AN INVESTIGATION
INTO
THE MATHEMATICAL EDUCATION OF ENGINEERING UNDERGRADUATES
IN
AUSTRALIAN COLLEGES OF ADVANCED EDUCATION

by

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IN MEMORY OF

MY PARENTS

For that which you love most in [them] may be clearer in [their] absence, as the mountain to the climber is clearer from the plain.

- Kahlil Gibran

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ABSTRACT

AN INVESTIGATION INTO THE MATHEMATICAL EDUCATION OF ENGINEERING UNDERGRADUATES IN AUSTRALIAN COLLEGES OF ADVANCED EDUCATION

by J. Gurcharan Singh

The context of this study is set principally in the DOCIT* colleges, the most significant subset of the Australian Colleges of Advanced Education in terms of engineering education, where the incidence of failure in mathematics courses amongst engineering undergraduates gives rise to fundamental questions about their mathematical education.

Starting with the student himself and his prior preparation at school, the mathematical education of engineers is surveyed in all its aspects.

Having established the indispensable groundwork that mathematics and mathematical modelling form in engineering education, attention is directed to the aims, objectives and underlying philosophy of mathematical education. The implications for content and methods of teaching and examining are considered.

The shortcomings in service teaching are signposted and remedies suggested. Examination techniques are critically reviewed. The need for, and the manner of, reducing the discrepancies between intention and achievement in teaching mathematics to engineers prescribe the substance of this study.

A chapter elucidates the distinctive aims and functions of universities and CAEs, since these inevitably imply a distinctive emphasis in the design and implementation of courses and lend perspective to the issues raised.

Quite apart from its great influence on the applications of mathematics, it is shown how the computer in particular and educational technology in general are valuable resources in making mathematics learning more meaningful, stimulating and illuminating, and for providing individualised instruction.

The "state of the art" is summarised by a number of surveys.

Recommended teaching syllabuses in mathematical methods, numerical analysis and statistics are preceded by the mapping of considerations that should influence the curriculum and its teaching.

The study urges a fundamental review of objectives and the methods of achieving them, and its conclusions and recommendations are stated.

* DOCIT: Directors of Central Institutes of Technology

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PREFACE

Special Concerns

The selection of the theme in this thesis has been governed by two interrelated concerns.

The broadening of entry requirements and the downward drift of the minimum Higher School Certificate aggregate entrance scores have meant that increasing numbers are attending colleges of advanced education (CAEs) and attrition rates are disturbingly high. Further, the increasing numbers induce anxieties about deteriorating academic standards which in turn may threaten the quality of advanced education that these colleges aspire to provide.

As a result, it was felt to be more than opportune to search underlying reasons for the high casualty rates in mathematics and to suggest ways of mitigating the problem. Since these cannot be meaningfully discussed in isolation from higher education in all its aspects, the thesis examines selection criteria, the mathematical background of engineering entrants, the purposes of higher education, the aims and objectives of mathematics courses, the methods of teaching and assessment and the interplay of research and teaching. To see clearly how these may exacerbate or alleviate the problem is, for us, to arrive at the heart of the matter.

The old Roman sage, Böthius, crystallised the obvious when he remarked: "If you want a doctor to heal you, you must lay bare your wounds". Accordingly, then, attention has been directed on content, and methods of teaching and evaluation.

Examinations are the subject of an extended discussion in the thesis since examination results are the main basis upon which decisions are made about the proportions of engineering students to be passed, the proportions to be failed and the proportions to be admitted to graduation. Quite apart from considerations of validity and reliability of examinations, such institutional characteristics as the extent of rigidity of assessment procedures and the policies on pass-failure rates have a direct bearing on success, failure and wastage in higher education. Formulation of correct criteria for use in mathematics examinations for engineering students is an all-important essential first step toward solving the problem of failure and wastage. This is the more significant when it is borne in mind that schools of engineering are dependent on such subjects as mathematics to act as a filter for their students.

The economic exigencies of the age have created fresh demands for new dimensions of power and effectiveness in both learning and teaching. As the expanding size of *Mathematical Reviews* testifies, the growth of mathematics has been exponentially rapid, and so it is not surprising that this growth should be reflected in mathematical curricula. Here we touch the very nerve of the educator's problem. What implications do the explosion of knowledge and the increasing vogue of specialisation have on content and methods of higher education? How do we teach for adaptation to continuous change? How do we prevent courses from becoming over-loaded? How do we develop courses and methods in teaching which will stress unifying principles and so enable students to integrate knowledge from various sources? These are fundamental, inescapable questions that merit responses, for part of the explanation of the unacceptably high failure rates is that students, burdened with excessive detail and left with little or no time to think, are unable to have an over-all grasp of the structure of knowledge which is needed to give a measure of clarity to their learning. The thesis argues that these considerations point to the need for a change in curriculum and method, not a cosmetic but a profound change, and it delineates the principles that should underlie the reform of the mathematical education of engineers.

Lest we raise premature expectations that may end in disappointment, it needs to be said that there is no easy by-pass road round this difficult problem and we have no neat solutions nor immediate remedies. However, to say that the problem is too ramified and hopeless and nothing can be done about it, is a counsel of despair. An effort has to be made so that the problem appears in a more genial light.

A promising clue to this vexing issue of high incidence of failure and wastage emerges from a growing awareness that a sizable part of it is due to causes which are either removable or amenable to modification. This thesis represents an attempt to state as clearly and simply as possible that a serious and professional concern with curriculum and the way it is taught and examined, besides making the problem less severe, can be instrumental in maintaining and raising academic performance.

Examples of measures considered which would reduce wastage and improve the productivity of colleges of advanced education include:

(a) drawing attention to the need to select methods of teaching and examining which are not dissonant with the aims and objectives of courses;

(b) making educators more sensitive to those serious deficiencies in planning and presentation of courses which give rise to student apathy and dilettantism; and,

(c) urging the utilisation of a repertoire of resources that extend, enhance and supplement the experience of learning.

Thus the first concern of this thesis is with the causes and effects of the high attrition rates in Australian advanced education.

The second concern that provided a powerful impetus to study this topic came from the realisation that to date no critical appraisal of the mathematical education of engineering undergraduates in CAEs has been attempted, despite the general unease with the incidence of failure and despite the clamour of complaint from students and employers that some courses provide inadequate preparation to meet the demands that will be made upon them in the practice of their professions. This neglect is understandable as the CAEs are of relatively recent foundation.

To give perspective to this appraisal, it is necessary in our view to consider the distinctive aims, educational philosophy and policies of the CAEs, and, more particularly, the paradox "equal but different" that has come to be applied to them. Advanced education was created, on the recommendations of the Martin Committee in 1965, as a distinct sector to provide for a range of additional activities in institutions complementary to the universities. Accordingly, we have devoted a section of the study to Universities and Colleges of Advanced Education .

This is not altogether unrelated to our first concern, for whenever a student fails, it is, in a sense, a reflection upon the goals and educational policies of an institution, its ethos and philosophy, and the quality of its staff. There are characteristics of institutions which decidedly impede or advance student performance and their output of graduates.

These, then, are the themes that give direction and purpose to our study and become its special concerns. In the simplest of words, the thesis is an endeavour to provide at least the beginnings of an answer to a dismaying problem by bringing together the sifted wisdom of wide research. Although what we have done is very theoretical, our preference has been to supply a mode of thought for the mathematics educator of engineering undergraduates.

The Scope of the Study

Engineering education at the professional level is offered in Australia by universities and colleges of advanced education. The scale of diversity of engineering education is so great as to defy simple classification or summary. As a consequence, the task of gaining an overview becomes difficult indeed.

The roots of the difficulty may be found in the complexity of the teaching pattern and in the absence of documentation of experience. Some CAEs provide courses on a full-time or part-time basis of study while others require a sandwich pattern of attendance. The subject matter, the methods employed in teaching and assessment, the number of contact hours, and the quality of education that CAEs give differ from one course to another, from one CAE to another, from one State to another. There are even differences in divisions of responsibility for the teaching of mathematics in service subjects between CAEs; some disciplines which are substantially mathematical can be given in one college by members of the mathematics department and in another by members of the engineering faculty.

The fact that very few studies exist, coupled with a dearth of comparative data, makes even a descriptive treatment of the "state of the art" an impossible task. Few, if any, CAEs have subjected their methods of teaching and evaluation techniques to a searching review or have seriously attempted to reduce the discrepancies between course objectives on the one hand and teaching procedures and assessment methods on the other.

Thus there are substantial obstacles set against the giving of a descriptive, let alone an analytical, treatment of the topic. The reality cannot be simplified easily: at best it can only be reported in general terms and even then these generalisations, being little more than "summaries of random experiences" can be hazardous and misleading. We see our attempt in the nature of a pilot study, no more than a preliminary review that may lay the basis for a possible future detailed study of a sadly neglected field. The aforementioned difficulties, which are formidable enough by themselves, are further compounded by the constraints of time and distance. There will be, therefore, no attempt to present a comprehensive picture of all advanced education, let alone all higher and further education. Instead, we have chosen to focus attention on those colleges of advanced education which constitute the DOCIT group (Directors of Central Institutes of Technology).

The DOCIT Colleges, consisting of the major multi-purpose institutes of technology and colleges of advanced education in Australia, have eight member institutions representing each of the mainland States, with two additional members from Victoria. The member institutions are: Royal Melbourne Institute of Technology (RMIT), Canberra College of Advanced Education (CCAE), South Australian Institute of Technology (SAIT), Western Australian Institute of Technology (WAIT), the Queensland Institute of Technology (QIT), Caulfield Institute of Technology (CIT), Swinburne College of Technology (SCT) and the New South Wales Institute of Technology (NSWIT).

The decision to confine our attention almost entirely to the DOCIT Colleges was influenced by a number of considerations, not least that they are substantial diverse institutions offering a range of studies at degree and post-graduate levels and that, situated as they are in major metropolitan areas of high population density, they attract a significantly higher proportion of student enrolment.

The tables in the appendix summarise the diversity of engineering courses given by the CAEs, both the DOCIT and the non-DOCIT colleges.

A table which gives the 1979 engineering enrolments in DOCIT institutions is also an attachment to the appendix.

Methodology

In addition to undertaking an extensive survey of the relevant literature, empirical material was gained from:

(a) observations of current practice and discussions with staff in a sample of colleges of advanced education.
The author visited the Queensland Institute of Technology, Royal Melbourne Institute of Technology and South Australian Institute of Technology, and is a lecturer at the New South Wales Institute of Technology.
The visits were generally arranged by personal letter and an excellent response was received to requests to visit the institutions. The academic staff associated with the mathematical education of engineers were available for interview and samples of syllabuses and examination papers were collected;

(b) documentary evidence as presented in the various handbooks and calendars of the universities and colleges of advanced education, the Tertiary Education Commission's *Recommendations for 1978*, and annual reports such as *DOCIT* and *Directory of Tertiary courses*;

(c) correspondence with the State Departments of Education, Board of Secondary School Studies (Queensland), Board of Senior School Studies (New South Wales), Victorian Universities and Schools Examination Board, Universities and Colleges Admissions Centre (Sydney) and Tertiary Institutions Service Centre (Perth).

As the selection of variables considered in the framework is necessarily limited, so also is the selection of research reviewed. Within the broad theme of this study may be found a whole spectrum of relevant issues, many of which have been the subject of individual research pursuits. Although the review of literature here will encompass much of this, this thesis is primarily concerned with the full range of the topic. Most of the research reviewed here has already been published; and, it is accepted that some useful unpublished material may have been overlooked.

The research and development reviewed is substantially Australian, although conceptual frameworks have been taken from foreign research. This has been done in full awareness of all the differences that exist between the Australian situation and that prevailing in other countries. That this is in one sense a limitation is undeniable, but the small scale on which research into teaching and learning is conducted in Australia, the small number of academic staff involved in the evaluation of the mathematical education of engineers and the small volume of published work have unavoidably led us to look to Britain, Europe and,to a lesser extent, the United States of America, where notable research has been, and is being, done. The British research is particularly comprehensive and covers in depth all the issues we wish to raise.

Though, it is admitted, one has to be necessarily hesitant and cautious in extending the findings across international boundaries, our dependence on overseas work, principally British, matters less than appears at first sight, because one is usually able to trace appropriate analogues in external experience which will enable us to view our problems and possibilities in a fresh perspective.

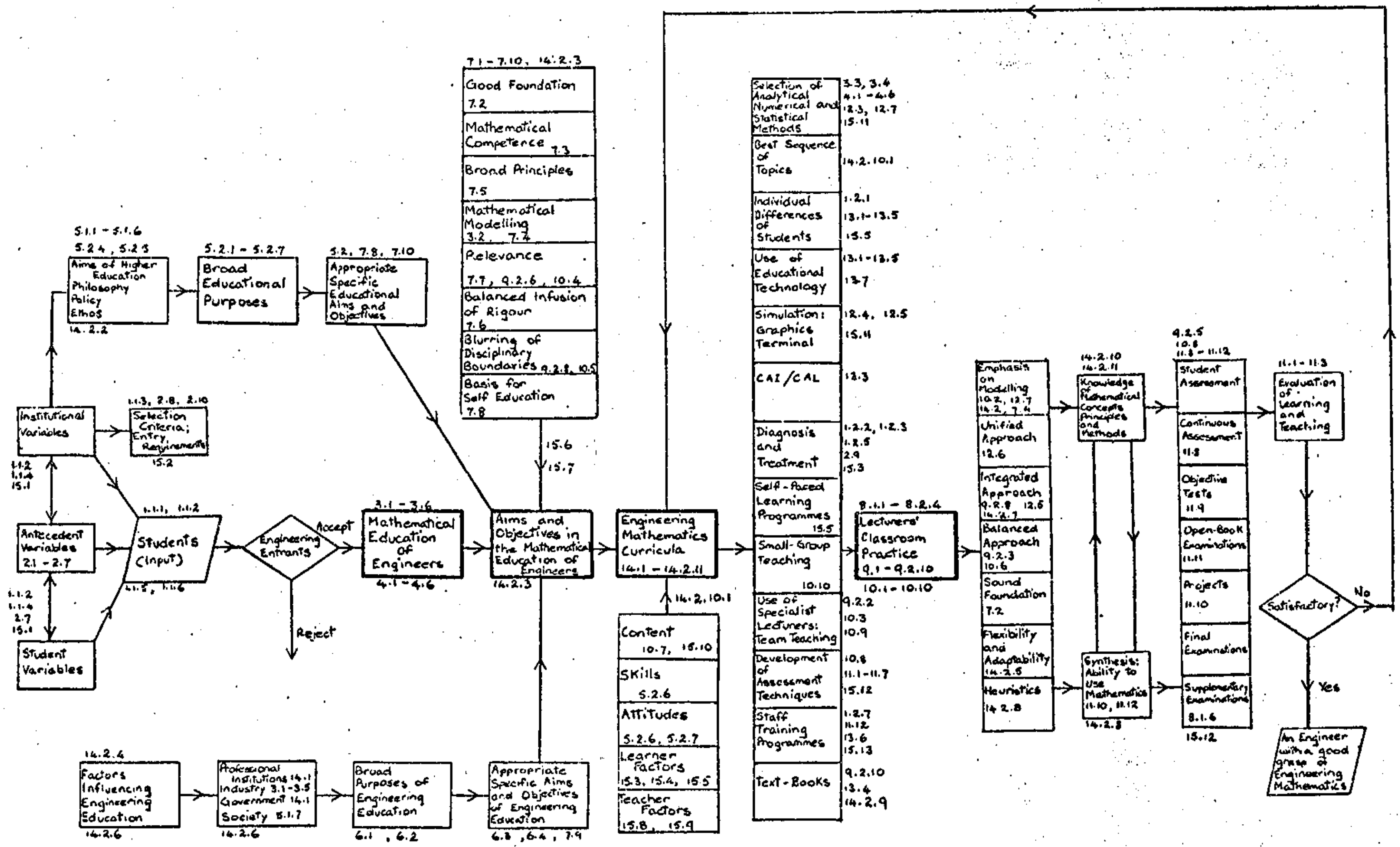
A comprehensive review of the major foreign literature in this study was not carried out because on many issues it was considered to be problematic as to the extent a direct transfer of experience may be valid.

A Blueprint

A major general recommendation that flows from the Williams Report is that access to education in areas too small to sustain specialised institutions should be extended by contract arrangements amongst post-secondary educational institutions. The Report envisages Technical and Further Education Colleges in provincial towns providing study centres and tutorial help for students enrolled in external courses offered by universities and colleges of advanced education.

It is precisely here, it is hoped, that the thesis provides a blueprint for the determination of the direction of the mathematical education of engineering undergraduates, a platform upon which other institutions will find it in their interest to build.

The hope is nurtured because the thesis attempts to establish the framework for the mathematical education of engineers and maps out the considerations which superlatively should shape the curriculum and the methods of teaching and assessment in the future.



A Flow Chart of the Main Ideas in the Thesis

(The numbers are keyed to the section headings used in the Thesis)

CHAPTER I

THE PROMISE AND PERFORMANCE OF ENGINEERING UNDERGRADUATES

Summary

This chapter introduces fundamental issues, provides the sombre background to the high attrition rate and focusses attention on such matters as selection of students and transitional difficulties. As well, it mentions some approaches in mathematical remediation and particularly the attempts by the Victoria Institute of Colleges and the Tertiary Education in Applied Mathematics Project. The fallacy of isolating a single variable from the flux of student, antecedent and institutional variables as the cause of failure is underlined. Some of the recommendations of the Williams report are examined.

1.1 FUNDAMENTAL ISSUES

1.1.1 Introduction

This has been an era of unprecedented expansion in Australian universities and colleges of advanced education. Professor Borrie (1972) estimated that the numbers of tertiary students would more than double during the period 1966-1986, with a period of particularly rapid growth towards the end of the 1970s. Students totalled 200,388 in 1973, their numbers were already approaching 282,000 in 1977 and, according to the Australian Bureau of Statistics, were 316,424 in 1979; courses have both multiplied and become more complex in content; the cost of tertiary education is rising very sharply. These developments coincide with a growing community expectation and need for better-trained and better-educated citizens : much has been said and written concerning the urgency and primacy of such educational provision.

Against this background the problems of universities and colleges of advanced education have come into greater prominence, with special attention to the rate of failure of students in their courses. When the output of graduates is regarded by the community as an urgent necessity, many look upon student wastage as a national extravagance that the community can ill afford - wasteful not only in terms of its financial cost to the community but also in terms of the time and effort of both the teachers and the taught. The failure rate of our students has become a major topic for discussion and investigation in recent years, particularly since in 1957 the Report of the Committee on Australian Universities (the Murray Report) commented adversely on the high failure rate. It said : "One of the most disturbing aspects of university education in Australia is the high failure rate". In commenting on the progress of entrants to the 1951 first-year examinations (35 per cent graduated in minimum time, 58 per cent eventually) the Murray Report (1957) said :

> *Such a high failure rate is a national extravagance which can ill be afforded. It gravely reduces the efficiency of the universities by causing many students to take a year or more longer than they should over their courses, thereby swelling the classes and getting in the way of the proper education of other students; it also seriously diminishes the national resources of trained university graduates by causing a loss of at least one year's service in the working lives of a very large number, and by excluding a further number from completing their courses at all.*

Wastage in higher education is an exceedingly ramified problem. Tertiary institutions which undertake earnest self-examination are faced with myriad questions such as : How do students perform in their examinations? To what extent is their success or failure either directly attributable to or in any way influenced by factors within the university or a college of advanced education? Do we make too many or too few demands on our students? Are our standards inconsistent or unusually high? What do the examiners know about the fundamental principles of educational measurement? Is the examining reliable? Can the content, organisation and presentation of courses of study be improved ? Does the university or college of advanced education provide adequate facilities - for example, in its laboratories and libraries? Is the quality of teaching impeachable or satisfactory? Are the classes small enough and is tutorial help sufficient? What influence do the relationships between staff and students and among students themselves have on performance? Do we provide adequate guidance and conselling both in academic and personal problems? What efforts are made to evaluate educational institutions in terms of the goals they set themselves or the needs of the community?

The attempt by tertiary institutions to answer such questions takes them far beyond self-examination in the narrow sense. What happens to the student at the university or a college of advanced education cannot be divorced from what happened to him before he came to it in school or at home or from whatever outside-university influences are currently operative. The student's transition from school to a tertiary institution is not easy. The hazards range from the social and financial, through the administrative, to the intellectual and emotional. They originate in home, in school and in a tertiary institution. They are not unconnected with the vagaries of examiners, the lure of the cricket field and the waywardness of sweethearts.

1.1.2 A Complex Set of Factors

Few will fail to appreciate that fitness for higher education depends on such factors as scholastic preparation, personal qualities, degree of motivation, maturity, and study habits, all of which are modifiable and all of which may exert together a greater influence than intelligence.

From the detailed and comprehensive investigations carried out by such eminent educationalists as Professor Fred J. Schonell and his associates, one finding towers clearly and conclusively, namely that it is virtually impossible to postulate a single cause for academic failure.

> *If we examine systematically the performance of each individual student, we find in every case a complex interplay of forces, intellectual, emotional and environmental; a unique combination of habits, attitudes, techniques and motives underlying achievement. Though one may suspect or attempt to isolate one or more major causes, it is essential to bear in mind that other factors are often inextricably bound up with them.*
>
> (Schonell *et al.*, 1962)

Furthermore, the same condition, intellectual, emotional or environmental, which appears to be an incapacitating or facilitating factor in one case may seem to be marginally influential in another.

The extent to which a single variable *per se* affects academic performance is obscure; indeed, the isolation of single factors in the study of academic failure or success is generally meaningless. It could simply be ascertained that the student with poor intellectual equipment, lack of persistence and application, bad home conditions, socio-economic handicaps, attending crowded lecture theatres and overwhelmed by the volume of work to be done is decidedly a poor risk; and that the student of high intelligence, excellently motivated, thorough and industrious, well provided with tutorials, without home or social distractions is highly unlikely to fail. There is, however, a huge uncharted area between these extremes. Indeed, it is possible to produce case studies of students whose performance at the tertiary level belied - though in opposite ways - their promise as shown by their matriculation results.

Any statistical enquiry into academic performance can aspire to encompass no more than a fraction of the complex of scholastic, psychological and environmental factors that are the determinants of success and failure. There is, however, one major finding of undoubted significance that emerges from the work of Schonell and his colleagues (1962) : the many high scores made by failing students in both verbal and non-verbal tests indicate with a definiteness that cannot be gainsaid, that an important proportion of students who fail do so because of factors other than lack of intelligence. That intelligence remains one of the principal factors of differentiation has been shown by the research executed by many educationalists; but, their work also affirms that intelligence results *per se* are a poor and unsafe guide. Hohne (1951) arrived at a similar conclusion : "More or less success at the University level does not seem to depend very greatly on the possession of more or less intelligence, but on factors such as interest, study habits, personality, socio-economic constraints". And yet evidence and experience suggest that there are minimum levels below which it is unsafe to commence tertiary studies. As Miller (1970a) puts it :

> *Ability is best regarded as a threshold variable. A certain level is required, but given that it is not absolutely critical, other intervening variables supplant it in importance.*

The problem of academic wastage cannot be considered realistically as one which is unaffected by *institutional variables.* Campus characteristics are just as critical a factor in academic attrition as student variables. Miller (1970a) writes : "While students' motivations and personality are clearly crucial, they cannot be considered properly in isolation. The way a student approaches his university or college work will depend upon how he has been taught and the manner in which he has been expected to study at school in the sixth form and earlier". Sanford (1956) has asked how can we discuss wastage without a clear perception of the aims of the institution. Sanders (1958) makes a similar point : "The problem of selection, wastage and failure cannot be considered apart from the university systems in which they occur, nor apart from the underlying theory which motivates academic action in particular systems. The point needs to be borne in mind that academic failure involves questions of aims and objectives" - that is, selection procedures; institutional ethos; curricula; teaching methods; counselling and guidance facilities; diversity, flexibility and rigidity of course arrangements; allowance of student mobility between courses and between institutions; pass rates and policies; examination and other assessment methods. These factors are amenable to manipulation by university and college faculty and it is possible to some extent to mitigate the influence of those factors that militate against success.

1.1.3 The Problem of Selection

The problem of selection is central. However, perfect selection, as Hohne (1951) maintains, is utopian : the problem is so involved that no method of selection - by academic examination, by interview, by psychological tests, by school report - will ever approach perfection.

The process of undergraduate selection, according to Christopherson, (1967), has two main objectives : first, the short-term view of simply selecting those who show promise in successfully qualifying in tertiary courses; and second, the distant long-term view of choosing those "whose value to the community will be enhanced by the possession of a degree".

As far as the short-term aim is concerned, the best evidence that a student has capacity to pass examinations of a particular kind is that he (or she) has previously demonstrated an ability to pass similar kinds of examinations. Therefore, the simplest method of selecting candidates, as examination passers, would be to base the selection process solely on performance at external HSC Examination or its equivalent : the work done in Mathematics and Physics and Chemistry at the inception of an Engineering degree course does not differ markedly from that attempted at the HSC level.

For the long-term objective, however, the situation is complex and unclear and there is an absence of unanimity and consensus among Universities and CAEs as to the selection instruments that may be used. However, as Christopherson points out, the following partial answers do receive substantial agreement :

(a) The really outstanding students, intellectually speaking, those who may become the real scholars, must always be accepted.

(b) Among the other candidates considered to be capable of graduating, a wide variety of other qualities such as curiosity, creativity, enthusiasm, practical sense, maturity of mind and general ability should be taken into consideration. These would surely be admissible attributes to use in selection of students. The Franks Report (1966), in outlining the limitations of existing selection techniques, states :

It is often supposed that their efficiency may be judged by the accuracy with which they enable the probable performance of candidates in the final examination to be predicted : the better the tests the higher the probability of an accurate forecast. The truth is not so simple for some important factors ... cannot be determined by tests at present in existence , but remain beyond the reach of any improvement in forecasting devices.

... within the broad groups marks are an uncertain guide to the forecasting of individual performance : yet selectors have to make positive choices. Preference for a candidate who obtained x marks on one particular set of questions over a candidate who obtained x-1 marks has no merit beyond that of simplicity. In selection for a university place, the choice between such candidates must hinge on other considerations, since on purely intellectual grounds they are indistinguishable. Weight can properly be given to evidence about their motivation, their background, the nature of their schooling, or their non-academic interests and activities.

If this general philosophy of selection wins acceptance, plainly it is unlikely that any one simple criterion of selection would prove satisfactory. In the long run what is needed is a comprehensive and complete assessment of personality, and personality is too complex a thing to submit to any simple measurement. It follows,therefore, that a selection based on a careful evaluation of evidence provided from as many sources as possible, and obtained in as many ways as possible, will always be preferable to one based on any single criterion. The practical problem, then, is to ensure that all the available and relevant evidence is collected in an efficient and economical way and that the various kinds of evidence are assigned appropriate weight. Statistical studies can, to some extent, help us decide what evidence is significant and the weight to be accorded to particular items. However, as Professor Christopherson (1967) reminds us, it is a mistake to assume that "such statistical studies, no matter how complete, can eliminate the element of judgement on the part of the selector or do anything to minimise the responsibility which is his".

The HSC Examination or its equivalent was always intended to serve in part as a qualification for University or CAE entry. Most Universities and CAEs state their matriculation and entry requirements in terms of passes at the HSC Examination and base their selection procedure on the attainment of specified standards at this examination.

The question of the suitability of the HSC Examination as a selection instrument attracts considerable discussion. Many educators view with concern the effects of present selection devices (public examinations) on the secondary school educational programmes. For it is plain that the HSC Examination cannot be perfectly adapted for the needs of all departments, and at the same time be adequate in its other equally important rôle as a school-leaving examination for those not proceeding to tertiary education. The effects of variations in the number of subjects taken, the differences in the standard of teaching and facilities available and the time devoted to specialist subjects further compound the problem.

A number of universities and CAEs are unable to admit all suitably qualified students. When demand exceeds places available, quotas are introduced. The selector has then the added task to decide which students would most benefit from tertiary education. In so far as this quality can be judged at all, it can only be judged by personal interview.

Some of the questions to which the interviewer directs his enquiries to ascertain which student would be of value to the community upon graduation are the following :

(a) *Has the candidate a real interest in and enthusiasm for Engineering? What steps has he taken to develop this interest and to make sure that he understands what the profession of Engineering involves? There are so many short courses, visits, lectures, etc., now made available to the schools by industry, that unless the candidate comes from a remote part of the country, it is a point against him if he has made no effort to learn anything of the industrial side.*

(b) *Has the candidate any experience, however slight, of the practical side of the job? A good many candidates have, of course, actually worked in industry for short periods; many others have spare-time interests with Engineering connections (cars and motor-cycles, radio, photography). By talking to them about these interests, an impression can be formed of the extent to which they have been able to relate their academic work to their practical interests - an important quality in an Engineer.*

(c) *Has the candidate the personal qualities which with experience will enable him to take responsibility, to direct the work of others, and generally to make a good contribution to an organisation?*

(Christopherson, 1967)

Some institutions study school reports in determining which candidates are selected. School reports can provide valuable information, because the school teacher is in a better position to assess the merits of the candidates by virtue of his long association. However, there are wide variations in the way school reports are compiled : some school teachers confine themselves exclusively to assessments of academic ability, while others, allowing examination results to speak for themselves, dwell chiefly on the candidate's contribution to the school. Moreover, as Christopherson (1967) rightly indicates, the use of the school recommendation contains two subjective assessments, that of the writer and that of the reader. The selector perforce has to read "between the lines, but what he reads there may depend a good deal on himself".

Hence, the problem of the selection of undergraduates defies a simple, neat solution.

> *For two-thirds of the candidates admitted and for three-quarters of those not admitted, the question of the relative importance to be attached to different sources of information does not arise : the really outstanding candidate is unmistakable, as is the really weak applicant. However, for the candidates near the borderline, some system of weighing the different items in the account has to be employed .*
>
> *When all the evidence has been duly weighed and assessed, the final responsibility remains with the selectors. The problem of the selection of undergraduates is essentially a human problem, and as such, in the last resort, not subject to quantitative analysis.*
>
> (Christopherson, 1967)

1.1.4 School Performance as Selection Criteria

The present method of selecting students for university or college education according to the standard of their performance at matriculation examinations seems to provide to date the best single predictor of academic performance.

There is an impressive body of evidence which suggests that matriculation results are pre-eminently the best single index of a student's performance at the tertiary level."The evidence is", Schonell *et al.*(1962) emphasise , "that matriculation results are undoubtedly the best single predictor of possible performance at the university (that is, excluding first-year university results which are the most effective predictor of ability to graduate)"..

The 1973 Report of the National Foundation for Educational Research arrived at a similar conclusion : "For most courses the best predictors of first-year performance and of final degree results were GCE A-level grades"..

Hohne, Hammond and Anderson of the University of Melbourne, Sanders in Western Australia, Schonell *et al.* in Queensland, Short and Gray at the University of New South Wales and Katz both there and at the University of New England - to name but some of the major investigations associated with the authors cited - have examined correlations between success and IQ, school examination performance, age, socio-economic status and other personality characteristics and background conditions of students entering universities. Unsurprisingly, there is substantial agreement in the findings of these studies that the best single predictor of performance in first-year examination at the university is performance at the final school examination and the best predictor of university success is success in first year. More recently, Professor O'Neil at the University of Sydney has found a very strong relationship between Higher School Certificate and first-year performance and between the latter and final graduation.

However, the research literature on selection procedures declares with a high level of consensus that this method is lacking in predictive validity : the correlations between secondary and tertiary academic performance are not very impressive. Miller (1970a) asserts : "Even though they appear to be the best single predictors, the inconsistency of the relationship casts doubt on their reliability and hence their predictive validity".

The research of Sanders (1948) in Australia impresses by the insight shown in the interpretation of the statistical work. After an exhaustive investigation he comes to the conclusion that the correlation coefficients between Leaving and University first-year examinations will lie between .50 and .65, with the best estimate being approximately .55. When more refined treatment is used, such as, limiting the number of Leaving subjects, grouping those which have greater relevance to first-year studies or excluding second attempts, the agreement is closer and the coefficients lie between .50 and .80, with the best estimate .63.

The general conclusion reached by Sanders (1948) is that

> *In every Australian University there is evidence in most faculties of a solid relationship between entrance examination results and the results in the university first-year examinations at least; but the extent of demonstrable relationships is not such as to give grounds for any educational complacency.*

Miller (1970a) arrives at a similar conclusion :

> *Improved student intake, using matriculation or A-levels as a criterion, must lead to diminishing returns in the improvement of selection procedures.*

Using matriculation examination as a principal criterion to discern promise is a hazardous procedure for other reasons. Some students are supervised, coached and spoon-fed to attain high marks in their matriculation examinations in ways that bear little relation to the more flexible, self-motivated and self-disciplined requirements of study in institutions of higher education. The words of Forster (1953) echo this :

> *Ideally, universities ought to be able to take for granted that all their students, by the time of admission, are able to plan and pursue studies without close supervision and direction; to exercise a measure of independent and critical judgement when faced with new material and ideas, to make full use of the library ... In practice, many students arrive at the university without this basic equipment, and even a promising school leaving examination may sometimes reflect an exceptional aptitude for cramming and rote memory rather than a developed capacity for thought and judgement.*

As well, some students make more than one attempt at matriculation examinations. Clearly students who require longer periods to satisfy admission criteria are not the best in university or college studies. Here, surely, is one of the most likely causes of error in the selection of students - an insufficient emphasis on the previous failure of those appearing for examination on a second or subsequent occasion. The high marks obtained by candidates can be positively misleading to selectors, if students have required repeated attempts to gain them.

The prediction of academic performance from matriculation scores is more likely to be achieved in Mathematics or in Physical Sciences than in other subjects. An important factor here may be that Mathematics particularly, but also Chemistry and Physics are the subjects in which examination performance may be more objectively assessed, whereas assessment of the more literary subjects is to some extent subjective.

The percentages of students passing a first-year mathematics subject at the New South Wales Institute of Technology (NSWIT) related to the students' HSC mathematics mark are presented in Table 1.1.

Table 1.1

Percentages of Students Passing a First Year Mathematics Subject at NSWIT Related to the Students' HSC Mathematics Mark

HSC Mathematics Mark (out of 100)	Percentage of Students Passing a First-Year Mathematics Subject
30 - 40	16
40 - 50	16
50 - 60	27
60 - 70	53
70 - 80	75
80 - 90	97
90 -100	100

Source : Shannon and Sutton (1979)

A study of the extent of correlations between engineering and science students' performance in mathematics at secondary school and the New South Wales Institute of Technology led Sutton and Matti (1979) to conclude :

> *There is a good correlation between students' performance in the first year mathematics subject, Calculus and Analytic Geometry, and their performance in the HSC examination . . . The correlation coefficients between the student 's mark in Calculus and Analytic Geometry and the student's aggregate mark and mathematics mark/unit are respectively 0.71 and 0.76.*

Universities and colleges of advanced education in Australia use academic criteria and academic tests to select their students. Minimum standards of achievement in school subjects are set and when there is an excess of applicants an "order-of-merit" is determined according to marks on a matriculation examination or some similar measure. Although the association between academic predictors and subsequent performance is not strong, marks from a matriculation examination or its equivalent (including assessments by schools) are assumed to provide the best single indicator of subsequent levels of performance. The reason for this is fairly obvious, since examinations of subjects taught at school are closest in nature to what is to be taught and tested at a tertiary institution. Furthermore, performance in school examinations is sensitive to all of those variables, such as study habits, personality traits and scholastic aptitude, which alternative measures attempt to isolate. "Examination marks represent", as Anderson (1972) asserts, "an encapsulation of the student's life history". Since examination marks reflect the influence of most variables that affect learning, composite predictors made up of examination marks plus other measures rarely improve the prognostication by a significant amount.

All of this, of course, does not mean that other devices in the administration of admissions may not be appropriate, especially if considerations additional to potential examination performance are taken into account. For example, an interview conducted by members of the academic staff is a means of informing the candidate about aspects of his future studies and the staff about personal attributes, interests and problems of their prospective students. These interviews can be a form of initiation and orientation : a good deal of apprehension and anxiety is induced in the candidates by having to confront academics and those students who are successful have a much greater feeling of acceptance and belonging than if their admission had been the result of postal correspondence and a computer calculation. For mature-age candidates who have been away from school for some time, aptitude tests and interviews, together with educational histories, are a much more appropriate means of selection than school results, which are severely out-dated and bear very little relation to university potential.

The prize of admission to a university is so great, especially to a faculty that has an intake on a quota basis, that applicants adapt their behaviour to maximise their chances. The introduction of tests of aptitude, for instance, would probably produce a proliferation of coaching schools offering cram courses. In Victoria, where about thirty per cent of university entrants have spent at least two years in sixth-form, it has been found that marks from a second year at matriculation over-estimate future performance levels. There is a real gain, in terms of probable future success, which follows a second year but the average increase in matriculation marks overstates this. In order to restore equity between those candidates who have had one attempt and those who have had two attempts at the annual matriculation examination, the admission formula introduced a debit which is applied to results from a second attempt. This debit is designed to remove only that amount of the average gain associated with over-prediction.

1.1.5 Transitional Difficulties

The student's transition from school to university or college is not easy. The requirements at tertiary and secondary level differ greatly and unless a student is able to modify his attitudes and study habits and adapt himself to his new environment, his undergraduate studies would be studded with many difficulties.

At school a student is guided and supervised closely and he has an obligation to attend set lessons and attempt the set homework. At the university or college, however, there may be a choice of possible lectures, where attendance is not compulsory. Nor is there any obligation on him to attend any remedial classes that may be convened. Thus students experience a sharp difference in the learning environments of school and university, a difference which may be minimised by making the last year at school nearer in intellectual temper and methods of study to universities or colleges.

Miller (1970a) points out that antecedents can be critical determinants of success and failure. He writes : "The relationship between performance and antecedent variables cannot be disregarded

. . . Among the many factors which may account for failure to fulfil high academic promise could be included the fact that students have not learned the art of independent study".

There can be no doubt that students in some schools experience the kinds of learning and atmosphere which are inimical to preparation for higher education. Modes of teaching and types of schools have a deep influence on the approaches to learning which some students bring to their undergraduate studies. The teacher who encourages a heavy dependence on himself and the school which stresses "rigorous, concentrated and convergent study attitudes", which imposes "straitjacketed thinking" and which sets a premium on the kind of teaching that is "relentless in the rejection of side issues" must together share the responsibility of producing a student destined to fail in the first year.

The onus to improve the quality of student intake rests, to a significant degree, on the quality of school-teachers. As Sir Geoffrey Crowther (1963) once said, using an expressive phrase, "What is extracted from the pool of ability depends less on the pool than on the pump", a remark which is equally applicable to primary, secondary and tertiary education.

Schools can help create quasi-tertiary conditions by helping students to think for themselves, giving them greater freedom in planning their time, allowing for more free study periods, teaching less formally and giving training in study habits and note-taking.

The universities and colleges, too, by exercising more pastoral care, can play a notable rôle in the minimisation of anxiety induced by transition. But, alas, few tertiary teachers have any knowledge of school conditions and any appreciation of the difficulties that can arise in the move from secondary to tertiary studies.

A few students are disturbed by the speed with which the syllabus is taught, having been used to a more gradual pace and to spoon-feeding. By modifying their teaching methods, universities and colleges can aid students in their adjustment. The modification of over-loaded syllabuses and the provision for pause and discussion during lectures would enable students to become "acclimatised" to the new situation, where the type of thought and the mode of working represents too abrupt a change from that obtaining in the final year of secondary schooling.

All but the very able are perturbed by the degree of abstraction and rigour found in tertiary mathematics. Generally, schools gloss over axiomatics and shy away from excessively rigorous analysis: certain results are accepted without proof. The insistence, at tertiary level, on giving lengthy and abstruse proofs for statements whose truth is self-evident or intuitively obvious is a major causative factor in the transitional trauma experienced by a considerable proportion of students. Ledermann (1972) observes: "Most university teachers will agree that analysis is a greater stumbling block than algebra for the majority of students". Thus, it is not the encounter with new types but the heavy tincture of abstraction and rigour that compounds a student's difficulties at the tertiary level. Striking a balance between abstract formulations and applications and between intuition and rigour, may lessen the difficulties.

Part of the problem can be traced, according to Budden (1972), to a lack of communication and interaction between secondary and tertiary educational institutions. Griffiths and Howson (1974) share this view: "With the rapidly increasing numbers flowing into the universities the high wastage-rate becomes politically important and some rapprochement between secondary and tertiary level has to be devised". By encouraging individual visits to universities and colleges by pupils while still at school, organising short vacational courses for sixth formers at universities and colleges, distributing pamphlets and literature delineating the aims and the objectives of tertiary mathematics offerings, extending invitations to academics to address secondary school pupils, and encouraging academics to spend their sabbatical leave in observing secondary schools in action, Budden is confident that bridges can be built over the chasm and its academic impact made less painful.

1.1.6 Pass Rates in General

The study of academic progress in Australian universities of all the 1961 entrants, initiated by the Commonwealth Department of Education and Science, is a very comprehensive one. It gives the history of a large number of students over a seven-year period.

Of 8,599 full-time students, 37.3 per cent graduated in minimum time; by the end of 1967, 63.9 per cent had graduated, 31.6 per cent had discontinued and 4.5 per cent were still enrolled. The rate of progress of part-time students is even more dismal. Of the 1680 who commenced in 1961, 29 per cent had graduated in minimum time, 63 per cent had discontinued and 8 per cent were still enrolled. The study affirmed a very strong relationship between matriculation results and academic progress and between performance in first-year examinations and ultimate graduation.

Table 1.2

The 1961 Study : Percentage of Full-time University Students Graduating

Number of Students in Faculty	Minimum Time Graduation Rates	Total Graduation Rates
All Faculties		
8599	37.3	63.9
Engineering		
887	37.2	64.2

Source : Commonwealth Department of Education and Science and the Australian Vice-Chancellors' Committee (1971)

Students at colleges of advanced education have an even more sorry academic record. The pass rates are more depressed and significantly lower than those experienced typically in universities. (This can be explained by the fact that the more able students elect to study at a university rather than at a college of advanced education.) For example, pass rates in first-year science ranged from 51 to 73 per cent in a sample of six advanced education colleges.

The graduation rates in Colleges of Advanced Education are given in Tables 1.3 and 1.4. Part-time students seem to have a higher propensity to withdraw than do full-time students.

Table 1.3

Percentages of Enrolees Graduating in Minimum Time from Colleges of Advanced Education

Field of Study	1974 Completing Students as a percentage of 1969 part-time plus 1972 full-time commencing students	1975 Completing Students as a percentage of 1970 part-time and 1973 full-time commencing students
Agriculture	61.69	67.22
Applied Science	55.01	56.05
Art and Design	47.57	46.69
Building	44.55	50.91
Business	40.07	41.95
Engineering	47.78	49.22
Liberal Studies	73.94	58.19
Paramedical	74.75	77.86
Simple Average	50.39	51.39

Table 1.4

Percentages of Diploma and Degree 1973 Full-Time Enrolees and 1970 Part-Time Enrolees Completing in 1975

Field of Study	Percentage Completing
Agriculture	45.44
Applied Science	50.54
Art and Design	47.53
Building and Construction	50.73
Business and Commerce	39.72
Engineering	51.88
Liberal Studies	45.61
Paramedical	67.17
Simple Average	47.86

Source of Tables 1.3 and 1.4: Gleeson (1978)

The Report of the Committee of Inquiry into Education and Training, under the chairmanship of Professor B.R. Williams, affirms that the attrition rates in universities and colleges of advanced education remain disturbingly high.

Table 1.5

Attrition Rates in Universities and CAEs

Type of Institution	Percentage of Students Discontinuing	
	Full-Time	Part-Time
Universities [1]	27	71
CAEs [2]	37	64

Source : 1 Williams (1979), Appendix D, Tables 1-6, 13-15;
2 Williams (1979), 1, 242

It is gross inefficiency that the resources of students and staff should be expended with no check on progress until the end of an academic year. An early warning system is much needed. "There is evidence of the value of setting an examination at the end of the first term as an early warning to those who are likely to fail later on" is an assertion by Beard (1976). According to an investigation by Nisbet and Welsh (1966), students most likely to fail in finals were in the bottom third of the class in two or more subjects at the end of the first term. Beard feels that rates of failure would be reduced by warning these students, "for feedback as to the probability of failure resulted in a redoubling of their efforts". The students themselves, as the National Union of Students indicated in its executive report on examinations at the 1969 Liverpool conference, are in favour of an early warning system in each subject. The unit courses currently being developed in some universities and the introduction of semester units by some colleges do in some measure meet this need for an early warning.

One reason for student failure in courses is that many are oblivious of the expectations of the courses and the standards they have to reach. According to Fuller (1976), syllabuses are often vague : "They are oftn not detailed in terms of content and sequential presentation and give little indication of the degree of comprehension and understanding required. The content of the syllabus was merely to meet the needs of staff teaching subject areas mostly in later years". The specification of the aims and objectives of a course in clear, unambiguous terms and the provision of trial tests which showed the student not just his relative position in the class but also precisely where his own attainment lay in relation to the standards required would do much to counteract this problem of vagueness.

The inflexibility of the present system is highlighted by the not infrequent usage of the term "minimum graduation time" in relation to assessment of students' academic progress. There are a large number of students whose attainments fall just short of the acceptable standard, but who are required to repeat a year or to withdraw from the course. Furneaux (1962b) observes that the practice of excluding a student from the next year's course as a result of a test at the end of the preceding year attracts criticism. He calculated correlations between examination results for students at the end of their first and second years of a course and found the correlations to be poor. As an instance, attainments in first-year applied electricity correlated more highly with second-year mathematics (0.69) than did first-year mathematics (0.47). Thus, a rigid insistence that a student should complete and pass in first-year work before proceeding to the second year may be unjustified, although a subsequent course might be developing higher cognitive skills.

Unreliability and invalidity of examinations would also account for a high proportion of errors. It may be expected that in a parallel test, even the next day, some border-line cases would change places, and even a few candidates from the extremes of the distribution might perform quite differently. Again, according to Parkyn (1967), there is frequently considerable unreliability in the marking of examination papers : variations occur between examiners or for the one examiner over a period of time. Parkyn has estimated that "marking 'errors' can cause around ten per cent of the annual academic casualties in New Zealand universities". Thus, sending a student down or holding him back a year on grounds of failure in such tests is unwarrantable, save in exceptional cases. Where such an action is considered necessary, it should be based on failure in a second test after an interval for revision, together with assessment of course work.

Thus, there is a need for making allowances to accommodate variations from normality as regards the progress rate of students. The report to the Australian Vice-Chancellors' Committee by Cochrane (1970) on the reorganisation of the academic year contains a number of proposals which would meet the needs of these students. One of the proposals worthy of special mention is the creation of a summer term which could be used for both recovery and acceleration.

1.1.7 Pass Rates in Mathematics

The problem of failure in mathematics amongst undergraduates, especially those enrolled in technological courses at universities and colleges of advanced education, is fairly ubiquitous : it is not the exclusive concern of any one institution.

The pass rates for students at the New South Wales Institute of Technology in Calculus and Analytic Geometry, Multivariable Calculus and Ordinary Differential Equations, for the 1977 Autumn Semester, were 68.8 per cent, 57.4 per cent and 67.2 per cent respectively. These percentages can be taken to represent, with small variations, the performance of students from semester to semester. The other tertiary institutions could provide an almost identical set of pass-rates. Not without reason the Faculty of Engineering and the School of Mathematical Sciences simultaneously expressed deep concern for the heavy casualties in mathematics.

Certain undergraduates have come to develop an aversion to mathematics as a consequence of their harrowing experiences at school. Mathematics takes on a formal flavour in secondary schools, as adolescent pupils learn to wrestle with quadratic equations, congruent triangles, logarithms, analytic geometry and calculus. These mathematical exercises, for some students, are a form of mental yoga; for many others, their high school encounter with mathematics is a deeply frustrating experience, devoid of any relevance or significance; not a few feel intimidated and confused. Too often mathematics in high schools is treated as a "spectator sport" rather than as a creative intellectual activity; too often, it is presented as a self-contained subject, divorced from other fields of human endeavour; and, too often, formal operational thought is required of pupils before they are properly into the concrete operational stage of cognitive development. It should not be surprising, then, that a significant proportion of students display adverse attitudes towards mathematics.

A deplorable attitude to mathematics is endemic to our society, an attitude which creates unfortunate mental blocks in nearly everyone. It is a compound of a somewhat envious admiration of those supposedly few, gifted with mathematical ability, and a pooh-poohing of mathematics as an airy-fairy irrelevance to "real-life" situations. A university lecturer in mathematics, when asked at a social gathering about his occupation, invariably said : "I teach big sums!".

Primary, secondary and tertiary teachers of mathematics are strategic factors both in the propagation of this unfortunate sense of unease and, hopefully, in correcting it. It should be their chief and overriding aim to provide the widest possible dissemination of an understanding of what mathematics is and what it is not. When this is done, the average citizen will understand that mathematics is not only a way of thinking which provides a powerful tool for analysing subtle and unobvious aspects of our experience, but also a cultural resource, which can add interest and enjoyment and a new dimension to life. Further, it will be apparent that the symbolism of mathematics - algebraic and graphical - constitutes an important language which is essential for communication of ideas and for the formulation of societal goals. When teachers communicate with the users of mathematics - the mathematical practitioners - they can begin to suffuse their teaching with practicality and regain the interest of some of their more sceptical students.

Several attitudinal variables were examined by means of a questionnaire and related to performance at ordinary and advanced levels by

Selkirk (1975). The survey confirmed the existence of unfavourable attitudes to the advanced course and provided strong evidence for the existence of too marked a change - whether in content, teaching method or pace - between ordinary and advanced level courses. (A change of a similar order occurs in the transition from school to university mathematics.)

Unless an attempt is made to arrest this sense of unease with accelerated realization, there is every danger that these antagonistic attitudes would be passed on and we would all be suffering from the effects of what Brown (1977) describes as a "feed-back loop". The primary task should be the annihilation of all the barriers between student and subject. Having achieved the desired attitudinal change, the second thrust should be in the direction of mathematical remediation. It cannot be too strongly stated that nothing succeeds like success and success in mathematics would stimulate tepid enthusiasm and interest.

It is a matter of regret that courses in mathematics remediation have many negative connotations attached to them. Many teachers feel that these courses lack academic status, prestige and creativity and therefore, it is beneath their dignity to become involved with them. Most students associate failure, inferiority complex and boredom with these courses. Bohigian (1973) observes that

> *The disdain and distress exhibited by faculty and students involved with such courses works against a meaningful approach to mathematical remediation. The use of endless and pointless drill exercises just adds to the demeaning and deleterious nature of the standard approach to mathematical remediation.*

Bohigian (1973) has designed and developed suitable models for the dignified eradication of one or more of the following student deficiencies :

(1) No formal training in classical mathematics.
(2) Fear of mathematics in an academic setting.
(3) Lack of respect for the importance of mathematics.
(4) Inadequate mathematical and computational skill.
(5) No knowledge of the philosophy or structure of mathematics.

In Bohigian's estimation, the guiding principles for the development of models suitable for the remediation, with dignity, of mathematical difficulties are :

(a) *The topic should have some research potential appropriate for students.*
(b) *The topic should lead naturally into at least an elementary aspect of some significant field.*
(c) *The topic should have potential interest for mathematicians.*
(d) *The topic should be easy and entertaining enough for all students to get involved in.*
(e) *The topic should be substantially different and new to most students.*

It is worth mentioning that the Canberra College of Advanced Education is going to develop tertiary level mathematical remedial programmes.

We have already observed that part of the cause of mathematics failures is that the syllabuses are vague and not sufficiently prescriptive with respect to the expectations of the courses and the standards to be attained. We have, in this connection, mentioned that a detailed, clear statement, delineating the aims and objectives of the courses and the degree of understanding and comprehension expected, would mitigate this problem.

We have seen too that a number of students are poor achievers in mathematics because their motivation is at a low ebb. Many engineering students voice an element of disquiet and discomfiture because mathematics is taught in a way that shows little, if any, relevance to their elected profession. The emphasis is that mathematics is a tool, to be used later. When mathematics subjects are presented in such a way that they act as a prologue to other professional courses, totally divorced from other fields of human endeavour, it is not surprising that students are poorly motivated. "A more effective functioning of interdepartmental liaison", advocates Fuller (1976), "would support the learning process by re-inforcing mathematical theory by applications". It is worth reiterating that only when teachers of mathematics begin to have a dialogue with mathematical practitioners would they be able to inject an element of realism into their teaching. This, together with an integrated approach to the teaching of mathematical topics and mathematical modelling, would aid in raising the level of motivation and interest. At the New South Wales Institute of Technology an attempt has been made to present a course which seeks to find the middle way between abstract formulations and applications which distract from the essence of the mathematics.

The relaxation of selection procedures and the broadening of entry qualifications for mature-age students contribute to the dismal academic performance of students. Although these mature students display a responsible approach to their work and are excellently motivated, they have been away from studies for a considerable period and often their background knowledge is sadly shallow. Syllabuses in schools, since they left school, have changed dramatically in depth and breadth. Thus these marginally qualified students commence their undergraduate careers with a distinct disadvantage. Attendance at bridging courses in mathematics and other disciplines should make an important contribution to the successful culmination of their university studies.

1.2 SOME APPROACHES TO THE PROBLEM

1.2.1 Modular Degrees in Engineering

Such institutions as the Oxford Polytechnic and Lanchester Polytechnic in Britain and the University of Sydney in Australia offer modular degree schemes in engineering.

These modular schemes of study enable a student to choose one or more subjects from a comprehensive list, providing the constraints of prerequisites for certain core subjects have been met.

Modular degree schemes vary considerably, but they do share an underlying philosophy. They seek to provide a greater variety of student choice and a means either of following broadly-based studies in several disciplines, or of specialisation by means of in-depth study of a particular subject.

To do this successfully necessitates the sub-division of each subject into parts or course-units. These course-units must be capable of being combined, not merely with each other, but with those from virtually every other subject. Thus, if units are to be fitted together in a wide variety of patterns it follows that

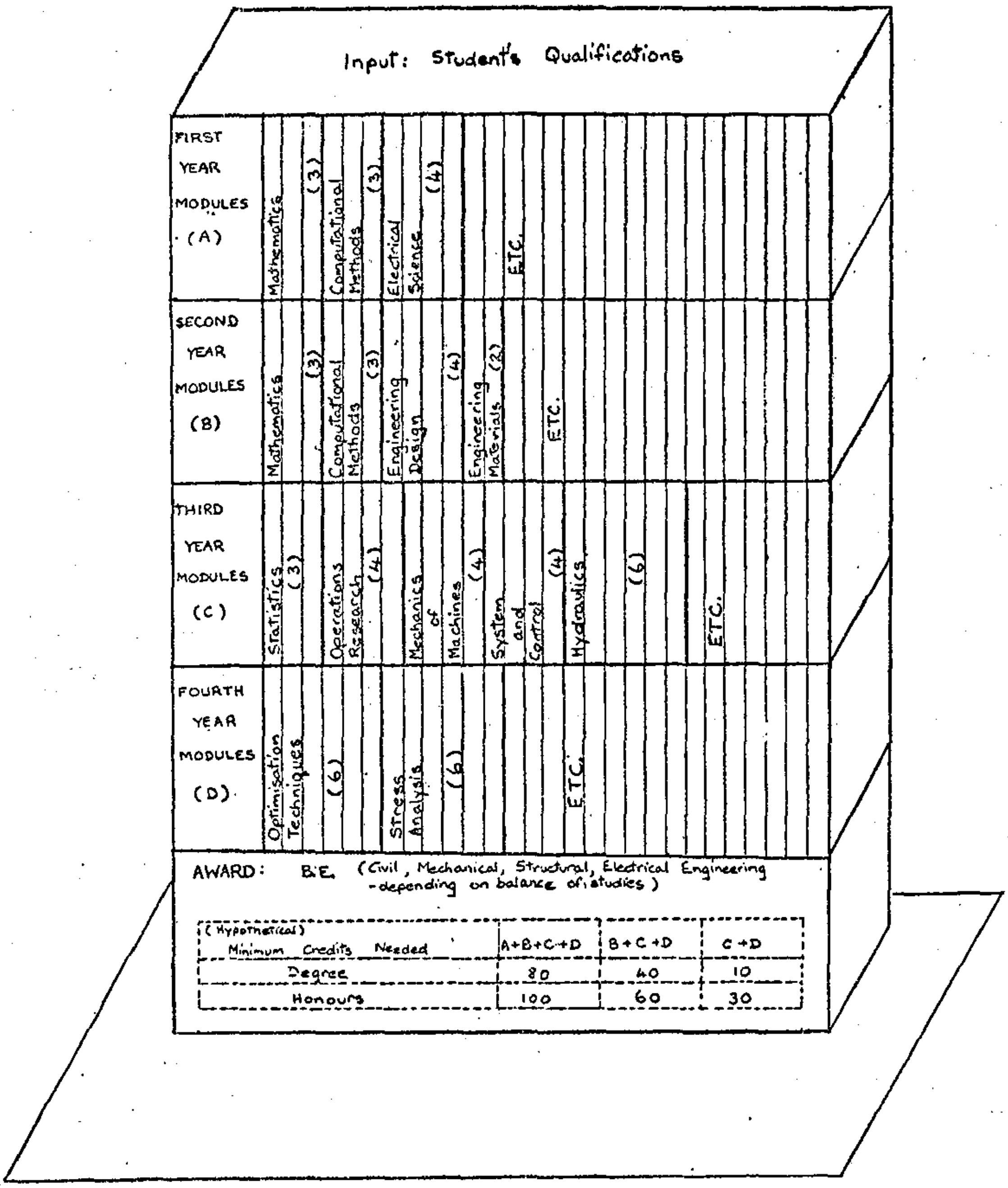


Figure 1.1

A Modular Degree Scheme in Engineering

(The figures in brackets indicate course-credits) they must all be of uniform size. Architects use the term "module" tp describe standardised dimensions : its extension into higher education, where the modules fit together like bricks in a wall, is useful, for it carries the implication that such an approach can lead, by careful selection of appropriate modules, to an infinite variety of programmes specially tailored to each student's abilities, aptitudes and career aspirations.

The flexibility of the modular structure gives the widest possible scope to pursue individually varying degrees of specialisation of a student's own choice. Under the guidance of academic advisers, a student is able to map out a programme of study that not only matches his interest, background and ability as closely as possible, but which provides him (or her) with the best opportunity of graduation. Thus, modular courses are flexible and student-centred, rather than structured and course-centred.

Since each module represents a separate course, it is examined as such, either immediately it is completed or at a later date. This means that only the method appropriate to the module need be considered, such as a project, a multi-choice paper, a dissertation, or a three-hour written examination. This leads eventually to the assessment of each student by methods that reflect his particular study programme. The over-all grade or class of honours awarded to a student is obtained by the summation of grades for each module.

1.2.2 The VIC Survey

Mathematics staff at various colleges of advanced education have been disenchanted with the paucity and inadequacy of suitable teaching/learning material, especially for remedial and refresher courses. Under the direction of the Victoria Institute of Colleges (VIC) Educational Technology Unit, a Working Group of lecturers was established in August, 1974 for the development of remedial and refresher packages. The Working Group's prior task was not only to visit the various colleges affiliated with the VIC for the purposes of ascertaining and discussing areas of mutual concern, but also to compile a detailed "mathematical skills" profile of the incoming students for the identification of special areas of mathematical weakness. This is the genesis of the report of the VIC Working Group in Mathematics (1975), a report dealing with the findings of the "Survey of Mathematical Skills" of the 1975 intake of students studying mathematics as a service subject. This survey was conducted at ten Victorian colleges with over 2500 students participating. Since a pilot survey, conducted in late 1974, revealed that these students displayed a spectrum of abilities, it was decided to use a stratified item sampling technique for the 1975 survey.

The Survey (VIC Working Group in Mathematics, 1975) makes an important observation:

> *Students proceeding from sixth form level mathematics subjects into Engineering/Applied Science studies with a substantial first-year mathematics content, whilst familiar with the basic concepts of the topics they have studied, will nevertheless not possess the mathematical skills and comprehension to the extent commonly assumed for such mathematical studies.*

As a consequence, the Survey has indicated a need for modification and reappraisal of existing first-year mathematics syllabuses in terms of assumed knowledge and skills for students undertaking Engineering or Applied Science studies. The Survey recommends the streaming of students and the reorganisation of the content and structure of first-year mathematics to allow for the wide spectrum of mathematical abilities of new students, perhaps even making provision for "catch-up" semester units. To assist in this, "it is advocated that a suitable 'interface' with secondary education be set up to ensure continuity of mathematical education of students proceeding to advanced studies".

Over recent months, the "New Maths" now being offered in schools has generated some controversy. This Survey, whilst it does not permit comparisons to support either side in the debate, shows that the "New Maths" notation presents a problem. As an instance, the Survey shows that although students can recognise the graph of a circle, only 57 per cent of students related the graph of a circle of radius 3 and centre at the origin to

$$\{(x,y) : x \in R ; -3 \leq x \leq 3 ; y \in R ; x^2 + y^2 = 9\}$$

Again, students display a measure of grasp and understanding of the function $\sin x$, but are perturbed by the notation

$$f : x \to \sin x$$

1.2.3 An Item Bank

A list of nineteen topics - general numeracy techniques, algebraic manipulation, analytic geometry and graphs, significant figures and rounding off of decimal numbers, surds, trigonometry, indices and scientific notation, logarithms, arithmetic and geometric series and binomial theorem, relations and functions, inequalities, matrices and determinants, vectors, probability, statistics and Venn diagrams, complex numbers, differentiation and integration, sigma notation, and limits - was used, so that the Survey was sufficiently wide-ranging. Within each topic items were written at each of four levels. Nearly seven hundred multiple-choice questions were written by the team and these were reduced to five hundred after detailed item-by-item discussion and consideration. Thus a large number of items was prepared so that several independent tests would be available at each level, thereby ensuring that there would be sufficient items to act as reliable indicators of student performance.

> *There are three major components of an item bank. The first is an aggregation of items, often referred to as the item pool. The second component is a physical system of storage and retrieval which may take one or more of the several forms, such as publication in a book, on loose sheets, on cards, on microfiche, on microfilm and through a computer system. The third component is the underlying philosophical structure, an intellectual system of storage and retrieval : for example, how shall questions be ordered and what steps should be taken to assist users by developing a logical retrieval system ?*
>
> *The general uses to which item banks have been put in the past include*
>
> *(a) for courses with defined objectives :*
> *the monitoring of individual and group performance on single objectives or groups of objectives.*
>
> *(b) for general courses :*
> *to stimulate student interest;*
> *to diagnose student weakness;*
> *to diagnose instructional weakness;*
> *to practise new skills and concepts;*
> *to provide models for teachers wishing to write items of their own; or,*
> *to act as a source of ideas for teachers.*
>
> (Foyster, 1976)

One further use for the existing items is that of assisting in guiding first-year students into appropriate service courses or, in the case of self-paced instruction systems, in indicating starting levels, by providing a "mathematics profile". Subject to the availability of appropriate items, the service can be extended to beyond the first-year.

1.2.4 The TEAM Survey

As a Tertiary Education in Applied Mathematics (TEAM) project at the New South Wales Institute of Technology, a questionnaire was distributed to all first-year students in the 1976 autumn semester, the dual purpose being

(i) to determine areas of difficulty within the first-year mathematics courses, and

(ii) to elicit the expectations of and general usage by students in regard to mathematics textbooks.

Mathematics has always been the bane of many students : nearly thirty per cent of first-year engineering students fail mathematics and only about seventy per cent of those sitting in second year mathematics subjects are able to pass at the first attempt. In the study of the diagnosis of students' mathematical difficulties, it was the intention to obtain a clear identification of those concepts and skills which presented above average difficulty, so that measures could be taken to provide the necessary remedial action. Also, much dissatisfaction had been expressed at the choice of some prescribed mathematical texts and it was the function of the survey to assess the expectations that students held with respect to their textbooks and to gauge any patterns of student usage of them.

Surveys, such as those designed to pin-point any major areas of inadequacy in a subject, provide a valuable feedback for the lecturer, by indicating to him segments of the syllabus that warrant special attention, more deliberate teaching and intensive revision, and by providing him with an insight into his own performance as a teacher. Hence, these surveys not only provide a profile of the student, but also, indirectly, a profile of the lecture's ability to teach effectively.

Mathematical topics that have posed above average difficulty in a number of courses include the binomial theorem, matrices and determinants, exponentials and logarithms, the chain rule for differentiation, trigonometric identities, graphs and curve sketching, the translation of literary statements into mathematical language and elements of integration. As most of these topics have been encountered by most students at secondary schools, the manifestation of difficulties makes the need for revision very compelling; without it, some students would be seriously disadvantaged.

Some students, especially those in first-year courses, are found to be bored with the syllabus, much of which they were covering for a second or even a third time. This happens when the subject matter of sixth form and first-year university or college overlaps and where students have spent two years in sixth form in order to gain a higher level of achievement, made necessary to win a place in a quota. Fuller (1976) makes a similar finding : "The same feelings of boredom and disinterest greet any watered-down approaches to some topics". On the other hand, irrespective of the degree of difficulty, new work is welcomed enthusiastically. Students often make a more determined effort to master more difficult work.

This view is supported by the analysis in Table 1.6. In their final examination in higher algebra, students were given a choice so that a comparison between the relative popularity of the various topics and student success in them as well as their degree of difficulty (as estimated by the students) may be made. Under popularity, A refers to most popular; under success, 1 refers to the question which was best handled in terms of relative scores; under difficulty, 1 refers to the easiest and 11 to the hardest. For example, the Table shows that eleven students in the class found topic 02 the most difficult and it was the 10th best done (or second worst) and one of the least popular. The topic 01 was also found to be quite difficult by 9 students, but it turned out to be the best done and one of the most popular.

Table 1.6

A Comparison between the Relative Popularity of Topics, the Degree of their Difficulty and Student Success in Them

Category	Popularity	Success	Difficulty
01	A	1	9
02	C	10	11
08	C	6	4
09	B	9	10
11	B	4	5
18	B	3	1
19	A	6	6
20	C	11	2
21	B	8	7
22	C	2	8
23	B	5	3

Source : Shannon (1977b)

Merely repeating difficult topics is no solution since the abler students become bored and disinterested and the less able are not helped significantly. According to Anderson (1972) greater flexibility should be shown with respect to the recruitment and placement of students. He says : "There should be opportunity for applicants who are selected to delay entry for two or three years; for students who are well prepared in given subjects to be given exemptions or advanced placement; and, for students who may be deficient in particular areas to receive intensive preliminary training".

There was wide agreement amongst students that textbooks which lacked an adequate number of good worked examples and graded exercises and which assumed a considerable depth of prior knowledge in the treatment of topics aggravated the difficulties which they experienced.

The economics of the publishing business militates powerfully against the production of good textbooks or other teaching material. Because what it means to mathematize is so little understood by curriculum planners, teachers or editors of publishing houses, the textbook market is an easy prey to fadism. The publisher tries to sell his books by a flashy use of colour, by transparency inserts, or by some other artifice. There are very few books that weave undergraduate mathematics and applications in engineering together, nor are there sufficient numbers of books stressing the importance and art of mathematical modelling.

A few years ago there was a sudden fad of "programmed" learning. The publishers feared and/or expected that this was the coming "thing" and hence many authors hastily produced programmed texts, which reveal to any critical informed reader many inadequacies. This is not to say, however, that programmed texts are without any merit.

The advent of the so-called "New Mathematics" in secondary schools triggered a publishing fiasco. The Australian publishers rushed in and flooded the market with books which appeared to meet the new desiderata. This was achieved by sprinkling the words "set", "associative", "multiplicative inverse", "pronumeral" and "solution set" liberally throughout the text. Perhaps the nonsense in some of these books has been the greatest single factor in bringing the new approach into disrepute and question : it is not improbable that this nonsense may have been conducive to the generation of antagonism and aversion that mathematics attracts.

1.2.5 Diagnostic Tests in Mathematics

Professor Dunn (1967), currently Chairman of the Education Research and Development Committee in Canberra, deplores that "too little attention is given to individual diagnosis of weaknesses in the secondary schools. It is recognised that large classes make remedial work difficult . . . The teacher is handicapped by the poverty of good diagnostic tests available". Keeves and Radford's (1969) contribution to the International Study of Educational Achievement in Mathematics and the more recent study of Literacy and Numeracy in Australian Schools by Bourke and Keeves (1976) indicate the importance of remedial work and the dearth of diagnostic testing.

The increasing clamour for the demise of an external Higher School Certificate Examination, particularly in some States, and the sympathetic demand for increased autonomy of secondary schools to design their own curricula will mean that the mathematical background of engineering entrants will be more hetrogeneous. Indeed, the existing practice which enables students to enrol in engineering studies after completing secondary mathematics courses at different levels clearly implies already that we cannot assume that all such students have uniform mathematical backgrounds. Further, there is already a general awareness of the limitations in present selection procedures and of the vulnerability of first-year students.

The early identification of weaknesses in the students' mathematical background and of "high risk" students in first-year studies is an essential first step toward lessening failure and wastage. The most direct method is to use diagnostic tests. Apart from fulfilling the outlined objectives, they are useful as a teaching strategy, since they may be used to pin-point areas of inadequacy common to many students so that the course can begin at an appropriate level.

Where the requisite mathematical skills and understanding are lacking in students commencing engineering studies, the schools from whence they came must bear some of the responsibility. However, having accepted the students, diagnosis and treatment should both form an essential part of the responsibility of all university or college mathematics departments towards their students.

The NSWIT has developed screening tests in mathematics since 1976 for the identification of basic weaknesses of students who enrol in first-year courses with a mathematics component. The production of these tests has been facilitated with the aid of a NSWIT research grant for assistance from an expert in educational measurement.

The analysis of student performance in these tests has shown grave deficiencies amongst many students in areas which we regard as fundamental.

Table 1.7 outlines the performance of the students admitted to the New South Wales Institute of Technology in the 1979 Autumn Semester. Most of the questions in these diagnostic tests deal with topics normally met in the first four years of high school.

Table 1.7

Percentages of Students Obtaining One, Two, Three and Four Correct Answers Out of Four Questions : NSWIT Autumn Semester Intake of 1144 Students

Topic \ Number of questions correct out of four	One or none Correct	Two Correct	Three Correct	Four Correct
Index Laws	14	29	25	32
Square Roots	14	25	45	16
Factors	13	15	26	46
Linear Equations	13	16	26	45
Simultaneous Equations	15	14	5	66
Quadratic Equations	11	19	32	38
Fractions	13	17	30	40
Functional Notation	23	19	26	32
Logarithms	33	21	19	27
Coordinate Geometry	21	14	26	39
Trigonometry	25	26	26	23

Source : Shannon and Sutton (1979)

The purpose of diagnostic tests is to pin-point areas of weakness or difficulty in mathematics so that effective remediation may be planned through the provision of tutorial assistance and suitable modules of study. The need for a carefully constructed diagnostic test is self-evident if it is to be an effective instrument. This generally entails task analysis of prerequisites and the construction of an item specifications matrix.

Since the main purpose of the diagnosis and the subsequent remediation is to ensure that as many of the students attempting tertiary courses acquire mastery in the mathematical skills that are prerequisites to their first-year studies, it is essential to determine the prerequisites by a careful study of the courses being serviced. This usually entails an examination of the curricula, prescribed textbooks and assignments, as well as past papers.

The final draft of an item specifications matrix dealing with content, levels of difficulty and the number of operations needed in one problem in the diagnostic tests at the New South Wales Institute of Technology was arrived at after discussions with staff and by revision of two preliminary diagnostic instruments. It was designed to test basic skills in the areas of fractions, logarithms, powers, factors, proportion, functional notation and verbal interpretation.

The selection of the items was based partly on the prerequisites necessary for the successful completion of a mathematics course and partly on common errors appearing in student assignments and end-of-semester examinations. A previous survey of the major difficulties experienced by the students also influenced the selection of the items.

A finding of major significance and concern is that an estimated sixty-five per cent of the marks which were lost on the two compulsory questions were forfeited due to weakness in high school algebraic techniques.

Some mistakes from actual scripts which exemplify that loss of examination marks was due mainly to a poor grasp of basic, high school algebraic techniques are as follows :

$$\int \frac{x}{4x^2+1}\,dx = \int \frac{x}{4x^2}\,dx + \int \frac{x}{1}\,dx \ ;$$

$$\int \frac{x}{(3x^2+4)^3}\,dx = \int \frac{x}{27x^6+108x^4+144x^2+64}\,dx$$

$$= \int (27x^{-5} + 108x^{-3} + 144x^{-1} + \frac{x}{64})\,dx$$

The latter, incidentally, had other variations; for example,

$$\int \frac{x}{(3x^2+4)^3}\,dx = \int x(3x+4)^{\frac{1}{3}}\,dx \; ; \quad \text{and,}$$

$$\int \frac{x}{(3x^2+4)^3}\,dx = \int x(3x^2+4)^{-3}\,dx$$

$$= \int x dx + \int (3x^2+4)^{-3}\,dx$$

The decision to use marked - sense data cards for student response, so that a performance profile for each student could be compiled, was taken. The cards were printed by Computer Resources Company for the MONECS system at the New South Wales Institute of Technology. The students were required to mark these cards with HB pencils (which were provided), appropriate hexagons being marked with firm vertical slashes.

Every student attempting a diagnostic test had his performance in various items graded as adequate, fair, weak and very weak. Follow-up action under the headings of none, quick self-revision, self-teaching and remedial class (optional, desirable and essential) was indicated. It is interesting to observe that a computer print-out of a student's performance profile has a definite psychological impact on him : a student accepts the verdict of a computer more seriously.

All entrants should be subjected to a test in their first few weeks at a tertiary institution and, as a result of this test, advised or compelled to attend a 'techniques','remedial' or some other levelling-up course. This is the view advanced by Professor Crank (1976), who says that "success usually depends upon the number of entrants with important gaps in their knowledge being small so that the levelling-up process can be intensive with almost individual tuition".

1.2.6 Core Syllabuses

The Faculty Board of the Faculty of Mathematics of Cambridge University (1977) says in its Report on a core syllabus for N-and F-level mathematics :

> *We have found wide agreement that syllabuses for mathematics at sixth form level should have a common core. On the one hand, there are some topics which are so fundamental that ignorance of them would handicap any pupil who went on to further education. On the other hand, teachers in universities, polytechnics, etc., will have their task made much more difficult - and unnecessarily more difficult - if they cannot assume some familiarity with a definite body of basic material.*

Professor Crank (1976) also presents cogent reasons for the development of a syllabus of core topics "in which we would like all entrants to the Department of Mathematics to be conversant".

There is a need in Australia for the adoption of a core syllabus in sixth form mathematics, since mathematics syllabuses do differ from State to State, thereby presenting added difficulties to students who attend universities or colleges in States other than those in which they completed their secondary schooling. Apart from enabling to cope with the wide discrepancies in the mathematical background of engineering entrants, the planning of a core syllabus is going to become increasingly important in New South Wales and elsewhere with the growth of school-based syllabuses. It may be said that the adoption of a core syllabus does not preclude one from teaching and examining other topics by setting a supplementary paper.

The task of designing and developing a core syllabus for sixth form students is fraught with problems and difficulties. Many sixth form pupils will not be progressing to a university or college of advanced education and many would not elect to read mathematics at a tertiary level. Thus there are competing aspirations and their resolution demands much wisdom. According to Professor Crank(1976), the core topics for entry into a mathematics undergraduate course should be :

1. *Algebra*
   - *(a) Sum of A.P. and G.P. , Σ notation, Method of finite mathematical induction applied to finite series.*
   - *(b) The quadratic function, quadratic equations; solution by factorisation and completing the square; graphical illustration in all cases including the case of no real root.*

(c) *Polynomials; remainder theorem; factorisation; division.*
(d) *Binomial theorem for positive integral exponent.*
(e) *Partial fractions.*
(f) *Inequalities in one variable.*

2. *Geometry and Trigonometry*

(a) *Cartesian coordinate treatment of conics in the standard forms*
$y^2 = 4ax,\ (x^2/a^2) \pm (y^2/b^2) = 1\ ,\ xy = c^2$

Focus-directrix property;
Distance and reflection properties associated with the foci.

(b) *Parametric treatment of curves; tangents and normals.*
(c) *Introduction to polar coordinates.*
(d) *Curve sketching; asymptotes : graphical solution of equations.*
(e) *Circular measure; formulae for sin* $(A \pm B)$, *cos*$(A \pm B)$*; simple trigonometric identities and equations; periodic properties; sine and cosine formulae for a triangle with elementary applications.*
Trigonometric problems in two and three dimensions.
(f) *Vector algebra in two dimensions including scalar product and orthogonality; vector treatment of theorems and results in plane geometry and trigonometry.*

3. *Calculus and Elementary Analysis*

(a) *The concept of a limit of a sequence of numbers and a function of a continuous variable.*
(b) *Derivative : application to tangents, rates of change, linear approximations and small changes.*
(c) *Differentiation of algebraic and trigonometric functions.*
(d) *The exponential and logarithmic functions; their graphical representation and derivatives.*
(e) *Inverse trigonometric functions; their graphical representation and derivatives.*
(f) *Differentiation of a function of a function, products and quotients, and of implicit and inverse functions.*
(g) *Maxima and minima.*
(h) *The integral* $\int f(x)dx = y$, *defined to be the primitive of* $dy/dx=f(x)$*. Examples of integration by inspection and the use of substitutions to facilitate this.*
(i) *Use of partial fractions and integration by parts to produce simpler integrals.*
(j) *The definite integral illustrated by a suitable limiting process. The fundamental theorem.*
(k) *Areas under plane curves; volumes of revolution; mean values.*

1.2.7 The Williams Report

The Report of the Committee of Inquiry into Education and Training under the Chairmanship of Professor B.R. Williams, the Vice-Chancellor of the University of Sydney, was tabled in Federal Parliament in March 1979. It is a long and far-ranging document and makes a number of major general recommendations in post-secondary education.

One of its principal areas of concern is with student progress. The Committee is disturbed by the high attrition rates and it advances a number of measures for the mitigation of this acute problem.

The Committee suggests that the Tertiary Education Commission (TEC) should not propose target numbers of students that encourage institutions to enrol considerable numbers with very little likelihood of graduating. It urges a review of the selection procedures and recommends that undergraduate entry to universities and colleges of advanced education should be related more closely to the statistical probability of success in degree studies.

Though the Committee is uncertain of the extent to which imaginative, improved methods of teaching and assessment can reduce the incidence of student failure, it advocates a review of the arrangements for teaching and examining and the provisions for personal and academic counselling. The Report urges that opportunities be made early for new students to improve their learning techniques and advises that not only more personal attention be given to first-year teaching but that each first-year student be assigned to a "general tutor".

As a prelude to initiating more effective programmes, the Committee recommends that a study of the reasons for the attrition rates of part-time and external students be commissioned.

There is little doubt that the problem of high failure and wastage rates would be ameliorated, were the recommendations to be put into practice. But where the Committee fails is in its conviction about the correctness of its analysis and proposals. To quote from the submission of the Australian Conference of Principals of Colleges of Advanced Education (1979) to the Federal Minister of Education,

> *The Committee's simplistic view of graduation rates as a measure of the efficacy of an institution's part-time or external courses is seriously questioned. Students in these courses may not necessarily intend to complete all of the award requirements, but may intend to select those parts of courses which are of personal value to them. The attrition rate of part-time students is also in many cases a product of a complexity of domestic, financial and employment pressures which are beyond the control of institutions. The Australian achievement in promoting access to post-secondary education for students of wide-ranging ages, levels of experience and qualification, and geographical distance may be jeopardised if institutions are required to constrain access on the basis of predictions about the likelihood of graduation. There is strong evidence that the well-motivated, mature age student group performs extremely well. In any examination of attrition rates, it will be important to ensure that 'graduation in minimum time' or even 'graduation' are not the only measures of the efficacy of institutions or the value of the student's educational experience.*

The Williams report is hopeful that the problems of academic wastage and failure would disappear by raising the minimum tertiary entrance scores. To say this, however, is an over-simplification of the problem. First, an intensive study of student progress at tertiary level by Schonell *et al.* has shown a complex set of circumstances associated with failure. Their study persuades them to conclude that lack of scholastic ability was not the only reason for failure and that the upgrading of matriculation criteria would not, by itself, significantly reduce the incidence of failure. Second, the research by Sanders has shown that prediction improves if different sets of predictors were used for different tertiary courses. To lay emphasis on a single measure is tantamount to a tacit endorsement of global rather than differential prediction and selection.

Again, inefficient teaching, particularly during the first year of undergraduate study, has received well-merited criticism. Unfortunately, the Committee's recommendation virtually ignores the problem of class sizes, a problem which can only be solved by restricting conditions of entry (the suggested reintroduction of fees may be helpful here) or by introducing small learning situations for which more tertiary teachers would be required. The recruitment of more teachers, however, conflicts with the recommendations of reduction in staff numbers.

The Williams report urges that staff should be properly trained for teaching, particularly those who teach first-year students. The recommendation has been made that the successful completion of programmes in the theory and practice of teaching, curriculum development and examining should become a normal condition of tenured appointments, but what if the percentage of tenured appointments keeps falling?

Reports emerging from committees of inquiry appointed by Governments often become, in the absence of other guidelines, blueprints for official, governmental action. Thus, although some of the recommendations are open to criticism and challenge, many, if not all, of the recommendations are adopted in the fullness of time by the government of the day. In so doing, such reports assume the aura of self-fulfilling documents, if only by default. And herein lies a real danger.

1.2.8 Directions for Future Research

Nearly nine-tenths of the research concerned with the correlates of performance is with the assessment of the student and his characteristics. Does this express the conception that where there is failure it is necessarily attributable to the student? There is a surprising dearth of research evidence on the interactive process, the interaction between the characteristics of students and the characteristics of tertiary institutions. There are no evaluative studies that provide a confident answer to such questions as whether by concentrating less on research and rather more on teaching, the pass rates may be elevated. There is little evidence that different approaches to teaching and examining have been scrutinised so that one may confidently assert whether, by taking thought, we could improve our teaching and examining arrangements and thereby the graduation rates.

There is need for a detailed study on such issues as whether we could reduce wastage by better selection of students *and* by better selection of staff. For, the extent to which universities and colleges of advanced education are prepared to appoint academic staff unseen, unheard, is a matter for grave concern. Reflecting on such appointments, Professor Williams, the Vice-Chancellor of the University of Sydney, says :

> *I think a little bit of the agricultural wisdom, that you do not buy pigs in pokes, has application here.*

Equally, there is need for a detailed study about the educator, how adequate in number is the teaching and support staff, what the natural resources and physical facilities of institutions are and how the impersonality of places of learning induced by poor student-staff ratios, a growing emphasis on research and an increase in the size of universities and colleges militates against the raising of success rates.

CHAPTER II

THE EDUCATIONAL BACKGROUND OF ENGINEERING UNDERGRADUATES

Summary

The discussion in this chapter is focussed on those antecedents which are capable of being observed at the school stage.

The details of mathematics courses completed by students prior to entering tertiary institutions to pursue professional studies in engineering are given. The extent of this mathematical preparation is assessed.

Admission criteria in terms of assessment reports from school authorities, a specified number and combination of subjects at an appropriate level or the achievement of a pre-determined aggregate of marks are considered. The difficulties of using the same examination to perform different and often conflicting functions are indicated, as are the heavily centralised systems of education and the associated deficiencies.

2.1 INTRODUCTION

This chapter attempts to provide an assessment of mathematics teaching and learning in Australian secondary schools in the seventies, by describing recent changes, trends and innovations, by examining the influences that shape the pattern of present-day mathematics education in the schools, and by giving an insight into the mathematical background of engineering undergraduates in universities and colleges of advanced education.

"In each Australian State", as Dow (1971) tells us, "the pattern of subject combinations within secondary schools and the system of examination which buttress the structure have evolved from the English practice which emphasizes specialisation in the senior secondary years". Sufficient variations between the States exist, necessitating a detailed study in each State, but when one reviews the implications and consequences of these variations, it strikes one that, despite the differing frameworks and rules, the overall abiding impression is "one of surprising similarity and uniformity".

The second impression that emerges, as one glances at the syllabuses and examination papers of the public examination authorities, is the astonishing degree to which mathematics syllabuses remained entrenched, statical and rigid for the first half of this century : few of the implications and little of the excitement which flowed from the scientific and technological advances of this century were reflected in the design of courses. It was during the sixties that mathematics syllabuses, as indeed syllabuses in other disciplines, were revised and updated, shattering the stability of mathematics courses over a a long period : school mathematics syllabuses were badly in need of revision, since the traditional courses were not only no longer an adequate preparation for further study but also indicated very little of the range of the contemporary uses of mathematics. Educators came to appreciate that it was no longer tenable that developments in mathematics were the exclusive preserve of the research mathematician.

Each of the six States which constitute the Commonwealth of Australia has its own Department of Education which controls the public schools, both primary and secondary, in that State. About three quarters of all school children attend these schools, the rest going to denominational and/or independent schools. Schooling is compulsory up to fifteen or sixteen years of age; primary schools run from ages five to twelve years approximately and secondary schools from twelve to seventeen or eighteen years approximately.

2.2 CONTROL AND RESPONSIBILITY IN AUSTRALIAN EDUCATION

Secondary education in Australia is State-controlled. The six centralised state systems of education impose a considerable degree of intra - and inter - state uniformity which some critics regard as oppressive. Many of these critics pay scant attention to the fact that without this centralism, it is doubtful if schooling could have been brought to remote areas at a time when older, more developed countries were finding it very difficult to do as much.

Eminent educationalists, both at home and abroad, have been quick to remark on the deleterious effect of centralised educational administration, which leads to the inevitable "rule by bureaucracy", sacrificing progress in education "at the altar of efficiency" and resulting in a "feeling of complacency". Centralism, Kandel (1961) observed, meant that inspection and seniority ruled the educational scene, and resulted in the triumph of mediocrity.

Undoubtedly, the best known overseas criticism of Australian education was that by Prof. R. Freeman Butts in 1954; much of what he said then is, regrettably, still valid. He was severe about the administration and control of Australian education, about the educational programmes and especially about teaching methods. He (Butts, 1955) echoed Dent's criticism (H.C. Dent, then editor of *The Times Educational Supplement,* visited Australia in 1952) that the Australian educational system seemed to "miss something of the vitality, initiative, creativeness and variety that would come if the doors and windows of discussion and decision were kept more open all the way the educational edifice". The assessment of a Canadian, R.W.B. Jackson, in 1960 endorsed much of what Butts had said a few years before.

Perhaps one of the earliest well-publicised criticisms of the Australian education from within the country came from the first Director of the Australian Council for Educational Research (ACER), Dr. K.S. Cunningham, who in 1936 made a plea for more autonomy, greater flexibility, more exploration and experiment.

In 1956, Cramer and Browne (the latter an Australian) in their *Contemporary Education* expressed a need for a clear statement of educational objectives in Australian education. Professor W.G. Walker of the University of New England, an authority on educational administration, has been particularly severe on the deleterious effects of centralisation. The Professor of Education at the University of Sydney, W.F. Connell, perceiving the changing nature of secondary education, with the growth in numbers of students staying longer at school, has spoken of the "heritage of behaviour" being as important as the "heritage of knowledge". "The purposes of secondary education", Connell (1967) wrote, "are more properly defined by a statement about the kind of person rather than the kind of knowledge that ought to be built up during adolescence". Professor P.H. Karmel, an economist, has pointed out that there exists considerable economic room for greater spending on educational services in Australia, although, it must be acknowledged, his views on education, viewed principally in terms of an input - output economic model, attract criticism from many quarters.

Although some brave and good attempts have been made towards decentralisation, in each State a rational-legal form of authority continues to exercise centralised direction of financial policy and school staffing, and a comprehensive set of official regulations fixes the boundaries for formal action and responsibility. The consequent insignificance felt by the individual teacher in such a large and impersonal system, the overlooking of individual needs in the machinery of centralised decision-making, restrictions of centrally devised courses of study, the external examination system and the inspectorial system are some of the baneful effects of centralised systems of education. However, in recent years, the last three complaints have certainly carried less weight. There have been laudable attempts in New South Wales towards regionalisation.

L.F. Neal (1969) makes a trenchant comment :

> *If it is too much to expect as yet to vest in the schools an autonomy and responsibility for their own fate, at least something should be done to break down the vastness of State organisations and place the centre of educational gravity more clearly in the schools themselves. We send our children to schools, not to systems of education.*

Prescribed state-wide curricula and standardised methods reinforced by a comprehensive system of inspection and public examinations have served in the past as distinctive mainstays of centralised authority. The pressure towards standardisation, bureaucratisation and routinisation is always, and inevitably, very powerful in any activity which involves large numbers and large-scale organisation. Yet all three are hostile to the spirit of education and can produce a diseducational effect.

2.3 THE NEW SYLLABUSES

Technological advance has brought a challenge to traditional modes of learning. The continual surge of new knowledge and more sophisticated ways of handling data have made obsolete the former stress on memorized content and mechanical function. Now the emphasis has shifted towards the need to understand and perceive the relationships between things. As a result, educators are currently engaged in trying to adapt or transform classroom practice in ways that can fulfil these more complex tasks. More groups of teachers work in concert with university teachers to fashion syllabuses more in touch with the rapidly changing contours of knowledge. The official state-wide curricula attempt to be less prescriptive than formerly, appreciating that too prescriptive a syllabus can discourage the free, spontaneous ranging of interests and intellectual curiosity - an essential phase of activity of education at every level - by providing a ready-made justification for ignoring everything that falls outside it and by reducing teaching to a formal cut-and-dried instruction. The present-day curricula also provide teachers with the opportunity to experiment and formulate their own teaching methods and assessment of student achievement. These measures reflect a new readiness and willingness on the part of administration to incur risks in order to promote reform.

However, one has to admit that many teachers *want* a prescriptive syllabus since they have neither the initiative nor the resources to be innovative : indeed, many teachers *need* a prescriptive syllabus since, without it, they are likely to omit or, at least, gloss over the duller aspects such as drill, a necessary undertaking for insuring that prerequisite skills are not lacking in tertiary study or employment.

2.4 EXAMINATIONS

H.C. Dent, then editor of *The Times Educational Supplement,* criticised "the baneful effects of the examination system upon teaching methods in all subjects", when visiting Australia in 1952. There is little doubt that public examinations exercise an almost vice-like grip on the work of the schools. Neal (1969) laments this stranglehold over education exercised by examinations :

> *There is nothing essentially wrong in having examinations for some people for some of the time. But examinations for almost everybody most of the time are bad. And examinations which are thought of as coterminous with the syllabus, prescriptions of method, and the very aim and object of education are pernicious in their effect. They stifle and cramp enquiry, curiosity, inventiveness; they destroy delight and pleasure in subject for its own sake; and worst of all, they convey a thoroughly false impression of what knowledge is.*

However, it needs to be said that such examinations do provide very specific goals for adolescents at a time when they are emotionally ill-equipped for freedom.

The effects of an over-examined secondary education can easily be seen in our universities and CAEs where rote learning is at a premium and inventiveness of thought almost eschewed. Students want to be taught and fed by a spoon, rather than to have to think. The future world of rapid change will place great stress on adaptation and inventiveness of thought, divergent as opposed to convergent thinking. As Neal (1969) points out :

> *It is not on doctrinaire, romantic, Rousseausque or Deweyfied grounds that examinations should be dislodged from their tyranny in schools, but precisely because they are a block, an obstacle, an impediment in the way of keeping up with the rapid change in knowledge and the quickening of intelligence to innovation and invention.*

There is a heavy swing away from formal external examinations to other assessments and the moderation of internal examinations. The phasing out of public examinations began at the middle school stage first in Victoria in the late sixties and since then has spread to other States. This process of abolition is being extended to the matriculation level. The Radford enquiry in Queensland recommended on educational grounds that all external examinations be abolished. From 1973 onwards, Queensland has relied on a scheme of internal school assessment combined with a state-wide system of moderating which is designed to ensure some uniformity of standards. Similar moves towards a more diffused form of assessment have been adopted by other States. At the same time, examination boards have reduced or removed requirements with regard to passes in certain subjects.

2.5 LEVELS OF STUDY

The process of rapid democratisation of education - the demand by a growing number of the young for more education, for access into the vocations and professions which offer great economic rewards, greater personal satisfaction, greater social standing and which require more preparation, more skill, a more prolonged and complex education - means that there is a broader spectrum of abilities, interests and aptitudes in Australian secondary schools. This marked diversity among individuals is complicated by an individual diversity of special talent and interest. Thus, there is to be found a measure of elasticity of organisation and curricular choice which goes beyond the provision of set "streams" within a secondary school, making provision for a variety of combinations of subjects in accordance with a pupil's special abilites and designing courses in the same subject at different levels so that the needs of the average and exceptional pupil may be met.

Many schools endeavour to provide a different ethos and atmosphere, more suitable discipline and improved staff-student relationships for the last two years of secondary schooling : many schools feel that these final two years of secondary schooling are a kind of ante-room experience to that of higher education. The chief question and the central educational dilemma is whether "Sixth Form" teachers should do their job strictly according to higher education specifications, where the "Sixth Form" is viewed as having a crucial rôle to play as the undercroft on which the whole new structure of higher education must be erected or whether these teachers have freedom to use their own genius to develop in their pupils habits of study and research, a sense of responsibility, an intellectual curiosity, an integrity and a capacity, so that from the point when they leave school they have both the power and the desire to continue their education and their growth in these qualities.

This latter function eclipses all others in importance. Yet increased pressures ham-string the best efforts of teachers: first, the mounting competitiveness of entry into higher education which entails forcing boys and girls up to an artificially high standard; second, the sheer increase of knowledge, especially in the sciences and in the applications of mathematics, which tempts teachers in higher education to demand that boys and girls should be pushed along farther along the road before leaving school; and third, secondary education must aim at giving greater occupational fitness in the curricula for those not proceeding to universities and the CAEs.

The inordinate influence wielded by matriculation requirements on courses taken by senior school students together with the social pressures which enhance the desirability of matriculating have resulted in many students selecting courses at levels of difficulty beyond their intellectual capacities. The reverse has happened in some cases : able students have selected courses of study at a lower level in order to maximise their aggregate score.

There is a high level of consensus on the aims and objectives of mathematics offerings to senior students at Australian schools. Most of the courses aim

> *to give the student an understanding of the methods and principles of mathematics and an ability to apply them; the capacity to model actual situations and deduce tentative properties from the model : an interest and ability in framing and testing mathematical hypotheses; the ability to express and communicate any results obtained; some knowledge of the history of mathematics and a general appreciation of its contribution to human progress; and encouragement to think independently and creatively.*

(Queensland Board of Secondary School Studies, 1978)

The preamble by the Queensland Board of Secondary School Studies (1978), for instance, goes on to say further : "These aims are very general and their achievement by the individual is realized at an appropriate level, as the student actually acquires new knowledge, new skills, new attitudes and new powers of analysis. These accomplishments constitute four categories of objectives leading to the fulfilment of the general aims".

This is an eloquent statement of aims and objectives which,though worthy and even inspirational, are difficult to implement and impossible to assess : they provide evidence of good intentions but are only of marginal help to the teacher and of less use to an external examiner. The aims are enunciated in grandiloquent, vague generalities : precision and particularity, essential guide-lines for action, are sadly lacking. They do not indicate the extent to which these aims could be achieved with students of a given age and degree of maturity. This is borne out by such statements emanating from the Queensland Board of Secondary School Studies as : "The teaching of mathematics should develop rationality and curiosity, objectivity and open-mindedness, an appreciation of some of the unique features of mathematics, preciseness and flexibility".

The Departments of Education invariably provide teachers with resource books, giving syllabus outlines and suggestions on the development of the topics. Whilst a definite reaction against such past practices as the dogmatic presentation of facts and principles, the compartmentalisation of knowledge and the undue emphasis on memorisation of facts can be perceived, the resource books remain eloquently silent on the methods to be used to introduce and sustain the enlightened approaches they suggest within the constraints and frustrations of a classroom.

2.6 MATRICULATION MATHEMATICS COURSES

The diversity in the mathematical background of engineering students, arising from their exposure to varied educational systems, makes it very difficult to gain an overview of their mathematical preparation prior to undergraduate study.

A careful reading of matriculation mathematics syllabuses in the several States suggests the presence of three broad levels of courses. The highest level, which is a basis for tertiary mathematics courses, is meant for students who have a flair, an aptitude and a genuine interest in the subject. The next level courses are designed for students who wish to proceed to those study areas where the application of skills rather than a deep understanding of the theoretical development is pre-eminently accented. The third level,where it is offered, is intended as a terminating study and is generally considered appropriate for those students who are either not undertaking tertiary studies or whose future undergraduate pursuits are not dependent on any formal mathematical education.

A resumé of the first two levels of mathematics courses is given in Tables 2.1 and 2.2. Since the second level courses are a subset of the first level (highest level) courses, only additional topics are listed in Table 2.2. The resumé in tabular form gives an insight into matriculation mathematics courses, given in the various States, in the most general terms only. Although the uniformity in the length of school courses to matriculation standard and the increasing coordination of curricula are diminishing differences between the States, differences in course content and organisation persist. The tabulated material does not provide details of interstate differences, where they occur, in either the material covered or in the depth of treatment. As a result, detailed fine comparisons are untenable.

In general terms, second level courses provide a broad coverage of traditional areas, such as algebra, analytic geometry, trigonometry and calculus, with matrices and probability included in some instances. The emphasis in these courses is on the application of skills and concepts to rather standard problems. This is reflected in the non-rigorous, informal manner of their presentation where the importance of proofs is de-emphasised.

The more demanding first level courses are made up of a mixture of traditional and modern topics, with algebra, analytic geometry, complex numbers and calculus, as well as transformations, matrices and elementary group theory. However, the treatment is more formal : a deeper understanding of the underlying principles and their use in more difficult and varied contexts are among the natural expectations of such courses.

Table 2.1

Resumé of Topics included in Second Level Courses, 1979

Courses	Qld.	NSW. (2U)	Vic. Gen. Math.	Tas. (Level III, Div. I)	S. Aust. Math 1S	W. Aust. Math I
Sets	+	+	+	+	+	+
Matrices			+		+	
Sequences	+	+	+	+	+	+
Indices and logarithms	+	+	+	+	+	+
Functions	+	+	+	+	+	+
Differentiation	+	+	+	+	+	
Integration	+	+	+	+	+	
Straight line	+	+	+	+	+	+
Circle	+	+	+	+	+	+
Parabola		+				
Trigonometry	+	+	+	+	+	+
Descriptive statistics	+	+	+		+	+
Elementary probability	+	+	+		+	+
Binomial	+	+	+		+	+
Normal	+		+			+

Table 2.2

Resumé of Topics included in First Level Courses, 1979

Courses	Qld.	NSW. 3U	NSW. 4U	Vict. Pure & Applied	Tas. Level III	S.Aust. Math 1,2	W.Aust. Math II,i
Number theory	+		+	+		+	+
Polynomials	+		+	+	+	+	+
Convergence			+		+	+	+
Induction		+	+	+	+	+	+
Applications of calculus	+	+	+	+	+	+	+
Transformations	+		+			+	
Matrices	+		+	+		+	
Group theory	+		+			+	
Three dimensional geometry	+	+	+	+	+	+	+
Central conic	+		+	+		+	+
Further trigonometry	+	+	+	+	+	+	+
Complex numbers			+	+	+	+	
De Moivre's theorem			+	+	+	+	
Mechanics	+	+	+	+			+

Note : The syllabus for Queensland students is presented in the form of modules or semester units, a variety being available. The Victorian students may study either Pure Mathematics or both Pure and Applied Mathematics. In New South Wales 3 unit and 4 unit courses are mutually exclusive.

2.7 ASSESSMENT OF ADEQUACY OF MATRICULATION MATHEMATICS

A measure of adequacy of matriculation mathematics courses may be obtained by developing screening tests for first year students.

A detailed analysis of the 1979 Mathematics Screening Tests pertinent to the Faculties in the New South Wales Institute of Technology (NSWIT) has produced the findings shown in Table 2.3

Table 2.3

Mathematics Screening Test, 1979 NSWIT - Average Percentage by School or Faculty

School/Faculty	Very Weak	Weak	Fair	Adequate
Mathematical Sciences	7.2	11.8	25.9	55.1
Life Sciences	8.6	16.8	28.4	46.2
Mechanical Engineering	15.7	18.2	26.3	39.8
Electrical Engineering	15.8	18.7	26.3	39.2
Business Studies	18.0	20.4	25.6	36.0
Civil Engineering	21.2	19.6	25.1	34.1
Science	33.6	27.7	21.7	17.0

Source : Shannon, Kaye, Fuller, Baskett and Edwards (1979)

The analysis of student performance in these tests has shown grave deficiencies amongst many students in areas which are regarded as fundamental. (The interested reader is referred to Table 1.7.) Table 2.4 shows, not surprisingly, that there is substantial correlation with the students' Higher School Certificate backgrounds. An analysis by Sutton and Matti (1979) shows that "students with a three-unit mathematical background have a probability of passing the subject of 0.74, compared with a probability of 0.35 for students with a two-unit mathematics background".

Table 2.4

Mathematics Screening Test, 1979 NSWIT - Average Percentage by HSC Background

HSC	Very Weak	Weak	Fair	Adequate
4U	7.9	14.3	27.2	50.6
3U	15.8	18.4	26.2	39.6
2U	19.6	20.0	25.4	35.0
2UA	27.4	23.7	23.4	25.5

Source : Shannon and Sutton (1979).

The lack of reasonable mastery of basic mathematical skills adversely affects the performance of students attempting tertiary mathematics courses. Table 1.1 illustrates this very vividly.

Though it is conceded that one has to be necessarily guarded in extending these findings at the New South Wales Institute of Technology to other colleges of advanced education, our discussions with the mathematics teaching staff at other CAEs, such as the Queensland Institute of Technology, the Royal Melbourne Institute of Technology, and the South Australian Institute of Technology, affirm our view that a parallel experience may be seen elsewhere. Every university and CAE we visited expressed a concern about the decline in mathematical attainment by aspiring engineers entering higher education. Furthermore, observing that J.M. Kaye is at the Sydney Teachers' College, M.L. Fuller is at the Capricornia Institute of Advanced Education, Jo Baskett and Jo Edwards are at the Canberra College of Advanced Education, we are able to assert that, to some extent, other CAEs were represented in the study.

From the screening tests and from our experience at teaching engineering mathematics to first-year students, some general comments on the areas of inadequacies may be made :

1. Students tend to place greater reliance on memorised formulae without adequate regard to their meaning. The way mathematical operations are performed betrays that basic concepts and underlying reasons are poorly, if at all, understood. This situation

is responsible for many incorrect answers and for failure to use these skills in unfamiliar areas.

2. Mechanical skills, especially in the fields of algebraic manipulation and numerical calculation, show all the signs of serious erosion. It is this lack of manipulative ability that mars, all too often, a student's otherwise perfect analysis of a problem.

3. Problem-solving ability is painfully weak. The verbalisation of a problem into mathematical terms, making a decision on the type of mathematical skill required for its solution, and the interpretation of results present extreme difficulties to students.

The same inadequacies are revealed by those students attempting the more demanding mathematics courses, notwithstanding their superior facility with, and understanding of, mathematical concepts and reasoning.

These deliberations lead us to urge upon mathematics educators four considerations.

The variety of mathematics courses, with a disparity of content and emphasis, means that engineering students have dissimilar backgrounds. As a result, at least for some students, there will be significant gaps in their mathematical knowledge. Bridging courses are essential to bring them to a common standard.

The initiative should be taken to find ways and means of securing a consensus within tertiary institutions on a minimum core of syllabus content at Higher School Certificate level which would provide the basis for service subjects in such mathematically-orientated disciplines as engineering.

Alternatively, if a common core commanding wide agreement across all mathematically-dependent disciplines is difficult to devise, a possible way forward is through the provision of modular degree schemes.

The direction of innovation in secondary mathematics teaching has been towards the earlier introduction of conceptual concepts, such as sets and matrices, at the cost of attention to manipulative skills and to certain traditional areas of mathematics. To teach for understanding and for skills to reach an acceptable threshold invariably calls for more time for drill. However, the push for coverage (at the expense of learning) is so great that many teachers and students have substituted memorisation for understanding. These observations have also been made by Dr. H.W. Straley (1978) from Virginia, U.S.A., where he serves as Chairman of the Mathematics Department at Woodberry Forest School and as Adjunct Professor of Mathematics Education at Madison University and the University of Virginia, and who has been participating in a teacher exchange. He asserts : "I find that abstract ideas are introduced too early for most secondary students ... the N.S.W. curriculum emphasises memory at the expense of understanding ... I have been very disappointed in the basic algebraic skills by my two-unit fifth and sixth formers ...".

The main mathematical inadequacies experienced by engineering entrants, lying in such areas as understanding underlying concepts, manipulative skill, application of skills to new areas and problem-solving, are a consequence of the direction of the thrust in mathematics education at secondary level. There is more than a little illusion in the belief of teachers that the study of calculus can be undertaken successfully, when many students do not have a firm base in such pre-requisites as algebraic and graphing skills.

Remedial programmes will be increasingly required if the sorry effects of this thrust are to be moderated and the mis-match between the expectations of tertiary institutions and the capabilities of sixth form students is to be resolved.

2.8 LEVELS OF ABILITY

When the Higher School Certificate (HSC) Examination was introduced in New South Wales in 1968 in accordance with a modified Wyndham Committee set of recommendations, each of the major subjects was to be studied in accordance with syllabuses designed for different levels of ability rather than study. The marks awarded by the examiners in each level of a subject were distributed in a defined pattern with the range of marks available; that is, 180 for Level 1, 130 for Level 2, and 100 for Level 3. A system of raw-mark conversion (scaling) was adopted to bring the marks awarded by the examiners within the permissible range to an approximately common denomination. This conversion was carried out on the basis of an external criterion of the ability or level of performance of the candidature in the course. Candidates for the Higher School Certificate Examination were passed or failed at the level at which they entered or were given a conceded pass at a lower level, in accordance with a Ministerial decree that eighty per cent at least should pass in each subject at some level.

This system had a distinct disadvantage as there was a tendency for students to opt for a higher level of study than perhaps their abilities warranted, in the hope of gaining a conceded pass at a lower level in the scramble for marks, often gaining a higher scaled mark than they would have gained, had they attempted the subject at the appropriate level.

The restructured Higher School Certificate curriculum, which involves the replacement of the level structure with the system of units, was examined for the first time in 1976. An emphasis was placed on catering not only for differences in the abilities of candidates, but also for differences of interest and purpose. The numerical unit value of courses was based on the number of teaching periods which the course required. The syllabuses corresponding to the various units of study reflected not only differences in volume of material and the interests and purposes of pupils, but also gradations of difficulty. Unit 4 and unit 3 courses are more demanding than unit 2 and are therefore meant for those with ability and aptitude. Each unit within a subject and between subjects is regarded as equally difficult and enjoys a "parity of esteem".

The HSC aggregate is 500, made up from the best ten units of study, each unit being worth up to 50 scaled marks.

An iterative method was devised for the standardisation of marks, whereby the means of all courses (after the means and standard deviations of the raw marks had been set to 25 and 10 respectively, on a one unit basis, range 1-50) were allowed simultaneously to float in accordance with marks in other courses. Finally, the adjusted marks were multiplied by the unit value of the course. The Pass-Fail categorisation was replaced by a system of grading, each subject being graded as grade 1, grade 2, grade 3, grade 4, and grade 5 in terms of best 10%, next 20%, middle 40%, next 20% and the last 10% respectively. This procedure has attracted incisive criticism from teachers, parents and students.

The arbitrary decision to make the initial standard deviation 10 has brought criticism. With an initial arbitrary mean of 25, a range of values 1 to 50 covers only 97.9% of the candidature, if the raw marks are normally distributed and the standard deviation is 10. This means that in larger samples, if the mean moves up or down significantly, a significant number of candidates will be pushed over one or other end of the range 1-50. This spill-over will be accented if the distribution of raw marks is leptokurtic.

The scheme was basically defective in allowing only the means and not the standard deviations also to float in the iterative process. As a consequence, top candidates, whom the examiners distinguished, were equated in respect of scaled marks in some courses; the best performers in lower unit courses were equated with the best performers in higher unit courses, whether or not there was any demonstrable evidence to suggest that they were of equal ability or competence. Thus the scaling system used did not adequately discriminate between candidates.

The second criticism centres on the concept of "parity of esteem". One may have the same degree of esteem for the value of sheep husbandry and wool technology for the boy or girl who aspires to be a farmer as one has for mathematics for a student wishing to qualify as an engineer, but it is difficult to concede that the intellectual demands of sheep husbandry and wool technology on the one hand and mathematics on the other can be equated. Further, though one may esteem equally the interests, purposes and abilities of those who appropriately present Mathematics 2 unit A, 2 unit, 3 unit and 4 unit, one cannot equally esteem the intellectual demands of these four different courses in mathematics nor the levels of ability of those who appropriately present them. The method has stopped people attempting work beyond their level of ability in the hope of a conceded pass : in fact, it may have gone too far the other way.

There has been considerable concern and disquiet among parents, pupils, teachers and universities since the publication of the 1976 Higher School Certificate Examination results, especially at the alleged anomalies produced by the grading and scaling system used. The difference in rankings between school examinations and the Higher School Certificate Examination, cited by many school teachers, has led to a diminution in the number of candidates presenting higher unit courses. Several letters in the newspapers have suggested that candidates offering fewer higher unit courses would be better placed in maximising their selection aggregate and therefore their chances of gaining admission to our more competitive Faculties : the advice being given is that it is better to attempt 2 x 2 unit courses rather than 1 x 4 unit course, so as to maximise the aggregate score. Perhaps all this serves to highlight the difficulties in using the Higher School Certificate Examination as an evaluative instrument for school achievement and matriculation to tertiary institutions.

As a result of inadequacies in the grading and scaling system, several changes in the manner in which scaled marks are determined have been introduced :

(1) The scaled marks for school candidates will comprise both moderated School Estimates and Examination marks on a 50-50 basis.

(2) The scaled marks in each course will not automatically be spread over the full range 0-50 on a one unit basis. The part of the range occupied by the marks will depend upon the examiners' judgement of the best and poorest performers in each subject.

(3) The use of a variable rather than a fixed standard deviation. This will overcome the problem of insufficient discrimination at either end of the mark scale.

(4) There will be a modification in the scaling of marks awarded in 4 unit Mathematics.

The New South Wales Board of Senior School Studies, concerned at the advice being freely proferred that it is more advantageous to read two 2 unit courses than one 4 unit course has issued a riposte by saying that 4 unit mathematics will be considered for purposes of scaling and subsequent determination of aggregate examination marks as comprising two 2 unit courses. The distribution of marks for the paper common to the 4 unit and 3 unit candidatures (Paper 1) will be scaled as a single group. The marks awarded on Paper 2 will not be combined with the Paper 1 marks for scaling purposes as in 1976, but will be scaled separately. In this way, the Board hopes that "candidates of comparable potential and who perform to expectation in the examination will be comparably rewarded in marks per unit irrespective of the number of units of the subject studied".

2.9 HSC EXAMINATION REVIEW

There has been general public disquiet over the HSC Examination since a number of changes were made to it in 1976. As a consequence the NSW Minister for Education, on the advice of the Board of Senior School Studies, appointed in June 1979 a panel to inquire into the procedures of the HSC Examination, to report on their appropriateness and to assess their fairness to candidates.

The members of the panel were two distinguished educationists : Dr. J. Keeves, the Director of the Australian Council for Educational Research, and Professor G. Parkyn, Emeritus Professor of Comparative Education in the University of London.

The panel presented its report *Higher School Certificate Examination in N.S.W. : Report of the Review Panel to the Board of Senior School Studies* in January 1980 and has recommended a change to methods of reporting HSC Examination results. The panel is anxious to promote an understanding of the marks gained by candidates, as it feels that a general lack of confidence in the examination procedures, which it regards as eminently fair and satisfactory, arose from a lack of understanding and from misinterpretations of the meanings of the final scores.

The panel recommends abandoning percentile bands and the inclusion of two scores : a "Course Score" which would give a more easily understandable indication of a candidate's performance on each course relative to the students taking that course, and a "Tertiary Entry Subject Score" which would provide a prediction of probable performance, rather than an indication of actual performance. The former would be on a scale from 0 to 100, with a mean of about 62 and a standard deviation of 12, whereas the latter would be calculated as at present.

This is a welcome recommendation as it is an attempt to separate the "leaving certificate" function of the HSC from its "tertiary selection" function.

The widely held belief that in some subjects, notably mathematics, 4 unit and 3 unit candidates have been at a disadvantage compared with 2 unit candidates has led many students to select 2 unit courses when their interests and their intention to proceed to higher studies would indicate that 3 unit courses would have been more appropriate. The panel suggests a general restructuring of the syllabuses so that there is a common content for those studying a 2 unit and those studying a 3 unit course within a subject. It is also hoped that candidates in different courses in the same subject might be more accurately assessed if there were common elements in their examination papers.

2.10 SELECTION AGGREGATES IN NEW SOUTH WALES

The Universities and Colleges Admissions Centre (UCAC) is a joint office for processing applications for enrolment in first degree and diploma courses in the twenty-one participating tertiary institutions. As far as engineering courses are concerned, the participating institutions are the New South Wales Institute of Technology, the University of New South Wales and the University of Sydney. The UCAC makes the applicant's task of selecting and applying much simpler than it would be if the applicant had to approach each tertiary institution separately.

To be selected into any of the institutions, a candidate must

(i) qualify for matriculation in the university or college concerned; and,

(ii) be accepted for enrolment in a degree course.

School leavers appearing for the 1979 NSW Higher School Certificate Examination are selected for enrolment in order of merit based on the aggregate of the marks in their best ten units in matriculation or admission subjects, called their selection aggregate.

The applicants are ranked in order of merit, the highest aggregate being numbered first. Each is considered in turn to see whether a place is available in the faculty or course listed as first preference. If no place remains available, the applicant is considered for a second preference and so on, until either an offer is made or the preferences are exhausted.

Other applicants are considered individually by each institution, making its own assessment of the relative merit of their qualifications. The decisions of each institution are considered so that a single offer of a place is made for the highest preference for which an applicant has been accepted.

Minimum HSC Aggregates for Selection

The lowest selection aggregates needed for entry into the various engineering courses offered by the three New South Wales institutions in 1977-1980 are given in Table 2.5. These are each institution's selection aggregate, based on the ten best units in matriculation or admission subjects. (Selection aggregates remain fairly constant and are raised only if a particular course is proving too popular or if there is a limited number of places available.) If a student presents at the HSC any subjects not accepted by one of the institutions, then the institution will not count the marks of those subjects in its selection aggregate. This may mean that the aggregate shown on a student's HSC result notice and an institution's selection aggregate may be different.

The minimum aggregates or cut-off points are not known until selection is actually made, the cut-offs being determined by the number of people applying for a course, the number of places available in a course, and the aggregates of the applicants. Some available places are offered also to applicants with qualifications other than the 1979 NSW HSC. For this reason, fluctuations in cut-off marks can occur from time to time and it is impossible to predict in advance what aggregates will be required for a particular course in the coming year.

The selection aggregates used in 1980 are slightly higher than those used in previous years, as institutions are anxious to raise entry standards. However, many universities and colleges are making a second round of offers at a much lower cut-off point than the first in

Table 2.5

Summary of Engineering Courses at Three NSW Institutions and Minimum Aggregates for Matriculation to Them

Name of Award	Estimated Quota or Enrolment Target 1980	Minimum Selection Aggregate for Entry in 1977	1978	1979	1980	Pattern of Attendance	Minimum Duration in Years
The New South Wales Institute of Technology							
Electrical Eng. - Bachelor of Engineering	96	237	230	233	270	PT	6
Electrical Eng. - BE	48	237	230	233	270	SW	6
Civil/Structural Eng. - BE	60	237	230	233	250	PT	6
Civil/Structural Eng. - BE	48	237	230	233	250	SW	6
Mech./Production Eng. - BE	96	237	230	233	250	PT	6
Mech./Production Eng. - BE	60	237	230	233	250	SW	6
University of New South Wales							
Bachelor of Engineering in Applied Science	290	250	258	245	270	FT	4
Bachelor of Engineering in Chemical Engineering	30	250	300	245	270	PT	7
Bachelor of Engineering (Other than Electrical)	280	268	288	245	270	FT	4
Bachelor of Engineering (Electrical)	170	268	288	245	320	FT	4
Bachelor of Surveying	90	250	245	245	270	FT or SW	4 6/8
Bachelor of Engineering (Civil Engineering)/Bachelor of Science (Engineering)	75	240	245	245	270	PT	6

Table 2.5 (cont.)

Name of Award	Estimated Quota or Enrolment Target 1980	Minimum Selection Aggregate For Entry in 1977	1978	1979	1980	Pattern of Attendance	Minimum Duration in Years
<u>University of Sydney</u>							
Bachelor of Engineering	350	270	270	270	275	FT	4

Source : Universities and Colleges Admissions Centre (1980) a bid to boost the number of students, as the funds received by the institutions are dependent on the EFTS (Effective Full Time Students). That there is an element of irresponsibility in such a practice as this is undeniable, especially when one considers the statement contained in a brochure issued by the University of New South Wales, as the year 1979 was drawing to a close, and entitled *Civil Engineering : Questions and Answers*. The statement reads :

> *Students with HSC aggregates below 300 have had difficulty with first year Civil Engineering. They should seriously reconsider their decision to embark on a career in professional engineering.*

2.11 AUSTRALIAN SCHOLASTIC APTITUDE TEST (ASAT)

The ASAT is designed to obtain information regarding students' aptitudes for higher education. To answer the test questions, students draw on the general verbal, numerical and related skills which they have acquired and not on the specific knowledge and techniques learned in the course of studying particular subjects. All the specific information required to answer the questions is provided in the test books. For example, questions on mathematical topics have been designed so that they do not assume a knowledge of secondary school mathematics beyond a familiarity with simple mathematical conventions.

The ASAT is essentially a general ability test, compiled by the Australian Council for Educational Research and used by nearly all States as a scaling model. It is generally administered by trained testers and psychologists and taken in two separate adminstrations, each of two hours duration. For this purpose the questions are provided in two separate books - Book I and Book II. Part A contains material and questions from a wide variety of scholastic areas including the humanities, mathematics, and social science. Part B contains material and questions testing particular numerical and verbal skills.

The purpose of ASAT is to obtain a measure of the scholastic aptitude of each subject-population within a school and of each school's total population. The information obtained relates to groups of students, not to individuals. The ASAT scores of all students within a subject population (for example, the group studying mathematics) collectively contribute to the average score of that group of students. It is this average score that is used to adjust for differences in standards between schools and for alleged differences of difficulty between subjects.

2.12 MATRICULATION AND ADMISSION CRITERIA

All universities and colleges of advanced education in Australia set minimum academic requirements which must normally be met before a student may be considered for admission to that institution. Generally, these requirements are related to the final school examination for the relevant State. In some institutions admission is based on school assessment reports. Students who pass a specified number and combination of subjects at an appropriate level or who achieve a pre-determined aggregate of marks are eligible to be considered for enrolment by the institution concerned.

However, while the majority of the students gains admission to universities and colleges of advanced education on the basis of formal academic qualifications measured in terms of the matriculation standard, all tertiary institutions make provision in their regulations for the admission of students who only partially satisfy the normal requirements. In some instances this may merely involve giving students on the border line the "benefit of the doubt" : in others, the university or CAE will take into consideration such factors as age and maturity, experience and performance in special entry or other professional examinations. Some tertiary institutions grant to such students "adult matriculation" status for which a lower age limit ranging from 21 to 30 years of age generally applies. Some institutions also confer "provisional matriculation" status : in these cases, matriculation is subject to confirmation after satisfactory performance in their first or second years of study.

At the tertiary level, individual institutions may specify certain subjects as prerequisites for admission to certain courses. This specification may be overt as in the Queensland Institute of Technology, where English, Physics, Chemistry, Mathematics I and Mathematics II are required for entry to engineering. In contrast, some institutions make the statement that no subject prerequisites are specified for entry to their courses in engineering. However, a more careful reading of regulations and subject syllabuses shows that first-year teaching assumes a certain level of knowledge only obtainable by taking specific subjects for matriculation : The Faculty of Engineering at the New South Wales Institute of Technology gives this advice on assumed knowledge and HSC course selection :

> *There are no formal HSC subject requirements for engineering courses. Students will, however, find less difficulty in engineering courses if they have a strong aptitude for mathematics and science. A combined minimum of five units of mathematics and science subjects at the HSC is recommended.*

(Universities and Colleges Admission Centre, 1980).

Although the University of Sydney has recently liberalised its matriculation requirements, a warning to engineering entrants is given in clear and unmistakable terms in a brochure entitled *Changes in Matriculation Requirements : Faculty of Engineering* and published by the University in 1978. The warning reads :

> *The major criterion for an HSC course suitable for an engineering degree is strength in the sciences. Thus while two Unit Maths, or for that matter no Maths at all, may still enable you to matriculate to Sydney University, your chances of passing the compulsory first year maths are remote. Only 15% of engineering students with two unit maths at the HSC level passed first year maths in 1977. Similarly the two unit courses in Physics and Chemistry provide a better grounding than do the four unit multistrand science courses.*

As the six States function under different systems of education, each with its own mode of examination and assessment for its secondary school courses, any attempt to describe a nationally measurable matriculation standard is destined to fail. Indeed, in view of variations between the States, an endeavour to provide complete details of current admission criteria is less than easy. In the expressive words of Professor K.L. Dow (1971) :

> *Finding a way through the regulations of six states is like working through six different countries. One sets out requirements in terms of points, another levels, another subjects. And yet, a careful analysis of the implications of the restrictions and requirements reveals a uniformity in the midst of diversity ...*

when the implications of the variations are examined, the final outcomes are surprisingly similar.

For these reasons, an attempt to be comprehensive has been side-stepped.

2.13 RESUME OF MATRICULATION REQUIREMENTS

2.13.1 New South Wales

When the New South Wales Board of Senior School Studies advises a candidate of his or her result, it also indicates by symbols the Universities in which the candidate has qualified to matriculate.

1. Matriculation Subjects

   The following subjects are recognised as matriculation subjects :

   (a) By the University of New South Wales and the University of Sydney.

   Group A : English

   B(i) : Chinese, Classical Greek, Dutch, French, German, Hebrew, Indonesian, Italian, Japanese, Latin, Modern Greek, Russian and Spanish.

   B(ii): Ancient History, Art, Economics, Geography, Modern History and Music.

   C : Agriculture, Industrial Arts, Mathematics and Science.

   D : General Studies.

   (b) by the University of Sydney

   The University admits Arabic as a subject under category B(i)

   (c) by the University of New South Wales

   The University recognises Farm Mechanics, Food and Textile Science, Home Science, Sheep Husbandry and Wool Technology, but only one of these subjects may be counted for purposes of matriculation.

2. Paths to Matriculation

   The subjects or units required to be taken for matriculation are :

   University of New South Wales : English and at least three other matriculation subjects.

University of Sydney : (a) Five subjects, including English and at least one subject from Group B, and one subject from Group C, or

(b) English, together with the 4unit Courses in each of Mathematics and Science and General Studies, or

(c) Four subjects (of which one must be English) including a 3unit course in any language other than English and a 3unit course in English or in any of the subjects listed in Group B(i) or Group B(ii) or in Mathematics, or

(d) Five subjects, including English, Agriculture and General Studies and two more subjects from Group C.

3. Restriction on the Number of 2 unit A Courses

University of New South Wales : English and not more than two additional units from 2 unit A and 1 unit courses may be included in the minimum aggregate for matriculation.

University of Sydney : English and not more than one of the remaining four subjects for matriculation may be taken as a 2 unit A course.

4. Required Performance

University of New South Wales : A minimum aggregate of the best ten units in at least four approved subjects and including at least 2 units of English.

University of Sydney : A minimum aggregate of marks based on the ten best units in those subjects which have been taken to satisfy the matriculation requirements (Matriculation Eligibility Aggregate).

5. Basis for Selection

University of New South Wales : Aggregate of the marks of the best ten units in matriculation subjects not necessarily including English and provided that not more than two units from 2 unit A and 1 unit courses may be counted except that where English 2 unit A is included one or two additional units from 2 unit A and 1 unit courses may be included.

University of Sydney : Aggregate of the marks of the best ten units in matriculation subjects.

The University of New South Wales

Matriculation is the minimum educational qualification necessary for entry to a bachelor degree course in the University, but it does not ensure admission to a course.

For students who are candidates of the NSW HSC Examination in 1977 or 1978,"matriculation to the University shall be the achievement of a minimum aggregate of marks equivalent to the aggregate attained or bettered by 58 per cent of the school candidates at the HSC and made up of marks in the best ten units from a minimum of four approved subjects, including at least two units of English".

Admission to all courses is subject to selection on the basis of academic performance. To gain admission to a particular course a student must :

(1) have matriculated, and
(2) have been selected on an aggregate of marks. The units to be used in making up the selection aggregate will be the candidate's best ten units in approved subjects.

There are supplementary provisions under which applicants who are graduates of other tertiary institutions, mature-age students or those who have a record of war service may be admitted.

Many of the subjects of degree courses are taught assuming that students will have achieved a minimum standard of preparedness and accordingly have as prerequisites certain levels of achievement (grades) in HSC Examination subjects. Some degree courses contain compulsory subjects that have prerequisites. The compulsory subjects of the engineering degree courses which have prerequisites based on the HSC subjects are set out in Table 2.6. The subjects of the engineering degree courses that have prescribed prerequisites in terms of the HSC subjects are given in Table 2.7, which also lists the grade required in each case. The grade achieved (1,2,3,4 or 5) in a particular subject at the HSC Examination will determine whether a student will be permitted to enrol in certain specified subjects at the University.

A student who is proposing to study for an engineering degree course is advised to refer to Table 2.6, note the compulsory subjects and refer to Table 2.7 to ascertain prerequisites for those subjects.

Table 2.6

The University of New South Wales Engineering Courses with Compulsory Subjects which have HSC Examination Prerequisites.

Course	Full-time or Part-time	Subject Identifying Number
Engineering-Aeronautical	FT or PT	1.001, 5.010, 10.001
Engineering-Ceramic	FT	1.001, 2.001, 5.010,10.001
Engineering-Chemical	FT or PT	1.001, 2.001, 5.010,10.001
Engineering-Civil	FT or PT	1.001, 5.010, 10.001
Engineering-Electrical	FT or PT	1.001, 2.001, 5.010,10.001
Engineering-Industrial	FT or PT	1.001, 5.010, 10.001
Engineering-Mechanical	FT or PT	1.001, 5.010, 10.001
Engineering-Mining	FT	1.001, 2.001, 5.010,10.001
Metallurgical Process Engineering	FT	1.001, 5.010, 10.001
Naval Architecture	FT or PT	1.001, 5.010, 10.001
Surveying	FT or PT	1.001, 5.010, 10.001

Source : University of New South Wales (1979)

Table 2.7

Subject Prerequisites and Grades Required in Engineering Courses at the University of New South Wales

University Subject	Prerequisite	Grade Required
1.001 (Physics 1)	3 Unit Mathematics or	1, 2 or 3
	4 Unit Mathematics	1,2,3,4 or 5⁺
	and	
	2 Unit Science (Physics or Chemistry)	1, 2 or 3
	or	
	4 Unit Science (Physics and/or Chemistry)	1, 2 or 3
2.001 (Chemistry 1A)	2 Unit Science (any Strand)	1, 2 or 3
	or	
	4 Unit Science (any Strand)	1, 2 or 3
5.010 (Engineering 1A)	Either 2 Unit Science (Physics) or	1, 2 or 3
	4 Unit Science (including (Physics) or	1,2,3 or 4
	2 Unit Industrial Arts or	1, 2 or 3
	3 Unit Industrial Arts	1,2,3 or 4
10.001 (Mathematics 1)	2 Unit Mathematics or	1 or 2
	3 Unit Mathematics, or	1, 2 or 3
	4 Unit Mathematics	1,2,3,4 or 5⁺

⁺Grade 5 at a standard acceptable to the Professorial Board.

Source : University of New South Wales (1979)

The University of Sydney

To be selected for enrolment in the University, an applicant must qualify for matriculation and be selected for enrolment. Entry to all faculties and courses is limited. In general, selection is based on a student's performance at the examination at which he becomes eligible to matriculate. A student becomes eligible for matriculant status if a minimum aggregate of marks at the HSC Examination in at least five approved subjects is achieved.

The minimum aggregate of marks for matriculation eligibility, known as the matriculation eligibility aggregate, is based on the ten best units in matriculation subjects and obtained in one of three ways, according to the manner in which subjects were chosen :

(a) For all candidates except those in (b) and (c) below, the ten units will be calculated using :
   (i) 2 units of English, and
   (ii) the 2 best units of a Group B subject, and
   (iii) the 2 best units of a Group C subject, and
   (iv) the 4 best remaining units.

(b) For candidates who offer 4 units of Mathematics, 4 units of Science and 1 unit General Studies, the ten units will be calculated using :
   (i) 2 units of English; and
   (ii) at least 2 units of Mathematics or of Science; and
   (iii) 1 unit General Studies, and
   (iv) the 5 best remaining units.

(c) For candidates who offer Agriculture and General Studies, the ten units, will be calculated using :
   (i) 2 units of English; and
   (ii) at least 2 units of Agriculture; and
   (iii) 1 unit General Studies; and
   (iv) the 5 best remaining units.

The University has several special provisions allowing persons in special categories, who are unable to matriculate by following the ordinary avenues, to satisfy the matriculation eligibility requirements : indeed, provisions exist for those who are able to substantiate their claim that their educational progress has been disadvantaged by circumstances beyond their control to be given admission to the University.

The University of Sydney announced in December 1977 a new set of matriculation regulations to be effective after 1978, with English remaining as the only compulsory subject so as to ensure essential standards of literacy. The main change is the abandonment of subject groupings : for years students have had to study at least five subjects chosen from four groupings to qualify for matriculation. The University will rely on the requirement of 11 units at the HSC

Examination, requiring at least one three or four unit course to be taken by every candidate seeking matriculation. Thus, the new rules would encourage specialisation in the last two years of secondary school and it would be possible to matriculate with courses in only three subjects.

The University cautions its incoming students that a high level of understanding of basic theory in Mathematics, Physics, and Chemistry is needed as a foundation for subsequent courses in Engineering. Students reading for the Bachelor's degree in Engineering study in their first-year, Mathematics, Physics and Chemistry, which are taught on the assumption that they have satisfactorily completed study of the syllabus prescribed for the Higher School Certificate Examination in the courses indicated as follows: Mathematics 3 unit course and, either the Science multi-strand 4 unit course, or the 2 unit Science course in Chemistry and the 2 unit Science course in Physics. A student who gains a place without a sense of adequacy in the three subjects is expected to undertake supplementary work in order to reach the level assumed. In mathematics, the level of knowledge required for supplementary work is the 2 unit course in mathematics at the HSC Examination.

The New South Wales Institute of Technology

(a) Category A Applicants

Applicants who hold the NSW HSC must gain at least the minimum aggregate of marks as determined by the Academic Board. This minimum aggregate is taken over a student's best ten units of approved subjects taken at the one examination. Attainment of eligibility for admission is indicated by the letter "T" appearing in the "matriculation box" on the official result notice.

Some Faculties require HSC applicants, besides satisfying the general admission requirements, to have completed certain subjects prior to admission. Although there are no subject prerequisites for the Faculty of Engineering, applicants are warned that a number of courses are taught on the assumption that a student has obtained an appropriate level of knowledge in certain subjects. Thus, it is indicated to students enrolling in the Engineering Faculty that English and a minimum of five units in Mathematics and Science will be helpful in Stage I courses.

(b) Category B Applicants

This classification embraces all applicants who do not possess the current NSW HSC. For the year or grade 12 high school students, the Institute uses the following selection indicators in determining eligibility for entry :

(i) Total performance of the student over two years;
(ii) Tertiary Entrance Score;
(iii) Principal's recommendation on the suitability of the student for tertiary studies;
(iv) Schools English language assessment; and
(v) Australian Scholastic Aptitude Test (ASAT) Scores.

Category B applicants are assigned a rank by Faculties and Schools and the ranking is used in the selection process. The system provides a means by which groups of Category B applicants are classified and ranked and one possible method of ranking is given below :

Rank	(1) Equivalent HSC* Aggregate Marks	(2) Other
1	401 - 500	Tertiary graduate
2	325 - 400	Preferred Certificate +
3	276 - 324	Good Other Certificate
4	251 - 275	Other certificate
5	Matriculation - 250	Under exclusion
6	Less than Australian HSC Matriculation	
7	Not Year 12 level	

* The term "Equivalent HSC" is not intended to suggest that there is a direct equivalence between these marks and those for the current HSC Examination; that is, it is not a direct attempt to match category A and B applicants in a given year.

\+ Students holding approved certificates of the NSW Department of Technical and Further Education will satisfy the general requirement for admission.

Bridging and Conversion Courses

The University of New South Wales offers a Bridging Course in Mathematics. The course is designed to assist students who have satisfied the mathematics prerequisites for enrolment in compulsory subjects of degree courses but feel that their understanding and use of mathematical concepts are poor. Additionally, a Conversion Course in Mathematics is offered, designed to assist students who have not satisfied the mathematics prerequisites for enrolment in compulsory subjects of degree courses. Examinations are held during the Conversion Course to assess the student's improved level of attainment in mathematics. Those who do not satisfy the above prerequisites for 10.001 (Mathematics 1, a compulsory first-year subject in Engineering), but who successfully undertake the Conversion Course are exempted from prerequisites.

(The subject Mathematics 1 is taken by students enrolled in mining engineering, civil engineering, chemical engineering, electrical engineering, mechanical engineering and aeronautical engineering courses and is offered at two levels : pass and distinction. The assumed standard of knowledge for the pass level is the 3 unit course at the HSC; for the distinction level, mathematics 4 unit course at the HSC with a high decile rating is required.)

2.13.2 Queensland

Tertiary Entrance Score

The assessment of a student's achievement in senior secondary school studies has two main functions :

(i) to provide a means by which the student may know how he has progressed over a course of study; and,
(ii) to provide a basis for evaluating a student's readiness and aptitude for post-secondary studies or fitness for specific tasks in employment.

Believing that the two functions should be separated, the Queensland Board of Secondary School Studies provides for school assessments to be reported in two distinct ways. The Board issues two statements to each student who completes Grade 12. They are :

(i) A Senior Certificate which reports a student's achievement in secondary school studies. The Certificate lists all subjects studied by the student in Grades 11 and 12 and records his achievement in each of the subjects as a numerical rating, 7 to 1; and

(ii) A Tertiary Entrance Statement which provides the student with information which enables him to establish his qualifications for tertiary entry. The principal basis for selecting entrants to tertiary institutions - a Tertiary Entrance Score - is reported on this statement as a single score in the range 990 to 690.

In evaluating a student's progress for success in a tertiary course of study, major consideration is given to the student's demonstrated ability and achievement in senior school studies. When, however, there are limits placed on enrolments and selection for the available places is based on merit, comparisons have to be made among those seeking enrolment. Such comparisons are facilitated if students are ranked in order of merit. Each year the Board of Secondary School Studies compiles an order of merit list on which all students who complete Grade 12 are ranked. A student's position on that order of merit list is reported as a Tertiary Entrance Score.

A student's Tertiary Entrance Score indicates a position on the State order of merit list and is not to be interpreted as a number of marks gained. Since individual ranking is not practical, recourse is made to ranking by groups or bands of students. Each score is a centile ranking based on the estimated 17-year old population in Queensland in 1975, that is 36,000. Tertiary Entrance Scores range from 990 downwards in intervals of 5, that is, 990, 985, 980, 975 ... The first 360 students on the order of merit list receive a Tertiary Entrance Score of 990, the next 180 a score of 985, the next 180 a score of 980 and so on.

Since schools are not regarded as operating on a common standard when they make their internal assessments, a moderating test called the Australian Scholastic Aptitude Test (ASAT) has been introduced. This test is also used for correcting differences of difficulty between subjects so that all students, irrespective of the subjects sat for, can be ranked on a common order of merit list.

Queensland Institute of Technology

To be eligible to enrol for courses leading to a first degree in engineering, an applicant must obtain a Tertiary Entrance Score of 810 or higher and a minimum of sixteen points in four semester units in each of the Board subjects - English, Physics and Chemistry, with a minimum score of three points in any semester unit, plus a minimum of thirty-two points over eight semester units of Mathematics, including a minimum of three points in each of the units I,II,III,VIII, and XI, that is, in Preparatory Mathematics, Algebra and Calculus 1, Algebra and Calculus 11, Mechanics, and Geometry and Calculus 3.

A student who has appeared for the external Senior Examination but who is not included in the Tertiary Entrance List must attain a minimum total score of twenty points in English, Mathematics 1, Mathematics 11, Physics and Chemistry, four points for each subject being required minimally.

Tables 2.8 and 2.9 summarise the entrance requirements for engineering courses at the Queensland Institute of Technology.

University of Queensland

An applicant for entry to any bachelor degree course must obtain a tertiary entrance score of 880 or higher and an aggregate score of at least 14 points in four individual semester units for each subject in the course.

2.13.3 South Australia

South Australian Institute of Technology

Registration, which is assessed on results achieved in the South Australian Public Examinations Board Matriculation Examination, is required for entry to degree and diploma courses.

A knowledge of Matriculation Mathematics 1 and 2, Chemistry and Physics is assumed in First Year subjects of the Bachelor of Engineering degree courses. Students who have studied Matriculation Mathematics 1S (a subject generally taken by those proceeding to tertiary courses in which mathematics is taken as an ancillary subject

Table 2.8

School of Engineering, Queensland Institute of Technology: Entry Requirements for Students Qualifying by External Examination

Mode* Offered	COURSE	Points Required: Normal Entry (Senior) — Total over five subjects unless stated	Points Required: Adult Entry (Senior) — Total over four subjects unless stated	PLUS Required Subjects: Specific Subjects	PLUS Required Subjects: Level in Specific Subjects
Degree (Bachelor) Level Courses					
F/P F/P F/P	B.Eng. Civil B.Eng. Electrical B.Eng. Mechanical	20	20 (in 5 Board subjects)	English Physics Chem. Maths 1 Maths 11	Grade of 4 in each required subject
S	B.A.S. Surveying	20	16	English Maths 1 Maths 11 Physics 1 other	Grade of 4 in each required subject
Associate Diploma Level Courses					
F/P F/P F/P	A.D. Civil Eng. A.D. Elect. Eng. A.D. Mech. Eng.	14 (in 4 Board subjects)	14	English Maths 1 Physics Chem.	Students must have sat for these subjects
P	A.D. Surveying	14 (in 4 Board subjects)	14	English Maths 1 Physics 1 other	Students must have sat for these subjects

* F = *Full-time;* P = *Part-time;* S = *Sandwich.*

Source: Queensland Institute of Technology (1979)

Table 2.9

School of Engineering, Queensland Institute of Technology: Entry Requirements for Grade 12 Students under the Semester Rating System

Mode* Offered	COURSE	Min. T.E.	Required Subjects	Min. Semester Units	Min. No. Pts	Specified Units
Degree (Bachelor) Level Courses						
F/P	B.Eng. Civil**		English	4	14	-
F/P	B.Eng. Electrical**	810	Physics	4	14	-
F/P	B.Eng. Mechanical**		Chem.	4	14	-
			Maths #	8	28	1,2,3,8,11
S	B.A.S. Surveying**		English	4	14	-
			Maths #	8	28	1,2,3,8,11
			Physics	4	14	-
			Other	4	14	-
Associate Diploma Level Courses						
F/P	A.D. Civil		English	3	-	-
F/P	A.D. Electrical	745	Maths #	3	-	1,2,3
F/P	A.D. Mechanical		Physics	3	-	-
			Chem.	3	-	-
P	A.D. Surveying	745	English	3	-	-
			Maths	4	-	1,2,3,11
			Physics	3	-	-

* F = *Full-time;* P = *Part-time;* S = *Sandwich.*

\# *Social Mathematics is not accepted.*

** *A total of 80 points must be attained over 20 semester units in these four courses.*

Source: Queensland Institute of Technology (1979) rather than as a major study) are advised to take Bridging Mathematics.

University of Adelaide

The educational requirements for matriculation may be fulfilled if a candidate :

(a) presents at one Matriculation Examination not less than five approved subjects selected from each of two groups; and,

(b) attains in five subjects so presented an aggregate of scaled marks not less than a figure determined from time to time by the Council, provided that if a candidate presents more than five subjects the aggregate of marks shall be his highest five scaled marks. (The minimum prescribed aggregate mark for 1978 was 225.)

There are provisions for candidates, who do not comply with the normal requirements for matriculation status, to be admitted to provisional and adult matriculation.

A school assessment component, provided by teachers, for each candidate and in each subject and moderated to compensate for differences between schools, has a weighting of twenty-five per cent in the composition of the total raw score, the examination mark having a weighting of seventy-five per cent.

To take cognizance of the possible variations in the quality of candidates from year to year and in the apparent difficulty of examination papers, the total raw score in each subject is suitably scaled. The aggregates of scaled marks so obtained are used for determining matriculation.

Although the University does not explicitly specify any prerequisites for first degree courses in the specialities of chemical engineering, electrical engineering, mechanical engineering and civil engineering, a knowledge of Matriculation Mathematics I and II is assumed in the first year mathematics subject.

2.13.4 Tasmania

University of Tasmania

The University recognises for purposes of matriculation certain subjects offered in the Tasmanian Higher School Certificate Examination. To qualify, a student must obtain four level 3 passes in acceptable subjects in any two sittings. For engineering courses Physics, Chemistry and Mathematics are considered as acceptable subjects.

2.13.5 Victoria

Royal Melbourne Institute of Technology (RMIT)

To satisfy the general entrance requirements for admission to the first year of degree courses a student must :

(1) obtain grades of D or higher in at least four subjects at the Victorian Higher School Certificate Examination including English, or
(2) satisfy the requirements of an approved Tertiary Orientation Programme or preliminary year at a Victorian technical school or college, or
(3) satisfy the general entrance requirements of an Australian university, or
(4) satisfy the Victorian adult matriculation requirements, or
(5) reach a standard approved as the equivalent of the above.

To satisfy individual course requirements a student must also have studied to a satisfactory level all of the subjects specified as prerequisites for the course. For undergraduate degree courses in engineering (Aeronautical, Civil, Chemical, Communication, Electrical, Electronic, Mechanical, Mining and Production) the prerequisites are Physics, Chemistry and a branch of Mathematics. Preference is given to students who have studied Pure and Applied Mathematics.

Caulfield Institute of Technology and Swinburne College of Technology

Applicants who satisfactorily complete the science/engineering course in the Tertiary Orientation Program (TOP) offered by the colleges are given preferred entry to the first year.

Standard entry to the first year of an undergraduate course in engineering requires satisfactory completion of year 12 (sixth form) in a Victorian secondary school.

It is clearly indicated that secondary students have a strong background in mathematics and the physical sciences if they are planning to read for an engineering degree. Students who will be presenting for the Victorian HSC Examinations are strongly advised to enter for subjects in the science - mathematics area. A suitable selection of HSC subjects, it is pointed out, would be English, Chemistry, Physics, Pure Mathematics and Applied Mathematics.

University of Melbourne

Normal entry is gained by passing the Victorian Higher School Certificate Examination, including any prerequisites, and being selected for enrolment.

The University has been gathering opinions on a particular selection proposal, designed to break the nexus between tertiary selection and the secondary school terminal examination, to reduce the pressure of external examinations on students, and to give teachers a greater say in the final assessment of their students. It consists of allowing students to take five subjects, of which at least three must be assessed externally and two may be assessed internally in any way a school desires.

On 31 March, 1979 the Victorian Universities and Schools Examinations Board (VUSEB) was succeeded by the Victorian Institute of Secondary Education (VISE), which is now responsible for the supervision of the Higher School Certificate Examination throughout Victoria.

A new style of HSC assessment is due to begin in 1981. The scheme proposed by VISE means schools may establish subjects which are totally devised and assessed within a single school or group of schools. Some students may then choose to make up their entire year 12-programme from such subjects (to be known as Group 2 subjects). However, the RMIT also will set up another group of subjects (Group 1) which will be externally examined, although up to half of the final mark may derive from school assessment.

2.13.6 Western Australia

The Western Australian Tertiary Admissions Examination (TAE) was introduced for the first time in 1975 to provide suitable information to tertiary institutions for use in qualifying candidates for tertiary admission.

The University of Western Australia

From 1975 onwards admission to the University of Western Australia and other tertiary institutions has been based on an aggregate of scaled marks in the TAE. This aggregate generally consists of the highest scaled marks in five subjects. The maximum possible aggregate from 1976 onwards is 540 and an aggregate mark of approximately 300 is needed to qualify for admission.

The Matriculation Regulations provide for a scaling test (ASAT) to be taken by all candidates taking one or more subjects in the TAE who are hoping to qualify for matriculation or provisional admission to the University.

In addition to TAE results the Matriculation and Admissions Committee may, in the case of marginal students, take into account teachers' estimates in determining whether a candidate has qualified for admission.

Early in January all candidates are sent a matriculation advice slip showing ASAT performance and qualification aggregate (where the candidate has qualified for matriculation). The candidate's advice slip does not show the individual subjects taken or the marks obtained. This may present problems for tertiary institutions outside Western Australia to which the candidate may be applying for admission. In such cases, the University undertakes to provide on a confidential basis details of subjects and marks if requested to do so by the institution concerned.

Western Australian Institute of Technology (WAIT)

(1) The normal avenues for entry to undergraduate courses at the Institute are :

(a) Entry through the Tertiary Admissions Examination (TAE). Eligibility for admission on the basis of the TAE is usually gained by attaining an aggregate of scaled marks exceeding a determined minimum. The aggregate is calculated on the basis of scaled marks gained in five subjects, one of which must be English or English Literature.

Candidates are selected in order of merit and, consequently, the attainment of the minimum entry requirement does not guarantee admission, particularly to competitive courses; where the demand for places exceeds the number available, candidates need to display a higher level of academic performance in order to be considered for selection.

Prerequisite studies (in addition to English/English Literature) may be specified for certain courses; for entrance to the Bachelor of Engineering course, the specified subjects are English, Physics, Mathematics 11, Mathematics 111 and Chemistry. Students with Mathematics 1 only can gain entry by successful completion of a bridging course run by the Department of Mathematics during the summer vacation.

(b) Candidates over the age of 21 at the beginning of the academic year become eligible for admission if

(i) they obtain a required aggregate under the TAE mature age provisions. The aggregate is determined on the marks obtained in two subjects and the scaling test, all of which being taken at the one Annual Tertiary Admissions Examination; and,

(ii) they meet the requirements of prerequisite studies, interviews and aptitude tests.

(2) Other Means of Entry

The Institute recognises other qualifications for purposes of determining eligibility for admission to undergraduate courses. The principal ways for selection, all of which require a decision on each applicant by the appropriate Head of Department, are as follows :

(i) School candidates may be admitted on the basis of evidence other than TAE performance, such as school assessments and reports.

(ii) A holder of a two-year full-time certificate course or diploma of the Technical Education Division is eligible for admission to an Institute course, provided the candidate satisfies the requirements for previous studies, interviews and aptitude tests.

(iii) Examinations (at TAE level) conducted by the Technical Education sector are accepted for the purposes of determining eligibility to the Institute . Such qualifying subjects are taken into account for the purpose of preparing rank order lists upon which selection is made.

(iv) Candidates over the age of 21 years who do not meet the prescribed entry requirements to any of the Institute courses and who have not enrolled for the TAE may sit for a special Mature Age Examination conducted by the Institute, provided they have satisfied the requirements for prior studies specified for the chosen course.

2.13.7 The Australian Capital Territory (ACT)

Neither the Australian National University nor the Canberra College of Advanced Education have Faculties or Schools of Engineering. As a consequence, one can expect a number of applications for admission to engineering courses in other States, and particularly NSW, because students in the ACT used to follow NSW HSC syllabuses and appeared for the NSW HSC Examinations.

The NSW HSC examination was held in the ACT for the last occasion in 1976, and ACT students are no longer required to attempt a public examination at the end of year 12; instead, they study school-based teacher-assessed curricula in the final two years of secondary education.

To help in the assessment of admissions-eligibility, the information which will be forthcoming on each ACT year 12 student is as follows :

(i) An Academic Profile, which will be a record of student achievement in accredited and registered courses; and,

(ii) a Tertiary Entry Profile, which will contain a record of information especially relevant to the selection of students for places in tertiary institutions.

The Academic Profile of each student, based on teacher assessments, will give, besides the name of the school or college attended by a student and his period of enrolment, an academic summary indicating the number of accredited and registered units and courses studied, details of assessed scores (recorded on a five point literal scale A to E) for all accredited units studied, registered unit scores recorded on the same or some other suitable scale, assessed scores for all courses (recorded on a 100 point scale with a mean of 65 and standard deviation of 15) and a key to the grading scales and course classifications used.

On the Tertiary Entry Profile of each student would appear, besides the name of the school or college attended and the name of the issuing authority (ACT Schools Accrediting Agency), a list of accredited tertiary entry courses studied by the student, a score for each accredited tertiary entry course studied (recorded on a 100 point scale, with a mean of 65 and a standard deviation of 15), the Australian Scholastic Aptitude Test (ASAT) total score and verbal and qualitative subscores, the Tertiary Entry Score recorded on a 360 point scale with percentile rankings based on the ACT age cohort in which year 12 students are found and the year 12 student population in the ACT, a statement on whether the student has a level of language skill to enable him to cope with tertiary study, and the statement "The pattern of courses studied satisfies the minimum requirements for consideration for entrance to the Australian National University".

The Tertiary Entry Score is based on achievement in courses identified by the Australian National University as having a high level of conceptualisation, suitable for use as a predictor of likely success in academic tertiary study. In the calculation of the Tertiary Entry Score, the scores obtained by students in four of these courses will be scaled using the ASAT total scores.

2.14 CONCLUSION

The effectiveness of mathematics programmes for engineering students is obviously determined to a large extent not only by the quality of the teaching staff but also, and perhaps more importantly, by the quality of the students.

Accordingly, we advance our belief that a student should not only exhibit some evidence of adequate achievement, but also display the necessary aptitudes for tertiary studies.

Every CAE should state clearly the *minimum* background in mathematics or physics or in any other subject that is expected of its engineering entrants. Any institution which admits students with less than these defined minimum expectations must be prepared to provide bridging courses or other remedial programmes.

The disturbingly high failure rates in the tertiary sector lead us to suggest that the admission of students with inadequacies should be restricted to those who give a strong indication of probable success. To improve the screening process, institutions of higher education should seriously consider using reliable standardised or local tests, validated by individual institutions.

CHAPTER III

MATHEMATICS IN INDUSTRY

Summary

The nature of mathematical modelling as well as the current and growing importance of mathematics in modern technology, an importance which has been accelerated by the expansion of the computer industry, are explained. Their implications for the education of engineers are considered.

3.1 MATHEMATICAL MODELLING

Mathematical modelling has come to win an increasing acceptance in engineering and industry. Many facets of engineering and theoretical physics devolve upon it.

More of an art than a science, mathematical modelling aims, to quote Beltzner *et al.* (1976), "to represent the behaviour of a real-world situation by means of a carefully contrived mathematical description, usually referred to as a mathematical model". Simple or complicated, deterministic or probablistic, such models use mathematics ranging from simple algebra to the most sophisticated mathematical theories.

Thus, for example, as Beltzner *et al.* (1976) point out:

> *Maxwell's equations for the electromagnetic field can be used to predict the behaviour of radio and radar signals to greater accuracy than one part in a million. Another physical model of astonishing accuracy is Newton's theory of the earth-moon system which made possible the landing of men on the moon within a few hundred yards of the pre-determined target.*

They go on to say:

> *Few subjects interest men and women more than the weather. Yet thirty years ago no one would have taken seriously the idea that we would be in a position to model the earth's atmosphere. The basic partial differential equations governing the motion of fluids have been known for a hundred years or more. However, the amount of data needed to specify the boundary conditions is so enormous and the problem of solving the equations is so involved, that even though a reasonable meteorological model could have been proposed, everyone knew it would never be of practical use. Now, with the help of a huge computer, owned by the U.S. government, into which masses of data from weather satellites and other stations is fed electronically, the model predicts the weather for the immediate future four times each day.*

The engineer (or mathematician) in industry has three main tasks. The first is the idealisation of the physical or engineering situation and its formulation in mathematical terms resulting in mathematical equations. The second is the manipulation of the mathematical symbolism to obtain a solution. And the third is the interpretation of the solution in physical terms. Implicit in this process is the function of the engineer to decide which problems are amenable to mathematical treatment.

The first of these, frequently referred to as the making of a mathematical model, is often the most difficult task. In a particular pilot questionnaire, sent to eighty-one engineers by members of the University of Salford, which asked which of the three stages they found the most difficult, seventy per cent said the formulation of the problem. The raw material for this is information which will almost certainly be incomplete or redundant, and occasionally even inconsistent. A physical or engineering problem is invariably so complicated that to attempt to write down, let alone solve, equations which take cognizance of all aspects is a daunting, formidable, if not an impossible, task. An appraisal of the situation at the outset is critical so that the mathematical model has features which are deemed essential, but from which irrelevancies have been meticulously removed. This appraisal demands a knowledge of the magnitude of the various quantities that occur in the problem and may necessitate some preliminary research. The crucial question concerns the disregarding of what is negligible : too much simplification and the solution is valueless; too little and the problem is intractable.

Mathematical modelling is an art, calling for the skilful use of approximations and judgement in striking the correct balance between too much detail and too little. Success in formulating a problem depends on an understanding of the physical situation, a good command of a sufficient range of mathematical techniques and the ability to relate the two. A contributory cause of the difficulty of formulation experienced by engineers (and others) may be found in the undue emphasis given to the manipulative aspect at the expense of the model-building aspect in the teaching of mathematics.

Mathematical modelling is used not only to gain an insight or understanding of some physical or engineering problem but also to make predictions: many models in engineering are designed to assist management in the making of decisions. Professor Synge puts it thus: "The use of applied mathematics in its relation to a physical problem involves three stages: a dive from the world of reality into the world of mathematics; a swim in the world of mathematics; a climb from the world of mathematics into the world of reality, carrying a prediction in our teeth ". The final predictions and conclusions need to be verified by further experimental work: a good agreement between theory and experiment means that the mathematical model is a good "fit" and can be used with confidence to make predictions. It goes without saying that an incorrect formulation of a problem is a more fruitful source of error than mistakes in the mathematical solution.

3.2 THE COMPUTER

The rapid advances in engineering practice occasion complicated mathematical analysis and equations, whose solution depends heavily on the development of new and more powerful mathematical techniques and computing machines. It is fortunate that parallel to the developments in mathematical methods has been the development of increasingly sophisticated computers, capable of larger and larger operations at speeds inconceivable before.

The advent of the digital computer has been the single most important precipitating factor which has influenced the use of mathematics in industry. Mathematics has become a tool of immense practical value to industry, and modern high-speed computers are the means of solving hitherto intractable problems, in many cases not so much because of their difficulty, but because of their complexity.

However, it is not only by increasing the capacity for mathematical analysis that the computer is affecting industry and engineering practice generally. Many of the more pedestrian tasks previously carried out by graduates are now performed by computers. This encroachment is already occurring in the design office and its extension to other fields may be anticipated in the forseeable future. As the direction of the past two decades of computer development is being reversed (that is, from large to small computers) the pervasive influence of computers is going to be more marked.

It is unrealistic to look only at the net displacement of staff to assess the disruptive effects of increasing computerisation. For, the penetration of computers into the mainstream of engineering means that some of the skills and methods are about to disappear, while others are changing in character, some marginally and some significantly. Engineers will thus be increasingly freed for creative work, such as more fundamental design, which may well require an extended use of mathematics.

The pre-eminent consideration for educators, then, is that some men would need to move across "lines of demarcation" sympathetically with the continuing inroads of computer techniques in engineering practice. Engineers would need to be a more flexible, more highly skilled professional work force. They would need to upgrade their mathematical apparatus and their mathematical education should reflect a matching breadth, flexibility and sophistication.

Webb (1972) makes a similar point:

> *The advent of the computer with its capacity for the efficient and accurate handling of vast quantities of arithmetic and data, in particular its ability to cope with large systems of linear equations and to compute eigenvalues and eigenvectors, has opened up exciting new possibilities for the construction industry. The engineer can now shed the restrictions of some of the old labour-saving methods. He can return to first principles and work on new designs which satisfy his creative instincts and result in novel structures pleasing to the beholder. He makes use of matrix methods of structural analysis, finite difference replacement schemes for ordinary and partial differential equations, and variational methods such as the finite element method for stress analysis, heat transfer and seepage problems. Linear programming has its place in the theory of plastic collapse and in minimum weight design.*

3.3 OPERATIONS RESEARCH

The field of operations research makes extensive use of mathematical modelling techniques to resolve problems of decision and control, particularly those involving the movement and allocation of goods and services within large systems.

A relatively new branch of applied mathematics, operations research was developed during the Second World War by scientists called upon to help with the analyses and solution of various tactical problems met by all branches of the armed forces. It may be defined as the scientific approach to the operational problems for the greater fulfilment of objectives and is a parallel development to an increased realisation of the usefulness of staff planning and analysis functions. As a result of different emphasis, operations research workers are known by various names such as systems engineers, industrial engineers, work study specialists, value analysts and methods engineers. The analytical tools of operations research fall into three broad categories: probability techniques, programming techniques and simulation techniques. The mathematical topics that are generally discussed in operations research are decision theory, linear and dynamic programming, Markov processes, network analysis, queueing theory, simulation techniques and Monte Carlo methods.

Many of the problems of management in industry are problems of optimisation; for example, how to deploy personnel and machines to achieve some kind of optimum crystallised in either maximum production or minimum cost. Mathematical optimisation techniques aim to find the maximum or minimum, subject usually to constraints of a "criterion function", which is a measure of the excellence of the system in question. The method of least squares, maximum likelihood, linear and dynamic programming and critical path scheduling are the principal methods used in optimisation.

More and more, problems in the field of industrial planning are being handled by linear programming methods, which are one aspect of operational research. Let Crank (1962b) enlarge on it:

> *To name only one or two, we have the problem of allocating jobs to different machines in a way which minimizes either cost or total time; and closely similar, the problem of job and salary evaluation: here we include the salaries of the managers and employers concerned with a job as well as the running costs of the equipment involved and decide how long each man should devote to his job in order to minimize cost or production time.*

Engineers in middle management and businessmen frequently encounter such problems. The difficulties in making decisions are to be found in the collection of facts, having to decide which are to be excluded and which taken into account and then interpreting them with a view to determining the best course of action. In the face of often voluminous and very complicated information which is difficult to assimilate, it is natural to construct a mathematical model which preserves the essential features of the situation but from which any irrelevance has been removed.

For example, linear programming has been used successfully in many apparently widely differing situations, situations which are basically similar and that can be represented by the same mathematical model. Many of the problems to which the techniques of linear programming can be applied come under the generic heading of transportation or distribution problems.

Professor Crank (1962b) indicates that

> *Although the methods for dealing with linear programming problems are relatively simple, much tedious and time-consuming computation is inherent in an involved problem. In most cases the time of solution rises by at least the square of the order of the matrix formed by the cost table. It is not surprising therefore that linear programming has proved a fruitful field for the application of high speed computing machines, mainly of the digital type. Cost matrices of the order of several thousand have been dealt with, but even bigger ones are arising from industrial problems. Apart from increasing machine calculating speeds, much is being done by taking advantage of special characteristics of the matrix.*

Another illustration is afforded by simulation. An important characteristic of a class of problems that are increasingly being met is variability. Analytical solutions are difficult, if not impossible, to determine and the usual practice is to reproduce or simulate the problem by a computer: a computer can be programmed to simulate the behaviour of the system with all its variability and physical constraints by developing a model in which these have been articulated in a numerical language. An excellent example of this may be found in mining. To quote Lee (1972):

> *The mining industry is concerned with the process of digging ore from a hill, but it is also involved with the optimization of an economic return function using geological data filed in a computerized data bank. A mathematical model adaptively guides the search pattern, decides if there is a profit to be made, designs the mining sequence and ultimate mine limits, optimizes the materials handling and plant location configuration, and uses game theory to select markets and pricing strategies.*

3.4 IMPLICATIONS FOR EDUCATIONAL PRACTICE

Professor Flowers, Chairman of the Science Research Council in Britain, sadly acknowledges that

> *not enough of our graduates from universities, still less their teachers, are attuned to the needs of industry. Industry does not successfully compete for its fair share of the best brains it so desperately needs. The achievement of a higher productivity and the marketing of new techniques are not matters for the second-rate.*

The situation in Australia, with notable exceptions, is no better and this is how, to use the incisive words of Weickhardt (1973), we are sometimes seen:

> *...feather-bedded through our easy purchase of technology from overseas, sheltering in craven fashion behind high tariffs, grumbling in surprise that university graduates are not tailor made to our requirement and unwisely myopic in our measure of research scientists.*

The need, then, for a close collaboration and interaction between industry and tertiary institutions is undeniably important. The Committee of the Mathematical Association (1966) maintains that.

> *Given a better relationship between industry and teaching, a large number of recruits to industry could be of much greater value to their firms if both sides realized the advantages of expressing situations and problems of the widest generality in mathematical terms. These recruits would not merely be more efficient from the start and be capable of greater things with further training; they would themselves obtain more satisfaction from their work, to the benefit of themselves and of their employers alike.*

There is a great need to spread this realisation among academics on the one hand and through industry on the other. An accompanying need is to espouse the view that both the appreciation of the potentialities of this situation and their transformation to actuality will be best fostered by much more interchange between tertiary institutions and industry.

It is difficult to discover what branches of mathematics are being used in industry. "Indeed, it is doubtful", Professor Crank asserts, "if such a general question is pertinent, having in mind the widely differing needs of many sectors of industry".

The need for increased use of mathematics is inherent in the general rapid advances in industry: equipment is being designed to far closer tolerances than would have been considered possible twenty years ago, while the synthesis of highly complex systems, at one time inconceivable, is commonplace today. Underlying all such developments is the economic necessity to minimise costs, so that techniques of optimisation have become important in all facets of industry, from research to management.

The increased need for mathematics has involved the use by engineers of mathematical theories formerly of academic interest only. This situation cannot fail to be reflected in the undergraduate mathematical curriculum of engineers and some universities and CAEs are undertaking extensive revisions in order to make the mathematical training respond to the needs of industry.

Unquestionably, industry depends upon skills in formulating a viable physical model in the first place and the mathematical expertise to obtain numerical or approximate analytical answers, or both, in the second. The over-all validity of the analysis in an industrial context depends upon the physical understanding that is put into the first stage than upon the mathematical niceties that go into the second; the results produced are only as good as the physical assumptions on which they are made. Insufficient attention is given to the aspect of formulation of physical problems in the training of engineers. At present too much emphasis is placed upon the solution of problems already formulated in mathematical terms and having clearly defined, elegant solutions in closed form.

While such problems are obviously necessary to reinforce the teaching of mathematics, their study alone does not equip students to deal with industrial work, which involves selection of relevant from superfluous data, choice of approximations and estimation of parameters. In fact, in many real-life situations the most difficult step is the stage that precedes the first stage of formulation - the identification of the problem. Dr. D. J. G. James of Lanchester Polytechnic has received a Nuffield Grant to begin a project in 1979 which will prepare numerous case studies of mathematical models. The project is entitled "Modelling Approach to the Teaching of Mathematics" and is in collaboration with Paisley College of Technology.

In all engineering analysis approximation plays a key rôle. The necessity to obtain a tractable mathematical problem should be explained in courses. More significance should be given to the obtaining of approximate solutions of real problems than to exact solutions of rather unreal problems of academic interest. Realistic problems not only provide motivation but are also a better preparation for a student's later career. Frictionless pulleys, light inextensible strings, inviscid fluids and many others of the same genre are concepts which have no real existence: they are abstractions, part of the modeller's idealisation which nevertheless can often usefully be regarded as a close approximation to real-life situations.

The academic attitude to mathematics, namely that it is a rigorous, intellectual discipline to be pursued in its minutiae for its own ends, is imposed, consciously or unconsciously, upon the student. Basically the capability of arguing rigorously is excellent and vital in industrial research when it is used to decompose a complex problem logically or to make physical judgements and approximations to complicated situations. However, if it is used in the industrial context solely to probe the mathematical minutiae and niceties of a problem then the time spent is usually wasted, the end-product being an erudite paper to a specialist, minority audience. Elegance has a place in all mathematical work, but not at the price of simple clarity. Again, too much emphasis on rigour stultifies the imaginative thinking that is often required in the model-building stage. For example, over emphasis on existence theorems (as opposed to the study of a few well chosen examples and a statement of the rest) gives a distorted impression of real world mathematics. There it is more usual to proceed on the basis that a well-posed physical problem ought to have a sensible result until proved otherwise. One has to think heuristically rather than rigorously.

These considerations should not detract from the prime importance that mathematics enjoys in the education of an engineer: command of a range of mathematical techniques is essential for success in this field and this implies a clear understanding especially of mathematical analysis, calculus, linear algebra , and computer programming.

If there is a simple mathematical technique which no industrial engineer should be without, it is perhaps matrix algebra. This is because the first act of descriptive mathematics in the face of a new and difficult problem is often to try to set up a linear model with constant coefficients. But if a man skilled in matrices is valuable, the man who realises that computers can perform matrix computations faster and more accurately than he himself, is more valuable; and the man who actually knows how to get a computer to do it is worth the two of them put together. Every industrial engineer should be acquainted with and made to appreciate the tremendous usefulness of computers.

An engineer who is faced with the task of controlling a complex system of interacting, inherently variable elements must learn to think in a statistical language. This is not to be equated with a knowledge of statistics, which is not a difficult subject for a competent engineering undergraduate, but rather with the realisation that uncertainty is an integral part of description: the variability of process-time, the feelings of machine operators, the physical capacities of the machines can complicate the problem of meeting delivery dates.

It must be admitted that there are dangers in constructing courses strongly orientated towards the applications of mathematics to a particular field. One of these is to reduce mathematics to a series of "tricks" for solving problems. Another is to give undue emphasis to current problems at the expense of preparation for the unknown problems which will arise in the future. These difficulties, however, can be avoided. Any tertiary mathematics course should provide the basis of mathematical knowledge required by those who intend to call themselves engineers, and should maintain a balance between pure and applied mathematics. Above all, it should give an understanding of the mathematical way of thinking and the importance of rigour in mathematics. It has been found in many tertiary institutions that, without sacrificing any of these requirements. it is possible to slant a first degree course in many different directions and there is no evident reason why a bias towards industrial applications of mathematics should be inferior to any other type of bias.

The subject of mathematical modelling is now so important for engineers that we might well introduce into the mathematics curriculum a section on "Mathematical Models for Engineers", where attention should be paid to the derivation of the mathematical equations describing a physical situation as well as to the interpretation of the solution obtained, which often involves investigation of the effect of variation of the parameters of the problem. Useful material for such a course will be found in the book by Professor J. Crank, entitled *Mathematics and Industry*, published by Oxford University Press. The topic of problem-formulation could possibly be introduced by a "case-history" approach, with the discussion group replacing the formal lecture. For a source of suitable material the best approach would be the participation of industrial firms in the course, allowing their engineers to expound suitable problems which they have encountered in their own work. This again presupposes an intimate partnership between industry and teaching. Perhaps, having "industrial professors" would make it easier to secure cooperation.

There are few courses which combine mathematics with a particular engineering discipline such as mathematics and electronics or mathematics and aeronautics. Also very few universities allow an engineering course to be taken as a subsidiary subject in a mathematics course. It might have been expected that the CAEs, whose main *raison d'être* has been to provide vocationally oriented courses could have taken a lead in the education of mathematicians and engineers in industry. However, their approach on the whole, in the preparation of mathematics syllabuses, has been disappointingly conservative. A feature most of them do have is the sandwich course, in which the student spends alternate semesters with an engineering firm or industrial organisation. This can obviously be valuable in giving the student a foretaste of the type of work he might do later. A number of CAEs offer courses in linear algebra, complex variable theory, differential equations, mathematical methods, numerical analysis, computing, statistics, linear and dynamic programming, simulation techniques, and operational research. Also, most students are expected to work on a project during their final year and present a short dissertation on it. This project will require, with the assistance of a member of staff, the formulation and solution of a moderately complicated mathematical problem related to engineering and will often involve the use of a computer.

Industry tends to organise itself in interdisciplinary teams of scientists, engineers, economists, mathematicians and statisticians. The mathematician working in this field should be able to understand the problems of, and exchange ideas with, his engineering colleagues and vice versa. A criticism frequently made is that engineers and mathematicians are unable to, or will not, speak each other's language and it seems worthwhile to bridge this communication gap.

One fact is inescapable: unless there is a change of heart by the majority of academic mathematicians in universities and CAEs, no suggestion will ever come to fruition and lead to the necessary changes in content and approach in graduate as well as undergraduate courses. This, in turn, will require a much greater involvement of mathematicians on a continuing basis with the real world and with other departments within their own university or CAE, such as the departments of physical and engineering sciences. In the difficult economic climate that currently exists in Australia, indeed throughout the world, industries will be reluctant to channel money to research unless they have the expectation of a much more positive return. It is a fact of life that academic freedom is an expensive luxury that universities and CAEs ought no longer to expect industry to support unconditionally.

To quote the memorable words of Beltzner *et al.*(1976):

> *Given the proper conjunction of real world problems, a resolute will to solve them on the part of management, and a team of imaginative persons familiar with the topic and having a bold wide-ranging mathematical competence - business, industry and government could profit enormously in many ways hitherto undreamt of by an effective exploitation of the mathematical sciences.*

CHAPTER IV

MATHEMATICS AND ENGINEERING

Summary

The content of mathematics is changing, but much more significant is the almost explosive growth of its usefulness in engineering. Little more justification is needed for the breadth of material generally included in mathematics curricula for engineering aspirants, but there are other cogent reasons, as this chapter shows.

Before 1900 only a few scientists and engineers were conscious of the key role which mathematics was playing in the technological progress of Western society. The only mathematics visible in everyday life was the arithmetic of finance and the geometry of design. The role of mathematics in business, in industry, in government, in the biological sciences or in the social sciences, could have been understood by anyone with a good grounding in the basic skills of addition, subtraction, multiplication, division and measurement. Only a handful of technical specialists - engineers, actuaries, physicists and mathematicians - felt the need for more advanced mathematical concepts and methods.

Today the situation is very different. There has been an expansion of mathematical activity during the last forty years, affecting an extraordinary range of human activities. These include economic policy planning, manufacturing and advertising, urban planning, agricultural and medical research, geological exploration, the behavioural sciences, genetics and ecology - not to mention such unlikely fields as anthropology, archaeology and linguistics! The influence of mathematics has never been so pervasive or so subtle as it is today.

(Beltzner *et al.*, 1976)

4.1 COMPLEXITY OF MODERN TECHNOLOGY

The engineering practice of today is becoming more of a science and less of an art, and a sound, broad mathematical base is imperative for any meaningful participation in this transition. Although, to most engineers, mathematics is a means to an end, the complexity and intricacy of modern technology provide a powerful impetus and compulsion for the pursuit of advanced mathematical studies. Problems in modern engineering are so complex that they cannot just be considered as an aggregation of elementary components, nor can they be solved solely by the use of physical intuition and past experience. The empirical approach has been successfully used in the solution of many past problems, but is hopelessly inadequate as soon as high speeds, large forces, high temperatures or other abnormal conditions are present. The situation is further aggravated when one ponders that various modern materials, such as alloys and plastics, have unusual physical attributes.

Experimental work in the present-day world of engineering has acquired a highly complex complexion and this precipitates an inordinate expenditure of time and money. Here again mathematics is singularly helpful in the planning of experiments, in evaluating experimental data and in curtailing the work and cost in the search for solutions.

The statistical nature of certain physical and engineering situations provides a different source of mathematical compulsion. Students of life sciences have long been at home with statistical techniques since the systems under study have not only shown an exceedingly complex disposition, but also because many of the inherent variables defy control. Engineers, on the other hand, have in the past been largely concerned with deterministic problems. However, the advent of complexity in modern technological development has led more and more to considerations of stochastic rather than deterministic behaviour.

Consider the problem of automatic fire control for anti-aircraft artillery. As Weller (1956) declares:

> *Here we find a radar set which automatically follows an aircraft and provides information as to its course and speed. These data are delivered to a computer that predicts the aircraft's position in the future and determines the proper azimuth and elevation of a battery of guns in order to obtain a hit. The guns are trained and fired by a servo system that derives data from the computer. Systems such as this cannot be considered simply as an aggregate of elementary components. They must be looked upon as a series of functional units, each of which performs some sort of transformation on its own input. The result of this transformation must constitute the proper input for the next functional unit. In considering such systems, one talks of transfer functions, impedance matches, and stability with respect to various forcing functions. To make appropriate calculations, one must be familiar with differential equations, functional transformations, matrices, and time series.*
>
> *In the case of this fire-control equipment, the computer selects the most probable future position of its target. If a maintenance engineer wishes to service such a system, he must stock those spare parts which are most likely to fail. If a calculation of lethality is to be made, the result is the expectation of destroying the target. These are all terms characteristic of the mathematics of chance and probability. One may, in fact, say much more generally that the experimental determination of the behaviour of a system must take into account the presence of manufacturing tolerances, variations in the behaviour of materials, individual differences in opinion of inspectors, and variety in the anticipated conditions of use.*

Such considerations not only serve to underscore the high degree of complexity resident in modern technological systems but they reduce all engineering test operations to a consideration of probabilities.

More and more in recent times, problems of ever-increasing complexity which have to be optimised with regard to such factors as cost, reliability, maintenance and design are unresponsive to pedestrian mathematics. Let Weller (1956) speak again:

> *The demand for efficiency in airplanes results in a 'one-hoss-shay' concept of design. No part may be excessively heavy. No part may be too weak. Hence it becomes necessary to compute stresses very accurately. But simple arithmetical formulae limit stress calculations to rough approximations, except for very simple shapes. It has been necessary to broaden the mathematical treatment of structural design to include such matters as elastic stability, flutter and stress concentrations. Efficient design has forced aeronautical engineers to adopt more refined methods of analysis and calculation.*

4.2 MODERN MATHEMATICAL METHODS

Engineering developments have successfully transformed such sophisticated mathematical-physical theories as quantum mechanics, fluid dynamics, relativity, electricity, and elasticity into technological realities, including our present-day communications networks, nuclear weapons, aerospace technology, and computers. Since World War II, however, there has been a phenomenal development of newer mathematical methods indicated by such terms as information theory, systems analysis, optimisation, decision theory, algorithmics and control theory. New developments in engineering require a broader mathematical training; new deveopments in mathematics appear especially apposite to modern engineering. The cool conclusion to be drawn, then, is the need, urgent and imperative, for a closer integration of the two powerful disciplines of engineering and mathematics.

Ever since the Second World War, there has been a growing awareness of the idea that many questions which have previously been answered on the basis of a hunch or prejudice may, in fact, be amenable to quantitative treatment. More specifically, if one can assign numbers to the possible outcomes of various courses of action, it may be possibble to select the optimum course analytically. We have observed already that the study of questions of this type is known as operations research. Other mathematical optimisation techniques include the calculus of variations, linear and dynamic programming, and various numerical "hill-climbing" procedures. Such mathematical disciplines bring with them a requirement for many mathematical techniques.

Many problems that confront engineers in their professional practice defy precise formulation, and only numerical solutions are possible. For example, as Weller (1956) points out, "we may desire to know the stresses in a tension member near a hole of arbitrary shape, the electric field gradient between electrodes of arbitrary shape, or the rate of fluid flow through a channel of arbitrary cross-section". The difficulty is crystallised in the word "arbitrary", which really means non-analytic in a mathematical sense. One may approach a solution by a series of successive approximations that will yield an answer in which the error may be reduced below any desired finite value. The use of the so-called "relaxation methods" and "iterative methods" and the replacement, in general, of differential equations by finite-difference equations lie in this field of utility: such procedures are applicable to problems with arbitrary boundary conditions. The use of numerical techniques has been encouraged by the astonishing development of digital computers.

4.3 STATISTICS

Statistics, mathematical modelling and computers pervade the new mathematical disciplines and their associated applications. Statistics is a mathematical science which draws heavily on the theory of probability and linear algebra. Any course in statistics includes the basic ideas on data processing, such as the use of graphs, measures of dispersion, and distributions. Most engineers indicate that these are the particular aspects of statistics most frequently used, an admission not totally unexpected since they are fundamental to any statistical enquiry. However, academic courses for engineers in Australia generally do not pursue the study of statistics in depth and perhaps greater use would be made of the more sophisticated statistical techniques if engineers had a greater knowledge of statistics. "The main factors in determining the sophistication of mathematical techniques employed to solve problems in business or government", according to Beltzner *et al.* (1976), "are firstly the knowledge and ability of the mathematical practitioner (engineer, statistician, etcetera) who has to solve them; and secondly, the capability of managers to appreciate the solution". This remark is very much to the point with respect to the range and degree of statistical techniques used by engineers.

The theory which exists on sampling techniques, sampling distributions, hypothesis testing, acceptance sampling and parameter and interval estimation is basic and necessary for the understanding of quality control. The theory of the design of experiments prescribes those systematic variations of parameters in an experiment which will, as far as possible, isolate the influence of the several factors controlling its outcome. A complementary analytical part is played by the theories of correlation, regression and analysis of variance. These techniques are used for the quantitative identification and characterisation of possible relationships between variables and for distinguishing the effects of various factors on a set of experiments; indeed, the theory of experimental designs depends largely on the analysis of variance. The theory of games and decision theory have applications in industry, as have reliability studies.

The theory of queues has particular relevance to industry, where decisions concerning the deployment of personnel and machines to achieve some optimum criterion, such as maximum production or minimum cost, have to be taken; specific examples of queues in the electrical industry are the demand for electrical power and telephone traffic. Again, auto-regression techniques have been used in filtering out the "noise" in signals received by aircraft engaged in geophysical exploration by means of sensitive magnetic devices.

The theory of stochastic or random processes unfolding in time may be described as the dynamic aspects of statistics. The most notable application of the theory of stochastic processes has been in the statistical theory of communications and control.

4.4. THE ROLE OF PROBABILITY IN ENGINEERING

The techniques of the statistician are becoming increasingly important to the scientist and the technologist. Apart from the widespread and increasingly necessary application of sound statistical methods to the great body of data garnered from experiment and research, an essential part of an engineer's problem in design and planning is that decisions are formulated irrespective of the state of completeness and the quality of information. The engineer is therefore faced with the problem of making decisions in the face of uncertainty, in the sense that the consequence of a given decision cannot be determined with complete confidence.

Quantitative methods of modelling, analysis, and evaluation are the methods of modern engineering, some of which are quite elaborate and include computer simulation, optimisation techniques and sophisticated mathematical modelling and analysis. However, despite the high degree of sophistication in the models, they are formulated under idealised assumptions and conditions and hence any conclusions or information inferred from these mathematical models may or may not reflect reality accurately.

Aside from the fact that information must often be derived from models that very imperfectly approximate actuality, many engineering problems involve processes and phenomena that are inherently random. These processes or phenomena are not deterministic in character and their state cannot be articulated in precise terms. For these reasons, decisions in the design and planning of engineering systems must be, and are, made in the presence of uncertainty.

The concepts and methods of probability and allied fields of statistics and decision theory offer the mathematical basis for modelling uncertainty and the analysis of its effects on engineering planning and design.

It is interesting to review briefly three major areas where probability methods are introduced for modelling engineering problems and evaluating systems performance under conditions of uncertainty.

First, the products designed and produced by engineers are usually used by large numbers of people differing in many ways. Some quantitative information about consumer behaviour becomes an important consideration in design.

If traffic studies and service facility studies are to yield significant, meaningful design information, a quantitative description of the behaviour of aggregates of people is again necessary. Such a description can only be attempted in probabilistic terms.

An early and highly successful application of such description may be seen in the design of automatic telephone exchanges.

Aggregate problems will play an increasingly important part in future engineering design since the presence of human population behaviour is a critical consideration.

Another engineering field in which large aggregates feature is industrial mass production. The interchangeability of parts is an essential requirement for the success of mass production. However, because of the great difficulty of specifying precisely machine wear, machine settings, and material properties, perfect duplication is impossible to achieve and some variability in dimensions has to be accepted. Statistical quality control, involving the use of quality control charts based on probability theory, marks another successful application of probability concepts in engineering.

Second, a certain haziness characterises the majority of problems met by engineers in their professional practice: in one manner or another, the problems are incompletely described and part of his problem is the inevitable state of ignorance in which he must work. The use of such expressions as engineering judgement, factors of safety and allowable limits not only reveal that engineers are accustomed to and aware of imprecision, but also they represent attempts by the engineer to cope with this lack of determinism. This is stressed by Freudenthal (1963) when he writes:

> *The pre-occupation, in recent years, on the part of designers with problems of structural safety is a sign of the growing realisation that a critical re-evaluation of the concept of safety and of the safety factor is a task of considerable urgency, if the elaborate and refined methods of stress analysis made possible by computer development are to be effectively utilised in structural design. The most careful and rigorous structural analysis is largely deprived of its merits if the accuracy of its results is diluted by the use of empirical multipliers, so called safety factors, on the basis of considerations which are not always relevant or even rational.*

The estimation of safety factors of structures requires that design calculations be related to material properties which are subject to fluctuations resulting from many chance factors. Attempts to rationalise the concept of factor of safety by accounting probabilistically for variability in material properties merit considerable attention from the structural engineer. This is well expressed, again, by Freudenthal (1963) when he says:

> *That the concept of probability must form an integral part of any rational design procedure follows from the realisation that the design, at present, of a structure for future use must necessarily embody prediction of expected performance of the structural material based on past experience (materials tests) as well as prediction of expected load patterns and load intensities extrapolated on the basis of past observations (load records). Since such predictions by their very nature, cannot be made with certainty but only with a certain degree of probability, any conceivable design condition of the structure is necessarily associated with a numerical measure of the probability of its occurrence. It is by this measure alone that the reality and structural significance of a specified design condition can be evaluated.*

Third, many problems in engineering involve processes or phenomena which are inherently random. Random forces or disturbances, noise of one kind or another, and the response of various systems to such inputs are engaging the attention of an increasing number of engineers. Here many of the loads, disturbances, noise or inputs met in engineering are usually such as to defy a precise description and they must therefore be approached in probabilistic terms through random functions.

Shu (1963) reminds us that

> *It is no longer practical nor even useful to try to list all topics in engineering which now employ, or are beginning to employ random disturbances in problem analysis. Such a list would certainly include signal detection in the presence of noise, information theory, automatic control theory, ship behaviour in a seaway, vehicle dynamics on hard surfaces, least-square smoothing and prediction, response of aircraft to taxing and gusts, turbulence, system transfer function determination, radar design, aseismic structural design, re-entry physics.*

The framework, then, within which the design and planning of engineering systems must be undertaken is hedged with many restrictions, such as limits on size, weight, shape, demands for a certain measure of reliability, life expectancy, optimisation of one variable or another, and economic constraints. Although modern scientific methods have done much to eliminate uncertainty fostered by ignorance, the inescapable reality is that uncertainty is still the environment in which an engineer does his work. The quantification of such uncertainty and the evaluation of its effects on the design and planning of an engineering system fall within the province of mathematical statistics.

Statistics is playing a widening rôle in industry; as a result, at least some engineers would need greater exposure to probability and statistics than present undergraduate courses provide. These courses are generally limited to an introduction to elementary probability, the basic statistical parameters of central tendency and dispersion, and the binomial, Poisson and normal distributions, with no elaboration of probability ideas leading to a consideration of stochastic processes or mention of sampling distributions. However, undergraduate mathematics syllabuses are already over-crowded and any inclusion of additional statistical topics would necessitate a major assessment and revision of mathematical priorities in the education of engineers. This is a difficult undertaking since the more advanced statistical techniques assume, for their understanding, that students have facility with a considerable fund of mathematical knowledge. The study of advanced mathematical statistics, in view of this observation, may well have to be deferred to post-graduate years.

4.5 THE COMPUTER

In an editorial, Professor Bajpai (1970a) asserts:

> *The advances in the last two decades in the use of computers in all branches of science and technology have brought about a new awareness and interest in mathematics and its applications.*

It is now a commonplace to observe that the advent of the electronic computer has greatly enhanced not only the possibility of making effective use of a wide range of mathematical techniques but also the usefulness and importance of statistics and mathematical modelling. Large tracts of pure mathematics which previously held only an academic, theoretical interest have suddenly been transformed and show the promise and potential of great practical serviceability. The advent of the digital computer, we believe, is likely to make the biggest single difference to the kind of mathematics engineers will need to know, notably the appropriate numerical techniques, and the way they tackle problems.

Using the computer's powers of data storage and manipulation at unbelievable speeds, the trend towards the mathematization of problems has been accelerated. The design of bearings is an example which may be cited as typical of those practical problems which have suddenly become susceptible to mathematical treatment, thanks to the computer.

The mathematical formulation is relatively straightforward. The basic equation is due to Reynolds and goes back to the second half of the last century. However, if you want to include any sort of reasonable approximation to physical truth - such as the variation of the lubricant's viscosity with temperature or the fact that the bearings themselves deform under pressure - you must bring in other equations which leave you with a very uncomfortable set of partial differential equations to solve. Today's computers have made it possible to overcome many of these difficulties and we have suites of computer programs in my own laboratory, for example, which will tackle a variety of meaningful bearing design problems.

(Wakely, 1970)

When mathematics and the computer run in harness with one another, the partnership offers a new approach to the solution of engineering problems in industry. A new approach, for example, has been made feasible to the problem of stress-analysis, of vital importance to industry, by the development of the so-called finite element method.

This is essentially an application to continuous structures of the procedure used in dealing with discrete structures merely by breaking down the continuous structure into a number of pieces - beams, plates, and so forth. The basic Hooke's law for each part together with the equilibrium and continuity equations provide a set of linear algebraic equations which can be run through the computer very nicely.

(Wakely, 1970)

4.6 PURE MATHEMATICS

The phenomenal development of the mathematical sciences has stemmed from the central core of abstract thought, usually referred to as "pure mathematics". The growth of this central core has been striking, especially in the present century, as mathematicians of stature apply themselves unstintingly to push back the frontiers of knowledge, their agile, incisive minds sluiced with the elegance and beauty of abstract mathematical thought. Nothing could be at a greater remove from the relevance and applicability to engineering and allied disciplines than these abstract mathematical systems. Yet history endorses the truth of A.N. Whitehead's (1926) penetrating observation:

The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete fact.

Indeed, it is striking, as Beltzner *et al.* (1976) would have us notice, that

> *many of the potential mathematical tools of today were first developed without reference to possible uses outside of mathematics. The English mathematician, Cayley, firmly believed that matrices, those rectangular blocks of numbers which he studied in the mid-nineteenth century, would never be applied to anything useful. They are now an everyday working tool of engineers, physicists, economists, statisticians and behavioural scientists. Complex numbers, involving the 'imaginary' square root of minus one, were at first regarded as mere mathematical whimsy. Now they play a crucial role in the theories of fluid dynamics and electrical circuits. Group theory, used by Galois in the early nineteenth century as a means of studying mathematical symmetries associated with the solutions of polynomial equations, has subsequently found significant applications in the study of subatomic particles, in crystallography, in information theory, in photochemistry, and in the elucidation of certain complicated marriage systems studied by anthropologists. Non-Euclidean geometry, one of the great triumphs of abstract logical thinking, was a forerunner of Einstein's celebrated physical theories, which imply that the universe we live in is 'curved' in the sense that parallel lines do not remain equidistant when extended into space. Graph theory, the mathematical study of abstract networks, was considered a rather esoteric kind of pure mathematics until recently, when it was applied to problems in transportation, communications, urban planning, electrical networks and sociology.*

Over the doors of Plato's Academy were inscribed the words "The Deity never stops doing mathematics", and it is well known that Plato had in mind several illustrations of the theme. It is, however, the work of the past years and more particularly the recent past that has brought the truth of the inscription to a far clearer degree of acceptance: a long series of discoveries establish the extraordinary precision with which qualitative measurements of the material world obey complex laws of a fundamentally and essentially mathematical character.

The astonishing expansion of the mathematical universe, accompanied by the sympathetic, dramatic expansion in the range of possible applications, has had the deleterious effect of unrelieved fragmentation and specialisation. A mathematical conference of contemporaneous times parallels the Tower of Babel, for few can profitably follow discussants from fields other than their own, and even there they are sometimes made to feel uncomfortably as strangers.

The particular economy of thought that mathematics lends, the generality of mathematical applications, the unity of mathematical structure -

this is the distinctive genius of mathematics. This should govern the direction and spirit of our teaching as we strive to affirm the privileged position that mathematics occupies in the education of engineers.

CHAPTER V

THE PHILOSOPHY AND EDUCATIONAL ETHOS OF HIGHER EDUCATION INSTITUTIONS

Summary

The distinctive aims and functions of universities and colleges of advanced education are amplified and viewed within the general framework of the purposes of higher education.

5.1 UNIVERSITIES AND COLLEGES OF ADVANCED EDUCATION

5.1.1 The Birth of the CAEs

Tertiary education in Australia includes three systems of institutions - Universities, Colleges of Advanced Education (CAEs) and Technical and Further Education (TAFE) Colleges. However, education within the higher education sector is the particular province of universities and CAEs, whose courses lead to awards above that of associate diploma, although some overlap occurs at this level with TAFE.

The development of the Colleges of Advanced Education stemmed from the recommendations of the special committee chaired by Sir Leslie Martin and appointed by the Commonwealth Government in 1961 "to consider the pattern of tertiary education in relation to the needs and resources of Australia and to make recommendations on the future development of tertiary education". The Martin Committee reported in 1964 recommending greater diversity in higher education, and the establishment in each State of tertiary colleges, now known as CAEs, with a strong technological emphasis: it stressed the need for a type of education that would be alternative to that provided by the universities.

The Federal Government, aware that individuals have an undeniable right to the highest level of education permitted by their capabilities and interests, that an educated nation is a better nation and that industry needs a well-educated work force, responded affirmatively to the recommendations.

If the genesis of the binary system of higher education is discoverable in the Martin Report (Martin, 1964), the Wark Committee's First Report (Wark, 1966) recorded its view on the differences which might be said to distinguish the new CAEs from universities, differences mainly of degree rather than of kind. It is true that the Martin Committee expected that the colleges would be distinguished by a greater diversity in kind, the promotion of equality of opportunity through the provision of higher education for all citizens according to their aptitudes and abilities, and the emphasis on vocationalism. The Martin Report emphasised that the teaching of college courses, having a direct vocational relevance, must be different from the teaching of allied courses in universities.

5.1.2 The Distinctive Emphasis

The colleges were established with the main object of providing specialised vocational training. Their principal *raison d'être* is to educate students in a wide variety of skills needed by our society. The necessary corollary to this is that colleges should have a more direct relationship with industry, commerce and the employing authorities. The colleges are expected to deploy a greater concentration upon part-time or sandwich pattern of studies for students who will generally be in a related employment.

If the essential characteristic of a CAE, a characteristic that differentiates its philosophy from that of a university, is its vocational bias or applied emphasis, another important distinction is the primary emphasis that colleges are expected to give on teaching, with post-graduate training and research playing a minimal humble rôle: the First Report goes so far as to record that there is comparatively little scope for research in a college operation.

The purpose of the colleges, primary and overruling, is stated to be the training of students to staff the various professions, the public service and industry. However, universities too are concerned with vocational training. Agriculture, commerce, dentistry, education, engineering, medicine and law - these are all examples of professional vocational training with which universities are concerned. The one-time Vice-President of the Victoria Institute of Colleges, Dr Phillip Law (1970) says that

> *Unquestionably the commonest motive in the pursuit of tertiary education is the vocational one, that which arises from the need to acquire adequate qualifications for reasonable success in a highly competitive society.*

He rejects the concept of a university which caters only for an élite group of scholars with liberal motivation: most of the students who enter universities are led by vocational motives.

Whilst it is conceded that universities, too, participate in the preparation for the professions, it is stated that they have a further, and presumably, dominating function as well - the disinterested pursuit of learning, the discovery and expansion of knowledge. The academic staff of universities have a commitment to research in that their academic duties include teaching *and* research, and they are expected to spend a substantial proportion of their time on research and scholarship.

Professor Partridge (1972) describes the function of university education in this manner:

> *...the teaching of basic humanistic and scientific disciplines in a thorough and advanced manner should be central to the conception of a university education: although all modern universities are necessarily chiefly occupied with certain types of professional training, yet, within universities, such training should be erected on the foundation of solid and advanced study of basic disciplines ... One can say, perhaps, that it should be a defining characteristic of university education that teaching is so conducted that the achievement of professional competence emerges from solid and disinterested study of basic disciplines: the study of the discipline is not tailored or curtailed to fulfil the conditions of minimum professional competence.*

The CAEs are exhorted to concern themselves with the provision of tertiary education to acceptable professional standards and/or tertiary education of a specialised nature not normally provided by the universities. The Committee recorded that its concept implies that the colleges will need to match a number of characteristics of the universities, such as a necessity to place a similar emphasis on the responsibility of students for their own learning, for staff to have qualifications and experience of a similar quality, though of a different nature, and for buildings and teaching facilities to be of similar standard.

There is a growing public concern that the CAEs are destined to be second-rate universities, that the colleges would fail to establish standards and thus debase the standing of Australian degrees and diplomas. The number of colleges has been growing steadily, the growth being more spectacular in the recent past. It is questionable whether the necessary academic standards can be established and maintained at all the tertiary colleges and whether equipment, physical facilities and staff can be provided to give all college students the fair and equal opportunities in the pursuit of their chosen field of study which Australian egalitarianism would appear to demand. There is, then, a growing apprehension that, with the advent of the CAEs, there might be a dilution of the concept of university education, an erosion of the accepted notions of the essence of a university.

Again, there will be obvious difficulties in maintaining the academic quality of the teaching staff in the CAEs if there were a prohibition of research or of reasonable opportunities for study leave, or if an over-pragmatic view is taken of the value of consultancy and industrial leave as a substitute for the more traditional outlets for academic refreshment and renewal. Here it is worth stating that governing college councils have wisely granted college teaching staff opportunities to take study or sabbatical leave. It will be interesting to see what serious effects the 1977 Federal Government restrictions on study leave will have, as the discouragement of using such periods for the improvement of professional qualifications will particularly affect CAE staff, who normally cannot pursue higher degrees (certainly at the doctoral level) in their own establishments.

5.1.3 The Wiltshire and Sweeney Committees

In the original concept of CAEs, teacher education was expressly excluded. The Federal Government made it abundantly clear that it would fund courses leading to diplomas but not degrees. However, quite early in the life of the colleges, in June 1968, the Federal Government announced the appointment of the Wiltshire Committee to enquire into advanced education awards. This Committee recommended that certain college courses should lead to bachelor's degrees and also that limited provision for post-graduate diplomas and master's degrees should be made. Though the Committee envisaged that diploma courses would continue to be the bulk of the work in the colleges, in fact after 1969 colleges rapidly developed bachelor's courses, which now form a very substantial part of their work, particularly in the larger and more diversified, multi-disciplinary institutions such as the Institutes of Technology in Brisbane, Sydney, Melbourne, Adelaide and Perth.

As well, in June 1968, the Sweeney Committee was appointed by the Commonwealth Government to inquire into academic salaries in colleges of advanced education. This Committee specified qualities required of lecturers and senior lecturers in CAEs and recommended that the Commonwealth Government should provide support for salaries equivalent to those of university lecturers and senior lecturers who met the defined standards.

The acceptance by Government of the major recommendations of the Wiltshire and Sweeney Reports has had a significant effect not only on the nature and quality of courses, the standing and recognition of the colleges, but also on the morale and self-esteem of their staff.

Following an earlier decision to support teacher education courses in colleges of advanced education, in 1972 the Commonwealth Government made the decision to include teachers' colleges as part of the advanced education sector.

5.1.4 Rationalisation and Cooperation

Admittedly, there are areas of overlap between universities and CAEs. For instance, both are engaged in the professional preparation of engineers, and, as far as New South Wales is concerned, both offer courses in law. However, despite this duplication, the college segment as a whole is more diverse, provides for a greater variety of students and is more unashamedly vocational in its teaching than are universities. The CAEs offer a different, alternative approach to higher education from universities, where the approach to teaching, even in their professional schools, is characterised by "deep scholarly interest and intellectual exploration".

More experimentation and more diversification in higher education, both because individuals are different and because we have a diversity of national needs, are made possible by the emergence of CAEs into the tertiary field. "Different but equal", the motto of the CAEs, is an exciting phrase. As Murray-Smith (1969) puts it:

> *It suggests a genuine spirit of educational innovation and adventure in a country where neither has previously been visible. It suggests a determination to establish flexible but demanding aims in higher education, to call up a responsiveness and a liveliness, an optimistic awareness of where we are going as a people, which we have not seen before.*

For these reasons, the advent of CAEs into tertiary education must be applauded.

The sheer number of universities and colleges of advanced education, not to mention other tertiary institutions, their decreasing homogeneity, and the correspondingly increasing variety of their offerings demand some central appraisal if uneconomic duplication is to be avoided and a reasonable degree of differentiation of function is to be achieved; it no longer makes sense, if ever it did, that each institution should seek to develop the range of its own offerings with no regard to the intentions and practices of sister institutions.

Though universities are accepting, to an increasing extent, CAE graduates into their post-graduate courses and there is an increasing number of university graduates undertaking post-graduate studies in CAEs, there is no tradition of cooperation amongst institutions of higher education as may be found in the United States of America. The Williams Committee of Inquiry into Education and Training (Williams, 1979), however, specifically recommends the use of contracting procedures between the three parts of post-secondary education which include Technical and Further Education (TAFE) Colleges, Colleges of Advanced Education and Universities. The Committee also recommends that institutions should explore ways of facilitating the mobility of students by increasing the variety of opportunities to gain credentials. Indeed, the Fourth Report on Advanced Education (Swanson, 1975), as far back as 1975, made a similar plea:

> *There should be ease of transfer of students between these three parts of the continuum. The colleges of advanced education should give credit for work successfully undertaken in Technical and Further Education (TAFE) colleges. There should also be as much ease of transfer as possible between colleges of advanced education and universities.*

The pressing need for a rationalisation of effort and for a redistribution of tasks on an Australia-wide basis between universities and colleges to achieve the optimum use of resources in particular locations presents a dilemma. If each university or college does that which is right in its own eyes with no regard for the totality of higher education provision or for national needs, there is a clear danger that anarchy and licence, under the universally respected name of academic freedom, will ensue. On the other hand, if the appraising authority becomes too *dirigiste*, too obsessed with overall planning, there is an equally clear danger that the free growth of academic institutions and the diversity in educational provision would be severely restricted. Here, as perhaps in other areas, the higher education planning authority has to adopt a Janus-like posture: its duty is two-faced - maintaining the delicate balance between uncontrolled exuberance and repressive direction.

5.1.5 Uncertainty and Confusion

There is a distressing lack of clarity about the precise functions, purposes and goals of higher education in general, as there is uncertainty and confusion about each of the main sectors of tertiary education. Perhaps the most serious aspect is the uncertainty and confusion about the respective rôles of universities and colleges of advanced education. The CAEs have not, as yet, evolved a clear and viable functional differentiation from the universities and they are still struggling for parity of esteem with the university institutions.

Professor Partridge (1972) declares that "Within the colleges and beyond, there is a considerable body of opinion that maintains there can be no intelligible distinction of character between universities and colleges". The enduring discussions of the "practical and vocational" versus the "pure and theoretical" dichotomy no longer seem relevant. The birth of the Deakin University by amalgamation of the Gordon Institute of Technology and the State College of Victoria at Geelong presents an example of the blurring distinction between the university and advanced education sectors.

Universities are no longer sure about their purpose in society and how to achieve it. Is it greater relevance or less subservience to industrial, commercial and other pressures which is required? Is the rôle of the university the extension and transmission of knowledge or to train people for the professions, or both? How should the university accommodate changes in society and the new demands made on it? What is the best form of internal government for ensuring the freedom essential to the best teaching, learning and research and what scope is there for effective and worthwhile student participation? What should be the balance between liberal and vocational education, between teaching and research, between undergraduate and post-graduate teaching?

A kindred confusion and uncertainty of purpose is evident in the CAE sector. Are colleges meant to be second class universities, or equal but different? Should they be complementary to, or competitive with, universities? Should they confine their attention to degree courses? There is no single conception prevailing throughout the whole country of what a university or a CAE is, of what are their distinctive sets of educational values.

5.1.6 The Binary System

The rapid development of science and technology and the increasing recognition and appreciation of its significance have contributed to the increasing importance of the universities and colleges as the source of the advancement and diffusion of science and technology in the national life. The emphasis on their rôle, in meeting national requirements for skilled manpower and high-level intelligence, together with a heightened social consciousness reflected in the social demand for more and better education, have been major factors underlying the development of tertiary institutions. However, it is the re-affirmation of the belief that all qualified students should have an opportunity to enter upon an appropriate form of tertiary education, that has led to the recognition and implementation of a greater diversity in educational provision and an increased diversification of tertiary institutions.

"We have a binary system of tertiary education", said Mr Fraser, the Prime Minister of Australia, "endeavouring to provide the kind of education which will meet both the inclinations of the different kinds of students seeking higher education and also the needs of employers and society at large for different kinds of qualifications. We have to face the fact that there are basic, inbuilt inequalities which have to be reckoned with and used".

The basic conception underlying this is one of justice to the individual - the claim that every person should be equipped to make as full a response as possible, commensurate with his ability, to his environment. It is an undeniable plea for justice in education: where there is capacity, there the capacity should be fostered. It underscores a significant fact of human experience: frustrations develop in proportion to expanding possibilities, since opportunity hardly ever keeps pace with possibility. The provision of educational opportunity allows the individual to maximize his own welfare and, incidentally, the national welfare as well.

The democratization of higher education - the demand by a growing number for more education, for access into the vocations and professions which offer great economic rewards, greater personal satisfaction, greater social standing, and which require more preparation, more skill, a more prolonged and complex education - makes it difficult to harness the pristine goals and aspirations of tertiary education. For, despite the pressure and clamour for higher education, few are prepared for the intellectual commitment which it demands. As the apex of our educational system, universities and CAEs must perforce be discriminatory, ensuring the maintenance of a climate conducive to scholarship and the elimination of all extraneous threats to the values which they espouse. A subtle manipulative process of redirecting the aspirations of the less able, by counselling, euphemistically-described alternative soft options and gradual disengagement becomes necessary.

Tertiary education in the past has been thought of largely as an education of an élite or specially talented segment of the community. The falsity of the position is becoming more apparent each day. The more scientific in character an industry is, the greater the ratio of highly educated people required and the higher the average standard of education among everybody engaged in the industry. As Templeman (1965) points out:

> *The more talented and productive the élite, the more it becomes necessary to have it built upon a highly educated community. If we are to educate even our gifted successfully, we must provide opportunity for as wide a segment of the community as possible to proceed to some form of higher education and at the same time raise the general level of educational achievement throughout the whole community.*

5.1.7 Our Industrialised Society

The growth and change of the industrial system is a significant force fostering and shaping the educational revolution. How much our economy depends on science, technology and lower forms of technical training is a commonplace. An important effect of industrialisation on education has been to underline the direct and close connection between education and the economy. As a consequence, vocationalism and professionalisation pervade modern education. "Industrialisation", exclaims Professor P.H. Partridge in his 1966 Buntine Oration, "has created an even more complex division of labour and more intricate pattern of specialisation, and, naturally, the educational pattern has tended to follow suit". The provision of a much wider variety of functionally diverse institutions at the tertiary level not only meets the need for highly trained manpower for specific vocations, but also permits the ultimate development of each individual's potential.

> *The pressure towards diversification, towards splitting, what we vaguely call education, into an ever-increasing number of different streams, is supported by the weight of a modern economy as well as by the more specialised and fragmented character of knowledge itself.*
>
> (Partridge, 1973)

Scientific industrialism swells the professional schools in universities and CAEs. At the same time, this diversity in tertiary education provision is able to respond more effectively to the spread of abilities, interests and motivations.

5.2 AIMS OF HIGHER EDUCATION

5.2.1 Introduction

After studying a parallel pattern of higher educational institutions in England, the Robbins Committee states that in its view there are four objectives essential for a properly balanced system of higher education: education in skills of a practical and vocational kind, suitable to play a part in the general division of labour; a kind of education that would develop in students a measure of general intellectual competence; a concern for the advancement of learning; and a responsibility for "the transmission of a common culture and common standards of citizenship" (Robbins, 1963).

Thus, assuming that we give endorsement to this statement of aims, Philp *et al.* (1964) delineate the implications for our places of higher learning:

> *We would expect to see that our places of higher learning are institutions for effective professional education, the education designed both to equip a person to earn a living, and to enable a society to achieve, in so far as production and cultivation of skilled intelligent people can do it, higher levels of economic and social development within the limits of its resources. At the same time we would expect that men and women who have had a higher education should be able to think accurately and penetratingly, to generalize pertinently and securely, and to use their intellectual skills constantly and habitually. Further, we would look to see that throughout the institutions of higher education there is a concern for scholarship and research, for the discovery and analysis*

of new facts and ideas. This is important in itself, and it gains an added importance when it illuminates the teaching in these institutions and stimulates the students to interest themselves in understanding and sharing in the processes whereby these discoveries are made. And finally, we would try to ensure that our institutions help to mediate to their students the foundations of cultural and social behaviour, and, at the same time, contribute to the development of the cultural and social life of the communities in which they are situated.

5.2.2 An Educated Person

Sureties (1974) says that one of the first requirements of the professional teacher is that "he or she should be an educated person", a requirement that applies equally to any professional person, be he a doctor, a lawyer, an engineer, or whatever. Acknowledging that the phrase is vague and difficult to define, most of the component factors not readily amenable to measurement, Sureties (1974) expands on it:

> *The words are to be taken as including some or all of the following: an appropriate intellectual standing in relation to other persons who have received tertiary education, together with the ability to communicate with them on a basis of mutual understanding and equality; a sufficient understanding of the intellectual tradition to see current problems in perspective; an adequate level of intellectual maturity expressing itself in soundness of judgement and the kind of mental balance in which commitment and rationality are not at variance; an incisiveness of thought enabling sharpness of focus and relevance in discussion; a genuine depth of understanding in at least one discipline or area of study.*

Recognising that the development of this maturity, understanding and commitment is not achievable by direct teaching, he continues to say:

> *It can be encouraged, and this is perhaps the strongest word that can or should be used. Hopefully it can be achieved by immersing the student in an atmosphere that, by way of staff example and a cordial staff-student relationship, gives clear evidence of the qualities it seeks to inculcate, notably a respect for learning, and a sense of professional responsibility.*

5.2.3 The Approaches to Higher Education

There are, generally speaking, three kinds of approach to higher education: the specialist interest in a subject, the broadly cultural and the vocational. The vocational shape to an academic purpose has always been a legitimate and worthy one, for there is a fundamental human value in setting out to acquire expertise of a high order for use in human society. "The primary function of tertiary educational institutions should be", according to Wark (1973), "to educate and inspire young men and women so that they may thereby render conspicuous service to the community and at the same time lead rewarding personal lives".

Granted that the fundamental concern of higher education is with the authentic personal transmission and reception of knowledge, whether this takes place in lecture theatre, laboratory, tutorial study or library, the aims of a university are different from those of other kinds of educational institutions, such as the CAEs, whose aims are strictly vocational. However high the intellectual demands it makes or the standards it sets, a vocationally oriented institution aims to equip students to do particular jobs, known beforehand, and the courses are designed to ensure competence. A university, however, is not regarded as an institution to turn out people equipped to step into particular jobs.

> *We are here to provide higher education so that people can choose where they can go in life to achieve their greatest happiness and achievement ... Nearly half of the university's efforts and ambitions go into research. Ultimately this is the work which produces the materials on which you are expected to work during your undergraduate courses.*

These are the sentiments of Professor David Derham, Vice-Chancellor of the University of Melbourne, addressing the 1968 intake of students.

University education has always been very much more than the mere transmission of knowledge: information alone has never been the concern of universities, but it has been the basis on which an educated understanding and a cultivated attitude could be developed. The ideal has, of course, outstripped reality, but the ideal has been there and has in itself been of importance to the activities and ethos of the universities. Although there are many who would despise the wider cultural concerns of universities in such pejorative terms as "frills" or "goodies", the ideal would still find widespread assent among academics, as a formulation of the task with which they are engaged. The aim of the university, as indeed that of the CAE, is to produce the educated man in the widest sense of the word. But, alas, in the course of time all institutions tend to suffer an attenuation of function and a dilution of values and there is a definite deflection of purpose and aspirations.

5.2.4 The Interplay of Research and Teaching

If there is an external mark of a university, it is the continued production of original work and the acquisition of knowledge.

> *In a fundamental sense universities have an unchanging duty, a duty so important that, if for any reason a university has failed to discharge it, then that institution has ceased to be a university in anything but name. This duty can be summarily defined as the task of adding to knowledge and disseminating it at the highest level as effectively as possible, freely and without external constraint.*
>
> (Philp *et al.*, 1964)

The debate about teaching and research is intense. The explosion of knowledge is so great, particularly in the sciences, that it must constitute an all-absorbing concern of some; gifted academics may be cut off altogether from teaching which is increasingly undertaken, at least as far as universities are concerned, by the more mediocre and pedestrian members of the profession. Graduate institutions flourish at the expense of undergraduate institutions. Glittering prospects of involvement which are highly rewarding in kudos and stimulus draw able university people away from the task which ought to be an immeasureably significant purpose of the university: the education in the fullest sense of the undergraduate.

The "frontier position" is not one that students can really cope with: many are not exhilarated, but simply oppressed by the thought of an expanding universe of knowledge. Yet, against this disillusionment that students feel, we must set the deep conviction of many academics that teaching and research must fertilize each other. The advancement of knowledge, they say, cannot be separated from its transmission and the most vital thing to communicate is precisely enthusiasm for the on-going process. Without research, undergraduate courses run the danger of becoming too stereotyped and unimaginative. For vitality in teaching (a necessary ingredient to sell your product) every teacher needs to be involved in some form of professional development.

The university's devotion to the aim of widening and deepening knowledge by discovery, by original, creative, critical thought is in danger of eclipse as the concept of the university's function as being principally concerned with the instruction of undergraduates becomes firmly embedded in the expectations of governments and the community.

A former Vice-Chancellor, Sir Robert Madgwick, has called attention to the view that over-emphasis on research has resulted in an under-estimation of the university's teaching function. He says:

> *I think the Australian universities must get back to the old idea that teaching is the vital function. Far too much emphasis in my view is placed upon research. Appointment to a university staff, promotion and advancement, are all determined to far too great an extent by the research record of the individual, and too little emphasis is placed on character, personality and the capacity for inspired teaching. My own attitude is that members of university staffs are appointed to teach. They are expected to research.*

Though a disproportionate immersion in research filches time from preparation that is critical for effective teaching, teaching and research should not be seen as unrelated functions, but as activities that may positively enrich each other. The positive effect that research can have on teaching has been vividly expressed by A.J. Scott - "He who learns from one occupied in learning, drinks of a running stream. He who learns from one who has learned all he is to teach, drinks the green mantle of the stagnant pool". In a situation where the rewards and prestige go for published work, it is easier to disregard the wider implications and obligations of teaching for which, no matter how well it is done, there is little recognition and small extrinsic reward: a heightened dedication to teaching may seriously jeopardise research and associated career prospects. Not surprisingly, as Mackenzie *et al.* (1972) would have us observe, academics speak positively of research *opportunities* and negatively of teaching *loads.*

The positive benefits that accrue from research are indicated by Philp *et al.* (1964):

> *The basic and enduring motivation for research, apart from prestige and reputation or of more tangible gain in the shape of an academic promotion and possible but rare and usually slight monetary reward, is the desire to look more widely and deeply into the subject around the study of which their whole life turns. As each member of the university staff pursues his interest, each subject becomes a moving frontier of facts and ideas which in due course are fed into schools, literature and professional practice and so into the activities and consciousness of the community. This process may not contribute to the community's material wealth, but it expands the community's horizons in a variety of directions, adds to its sophistication, deepening and sharpening its understanding of many of the possibilities and facets of its life.*

In their view, while the subordination of research would not be disastrous, "it would in time produce a pedestrianness, a general blunting of sensibilities, and a community which lumbered forward but did not reach out".

5.2.5 Research and Consultancy in CAEs

The increased community demand both for trained research personnel in many fields and for research-based solutions to immediate problems in industry, engineering, agriculture and in the economic and social sphere, as well as the awareness of the benefits that flow from research - these underscore the importance and legitimacy of the CAEs' participation in *applied* research, that is, research determined by external needs. Theoretical, basic, fundamental research is the legitimate province of universities, whose *raison d'être* is to further the ends of study and investigation for their own sakes and not as a means to anything else; however, short-term research related to the needs of the community is a valid activity for the CAEs.

Thus research is no longer confined to universities, but is being increasingly pursued in CAEs where teaching is still pre-eminently important. The balance of opinion and present trends endorse the view that higher education must be kept in close contact with the advancement of learning. The realization is dawning on the CAEs that, if professional work is to be kept alive and to be moved forward, diligent efforts should be made to interest adequate numbers of students of high quality in research and to ensure that they and the staff are induced to look more rigorously and penetratingly at the problems the community at large experiences.

Research and consultancy are assuming increasing significance in the life and work of the DOCIT Colleges. Research and other forms of professional development are seen as essential accompaniments of effective teaching, not least because they ensure that courses are kept abreast with recent advances in the respective disciplines and because staff are provided with opportunities for professional stimulation. However, consultancy is seen as the particular province of the DOCIT Colleges, permitting staff, as the *DOCIT: 1977 Annual Report* maintains, "to disseminate research developments in an appropriate form to the community and at the same time become aware of the developing needs and problems of the factory and the market place".

Research and consultancy in CAEs is financed and facilitated through such sources as research grants, private contracts with staff through agencies set up by the Colleges, or funds provided by the institution under staff development.

Five of the eight DOCIT Colleges have now established investigation and consultancy agencies, namely Technisearch (Royal Melbourne Institute of Technology), Techsearch (South Australian Institute of Technology), WAIT-AID (Western Australian Institute of Technology), Swinburne Applied Research and Development Division (Swinburne College of Technology) and Insearch (The New South Wales Institute of Technology). Their purpose is to organise the use of knowledge, technical expertise and equipment for use by organisations in the community. They seek to provide services on a project basis and in a client relationship to government departments and instrumentalities, to primary, secondary and tertiary industries, to business, educational, professional and other community organisations.

As well, Dr B.S. Thornton, Head, School of Mathematical Sciences at the New South Wales Institute of Technology, set up a non-profit, voluntary foundation in 1975. Named the Foundation for Australian Resources (FAR), it was conceived "to foster the evaluation of Australia's resources against the needs of our time". The Foundation undertakes "defined objective" research in selected resource projects that may involve natural, physical and human resources. Among the projects undertaken and completed may be mentioned:

(i) The application of a new geophysical method for the evaluation of the quality of water resources, in coordination with the Irrigation and Water Supply Commission of Queensland.

(ii) The evaluation of possible alternative energy policies, including the rôle of solar energy in Australia - published as *Solar Australia* in 1977;

(iii) The development of a new design for a low cost solar absorber. A post-graduate scholarship was awarded to research this work in 1975, and led to the award of a Master's degree at the New South Wales Institute of Technology.

(iv) The publication of *Computers in Australia: Usage and Effects*, the development and extension of the 1975 FAR report on the computer population in Australia; and

(v) The study of factors impairing the optimum efficiency and economics of St Vincent's Hospital's Radiology Department. A publication has flowed from this study and it indicates that the methods developed and applied have a more general validity to other service departments featuring a network of queues and different kinds of service channels.

The initiatives taken by Dr Thornton over the past years have resulted in a Foundation membership which embraces distinguished citizens and leaders of industry and science from throughout Australia. Finance and facilities have been pooled and used to encourage resource research and the teaching of related mathematical and computing methods at the Institute and elsewhere.

The establishment of investigation and consultancy agencies and of the Foundation is a good example of an initiative appropriate to the CAEs, which are urged to undertake research with an applied bias.

5.2.6 Education for Change

We live in a world in which the frontiers of knowledge are expanding continuously. The sum total of man's knowledge, it has been said, has doubled in the past ten years. Scientific and technological education, investment, publications, the number of men trained, percentage of gross national product committed to research and development - these critical indicators are doubling every seven to ten years.

> *No previous period in history offers any parallel to the current exponential growth in the rate, multiplication, and effect of scientific and technological advance.*
>
> (Former Prime Minister of Australia, Mr Gough Whitlam)

This astonishing proliferation of knowledge is producing newer, more specialised disciplines and the seamless garment of learning has been rent and tattered to an extent where it is beyond mending. These specialisations give rise to an increasing measure of differentiation within departments and faculties as well as within the courses offered for study.

The ever-increasing weight of what we must transmit raises problems of curriculum design, such as, what is to be taught, and what is to be left out? Are courses to be extended in scope and in length as the subject matter increases, or is our technical tertiary education to concentrate more and more on fundamentals and postpone specialised studies for treatment either by in-service training in industry, or by post-graduate courses? How are we to provide the breadth and depth of education? A selection of topics must necessarily be made: there is no practical alternative.

The accelerating rate of scientific and technological advance is producing marked changes in society. Indeed, to say that we live in a time of rapid change has become one of the most pervasive of clichés. The effective life of acquired technical skills, expert knowledge, and job know-how has become halved and quartered by change and innovation.

This speed of modern progress has raised new problems in professional education. It is difficult for people to pick up new concepts, learn new techniques. How do we equip our students to meet the challenge of change? How do we induce a flexibility of mind that will enable our students to adjust continuously to their changing environment?

Professional education is not to be considered as taking place once and for all in the pre-service years, but rather

> *as that part of life-long education that establishes a platform of fundamental knowledge and skills upon which later learning can build, and that begins to develop those qualities of mind - mental discipline, flexibility, discrimination, critical ability, objectivity - that distinguishes a trained from an untrained intellect.*

(Law, 1969)

Professional growth demands continuing education throughout the professional life and therefore pre-service and in-service education should be seen as complementary phases of the whole process of professional education. Dr I.W. Wark, giving the Sir John Morris Memorial Lecture, said:

> *The graduate, whatever his subject, will not find life unchanged in twenty or forty years' time, and only the person of flexible outlook will thrive. Education must come to be regarded as a continuous process requiring effort and self-discipline. His formal qualification is but a card of entry. His further advancement will depend upon self-education, self-discipline and hard work.*

Sir Eric Ashby has spoken of a degree or like qualification as analogous to a passport: good for a limited time and then expiring, and in need of renewal. The difference is that the passport may be renewed automatically; the certificate or degree only on a showing of present fitness in the state of current knowledge.

Since no pre-service course at a university or a CAE can produce the finished practitioner, an organisational principle that will receive more emphasis and general acceptance is to make provision for students to leave and return to courses: professional people will need repeated doses of retraining to keep abreast with developments in their specialised callings. Systematic attendance at post-graduate refresher courses becomes mandatory; going overseas on study leave and inviting distinguished men and women to teach for a period in graduate schools, apart from private reading, are other approaches to the problem of keeping pace with change and the explosion of knowledge.

The responsibility for the provision of continuing education in engineering at the professional level is generally shouldered by the universities, the CAEs and the professional bodies themselves. The need to strengthen the ties between the educational institutions and industry is clear if we are going to devise a pattern of education to meet the challenge of change. The universities, being inward-looking, pose a particular problem in this regard, and it is here that the colleges of advanced education can play a prominent rôle, largely because of the vocational emphasis in their courses and because not a few of them were formerly technical colleges with close industrial links.

McLaren (1974), advocating specialisation as the only effective riposte to the problems arising from knowledge explosion, says forcefully:

> *A smattering of study over many fields is more likely to lead to the quizmaster's trivialization of learning, or to the confirmation of ignorance through the accumulation of facts, than to develop any real understanding. On the other hand, immersion in a single field of study can lead to an awareness both of the limitations of knowledge and the fundamental unity of all understanding. On this basis he can engage in that co-operation with people from other fields of learning which is the only possible answer to the problems of the knowledge explosion.*

Study in depth is essential for the achievement of mastery.

5.2.7 A Modern Dilemma

Universities and Colleges of Advanced Education have a large, albeit unequal, share not merely in producing new knowledge, but above all in training those who can be both the producers of new knowledge and the agents by which it is put to practical use.

The magnitude of our present dilemma can be appreciated when we consider that, though power came to an engineer, say, by virtue of the kind of knowledge and background required to manage society in its infinitely more technically developed form, this does not mean that he is thereby automatically fitted to undertake the full responsibilities of those who will have not merely to say how to do things but will have to make the infinitely harder decision as to what ought to be done and what is worth doing. Mr Gough Whitlam, former Prime Minister of Australia, says with characteristic eloquence:

> *With this growth in knowledge, there is a matching growth in bafflement and frustration. The problems faced not just by us, but by humanity, are of a new order and scale: growing urbanization, mounting population pressures, rapidly diminishing resources, widespread hunger and pollution, a rampant technology heedless of our natural environment and delicate ecological balance, the vast destructive potential of our modern armaments, the challenge to human values and human freedom by a growing multi-national industrial technocracy. Against these threats to civilization, the universities will be man's chief ally in the struggle to preserve freedom and our species from destruction.*

"The real menace of our present situation", writes Geoffrey Templeman (1965), when considering the responsibilities of institutions of higher education,

> *is that the graduate adequately trained in his specialty is likely to find himself forced by the exigencies of our new social and economic circumstances to a dominant situation in society where he will also be required to act as a leader of opinion. The likelihood is that his academic training and his technical expertise will by themselves be of little use to him in facing this new range of problems. He will then rightly look back upon his university training and reflect, when it is too late, that while it gave him much in the way of professional preparation, it almost entirely neglected to equip him in a more general way; in fact, to educate him in the wider sense of the word.*

All this is simply a plea for an enlargement of our existing notion of the duties of universities and colleges, an enlargement which is difficult to make, but one which is inescapably necessary. Universities and colleges must feel themselves to have larger obligations to the community at large. They must somehow contrive things so that they give the appropriate specialist training which must be provided, but to that they have also to add some measure of what, for want of a better word, must be called education. "I would like to see", exclaimed Portus (1939), " a composite course which would be compulsory on every student before graduation in any faculty ... the aim would be to lead these young doctors and teachers and engineers and lawyers and scientists to an intelligent interest and understanding of the world they are going out to serve". Thus, universities and CAEs have a particular obligation to educate *through*, as well as *in*, engineering.

CHAPTER VI

A PHILOSOPHY OF MATHEMATICAL EDUCATION OF ENGINEERS

Summary

The basic philosophy underlying the mathematical education of engineers is formulated. The implications of the rapid changes in technology and of the growth and diversification of knowledge for effective teaching and learning are observed.

6.1 INTRODUCTION

There are many reasons why so much attention is concentrated on mathematics in engineering education. Mathematics clearly provides the best training in rational thinking; it is one of the principal tools that engineers use for the derivation of quantitative information about natural systems; it is really basic to methods used in the analysis of natural phenomena; it is important in enabling the engineer to generalise from his experience; and, one of the most important considerations of all, it is a training for adaptation to the future. The most valuable effect of studying mathematics is seen to be the inculation of a way of thinking, and a power of analysis which seldom ensues from other forms of training.

6.2 THE CHALLENGE OF CHANGE AND KNOWLEDGE EXPLOSION

The salient feature of the world of today and tomorrow, one which is perceptively critical of our education system, is change. Though change itself is not a new phenomenon, the rate of change is. Therefore, any consideration or assessment of the professional education of engineers must squarely face the fact that knowledge explosion and the rapid changes in technology must ultimately imply changes in what we teach and how we teach. The question is what are the educational needs of a world characterised by rapid technological change? What are the implications of knowledge explosion and change for educational practice?

Already, the framers of a curriculum in any subject are seriously embarrassed by the knowledge content growth in that subject. They are prone to overload, to add and add to the burden of knowledge and techniques required so that it becomes almost intolerable for the teacher and the taught alike to complete the course. Yet it is obvious that a curriculum must be devised.

Granted that content or subject matter is certainly important to educational processes, how can we teach the undergraduates all of this material and also anticipate what they will need to know in relation to changes in knowledge?

6.3 EDUCATIONAL IMPLICATIONS

6.3.1 Broad Principles

Quite clearly, in relation to the curriculum, we must develop in the students techniques and principles which will help them to learn how to learn and which will motivate them to want to go on learning as it is obvious that this will be a life-long necessity. There will have to be a great deal of pruning of the knowledge that we require of students, particularly knowledge of facts. In its place we will need to synthesise knowledge into broad ideas and general principles and the student led to a fundamental creative understanding of these. As Jenkins (1969) expressed it:

> *Education is at present devised with an emphasis on courses and content and the acquisition of regurgitative knowledge. We must replace facts to be examined with the development of an attitude of inquiry and methodologies for using that inquiry effectively.*

The different branches of engineering - aeronautical, chemical, civil, control, communications, electrical, electronic, highway, mechanical, mining, municipal, nuclear, production and structural - prompt the question how a single profession may claim to have so wide a range of application. And the plain answer is to be found, paradoxically, in the unity in its diversity. Let Professor Morton (1976), Professor of Mechanical Engineering at the University of Manchester, enlarge on it:

> *The answer is that, despite their outward variety, they are all subject to the same natural laws governing stress, motion, vibration, temperature and other fundamental factors, and thus the same types of calculation are applicable to all. The laws of vibration, for example, apply equally to a bridge, an internal combustion engine or a tuned radio circuit. Similarly the laws governing the force exerted by a fluid against a solid body are applicable to such widely differing cases as the force of waves on an underwater structure, the force of steam on the blades of a turbine or the force of wind on overhead cables and pylons. Thus the scientific bases of all branches of engineering have much in common; outward forms and details differ but the underlying principles are universal.*

The unification of engineering theory by the generalised treatment of principles applicable to several branches and the rationalisation of course material so that topics are not covered more than once in separate subjects would ensure that time is used more effectively in modern courses.

6.3.2 A Sound Foundation

Unquestionably, there is an increasing need for people with cognitive rather than manual skills, particularly in the scientific and technological fields. This means that more mathematics and more science will have to be taught to provide a sound basis on which to build tertiary courses in engineering and allied sciences. The emphasis must shift to understanding and away from rote learning in these foundational disciplines.

The rapidity of technological change likely to be experienced in the future has strengthened the case for giving more time to basic disciplines, such as mathematics, and less to specialised vocational skills, as a foundation for professional education.

Unless existing syllabuses can be sufficiently pruned of all "dead-wood" and/or teaching effectiveness is enhanced dramatically, a natural response to the ever-increasing heritage of knowledge to be passed on is the lengthening of many professional courses. The alternative, a greater degree of specialisation, has the drawback of not providing as good a foundation for future retraining as the more generalised courses.

6.3.3 Synthesis

Educational problems posed by the increase in the total sum of knowledge are compounded by the accelerating speed with which fresh discoveries are found an application in engineering practice or in some other field. This increase in the rate of application paralleling the increase in quantity has implications for education. As Hughes (1969) points out:

> *The major emphasis in educational objectives has been on the capacity to retain or reproduce particular facts and skills. Any analysis of tertiary examinations will demonstrate this. The flood of information makes obvious the need for higher level objectives. Information may be stored in a variety of ways - written, mechanical, electronic: it is the system of organisation of storage which is vital for practical use and this is essentially a product of the human mind. Thus, separate and uninterpreted facts become merely a step to forming concepts, and concepts to forming theories. It is the process of linking, of synthesis, of theorising if you wish, which assume special importance. Paradoxically, therefore, while facts are permanent and theories temporary, our emphasis in objectives must extend from facts to theories, if they are to have long-term value. It is the ability of man to give meaning to facts through theories which leads to the advance of knowledge and which must receive due emphasis in our educational practices.*

6.3.4 Motivation

Implicit in the consideration of the educational challenge of change and knowledge explosion is the question of motivation. The increasing education called for by technological change means that many more people will be involved in formal learning for a longer period than in the past. This increase will include many adolescents who have little or no innate love of intellectual study for its own sake. How is their interest to be maintained at a high enough level for a long enough period? For motivational reasons, learning must be given meaning by linking it to reality. The extent to which what is spoken, read, practised, demonstrated and discussed represents the best of current knowledge and understanding in a field of inquiry and can be shown to have a relevance to a future occupation and personal identity will determine the degree of our success in inducing a favourable attitude to the learning process.

The problem of motivation is much deeper and wider and implies not only motivation during formal studies but the motivation to remain willing to learn during the whole of one's professional life. The solution rests in the method of presentation, in relating each facet of the subject to a reference point in reality, some vocational or professional application that will command the interest of the individual.

6.3.5 Flexibility and Adaptability

A consequence, of no mean significance, of the accelerating technological change is the distinct probability that most people will have to face several occupational changes over their working lives. This means that virtually all professional people will be in the position of having continually to update their knowledge: professional education cannot be considered as terminal. Weber (1963) puts it very vividly:

> *... man must go through life with his educational umbilical cord uncut.*

The speed of technological change underlines the need to develop flexibility and adaptability. CAE programmes should not be regarded as training schemes to produce a neatly embossed product to fit perfectly into a predetermined slot. Nor should mathematical education be tailored to meet the immediate needs of a particular profession. As the store of knowledge expands and as technology alters, most narrow specific trainings will become dated. Therefore the greatest need is for people with flexible minds and the need to avoid obsolescence by teaching techniques of problem-solving, rather than ready-made solutions to current problems, and emphasising broad principles. The pressures exerted by a changing technology and the increase in the sum of knowledge on institutions of higher education call for a shift of emphasis in both what is taught and the way it is taught. An education that stresses knowledge rather than techniques, understanding rather than memory and that strives to provide sufficiently broad foundations would ensure that graduates begin their professional careers with minds which are not only agile, flexible and adaptable, but also alert and hospitable to new ideas and new mathematical methods. Education for vocational adaptability must provide a basis upon which skills can be developed.

> *This trend is already apparent; for example, statistical thermodynamics has replaced heat engines in many engineering courses. Even when education is for specific vocations, there are obvious advantages in training for occupational clusters than a single occupation.*
>
> (Dufty, 1969)

6.4 PROFESSIONAL EDUCATION

The two characteristics of most branches and phases of engineering are heavy capital outlay and large-scale production, which inevitably mean the existence of large and complex organisations over which engineers exercise direction and control. How may their professional education prepare them for such responsibility? The intricate authority relations in complex organisations highlight, more than ever, human relations which often require in the formulation of decisions a capacity for balanced, mature judgement from engineers. How may their professional education equip them for such maturity?

A knowledge of such subjects as psychology and social studies becomes vocationally essential for all supervisors, technologists, engineers and scientists as well as managers. Also, such mathematical courses as statistics, operations research and linear programming become appropriate offerings for engineers.

However, it has to be conceded that no matter how detailed and comprehensive the engineer's learning has been, his capacity to make mature, wise, managerial decisions depends directly, as Aston (1968) indicates,

> *on his ability to transfer to other problems the knowledge and skills and habits of thought which he has derived from his study and practice in the course subjects of mathematics, science and design.*

Though colleges of advanced education have a clear commitment to provide courses that have a specific vocational orientation, they should, in the light of these considerations, move from their severely narrow and practical curricula towards broader educational attitudes, which set the parameters for a climate in which professional education, as distinct from technical training, may flourish.

Wickenden (1949) draws the distinction between professional education and technical training:

> *Mere technical training, at any level, is vocational rather than professional in nature. The difference between technical training and professional education is no simple matter of length ... nor is it a mere matter of intellectual difficulty. It is rather a matter of spirit and scope. More specifically, it can be described as an overplus beyond the knowledge, however intricate, a man needs to master his daily tasks.*
>
> *... for an engineer, technical training aims at skills in applying mathematics and the physical sciences to concrete problems of design, construction and operation, but professional education looks beyond to philosophic insights into the relations of mathematics and science as modes of individual human experience and to competent understanding of the social and economic forces set in motion by technological achievement ... no professional body can be strong and effective unless it contains a substantial nucleus whose intellectual attainments far exceed in depth and breadth the technical demands of its practice.*

The growing realisation that educationists would need to concentrate increasingly on teaching students to think from a broad platform of fundamentals and that industry needs people whose studies include engineering and social sciences led Dr. Phillip Law (1970), the then Vice-President of the Victoria Institute of Colleges, to assert a decade ago:

> *I see a tendency, therefore, over the next ten years or so, for vocational courses in universities and colleges to converge.*

Like the universities, the CAEs can, and should, represent through their life and work a commitment to values and standards in scholarship and teaching that have deeper roots and longer time spans than any ephemeral technical training signifies.

CHAPTER VII

AIMS AND OBJECTIVES IN THE TEACHING OF ENGINEERING MATHEMATICS

Summary

The aims and objectives in teaching mathematics courses to engineers are clarified: both the implicit and explicit goals of engineering education are called into account. The need for relevance to their current studies and future careers, for a balanced infusion of rigour, for teaching general principles at the expense of excessive details, for a consistent emphasis on mathematical modelling, and for the development of mathematical adequacy are among the aims and objectives mentioned.

As an instrument of national purpose, engineering education should provide that formal basis for preparing the nation's engineering manpower with the capability, knowledge, understanding and insight to fulfil the technological needs of society on a timely and effective basis. Such preparation should include the foundation for breadth of vision, leadership, and statesmanship with appropriate disciplinary balance and flexibility to meet new tasks and new opportunities.

(*Assessment of the Goals of Engineering Education in the United States*: Report of the Panel on Engineering Education, 1966)

7.1 INTRODUCTION

Many problems confront engineering educators, not least the growing difficulty of arriving at a consensus on what is the rôle of the engineer. Complex at any time, questions bearing on the function of the engineer become even more complicated in our times as a result of three realities.

The first of these is that engineering is becoming an increasingly specialised profession. It comprises a complex of branches, each with its own specialisms and phases of practice. The most dramatic advances which have recently been made have been in engineering: we refer here to more than just sputniks and the space age - profound engineering achievements to be sure, but far from being the only ones. The "spin-off" of research from the space programme has produced great advances in telecommunications, electronic circuitry and computer control.

The mechanism of coal cutting and loading, the desalination of sea-water, the development of under-water technology and off-shore drilling, the development of mechanisation and automation and computerized automation - these are all, as Lord Hinton (1970) of Bankside reminds us, engineering achievements which are no less spectacular and which point to the growing complexity of engineering. Furthermore, an unquestionable evidence that the theory of engineering has become more complex is provided by the multiplication of the branches of engineering, with separate departments of aeronautical engineering, electronic engineering, nuclear engineering, control engineering and environmental engineering.

Not unrelated to the first, the second feature of the present-day reality is the unprecedented growth and diversification of knowledge, particularly in the fundamental sciences on which engineering is based.

An acute awareness that engineers have broader responsibilities in professional practice is the third reality. More and more, engineers have an obligation and a responsibility to be informed and intelligent about an ever widening range of matters of public policy which impinge upon their day-to-day professional activities.

Any attempt to formulate the aims and objectives of the mathematical education of engineers must squarely face these realities. How does their mathematical education fit into their overall education and thus help to prepare them for the stern demands of their specialised calling? How does their education give them a sense of adequacy in a period when the explosive growth of knowledge continues unabated? How does their education inculcate in them a sense of personal and professional autonomy and the attendant responsibilities for decision making, flexibility of mind, ability to identify and solve problems and ability to communicate? Questions such as these are germane to any determination of the aims and objectives of mathematics courses for entrants to the engineering profession.

As a prelude, then, to the formulation of the aims and objectives of mathematics programmes for engineering undergraduates. a compelling responsibility of educators is to consider, in the light of the stated realities, what knowledge, skills and attitudes might be expected of engineers as professional and educated persons. The problem is not only to give a student adequate technical competence, but also to establish habits of thought that will underpin his professional and personal development.

Happily, on the general pattern of mathematical provision for students in engineering schools, the degree of agreement is impressive. The aim of pre-service professional education is to ensure that a fledgling engineer is grounded in the basic principles of his specialty, capable of updating his knowledge and skills by experience and further education and is generally educated for "the common responsibilities of life which are today the inescapable lot of those to whom society has given the privilege of a tutored mind". More than this is considered to be outside the ambit of undergraduate education.

However, to say that we have to educate students in a general sense and train them in the specialist skills which a modern engineer requires and nothing else is to speak at a level of abstraction which has no meaningful sense and leaves the educator adrift without guidance or direction.

Accordingly, we focus attention on a number of matters that seem to us critical in the mathematical aspects of the preparation of engineers.

7.2 GOOD MATHEMATICAL FOUNDATION

> *Any professional education, because of its very nature, must obviously inculcate the corpus of knowledge, the complement of skills, and the traits of personality and character which constitute the distinctive features of a particular craft. It is these characteristics which give the profession its cohesiveness and identity.*
>
> (McGrath,1960)

The application of scientific and mathematical principles for practical ends - and this is essentially what engineering is - is developing at such a rate and in such diverse fields that nothing but a broadly conceived, and yet thorough, study of mathematical sciences would meet the need of the neophyte.

To attempt to include detailed knowledge of techniques in a degree course, apposite to a particular branch of engineering, is an exercise in futility, as there are many specialisms. Nor is it practical to endeavour to provide a comprehensive coverage of all disciplines. And plainly, the more specific and the narrower the education that an engineer has received, the less adaptable will he be to evolving techniques and widening professional responsibilities.

Thus, engineering educators are showing a growing appreciation of the rising importance of a prospective engineer being sufficiently steeped in such fundamental sciences as physics and mathematics on which the profession rests. A proper knowledge and understanding of these foundational disciplines is an essential preparation for the varied patterns of professional exigencies and the infinitely varying problems of day-to-day practice. "Unless the [development] engineer of to-day is given sufficient insight into mathematics", asserts McTernan (1964),"he will find difficulty in educating himself in the disciplines of tomorrow".

Taylor (1959), in a lecture to the Mathematical Association, urges that engineers should be given a sound mathematical foundation. He says:

> *With the pace at which engineering is developing, an engineer has much need of continuing to learn mathematics after ending his formal training, which he will have to do by studying mathematical text-books. In his undergraduate days he should be taught how to do this. It is suggested therefore that it is more, rather than less, necessary that the engineering student should be told something of mathematics as a language and of its philosophy.*

7.3 THE DEVELOPMENT OF MATHEMATICAL COMPETENCE

The development of mathematical competence must not be viewed primarily as the feeding of information and knowledge, an important function to be sure. The mere possession of knowledge, however detailed, in no way guarantees the ability to utilise and act upon it effectively, nor the incisive and imaginative use of the mind. Cardinal Newman (1955) states:

> *Knowledge then is the indispensable condition of expansion of mind and the instrument of attaining to it; this cannot be denied, it is ever to be insisted on: I begin with it as a first principle; however, the very truth of it carries men too far ... The end of a Liberal Education is not mere knowledge.*

The range of mathematical techniques actually utilised appears to be limited chiefly by the dearth of persons with adequate mathematical competence to assist in the formulation and analysis of the problems at hand.

Therefore, a major responsibility of engineering educators should be, in the words of McGrath (1960),

> *to cultivate those skills and habits of reasoning which constitute intellectual competence, the capacity to think logically and clearly, the ability to organise one's thoughts on the varied subjects with which the citizen today must unavoidably concern himself. In a sentence, these faculties might collectively be described as the capacity to order and interpret a complex set of circumstances in the physical, social, or artistic world, and to bring one's full intellectual resources skilfully to bear on the solution of a problem.*

As far as engineers are concerned a suitable level of mathematical competence includes not only the ability to use basic mathematical techniques, to understand the nature and limitations of these techniques, but also the ability to know when a particular mathematical technique is not apposite to a given situation.

Sufficient stress should be placed on problem-solving and giving the undergraduate a feeling of confidence when faced with an unfamiliar situation. The student should receive such training that will enable him to weigh the pros and cons of various lines of approach to a particular problem and then to decide on the best course for its resolution. Further, the qualified engineer should not only be able to follow a mathematical argument, but also generate one.

There are many drawbacks of courses which are deliberately tilted toward utilitarian ends, not least that such courses may give the impression of reducing mathematics to a series of devices for solving problems and of placing a disproportionate emphasis on current problems at the cost of preparation for the unknown problems of the unborn tomorrow.

The insights of mathematics are seen to be invaluable to a growing proportion of an engineer's experience. Important as knowledge and techniques are, our foremost objective should be to give students an understanding of principle over a wide field of mathematics, an appreciation of what mathematics is and does, so that it comes to be seen as part of the necessary equipment of a trained mind.

7.4 MATHEMATICAL MODELLING

The idea of a mathematical model is absolutely fundamental at any level for engineers and non-mathematicians who wish to use mathematics. H. Graham Flegg (1974) says that

> *The philosophy and the practicalities of mathematical modelling should be an integral part of the education of scientists and technologists.*

Professor Bajpai urges that students should appreciate the concept of a mathematical model and the methods of obtaining solutions to such a model. Scott(1972) calls attention to the "ability to formulate a physical problem in mathematical terms and to interpret a solution".

The majority of engineers experience their greatest difficulty in the formulation of the mathematical model, while others consider solution and interpretation the most difficult stages. This is because undergraduates are accustomed to dealing with completely formulated problems which possess neat solutions that are all too uncommon in reality.

This comparative neglect of a duty - to teach students how to formulate, solve and interpret - leaves engineers seriously disadvantaged. Our present courses are mainly preoccupied with the consideration of unreal problems that possess elegant, exact solutions.

> *Applied Mathematics has in most cases lost contact with engineering and become either modern theoretical physics or a formalised treatment of the classical theories of particle and rigid body mechanics, electromagnetic theory, fluid mechanics and occasionally, elasticity,... The emphasis tends to be on the exact solutions of rather unreal problems rather than on the approximate solutions of real ones; both are important, but the balance is at present heavily weighted on the side of production of elegant solutions.*
>
> (Spenser, 1967)

Mathematical modelling should form the core of any curriculum for the teaching of applied mathematics; indeed, only in this way can one assign meaning and purpose in dealing with point masses and weightless strings.

7.5 EMPHASIS ON BROAD PRINCIPLES

A notable trend in engineering education is the decreasing emphasis on techniques and a turn toward basic and general principles.

The time lag between the discovery of new items in science and mathematics and their applications to engineering systems has been dramatically reduced. The engineer of tomorrow, therefore, must be equipped to effect this rapid transition. The rapidity of technological change, the shrinking interval of time between discovery and application, together with the ever-increasing fund of knowledge results in an unquestioned need to reorganise training methods.

> *The growing demands upon the profession of engineering to assume leadership in the constructive integration of technological change and human fulfilment in a time of accelerating change requires a total reassessment of engineering education.*
>
> (*Assessment of the Goals of Engineering Education in the United States:* Report of the Panel on Engineering Education, 1966)

This includes the pre-eminent necessity to expose the student to a series of fundamental principles as opposed to details of devices. The life of a device before obsolescence varies greatly depending upon the individual case. However, it has been estimated that an average figure of somewhat less than ten years may safely be applied - certainly a short time compared with the useful professional life of the engineer. A detailed knowledge of devices, therefore, is not an education of any great permanency. A knowledge of fundamentals - of basic, broad principles - is, on the other hand, more enduring. The student who grasps these can project his knowledge to the next device; he is prepared to cope with, adapt to, and understand change, as well as to deal with circumstances for which there is no direct precedent.

The basic need is for engineers who are sure-footed in mathematics, whose mathematical education rests on a sufficiently broad foundation to allow the easy assimilation of new techniques and who possess a sufficient armoury of techniques for their initial needs.

There seems one particularly important way in which mathematicians could help engineers. This is by adding generality, by developing methods applicable to other engineering areas or by carrying over a technique developed in one area into another; in view of the frequency with which the same equations appear in different areas of engineering, this is not unimportant. Bajpai *et al.* (1976) mention that the vibration of a two-story frame, the rotational motion of a set of discs mounted on a central shaft and an electrical circuit - all these different physical problems lead to the same mathematical model, governed by the eigenvalue equation,

$$A\underset{\sim}{x} = \lambda \underset{\sim}{x}$$

demonstrating the power of mathematics to provide generality.

The process of technological advance has created a desperate need for a more general and less specialised education for engineers. A mathematics educator can stress synthesis by showing how a particular principle or analytical theory has a relevance to more than just one aspect of engineering practice.

Many useful analogies exist between different fields. Lloyd (1968) points out that by showing

> *how the laws that govern oscillations are essentially the same whether they be applied to variations of current in an electrical circuit, vibrations in a machine or in a structure, or even to pulsating flows of fluids; how the derived theory of control systems applies equally well in electrical, mechanical or chemical systems; and how the study of fields finds applications in electrostatics, electromagnetism, stress and strain in materials, and in flow patterns in liquids and gases,*

a particular problem of teacher and taught in a climate of increasing knowledge and technological change is in part resolved.

It is desirable to give engineering students opportunities to see common mathematical principles in a variety of situations. Such an approach enables students to integrate knowledge from various sources and helps them to an appreciation of the salient features of a course. As well, students are able to appreciate what Professor Sir James Lighthill (1979) describes as "the flexibility of applied mathematics and its methods; flexibility in the sense that the same method can be applied in many different areas of engineering".

The concentrated demands of time, too, impose a need to develop and emphasise broad principles and generic theories if adequate coverage of material is to be achieved. This facilitates an unhurried absorption of knowledge, reflective analysis of what has to be learnt, and an orderly expansion of mind.

7.6 THE PLACE OF RIGOUR

Many mathematicians, and more especially pure mathematicians, deeply deplore the absence of rigour in the engineer's approach to mathematics. It is not unimportant to appreciate that this springs not from any slipshod, careless way of thinking but from a fundamental difference of approach to the subject.

Scott *et al*. (1966) draw the distinction between a pure mathematician and an engineer:

> *Modern mathematics is concerned with logical deductions from a set of basic axioms which need have no relation to any physical world and in which internal validity is paramount, and there is no question of their being right or wrong, although they may be inconsistent or redundant. At no stage has the pure mathematician any touchstone outside his own deductive processes whereby he can confirm the validity of his deductions, so it is essential that he*

should proceed with the utmost rigour at every step. On the other hand, the engineer starts from a set of axioms which are mathematical abstractions from the behaviour of the physical world with which he is familiar. He then proceeds to test the correctness of his axioms by comparing deductions based on these axioms with the physical world which he can examine. In so doing he tests also his own deductive processes and establishes confidence in his methods without the necessity for the support of rigour upon which the mathematician must rely. Admittedly, there are situations where the engineer may not be able to test the results of his prediction or where he may find they do not agree with the physical world. He may then wish to call in the services of a mathematician to verify his reasoning, though no amount of rigour will avail him if his initial formulation is false.

The future engineer needs to be exposed to all "good" courses in mathematics. That does not mean epsilon and delta courses necessarily. Rather, as Zadeh (1966) indicates,

> *It means courses in which he sees the way in which mathematical ideas evolve, in which techniques come out of that evolution and in which one extends both the range of validity and the scope of applicability of such techniques. He needs instruction which emphasizes heuristic arguments that lead him to anticipate procedures just as much as he needs instruction emphasizing the arguments that lead him to conclude that the procedures are rigorously correct.*

To the question, then, what is the place of rigour in the mathematical training of engineers, the answer is that, as the Committee on the Undergraduate Program in Mathematics (CUPM, 1967) puts it,

> *mathematical rigour - by which we mean an attempt to prove essentially everything that is used - is not the way of life of the physicist and the engineer. On the other hand, mathematical sophistication - which means to us careful and clear mathematical statements,*

a clear knowledge of the restrictions on the mathematical processes used, an ability to follow a given mathematical argument critically, and generally speaking, a broad appreciation of mathematical techniques used in modelling - "is desired by, and desirable for", all students of engineering sciences.

The worst abuses of mathematical methods occur when engineers try to solve problems by substituting values into mathematical formulae which they do not understand. McTernan (1964) asserts that any engineer who uses techniques he does not fully understand lays himself open to two dangers:

(1) *He may use the technique in circumstances which render it invalid.*

(2) He may use the wrong technique anyway.

Any mathematics lecturer can cite occasions when students persist in trying to sum a divergent series or to invert a singular matrix.

Hence it should be the foremost objective of the mathematics educator to present the substance of the mathematical methods together with clear and understandable statements of the conditions which must be satisfied in order that the method or mathematical manipulation in question can be validly carried through.

On cultural and educational grounds, the mathematics an engineer learns should be characterised by a rigour and a precision that at once stimulate him to an excitement in grasping the essentials of mathematical process and enable him to understand the peculiar qualities of mathematical thinking. On more practical grounds, the mathematics the engineer learns should be seen by him not to be a remote, esoteric exercise but to have direct application to his engineering studies.

> *He may never become wholly convinced that as an engineer his ultimate aim is to translate the observable configuration of the universe into mathematical terms. But he may be expected to recognise that in much of his work, mathematics is not merely a helpful adjunct to his engineering thinking but is inherently the basis of the way in which he is able to order his observations - that indeed he would find his observations unmanageable and himself helpless in manipulating them unless he expressed them or aspects of them in mathematical language.*
>
> (Committee on the Undergraduate Program in Mathematics, 1967)

Many problem areas in engineering turn out to have only a facade of precision behind which lies a great deal of fuzziness and uncertainty. In relation to such problems, classical mathematics is over-precise: it makes no provision for partial truths or fuzzy sets.

> *A major weakness of modern mathematics is that it fails to come to grips with the reality of fuzziness. By assigning precise meanings to fuzzy concepts and then proceeding to prove theorems about them, a mathematician can, and frequently does, construct a beautiful theory which may or may not have any relevance to the problem which motivated it; the domain of applicability of such theories is severely restricted by the neglect of fuzziness and the resultant exclusion of partial truths.*
>
> (Zadeh, 1966)

This question of fuzziness has an indirect bearing on the mathematical training of engineers. There is a point beyond which the exposure of an engineer to mathematical concepts and techniques can have harmful effects on his attitudes and, in particular, on his creativity. Thus, a mathematically over-exposed engineer might become excessively preoccupied with rigour or stay away from problem areas which do not lend themselves to mathematization. In this way, he may lose interest in doing otherwise creative work which does not involve making precise definitions of various concepts and proving theorems about them.

How does one choose what is actually to be proved? This is related to the plausibility of the desired result. The Committee on the Undergraduate Program in Mathematics (1967) explains:

> *It is unwise to give rigour to the utterly plausible or the utterly implausible, the former because the student cannot see what the fuss is all about and the latter because the most likely effect is a rejection of mathematics. The moderately plausible and the moderately implausible are the middle ground where we may insist on rigour with the greatest profit: the greatest danger in the over zealous use of rigour is to employ it to verify only that which is utterly apparent!*

7.7 MAKING MATHEMATICS RELEVANT

Scott *et al.* (1966) carried out a survey to determine firstly whether engineers were provided with an incentive to study a mathematical topic, and secondly whether engineers found it easier to understand a mathematical topic which is closely related to an engineering application. Their findings revealed that more than two-thirds of the respondents admitted that not only the applications of mathematics motivate them to tearn the subject, but also that a particular topic of mathematics is understood better by its applications.

The latter has special significance for mathematicians involved in the provision of service courses for engineers, as it runs directly counter to their usual attitudes as mathematicians. "All too often", André Lichnerowicz (1966) sadly reflects,

> *mathematics is presented as a haughtily independent discipline, interacting only tenuously with any application; mathematics as taught is dissected like a dead specimen or exhibited like a work of of architecture, in finished and final form, and good minds not infrequently reach the point of wondering if it is still possible to do any creative work in mathematics.*

McWeeny (1965), giving an Inaugural Lecture at the University of Keele, makes a similar point. Speaking of mathematics as a mode of expression and communication for much of the physical and engineering sciences, he says:

> *The widespread reluctance to employ such symbolic modes seems to me to arise from a misunderstanding of the role of mathematics in science, and perhaps also from the fact that in our schools higher mathematics tends to be taught in complete isolation from other subjects, rather than in relation to the world around us.*

An engineering course requires a correlated mathematics, integral and complementary, but not accretive. Mathematics should not be viewed as an alien and discordant element in the engineering studies. The mode of teaching mathematics as an abstract, shut-off subject, having no contact with what they study elsewhere, leaves them without a facility in discerning the many apposite ways in which it may be applied to the better understanding of most of the engineering sciences. This is a pedagogical failure: an academically unrestrained mathematics, hedged with functionlessness, is out of place for engineers, for whom mathematics is a means to an end and not an end itself. The lack of a perceived connection between applications and theoretical studies means that learning becomes a sequence of activities, arid and superficial, without purpose or significance.

Speaking in dramatic terms of a "crisis of relevance", Niss (1977) counsels that

> *The mathematical treatment of a problem must aim both at solving the problem itself and at producing insight that can be mobilized in other problem-situations. Thus the insight must be raised over the concrete situation with its specific restrictions, to an understanding of the structure of the problem. This understanding, however, must be anchored in concrete matters and its mathematical treatment must find its inspiration from them.*

Scott *et al.* (1966) suggest four possible methods by which mathematics may be closely linked to its engineering applications. They are:

(a) *Introduction of the applications in the mathematics class.*

(b) *Examples classes taken by engineers or mathematicians, following the introduction of each mathematical topic.*

(c) *The subsequent introduction within a few days of the applications of each mathematical topic in an engineering class in order to clarify and reinforce the mathematics teaching.*

(d) *The introduction of the mathematics as required in the engineering lectures.*

Of the methods outlined above, Scott *et al.* (1966) make the following observations:

Method (c) would make impossible demands on the planning and coordination of lectures. Method (d) is unsuitable as students will not get a picture of mathematics as a coherent whole, although it may be the best method of presenting isolated mathematical topics e.g. Boolean algebra or linear graph theory. Method (b) is adopted in one or two institutions, but provided suitable lecturers can be obtained, method (a) would appear to be the most satisfactory.

Professor Bajpai *et al.* (1975), acknowledging that it is an ambitious task, stress that

The student should see his complete course as an integrated entity with various facets. He should not only see mathematics as a language for engineering but also appreciate how mathematics pervades the whole fabric of his engineering subjects.

Neumann (1960) declares that

The most vitally characteristic fact about mathematics is, in my opinion, its quite peculiar relationship to the natural sciences, or, more generally, to any science which interprets experience on a higher than a purely descriptive level... that much of the best mathematical inspiration comes from experience and that it is hardly possible to believe in the existence of an absolute immutable concept of mathematical rigour, dissociated from all experience.

Every future engineer should be made aware of the applicability of mathematics and of the constructive interplay between mathematics and engineering disciplines. The content and procedures in the mathematical education of engineers should reflect, as much as possible, demonstrable relevance.

7.8 BASIS FOR SELF EDUCATION

The commonplace experience of evolving knowledge and practice persuades us that we must abandon the assumption that professional education concludes at the age of twenty-one or upon graduation. The realisation is fast dawning on the minds of engineers and other professional men and women that they are going to have to remain as scholars throughout their professional lives. No one denies the importance of what Johnston (1971) has termed "the continuum of professional education".

Mathematics will play a key rôle in the technological society of tomorrow. We must train our engineers to be able to attack and solve problems that did not exist in their undergraduate years. This requires an educational system that teaches not only the fundamental mathematical principles and techniques, but stresses understanding and originality of thought in its mathematics courses. Professor Bruner (1964) urges a shift of emphasis in teaching:

> *We teach a subject not to produce little living libraries on the subject; rather, to get a student to think mathematically for himself, to consider matters as a historian does, to embody the process of knowledge-getting. Knowing is a process, not a product.*

The encyclopaedic approach to the problem - an attempt to provide an acquaintance with or a conspectus of all learning - has slender, if not nil, chance of success. The general consensus of opinion is that sufficiently broad mathematical foundations must be laid in mathematics which will carry them to the stage where they can learn for themselves. However, it is desirable that at least one topic which is of pivotal significance to his particular engineering discipline should be studied in depth, such as complex variables for control engineers.

If the provision of skill in self education is our objective, then our present methods of teaching, which place an unduly heavy emphasis on lectures and which give little encouragement for self learning, are open to question. It is difficult to see, Scott (1972) maintains, how students would acquire

> *the finer points of the techniques of problem solving by watching the brilliant and carefully premeditated intellectual efforts of their instructors.*

He goes on to make the point that few educators have concerned themselves with the task of providing

> *a suitable course in problem solving as opposed to a set of problems to be solved.*

The pace of technological change and the accumulation of knowledge necessitate the continuing review of the curriculum. To train engineers to meet the problems of the unborn days ahead, what we teach and how we teach should have a timelessness and a timeliness in combination. Thus in planning and implementing a curriculum for the mathematical education of our future engineers, a basic objective should be, to use the words of Johnson (1963),

> *to teach the student to think; to expose him to a series of fundamental principles; to utilize current developments as illustrations or vehicles to demonstrate the manner of applying these principles to the ultimate production of a desired device. Such a student, or rather graduate, should be capable of interpreting the physical significance of a problem, to reason through the various ramifications of the situation facing him, and be able to evolve a reasonable solution based on his composite knowledge, interpretative ability, and reasoning skill.*

The engineering of the past has not demanded this unified training. It is therefore imperative in presenting mathematical fundamentals that real attention be given to injecting a feeling for the physical significance of the mathematical symbols and equations, their relation to other fundamental principles, and the manner in which these relationships may be combined for application to broader situations. Early in the instructional curriculum, a start must be made in giving the student problems which will require his reasoning through a situation, and one which, while solvable by use of known formulae, represents a need for him to interrelate fundamental principles. As the student progresses, the student should be encouraged to be alert to ways of correlating and integrating fundamental knowledge.

Beltzner *et al.* (1976) argue that more emphasis should be placed on:

(a) *the origin and background of problems;*

(b) *the intuitive trial-and-error methods which all mathematicians employ to actually solve problems rather than swift exposition of a neat solution.*

(c) *problem solving by students in groups and in tutorial sessions; and,*

(d) *independent study projects by individuals or groups of students.*

7.9 DIFFERENTIATED MATHEMATICS COURSES

According to Wakely (1970), industry has need of two kinds of engineers. The first is the generalist or the "compleat engineer", that is, one who has an appreciation of the various disciplines that impinge upon engineering and who should have a feel for certain major branches of mathematics. The second kind of engineer that industry needs is the specialist, who would be a mathematical engineer: his qualifications should be very similar to those necessary for the generalist, except that one would clearly expect less appreciation and more understanding in depth. Wakely suggests that the generalist's knowledge of mathematics should include matrix algebra, differential equations, numerical analysis and computer science, as well as a feel for operational research, statistics and the calculus of variations.

Research engineers use a great deal more mathematics than their counterparts in, say, design or production. Indeed, the mathematical requirements of the production engineer are different from those of the design engineer.

These reflections persuade us that there is a definite need and merit in the provision of flexible, differentiated mathematical courses which could be tailored to meet the differing demands of different branches of engineering specialties such as civil, chemical, electrical and mechanical.

The argument we wish to put forward here is that there should be two groups of mathematical studies available in engineering faculties of CAEs; a common core curriculum for all students in the earlier years, and a substantial number of electives. This arrangement would recognise a modern trend in engineering education, namely that,more and more, schools have adopted a core programme of professional subject matter as the common basis of the various branches of engineering (convincing illustrations of how greater stress on general principles can be integrative). The provision of electives would serve to confirm the realisation that a single, undifferentiated mathematics curriculum is unrealistic because it transcends the natural divisions in engineering practice and the varied special interests of its practitioners.

7.10 EDUCATION, NOT TRAINING

If, as it seems, universities have come to see themselves as a part - a clearly distinguishable and very distinguished part, but still a part - of the nation's provision of higher education, the colleges of advanced education cannot leave to universities the difficult task of responding to the more inclusive purposes of higher education.

The meaningful question to be asked about the professional schools in colleges of advanced education is how their offereings can be patterned to meet the needs of youth who have at least a tentative vocational objective but whose education must be so designed as to prepare them to act as intelligently in the broader contexts of life as in their own work.

The disciplines that are usually classified under the caption "liberal arts" are considered to give the necessary preparation for the broad activities of life. However, as Hancher (1953) holds, much of the instruction in the engineering courses can, and should,be taught so that the realisation of the ends of liberal education becomes a distinct possibility. Hancher (1953) declares:

> *We forget that it is possible to become liberally educated by the teaching and study of professional or specialised subjects in a liberal manner ... While in general I would support the proposition that there are some things which every liberally educated man should know, I fear that we have been led into error sometimes by believing that the study of certain subject matter inevitably results in a liberal education. This is a doubtful proposition. It is nearer to the truth to say that there is no subject matter worthy of a place in the curriculum of a modern Land-Grant College or state university, which cannot be taught either as a professional specialty or as a liberal subject.*

Echoing the sentiment that it is possible to provide a broad educational experience through the teaching of professional courses, McGrath (1960) says that

> *It is obvious that courses in engineering or pharmacy, if properly taught, acquaint the student with a wide range of scientific facts and cultivate the intellectual skills of the scientist. They also instill a respect for truth, a humbleness of spirit, a desire to learn, and the habit of philosophical reflection about the place of man in a limitless cosmos.*

The student of engineering should be given the vocabulary and basic principles of his calling, but he should be put in a position in which he is unable to avoid being educated in the process.

Richard Courant opposed the over-development in mathematics which emphasised abstraction, generality and axiomatisation:

> *Living mathematics rests on the fluctuation between the antithetical powers of intuition and logic, the individuality of 'grounded' problems and the generality of far-reaching abstractions. We ourselves must prevent the development being forced to only one pole of the life-giving antithesis.*
>
> (Reid, 1976)

To develop mathematics courses which, by choice of topic and treatment, reflect such aims and hopes is not an enviable task, but one which presents the educator with an unparalleled opportunity.

CHAPTER VIII

SERVICE TEACHING OF MATHEMATICS

Summary

The "state of the art" is delineated in this chapter. To the questions: Who does the teaching? What is taught? How is the selection of topics made? How is it taught? and, How is it assessed? - answers are given. As well, an attempt is made to articulate principles that should ideally govern educational practice.

8.1 THE "STATE OF THE ART"

8.1.1 Introduction

The service teaching of mathematics to engineers raises a number of pertinent questions which invite answers.

In visiting the Queensland Institute of Technology, the Royal Melbourne Institute of Technology and the South Australian Institute of Technology, we asked the following questions that would give us a glimpse of engineering mathematics teaching today:

Who does the teaching?

Are various branches of engineering taught together, or separately?

What is taught?

How is the selection of topics made?

When is mathematics taught?

How is it taught? Is modelling, for example, given prime emphasis?

Is the axiomatic approach avoided or given pride of place?

To what extent are case studies and projects used?

The examination of service teaching of engineering mathematics has certain limitations in scope. As our experience has been exclusively in the DOCIT Colleges, our discussion draws heavily from, and is most applicable to, that setting. We aim to delineate the practice at some of the seven DOCIT Colleges, as the overwhelming impression, despite an element of variability, is one of astonishing similarity and uniformity. Though we believe that what we have to say is very applicable to other CAEs and to those who teach there, we make no attempt to discuss such applicability.

The seven DOCIT institutions providing engineering education are: Caulfield Institute of Technology (CIT); The New South Wales Institute of Technology (NSWIT); The Queensland Institute of Technology (QIT); Royal Melbourne Institute of Technology (RMIT); South Australian Institute of Technology (SAIT); Swinburne College of Technology (SCT); and, Western Australian Institute of Technology (WAIT).

8.1.2 Who Does the Teaching?

At all seven DOCIT Colleges, mathematics for engineers is serviced by mathematics departments in the earlier years. However, the three Melbourne DOCIT institutions, namely CIT, RMIT and SCT, have a general policy whereby the mathematics departments service all mathematics for engineers in all years.

In its service rôle, the Department of Mathematics and Computer Science at QIT is responsible for teaching mathematics and computing subjects in the School of Engineering. Some subjects, such as Integral Transforms, Applied Mathematics and Vector Calculus have been "annexed" by the School of Engineering. Some members of the mathematics staff have an engineering qualification or industrial experience.

At SAIT, an attempt at team teaching may be seen: Engineering Mechanics and Dynamics is taught jointly by members of staff from schools of engineering and mathematics. The staff from the School of Engineering participate with the mathematics staff during tutorial sessions.

Mathematics staff of DOCIT institutions are in general agreement that all mathematics courses in all years should be serviced by mathematics departments. However, a number of mathematics departments are under pressure from schools of engineering and commerce to relinquish their service teaching responsibilities. Though these pressures stem not from any consideration of educational grounds, they are understandable, since they arise from the compelling need for sheer departmental survival in the so-called "steady-state" period of more recent times, a period of relative contraction or, at best, one of very limited growth: great emphasis is placed on preserving and gaining EFTS (Effective Full-Time Students) entitlements, since it is on this basis that funds are distributed and staff requirements determined.

8.1.3 Taught to Whom?

At NSWIT and WAIT, several departments are taught a common mathematics syllabus in calculus and analytic geometry, multivariable calculus and ordinary differential equations. In the first year, the classes are "mixed", but the second-year classes are such that each department is taught separately the same syllabus and given a common examination.

At RMIT and SAIT, the first year syllabus in mathematics is the same for all engineers; however, from first year onwards, different branches of engineering are taught in separate groups, the various Schools of Engineering specifying the number of mathematics units and the subjects to be studied.

At SAIT, the first-year mathematics course is an all-purpose one, taken by engineers and applied scientists.

At both NSWIT and RMIT, provision is made for students with marginal competence and adequacy in mathematics.

At QIT, the civil, electrical and mechanical engineering students attend the same mathematics lectures at the same time. However, electrical and mechanical engineering students study Applied Mathematics concurrently with their fourth semester mathematics subject.

8.1.4 What is Taught?

At NSWIT and WAIT, all engineers are given courses in calculus and analytic geometry, multivariable calculus, and ordinary differential equations in the first two years of the course, by staff from Mathematics Departments. For some branches of engineering, it is necessary to undertake further mathematical studies. Thus, in electrical engineering, students are required to take field theory, operational methods, feedback theory and engineering computations; mechanical and production engineering students are expected to take numerical analysis, engineering statistics and computer programming; fluid mechanics and computation must be studied by civil engineering students. Such further mathematical studies are normally presented by staff from Engineering Departments.

At SAIT, after a common course in first-year mathematics, engineering students split into different streams, according to their chosen branches of engineering. Table 8.1 displays the subjects studied by engineering students.

At RMIT, all first-year engineering students take the subject MA250, a

Table 8.1

Mathematics Courses for Engineering Students at South Australian Institute of Technology

Branch of Engineering	Code	Units									
		201	202	211	213	215	216	220	P1	P2	P2A
Second Year: Mathematics 2											
Civil	2CE	+	+						+		+
Electrical	2EE	+	+	+	+						+
Electronic	2ET	+	+	+	+						
Mechanical	2ME	+	+								
Mining	2MN	+	+								
Third Year: Mathematics 3											
Civil	3CE			+	+						
Electrical	3EE						+	+			
Electronic	3ET					+		+			+
Mechanical	3ME				+					+	
Mining	3MN				+						

201 Advanced Calculus
202 Differential Equations
211 Matrix Applications
213 Statistical Inference and Reliability
215 Analysis of Random Signals
216 Complex Variable Analysis and Transform Theory
220 Numerical Mathematics
P1 Programming
P2 Programming
P2A Programming

Source: South Australian Institute of Technology (1979) broadly-based course which revises and extends topics in calculus, geometry and algebra and leads into a bank of specialised units. The unit bank is displayed in Figure 8.1, which also indicates the appropriate prerequisites. Second-year engineering students select any four units from the unit bank, subject to appropriate prerequisites. However, individual departments of engineering usually specify the units most suitable to their courses. Additional units of study, in some cases as electives, are taken in various selections by third and fourth year students from different engineering departments. An outline of subject streams is shown schematically by Figure 8.2 . The subjects studied by different categories of engineers during their undergraduate years are shown in Table 8.2 .

RMIT is becoming increasingly involved with graduate programmes and the need for the inclusion of appropriate mathematical courses is growing. Students who have a degree which involved courses from the unit bank (or equivalent subjects) can take either

(a) suitable approved units, not previously taken; or,

(b) new subjects or units in other topics, or in existing topics at a more advanced level.

Thus the unit bank, with suitable extensions, enables courses to be devised to suit post-graduate students.

At QIT, students reading for Bachelor Degree courses in civil, electrical and mechanical engineering have to study Preparatory Mathematics (a self-paced course), and Engineering Mathematics II to VI, one subject in each of the six semesters. The main mathematics topics studied are: differentiation and integration, partial differentiation, ordinary differential equations, linear algebra, probability, statistics, numerical methods, operations research, linear programming, complex variables and boundary-value problems.

The mathematical syllabuses for undergraduate engineering students at CIT and SCT have a striking resemblance to those taught elsewhere, although there are differences in organisation of courses.

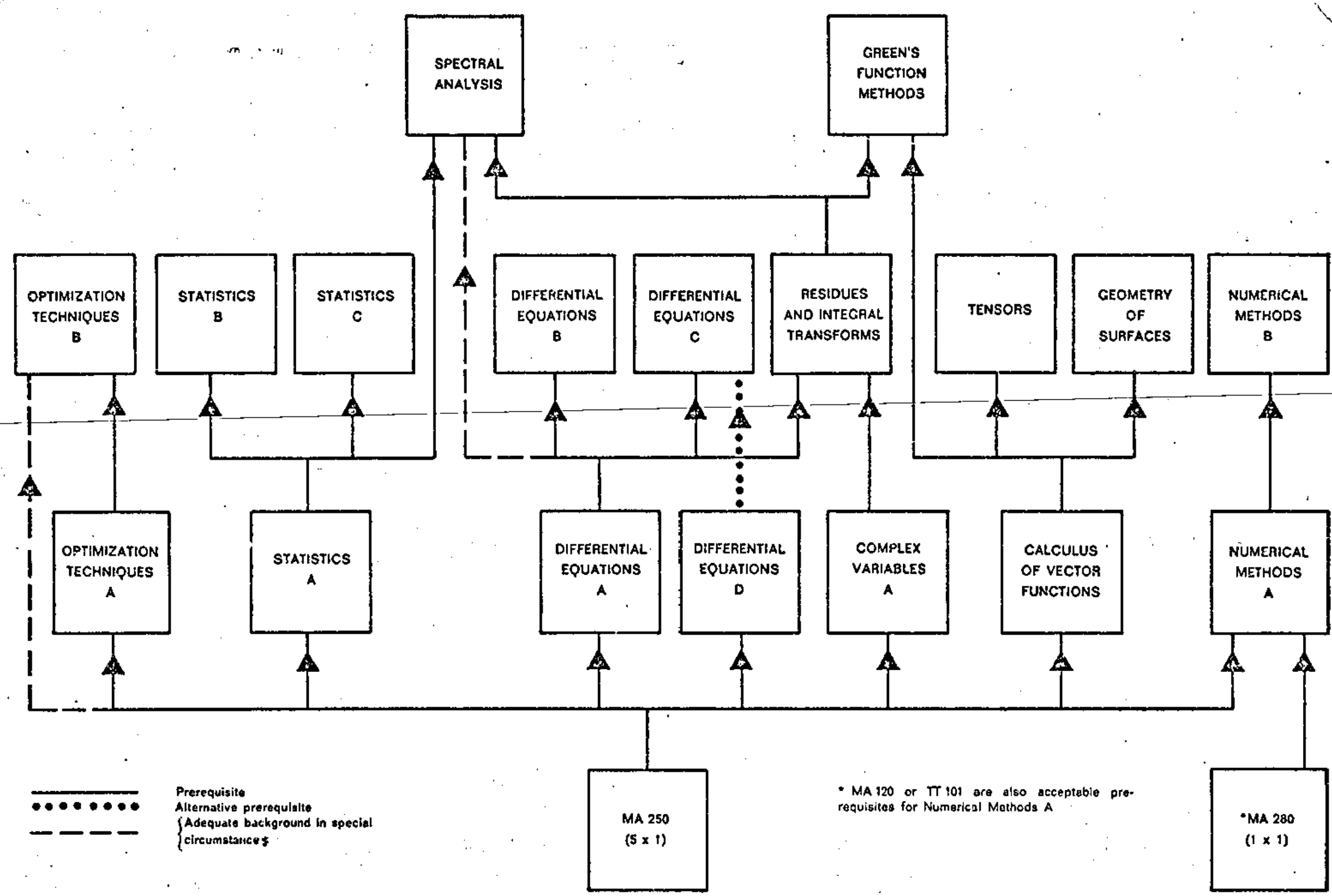


Figure 8.1

Schematic Outline of Unit Bank at Royal Melbourne Institute of Technology

Source: A brochure entitled *Service Subjects*, produced by the Department of Mathematics and Computing Sciences, 1979

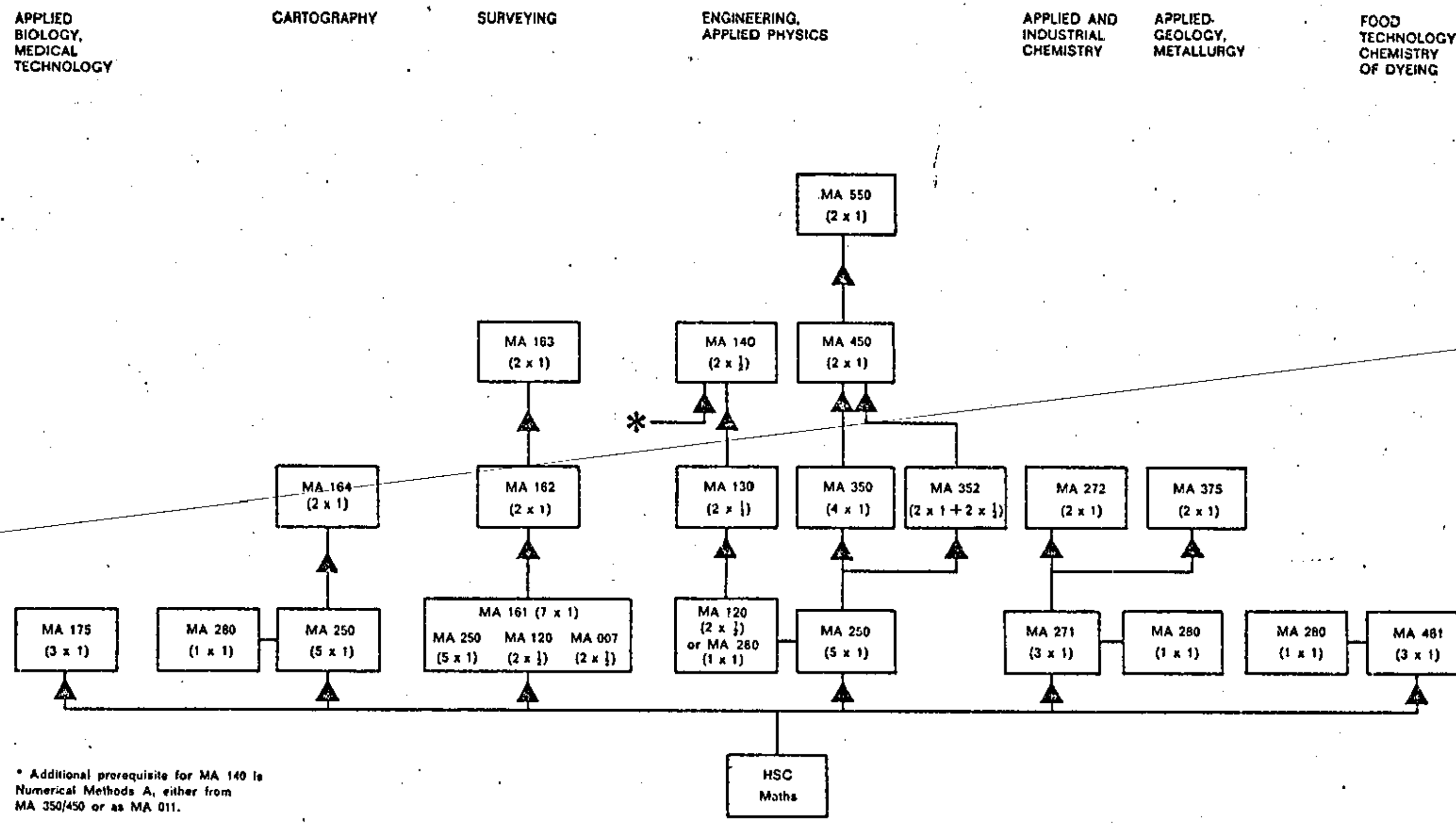


Figure 8.2

Schematic Outline of Subject Streams based on HSC Mathematics at Royal Melbourne Institute of Technology

The duration of each subject is described by (hours/week x fraction of 1 year)

Source: A brochure entitled *Service Subjects*, produced by the Department of Mathematics and Computing Sciences, 1979

Figure 8.2 (Cont.)

MA120	Computer Programming 1
MA130	Computer Programming 2
MA140	Computer Programming 3
MA250	Mathematics 2 (for First Year Students)
MA280	Computer Programming for Technologists
MA350	Mathematics 3 (for Second Year students): four units from the unit bank
MA352	Comprises the units Differential Equations C, Differential Equations D, Statistics A
MA450	Mathematics 4: two units from the unit bank, which have not been taken previously
MA550	Mathematics 5: two units from the unit bank, not previously taken

Table 8.2

Subjects Studied by Undergraduate Engineering Students at Royal Melbourne Institute of Technology

Department	1st Year (MA 250) Semester 1 2	2nd Year (MA 350) Semester 1	2nd Year Semester 2	3rd Year (MA 450) Semester 1	3rd Year Semester 2	4th Year (MA 550) Semester 1	4th Year Semester 2
<u>Engineering</u>							
Chemical	MA 250	CVF STA	NMA STB	OTA	DEA		
Civil	MA 250	CVF NMA or DEA	STA DEA or NMA	OTA	STC	DEC	OTB(1) [alt. pref: DEB(2) COV,RIT(3)]
Aeronautical	MA 250	CVF NMA	DEA COV	OTA	STA	STC	OTB(1) [alt. pref: SAN(2) DEC,RIT, GEO,TEN(3)]
Communication	MA 250	CVF DEA	NMA STA	COV	DEC	RIT	SAN or OTB
Electrical	MA 250	CVF DEA	NMA COV	STA	DEB		
Mining	MA 250	CVF NMA or DEA CVF DEA	NMA or DEA STA NMA STA or COV				
Mechanical } Production	MA 250	DED STA	DEC	NMA	COV	OTA	STC

Source: A Leaflet produced by the Department of Mathematics and Computing Science, 1979

Table 8.2 (Cont.)

COV	Complex Variables
CVF	Calculus of Vector Functions
DEA	Differential Equations (A)
GEO	Geometry of Surfaces
MA 250	Mathematics (2)
NMA	Numerical Methods (A)
OTA	Optimization Techniques (A)
RIT	Residues and Integral Transforms
SAN	Spectral Analysis
STA	Statistics (A)
TEN	Tensors

8.1.5 When is it Taught?

At the NSWIT, for a six-semester scheme of study, the common core mathematics, consisting of calculus and analytic geometry, multivariable calculus and ordinary differential equations, occupies the first two semesters; additional mathematical units such as numerical methods, engineering statistics, operational methods, field theory and feedback theory are deferred to third, fourth and fifth semesters.

An almost parallel pattern can be seen at RMIT and SAIT. At RMIT, all branches of engineering require in the first year the study of the unit MA250, which revises and extends topics in calculus, geometry and algebra, and introduces the major subject streams available in the unit bank. The study of statistics, calculus of vector functions, numerical methods, differential equations and complex variable analysis is deferred to the second year, and the more demanding mathematical topics, such as Green's Function methods, residues and integral transforms, optimisation techniques and tensor analysis are generally prescribed during the third and the fourth year of a four-year degree course.

At SAIT, students from all branches of engineering study the common Mathematics I during their first year of a four-year course: Mathematics I deals with such aspects as differential and integral calculus, line integrals, difference and differential equations, power series, vector spaces, matrices, eigenvalues, and statistical inference. In the subsequent two years, civil, mechanical, electrical and mining engineers probe more deeply into those aspects of mathematics which have been introduced in Mathematics I. The details are given in Table 8.1 .

Students undertaking first degree courses in engineering at QIT study Preparatory Mathematics in the first semester and Engineering Mathematics II to VI in the second, third, fourth, fifth and sixth semesters respectively.

There are differences among institutions concerning the number of hours per week devoted to lectures and tutorials, as there are differences in the ratio of lecture hours to tutorial hours. However, the abiding impression is one of surprising similarity.

8.1.6 How is it Taught?

The most usual method of teaching at NSWIT, QIT, RMIT, SAIT and WAIT, is by lectures, using chalk and talk. Audio-visual aids are seldom used during lectures, and many lecturers expressed an undisguised aversion for programmed texts. Team teaching is only evident on a very restricted basis at SAIT.

Every effort is made to make mathematics plausible, and to resist a "cook-book" method of presentation. The proofs of mathematical results, which are considered of crucial significance, are established but there is no desire, nor is it the practice, to have students lost in a welter of rigour. Some lecturers, especially those who have an engineering background or previous industrial experience, are able to give an applications approach to their lectures. An approach utilising mathematical modelling is currently being developed at WAIT.

At all five places, it is normal for formal lectures to be backed up by tutorials which are led by tutors and devoted to solving standard set problems.

SCT has most of its first-year mathematics by self-paced instruction, and CIT has two-thirds of its first-year course developed in this manner. The development costs are too prohibitive to attempt its completion in the current economic climate.

QIT, too, has developed a course in self-paced instruction for its first-semester engineering students.

At most institutions, close contact is maintained between the servicing and serviced departments, so that appropriate emphases may be laid on both syllabi and teaching methods. Some of the channels of communication have been formalised by the establishment of departmental liaison committees, advisory committees and the appointment of Service Course Coordinators. At an informal level, staff liaise through personal contacts and discussions.

8.1.7 How is it Assessed?

The examination question papers in service mathematics courses for engineering undergraduates at the colleges of advanced education are set by the teaching staff of mathematics departments and are submitted for approval to assessors. Each paper is accompanied by model solutions and a detailed marking scheme.

There are mini-tests and mid-semester examinations, and they form the basis of continuous assessment. These tests are principally designed to test the student's mastery of the concepts in a particular area. Since these are returned to the student, they also perform a teaching function. The marks obtained throughout the year for coursework contribute a component of the percentage mark in the final assessment.

The coordinating examiners decide how the continuous assessment grade is to be combined with the grade the student achieves in the final examination for the course. The usual procedure has been to weight the final examination a little higher than the continuous assessment, roughly in the proportion of three to two. The limited analyses carried out show that there is good correlation between the over-all continuous assessment grade and the examination grade.

At the NSWIT, it is customary to use different schemes of assessment for each of the three basic courses: calculus and analytic geometry, multivariable calculus and ordinary differential equations. For example, in calculus and analytic geometry, the final assessment is made up of three components: five quizzes or mini-tests aggregating to 40 per cent, a mid-semester examination worth 20 per cent, and an end of semester final three-hour examination having a weighting of 40 per cent. The purpose of the mini-tests is to induce diligence in students and the students are cautioned that an unsatisfactory performance at the final examination would automatically constitute a failure.

At SAIT, no attempt at continuous assessment is made. External markers are used to assess "homework" in Mathematics I, and the "homework" component generally contributes between five and ten per cent of the final mark. Mathematical units during the second and third years of study are graded as α, β, γ, and, in all cases, heavy reliance is placed on the final examination at the end of the year for assessment.

At RMIT, three modes of assessment are used for MA250, the first mathematics unit taken by all engineers: lecturer assessment contributes 10 per cent, the mid-year examination, 20 per cent, and the final examination at the end of the year, 70 per cent. In statistics, class assessment is allocated 15 per cent and the final examination, which is an open-book examination, is worth 85 per cent.

SCT and CIT both make use of mastery tests for first-year students. At SCT, which has thirteen mastery tests in its first-year courses, students are required to score 100 per cent in each of these before they progress to the next one. An unsuccessful student attempts another parallel test, until successful. The successful completion of thirteen mastery tests ensures a pass in the course. Higher grades of success can be achieved by attempting enrichment work. The total assessment of a student at SCT is by means of these mastery tests.

CIT, however, assesses students in mathematics courses by giving a weighting of 65 per cent to the mastery tests and 35 per cent to the final examination. The scoring of mastery tests at CIT is not as stringent as at SCT: each one is out of 15 and the procedure is as follows:

14/15 or 15/15	5 marks
12/15 or 13/15	3 marks
10/15 or 11/15	1 mark
≤ 9/15	0 mark

The advantage of these mastery tests is that they encourage a thorough grasp of the essentials of a course.

QIT requires its first semester engineering students to undertake Preparatory Mathematics, which is run as a unit of self-paced instruction and is assessed entirely by results obtained in seven separate modules. Each module of the unit has to be mastered at a very high level by the student prior to an attempt on the next sequential module. The other five mathematics courses for engineers, namely, Engineering Mathematics II to VI, are assessed wholly by results obtained in final examinations. Although assignments are given during the semester, they do not contribute to the assessment of a student. Supplementary examinations are set and these have an ameliorative effect on progress and pass rates of students.

The method of assessment at WAIT provides for formal annual examinations as well as assignments and written tests during the year. The over-all examination mark is awarded on the weighted average of these.

A cursory glance at the examination papers set at various institutions affirms that examination methods are themselves liable to be defective. The common assumption of examination papers that the only important thing is information, exercises a most deleterious influence over the whole student experience. There have been times when examiners have been testing the ability of students to apply standard techniques to standard problems, problems which require no innovative mathematical skill. The student's ability and skill for problem-solving and exposition are marginally examined.

Lecturers find it difficult to set questions which give the student opportunity to exercise his judgement, his originality, or to display the width of his reading in mathematics. The pattern that examination papers help to affirm is a process of feeding information to students in lectures so that they can feed it back in examinations: there is an excessive concentration on mere technique.

Thus the pattern of instruction becomes a "feed-in feed-back" system, an attempt to cover a curriculum without sufficient attention to training in thinking. As the system stands, it permits smartness to replace wide knowledge and genuine ability. The occupational skill of students ceases to be intellectuality and becomes the ability to pass examinations without being exposed to a mental discipline. The discrepancy between the kinds of problems appearing in examination papers in engineering mathematics and the kinds of problems engineers are faced with is, to be charitable, marked.

8.2 SOME GUIDING PRINCIPLES

8.2.1 Who Should Teach?

Although we attach such importance to mathematics in engineering education, educators are not all agreed about the kind of mathematics we expect engineers to learn, nor even the kind of mathematicians who should teach them.

In some universities, all the mathematics teaching of engineering students is done by members of the staff of the Engineering Department, who would certainly regard themselves as being engineers rather than mathematicians. In other universities, like the Loughborough University of Technology, there are separate Departments of Engineering Mathematics for this purpose. At the other end of the spectrum come those cases in which it has been decided that engineers are not different from others in their mathematical needs, and so they can perfectly well join in the general mathematical classes, even those designed for proper mathematicians.

The traditional method, however, in universities and CAEs, is for engineers to be taught mathematics in the mathematics departments, but there is occasionally evidence of some dissatisfaction with the system and many engineers would welcome the transfer of the mathematics teaching function to the engineering department. This is especially so when the kind of mathematics taught is mathematics for mathematicians - a detached, self-sufficient, rationalised mathematics seemingly restricted to the mathematics classroom and having no relevance to the science or practice of engineering.

Since in most universities and CAEs, engineering students are not a small minority, it seems realistic to suggest that engineers should be taught by mathematicians specially selected, or even specially appointed for the purpose, as members of the Department of Engineering Mathematics. Such lecturers would be mathematicians, who would understand and accept the engineers' attitude to mathematics, and who would be able to relate the mathematics to the engineering disciplines of the students. An interest in mathematics would be stimulated by this means among students who often regard mathematics as an ancillary subject which has to be studied for an examination, but can be conveniently relegated to the past, once assessment has been completed.

As nothing can deflect us from our conviction that mathematics has a central place in the training of a professional engineer, it is essential that a student's contact with mathematics in the undergraduate course should engender a wholesome, lively interest in the subject. This can only be achieved if mathematics is taught in the right spirit and with simultaneous emphasis on the utility of mathematics in engineering. It is recognised that this is a very recent florescence, disturbing to long-entrenched routine, and demands a re-orientation.

In a university, the mathematics department exists for the training of mathematicians and for the pursuit of mathematical knowledge, and staff are quite properly selected with this end in mind. The training of engineers is an ancillary duty, which has to be arranged so as to produce the minimum interference with the normal running of the department. It is true that the scope of such departments has been widened in recent years with increased emphasis on statistics, numerical methods and various branches of applied mathematics; yet any departure from the true spirit of mathematical thought would conflict with the primary purpose of the department.

At present there is a shortage of mathematicians who are well equipped to teach engineering students, but we may hope that as the proportion of mathematics graduates entering industry increases, many more mathematicians with a direct interest in applications will be trained. In particular, the Mathematics Degree Courses at the CAEs, such as those given at the NSWIT, should provide mathematicians who are able to feel at home with applications of mathematics. The courses given by the mathematics departments of the CAEs for their mathematics students are characterised by a less abstract approach and by an emphasis on the use of numerical methods, statistics and the digital computer.

However, even if an adequate supply is available, such mathematicians will not come forward if they are regarded in the mathematics departments as of inferior status. This raises the fundamental difficulty that in a university and, to an increasing extent, in a

CAE, prestige and promotion depend on published research and the incentives for improvements in teaching are not adequate.

The problems of teaching mathematics to engineers cannot be side-stepped by mathematics departments unless they would welcome the transfer of such teaching to the engineering departments.

To the question, then, who should perform the service teaching of mathematics for engineers, our answer is that a lecturer who is sympathetic to the needs of the engineer is the best person; while we agree that the language and illustrative examples of most mathematicians would not be appropriate for engineering mathematics teaching, we feel strongly that few engineers have the necessary background in mathematics for effective mathematics teaching. Professor T. Neville George (1966) writes:

> *The practice of having mathematics taught by a competent geologist, adopted in some universities, gives to the mathematics an immediacy particularly convincing to the students, who are always inclined to identify the propriety of what is taught with the teacher; and it allows exceptionally close integration of mathematics with geology. But only the occasional geologist is competent enough in mathematics to teach mathematics, and even when he can be found it must be conceded that the mathematical scholarship and principles lying behind the instruction are more likely to be sustained by a sympathetic mathematician sufficiently prompted in the geologist's needs.*

These words are just as apposite for engineering as they are for geology.

Professor Bajpai affirms the point made by Professor George. Says Professor Bajpai (1970b):

> *The best man for the task is, in my view, the mathematician who is sympathetic to the needs of the engineer, provided he has the support and advice of his engineering colleagues. The pure mathematician will almost certainly fail to motivate his students. The engineer may, in certain cases, be the ideal choice, but may tend, in general, to concentrate on his narrow speciality and will probably not have the patience to develop the idea of the unity of mathematics, which is often met with in the mathematical representation of physical situations arising from different branches of engineering. A welcome move in this direction is the creation of engineering or industrial mathematics.*

The introduction of a course in which mathematics and engineering play equal rôles - a joint honours course - may mitigate, to some extent, the difficulties associated with the finding of the right person to teach engineering mathematics. Joint honours courses are common in mathematics and physics, and it should be possible to design a course in mathematics and engineering to produce the type of engineering mathematician or mathematical engineer who is required not only in in industry, but also in tertiary institutions as a teacher.

The disenchantment expressed by engineers, when pure mathematicians have tended to present delta and epsilon courses, has led in some instances to the mathematics teaching being taken by mathematicians, who had transferred, at least for a period of time, their allegiance to the non-mathematician's field. This, too, is not an answer, since it tends to result in *ad hoc* syllabuses that lack generality. It is now realised that what is needed is a collaboration on equal terms between the two sides, a collaboration personified in team teaching. H. Graham Flegg (1974) pleads for team teaching:

> *The current crisis in the mathematical education of scientists and technologists should be resolved by replacing service courses by integrated courses whose theme is mathematical modelling and which are prepared and taught by joint teams of mathematicians, scientists and technologists.*

8.2.2 What Should be Taught?

A typical syllabus occupying the first two or three years of an undergraduate course in engineering mathematics would contain an introduction to differential and integral calculus, ordinary differential equations, numerical methods, mathematical statistics, calculus of several variables, vector analysis and matrix applications. There are optional courses given at a higher level to meet the requirements of individual departments.

In deciding on what we teach, we have been putting too much emphasis on choosing those parts of mathematics which seemed to us to be immediately applicable to the purpose on hand, and too little on those which form the best material for broadening our minds, for teaching us mathematical modelling, and the logical approach to the formulation and solution of physical problems. Writes Professor Christopherson (1967):

> *Part of the difficulty arises from the fact that throughout our educational system, too much emphasis has been placed on mathematics as a means to an end - on the particular bits of mathematics that are assumed to be relevant to certain parts of physics or engineering, or indeed accountancy or life insurance, and*

not enough attention has been paid to the main object of the exercise, to give people an understanding of what mathematics is and does - not for utilitarian ends, but simply as part of the equipment of a trained mind.

What, then, constitutes an adequate body of mathematical experience and a sufficient technical apparatus? The answer is directly related to the aims and objectives of teaching service mathematics for engineers. They are, as Bajpai succinctly expresses them:

(a) to teach the student to reason properly, logically, and with economy of thought, and to generalise;

(b) to provide him with a useful tool for tackling problems arising in engineering disciplines, and

(c) to give him facility in understanding new engineering topics with a significant mathematical content.

Some of the recent developments in mathematics which have become recognised as part of the apparatus of engineering were, interestingly enough, developments in what would normally be regarded as pure mathematics. "John von Neumann", Christopherson reminds us, "brought out his magnum opus *The Theory of Games and Economic Behaviour* in 1944, and from it comes the conception of linear programming, which provides the grammar of operation for so many industrial processes".

A.L. Allen and A.G. Shannon (1972) emphasise the power of pure mathematics:

The idea that pure mathematics is not useful betrays an intellectual hardening of the arteries ... Any dichotomy between pure and applied mathematics is born of a faulty mathematical education rather than the existence of a real obstruction between theory and practice.

The authors go on to draw a distinction betweem mathematical education and mathematical training:

When the bulk of mathematics education aims to educate an intelligent user of mathematics in some field of its application, we feel that this will rather be mathematics training, unless the unifying aspects of modern mathematics are the background of such courses.

Indeed, Christopherson stresses that examples in engineering abound in which all the complete certainty and rigour, which is the characteristic of the pure mathematician's approach, is needed.

The mathematical experience of an engineer should not just be an exposure to a series of mathematical techniques. For, to reduce his mathematical education to this is to see mathematics denuded of all its significance, interest and most of its value. Graham Flegg (1974) dissuades us from presenting a course consisting of nothing but techniques:

> *The users of mathematics have done themselves and their subject a disservice whenever they have approached mathematics purely from a conventional user point of view - the 'maths as a tool' approach.. This approach leads to the prostitution of mathematics. It is a fact of history that where mathematics has been relegated to the status of a mere computational tool, sterility in the development of science [and, he may have added, engineering] has very quickly been all too apparent.*

To concede that mathematical techniques must be presented within the framework of mathematical principles is not to be unmindful that many young students are best able to absorb the abstractions of mathematics through its applications and the present emphasis on elegant techniques rather than practical purposes has driven many of them away from the subject.

There is obviously a need for a balance between theory and practice. "We are not afraid", Seidel (1973) tells us, "of abstraction in the good sense, when it serves better understanding and overview, but we are allergic to general abstract nonsense".

A course in engineering mathematics must be recognised as a challenging course in mathematics, but constrained to mathematical, statistical and numerical methods which are usable in subsequent engineering careers. It is not easy to prescribe the substance of what is to be taught in service mathematics for engineers since its usefulness is increasingly recognised in divergent and increasingly specialist branches of rapidly developing engineering disciplines; still less is it possible in a present instruction of undergraduates to foresee what they are likely to need in five or ten years after entering professional life. A service mathematics that is immediately appropriate thus should not be conceived of merely as being useful *ad hoc*. It should allow the students themselves, when the future occasion arises, to build upon it and to apply their mathematical knowledge with some originality and independence in novel ways and novel situations.

According to Professor L.C. Woods, "The basic philosophy of the

Institute [he was referring to the NSWIT] concerning service teaching appears to be that the School requiring the service is entirely free to specify the syllabus of the subject it wants taught". Like all generalisations, it has an element of hyperbole in it. The usual procedure for drawing up the mathematics syllabus for engineers at, for example, the NSWIT, QIT, RMIT and SAIT is a series of discussions between the servicing and serviced departments.

Writing as an engineer, D.G. Christopherson (1967) advocates:

> *In my view, our approach to our mathematical colleagues should not be to say, 'We want our men taught this, that, and the other', but to say, 'We want you to give us a list of items which in your view will be useful in giving our men an understanding of what mathematics is, and how mathematicians think and which will develop to the full whatever mathematical ability they have'.*

H. Graham Flegg (1974) puts forward an alternative basis for the determination of mathematics syllabuses suitable for service teaching. He writes:

> *It is my belief that it is perfectly reasonable to let the needs of science and technology determine those areas of mathematics which should be included initially in any syllabus designed for students of these disciplines, but that once these areas have been appropriately identified the syllabus must allow for them to be explored properly and must be extended to allow for the inclusion of other and more basic mathematical concepts.*

According to Professor Neville George (1966), such

> *an enlarging comprehension of mathematical potentialities emphasises in the students' minds something of the qualities of a mathematician's mathematics and helps to break down the unscholarly view of a service mathematics that is no more than a convenience available in stereotyped contexts for mechanical application. It is to be encouraged if only because as he needs the mathematics the student becomes technically more competent in the freedom of his applying it.*

Elton (1971) emphasises that

> *The significant point about mathematical modelling and about mathematics as the language of science and engineering is that they arise at the interface between mathematics and the applied subject.*

For this reason, their proper treatment at any level requires the cooperation of the mathematician and the scientist or the engineer, as the case may be. Graham Flegg (1974) shares this view:

> *The students need, not a course in mathematical techniques, nor a course in selected mathematical topics alone, but an integrated course in mathematical modelling which will include some techniques, the mathematical concepts necessary for the understanding*

> *of such techniques and which will discuss mathematical modelling as such in the total context of the modelling process as a whole. The design, construction and execution of such a course requires a joint effort on the part of mathematicians and scientists and technologists: it cannot be successfully developed by any individual discipline. It must be planned from the start and operated throughout by a joint team of academics from both the serviced and the servicing departments.*

Professor Bajpai (1971), too, is of a similar mind:

> *Our own view is that syllabuses should be worked out in consultation with the various engineering departments, without sacrificing fundamental requirements in mathematics.*

Such integrated, interdisciplinary courses would not only provide the necessary motivation for students, but they would induce a disintegration of barriers of communication and language between departments and faculties.

There are no formal channels of communication between the departments of mathematics and engineering at the NSWIT, RMIT and SAIT, but there is rapport between the two departments on an informal basis. However, it is pleasing to report that the QIT has established rapport between the engineering and mathematics departments at both the informal and the formal levels.

8.2.3 How Should it be Taught?

Some mathematicians feel that mathematics should be taught by building up fundamental structures of mathematics axiomatically and abstractly, and that this discipline is self-sufficient. Other mathematicians feel that the mathematical structures should be reinforced by applications whose solutions use the structures to give answers that are analytical on the whole. Yet other mathematicians consider that the principles and techniques of mathematics should be taught with their applications in practice, without the artificial limitations of analytical solutions.

As far as the engineer is concerned, mathematics has two important functions. The first concerns his general education: the study of mathematics should teach him to reason properly, logically and with economy of thought, and to generalise prudently. The second function is as a useful working tool for tackling problems encountered in his discipline. This dual purpose needs to be constantly borne in mind and must influence curricula and teaching methods.

Engineers require not only a critical understanding of mathematical techniques, but also to see the applications of these techniques. What is needed, then, is the fusion of these two facets, by interspersing the applications with the theory.

It may be quicker to teach mathematics in terms of morphisms, say, for the simple reason that theories and powerful generalisations are the very means by which we achieve an economy of thought. The approach through established theories and other unifying concepts, however convenient, is not very helpful when it comes to applying mathematical concepts and indicating modes of thinking in ways that are going to be helpful to engineers.

Mathematics is a language by means of which knowledge finds expression and through which it is communicated. It is worthwhile to bear in mind, as a writer has pointed out, that being versed in the formal intricacies of grammar, regardless of purpose and meaning is one thing, being able to grasp the intended significance of linguistic statements is another, whilst using the language to express thoughts satisfactorily is the most difficult task of all. The use of mathematics as a descriptive language of engineering science is an art for which few serve an adequate apprenticeship.

With this caution in mind, we believe that the lecturer must be prepared to sacrifice for the time being such lofty ideals as generality, elegance and rigour, all of which are so highly esteemed by pure mathematicians, and instead interweave theory and application and prove rigorously those aspects of mathematics where it serves better understanding and overview. The plea is not for a traditional techniques course in which the student is able "to differentiate $\ln \ln \cos x$ by purely mechanical means without so much as a single thought as to whether the result of his labours is defined for any real number" or in which the student tries to integrate a divergent integral.

Professor L.C. Woods of the University of Oxford, as well as Associate Head of the School of Mathematical Sciences at the NSWIT, coining the catchphrase "Vigour, not Rigour", pleads for a pragmatic approach to the teaching of applied mathematics. "Applied mathematics", he reminds us, "is essentially interdisciplinary and a cross-fertilisation between the disciplines shows itself in almost every investigation". E.J. Richards (1973) says: "The value of mathematical methods is enhanced greatly by a truly realistic approach to mathematical modelling, and those who plan to use their mathematics in science, industry and commerce cannot afford to confine their talents to mathematics alone". This calls for close collaboration between the departments of engineering and mathematics if the mathematics is to be genuinely rooted in practical engineering problems.

The unprecedented expansion of the mathematical universe has meant increased compartmentalisation in teaching syllabuses. Separate courses by different lecturers are presented in analytical methods, numerical mathematics and statistical techniques. This process of excessive differentiation of function needs to be arrested and students should be made to appreciate the essential unity of mathematics. The minimization of division of mathematics into rigid, inflexible compartments should be encouraged through the use of an integrated approach to teaching, in which the purely analytical and statistical methods are considered in juxtaposition with those needing the use of analogue and digital computers. The simultaneous treatment of the analytical, statistical, numerical and computer methods provide the student with an opportunity to compare the merits and demerits of the various procedures. Each method has a place in an engineer's mathematical arsenal.

The parallel exponential growth in knowledge in other disciplines has resulted in heavy, rigid lines of demarcation between them. Yet the need for blurring the lines of demarcation between engineering and mathematics is urgent and pressing, if we are to motivate our students by lifting their sights to an appreciation of the commanding relevance of mathematicas to their engineering disciplines. Bajpai (1971) urges that applications should both precede and follow the mathematical development.

> *One or two 'motivational' lectures given before the mathematics of a particular topic is taught help the engineering student considerably, and give him a desire to learn the mathematics involved. Applications during and after the set of lectures further*

help the student in understanding and appreciating the topic ... We believe that lectures should be planned in conjunction with one or more members of staff from the engineering departments and be given a strong backing with carefully prepared tutorials, these being given by a joint team of staff drawn from the mathematics department and the engineering departments.

Bajpai *et al.* (1976) have called attention to the use of case studies as an effective teaching strategy in the mathematical education of engineers. Case studies have been successfully and extensively used in teaching law, medicine and business; however, in engineering education they have been used sparingly.

Case studies not only help to inject a note of realism in an engineer's education, but the solution of the problem presented by a case study generally requires a synthesis of mathematical techniques: it is never a problem that would fit into a standard mould to which a standard technique may be applied, but one which provides students with a growing appreciation of the need to knit together a number of concepts and techniques learnt in their earlier mathematics courses.

The case method of study provides

opportunity for a shared lecture scheme, whereby an engineering lecturer can present the problem in an engineering context, the mathematical colleague provides the solution to the mathematical model, and the solution is interpreted by the engineer.

(Bajpai *et al.*, 1976)

This teaching strategy not only serves to emphasise the strong ties between mathematics and engineering faculties, but also underscores the importance of interdisciplinary partnership for a more relevant, more significant, more meaningful professional education. The degree of success that this particular mode of teaching enjoys depends directly on the measure of rapport that exists between mathematicians and engineers. Here, as in all aspects of service teaching of mathematics, it is the measure of cooperation and coordination that exists between the serviced and servicing departments that is the exclusive determinant of success.

8.2.4 How Should it be Assessed?

Scott *et al.* (1966) make a powerful plea for open-book examinations. They write:

> *One of the criticisms we often hear frequently is that the graduate entering industry is conditioned to textbook problems in which the data are both 'necessary and sufficient', and that students have little experience in the selection of relevant information. While the introduction of projects into the course has done much to ensure that each student encounters at least one problem which is closely related to real life, the present methods of examination tend to perpetuate the current type of training. We should find out whether examinations test the qualities we wish to find in an engineer, and more important still, whether they encourage the development of such qualities. One may well question the wisdom of conducting examinations with closed books - a situation which is surely the antithesis of attitudes we are trying to encourage. It is more important that students learn to recognise a situation and know where to seek the relevant information than that they should become prodigies of retentiveness or walking encyclopaedias. The introduction of textbooks into examination would make the task of the examiner harder, but it would also encourage a different outlook among the students.*

The question of assessment in engineering mathematics is the subject of an extended discussion in a subsequent chapter. Here we simply state that no single method of assessment can be recommended: it is important only that the forms of assessment should be compatible with the aims and objectives of the mathematical education of engineers.

CHAPTER IX

AN IDENTIFICATION OF SHORTCOMINGS IN ENGINEERING MATHEMATICS TEACHING

Summary

This chapter dwells on those inadequacies which can mar the effective service teaching of mathematics and which need to be minimised, if not completely eradicated, in order to bridge the aims and achievement in the mathematical education of engineers.

9.1 INTRODUCTION

In no area of higher education is the expressed dissatisfaction keener than in relation to service subjects for undergraduate courses. There is a widespread feeling within and outside of the mathematics departments that all is not well. The users of mathematics in the other sciences, in engineering, in business and government, and in industry, whose need of mathematics has seen a steady escalation, complain that mathematicians remain indifferent and unresponsive to their pleas for assistance. Sir David Pye entitled his 1952 Presidential Address to the Institution of Mechanical Engineers very aptly when he called it "The Yawning Gap between the University Graduate and the Competent Engineer".

The general malaise in the mathematical education of engineers, in particular, has been admirably caught by Professor Bajpai's trenchant statement:

> *The teaching of mathematics to engineers is regarded as a necessary evil and the learning of mathematics by engineers is seen as an unnecessary evil. Lack of inspiration from the teacher means lack of motivation and response from the student. Examinations are set which merely require from the student regurgitation of standard techniques applied to standard examples. And so the system stagnates... The present situation does not match up to a set of aims and objectives that we think is desirable. There is a general opinion amongst staff of engineering departments that students do not easily see how to apply mathematics to the non-standard problems that arise in design projects during their scheme of study.*
> (Bajpai *et al.*, 1975)

The more we delve into the problem of providing service mathematics courses for prospective engineers (by talking to educators, by observing classroom practice, and by scrutinising the content and procedures), the more convinced we become of one thing: the felt deficiencies of the mathematical education of engineers cannot simply be explained by focussing attention on one variable, but by considering a complex of factors.

9.2 SHORTCOMINGS

9.2.1 Lack-lustre Teaching

Sir Samuel Dill, sometime Professor of Greek in Queen's College, Belfast, once remarked:

> *I am here to profess Greek, not to teach it.*

Such an utterance would strike a responsive note in the hearts of many academics who express themselves contemptuous of teaching. The infamous slogan "publish or perish" points to an attitude which dominates the spirit of most mathematicians: their highest loyalty is to their discipline and the only item of real significance in determining their prestige and promotion is the number of papers they publish. As a consequence, able and gifted academics pursue research almost exclusively and the teaching of service mathematics, viewed as a chore and an intrusion, is usually relegated to the young, inexperienced members, who present the course in an unimaginative manner. There is more than an element of truth in the aphorism attributed to Bernard Shaw: "Those who can, do; those who cannot, teach".

Teaching is viewed primarily as the feeding of information and knowledge. More often than not, engineering students are seen to be viewed as computers in which we store information so that it can be recalled upon certain signals. At best, students are exposed to facts, ideas and theories - an exposure which we do not criticise. What we do criticise, however, is the failure to conduct mathematics courses in ways that would increase the possibility that the knowledge acquired will be thought about and used in appropriate ways in subsequent problem-solving. What is so distressing is that often teaching proceeds in a manner which does not seek to develop in students new perspectives and habits of critical thinking, nor does it provide a secure basis for independent learning.

If curriculum is to be relevant and courses are to be taught meaningfully, academics need first-hand experience in the situations in which mathematics is deployed. This usually necessitates on the part of the lecturer an enormous investment in time so as to forge a link with one engineering discipline. The reward system in universities and CAEs needs to be dramatically revamped to give due recognition to excellence in teaching.

9.2.2 Inadequate Liaison

The communications system between the servicing and serviced departments is often in a precarious state and there is little or no feedback. The lack of effective interaction among engineers and mathematicians results in a hodge-podge of uncoordinated efforts acting at cross-purposes. There is a poor phasing of the mathematics in relation to the engineering disciplines in which they may be used, and the students are often wrongly blamed for not appreciating the mathematics involved in their other subjects. There is an apparent unwillingness and reluctance to assist the mathematics lecturers with the provision of meaningful applications of mathematical methods in engineering subjects. This absence of rapport is partly responsible for the dilettante interest in the teaching of service courses.

9.2.3 The "Cook-book" Approach

The teaching of service mathematics to engineers degenerates not infrequently into a sketchy "cook-book" approach. Unfortunately, the spirit of applied mathematics as expressed in mathematical modelling is noticeably absent. Those who teach service courses frequently seem to think that it is sufficient to teach a bagful of handy little tricks with little or no emphasis on creative mathematical thinking. The student seldom learns to "mathematize": the training of engineers does not develop in them the ability to venture upon the mathematical modelling of the vaguely defined and messy problems that are thrown up by reality.

The "cook-book" approach is characterised by the unthinking, repeated application of standard techniques to standard problems. The impression is conveyed that mathematical formulae can be applied almost blindly and unthinkingly to problems of engineering practice: these formulae, like simple recipes, can be used without any understanding of how they came about or why they work.

The reason why this "cook-book" or "plug-in" approach often succeeds in engineering is because the mathematical equations representing physical and chemical processes are such remarkably exact representations. For all practical purposes, the mathematical models parallel reality.

However, modern engineering practice increasingly demands a thorough, clear understanding of the mathematical basis of the formulae involved, as well as a sound understanding of their applicability and limitations. Moreover, the advent of the computer has made its notable contribution to the resolution of engineering problems in a manner that renders the cut-and-dried formula approach untenable: the simple rule-of-thumb methods are hopelessly inadequate, an ever-increasing mathematical sophistication being necessary to keep abreast with, and promote,technological advance. Unfortunately, graduates exposed to the "cook-book" approach of teaching are unable or afraid to take initiative in unfamiliar situations or to model ill-defined real-life problems.

9.2.4. Undue Emphasis on Rigour

Some lecturers, having little or no direct experience of applicable mathematics, place an exaggerated emphasis on topics of purely mathematical interest and tend to view mathematics as a discipline worth studying for its own sake, appreciating its beauty and elegance. There is an overemphasis on rigour, such as the need to prove existence theorems, rather than attitudes and ability to mathematize.

Implicit in the presentation and structure of many mathematics programmes has been the guiding principle which Beltzner *et al.* (1976) identify as "...the supreme good is a replication of the species mathematicus academicus". The effect of this is to direct the manner and substance of what is taught as though all engineers are destined to be research mathematicians or research and design engineers. The mathematician's approach to the teaching of mathematics is conditioned by his devotion to its logical and abstract qualities as well as his concern for ensuring an academic succession.

While the professional mathematician must concern himself with the foundations of his discipline, less rigour is required of those who use mathematical language to investigate, summarise and communicate. Professor Bayliss (1963) has admirably crystallised the flaws in mathematics teaching when he says sharply:

> *Our teaching of mathematics is dominated by the professional attitude and by the demand to solve puzzles; thus taught it repels and mystifies many who could be taught to understand mathematics as a means of communication.*

It is much easier to teach mathematical theories of purely academic interest than to teach students to tackle unfamiliar problems with zest and courage or to model poorly defined real-life situations. Also, it is much easier to examine students on whether or not they can solve a well-posed problem or prove a particular theorem than to examine them on their ability to formulate models of vaguely defined situations. All this is not surprising since academics seek personal enjoyment and fame among their peers through the single-minded pursuit of research in abstract mathematics and regard service teaching as an imposition.

The warning given by Beltzner *et al.*(1976) is salutary, when they say:

> *A topic such as the theory of complex functions, Galois theory of fields, convex programming of C*-algebras can have extraordinary elegance and the beauty of a Bach fugue. There is great joy for the lecturer in expounding and for the student in witnessing the development of such a topic. But contemplation of the beauty of great mathematics is not the active, demanding work of mathematizing. Indeed, contemplation can cut the nerve of action.*

9.2.5 Unimaginative Examinations

A major weakness lies in the methods of assessment, the most usual one being the end of session three-hour examination.

A general criticism of examinations is that they concentrate on those aspects of a subject which can be treated within the temporal limits of forty-minute question and three-hour examination. They are essentially artificial in the face of the normal requirements of a particular vocation or profession in that they test knowledge, not only in a fragmentary fashion, but also under abnormal constraints of time and availability of reference material.

The majority of questions call for the kind of manipulative skills that may easily be acquired by constant practice on similar problems or by attempting drill exercises. Absorption and regurgitation is the method which demands very little creative effort. Indeed, it is not unusual for students to protest at the inclusion of a topic in examinations that has not been fully treated in lectures and to study with the purpose of dishing back on examination more or less verbatim the material discussed. Exposition-regurgitation makes minimal demands on both the teacher and the taught and is therefore an irresistible universal temptation. This process fails to develop in students any confidence in their own ability and nor do they appreciate the meaning and significance of what they are learning.

This factor of meaning and significance of what is going on is an important one in the minds of students, and one which has been neglected. The need to integrate his learning into himself is critical. The diligent learning and faithful reproduction of facts, without knowing why, or what they have essentially achieved, or where this may lead them or help them in the future, is a practice which students should be seriously constrained to leave. Educators have a compelling obligation to make mathematics meaningful, relevant and significant.

9.2.6 Lack of Relevance

There is probably no more critical factor in productive learning than to perceive meaning and significance in what is taught. And yet, the most serious flaw in mathematics teaching is the failure to clothe learning with meaning and significance.

What is currently being questioned is not the standard of the academic aspect of the mathematics curricula, but rather the professional relevance to the engineer's future needs of much that is taught in the colleges, and the neglect in many cases to relate one element of the course to the others. The lack of a perceived relation between mathematical studies and engineering experience means that mathematics becomes a sequence of activities, denuded of meaning, purpose and significance.

The concern of most educators and learners derives from their recognition that mathematics courses, both in their content and teaching procedures, reflect the sterile scholasticism of the ivory tower, as though their existence *per se* may be justified. Little or no attempt is made to show the models of thought and experience peculiar to mathematics, nor by choice of topic and treatment to stimulate the imagination and show the subject in its personal and professional relevance. In defence of their failure to present mathematics in the nexus of engineering situations, quite a few lecturers unhesitatingly and unashamedly point out:

We lecture <u>in</u> mathematics, not <u>to</u> engineering students.

The nett result, unfortunate in the extreme, is that interest and motivation are quickly extinguished.

9.2.7 De-emphasising of Numerical Techniques

Analytical techniques have always enjoyed a pre-eminence in the mathematical education of engineers, while the importance of numerical techniques has been much discounted. The use of computational aids including the familiarity with digital computer techniques are not considered as critical as modern trends would suggest was desirable. Stressing the need for digital computer techniques to become an integral part of the education of an undergraduate civil engineer, Constantine (1972) indicates that a valuable by-product of designing a flow chart for a computer programme is that the thought processes involved are very similar to those undertaken in design work.

9.2.8 Excessive Compartmentalisation

The current technological revolution has made existing mathematical, scientific and engineering studies more intricate and sophisticated, has led to the establishment of new fields of study and has set up requirements for increased specialisation. The "spin-off" from the proliferation of specialised disciplines has been an increasing tendency for knowledge to be compartmentalised by inflexible demarcation lines.

An unhappy consequence of this is that studies are seldom integrated and students become blinkered to their own narrow learning. Subjects are not taught across conventional discipline boundaries and/or are not slanted towards understanding particular substantive issues from a variety of perspectives. Engineers are taught analysis at the expense of synthesis. The effective study of a problem is in some ways hindered when knowledge is fragmented artificially into subject departments.

It is usual for an institution to give separate courses on analytical methods, numerical methods and statistics, which are often taught by different members of staff. This arrangement tends to emphasize a rigid compartmentalisation of mathematics and, as a result, students are unable to synthesise their learning in their search for solutions to problems.

The advent of the high-speed computer and of problems which demand its use for their solution calls for a re-assessment of the mathematical subjects provided for engineers. The emphasis is now on general methods of wide validity, rather than on special solutions of more limited scope. This change of emphasis should be reflected in the way we present mathematics: we should emphasise the unity rather than, as at present, the diversity of mathematical methods available to the engineer and adopt the integrated approach to teaching to show that numerical, statistical and analytical procedures are all of a piece, a seamless robe. Indeed, to motivate the students and to answer their general call for relevance, the boundaries between engineering and mathematics must perforce be trespassed.

The mathematics staff are equally capable of listing their own perceptive criticisms of the manner in which mathematics is presented to engineers. Professor Bajpai *et al.*(1975) draw attention to two:

9.2.9 Relegation of Mathematics

The engineering departments may set out to undermine the significance of mathematics by discussing it in a cavalier fashion and stressing the importance of the final answer rather than the procedures used. The peripheral rôle of mathematics may at times be reflected in the schedule of assessment for a course. All this accentuates the isolation of mathematics and the feeling on the part of the mathematics lecturer that he is not regarded as being part of the teaching team.

9.2.10 Dearth of Suitable Textbooks

The production of suitable textbooks that emphasise mathematical modelling and an integrated approach to teaching, and that reflect a balanced emphasis on rigour, has, with a few notable exceptions, generally lagged behind.

To us, at least, the future is not as dark as it may appear. Engineering educators are becoming increasingly aware of the many inadequacies in the mathematical education of engineers. The only basis, then, for our optimism, if indeed it is not pure quixotism, is that the deficiencies which have been the focus of attention are being thought about and discussed in some CAEs.

CHAPTER X

MEASURES FOR EFFECTIVE SERVICE TEACHING OF ENGINEERING MATHEMATICS

Summary

To meet the demands of students for more relevant and imaginative teaching and the requirements of more effective learning, this chapter discusses the ways in which the CAEs may begin to reconsider and improve their methods of teaching and to modernize the curriculum where appropriate.

10.1 INTRODUCTION

Among the aims and objectives of teaching service mathematics to engineers, we have cited the importance of appreciation of mathematical modelling, the development of mathematical competence, the relevance of mathematics to the study of engineering disciplines and a balanced injection of rigour.

An examination of engineering mathematics teaching at some colleges of advanced education has revealed a number of serious shortcomings such as insufficient or, worse still, no stress on mathematical modelling, a failure to show intimacy between mathematics and engineering subjects by presenting mathematics divorced from reality, staff not being sufficiently *au fait* with the analytical, numerical, statistical and computer techniques to be able to demonstrate the unity of mathematics, and an undue preoccupation with rigour for its own sake, especially by those staff members who have had no engineering or industrial experience.

All of us in higher education have an obligation to develop and present courses that are defensible against such criticism. We recognise that it will not be easy for mathematics departments to give up all their long-entrenched procedures and modify their programmes and presentation so as to effectively meet all the desiderata suggested below by us; perhaps the most hopeful direction of effort lies in the internalising of some, if not all, of the recommendations into our teaching, if we are to arrest the drift to a sterile mathematical education.

10.2 MATHEMATICAL MODELLING

Lectures should be so slanted that they give a primary emphasis to mathematical modelling, dealing with the whole range of aspects from abstraction and formulation of a mathematical model to the solution and interpretation. "The student should be encouraged," urge Bajpai *et al.* (1975), "to appraise critically the usefulness and limitations of the models he encounters".

The stress on mathematical modelling will help to give the student a feel not only for the relevance of mathematical processes he learns, but also the spirit of applied mathematics; it is our view that the art of applying mathematics is expressed, in its quintessence, in mathematical modelling.

10.3 THE USE OF SPECIALIST LECTURERS

Courses should be organised on an interdisciplinary basis. This, of course, has implications about the way in which courses are planned and presented and should lead us to develop "planning teams". The planning team is a group of people, deliberately selected from various areas of interest, who operate on an integrated basis, both in the development and in the presentation of the courses.

"An engineering lecturer could set the background and lead up to the framing of the problem in mathematical terms and state what is required from the mathematics. His mathematical colleague could develop the necessary techniques and hand back to the engineer to interpret the solutions obtained." This is a course of action advised by Professor Bajpai.

This approach - the presentation of topics by a team approach, using the complementary strengths and skills - makes the man more convincing, more authoritative and more stimulating as a lecturer, and simulates the situation in industry which generally has multi-disciplinary teams of experts. The joint teaching approach helps to lessen the *ex cathedra* nature of lectures.

10.4 MOTIVATION

The mathematical techniques developed in the courses should be introduced, for motivational purposes, by relating them to an engineering application. The case method of study is a particularly effective strategy, for it permits a team approach to teaching and stresses mathematical modelling; we should provide case studies of real-life mathematical problems occuring in industry, which are neither too trivial nor demanding an unacceptably high level of sophisticated, mature mathematical thinking nor extensive engineering knowledge.

10.5 AN INTEGRATED APPROACH

The approach to service teaching of mathematics must at all times affirm the essential unity of mathematics: analytical, statistical, numerical and computer techniques should be given equal emphasis and should constitute the engineer's mathematical apparatus. The peculiar merits and demerits of a particular method should be indicated, enabling appropriate comparisons between methods to be made; wherever possible, guidance on the most apposite method for a particular type of problem should be given.

10.6 A BALANCED APPROACH

Although the engineer understands and appreciates the nature of a rigorous demonstration, he cannot be made inactive by these considerations. His primary emphasis is always directed toward the ultimate solution, and he frequently uses heuristic, scientific reasoning to achieve this end. In this way, an analysis might be made more amenable to solution, approximations introduced or arguments made plausible. His treatment is at all times responsible and disciplined, but he is not a deductive logician, interested solely in the beauty and the power of abstraction.

This is not to say that the courses should degenerate to a mere techniques course, without any emphasis on understanding. On the contrary, courses must reflect an academic stature and strike a balance between the rigorous, pure mathematics line and the "cook-book" approach. The strict axiomatic approach which is favoured by some pure mathematicians should be sedulously avoided but this is not to be equated with a plea for a dilution of the standard of mathematical argument.

10.7 CONTEMPORANEOUS SYLLABUSES

The education of the future engineers must be particularly sensitive to the developments happening in mathematics today and to the projected future developments.

If our aim is to produce the sort of engineers whose minds are open to future possibilities, new insights and who have a clear analysis of professional practice, it means a rigorous and continuous review of our curricula. Syllabuses should be subject to periodical appraisal and criticism and efforts for better programmes, syllabuses and courses must continue. This implies a flexibility of approach that can accommodate change as well as an awareness that change itself does not constitute progress. "In the development of courses", exclaims Phillip Hughes (1974), Head of the School of Teacher Education at the Canberra College of Advanced Education, "staff should see themselves as being in a fluid situation, with a continuing ebb and flow of invention, evaluation and revision ".

Thus, we need to cast away the strait-jackets of course structures, and examine whether certain topics have a justifiable inclusion on the syllabus and whether more recent and useful topics such as optimisation techniques do not exert a more valid compulsion for their incorporation into the teaching syllabus. One must avoid the danger of sterility by an over-emphasis of traditional subject matter. There is a compelling need for a redisposition of the elements of a course to obtain much more significant interrelationships between theory and practice.

10.8 IMPROVED METHODS OF ASSESSMENT

There seems to be no simple way of removing the stultifying influence of the examination system: those of us who have taught and examined undergraduates will know how barren and superficial knowledge acquired to pass muster at an examination can be. Yet, formal examinations can force students to tune their knowledge up to a certain level of precision.

Some form of assessment is obviously necessary for motivational reasons and as convenient tests of the possession of qualifications for vocational and professional recognition: the fault lies chiefly with the "sudden-death" three-hour examination. "It is high time," counsel Professor Bajpai and his colleagues, "to consider departures from the total reliance on end-of-session examination".

A regular assessment of the student's advance towards professional acceptability, based on open-book examinations that require the student to place less reliance on memorised learning and on projects which require the student to apply mathematical principles to practical, real-life situations, need not be less rigorous and may be more exacting than other evaluative instruments used in monitoring student progress. Yet at the first-year level there can be too much collaboration, and at this level there are certain points which these students need to know inside out so that they can have them at their finger tips and operate efficiently later.

10.9 HIGH LEVEL OF RAPPORT

The *de facto* policy of most mathematics departments is one of isolationism: sometimes there appears a great gulf between the mathematics and engineering faculties. "The problems of the CAEs are in one sense the same as those of any other tertiary institution", writes J. McLaren (1974): "To overcome them we will need to recover the universality which was inherent in the original meaning of university, and should be inherent in any education. This must occur first at the staff level where a spirit of cooperation must be developed that leads to each individual both developing and sharing his special interests ".

The teaching of service mathematics to engineers should be viewed as a cooperative educational enterprise, for only when there is a real partnership between the schools in the pre-service education of engineers will there be a real hope of linking theory and practice in a meaningful manner.

There is general agreement that interaction and liaison are absolutely crucial for the design and presentation of courses as well as for the effective deployment of mathematics: only with depth of understanding of mathematical structures and processes *and* real contact with other disciplines can one do meaningful work in engineering. An invaluable by-product of such close ties would be the provision of fresh, real-life, less-laboured examples to highlight the ubiquitous relevance of mathematics.

There exist several practical mechanisms which facilitate cooperation and interaction between the mathematicians and the users of mathematics:

(a) Fostering interchanges of personnel or joint projects;
(b) Establishing committees and special seminars involving industrial organisations, mathematicians and engineers;
(c) Encouraging academic mathematicians to spend leaves of absence and sabbaticals in industry or in engineering departments;
(d) Inviting industrial experts and engineers as guest lecturers for seminars and workshops in mathematics departments;
(e) Encouraging lecturers to work in other milieux before becoming academic mathematicians;
(f) Recruiting mathematics staff who have sound, solid mathematical background, but whose primary training is in another field;
(g) Encouraging mathematicians to undertake consultancy work; and,
(h) Providing opportunities for academics to obtain multi-disciplinary qualifications, such as a bachelor's degree in engineering followed by a master's in mathematics.

10.10 SMALL GROUP SITUATIONS

The quality of higher education can be enhanced by an extended use of tutorial or small-group methods. Tutorial services, by providing the closeness of liaison with staff to which matriculants have been accustomed, can and probably do more than any other agency to reduce the bewilderment and uncertainty of a number of first-year students as well as to ease the pain of transition. "Here", asserts Frederick (1958), "with a small group, with three-way communication - tutor to student, student to tutor, and student to student - are conditions ideal for teaching".

Sureties (1974) argues the case for small group situations:

> *Although the lecture is an economical, and may be effective, vehicle for some aspects of the teaching process, it is essential that a good deal of learning should be handled in the 'small group' situation. This not only ensures close contact between staff and students - with all that this implies in terms of involvement, immediate feedback, early identification and treatment of learning difficulties, student awareness of staff concern - but also allows the development of learning patterns based on the general method of discussion.*

He may have added that if the small group consists of students from a particular engineering department the task of providing model examples germane to that particular branch of engineering becomes easier.

One can use final-year students as tutoring assistants to make small group work economically viable.

CHAPTER XI

EXAMINATION OF EXAMINATIONS IN ENGINEERING MATHEMATICS

Summary

The techniques of examining and such technical characteristics as validity and reliability are discussed. This chapter shows that to ask which of these techniques is the best is to pose a false question. The point is made that an important prior requirement of an educator, who is contemplating the use of a particular evaluative instrument, is an understanding of the aims and objectives of the mathematical education of engineers: assessment techniques have to be selected not in isolation but in relation to the purposes of education. The art of examining seems to be a sadly neglected aspect of lecturer education and deserves careful attention.

11.1 INTRODUCTION

In education, as in most other spheres of purposeful human activity, it is necessary and desirable to assess periodically the progress that is being made towards objectives and goals, and to determine whether such progress is sufficient or satisfactory.

Evaluation is an integral part rather than the end-point of an educational programme. Hall emphasises that examinations are not an external feature of the system:

> *Examinations are not extrinsic to the process of teaching and learning, education and training; they are an integral and inseparable part of these processes, interlocking with them.*
> (Hall, 1971)

To contribute fully to education, assessment procedures must be carefully planned. This implies that examiners must themselves be clearly aware of what they want to do and why. The answer to the question "Why do I want to test?" usually indicates the type of test to be used. Having decided on the aims, one still has to make a decision on the skills and abilities to be tested in a particular examination. Once all of this has been determined, it is then, and only then, possible to consider what type of examination instrument or combination of instruments is most suitable for one's requirements.

According to H.G. Macintosh (1974), the basic components of any assessment are four in number:

(a) The objectives of the course proposed for assessment.
(b) The skills to be developed and the content to be covered - the syllabus.
(c) The techniques of assessment to be used.
(d) The weightings of the various parts of the assessment.

11.2 EDUCATIONAL OBJECTIVES

The first task of teachers, individually or in subject panels, is to state, in clear, precise terms, the objectives of their courses: decisions taken with skills, content, form of assessment and weighting must flow from a consideration of these objectives. Hall (1971) declares that

> *The crucial question is not what kind of a test, but a test of what? What do we want to measure? Can we test observable and measurable qualities of behaviour which express the qualities we want to see in our good students? A long list will imply a more diverse system of testing than we usually use. Thus, knowledge can be of facts, technical terms, conventions, trends, categories, criteria, principles, generalisations, theorems, ideas; whilst abilities and skills can include comprehension, understanding, ability to organise and to extrapolate; application of general principles and theories to old and new situations; analyses of problems; synthesis of facts and ideas and production of plans; evaluation and judgement; and computation.*

Thus examinations follow rather than determine the content and the methods of teaching. Evaluation is most effective educationally when it is related closely to the educational programme.

When the teaching process is directed towards the examination instead of the educational objectives, when tests are based on curricula only and not on the objectives of a course, when teachers become too impressed with the importance of examination results, then examinations begin to exert a pernicious influence on teaching and learning: content and methods of teaching experience a constricting influence. Scorer (1971) underlines this distortion in teaching when assessment procedures are not related to our educational aims:

> *Once as an external examiner, I criticised a particular question on Bessel functions as being rather pointless and setting wrong objectives for the course. The setter replied by quoting the recent occasion on which the identical question had been set in a London honours examination. So you have a syllabus containing the phrase 'Bessel functions': the lecturer looks through past examination papers to discover what this means; if he teaches his class to be able to do all the questions set on Bessel functions, who is entitled to complain? Thus it happens when a pure mathematician is set to teach engineers! The functions were defined as infinite series because the lecturer saw this the most convenient logical starting point for the purpose of the questions he observed to be conventional; and the students never knew their relationship to the sine of a sine, or their origin in cylindrical physical systems.*

T. J. Rogers (1974), bemoaning the dichotomy that exists between examinations and education as a sad consequence of examinations exerting undue pressure on what we teach and how we teach, makes a trenchant comment on the (English) secondary education which has a relevance to our own secondary education:

> *The development of examinations has been such that, despite the stirring and high-sounding claims of educationalists, the history of our secondary education in this century is, in no small*

measure, the history of certain examinations.

However, as I. J. Goddard (1973) reminds us: "Examinations, of themselves, are neutral: it is the way they are used that carries a beneficial or perjorative effect".

To ensure a greater degree of coincidence between teaching objectives and assessment objectives, it is essential that the statements of objectives for both teaching and assessment be made in a form which makes them susceptible to observation, description and assessment. The need for precision in the statements and for a finely structured taxonomy of objectives is imperative. As long as educational objectives are not specified in operational and measurable terms, the criteria employed are bound to be very general and subjective, and therefore unsatisfactory, though this should not preclude the inclusion of less tangible, less readily measurable affective aims.

Thus no one can gainsay that there are many intangible, long-term outcomes of instruction. The University Grants Committee (1964) believed that the primary aim of a first degree course should be "to train and equip the student to think for himself and to work on his own". The Colleges of Advanced Education would be hospitable to the whole range of purposes and activities of university education that Sir James Mountford (1966) has in mind:

> *This brings us to what is the most important point about the nature of teaching and education at a university level. It must necessarily provide the student with a body of positive knowledge which enhances his store of learning and in part equips him for his career in later life. But it also has another and more notable attribute. It inculcates in the student an attitude of mind which regards the critical assessment of facts and values as more important than dogmas, and which holds that a grasp of underlying principles is more valuable than the accumulation of information or the acquisition of skills and techniques. A university expects that at the end of their courses its students will not merely be able to comprehend the extent and significance of what is already known within their own field, but will be receptive to what is new, eager to explore it, show the ability to cope with it and - above all - be able to work confidently on their own. By entering a university a student has undertaken to accept a rigorous intellectual discipline and to be more than a passive receptacle for information, much of which in many subjects may be out of date within twenty years. To the limit of his capacity he is trained to collect evidence for himself and form a balanced judgement about it. He fortifies his ability to think for himself; he refuses to accept orthodoxies simply because they are orthodox; and when*

he dissents, he does so on the basis not of prejudice but of reason. This is what good teaching achieves in a university.

How does one translate into operational and behavioural objectives such long-term and highly philosophical aims - to produce such intellectual capacities and habits and attitudes as a concern with knowledge as a means to understanding, with enquiring or exploratory habit of mind and the faculty of critical discrimination - into immediate realities through the medium of an academic discipline? How do we examine the students in such a way that we can determine whether or not the students have acquired these qualities at the end of the course?

The short answer is that it is impossible to measure such long-term, intangible objectives in a formal written test. In practice, a sensitive educator is continually observing the behaviour of his pupils for signs that he is transmitting these intangibles. The further a teacher moves from the limited field of objective tests in an endeavour to test more subjective qualities, attitudes and interests, the more difficult he finds it to make an objective estimate. To find out whether as a result of his training in mathematics, a student is more cooperative, more self-reliant, more ready to submit prejudices to the tests of reason is an entirely difficult and different problem. In the last resort no quality can be adequately represented by a number, a letter, or a percentage. Yet unless certain qualities have in fact been developed, the teaching of mathematics, or indeed any other subject, has been abortive. Whether a student derives any educational benefit from his course or not will depend on the way in which the syllabus of his course has been framed, and on the nature of the questions customarily set in the examinations with which it ends, as much as on the way in which he has been taught.

The problem for those who determine the scope and character of examinations and who sit and mark them is to avoid creating a conflict between what the career-minded student regards as his interest and what an enlightened teacher would regard as educationally desirable. The student whose main object is to pass his examinations should of course be put into a position to do so, but he should be unable to avoid being educated in the process.
(University Grants Committee, 1964)

This suggests that it is very necessary to formulate short-term behavioural objectives, which are assessable more readily with precision and clarity. The phrase "to teach students to think mathematically" is imprecise and vague. Flett (1973) wonders if thinking mathematically simply means

> *'to think logically'; for example, to know the difference between necessary and sufficient conditions and between 'if' and 'only if', to know the permissible ways of proving 'p implies q' and to be capable of logical deduction, step by step, from carefully stated premises. Or is there something more? Does 'thinking mathematically' refer to the ability to apply mathematics to physical, biological or economic problems - the ability to translate the problem into mathematical terms and to interpret the solution in terms of the original problem? Or does it refer to that mode of thinking which combines reliance on one's mathematical intuition, an intuition sharpened by training and guided by investigations of many particular situations, with the ability to give certainty to the intuitive ideas by the creation of a logical proof? And, equally, to what extent can students be taught to think mathematically, and how far can one assess by examinations the students' success in learning to do so?*

The difficulty in measuring affective aims does not mean that no attempt should ever be made to gauge them, although such attempts have to be exercised with caution in that they are more likely to intrude into the educational process than attempts to measure in the cognitive and psychomotor domains.

This clarity and precision, this fineness of structure in the statement of objectives is achievable in practice by answering three questions:

(1) What do we want to do?

(2) How can we achieve what we want to do?

(3) How can we assess whether we have achieved what we want to do?

What is required is a statement of objectives which stresses the essential interrelationship between content and skills.

11.3 TAXONOMIES

Many educators engaged in the formulation of statements of aims and objectives find that a taxonomy of objectives is helpful since its usage compels a more systematic approach to the classification of educational objectives. A number of taxonomies have been published recently, the first and best known of these was prepared by Benjamin Bloom and associates. The original publication dealt with the cognitive domain, grouping in ascending order of complexity six major classes of cognitive objectives: knowledge, comprehension, application, analysis, synthesis and evaluation.

R. L. Ebel has suggested a somewhat simpler taxonomy:

(1) Understanding of terminology.

(2) Understanding of fact and principle.

(3) Ability to explain or illustrate (understanding of relationships).

(4) Ability to calculate (numerical problems).

(5) Ability to predict (what is likely to happen under specified conditions).

(6) Ability to recommend appropriate action (in some specific practical problem situation).

(7) Ability to make an evaluative judgement.

For the mathematical education of engineers, the following taxonomy may provide a stimulus to thought about educational problems and objectives as well as a source of ideas for teachers, both in teaching and assessing:

A. Knowledge and information: definitions, notation, concepts.

B. Techniques and skills: solutions.

C. Translation of data into symbols or schema, and vice-versa.

D. Comprehension: capacity to analyse problems, to follow reasoning.

E. Inventiveness: reasoning creatively in mathematics.

The value of a list such as this is that one can use it to clarify what are the most important abilities to be developed and therefore to clarify what should be evaluated. This underlines the point made earlier that assessment should be placed at the end and not at the beginning. A taxonomy need not be regarded as prescriptive, nor should one necessarily follow it comprehensively, but it can serve the function of sensitising a teacher to objectives so that if some are omitted this is based on a conscious decision and not ignorance.

11.4 FUNCTIONS OF ASSESSMENT

Examinations serve different functions. According to Flett (1973), the primary purposes of examinations are "gate-keeping" and "labelling". Examinations are used to provide information about students: information that may be used by teachers to determine the fitness of the student for a subsequent course, either at school or at an establishment of higher education; information that may aid in the determination of a student as an acceptable entrant to certain professions; information that may be used by employers for recruitment and in allocating an individual to a particular niche in industry or commerce.

The gate-keeping function is synonymous with the selection or filter function of examinations. The principal gates, as far as engineers are concerned, are the Higher School Certificate and the final degree examinations. The labelling aspect of examinations aims to provide a criterion of the fitness of a candidate for the next stage of his career: the HSC and final degree examinations perform labelling rôles as well.

Examinations may be used to obtain a measurement of attainment or achievement in keeping with the aims of the course. These attainment tests seek to determine how much knowledge and understanding of a particular subject a student has and how far he can apply it. Some attainment tests, such as the Matriculation and Higher School Certificate Examinations, are used predictively by universities and colleges of advanced education to select students who are likely to benefit from tertiary studies.

The secondary purposes of examinations are:

(a) motivational. Examinations are used both as a stick and a carrot. They provide a reward and a sense of achievement for the disciplined student; for the less disciplined, they provide a form of compulsion to study; and,

(b) feedback from the teacher to the student about the student's progress and from the student to the teacher about the effectiveness of his teaching. A further aspect of this feedback is found in the examination papers. To the lecturer, past examination papers can give a glimpse of the significant topics and an insight into the required standard. To the student, the papers clearly indicate the more significant parts of the syllabus, the degree of emphasis given to the derivation of results as opposed to their application. In view of this, it seems very desirable that examiners should decide what are their aims of teaching a particular subject and indicate to the students the concepts, skills and qualities which are to be fostered and reflected in examination papers.

11.5 DIFFERENT TESTS FOR DIFFERENT PURPOSES

The discussion about functions of assessment makes it clear that different types of tests are likely to be needed for different purposes.

Godwin and Shannon (1975) are highly critical of the many different uses that are made of the results of an examination. They declare:

> *Taxonomies of objectives have rightly received plenty of attention in the past decade; yet, one still sees tests which are apparently meant to be servants of many different and perhaps incompatible masters simultaneously. For instance, a test of the suitability of a particular course might not be appropriate as a measure of some aspect of a student's knowledge or of a teacher's effectiveness, or as a diagnostic test; or, to what extent should a school leaving test correlate with an aptitude test or university examinations in the same subject?*

The failure to formulate explicitly the objectives of a course and to clarify the rôles of the methods of assessment chosen is reflected when an examination designed for one purpose is used at the same time for another purpose. One clear implication of the integration of evaluation with teaching and with the general flow of education is that a range of assessments need to be made.

Personal interviews as well as a perusal of the latest handbooks reveal that one of the clear trends in examinations in engineering mathematics in the CAEs is an increasing diversity in methods of assessment and examination. The assessment of students on the basis of a single three-hour paper and nothing else, in such mathematical subjects as calculus, multivariable calculus and ordinary differential equations, is fast disappearing. Academics are searching for a wider range of measuring instruments that can give a broader and fairer assessment of the student in relation to the aims of the courses they present. They have begun to heed Dunn's (1964) advice:

> *Arguments about the merits of objective answer versus essay-type answer questions, or the virtues of external examinations versus accrediting, cannot be conducted in general terms but must be related to specific situations. When the situation is specified, the either-or nature of the argument is often found to be a false antithesis.*

There is no reason at all why an examination should be conducted by only one method of assessment. As the emphasis in engineering mathematics shifts from skills in performance to a fuller grasp of the principles underlying the subject, the assessment of a candidate's understanding may well call for techniques of examining which differ from those needed to test performance skills. A good examination is surely likely to employ more than one method of assessment.

> *A variety of test techniques (essay questions, multiple choice questions, questions requiring short answers, questions requiring longer answers), combined with internally assessed course work (controlled assignments (that is, short undertakings closely controlled by the teacher), experimental investigations (that is, longer ones with more discretion for candidates),projects (that is, major undertakings with maximum freedom on the part of the candidate in planning and operation)), can be used to good effect within one and the same subject examination.*
>
> (Christopher, 1973)

11.6 PROFILE

People have many different attributes. There is a range of concepts, skills and differing qualities which the mathematical education of engineers seeks to nurture. No single test could adequately assess a candidate's attainment, understanding or potential in mathematics. Nor can a single statistic encompass a meaningful indication of a student's stage of development and achievement, his strengths and weaknesses. Rather does a valid meaningful description of his mathematical development depend on a profile of assessments and of professional judgement of his progress in garnering for himself the skills and abilities and methods that should be part and parcel of an engineer's mathematical apparatus.

Assessments should be descriptive of students' differential strengths and weaknesses. The expression of a candidate's performance can be better delineated by evolving a system of "multi-dimensional" reporting of examination results, a system which has become generally known as profile-reporting.

Stroud (1973) feels that the traditional methods of recording results in terms of simple pass or fail or a more refined grading system are inadequate when the range of methods of assessment has widened considerably in recent years. He says:

> *The limitations of single-scale reporting of the traditional kind are likely to become even more noticeable in the context of the wide range of assessment procedures and the wide differences in curricular approach which are certain to characterise the new situation.*

11.7 VALIDITY AND RELIABILITY

Since assessment plays so many important rôles in the teaching/learning process, it is vital that it be as effective an instrument as possible. By what criteria can the quality of an assessment technique be judged?

One criterion might be content. Did the questions come within the scope of the syllabus? Another might be difficulty. Were the questions too easy or too difficult? But above all, it must be asked whether the questions measured the objectives or desired outcomes of a particular course of study. Assessment and evaluation are seen to form an integral part of the learning process, and an examination can be fully valid as a description of what has been learned when it is structured to ensure that it can reflect, and allow assessment in, as full a range as possible of the aims of the teaching. If the purpose of a test is to measure the student's understanding of mathematical modelling, it is senseless to provide him with a test that claims to assess this understanding when really testing mathematical techniques. Validity or suitability for a given purpose is the most important characteristic of an evaluation instrument. The lack of relevance or validity destroys the value of any measuring procedure, despite the presence of other favourable features.

Many tests and assignments have fairly low validity. At the heart of the problem of low validity, there are two explanations. The first is that we, as examiners, are often unclear about what it is we are trying to measure. This is tantamount to saying that the objectives of courses are sometimes unstated or couched in terms which make them unintelligible or at least immeasurable. The objectives of a course may be quite involved but the more finely the objectives are structured the more likely it is that valid assessment of the student's achievement will be possible.

The second explanation of low validity is that goals set are neither realistic nor susceptible to measurement. No matter how noble an aim may be, if it is unattainable or unmeasurable, it is not much use in assessing student performance. However, as Godwin and Shannon (1975) remind us, an "overemphasis on behavioural objectives may cause the neglect of the wider issues at stake".

There is a natural tendency to concentrate on teaching those things that can be satisfactorily examined in a normal examination paper. This is why many mathematics papers in engineering studies are studded with questions requiring the application of standard techniques to standard problems.

A second characteristic to be considered is reliability or consistency. Any measuring device used must be reliable. Provided that a situation remains relatively unchanged the measuring instrument when used repeatedly should produce similar results. Thus reliability is a matter of experimental error: obviously, in every experiment we are concerned with not only to reduce the error of our result but also to have some reasonable idea of the accuracy of that final result. The following factors are useful in improving reliability:

(a) Make the measure as objective as possible, with as small a range of marks as possible.
(b) The same operator should be used with the particular instrument to reduce inconsistencies. This means that a particular test item or assignment should be marked by only one person.
(c) Do not rely on a single measurement. The implication of this is that progressive assessment procedures are to be preferred to single examinations at the end of courses.

Ease of construction, administration and correction are other considerations that must be taken into account when designing a test, for , despite its importance, evaluation is only one of an educator's tasks. However essential it is to keep the time involved in planning and correcting at a minimum, the measuring instrument's relevance and consistency must be maintained.

11.8 PROGRESSIVE OR CONTINUOUS ASSESSMENT

The measurement of a student's attainment, when restricted to the consideration of evidence based wholly on a written script or scripts completed in time-constrained situations on certain dates, can hardly be accurate and dependable. (All of this is an aspect of sampling, and so the errors associated with it should not be ignored.)

Mathematical attainment, as far as engineers are concerned, has many dimensions. The outcomes of courses of instruction demand more than the pedestrian ability to recall and use information, but rather encompass the acquisition of a broad spectrum of abilities and skills and the development of attitudes that are part of the "art of engineering" — the discernment of an underlying structure in a physical system, the skill in rapidly grasping the essentials of a problem by a specialist in a problem field and translating these into mathematical language, the development of a "mathematical discretion" that leads to effective decisions in simplifying models, the choosing of optimal solution methods, appreciating the significance of the solution obtained and making valid inferences from it. Obviously, no single test can enfold such a range of outcomes: the need for multiple measuring instruments becomes very plain.

Mathematical subjects are susceptible to variations in the performance of a candidate. Some of the sources of this variability are: sample of knowledge examined, variable difficulty of the questions set, unequal penalties for errors in scripts, tension induced in the examination, and the physical and psychological states of the candidate. In addition to the sources of variability in the performance of students listed here, Scott (1973) says, "there are sources of variability in the performance of the marker and interaction between student and marker, for example 'halo effect'".

No examination result should be used in isolation as a decision-making basis: the best evaluation of a pupil's progress is made up of a large number of separate judgements. "Assessment that is periodic or continuous, rather than terminal", Stroud (1973) asserts, "reduces the dangers of chance failure or success by increasing the number of occasions on which an assessment is made". Thus, continuous assessment gives the teacher an opportunity of building up a growing, documented description of each pupil's experience, understanding, achievements and attitude.

Western Australian Institute of Technology has undertaken valuable pioneering work in the adoption of continuous assessment in electrical engineering; it is used at South Australian Institute of Technology, Royal Melbourne Institute of Technology, and the New South Wales Institute of Technology and forms part of the final assessment of engineering students in mathematics. What is meant by continuous assessment? As a term of art it is more amenable to description than to definition. An information leaflet of the West Yorkshire and Lindsey Regional Examinations Board (Macintosh, 1974) puts it thus:

(a) It represents continuing awareness, by the teacher, of the development and knowledge of his pupils; it is a process which extends over a period of time; the gradual build-up of a cumulative judgement about performance.

(b) A teacher making use of continuous assessment is looking for signs which show the growth of thinking processes and the development of those varying abilities towards which the teaching is aimed; he is more concerned with signposts than with the whole itinerary.

(c) An end-of-course examination will test achievement at one point of time within the limits of the test; under continuous assessment there is knowledge not only of this achievement but also of progression towards it; not merely of where he has got to but also of how he got there.

Continuous assessment provides a reasonable feedback mechanism that gives the student a reliable guide to his progress and enables teachers to monitor student progress. Rogers (1974) declares that a comparable procedure to continuous educational assessment is to be found in the study of control systems:

A missile, for example, which 'homes' on the target does not, necessarily, move along the path initially planned for it. Its course is constantly modified in accordance with 'feedback' information it provides its controllers. The missile's programme, in turn, receives information which makes it take compensatory movements. A similar process can be recognised in the sequence of learning experiences provided to bring about a pupil's attainment of the knowledge, skills, and abilities his course intended he should acquire. The objectives must be clearly recognised by the teacher and by continuous evaluation of the pupil's progress, the teacher is provided with the feedback information which is necessary for individual adjustment.

The feedback facet of continuous assessment implies that the teacher's focus is shifted from the class as a single entity and directed towards developing the full potential of each individual to his/her fullest level of human worth. This is particularly important when teaching part-time students, who represent a very sizable proportion of those pursuing engineering studies at Australian CAEs. The continuous assessment should be so planned to permit the early identification of potential mathematical casualties before they give up working on the course and provide them with tutorial help and remedial work to encourage them to persist in their undertakings.

Continuous assessment demands steady work by a student during a course rather than a burst of intense activity toward the end with the traditional examinations. It also permits the observance of qualities such as persistence, diligence and enthusiasm. To say this, however, does not mean that we advocate the abolition of terminal tests, as they permit the synthesis of diverse topics.

Sellers (1973) calls attention to the positive value of coursework assessment in the new and developing areas of interdisciplinary studies. He says:

> *It is possible, by the judicious use of teaching teams of lecturers from different disciplines, for a student to be guided on the interleave between two specialist disciplines and to develop further this necessary link through the medium of coursework assessment, backed by seminar discussion. In the areas of engineering systems and mathematics there has been a great deal of work done at the Lanchester Polytechnic between applied mathematicians, statisticians and mechanical and production engineers, so the students who are reading the Honours Degree in Mathematics in which there is an essential stream of engineering systems work, readily can see for example the necessity for the development of automatic control techniques in the mass production of various engineering parts, such as machine tools. The work between the statisticians and the production engineers has not only led to an interesting coursework development and assessment of students' ability, but also to a cross-fertilisation of work between the two departments, which in itself has led to a number of successful research projects.*

Thus lecturers and students concerned have been the richer as a result of the practice of group presentation of coursework problems between the lecturers of the two departments and the assessment of the coursework has been enhanced by its subsequent use as the basis for seminar discussions in the interdisciplinary area.

The technique of continuous assessment is not as simple as at first appears. The procedure still lacks the rigour and definition which will make it widely acceptable and give it equal status with the more conventional techniques usually used as end-of-course procedures. The first difficulty concerns the problem of devising a continuous assessment component which will give a reliable measure of student progress: the grades that are achieved must mean something when viewed as a contribution to the student's grade for the course *and* they must provide feedback. The second difficulty is associated with the necessity of producing high quality questions that reflect not only the substance of what we teach but the spirit in which it is taught.

There are one or two negative features of coursework or continuous assessment. It can be a laborious and demanding activity for the teacher.

"The advantages it has", Stroud (1973) declares, "must be weighed against the increased demands made when the teacher and examiner are not only the same person but are carrying out the two rôles simultaneously".

The second disadvantage affects the student. One of the effects of progressive assessment has been to put students under considerable pressure all the time. It is very easy to arrive at the position where the "tail" of assessment begins to wag the "dog" of learning: that is, assessment becomes the all-important objective and all else pales into insignificance. This possibility is real and calls for good judgement on the part of lecturers if it is to be kept in check. We can hardly afford to improve the statistics of our assessment at the price of removing all incentive to learn anything not immediately assessable: an open-ended commitment that centres on his assessment questions rather than on the rest of his study materials needs to be studiedly avoided.

11.9 OBJECTIVE TESTS

Since 1972, multi-choice tests have been incorporated into the major mathematics subject, Mathematics 1A, which is the basic first-year mathematics subject in the engineering and applied science courses at the South Australian Institute of Technology. Eight multi-choice answer tests were given throughout the year, on approximately a three-weekly basis, and run during the tutorial period. These multi-choice tests contributed to the final assessment and there was a good correlation between multi-choice testing and other forms of assessment.

An examination syllabus can often traverse considerable ground, necessitating a wide range of questions to be set and allowing students to make a choice. Thus the use of optional questions encourages neglect of some aspects of a given course. This is not without its pedagogical implications, as Dunn (1964) reminds us:

> *In subjects where sequential content is important, as in mathematics and the sciences, examination papers which employ optional questions are likely to produce students who have studied intensively some parts of the course only. This makes it unwise for a lecturer to proceed on the assumption that the students have a common minimum knowledge of all areas of the syllabus for the preceding year.*

Again, if tests are being used to compare the ability or attainment of pupils, it must be borne in mind that any choice of questions implies that different pupils are in fact taking different tests.

Educators will recognise that the precision of an examination, if the quality of each question is the same, is related to the size of the sample, namely the number of questions used. Objective tests make it possible for a relatively large number of questions to be answered in a single paper: this in turn permits the sampling of a wide range of subject matter and abilities. This makes for a more reliable examination since the element of chance in a candidate having only questions that he can answer well is reduced. Also, the examination is more valid since the assessment is based on the whole syllabus rather than a part of it.

Apart from the fact that a much wider range of the syllabus can be sampled than is possible by the use of conventional papers, objective questions can secure greater detachment and objectivity on the part of the teachers in relation to the assessment of their pupils. Since there can in each instance be only one acceptable answer, the marking of the scripts is not burdensome and problems of examiner reliability do not arise: indeed, an objective test can be scored mechanically.

It is not to be assumed that all objective test items are necessarily multiple-choice; some elements of an objective test can be constructed as short answer items, true/false items, multiple completion items, situation matching items, and assertion/reason items. The first question of the Multivariable Calculus Examination paper, which is taken by all engineering students at the New South Wales Institute of Technology, consists of short answer items covering the whole syllabus.

The process of pre-testing, that objective tests facilitate, helps to identify those questions which are faulty or ambiguous, too easy or too difficult, or with distractors which may be readily discarded by the weaker students. Thus poor questions can be eliminated and this makes for a much more reliable paper.

Every form of assessment has limitations and objective tests are not immune to disadvantages. Although their introduction reduces the burden of the examiner in the process of marking, they are much more difficult to prepare: the writing of items is a highly skilled task. The building of item banks makes the compiling of tests for future use comparatively simple.

One of the arguments that is put most strongly against objective testing is that this procedure cannot be used to test all skills and abilities that need testing. For instance, coherence of expression certainly cannot be tested, nor the ability to write creatively. "The backwash effect on the curriculum", Stroud (1973) cautions, "where all testing consists of underlining, ringing one of five possible answers, or one word completion, might result in the atrophy of writing facility". Also, objective tests are expensive to produce and susceptible to guessing.

However, the most serious defect of objective testing is that it is difficult to assess synthesis. A successful accumulation of facts is no evidence of capacity to use a series of skills or processes, no proof of the ability to integrate those facts into a meaningful comprehension of the subject matter, nor of the ability to communicate the understanding one has achieved. Shideler (1963) comments:

> *A major problem in testing is that of determining the extent to which the student has learned to integrate the material into a meaningful whole. Even though it can be argued that a sufficiently detailed true-false or multiple-choice test does indicate whether or not the student can relate one item to another ..., such tests give few clues to the student's difficulty in integrating the material.*

There is a paradox associated with objective tests, since objective tests are subjective. The adjective "objective" in "objective tests" pertains to the marking. There is still a large subjectivity in the composing of these tests.

> *The examiner decides what parts of the syllabus to sample; he decides the way in which questions should be phrased and the ease or difficulty of the questions; he decides the way in which the correct answer should be stated. All these are subjective decisions made by the examiner and there is no way in which all subjectivity can be excluded from this type of examination.*
>
> (Goddard, 1973)

As part of the final assessment of the mathematics course and augmented by the more conventional types of tests, objective tests have a valid rôle in the mathematical education of engineering undergraduates. The long-term goal of any multiple-choice objective paper is to achieve a valuable diagnostic test which not obly measures student performance but provides a basis for helping students with their difficulties and for suggesting programmes of remedial work.

11.10 PROJECTS

The skills acquired by most applied mathematics and engineering undergraduates fall below expectations, such as the abstraction of the engineering problem, the formulation of a mathematical model, the progression to a mathematical solution, the development of a mathematical discretion that leads to effective decisions in the simplification of a model, the selection of an optimal solution method and the critical evaluation of the solution so as to extract the maximum of interpretation and inference.

Speaking from personal knowledge, Clements (1978) says:

> *It is my experience that such students compartmentalise their learning very rigorously. They view mathematics as a subject apart in which they learn to solve well defined problems leading to unique solutions. Each problem is expected to be complete in itself, to contain all the required information, to involve the routine application of one or other of the mathematical teachniques they have learnt and to have a correct solution. When attempts are made to pose problems in which students are required to combine mathematical knowledge with material from other engineering courses, or even with the use of common sense, they rapidly become disorientated and give up the attempt to find a solution. Often it is found that students, whilst learning one technique for the efficient solution of a particular type of problem in mathematics lectures, will use a different technique when faced with such a problem during engineering courses. Students lack confidence in their ability to use the mathematics they learn in a constructive manner to assist in solving problems in other contexts.*

These wide-ranging inadequacies are fostered and nurtured by current examination practice. A candidate appearing for the conventional type of examination is usually asked, in the words of Deare (1974), "to consider a task which is more or less circumscribed, is shorn of most of its ambiguous fringe, and leads to an answer which is tightly defined. Further to this, the task is based on a single packet of subject material ". Additionally, the questions are framed so that they can be solved in thirty to forty-five minutes.

Examinations do exert a tyrannical influence on the content and method of our teaching. This is a truth which educators may wish to deny, but it is inescapable. This seems to be the essential criticism that Scorer (1972) directs to the present-day form of three-hour examinations. He claims that

> *in the process of formal preparation for examinations, mathematical thought is presented as a series of theorems that confer knowledge followed by applications of the knowledge in rather stereotyped situations. Afterwards they [students] have to learn that real situations do not conform to the conditions of the theorems, and the whole process of understanding what is known and unknown has to begin again ... Instead of provoking grand thoughts about energy, integration, parameterisation, equations and so on, mathematics is presented as a collection of little bits and techniques, definitions, theorems, corollaries and exercises, each of which can be done in a few minutes ... we have to create a subject by means of questions that can be done in half an hour.*

Many successful examinees have painfully to unlearn the attitudes and skills garnered while preparing for examinations. Recent engineering graduates have to learn that problem-solving needs time to ponder over the problem and hence assimilate a new time scale of perhaps months instead of minutes. They need to see for themselves the special significance of mathematical modelling and realise that at times they may have to work without the knowledge that their problem is capable of resolution. They have to appreciate that practical problems contain much irrelevant and redundant information, that in practice it may require some preliminary analysis even to select the branch of mathematics that would be suitable for solving the problem. They have to become accustomed that in a practical situation there may not exist one correct answer and that it is their function to choose a compromise solution from a set of different answers produced by a number of conflicting criteria. They should have a growing awareness that they will never have the pleasure of proving that their model is right: at best it is vindicated, when their predictions are tested by appeal to experiment.

With the proliferation of new technology, the present practice of assessing students by means of three-hour examination papers may even be dysfunctional. There are very clear trends for professional services, as in the legal and medical fields, to be provided by teams. Alluding to the needs of industry, Wakely (1978) writes:

> *We must recognise that manufacturing industry is far from homogeneous. It spans many branches: mechanical engineering,*

electrical engineering, civil engineering, etc., and these may split into many subdivisions ... The interests of all these industries are product orientated rather than discipline orientated, i.e., industry's interests are multi-disciplinary, usually centring on one branch of engineering but embracing many other disciplines, including mathematics, computer science, materials engineering, foundry technology, electronics, management science, accountancy and psychology. Faced with this situation, industry will, in many cases, resort to the use of the interdisciplinary project team, i.e., a team charged with the completion of a specific project and including the more important of the disciplines needed for this.

And yet, in sharp contrast, the stress in education at universities and colleges of advanced education is on individual effort, and the emphasis is heightened by present examination procedures, which provide little opportunity for collaboration in learning and problem-solving.

The assessment of projects should form an integral part of examination procedures. This is particularly important when a student in tertiary education is nearing the successful completion of a given course of study.

A project may be defined as an investigation which normally does not have a unique right answer. It is usually concerned with a realistic situation and often is clothed in considerable information, involving the student in judgements, compromises, and decisions in relation to his use of the material. The project provides the student with the opportunity to synthesise the skills and knowledge that were acquired throughout all aspects of his professional education, so as to produce over a period of time a documented report on a particular problem.

Examinations in mathematics for scientists, technologists and engineers must clearly be concerned with realistic practical situations, testing a student's comprehension of the interleave between mathematics and science, or technology, or engineering. The standard types of examinations examine what is most reliably and easily tested, such as the recall of factual information and the application of well-practised standard techniques to standard problems: seldom do they assess what most teachers consider to be important attributes of mathematical thinking and ability, namely, flair and imagination. The constraints of the conventional time-constrained examination paper should not deflect us from preparing students for the type of environment in which they will practise the art of their calling.

No examination can hope to reproduce completely the actual conditions of professional practice. However, projects provide one of the more hopeful aspects of professional preparation for, in effect, the student, in carrying out a project, assumes the mantle of a researcher or an engineer. The general abilities and attitudes developed in the use of projects as a form of professional simulation would include the confidence to make judgements and take decisions, the ability to generate and consider several possible avenues to approach a problem rather than the pursuit of the obvious one, the capacity to integrate the discrete packages of knowledge and skills given during the course of study, the need to bring a measure of organisation, resourcefulness and intelligence to the undirected structure and open-ended content that characterise the undertakings, and the ability to express himself clearly and cogently in documented form.

Thus, as a project confronts a student with a situation which calls for him to appraise it, appreciate its implications in mathematical terms, plan its investigation, pursue any leads which subsequently arise and interpret conclusions, the context of the examination is brought closer to the work content of the life of the professional engineer. This implies, as Sellars (1973) says, that a project form of assessment "greatly enhances our ability to draw a fuller, more meaningful assessment profile of the student's progress on a course than by using formal examinations on their own".

It is acknowledged that it is not easy to formulate a project that is at once neither too trivial nor so difficult that it would more properly form the basis of a post-graduate thesis; also there is the very serious risk that if the problem is very complex, this would necessitate a more frequent intervention by the supervisor, making it impossible to separate the student's own contribution for purposes of assessment.

The University Grants Committee (1964) urged that the power to deal with a subject or problem requiring more extensive treatment and longer reflection than are possible under the limiting circumstances of the traditional examination setting should be taken into account in assessing a student's quality. The difficulties attendant upon the use of a project form of assessment are many and include:

(i) uniformity of assessment of diverse projects, namely difficulty with achieving some uniformity in the level of difficulty between projects;

(ii) finding enough topics at the right level: an ill-chosen topic undermines the whole notion of assessment of project activity;

(iii) identifying the contribution of a single candidate within the group, when projects are pursued as a group activity;

(iv) the issue of teacher participation: the final assessment is as much a measure of him as it is of his students. Teacher's rôle as planner, consultant, and assessor are all interdependent; and,

(v) monitoring a student's commitment to it. Sellars (1973) writes:

> *The students' enthusiasm for this form of education can be such that they spend an inordinate amount of time in the presentation and prosecution of their coursework and projects. This excessive zeal for what is only one part of the assessment procedure must be carefully monitored so that a student's balanced programme of education is not disrupted.*

These difficulties notwithstanding, a definite bonus accrues to courses, such as engineering, where mathematics is a subsidiary subject, when projects are used to augment teaching and to serve as a form of assessment. This is particularly so when engineering undergraduates have to undertake project work whose themes are centred in some engineering disciplines. The projects superbly highlight the relevance of mathematics to engineering, motivate the learner, and help considerably the integration of the mathematics in the course.

The philosophy of assisting students to a greater independence implies the need for decisively different emphases, methodologies and assessment techniques. An American researcher, R.H. Brown, writes succinctly:

> *Students learn best what they discover for themselves ... the most important thing for them to learn, through practice, is how to learn, in order that they may go on learning through life long after today's facts are outdated ... This means that students learn best and most usefully when they are asked to play the role of scholars themselves, rather than when they are asked to master the conclusions of scholars about questions which they themselves may only dimly perceive and may be quite irrelevant to them. In short, students should be given the new data of the discipline insofar as possible, learn to ask their own questions and move to their own conclusions.*

11.11 OPEN-BOOK EXAMINATIONS

The usual procedures used for assessment in service courses do not reflect the student's problem formulation ability or initiative, save at a very elementary level. The emphasis is predominantly on the part of the job requiring proof and the application of well-practised techniques to standard, well-rehearsed, unrealistic problems. "There is a tendency", declares Willmore (1973),"to set questions which test the candidate's ability to use the technique, without testing his understanding of the real subject of study ... Special topics tend to be examined by 'bookwork' questions which can easily degenerate into tests of memory alone".

The traditional lecture course and its appendage, the unseen examination, are eminently suited to each other, as the primary emphasis in the design of many tertiary courses is enshrined in the guiding principle, "knowledge to be transmitted". This is a view supported by John Elliot (1978), and he writes:

> *It is difficult to see what kind of assessment other than an unseen examination which requires the student in some way to give back the received knowledge is feasible at the end of a course whose content is principally derived from a lecture programme and where the basis of planning in the majority of Higher Education courses is the 'knowledge to be transmitted'.*

The unseen examination is the complete antithesis of the work content of a professional man, such as an engineer, who should have the ability to carry out non-routine applications of mathematical techniques, to discern relationships, to organise and use his knowledge of mathematical concepts and operations in a non-practised way. No professional engineer attempts to answer three or four disparate problems almost simultaneously, in total silence and time-constrained conditions, without having access to books, reference material or the considered views of his colleagues, as do undergraduate engineers. And yet, it is on the basis of a student's performance in such unnatural, constrained conditions that his fitness to practise engineering is determined.

The whole question of assessment needs to be seen within the framework of our concern with the philosophy of emphasis on mathematical modelling and on exposure to ideas rather than remembering details. The aim of open-book examinations is to reduce the amount of information to be remembered by the student and to make the examination a better assessment of analytical thinking and problem-solving ability. Ideally, of course, a certain amount of detail needs to be "on tap", and a certain amount needs to be "accessible" (so that obliterative subsumption can occur).

From the student's point of view, open-book examinations will usually be harder than conventional ones. The questions which aim to test genuine insight are often difficult: they demand a deeper understanding of the subject and its importance to the whole field studied.

11.12 TRAINING IN THE ART OF EXAMINING

Although the importance of examinations, both in the educational system and in the influence which they have on people's careers, is unquestioned and growing with the passage of time, it is a curious fact that more attempts are not being made to delineate and understand the function of assessment and to improve its procedures. Nor is there much being done to provide a systematic training for examiners.

With so much depending on examinations, it is plainly a matter of singular importance, as the University Grants Committee (1964) maintains, that

> *they should measure as well as possible, not only the knowledge which the candidate has acquired of his subject or subjects, but also the qualities and habits of mind for which a degree may be expected to vouch.*

Unfortunately, the time devoted to training teachers to examine effectively is all too brief. Indeed, in universities and colleges of advanced education, arrangements for the appointment of examiners are very casual: a young lecturer without previous experience of examinations, save as an examinee, has often found himself assuming a major part in an examination of the first importance with very little in the way of advice or guidance from his more experienced colleagues. Dunn (1964) points out that

> *Until we are prepared to treat examinations as an education problem worthy of serious study, we must expect to operate with a system which at best gives rough justice and at times it is very rough indeed.*

After attracting years of criticism, universities and colleges of advanced education have accorded increasing acceptance to the principle that, since they are appointed to teach but expected to research, academics should receive training in the art of teaching. In view of the fact that evaluative instruments are often in the hands of the untrained, and often serve a number of incompatible purposes, the training of tertiary teachers should embrace not only the art of teaching but the art of examining also. The Report of the Committee of Inquiry into Education and Training has underscored, as we have seen, the need for lecturer education, and has recommended the setting-up of research centres of higher education in universities.

How can one improve the art of examining? Hall (1971) makes four suggestions:

(a) *Much more research, coupled with exchange of data and experiences and feedback.*

(b) *Systematic training of examiners.*

(c) *Insistence always on the explicit formulation of educational objectives. What qualities or abilities or behaviour do we want our good students to acquire?*

(d) *Reform traditional types of examination and also devise and use more varied forms of assessment designed specifically to fulfil the objectives of the particular educational policies of which they are an aspect.*

CHAPTER XII

COMPUTERS IN ENGINEERING EDUCATION

Summary

The use of a computer as a teaching and learning tool in engineering mathematics is described. The ability of the computer to provide accessible approaches to the solution of complex, unassailable problems has many implications, not least the need for an integrated approach to the teaching of analytical and numerical methods and the necessity to subject to critical analysis the concept of a solution. The rôle of the computer as a valuable educational resource in a demonstrative mode, let alone the significance of simulation techniques, is indicated. The advantages of a laboratory-based mathematics course are seen.

12.1 INTRODUCTION

It is comparatively recently that mathematics has been winning an increasing acceptance in industry and technology. This is principally due to the advent of high-speed computers which offer to mathematicians and engineers an unprecedented power of technique to come to grips with problems which previously proved elusive and unyielding, not so much because of their difficulty, but because of their sheer complexity. Alluding to the dramatic changes destined for the practice of mathematics and engineering, as early as 1954, Dr. F. J. Weyl was led to comment:

> *The advent of the high-speed computer has opened the way for an unprecedented mathematization, not only of fundamental scientific research in the physical and biological sciences, but also in the management of our industrial and social systems. This is about to assign to mathematics an entirely new part in our civilization, with far reaching implications on what should be taught, how it should be taught, and to whom.*

Giving his opening address at the Conference on "The Mathematical Education of Engineers - Where Next?" at Loughborough University of Technology, Dr. J. A. Pope (1978) said:

> *Quite frankly, I feel as far as fundamental mathematics is concerned, the engineering courses need very little expansion of their mathematical content. Where experience is needed is in the use of computers and the understanding of the type of mathematics and problems that can be coped by computer modelling.*

The significance of these and comparable statements is difficult to escape: for, any tertiary institution charged with the task of educating tomorrow's mathematicians, scientists and engineers for employment in industry cannot fail to draw important pedagogic conclusions and implications.

The successful introduction of computers into a course is possible only when the objectives of the course have been re-assessed in terms of the possible contribution of the computer. The usage of computers in the mathematical education of engineers should be aimed directly at the achievement of the course objectives, which, according to Bajpai *et al.* (1974a) are:

- *(i)* *The formulation and use of mathematical models.*
- *(ii)* *The understanding of fundamental mathematical processes.*
- *(iii)* *The appreciation of the links between mathematics and other engineering subjects.*
- *(iv)* *The ability to compare methods of solution to problems.*

(v) The development of a logical approach to problem solving.
(vi) The acquisition of mathematical competence.

The computer is now a significant and an indispensable tool in the hands of researchers, applied mathematicians and engineers. Whereas in the past analytical techniques in the main were considered, with the increasing use of digital and analogue computers, a definite shift of emphasis from analytical to computer approaches is plainly evident. There are significant ways in which computers can be used to improve the content as well as the method of teaching; indeed, one of the striking features is the motivation experienced by the students who display a great deal of excitement in using the terminal and developing solutions to their problems.

12.2 NEW APPROACHES TO CONTEMPORARY PROBLEMS

The facility offered by the computer is staggering. The digital computer can perform vast numbers of calculations and make logical decisions at high speeds with a very small probability of error. Also, the sequence of calculations or programme performed by the computer can be readily varied. It is this remarkable power of technique, which, transcending all previous capabilities of analytical processes, enables the student to investigate important, realistic,contemporary problems and which encourages successive approximation approaches to optimal solutions.

The engineer can attack a problem using one technique, study the results and then try another technique. This gives the engineer a different outlook when he approaches a problem involving mathematics, making simple problems that were difficult, and making complex problems less intractable.

Professor E. M. Edwards of the University of Alberta recounts his experience of using a computer in electrical engineering education as follows:

> *The information explosion presents a serious challenge to today's education. One is caught between the fire of teaching too many topics too poorly and the frying pan of overspecialization. One effective approach is to teach general principles using specific examples. Properly done, this technique will leave a student with an ability to cope with today's technology and the background to understand tomorrow's.*

The teaching of servos requires either highly idealized examples of a great deal of calculation (often many iterations with different values of parameters). In the first approach the whole concept of the 'engineering compromise' is missed. Engineering compromise is the analytic or intuitive selection of an optimal operating point or region within the multidimensional region of the system space in which the system operation is possible.

One of the main differences between the experienced professional and the fresh graduate is that the experienced man has acquired the intuitive ability to arrive at a workable engineering compromise. It is extremely difficult to develop intuition, so we must therefore convert the process to an analytic one if we are to close the gap. The 'laborious calculation' approach loses the student among the trees of calculation to the extent that the forest goes unseen. Enter the computer. The student can now manually work an idealized example to acquire the concept being taught. With the aid of the computer he is then able to examine the possible solutions of real problems and thus acquire a better understanding of real system design.

(Quoted by Rockart and Morton, 1975)

Thus, the better calculation techniques of the computer, relieving the drudgery and reducing the probability of error, make possible the application of certain mathematical techniques which would otherwise be unavailing. This means that students can attempt problems which were prohibitively complex prior to the advent of the computer. Indeed, the computers now offer us the possibility of conducting mathematical experiments on hypothetical models on a scale hitherto without precedent. This is an aspect of the computer that needs to be fully exploited; indeed, as Hall (1962) reminds us, "Computers bring the possibility of discovery within reach of our students and give inductive reasoning a role in mathematical education more nearly equal to that traditionally occupied by deductive reasoning". As an example, "students undertaking traffic or highway engineering, who are engaged in evaluating an existing mass transit system, could investigate the effects of adding more or larger subway cars, changing the routes, staggering working hours" and so develop their own criteria for improving public transportation.

The educational consequences of this perspective - the ability of the computer to provide accessible approaches to complex, contemporary problems - are profound. It underlines the need to develop in students of engineering mathematics not only deductive and inductive powers, but also a high level of algorithmic ability to change the mathematical form of the problem to be solved in such a way that solutions can be found easily. Abelson (1976) writes:

> *In mathematics, for example, the increasing emphasis on algorithms reflects a tendency to cast mathematical descriptions in procedural rather than axiomatic terms and... there are indications that the procedural formulation is more accessible to most students.*

The increasing stress on algorithms provides an opportunity for introducing students to heuristic approaches. To quote Abelson (1976) again:

> *The increased sophistication of procedural models also opens the possibility of formalising, and formally teaching, those aspects of a subject that are usually regarded as tacit or heuristic.*

Further, students need to be taught to produce flow charts and code them for a computer. The discipline imposed on the student by having to translate mathematical algorithms into computer programs has beneficial results, as it encourages logical analysis and in-depth understanding of any given problem. Professor Perlis has put it well: "Whereas we think we know something when we learn it, and are convinced we know it when we can teach it, the fact is that we don't really know it until we can code it for an automatic computer ". In the expressive words of Forsythe (1959): "The automatic computer really forces that precision of thinking which is alleged to be a product of any study of mathematics".

12.3 NUMERICAL APPROACHES TO PROBLEM-SOLVING

The second major advantage is that computers allow us to explore and practise numerical approaches to problem-solving, thereby complementing traditional analytical techniques. The computers are introducing into the mathematical relationships and techniques a numerical aspect and these numerical methods enhance "the scope of existing techniques as well as suggest new lucid approaches to otherwise difficult areas". Not only has the digital computer enlarged the range of problems in applied mathematics and engineering which can be solved, but it has provided an alternative to analytical methods for conveying understanding of the nature of a problem.

Traditionally mathematicians have been trained in analytical techniques and this skill has shaped both the substance of their courses and the style of their teaching. Analytical methods often produce mathematically neat, elegant solutions which are neither meaningful nor useful to applied mathematicians and engineers. Thus, an implicit solution such as

$$\log_e a(x^2 + y^2) - 2 \tan^{-1}\left(\frac{y}{x}\right) = 0$$

of the first-order homogeneous differential equation

$$\frac{dy}{dx} = \frac{x + y}{x - y}$$

is in fact a solution in nothing but name: it has no practical significance to a user of mathematics. Not without reason are engineering students scornful of classical mathematics which they know can only be applied to over-simplified problems, selected not because of their interest but because they can be solved.

Again, in studying the steady state heat flow problem in a rectangular sheet of size L x M whose faces are insulated and whose edges are subject to temperature control, the solution of Laplace's equation subject to appropriate boundary conditions, by the method of separation of variables, describing the temperature u(x,y) of a point (x,y) on the plate is given by the expression

$$u(x,y) = \frac{400}{\pi} \sum_{n=1}^{\infty} \frac{\sin[(2n-1)\pi x/L]\sinh[(2n-1)\pi y/L]}{(2n-1)\sinh[(2n-1)\pi M/L]}$$

The usefulness of such a result is open to question. One may well ask what, if anything, does this solution mean to an engineer? What does it convey to him about the temperature distribution?

Numerical methods and their applications in solving differential equations are particularly commendable where analytical techniques are either distressingly difficult or altogether abortive. Without the computer a course must avoid treating those problems which cannot be solved analytically. This has often produced a distortion in the mathematical education of engineers and has led to courses that are full of artificial examples, avoiding topics that just happen to be insoluble by analytical means. Not a few problems in research or industry belong to the class of analytically insoluble problems. Dr. J. A. Pope (1978) echoes this:

> *To the uninitiated the design engineer works to precise calculations to find the right numerical answer to each problem. Nothing is further from the truth. One designs for stability; many technological phenomena are too complex to provide precise analytical solutions. It is only the easy ones that are*

solved that way.

There is, additionally, the distinct danger that "when explicit formulae are used to evaluate data, they may be taken to confer upon the data something of their own precision, a precision the data may well in no way merit ". A parallel concern is expressed by Dr. Topping (1951):

> *In the world of experiment, precise data are unknown. Errors of observation and the like are always present, so that calculations involving measured data lead to results which are inevitably inaccurate, even though the technique of calculation has been correctly applied. The accuracy of the result is dictated by the accuracy of the data, and consequently the complete process of calculation is often neither justifiable nor necessary. An approximate solution is usually adequate.*

These are salutary words.

These considerations have a number of compelling implications for the mathematical education of engineers. Digital computers demand a distinction between "pure theory" (that is, theory regardless of actual numerical processes) and "practical theory" (that is, theory valid for actual numerical processes). Thus, in the planning of syllabuses and in their interpretation, this implies a recognition of the distinction, in the expressive words of J.W. Tukey, between "monastic and secular mathematics".

The second implication that flows from the above considerations is that the concepts of a solution and of a problem need to be subjected to critical analysis. There is a need to develop in students a measure of breadth as to what constitutes a solution: computers have the capacity to provide the numerical calculation of a table, giving the values of some unknown function defined, for instance, by a differential equation whose solution is inexpressible in terms of standard functions. Further, students need increasingly to appreciate the existence of problems other than the familiar deterministic ones, such as stochastic systems.

The mathematical analysis of the model has gained in no small measure from the matchless power of the computer. With this goes the need, as Hall (1962) informs us, to teach students

> *how to set up mathematical models and how to interpret the results of mathematical analysis of such models in a useful way. These are difficult arts and teaching them successfully is even more difficult.*

This is not new: students have long had difficulty in applying mathematics to physical problems. What is new is the complexity of the problems now being tackled with the aid of computers and the kinds of 'solution' the machines produce.

The digital computer is having a profound effect on mathematics. Not only, as we have observed, does it make it possible to perform major numerical experiments on hypothetical models and to introduce into mathematical relationships and techniques a numerical aspect, but also it is bringing to the fore finite difference techniques and discrete variable methods. We need to teach more about the calculus of finite differences, since the digital computer deals not with differential equations but with algebraic relations. Also, the digital computer is giving to such concepts as mathematical stability, approximations and convergence a new significance.

These observations imply two tasks. The first concerns the clear obligation of teaching not less but more mathematical analysis.The computer is a powerful instrument and a sound foundation in mathematics is essential, not as an end in itself, but directed to the analysis of some situations in which some of the physical factors are known in order to facilitate the right choice of operating conditions and control. The computer, despite its astonishing capabilities, does not possess inherent potential for mathematical creativity.

The second imperative is integration, a fusion with the general mathematics teaching. The usage of computers should be fused with the other methods of teaching engineering mathematics. The superposition of computers on existing courses should not be the objective, but rather the weaving together of the teaching of basic analysis and numerical analysis so as to reinforce and strengthen each other. Bajpai *et al*. (1970b) have provided an excellent example of an integrated approach to the teaching of differential equations: analytical, numerical and analogue methods of solution of differential equations are introduced at the same time, permitting a critical appraisal of the merits and demerits of each method.

Students come to appreciate the relative merits of a particular numerical solution of a non-linear system and a general analytical solution of a linearised simplification of the system.

Professor Crank (1962a) is not averse to the integrated approach. He says:

There is much to be said for the handling numerically of a simple differential equation, for example, to which the formal solution can also be obtained and compared with the numerical one. The important thing is that the student should recognise on the one hand the generality of the numerical methods, and on the other that the formal approach is essentially a device of limited application - but worth knowing because certain types of equation tend to recur.

An important corollary of this concerns the teaching of computing in the context of engineering. The engineer, in the exercise of his profession, is interested in computing as a means to an end; computation should not be amputated from the situation which gave rise to it. The use of the digital computer to control manufacturing processes, to perform air, automobile, and railroad traffic control should comprise one of the most effective applications of the computer in the future. These and comparable situations should be used to teach students to design and conduct numerical experiments to test mathematical models and hypotheses. And, these activities could provide a forum for discussing such mathematical techniques as operations research, queueing theory and optimisation, which many, if not most, engineers would find useful as they move into the administrative positions of middle level management, often within a very few years of qualifying professionally.

12.4 A GRAPHICAL DISPLAY

There is another aspect to the place of the computer in the mathematical education of engineers, and that is, its importance as a teaching aid. The computer is a valuable educational resource in a demonstration mode: a complex calculation can be carried out with ease and speed and the result displayed effectively and simultaneously in diagrammatic form.

The use of graphical display facilities to produce results in diagrammatic form can be of great help in overcoming mathematical complexities that often arise in teaching engineers and prevent principles from being understood. They can assist the student, for example, in his appreciation of the convergence of a Maclaurin Series or of the behaviour of the solution of a differential equation as parameters are varied. They may be used by students of introductory differential equations to generate direction fields and thus to gain insight into the global character of the solutions to these equations. Cox and Elton (1974), investigating the advantages of different computer configurations in an introductory quantum mechanics course, write:

The experience with the plotter has strongly confirmed our previously held opinion that at the undergraduate level a graphics display has very substantial advantages over a print out.

The oscilloscope of the computer promises to be effective as a kind of animated blackboard if it becomes part of an educator's teaching apparatus. The computer itself can serve to make films and permits the development of a natural and perspicuous presentation of topics traditionally reserved for more advanced treatment. Thus, computer-produced movies showing the effect of increasing the number of terms in the partial sum of the series can not only assist in understanding that any periodic function can be approximated by the sum of a series of terms that oscillate harmonically, converging on the curve of the function, but this demonstration can motivate a more formal treatment of non-uniform convergence. Apart from illuminating mathematical concepts and processes and kindling an interest in them, such a procedure can help to blur the aberrant but currently fashionable division between pure and applied mathematics.

The digital computer with plotter can be used to solve an analytically insoluble differential equation by numerical methods, the graphical display unit illustrating the graph of a functional relationship.

However, displays need not be static but may be time-dependent: they may show on the screen a representation of a physical event changing with time in the way that it would in reality. Thus the computer is a simulator.

12.5 SIMULATION TECHNIQUES

Simulation techniques are becoming increasingly necessary in areas where genuine experiments are expensive, dangerous, time-consuming or impossible. The underlying principle used in simulation is that one system (the computer) is being examined physically in order to determine the behaviour of the other: put in a word, the analogue computer in a real sense behaves like the original system. Research engineers engaged in the evaluation of new processes or plants, and particularly their dynamic behaviour and control, use the analogue computer, because it is possible to vary the values of the parameters or change the form of the empirical functions and to assess their influence on the graphical display unit with dramatic immediacy and perspicuity.

This technique of simulation, of producing a confrontation between model and reality and using the former to gain an insight into the latter, is used extensively in the study of nuclear power plants, chemical plants, guided weapons and space craft, control systems in general and aircraft performance.

The analogue computer is suited to the solution of differential equations. The mathematical education of engineers would be seriously deficient if the analogue computer approach to the solution of differential equations does not become a major teaching focus. This view is also espoused by Bajpai *et al.*(1970b). The worth of simulation as a mechanism for understanding the action of complex systems is not inconsiderable, and by using an analogue computer to solve a differential equation, we are at the same time simulating the physical system of which the differential equation is the mathematical model. For example, the analogue circuit may be used to provide a solution of the differential equation

$$\ddot{x} + 2k\,\dot{x} + n^2x = f(t)$$

and the different types of solution (x against t), for varying values of k and n and for the case when f(t) is zero, $a\cos wt$ or $a\sin wt$, may be portrayed on the visual display unit. Further, the students need to be emphatically reminded that the oscilloscope is, in effect, displaying the behaviour of three distinct physical systems which have the same mathematical formulation:

(a) the bending of a beam under a load, showing deflection against distance;
(b) the oscillations of a mechanical system; and,
(c) an electrical circuit consisting of an inductance, a resistance and a capacitance.

This scheme has the added advantage in that it serves to emphasise that mathematics can and should be the unifying factor between superficially diverse parts of the engineering science curriculum, such as vibrating systems or electric circuits.

12.6 THE UNIFIED APPROACH

The motion of a planet revolving about the sun, the motion of an electron in its orbit about the nucleus of an atom and the motion of a satellite about its parent planet are all described by similar mathematical equations. The differential equations for electrical and mechanical systems are also identical in form. This is one of the most striking examples of the way mathematics helps to show a fundamental relationship between apparently unrelated branches of engineering science.

The similarity between the equations governing electrical circuits and mechanical systems has many important ramifications. The most obvious is the fact that the differential equation

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = E(t)$$

need not be solved afresh, since its solution can be inferred from the solution of

$$m \frac{d^2x}{dt^2} + r \frac{dx}{dt} + k\, x = F(t)$$

In addition to this pleasant circumstance, the similarity of these equations enables us to interpret electrical phenomena in terms of their more concrete mechanical analogues. Finally, engineers sometimes utilise this similarity by determining the properties of a proposed mechanical system from experiments with a simple electrical analogue.

The correspondence between the above differential equations depends on the following correspondence between the quantities in the two systems. This correspondence enables one to simulate mechanical systems which might be very expensive to construct - an airplane, for example - by electrical systems which are relatively inexpensive: by measuring the response of the actual electrical system one determines the response of the hypothetical mechanical system.

Mechanical Systems		Electrical Systems
displacement x	$\leftrightarrow$	charge Q
mass m	$\leftrightarrow$	inductance L
damping r	$\leftrightarrow$	resistance R
spring constant k	$\leftrightarrow$	reciprocal of capacitance $1/C$
driving force $F(t)$	$\leftrightarrow$	impressed voltage $E(t)$
velocity $v = \dot{x}$	$\leftrightarrow$	current $I = dq/dt$

Thus, the underlying mathematical unity exhibited by many physical systems may be used by teachers to permit the telescoping of the engineering syllabus on the one hand and, on the other, to provide instructive analogies between different applications of the same mathematics.

12.7 LABORATORY-BASED ENGINEERING MATHEMATICS

There are quite a few engineering students who display a feeling of uneasiness towards mathematics. The precise formulation of a mathematical description of a physical system and the making of inferences are two arts to which they feel unequal.

This uneasiness and inadequacy may be explained in part by the abstract approach to the teaching of engineering mathematics, by the failure to teach mathematics in the real world context and by the neglect to emphasise the legitimacy of iterative and heuristic processes.

To overcome this damaging criticism, it is felt that mathematics should be taught in a laboratory. "The laboratory is included", declare Schey *et al.* (1970), "to stimulate in the student that most important of all prerequisites: the need and desire to know. These will arise in a student working on a laboratory project when he realises that to move further in his investigations, he will need to know more about mathematics". Hence the laboratory motivates the student's study of mathematics.

The laboratory is not concerned with mathematics *per se*, but rather with the relationship between the physical world and mathematics. As a result of his measurements and observations of some simple system, a student attempts to discover any regularities of the system and makes a mathematical model that can predict the future behaviour of the system. The subject matter of the course then develops in response to the student's need to operate on his mathematical model. The student has the experience of translating the behaviour of a physical system into mathematical language and finally retranslating his mathematical model back into observable predictions. The laboratory would have models for which relatively simple mathematical descriptions may be found.

The laboratory-based course in mathematics, with its emphasis on measurements, is eminently suited to an exploration of numerical methods and their application to the evaluation of the derivatives, definite integrals and solutions of differential equations, where analytical techniques are discouragingly difficult or fail utterly.

An experimental project should raise a multiplicity of questions which may motivate the pursuit of other branches of mathematics. As an example,

> *A student studying exponential functions by making measurements on radioactive decay may discover that for small values of the time his data can be fitted with a straight line rather than the more complicated exponential. This can lead to a study of Taylor Series expansions and their use in approximating and calculating functions; and as the student becomes knowledgeable he can turn to other kinds of expansions and approximation techniques and be led eventually, for example, to Fourier Series and integrals and to the general question of ortho-normal sets of functions.*
>
> (Schey *et al.*, 1970)

The planned use of computers in laboratory work is dictated by several considerations. At his opening address at the conference on "The Mathematical Education of Engineers - Where Next?", held at the Loughborough University of Technology, Dr. J. A. Pope (1978) said:

> *The computer has made possible many detailed calculations which heretofore were impossible to undertake due to the time involved. It has also enabled these calculations to be repeated many times in order to optimize the calculation or indeed to program the computer so that it automatically optimizes.*

Schey *et al.*(1970) have put the case for computers forcefully:

> *They will increasingly affect the formulation and analysis of intellectual problems in every field. Computers, more than any other single thing, impose upon students (and no less upon trained professionals) the need for logical thinking and for explicitness and precision. But most important of all, computers take the drudgery out of mathematics and thereby make it possible to attack real, scientific and technological problems rather than the uninteresting ones forced upon us when we are compelled, for the sake of calculational simplicity, to an artificial and sterile world of linear, quadratic and (occasionally) cubic laws.*

Arguing the case for the inclusion of computers, Oettinger (1969) declares:

> *The desire to keep labour within reasonable bounds generally leads to oversimplified and superficial experiments in student laboratories. Where the observation and intelligent interpretation of a variety of significant phenomena are the primary objectives of a laboratory exercise, using a transparent computer should reduce unnecessary drudgery to the point where judgement and interpretation, even of realistic experiments, can prevail.*

Thus, the potential of computers is so great we have an inescapable obligation to include their use as an integral part of the laboratory-based mathematics course.

Reinforcement of Learning

Reinforcement of learning becomes possible with the provision of a number of "entry points" for a given topic. A student may become acquainted, for example, with exponential functions "by making measurements on a simple radioactive material, by determining the intensity of light as a function of path length through an absorptive medium, by investigating growth and death rates of bacteria in a culture, or by determining through direct measurement the equation of a chain hanging under its weight". Thus a variety of ways explicate one topic: and, by examining a topic in several ways, a student's learning is reinforced in the process.

Among the many pathways that lead to a given mathematical topic, the presence of one that has for the student vivid interest, grip and appeal means that it is possible for the student to enjoy his study of mathematics: he willingly and easily learns the background material as the need arises, once the subject is made interesting and captures his imagination.

The mathematics laboratory, it is hoped, would have not only models which have relatively simple mathematical descriptions, but analogue devices such as conducting paper used for field plotting where Laplace's Equation holds: electrostatic and magnetic fields; heat flow, diffusion and fluid flow in the steady state. Such conducting sheet analogues are very valuable: the student sees what the solution looks like and so gets the "feel" of it in a way he never will by looking at an infinite series. Additionally, analogies are useful in linking together different physical systems having a common mathematical background. This underlying unity of different physical systems, emphasised by mathematics, is of considerable value to engineering students.

There are, as we have seen, many advantages that flow from a laboratory-based mathematics course: qualities of directness, liveliness, verve and immediacy, which are so essential. However such a course would need not only laboratories but laboratory time as in physics and chemistry. As the editor of the book *Computers in Education* remarked:

Mathematics is now a laboratory subject. Its teaching, therefore, requires not only laboratories but laboratory time. This needs to be provided on the same scale as for other laboratory subjects.

(Hall, 1962)

CHAPTER X111

EDUCATIONAL TECHNOLOGY IN THE SERVICE OF ENGINEERING MATHEMATICS TEACHING

Summary

Taking educational technology to include audio-visual media, programmed instruction and computer-assisted learning, this chapter stresses its use as a means of enhancing the quality of the mathematical education of engineers, based on a recognition of the individual differences of the learner. The need for lecturer education in the potential applications of the computer in the educational mode is emphasised.

13.1 INTRODUCTION

There appear to be three trends in higher education which are likely to shape the needs of the CAEs of the future:

1. Following a period of unprecedented growth, the flow of students has reached a "plateau" or a "steady state". Though this rate of flow of students has slowed down, the number of students will continue to increase. The formal educational system is certainly going to expand and to encompass growing numbers of students. The trend towards increasing specialisation will bring increasing number of adults back into universities and CAEs for continuing education and keep others for longer periods. For the CAEs, as indeed for all institutions of learning, this expanding enrolment will produce increasing demands on all of their resources: physical, financial, instructional and administrative. There will be a great pressure to optimise the use of available facilities and to automate whatever tasks are amenable to automation.

2. There will come a parallel expansion in the range of subjects offered. The knowledge explosion, which is a characteristic of our times, will be reflected by a proliferation of new disciplines, of new specialities within disciplines and of interdisciplinary studies. The efficient management of this expanded range of courses would require from higher educational institutions improved methods of resource allocation, better ways of assessing the demand for new courses and means of ensuring that study programmes are suitable for the employment that graduates will seek. For students, the vast array of courses will pose serious problems in selecting a realistic course of study. Counselling and guidance, now inadequate, will become crucial if the educational system is to function smoothly.

3. Individualised instruction, in one form or another, will become common.

> *The age in which we live is one which requires the maximum development of each individual's special talents. And this demand requires that students be given the opportunity to pursue their own unique objectives in their own unique ways. This is the mark of the times as opposed to teaching for identical objectives for all students in the same way. This makes planning for instruction a tremendous task for even the very best of teachers. It multiplies the decisions about what to teach and how to teach by as many times as there are students in any school.*
>
> (Harnack, 1971)

Again, individualised instruction in an automated or programmed form may be the only practical way for higher educational institutions to offer an expanded range of courses with the relatively restricted faculty they are likely to have available.

For teaching and administrative staff, individualised instruction will pose a major problem, that of monitoring large numbers of students, each of whom is proceeding at his own pace on his own unique course of study.

To these primary factors, we may add that inflation will be a continuing part of the economic scene and higher educational institutions would face sharply escalating costs. These considerations, in their totality, establish a *prima facie* case for the use of educational technology in higher education.

13.2 THREE COMPELLING FACTORS

The computer has the capability of storing large amounts of data, about an individual or about a system, which is available for almost instantaneous retrieval. It is this property of the computer - an efficient and effective instrument for identifying, collecting and summarising data - that makes its use possible to assist in the management of the educational process. Apart from the usual tasks such as budgeting and planning and time-tabling that a computer can handle, proving a boon to harassed educational administrators, it can aid in counselling and monitoring student progress as well as making education less labour intensive.

The use of computers in education is made compelling by three identifiable factors:

(a) The use of the computer may essentially be the only method of obtaining any worthwhile results. There are two areas in which the use of computers is essential if realistic teaching is to take place. The first is in the area of automation: the computer is indispensable in giving instruction in the development of automated systems. The second area in which the use of computers may be an absolute or practical necessity is in the area where instruction concerning the operation of unavailable systems or the exploration of real-world phenomena in which a mistake can lead to a physical or economic disaster has to be given.

(b) The computer contains certain unique characteristics which can confer unique benefits on the educational process: versatility of function, rapid real-time responsiveness, and problem-solving and computational aid. Milner and Wildberger (1974) indicate that:

It is now possible to have a single system in which (1) instruction can be delivered (for example, drill and practice, tutorial) and manipulated (for example, learner-control, interactive graphics); (2) real world phenomena can be simulated; (3) information retrieved; and (4) the full computational power of the computer can be used by the student to solve problems. Additionally, it can generate problems, derive solutions to these problems, evaluate student-proposed solutions, and generate remediation. Instructional management functions such as testing, prescribing, record keeping and resource allocation would also exist.

The use of the computer in this manner will enable the student to work at his own pace on problems generated either by the teacher or himself, in a situation which closely parallels the one in which the problems are met and solved in the real world.

(c) The computer may provide the most economical way of providing instruction. The routine use of computers makes education less labour intensive and facilitates treatment of students with special handicaps or widely differing backgrounds. Such technical developments as mini-computers will have a marked effect on the extension of the available interactive facilities.

13.3 EDUCATIONAL MODES OF THE COMPUTER

There are various modes in which the computer can be used in the instructional process:

(a) The student aid mode.

This mode of operation involves the direct interaction between the student and the computer which may present learning material in a number of ways: by conventional drill and practice, by a tutorial mode, using an enquiry and Socratic approach, by problem solving, or using simulation and gaming.

Computer assisted instruction (CAI) refers to this kind of situation where the computer interactively directs a student through a series of exercises, either in straight drill and practice mode or with some degree of tutorial dialogue.

(b) The teacher aid mode.

This mode of operation involves the computer's capacity to assist the teacher in the preparation of course material for computer-based learning systems, and to assist the teacher in demonstrating material during a lecture or tutorial.

(c) The management aid mode.

> *This mode of operation concentrates on the monitoring and the recording of a student's performance, so that the teacher and the taught have access to an evaluation of an individual student's performance.*
>
> (Hawkins and Nimmervoll, 1977)

It also suggests the appropriate next stage of instruction to the teacher. Such a use of computers to assist and manage the educational process is generally referred to as computer managed instruction (CMI).

> *The aim of CMI systems is to facilitate the provision of individualised instruction. The computer scores diagnostic tests, compiles reports for the teacher on student performance, then diagnoses learning difficulties and prescribes new learning tasks by relating performance profiles to a data bank of modules of instruction according to pre-programmed decision rules.*
>
> (Marsh, 1975)

Hunter (1978) counsels that "Our approach to mathematics for engineering students should be flexible and should depend very much on the interests, background, likely future careers, etc. of these students. It is better to try to develop as much mathematical confidence and expertise as we possibly can in the training of engineers". The computer now makes it possible to move toward a completely individualised kind of instruction: the major advantage derived from the use of the tutor function of the computer is its ability to do elaborate branching. The CAI system is particularly suited to a general class of engineering students whose interest and abilities and skills in mathematical methods reveal a detectable diversity: it embodies no significant educational insights not contained in other programmed instruction techniques which do not use a computer.

13.4 PROGRAMMED INSTRUCTION

Programmed instruction (PI), although its present fame derives primarily from computerization, is also available in paper form. Professor A. C. Bajpai and his colleagues at the Loughborough University of Technology have produced a series of texts on mathematics for engineers and scientists, using a programmed method of presentation. The authors indicate that such a method of presentation has many advantages:

> *The development of the subject progresses in carefully sequenced steps, with the student proceeding at his own pace. At each stage he has active part to play by answering a question or solving a problem, and thus learns by doing. By comparing his own answer*

with that given in the text, he obtains a continuous assessment of his understanding of the subject up to that point.
(Bajpai *et al.*,1973)

Thus the student moves through a body of data in small steps with instantaneous feedback on his ability to understand the meaning of a fact, internalise a skill or grasp a concept; both the act of writing out the answers and learning their correctness are viewed as reinforcing.

13.5 PEDAGOGICAL USES

As a tool of this period, the electronic computer as an adaptive device is unmatched in its potential to help teachers achieve genuinely individualised instruction . The student needing special education may, according to Carpenter (1970), require:

(a) idiosyncratic programs;
(b) an unusual degree of isolation from distraction;
(c) close and detailed attention to his progress;
(d) wide and rapid variation of stimulus output;
(e) extraordinary patience.

Each of these is available via CAI. The concentration required at a computer-based terminal precludes the student's attention from wandering and achieves a degree of efficiency which would be difficult to match in the best organised lecture room.

The other benefit deals with the heightened motivation which is detectable among students using CAI. This may partly be explained by a "Hawthorne effect", which refers to an improvement in a subject's performance that can be attributed to the subject's enhanced sense of importance resulting from the care and interest of the experimenter.

The primary clear target of opportunity for the computer in the mathematical education of engineers is in enrichment activities: problem solving, games and simulation can provide the learner with better methods of integrating and testing the knowledge that he has acquired. Since the computer can perform sophisticated calculations with alacrity, realistic problems may be introduced. The discipline of programming forces the learner to understand, or at least to know, a sequence of steps necessary to solve a problem. Computer simulation or computer augmented learning (CAL) represents one of the most important pedagogical uses of the computer. Students are given an insight into the importance and usefulness of the computer as a tool for decision making. "Simulated practice" should occupy a central niche in the education of engineers, as the viability of experiments is affected by such factors as cost, danger, and the time involved. As an example,

> *to test onè's concepts of bridge design in a laboratory by building toy bridges and loading them to see where they fail is an obviously powerful method of testing. This slow, expensive, mechanical means of testing is now being replaced by conceptual testing whereby the student draws a bridge on a computer-driven visual display and simulates the collapse. This sharply reduces the setup time and busywork (building the bridge) that the student must employ and is thus much more cost effective.*
>
> (Rockart and Morton, 1975)

13.6 LECTURER EDUCATION

A striking feature of the teaching of engineering mathematics in CAEs is the very limited and sporadic use of audio-visual aids. For most CAE teachers involved in the mathematical education of engineers, the use of any resource other than chalk, talk and book is regarded as something special, if not as a novelty. The relative lack of use of the computer as a teaching and learning tool for engineering students in terms of both content and techniques is a reflection on the rate of innovation in the mathematical education of engineers in the CAEs. Only one or two CAEs have acknowledged the visual aspect of analogue computing as a particularly useful aid in the teaching of such topics as ordinary differential equations to engineers.

If the prime thrust in the utilisation of the computer in the mathematical education of engineers will be toward extending, enhancing and supplementing the student's experience as well as individualizing education for large numbers of students with very diverse characteristics, there is a concomitant need: lecturer education in the potential application of the computer in the educational mode is desirable and essential. Enthusiasm and competence will be among the major determinants of the degree of success enjoyed by computer-based instruction, which is still in its infancy. The importance of giving some computer training to lecturers is explicitly expressed by Miller (1972):

> *... at least that some kind of orientation in computers should be given to all teacher trainees. They would not need to be programmers, but they should at least be conversant with what computers are, what they do and how they can be used as a learning resource by students.*

The future increasing use of the computer's potential as a teaching tool is intimately linked to the degree of exposure that future educators have with respect to the computer's capacity as a tutor, as a simulator and as a calculator and problem solver. In their assessment of computer education needs in Australia over the next decade, Smith and de Ferranti (1975) have clearly enunciated the vital importance of teacher education in computers:

> *The consequent needs for the study of informatics (or computing) in teacher education represent the most significant single conclusion of the project.*

13.7 MIX OF LEARNING MECHANISMS

Learning takes place in a variety of ways. The recognition of this fact makes it appropriate to utilise a multiplicity of methods, resources and systems. Rockart and Morton (1975) would have us notice that

> *...'acquiring', 'embedding', 'integrating', and 'testing' are not the same things, and there is no reason to suppose that each stage will be supported equally well by the same pedagogic device. Added to this is the fact that students differ sharply in maturity, intelligence, background, and style of learning. Poets do not learn in the same fashion as engineers; it does not seem reasonable that they would find the same pedagogic tools useful... These arguments suggest that a diversity of pedagogic devices is not only desirable but necessary if we are to be effective. At the moment such diversity must look like a smorgasbord - an array of teaching devices from lecture, film and television to multimedia computer-based labs. As we learn more, this diversity can be managed and take on the aspects of a portfolio, a collection of techniques gathered together for a purpose.*

No single technology can adequately serve the learner in all stages of the learning process, or across all types of material. To effect integration of new skills and established concepts into an individual's existing knowledge base calls for a total repertoire of available resources. Professor A. C. Bajpai (1972) indicates that in teaching mathematics to non-specialists, students prefer an array of teaching devices. He writes: "Generally, students prefer a combination of new teaching ideas with conventional methods". He advocates the use of integrated packages for teaching mathematics to non-specialists, defining integrated packages as "a combination of conventional methods together with some or all of the forms of teaching material mentioned (handouts, overhead transparencies, slides, audio tapes, films and film loops, video tapes and programmed texts)".

An important prior requirement of an educator contemplating the introduction of any educational aid is an understanding of the taxonomies and theories of learning. Within the framework of those theories, he must select those strategies which are best suited to the students.

CHAPTER XIV

MATHEMATICS PROGRAMMES FOR ENGINEERING UNDERGRADUATES

Summary

Having surveyed mathematics courses for engineers in the DOCIT institutions, this final chapter pulls together into a coherent whole the ideas which have appeared in the preceding chapters. Some of the factors that influence the drawing up of a blueprint for content and method are considered, as are the constraints, both explicit and implicit, under which courses are designed and articulated. A rationale which justifies the study of heuristics is given. Typical teaching syllabuses in mathematical methods, numerical analysis and statistics are qualified by several underlying assumptions, not least that there are no static programmes and that how we teach is as important as what we teach.

14.1 A SURVEY OF MATHEMATICS COURSES FOR ENGINEERS

This survey is an attempt to describe the present position in broad terms only, as the differences that exist between one institution and another and between one branch of engineering and another are not inconsiderable.

An array of electives offered to students by universities and colleges of advanced education (CAEs) as well as the variety of courses of substantive mathematical content presented by engineering departments make direct comparisons of courses difficult. There are differences among various departments in terms of the content, the emphasis and the organisation of undergraduate mathematics courses. Indeed, a comparative study of the mathematical education of engineers is fraught with an additional difficulty, namely, that the standard of mathematics achieved by the students at the secondary level is clearly an important determinant of the specific offerings at the tertiary level: at the Queensland Institute of Technology, the first unit of mathematics taken by all engineering students is entitled "Preparatory Mathematics", a self-paced course which revises and extends the material of the Senior syllabus in trigonometry, basic algebra, coordinate geometry, differentiation, integration, complex numbers and basic techniques of calculation.

Table 14.1 gives the main colleges of advanced education in the various Australian States that provide engineering courses and the minimum length of their full-time study in years. The column giving the total number of hours in mathematics, as a percentage of the total course time, is broadly indicative of the hours (averaged for the three disciplines: civil, mechanical and electrical) time-tabled in the main colleges of advanced education. Two universities have also been included to assess any significant degree of variability between universities and the CAEs.

The academic year in the colleges of advanced education is longer than in the universities, resulting approximately in the same number of teaching weeks in the full-time study courses in engineering provided by each kind of institution. Electrical engineering courses, both at the universities and the CAEs, tend to cover more mathematics than others; civil engineering courses have generally the least time devoted to mathematics courses, whilst electrical engineering courses have the most.

Table 14.2 lists typical mathematical topics and indicates the institutions that teach them to engineering students. It needs to be emphasised that this is a crude analysis, designed to present an overall view and give a good first approximation for comparative evaluation purposes. What is tabulated is information about the printed syllabuses. This does not take into account the way lecturers interpret the syllabuses: the same topics appearing in different syllabuses are susceptible to varying interpretations. Also, the printed syllabuses provide little insight into the depth of study intended: the inclusion of a topic in the table may indicate anything from a superficial coverage occupying a few lectures to an extensive and intensive study spread over a major part of an academic year.

The analysis of Table 14.2 shows that most institutions preparing students for first degree qualifications in engineering aim to give a deeper and broader understanding of fundamentals, a solid base for the technical sciences of their speciality. The table indicates a conformity of pattern between courses in various institutions in the earlier undergraduate years. However, as we have said, this is not necessarily to say that all courses, which are similar in content, reach the same standard.

There is no doubt that university courses in general attain a somewhat higher standard of mathematics than that in most CAE courses, although some individual courses in CAEs can be more demanding and rigorous. The university courses give more emphasis to generalisations and abstractions, and tend to equip their graduates with a greater range of highly analytical, sophisticated mathematical apparatus.

This is wholly consonant with the differing aims of each type of institution. As Lloyd (1968) expresses it:

> *It is in keeping with the aims of each type of institution that the universities, concerned with research and scholarship, should include more basic sciences (such as mathematics) to permit a more fundamental treatment of engineering science and advanced engineering subjects. The colleges, on the other hand, being concerned more with the application of knowledge, concentrate less on basic sciences, but have greater emphasis upon the practice of engineering.*

Table 14.1

Duration of Courses and Allocation of Hours

Engineering Course at:	Minimum Length of Course in Years	Percentage Mathematics Average of Courses
University of Sydney (US)	4 FT	Modular degree scheme
University of New South Wales (UNSW)	4 FT	11.1
New South Wales Institute of Technology (NSWIT)	6 academic years or stages (PT or sandwich pattern)	12.5
Royal Melbourne Institute of Technology (RMIT)	4 FT	12.7
Caulfield Institute of Technology (CIT)	4 FT	12.2
Swinburne College of Technology (SCT)	4 FT (Sandwich or integrated pattern)	14.4
Queensland Institute of Technology (QIT)	4 FT	9.2
South Australian Institute of Technology (SAIT)	4 FT	12.8
Western Australian Institute of Technology (WAIT)	4 FT	12.3

Source: 1979 *Calendars* and *Handbooks* of Universities and Colleges of Advanced Education

Table 14.2

Comparison of Engineering Mathematics Syllabuses

A: Mathematical Methods

+ Means this subject is taught to the engineering students	US	UNSW	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT	CEI
Differentiation and applications	+	+	+	+	+	+	+	+	+	+
Integration and applications	+	+	+	+	+	+	+	+	+	+
Convergence of series	+	+	+		+	+		+	+	+
Fourier series	+	+	+	+	+	+	+	+	+	+
Taylor and Maclaurin series	+	+	+	+	+	+		+	+	+
Functions of several variables	+	+	+	+	+	+		+	+	+
Partial differentiation	+	+	+	+	+	+	+	+	+	+
Vector analysis	+	+	+	+	+	+	+	+	+	+
Vector field theory	+	+	+	+	+	+	+	+	+	+
Multiple integrals	+	+	+	+	+	+	+	+	+	+
Complex numbers	+	+	+	+	+	+	+	+	+	+
O.D.E.s first-order	+	+	+	+	+	+	+	+	+	+
O.D.E.s linear, constant coefficients	+	+	+	+	+	+	+	+	+	+
O.D.E.s Laplace transforms	+	+	+	+		+	+	+	+	+
Series solution of O.D.E.s	+	+	+	+	+	+	+	+	+	+
Matrix and determinant theory	+	+	+	+	+	+	+	+	+	+
Special functions	+	+	+	+	+	+	+	+	+	+

Table 14.2

Comparison of Engineering Mathematics Syllabuses

A: Mathematical Methods (cont.)

+ Means this subject is taught to the engineering students	US	UNSW	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT	CEI
Partial differential equations	+	+	+	+	+	+	+	+	+	+
Integral equations	+	+								
Integral transforms	+	+	+	+		+	+		+	
Complex variable, functions and conformal transformations	+	+	+	+	+	+	+	+	+	+
Complex integration	+	+	+	+		+	+	+		+
Calculus of variations	+	+				+		+		
Tensors						+		+		
Perturbation methods	+									
2-D Analytic geometry			+	+					+	+
3-D Analytic geometry			+	+					+	+
Dimensional analysis									+	
Group theory	+									
Metric spaces	+									
Combinatorial theory	+									
Linear algebra	+			+	+	+	+	+		+
Finite element methods	+						+			

Table 14.2

Comparison of Engineering Mathematics Syllabuses

B: Statistics

+ Means this subject is taught to the engineering students	US	UNSW	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT	CEI
Data processing	+	+	+	+		+		+		+
Descriptive measures	+	+	+	+		+		+		+
Discrete and continuous distributions	+	+	+	+	+	+		+	+	+
Sampling distributions	+	+	+	+	+	+	+	+	+	+
Hypothesis testing	+	+	+	+	+	+	+	+	+	+
Estimation	+	+	+	+	+	+	+	+	+	
Correlation	+		+	+		+	+	+	+	+
Regression	+		+	+		+	+	+	+	+
Design of experiments	+						+			+
Analysis of variance	+		+	+	+	+	+	+	+	+
Multivariate analysis	+					+				
Probit analysis										
Probability theory	+		+		+		+	+	+	+
Stochastic processes	+									
Information theory										
Time series										
Reliability studies	+		+	+			+			

Table 14.2

Comparison of Engineering Mathematics Syllabuses

B: Statistics (cont.)

+ Means this subject is taught to the engineering students	US	UNSW	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT	CEI
Process control	+									
Acceptance sampling	+		+			+	+			
Production control	+					+				
Budgetary control										
Econometrics										
Market research										
Monte Carlo methods	+				+			+		
Markov processes	+				+					

Table 14.2

Comparison of Engineering Mathematics Syllabuses

C: Numerical Analysis

+ Means this subject is taught to the engineering students	US	UNSW	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT	CEI
Interpolation	+	+	+	+		+		+	+	+
Error analysis	+	+	+	+	+	+			+	+
Matrix eigenvalues	+	+	+	+	+	+			+	+
Difference equations	+	+	+	+	+	+	+	+	+	+
Numerical solution of systems of linear equations	+	+	+	+	+	+	+	+	+	+
Numerical solution of ordinary differential equations	+	+	+	+	+	+	+	+	+	+
Nonlinear differential equations	+	+	+	+	+	+	+	+	+	+
Numerical solution of partial differential equations	+	+	+	+		+	+	+	+	+
Digital computer programming	+	+	+	+	+		+	+	+	+
Analogue computers	+	+	+		+		+		+	+

Table 14.2

Comparison of Engineering Mathematics Syllabuses

D: Operations Research

+ Means this subject is taught to the engineering students	US	UNSW	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT	CEI
Linear programming	+	+		+	+	+	+	+		+
Dynamic programming	+	+		+		+	+	+		+
Decision theory and games	+	+								
Inventory models	+	+					+	+		+
Queues	+	+		+		+	+	+		+
Simulation	+	+		+			+	+		+
Network analysis (CPM/PERT)	+			+		+	+	+		+
Project evaluation	+			+				+		
Heuristic problem solving	+									

Source: a) 1979 *Calendars* and *Handbooks* of the Universities and Colleges of Advanced Education

b) Information brochures of the Council of Engineering Institutions

The greater stress on such courses as Group Theory, Combinatorial Theory and the Theory of Stochastic Processes by universities is simply a natural outcome of their principal concern, namely, the *development* rather than the *application* of knowledge. We use the word "principal" advisedly for we recognise that universities and the CAEs have overlapping concerns, such as the development of professional skills, where neither has a monopoly.

There have been many changes in the patterns of higher education such as the development of various forms of credit-systems or modular schemes. Although considerable differences with respect to required versus elective courses exist, most engineering programmes allow students some freedom in mapping their own programmes of studies from a wide range of possible electives. At the upper undergraduate and graduate level, it is not uncommon for institutions to provide flexible schemes of study, whereby each student may match his education to his abilities, aptitudes and career aspirations by selecting courses from a variety of mathematical and technical electives. The more advanced topics in mathematics are usually presented as optional subjects in the latter stages of the degree course.

The Council of Engineering Institutions (CEI) aims at setting the standard for the qualification of professional engineers. Although the Constituent Institutions remain responsible for the conditions of admission to their own membership, and may require qualifications supplementary to those called for by the Council, the Council itself determines the standards to which corporate members of the Constituent Institutions must conform for use of the style and title of "Chartered Engineer".

A number of professional institutions such as the Institution of Electrical Engineers, London, and the Institution of Engineers, Australia, have recognised or given provisional recognition to several CAE courses. The Council of Engineering Institutions encourages students to acquire qualifications of an acceptable standard by full-time or sandwich attendance pattern at universities and elsewhere.

However, the Council administers its own examination, to serve as a datum against which the standard of other academic qualifications can be assessed and also as an educational test for those who may have prepared themselves for professional recognition in other ways. Table 14.2 gives an indication of the mathematics syllabus for examinations of the Council.

The Institution of Engineers, Australia, provides interstate co-ordination at the professional level and has articulated the minimum level of education acceptable to the engineering profession. Aside from fostering professional ethos, it has the rôle of accrediting both standards and content of engineering courses, and the education, training and experience appropriate to Chartered Engineer (Australia).

14.2 RECOMMENDATIONS ON UNDERGRADUATE MATHEMATICS COURSES FOR ENGINEERS

14.2.1 Introduction

A sense of dissatisfaction stemming from several sources is detectable in regard to the mathematical provision for undergraduate engineers. Students complain of poor teaching, boring, ill-prepared and ill-delivered lectures, of the impersonality of large classes and the marginal contact that exists between faculty and students. There are also strictures about obsolete or irrelevant material in the curriculum, about the undue emphasis placed upon formal instruction and traditional assessment procedures. There are criticisms about the insufficient attention given to the ways in which students learn, and to new techniques for evaluating their performance.

The Report of the University Grants Committee (1964) under the chairmanship of Sir Edward Hale, warns that the quality of undergraduate education may be imperilled by the increasing volume and complexity of knowledge and the resulting tendency to overload the degree courses. It reiterates the importance of first-degree courses as a preparation for professional life, but also cautions against an over-emphasis on the acquisition of knowledge to the detriment of what is regarded as the main purpose of a first-degree course, namely, to train students how to learn, to learn to think, to understand, to appreciate, to make use of knowledge and to discover the values of it - its usefulness, its clarifying and revealing powers, its insight, its truth. The danger that overshadows all our higher education is that we may fail to educate in the true sense of the word because we have an over preoccupation with stuffing students' minds with examinational gobbets of specialised information.

The construction of a curriculum, that is, the selection and organization of content and methods, includes consideration of such items as the educational philosophy of the institution, professional determinants, the objectives to be achieved, the organising principles around which instruction is built (the development of a range of competencies such as problem-solving, mathematical modelling) and the sequence in which instruction is offered.

14.2.2 Educational Philosophy of the CAEs

The curriculum of an institution has to be related to its purpose.

As already observed, a distinctive function of colleges of advanced education is that they provide predominantly professional education. Their courses tend to give an applied emphasis.

However, this is not to say that the CAEs are unconcerned, unlike the universities, with the development of what Professor Partridge identifies as, "certain intellectual capacities and habits, and certain attitudes and affections to which we attach high value - a concern with knowledge for its own sake, and especially with knowledge as a means to understanding, with enquiring or exploratory habit of mind, and the faculty of critical discrimination". Rather, it is a question of the balance or mix in the total activities of the institution.

The CAEs are institutions whose programmes of teaching are pretty directly associated with vocational or professional preparation, and the other aims are conjoined with, and perhaps subordinate to, the aim of developing professional competence.

Although the distinction between university and advanced education sectors is becoming increasingly blurred, it is this functional differentiation between universities and CAEs which should be the harbinger of a major shift of emphasis in the design and articulation of courses given at the CAEs.

14.2.3 Aims and Objectives

The prior task of any educator engaged in the formulation of courses and their presentation and assessment, is the enunciation of the aims and objectives of the courses. Aimlessness is the single most important cause of ineffectiveness in teaching and of frustration of effort; again and again one looks for evidence of purpose in the lecture theatre and the laboratory. "The 'clarification of objectives'", Mackenzie *et al.*, (1972) remark,

> *is not a phrase that will be familiar to most teachers in higher education and many will be tempted to dismiss it as meaningless pedagese. Yet it remains a process for making explicit a problem which is normally implicit; the problem of deciding what to teach. It has been traditional to list topics in the form of a syllabus, to make general declarations of intent such as 'we try to teach our students to think' or 'we want our students to be good engineers', and to leave the rest to the individual teacher. The teacher then has to decide how much of a topic to include and to what level to take it; and at the same time, somehow to relate his decision to his 'inspirational' aims of 'teaching his students to think', 'developing good engineers', etc. The relationship between syllabus and its aims remains implicit and individual.*

A perusal of the handbooks shows that many of the aims and objectives of mathematical education are expresed in general terms; such general aims reflect an attitude or state of mind on the part of academics, rather than an operational description of the curriculum. It cannot be too strongly emphasised that if teaching-learning is to be a purposive endeavour, it will have to be based on precisely formulated statements of objectives: it is imperative for educators to perceive clearly the end from the beginning. Any mis-match between intended objectives and actual outcomes may be resolved either by modifying the objectives or by introducing changes in the curriculum, namely, what is taught and how it is taught.

As recently as March 1973, a leading Australian engineer and educator (Roderick, 1973) called for an enquiry into engineering education in Australia to establish, *inter alia*, the goals of engineering education. The American Society for Engineering Education published an authoritative report on the subject five years earlier (Walker, Pettit and Hawkins, 1968) and stated the cognitive objectives of scientific and technological studies in undergraduate engineering courses as "(1) Mastery of the fundamental scientific principles and command of basic knowledge underlying a branch of engineering..." and "(2) Thorough understanding of the engineering method of problem-solving and elementary competence in its application...".

There is a regrettable element of imprecision in the phrase "the engineering method". However, in the context of engineering mathematics, there should be a growing concern to give the students a clear idea of the methodology used in mathematical activity. This includes understanding the nature, the power and the limitations of the process of mathematization, and hence the process of symbolization (building a mathematical model from a given situation) and interpretation (going from a model to actual situations). The different complementary rôles of induction and deduction in mathematical activity need particular stress. The aim of mathematics programmes should be to produce graduates trained in mathematical methods of wide validity, with a particular appreciation of, and facility for, their applications to engineering situations.

14.2.4 Three Influences

Undergraduate curriculum in engineering mathematics has been mainly influenced by three factors:

(a) High school curriculum in mathematics has been strengthened and improved in recent years. Many students would have had an exposure to calculus and analytic geometry and perhaps, more importantly, at least "some students would be accustomed to the care and precision of mathematical thought and statement", a hope expressed by the Committee on the Undergraduate Program in Mathematics (CUPM). Students now receive a better preparation and we should be able to plan on this basis: it means, for instance, that the first year calculus course could be presented in a more sophisticated vein.

(b) There is an increasing mathematization of the management, social and engineering sciences. The steady escalation in the applications of mathematics in engineering is generating, in the words of the CUPM (1967) "several trends heading in different directions and visible simultaneously. One is a trend toward basic science. The mathematical aspect of this trend is a strengthening of interest in more algebraic and abstract concepts. An orthogonal trend is one toward the engineering of large systems. These systems, both military and non-military, are of ever-increasing complexity and must be optimized with regard to such factors as cost, reliability and maintenance. A further trend, in part a consequence of the preceding two, is a real increase in the variety and depth of the mathematical tools which interest the engineer. In general, engineers are finding that they need to use new and unfamiliar mathematics of a wide variety of types". Thus we witness the emergence of such new tools within engineering mathematics as combinatorics, logic, operator theory and functional analysis.

(c) The advent of the digital computer is affecting every phase of engineering, from basic research to the production line. It has precipitated significant changes in analysis procedures, with numerical methods assuming an important rôle. Its great speed and accuracy makes it possible to solve problems previously regarded as unsolvable; new methods of solution have been formulated and the older ones have been revitalised. (One can include under this heading, of course, the hybrid and analogue computers.)

14.2.5 Extension of Knowledge

The unprecedently vast growth in knowledge is a contemporary phenomenon which bears directly on the content and methods of higher education. Since factual knowledge is too extensive and too susceptible to obsolescence, the ability to memorize can become increasingly irrelevant to the central tasks unless used judiciously. Flexibility and adaptability, the capacity to respond effectively to novel situations - these have always been important but are commensurately more important in periods of rapid change. The growth and diversification of knowledge make it mandatory that every student should remain a scholar throughout his professional career. Such a shift of emphasis has inevitable implications, not only for what is taught and how it is taught, but also for the procedures we use to assess students' skills and behaviour. Elliott (1978) warns that "The student who is taught only knowledge and not its sources, its methods of retrieval or the 'scientific' methodology of the discipline will not acquire the ability either to understand those significant developments which he encounters or to update his knowledge when the course is over". As the Hale Committee (University Grants Committee, 1964) remarked, "a merely factual knowledge of the latest developments in a growing subject is a wasting asset".

The expanding universe of knowledge has a number of implications for course planning. In the words of Elliott (1978):

> *First, factual knowledge must be seen merely as a vehicle for the promotion of an understanding of concepts, the shaping of attitudes and the learning of methodologies and skills. The accumulation of factual knowledge is merely a by-product of these activities and is not an end in itself. Second, teachers should strive to acquaint students with the nature and sources of the raw material of their disciplines and how to use this material rather than with the generalised conclusions of other scholars - unless the latter can be shown to promote the self-reliance and abilities of the student through critical study of conclusions, methodology or case histories. This means fewer lectures and these with a very different emphasis. Third, as the understanding and capabilities of the students develop, they should apply themselves to progressively more difficult problems. Fourth, assessment should be by project reports and dissertations.*

A concomitant implication, as the store of knowledge expands and alters, is that the curriculum demands continual revision and adaptation: "new 'maps of learning' must be drawn". It is this that prompts the Committee on the Undergraduate Program in Mathematics (CUPM, 1967) to write, by way of an introduction to its recommendations on the undergraduate mathematics programme for engineers and physicists:

> *This is a program for today, not for several years in the future. ...Five or ten years from now the situation will undoubtedly be different in the high schools, in research, in engineering practice, and in such adjacent areas as automatic computation. Such differences will necessitate changes in the mathematics curriculum, but a good curriculum can never be static, and it is our belief that the present proposal can be continually modified to keep up with developments.*

According to Mackenzie *et al.* (1972) "...many of the growing points of knowledge in recent years are found at the intersection of traditional disciplines". An important consideration of the confluence of various disciplines, therefore, is to reflect in the structure of courses the constructive interplay between mathematics and the engineering disciplines, a consideration best reflected by team or interdisciplinary teaching.

14.2.6 Course Determinants

It is generally held that the content and methods of higher education are the province of the tertiary educational institutions. However, Graycar (1975) says: "...in the design of a professional course a number of non-academic interests must be taken into account. Influence is exerted upon those responsible for determining course structure and content by government, which provides the financial resources for higher education and which also sees itself as responsible for economic growth and development; by industry, which employs the student upon completion of his course; and by the professional association, into whose ranks the graduate is eligible to pass, and which has its professional status to protect and maintain". He (Graycar, 1973) observes that "much of the influence that is brought to bear is often informal, and not often perceived by the academics as influence" and critically examines how "the status seeking of a professional body", such as the Institution of Engineers, Australia, "the interests of industry" and "the interests and actions of Government are determinants in educational policy making".

The Institution of Engineers, Australia, is concerned with giving recognition to particular engineering courses which, on the basis of their length, standards of entry, course content and the quality of their teaching staff are considered to provide a basic minimum standard for entry into the profession. Apart from this, it is well-known that in 1967 the Institution of Engineers, Australia, unilaterally decided that from 1980 only those students who followed an approved four-year course would be accepted into membership, a decision which is often referred to as the "1980 Rule". Thus, because of the status that attaches to its accreditation, the Institution of Engineers, Australia, is seen by many engineering academics to have a major influence in university and CAE engineering schools.

14.2.7 Methods of Teaching

The Chairman and Managing Director of Associated Engineering Developments Limited, Dr P.G. Wakely, addressing a conference in Britain on "The Mathematical Education of Engineers - Where Next?" said:

> *I am less concerned about how much is included in the mathematical syllabus than that the foundations be properly laid, ... less worried*

about what is taught than about how it is taught.

(Wakely, 1978)

Mathematics education depends not so much on the syllabus (although it is important) as on the teacher and the methods of teaching used.

> *Let us make no mistake: any syllabus, no matter how sensible, modern and balanced it may be, can degenerate into mere dogma in the hands of a dogmatic teacher.*

(The International Commission of Mathematical Education, 1966)

The actual curriculum of subjects is less important than the manner of its teaching. The same syllabus may be presented in a way that it becomes the teaching of inert husks of knowledge, a barren and futile exercise, inimical to educative development; or, the syllabus may become the vehicle through which mathematical modelling - the art of mathematization - may be taught, thereby enabling knowledge to become an organic part of a student's experience.

How, then, should mathematics courses be taught to engineering students? Our deliberations in the preceding chapters have been conclusive on a number of points, and here we collate them.

14.2.7.1 An Integrated Approach

The spirit of unity of mathematics should permeate the education of applied mathematicians, scientists, technologists and engineers. Some universities and colleges of advanced education persist in drawing dividing lines between pure and applied mathematics, despite the fact that such a division of mathematics into pure and applied is difficult to sustain.

> *Fourier considered mathematics as a tool, for describing nature. But the impact of 'Fourier series', crucially important as these series are in physics and engineering, has been particularly felt in some of the 'purest' branches of mathematics. Cayley, on the other hand, believed that matrices, which he invented, would never be applied to anything useful (and was happy about it). They are now an everyday working tool of engineers, physicists, economists and statisticians. These examples show the futility of attempting to draw dividing lines between pure and applied mathematics... Also the name 'pure mathematics' is unfortunate since it implies a monastic aloofness from the world at large and an isolation from its scientific, technological, and social concerns. Such an aloofness may be characteristic of some mathematicians. It is certainly not characteristic of mathematics as a collective endeavour. In fact, many of the greatest mathematicians have*

attacked, with equal vigour, enjoyment, and success, problems posed by nature and problems arising from mathematics itself. For example, Hermann Weyl contributed in equal measure to the theory of groups as a pure mathematical discipline and to the effective uses of this theory in the theoretical constructions of atomic physics.

(Committee on Support of Research in the Mathematical Sciences of the National Research Council, for the Committee on Science and Public Policy, National Academy of Sciences, 1971)

This eloquent statement serves to underline the fact that a specious compartmentalisation of mathematics not only induces an incomplete understanding of, and a distorted attitude towards, mathematics, but can inhibit the creation of mathematical models of practical engineering problems. Such unifying themes as mappings and functions should be used to give students a synoptic view of mathematics. Indeed, as we have argued earlier, an integrated approach to the teaching of engineering mathematics of the kind suggested here, would include an emphasis on the complementary nature of analytical and numerical methods. Bajpai *et al.* (1974b), in the preface to their textbook remind us:

We feel that separation is artificial ... numerical methods, now well established, often give a solution where the analytical techniques have failed and it is unrealistic to treat them as second best ... Indeed, in most practical cases met by the engineer and scientist the desired answer is a set of numbers; even if the solution can be obtained completely analytically, the final process is to obtain discrete values from the analytical expression.

14.2.7.2 Level of Rigour

The attitude of the engineer to mathematics is quite different from that of the pure mathematician. The structure of pure mathematics is based on axioms, but an axiomatic approach has little attraction for the engineer, since, as an engineer, he is not intrinsically interested in mathematics, *per se*. An engineer will take for granted the existence of a solution to a differential equation, where the mathematician would prove an existence and might not care about the solution.

It needs to be said that one is not trying to create mathematicians out of engineering students. The emphasis in the course should be on the applicational aspects of mathematics. However, it should not be a "cook-book" of ready-to-apply formulae nor the teaching of a series of techniques, but one in which the basis of standard procedures has been revealed. As Rosenbrock and Storey (1966) put it: "We are not opposed to mathematical standards of rigour ... What we object to is an attitude which concentrates on mathematical subtleties arising in a non-essential way".

A similar point of view is put forward by Professor Sir James Lighthill (1979). Commending a balanced approach, he asserts:

> *I cannot agree with those extremists who would teach applied mathematics as a collection of empirical formulae. Equally unsuitable is the rigorously logical development of the subject as a series of theorems and proofs. A middle course is needed, retaining important aspects of the logical structure of mathematical analysis without swamping the students in a mass of deductive detail.*

Making his contribution to the Conference on "The Mathematical Education of Engineers - Where Next?", in ringing words, Craggs (1978) reminds us that, as for as mathematical standards of rigour are concerned,

> *An engineer, in mathematics, should be like a GP in medicine. He should have a working knowledge of relatively healthy patients and the ability, and humility to recognise the pathological conditions which are beyond his resources.*

The conclusion of his article delineates the level of abstraction and rigour that should be found in mathematics courses for engineers:

- *(i) Accuracy in manipulation.*
- *(ii) Enunciation of firm theorems, under strong conditions, with rigorous proof where it is easy and heuristic justification otherwise.*
- *(iii) Illustration of the possible profits and losses of speculative work outside the firm theorems, and*
- *(iv) Respect for the extra expertise of the fully trained mathematician, who can guide the engineer through the difficult country which an engineering education is unable to survey.*

<u>14.2.7.3</u> Relevance

The austerity of abstract mathematics and the drift away from the applicational moorings of the subject lead to poor student attitudes as well as a failure of the student to perceive the utility of mathematics in problem-solving and modelling. Dr J.A. Pope (1978), delivering the opening address to the Conference on "The Mathematical Education of Engineers - Where Next?", has reminded us:

> *I believe it is only right and proper that the engineer is primarily interested in engineering and not intrinsically in mathematics, but only uses mathematics as a tool. For this reason I believe it is important to tell about the applications of the mathematics before one deals with the mathematics.*

However important the development of abstract, axiomatic mathematics *per se* may be, the strength and finest motivation of the science comes, and has always come, to an engineer from a direct involvement with the immediate problems that arise in the practice of his profession. Therefore, his interest in the course will only be maintained if the mathematics which he learns is seen to form an integral part of a course specifically designed to meet his needs as a non-specialist mathematician. Richard Courant (quoted by Woods, 1973) has said:

> *Mathematical material presented as a closed, linearly ordered system of truths without reference to origin and purpose certainly satisfies a philosophical need. But the attitude of introverted science is unsuitable for students who seek intellectual independence rather than indoctrination; disregard for applications and intuition leads to isolation and atrophy of mathematics.*

Professor Partridge (1973) underscores this:

> *The ideal of devotion to 'knowledge for its own sake' is a treacherous one: it leads to a great deal of university teaching becoming clogged with pedantry, triviality, the teaching of inert husks of knowledge. The issue of 'relevance' which student rebels have been raising is, in my opinion, a quite real issue that is not often or seriously enough faced within modern universities.*

We do not wish our attitude to mathematical relevance to be misunderstood. In urging educators to provide applicationally motivated mathematics courses we are *not* pleading for just *tools*. The students need to be given not only standard problem-solving techniques, but helped to an understanding of some important mathematical ideas upon which they can build. However, we need to be critically on guard, for we may forget that the precise meaning of general mathematical concepts can be comprehended only in their application to a real situation. "For the undergraduate engineer", Creese (1979) says, "the remembering and even the learning of any mathematics seems to be very much connected with the use of it".

There are tendencies in tertiary institutions to win academic respectability for professional courses by simply increasing their theoretical content, according to Lewis (1974); he warns: "Unless carefully watched, such trends may result in unbalanced courses and poorly educated professionals".

14.2.7.4 Mathematical Modelling

The engineer is most typically engaged in the formulation, analysis and interpretation of mathematical models. Mathematics has become a tool of major importance in physical and engineering sciences for the formulation of models and the development of penetrating analyses. However, surprisingly enough, this fundamental rôle played by the queen of sciences is not stressed significantly in most mathematics curricula.

Sir Cyril Hinshelwood (1957), sensing the crucial importance of mathematical model building and analysis in the practice of applied mathematics, pleads the case for mathematical modelling: "My thesis is not that there should be an enormous range of mathematics taught, but an early and rather intensive cultivation of the power of thinking about real things and the application of mathematical symbolism to physical ideas".

Engineering mathematics courses should be designed to develop, stimulate and nurture the attitudes and practice of applied mathematics. Their main themes should be the formulation of technological or practical problems in terms of abstract mathematical models, the solution of the resultant mathematical problems and the discussion, interpretation and evaluation of the results of the analysis.

Educators should make an increasing use of problematic situations to initiate the learning of new mathematical topics. This means that pupils are induced to explore

> *a state of affairs, concrete or abstract, which by its presence constitutes a challenge to their powers of discernment or invention ... Using problematic situations in teaching is an excellent way of engaging mathematical activity and fostering understanding of the process of mathematization or building mathematical models. It demands an appreciation of the complementary roles played by induction and deduction and development of heuristics in problem solving.*
>
> (The International Commission of Mathematical Instruction, 1966)

14.2.7.5 Team Teaching

The use of guest lecturers, be they from industry or from the faculty of engineering, aids in demonstrating the vital and significant interdependence between mathematics and engineering disciplines: it vivifies the interplay between applications and mathematical ideas.

There is yet another compelling reason for using interdisciplinary teaching. Mathematics can be portrayed as a unifying factor by showing that diverse applications have many unifying threads. This aspect of mathematics - its unifying feature - permits the telescoping of the syllabus, which would be a great boon as teaching syllabuses are already over loaded. Shercliff (1978) asks a rhetorical question:

> *At Warwick we have a first year course in linear systems modelling which aims to teach a mathematical and intuitive grasp of all systems which obey simple, ordinary differential equations, be they electric circuits, vibrating systems or simple systems involving heat flow and heat capacity, etc. Why teach essentially the same material several times in separate courses on circuits, vibrations and differential equations?*

The mere accumulation of facts leads to a surfeit of inert, uncodified knowledge, that leads nowhere. However, team teaching can dramatically enhance the crystallisation of our principal objective, namely, to impart mathematical knowledge and to develop the ability to use it. Sharpe and Moore (1977) have given several examples of the use of a team of engineers and mathematicians in teaching mathematics to civil engineers. These are indicated in Table 14.3 .

Mustoe (1978) indicates that "work on matrices is woven into a package of lectures on finite elements in structures, presented with an engineering lecturer ... The work on eigenvalues is preceded by a lecture from a structural engineer on vibrations of framed structures, using a laboratory model".

Such a team approach to the teaching of engineering mathematics makes for authoritative, but not dogmatic, teaching. Authority is born of personal apprehension of the truth and when an engineer, in partnership with mathematicians, addresses engineers-to-be, teaching is based, not on rumour, hearsay and second-hand experience, but throbs with the vitality of first-hand knowledge, experience and conviction.

Table 14.3

Illustrative Engineering Examples

Engineering examples	Mathematical formulation
Ordinary differential equations: The oscillation of water level in a surge tank was computed using the equations developed from considerations of continuity and momentum.	$\frac{L}{g}\frac{dv}{dt} + z + Fv^2 = 0$ $va - Q - A\frac{dz}{dt} = 0$ where $v = v(t)$ = velocity in tunnel $z = z(t)$ elevation of water level in surge tank All other parameters taken to be constant.
Partial differential equations: The solution of hyperbolic equations was demonstrated using the method of characteristics applied to the partial differential equations of the waterhammer.	$\frac{\partial H}{\partial t} + \frac{a^2}{g}\frac{\partial v}{\partial x} = 0$ $g\frac{\partial H}{\partial x} + \frac{\partial v}{\partial t} + \frac{fv^2}{2D} = 0$ where $v = v(x,t)$ = velocity in pipe $H = H(x,t)$ pressure in pipe All other parameters taken to be constant.
Linear programming: Minimize the transportation costs of supplying *m* concrete users from *n* concrete mixing plants.	Minimize $vb = g$ subject to $va - u = c$, $v \geq 0$, $u \geq 0$ where v, u, b, g and c are matrices.
Fourier series: The tidal ice forces on a simply supported bridge pier are represented by a Fourier series so that the deflection may be computed using a typical term of the Fourier series input.	$\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2} + b^2\frac{\partial^2 y}{\partial x^2} + \phi(x,y)\sin\omega t$

Source: Sharpe and Moore (1977)

14.2.8 Heuristics

The accelerating increase in scientific knowledge, particularly after the Second World War, produced in engineering courses a heavy emphasis on the acquisition of knowledge and the techniques of analysis it made available. The present pattern of education reveals an identical emphasis and engineering graduates feel unequal to the challenge posed by complex contemporary problems, which call for creativity in thinking. "Today the great problem in engineering education is the trend towards the academic in our educational system", Conway (1967) tells us. He goes on: "While some schools are struggling to give rein to creative and three-dimensional thinking in young children, few if any universities, bent on obtaining first-class honours results, are doing the same with engineering students". De Simone (1968) sadly reflects that "The art of creative engineering has been orphaned in the engineering schools".

The low correlation between academic achievement and subsequent professional practice, claimed by disenchanted employers, emphasises that the development of creative problem-solving techniques should be a major concern of mathematics educators.

A course in heuristics greatly adds to the students' problem-solving repertoire. A rationale justifying the study of heuristics is given in a recent study by Woditsch (1977) who classifies the skills developed during education as just five:

1. *Winnowing: the process of selecting information which relates to the problem in hand from all the available information on that issue.*
2. *Sustained analysis: the ability to break a problem into components, then to follow through the consequences of hypotheses made about the system in question.*
3. *Forming analogies: transferring patterns of analysis established in one subject area for use in another.*
4. *Suspension of closure: maintaining an awareness of alternatives to the hypotheses used in analysis of a problem and of the possible consequences of these alternatives: maintaining a tolerance for ambiguity.*
5. *Autocensorship: finding checks with which to detect incompatibilities between the logical consequences of a hypothesis and the circumstances to which it relates.*

According to Blackburn (1978), students are asked to develop these skills in a very uneven way when learning mathematics. He says:

> *Pure mathematics is a vehicle through which skill in sustained analysis is developed to the very highest degree, but it neglects other skills. Applied mathematics could bring in most of the other skills but, as conventionally taught, makes some demands in autocensorship then asks little more than the use of a few standard analogies. The skills of suspension of closure and of winnowing are not used.*

Undeniably, mathematics has often been presented as a ready-made, prefabricated body of knowledge; some topics have been so taught that they are little more than a collection of recipes to be applied mechanically. Mathematicians like Polya have long been saying that presenting mathematics this way gives a very distorted picture of the subject and, above all, of genuine mathematical activity:

> *Mathematics is regarded as a demonstrative science. Yet this is only one of its aspects. Finished mathematics presented in a finished form appears as purely demonstrative, consisting of proofs only. Yet mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to guess the idea of a proof before you carry through the details. You have to combine observations and follow analogies; you have to try and try again. The result of the mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible reasoning, by guessing. If the learning of mathematics reflects to any degree the invention of mathematics, it must have a place for guessing, for plausible inference.*
>
> (Polya, 1954)

There is a wealth of significance in the term "engineering solutions", for the solution to any real engineering problem must always be a compromise. The art of engineering is embedded in realism: whether it be because of time or financial constraints or simply because of the sheer complexity of the problem, a perfect solution is seldom, if ever, achievable.

As an engineer invariably has to decide on a course of action on the basis of some certain knowledge, some partial knowledge and some unknowns, many practising engineers speak of "the art of engineering", not least Lord Hinton (1970) of Bankside. He writes: "Engineering is not a science; it is an art. In the sciences there is normally one single correct answer to every problem; in the arts there are no such single answers to the problems one seeks to solve: there are many alternative answers and each one of them is a compromise between conflicting advantages and disadvantages".

Many attempts have been made to articulate a methodology for the problem-solving process, and have failed. For problem-solving is a difficult art, demanding some degree of independence, judgement, originality and creativity. It is an acquired skill in which the interplay of thinking, reasoning, ingenuity and intuition is harnessed in the determination of engineering solutions to engineering problems. The very imprecision of an engineer's science renders any attempt to define a problem-solving process almost self-defeating. Lewis (1974) underlines this with vividness when he describes his observations of problem-solving by engineering students:

> *In design exercise (2), students had to consider the behaviour of a certain class of structural member at a comparatively early stage of their course before any relevant engineering theory had been covered. One student explained that he had used a hollow cylinder in his design because 'The hollow cylinder is used in nature most effectively in reed stalks, bamboo shoots, and animal bones, and is approximately as strong under compressive load as a solid cylinder of comparative diameter'. The analogy led to a solution which was technically sound; the performance of this student's structure exceeded the group average ...*

However, this is not to say that nothing can be done to aid the students in bridging the gap between knowing a set of principles and solving the problems of engineering. The best way of developing in students a sense of adequacy in the elusive art of problem-solving, according to Weinstein and Angrist (1970),is to devise "courses, perhaps under the generic designation of Engineering Analysis, which have as their basic premise the exposure of the student to a myriad of realistic engineering problems without the teaching of new content material". Students should be provided with opportunites to acquire know-how, as distinct from the mere possession of knowledge: more open, unsolved, realistic problems should be used in teaching, problems which require students to winnow the essential from the inessential details, to make their own analogies and to think about parallel or similar situations in which, in D.A. Blackburn's phrase, "their own external experience might give them cross-checks on their analysis". Standard, routine problems amenable to standard, routine techniques are not reflective of the universe of realistic engineering problems. The ability to obtain correct answers to exercises at the end of a chapter in a textbook does not necessarily mean sufficient capacity to solve the real problems that the engineer will inevitably face. Weinstein and Angrist (1970) stress this difference when they say:

> *You cannot acquire this depth of understanding [to become proficient in problem-solving] from a sterile, unproductive statement of a principle. You must use it continually in different situations - in realistic problems, not exercises, where it is not immediately obvious what principles should be applied or how they must be used. Only by trying, making mistakes, and then trying again to solve problems will this very necessary understanding be achieved.*

The whole area of problem-solving is one to which mathematical educators might well devote more attention in their research.

14.2.9 Textbooks

"The curriculum content of a course is often dictated by what is available in textbooks and, conversely, what is available is dependent on what we teach in our courses." This "chicken and egg" syndrome was pointed out by Dr A.G. Shannon.

Accordingly, we selected a total of twenty commonly used textbooks (listed separately in the appendix) dealing with various branches of engineering mathematics and analysed the topics listed withing their covers. The analysis produced the results which are displayed in Tables 14.4, 14.5, 14.6 and 14.7 .

This survey has served to indicate that what is conspicuously lacking is books which contain the appropriate intertwining of mathematics and engineering applications and written in a spirit that emphasises the unity rather than the diversity of mathematical methods, as well as general methods of wide validity, rather than special solutions of more limited scope.

Although there are several textbooks that provide a satisfactory framework for the teaching of engineering mathematics to undergraduates, we are aware of only a very limited number that achieve the kind of emphasis we wish to see: how to apply mathematics to the solution of engineering problems, the art of mathematical modelling, the significant advantage in combining analytical insight with numerical work.

Table 14.4

Frequency Table of Engineering Mathematics Topics

Topic	Frequency
Functions and sets	5
Limits and continuity	2
Vectors	6
Tensors	2
Complex numbers	5
Determinants and matrices	7
Differentiation and applications	7
Integration and applications	7
First order differential equations	4
Second order differential equations	3
Partial differentiation	2
Plane coordinate geometry	1
Solid coordinate geometry	1
Introduction to abstract algebra: equivalence, invariance, homomorphisms and isomorphisms	2
Rings, integral domains and fields	1
Linear vector spaces	1
Topological spaces and networks	2

Table 14.5

Frequency Table of Advanced Engineering Mathematics Topics

Topic	Frequency
Linear algebra (solution of simultaneous equations)	3
Boolean algebra	2
Eigenvalues and eigenvectors	3
Linear programming	1
Dynamic programming	1
Special functions (Bessel and Legendre)	6
Fourier series	4
Partial differential equations	4
Integral transforms: Laplace, Fourier	6
Multiple integrals	4
Vector calculus	2
Complex variable theory and conformal mapping	5
Calculus of residues and complex integration	5
Introduction to finite element methods	1
Mathematical theory of control processes	1
Calculus of variations	1
Optimisation techniques	1
Integral equations	1
Information theory	1

Table 14.6

Frequency Table of Topics in Numerical Methods

Topic	Frequency
Errors	3
Simultaneous linear equations	3
Solution of non-linear equations	3
Matrix algebra: eigenvalues and eigenvectors	4
Finite differences	6
Least squares	2
Interpolation and extrapolation	4
Numerical differentiation	3
Numerical integration	4
First order differential equations	7
Simultaneous and second order differential equations	2
Partial differential equations	2

Table 14.7

Frequency Table of Topics in Statistical Methods

Topic	Frequency
Introduction to statistical methods	5
Measures of central tendency	6
Measures of variability	6
Probability	6
Expectation	4
Discrete probability methods (binomial, Poisson)	7
Continuous probability distributions (Normal)	7
Sampling distributions	5
Testing statistical hypothesis	7
Tests of significance	6
Correlation and regression	7
Analysis of variance	6
Quality control	4
The design of experiments	2
Maximum likelihood	1
Order statistics	1
Non-parametric tests	1

Professor A.C. Bajpai and his staff at the Loughborough University of Technology have produced several programmed textbooks which not only have the correct level of rigour, but stress the importance of mathematical modelling and the welding together of analytical and numerical procedures. Equally, we wish to draw attention to an excellent book, *Applications of Undergraduate Mathematics in Engineering*, written and edited by Ben Noble, published in January 1967 by The Mathematical Association of America and The Macmillan Company, and based on a collection of problems assembled as a joint project of the Committee on the Undergraduate Program in Mathematics and the Commission on Engineering Education.

14.2.10 The Essential Mathematics Core Curricula for All Engineers

14.2.10.1 Underlying Assumptions

Any mathematics syllabus, however carefully planned and balanced it may be, can never aspire to meet the needs of all students, all colleges of advanced education and all times. It is therefore necessary for educators, seeking guidance and inspiration from the recommendations which follow, to understand clearly the assumptions which underlie them.

1. This is a contemporary syllabus, designed to meet present day needs: it is not a prescription for posterity. A few years from now, changes in high school mathematics and in engineering practice may perhaps necessitate sympathetic changes in engineering mathematics syllabuses.

2. The syllabus is intended to be a common core curriculum for all engineers and its prescriptions should be regarded as minimal for a sound introduction to the mathematics necessary for all engineers commencing their professional careers in a few years from now: it should be supplemented by elective courses for meeting the special needs of different branches of engineering and of specialist engineers.

3. Everyone appreciates that certain aspects of mathematics are sequential in nature and the order of presentation of topics cannot be altered. This is not to say, however, that teachers have no

latitude to vary, whenever possible, the sequence and depth of treatment to accommodate the diversity of ability and achievement of engineering students and their own personal preference.

4. We have not put a time-scale on our suggested core curricula for several reasons, not least because we did not wish to be prescriptive to an extent that would infringe on an educator's professionalism and because, to do it properly would have necessitated the consideration of other issues which are tangential to our main theme.

5. The syllabus is structured for the mathematical education of engineers, not of mathematicians. In saying this, we are not pleading for a dilution of the mathematics, but that, in the implementation of the teaching programme, stressing the complementary nature of analytical and numerical methods, the art of mathematical modelling and the development of problem-solving skills via heuristics, engineering mathematics may be shown as a subject in its own right, having an ethos, motivation and attitudes all of its own. This should be the *leit-motiv* of all our teaching of mathematics to engineers: not an exhaustive mathematical emphasis, nor an exclusive concern for mathematical generality and abstraction, but the conveying of the essential ideas together with a suitable statement of the conditions and limitations of the applicability of mathematical methods for the solution of engineering problems.

6. As motivation is the essence of the matter in teaching mathematics to incipient engineers, the need to kindle and sustain a student's interest in mathematics by showing its relevance to his main discipline becomes fundamentally important. It leads us to suggest, in a phrase that has become now rather threadbare, that mathematics should be taught in relation to engineering applications as soon and as much as possible. A problem-oriented approach is more likely to pique a student's interest in mathematics and provide the necessary motivation for its study.

7. We can put it no more strongly than to say that the ultimate success of the mathematics programmes we shall outline hinges, in no uncertain manner, upon the meticulous maintenance of proper intellectual attitudes to the methods of teaching we have described. These

methods enshrine our whole philosophy to the teaching of mathematics to engineering aspirants: they should not peep out here and there in our teaching, fugitively and marginally, but rather our approach to teaching should be underpinned, overarched and suffused by these methods.

8. The course recommendations are not to be viewed as the final word. The desire to lull educators into uncritical acceptance of what we indicate in the core syllabuses is alien to us. There is nothing in these recommendations that should dissuade an educator from designing another course or from further experimentation.

14.2.10.2 Mathematical Methods Syllabus

Linear Algebra

Vector spaces; linear transformations; matrices and determinants; the solution of simultaneous linear equations; eigenvalues and eigenvectors.

Functions and Sets

Real numbers; sets, relations and functions; the standard functions of calculus.

Elements of Differential Calculus

Limits and continuity; techniques of differentiation; differentials; partial derivatives; total differential and total derivative; maxima and minima of functions; errors; Maclaurin and Taylor series; indeterminate forms.

Elements of Integral Calculus

Indefinite integrals; integration methods; definite integrals; improper integrals; multiple integrals; line integrals; differentiation under the integral sign; applications to areas, volumes, centres of gravity, moments of inertia; Jacobians.

Ordinary Differential Equations

Initial-value and boundary-value problems; first-order differential equations; the use of the D-operator; Wronskian; linear differential equations; linear equations with constant coefficients; reduction of order; systems of differential equations.

Operational Calculus

General principles of the Laplace transformation; methods of finding Laplace transforms and inverses; step and impulse functions; solution of ordinary differential equations; Fourier transforms.

Nonelementary Solutions of Differential Equations

Solution by Taylor and Maclaurin series; Frobenius' method of solution; Legendre and Bessel functions; elliptic integrals.

Partial Differential Equations

Derivation of partial differential equations; Fourier series; separation of variables; the Fourier integral; solution by Laplace transform; solution by orthogonal expansions.

Vector Field Theory

Vector algebra; vector functions; differentiation of vectors; the gradient of a scalar field; divergence and curl of a vector field; theorems of Gauss, Green and Stokes.

Functions of a Complex Variable

Fundamental properties of complex numbers; the Argand diagram; complex algebra; analytic functions; the Cauchy-Riemann equations; complex potential; conformal transformation; Schwarz-Christoffel transformation; integrals of a complex variable; Cauchy's integral; series expansions; zeros and singularities; Cauchy residue theorem; contour integration.

Optimisation Techniques

Description of the optimisation problem; local and global extrema; formulation of linear programming problems; the simplex method and its variants; duality theory; calculus of variations, including extrema of an integral with fixed and variable limits; conditional extrema of an integral; constrained extrema; reduction to unconstrained form; Lagrange multipliers.

14.2.10.3 Statistics Syllabus

The object of this course is to provide a basic knowledge of descriptive and mathematical statistics and the techniques of statistical inference.

Introduction to Statistical Methods

Graphical representation of data; frequency distributions; measures of central tendency; measures of variability.

Discrete Probability Models

Probability models; simple probability; compound probabilities; expectation; the binomial distribution; the Poisson distribution.

Continuous Probability Distributions

Mean and variance of continuous random variables; the moment generating function; the normal distribution; normal approximation to binomial distribution; the law of large numbers and the central limit theorem.

Statistical Tests of Significance

Extimation of population parameters from a sample; testing statistical hypotheses; confidence intervals and levels; significance tests; the student's t-test; Chi-squared test; goodness-of-fit test; the F-test.

Analysis of Variance

Introduction; one-way experimental layouts; two-way experimental layouts; Latin square layouts.

Correlation and Regression

Scatter diagrams and the least squares model; estimation of the parameters of the regression line; confidence intervals for predictions (α, β, $E(y/x)$); correlation; variance analysis approach to regression; multiple regression.

Quality Control

Acceptance sampling; types of sampling; process control; control charts; the warning and action limits on average control charts; control limit factors; range control charts; life-testing;

reliability; failure time ; the Weibull distribution.

Non-Parametric Tests

The sign test; the Wilcoxon test; run tests.

Order Statistics

Distribution of the largest element in a sample; distribution of the smallest element in a sample; distribution of the median of a sample and of the k th order statistic; distribution of the range of a sample; distribution-free tolérance limits.

Experimental Designs and their Analyses

The factorial designs; unreplicated 2^k factorial designs; blocking the 2^k factorial designs; the two-level fractional factorial designs; response surfaces; fitting a second order model.

14.2.10.4 Preliminary Course to Numerical Methods

For many engineering problems encountered in practice,explicit analytical solutions either do not exist or are not well adapted for numerical calculations. The search for efficient and effective techniques to determine real solutions to real problems is becoming increasingly important.

The course recommendations for "Numerical Methods" contain introductory material on numerical techniques for solving engineering problems, applicable to all fields of engineering.

Appreciating the strong interdependence between computers and numerical analysis, we have attempted to focus attention, primarily, on computer-oriented numerical methods, procedures which allow effective algorithmic realisation on modern machines. It is, therefore, essential for engineering students to achieve a working knowledge of digital computer calculations and programming methods.

The objective of the course is to give the student an introduction to modern computational methods with an emphasis on algorithms and computer programming.

The Report of the OECD Seminar on "The Mathematical Education of

Engineers" places particular stress on giving students adequate experience in flow charts. The Report (OECD, 1965) says: "Although modern languages may reduce the importance of flow-diagrams for the experienced programmer, flow-diagramming should be taught in the programming course to enhance the improvement in the logical analysis of problems which is a by-product of this training".

The course on numerical methods should be preceded by one providing experience not only in devising algorithms and flow charts for solving problems on the computer, but also in expressing the problem-solving process in a computer language for a meaningful solution of a problem. This preliminary course should consist of:

1. Introduction

   Characteristics of digital computers; functions of computer components.

2. Algorithms

   Problem analysis; algorithm development.

3. Flow Charting

   Flow charting symbols; flow charting guidelines; the construction of flow charts; making decisions; looping; nested loops; iteration; adding notes to the flow chart; subroutines.

4. FORTRAN IV Semantics and Syntax

   Arithmetic Statements

   Constants, variables; variable naming; fixed and floating point numbers and their description as arguments; operations and expressions; library functions.

   Control Statements

   The PAUSE, STOP and END statements; transfer of control: GO TO, the arithmetic IF and logical IF.
   The CONTINUE statement; the DO statement, nested DO's.

   Logical Statements

   The NOT, AND, OR operators; relational operators; logical constants; logical expressions.

Subscripted Variables

Linear, planar and spatial arrays; arrangement of arrays in core storage.

Input/Output Statements

List specifications; the format statement; specifications for numerical, alphanumeric, input/output, skip specification, carriage control; input and output on card, printer, magnetic tapes and console typewriter.

Specification Statements

The DIMENSION, COMMON, EQUIVALENCE and DATA statements; the type statement.

Subprograms - Function and Sub-Routine

Statements:
Naming sub-programs; the FUNCTION, SUBROUTINE, CALL and RETURN statements.

5. Libraries and Packages

The students will be introduced to the scope and use of library programmes.

6. The Analogue Computer

Analogue computer components; analogue computer solution of differential equations; analogue simulation of physical systems.

14.2.10.5 Numerical Methods Syllabus

Errors and Error Studies

Introduction; types of errors; computational and measurement errors; error studies.

Interpolation and Polynomial Approximation

Graphical interpolation; linear interpolation; higher-order interpolation; difference tables; Gregory-Newton forward and backward interpolation; Taylor polynomials; Lagrange polynomials; Lagrange interpolation; other methods of interpolation.

Approximation Theory

Introduction to approximation and norms; orthogonal polynomials and least-squares approximation; Chebyshev polynomials; economisation of power series; FORTRAN program for least-squares polynomial fit; Chebyshev-polynomial curve fitting with a computer.

Roots of Equations

Types of methods; properties of polynomial equations; direct solutions of polynomials; transcendental equations; searching; secant method; iteration method; Newton's iteration; complex roots; derivation of convergence criteria and rates of convergence; Computer program for Lin-Bairstow's method.

Determinants

Introduction; evaluation of a determinant; special properties of determinants; pivotal condensation; flow diagram for pivotal condensation.

Matrices

Introduction; special matrices; elementary matrix operations; partitioning of matrices; flow diagrams for elementary matrix operations.

Simultaneous Linear Algebraic Equations

Introduction; Cramer's rule; elimination methods; iteration methods; relaxation methods; computer solutions.

The Inverse Problem

Introduction; Cramer's rule; elementary row operators; elimination methods; inversion of matrices by iteration or relaxation; matrix inversion by partitioning; FORTRAN program for matrix inversion.

Eigenvalue Problems

Introduction; examples of eigenvalue problems; determination of eigenvalues by iteration; modification of the coefficient matrix; FORTRAN program for reduction of eigenvalues and eigenvectors.

Numerical Differentiation

Graphical differentiation; numerical differentiation by secant line approximation; numerical differentiation by use of interpolating polynomials; numerical differentiation by means of smoothing polynomials; higher derivatives.

Numerical Integration

Graphical integration; numerical integration using interpolating polynomials; numerical integration using smoothing formulae, Newton-Coates integration formulae; Romberg integration; Romberg's and Gaussian quadrature in FORTRAN language.

Initial-Value Problems for Ordinary Differential Equations

Semi-numerical methods; Euler's method; modified Euler's method; predictor-corrector methods; Runge-Kutta methods; error analysis; semi-analytic solutions of differential equations; solution of higher-order differential equations; stability of numerical solutions and choice of method; digital computer program for the solution of initial-value problems.

Boundary-Value Problems for Ordinary Differential Equations

Linear shooting method; shooting method for nonlinear problems; finite-difference methods for boundary-value problems; computer program for the solution of a boundary value problem.

Numerical Solutions to Partial Differential Equations

Categories of partial differential equations; numerical solution of partial differential equation problems by the finite-difference method; iterative solution techniques; numerical solution of partial differential equation problems by the finite element method; a program for the simple wave equation.

14.2.11 Advanced Elective Courses

The exponential growth of knowledge and its increasing diversification place the profession of engineering in a state of active change.

The realisation is upon us that there is a need for pre-service professional education to instil an attitude of adaptability to change and readiness for retraining. Although it is difficult to forecast precisely how the functions of the engineer would change in sympathy with the continuous technological upsurge, it is, however, becoming increasingly plain that the mathematical education of an engineer must extend beyond the relatively basic core curricula in mathematical methods, numerical analysis and statistics that we have indicated, as the emerging engineering disciplines imply an understanding of sophisticated mathematics.

The process of change can be a threatening and uncomfortable experience and in-service training can provide a supportive climate for the change. Thus, educational establishments such as colleges of advanced education have a clear obligation in helping students and practising engineers to meet their on-going educational needs, by making available a selection of courses that would enable them to broaden and enhance their range of mathematical competencies apposite to their areas of specialisation and "the changing demands of current and foreseeable technologies". Both integrated courses leading to post-graduate degrees and separate units for special needs should be made available.

A practice followed at the New South Wales Institute of Technology is to offer a "cafeteria style" Graduate Course in Engineering, made up of subjects drawn from both existing undergraduate and graduate courses offered by the Faculty of Engineering, as well as by the other faculties within the Institute. The objective is to let students put together a programme of study which meets individual needs, although guidelines are set to ensure prerequisite knowledge for subjects included in any programme and that each individual student's programme has an adequate engineering emphasis. The Queensland Institute of Technology not only provides Graduate Diploma courses in Environmental Engineering and Automatic Control, but conducts, where the demand is indicated and there are sufficient enrolments, a number of specialist subjects for the development and continuing education of engineers.

As far as the continuing mathematical education of engineers is concerned, colleges of advanced education should ensure, as the Report of the OECD (1965) Seminar on "The Mathematical Education of Engineers" urges, that:

(a) *faculties continually evaluate the future mathematical demands of the specialised fields of engineering and provide elective courses designed to meet these demands;*

(b) *the student be guided by experts in his chosen speciality to select electives that both will be directly useful to him and will add to his basic mathematical understanding.*

Without attempting to be comprehensive, we simply state that the present-day engineer requires an extensive comprehension of such topics as functions of a complex variable, differential equations and probability in many specialisations. Topology simplifies analysis of complex networks, whether electrical circuits or structural frames. Boolean algebra, besides providing a theory of propositional functions and of the large circuits of computers, may be projected as a model of a class of simple switching circuits.

The ever more extensive and diverse applications of modern mathematical techniques to theoretical and applied investigations and the rapid development of modern computers have changed dramatically the mathematical training requirements of scientists and engineers. They must now be conversant with many areas of modern mathematics and must have, primarily, a firm grasp of the methods and techniques associated with at least one of the following areas:

Operations research: linear programming, games theory, decision theory, queueing theory.

Control engineering: feedback, performance and stability, noise and random processes.

Advanced statistics: estimation, variance and co-variance, multiple regression, stochastic processes.

Optimisation: calculus of variations, constraints, dynamic programming, gradient methods.

Information theory: communication and signal processing, sampling theorems and applications.

Computing science: advanced numerical and digital techniques, integral equations, approximation.

Systems engineering: the over-all design problem, analysis and synthesis, networks, applications.

Field theory: vector and tensor analysis, applications in elasticity, electrodynamics and fluid mechanics.

Modern computer capabilities have permitted an extension of the realm of computational work, so that specialised divisions of mathematics, such as non-linear differential equations, functional analysis, and probability methods, have become increasingly studied and utilised.

Professor Sir Charles Inglis, one of the brilliant line of Professors of Mechanism and Applied Mechanics in the University of Cambridge, said:

> *The spirit of education is that habit of mind which remains when a student has completely forgotten everything that he has been taught.*

(Quoted by Hinton, 1970)

There is a serious obligation on educators to ensure that we teach and develop the right habit of mind.

CHAPTER XV

CONCLUSIONS AND RECOMMENDATIONS

From the considerations of diverse issues in this thesis, a question that is deserving of an answer is, what general conclusions and recommendations can be formulated concerning the mathematical education of engineers? Denuded of all documentation and reduced to their essence, these may be summarised as follows:

15.1 STUDENT ATTRITION AND FAILURE

The generalised claim that students fail because they are indolent may well be applicable to some students, but it is very unwise to disregard the complex interaction of antecedent, institutional and other student variables.

Though some CAEs have undertaken investigations into the causes of high attrition rates, these have been college or discipline specific. As a prelude to initiating more effective programmes, further study of student attrition and graduation rates in CAEs is all-important: an essential first step is the systematic collation of detailed statistics from numerous colleges concerned with engineering education over a period of years.

15.2 STUDENT SELECTION

The prerequisites of engineering courses are not,as a general rule, clearly stated.

Every CAE should make explicit the assumed minimum background in mathematics for engineers in its official publications.

To improve selection procedures, CAEs should seriously consider the use of other instruments, such as aptitude tests, standardised tests and interviews, to complement the matriculation examinations.

15.3 THE WEAKER STUDENTS

There are considerable divisions of opinion among CAE teachers about the degree of their responsibility to marginal students. Some CAE mathematics departments devote a good deal of time and thought to the design of diagnostic tests and remedial work, others do not consider this to be any part of their responsibility.

Having accepted the students, diagnosis and treatment should become an essential part of an institution's responsibility towards them. The practice of using diagnostic tests to identify high-risk students and of providing remedial or bridging programmes should become more widespread.

15.4 FIRST YEAR STUDENTS

The quality of a student's work, the techniques of study acquired and the attitudes developed during the first year are critical to his subsequent performance. A student may scarce survive the handicap of imperfectly laid foundations during the first year. The allocation of staff to first-year mathematics units calls for care: the most effective and experienced teachers should be assigned.

15.5 INDIVIDUAL DIFFERENCES

Students differ greatly in their interests and abilities, purposes, learning styles, backgrounds and personalities. Teachers should be alert to the possibilities offered by a repertoire of resources in the individualisation of the instructional process. Computer-assisted learning may be used to supplement lecture and tutorial classes, while graphics terminals are invaluable in giving students with a poor mathematical background a "picture" of the equations and concepts.

Three of the DOCIT Colleges have introduced self-paced instruction (SPI) courses in mathematics, based on the Keller Plan, for their first-year students. These self-paced learning programmes, which are not to be confused with remedial courses, are admirably suited to ensure an acceptable level of attainment in mathematical basics in students who have begun engineering studies with varying levels of mathematical facility, despite the rigidly defined entrance requirements.

It is a measure of the rate of innovation in the advanced education sector that self-paced learning programmes are still regarded somewhat of a novelty in some quarters, although their rationale and theory were enunciated by F. S. Keller as far back as 1968.

Notwithstanding the paucity of experience and the heavy investment in time, effort and money entailed, we would urge that CAEs display a spirit of vanguardism and give careful and urgent consideration to their adoption, where appropriate, in first-year teaching.

15.6 ENGINEERING MATHEMATICS

Examples of basic premises that should underlie any well-structured mathematics course for engineering students include:

(a) stress on the art of mathematical modelling;
(b) provision of adequately laid foundations;
(c) development of mathematical competence;
(d) a conscious effort towards blurring disciplinary boundaries amongst analytical, numerical and statistical methods;
(e) emphasis upon broad principles at the expense of techniques;
(f) a balanced infusion of rigour;
(g) presentation in the context of engineering; and,
(h) a basis for self-education.

These premises, upon which there is wide consensus, would have the most far-reaching and welcome consequences if they are reflected in the design and implementation of mathematics courses for engineers.

15.7 TWO SERIOUS OMISSIONS

There are two omissions which singularly fail to assist the engineering student in the way they should.

(a) Mathematical Modelling

No area of the mathematical education of engineers has been singled out for criticism more than the area of mathematical modelling. Mathematical modelling is not sufficiently emphasised. It deserves to be given careful consideration in the general mathematical heart of the curriculum instead of being touched on spasmodically and superficially.

(b) Cross-disciplinary Approach

All too often, we feel constrained to say, analytical and numerical methods are taught in a way that does not illuminate for the students their complementary nature and the engineering disciplines.

Effective study is in some ways hindered when knowledge is fragmented artificially into subject departments. A more determined effort is called for to show not only that analytical and numerical methods are opposite sides of the one penny but also that mathematical methods - analytical, numerical and statistical - have a commanding relevance to engineering sciences. Team teaching can provide valuable aid in the latter context and is commended.

15.8 WHO SHOULD TEACH

The Department of Mathematics should assist in the planning of mathematics courses prepared for engineers. Engineering Mathematics is different and needs a different approach. Some care is needed in the selection of staff to teach it. The teaching of mathematics to engineers must be by mathematicians who have an empathy with the engineering profession and, where possible, an interest or a background in engineering.

15.9 LIAISON

The importance of curricula and teaching methods in the mathematical education of engineers leads us to emphasise the necessity of securing strong liaison between the mathematics and engineering departments. The selection of mathematical topics and the best sequence in which to teach them must be determined by coordinated efforts of mathematicians and engineers.

At most CAEs, the channels of communication are at an informal level, such as through personal contacts. Though we do not minimise the importance of personal contacts, we have reason to believe that very often an informal process shows dismayingly little of anything that could properly be regarded as interdepartmental liaison.

Accordingly, we endorse the concept of formal liaison between the serviced and servicing departments, a concept which may be pursued via departmental liaison committees and the appointment of service course coordinators. These committees can become the forum for seeking, from time to time, a clarification of the philosophy of service teaching, a clarification that *inter alia* could examine the reasons for pass-failure rates, the efficiency of departments in pursuit of formulated objectives and the possibilities for close cooperation and collaboration between engineering and mathematics staff.

15.10 ADVANCED STATISTICAL AND NUMERICAL METHODS

Statistical and numerical methods are becoming increasingly important as part of the mathematical apparatus of an engineer. Undergraduate mathematics syllabuses are already over-crowded and any inclusion of *additional* topics in statistics and numerical analysis would necessitate a major assessment and revision of mathematical priorities in the education of engineers. This is a difficult undertaking since the more advanced statistical and numerical techniques assume, for their understanding, that students have facility with a considerable fund of mathematical knowledge. The study of advanced statistical and numerical methods, in view of this observation, may well have to be deferred to post-graduate years.

15.11 THE COMPUTER

The potential of computers in the solution of engineering problems is so great we have an inescapable obligation to include their use as an integral part of the mathematical education of engineers.

There is evidence that when the computer is used in engineering courses it leads to an enrichment of these courses.

The introductory course on computations should be given quite early in the first or second year. The use of flow diagrams helps the students to develop good techniques for setting up and solving analytical problems. The choice of language is critical to the ability of a student to develop an understanding of computer science. Though this choice is likely to be constrained to those languages accessible to the available computing equipment, the best available procedure-oriented language should be taught, as procedure-oriented languages facilitate the representation of mathematical procedures.

Of course, the effective teaching of computational techniques presupposes that cooperative liaison alluded to earlier so that the other subjects utilise these skills and teach the numerical as well as the functional aspects of mathematics.

15.12 ASSESSMENT

The practice of having annual or terminal examinations is receiving decreasing emphasis among CAEs. The over-all assessment of a student in engineering mathematics is based, with one exception, on the weighted average of class tests, assignments and final examinations. However, it is regrettable that these place too high a premium on the recall of mathematical knowledge and the application of standard techniques to standard problems.

The recommendation arising from this is that staff concerned with the mathematical education of engineers should be made aware that a variety of techniques, used together, enhance the reliability of assessments by sampling several attributes. This is not to deny that some CAEs already do this; it is merely to urge that the CAEs do it *better*. In particular we advocate:

(a) The adoption of some open-book examinations not only to test a candidate's ability to solve problems, to synthesise learning and to use knowledge but also to reduce the premium on mere memory; and,

(b) The assessment of projects should form an integral part of examination procedures.

These are particularly important when a student is nearing successful completion of his engineering studies.

The practice of having supplementary examinations for borderline candidates, as a means of reducing wastage, is one which is not fully exploited by CAEs, save for one notable exception. Supplementaries give students a second chance and save repeating the whole year or semester. The compelling finding that nearly thirty-three per cent of engineering students saved a year through supplementaries in the CAE that uses them prompts us to suggest that much good might come from an extension of this practice.

15.13 LECTURER EDUCATION

It is generally true that CAE staff in DOCIT colleges have no preparation in teaching methods, having been selected on account of their scholarship and research.

Since the quality of the student and the quality of the lecturer are interrelated, there seems every reason for advocating a programme of preparation to initiate new lecturers. However, given the prevailing economic climate, there are not likely to be many new appointments.

What is needed is an incentive scheme to get lecturers to attend in-service courses and teaching workshops in the theory and practice of teaching, curriculum development and examining. The satisfactory completion of staff training programmes should become not only a normal condition of tenured appointment but it should be one of a range of factors to be taken into account in deciding study leave and promotion.

Attendance at such training courses should be recognised by making appropriate adjustment to the work-load.

Educational units have been established in a number of CAEs. They should develop a number of activities designed to assist academic staff to improve and to evaluate learning and teaching. They should also provide a consultant service for newer members of staff in such matters as defining their objectives, student-lecturer rapport, styles of presentation, delivery, structuring of lectures and new media.

Academic staff should acquire a concern for excellence in teaching. However, this concern cannot be engendered by compulsory staff training programmes. In the final analysis, it will be acquired most readily by staff in a higher education institution which has developed a respect for its teaching function, a respect articulated in the selection and promotion criteria of its staff and discernible in its spontaneous concern for its students. Thus, CAEs have a clear duty to examine their priorities.

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Appendix A - Engineering Courses Offered by Colleges of Advanced Education

Table A.1

Engineering Courses Offered by DOCIT Colleges in 1979

Course	Level	CCAE	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT
Civil Eng.	PG2		+	+				+	
Civil Eng. (Research)	PG2						+		
Control Eng.	PG2			+					
Electrical Eng.	PG2		+	+				+	
Electrical Eng. (Research)	PG2						+		
Engineering (Research)	PG2				+				
Local Govt Engineering	PG2			+					
Mechanical Eng.	PG2		+	+				+	
Mechanical Eng. (Research)	PG2						+		
Metallurgy	PG2								+
Metallurgy (Research)	PG2						+		
Production Engineering	PG2							+	
Surveying (Research)	PG2						+		
Surveying and Mapping	PG2								+
Air Conditioning	PG1							+	
Automatic Control	PG1				+	+			
Biochemical Engineering	PG1							+	
Chemical Engineering	PG1					+		+	+
Civil Engineering	PG1							+	
Control - Measurement Systems	PG1						+		
Digital Electronics	PG1							+	
Electronics	PG1								
Electronic Instrumentation	PG1					+			+
Engineering Geology	PG1					+			
Environmental Engineering	PG1				+				
Fuel Energy Utilisation	PG1					+			
Highway and Traffic Eng.	PG1		+						
Industrial Ergonomics	PG1					+			
Industrial Management	PG1							+	
Local Govt (Eng.)	PG1				+				
Maintenance Engineering	PG1							+	
Metallurgy									
- Extractive	PG1								+
- Engineering	PG1								+
Metallurgy and Materials	PG1								
Photogrammetry	PG1					+			
Primary Metallurgy	PG1								
Process Engineering	PG1					+			
Production Engineering	PG1								
Production Management	PG1					+			
Quality Technology	PG1					+			
Refrigeration and Air Conditioning	PG1			+					
Remote Sensing	PG1						+		
Surveying	PG1					+			

Table A.1 (Cont.)

Course	Level	CCAE	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT
Surveying and Mapping	PG1								+
Urban Systems	PG1							+	
Welding Technology	PG1					+			
Aeronautical Engineering	UG1					+			
Cartography	UG1					+			
Chemical	UG1					+			
Civil	UG1								+
Civil Eng.	UG1		+	+	+	+	+	+	+
Communications	UG1								+
Communications Eng.	UG1					+			+
Construction	UG1								+
Construction Eng.	UG1								+
Electrical	UG1								
Electrical Eng.	UG1		+	+	+	+	+	+	
Electrical Power	UG1								+
Electrical Power Eng.	UG1								+
Electronic	UG1								+
Electronic Eng.	UG1					+	+		+
Engineering Metallurgy	UG1								+
Extractive Metallurgy	UG1								+
Mechanical	UG1								+
Mechanical Eng.	UG1		+	+	+	+	+	+	+
Metallurgy	UG1					+	+		
Mining	UG1								+
Mining Eng.	UG1						+		+
Planning	UG1					+			
Primary Metallurgy	UG1								
Production Eng.	UG1			+				+	
Secondary Metallurgy	UG1								
Structural Eng.	UG1			+					
Surveying	UG1				+	+	+		
Surveying and Mapping	UG1								+
Technology									
- Civil	UG1				+				
- Electrical	UG1				+				
- Mechanical	UG1				+				
Aeronautical (Eng.)	UG2					+			
Cartography	UG2					+			
Chemical	UG2					+		+	
Civil	UG2								
Civil Eng.	UG2		+			+		+	+
Communications (Eng.)	UG2					+			
Electrical	UG2								
Electrical Eng.	UG2		+			+		+	
Electronic Eng.	UG2		+			+		+	
Mechanical	UG2								
Mechanical Eng.	UG2		+			+#		+	+
Metallurgy	UG2					+			

Table A.1 (Cont.)

Course	Level	CCAE	CIT	NSWIT	QIT	RMIT	SAIT	SCT	WAIT
Metallurgy									
- Extractive	UG2								+
- Physical	UG2								+
Mining Eng.	UG2					+			+
Naval Architecture	UG2					+#			
Primary Metallurgy	UG2					+			
Process Science	UG2					+			
Production Eng.	UG2					+#		+	
Secondary Metallurgy	UG2					+			
Surveying	UG2					+			
Cartography	UG3						+		
Civil Engineering	UG3				+		+		
Digital Systems and Computers	UG3								+
Electrical Eng.	UG3				+		+		
Electronic Eng.	UG3						+		
Engineering (Instrumentation)	UG3								+
Mechanical	UG3						+		
Mining Eng.	UG3				+				
Mining Technology									
- Mine Ventilation	UG3								+
- Mine Engineering	UG3								+
Surveying	UG3				+		+		

PG2 *Master Degree*
PG1 *Post-graduate Diploma*
UG1 *Bachelor Degree*
UG2 *Diploma*
UG3 *Associate Diploma*
•• *No students enrolled in 1979*
\# *Phasing out*

Source: Data Collection for *DOCIT Annual Statistical Report 1979.* The DOCIT Research Secretariat, the New South Wales Institute of Technology, Sydney

Table A.2

Engineering Courses Offered by non-DOCIT Colleges in 1979

Course	Level	Ballarat CAE	Bendigo CAE	Capricornia IAE	Darling Downs IAE	Footscray IT	Gippsland IAE	Preston IT	Tasmanian CAE	Warrnambool IAE
Civil Eng.	PG2			+						
Civil Eng. (Research)	PG2									
Control Eng.	PG2									
Electrical Eng.	PG2			+						
Local Govt Engineering	PG2									
Mechanical Eng.	PG2			+						
Mechanical Eng. (Research)	PG2									
Metallurgy (Research)	PG2									
Production Eng.	PG2									
Surveying (Research)	PG2									
Air Conditioning	PG1									
Automatic Control	PG1									
Biochemical Engineering	PG1									
Chemical Engineering	PG1									
Civil Eng.	PG1					+				
Electronics	PG1					+				
Electronic Instrumentation	PG1									
Environmental Engineering	PG1									
Fuel Technology	PG1									
Geology (Eng.)	PG1									
Highway and Traffic Eng.	PG1									
Industrial Management	PG1									
Local Govt (Eng.)	PG1					+				+
Metallurgy										
- Extractive	PG1									
- Engineering	PG1									
Metallurgy and Materials	PG1									
Photogrammetry	PG1									
Primary Metallurgy	PG1									
Process (Eng.)	PG1									
Production (Eng.)	PG1									
Quality Tech.	PG1									
Remote Sensing	PG1									
Surveying	PG1									
Urban Systems	PG1									
Aeronautical (Eng.)	UG1									
Chemical	UG1									
Civil Eng.	UG1	+	+	+	+	+	+	+	+	
Communications	UG1									
Communications Eng.	UG1									

Table A.2 (Cont.)

Course	Level	Ballarat CAE	Bendigo CAE	Capricornia IAE	Darling Downs IAE	Footscray IT	Gippsland IAE	Preston IT	Tasmanian CAE	Warrnambool IAE
Construction Eng.	UG1									
Electrical	UG1									
Electrical Eng.	UG1	+	+	+	+	+	+	+	+	
Electronic	UG1									
Electronic Eng.	UG1	+	+			+	+		+	
Mechanical	UG1									
Mechanical Eng.	UG1	+	+	+	+	+	+	+	+	
Metallurgy	UG1									
Mining Eng.	UG1	+								
Primary Metallurgy	UG1									
Production Eng.	UG1									
Secondary Metallurgy	UG1									
Structural Eng.	UG1									
Surveying	UG1									
Technology										
- Civil	UG1									
- Electrical	UG1									
- Mechanical	UG1									
Aeronautical (Eng.)	UG2									
Cartography	UG2									
Chemical	UG2									
Civil	UG2									
Civil Eng.	UG2	+	+			+	+	+		
Communications Eng.	UG2									
Electrical	UG2									
Electrical Eng.	UG2	+	+			+	+	+		
Electronic Eng.	UG2	+	+			+	+			
Mechanical	UG2									
Mechanical Eng.	UG2	+	+			+	+	+		
Metallurgy	UG2									
Metallurgy										
- Extractive	UG2									
- Physical	UG2									
Mining Eng.	UG2	+								
Primary Metallurgy	UG2									
Process Science	UG2									
Production Eng.	UG2									
Secondary Metallurgy	UG2									
Surveying	UG2									
Cartography	UG3									
Civil Eng.	UG3		+		+					
Digital Systems and Computers	UG3									

Table A.2 (Cont.)

Course	Level	Ballarat CAE	Bendigo CAE	Capricornia IAE	Darling Downs IAE	Footscray IT	Gippsland IAE	Preston IT	Tasmanian CAE	Warrnambool IAE
Electrical Eng.	UG3		+		+					
Electronic Eng.	UG3									
Mechanical	UG3		+		+					
Mining Eng.	UG3									
Mining Tech.										
- Mine Ventilation	UG3									
Surveying	UG3									

Ballarat CAE *Ballarat College of Advanced Education*
Bendigo CAE *Bendigo College of Advanced Education*
Capricornia IAE *Capricornia Institute of Advanced Education*
Darling Downs IAE *Darling Downs Institute of Advanced Education*
Footscray IT *Footscray Institute of Technology*
Gippsland IAE *Gippsland Institute of Advanced Education*
Preston IT *Preston Institute of Technology*
Tasmanian CAE *Tasmanian College of Advanced Education*
Warrnambool IAE *Warrnambool Institute of Advanced Education*

PG2 *Master Degree*
PG1 *Post-graduate Diploma*
UG1 *Bachelor Degree*
UG2 *Diploma*
UG3 *Associate Diploma*

Source: (a) *Calendars* and *Handbooks* of Colleges of Advanced Education

(b) Department of Education (1978) *Directory of Tertiary Courses.* Canberra: Australian Government Publishing Service

Appendix B - Engineering Enrolments

Table B

Engineering Enrolments as at 30th April 1979 by Level and Attendance Pattern in DOCIT Colleges

I: Post-graduate

Engineering	College	PG2 FT	PG2 PT	PG2 EX	Total	PG1 FT	PG1 PT	PG1 EX	Total
	CIT	1.0	5.0	-0	6.0	-0	16.0	-0	16.0
	NSWIT	1.0	47.0	1.0	49.0	-0	-0	-0	0
	QIT	-0	3.0	-0	3.0	-0	57.0	-0	57.0
	RMIT	6.0	12.0	-0	18.0	-0	147.0	-0	147.0
	SAIT	1.0	7.0	-0	8.0	4.0	22.0	-0	26.0
	SCT	8.0	13.0	-0	21.0	-0	262.0	-0	262.0
	WAIT	-0	8.0	2.0	10.0	7.0	54.0	11.0	72.0
H.C.		17.0	95.0	3.0	115.0	11.0	558.0	11.0	580.0
EFTS		17.0	47.5	1.5	66.0	11.0	279.0	5.5	295.5

PG1 *Post-graduate Diploma*
PG2 *Master Degree*
H.C. *Head Count*
EFTS *Effective Full Time Students*
EX *External*

Source: Data Collection for *DOCIT Annual Statistical Report 1979*. The DOCIT Research Secretariat, the New South Wales Institute of Technology, Sydney

Table B (Cont.)

Engineering Enrolments as at 30th April 1979 by Level and Attendance Pattern in DOCIT Colleges

II: Undergraduate

Engineering	College	UG1 FT	UG1 PT	UG1 EX	Total	UG2 FT	UG2 PT	UG2 EX	Total	UG3 FT	UG3 PT	UG3 EX	Total
	CIT	323.0	31.0	-0	354.0	102.0	184.0	-0	286.0	-0	-0	-0	0
	NSWIT	563.0	1059.0	-0	1622.0	-0	-0	-0	0	-0	-0	-0	0
	QIT	571.0	247.0	-0	818.0	-0	-0	-0	0	126.0	370.0	-0	496.0
	RMIT	1225.0	405.0	2.0	1632.0	69.0	76.0	11.0	156.0	-0	-0	-0	0
	SAIT	457.0	223.0	-0	680.0	-0	-0	-0	0	86.0	257.0	-0	343.0
	SCT	529.0	264.0	-0	793.0	70.0	99.0	-0	169.0	-0	-0	-0	0
	WAIT	538.0	363.0	28.0	929.0	-0	19.0	2.0	21.0	48.0	49.0	5.0	102.0
H.C.		4206.0	2592.0	30.0	6828.0	241.0	378.0	13.0	632.0	260.0	676.0	5.0	941.0
EFTS		4206.0	1296.0	15.0	5517.0	241.0	189.0	6.5	436.5	260.0	338.0	2.5	600.5

UG1 *Bachelor Degree*
UG2 *Diploma*
UG3 *Associate Diploma*
H.C. *Head Count*
EFTS *Effective Full Time Students*
EX *External*

Source: Data Collection for *DOCIT Annual Statistical Report 1979.* The DOCIT Research Secretariat, the New South Wales Institute of Technology, Sydney

Appendix C - Textbooks Used in the Analysis of Mathematical Topics

1. Bajpai, A.C., Calus, I.M., and Fairley, J.A. *Mathematics for Engineers and Scientists*. 2. John Wiley, 1973.
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7. Heading, J. *Mathematical Methods in Science and Engineering*. Edward Arnold, 1963.
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13. Rainville, E.D., and Bedient, P.E. *Elementary Differential Equations*. 5th edn. Collier Macmillan, 1974.
14. Scheid, F. *Numerical Analysis*. McGraw-Hill, 1968.
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16. Spiegel, M.R. *Advanced Mathematics for Engineers and Scientists*. McGraw-Hill, 1971.
17. Spiegel, M.R. *Theory and Problems of Complex Variables*. McGraw-Hill, 1964.
18. Thomas, G.B., and Finney, R.L. *Calculus and Analytic Geometry*. 5th edn. Addison Wesley, 1979.
19. Walpole, R.E., and Myers, R.H. *Probability and Statistics for Engineers and Scientists*. Collier Macmillan, 1972.

20. Wylie, C.R. *Advanced Engineering Mathematics.* McGraw-Hill, 1975.

Appendix D - Related Works by the Author

Bajpai, A.C. and Singh, J.G. (1980) *A Survey of Mathematics Courses for Engineers in Australia.* Report Number 15. Sydney: Tertiary Education in Applied Mathematics Project, the New South Wales Institute of Technology.

Singh, J.G. and Shannon, A.G. (1980) "Advanced Education and Technological Change", *Unicorn: Bulletin of the Australian College of Education.* 6, 1, 35-43.

Singh, J.G. (in press) "Letter to the Editor", *International Journal of Electrical Engineering Education.*

Singh, J.G. (submitted) "New Responsibilities in the Mathematical Education of Engineers". Paper for the *1981 Engineering Conference* of the Institution of Engineers, Australia, to be held in March at Canberra.