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1 **Modelling strain localization in Ti-6Al-4V at high loading rate: A** 2 **phenomenological approach**

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8

9 10 **Abstract**

11 A phenomenological approach based on a combination of a damage mechanism and a crystal-
12 plasticity model is proposed to model a process of strain localization in Ti-6Al-4V at a high
13 strain rate of 10^3 s^{-1} . The proposed model is first calibrated employing a 3D representative
14 volume element model. The calibrated parameters are then employed to investigate the process
15 of onset of strain localization in the studied material. A suitable mesh size is chosen for the
16 proposed model by implementing a mesh-sensitivity study. The influence of boundary
17 conditions on the initiation of the strain localization is also studied. A variation of
18 crystallographic orientation in the studied material after the deformation process is
19 characterized, based on results for different boundary conditions. The study reveals that the
20 boundary conditions significantly influence the formation of shear bands as well as the
21 variation of crystallographic orientation in the studied material. Results also indicate that the
22 onset of strain localization can affect considerably the material's behaviour.

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25
26 **Keywords:** Ti-6Al-4V; localization; crystal plasticity; finite-element analysis
27

28 **1. Introduction**

29 Titanium alloys are widely used in high-performance applications thanks to their high specific
30 strength. Among titanium alloys, the largest market share is owned by Ti-6Al-4V (Ti64), with
31 applications including sporting goods, biomedical devices and aerospace parts [1,2]. As a result
32 of its prevalence across a wide range of applications, Ti64 was extensively studied over the
33 years. A typical Ti64 alloy consists of a hexagonal close-packed (HCP) α phase and a body-
34 centred cubic (BCC) β phase, generally displaying Widmanstätten $\alpha+\beta$ colonies [3]. There are
35 numerous studies on the mechanical behaviour of, and microstructural changes in Ti64 alloy
36 and its variants at various temperatures and strain-rate processing regime [4,5]. Multiple
37 experimental studies [4–10] showed that deformation microstructure of Ti-alloy, below their
38 recrystallization temperature, is inherently marked by the presence of regions with localized
39 strain leading to instability with a widespread formation of shear bands. The process of
40 formation of such adiabatic shear bands induces localised softening, which may further
41 promote the generation of voids and cracks [11]. The initiation, evolution and behaviour of these
42 localised instabilities are influenced by the underlying microstructure [5,12]. Wagoner et al.
43 [5] reported that the instabilities (shear band) formed at lower strain values in Ti64 with a
44 Widmanstätten microstructure than in Ti64 with an equiaxed one. Instabilities were observed
45 to form at boundaries of α colonies with 90° misorientation. The addition of boron in small
46 quantities was reported to delay the strain localization and shear-band formation in Ti64
47 [3,11,12].

48

49 A study of strain-localization phenomena is essential as it is a precursor to crack formation and
50 eventual material failure. Most microstructure-based experimental studies dealing with shear-
51 band formation are typically carried out post-failure, and conclusions related to the course of
52 strain-localization events are principally hypothesised. Certainly, a study of the phenomena of

53 strain localization is crucial in developing a predictive capability for analysis of nucleation of
54 instability in the microstructure and material failure. Some attempts at modelling strain
55 localization and shear band phenomenon were reported in the literature [13–17]. Constitutive
56 isotropic material description, employing a modified form of a Johnson-Cook model, is
57 popularly used in such simulations [13,14], which do not account for the underlying anisotropy
58 related to the crystal orientation. It is expected that a change in local material behaviour due to
59 strain localisation will also affect its behaviour in adjacent areas. Surprisingly, this was not
60 investigated in detail in previous studies. This work aims to develop a modelling framework,
61 which captures strain localization with a minimal set of parameters.

62

63 The paper is organised as follows: A constitutive description of the crystal-plasticity model is
64 introduced in Section 2. Section 3 presents the details of the suggested computational
65 framework including the finite-element model and calibration of the model parameters. A
66 damage-based mechanism in each slip system was also introduced into the crystal plasticity
67 model to capture the onset of strain localization in Section 3. Numerical simulations of the
68 studied alloy in uniaxial compression are conducted in Section 4 for different boundary
69 conditions with the focus on assessment of strain localization accounting for the boundary-
70 condition effects as well as the influence of strain localization on the adjacent parts of the
71 material. We conclude our study in Section 5.

72 **2. Crystal plasticity theory**

73 In this section, a classical crystal-plasticity (CP) theory adopted in this study is briefly
74 discussed. Here, a deformation gradient, \mathbf{F} , can be decomposed into its elastic and plastic parts
75 as,

$$76 \quad \mathbf{F} = \mathbf{F}_e \mathbf{F}_p, \quad 2.1$$

77 where the subscripts ‘e’ and ‘p’ denote the elastic and plastic parts, respectively. By applying
 78 the product rule of differentiation, one can obtain the rate of the total deformation gradient, $\dot{\mathbf{F}}$.
 79 Therefore, the velocity gradient, \mathbf{L} , can be introduced following its definition $\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}$ as

$$80 \quad \mathbf{L} = \dot{\mathbf{F}}_e\mathbf{F}_e^{-1} + \mathbf{F}_e(\dot{\mathbf{F}}_p\mathbf{F}_p^{-1})\mathbf{F}_e^{-1} = \mathbf{L}_e + \mathbf{L}_p. \quad 2.2$$

81 It is assumed that the plastic velocity gradient, \mathbf{L}_p , is induced by shear on each slip system in
 82 a crystal. Hence, \mathbf{L}_p is formulated as the sum of shear rates on all the slip systems, i.e.

$$83 \quad \mathbf{L}_p = \sum_{\alpha=1}^N \dot{\gamma}^{(\alpha)} \mathbf{s}^{(\alpha)} \otimes \mathbf{m}^{(\alpha)}, \quad 2.3$$

84 where $\dot{\gamma}^{(\alpha)}$ is the shear slip rate on the slip system α , N is the total number of slip systems,
 85 and unit vectors $\mathbf{s}^{(\alpha)}$ and $\mathbf{m}^{(\alpha)}$ define the slip direction and the normal to the slip plane in the
 86 deformed configuration, respectively. Furthermore, the velocity gradient can be expressed in
 87 terms of a symmetric rate of stretching, \mathbf{D} , and an antisymmetric rate of spin, \mathbf{W} , as,

$$88 \quad \mathbf{L} = \mathbf{D} + \mathbf{W} = (\mathbf{D}_e + \mathbf{W}_e) + (\mathbf{D}_p + \mathbf{W}_p) = \dot{\mathbf{F}}_e\mathbf{F}_e^{-1} + \sum_{\alpha=1}^N \dot{\gamma}^{(\alpha)} \mathbf{s}^{(\alpha)} \otimes \mathbf{m}^{(\alpha)}. \quad 2.4$$

89 Following the work of Huang [18], a constitutive law is expressed as the relationship between
 90 the elastic part of the symmetric rate of stretching, \mathbf{D}_e , and the Jaumann rate of Cauchy stress,
 91 $\overset{\nabla}{\boldsymbol{\sigma}}$, i.e.

$$92 \quad \overset{\nabla}{\boldsymbol{\sigma}} + \boldsymbol{\sigma}(\mathbf{I} : \mathbf{D}_e) = \mathbf{C} : (\mathbf{D} - \mathbf{D}_p), \quad 2.5$$

93 where \mathbf{I} is the second-order unit tensor, \mathbf{C} is the fourth order, possibly anisotropic, elastic
 94 stiffness tensor. The Jaumann stress rate is expressed as

$$95 \quad \overset{\nabla}{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} - \mathbf{W}_e\boldsymbol{\sigma} + \boldsymbol{\sigma}\mathbf{W}_e. \quad 2.6$$

96 On each slip system, the resolved shear stress, $\tau^{(\alpha)}$, is expressed by a Schmid law,

$$97 \quad \tau^{(\alpha)} = \mathbf{sym}(\mathbf{s}^{(\alpha)} \otimes \mathbf{m}^{(\alpha)}) : \boldsymbol{\sigma}. \quad 2.7$$

98 The relationship between the shear rate, $\dot{\gamma}^{(\alpha)}$, and the resolved shear stress, $\tau^{(\alpha)}$, on the slip
 99 system α is expressed by a power law proposed by Hutchinson [19]:

$$100 \quad \dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \left| \frac{\tau^{(\alpha)}}{g^{(\alpha)}} \right|^n \text{sgn}(\tau^{(\alpha)}), \quad 2.8$$

101 where $\dot{\gamma}_0$ is the reference shear strain rate, $g^{(\alpha)}$ is the slip resistance and n is the rate-
 102 sensitivity parameter. The evolution of $g^{(\alpha)}$ is given by

$$103 \quad \dot{g}^{(\alpha)} = \sum_{\beta=1}^N h_{\alpha\beta} |\dot{\gamma}^{(\beta)}|, \quad 2.9$$

104 where $h_{\alpha\beta}$ is the hardening modulus that can be calculated in the form modified from that
 105 proposed by Asaro [20],

$$106 \quad h_{\alpha\alpha} = h_0 \text{sech}^2 \left[\frac{h_0 \gamma}{\tau_s - \tau_0} \right], \quad h_{\alpha\beta} = q h_{\alpha\alpha} (\alpha \neq \beta), \quad \gamma = \sum_{\alpha} \int_0^t |\dot{\gamma}^{(\alpha)}| dt. \quad 2.10$$

107 Here, h_0 is the initial hardening modulus, q is the latent hardening ratio, τ_0 and τ_s are the
 108 shear stresses at the onset of yield and the saturation of hardening, respectively, and γ is the
 109 accumulative shear strain over all the slip systems.

110

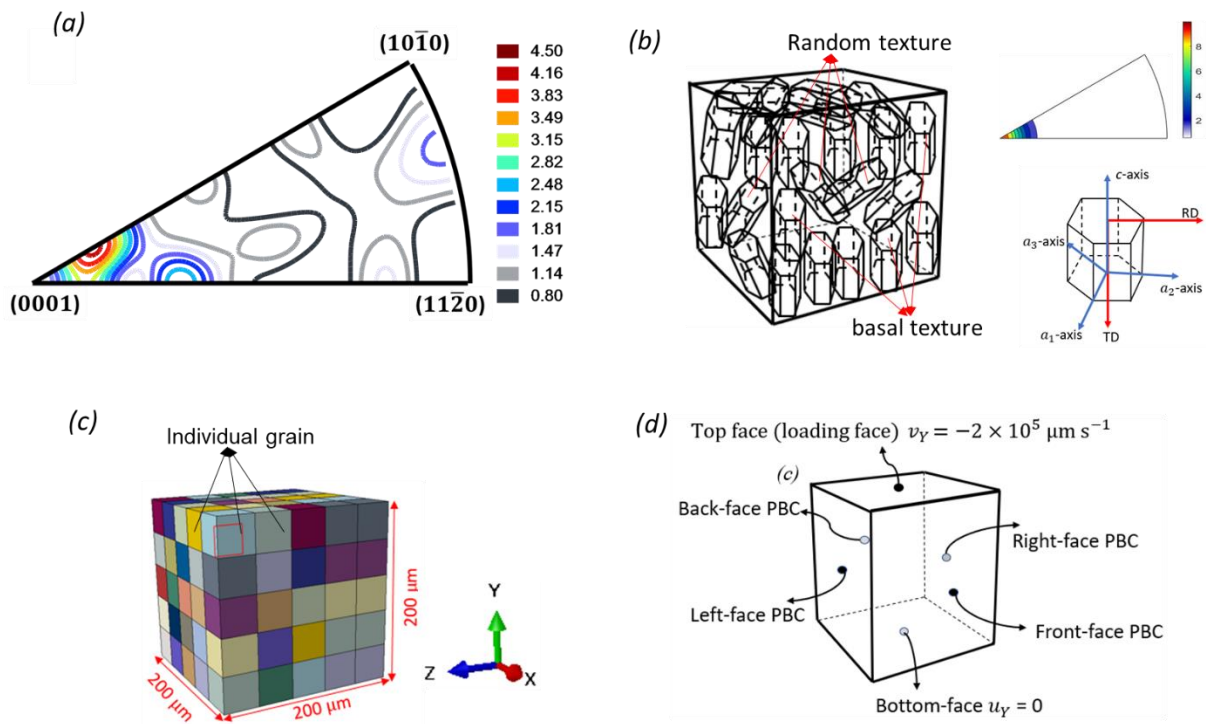
111 **3. Computational framework**

112 **(a) Finite-element model**

113 Here, the CP model was implemented in general-purpose finite-element software,
 114 ABAQUS/Explicit, by employing the user subroutine (VUMAT). Note that the stress update
 115 algorithm was based on the Green-Naghdi stress rate in ABAQUS/Explicit environment.
 116 Therefore, a conversion algorithm was required to evaluate a stress update based on the
 117 Jaumann stress state defined in [21]. A modelling approach based on a 3D representative

118 volume element (RVE) was used to investigate the deformation mechanisms of the studied
119 alloy at room temperature (293 K) with a strain rate of 10^3 s^{-1} . The RVE model (Figure 1(b))
120 have a side length of 200 μm , with 5 grains in each direction, as shown in Figure 1c where each
121 cube of a different colour indicates an individual grain. Construction of the RVE model was
122 based on input from experimental data. The inverse pole figure (IPF) of the sample
123 microstructure along the compression direction is shown in Figure 1a, plotted using ATEX
124 software [22]. A strong intensity was observed around the (0001) pole in the IPF indicating
125 basal planes oriented along compression direction. The rest of the IPF show random intensity
126 (intensity close to 1). The volume fraction of the basal orientated grains about the compression
127 axis is $\sim 57\%$ as calculated from ATEX. To mimic the experimental condition, the RVE model
128 for simulation was constructed with $\sim 60\%$ of grains with basal orientation and the rest with
129 random orientation. The random orientations were generated via random number functions for
130 Euler angles incorporated in python codes. Figure 1(b) represents the crystal orientations of the
131 grains in the model domain, which also show the IPF of the RVE model. Each grain was
132 meshed with $2 \times 2 \times 2$ first-order hexahedral reduced-integration elements (C3D8R elements in
133 ABAQUS). A mesh convergence study was performed with a refined mesh where each grain
134 was meshed with $4 \times 4 \times 4$ elements which indicate that the chosen mesh is a reasonable balance
135 between computational efficiency and numerical accuracy.

136 Uniaxial compression deformation was imposed with the use of a velocity constraint applied
137 along the Y-axis as marked in Figure 1d with periodic boundary conditions applying on the
138 front and back faces and left and right faces, while the bottom face was constrained in
139 displacement in the Y-direction.



140

141 **Figure 1.**

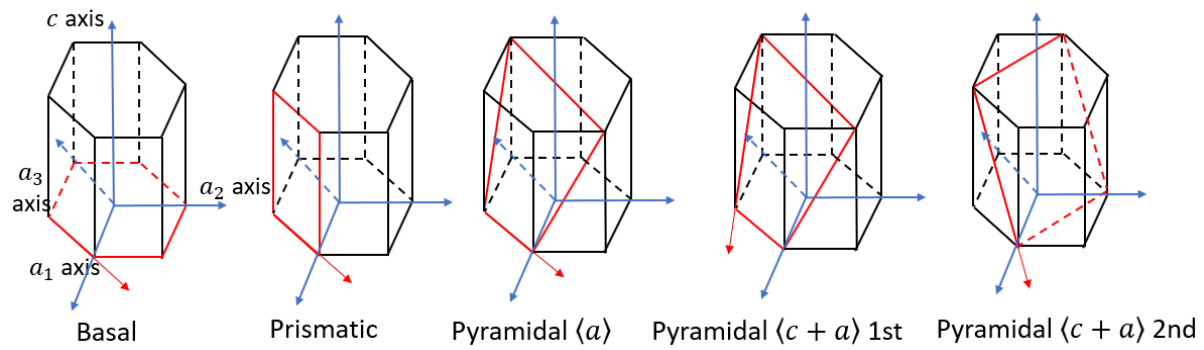
142 (a) inverse pole figure of the as-received microstructure ; (b) crystal orientation of grains in the RVE (rolling (RD)
 143 and transverse (TD) directions); the inverse pole figure shows the numerically modelled texture; (c) the RVE
 144 model with each colour cube indicating an individual grain; (d) imposed boundary conditions.

145 An HCP crystal orientation is considered for Ti64, with five slip systems active in the material.
 146 Here, the basal, prismatic, pyramidal $\langle a \rangle$, and pyramidal $\langle c + a \rangle$ 1st and pyramidal
 147 $\langle c + a \rangle$ 2nd slip systems are considered (Table 1), with the corresponding slip planes shown
 148 in Figure 2.

Table 1. Slip systems in Ti-6Al-4V

| Slip system | Slip plane | Slip direction | Number of modes |
|---------------------------------------|------------------|------------------------------|-----------------|
| Basal | $\{0001\}$ | $\langle 11\bar{2}0 \rangle$ | 3 |
| Prismatic | $\{10\bar{1}0\}$ | $\langle 11\bar{2}0 \rangle$ | 3 |
| Pyramidal $\langle a \rangle$ | $\{10\bar{1}1\}$ | $\langle 11\bar{2}0 \rangle$ | 6 |
| Pyramidal $\langle c + a \rangle$ 1st | $\{10\bar{1}1\}$ | $\langle 11\bar{2}3 \rangle$ | 12 |
| Pyramidal $\langle c + a \rangle$ 2nd | $\{11\bar{2}2\}$ | $\langle 11\bar{2}3 \rangle$ | 6 |

149

151 **Figure 2.**

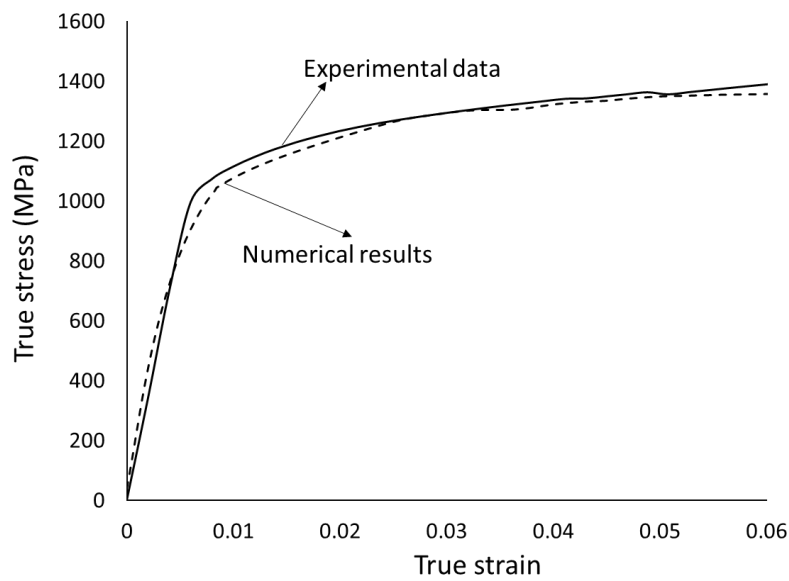
152 Slips systems and slip planes in Ti-6Al-4V.

153 **(b) Model verification and parameter calibration**

154 A high strain-rate compression test at a strain rate of 10^3 s^{-1} and room temperature (293 K) was
 155 performed using a gas-gun-driven split-Hopkinson pressure bar (SHPB) at Indian Institute of
 156 Science Bangalore. For the SHPB test, cylindrical samples were extracted from the as-received
 157 hot extruded Ti64 rod with compression axis parallel to the extrusion axis. A cylindrical sample
 158 of 3 mm diameter and 3 mm height was used for the test. The stress-strain data for the
 159 compression test were extracted from the recorded transmitted and reflected wave signals
 160 [8,23,24].

161 The CP model was calibrated with the experimental SHPB data. As can be seen from Figure 3,
 162 a reasonable correlation between the experimental and numerical stress-strain curves for the
 163 loading case was achieved after calibration. The parameters related to the elastic constants of
 164 the studied alloy are listed in Table 2 while model parameters for the slip deformation modes
 165 in the CP model are summarised in Table 3. The reference shear strain rate $\dot{\gamma}_0$ and the rate-
 166 sensitivity parameter n were set to be 10^{-3} s^{-1} and 50, respectively. The validity of the
 167 parameters should be tested once more experimental results, potentially under complex loading
 168 states are available. A modelling approach described in [25] may be attempted once relevant
 169 experimental data is available. We leave this as a future work.

170



171

172 **Figure 3.**

173 Model verification with experimental results

174 **Table 2.** Elastic constants of Ti-6Al-4V (in GPa)

| C_{11} | C_{12} | C_{13} | C_{33} | C_{44} | C_{55} |
|----------|----------|----------|----------|----------|----------|
| 162.4 | 92 | 69 | 180.7 | 70.4 | 49.7 |

175

176 The proposed CP model can be suitably modified to account for effects of temperature and
 177 strain rate on the mechanical behaviour of the studied alloy. To accomplish this, the initial
 178 critical resolved shear stress, the resistance from grain boundaries and from the interaction of
 179 deformation mechanisms, in the hardening law should be defined as a function of temperature
 180 and strain rate. In this study, the focus is on the shear-band formation and its effects on material
 181 behaviour at high strain rates and room temperature only; thus, the development of
 182 temperature- and strain-rate-dependent functions in the hardening law is left for future studies.
 183 More details of the theoretical implications are available in [21].

184

185

Table 3. Related parameters (stress in MPa) at reference conditions: $T = 293 \text{ K}$, strain rate 10^3 s^{-1}

| Slip system | τ_0 | τ_s | h_0 |
|---|----------|----------|-------|
| Basal | 352.3 | 410 | 1400 |
| Prismatic | 314.3 | 360 | 1200 |
| Pyramidal $\langle \mathbf{a} \rangle$ | 428.6 | 480 | 1100 |
| Pyramidal $\langle \mathbf{c} + \mathbf{a} \rangle$ 1st | 365 | 410 | 4500 |
| Pyramidal $\langle \mathbf{c} + \mathbf{a} \rangle$ 2nd | 423 | 480 | 4500 |

186

187 **(c) Phenomenological approach for modelling strain localization**

188 To model the initiation of strain localization in the material, a phenomenological damage
 189 mechanism was introduced into the CP model. Here, a damage parameter (D) was introduced
 190 into the evolution of strength in a slip system, $g^{(\alpha)}$, so that the strength evolution is defined by
 191 the following law:

$$\begin{aligned}
 \dot{g}^{(\alpha)} &= \sum_{\beta=1}^N h_{\alpha\beta} |\dot{\gamma}^{(\beta)}|, \gamma^{(\alpha)} \leq \gamma_{cr}^{(\alpha)} \\
 \dot{g}^{(\alpha)} &= \sum_{\beta=1}^N h_{\alpha\beta} |\dot{\gamma}^{(\beta)}| - \dot{D}(\gamma), \gamma^{(\alpha)} > \gamma_{cr}^{(\alpha)},
 \end{aligned}
 \tag{3.1}$$

193 where $\gamma_{cr}^{(\alpha)}$ is the critical shear strain at α slip system. We choose $\gamma_{cr}^{(\alpha)} = 0.1$ in this model,
 194 which implies that softening is induced once the net strain in a slip system exceeds 0.1. Next,
 195 the damage in the material ($\dot{D}(\gamma)$) evolves as

$$\dot{D}(\gamma) = (\tau_s - \tau_0) m \dot{\gamma} e^{-m \dot{\gamma}}
 \tag{3.2}$$

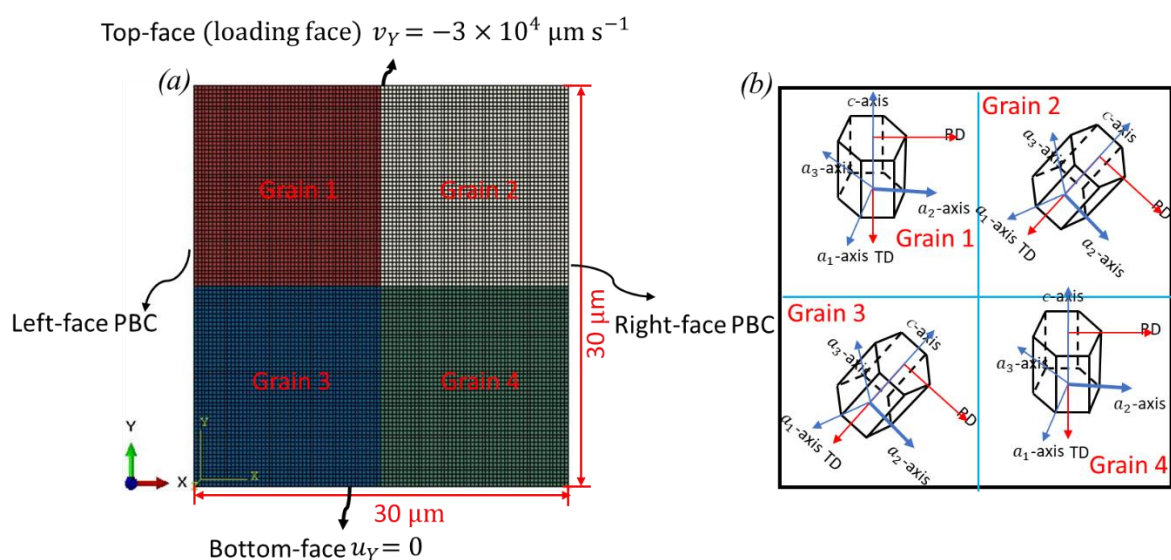
197 where m is a calibration parameter with a value of 100.

198 The proposed damage-based softening mechanism in each slip system (Eq.3.2) is
 199 phenomenological. The underlying justification for this is that a given slip system can sustain

200 a critical amount of deformation slip before incurring damage which will allow for a rapid
 201 evolution of plastic slip which lead to strain localization

202 (d) Mesh-size sensitivity

203 Numerical analysis incorporating strain localisation is typically mesh-dependent. Thus, a
 204 mesh-sensitivity analysis is key in determining the efficacy of such a phenomenological
 205 approach. Here, simulations with three different mesh sizes, with a nominal element size, h , of
 206 0.15, 0.3 and 0.6 μm were conducted for a specimen with a dimension of $30\ \mu\text{m} \times 30\ \mu\text{m} \times 3\ \mu\text{m}$.
 207 The domain was formed by 4 grains as shown in Figure 4*b*. Without any loss of generality,
 208 Grain 1 and Grain 4 were assumed to be identical; a similar assumption was made for Grain 2
 209 and Grain 3 as well. The top face of this model was deformed by imposing a velocity boundary
 210 control, corresponding to a strain rate of $10^3\ \text{s}^{-1}$. A periodic boundary condition (PBC) was
 211 applied on the left and right surfaces of the model. The bottom face was constrained in
 212 displacement in the Y-direction (Figure 4*a*). Plane-strain conditions were imposed on the front
 213 and back faces of the modelled domain.



214
 215 **Figure 4.**

216 Simple RVE model for mesh-sensitivity study: (a) loading and boundary conditions; (b) grain size and orientation
 217 in the model domain.

218

219 Due to imposed deformation, strain localisation occurred in the material, represented by the

220 total accumulated shear strain overall slip systems ($\sum|\gamma^{(\alpha)}|$) in every grain. Bands of strain

221 localisation, similar to shear bands are shown in Figure 5a for 3 mesh densities. As expected,

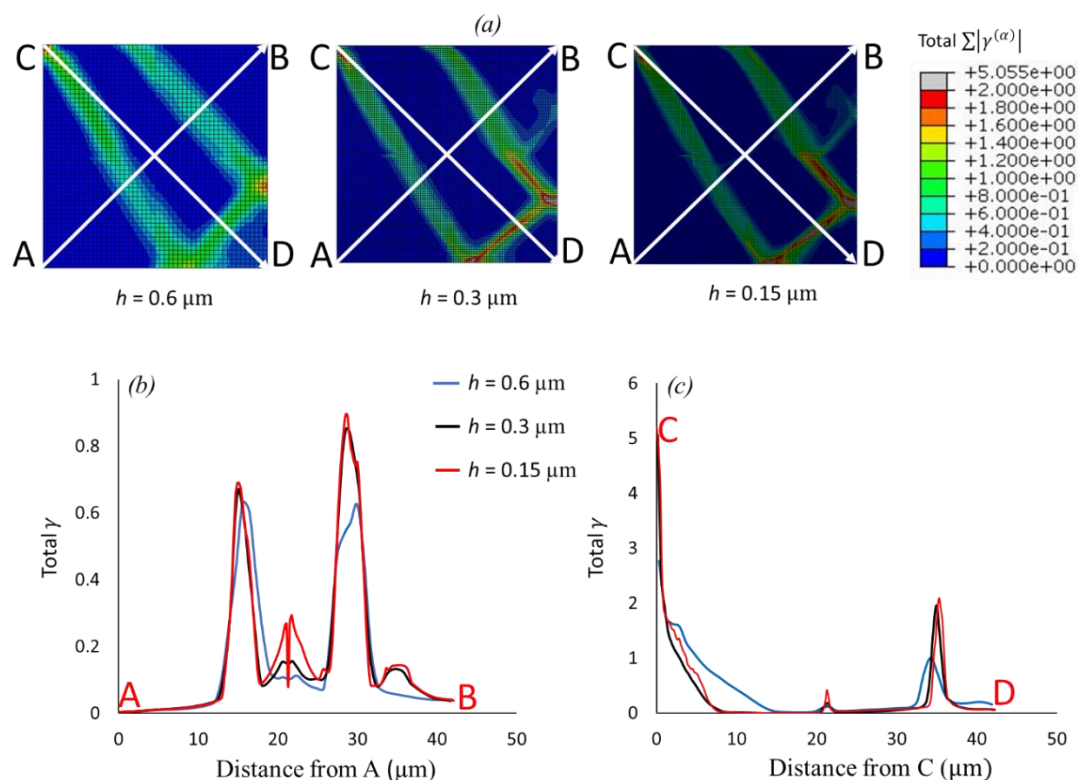
222 these bands were better resolved with a finer mesh; however, the overall features were similar

223 and comparable across all the 3 meshes. For further analysis, two diagonal paths were chosen

224 in the meshed domain, indicated by paths A-B and C-D in Figure 5a and the total accumulated

225 $\sum|\gamma^{(\alpha)}|$ was traced along these paths in Figure 5b, c. The results indicate that an element size226 corresponding to $h = 0.3 \mu\text{m}$ is reasonable, when compared to the those for the finer mesh.227 Hence, a mesh size of $0.3 \mu\text{m}$ was chosen for the investigation of strain localization in the

228 developed crystal-plasticity finite-element model hereinafter.



229

230 **Figure 5.**

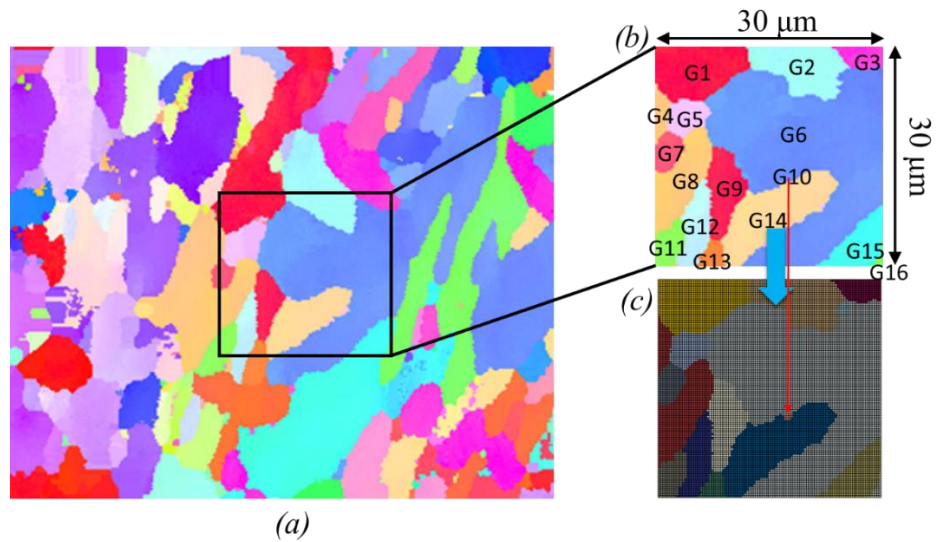
231 Mesh-size sensitivity study: (a) effect on the pattern of strain localization; (b) evolution of accumulated shear

232 strain along path A-B; (c) evolution of accumulated shear strain along path C-D.

233 (e) Microstructural model

234 Next, the effect of strain localisation in a representative microstructural ensemble was studied.
235 A Ti64 sample was sectioned to expose the transverse plane. An electron back-scattered
236 diffraction (EBSD) technique was employed to obtain the character of microstructural
237 orientation in the sample. The EBSD data obtained from the scan was further analysed with
238 DREAM.3D, and the reconstruction of the EBSD data was performed with the Abaqus
239 Hexahedron Writer available in the software. An area with a random grain structure with
240 dimensions $30\ \mu\text{m} \times 30\ \mu\text{m} \times 3\ \mu\text{m}$ was selected at the centre of the scanned area (Figure 6). 16
241 crystallographic orientations were identified in the selected area. A list of the grains with their
242 orientations is presented in Figure 6 and Table 4. The selected area was converted into an RVE
243 model of the Ti64 sample. Steps followed to obtain the FE representation of the microstructure
244 model are shown in Figure 6. Here, due to the inherent limitation of the available EBSD scans,
245 the microstructure at the sample surface was extracted. The in-depth grain morphology data is
246 not available. For this reason, a 2D plane-strain condition with only one grain in the depth
247 direction. is modelled in the current study. The plane-strain state considered in the present study
248 is consistent with the real material state.

249 Obviously, the grain structure in the depth-direction should influence the macroscopic as well
250 as the local response of the studied alloy. Should a true 3D grain morphology data be made
251 available the proposed modelling approach can capture the nuances of strain localization in the
252 depth direction. The mesh density used was identical to the converged mesh obtained earlier.
253 The boundary conditions imposed on the RVE were identical to those employed in Section 3.4.



254

255 **Figure 6.**

256 Generation of the finite-element model. (a) microstructure from EBSD data; (b) chosen domain for RVE model
 257 showing the grains identified ranging from G1 through G16; (c) FE mesh of the RVE model.

258 **Table 4.** Initial crystal orientations with three Euler angles (unit degree) for the identified
 259 grains

| Grain number | Euler angle φ_1 | Euler angle φ | Euler angle φ_2 |
|--------------|-------------------------|-----------------------|-------------------------|
| G1 | 88.11 | 155.84 | 215.52 |
| G2 | 331.14 | 69.30 | 314.12 |
| G3 | 50.71 | 11.73 | 121.03 |
| G4 | 145.05 | 131.96 | 280.51 |
| G5 | 37.23 | 42.17 | 81.29 |
| G6 | 224.84 | 79.83 | 127.18 |
| G7 | 91.87 | 155.19 | 229.36 |
| G8 | 144.92 | 131.93 | 280.70 |
| G9 | 88.12 | 155.84 | 215.53 |
| G10 | 283.67 | 92.84 | 315.59 |
| G11 | 44.73 | 101.07 | 232.04 |
| G12 | 330.79 | 69.53 | 314.26 |
| G13 | 235.65 | 138.32 | 121.35 |
| G14 | 143.36 | 126.94 | 252.79 |
| G15 | 331.15 | 70.15 | 306.29 |
| G16 | 45.81 | 100.40 | 293.00 |

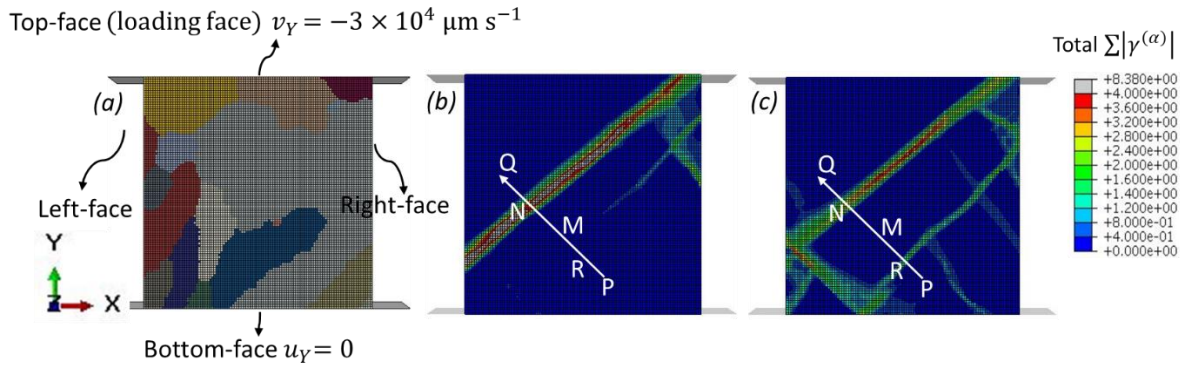
260 **4. Results and discussions**

261 **(a) Effects of boundary conditions**

262 First, the effects of imposed boundary conditions on the formation of the strain localization
263 were analysed in the studied alloy. Here, two different boundary conditions, namely, the effect
264 of imposing periodic boundary conditions (PBC) and traction-free surface boundary conditions
265 (FRE), are studied in detail. Loading was applied on the top surface with a velocity boundary
266 condition, while the bottom surface is fixed in displacement in the Y-direction (Figure 7).

267 Strain localisation was significantly influenced by the imposed boundary conditions. The
268 pattern of the strain localization is pronounced, with an imposed periodicity at the left and right
269 surfaces. This boundary condition is typically imposed when the spatial range of a sample in
270 the X-direction is large. In contrast, when traction-free boundaries were assumed, a single
271 dominant shear band was observed in the material volume. This indicates that shear bands are
272 denser in a constrained material domain corresponding to the interior of a bulk crystalline
273 material. By a similar argument, less pronounced localisation and shear- band formation are
274 expected near the free surface of a deformed component. This conclusion is consistent with the
275 experimental observations in [26–30], indicating that the shear band density was higher in
276 central parts of specimens than elsewhere.

277



278

279 **Figure 7.**

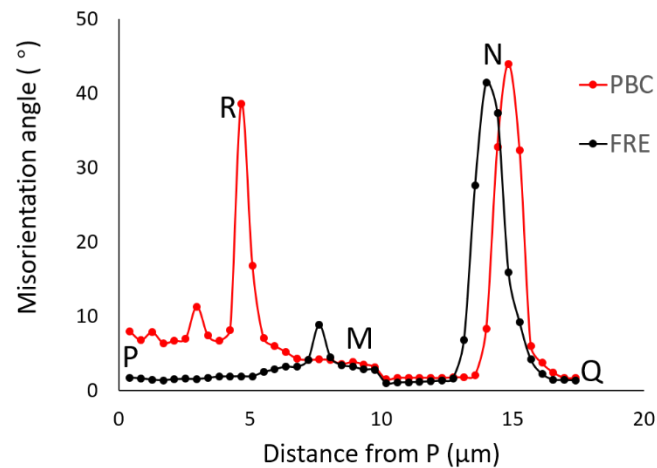
280 Effects of boundary conditions on strain localization: (a) microstructured model and boundary conditions; (b)
 281 case with FRE boundary conditions (right and left face are traction-free); (c) case with PBC boundary conditions
 282 (right and left face are constrained via periodic boundary conditions).

283 (b) Misorientation angle

284 To characterize the variation of crystallographic orientation of the studied alloy after the
 285 deformation process, the concept of misorientation angle was invoked. This is defined as

$$286 \quad \theta = \left| \cos^{-1} \left\{ \frac{\text{tr}(\mathbf{g}_A \mathbf{g}_B^{-1} - 1)}{2} \right\} \right|, \quad 4.1$$

287 where θ is the misorientation angle, \mathbf{g}_A and \mathbf{g}_B are the orientation matrices at chosen spatial
 288 locations A and B, respectively. In this paper, the undeformed crystallographic structure was
 289 considered as the reference configuration for calculation of the misorientation angle. The
 290 variation of the misorientation angle along the path P-Q (perpendicular to the strain-
 291 localization band) is shown in Figure 8. Along the path P-Q, three other points, -R, M and N-
 292 were selected (see Figure 7); They respectively represent the regions in the shear band near
 293 point P, between two shear bands and near point Q.



294

295 **Figure 8.**

296 Misorientation angle along path P-Q across shear bands

297

298 The change in the misorientation angle after deformation was compared for the two selected

299 boundary conditions. The character of misorientation angle along the path P-Q varies for the

300 two boundary conditions, with higher values corresponding to the constrained domain. Two

301 obvious peaks for the PBC were observed, while there was only one distinct peak for FRE

302 boundary conditions. This is consistent with the number of shear bands along the path P-Q

303 (Figure 7). This suggests that strain localization can strongly influence the material behaviour

304 in local areas by changing significantly the crystal orientation post-deformation.

305

306 **5. Concluding Remarks**

307 A crystal-plasticity finite-element modelling approach for the formation of the strain

308 localization in Ti-6Al-4V was presented, incorporating a strain-softening/damage description

309 for each slip system. The parameters of the crystal plasticity model were calibrated against the

310 experimental studies in terms of the stress-strain curve at room temperature and high strain

311 strain rate of 10^3 s^{-1} . The model is shown to have the ability to capture the formation of the

312 strain localization in the studied alloy. The effect of boundary conditions on strain localization

313 was analysed. The study revealed that boundary conditions significantly influenced the
314 formation process of strain localization including the pattern and the number of shear bands
315 formed. Additionally, it was demonstrated that strain localisation can affect the local
316 mechanical response of Ti64 as a result of a change in the misorientation angle of the
317 polycrystalline ensemble. It was observed that the magnitude of the misorientation angle was
318 higher in the interior parts of the specimen than near its free surfaces.

319

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323

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