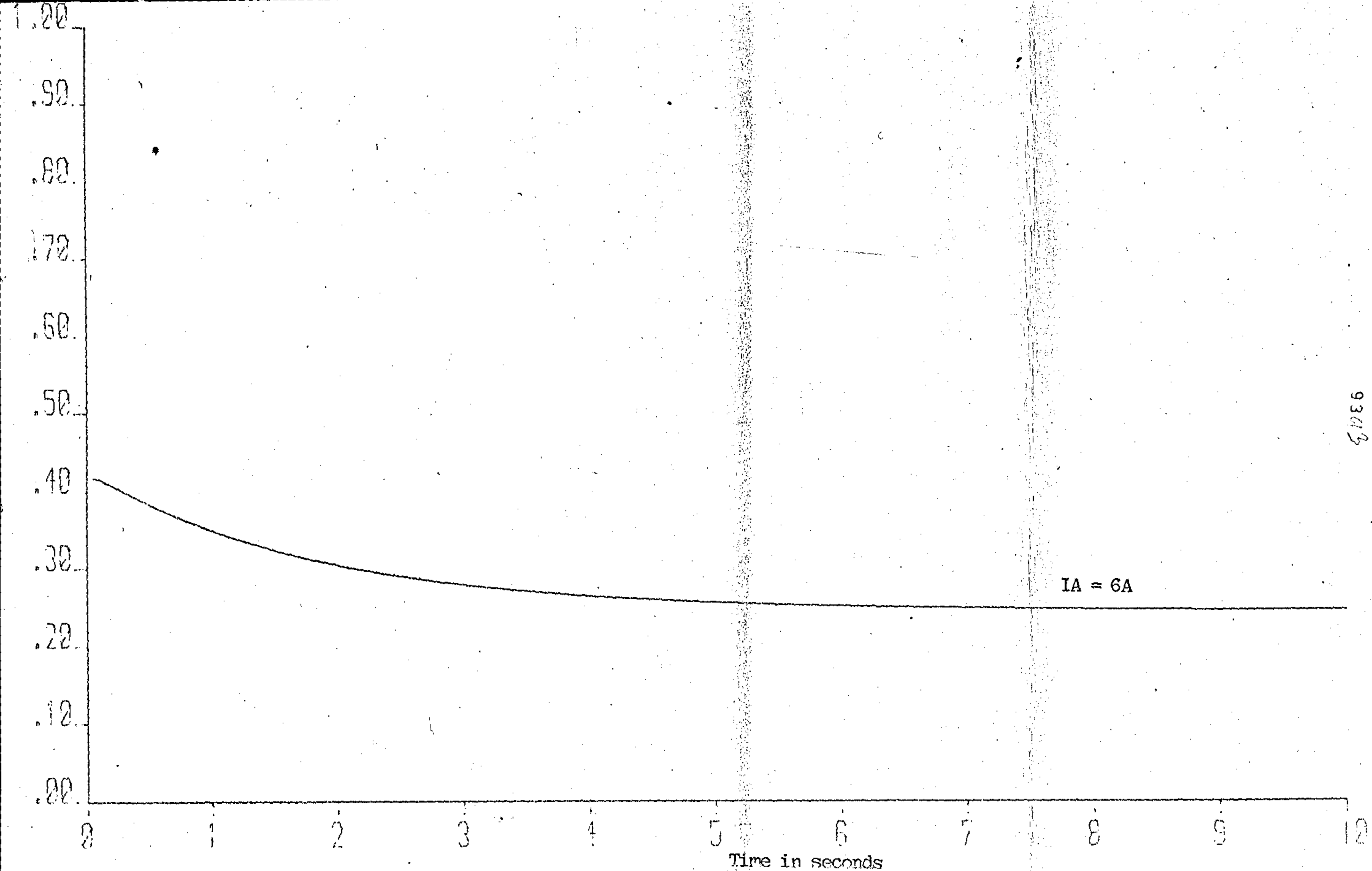


FIG. 7.2.14 Machine Model Response

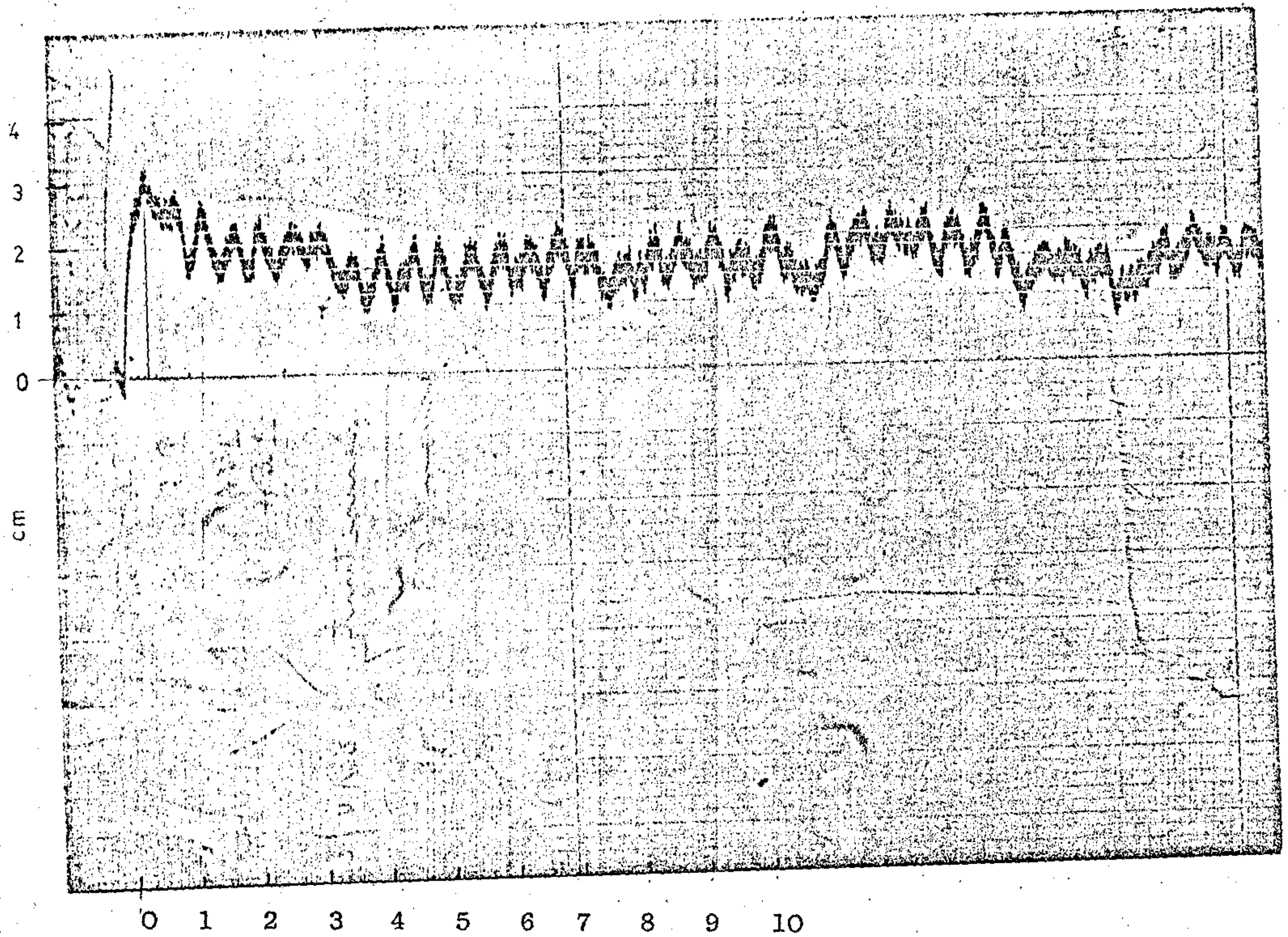


FIG. 7.2.15 Machine response

TABLE 12

Steady state field current 2.2A
Change in series field resistance from 0.05 to 0.1 ohm
Comparison of change in armature current

Time in Secs	Steady State Armature Current A	Theoretical Change in Armature Current, A	Practical Change in Armature Current, A	Percentage Difference Between Practice and Theory
0.1	4A	0.0162	0.015	- 6.66
0.2		0.0210	0.019	-10.55
0.3		0.0230	0.021	- 9.25
0.5		0.0244	0.024	- 2.50
1.0		0.0500	0.026	3.84
5.0		0.0284	0.030	5.30
10.0		0.0298	0.032	6.87
0.1	5A	0.0200	0.020	0.0
0.2		0.0260	0.023	-13.04
0.3		0.0290	0.028	- 3.57
0.5		0.0300	0.029	- 3.45
1.0		0.0310	0.030	- 3.33
5.0		0.0356	0.036	1.00
0.1	6A	0.0240	0.024	0.0
0.2		0.0316	0.0300	- 5.33
0.3		0.0340	0.0320	- 6.25
0.4		0.0350	0.0340	- 2.94
0.5		0.0357	0.0360	0.83
1.0		0.0370	0.0370	0.0
5.0		0.0426	0.0450	5.33

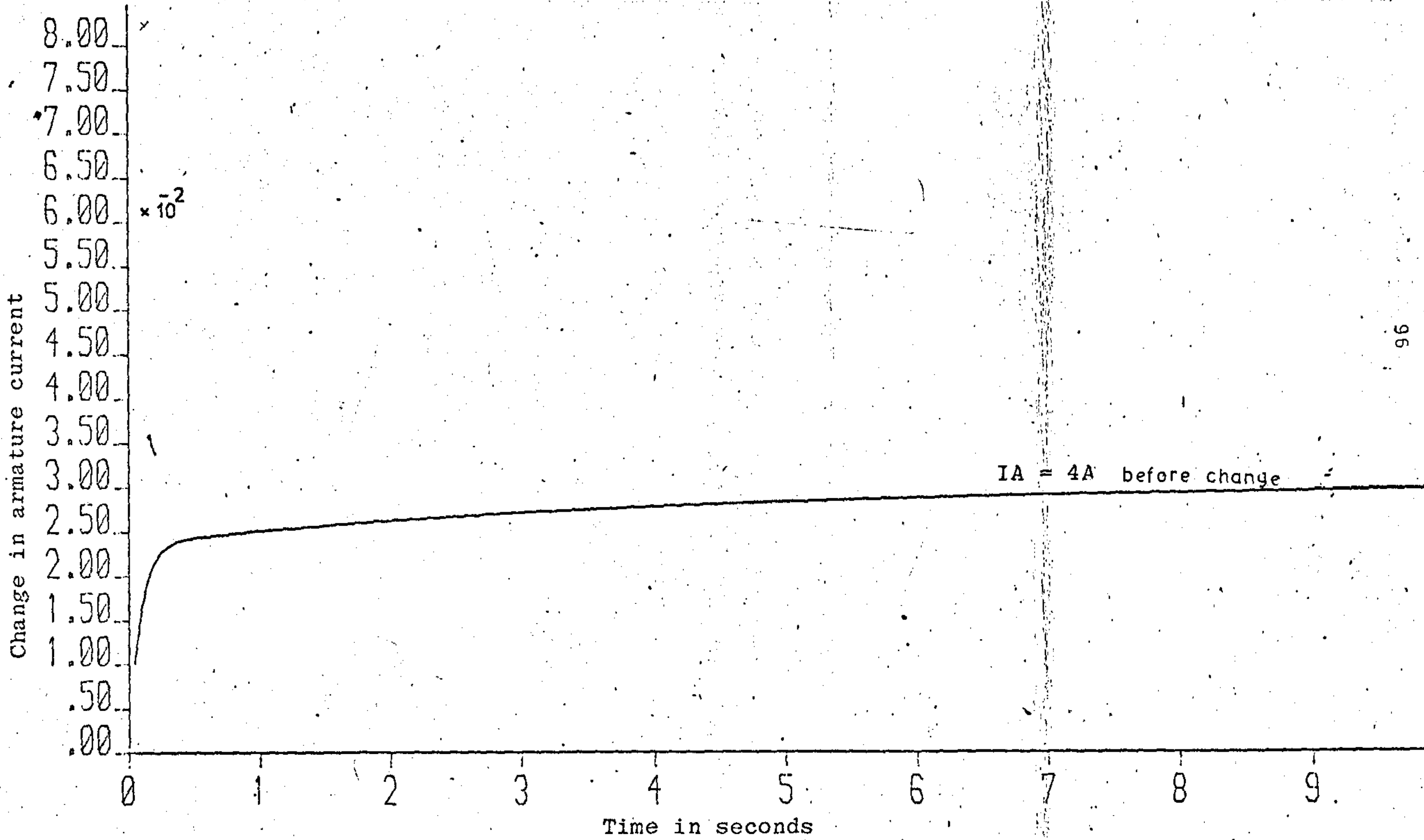


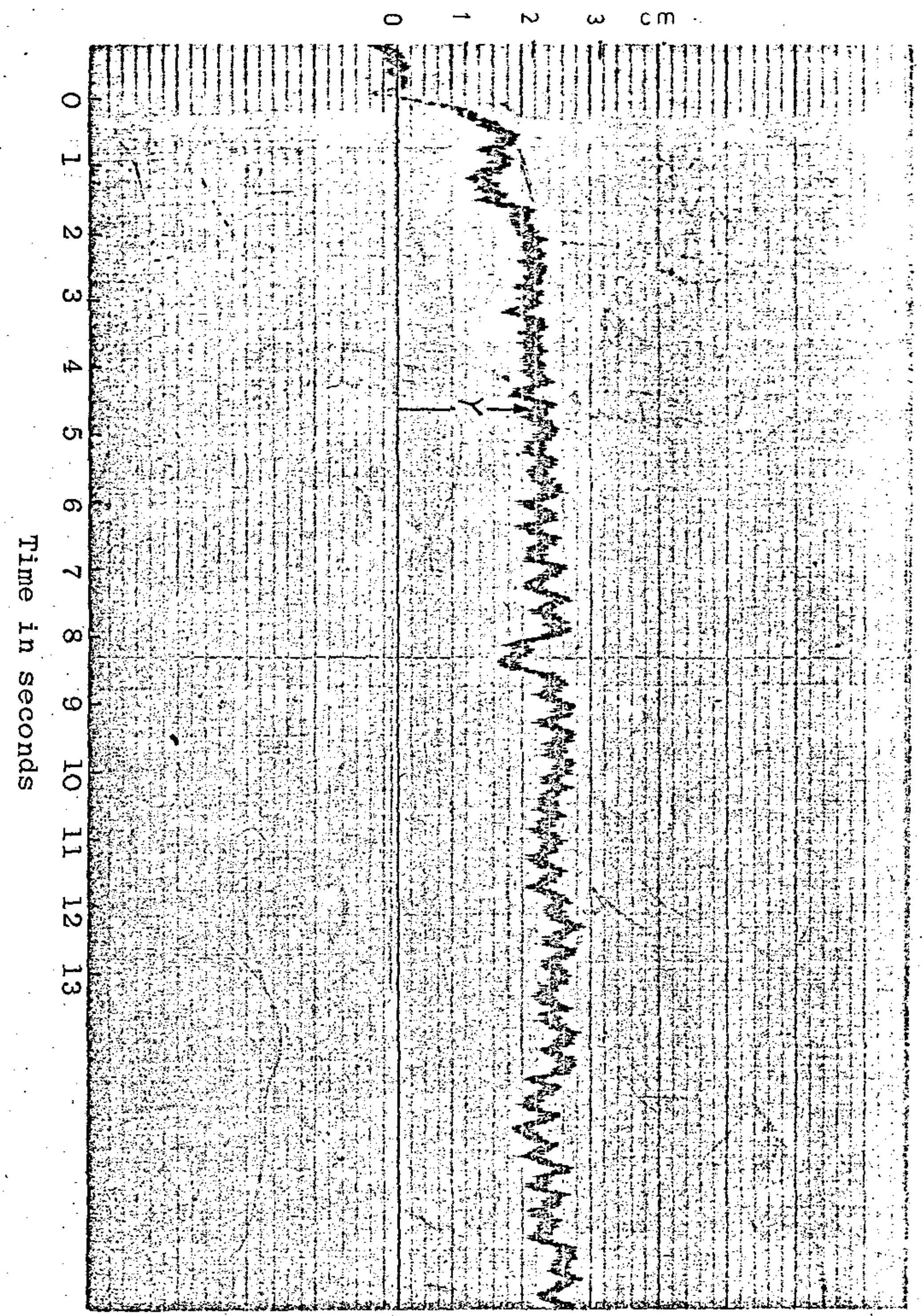
FIG. 7.2.16 Machine model response

after a change in the series field resistance

Change in voltage across 35Ω resistance

Sensitivity 0.5V/cm

$$\text{Change in armature current} = \frac{\text{Y.Sensitivity}}{35}, \text{A}$$



IA = 4A

FIG. 7.2.17 Machine response

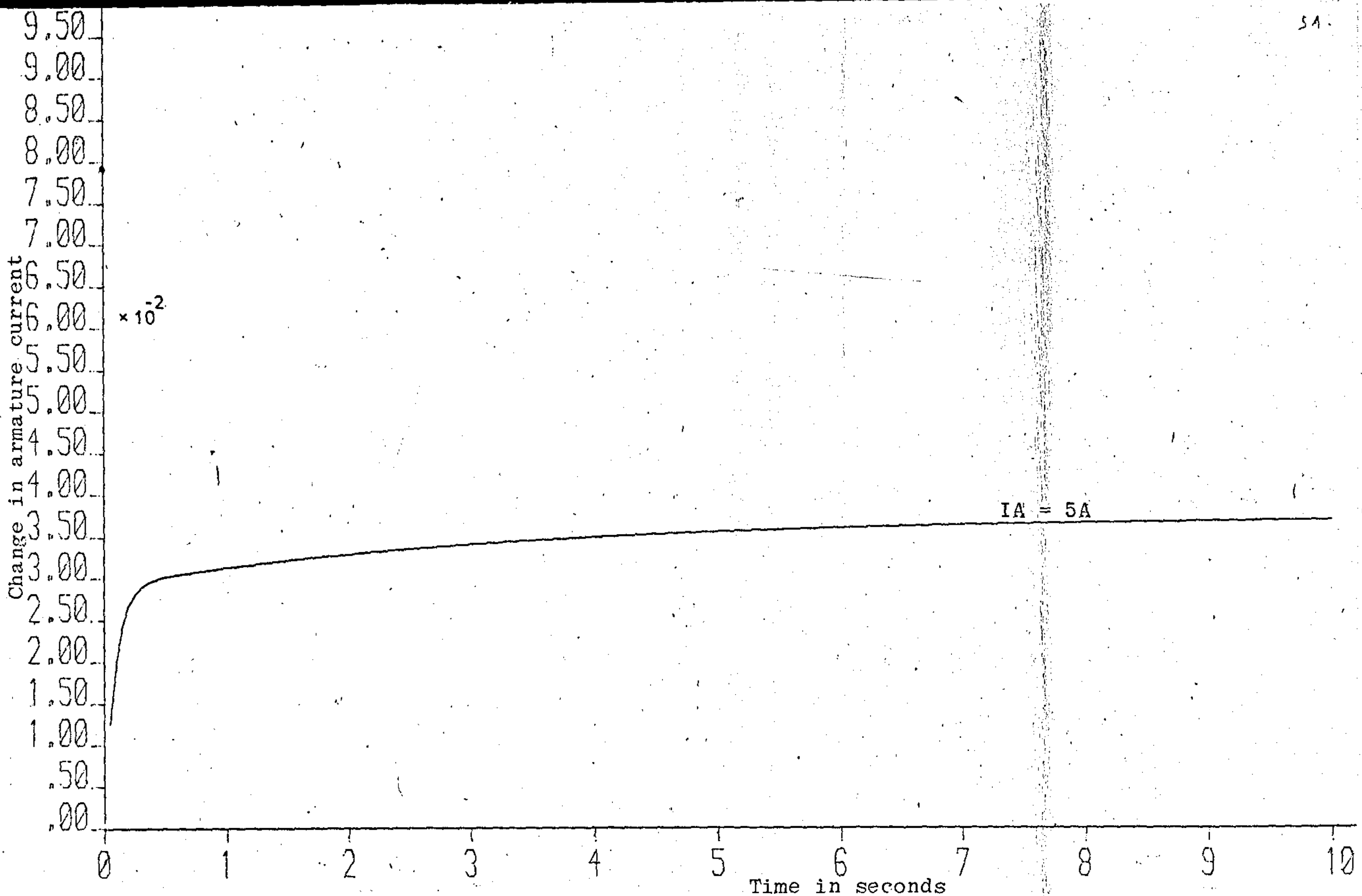


FIG. 7.2.18 Machine model response

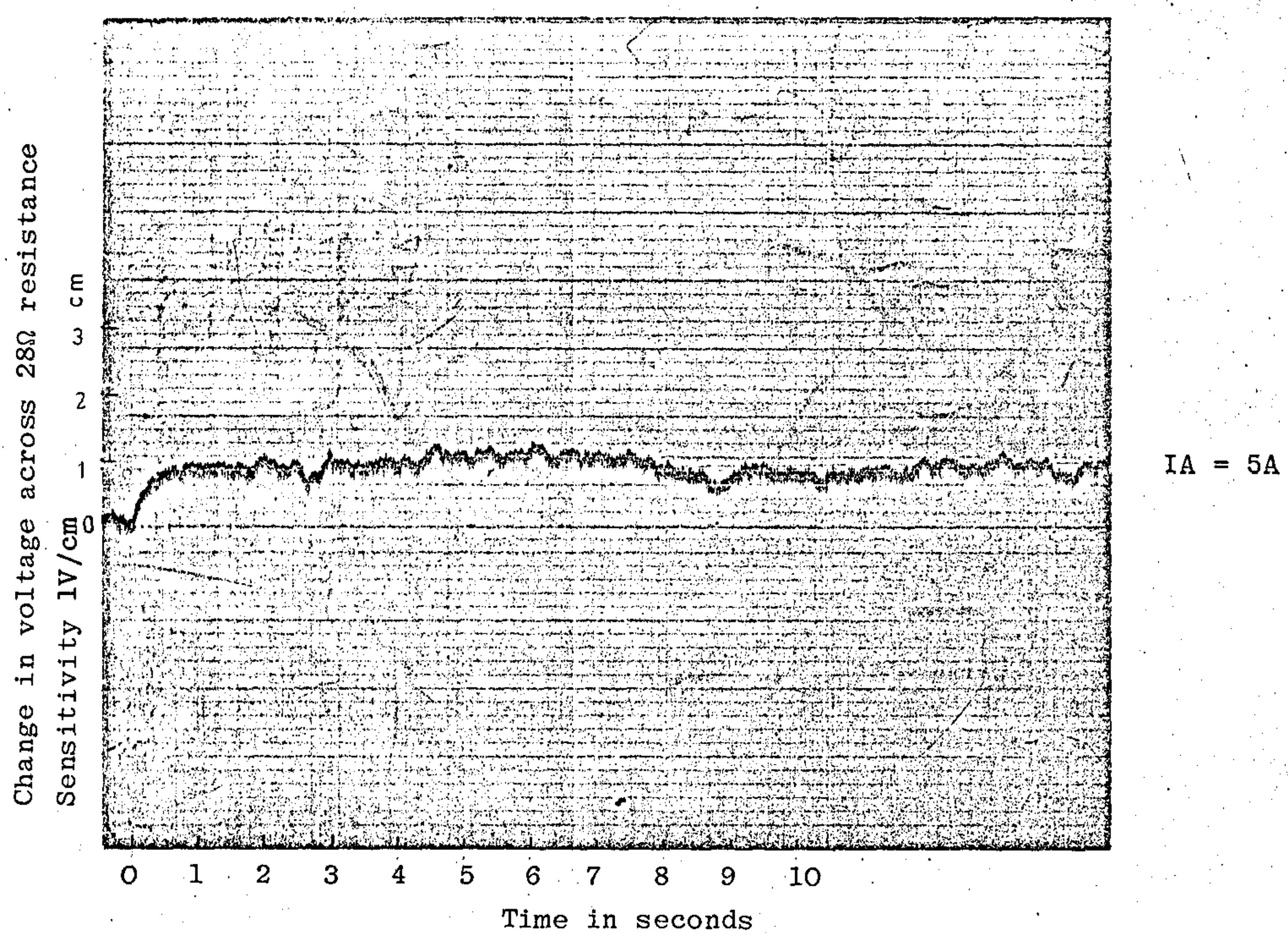


FIG. 7.2.19 Machine response

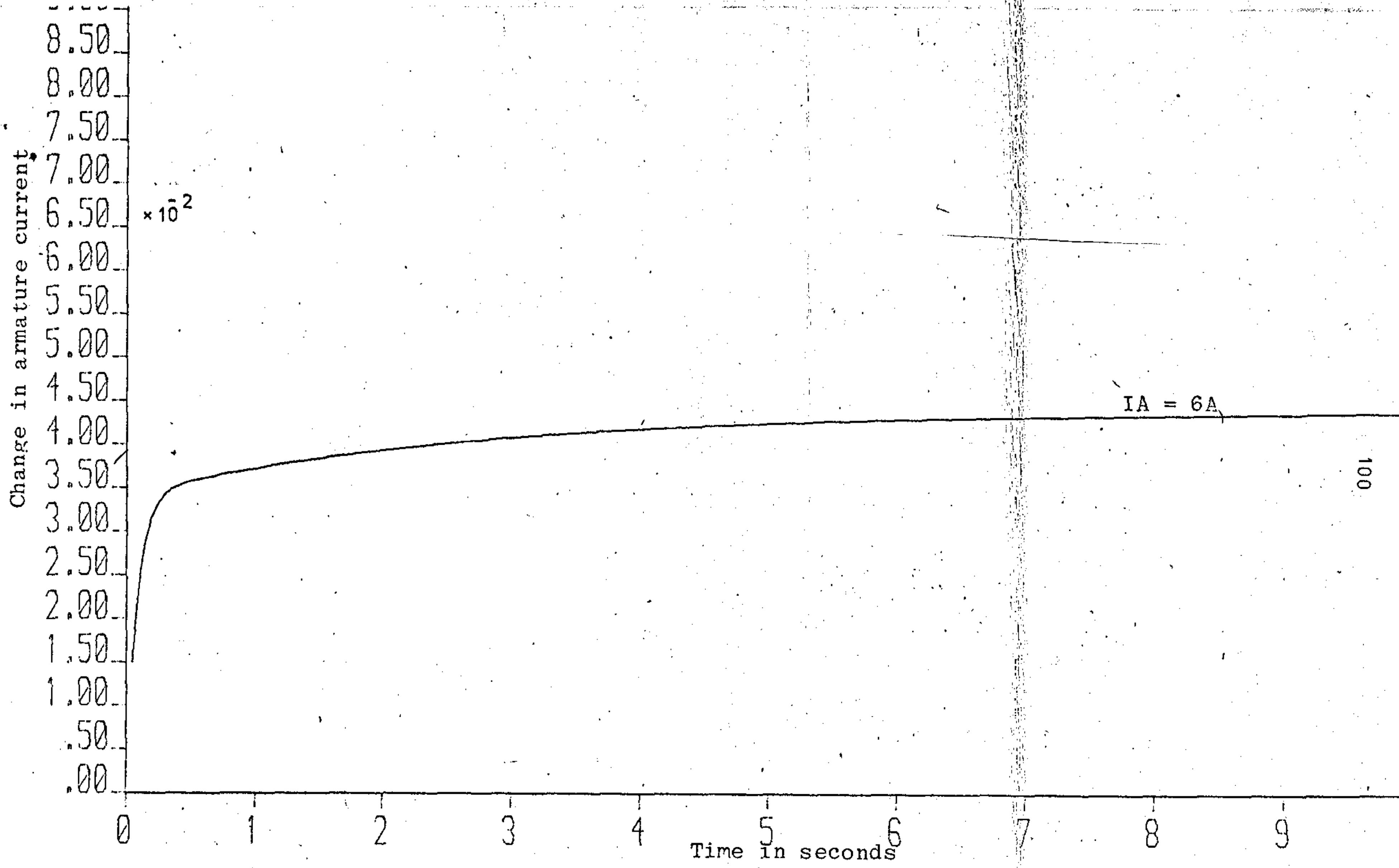


FIG. 7.2.20 Machine model response

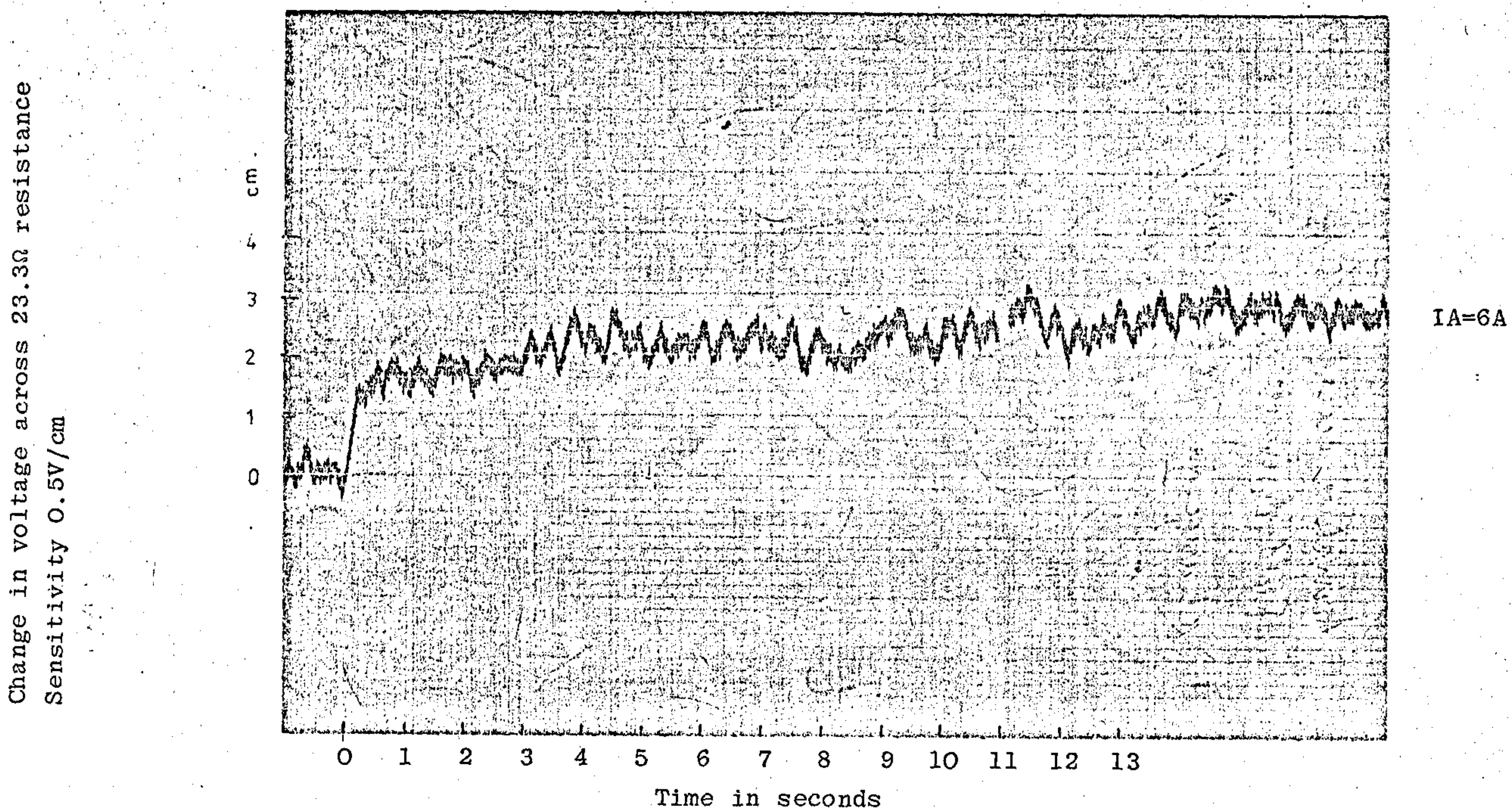


FIG. 7.2.21 Machine response

TABLE 13

Steady state field current 2.2A
Change in series field resistance from 0.05 to 0.1 ohm
Comparison of change in speed

Time in Secs	Steady State Armature Current A	Theoretical Change in Speed rev/min	Practical Change in Speed rev/min	Percentage Difference Between Practice and Theory
0.5	4	-0.69	-0.63	- 8.69
1.0		-1.44	-1.37	- 5.11
2.0		-2.69	-2.75	2.18
3.0		-3.68	-4.06	9.36
5.0		-5.10	-6.87	25.76
10.0		-6.69	-7.81	14.34
0.5	5	-0.86	-1.25	31.20
1.0		-1.75	-2.50	30.00
2.0		-3.19	-3.75	14.90
3.0		-4.25	-4.69	9.38
5.0		-5.61	-6.25	12.19
10.0		-6.89	-8.75	21.25
0.5	6	-1.02	-1.0	- 2.00
1.0		-2.06	-2.19	5.93
2.0		-3.66	-4.06	9.85
3.0		-4.76	-5.00	4.80
5.0		-6.07	-8.08	24.87

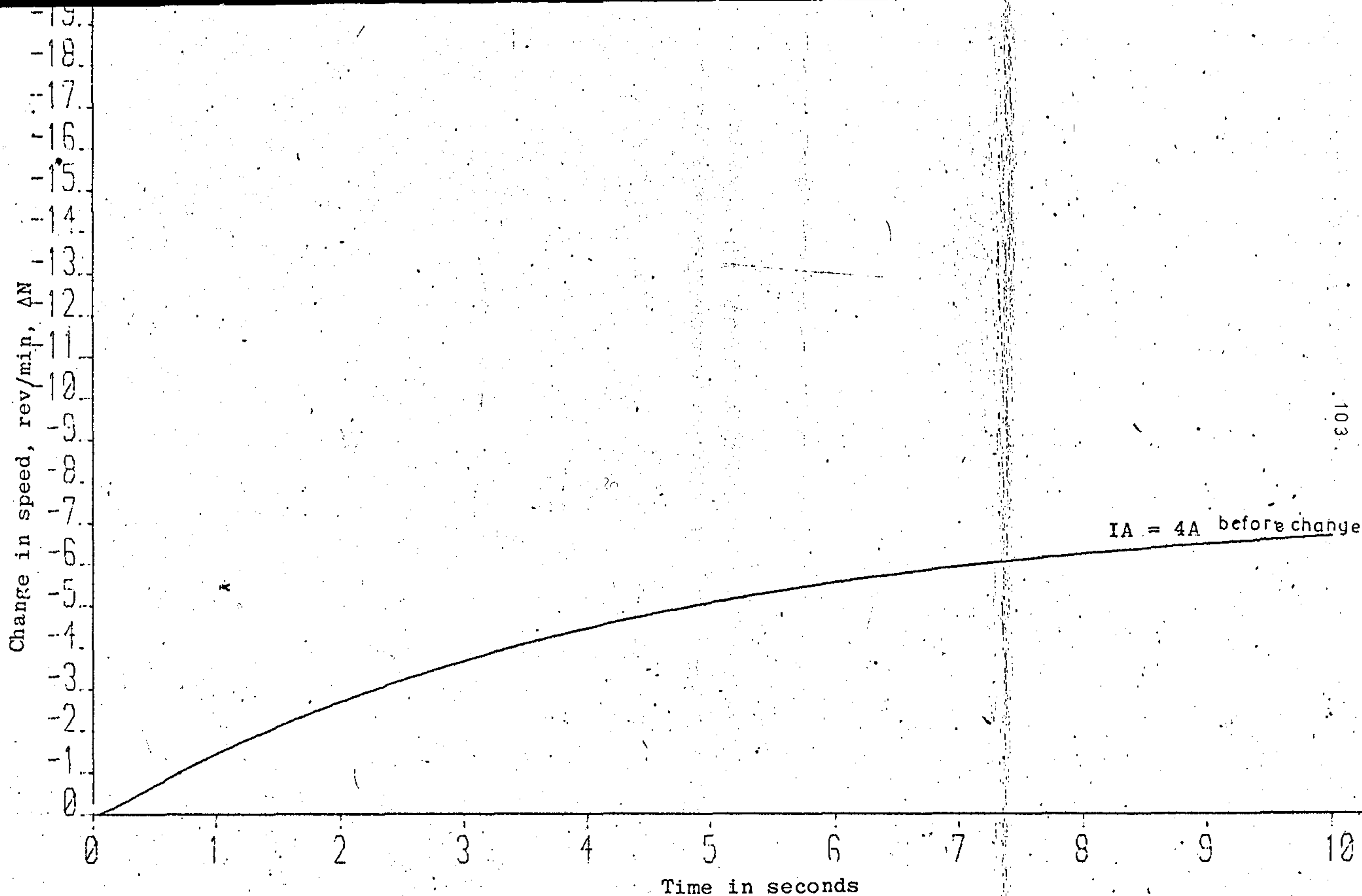


Fig. 7.2.22 Machine model response
after a change in the series field resistance

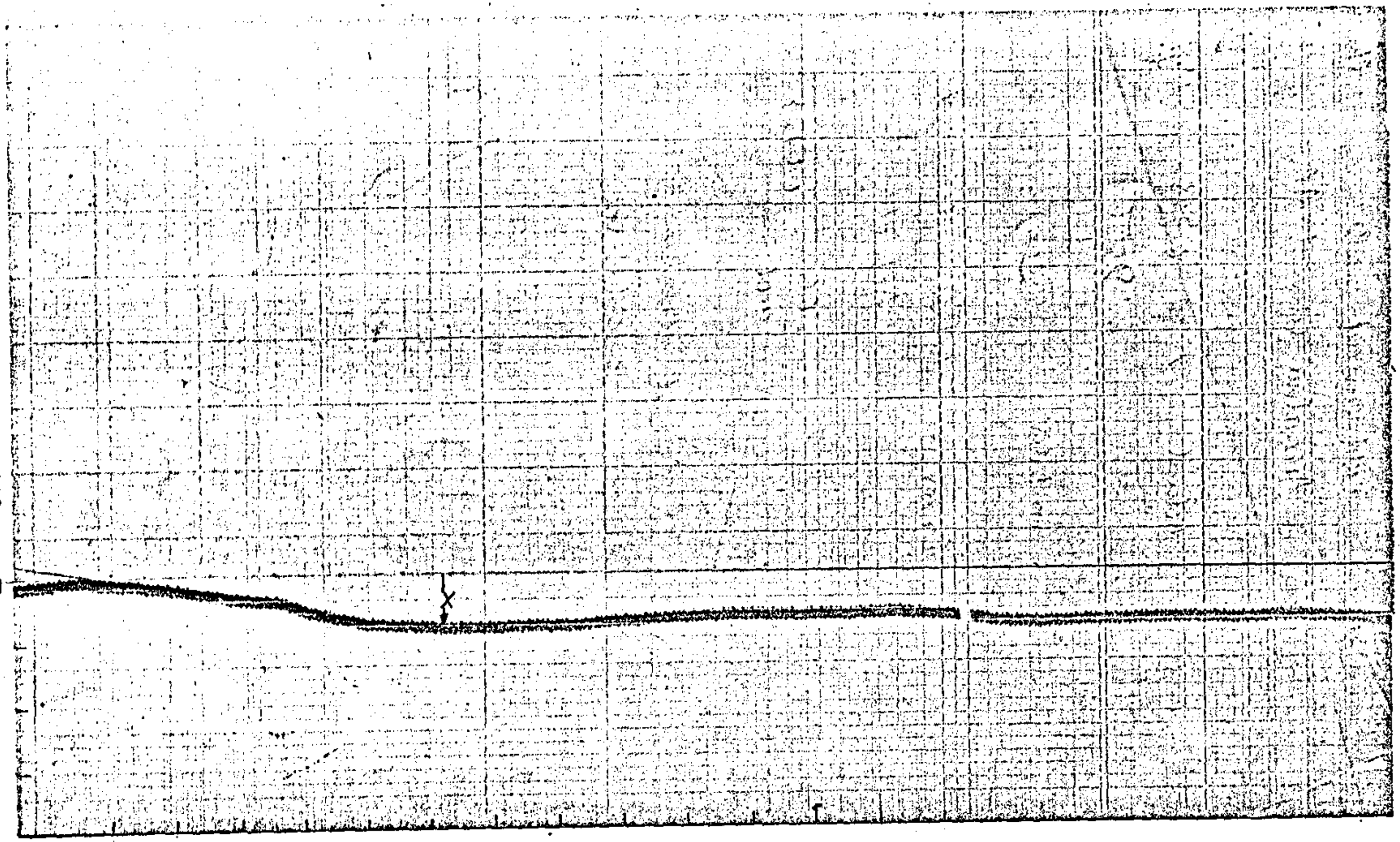


FIG. 7.2.23 Model response

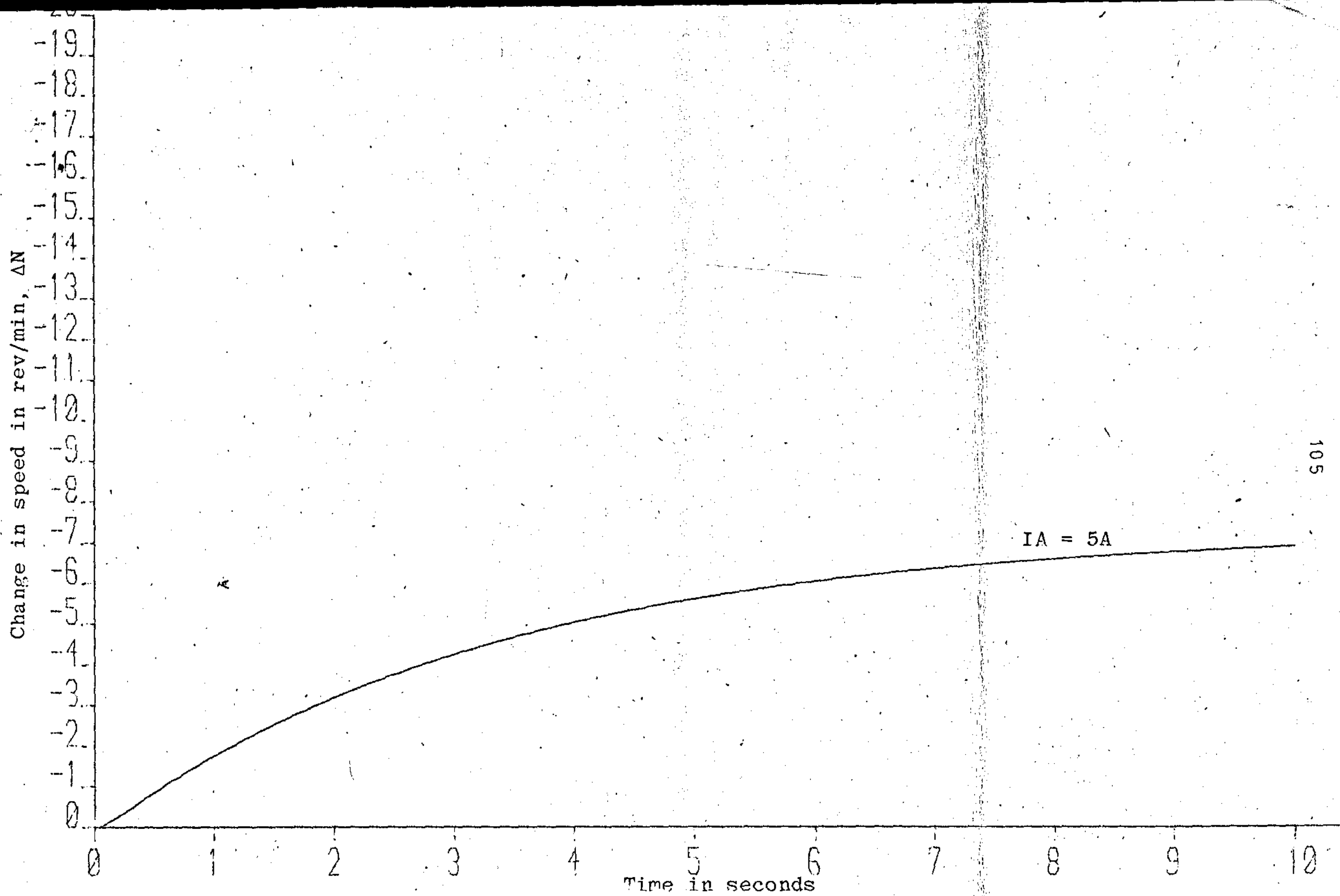


FIG. 7.2.24 Machine model response

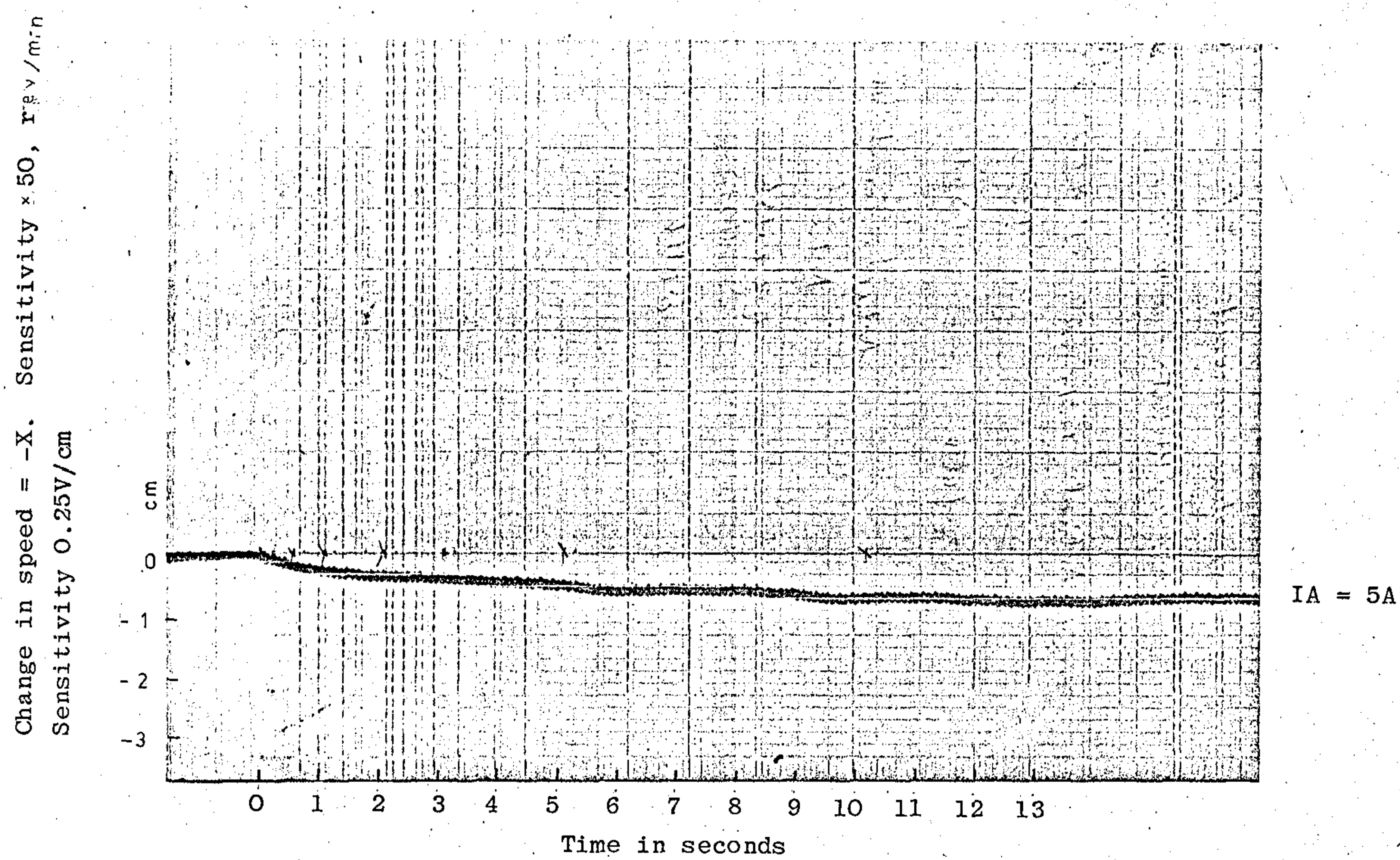


FIG. 7.2.25 Machine model

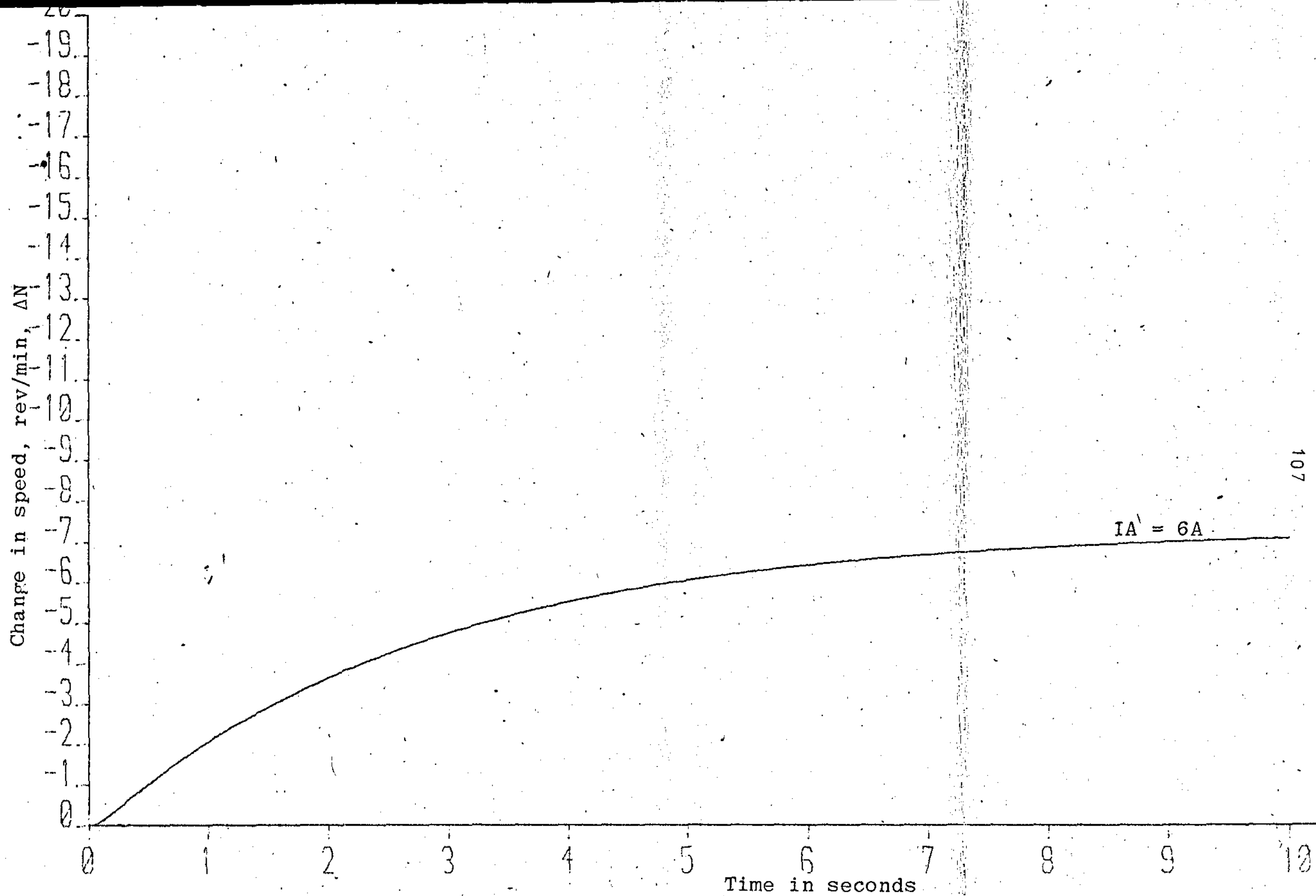


FIG. 7.2.26 Machine model response

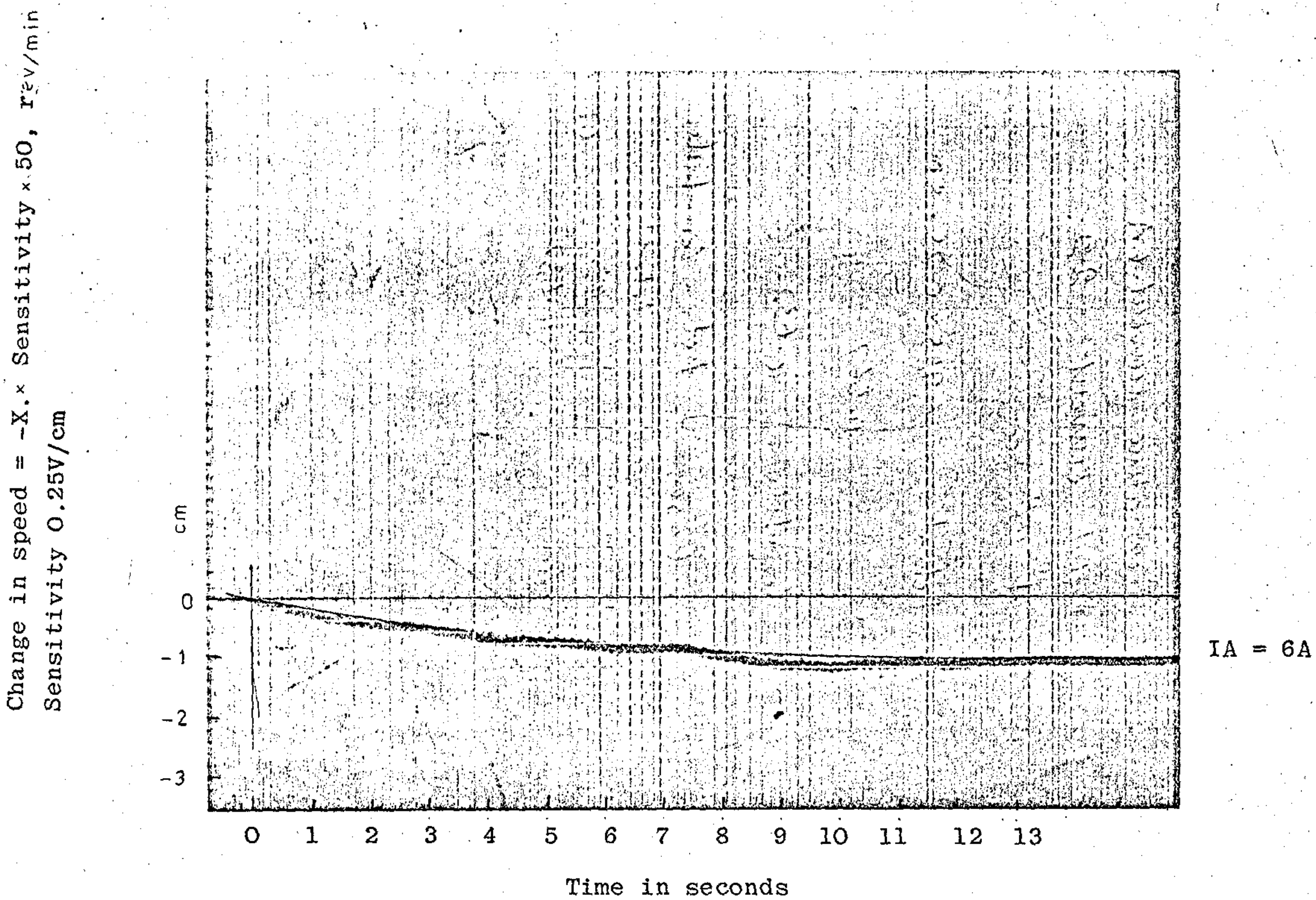


FIG. 7.2.27 Machine response

TABLE 14

Steady state field voltage 2.2A
Change in armature series resistance
Comparison of change in speed

Time in Secs	Steady State Armature Current A	Theoretical Change in Speed rev/min	Practical Change in Speed rev/min	Percentage Difference Practice to Theory
0.5	4A 4900 Ohms resistance switched in parallel with 35 ohm resistance	1.60	1.75	8.57
1.0		3.04	3.50	13.14
1.5		4.32	5.00	13.60
2.0		5.46	6.25	12.60
3.0		7.40	9.37	21.62
5.0		10.17	13.12	22.48
0.5	5A 3420 Ohms resistance connected in parallel with 38 ohm resistance	1.93	2.12	8.96
1.0		3.61	4.00	9.75
1.5		5.06	5.75	12.00
2.0		6.31	7.50	15.86
3.0		8.34	10.50	20.57
4.0		9.86	12.75	22.66
10.		13.61	18.75	27.78
0.5	6A 3266 Ohms resistance connected in parallel with 23.33 ohms resistance	2.23	2.50	10.80
1.0		4.11	4.37	5.95
1.5		5.69	6.25	8.96
2.0		7.01	7.50	6.53
3.0		9.07	10.0	9.30
5.0		11.56	15.0	22.90

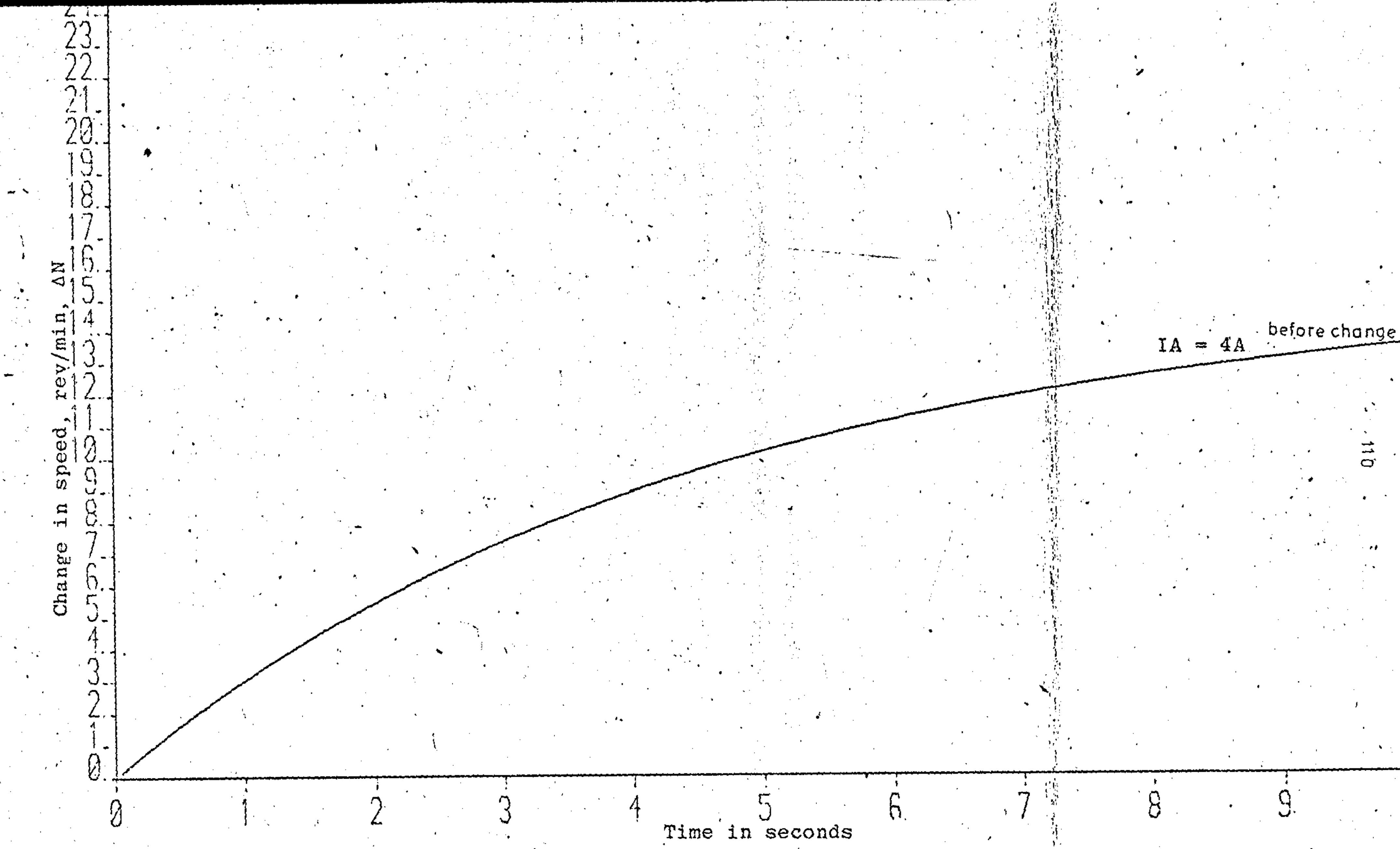


FIG. 7.2.28 Machine model response

after a change in the series armature resistance

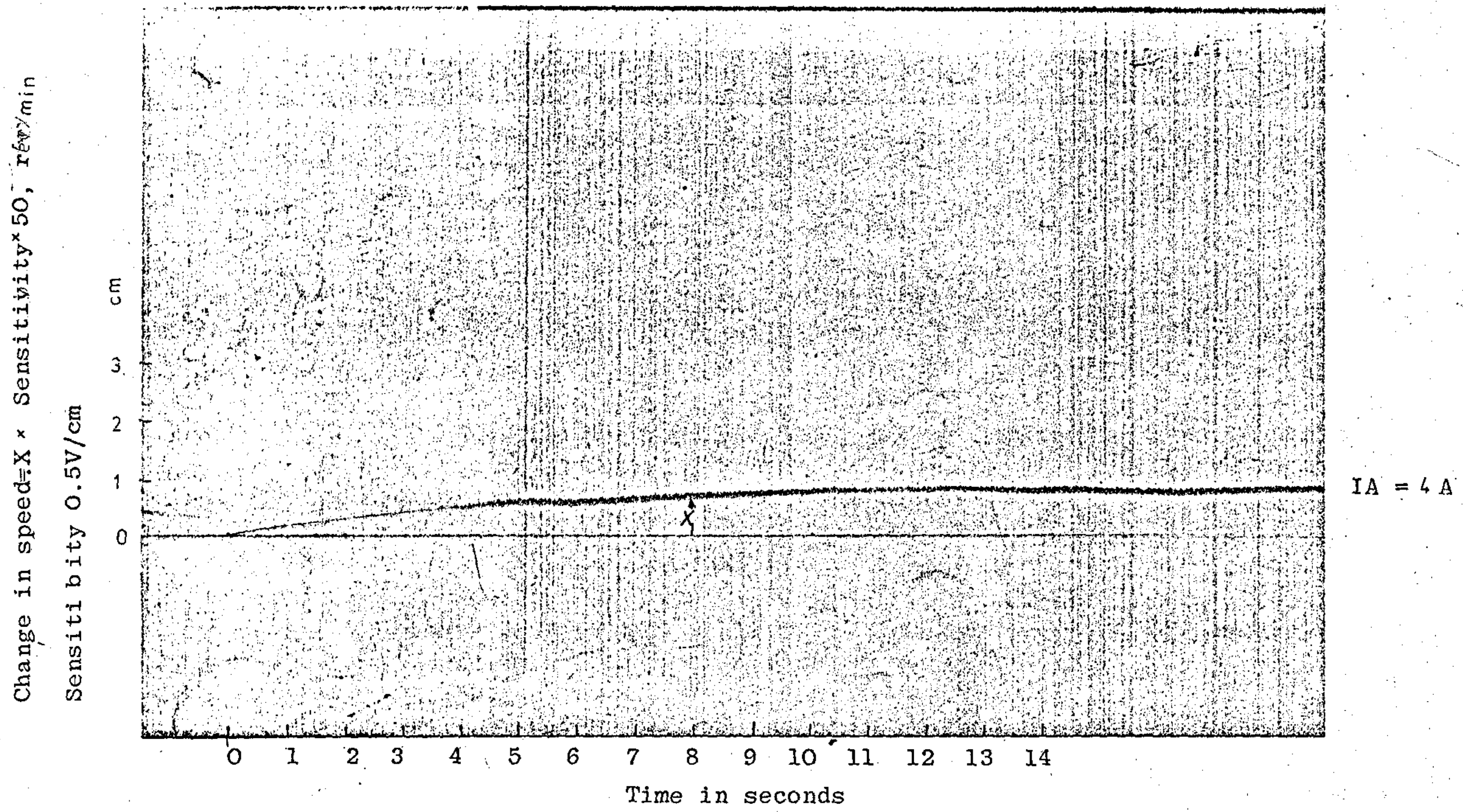


FIG. 7.2.29 Machine response

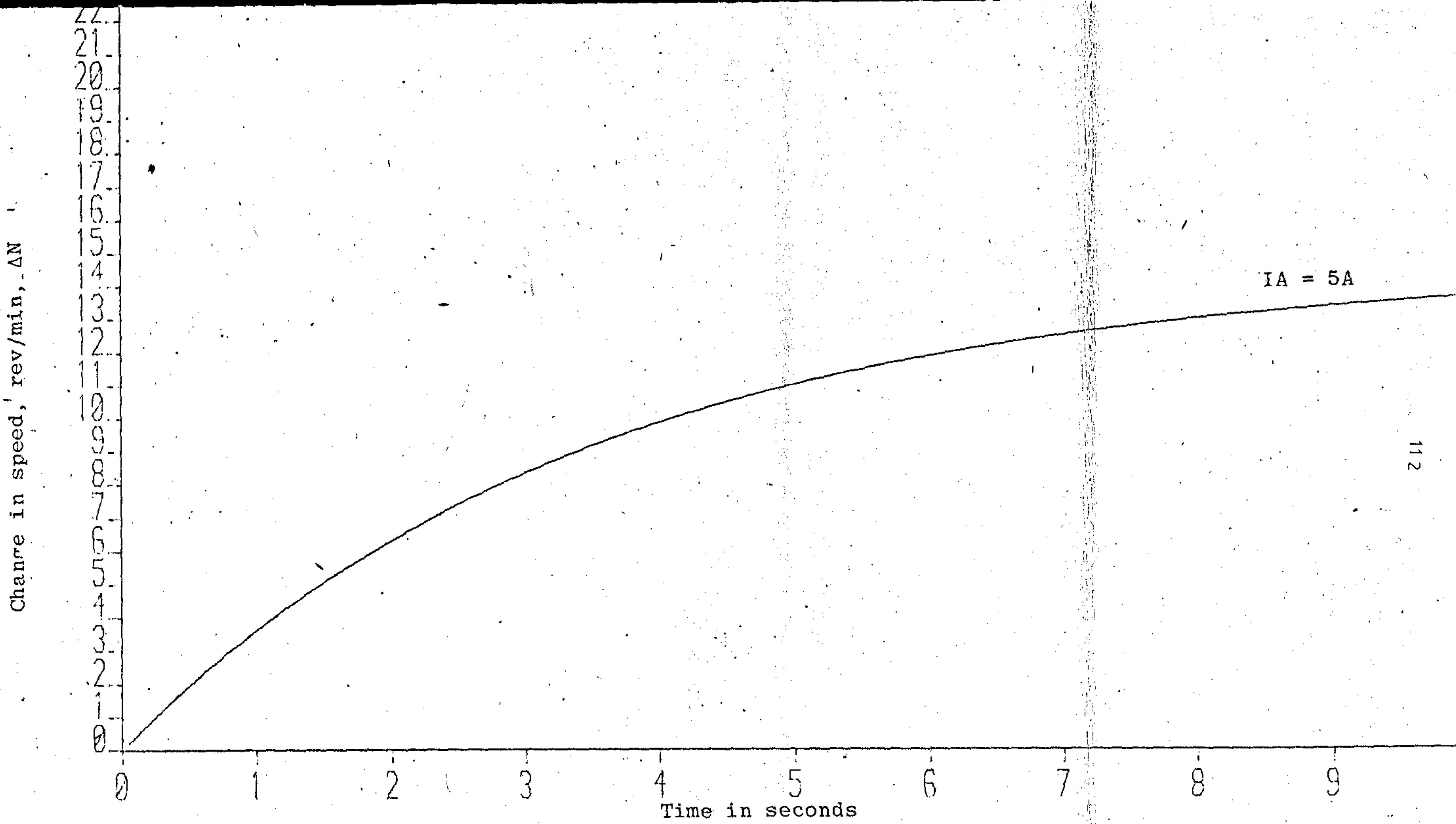


FIG. 7.2.30 Machine model response

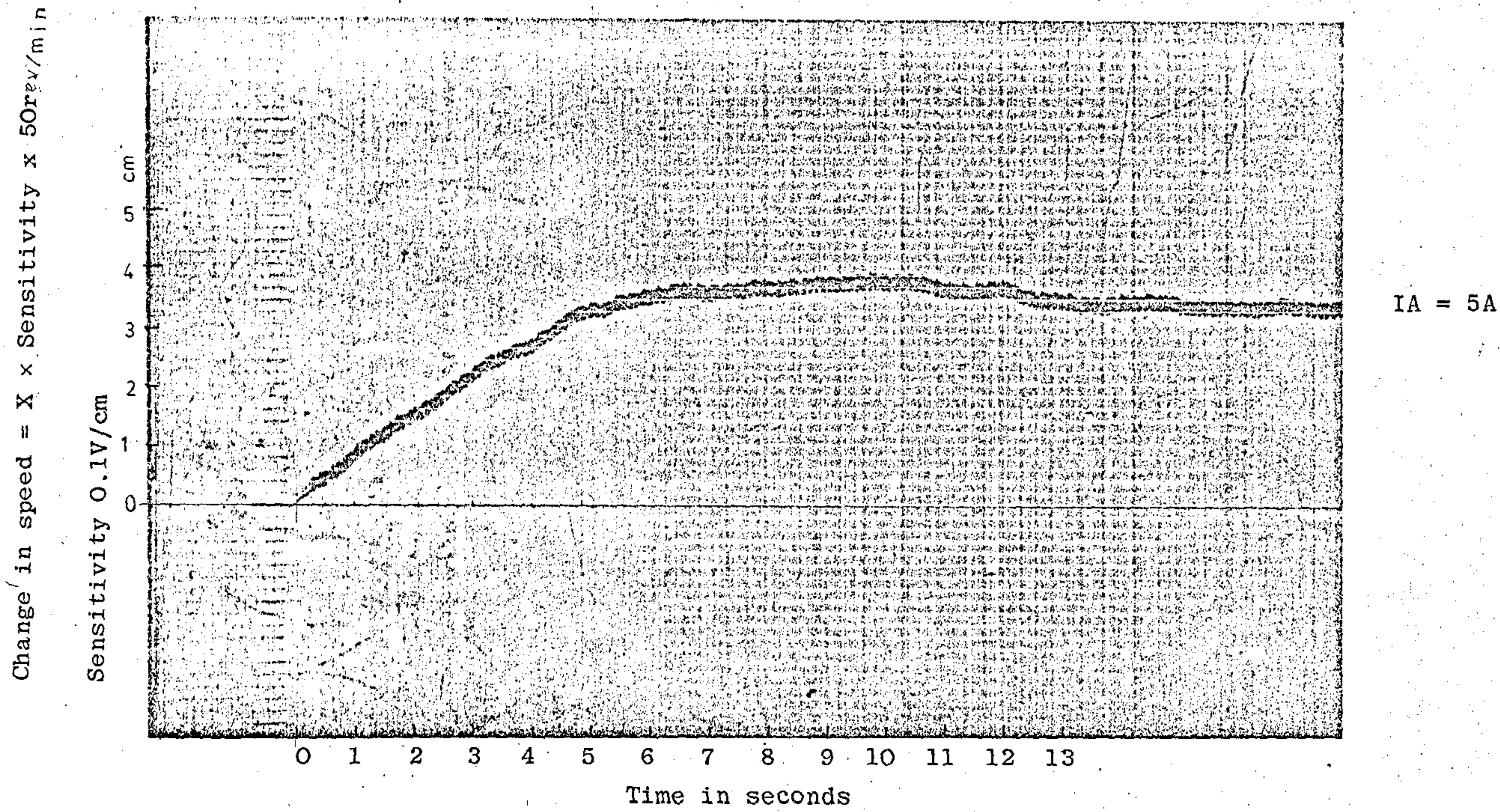


FIG. 7.2.31 Machine response

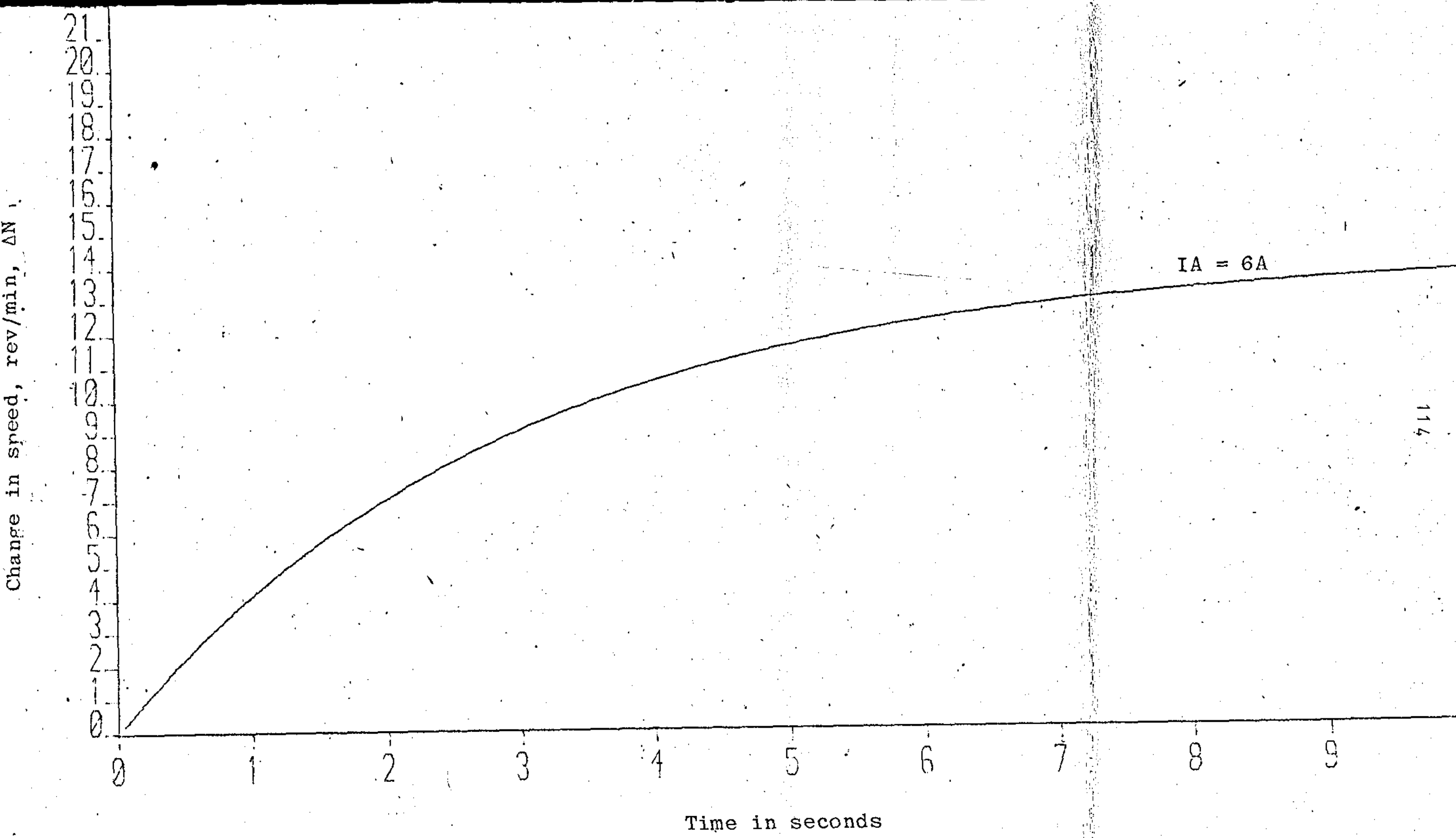


FIG. 7.2.32 Machine model response

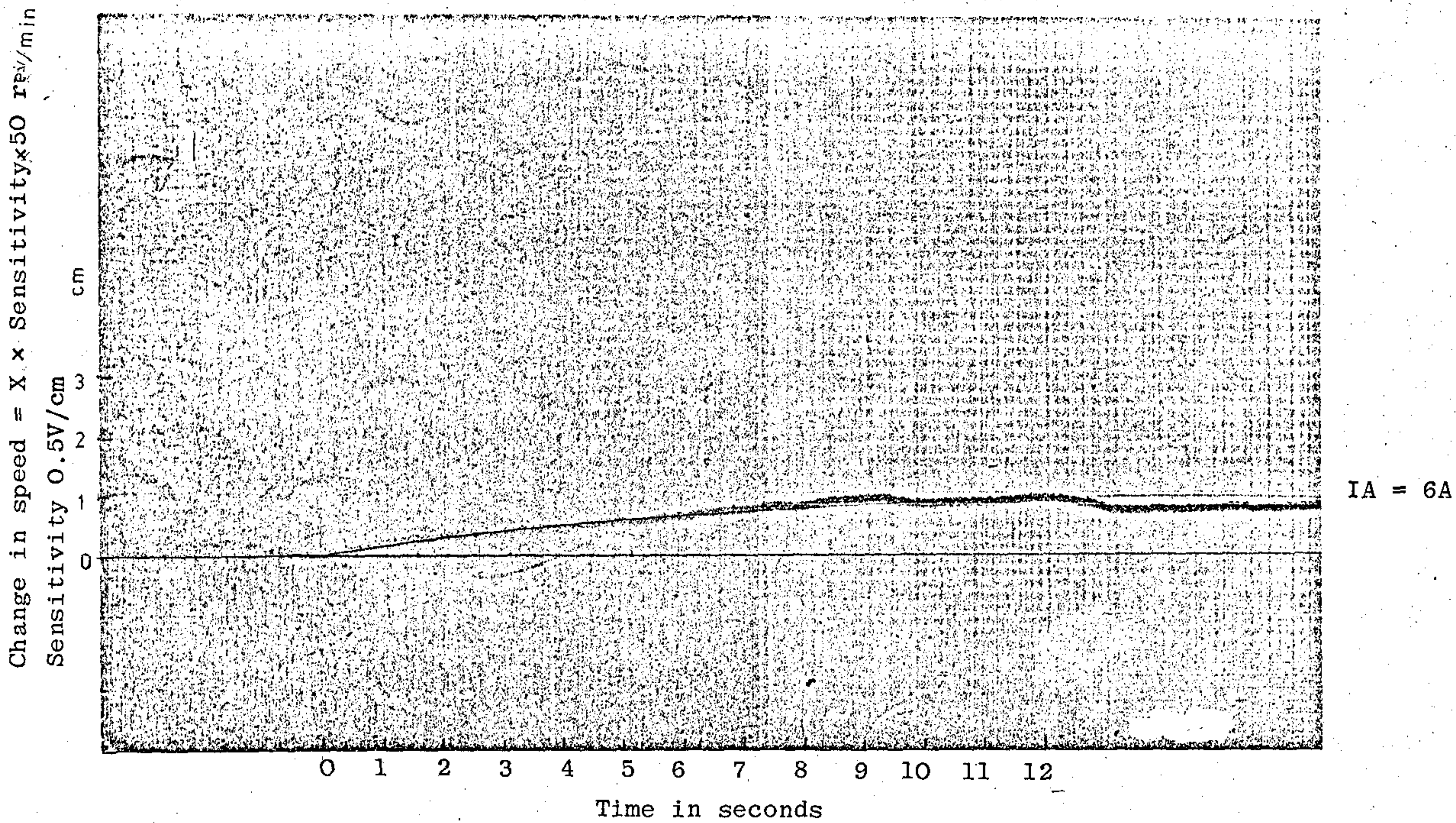


FIG. 7.2.33 Machine response.

Change in speed from no load to full load and armature voltage are two important factors on which the main parameters G1 and G2 depend.

Also the variation of voltage across the armature due to sparking, makes armature current vary as well. It can be seen from the machine response of change in armature current, that the output is varying around some average value, which makes it difficult to determine the change in armature current. There are additional reasons for not having closer comparison of theoretical and practical results.

Parameters G1 and G2 depend upon ΔN, $\frac{\partial \Delta N}{\partial IF}$ and $\frac{\partial NO}{\partial IF}$. It can be observed from Figure 4.2 that for weakened field currents the machine cannot be run for full load current because of heavy sparking and therefore the N Vs IA characteristic has to be dependent largely on the few points available. Therefore ΔN for low values of field current has a large margin within which its actual value may lie and chances of error are more. If a salient pole machine, preferably with interpoles, is used then N Vs. IA characteristic can be obtained by running the machine up to full load current and therefore the value of ΔN will be more accurate.

Also the gradient $\partial NO/\partial IF$ at lower field current has high sensitivity and therefore the tangent at any point should be drawn more accurately.

Near rated field current, $\frac{\partial \Delta N}{\partial IF}$ becomes comparable to $\partial NO/\partial IF$ and therefore accurate drawing of the tangent at any point on $\Delta N/IF$ characteristic is also an important factor contributing towards the accuracy of the result

The temperature of the room in which the machine is run, always remains nearly constant and therefore any change in the resistance of the field and the armature winding due to the change in ambient temperature is small. Also due to a high resistance in series with the armature winding, any change in the armature resistance, due to the change in ambient temperature, will not make any effect on transient and steady state response. However in the absence of large resistance, where rated armature voltage is equal to the supply voltage, the brush contact resistance and the armature reaction becomes important along with ambient temperature and rise in temperature, It is shown in appendix 3 that, appoximately, for the stability $RT > -Ke.N \frac{\partial \Phi}{\partial IA}$. Therefore accurate measurements of armature resistance, taking effect of temperature in consideration, becomes an important factor, not only when comparing the machine model and machine response, but also for stability as well.

The model has taken armature reaction into account by considering flux per pole as a function of armature and field current. By comparing the results obtained from the machine and the model, it could be claimed that shapes of the two sets of responses are similar and transient and steady state responses generally close enough and could be useful for control applications.

This model needs the gradients G1 and G2 as its main parameters. Both G1 and G2 are functions of flux per pole, which in turn is a function of armature and field current. These parameters are affected by armature current, as can be seen from equations 4.2.4 and 4.3.5. The success of the model as the representation of the machine demonstrates not only that the analysis is correct, but the special method of obtaining the parameters from the characteristics N Vs. IA, $\partial\Delta N$ Vs. IF and ∂NO Vs. IF and the relationship for calculating G1 and G2 are also correct. These parameters are derived from a set of N Vs. IA characteristics at different constant field currents. The experimental set is simple and performance of experiments is easy. The success of this model, after taking into account the reasons given above, shows that the new approach of the model is comparable to any other approach as far as response is concerned. It is advantageous because of the ease with which parameters can be obtained from a set of N Vs. IA characteristics. The computer program for the model response gives change in armature current, field current and speed simultaneously. Their derivatives are also calculated. This gives a great advantage because states and their derivatives are known, for any instant, and can be used for designing a flexible control circuit.

To explain the time versus change in the speed and time versus change in the armature current characteristics, when the field voltage is changed by changing a series resistance in the field circuit, the following equations can be used.

$$E = K1.N.\Phi \qquad (7.2.1)$$

$$T = K2.IA\Phi \qquad (7.2.2)$$

$$V = E + IA.RC + LA\frac{dIA}{dt} \qquad (7.2.3)$$

$$T - T_{\ell} = K.\frac{dN}{dt} \qquad (7.2.4)$$

where K1, K2 and K are constants.

LA - inductance of armature winding

RC - total resistance in the armature circuit.

There will be a transient change in the field current when the field voltage is reduced. The change in the field current is shown in Fig. 7.2.1a. and will depend upon the series resistance and the time constant of the field winding. The flux per pole will reduce as well and will follow the field current, if changes in the magnetic properties of iron due to saturation are ignored. The reduction of flux per pole, shown in Fig. 7.2.1b, will reduce the back emf E, shown in Fig. 7.2.1c. Due to the reduction in the back emf E, the armature current will increase as shown in Figs. 7.2.2 to 7.2.9 and Figs. 7.2.16 to 7.2.21 .

It can be seen from the time versus speed characteristics of the machine and the model, shown in Figs. 7.2.22 to 7.2.27, that the speed is reducing. If the load torque is assumed to be constant, the reduction in the speed is due to the reduction in developed torque because the product IA.Φ of equation 7.2.2, after the change became less than what it was before.

Initially the flux per pole will fall very rapidly and the increase in the armature current will be very fast as well. The decrease in the speed will be very slow due to the large mechanical time constant, compared to either the field winding time constant or the armature winding time constant. Therefore, as the flux per pole attains the steady state value, the speed will still continue to decrease. The back emf will decrease according to the decrease in speed. The armature current will be increasing but very slowly and because the flux is no longer changing, the torque given by equation 7.2.2 will increase until it is again equal to the load torque when the speed and the armature current will attain the steady state value.

The time versus change in speed and time versus change in armature current characteristics are explained below when the change in armature input voltage is made.

The armature voltage is changed by changing the series resistance so that the armature current is increased. This increase in the armature current will depend upon the time constant of the armature circuit. Figs. 7.2.10 to 7.2.15 show the time versus armature current characteristics of the model and the machine, when a change in the series resistance of the armature circuit is made. speed versus time characteristics shown in Figs. 7.2.28 to 7.2.33 are rising. It means that the developed torque, as given by equation 7.2.4, is exceeding the load torque and therefore accelerating the motor. Due to the change in the armature input voltage, the armature current will rise very rapidly, according to the time constant of the armature circuit, because initially the change in the back emf will be small due to slow rise in the speed and constant flux per pole.

It can be seen from Figs. 7.2.10 and 7.2.15 that the armature current starts falling after attaining maximum. This is due to increase in the back emf. The developed torque will reduce as well and become equal to load torque when both the armature current and the speed will attain the new steady state value. It can be observed that the armature current is falling to the new steady state value very slowly. This is due to the fact that the speed is increasing slowly and the back emf is following the rise in speed.

REFERENCES

1. H. Winkler, "Development of standard thyristor drives", Electrical Times (GB), 21/28, Dec. 1972, pp 29-33.

2. N.K. Sinha, C.D. di Cenzo, B. Szobodos, "Comparison of mathematical model for electromagnetic and D.C. motor" Engineering Journal, Canada, Vol. 55, 1972 and transactions.

3. J.M. McTaggart, "Application of controlled D.C. devices" Hydraulic Pneumatic Mechanical Power, pp 139-143, April 1975.

4. D. Smith Gall, "A phase locked loop motor control system", IEEE Transaction, Industrial Electronics and Control Instruments, Vol. IECI22, Vol. 4, pp 487-490, 1975.

5. J.J. Pollack, "Some guidelines for the application of adjustable speed drives", IEEE Transactions Industrial Application, Vol. IA9, pp 704-710, Nov/Dec 1973.

6. Alan London, "A.C. motors, A step by step guide to types and applications", Control and Instrument, Vol. 7, No. 11, Dec. 1975, pp 24-27.

7. P.C. Bracher, "Trends in three phase D.C. thyristor drives", DEM Design, April 1976, pp 65-67.

8. Gunnar Kironberg, "Industrial D.C. motor drives", ASEA Journal, 1975, Vol. 48, No. 5.

9. Per-Ake Diborn and Olof Shuncke, "Modern thyristor converters for D.C. motor drives", ASEA Journal, Vol. 5, 1975.

10. Gèza Joes and Thomas H Benton, "Four quadrant D.C. variable drives", Proceedings IEEE, Vol. 63, No. 12, Dec. 1975.

11. Timothy A Maloney and Franando L. Alvarado, "A digital method for D.C. motor control", IEEE Transaction, Industrial Electronics and Control Instrument, Vol. IECI-23, No. 1, 1976.

12. A.N. Moore, "Phase locked loop for motor speed control", IEEE Spectrum, pp 71-67, April 1973.

13. W. Wood, "Making the most of D.C. modelling", Electrical Review, Vol. 199, No. 6, 6 Aug. 1976.

14. Del. Taro "Electromechanical devices for energy conversion and control systems", Prince Hall, EE Series, 1968.

15. N.N. Hancock, "Matrix analysis of electrical machines", Pergamon Press, 1964.

16. Bernard Atkins, "The general theory of electrical machines", Chapman and Hall, 1964.

17. Michael Lipschitz, "Direct current machines", D. Van Nostrand Company Inc., 1959.

18. N.K. Sinha, C.D. De Cenzo, B. Szabodos, "Modelling of D.C. Motor for control applications", IEEE Transaction, Industrial Electronics and Control Instruments, Vol. IECI-21, No. 2, pp 84-88, 1974.

19. J. Hindmarsh, "Electrical machines and their applications", The Commonwealth International Library, Pergamon Press, 1970.

20. E.B. Moulin, "Electromagnetic principles of dynamo" "Clarendon Press, 1955.

21. Samuel Seely, "Electromechanical energy conversion", McGraw Hill, 1962.

22. A. Fitzgerald and L. Kingsley, "Electric Machinery", 1961, McGraw Hill.

23. A.E. Clayton, "Performance and design of direct-current machines", Pitman 1959.

24. C. Siskind, "Electrical machines, direct and alternating current", McGraw Hill Book 1959.

25. A.S. Langsdorf, "Principles of direct current machines", McGraw Hill, 1959.

26. Manual of Westinghouse Generalised Machines.

27. J. Rosenblatt Freedman, "Direct and alternating current machinery", McGraw Hill, 1963.

28. Dr. G.K. Creighton, "Separately excited D.C. motor model", Personal notes.

APPENDIX 1

COMPUTER PROGRAM

```
      MASTER PROGRAM
      REAL IFL
      REAL KE
      REAL MI
      REAL N
      REAL KT
      DIMENSION X(700),XA(700),Y(700),YA(700)
      DIMENSION A(4,4),C(4,4),B(4,4)
      DIMENSION DY(4),DDX(4),DX(4)
      DIMENSION DY1(4)
      COMMON/BLOCK1/A,C
      COMMON/BLOCK2/X,XA,Y,YA
C     DY1(2) IS CHANGE IN ARMATURE INPUT VOLTAGE
C     DN IS CHANGE IN SPEED FROM NO LOAD TO FULL LOAD
C     S1 IS TENGENT AT ANY POINT ON FIELD CURRENT VS ON CURVE
C     S2 IS TENGENT AT ANY POINT ON IF VS NO CURVE
C     ZN IS NO LOAD SPEED,NO
C     DX(1) IS TIME IN SECONDS
C     DX(2) IS CHANGE IN ARMATURE CURRENT,A
C     DX(3) IS CHANGE IN FIELD CURRENT,A
C     DX(4) IS CHANGE IN SPEED
C     DDX(1) IS DERIVATIVE OF TIME
C     DDX(2) IS DERIVATIVE OF CHANGE IN ARMATURE CURRENT
C     DDX(3) IS DERIVATIVE OF CHANGE IN FIELD CURRENT
C     DDX(4) IS DERIVATIVE OF CHANGE IN SPEED
      IF=2.2
      CALL UH7934
      P=2
      AC=2
      Z=560
      KT=Z/(2*3.141592654)
```

C1= 94.

C1= 30.

```
      KE=Z/AC*P/60
      DY1(2)=1.
      IFL=8.
      MI=0.02412
      PI=3.141592654
      DVT=2.4
      ZN=55.1*50+2.*50
      DO 11 JJ=1,5
      READ(1,10)IA,RA,RA1,RFF,PA
   10 FORMAT(I4,3F7.3,F10.5)
      DN=DVT*50
      R2=140./IA
      R1=IA/DY1(2)*(R4**2)-R2
      WRITE(2,99)R1
   99 FORMAT(1H ,E10.5)
      VA=240-RA1*IA-R2*IA
      N=ZN+DN*IA/IFL
      D1=PA/((2*PI*N/60)**2)
      S1=-40.37*IA/IFL
      S2=-604.84
      G1=-(((ZN+DN*IA/IFL)*RA)+((VA-IA*RA)*DN/IFL))/(KE*(ZN+(DN*IA/IFL)
     1**2)
      G2=-(VA-IA*RA)/(KE*(ZN+DN/IFL*IA)**2)*(S1+S2)
      NS=Z/2*1/P
      F1=0.05
      F2=0.
      HF=324
      H1=HF*G1*11.[illegible]*0.7915
      H2=HF*G2*1.1[illegible]*0.7913
      PHI=(VA-IA*RA)/(Z*N/60)
        KA=KT*PHI*IA*60/N- 2*π*D1-(C1+C2)/(2*π*(N/60)²)
      R=RA+RA1+KE*N*G1+R1*R2/(R1+R2)
      RF=KE*N*G2
      RTA=KT*((G1*IA)+PHI)
      RTF=KT*IA*G2
      EL=F1*H2-H1*F2
      B(1,1)=S1
      B(1,2)=S2
      B(1,3)=G1
      B(1,4)=G2
      B(2,1)=F1
      B(2,2)=F2
      B(2,3)=H1
      B(2,4)=H2
      B(3,1)=R
      B(3,2)=RF
      B(3,3)=RTA
      B(3,4)=RTF
      B(4,1)=T
      B(4,2)=PHI
      B(4,3)=C1
      B(4,4)=D1
      WRITE(2,92)((B(I,J),J=1,4),I=1,4)
   92 FORMAT(1H ,4(/1X,4(4X,E20.10)))
      A(1,1)=0.0
      A(1,2)=0.0
      A(1,3)=0.0
      A(1,4)=0.0
      A(2,1)=0.0
      A(2,2)=-R*H2/EL
      A(2,3)=-RF*H2/EL+RFF*F2/EL
      A(2,4)=-KE*PHI*H2/EL
```

```
      A(3,1)=0.0
      A(3,2)=H1/EL*R
      A(3,3)=RF*H1/EL-F1*RFF/EL
      A(3,4)=KF*PHI*H1/EL
      A(4,1)=0.0
      A(4,2)=RTA/(2*PI*MI)*60
      A(4,3)=RTF/(2*PI*MI)*60
      A(4,4)=-D1/MI-KA/(2*π*MI)
      WRITE(2,91)((A(I,J),J=1,4),I=1,4)
   91 FORMAT(1H ,4(/1X,4(4X,E20.10)))
      C(1,1)=0.0
      C(1,2)=0.0
      C(1,3)=0.0
      C(1,4)=0.0
      C(2,1)=0.0
      C(2,2)=H2/EL
      C(2,3)=-F2/EL
      C(2,4)=0.
      C(3,1)=0.
      C(3,2)=-H1/EL
      C(3,3)=F1/EL
      C(3,4)=0.0
      C(4,1)=0.0
      C(4,2)=0.0
      C(4,3)=0.0
      C(4,4)=-60/(2*3.14159265*MI)
      DO 79 I=1,4
      DX(I)=0.0
      DDX(I)=0.0
   79 CONTINUE
      WRITE(2,251)
  251 FORMAT(/8X,5HDX(1),5X,5HDX(2),5X,5HDX(3),5X,5HDX(4)/10X,6HDDX(2),5
     1X,6HDDX(3),5X,6HDDX(4))
      CALL INTEG(4,DX,DDX,DY,0,0004,35000,K2,J)
      CALL AXISCA(3,15,0.,15.,1)
      CALL AXISCA(3,30,0.,0.15,2)
       CALL AXIDRA(1,1,1)
      CALL AXIDRA(-1,-1,2)
      CALL GRAPOL(X,XA,700)
      CALL PICCLE
      CALL AXISCA(3,15,0.,15.,1)
      CALL AXISCA(3,30,.0029,-.0001,2)
      CALL AXIDRA(1,1,1)
      CALL AXIDRA(-1,-1,2)
      CALL GRAPOL(X,Y,700)
      CALL PICCLE
      CALL AXISCA(3,15,0.,15.,1)
      CALL AXISCA(3,25,0.,25.,2)
      CALL AXIDRA(1,1,1)
      CALL AXIDRA(-1,-1,2)
      CALL GRAPOL(X,YA,700)
      CALL PICCLE
   11 CONTINUE
      CALL DEVEND
      STOP
      END
```

```
ENT, LENGTH    659, NAME  PROGRAM
```

```
      SUBROUTINE INTEG(N,DX,DDX,DY,H,NI,K2,J)
```

```
      DIMENSION X(700),XA(700),Y(700),YA(700)
      DIMENSION DX(4),DDX(4),DY(4)
      COMMON/BLOCK2/X,XA,Y,YA
      KK=1
      DDX(1)=H
      DO 119 J=1,M
  119 DY(J)=0.0
      DO 117 K2=1,NI
      DO 117 J=1,4
      CALL EQUAT(4,DX,DDX,K2,J)
      DO 114 I=2,M
  114 DDX(I)=DDX(I)*H
      DO 102 I=1,M
      GO TO (104,105,106,107),J
  104 R=(0.5*DDX(I)-DY(I)+DX(I))-DX(I)
      GO TO 108
  105 R=(0.2928932188*(DDX(I)-DY(I))+DX(I))-DX(I)
      GO TO 108
  106 R=(1.7071067812*(DDX(I)-DY(I))+DX(I))-DX(I)
      GO TO 108
  107 R=((DDX(I)-2*DY(I))/6.+DX(I))-DX(I)
      GO TO 108
  108 DX(I)=DX(I)+R
      S=DY(I)+3.*R
      GO TO (109,110,111,109),J
  109 DY(I)=S-0.5*DDX(I)
      GO TO 102
  110 DY(I)=S-0.2928932188*DDX(I)
      GO TO 102
  111 DY(I)=S-1.7071067812*DDX(I)
  102 CONTINUE
      IF(J-4)117,0,0
      IF(K2-50*KK)117,0,0
      WRITE(2,82)(DX(I),I=1,M),(DDX(I),I=2,M)
   82 FORMAT(1H ,4F12.5,5X,3F12.5)
      X(KK)=DX(1)
      XA(KK)=DX(2)
      Y(KK)=DX(3)
      YA(KK)=DX(4)
      KK=KK+1
  117 CONTINUE
      RETURN
      END
```

GMENT. LENGTH   336, NAME INTEG

```
      SUBROUTINE EQUAT(M,DX,DDX,K2,J)
      DIMENSION A(4,4),C(4,4)
      DIMENSION DY1(4),DDX(4),DX(4)
      COMMON/BLOCK1/A,C
      DY1(2)=1.0
      DO 5 I=2,M
      DDX(I)=0.0
      DO 5 K=2,M
    5 DDX(I)=DDX(I)+A(I,K)*DX(K)+C(I,K)*DY1(K)
      GO TO 55555
55555 RETURN
      END
```

```
END OF SEGMENT, LENGTH    92, NAME  EQUAT

0217                    FINISH

END OF COMPILATION  -  NO ERRORS
```

APPENDIX 2

MACHINE SPECIFICATION

Generalised Machine

Rotor: Standard two pole, continuous lap wound, armature with commutator.

Moment of inertia	1.1202×10^{-2} kg m^2
Number of rotor inductors	560
Number of rotor circuits	2
Number of turns per single coil	5
Winding scheme	Full pitch
Rotor resistance at 25°C	0.46Ω

Stator: Standard two pole, two phase, distributed winding

Number of slots	36
Winding scheme	4 winding groups
	9 coils per group
	18 turns per coil

Rotor Drive Motor

D.C. shunt motor

Voltage	240 volts
Rated field current	0.562 amps
Rated armature current	10.8 amps
Armature resistance at 21 °C	0.72Ω
Number of poles	2

Number of interpoles	2
Moment of inertia	1.2925×10^{-2} kg.m^2
Tachometer	50 rpm/volt

APPENDIX 3

APPOXIMATE STABILITY CONDITION

If a change is made in the armature input voltage, assuming $\Delta T\ell$ and ΔVF zero, then according to the set of equations 3.3.17: and with F2=0, H1=0 due to negligible magnetic coupling between the field and the armature windings.

$$\Delta VA = R\Delta IA + Ke.\Phi.\Delta N + F1.D\Delta IA \qquad (A3.1)$$

$$0 = RTA.\Delta IA - \frac{2\pi D1}{60}.\Delta N - \frac{2\pi J}{60}.D\Delta N \qquad (A3.2)$$

$$0 = RF.\Delta IF + H1\,D\Delta IA + H2\,D\Delta IF \qquad (A3.3)$$

Taking Laplace transform of A3.1 and A3.2:

$$\frac{\Delta VA(S)}{S} = R.\Delta IA(S) + Ke\Phi.\Delta N(S) + F1.S.\Delta IA(S) \qquad (A3.4)$$

$$0 = RTA.\Delta IA(S) - \frac{2\pi D1}{60}.\Delta N(S) - \frac{2\pi J}{60}.S.\Delta N(S)$$

$$\Delta N(S) = \frac{RTA.\Delta IA(S)}{(\frac{2\pi}{60})(D1+JS)} \qquad (A3.5)$$

Substituting A3.5 into A3.4

$$\frac{\Delta VA(S)}{S} = R.\Delta IA(S) + \frac{Ke.\Phi.RTA}{(\frac{2\pi}{60})(D1+JS)}.\Delta IA(S) + F1.S.\Delta IA(S)$$

$$(\frac{2\pi}{60})(D1+JS)\frac{\Delta VA(S)}{S} = [(\frac{2\pi}{60})R(D1+JS) + \frac{2\pi F1 S}{60}(D1+JS) + Ke.\Phi.RTA]\Delta IA(S)$$

$$\Delta IA(S) = \frac{(\frac{2\pi}{60})(D1+JS).\Delta VA(S)}{S[\frac{2\pi}{60}.R(D1+JS) + \frac{2\pi}{60}.F1S(D1+JS) + Ke.\Phi.RTA]}$$

(A3.6)

For stability the real parts of the roots of the denominator should be negative:

$$\frac{2\pi}{60} R(D1+JS) + \frac{2\pi}{60}.F1.S(D1+JS) + Ke.\Phi.RTA = 0$$

$$\frac{2\pi}{60} RD1+Ke.\Phi.RTA + \frac{2\pi}{60}(R.J+F1.D1)S + \frac{2\pi}{60} F1.JS^2 = 0$$

$$S1,S2 = -\frac{\frac{2\pi}{60}(RJ+F1.D1)\pm\sqrt{((\frac{2\pi}{60})^2(R.J+F1.D1)^2-4.\frac{2\pi}{60}F1.J(\frac{2\pi}{60}R.D1+Ke.\Phi.RTA)}}{2.\frac{2\pi}{60}.F1.J}$$

(A3.7)

Second term in the square root of the equation A3.7 cannot be negative unless RTA is negative and large such that Ke.Φ.RTA becomes more than $\frac{2\pi}{60}$.RD1 in that case. Instability is a certainty because then one of the roots will become positive. In the present case RTA is positive and large so that Ke.Φ.RTA > $\frac{2\pi}{60}$.RD1.

The first term in the under root factor is always positive. From the above it can be concluded that the under root term, if positive, will always be less than:

$\frac{2\pi}{60}$ (RJ+F1.D1) and if negative then it will be the imaginary part of complex roots. In either case now the stability depends upon the factor RJ+F1.D1, which should be more than zero:

$$RJ + F1D1 > 0$$

$$R > -\frac{F1D1}{J}$$

$$RT + \frac{\partial\Phi}{\partial IA}.\ KeN > -\frac{F1D1}{J}$$

$$RT > -Ke.N.\frac{\partial\Phi}{\partial IA} - \frac{F1D1}{J} \qquad (A3.8)$$

Factor $\frac{F1D1}{J}$ in equation A3.8 is small and positive and contributes marginally to the stability. Therefore for stability:

$$RT > -Ke.N.\ \frac{\partial\Phi}{\partial IA} \qquad (A3.9)$$

Accurate measurement of armature resistance, taking effect of heating into account and accurate measurement of brush contact resistance is very important when there is no series resistance.