Why the details matter: Learning from Japanese *Kyouzai Kenkyuu*

by **Tatsuhiko Seino** and **Colin Foster**

Have you ever heard someone say, "It's exactly the same question, just with different numbers"? Sometimes a teacher might say this about a practice test or a set of exercises in a textbook. The implication is that the details of the numbers make no substantive difference to the problem. In mathematics task design, careful thought is not always given to the particular numbers used in questions; in some circumstances, these might even be randomly computer-generated. However, in Japanese mathematics teaching, the numbers used in questions (and the details in general) are taken extremely seriously. In this article, we will explain why the details of tasks matter so much. We will show across four mathematics topics with different ages of students how the detailed choices of examples can be critical for students to learn what is intended.

Subtraction with Regrouping

In Grade 1 (age 6–7), Japanese students learn subtraction with regrouping, which is typically (2-digit) – (1-digit) across 10, where the minuend is less than 20 and the subtrahend is less than 10. When designing an introductory example, a Japanese task designer, rather than picking the numbers in a haphazard fashion, would consider the 36 possible calculations that satisfy this condition (Fig. 1). Which calculation would you choose for the introduction of subtraction with regrouping?

We will compare the numbers used by all six textbooks approved by the Ministry of Education, Culture, Sports, Science and Technology in 2015, extracts from which are shown in Figure 2. Even without being able to read Japanese, you can see that four of the books used 13 – 9 and the other two went with 12 – 9. These were the only examples used. Why might this be?

The reason relates to the four main methods for subtraction with regrouping, which are illustrated below for the example of 13 – 9:

1. *Counting down from 13*: 12, 11, 10, 9, 8, 7, 6, 5, 4, so it's 4.
2. *Counting up from 9*: 10, 11, 12, 13, so it's 4.
3. *Subtraction – subtraction*:
   (13 – 3) – 6 = 10 – 6 = 4.
4. *Subtraction – addition*:
   (10 – 9) + 3 = 1 + 3 = 4.

Methods 3 and 4 are the more sophisticated methods, which the books wish to emphasize. Method 4 is particularly powerful, as it works well even for larger numbers, and may involve less cognitive load than method 3. In method 3, it is necessary to partition the subtrahend (9) according to the minuend (13); in the example above, 9 must be partitioned into 3 and 6, and then the 3 is subtracted from the 13 and the remaining 6 from the 10. This process may involve higher cognitive load for students than method 4, where it is the *minuend* (13) that is partitioned into 10 and 3.

The reason for choosing 13 – 9 and 12 – 9 is that these problems are highly suited to method 4. To subtract 9 from 10, we need to see 13 as '10 and 3', and students have, in their lessons immediately before this, learned to decompose numbers into '10 and something'. Method 4 is usually dominant when solving a problem such as 13 – 9, because 9 is so close to 10 (Doig *et al*., 2011, p. 192).

Later on in the books, when the intention is to focus students on method 3 instead, the numerical choices made by the same books are different (Fig. 3). Method 3 is preferred when the number in the units column of

11 – 2							
11 – 3	12 – 3						
11 – 4	12 – 4	13 – 4					
11 – 5	12 – 5	13 – 5	14 – 5				
11 – 6	12 – 6	13 – 6	14 – 6	15 – 6			
11 – 7	12 – 7	13 – 7	14 – 7	15 – 7	16 – 7		
11 – 8	12 – 8	13 – 8	14 – 8	15 – 8	16 – 8	17 – 8	
11 – 9	12 – 9	13 – 9	14 – 9	15 – 9	16 – 9	17 – 9	18 – 9

Fig. 1

どんぐりが 13こ あります。 9こ つかいました。 どんぐりは，なんこ のこって いますか。 Tokyo shoseki (2015) There are 13 acorns. We used 9. How many acorns are left?	かきが 13こ なって います。 9こ とると，なんこ のこりますか。 Keirinkan (2015) There are 13 persimmons. If we take 9, how many persimmons are left?
どんぐりが 12こ あります。こまを つくるのに 9こ つかいました。 のこりの どんぐりは，なんこでしょうか。 Gakkou tosho (2015) There are 12 acorns. We used 9 to make the [spinning] tops. How many acorns are left?	ぎゅうにゅうが 12本 ありました。 9本 くばりました。 のこりは なん本 でしょうか。 Kyouiku shuppan (2015) There were 12 bottles of milk. We distributed 9. How many bottles of milk are left?
こうえんに 13にん いました。 9にん かえりました。 こうえんには なんにん のこって いるでしょう。 Dainippon tosho (2015) There were 13 people in the park. 9 people left. How many people are left in the park?	かきが 13こ なって いました。9こ とりました。 かきは，なんこ のこって いますか。 Nihonbunkyou shuppan (2015) There were 13 persimmons. We took 9. How many persimmons are left?

Fig. 2

Tokyo shoseki (2015)	12 – 3
Keirinkan (2015)	13 – 4
Gakkou tosho (2015)	11 – 2
Kyouiku shuppan (2015)	12 – 3
Dainippon tosho (2015)	12 – 3
Nihonbunkyou shuppan (2015)	12 – 3

Fig. 3

the minuend is close to the subtrahend number (Doig *et al*., 2011, p. 192), so examples such as 12 – 3 and 11 – 2 are natural choices. In both cases, examples are chosen to focus students' thinking on the mathematical method that the teacher is emphasizing at a particular time.

Division of Fractions

In Japan, students learn about division of fractions in Grade 6 (age 11–12). Figure 4 shows the examples used by the six textbooks to introduce division of fractions. All six books use the scenario of painting of a board, walls, fences or desks, and the numerical values used are:

$\frac{2}{5} \div \frac{3}{4}$ (4 books)

$\frac{5}{8} \div \frac{2}{3}$ (1 book)

$\frac{3}{5} \div \frac{2}{3}$ (1 book).

In addition, Keirinkan (2015), Kyouiku syuppan (2015) and Nihonbunkyou shuppan (2015) first consider the problem of dividing fractions with unit fraction divisor ($\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{3}$), before using the non-unit-fraction divisor ($\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{3}$). So, again, we ask, why do four of the books use exactly the same calculation? You might like to think about what might be so good about this particular example before reading on.

One reason for using $\frac{2}{5} \div \frac{3}{4}$ is that both $\frac{2}{5}$ and $\frac{3}{4}$ can be expressed as decimals, meaning that the answer to $\frac{2}{5} \div \frac{3}{4}$ can be obtained from the calculation 0.4 ÷ 0.75 using students' prior knowledge of decimal division from Grade 5. Also notice that this calculation leads to a recurring decimal answer of 0.5333..., so the goal of the subsequent thinking can be to find a way to answer the question *precisely in fractions*. Another important feature of $\frac{2}{5} \div \frac{3}{4}$ is that the four numbers used as numerators and denominators are all different, which may help students to generalize from this one example. By contrast, a calculation such as $\frac{3}{5} \div \frac{3}{4}$ would be less helpful in this

<table>
<tr>
<td>$\frac{3}{4}$dLのペンキで，板を$\frac{2}{5}$m²ぬれました。
このペンキ1dLでは，板を何m²ぬれますか。

Tokyo shoseki (2015)

With $\frac{3}{4}$ dL of paint, we could paint $\frac{2}{5}$ m^2 of board. What area of board can we paint with 1dL of this paint?</td>
<td>$\frac{3}{5}$m²のかべを$\frac{1}{3}$dLでぬれるペンキがあります。
このペンキ1dLでぬれる面積を求める式をかきましょう。

Keirinkan (2015)

With $\frac{1}{3}$ dL of paint, we can paint $\frac{3}{5}$ m^2 of wall. Let's write an expression for the area of wall that we can paint with 1 dL of this paint.

Then…

$\frac{3}{5}$m²のかべをぬるのに，ペンキを$\frac{2}{3}$dL使いました。
このペンキ1dLで何m²ぬれますか。

We used $\frac{2}{3}$ dL of paint to paint $\frac{3}{5}$ m^2 of wall. What area of wall can we paint with 1dL of this paint?</td>
</tr>
<tr>
<td>$\frac{2}{5}$m²のへいをぬるのに，黄色いペンキを$\frac{3}{4}$dL使います。このペンキでは1dLあたり何m²ぬれるでしょうか。

Gakkou tosho (2015)

We use $\frac{3}{4}$ dL of yellow paint to paint $\frac{2}{5}$ m^2 of fence. What area of fence can we paint with 1dL of this paint?</td>
<td>$\frac{1}{4}$dLで$\frac{2}{5}$m²の板をぬれるペンキがあります。
このペンキ1dLでは，何m²の板をぬれるでしょうか。

Kyouiku shuppan (2015)

With $\frac{1}{4}$ dL of paint, we can paint $\frac{2}{5}$ m^2 of board. What area of board can we paint with 1 dL of this paint?

Then…

$\frac{3}{4}$dLで$\frac{2}{5}$m²の板をぬれるペンキがあります。
このペンキ1dLでは，何m²の板をぬれるでしょうか。

With $\frac{3}{4}$ dL of paint, we can paint $\frac{2}{5}$ m^2 of board. What area of board can we paint with 1dL of this paint?</td>
</tr>
<tr>
<td>$\frac{3}{4}$dLで$\frac{2}{5}$m²の板をぬれるペンキがあります。
このペンキ1dLでは，何m²の板をぬれるでしょう。

Dainippon tosho (2015)

With $\frac{3}{4}$ dL of paint, we can paint $\frac{2}{5}$ m^2 of board. What area of board can we paint with 1 dL of this paint?</td>
<td>$\frac{1}{3}$dLのペンキで，机(つくえ)を$\frac{5}{8}$m²ぬれました。
このペンキ1dLでは，机を何m²ぬれますか。

Nihonbunkyou shuppan (2015)

With $\frac{1}{3}$ dL of paint, we could paint $\frac{2}{5}$ m^2 of desk. What area of desk can we can paint with 1 dL of this paint?

Then…

$\frac{2}{3}$dLのペンキで，机(つくえ)を$\frac{5}{8}$m²ぬれました。
このペンキ1dLでは，机を何m²ぬれますか。

With $\frac{2}{3}$ dL of paint, we could paint $\frac{5}{8}$ m^2 of desk. What area of desk can we paint with 1 dL of this paint?</td>
</tr>
</table>

Fig. 4

respect, since, if students were to write $\frac{3}{5}\times\frac{4}{3}$, there would be ambiguity over which of the 3s here had come from the numerator of $\frac{3}{5}$ and which from the numerator of $\frac{3}{4}$. This could lead to a faulty generalization of

$\frac{a}{b}\div\frac{c}{d}=\frac{c}{b}\times\frac{d}{a}$ instead of the correct $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$

So, keeping all four numbers distinct is extremely useful.

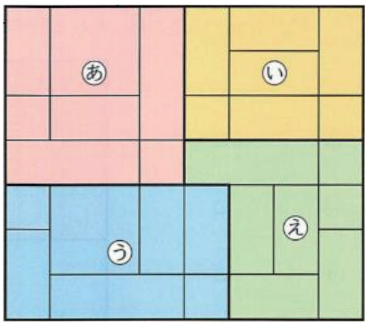
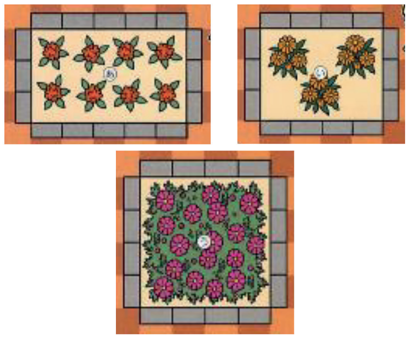
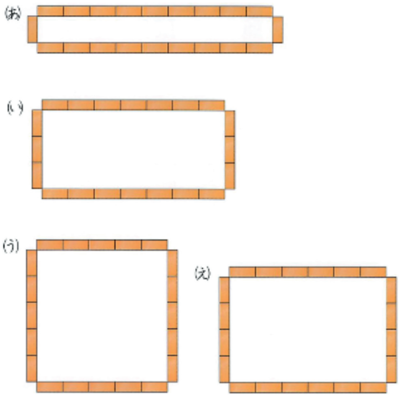
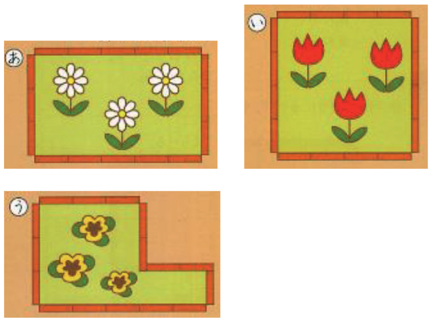
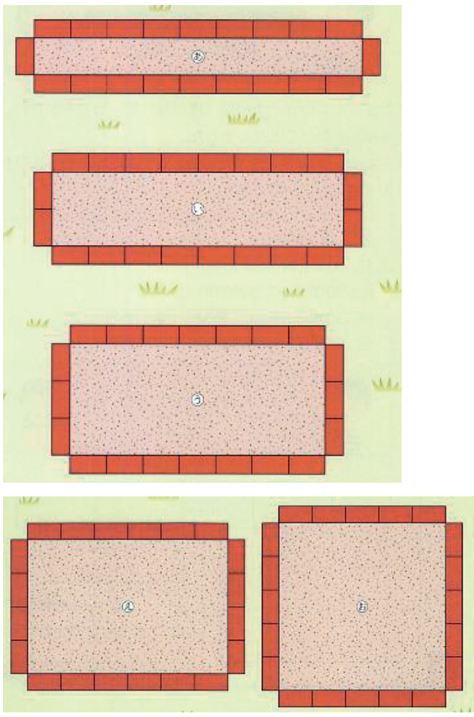
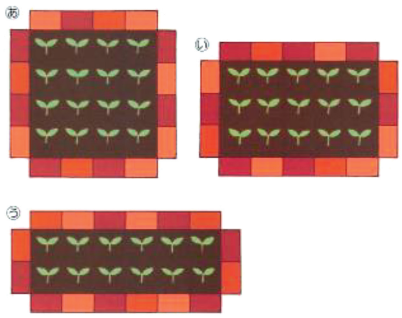
 Tokyo shoseki (2015) The perimeter is 16 in the upper left, 14 in the upper right, 16 in the lower left, and 16 in the lower right.	 Keirinkan (2015) The perimeter is 16 in the upper left, 14 in the upper right, and 16 in the lower drawing.
 Gakkou tosho (2015) The perimeters are all 20.	 Kyouiku shuppan (2015) The perimeters are all 20.
 Dainippon tosho (2015) The perimeters are all 20.	 Nihonbunkyou shuppan (2015) The perimeters are all 16.

Fig. 5

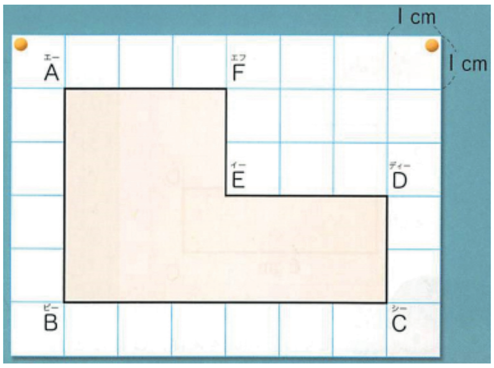
Area and Perimeter

In Grade 4 (age 9–10), when students learn about area, all of the six textbooks introduce this by presenting figures of equal perimeter and asking for which figure the amount of space is largest, and the numbers used are quite similar across the textbooks (Fig. 5). Students may think that the longer the perimeter, the larger the area, and may also think that when the perimeter is the same, the area is the same (Kaji, 1983). These examples highlight this misconception and challenge it.

But why the similarity in the numbers chosen? In the case of figures of equal perimeter, we obtain the maximum area when the rectangle is a square. Therefore, it is important to compare oblongs with squares. So, we start by creating the square. A side length of 3 is too short, and 6 feels a little too long. An appropriate length is 4 or 5, meaning that the perimeter will be 16 for a 4 × 4 square or 20 for a 5 × 5 square, and these values dictate the perimeters used.

Calculating the Area of Compound Shapes

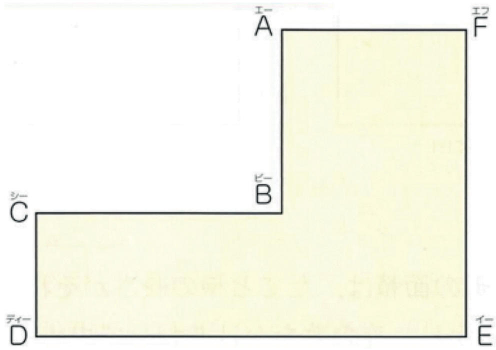
In Grade 4 (age 9–10), students in Japan learn how to calculate the area of rectangles and squares. They also learn how to calculate the area of rectilinear compound figures consisting of juxtaposed rectangles. Figure 6 shows how each textbook presents compound figures.



Tokyo shoseki (2015)

$$4 \times 3 + 2 \times 3 = 18$$

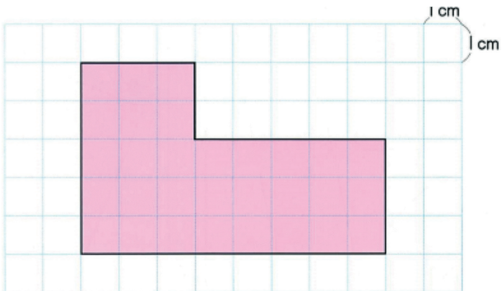
Answer $18cm^2$



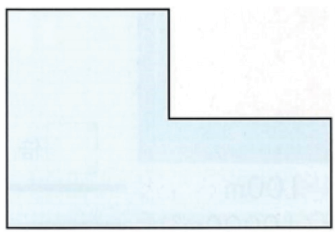
Keirinkan (2015)

$$2 \times 4 + 5 \times 3 = 23$$

$23cm^2$

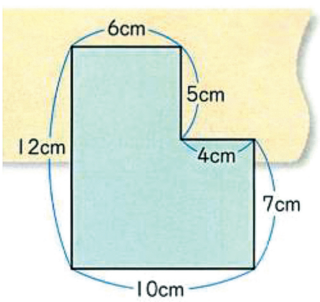


Gakkou tosho (2015)



Kyouiku shuppan (2015)

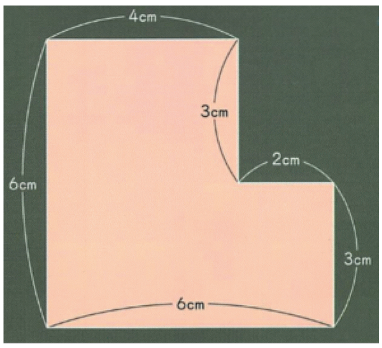
$4 \times 6 - 2 \times 3$



Dainippon tosho (2015)

$$5 \times 6 + 7 \times 10 = 100$$

Answer $100cm^2$



Nihonbunkyou shuppan (2015)

$$3 \times 4 + 3 \times 6 = 30$$

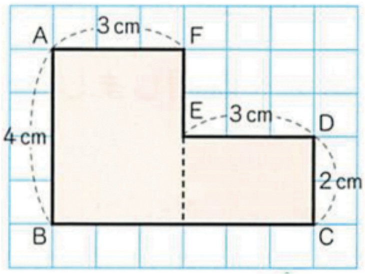
Answer $30cm^2$

Fig. 6

The dimensions are presented in various ways: the figure is placed on the grid and the length is read off (Tokyo shoseki and Gakkou tosho); the length is stated alongside the edges of the figure (Dainippon tosho and Nihonbunkyou shuppan); and the figure has neither grid nor stated measurements and the length is measured by the student (Keirinkan and Kyouiku shuppan). Why these differences? And why are the numbers used in Tokyo shoseki and Gakkou tosho the same – and different from those used in the other textbooks? Again, the answers lie in the different possible methods; in this case, for calculating the area (Fig. 7).

When we calculate the area, it is important to understand which edges we need to measure. *Keirinkan* and *Kyouiku shuppan* emphasize this point. But, for children, it is

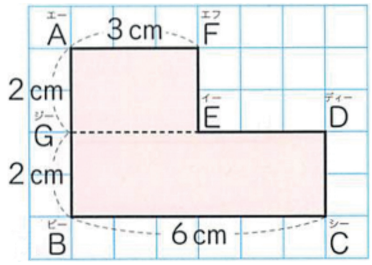
1. Vertical split



$4 \times 3 + 2 \times 3 = 18$

Answer 18cm²

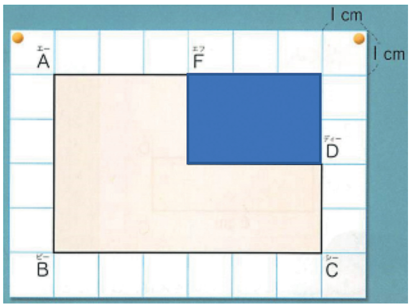
2. Horizontal split



$2 \times 3 + 2 \times 6 = 18$

Answer 18cm²

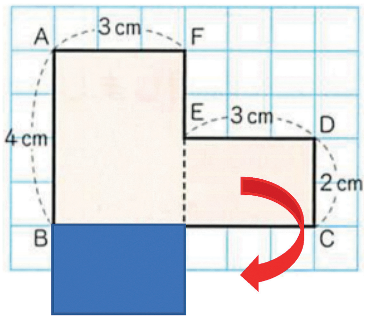
3. Large rectangle – small rectangle



$4 \times 6 - 2 \times 3 = 18$

Answer 18cm²

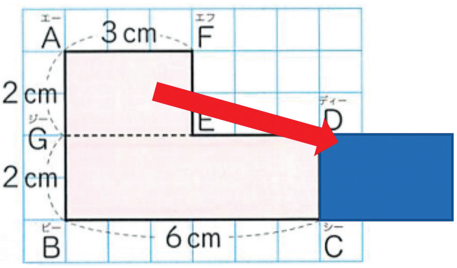
4. Rearrangement 1



$(2 + 4) \times 3 = 18$

Answer 18cm²

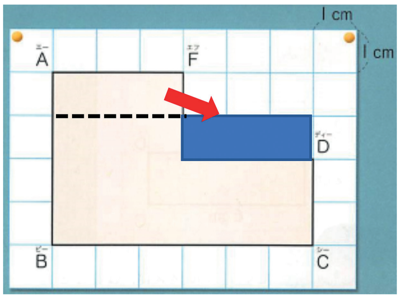
5. Rearrangement 2



$2 \times (3 + 6) = 18$

Answer 18cm²

6. Rearrangement 3



$3 \times 6 = 18$

Answer 18cm²

7. Duplication

$4 \times (3 + 6) \div 2 = 18$

Answer 18cm²

Fig. 7

difficult to measure the length of the edges themselves, and the textbooks come to different conclusions about the balance between the difficulty and importance of measurement.

The figures and numerical values in the Tokyo shoseki and Kyouiku shuppan books allow all seven methods to be used, whereas the Keirinkan and Dainippon tosho books use examples where it is difficult to deform the compound figures into a single rectangle. The method of duplication works only for the Tokyo shoseki, Kyouiku shuppan and Nihonbunkyou shuppan books. So, the choices of these examples reflect the methods that the textbooks wish to prioritize. Indeed, the Tokyo Shoseki book (2011) goes so far as to provide a blank space adjacent to the figure to allow a duplicate shape to be drawn in (Fig. 8).

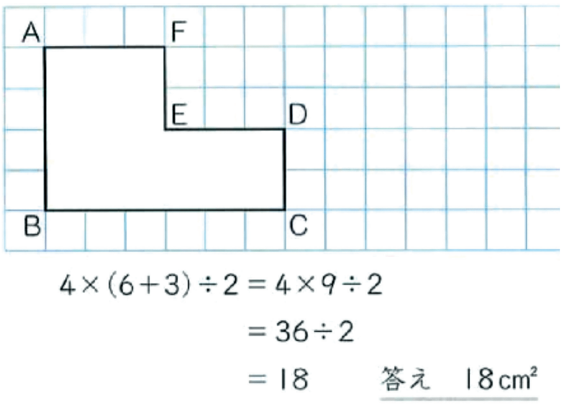


Fig. 8

Notice also throughout that area is always expressed as 'height × width'. This consistent approach to expressing calculations facilitates here the teacher–student and student–student discussions about, and understanding of, the different methods used by different students.

Conclusion

The examples used in these four topics show the care that is taken in Japan over the details of the mathematical examples and questions used in textbooks. They demonstrate how the tasks are designed to ensure that student thinking carefully builds from prior knowledge and takes into account how current thinking needs to inform future learning. Considering in detail the possible methods that students might use, and mapping these onto possible examples, allows the task designer to select numbers that will promote or elucidate the use of a particular method. This then enables tasks to be constructed that really help students to focus on the deep features of the mathematics to be learned.

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Keywords: Japanese textbooks; Kyouzai Kenkyuu; Task design.

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