# Incorporating Risk in Field Services Operational Planning process 

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#### Abstract

This paper presents a model for the risk minimisation objective in the Stochastic Vehicle Routing Problem (SVRP). In the studied variant of SVRP, service times and travel times are subject to stochastic events, and a time window is constraining the start time for service task. Required skill levels and task priorities increase the complexity of this problem. Most previous research uses a chance-constrained approach to the problem and their objectives are related to traditional routing costs whilst a different approach was taken in this paper. The risk of missing a task is defined as the probability that the technician assigned to the task arrives at the customer site later than the time window. The problem studied in this paper is to generate a schedule that minimises the maximum of risks and sum of risks over all the tasks considering the effect of skill levels and task priorities. The stochastic duration of each task is supposed to follow a known normal distribution. However, the distribution of the start time of the service at a customer site will not be normally distributed due to time window constraints. A method is proposed and tested to approximate the start time distribution as normal. Moreover, a linear model can be obtained assuming identical variance of task durations. Additionally Simulated Annealing method was applied to solve the problem. Results of this work have been applied to an industrial case of SVRP where field engineering individuals drive to customer sites to provide time-constrained services. This original approach gives a robust schedule and allows organisations to pay more attention to increasing customer satisfaction and become more competitive in the market.


Keywords: Vehicle Routing with Time Windows; Stochastic Service Time and Travel Time; Risk Minimisation.

## 1 Introduction

Increasing customer satisfaction is always an exciting topic for managers and researchers in order to build a more customer-oriented business. It is in particular true when planning geographically distributed services on customer sites. Therefore, the consideration of visit time windows, the stochastic service time and travel time in the workforce scheduling and vehicle routing problems (VRP) becomes crucial for ser-
vice providing organisations [1]. Specifically, in the studied application domain, a start-of-day planned tour of visits is created overnight, and then updated throughout the day as more and new information becomes available. The start-of-day schedule has to make certain assumptions such as technician availability, travel times, how long certain engineering tasks will take, whether technicians will be able to successfully complete work or whether they have to come back again, how much additional work will arrive during the day, and so on [2]. In terms of traffic networks, the travel time varies due to traffic congestion especially in big cities, which affects not only the service quality but also the air pollution [3].

The environment in which services need to be delivered is inherently dynamic and subject to disruption in the workstack estimates, in the execution of jobs by workforce as well as the travel conditions [4, 5]. Our research studies how elements of the risk can be incorporated into the scheduling approach, i.e., how the scheduling can manage and address the aforementioned sources of the risk to build both an optimal but also a robust schedule that minimises the risks and increases the likelihood of successful service delivery. A better schedule can help to improve the level of customer satisfaction as well as the work efficiency of technicians. Consequently, the company may get more customers and become more competitive in the market.

This research focuses on the Stochastic Vehicle Routing Problem (SVRP), in which technicians drive to customer sites to provide services. In the problem, we assume that service times and travel times are stochastic, and a time window is associated with the start time of the service.

Most previous relevant researches on VRP consider time windows and stochastic demands, $[6,7,8,9,10,11$, and 12] use a chance-constrained approach to the problem. Limited researches [1, 3, 5, and 13] investigate the routing problem with time windows and stochastic travel time. We note that in these approaches, the objectives of the problem are related to traditional routing costs. In this paper, we introduce a new risk model that can be incorporated into the set of objectives to be minimised during the optimisation process.

In this paper, we present an application of the proposed model to a real scheduling problem in the field-engineering-service world. In that context, technicians offer services to customers, associating with a time window to each visit, and services are subject to disturbance in delivery causing the actual service time to be inherently stochastic, as well as the travel time to be uncertain. Section 2 gives a short description about the risk according to our previous research; while in section 3, a complicated estimation of risks is proposed in order to prove that the normal distribution can be used in the risk calculation. In section 4 a mathematical model is constructed by considering risks in the objective and can be solved by an exact method. Due to the complexity of the problem, heuristic methods show advantages while solving massive size problems. Thus section 5 gives several a heuristic method - Simulated Annealing method - for this problem, followed by some results and discussions in section 6.

## 2 Risk definition

The risk in the problem is defined as the probability that the arrival time is after the upper limit of the time window [14]. More specifically, as it is shown in the Fig.1, given a schedule, with a sequence of tasks $\left\{i_{1}, i_{2}, i_{3}, \cdots\right\}$ allocated to technician $k, T_{1}, T_{2}, T_{3} \ldots$ are the task duration times, $d_{k i_{0} i_{1}}$ is the travel time from the depot of technician $k$ to his first task, $d_{k i_{1} i_{2}}, d_{k i_{2} i_{3}}, d_{k i_{3} i_{4}}$ are the travel times between tasks (e.g., $d_{k i_{1} i_{2}}$ from $1^{\text {st }}$ task to $2^{\text {nd }}$ task). Suppose the arrival time follows a normal distribution, then we define the risk of missing the appointment for task $i, R_{k i}$, as the probability of the arrival time $A T_{k i}$ being later than the upper limit of the time window $b_{i}$. The stochasticity of the arrival time arises from the uncertainty of the travel time and task duration. Moreover, an important property is that the risk increases simultaneously as it propagates for each technician, it is reasonable because the uncertainty aggregates as there is more uncertainty of the travel time and task time. In addition, from a previous study [14], the mathematical expression of risks can be derived as

$$
\begin{equation*}
R_{k i_{n}}=\mathrm{P}\left(A T_{k i_{\mathrm{n}}}>b_{i_{\mathrm{n}}}\right)=1-\int \cdots \int_{D} \prod_{l=1}^{n-1} f_{k i_{l}}\left(X_{l}\right) d X_{1} d X_{2} \cdots d X_{n-1}, \tag{1}
\end{equation*}
$$

where $X_{1}$ denotes the arrival time $A T_{k i_{2}}, X_{l}=\delta_{k i_{1}}+d_{k i_{1} i_{1+1}}$ for $l \geq 2,, f_{k i_{l}}\left(X_{l}\right)$ represents the probability density function of $X_{l}$, and $D=\left\{\left(X_{1}, \ldots, X_{n-1}\right): \sum_{l=1}^{n-1} X_{l} \leq\right.$ $\left.b_{i_{n}}, \sum_{l=2}^{n-1} X_{l} \leq b_{i_{n}}-a_{i_{2}}, \sum_{l=3}^{n-1} X_{l} \leq b_{i_{n}}-a_{i_{3}}, \ldots, X_{n-1} \leq b_{i_{n}}-a_{i_{n-1}}\right\}$.


Fig. 1. Description of risks

## 3 Estimate of risks

To begin with, by analysing 72114 task data over a 12 month period [15], according to the task types that have a large number of samples, we found that the distributions of the actual time spent on the task, for each task type mostly are normally distributed or follow gamma distributions. Therefore, it is reasonable for us to use a normal distribution to calculate risks.

In the problem, due to the effect of time windows, technicians have to start work after the lower limit of the time window, so the distribution of the start time will be of the format in Fig. 3, which is not a normal distribution shape. More specifically, given a technician, suppose the arrival time at his/her 1 st customer service spot $A T_{k 1}$ fol-
lows the normal distribution in Fig. 2 and the lower bound of the task time window $a_{1}$ is $9: 00$, then the distribution of the task start time $S T_{k 1}$ will turn out to be the distribution shown in Fig. 3, the probability at 9:00 will be the sum of the probability of that arrival time to be before 9:00. From the figure, it can be seen that the distribution of the start time $S T_{k 1}$ does not align with a normal distribution. Then combined with the 1st task duration normally distributed as shown in Fig. 4, the arrival time at the 2nd customer site looks like the distribution illustrated in Fig. 5. Theoretically, it is not normal, but the risk is intuitively defined as the right tail of the arrival time distribution. Also, it is observed that a normal distribution may fit the arrival time well, especially regarding the right tail. Therefore, the idea of using a normal distribution to estimate the skewed start time distribution comes naturally.

Furthermore, it is easy to conclude that the closer the average arrival time at the $1^{\text {st }}$ customer site $\mu$ is to the lower limit of $1^{\text {st }}$ task time window $a_{1}$, the more the start time distribution changes. From the Fig. 6, it is easy to observe that if the time window $a_{1}=\mu-2 \sigma$, where $\sigma$ is the standard deviation of the arrival time at the $1^{\text {st }}$ customer site, the effect of the time window is small as shown in (b), compared to the original arrival time distribution shown in (a). In terms of the time window closer to the average arrival time, such as $a_{1}=\mu-0.5 \sigma$, the effect of the time window is shown in Fig. 6 (c), the shape of the start time is completely different from the arrival time. As for the scenario where the mean of the arrival time $\mu$ is much earlier than the lower limit of time window $a_{1}$, i.e., $a_{1}=\mu+2 \sigma$, the task is likely to start at time $a_{1}$ with a high probability so that the variance of the arrival time can be omitted.


Fig. 2. Arrival time


Fig. 4. Work time


Fig. 3. Start time


Fig. 5. Arrival time at the next task


Fig. 6. Time window effect on the start time
Therefore, the estimation consists of three scenarios. To start with, if the mean $\mu$ of the arrival time $A T_{1}$ is much later than $a_{1}$, i.e., $a_{1} \in[\mu-3 \sigma, \mu-\sigma)$, it is reasonable to use the normal distribution estimation model I below for the arrival time $A T_{2}$ that

$$
\begin{align*}
& \mu\left(A T_{2}\right)=\mu\left(S T_{1}\right)+\mu\left(T T_{1}\right)  \tag{2}\\
& \sigma^{2}\left(A T_{2}\right)=\sigma^{2}\left(S T_{1}\right)+\sigma^{2}\left(T T_{1}\right) \tag{3}
\end{align*}
$$

where $T T_{1}$ is the operation time of the $1^{\text {st }}$ task plus the travel time from the $1^{\text {st }}$ task to the $2^{\text {nd }}$ task. In this scenario, the effect of the time window is so little that can be ignored. In contrast, if the mean $\mu$ of the arrival time is much earlier than $a_{1}$, i.e., $a_{1} \in$ ( $\mu+\sigma, \mu+3 \sigma]$, which is shown in the example of the Fig. 6 (d), the variance of the start time can be ignored because of the long waiting time till $a_{1}$. Then the estimation model II for the arrival time $A T_{2}$ is as below

$$
\begin{align*}
& \mu\left(A T_{2}\right)=a_{1}+\mu\left(T T_{1}\right)  \tag{4}\\
& \sigma^{2}\left(A T_{2}\right)=\sigma^{2}\left(T T_{1}\right) . \tag{5}
\end{align*}
$$

Then for the complicated scenario where $a_{1} \in[\mu-\sigma, \mu+\sigma]$, the calculation may follow the approach from Madarajah and Kotz [15] who investigated the exact distribution of the maximum and minimum of two Gaussian random variables. Suppose $X_{1}$ and $X_{2}$ are Gaussian random variables, then the mean and variance of $X=$ $\max \left\{X_{1}, X_{2}\right\}$ are
$E(X)=\mu_{1} \Phi\left(\frac{\mu_{1}-\mu_{2}}{\theta}\right)+\mu_{2} \Phi\left(\frac{\mu_{2}-\mu_{1}}{\theta}\right)+\theta \phi\left(\frac{\mu_{1}-\mu_{2}}{\theta}\right)$,
$E\left(X^{2}\right)=\left(\sigma_{1}^{2}+\mu_{1}^{2}\right) \Phi\left(\frac{\mu_{1}-\mu_{2}}{\theta}\right)+\left(\sigma_{2}^{2}+\mu_{2}^{2}\right) \Phi\left(\frac{\mu_{2}-\mu_{1}}{\theta}\right)+\left(\mu_{1}+\mu_{2}\right) \theta \phi\left(\frac{\mu_{1}-\mu_{2}}{\theta}\right)$,
where $\theta=\sqrt{\sigma_{1}^{2}+\sigma_{2}^{2}-2 \rho \sigma_{1} \sigma_{2}}, \rho$ is the correlation between $X_{1}$ and $X_{2}, \Phi(\cdot)$ and $\phi(\cdot$ ) are the probability density function and the cumulative distribution function of the standard normal distribution respectively. Nadarajah and Kotz also state that if the standard deviation $\sigma_{1}, \sigma_{2}$ of the two Gaussian random variables is identical, the Gaussian random variable with the mean and variance of $E(X)$ and $E\left(X^{2}\right)-E^{2}(X)$ can well approximate the distribution of $X=\max \left\{X_{1}, X_{2}\right\}$. Moreover, with the higher difference between $\sigma_{1}$ and $\sigma_{2}$, the estimate gets worse.

In terms of our engineering services risk model, a constant which is the lower limit of the time window replaces one of the Gaussian random variables and the correlation $\rho$ is set to $0(\rho=0)$. Therefore the parameters of the normal distribution estimation for $S T_{1}=\max \left\{A T_{1}, a_{1}\right\}$ are
$E\left(S T_{1}\right)=\mu_{1} \Phi\left(\frac{\mu_{1}-a_{1}}{\sigma_{1}}\right)+a_{1} \Phi\left(\frac{a_{1}-\mu_{1}}{\sigma_{1}}\right)+\sigma_{1} \phi\left(\frac{\mu_{1}-a_{1}}{\sigma_{1}}\right)$,
$E\left(S T_{1}^{2}\right)=\left(\sigma_{1}^{2}+\mu_{1}^{2}\right) \Phi\left(\frac{\mu_{1}-a_{1}}{\sigma_{1}}\right)+a_{1}^{2} \Phi\left(\frac{a_{1}-\mu_{1}}{\sigma_{1}}\right)+\left(\mu_{1}+a_{1}\right) \sigma_{1} \phi\left(\frac{\mu_{1}-a_{1}}{\sigma_{1}}\right)$,
where $\mu_{1}$ and $\sigma_{1}$ are the mean and standard deviation of the arrival time $A T_{1}$. Then the estimated risk of the $2^{\text {nd }}$ task is obtained via the $S T_{1}$ estimation and the normal distribution of the mixed time $T T_{1}$. Thus the normal distribution estimation model III for the arrival time $A T_{2}$ is as below

$$
\begin{align*}
& \mu\left(A T_{2}\right)=E\left(S T_{1}\right)+\mu\left(T T_{1}\right)  \tag{10}\\
& \sigma^{2}\left(A T_{2}\right)=E\left(S T_{1}^{2}\right)-E^{2}\left(S T_{1}\right)+\sigma^{2}\left(T T_{1}\right) \tag{11}
\end{align*}
$$

If we suppose that

$$
\text { error }=\text { real risk }- \text { estimate risk },
$$

the estimation gives relative small error terms based on a 5000 -sample test, which are shown in Fig. 7, in which the vertical axis represents the error value according to the number codes of 5000 samples shown in the horizontal axis.


Fig. 7. Error of the estimation model

## 4 Mathematical model

Let $G=(V, A)$ be a complete graph, where $V=V_{0} \cup V_{c}$, and $V_{c}=\{1, \ldots, N\}$ is a set of vertices which denote customer locations and $A=\left\{(l, j): l, j \in V_{c}, l \neq j\right\}$ is a set of arcs. While $V_{0}=\left\{D_{1}, D_{2}, \ldots, D_{k}\right\}$ represents the depots. Each customer $j \in V_{c}$ has a time window $\left[a_{j}, b_{j}\right]$. If the technician arrives at customer $j$ before $a_{j}$, it is necessary for him/her to wait until $a_{j}$. In this model, we suppose the task duration time and the travel time follow normal distributions. The following notations are defined:

- $M$ a large number;
- $K$ the set of required technicians in a feasible solution $K=\{1, \ldots, K\}$;
- $I$ the set of tasks order for each technician, $I=\{1, \ldots, C\}$ and $C$ is the maximum number of customers served by each technician;
- $d_{k l j}$ the travel time of technician $k$ between customers $l$ and $j$;
- $t_{k j}$ the travel time of technician $k$ between his/her depot to customer $j$;
- $\mu_{k j}$ the mean of the task duration time of technician $k$ spending at customer $j$;
- $\sigma^{2}$ the variance of the task duration time, suppose it is identical for all tasks;
- $x_{k i j}$ a binary variable equal to 1 if technician $k$ serves customer $j$ as his/her (i)th task and 0 otherwise;
- $A T_{k i}$ the arrival time of technician $k$ at his/her (i)th task;
- $S T_{k i}$ the start time of technician $k$ to serve his/her (i)th task;
- $Z_{k i}$ the standard score of the risk probability for technician $k$ 's $(i)$ th task;
- $Z$ the lower bound of the standard score of the risk probability for all tasks. The objective for this model is to minimise the maximum risk in the schedule and the mathematical model for the problem is formulated below:

$$
\begin{equation*}
\max Z \tag{12}
\end{equation*}
$$

Subject to:

$$
\begin{gather*}
\sum_{k} \sum_{i} x_{k i j}=1, \forall j \in V  \tag{13}\\
\sum_{j} x_{k i j} \leq 1, \forall k \in K, \forall i \in I  \tag{14}\\
\sum_{j} x_{k i j} \leq \sum_{j} x_{k i-1 j}, \quad \forall i \geq 2, \forall k \in K  \tag{15}\\
S T_{k i-1}+\mu_{l} x_{k i-1 l}+t_{l j}+M\left(x_{k i-1 l}+x_{k i j}-2\right) \leq A T_{k i}, \forall k \in K, i \geq 2, j \in V, l \in V
\end{gather*}
$$

$$
\begin{gather*}
A T_{k i} \leq S T_{k i}, \forall k \in K, \forall i \in I  \tag{17}\\
\sum_{j} a_{j} x_{k i j} \leq S T_{k i}, \forall k \in K, \forall i \in I  \tag{18}\\
A T_{k 1} \geq S T_{k 0}+\sum_{j} t_{k j} x_{k 1 j}, \forall k \in K  \tag{19}\\
A T_{k i} \leq \sum_{j} b_{j} x_{k i j}, \forall k \in K, \forall i \in I
\end{gather*}
$$

$$
\begin{gather*}
\sqrt{i-1} \cdot \sigma \cdot Z_{k i} \leq \sum_{j} b_{j} x_{k i j}-A T_{k i}, \forall k \in K, \forall i \in I  \tag{21}\\
Z \leq Z_{k i}, \forall k \in K, i \in I \tag{22}
\end{gather*}
$$

Constraints (13) indicate that each customer is served by one technician. Constraints (14) and (15) make sure that the task list of each technician is consecutive. Constraints (16) show that for each technician the arrival time of the current task is the previous task start time combined with the previous task duration and the travel time to the current task. Constraints (17) and (18) endure that the start time is after both arrival time and the lower limit of the time window. Constraints (19) state that the arrival time of the first task for each technician is the travel time from his depot to the first task based on the technician start work time. Constraints (20) make sure the expected arrival time is before the latest time window. Constraints (21) calculate the z -score corresponding to the probability of the risk, the risk is defined as the probability of the value which is greater than $Z_{k i}$ for the standard normal distribution, while $Z$ denotes the lower bound of $Z_{k i}$ as shown in constraints (22).

Note that in order to obtain a linear model, the variance of the duration for different tasks is supposed to be equal, and the $z$-score is introduced to present the same trend of the probability for risks, instead of calculating risks which are not linear.

## 5 Simulated Annealing method

Due to the fact that the risk is defined as a probability and considered in the objective of the model, the exact method may solve a small size problem as shown in the previous section. However, there are some limitations such as the variance for the uncertainty needs to be identical for all tasks. Also, it cannot solve the problem if the objective is to minimise the average risk in the schedule, because the average z -score is not the same as the average value of risks. Therefore, one of the heuristic methods, Simulated Annealing (SA) is applied to solve problems of a larger size and with multiple objectives. Before the illustration of the SA method, two search operators that are used in the searching process are explained first.

Given a specific task, the swap operator swaps the task with another task, while the insert operator withdraws this task and inserts it to another technician's task list according to the order of the lower limits of these task time windows. To be more specific, given a task $m$ of technician $p$ and task $n$ of technician $q$, the swap operator exchanges the task $m$ of $p$ and task $n$ of $q$. The insert operator withdraws the task $m$ from technician $p$ and assigns it to technician $q$.

Simulated Annealing (SA) is a probabilistic method proposed for finding the global minimum of a cost function that may possess several local minima [16, 17]. While Burkard and Rendl [18] first applied SA method to solve quadratic assignment problems, computational results indicated that they could obtain the best-known solution with a relatively high probability.

The Simulated Annealing algorithm was originally inspired from the process of annealing in metal work, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects. Both are attributes
of the material that depends on its thermodynamic free energy. While the same amount of cooling brings the same amount of decrease in temperature, it will bring a different decrease in the thermodynamic free energy depending on the rate that it occurs, with a slower rate producing a more prominent decrease.

In Simulated Annealing, a temperature variable is used to simulate this heating and cooling process. We initially set it high and then allow it to slowly 'cool' as the algorithm runs. While this temperature variable is high the algorithm will be allowed, with a higher probability, to accept solutions that are worse than our current solution. It gives the algorithm the ability to jump out of any local optima as it explores the solution space. The chance of accepting worse solutions is reduced due to the decline of the temperature, which allows the algorithm to gradually focus in an area of the search space close to the optimum solution.


Fig. 8. Simulated annealing method

## 6 Experimental results and discussions

### 6.1 Basic experiments

In our experiments, the risk is a result of the uncertain task duration and the fluctuating travel time. Besides, several factors are considered during the scheduling process. On the one hand, tasks have different time windows, different means and variances of
the estimated duration time, different necessary skill levels and different priorities. On the other hand, technicians have different depots and different skill ability. Moreover, the travel time is also treated as an uncertain factor and the variance of the travel time is distinct in the morning or the afternoon. In addition, the distributions of uncertain factors in the model are all supposed as normal distributions.

In the basic experiment, the testbed is based on 120 tasks and 20 technicians. In order to see the effect of considering risks in the scheduling, as well as the effect of considering task priorities, we suppose that the required skill level for tasks are all equal and the priorities are distinguished as two levels: high and low.

Hence there are three scheduling models: the travel time model is of a traditional scheduling problem that the objective is to minimise the total travel time; while the risk model and priority risk model aim at minimising the average risk of all tasks. Moreover, in the risk model, all tasks are treated as the same importance while for the priority risk model each task has one of the two different priorities. Fig. 9 shows the average risks of high and low priority tasks for the three models. From the comparison of the travel time model and the risk model, the average risk for all tasks in the case of travel time model is much higher than that obtained when minimising task and travel risk; it is reasonable because we did not consider risk when minimising the total travel time during scheduling. Meanwhile, Fig. 9 shows that the average risk for the travel time model is not significantly high in value; this is because risks are limited by the time window threshold constraints in the travel time model.

Furthermore, in the real world, tasks appear to have different importance or priority according to the business objectives. If a technician fails to start a high priority task in time, then the penalty should be higher. Therefore, the priority risk is introduced in the scheduling where the priority risk of a particular task is defined as the risk of the task multiplied by an adjusted task importance score, in order to force high priority tasks to possess low risks. As we can see in Fig. 9, the average risk of high priority tasks is smaller at the cost of the increased average risk for low priority tasks.


Fig. 9. Risks for different models
Additionally, a comparison of the average travel time is shown in Table 1. The average travel time is the total travel time spent by all technicians divided by the total number of tasks. As expected, the travel time model results in the smallest travel time
among the three models; but the risk model and priority risk model also show relatively short travel time. An explanation could be that by minimising the risks, there is a side effect of minimising the travel time simultaneously. Specifically, the risk is considered as the area of the distribution of the arrival time fell after the latest time of the time window, and the mean of the arrival time is associated with the estimated operation time of all previous tasks and the travel time between the depot to the $1^{\text {st }}$ customer and between previous tasks for each technician. Therefore, during scheduling, when we try to minimise risks we also minimise the travel time simultaneously.

Table 1. Travel time for different models

| Model | Travel time model | Risk model | Priority risk model |
| :---: | :---: | :---: | :---: |
| Travel time (mins) | 19.56 | 31.65 | 21.46 |

### 6.2 Structures of the task priority

From the definition of risks, a conclusion can be drawn that the risk increases as it propagates because the variance of the arrival time increases along the task list for each technician. The position of the task in the planned tour of visits is the essential information for the robustness of the plan during the day against disturbances. From Fig.9, we notice that the risk for high priority tasks becomes smaller in the priority risk model. By analysing the structure of the task priority at each position in the task list for every technician, we can find that high priority tasks are completed at the early position in the tour of visits. It models the real-world fact that technicians prefer to do the important task first to make sure its completion will be achievable.


Fig. 10. High priority task position composition
In our application case, the average number of tasks for each technician is 6 , which is derived from 120 tasks divided by 20 technicians. Fig. 10 illustrates the number of high priority tasks as the vertical axis (figures on lines) according to each position number in task lists for all technicians on the horizontal axis. For instance, there are 13 high priority tasks assigned to technicians as their $1^{\text {st }}$ task in the schedule obtained
by priority risk model, whereas 11 tasks for the risk model and 10 tasks for the travel time model. In other words, there are 13 technicians scheduled by a high priority task as their $1^{\text {st }}$ task in the priority risk model, and 11 technicians and 10 technicians processing a high priority task in their $1^{\text {st }}$ task position for the risk model and the travel time model respectively. We can notice that the graph shows in the case of the travel time model, some technicians may have more than 8 tasks which are really tense for them and accordingly the risks for missing them may be much high. In both risk models, technicians may have at most 7 tasks.

Moreover, the high priority tasks at position 1 and 4 are more in the priority risk model than those in the risk model, which explains the high priority tasks are executed earlier both in the morning and in the afternoon. Also, there is a cost of focusing on high priority tasks: the low priority tasks are pushed to late positions as is shown in Fig. 11. In addition, we may observe that the number of tasks in the travel time model and risk model did not fluctuate as much as that in the priority risk model.


Fig. 11. Low priority task position composition

### 6.3 Structures of the task priority

In order to study the technicians' behaviour, the productivity for a technician is introduced, it can be defined as

$$
\begin{equation*}
\text { technician's productivity }=\frac{\text { number of tasks }}{\text { work hours }} \cdot \text { roster hours. } \tag{23}
\end{equation*}
$$

First, we get the number of tasks done per hour (number of tasks / total work hours), then we get the maximum number of tasks achievable per day, by multiplying by the technician daily rostered hours.
From the definition, the productivity states a certain task number for a technician only considering the work time factor. Then Fig. 12 shows the number of technicians (vertical axis) according to the value of productivity (horizontal axis). For example, there are around 7 technicians whose productivity is around 12 tasks when building the start-of-day service visits plan with the priority risk model. Thus from the graph, we
may conclude that by introducing risks in scheduling, technicians will have even workload among them, which also means a robust schedule can be obtained.


Fig. 12. Productivity distribution

## 7 Conclusion

This research aims at modelling the risks observed in field force services delivery operations and incorporate risks in the operational planning process from a new perspective. The risks, which interpret the possibility of missing appointments regarding time windows, arise from the stochastic service time and travel time. Moreover, the model also demonstrates the risk increases simultaneously as it propagates for each technician. In addition, this model has been applied to a real-world problem in the telecommunication sector. Results have shown that the schedule generated is more robust while minimising the risk of failure, pushing high-priority tasks earlier in the schedule to avoid failure of these tasks.

In a risk assessment tool or a risk-based scheduling engine under a dynamic environment, the calculation needs to be fast enough but also realistic. In the first step, we proposed an addition method to estimate risk distributions and test the accuracy. Then a linear model is built for this problem by limiting some factors and can be solved by exact methods. Concerning the application area, the Simulated Annealing method is utilised in the scheduling process to obtain a good solution in an acceptable time.

As for the future work, the task duration which follows a gamma distribution will be considered; even the combination of gamma and normal distributions will be taken into consideration. Furthermore, it is worthwhile to work on a simulation which mimics a real-time task process in one day to demonstrate the advantage of considering risk in the scheduler. Additionally, in the simulation we may study how to re-schedule at specified time points based on the information from completed tasks, in order to obtain the pros and cons that rescheduling or updating technician task lists several times in a day.

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