PhD Thesis

Cost Optimal Charging of Electric Vehicles, using Real Time Pricing

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Abstract

The large-scale adoption of EVs presents both potential benefits and difficult challenges. The already stressed electricity grids will have to manage the influx of EV charging requirements, which is especially difficult at peak times. This calls for smart solutions to optimally charge EVs in a grid-friendly way, using demand response where possible. In line with the demand, the electricity prices at peak times can be very high and it would also be advantageous for the user to avoid charging at these times. Therefore, the goal of grid friendly charging is twofold: to avoid putting additional load on the electricity grid when it is heavily loaded already, and to reduce the cost of charging to the consumer.

Along with the technological progress in the EV field, the electricity grid is evolving toward a smart-grid. One of the changes a smart-grid will bring is smart-metering. In such a system, Day Ahead tariff (DA) prices are announced in advance for the next day. However, the balance of supply and demand is not fully known in advance and therefore, the Real-Time Prices (RTP) are more reflecting of the actual grid situation, but unknown in advance.

This thesis presents control strategies for *Cost Optimal Charging of Electric Vehicles*, from the point of an EV user connected to a real time pricing tariff system. Firstly, since there are differences in the DAP and RTP, the thesis proposes a predictor to create an unbiased estimate of the RTP tariff based on the available factors in the pricing data. It uses a linear regression on historical data to find the best prediction of the expected price. The results find that the predictor achieves a slight reduction in prediction uncertainty with the used data set and has a negligible effect on overall cost. It means that the DAP can be used as a fair prediction of RTP.

The first charging strategy proposed, uses the available DAP (price-prediction) for optimisation and follows a deterministic approach, to achieve the lowest charging cost. It achieves a sub optimal solution in which the controller successfully picks the times of lowest electricity cost from the prediction and provides a full charge to the vehicle by the time the user requires it. Since the electricity prices are affected by random disturbances and therefore the RTP can be different, it makes the charging process less predictable and introduces a stochastic element into the problem.

A second optimal controller is presented which takes this problem into account by following a stochastic optimisation approach, specifically based on a stochastic dynamic program (SDPM). It uses a stochastic optimisation algorithm to minimise total cost of charging over a given time-period, whilst still providing required state of charge (SoC) in the EV battery. The controller does this by predicting future prices changes from available

data, based on a probability. It takes into account price variability via a simple grid model that allows for unexpected price rises and a gradual return to a normal grid price.

Finally, a case study is presented based on the price data available from the Illinois Electricity Grid (USA), to validate the optimal controllers over a year. The *Stochastic Dynamic Programming* optimal controller, can save up to (US) ***$112.88*** over a year, compared to charging directly. This is very close to the theoretical optimum (full knowledge of real prices in advance) of ***$119.76***. The controller uses the DAP and RTP prices effectively in simulation and optimisation stages, to avoid times of high price and price spikes. The result is, lower charging cost over the year which is achieved by shifting the charging over to the off-peak hours.

Both strategies demonstrate significant advantages against conventional charging. The simple optimisation can realise most of the benefits and may therefore be the preferred strategy in practical terms, while the stochastic optimisation does offer slight further benefits at more significant complexity. This may change as the smart grid matures, the billing periods become shorter, and the processing capability of chargers increases.

1 Introduction

The goal of reducing carbon and greenhouse emissions has created a revolution in the automotive industry leading to the popularity of Electric Vehicles (EVs). A report by the Royal College of Physicians estimates that 40,000 deaths a year are caused by particulates and nitrogen dioxide exposure [1]. Both pollutants are present in internal combustion engine (ICE) tailpipe emissions. Future transport policies in most countries now are geared toward expanding alternative networks of transport rather than the reliance on ICE vehicles. France has declared a ban on petrol and diesel ICE vehicles by 2040 and UK has followed suit on the basis of the report presented by the Department of Energy Change, UK in 2011 [2]. Although initial adoption of EVs was mild, the introduction of better battery technology and more mainstream vehicle manufacturers, like Nissan and BMW deciding to sell EVs, has made the number of vehicles grow quickly. Government initiatives, policies and aims to meet carbon reduction targets has not only driven manufacturers to make better and cheaper EVs but also has made EVs and charging infrastructure more accessible. As a result of all the efforts, the price of EVs has fallen drastically. Between 2010 and 2017, the prices have fallen by 65% [3].

As EVs become mainstream, they will have a big impact on the already stressed electricity grid when they are charging [4]. The electricity grid and systems must be prepared for the extra demand required for these vehicles which require long times, high power and high currents for charging. For most domestic users, charging will take place in the evening at home which is already a peak time for electricity demand [4]. Numerous studies like [5]–[9] highlight the impact of EVs on electricity distribution in the future, in elaborate detail. Whilst the recharging load for a small number of EVs is likely to be buried in the baseline load fluctuation, a large fleet of EVs charging at the same time could have an overwhelming effect on the grid during peak hours [10]. The latest National Grid report estimates at least a 30% increase in peak electricity demand due to EVs by 2050, when the deadline to shift to zero-emission vehicles is to be met [3]. Therefore, if the electricity system is left unmanaged and is not prepared for this demand, it will be very difficult for distribution networks and system operators to satisfy the demand.

Subsequently, the electricity market, infrastructure and pricing structure is changing, both due to the market requirements and technological progress. Investments in physical infrastructure are slow and expensive but information technology offers an opportunity. *Smart-grids and smart-metering* are being deployed in many countries already and the Energy Networks Association (ENA) has set a target of 2020, to effectively roll out the smart-grid for UK [11], [12]. *Smart-grid* generally refers to an energy network that can use various technology with digital control and automation to monitor the energy flow in an electric grid and adjust the energy supply and demand accordingly. Ideally, it is more efficient and able to manage problems in a more 'real-time' manner by allowing elastic behaviour from households, letting them manage electricity supply and demand in a way that is more cost-effective and environmentally friendly. Smart-metering or advanced metering infrastructure (AMI) can change the way users are billed. These meters record hourly consumption and alert the users about future prices. The smart grid is being promoted and provided to consumers in the USA for some time now. According to the Energy Information Association (EIA), 38% of all meters in the USA are already smart-meters [13]. In real-time pricing (RTP) tariffs[1], users are provided with a day ahead price prediction (DAP) and then a second set of prices hourly, on the day, which are the prices they are billed on. The idea is to offset electricity peak demand by encouraging behavioural change. This means that the users will have different prices for each hour which is very much like the 'spot-electricity' market prices most industrial sectors are charged on.

Following on from the discussion, we can conclude that the electricity grid and EV will be interdependent in the future. Firstly, it will be important to manage the electricity loads and peak demands due to the user profile of EV charging. Some load may have to be shifted from peak hours either by persuading or enforcing the consumer to charge earlier or later. There will be a big push for smart demand-response solutions which can be integrated into the smart-grid. Secondly, charging the vehicle without control might also be a disadvantage for the consumer due to the possibility of a future with RTP. There is a good argument for vehicle to grid solutions (V2G) to help reduce the impact on the electricity grid in a future where there are high numbers of EVs charging at the same time. V2G technology can provide electricity from the vehicle back to the grid, to level any load fluctuations and gives the opportunity to sell surplus electricity in the EV to national grids. However, the more immediate solution to consider in the same scenario is a smart automated charging control. The goal would be to reduce cost of charging to the vehicle user, by delaying the charging to the hours when there is already less stress on the electricity grid.

This thesis presents a number of optimal charging algorithms for EVs in a real-time electricity market, for a user with AMI on the smart-grid. The idea is to automatically manage the charging time once the vehicle is plugged in, in order to provide a required

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[1] Real time pricing tariff is a type of flexible electricity pricing provided by certain electricity utilities over the world. In such a tariff, users are billed on prices which change hourly. The information is provided via a smart or advanced metering system.

amount of charge by the time the vehicle is needed, but at the lowest possible cost. This means that the charging would be shifted to off-peak hours when the prices are lower. The subsequent advantage of this can be to the grid in a way of load shifting. The research follows the approach of a control problem, trying to solve an optimization problem from the point of view of the vehicle user. Firstly, a basic time-discrete solution is explored which assumes that the known prediction (DAP) is accurate. Secondly, dynamics and the unpredictability of the real electricity prices is introduced as a stochastic element. The more complex problem with the stochastic nature of the prices is solved using stochastic control and dynamic programming.

The thesis is presented as a problem, followed by its solution. A literature review is presented in chapter 2, which looks at EVs, driver charging behaviour, the RTP market and EV charging solutions in brief. The problem overview and statement is summarized in chapter 3 which presents a highly abstracted version of the optimisation problem: the problem this research is set out to solve. Chapter 4 introduces the RTP market data from the Illinois grid and an analysis in terms of a linear regression predictor, with the aim to quantify the validity of the provided DAP versus the RTP.

The basic problem which assumes that the DAP provided are accurate and hence the prices are known is presented in chapter 5. It also discusses variations to this problem and the reason why the dynamic nature of the electricity grid needs to be considered to achieve optimality in the solution. It is concluded with a simulation result and a short example based on real data from the Illinois grid to put the findings into perspective. The work done in this chapter has also been published as a journal paper in the SAE [14].

Chapter 6 approaches the problem by using stochastic control and dynamic programming to account for the dynamics of it. It presents an algorithm developed on the basis of this approach which achieves an optimal charging solution. A simulation study is presented to validate the functioning of the strategy. The work done in this chapter is presented in another SAE journal paper [15].

Lastly, chapter 7 discusses a case study based on the real time price data from Illinois grid which proves the requirement and benefit of using the optimal charging strategies developed during this research. It highlights the potential financial benefit of using this approach toward the vehicle owner. An initial version of the case study is presented in a conference paper at the SAE World Congress 2017 [16]. Chapter 8 concludes the thesis by presenting and overview, summarizing and analysing achievements and discussing the outlook of optimal charging of EVs.

2 Literature Review

This chapter discusses the motivation and relevant research in the field of EV charging. Section 2.1 through 2.3 present a review of adoption of electric vehicles, expected driving behaviour and the impact of EVs on the electricity grid. Section 2.4 discusses smart-grids with flexible pricing and section 2.5 presents a review of studies on electricity price prediction. Section 2.6 examines studies and solutions for automated EV charging. Lastly, section 2.7 presents a summary of the literature review.

2.1 Electric Vehicles

The power and transport sectors are the largest contributors to global greenhouse gas emissions (GHG). The transport sector contributions to both emissions and energy use are growing quickly by the year and World Energy Outlook projections predict that they will overtake the power sector by 2035 [17][18]. In 2007, the road transport sector accounted for 71% of the total emissions attributed to the sector as a whole, with 63% of them generated by passenger cars [19].

The world community has set ambitious targets for GHG reduction in the future and many countries (especially the developed nations) have registered emission reduction targets or commitments to the actions by 2020. As an example, the European Union's (EU) ambitious target is set for 2020 to reduce GHG emissions by at least 20%, improve energy efficiency by 20% and ensure the contribution of renewable energy sources in gross energy consumption is 20% [20].

The pressing requirement for a 'greener future' has brought electric vehicles (EV) forefront. The opportunity that EVs and related technology vehicles (Hybrid EV (HEV), Plug-in HEV (PHEV)) have, is to revolutionize the transport sector and other related infrastructures in the next decade. They can significantly reduce the CO_2 emissions if electricity sources with low carbon intensity can be used. CO_2 savings are offset by generation, which reduces the saving effect. Studies [21] have shown that PHEVs would emit less CO_2 per mile than a conventional petrol vehicle, even if the electricity is produced at 100% using coal plants which emit the maximum CO_2-eq/KWh. If organisations began converting their conventional vehicle fleets to EVs, they could not only gain a greener footprint but also help in lowering the emissions from the light-duty transportation sector.

It seems obvious that the charging of these vehicles will be an added load on the already stressed electricity grid. New and emerging research has shown the possibilities of a two-way electrical system, connecting EVs through grids to homes, commercial establishments or other facilities when they are being charged. The National Grid's key insights include the acknowledgement that technology progress in EV infrastructure can help reduce the peak demand with the help of smart applications [3]. New technologies like the 'smart-grid'[2] could integrate with load management services and allow charging of EVs individually or collectively in a manner that may benefit both the consumer and the electricity grid. This synergy between EVs (as electricity storage banks) and the grid will be able to smooth the demand profiles and help balance contingency needs. In a future which promises more renewable energy use, the surplus energy balancing can also be smoothed by charging or discharging of a large number of EVs.

The EVs currently in the market can be broadly classified as follows. Hybrid electric vehicles (HEV) are a combination of the typical internal combustion engine (ICE) and a battery electric vehicle (BEV), with the electric motor supplying auxiliary power when the ICE is not in use. Plug-in hybrid vehicles (PHEV) combine the advantages of the HEV and BEV. They work in two modes: fully electric or hybrid.

HEVs have been mildly successful in the past decade with the major producer, Toyota (Prius) selling 2 million units by 2009. Whilst it is widely accepted that market forces alone have not been able to make the EV a first choice for many consumers, government policy support, research to make EVs less expensive and economy improvement will improve their market significantly [22][23].

The barriers in EV penetration into the main market have been limited driving range and high cost of electric technology. Although electric driving ranges are limited for all three HEV, PHEV and BEVs, surveys have indicated that 47-55% of single vehicle usage in a single day is less than 20 miles, with 82-88% of vehicles travelling less than 60 miles [24]. Kang and Recker's 2009 [24] study concludes that it is possible to convert between 80% to 90% of daily mileage to electric when using PHEV with a 60 mile range in California, under the condition that both home and public place charging stations are in use. These numbers indicate that EVs are more feasible than previously thought.

JP Morgan performed a study in 2009 which forecasted 11.28 million EVs worldwide by 2020 and 20% of the total cars sold in North America [25]. In 2008, ARUP [26] forecasted that there will be between 0.5 to 5.8 million EVs in the UK by 2030. National Grid's (UK) *Future Energy Scenario* [27], predicts an even higher number of up to 7 million EVs by 2030.

[2] Smart-grid technology includes a smart- interface for electricity consumers by which they can be informed of future electricity prices. Technology research in this area is gearing to provide automatic and helpful control systems to enable smart load shifting and possible electricity bill savings.

National Grid's 2015 study [28] states that battery costs need to come down for sales of EVs to grow alongside the proliferation of fast charging points, in order to battle 'range-anxiety'. The UK government has committed to 35% discount on the purchase of new EVs for a car and 20% discount for light goods vehicles. In addition to this, funding is available for on-street charging points and the Highways Agency has installed charging points on most motorway service stations which is a great step forward in producing infrastructure for an EV future. These studies indicate that EVs are more feasible than previously thought.

2.2 Driving and Charging Behaviour

The biggest variable in all the studies performed and being performed on EVs is driver behaviour analysis. Whilst many others can be predicted accurately using scientific formulae and principles, driver behaviour regarding selection of vehicles, driving styles and charging EVs is still unpredictable to a degree. The concern from the data already recorded is the viability of the behaviour of early EV adopters, which could change easily as EV technology, numbers and prices change.

Conventionally, drivers refuel their car when petrol/diesel runs out after the mileage the vehicle normally provides. With regards to any kind of EV, this can change because it depends on the battery size (like the fuel tank size) however, so far, the range of these is very limited compared to conventional vehicles. This introduces an unknown about the charging behaviour the driver or owner might adopt. Will the vehicle be charged at the end of the day? Or will it be charged at every stop it makes during the day? To be able to answer these questions based on behavioural analysis, many variables must be considered. Total distance travelled during the day matters most if the vehicle is only charged at night. This may depend on the type of user, for example an organisation fleet user may charge the vehicle during the day if the workplace provides free charging. This introduces another variable- charging infrastructure and cost incentives.

Regarding the usage of vehicles in the UK, the government Department for Transport (DfT) releases annual data on trends in personal and public transport. The 2012 analysis [29] shows that the distance travelled per person per year is 2% lower in 2011 compared to 1995-97, however 89% of all trips are still made by private transport modes (including van, car, walking and cycling). Car travel formed the largest proportion of transport method in 2011 as 79% of total distance was travelled by car. The average trip length for car or van personal transport is 8.5 miles and 21 minutes.

Similarly, the Department of Transport for the USA's National Household Travel Survey in 2009 analysed the driving usage of American drivers. Haaren et al.'s study [30] took the

748,928 samples in the survey and concluded that 95% of the single-trips in a car were under 30 miles and less than 98% were 50 mile trips. Most people's travel to work was not over 40 miles with an average work trip distance of 13.6 miles. They concluded and pointed out that with a public charging infrastructure which could support EVs, the range anxiety plaguing the changeover to EVs would not be significant.

This shows that the car is the chosen mode of transport for very modest average trip lengths, and has been even through changing trends. The changes over the years have been attributed to variables like car availability, household income and policy changes and incentives, which themselves can be all interconnected. It is also evident from the numbers that the prediction of higher EVs in the future is a valid one because most daily trips can easily be made with the current battery technology available. It must be noted that there are, however minor, differences between driving behaviour in different geographical regions and this is also a variable connected with behavioural effects.

With EVs being a new influx in the automotive market, real world data on driving and charging behaviour isn't well populated. It is known that understanding consumer behaviour on charging EVs is crucial to understanding whether EVs will be viable and our current electrical infrastructure will be able to cope. Most theoretical or real-world studies that have been performed concentrate on two basic scenarios: Uncontrolled charging- where users charge their vehicle where and when they please; Controlled charging- where devices like the smart-meter are predicted to control times of charging to manage electricity grid peaks and to save cost for the consumer.

Morrow et al. [31] performed a study for the U.S Department of Energy on PHEV charging and infrastructure. They looked at two main charging scenarios- at night home charging and opportunity charging at public facilities. The conclusion was that there is a peak in EV charging during the evening when users plug-in at home between 19:00 and 23:00. Wang et al. [32] simulated four charging scenarios which included uncontrolled charging, delayed charging (to promote load shifting), smart charging and smart charging with demand response (where the charging is optimally controlled by the electricity distributor). They concluded that smart and delayed charging can significantly reduce the total cost of the system both on the electric and charging side.

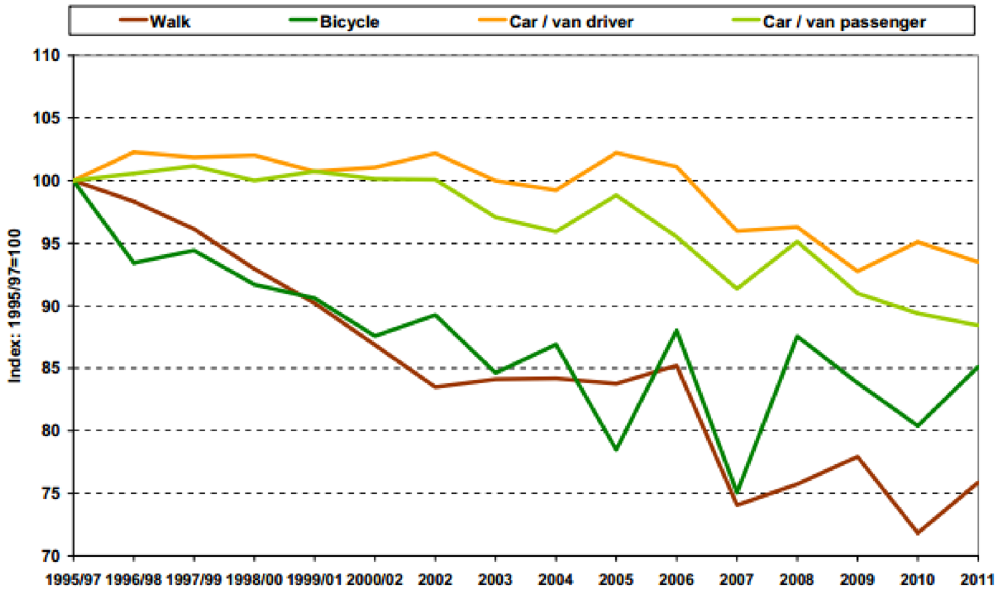


Figure 1: Personal Travel Trends, UK (DFT-NTS 2012)

Kang and Recker [24] developed four theoretical scenarios of uncontrolled home charging, end of day travel charging, controlled (after 10 pm) charging and public infrastructure charging. They concluded that it was important for the electricity charging infrastructure's circuits to be upgraded for faster recharge times, which would allow more of the daily mileage to be electric. Although day-time public charging would allow for more trips in this manner, it would also cause higher spikes at peak time and this needs to be considered. Clement and Hasen [33] and Parks et al. [34] have also considered both 'uncontrolled charging' and 'charging with a time delay scenario' in their studies. However, Clement and Hansen have suggested a 'coordinated controlled charging' scenario which should help minimise grid power losses to be a more optimum step towards EV charging.

Weiller [35]studied the effects of different charging behaviours in terms of time of day and location of charging. She highlighted how the time of the day of charge is influenced by the location of the driver. The model developed helps determine how access to different recharging locations can impact the recharge profile of PHEVs. It was concluded that enabling charging in places other than the home (like workplaces and public areas) can significantly increase the mileage covered by electricity over that covered by fuel.

Mullan et al. [36] also took into account three scenarios: evening time charge (16:00-23:00), night time charge (22:00-7:30) and controlled night time charging using smart meters. They concluded that shifting the EV peak to later in the night can benefit the base-load utilisation in Australia up to a limit governed by other effects in daily electricity-grid/transformer maintenance.

A study and report produced by the Consumer-led Network Revolution in UK [37] shows a similar profile to the academic studies discussed earlier. The study was performed with real world EV drivers in the UK and show EV charging peak demand results coinciding with the peak demand hours projected by the National Grid for current consumer usage. The report assumes and then states that consumers will charge their vehicle when it is convenient to them which is mostly when they arrive home or when work finishes.

The National Grid report has based a simulation on this in their report [28] by applying a prediction of consumers switching to TOUTs. It shows that by shifting the demand by just 2 hours from the peak at 17.30, it is reduced to 38% at this bad time for the grid. Figure 2 explains this very well and almost makes a good case for charging automation applied with the uptake of RTP or TOUT.

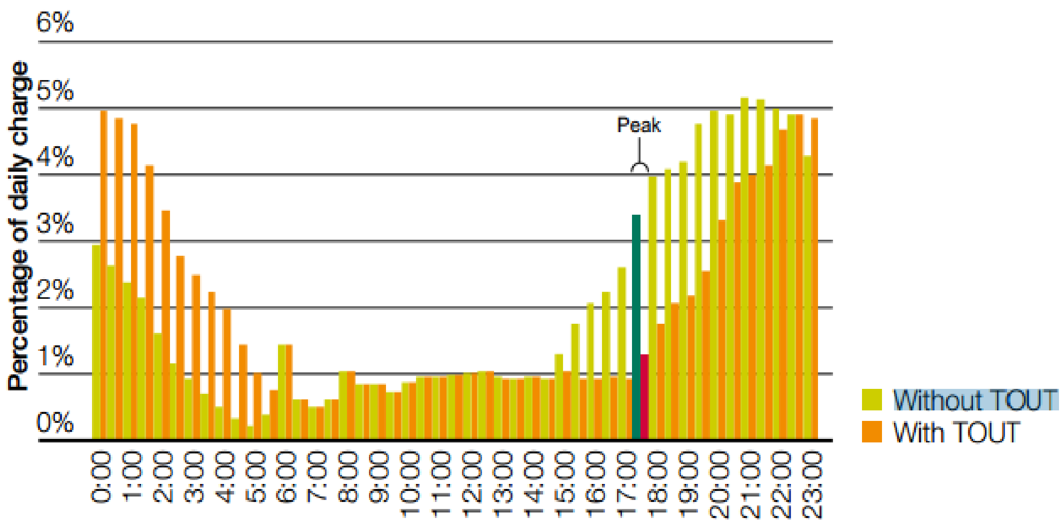


Figure 2: National Grid Report 2015, EV Charging Profile on a January day with and without TOUT

This shows charging profiles can be a huge variable in determining the specifics of EV charging and usage in the future. Most studies more or less conclude that the tendency to charge vehicles as soon as users reach home is extremely high. It will not be enough to allow users to charge their vehicles as they please without serious effects on the grid's health and power quality or simply the capacity. Most of them highlight the requirement of a 'smart' or 'controlled' way of charging the vehicles, the control of which may be shared between the user and the electricity provider.

The charging behaviour of users is difficult to predict, although it can be derived from their usage profiles. Charging locations and times will vary with user type, infrastructure and geography. For instance, a company fleet user may opt to do most charging at work, where infrastructure may be provided. However, an individual user would choose to charge both at home and public infrastructure. It is likely though that human behaviour will compel users to plug-in their chargeable appliance as soon as they are home so 'it remains charged' by the time they need it. Studies like [35], [38], [36] and [39] conclude that the spike in charging requirements is pronounced between 5:00 pm and 7:00 am based on the time majority vehicles arrive home. What is more interesting is that these studies are done in different geographical regions of the world and yet this time period is of concern whether or not the morning period at work may be of charging demand (assuming infrastructure is provided).

2.3 The Impact of EVs on the Electricity Grid

A change from ICEVs to EVs means a change for consumers in the context of refuelling patterns. However, their main impact will be on the already stressed electric grids because the batteries of these vehicles require long times and high currents for charging. Whilst peak load times would be different in different parts of the world, depending on many factors including weather, some kind of load shifting to accommodate EV charging may be required.

The United Kingdom (UK) National Grid's simulation study in 2015 has shown a steady and then accelerated increase in electricity demand with influx of EVs between 2010 and 2035 [28]. These projections show that there are 20,000 PHEVs in Britain but the number could surpass a million as early as 2022, adding 2TWh/year to the electricity demand of the country. By 2035, EVs alone would account for 14TWh/year. Although charging will take place at the workplace or at service stations, most domestic users will also charge overnight at their homes. Consumer tendency to plug the vehicle in for charging as soon as they reach home will be high. If they charge the vehicles after every trip: for example, every time they reach office or a supermarket or come home and leave again, this would be of even higher concern to electricity peaks.

The latest report by the National Grid [3] concludes that it is most likely by 2050, 90% of all vehicles sales will be of EVs. They perform the study in terms of four scenarios: 1) Steady State: Business as usual where focus is on providing low cost electricity supply rather than investing in long term low carbon technologies. 2) Two Degrees: A prosperous future with increased investment in low carbon energy with consumers making conscious choices and being able to afford technology. Additionally, there are effective low carbon policies in place. 3) Slow Progression: Low economic growth and affordability competes with the desire to be greener and reduce carbon emissions. 4) Consumer Power: High economic growth and more money available to spend. However, consumers have little incentive to be environmentally friendly and their appetite for latest gadgets drives innovation. In all these cases, the number of EVs and the infrastructure needed to support them grows dramatically. Figure 3 is from the report and shows the results of the study in terms of annual electricity demand rise from EVs.

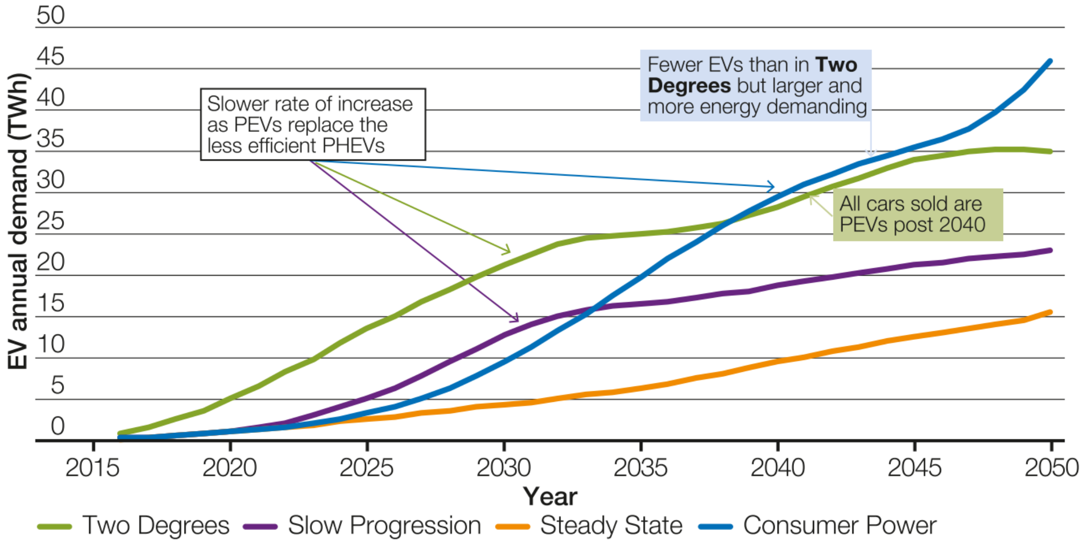


Figure 3: Annual Demand from EVs (Future Energy Scenario, National Grid, UK 2017)

Studies on the effect of EV penetration to the electricity grid date back to the 1980s and have been performed in all developed and developing parts of the world. In [40], Heydt discusses the effects on and of electric load management due to EV penetration. The study concludes that it is likely that charging will occur at peak demand times and some form of load management must be introduced to manage the additional EV charging load. Webster's review of electric infrastructure in the UK concludes that in case of high EV penetration, it is likely that battery recharging times will coincide with peak electricity demand [41]. Measures must be taken to avoid this although the electric networks may be able cope with the additional load. Both the above studies consider user driving profiles to have the primary impact on recharging times.

The major UK energy provider, E.ON, projected that if PHEVs represented 5-7% vehicle miles travelled in 2020, it would be equivalent to 7.5 TWh of electricity [42]. The article concluded that this represented only 2.2% of the current electricity capacity and UK's electric networks would cope with the influx of EVs. However, considering EV loads are mobile and very unpredictable, an increase in contingencies may be needed because nuclear power is not flexible enough and wind power is too unpredictable to meet EV recharging loads. This leaves thermal plants complimented by smart-meters and tariff incentives to minimise spikes.

Clement and Hasen [33] talk about the effect of EV penetration in 2030 on the Belgian electricity grid. They propose that coordinated charging of PHEVs will be needed to avoid power losses and maximise the main grid load factor. In their simulation they show that uncoordinated charging (whether immediate plug-in when home or delayed to avoid spikes) both can cause problems for the local grid.

In a much more recent study, Camus et al. [43] simulate a 2020 scenario of 2 million EVs in the Portuguese spot electricity market, considering different mixes of renewable power generation. They conclude that with low renewables and high cost, charging of EVs during peak times can lead to electricity prices of 17 Euro cents/KWh. This can be brought down to 7 Euro cents/KWh with off-peak charging and with higher renewables and low general costs, down to 5.6 Euro cents/KWh. Mahalik et al. [44] performed a simulation to realize EV impacts on the Illinois grid in 2020 and concluded that on-peak uncontrolled charging would require an additional 400 MW unit to support the state's reserve margin. If off-peak and controlled charging is facilitated, no additional supporting grid would be needed. The additional electricity required could be provided by reducing the electricity exports Illinois makes. A study for South Californian Edison [45] looking at the impact on its local grids concluded that, without planning load management substation and circuit rebuild costs could be very high due to the randomness of the EV load. All the above research recognizes the problems related to on-peak charging in a high EV penetration scenario, but no mathematical formulations are presented for optimal charging.

Acha et al. [46] present a time coordinated optimal power flow (TCOPF) tool for distribution networks to decide on load control approaches for EVs in the future. The algorithms concentrate on showing different charging strategies to the electricity providers to see how they may have to change energy production to reduce carbon emissions and cost. They conclude that, the UK will need to introduce more renewables or non-carbon fuel mix to offset costs and emissions for high EV charging scenarios. Kristoffersen et al. [47] use linear regression to minimize charging costs based on the Danish (Norpool) electricity market prices. The study assumed an EV fleet controller who managed the participation of EVs during charging or providing electricity to the grid, based on fleet driving patterns and electricity prices. It concluded that EV driving patterns and hence charging time is highly flexible during the day but not from day to day. In [48], Rotering and Ilic present two time-discrete algorithms for optimal charging; one considering only minimizing cost and the other also taking into account V2G support. They perform a case study based on the California day ahead electricity price market and conclude that optimal/smart charge reduces the charging cost from 0.43 USD to 0.2 USD daily. In case of V2G support, the profit amounts to 1.71 USD including charging.

The charging of EVs can have an impact on the electricity grid on a number of time scales, ranging from the millisecond range to hours and days. Schirmer et al. [45] concluded that multiple EV chargers connected to the grid could lead to power quality issues by utilities on a large-scale. They mentioned another study which estimated that harmonic levels could reach 90% at some locations if the batteries were being charged fully at the same time. There is potential for all these effects to not be a problem to the grid with today's research and technology: for example, active inverters can absorb harmonics; they can apply droop control to enhance grid stability and they can pick times for charging when excess electricity is available. However, very little progress has been made towards these goals so far and regulations often only aim to prevent harmful effects, not to leverage potential benefits.

2.4 Smart Grids and Real Time Pricing

The subsequent effect to consider is the logical change to electricity generation, distribution and pricing in the future. The higher peaks, consumption and addition of more renewable energy might change the prices in markets significantly. Whilst EVs have been much the topic of recent research and significant progress in the transport world, the electricity grids are also about to undergo a landmark change from their historical method of exchange. This can be rather beneficial because the simultaneous and synergetic progress of research ideas with regards to EVs and electricity grids can shape a reliable future.

Traditionally, the electricity grids in the world have been *distribution grids.* A few central power generators or stations broadcasting electricity via a large network of transformers and cables to a country or region. Over the years, load forecasting has developed significantly allowing generators and distributors to forecast requirement by the use of sophisticated models. However, no prediction can account for a hundred percent balance and therefore demand is more often than not over-provisioned with respect to peak load. The demand is then managed by using backup plants which use non-renewable (traditional sources like coal in large quantities) sources of energy to balance the load on the grid and cope with demand. This approach has proved necessary but wasteful at the same time because when the average demand is lower than peak, surplus electricity that is produced has to be consumed due the high expenses often involved in energy storage [49].

As the electricity demand increases with scenarios and studies discussed previously, a solution more widely accepted and currently in the process of introduction all over is a

'Smart Grid'. The idea is to manage and match the demand to the available supply by two-way communication between the grid and customers (and other stakeholders). The benefit of a grid-wide communication system also offers a method of providing the customer with incentives like variable pricing (with an aim to urge them to shift their electricity use to off-peak times).

With the rapid research and progress that has taken place in regard to Smart Grids, there is no one solid definition given by regulators and nations, but the two important statements given by the European Union (EU) and the United States Department of Energy (US DoE) are reproduced here.

1. The EU defines the Smart Grid as an electricity network that can intelligently integrate the behaviour and actions of all users to ensure sustainable, economic and stable electricity supply [50].
2. The US DoE states that smart grids use digital technology to improve reliability, security and efficiency of the electricity system [51].

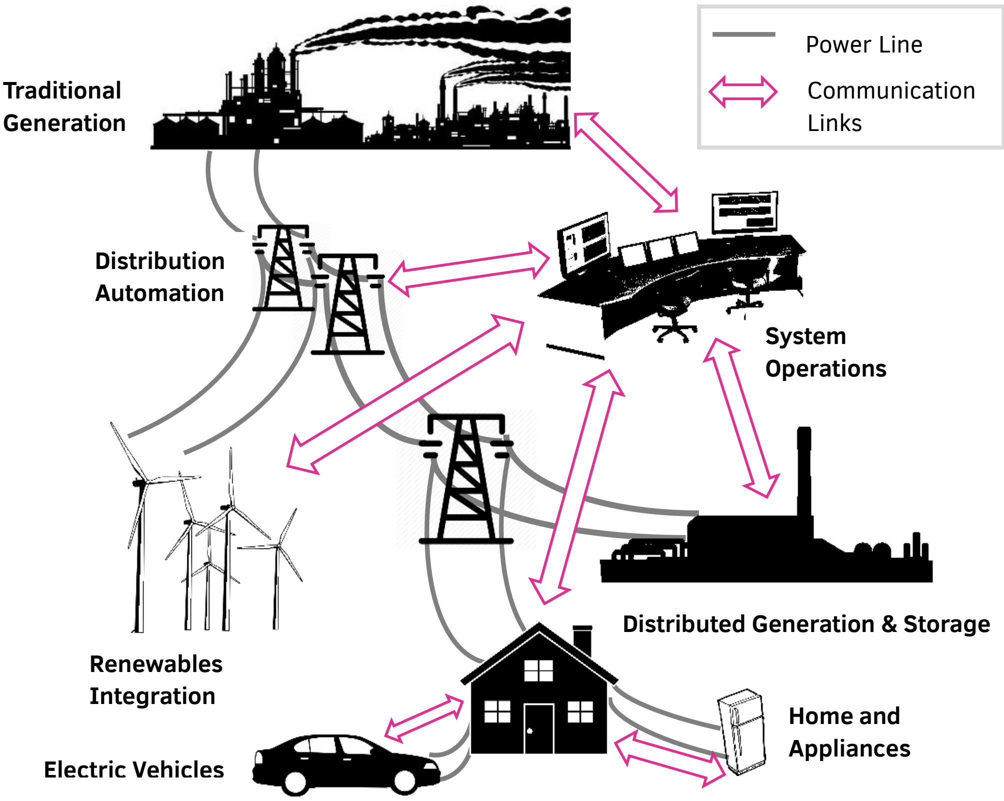


Figure 4: Typical Smart Grid Architecture

Figure 4 shows a typical smart grid architecture which clearly indicates integration of all players in the electricity grid as interconnected, at least via the communications link. If utilised and planned properly, each of these can both consume and provide to the central transmitting network of the electricity grid. A proper demand management through smart grid technology has the potential to yield significant savings in both the generation and transmission of energy [52].

The Smart Grid revolution has many aims according to the EU and US DoE, but the most relevant change with regards to general and EV consumers will be pricing incentives for demand response. Smart Grids are expected to play a significant role in shifting and customising consumers' demands to the effect of load balancing and prevention of peak loads. Load profiles of electricity grids have changed significantly over time and today's situation is more complex but, in some ways, predictable. This is due to change with newer loads like the EV coming into play.

Figure 5 shows a typical winter day power demand profile[3] for the UK (precisely 11th January 2015), obtained from the archives of the National Grid and charted in Excel.

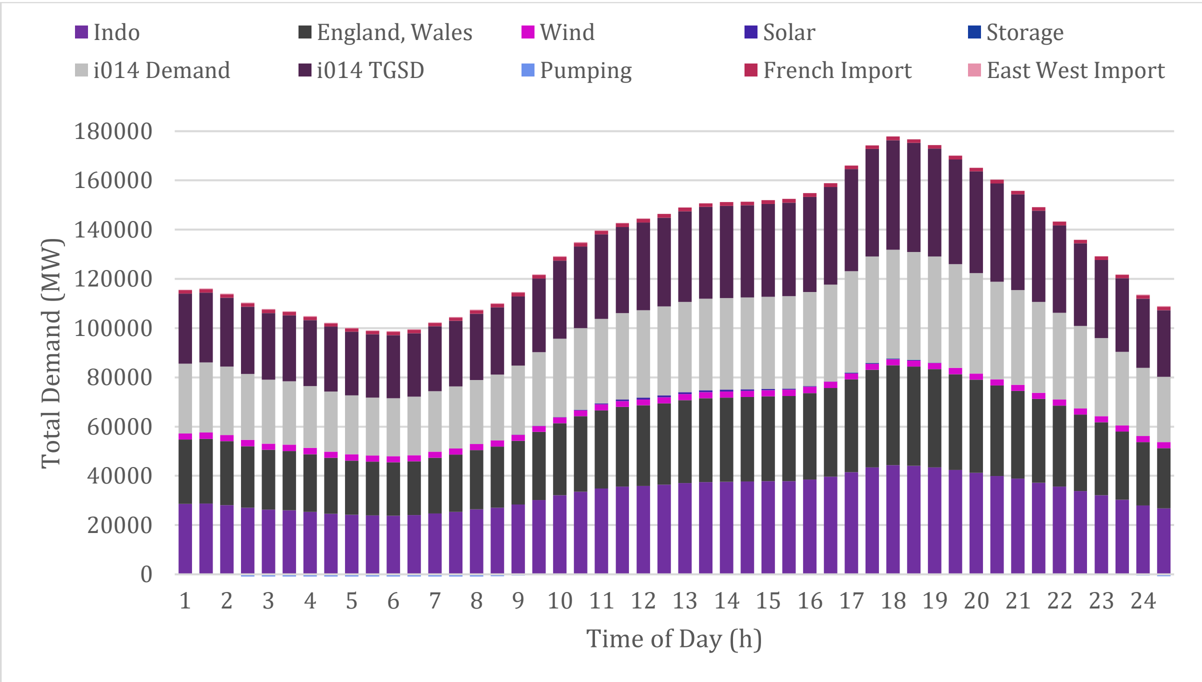


Figure 5: Winter Day Power Demand for UK (National Grid 2015)

Most countries' load profiles follow similar curves owing to the routines of people: there is a rise in demand from the morning till noon when people wake up, use appliances, and go to offices; the demand rises steadily after this period up to the point when people return

[3] INDO refers to published BMRA Initial Demand Outturn based on National Grid operational generation metering, EXCLUDES Station Load, Pump Storage etc.; i014 DEMAND: This is calculated from Elexon SO_I014 generation data. It is the sum of the same generation meters as INDO; i014 TGSD refers to Total Gross System Demand calculated from Elexon SO_I014 generation data and INCLUDES Station Load, Pump Storage Pumping and Interconnector Exports.

home and start using appliances again at which point it peaks. The key in these profiles is the commonality of times at which people require to use appliances.

It is important to note here that this will be different in different geographical areas (owing to weather differences) but not so much that the overall curve would differ. For example, winter in the UK would demand higher lighting and heating loads in residential areas when people return home. As confirmed by the National Grid [28] the peak demand is reached at 17.30 more commonly as the industrial loads settle and residential ones take over. If we were to take the example of a hot state in the USA during summer, the loads might be different i.e. air conditioning and fan loads but the curves are more or less comparable. The key once again is commonality of timing, the loads tend to increase at the time of returning home. Therefore, the promise of Smart Grids to integrate the consumer side demands and consumers themselves into the ecosystem to balance the peaks is of great significance.

In most countries the industrial sectors are on wholesale electricity prices- buying electricity at lower rates during off-peak hours. There is a possibility for such 'spot-markets' even for domestic electricity consumers as is the case in Portugal, Germany, some parts of continental Europe and a few states in the USA. In such markets, the consumers are encouraged to shift their electricity usage to off-peak hours through high-price updates/alerts either hourly or daily. Many studies like [53], [54] on smart grids have been carried out and the research agrees that one of the solutions for load shifting is going to be the manipulation of consumers' demands using intelligent real-time pricing models (RTP) (a term mostly used in the US grid).

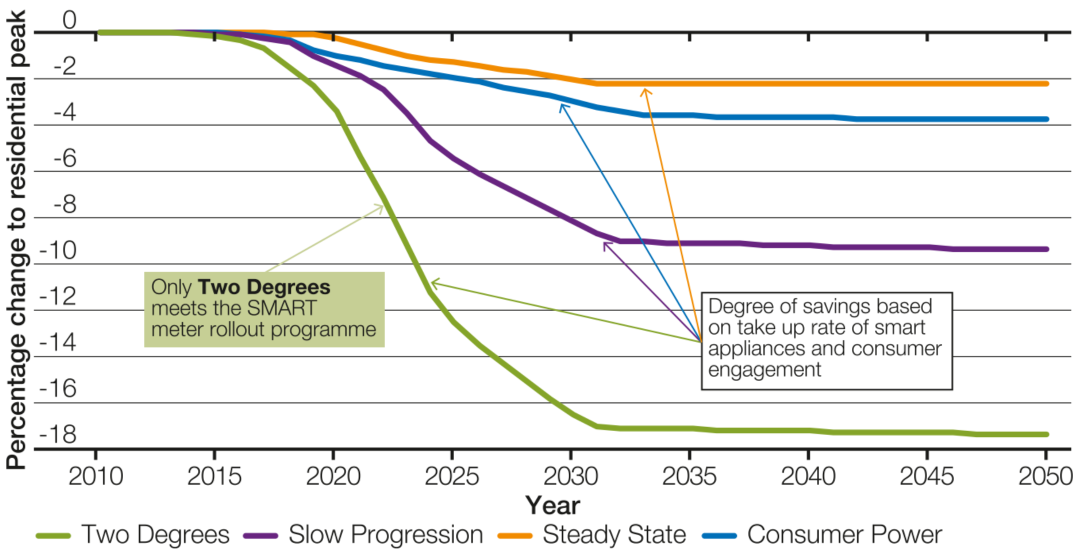


Figure 6: National Grid Future Energy Scenario 2017, Predictions for Time of Use Tariffs

The integration and implementation of such tariffs is the near future and will be done in tandem with smart meters, distributed energy resources and energy management units. The energy price of the *time of use* (TOU) (a term used by the UK national grid) including peak pricing would be shown to consumers with an attempt to urge them to shift their loads to a lower price time. Figure 6 shows the potential percentage change to residential peak, if TOU tariffs are rolled out to UK customers. The natural thought here is, could this be done by automation? The answer is a possible yes, especially for loads which don't need to be constantly used much like the EV: strategies like the ones which are the aim of this research- to automatically help the consumers and grid to benefit from the situation.

One of the other benefits of recharging during off-peak times is that it can reduce the carbon content of the electricity generation and lead to financial savings. In winter, power generation from coal-fired plants increases from 34% of total generation to 41% during on-peak. Coal has the highest carbon content (910gCO_2/KWh compared to natural gas' 390 gCO_2/KWh) of all electricity generation profiles (in the UK) and as power demand increases the carbon content therefore increases [55]. Therefore, recharging off-peak has this major positive effect on the environment. The study in [56] shows that by deferring the peak demand to off-peak, the capacity transmission cost could be reduced up to 67 billion Euros in Europe. The McKinsey report [57], [58] from USA shows a potential value generation of 130 billion dollars by 2019 by deployment of a successful smart grid infrastructure.

Smart metering is being promoted and provided to consumers in the U.S.A. for some time now. Electric utility providers in California, Colorado, Florida, Illinois, Indiana, Ohio, Texas, Washington and some other states have already been introducing smart meters to many customers. There is also a strong financial incentive being provided for both smart grid research and introduction via the Energy Independence Act of 2007 and the US Stimulus Package of 2009 [59]. As a result of this by 2015 38% of meters in the USA are smart meters or advanced metering information (AMI) enabled [13]. The report [60], claims that 9% of peak demand could be offset using just small-incentive demand response programs, if they were used all over U.S.A.. Moreover, if dynamic pricing programs are introduced to all electric consumers, 20% of the peak demand could be offset. This strongly indicates that smart grids and RTP are the future of electric pricing and management.

The state of Illinois is a good example where RTP has been available to customers since 2003. The RTP programs have been successful in reducing the participating consumers' electric usage and bills and shifting usage to non-peak times of the day [61]. The two electricity providers which allow the choice of RTP are Amaren and ComEd. Amaren's Power Smart Pricing (PSP) and ComEd's Residential Real Time Pricing (RRTP) programs have reduced their peak demand in the range of 15% and achieved participant bill savings between 10-15% [61].

The UK hasn't been lagging in their plans to introduce such tariffs and advancements, a lot of it though is still in its research and planning stages. National Grid survey of July 2015 [28] mentions three potential implementation plans for the smart metering change in the country, the least of which is 1 million units a year by 2017-18. The other two more optimistic plans take the number up to 6 million units a year by 2017-18. Along with smart-metering, there is an effective roll out plan for Time of use tariffs (TOUTs)[4] which are expected to significantly reduce peak demands by urging consumers to become a part of the ecosystem. The optimistic prediction is that of rapid uptake of such tariffs by consumers and close to a peak demand reduction up to 4GW by 2035. The more realistic predictions still claim a reduction of at least 1.2GW up to a potential of 2GW.

2.5 Day Ahead Price Prediction

Many factors influence electricity prices: the cost of fuel used to produce it; the transmission and distribution systems of the power plants; weather conditions; the load on the grid; location and location specific regulations are just some important considerations. Even basic factors like day, hour, week and month are of importance, when the reaction of consumers is considered.

Additionally, electricity grids can suffer from transmission congestion which may prevent free power exchange between control nodes. This creates complex non-linearity in the electricity load and prices, making them even more difficult to predict. This volatility can give rises to unexpected electricity price spikes.

The prediction of electricity prices is an important science but in spite of the numerous methods in use and ongoing research studies, short term price forecasting isn't a mature science, especially with price sensitive loads being introduced in the system [62]. Weron, R [63] presented a thorough review of state-of-the-art price forecasting techniques being used in the electricity market, aiming to explain the complexity and effectiveness of these techniques. It covers over fifteen years of studies and provides a detailed breakdown of them.

[4] This is the United Kingdom terminology for the real time pricing tariffs (RTP) as used in the United States of America.

Once, a significant share of electricity is generated from (generally uncontrollable) renewable sources, load shifting will be an important measure needed to align generation and consumption. Therefore, with a good next day forecast, a stakeholder in the 'Electricity System' would be able to make better financial decisions. A power producer could develop strategies to maximize its pay off and the consumers could minimize their utilization costs [64]. As such, there are three types of price forecasting, short-term, mid-term and long-term. The short-term time scale (hourly up to 24 hours) is the important one to consider for load shifting and consumer usage.

The wholesale spot-electricity markets rely upon price-forecasting techniques for bidding purposes and hedging against volatilities. Electricity is traded like wholesale and retail markets for other products. Generators connect to the grid and produce electricity, which is sold in the wholesale market to resellers (electricity providers). The resellers sell electricity in the retail market, to meet end user demand. The wholesale price can be pre-set via a contract between the generator and reseller or bought hourly in an auction style market. The clearing price for every hour is determined by these auctions, the cheapest resource clearing the market first. When the supply matches the demand, the market is cleared [65].

The trouble with forecasting arises from this nature of the electricity market. It does not allow for continuous trading, instead it must be cleared and the market clearing price (MCP) is set the day before by the bids submitted hourly. This arises from the need of the system operators requiring advanced notice to verify the supply schedule for the next day can be met by the transmission constraints. The prices for all contracts cleared for the next day would be determined at the same time using the same available information [66].

Complexity arises when there is transmission congestion and local and zonal prices might differ from the actual MCP. For a large market like North America, *PJM* zonal prices are computed and used. However, a study done in 2012 by Loland et al. [67] shows that transmission congestion can be predicted in short-term. For the short time, just before delivery, the transmission system operator operates the real-time market. This is used to adjust and deviate price in the day-ahead market. The system operator must react to any shortfall in production at very short notice, in order to ensure system balance [63]. To minimize this time, the system operators run an ancillary market for reserves. The day-ahead and real-time market which we are interested in here, is complementary to the ancillary services market for which the forecasting techniques used are different.

Some markets in the world are more volatile than others but price spikes are not rare across the board, when the ancillary generation markets are used to balance the shortfall. The complexity of price-spikes in the real-time market is such that Aggarwal et al. [68] conclude in their review paper: the accuracy levels achieved by models of day-ahead forecasts can be higher than those of real-time forecasts.

There are a multitude of approaches employed over the last fifteen years for day ahead price forecasting. Han et al. [69] propose a composite approach for ultra- short term load forecasting using two well-known methods: recursive least square support vector machine algorithm and Takagi-Sugeno fuzzy control. Hong, Wilson and Xie [70] propose statistical methods to predict long term probabilistic forecasts, also including linear regression with multiple factors. They use hourly information to create a more robust forecast using predictive modelling, scenario analysis and weather data. Singhal et al. [71] use back propagation with neural networks. Their basic idea uses history and other estimated factors in the future to fit and extrapolate, to achieve a price prediction. Many of these studies and the strategies they propose, are already being used by power utilities and claim to have less than 5% error in their predictions [69]–[71].

Weron [63], [72] classifies the various different approaches as follows:

- Multi-agent Models which simulate the operation of the electricity system (generators and companies) and build a price process by matching supply and demand.
- Fundamental models, which describe price dynamics by modelling the impact of different factors, economic and otherwise, which affect the prices.
- Quantitative and stochastic models, which characterize the statistical properties of the market over time.
- Statistical models which are direct applications of the techniques of load forecasting.
- Artificial intelligence and non-linear techniques, which combine several elements of learning and fuzzy logic.

Regression is one of the most widely used statistical techniques in literature. Multiple regression can bring out the relationship between several independent variables on the criteria of prediction. However, many of the modern regression-based techniques are usually combined with other methods. For example, Conejo et.al [73] combine multiple regression with wavelet decomposition to predict day ahead prices in the *PJM* market. Schmutz and Elkuch [74] use multiple regression with gas prices, nuclear generation capacity and weather factors as predictor variables and a mean-reverting stochastic process for residuals. There are several more, using varied regression techniques. The accuracy of historical data-based regression models depends on the efficiency of the algorithms and the quality of data analysed. The inclusion of relevant factors like weather forecasts, fuel prices, historical demand etc. is a difficult task as there are so many factors, yet some might not be relevant. However, the seasonality or predictability that exists in the electricity market can tend to make technical analysis of the data based on regression useful.

The inevitable price-spikes are what makes it difficult for statistical methods to perform well; stochastic models are more effective at capturing this clearly 'random' behaviour. Literature is unclear on whether a combination of both techniques works better and if it is useful to include the stochastic analysis in the final statistical predictor, or the spikes should be filtered out as outliers. Logically, this depends on the periodicity of the spikes, which is also quite unpredictable but also some markets are more volatile than others, making it difficult to conclude on a perfect technique. As such, there is literature and methods devoted to just spike forecasting for the more volatile electricity markets, like Australia [75], [76].

2.6 Demand Response and Optimal EV Charging

The evolving concept of smart-grid aims to develop a strong integration between electricity generation and demand. Demand response (DR) is a term which relates to any program which encourages shift of energy demand by end consumers. The response is due to incentives like flexible tariffs or greater awareness and their participation may involve passive responses or active behavioural changes [12].

Load management problems have received significant attention from researchers for industry and grid-side management. In the last decade, there have been DR or demand side management (DSM) programs proposed for industrial electricity usage. In [77] Ashok proposes a peak load management strategy for mini steel plants in India. Using a process industry load model coupled with integer programming, an optimisation formulation is developed. The goal is to reduce peak demand and electricity cost by optimal DSM. Vadabhat and Banerjee [78] propose three DSM options for global temperature adjustment, chilled water storage and variable air volume systems. The models proposed show a potential for shifting peak electricity in commercial applications for heating, cooling and ventilation systems.

Similar load management techniques would apply to EVs as well. As established in previous sections, there is a clear need for EV load management when they are used abundantly. In

[79] Richardson et al. investigate impact of different levels of EV penetration on low voltage distribution systems. They assess modest to worst case scenarios by setting up an optimization problem and doing load flow analysis. The idea is to maximize energy for charging operations while keeping factors like network congestion and thermal loading in check. They propose a weighted objective function to penalize charging points with high sensitivity. Overall, they propose that optimisation techniques for charging EVs should be explored and demand side operators could potentially control EV charging with smart-metering. However, in this work user requirements are not modelled.

There are some control approaches in literature which aim to propose control of the EV charging process but with aims that are not aligned with user benefits. Bashash and Fathy [80] propose a robust feedback strategy for controlling aggregate grid power demanded by an plug-in EV fleet. They apply sliding mode control principles to achieve stability in the case of using renewable generation in tandem with a number of grid-connected vehicles. The outcome is a centralized control strategy which attenuates the imbalance between supply and demand in the above case. Once again, user requirements are not discussed.

In 2012, Druitt and Fruh [38] investigated the integration of additional wind power and electric vehicles in the future electricity network. They suggested a stochastic model for both wind power generation and electricity price market. The study's aim however, was to investigate the role of a 'fleet' of EVs in a future grid to load management and energy storage potential (essentially V2G), with the integration of more wind power.

Yunus, Parra and Reza [81] presented a paper on distribution grid impact of fast charging with a stochastic charging model (2011). The stochastic model they used was for a simulation of the effect of many PEVs loading the grid, when charging at high power (fast charging). The model results led to a conclusion that widespread fast charging affects the quality of electricity supply and necessary actions need to be taken to continue the use and deployment of such stations for EV charging.

Zheng and Wang [82] proposed an aggregation model for large number of EVs charging, to control power fluctuation problems. They consider the randomness of the number of EVs charging at any time as a stochastic disturbance and employ a genetic algorithm to obtain these. They simulate this model for a parking lot scenario and show the stochastic feature of the charging characteristics. Their proposed charging strategy is to control (minimize) the influence of many EVs charging simultaneously to the grid power. They conclude their updateable model reduced power fluctuation levels in the residential district where EVs in parking lot are being charged. However, their idea does not consider the effect to SoC and user needs.

The study by Deforest et al. [83] suggests adopting a smart strategy to mitigate issues which would schedule the EV charging to fill the overnight valley in power demand. Many studies propose centralized and decentralized 'valley-filling' schemes but they all concentrate on the problem from the grid point of view. 'Peak clipping' and 'valley-filling' are two concepts in DSM which concentrate on reducing the difference between maximum and minimum power demand [84]. This essentially means that at certain times peak load can be shifted from peak to off-peak, performing the function of filling the low-demand 'valley' in the graph of power demand (Figure 7).

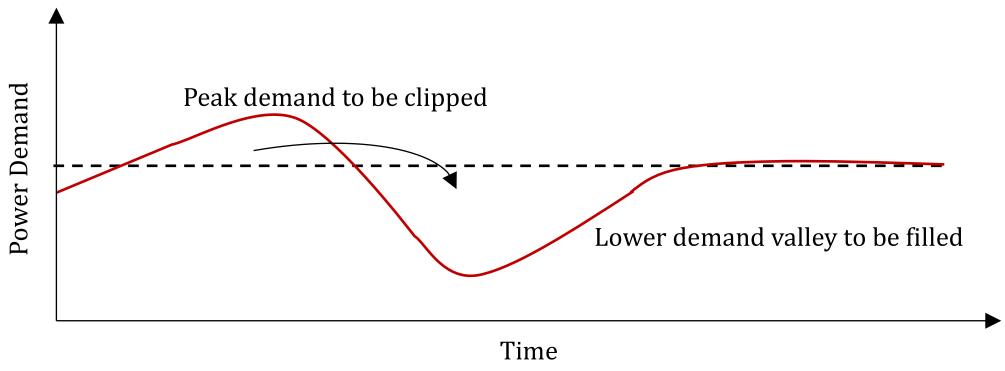


Figure 7: Example of Valley Filling

Ma et al. [85] propose such a decentralized charging strategy which optimizes the charging profile through a day-ahead negotiation between the electricity utility and EV users based on predicted load profiles. These charging profiles optimally minimize load variance by filling the 'valleys' of the load curves. In other words, the charging is offset to periods when there is a dip in the electricity demand. It assumes that all the EVs participate in the negotiation together and their load profiles must be known beforehand. A strategy like this could potentially address the peak demand issue but the users may not get the best out of it. Moreover, the control solution is optimal only if it is assumed that the predicted load profiles are accurate.

In [86], Kang, Duncan and Mavris present a real time scheduling (RTS) concept which promises to extend the 'valley-filling' strategies benefits by also guaranteeing the satisfaction of EV users in terms of complete charging and the time they would like that charge. Their strategy is a centralized RTS where EVs are charged in a distributed manner and different queues for charging are maintained based on the user requirements and electricity utility's main functional need (minimizing total load variance).

Mody and Steffen [87] presented a study in 2013 describing the need to use automated control for 'optimal' charging of EVs. They observed that the increasing EV population will lead to electricity grid problem if it is treated like a normal appliance. It would be necessary to use a smart method to offset EV charging to lower demand hours by automatically charging the vehicle rather than let it charge instantly when it is plugged in. The paper presented a sub-optimal controller for automated charging based on real time tariffs provided by electricity grid distributors in Illinois (Chicago). The first conclusion was that if prices are known in advance (day-ahead pricing), the optimization only requires picking the cheapest time slots for charging the battery. Further savings can be made by using real time prices that are not known in advance.

Scholer and Glynn [88] presented a charging solution in the technical paper (2014) with a similar idea to [87]. However, it was mainly a part of a series of technical papers written by the SAE PEV task force. The main theme of their paper was 'smart-charging standards' but their main conclusion was that it is not necessary to charge the PEV immediately when plugged in, and smart charging is required to balance the load and prevent problems in the local distribution circuit. They presented a price-based smart charging idea based on RTP and smart grid communication very similar to the one presented by Mody and Steffen in [87]. It reacted to the price information and offset charging to lower demand times, still making sure that full charge was provided when needed.

Various studies including the ones discussed here which propose the need for a solution or a solution for optimal EV charging. However, most of them stress on the need to find an optimal control solution from the electricity utility and grid point of view. They concentrate on the system view, including vehicle-to-grid (which may be valid in the future and useful) and on assumptions about grid requirements regarding future regulations.

2.7 Summary

The above discussion urges us to consider two things when EV penetration becomes high in the future. Firstly, it will be important to manage the electricity loads and peak demands due to user profile of EV charging. Some load may have to be shifted from peak afternoon and evening times in some manner either by persuading or enforcing the consumer to charge earlier or later. Secondly, charging the vehicle without control might also be a disadvantage for the consumer due to the possibility of a future with RTP for electricity. However, both these problems can be looked as an opportunity for EVs. The flexibility of charging time can be viewed as an advantage for load shifting opportunities.

Majority of the research in the area of optimal EV charging looks at the user point of view in a limited manner. Most of the work done concludes that in any centralized or decentralized charging strategy, the users' requirements might be secondary. For example, the studies proposing centralized control by negotiating requirements with utilities in advance, do not acknowledge the basic user requirement of state of charge deadline being met for a high percentage of EVs. Moreover, there is not much stress on 'optimality' in finding the control inputs in the majority of studies. Although some studies like [89] and [90] do take into account minimising cost of energy, there is no direct link to electricity prices or the benefit or loss to the users who might be on a smart-tariff.

The approach taken in this thesis tries to fill this gap in the field. The aim is two-fold: to reduce cost of charging the EV for the user and to shift electricity demand from peak to off-peak. A control engineering approach which views the problem from a user point of view first, is the main concept behind the strategies developed here. Data of flexible pricing tariffs like RTP can be used to decide when it is the optimum time to charge. Chapter 3 presents and overview of the problem and scope of the solutions presented in this thesis. The optimal charging strategies presented in this thesis have been published in three journal and conference papers [14], [15], [91].

3 Problem Overview and Statement

A problem statement is formulated in this chapter, on the basis of the discussion in chapter 2. The research question is put forward clarifying the scope of the optimal EV charging solutions presented in this thesis.

The interests of the grid are obvious in a scenario with high EV penetration. It will be essential to manage the electricity loads and peak demands due to user profile of EV charging, or the additional load could lead to power outages. Some load may have to be shifted from peak afternoon and evening times in some manner either by persuading or enforcing the consumer to charge earlier or later.

This has to be traded against the goals of the EV owner. Charging the vehicle without control might be a disadvantage for the consumer, especially when smart-grids are in place and most customers are on the real-time pricing tariff (RTP). Essentially with users being billed upon changing prices, the nature of the electricity market and possible price spikes might lead to a financial disadvantage from the user's point of view.

The flexibility of charging time can be looked as an advantage (discussed in chapter 2 section 2.2) for grid-side load management and for the user saving on charging cost. If the daily mileage required is for urban travel, a desired state of charge (SoC) requirement can be achieved in less time and times of charging can easily be varied. The question of when the right time to charge is the core of the problem. Up-to-date pricing information from the smart-metering system could be a starting point which aids automation strategies to decide charging times.

The charging of electric vehicles can impact the electricity grid on a number of time scales, ranging from the millisecond range to hours and days. The potential is there for all these effects to be beneficial to the grid: active inverters can absorb harmonics; they can apply droop control to enhance grid stability [92], and they could pick times for charging when excess electricity is available. However, very little progress has been made towards these goals so far, and regulations often only aim to prevent harmful effects, not to leverage potential benefits.

This thesis concentrates on the slow time scale, and what kind of effects smart metering has on the optimal charging timing. Currently, prices are set in intervals of 5 to 15 minutes depending on markets, and it is difficult to see a price-based approach being able to respond faster than about 30 seconds. On the other hand, using prices allows sophisticated optimisation strategies to schedule the charging. The aim is to look at the potential benefit a combination of real time pricing and charge automation strategies can achieve for the user.

3.1 Context and Terminology

Figure 8 shows a typical representation of North-American and European distribution grids. The urban, low-voltage distribution highlighted in purple is of significance for the problem described in this thesis. More specifically, the snapshot of a single home and EV connected to the grid.

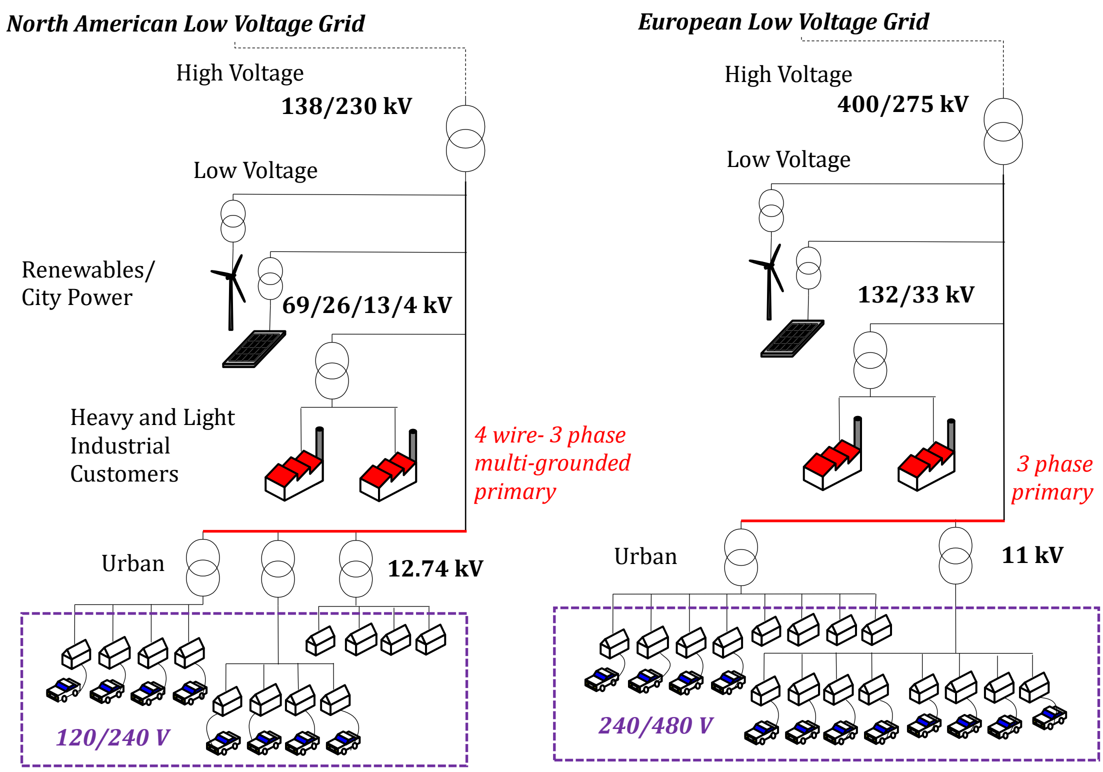


Figure 8: Single Line Diagram representation of Low Voltage Distribution Grid

There are many differences in European and North-American grid designs. Primarily, European systems have larger transformers and more customers per transformer, than North-American systems do. European transformers are three-phase and 300-1000 kVA, compared to North-American 25-50 kVA, single-phase transformers. In terms of secondary voltage, North-America has a standard of 120/240-V, whereas European systems have the 240/480-V standard. European systems can have more customers per transformer because they are three-phase and have twice the voltage. This means they can reach eight times the length of an American system for a given load and voltage drop [93]. There are pros and cons with either system but with respect to significant number of EVs being charged, having less customers per transformer can be an advantage.

In terms of secondary supply (urban and low voltage customers), European systems are more flexible. For example, transformers can be sited more conveniently. Conversely, North-American systems have more flexible primary distribution layout, which means it is easier to extend and supply to new loads in rural areas. On average, North-American designs result in fewer customer interruptions. European systems have less primary but all of it relies on the main feeder backbone. The loss of the main feeder would result in an interruption for all customers on the circuit [94]. The context of the North-American grid is used in this thesis because there are smart-metering programs in place in some states, and price data is available from the electricity distributors for research.

We begin by foreseeing a future with wide spread adoption of battery electric vehicles (EVs), although it is not clear when exactly this will happen. The cost of EVs has fallen by 65% since 2010 already, as discussed in chapter 2. If the decline continues the possibility of electric mobility in the future is a given. Many drivers have already opted for EVs as second vehicles but will likely have one or more EVs as the main household vehicle in the future. This influx of EVs will cause additional load on electricity grids. One solution for that problem is smart EV charging control which shifts this load from peak to off-peak times.

The electrification of transport will be aided by charging infrastructure that can support it. There is already a network of charging points in countries like the U.K. and U.S.A., and this will only get better, including the availability of chargers at public and work places. However, users are likely to charge when it is cheapest and most convenient for them, unless they absolutely need to. Especially, the availability of fast chargers to install at homes is a great incentive to charge conveniently at home, like users would their smartphone every evening at home. Although fast chargers are currently expensive, they are aided by government grants and tax reliefs in many countries and like EVs themselves (chapter 2), the price of this infrastructure will undoubtedly fall to more affordable levels.

Overall, the driving behaviour is assumed generally unchanged in this future, where most drivers still use the vehicle for urban commute like daily trips to the workplace and back and some leisure driving on the weekends. As reviewed in section 2.2 most single vehicle usage is likely to be in between 20-80 miles a day. Based on this, the users travel to and from work back to home every day and do some leisurely driving on weekends with a matching mileage to the weekdays. The EV is charged using a fast charger installed at home when it is most convenient, i.e. when they return from work which is a peak time for electricity use.

The users' homes are connected to the smart-grid and most importantly have a smart-metering connection which makes the day-ahead prediction (DAP) of price and real time price (RTP) tariff available. The users are also billed on the real time price tariff. These tariffs are explained in chapter 2, section 2.4.

The key question to answer is: when is the best time to charge the EV? This question is surprisingly difficult to answer because of the number of parameters which could affect this decision. The most important parameters would be location; weather; charge requirement, and price of electricity (in turn dependant on the time). Although, the intensive discussion in the Literature sections (chapter 2 section 2.2) reveals that charging times can be somewhat predicted, this is not strictly true all the time for all locations in the world. How much charge will be needed and when will depend on the use profile of the EV. This is a significant variable here; is the vehicle used for leisure, work, urban driving, extra-urban etc. This makes predicting a charging profile tedious. The consumer themselves will have to intervene to decide when exactly they would want to charge, unless there is an intelligent automatic system which can do this.

Moreover, users are likely to plug-in at their convenience and the price of electricity (and demand) at these times might be very high. More vehicle owners will obviously choose to charge where the service may be provided for free, or at lower rates. So, with the narrowing down of some abstraction, the two parameters which will always affect the decision of when to charge, will be State of Charge (SoC) and Price of Electricity; which in effect would be dependent on the time at which charging will take place and time at which it is needed i.e. Time of Charge (ToC).

3.2 Problem Breakdown

There are two main stakeholders in the electricity system who would be affected by a high number of EVs in the future: 1) the electricity grid and 2) EV users. This research concentrates on the problem from the second point of view, i.e. the vehicle user's point of view. To make it easier to explain, Figure 9 and Figure 10 try to visualise the problem.

The user comes home at 18:00 and plugs the vehicle in. Until 7:00, it is available for charging, and then it needs to have a certain minimum state of charge (SoC) for the daily drive. The vehicle battery is an integrator of power, so it can be charging at any point during this time. The charging time does not have to be continuous.

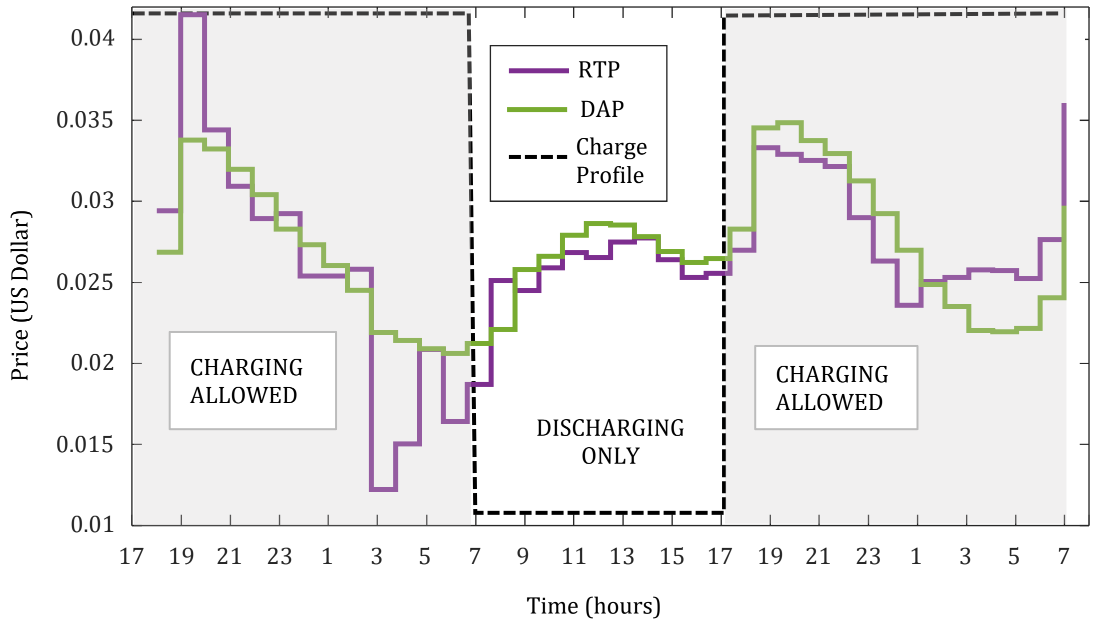


Figure 9: Charge profile and DAP-RTP Price curves

A simple strategy (and in widespread use current) is to charge the vehicle immediately. This makes the vehicle available as soon as possible, and it is simple. However, this approach leads to very high prices for charging, and it causes extra load for the grid at a very busy time.

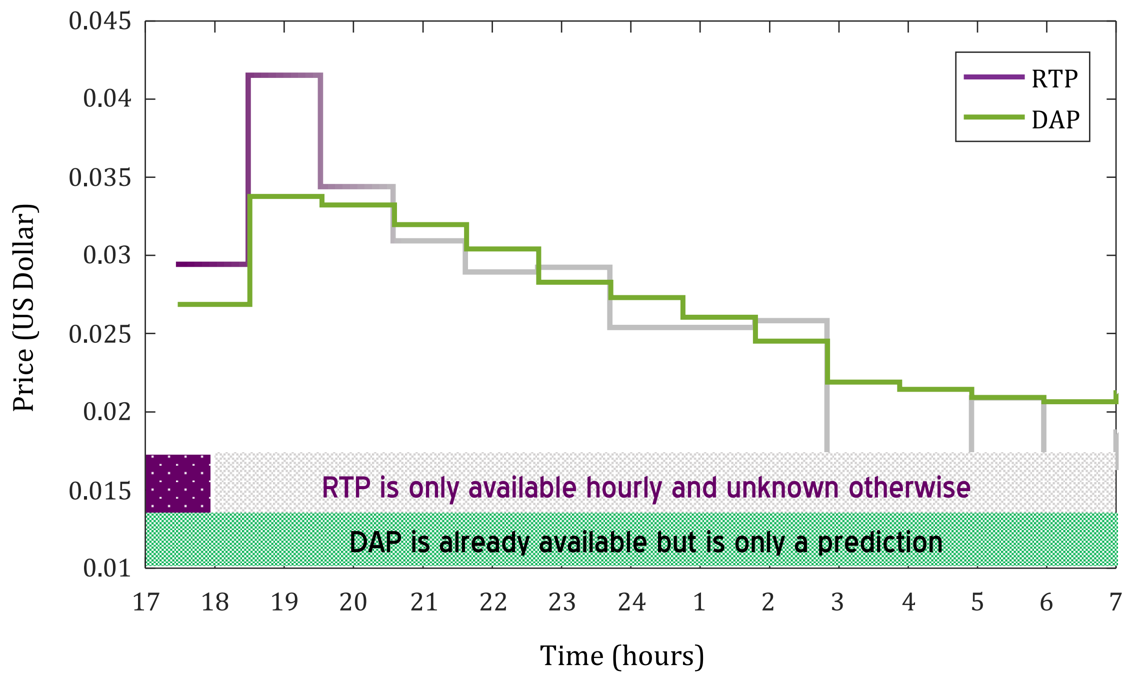


Figure 10: Availability of DAP and RTP

At 17:00 in the evening the day ahead prices (DAP) are released for the rest of the day and the next day. Based on these prices, it is relatively easy to come up with an optimal charging strategy, as will be discussed in Chapter 5. The real time prices are of course unknown, and they only become available hourly as time progresses. Because the prices are unknown, it is much more difficult to identify the right time for charging. Figure 10 shows this problem: the purple section is the hourly price which is known, and the grey section represents the hours for which only a prediction is available.

Part of the problem is about the user's requirements; the time when the vehicle is needed again, and how much charge is required in the vehicle to manage the mileage for the day. Based on this information, the time required to charge the vehicle can be known. Two variables of concern to the user are SoC and ToC, i.e. how much charge is required and when is it required by.

The solution needs to answer: when is the best time to actually charge the vehicle between now and then (the point in time when the vehicle is needed)? It must be able to balance the trade-off between the user's requirements and delaying the charging to the correct hours to achieve the lowest possible cost.

Answering the question of when to charge is not trivial because the real prices are not available in advance when they would be helpful to make the decision, and the prediction is not likely to always be accurate or can even be far off. The question can then be posed as: What will be the cost of charging now versus charging later? The dilemma can be explained well using the following example.

The DAP price at the hour, *say 23:00*, is low compared to other hours and we decide to start charging. This reality might change at *the hour*, when the RTP is available, i.e. the RTP might be higher. This poses more than one problem. Firstly, we may have missed the opportunity to charge earlier at a lower cost in the earlier hours. Secondly, we now must decide if to delay the charge further and rely upon the known DAP and hope we can we rely on them. Lastly, the charging cannot be delayed indefinitely i.e. we cannot keep waiting. The battery will require a known amount of time to charge and that threshold would be most important because the user cannot be denied a charged vehicle. Basically, the nature of the problem is such that the opportunity cost of charging later is unknown compared to charging now. The trade-off between saving cost and achieving the outcome of a charged EV must be balanced, which is difficult because the time dependency of the problem is on the environment and not just the cost.

The problem now takes shape on the basis of the change or delta of price over time. This is an important and largely unpredictable variable. Smart meters or AMI can deal with frequently changing tariffs and typically the cost of electricity changes every hour. The power provider makes an hourly prediction for the next day in form of DAP, but these prices are merely a best possible prediction. DAP then, is a known factor but also a ‘known unknown’, as they are likely to be unreliable to a certain degree. The problem becomes simple, if we were to assume that DAP are accurate and therefore the known price but this cannot achieve optimality in the solution. RTP on the actual day and hour can be very different due to the nature of the market because the real prices fluctuate based on the supply and demand and are prone to random disturbances. This is evident from the Figure 9 above as well.

3.3 Model Framework

Now, taking the important aspects discussed above in account, the basic plant model of the problem can be represented as shown in Figure 11.

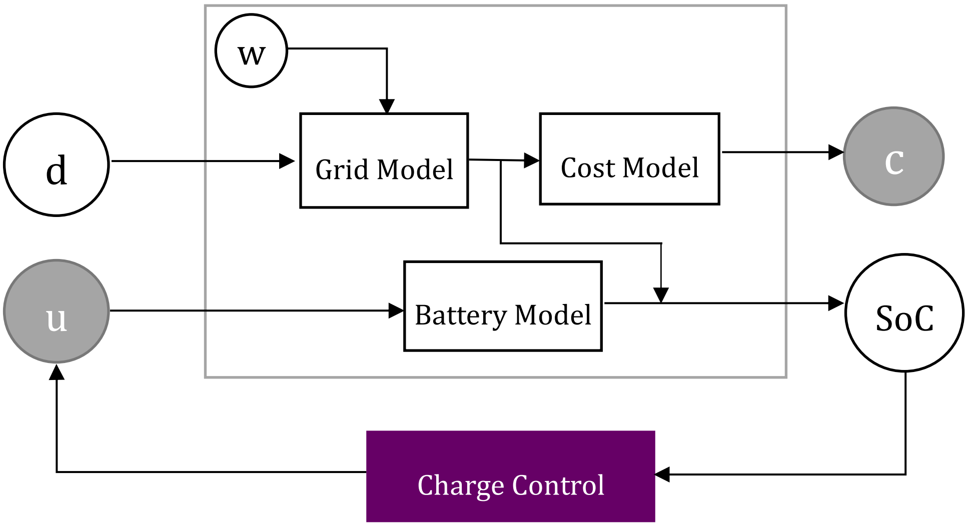


Figure 11: Basic Plant Model showing the Minimisation Problem

- **u:** The charging power will be the ***input*** which has to be regulated or switched on an off based on the control solution.
- **Battery Model:** We want to integrate charge inside the battery over time, i.e. the SoC of the battery is a state of the problem. The SoC is basic and internal state which is explained by its dependence on the control input and input alone.
- **Grid Model:** The Electricity Grid cost is affected by external disturbances and is an external state which will affect the output but there are several approaches which can be taken to model this. Further analysis is performed to understand electricity price data in chapter 4.
- **Cost Model:** The cost is added up at every unit of time when the input is applied and is affected by the input and grid cost.

- **w:** This is the known price disturbance i.e. the day ahead price prediction (DAP).
- **d:** This is the unknown price disturbance i.e. the factors which are unpredictable but change the electricity price.
- **c:** This is the ***output*** is the final cost which the solution aims to minimise.

The nature of the external state, the grid cost, divides the problem into two distinct parts, one is deterministic and the other is the disturbance which changes the basic problem into a stochastic one. The 'Charge Control' in Figure 11 can be developed with a variety of methods. The deterministic part can be resolved by linear programming. However, the stochastic part may require methods like model predictive control (MPC) or dynamic programming (DP). These are methods to solve classic optimisation problems, but they have the advantage that they can deal with stochastic elements.

The deterministic approach and controller are discussed in chapter 5. The stochastic control approach is presented in chapter 6. It is the combination of these approaches which leads to an understanding that has ultimately resulted in an optimal controller for EV charging.

3.4 Summary

Thus, to summarize the problem and requirements:

1. The EV user drives in an urban area and plugs in the vehicle to charge as soon as he or she is home. The vehicle needs to be charged by morning, ready for use again.
2. The condition is to *provide the required charge at the required time* and to *minimise the total cost of charging*.
3. The charging of the vehicle should be offset to lower demand hours which in turn benefits the user because he or she is charged less for electricity.
4. Retrospectively, there is DAP and RTP price data which is available from some electricity providers in the world where these tariffs are active. This data can be used to formulate (by training) a grid model which affects the total cost.
5. The vehicle battery can be modelled as an integrator of power. Without any losses or effects of temperature or other chemistry being considered, any charge stored in it is not lost unless used. Therefore, charging can be done in stages if required.

3.5 Research Question

This research aims to provide an optimal solution to the problem of ***'when to charge?'*** for an EV connected to a smart-metering system. The idea is to provide a unique approach in which the system of control is not hierarchical, but the problem is viewed from the perspective of the vehicle owner instead. The aim is to minimise charging cost over a given period of time while still providing the required amount of SoC.

This discussion leads to ***the main research question*** this thesis presents a solution to:

From the perspective of a battery electric vehicle owner on real time tariff, what kind of control law for vehicle charging leads to the lowest cost for charging the vehicle?

Although this may seem like a forgone conclusion, in that an optimal control will deliver the lowest cost, this problem is not specific enough to solve yet. A number of assumptions need to be formalised before a solution can be attempted and the nature of the assumptions will imply the appropriate algorithm. Therefore, the remainder of the thesis approaches this very question from a number of different points of view.

4 Data Analysis and Price Prediction

This chapter begins by assessing the data from the RTP programs in the state of Illinois, USA, because this is used for the case study which validates the controller strategies proposed in this research. Since there are differences between these two prices, i.e. the day-ahead (DAP) being the prediction provided by the electricity company, compared to the real-time (RTP) and therefore unpredictability in them, we proceed to analyse this further.

We explore a linear regression predictor to create an unbiased and improved estimate of the RTP tariff based on the available data, specifically the available DAP trajectory. The aim of this basic predictor is to understand how useful DAP is in predicting RTP, and whether there is any systematic difference between the two that would have an impact on a charging control strategy.

As discussed in chapter 2 section 2.5, predicting the real time price is a tedious endeavour. The concept of using linear regression with the available factors is to see how close the DAP come to the RTP, and whether by using the available information (the DAP themselves along with the hour, week, month and day), we can achieve a better prediction than that is already provided. The other question this analysis could answer is: Is there a clear alteration in the DAP to already encourage load shifting?

The chapter is arranged as follows: Section 4.1 introduces the pricing data which is being used for training and verification purposes. Section 4.2 explains the methodology behind the linear regression predictor. Section 4.3 discusses the various models and 4.4 presents an analysis of them. Finally, the results are discussed, and a conclusion is presented.

4.1 Pricing Data

In Illinois, U.S.A., RTP tariff has been available to customers since 2003. These programs have been successful in reducing the participating consumers' electric usage and bills and shifting usage to non-peak times of the day [61]. The two electricity providers which allow the choice of RTP are Amaren and ComEd. Amaren's Power Smart Pricing (PSP) and ComEd's Residential Real Time Pricing (RRTP) programs have reduced their peak demand in the range of 15% and achieved participant bill savings between 10-15% [61].

In the Midwest area of U.S.A., *'PJM'* is a neutral and regulated organisation which directs the operation for different generators. They serve as an agent, to regulate fair access to transmission systems for different electricity suppliers. Illinois, is one of the states in which *PJM* regulates the spot-electricity market and *'ComEd'* is one of the electricity resellers to the retail market. The reseller, *ComEd*, charges the end users based on the 'real time price' (RTP) from *PJM* which is determined by the average of twelve 5-minute prices from that hour, without any mark-up. The 'day-ahead price' (DAP) which is the prediction based on weather, capacity, generation factors and other variables, is also provided by *PJM* [95].

There has clearly been great progress in the last five years. Price information was available on an hourly time-scale in 2012 but now is available at both fifteen and five minutes. Although these programs have been successful, smart-grids and AMI are in their infancy. This is evident by these changing policies and non-standard data availability across the same distribution network PJM. A recent review paper in the area concludes something similar about AMIs. The data clarity and frequency is questionable at times and therefore unreliable [96].

The data available from ComEd also has been incomplete for many years which have been discarded. Secondly, it is now claimed that the RTP is only available after the hour is passed. This reflects a grave issue because the whole idea of encouraging behavioral change on part of the users' electricity usage is defeated if the real price isn't available before the billing interval. The availability of the five-minute prices makes it better but even then, the last five minutes in the hour could change everything if there was a 'spike'. If the price is ultimately unavailable before the billing interval (whether it is an hour or five minutes), demand response solutions will be limited to using unreliable DAP.

Retrospectively, both the DAP and RTP are available from the electricity provider. The data available from Amaren, Illinois (year 2010) was first used as the baseline. However, due to changes in their company policy, further data was not available to download and ComEd data was used instead. As a result of this, the example in the first part of the research (chapter 5) has been conducted using the Amaren data and the subsequent case study in the latter part (Chapter 1) is conducted on the basis of ComEd data. An analysis is performed here on the basis of both, but the predictor is explored on the basis of ComEd data.

The data from years 2010 to 2014 is used, from 1st of January to the 31st of December for each year [97]. These prices exclude the distribution cost, which is constant and therefore not relevant for comparison purposes. So, that question, 'When to charge?' comes down to answering this one, 'How much can we rely on the DAP data for the prediction of real prices the next day?'

Table 1 left (Amaren, 2010) and right (ComEd, 2010) show the pricing data from a statistical point of view. First of all, both the day-ahead price and even more so, the real-time price become negative at times. This happens because bids with negative prices are allowed in the spot market, when the demand is very low or the production from renewable sources like wind is very high. The costs of shutting down and ramping up a power plant can exceed compared to the losses due to negative prices, so the existence of negative prices is not odd [72].

Secondly, the standard deviation of the real-time price is much higher than the day-ahead price although the means are quite similar (highlighted in *Table 1*). There is a remarkable difference in the standard deviation between DAP and RTP; in the Amaren data it is more than double for RTP and in the ComEd data slightly less but still significant.

Table 1: Statistical Properties of Amaren data (2010) (left) and ComEd data (2010) (right) (SD-Standard Devaition)

Amaren	**Mean**	**SD**	**Min**	**Max**
Day-Ahead (US cents)	2.73	1.04	-0.19	19.81
Real Time (US cents)	2.63	2.20	-8.85	107.58
Difference (US cents)	-0.10	1.99	-99.19	13.19
Ratio	1:0.96	1:2.11	1:6.92	1:5.43

ComEd	**Mean**	**SD**	**Min**	**Max**
Day-Ahead (US cents)	3.61	1.60	-0.44	12.38
Real Time (US cents)	3.61	2.19	-12.58	33.30
Difference (US cents)	0.13	1.54	-23.85	13.31
Ratio	1:1	1:1.37	1:28.5	1:2.69

The difference between the day-ahead price and the real-time price can be seen as prediction error and interestingly its standard deviation is only slightly lower than standard deviation of the real-time price. The RTP are much more volatile, and therefore offer greater potential for load shifting. The correlation coefficient between day-ahead prices and real time prices of the Amaren data is ***0.43***, which indicates that day-ahead prices have only moderate value as a prediction of the real-time prices. However, the correlation coefficient of the ComEd data is ***0.71***, indicating that their day-ahead prices have a higher value as a prediction for the real prices.

To understand further, the autocorrelation property of the data can be analyzed. Autocorrelation is a characteristic of the data which shows the degree of similarity between the values of the same variables over successive time intervals. As such, it measures the correlation in the data of the same series, between y_t and y_{t+k} where $k = 0, \dots, K$ and y_t is a stochastic process.

Lag k is defined as

$$r_k = \frac{c_k}{c_0} \tag{1}$$

Where

$$c_k = \frac{1}{T-1}\sum_{t=1}^{T-k}(y_t - \bar{y})(y_{t+k} - \bar{y}) \tag{2}$$

c_0 is the sample variance of the time series

The estimated standard error for the autocorrelation lag k is

$$SE(r_k) = \sqrt{\frac{1}{T}\left(1 + 2\sum_{j=1}^{q} r_j^2\right)} \tag{3}$$

Where q is the lag beyond which the theoretical ACF is effectively 0. If the series is completely random, the standard error reduces to $1/\sqrt{T}$ [98].

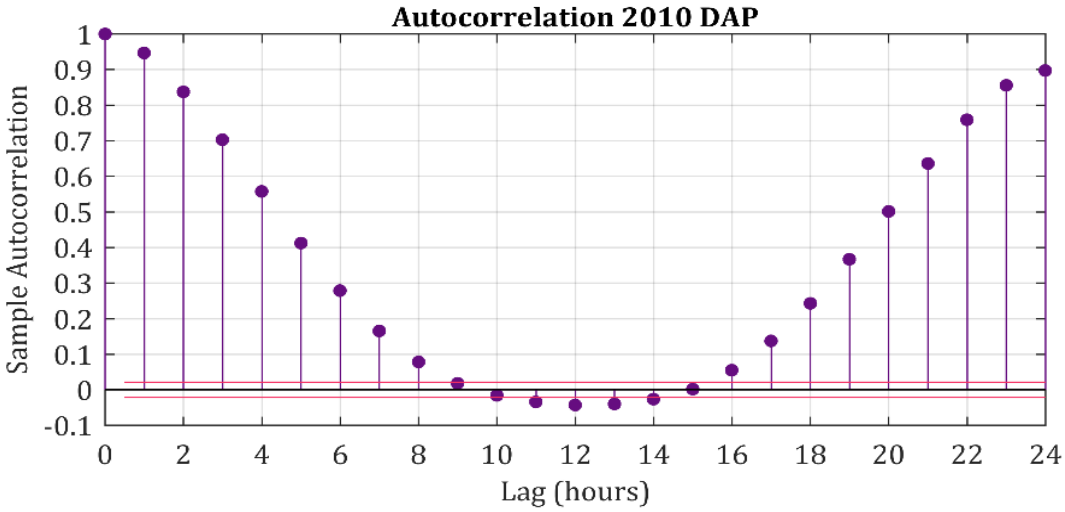


Figure 12: Autocorrelation of DAP

Figure 12 shows the autocorrelation of DAP data for the 2010-year data. The DAP data decays quickly to lag 10, reaching zero and then going negative before rising back up. Figure 13 shows the autocorrelation of RTP data for the same year. The RTP data stays positive nearly converging to zero but stays between 0-0.1 before rising again. The data is predictable starting with a high autocorrelation, but the randomness is stronger at the central lags, near 12. There is likely to be more randomness during the hours when there is higher use of electricity. The DAP seem to show a higher autocorrelation, as they converge more quickly to zero. Of course, both tariffs are a combination of regular daily changes in price, and random fluctuations caused by specific supply or demand situations.

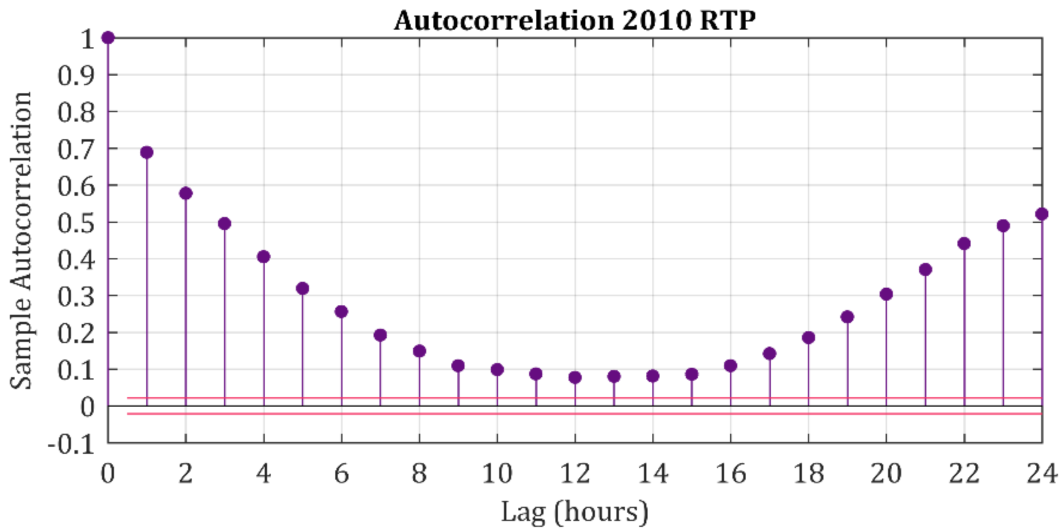


Figure 13: Autocorrelation of RTP

Figure 14 shows the autocorrelation of the error ($DAP - RTP$). This removes the regular daily changes from consideration and makes interpreting the data easier. The exponential fit ($0.83e^{-0.33x}$) shows a convergence to zero by lag 15.

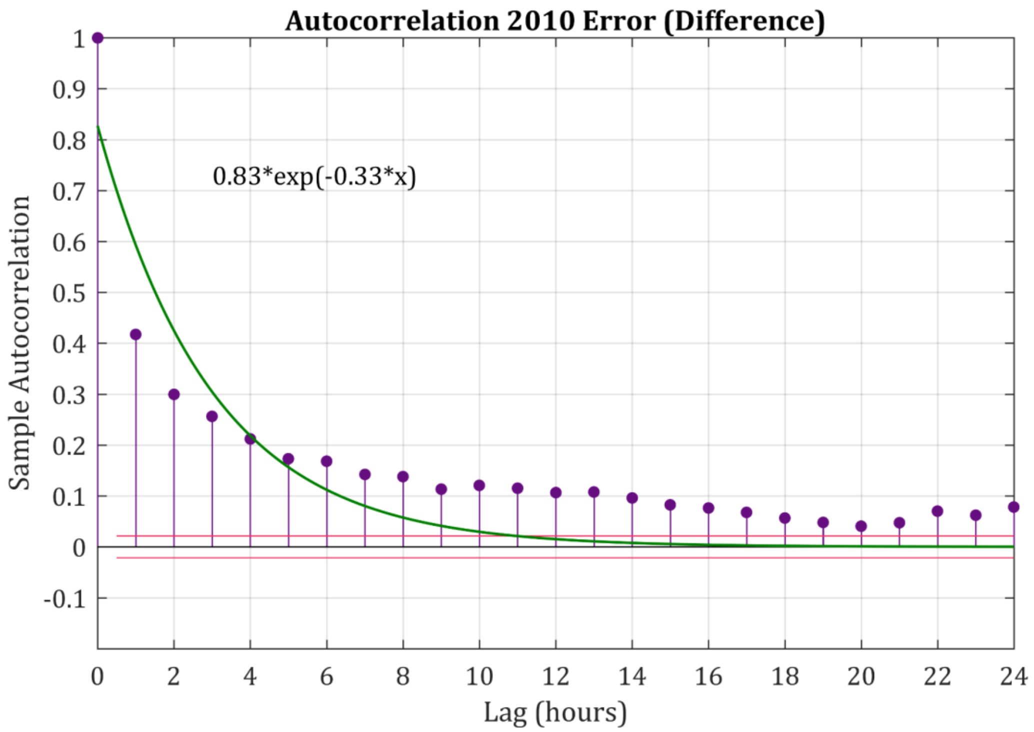


Figure 14: Autocorrelation of the error (DAP-RTP)

The plot starts with a high autocorrelation at lag 1, which quickly decreases but then decays exponentially. It does not quite converge to zero itself indicating a moderate autocorrelation. So, the correlation shows a long tail, which is best explained as a combination of exponential functions. This means that a stochastic model would need to include hidden variables – a first order system cannot produce this autocorrelation with precision. Still, for the purpose of this research, the first order approximation is considered to be close enough to be useful.

Figure 15 shows the normal probability plots for DAP and RTP. Firstly, the key observation that can be made, is that there is much more variability in the RTP prices compared to the DAP. The RTP plot clearly shows the trend of the electricity market, discussed in sections 4.1 and 4.2. The 'normal' prices all fall on the linear line, but there is a long tail on either end. The non-Gaussian nature of electricity prices is the cause of this shape. The higher prices, which do not fall on the line and tend to elongate the right end, can be attributed to the large-scale unforeseen events, which are difficult to predict. The extreme high prices indicate failure of transmission in some cases. Therefore, it is essential that heavier loads like EVs are encouraged to shift their charging away from peak times.

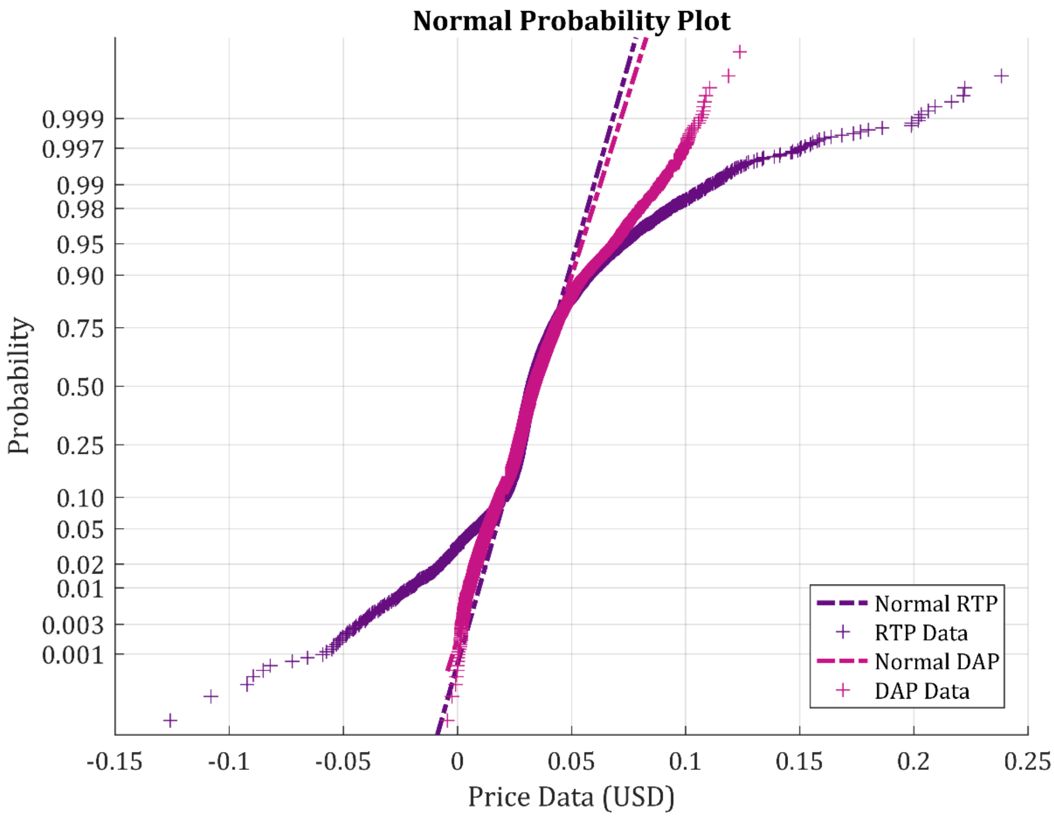


Figure 15 : Normal Probability plot for DAP and RTP

The lower end of the tail, indicates the negative prices which are the cause of negative bids in the wholesale market. These are allowed when the production is surplus (usually from renewable sources) or when the demand is too low, or even a combination of both. This end of the scale is interesting and an opportunity to make the most of, for a stakeholder, in this case the EV owner. However, as it is clear, the prediction of this is difficult and the DAP do not show the negative tail. It is possible that with the large-scale change to smart-grids in the future, the negative tail might come under control. The transparency and communication should help all stakeholders to balance electricity transmission and usage with the aim to not 'waste' any surplus.

4.2 Predictor Methodology

There are many in-depth studies on 'predictors for hourly electricity prices', as discussed in chapter 2 section 2.5. Many of these predictors proposed are already being used by electricity providers themselves. Use of a sophisticated algorithm to generate the DAP is crucial, with multiple underlying factors involved including history and weather predictions. Therefore, the exercise to create this linear regression to make a predictor is to check validity of the DAP provided, and to see if a marginally better prediction can aid the controllers proposed in chapter 6 and 7, in making a control decision of 'when to charge?' Secondly, there is an underlying question to consider: are the DAP provided already tailored in some way by the electricity providers to encourage the users to shift electricity?

Proper multiple linear regression using least square technique is used to create a model for RTP based on the provided DAP. A study has been performed using the standard methodology, to achieve possible predictions on this basis. The value of the predictions is then analyzed. Data for the year 2010 has been used as training data, and the years 2011-2014 have been used for verification.

***RTP** is treated as the **dependent or response variable**, which is dependent on the available factors (available from the data provided by the electricity company): **DAP; day of the week; hour of the day, and month of the year**. These are treated as the **explanatory or predictor variables**.*

The linear model underlying the least square regression analysis is:

$$\boldsymbol{y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip} + \varepsilon_i, \quad i = 1, \dots, n,} \qquad (4)$$

- y_i is the i[th] response
- β_k is the k[th] coefficient, where β_0 is the constant term in the model
- X_{ij} is the i[th] observation on the j[th] predictor variable, *j = 1, …, p*
- ε_i is the i[th] noise term indicating the random error

Using the *least square method* [99], [100], the unknown parameters are estimated by minimizing the sum of the squared deviation between the data and the model. This reduces any over-determined parameters and leaves functional parameters in a system of equations which is finally solved.

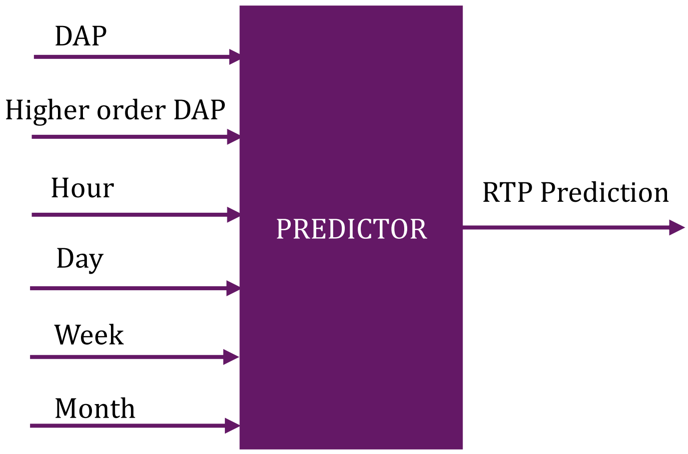


Figure 16: DAP based Linear Predictor for RTP

As is well-known and established, linear least squares can make use of any reasonable size data sets efficiently in-turn providing good results for the purposes of prediction. In this case, the data-sets are not very small; considering that we are concerned about the hourly time-scale, each year's data has price data for every hour of the year. The main concern with using this method of regression is generally the limitation in shapes linear models can take over long range data. Secondly, this method of regression is also sensitive to outliers in the data-set.

Unusual data points can give very misleading results and cause errors in further extrapolation from these points. However, if the model is linear to a high degree, especially in the data region of interest, this method is most reliable. In the case of electricity prices, we mainly need to find the parameters that most affect the RTP.

The following method was used to find and assess a regressive predictor in MATLAB:

1. Data imported and sorted to make sure known factors are available for study (2010-year prices for training the linear-regression model and 2011-2014 for verification of the model)
2. Fitted models are created.
    2.1 Using DAP as the main factor of concern.
    2.2 Using higher order DAP-based models.
    2.3 Other factors added one by one.
    2.4 All factors are added together (*Full Model*).
3. Results of each model are compared and test plots are created to understand predictor effects
4. An attempt is made to find if there are any outliers of concern and to test the quality of the models
5. Ineffective models are eliminated based on their plot or they are simplified by removing factors (coefficients) which did not prove significant based on their p-value.
6. The most-effective model is verified with new data (*verification with 2011-2014 data)*

Important Definitions

- **Null Hypothesis:** It represents a theory put forward which is believed to be true but is not proven. It is the starting point of analysis with no effect or no difference.
- **Alternative Hypothesis:** It is the hypothesis that sample observations are influenced by some non-random cause.
- **P-Value:** The P value is defined as the probability under the assumption of the null hypothesis. The 'P' stands for probability and can take a value between 0 and 1. It measures how likely it is that any observed difference is unlikely to be due to chance. A value closer to 1 suggests no difference between observations other than chance.
- **T-Stat:** The t-stat value is a test between the null hypothesis and alternative hypothesis. Along with the p-value it is used to make inferences about the regression coefficients. A higher t-value usually corresponds with a lower p-value.

4.3 Models

The training data for all models is the *ComEd* pricelist for 2010. The file included DAP and RTP with the dates and times. The tables were adjusted to reflect the other factors which can be derived from the information i.e. day, week, hour and month. The same method was applied to the verification data years (2011-2014). This section shows the final models used to obtain fitted results.

4.3.1 First Order Model

Since DAP is provided as a reliable prediction by the electricity company, it is considered to be the most important factor in the prediction of RTP for the study. Therefore, starting with the simple model, which uses only DAP as the predictor the following model is obtained for the training data.

Figure 17 shows the scatter plot of DAP and RTP. The linear fit line explains the first order model shown below. The scatter shows how similar the DAP and RTP are. The colors in the plot are varying from the first to last point in the data (8760 points). The function of the varying color is to make it easier to read the plot.

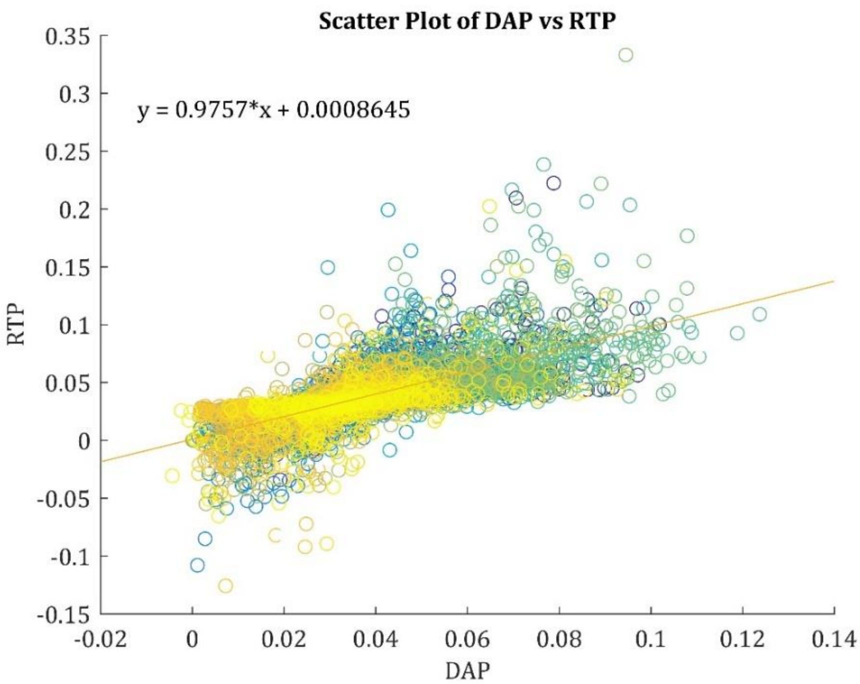


Figure 17: First order DAP Model

Referring to equation (4), Y is RTP and X_1 is DAP,

$$\boldsymbol{RTP = 0.0008 + 0.9757(DAP)}$$

Table 2: Coefficients of the Simple Model (DAP Predictor)

	Estimate	**Std Error**	**tStat**	**pValue**
$\boldsymbol{\beta_0}$	0.00086454	0.0004056	2.1315	0.33075
DAP (X_1)	0.9757	0.010272	94.988	0

4.3.2 Second and Third Order Model

An attempt to make the 'prediction' better, i.e. attempt to make the linear fit better, is made by trying to use a higher order DAP factor model. An additional factor is added to the model by squaring and cubing DAP. This is still a linear coefficient model.

Referring to equation (4), Y is RTP, X_1 is DAP and X_2 is DAP^2

$$\boldsymbol{RTP} = -0.0025774 + 1.1602(\boldsymbol{DAP}) - 2.065(\boldsymbol{DAP^2})$$

Table 3: Coefficients of the second order DAP model

	Estimate	Std Error	tStat	pValue
$\boldsymbol{\beta_0}$	-0.0025774	0.00073537	-3.5048	0.00045918
DAP (X_1)	1.1602	0.034463	33.665	9.46E-234
DAP^2 (X_2)	-2.065	0.36829	-5.6069	2.12E-08

Adding third order DAP as a factor:

Referring to equation (4), Y is RTP, X_1 is DAP, X_2 is DAP^2 and X_2 is DAP^3

$$\boldsymbol{RTP} = -0.00094789 + 1.0116(\boldsymbol{DAP}) + 1.6321\left(\boldsymbol{DAP^2}\right) - 25.624\left(\boldsymbol{DAP^3}\right)$$

Table 4: Coefficients of the third order DAP Model

	Estimate	SE	tStat	pValue
$\boldsymbol{\beta_0}$	-0.00094789	0.0010821	-0.87597	3.81E-01
DAP (X_1)	1.0116	0.080197	12.613	3.67E-36
DAP^2 (X_2)	1.6321	1.8386	0.88767	0.37474
DAP^3 (X_3)	-25.624	12.485	-2.0524	0.040165

4.3.3 Full Model

The *full* model includes all the available factors (hour, weekday, month, DAP), including the interdependencies between them. This makes it a linear formula with 11 terms and 4 predictors.

$$\begin{aligned}\boldsymbol{RTP} = \boldsymbol{\beta_0} + \boldsymbol{\beta_1}(DAP) + \boldsymbol{\beta_2}(HOUR) + \boldsymbol{\beta_3}(WEEKDAY) + \boldsymbol{\beta_4}(MONTH) + \boldsymbol{\beta_5}\,(HOUR \times DAP)\\ + \boldsymbol{\beta_6}(HOUR \times WEEKDAY) + \boldsymbol{\beta_7}(HOUR \times MONTH) + \boldsymbol{\beta_8}(DAP \times WEEKDAY)\\ + \boldsymbol{\beta_9}(DAP \times MONTH) + \boldsymbol{\beta_{10}}(DAP \times HOUR) + \boldsymbol{\beta_{11}}(MONTH \times WEEKDAY)\end{aligned}$$

The ***RMS Error=0.0149***, which at first glance confirms the dependence of the output on these factors in addition to DAP.

Initially, models with one factor added at a time were explored, however the improvement or change was so negligible compared to using all the factors together that these models were discarded as insignificant. This may be due to the fact that each of these factors represents various factors. For example, 'HOUR' is actually 24 different hours.

Running the simplification method within MATLAB, and analyzing results and plots, no significant factor reduction is obtained. For example, MATLAB removes *HOUR×WEEKDAY* which has a *p-value=0.40522* (average of all *hour x weekday* combinations*)* which is not very small (only <0.5). Moreover, there is no change in RMS error.

4.4 Analysis

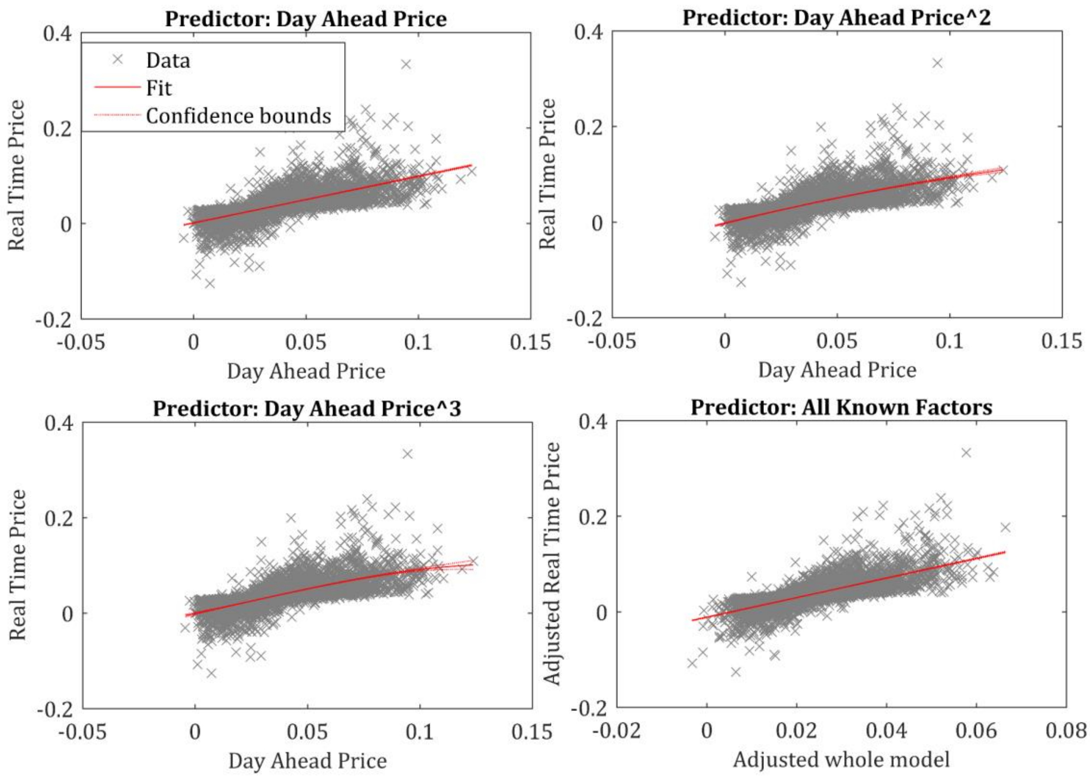


Figure 18: Plot of Linear Models (from top left: DAP, DAP2, DAP3 and Full Model)

The results show that DAP is very closely related to RTP in this set of data. As such, DAP is a good prediction of the RTP except in cases of 0.05 cents or less, or in case of very large price spikes. Figure 18, shows the fit of each model as a whole and it is clear to see that, the bounds don't come too close to containing a horizontal line. For the most part, the fit is linear with good confidence bounds which conform to the linearity. There is very little difference in between first order and higher order models and we can see the confidence bounds start to diverge post 0.12 when it comes to the third order. The full model with all factors on the other hand shows a good fit like the basic DAP model but since it contains DAP as a factor, the effect of DAP is not at all diminished.

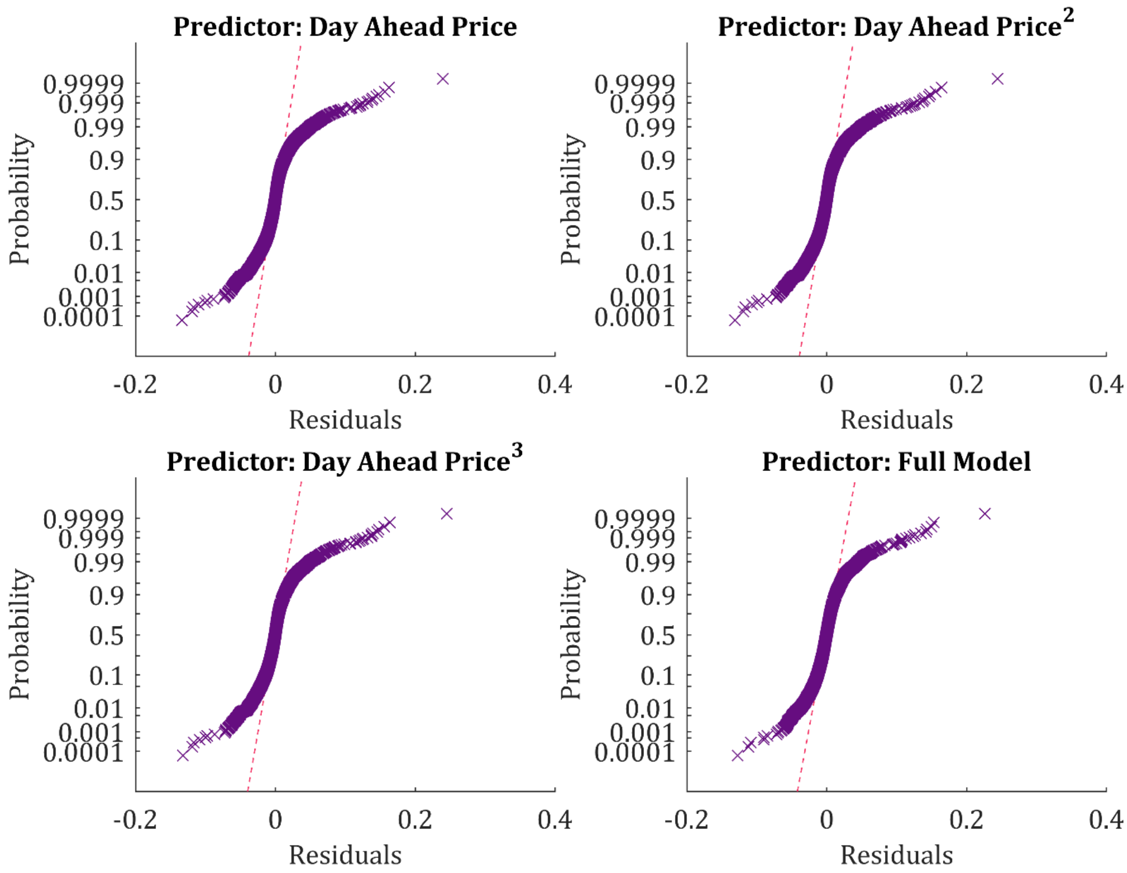


Figure 19: Probability Plot of Residuals (Cumulative Histogram)

Some outliers and deviations are expected since the *electricity price is non-Gaussian in nature* (Figure 19). It exhibits a rather long tail at both, very high and very low probability bounds. This can make relying entirely on the linear regression data analysis, quite difficult. It shows high similarity to figure 4, the probability plot for the real RTP data.

In Figure 19, the probability plot of the residuals shows a good straight plot for the region of interest, the region where most of the average price data lies. On each end of the data, there is an expected long tail which makes prices less predictable and more 'random', indicating the stochastic nature of the electricity prices. Additionally, this plot helps find outliers and there seems to be only one outlier in approximately 8760 data points in each year.

On a shorter time-scale i.e. hourly, and the one of concern in this project, electricity use is logically and statistically higher during office/work hours including usage by industry, compared to that at night. The market prices are directly affected by this trend as discussed previously in the literature. Naturally, the addition of these factors in the DAP only model becomes a significant step forward and we expect to see a reduction in the prediction (RMS) error of the model.

While all the different coefficient values obtained are helpful in making a decision about the predictor's usefulness, the most important observations for our purpose are the *intercept, coefficient* of the DAP factor and the *root mean squared (RMS) error estimate.* Clearly, the RTP prediction is almost closely based on the DAP with a small error.

4.5 Discussion of Results

The reduction in the RMS error between the normal and higher order models is negligible, although as expected, it is slightly lower for the higher order models. The coefficients of the model now show a negative proportionality with the higher order DAP factor. Figure 20 shows the RMS error of the different models compared to that of the original training data. The first order DAP model shows no improvement. The second and third order models both improve by ***0.65 %***, and the full model shows a ***6.5 %*** improvement.

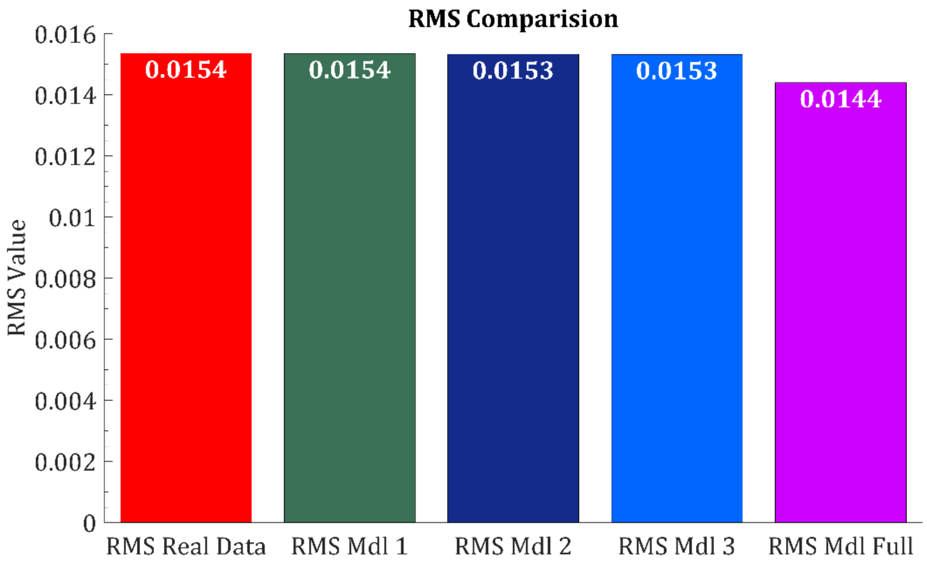


Figure 20: RMS errors of models for training data (2010)

There seems to be a significant improvement in the model when multiple factors are added, at least based on the RMS error. Although it would not be correct to conclude that all factors are fully important (because their interactions can be redundant at many values and their coefficients are highly negative in some cases), it would be wrong to assume they are unimportant because the effect of hours, days and months is significant to weather and other user/consumer related interactions which do affect electricity prices. However, it can be assumed with some certainty that the electricity companies already use these factors in their models to provide the DAP prediction. Secondly, using higher order DAP models only marginally improves on the RMS error, with second and third order showing the same.

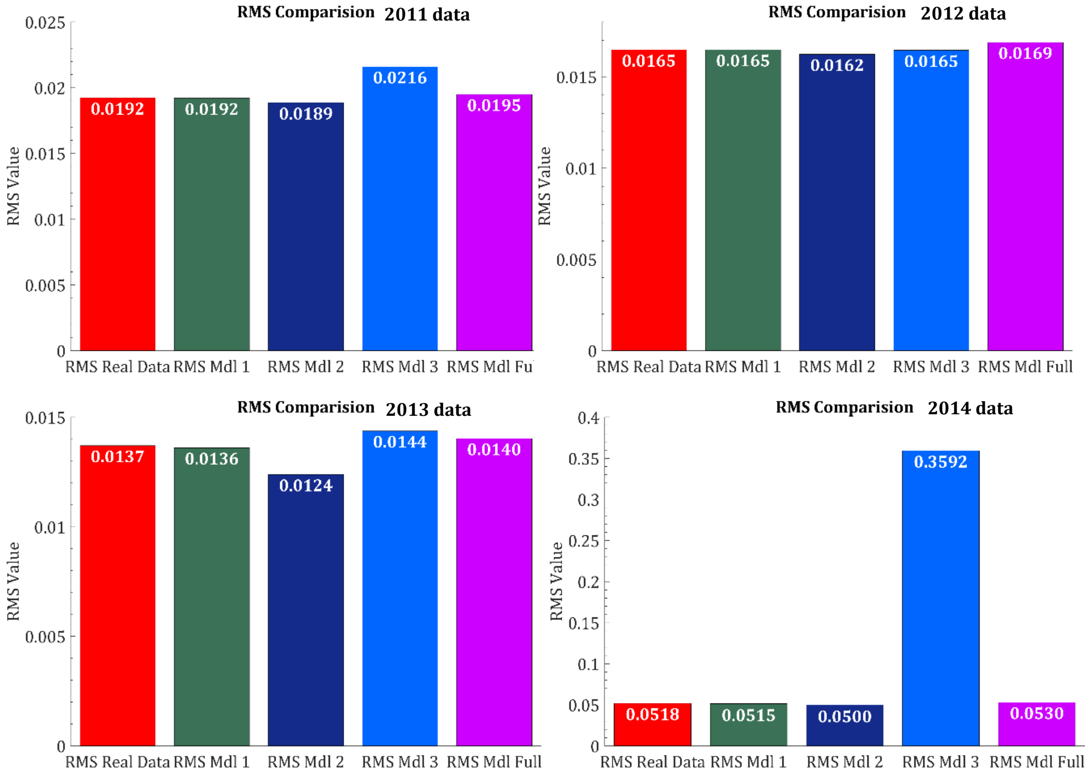


Figure 21: RMS error comparison for years 2011 (Top-Left), 2012 (Top-Right), 2013(Bottom-Left), 2014 (Bottom Right)

Figure 21 shows the results from the verification data from years 2011-2014. The data for 2014 was incomplete, so it should be disregarded. For the other years, the common trend is a small decrease of the RMS error with the first order and second order DAP, which means that the predictor is working as expected. However, starting with the third order models the error for the validation data set is increasing, which indicates overfitting to the training data. This also applies to the model with all factors included. Therefore, it is not recommended to rely on the third order or the full model for prediction.

4.6 Model Verification

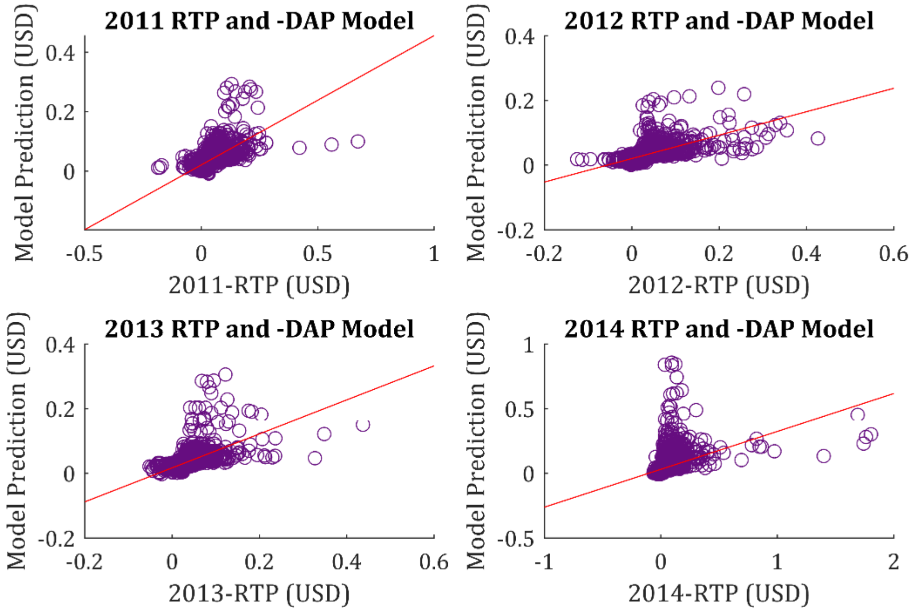


Figure 22: DAP Model verification

Figure 22 and Figure 23 show the verification of the linear regression predictors. Figure 22 shows the fit of the DAP only model and Figure 23 shows the verification of the full model. There is negligible difference between the fits but as the RMS error comparison has shown that there is chance of overfitting in the full model. These plots also clearly show the skewness in the 2014 data.

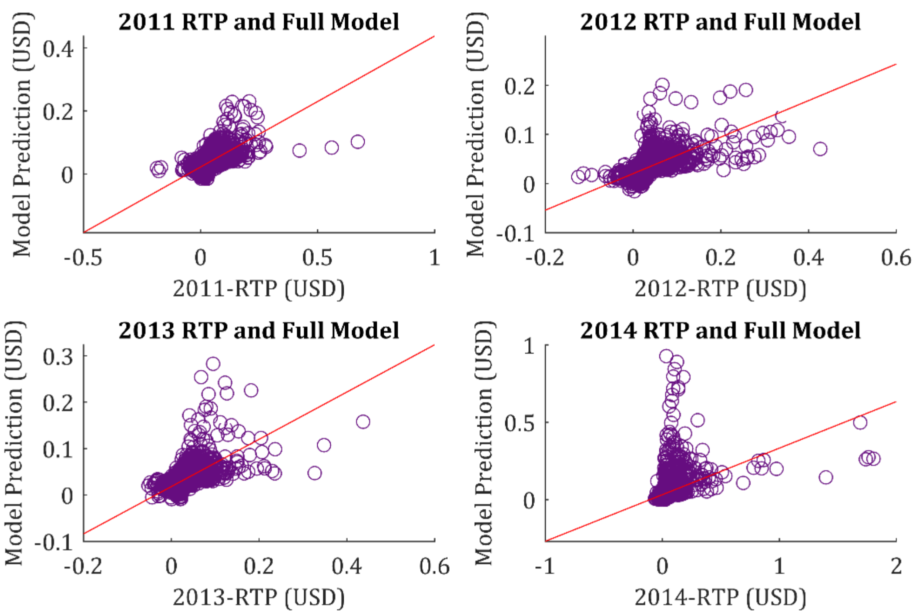


Figure 23: Full Model Verification

4.7 Conclusion

It is clear when all the data results are analyzed, that it is possible to use DAP as an appropriate prediction for the RTP. Using linear regression, it may be possible to gain a very slight improvement using the first or second order predictor. The simple linear regression predictors, based on known factors from day ahead price data, show that DAP is already a good estimation of RTP. Some of the predictors show slight improvements in prediction, but there is also evidence of overfitting for the more complex predictors, resulting in worse performance on the verification data.

5 Deterministic Controller

This chapter presents a deterministic charging controller. Firstly, the basic problem and its solution are introduced in the mathematical form. The solution follows a deterministic approach reflecting the symmetry assumed in the basic problem. A simulation is performed with arbitrary prices to prove the effectiveness of the solution. Lastly, variations to the problem are discussed and an application example is presented to put the findings into perspective.

5.1 Basic Problem

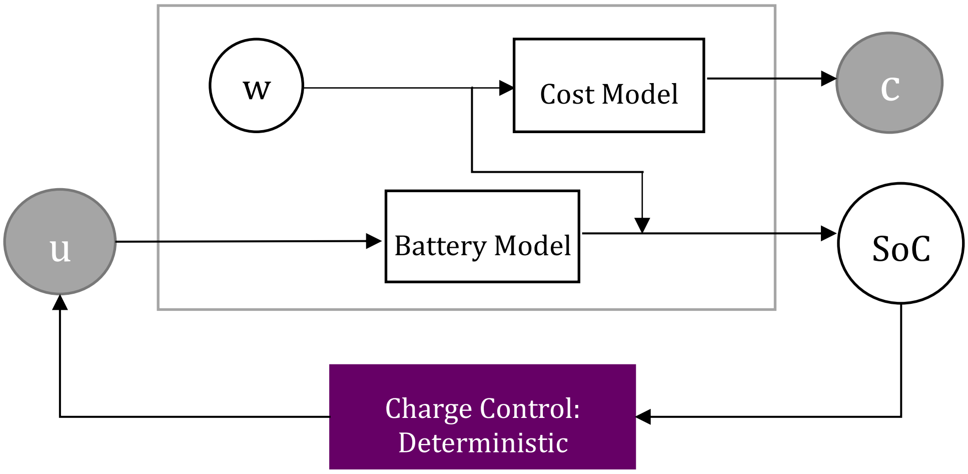


Figure 24: Block diagram of the controller for a known price disturbance

List of Notations

$\boldsymbol{x_1}$: Battery SoC (state)
$\boldsymbol{w}$: Known cost prediction (DAP)
$\boldsymbol{w'}$: Threshold Price
$\boldsymbol{u}$: Input (charge power)
$\boldsymbol{u'}$: Threshold charge power
$\boldsymbol{u_{max}}$: Maximum available Charge Power
$\boldsymbol{d}$: Random disturbance (stochastic variable)
$\boldsymbol{c}$: Cost
$\boldsymbol{c_0}$: Initial Cost
$\boldsymbol{\alpha}$: Decay constant (grid price decay)
$\boldsymbol{k}$: Current Time-Step
$\boldsymbol{N}$: Final Time-Step
$\boldsymbol{E}$: Expected Cost
$\boldsymbol{x_{1_0}}$: Initial SoC
$\boldsymbol{x_{1_N}}$: Required SoC
$\boldsymbol{\beta}$: Disturbance Scaling Factor
$\boldsymbol{T}$: Time Period (assumed 1 unit)
$\boldsymbol{T_s}$: Step-size

Figure 24 shows the block diagram of the controller. It is based on Figure 11 (chapter 3), but with the external price (unknown) disturbance *(d)* removed. The grid model is based simply on the known prices *(w)* (day ahead prediction), which are assumed accurate.

The basic optimal charging problem is defined in discrete time with step size T_s. It has one control variable: the charging power u. The power is subject to two constraints: it cannot be negative and there is a constant maximum power u_{max} such that $u \in [0, u_{max}]$.

The behaviour of the system is determined by two separate dynamics: the battery state (x_1) and the total cost (c). Both accumulate (integrate) over time and the only difference is the coefficient.

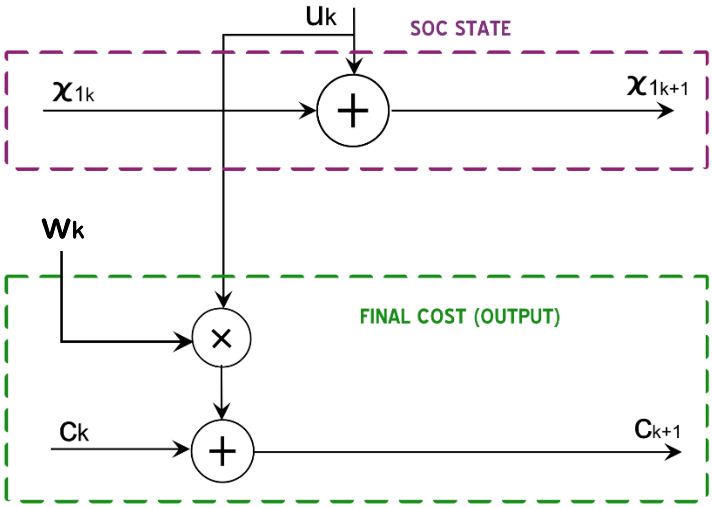


Figure 25: Plant Model of the Optimisation Problem (Deterministic Approach)

The diagrammatic representation of the problem is shown in Figure 25.

The battery state x_1 is an integral of the charge power over time and assuming that the self-discharge and charging losses are negligible, it can be written as:

$$x_{1_{k+1}} = x_{1_k} + Tu_k \tag{5}$$

The time period T is assumed as 1 unit, for the ease of explanation. Therefore, equation (5) becomes:

$$x_{1_{k+1}} = x_{1_k} + u_k \tag{6}$$

The cost c_k is also an integral of the charge power but weighted by the known cost disturbance (w). In the deterministic approach, we assume the price is known and we can rely on the prediction. The cost function is written as follows and the initial condition is that the cost is zero.

$$c_{k+1} = c_k + w_k u_k \tag{7}$$

$$c_0 = 0 \tag{8}$$

The total number of steps N to consider with $k = 0 \ldots N$ are also defined in advance.

The basic optimal charging problem is defined by the cost function $J = c_N$ representing the total electricity cost and the boundary condition $x_{1\,N} = x_{1\,full}$, which requires the battery to be fully charged at the end of the charging process.

Because no discharge is allowed, it is not necessary to impose limits on the charge state. The advantage of using this simple model is that the final state and cost can easily be calculated as:

$$x_{1_N} = x_{1_0} + \sum_{k=0}^{k=N-1} u \tag{9}$$

$$c_N = \sum_{k=0}^{k=N-1} w \times u \tag{10}$$

5.2 Solution to the Basic Problem

Control Law for Optimal Charging

The solution to basic optimal charging problem is:

$$u_k = \begin{cases} 0, w_k > w_k' \\ u_{max}, w_k \le w_k' \end{cases} \tag{11}$$

The threshold price w_k' is one of the prices w_k and the threshold charging power u' can be found from the boundary condition using a linear equation. There may be more than one solution if several time steps have the same price $w_k = w_k' = w'$ and for now it is assumed that is not the case.

Algorithm for Optimal Charging

1. Determine the required charge as intervals (n)
2. Sort the electricity prices in ascending order
3. Set the price, in the ascending ordered list at index *1 + n* (rounded up), as the threshold price *(w_k')*
4. Set the threshold power *(u')* by subtracting maximum available power over *n,* from required charge power for desired SoC

The method involves picking the lowest from a known set of prices for the given charging period. Once, the time required to charge the vehicle battery is known (intervals n), the *'n'* lowest costs can be picked. Charging should begin at the highest (n^{th}) cost hour and continue at the hours with the other costs in this set.

Figure 26 further clarifies the process. For an example where $n = 4$: the top plot shows the known price prediction for a given charging period between 17:00 and 07:00, and the bottom plot shows the process of selection. The prices are arranged in ascending order and the threshold price is set at $0.025, which is at 03:00. At the hours (intervals) where the price is lower, the vehicle is charged (in this case, 03:00-07:00).

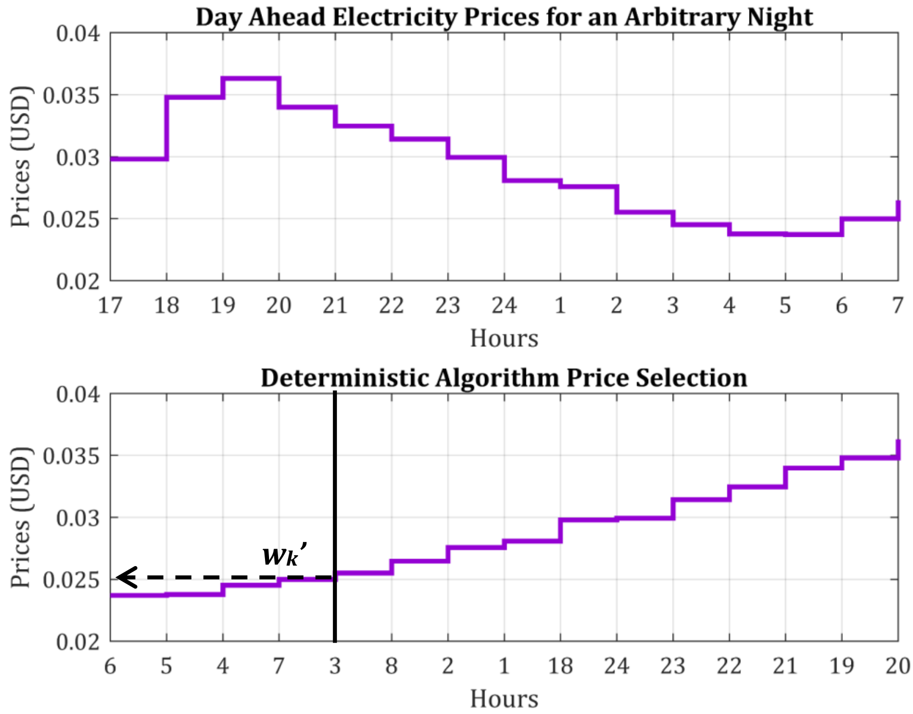


Figure 26: Visual Representation of Price Selection

Theorem for Optimal Charging

The given control law with the parameters set by the Optimal Charging Algorithm leads to the lowest cost charging process.

Proof for Optimal Charging

The proof has two parts. The first step is to demonstrate that the presented from is an admissible solution to the problem and the second is to show that it is indeed the only optimal solution when the assumption that DAP are accurate is made.

With the given control law, the final charge state x_{1_N} is a function the initial state x_{1_0}, the threshold charging power u' and the number of full charging cycles n that satisfy $w_k < w_k$'. As long as $x_{1_N} \geq x_{1_0}$ and $x_{1_N} \leq x_{1_0} + T_N u_{max}$, the problem has a solution.

Assuming that the prices are different at each time step, there is exactly one solution, which is given by the following two equations when $T = 1$:

$$n = \text{floor}\ \frac{x_{1_N} - x_{1_0}}{u_{max}}$$
$$u' = x_{1_N} - x_{1_0} - nu_{max} \qquad (12)$$

This solution is not just admissible but also optimal for the assumption that DAP are accurate because any deviation from this solution within the charging power limits leads to a higher cost. In order to maintain the same final charge state x_{1_N}, an alternate solution needs the charging power to be decreased at some time step $u_i = u_i^* - \Delta u$, and increased it at another $u_j = u_j^* + \Delta u$. This maintains the same integral and therefore satisfies the boundary condition. But increases are only admissible when $u_j < u_{max}$, and decreases only when $u_i > 0$. It follows that $w_i < w' < w_j$, and therefore the net effect is an increase of charging cost by $\Delta J = \left(x_{2_j} - x_{2_i}\right)\Delta u$.

5.3 Results

5.3.1 Assumptions

The example assumes a typical electric vehicle that is being used for a regular commute to work during the week and for reduced driving during the weekend.

The car is driven to work at 07:00 and driven back home at 17:00. Charging is possible at home between 17:00 and 07:00 using a smart meter. The electricity is provided by Ameren, and two tariffs are considered: the day-ahead tariff, where prices are set at 5 pm for the following day and real time pricing. The prices for Ameren Illinois Zone have been taken from the Ameren web site [101], for the period from September 1st 2011 to September 1st 2012. These prices exclude the distribution cost, which is constant and therefore not relevant for comparison purposes.

In addition, the following assumptions (Table 5) are used for the simulation. No specific vehicle is used as a reference, since electric vehicles are still at a very early stage. The GM Volt and the Nissan Leaf for example both have a smaller battery than assumed here (The Nissan Leaf has a battery with nominal 24 kWh capacity but not all of that is actually usable.) Instead these figures are based on a slightly longer than average commute of about 35 miles one way, where the savings of an electric vehicle should be more pronounced than on a shorter commute.

Table 5: Assumed Constants

Constant	Symbol	Value	Unit
Usable Battery Capacity	E_{Total}	24	kWh
Weekday Consumption	E_{WD}	16	kWh
Weekend Consumption	E_{WE}	8	kWh
Charging Power	P_{Slow}	2	kW
Charging Efficiency	η	90	%
Charging Period	T_{Charge}	14	h

5.3.2 Simulation Result

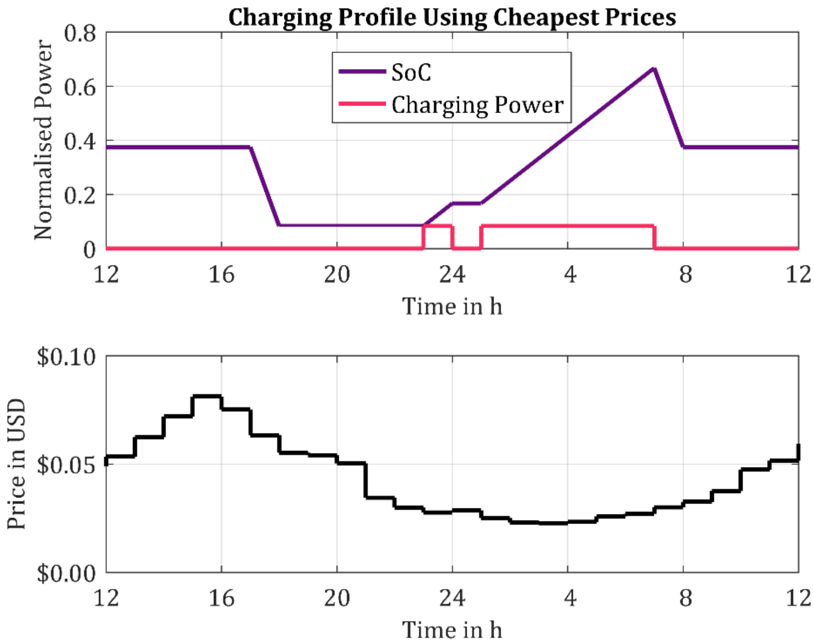


Figure 27: Simulation Results

The charging strategy is based on the basic optimal charging problem. The day-ahead prices are used as an indication for the electricity prices during the charging period, and the cheapest prices are used to charge the battery. In the simulation, the controller is given an arbitrary set of prices from one night in the year. The effect of this can be seen in Figure 27 – charging takes place during the hours of the night when the electricity has its lowest price. The controller fulfils its aim perfectly, assuming the DAP data it has for the given time in the night, it waits once the car is plugged in for a low price. It delays the charge from 17:00 till 23:00 when it calculates the time required for a full charge against the lowest average price. It then stops the charging at 24:00, when the price goes above threshold. It restarts charging at 01:00.

5.4 Application Example Results

The car uses a total of 4928 kW during the simulated year and with the assumed 90% efficiency this means 5476 kW of electricity is used from the grid. With a traditional tariff at an average electricity price of 2.73 cent per kWh, this would cost 149.4 USD.

5.4.1 Charging Strategies

Several different strategies and assumptions are tested with the price data for 2010 obtained from Amaren, to test the effect of the controller on electricity costs for charging. For comparison purposes, two dumb strategies are considered first: charging as soon as the car is plugged in ("fixed early") and charging as late as possible while still filling the battery before setting off ("fixed late").

The optimal strategy[5] implements the selection of the cheapest tariffs while still filling the battery before setting off. For this purpose, a price prediction horizon of 24h is used, of which only the 12h covering the charging period are relevant.

As a variation, a second strategy looks ahead to the next night and decides whether it is cheaper to charge the battery fully or to fill the battery only as much as required for the daily commute, followed by an expected complete charge during the following night. Ideally this requires a prediction horizon covering two nights (48h), but this is not actually feasible because the prediction only extends to the end of the next day (indicated by a star in the graph). Therefore, a realistic horizon of 31h is also added to the comparison.

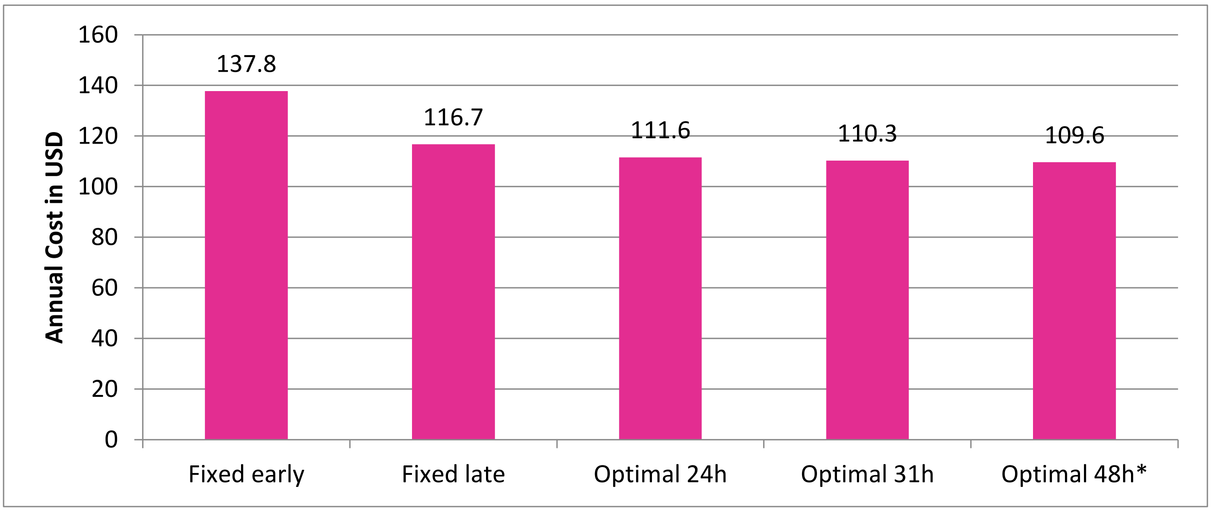


Figure 28: Comparison of Charging Strategies using DAP

[5] In the deterministic approach, the optimal strategy refers to the best possible solution of the problem within the bounds of the assumption that DAP are perfect i.e. the Deterministic Controller.

As can be seen in Figure 28, the annual electricity cost of charging an electric vehicle is highest when it is charged as soon as the owner gets home. Charging late in the morning (just in time) is ***15.31%*** more cost-effective. Using an optimal charging strategy further reduces the cost by ***4.37%***, and the benefit is increased by ***1.17-1.79%*** when an extended prediction horizon is used. The maximum cost-saving that can be achieved is ***20.46%***.

5.4.2 Real Time Pricing

As has been shown in the data analysis chapter (chapter 4), the DAP tariff is not ideal for charging purposes. The RTP tariff offers much greater variations of prices, and thus better opportunities for load shifting and for saving costs. The difficulty is of course that the RTP prices are not known in advance, and therefore the optimal algorithm is not applicable. But there are a number of way that the algorithm can be adapted to extrapolate from the DAP prices to the ideal time for charging on the RTP tariff.

To show the significance of more accurate prices than DAP, i.e. the ideal situation where we would know the RTP, a few comparisons are made. This effectively shows why it is necessary to build on the deterministic controller explained in this chapter and resolve the stochastic element which may bring us closer to predicting the price. Accounting for price changes could lead to significantly more savings and, by implication, more accurate load shifting.

The first approximation is using the day-ahead price information to schedule the charging of the electric vehicle but in fact real time prices are used to calculate the cost ("RT Rate"). This can be achieved with minimal effort by changing electricity tariffs. In this case, the day-ahead price becomes effectively a disturbance model for the real time price development.

The second approximation uses the threshold cost x_2' as calculated using the day-ahead price information, but it compares it to the real time price of electricity to decide whether charging takes place or not ("RT Trigger"). Again, this is simple to implement, although special care needs to be taken to ensure that the car always has sufficient charge at the beginning of the commute. The simulation does this by starting to charge irrespective of price if this is necessary to reach sufficient charge.

The final approximation assumes complete knowledge of real time prices ahead of time – otherwise it is identical to the day-ahead optimization ("RT Optimal"). Obviously, this is only possible to simulate in retrospect and it is not implementable because it uses knowledge of future events. But the simulation provides an upper limit for the potential savings possible using a perfect price prediction model. It is worth noting that even using an optimal model, the savings may be significantly less than this upper limit.

It can be seen in Figure 29 that these algorithms provide significant reductions in cost. The more sophisticated the algorithm is, the bigger the savings. The effect of real time prices is distinctly more pronounced than the effect of different prediction horizons discussed before.

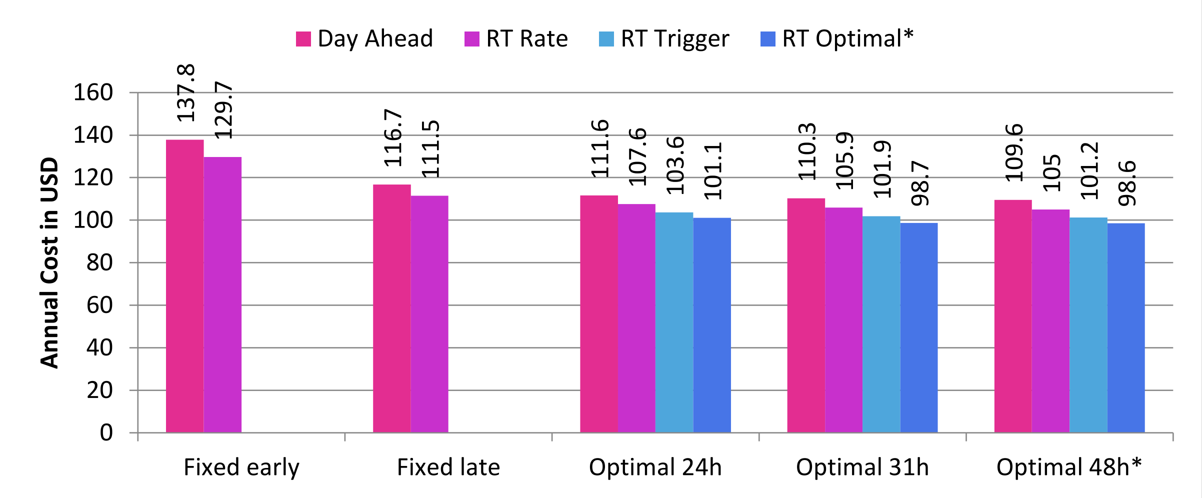


Figure 29: Comparison of Charging Strategies Assuming RTP is KNOWN

5.4.3 Fast Charging

All previous simulations are performed with a moderate charging power of 2kW, which is approximately the amount of power that can be provided by a standard electricity outlet. If a smart charger is used, it is reasonable to assume that it will be a dedicated fast charging unit, which can provide higher power levels. A higher charging power means a shorter charging duration and therefore load shifting is expected to become more effective.

To study this effect, charging powers of 2kW, 4kW and 8kW are simulated using the charging strategies and pricing schemes introduced above. The same charging efficiency is assumed for all charging powers, which may not be quite realistic depending on battery technology. When the infrastructure is advanced enough to support fast charging, the realistic powers could be higher than 10kW; it is important to remember that higher powers means a trade-off between charging efficiency and price optimization.

The result shown in Figure 30 paints an interesting picture. Firstly, it is worth noting that fast charging is more expensive if a bad fixed time charging strategy is being used. This is because the cheapest prices are typically found during the middle of the night and neither the early nor the late charging times make use of them. Charging once the vehicle is home can get quite expensive. Setting an early morning time for the start of charge, for example 3 am, provides much better results, leading to a cost of approximately 100 USD.

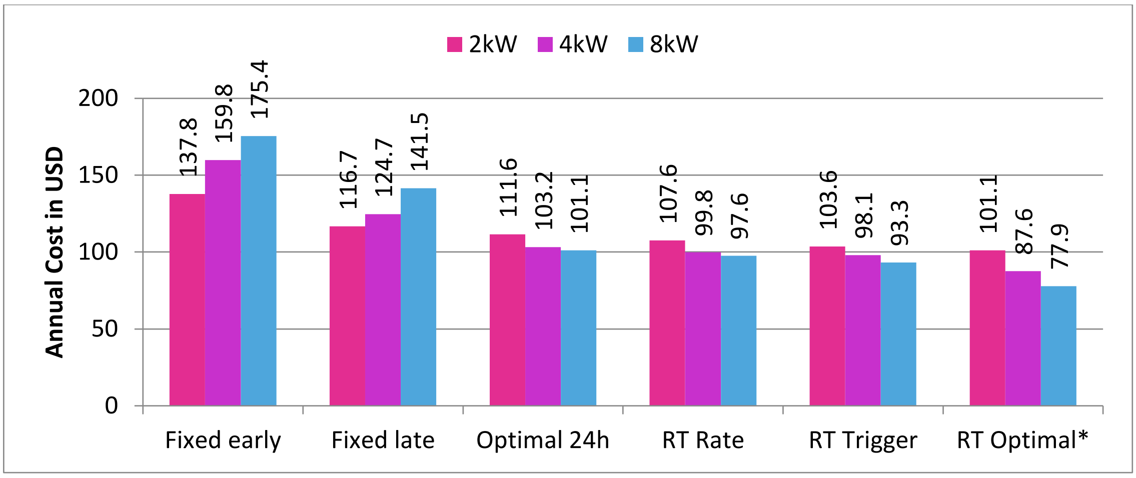


Figure 30: Impact of Using Fast Charging

The next interesting conclusion is that fast charging does indeed provide better load shifting and a further reduction in electricity costs. The benefit depends on the charging strategy but it is in the order of 10 USD or more and it certainly is higher than the potential loss of efficiency due to the faster charging. Whether it is also high enough to compensate for the increased wear of the battery and the investment cost of the fast charger remains very much doubtful.

Finally, the difference between the best feasible charging strategy ("RT Trigger") and the retrospect theoretical optimum ("RT Optimal") increases significantly with higher charging powers. It is only about 2.5 USD at 2 kW but it increases to over 15 USD at 8 kW. This means that fast chargers create significant demand for better real time price prediction strategies.

Of course, fast chargers may also provide an opportunity to perform load shifting on the faster time scales mentioned above. For example, they can help to absorb harmonics and noise to improve local power quality, or they could apply droop control to improve frequency stability of the electricity grid. But so far there is no business case for these measures and in fact it would cost the consumer both in terms of investment and in terms of loss of cheap electricity.

Assuming that electricity prices are known in advance, a simple solution to this problem can be found and implemented. According to numerical simulations using real price information from Illinois, this leads to very moderate savings in the order of about 30 USD a year compared to immediate charging, and 10 USD compared to fixed time charging.

Larger savings can only be achieved by combining two measures: using a fast charger and changing to a real-time price tariff, where electricity prices are not known in advance. The problem of identifying the cheapest charging times becomes much more complicated in this case, because it depends on the prediction of future prices, which is not reliable. Using the same simple approach informed by day-ahead price information, a reasonable solution can be found that saves another 20 USD. The potential for further savings in the order of 15 USD exists but it would rely on an operation model for predicting future electricity prices. It is therefore worth progressing this approach by exploring the dynamic problem of 'when to charge?' by respecting the stochastic element of the grid prices.

5.5 Variations to the Basic Problem and their Solutions

The basic problem uses a highly abstracted model of the battery, the grid, and the charging cost. It does not take into account anything but the basic constraints of power and the requirement of minimising the cost, hence the grid and battery model just worry about these. There are several extensions that can be made to make this problem more applicable. However, since the main assumption for the prices being known i.e. the DAP being a perfect prediction remains, it is possible to reduce these variations to the basic problem and solve it in a similar way.

Varying Charge Power Limit

The amount of available charging power u_{max} may change over time, for example due to electricity use restrictions at peak load periods. This means a new time series $\overline{u}_k$ has to be introduced. The closed formulation of the optimal solution is no longer applicable but a simple iterative algorithm can still find the best solution.

The proof applies appropriately.

Algorithm

1. Determine the required energy $x_{1_N} - x_{1_0}$
2. Sort the costs w_k
3. Iterate starting from the lowest cost: add up the energy per time step $w_k\overline{u}_k$ until the required energy is exceeded.
4. Reduce the power u' for the last time step as required.

Self-Discharge and Losses

Every battery has losses which are associated with both charge and discharging. It can be assumed that the efficiency of the battery pack therefore will be less than 100%, and that the battery loses a certain part of its charge at every time step. This modifies the basic battery model:

$$x_{1_{k+1}} = \delta x_{1_k} - \mu + \varphi u_k \tag{13}$$

Where 1-δ is the relative discharge coefficient for each time-step, μ is the absolute discharge energy per time step and φ is the charging efficiency. This leads to the following battery SoC:

$$x_{1_N} = \delta^N x_{1_0} - \mu \sum_{0\ldots N-1} \delta^k + \varphi \sum_{0\ldots N-1} \delta^{N-k-1} u_k \tag{14}$$

Resistive losses within the battery (and the electricity supply) can be the dominating factor for charging losses. These resistive losses are proportional to the square of the current, assuming a constant voltage:

$$u_{in} = u_{out} + R u_{out}^2 \tag{15}$$

where R is the resistance normalised for the charging power. In terms of the optimization problem, the losses can be included either in the power going into the battery or in the cost of the electricity depending on whether u_{in} or u_{out} is the wanted variable. The latter produces an easier problem definition:

$$c_{k+1} = c_k + w_k\left(u_k + R u_k^2\right) \tag{16}$$

These resistive losses in effect will tend have an influence in the decision to charge because practically the battery will only receive a part of the charging power. Secondly, losses are proportional to charging power, therefore they will be higher at higher charging powers (fast charging).

5.6 Conclusions

The deterministic approach provides a sub-optimal solution to the optimisation problem for the cost-effective charging of electric vehicle. Assuming a day-ahead tariff, electricity prices are known in advance and a simple solution to this problem can be found and implemented. According to numerical simulations using real price information from Illinois, this leads to very moderate savings in the order of about 30 USD a year compared to immediate charging, and 10 USD compared to fixed time charging.

Larger savings can only be achieved by combining two measures: using a fast charger and changing to a real-time price tariff, where electricity prices are not known in advance. The problem of identifying the cheapest charging times becomes much more complicated in this case, because it depends on the prediction of future prices, which is not reliable. Using the same simple approach informed by day-ahead price information, a reasonable solution can be found that saves another 20 USD. The potential for further savings in the order of 15 USD exists but it would rely on an operation model for predicting future electricity prices.

Clearly, the deterministic solution assumes the problem is symmetric and achieves a sub-optimal solution because of its main assumption, that the prices are known in advance or that the DAP are considered as 100% reliable. The real price is a known unknown because we know the RTP will indicate the correct price but it is not available until the hour the decision needs to be made. The disturbance responsible for changing the RTP compared to DAP is random and therefore stochastic which adds a dynamic element to the minimisation problem. To achieve optimality, the RTP must be considered as the price which actually matters and the stochastic element of the prices must be accounted for. The problem becomes complex since the varying nature of the grid cost will affect the key decision the controller has to make. Chapter 6 explains the stochastic problem and approach to the solution in depth.

6 Stochastic Dynamic Programming (SDPM) Controller

The optimal charging controller in the previous chapter was simple and successful because it assumed completely knowledge of future electricity prices. However, in reality, the best price for electricity in only available on the real time (RT) tariff, which is not known in advance. This uncertainly of available prices makes the problem much more complex to solve. It is necessary to introduce a stochastic model for future prices to define an optimisation problem.

This chapter approaches the problem using stochastic optimisation, specifically stochastic dynamic programming (SDPM), which results in a controller that achieves on average the lowest optimal charging price. Firstly, the problem is explained and a grid cost model and possible approaches are discussed. Secondly, the choice of dynamic programming with a stochastic element is justified and a solution is shown with the algorithms used. Finally, a small simulation study is used to illustrate the key features of the optimisation and the resulting controller.

6.1 Problem

6.1.1 Dynamic Problem and Random Disturbance

The charging problem has been solved using a deterministic approach in Chapter 5, but this approach fails to appreciate the stochastic nature of electricity prices on the real time tariff. When considering the random effects on prices, the varying nature of the grid-cost is important to the decision made by the controller. It has been demonstrated in Chapter 4 that these variations are correlated over time: a higher price at one point is likely to lead to higher prices in the near future.

To keep the model simple, a first order dynamic stochastic process (or Markov process) is used to model the electricity price. Therefore, this problem turns into a 2-state dynamic problem (state of charge and electricity price) with a stochastic disturbance *d.*

By definition a dynamic programming problem is one which has both inputs and outputs which change dynamically with time [102]. They are connected via both the system model and the control law. The dynamic programming approach also fits this problem well because it provides a systematic way to establish a control law for the decision of 'when to charge', that takes both states (battery state of charge and grid cost) into account. The time varying grid cost (and potential unpredictable disturbance) make the decision requirement dynamic. Dynamic programming allows to balance the trade-off between current cost and future cost (discussed in the *Problem Statement*, chapter 3) by identifying the value of charge and the periods of most cost-effective charging. The aim is of course to minimize the resulting final total electricity cost over the time horizon.

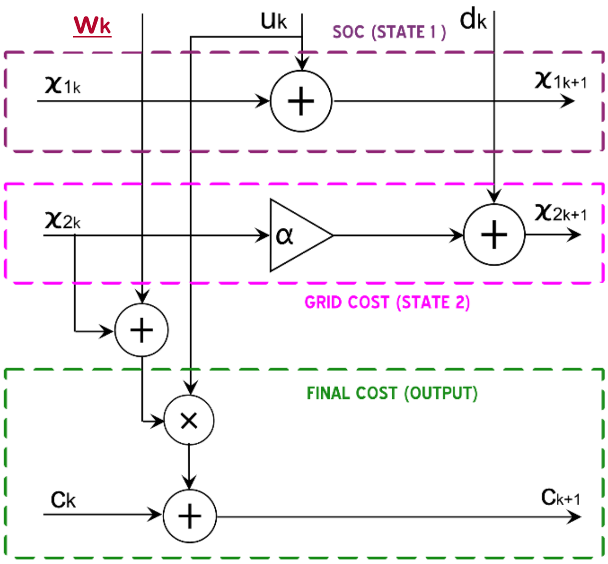


Figure 31: Plant Model of the Problem

List of Notations

$\boldsymbol{x_1}$: Battery SoC (state)
$\boldsymbol{x_2}$: Grid cost (state)
$\boldsymbol{w}$: Known cost prediction (DAP)
$\boldsymbol{u}$: Input (charge power)
$\boldsymbol{d}$: Random disturbance (stochastic variable)
$\boldsymbol{c}$: Cost at each time step
$\boldsymbol{\alpha}$: Decay constant (grid price decay)
$\boldsymbol{k}$: Current Time-Step
$\boldsymbol{N}$: Final Time-Step
$\boldsymbol{x}$: All states
$\boldsymbol{E}$: Expected Cost
$\boldsymbol{x_{1_0}}$: Initial SoC
$\boldsymbol{x_{1_N}}$: Required SoC
$\boldsymbol{u_{max}}$: Maximum available Charge Power
$\boldsymbol{\beta}$: Disturbance Scaling Factor
$\boldsymbol{T}$: Time Period (assumed 1 unit)

Figure 31 shows the plant model of the discrete-time system model. The dynamics of state x_1 (battery state of charge) are simple: it just integrates the charge over time. The grid model x_2 on the other hand is a bit more complex, since it consists of a first order decay of the deviation and a stochastic change of price d_k.

$$x_{1_{k+1}} = x_{1_k} + u_k \tag{17}$$

$$x_{2_{k+1}} = \alpha x_{2_k} + d_k \tag{18}$$

It is possible to separate the model into two distinct parts: a deterministic part that models the known dynamics, and a stochastic part that adds the random element in the form of the disturbance d_k. The deterministic or basic problem is symmetric and essentially independent of time, as discussed in the previous chapter.

The sum of the cost of the electricity used at each time step leads to a final cost, say $J_{N,}$ where N is the final time-step. The final cost (which we want to minimize) is the output of the problem. The cost at each time-step can be calculated as follows:

$$c_{k+1} = c_k + u_k(x_{2_k} + w_k) \tag{19}$$

The solution is subject to constraints on the state of charge according to the needs of the user. These are the given or known variables: the State of Charge (SoC) at the beginning of the charging period (initial SoC) x_{1_0}, final SoC or required SoC x_{1_N} and time of charge which is essentially the end of the time period (ToC) (N is the final time-step). Together, they define how much charge is required, and when is it required by. The disturbance w, is the known prediction of electricity price (DAP or output of the Linear Regression Predictor (chapter 4)).

The disturbance d, is not given and is the uncorrelated stochastic variable with a known distribution. It describes the price 'spikes' or fluctuations which can change the trajectory of grid-cost and is dependent on market forces and electricity demand. The nature of the electricity market (chapter 4) leads to unpredictable real time costs. This is what causes the difference between day-ahead prices and real-time prices when a smart-grid and smart-metering tariff is in question.

There are two main range constraints:

$$0 < u \leq u_{max} \tag{20}$$

The charge power is always positive because we don't consider vehicle to grid and therefore no charge power can go to the grid instead of the battery and u_{max} is the maximum allowable charge power (dependant on the charging station), $u \in [0, u_{max}]$.

$$0 \leq x_1 \leq 1 \tag{21}$$

The battery SoC can never be negative and never above 100% which means there is no overcharging.

By adding the stochastic element to the dynamic program, a solution for price prediction in context of EV charging (to try and minimize final cost) can be obtained as explained in further sections.

6.1.2 New Grid Cost Model and Possible approaches

To describe this nature of the electricity grid we can model the price 'spikes' as stochastic events and associate a probability to the occurrence of these events. For the sake of simplicity, the first order linear dynamics are assumed for the cost model, resulting in a first order linear stochastic process. The process models the price deviation between the predicted price (DAP tariff) and the actual real time cost (RTP tariff).

Grid Model Definition

$\boldsymbol{d}$: A temporally uncorrelated random variable. Different distributions can be assumed. Here it is used to implement the event probability, and therefore a uniform distribution in the interval [0,1] is assumed.

$\boldsymbol{c_{k+1}}$: The cost deviation at time-step *k+1*

$\boldsymbol{\alpha}$: The decay factor of the dynamic process. It is informed by the autocorrelation of historic data.

$\boldsymbol{\beta}$: A scaling factor for the disturbance. It is used to match the standard deviation observed in the data.

$\boldsymbol{P_{ev}}$: The probability of an event occurring, separated into the positive $Pevp$ and negative $Pevn$

We get:

$$c_{k+1} = \begin{cases} \alpha c_k + \beta \mid \text{positive event } \left(d < P_{evp}\right) \\ \alpha c_k - \beta \mid \text{negative event } (d > 1 - P_{evn}) \\ \alpha c_k \mid \text{no event (otherwise)} \end{cases} \tag{22}$$

This model defines a simple first order stochastic process with a discrete distribution of the disturbance. Different assumptions could be made for the probability, but it turns out that the optimisation is not very sensitive to the specifics of the distribution, because it evens out over time due to the low pass nature of the dynamics.

Based on the analysis in chapter 4 section 4.1, a probability value can be selected as a beginning point. Figure 32 (page 75) shows the normal probability plot of RTP and DAP. It is clear that the predicted prices conform to real prices between 0.05-0.95 (5-95%). The upper and lower 5% show long tails and the RTP are very different from the DAP. This can be selected as the initial value of P_{evp} and P_{evn}. Different values are tested as part of the case study, in chapter 7.

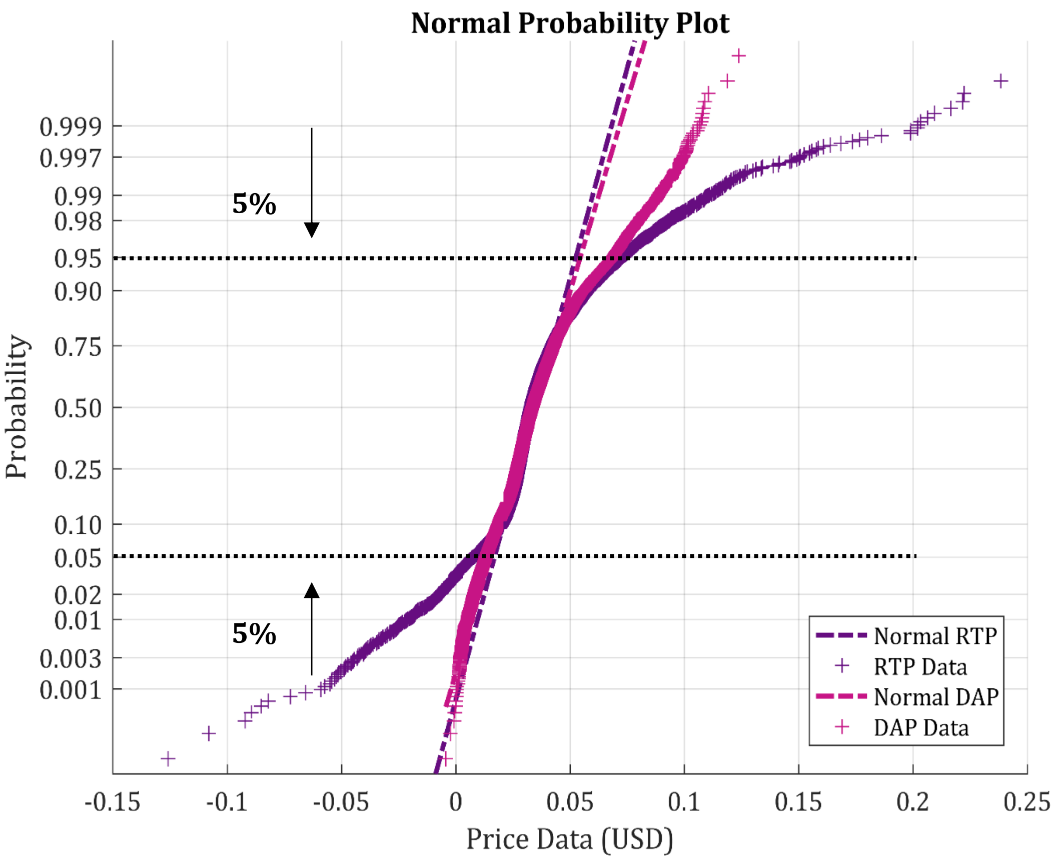


Figure 32: Probability Selection

If $Pevp = Pevn$, the distribution is symmetric, which means that the expected difference between predicted and actual price is zero – there is no bias in other words. Figure 33 shows the selected probability distribution. If the selected probability is 10%, then $P_{evp} = P_{evn} = 5\%$, and $1 - P_{evp} - P_{evn} = 90\%$.

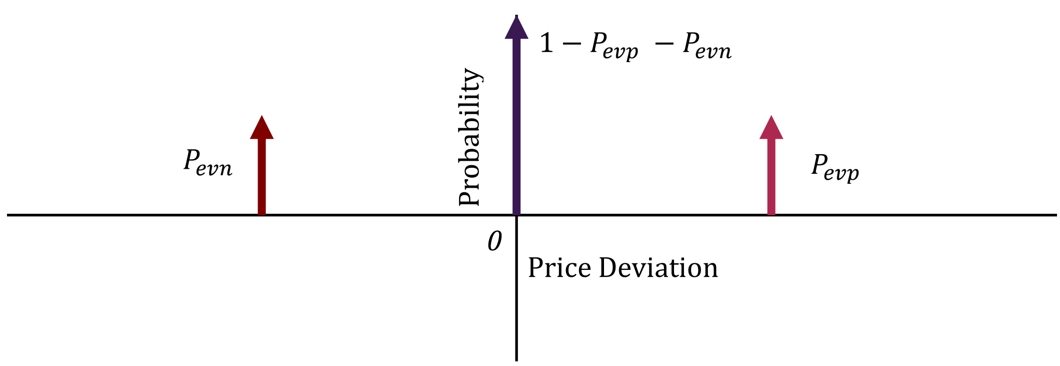


Figure 33: Representation of Probability Distribution

Problem Statement

The optimal solution for a stochastic process is defined as providing the lowest overall expected cost *E<J(u)>*. Due to the stochastic nature of the problem, the decision when to charge is a compromise between using the cheapest electricity and mitigating the risk of future price changes. The charging controller can take the current cost into account, but not the future cost:

$$\min_{u_k(c_k)} E < J(u_k) > \tag{23}$$

Therefore, the following summarizes the minimization problem:

Problem	$\min_{h} E < J >$	*(24)*
Control Law	$u_k = h_k\ (x_{1,k}, x_{2,k})$	*(25)*
Battery Model	$x_1 = i(u)$ $x_{1_{k+1}} = x_{1_k} + u_k$	*(26)*
Grid Model	$x_2 = j(d)$ $x_{2_{k+1}} = \alpha x_{2_k} + d_k$ d_k is Gaussian with $E < d_k > = 0$ $E < d_k^2 > = \beta$	*(27)*
Cost Model	$c = k(x_2, u, w_k)$ $c_k = (x_{2,k} + w_k)u_k$ w_k known $J = \sum_k c_k$	*(28)* *(29)*
Constraints	$0 \leq x_{1,k} \leq 1$ $0 < u_k \leq u_{max}$	*(30)*
Boundaries	$x_{1_0}\ (Initial\ SoC), x_{1_N}(Required\ SoC)$	

A finite horizon problem definition (as above), requires a finite or receding horizon approach. One popular approach is model predictive control (MPC), specifically stochastic model predictive control (SMPC). While it deals well with the finite horizon, the limits, and the dynamics, MPC is rooted in a linear quadratic Gaussian problem definition; it struggles both with linear cost of charging, the stochastic cost, and the resulting interaction between limits and stochastic variables. MPC with a stochastic weight models could be used [103], but most stochastic MPC approaches will consider only stochastic limits, not weights. The reason is that the stochastic cost together with the input limits turns the Gaussian probability distributions into piecewise Gaussian distributions, which are complex to handle numerically.

This problem is more complicated than typical control problems, because it asks the question whether it is better to charge at current electricity prices, or whether it is worthwhile to wait, based on the chance (not certainty) that price might fall in the future. The central question "are prices going to go up or down?" lies at the very heart of economic markets and market theory. And although it is not possible to come to a deterministic answer, based on the stochastic process model it is possible to calculate which one results in the lower expected cost (the average cost over a sufficiently large number of scenarios). This trade-off is addressed using the Hamilton-Jacobi-Bellman (HJB) equation [104], which minimises the expected cost based on the stochastic pricing model.

Exact solutions of this equation are typically not feasible, but many reasonable numerical approximations exist. Dynamic programming uses quantization of continuous states, and specifically stochastic dynamic programming is well suited for addressing the problem at hand. Mixed integer algorithms may also be able to find the expected cost benefit of charging at specific times [105] with relative ease and accuracy. Finally there are a number of industry specific approaches coming from operations research, that deal with the question of optimal load shifting and scheduling using a limited capacity [106][107].

6.2 Stochastic Dynamic Programming

Based on an analysis of available options, dynamic programming (DP) was picked as the most appropriate way to find the solution to the optimization problem. Classic dynamic programming requires knowledge of all disturbances in advance, and there it does not produce a causal controller. In a typical deterministic DP, decisions are taken backwards in time at each stage, based on the summation of present cost and expected future cost, assuming optimal decision making for future stages [102].

The symmetric part of the problem discussed in chapter 5 can be successfully dealt with in this manner. When the stochastic element is taken into account, dynamic programming can be extended to solve the stochastic problem in an effective way [108]. The cost model defined in equation ((*19*) is a Markov process because decisions are only based on the current state, but not any past decisions. This makes it easy to integrate into the dynamic programming framework.

The typical structure of dynamic programming is shown in the following equations based on the definitions and the algorithm in [109].

The dynamic programming problem is based on a non-linear **discrete-time plant model**:

$$x_{k+1} = F_k(x_k, u_k) \text{ } where \text{ } k = 0,1, \dots, N-1 \tag{31}$$

Here, x_k is the state which uses information from the previous step for future optimization, u_k is the control variable which is also the decision variable. N is the length of control horizon. The problem assumes a **cost-function** which is additive over time:

$$J_\pi = E \langle F_N(x_N) + \sum_{k}^{N-1} F_k(x_k, u_k) \rangle \tag{32}$$

Here π is the control policy- $\pi = \{\mu_0, \mu_1, \dots, \mu_{N-1}\}$, where μ_k is the control law for time step k based on state x_k.

For a given initial state x_0, the **expected cost of $\boldsymbol{\pi}$** can be determined as:

$$J_\pi(x_0) = E \langle H_N(x_N) + \sum_{k=0}^{N-1} G_k\big(x_k, \mu_k(x_k)\big) \rangle \tag{33}$$

but it is important to notice that the dynamic programming results in a control strategy, and this strategy can be calculated without knowing the initial state. (It is only when applying the strategy that the knowledge of the state becomes relevant.)

The **optimal control policy $\boldsymbol{\pi^0}$** is the policy that minimizes J_π for $k = 0,1, \dots, N-1$

$$J^0(x_0) = \min_{x \in S} J_\pi(x_0) \tag{34}$$

Where S is the set of all admissible policies.

The framework can be extended in a few ways to include stochastic elements. Assuming a random stochastic element d that affects the cost, (11) becomes:

$$J_\pi(x_0) = E \langle H_N(x_N) + \sum_{k=0}^{N-1} G_k(x_k, \mu_k(x_k), d) \rangle \tag{35}$$

However, in this specific problem, the price (and the Markov process that models its behaviour) is part of the plant state:

$$x_{k+1} = F_k(x_k, u_k, d) \tag{36}$$

The implementation becomes easier if the stochastic element can be separate from the input, in the form:

$$x_{k+1} = F'_k(F_k(x_k, u_k), d_k) \tag{37}$$

The definition and the significance of the disturbance d_k depend on the specific problem and the stochastic model used. There are some assumptions central to this theory: the set of values which control the input u_k depend only on the state x_k at time k, and the disturbance d_k is uncorrelated in time (Markov property). As typical for finite horizon problems, it is not necessary to assume time invariance. Although many of the elements of the problem are time invariant, the available charging power and electricity cost certainly are not.

As can be seen by comparing this problem statement with the one above in section 6.1, the dynamic programming algorithm is a perfect fit for the optimal charging problem.

6.3 Solution

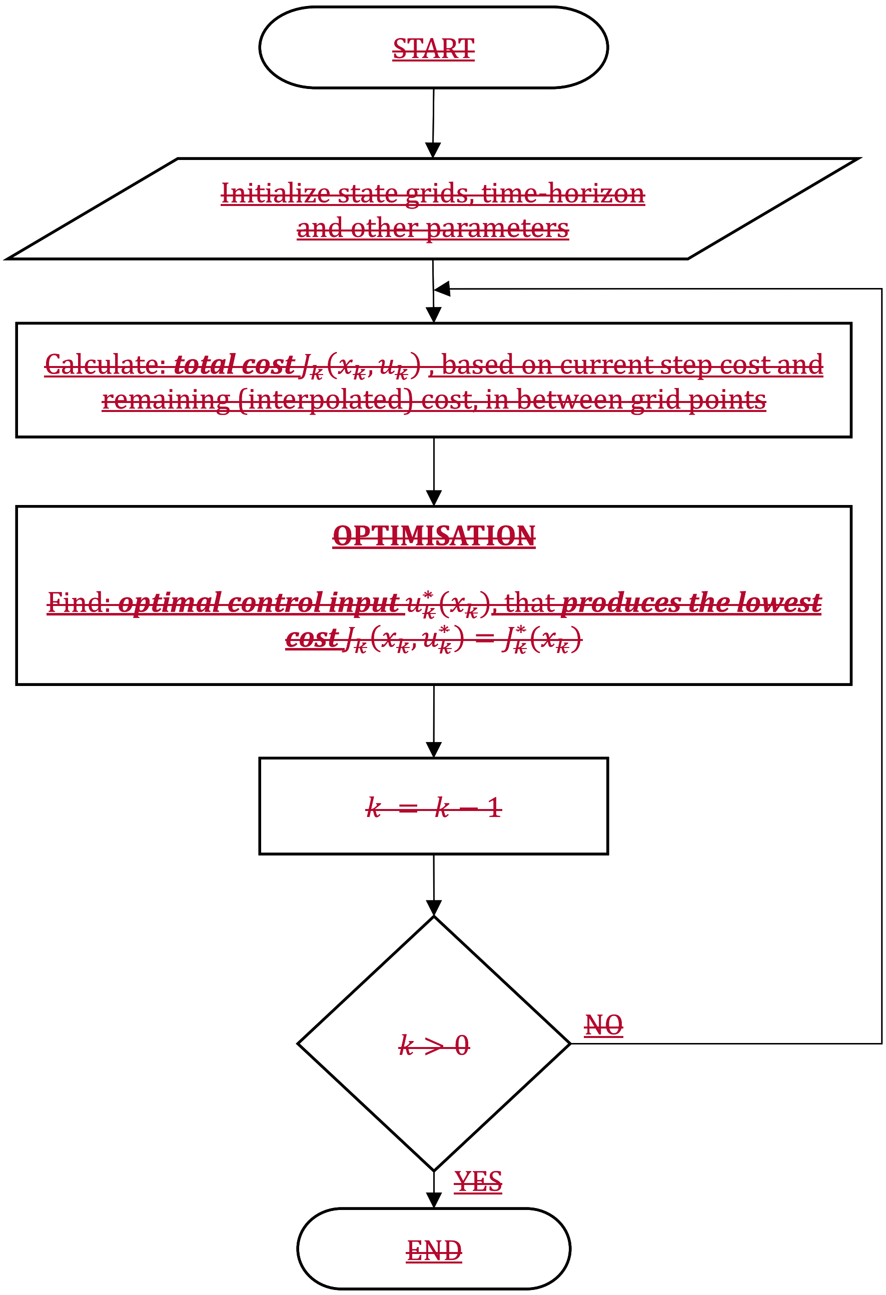
This section presents the solution to the problem in 6.1 using stochastic dynamic programming. Section 6.3.1 explains the dynamic programming (DP) algorithm in the context of EV charging. The dynamic programming algorithm is extended with a step to resolve the stochastic disturbance in section 6.3.2. Lastly, section 6.3.3 presents the stochastic dynamic programming (SDP) theorem.

6.3.1 DP algorithm for the EV charging problem

The standard way of solving the dynamic programming problem (DP) is through discretisation of continuous variables. A tested implementation of this approach is published in [108]. The dynamic programming function solves the discrete-time optimal control problems backwards in time using Bellman's principle of optimality. The paper by ETH Zurich affiliated Sundstrom and Guzella, describes a generic deterministic DP function implemented in MATLAB. This proved to be a good starting point as they have shown their function to be applicable to a hybrid EV energy management system.

The charging problem described here is different due to the stochastic plant definition, and therefore the algorithm had to be rewritten to include the stochastic computational elements added to the overall program. The final code is much shorter, because it lacks the generic applicability of the dynamic programming function, and it differs significantly from it. It is based on the algorithms explained here and in the SDP algorithm section. Although this function is much more specific (supporting only two variables, and only specific stochastic plant models), it would provide a good starting point for a more generic Stochastic Dynamic Programming function.

Figure 34 shows the flowchart of the modified algorithm. Followed by this is the detailed description of it. It is clear from the flowchart, that the calculation is done backwards over the time horizon ***N.***



~~*Figure 34: Deterministic Dynamic Programming Algorithm (algorithm 1)*~~

Algorithm 1: Deterministic Dynamic Programming

Variables:

- State: $\boldsymbol{x}$ (here x_1 – SoC and x_2 – grid-cost)
- Control: $\boldsymbol{u}$ (here Charge Power)

Inputs:

- Input and state grid
- Plant model function: $F(x, u)$
- Cost functions $G(x, u)$
  here (where w_k is the predicted cost):

$$
\begin{gathered}
x_{1_{k+1}} = x_{1_k} + u_k \\
x_{2_{k+1}} = \alpha x_{2_k} \\
G(x, u) = \left(x_{2_k} + w_k\right) u
\end{gathered}
\tag{38}
$$

- Final state cost: $H(x)$ for all states $\boldsymbol{x}_{k+1}$ on the grid
  (A penalty function for inadmissible SoC states is applied)
- Time horizon: $\boldsymbol{N}$

1. Initialise the final state cost $J_{N+1}(x)$
2. **Iterate** over the time horizon backwards $k = N \dots 1$

   2.1 ***For each state*** $\boldsymbol{x}_k$ in the grid

   2.1.1 ***For each input*** $\boldsymbol{u}_k$ in the grid

   2.1.1.1 <u>Evaluate the Model</u>

$$
\begin{gathered}
x_{k+1} = f(x_k, u_k) \\
c_{k+1} = c(x_k, u_k, w_k)
\end{gathered}
\tag{39}
$$

   2.1.1.2 <u>Interpolate the remaining cost</u>

   $J_{k+1}^{*}(x_{k+1})$ based on x_{k+1} and the cost at the nearest state grid points.

   2.1.1.3 <u>Calculate the total cost</u>

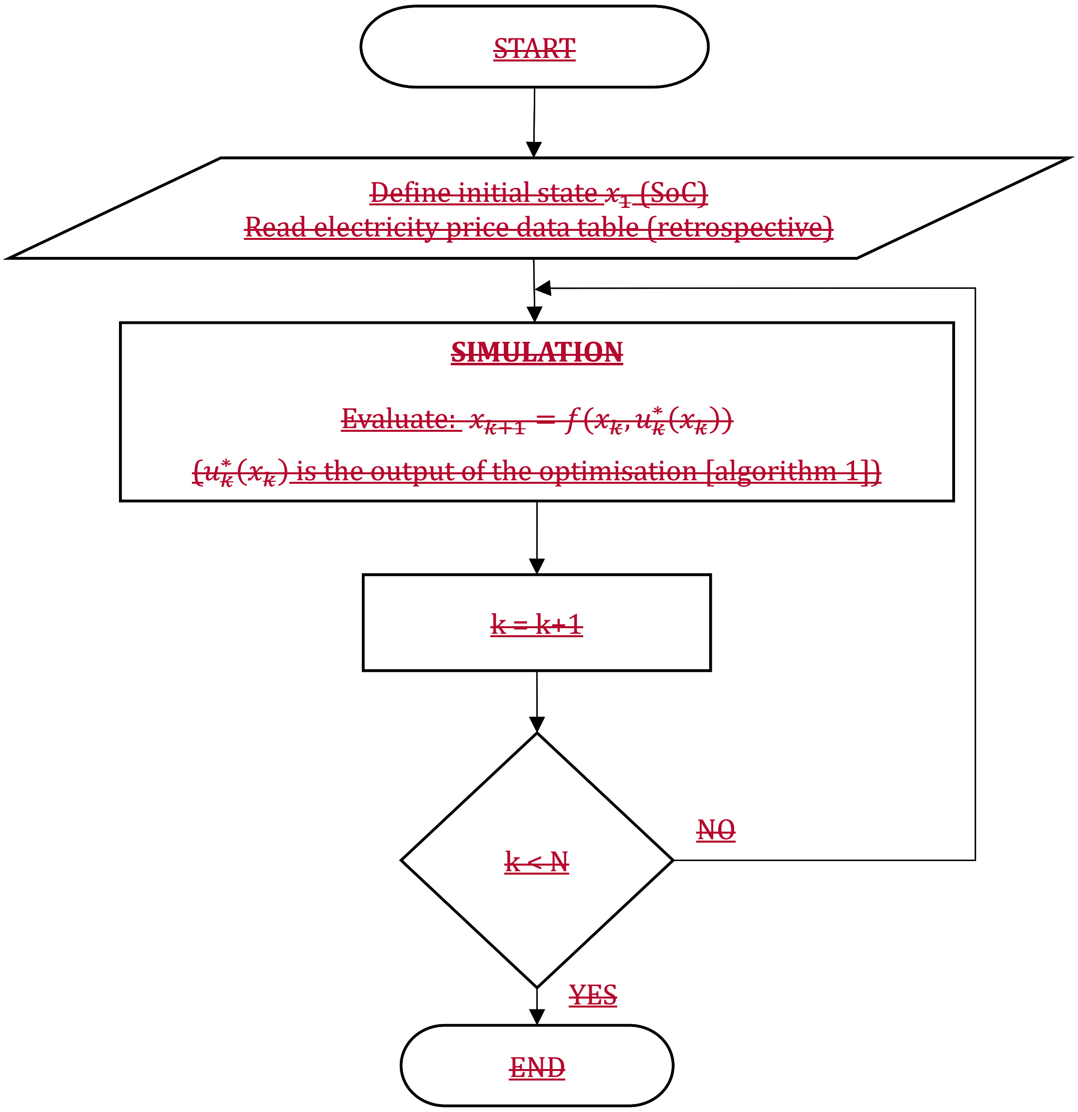
   Based on the step cost and the remaining cost

$$J_k(x_k, u_k) = c_k(x_k, u_k, w_k) + J^*_{k+1}(x_{k+1}) \qquad (40)$$

2.1.2 Find the best input $u^*_k(x_k)$ that produces the lowest cost $J_k(x_k, u^*_k) = J^*_k(x_k)$

2.2 For visualization purposes, a cut-off grid-cost is determined – it is the grid cost for a given state of charge at which the charging power reaches 50%. (optional)

This algorithm produces the optimal charge strategy. Once it has been determined, a simulation algorithm is used to find the charging cost. This algorithm has to be deterministic and uses real data (known retrospectively). Figure 35 shows the flowchart for forward simulation.



~~Figure 35: Optimal Charging (forward) Simulation Algorithm~~

Algorithm 2: Optimal Charging Simulation

1. For the forward simulation, define the initial state x_1
2. Iterate over the time horizon forwards $k = 1 \dots N$

    2.1 Evaluate the model

$$x_{k+1} = f(x_k, u_k^*(x_k)) \tag{41}$$

6.3.2 SDP version of the Algorithm

The previous algorithm solves the deterministic dynamic programming problem. The following algorithm is extended to deal with the stochastic element of the plant model, (Figure 36). The stochastic step is separated in the model, and therefore does not have to be included in the input loop. This makes the final algorithm more elegant and much computationally less intensive. The separation is possible because the model isolates the stochastic element from the system input and thus from the optimization, eliminating the need to calculate it in the nested iterations.

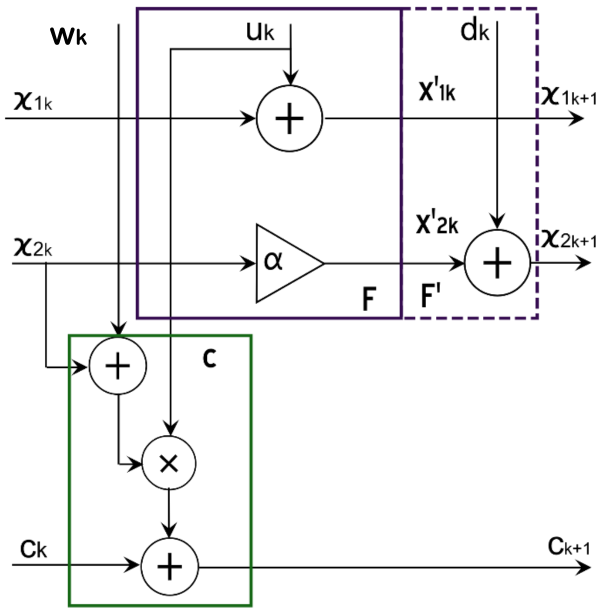


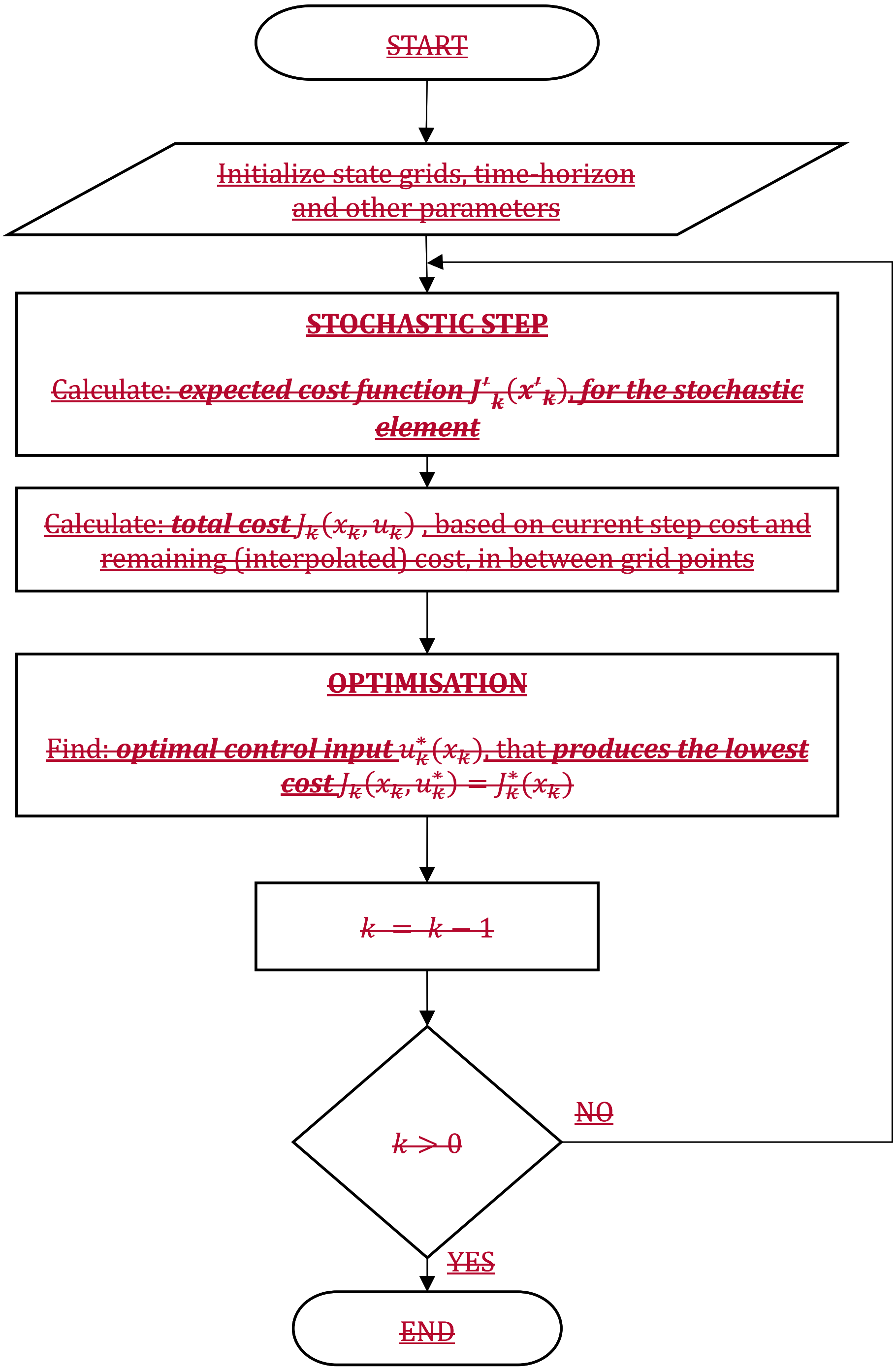
Figure 36: Charging controller showing stochastic function

The separation is achieved by formulating the model in two steps:

$$x_{k+1} = F'_k(F_k(x_k, u_k), d_k) \tag{42}$$

Where $F_k(x_k, u_k)$ deals with the control input u, and $\mathrm{F}'_k(x'_k, d_k)$ with the stochastic input d. Specifically, the disturbance only acts on x_2, the deviation of the electricity cost from the prediction.

The algorithm can now be extended as follows. Figure 37 shows the flowchart, and this is followed by the detailed description of the algorithm. The differences are highlighted in purple.



~~*Figure 37: SDPM Algorithm (algorithm 3)*~~

Algorithm 3: Stochastic Dynamic Programming

- State: $\boldsymbol{x}$ (here x_1 – SoC and x_2 – grid-cost)
- Control: $\boldsymbol{u}$ (here Charge Power)
- Grid size for all states
- Model functions: F and c

  here:

$$
\begin{aligned}
x'_{1_k} &= x_{1_k} + u_k \\
x'_{2_k} &= \alpha x_{2_k} + d_k \\
c_k &= \left(w_k + x_{2_k}\right)u
\end{aligned}
\tag{43}
$$

- Final state cost: J^*_{N+1} for all states $\boldsymbol{x}_{k+1}$ on the grid

  (here: $J_{N+1} = c_{N+1}(x_{N+1}) \times k$, where c_N is the distance from the admissible set, and k is a penalty factor)

- **The stochastic model $\boldsymbol{F'(x', d)}$**

  here:

$$
\begin{aligned}
\boldsymbol{x}_{1_{k+1}} &= \boldsymbol{x'}_{1_k} \\
\mathbf{x}_{2_{k+1}} &= \boldsymbol{x'}_{2_k} + \boldsymbol{\Delta(d_k)}
\end{aligned}
\tag{44}
$$

- Time horizon: N

1. State and input grids are created from discretization limits
2. **Iterate** over the time horizon backwards $k = N \ldots 1$

   2.1. The expected cost function for the stochastic element is calculated using a folding integral or folding sum:

$$
\begin{aligned}
\boldsymbol{J'_k(x'_k)} &= \boldsymbol{E\langle J^*_{k+1}(x_{k+1})\rangle} \\
\boldsymbol{J'_k(x'_k)} &= \sum_{\boldsymbol{i}} \boldsymbol{P(i)\, J^*_{k+1}(x^i_{k+1})}
\end{aligned}
\tag{45}
$$

   Iterating over all possible cases *i*

   2.2. ***For each state $\boldsymbol{x}_k$***

   2.2.1. ***For each input $\boldsymbol{u}_k$***

   2.2.1.1. <u>Evaluate the Model</u>:

$$x'_k = f(x_k, u_k)$$
$$c_k = c(x_k, u_k, w_k) \tag{46}$$

2.2.1.2. Interpolate the remaining cost

$J'_k\ (x'_k)$ Based on x'_k and the cost at the state grid points.

2.2.1.3. Calculate the total cost

Based on the step cost and the remaining cost

$$J_k(x_k, u_k) = c_k(x_k, u_k, w_k) + J'_k(x'_k) \tag{47}$$

2.2.2. Find the best input $u^*_k(x_k)$ that produces the lowest cost $J_k(x_k, u^*_k) = J^*_k(x_k)$

2.3. A cut-off grid-cost line is mapped for visualization purposes. This is the grid cost for a given state of charge at which the charging power reaches 50%. (optional)

Once the ideal control strategy has been found, it is simulated to find the projected cost using Algorithm 2. Note that unlike Algorithm 3, Algorithm 2 is forward facing, so it has to be deterministic, relying either on data or a Monte Carlo simulation of the stochastic model. A probabilistic forward simulation may not be possible [102].

6.3.3 Theorems

Theorem 1

The optimisation problem is separable and can be solved as a series of optimisation problems backwards in time $\min_{h_k} E < J_k >$ iterating over $k \coloneqq N, k-1, \dots 1$.

Proof

This follows from the causality of the problem. Firstly, the control law is the optimal solution, because it has access to all information relevant at time step k. Previous time steps have no impact that is not captured in the system state x_k (Markov property), and future time steps are not known yet, specifically d_{k+1} is unknown.

This allows the separation of the total cost at any point into the step cost and the remaining to go cost (Bellman's principle of optimality):

$$\min_{h_k \cdots N} E < J_k > = \min_{h_k} E < J(u_k, x_k) > + \min_{h_{k+1} \dots N} E < J_{k+1} >$$

Assuming the last term has been solved, finding the minimum for the current step does solve the optimisation problem now. Recursive application solves the initial problem over the full horizon.

Theorem 2

The algorithm does identify the optimal control law for each time step $\min_{h_k} E < J_k >$ in approximation.

The algorithm solves this problem in two steps. First, it solves for the expected cost function (equation (45) which corresponds to the definition of the stochastic model, and then the optimal input is found in step 2.2.1 of algorithm 3, based on the deterministic part of the model and the stochastic part that is already included in the cost function.

In selecting the lowest possible cost for every state, it uses three approximations:

1. It interpolates the remaining cost J_{k+1} which is only known for grid points
2. Input u is only ever returned on the points of the input grid but not in between the grid points.
3. The optimal u is only ever calculated for points on the state grid x

All these errors decrease with reducing grid size and eventually converge to zero for any smooth cost function.

6.4 Results

6.4.1 Scenario

The baseline scenario provided to the controller assumes fast-charging at ($0.5\ x\ batt\ capacity$) and no resistive losses. The user drives an EV back home from work and plugs in the vehicle at 17:00. The vehicle is required by 07:00 with 80% SoC ($0.8\ x\ batt\ capacity$). During this 14 hour charging period, no discharging is allowed. Different cases are explored within this scenario to see the decisions the controller makes and this tests the SDP output. The baseline assumes no resistive losses in the battery or charger.

6.4.2 Assumptions

A number of assumptions about the boundary conditions and the model have to be made to test the optimization algorithm. The aim of these assumptions is to test the function of the controller, specifically its ability to pick the best hours to charge by using a prediction for price changes. The test requires exercising different scenarios and eliciting different responses from the controller.

A DAP profile for an arbitrary evening is selected from the ComEd tariffs for Illinois, Chicago, USA [95]. A hypothetical event is triggered at different times (or not triggered at all), which changes the price compared to the predicted cost. This set up allows to see the reaction of the controller for the different scenarios.

Further assumptions are:

- The grid model described is a Markov process as detailed above.
- The *decay factor (α), disturbance (β) and price-event change* are all assumed as constant for simulation (stated in each scenario results). α is set at 0.8 and derived from the analysis in section 4.1. β is set at 10 cents.
- The P_{ev}, probability that an event might occur is constant and has a discrete-distribution (Figure 33). The baseline value is set at 10%. Different values are tested to verify the controller functioning in different probability situations (only positive values are considered).
- The dynamics of the grid model are linear.
- The EV specifications have no effect on the charging process. The only relevant factors are the battery size, the required level of charge, and the time available.
- EV battery temperature, ambient temperature effects are not taken into account.
- The available charge power is assumed to be constant.

6.4.3 Results

6.4.3.1 No Event Case

Table 1 shows the input parameters: for the first baseline case, we discuss the results for price-selection, charge-decision, optimal-control map, cost-of-control decision and a snapshot of the cut-off prices. There is a **positive event probability** for a price spike, but in the simulation **no price-event** happens.

Table 6: Input Parameters for Basic Scenario

	Current Parameters
Time Step	1
Number of Hours	14
Alpha	0.8
Max Available Charge Power	0.5 (of batt capacity)
Resistive Penalty	0
Event Probability	0.1 (10%)
Event Time	None

Figure 38 shows, the controller decides to start charge at 21:00 (allowing full power at 22:00) and realizes it may need lesser time than initially calculated to charge. The charge power drops after 22:00 and a full charge is achieved by 00:00.

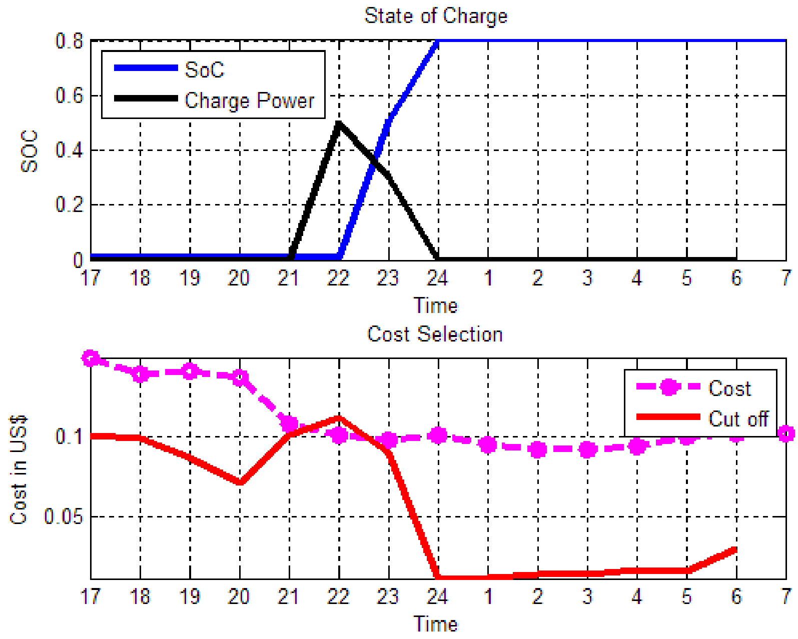


Figure 38: Controller output for Base Scenario

In this case, the controller picks a slightly higher cost than the lowest possible in the 14 hours. This is because the stochastic model includes a 10% probability per hour for price spike, which would make later charging more expensive. Therefore, the certainty of a slightly higher price is considered superior to the uncertainty of a later price, which may be slightly lower, or significantly higher. So the controller decides that it is better to provide a 'full' charge at a minimal penalty (low enough cost at 22:00) than to wait for the lower cost (at a later hour) and risk having to charge during a price-event. This shows that with a probability of an event occurring at ($Pev = 0.1$), the controller works intelligently to provide a full charge (which is a boundary condition $x_1 = x_{1_N}$) whilst picking the lowest expected cost according to the stochastic model.

Figure 39 shows the cut off cost selection for odd hours for the 14 hour time period. It is visible that as time passes and SoC has not been met, the controller is ready to pick a higher cost to make sure charge is provided in time. When possible, the lowest cost is chosen proving that the controller works to provide 'the optimal cost charging' within the set parameters (SoC and ToC) by predicting an event based on probability.

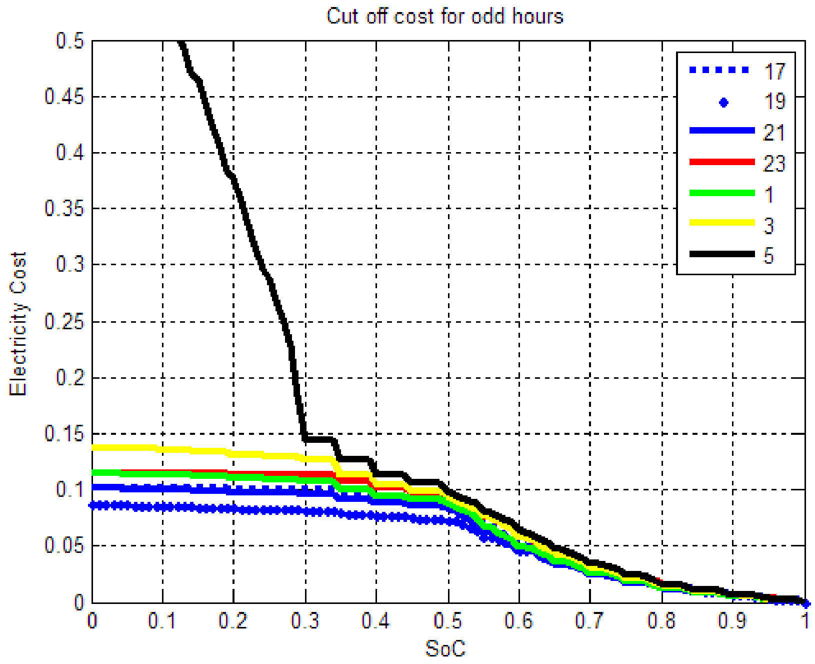


Figure 39: Cut-off Cost Trajectories

This trade-off can be seen in the cut-off cost (deviation) plot: a very low cut-off cost is set in the early hours of the evening. This means charging is unlikely, and it would only happen if the cost is unexpectedly low, because they expectation is for cost to fall. As time progresses, and the freedom of the controller is reduced, the cut-off cost rises. Around 20:00 and 00:00, it is expected to reach the grid cost, and then charging would happen. The higher the SoC, the less need there is for charging, and the lower the cut-off cost can be set – making charging less likely. Finally, at 5 am the cut-off cost rises significantly, especially at low SoC. This is to ensure the required level of charge, in case the grid price has prevented charging before then. Obviously, this would incur a high cost, so it is discouraged by the control strategy. So the rise of the cut-off cost is initially because of the expected cost, and later because of the penalty of insufficient SoC.

Figure 40 shows a snapshot of the optimal control map where the colour bar represents charge power for 21:00 and 23:00 (covers one charge decision and one discharging decision for this scenario). This shows more detail of the control law than the cut-off plot, but only for specific points in time. The control action follows the expectations: the charge power is increased if the cost and SoC are lower. As both increase, charge power is decreased. In this case the most important area is between 0 and 37% SoC, where the decision to charge or not falls within a small range of grid-cost. The map suggests that once the cost is near $0.1, the controller will charge because the cost before is very high and $0.1 is a low-enough cost to pick and charge to achieve full SoC. The black line shows half the available charge power, which is exactly the cut-off cost discussed earlier, highlighted here for comparison. At 23:00 time has passed (SoC has increased), the controller selects a slightly higher charge power, because less time is left for charging, and therefore the urgency has increased slightly.

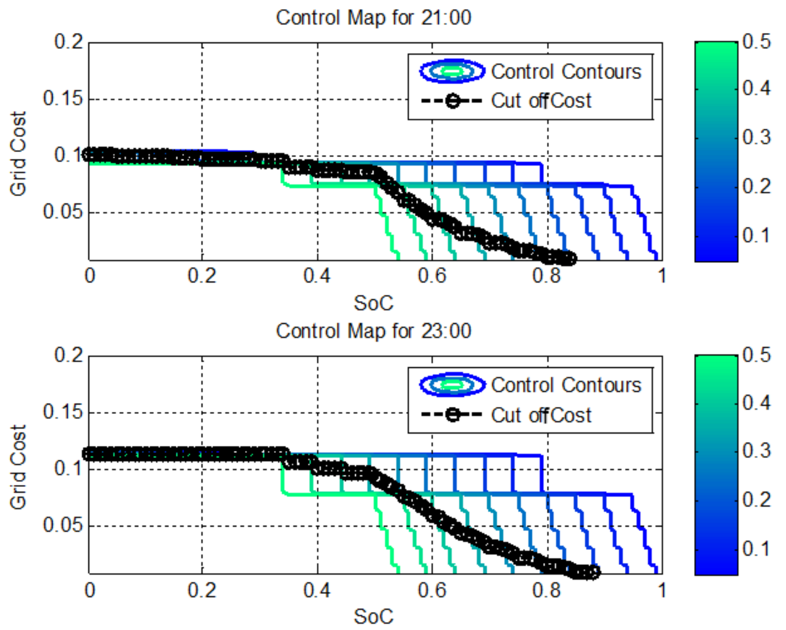


Figure 40: Controller Maps Base Scenario with contours showing charge power

Figure 41 shows the map of the expected cost at the start of the horizon at 17:00. The colour bar indicates the cost. The plot shows that cost is roughly proportional to the SoC missing in the battery.

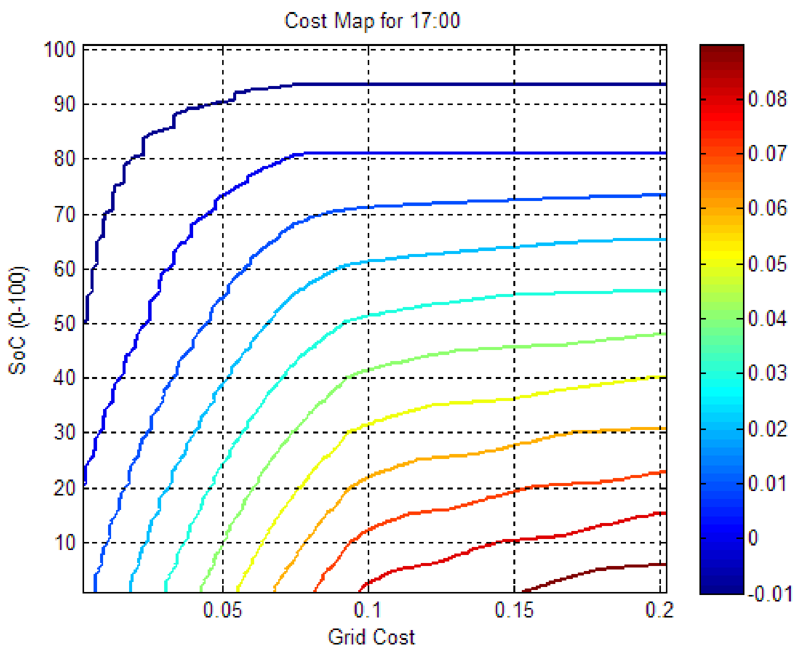


Figure 41: Cost (J) map for 17:00

The cost shows a large decrease for very low grid costs (towards 0), because this would indicate that the battery can be charged immediately for little cost. There is little corresponding increase for a high grid cost, because according to the grid model, the cost should normalise throughout the night, and allow charging at a moderate rate.

6.4.3.2 No Probability

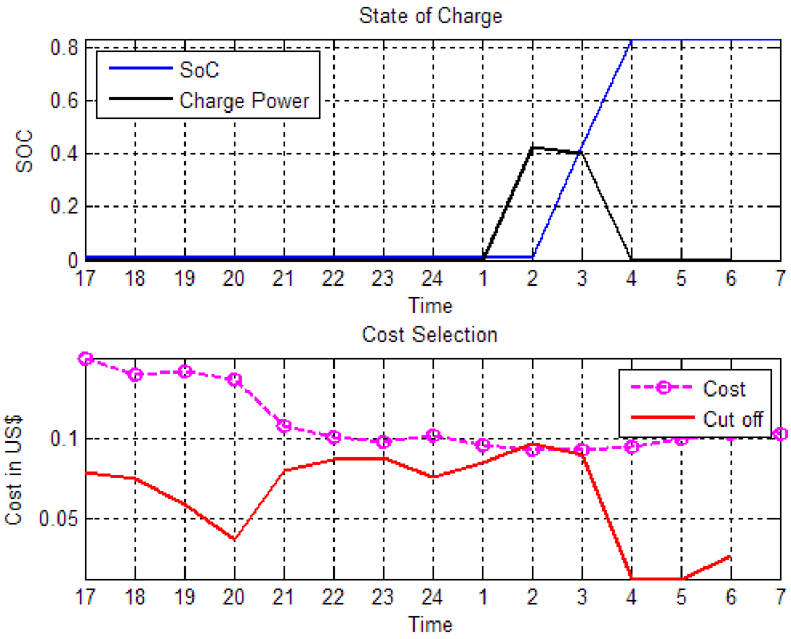


Figure 42: Controller output with Pev=0

For comparison, an optimisation is performed without the stochastic element in the grid model. This is achieved by setting the probability of an event to zero, which matches the simulation used for testing.

Figure 42 shows that the controller reacts subtly differently in this scenario. Because the grid is assumed to be predictable, there is no rush to charge early to reduce the risk. Instead, the controller waits until the price is at its lowest point. It picks the hours with lowest price to charge at full power, making sure that it can provide a full charge in the remaining hours. It is remarkable how constant the cut-off cost is throughout most of the night. The controller approximates the constant cut-off cost solution described in the previous chapter for deterministic grid models, which has been shown to be the optimal solution. The difference between both scenarios both in terms of charge time and incurred cost reinforces the requirement for accurate probability values to be passed from the predictor.

6.4.3.3 Scenarios with Events

The same simulations can be repeated with different events. Note that the controller is the same as before, but "as if by chance" different times for an event are tested. Table 2 again shows the input parameters: There is a **positive event probability** and **early, middle, late hours and long events.** The reactions of the controller in each case are compared.

Table 7: Input Parameters for Base Scenario

	Current Parameters
Time Step	1
Number of Hours	14
Alpha	0.8
Charge Power	0.5
Resistive Penalty	0
Event Probability	0.1
Event Time	3, 6, 12, 6+

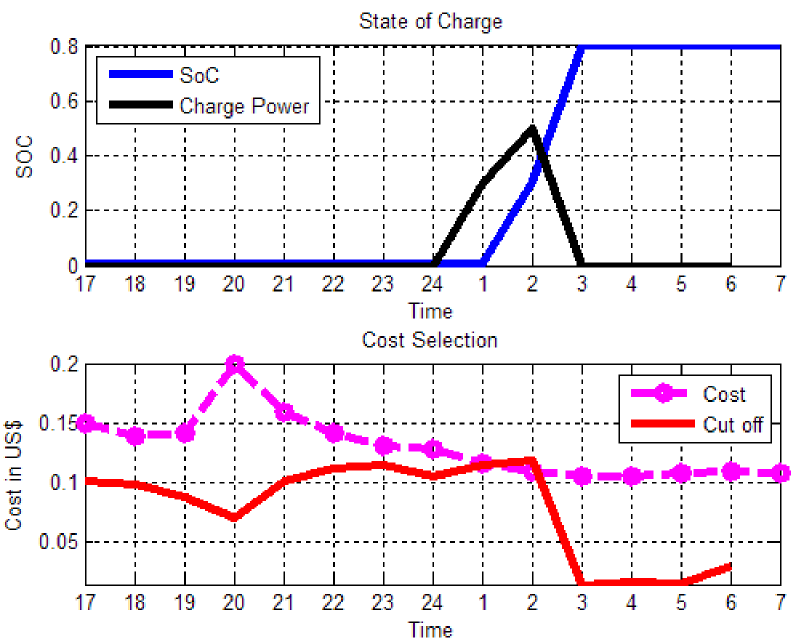


Figure 43: Early Event (base)

Figure 43 shows the early event case; at 20:00 the price event occurs and alters the trajectory for future hours. The controller detects this from the increased price, and it delays charging until the price has come down again to a more reasonable level. The price does fall, and it reaches low enough at 1:00 to warrant a charging phase to complete the charge. The cut-off prices increase slightly (indicating the need to charge in any cost situation) after 00:00 because the battery is empty, and the time left to charge it is reducing. The controller successfully picks low price hours till 3:00 to charge leaving no risk for later in case there is another event. Overall, the controller works very well in this case.

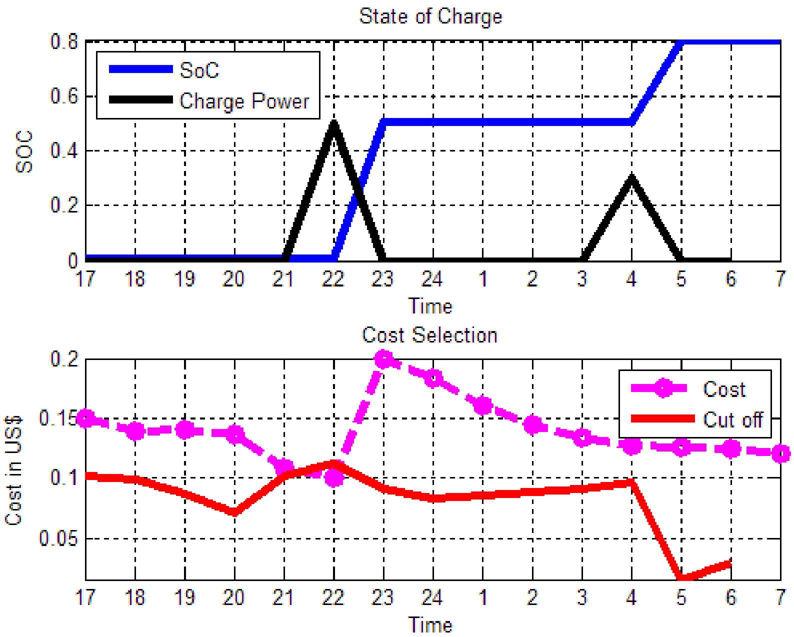


Figure 44: Middle Event (base)

Figure 44 shows the middle event case, which is much more disruptive. The controller sees a low price point at 21:00-22:00 where it decides to charge. But this is followed immediately by a price spike hits, and it increases the cost significantly after 23:00. The controller responds and stops charging as the price reaches a high at 23:00. The battery is 50% charged by then so the controller is comfortable to wait for a lower price. This is reached around 3:00-4:00 when it decides to charge again to provide the SoC as required. In the second charge phase it does not use all the charging power (only 0.3 compared to 0.5). This shows it meets the goals of avoiding high demand and cost, but still provides a full charge by ToC. The charging cost in this scenario is higher, because the price never really falls to the expected levels again. It would have been better to charge earlier in this case, but because the event is stochastic in nature, this has to be traded against the chance of an event not happening. As can be seen here, the controller provides a balanced trade-off for these conflicting possibilities.

Figure 45 shows the late event case, where the event happens after the charge is completed, which means that it has no effect at all. The controllers picks to charge exactly like the no event case in Figure 38. This proves that it predicts the possibility of an event due to the positive Pev and decides to charge with a minimal penalty, to be safe and provide SoC at ToC. The deterministic controller may still be charging when the event happens, and incur a higher cost. So again, charging slightly earlier than the lowest price pays off, and leads to a lower charge cost. The resulting cost is close to the minimum possible.

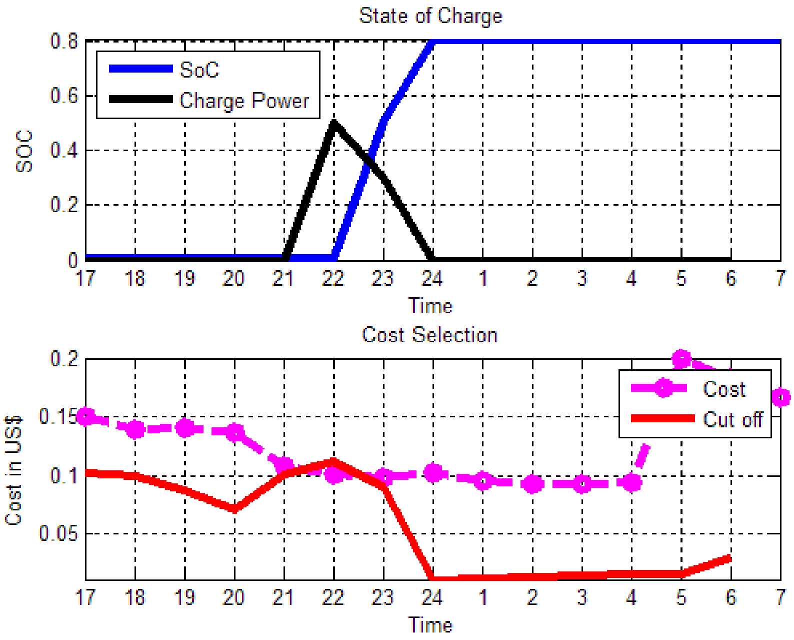


Figure 45: Late Event (base)

Figure 46 simulates an extreme case of a sustained event, where the grid cost jumps up at 23:00 and remains high for the remainder of the night. This is not in line with the stochastic grid model, but it has to be tested to see whether the controller always delivers the required charge, independent of the grid price trajectory.

Initially the controller behaves similarly to the middle event case, where it charges between 21:00 till the event alters the costs. It then stops and waits for a time of low cost to try and save both demand and price. However, the sustained event continues to keep the prices high all through the remaining hours.

At 6:00 am, the controller realises that this is the last opportunity to charge the battery, and it decides to request a charge up to the minimum 80%. Although this comes at a high cost, it is still preferable to the penalty of not reaching the necessary charge level. The cut off cost trajectory shows this decision behaviour: it increases at the last opportunity for charging.

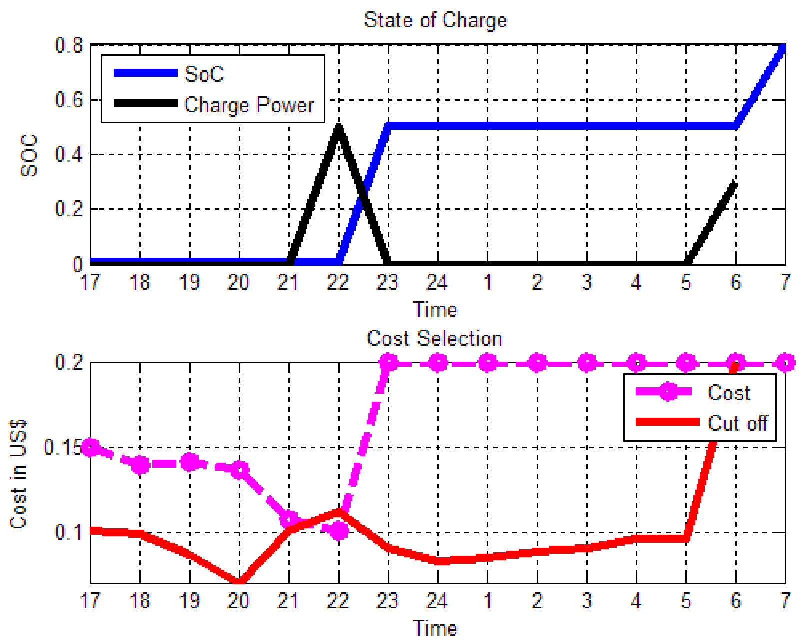


Figure 46: Long Event (base)

6.4.4 Scenario with Resistive Losses

The battery model does contain a term to model resistive losses. Just like the power loss on an Ohmic element grows with the square of the current ($P = RI^2$), this term grows with the square of the charge power. Without the square term, the optimal solution is usually an on-off control, that either charges at full power or not at all. Introducing a square term of the control input is well known in Linear Quadratic control theory, because it leads to a less aggressive and typically more robust controller.

Table 8: Parameters for Scenario with Losses

	Current Parameters
Time Step	1
Number of Hours	14
Alpha	0.8
Charge Power	0.5
Resistive Penalty	0.03
Event Probability	0.1
Event Time	6

Applying a penalty for losses, the cost model in equation (19) can be re written as:

$$c_{k+1} = c_k + \left(x_{2_k} + w_k\right)u_k + Ru_k^2 \qquad (48)$$

The effect of this term on the behaviour of the optimal control is analysed here. The expected effect is that to minimise the square term, charging is happening at lower power, spread over a longer time duration. Predictions, charging decisions, and risk mitigation will change accordingly.

Figure 47 shows the same scenario as Figure 44, but with the square penalty. The expected effect is clearly visible: the controller charges at lower power, and for longer. To compensate for the lower charge power, it also starts earlier. It starts to provide charge after 20:00 and more power by 22:00 when price is even lower but the power is clearly affected by the resistive losses and effectively reaches only a value of 0.3.

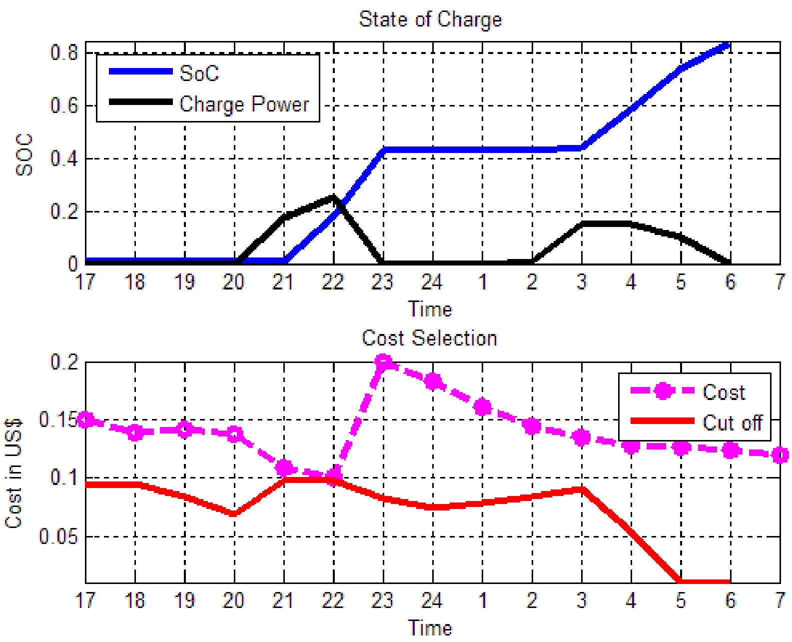


Figure 47: Controller output with Resistive Losses

The event occurs at 23:00 and the price shoots up, the controller stops charge and waits. Unlike the choice in scenario without resistive losses (Figure 44), the controller picks a slightly higher price at 2:00 because more time is needed to charge owing to low net charge power. This proves the intelligence in the controller works both for picking a low cost and avoiding the square penalty. This can also be seen in the cut-off cost trajectory, where the risk cost is higher at 2:00.

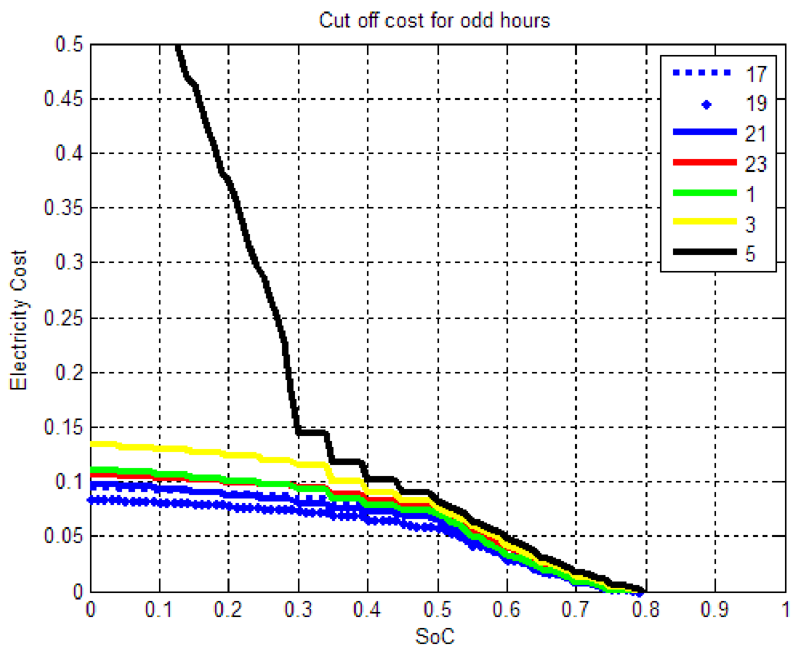


Figure 48: Cut-off cost trajectories for Losses

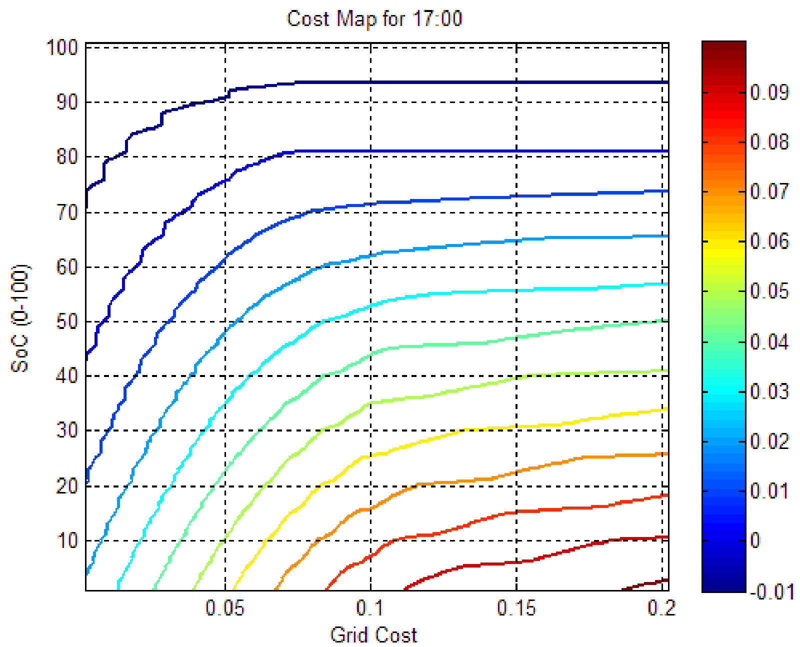


Figure 49: Cost (J) Map for 17:00

The cost of control map in Figure 49 shows only minor differences, but it is noticeably smoother than the previous version. It indicates clearly that the penalty of charging at the last moment at full power is significant, this being the main difference. This scenario and the reaction to it shows that, with losses considered, any controller will have to make compromises but the SDP controller makes these whilst still striving to find the lowest possible expense.

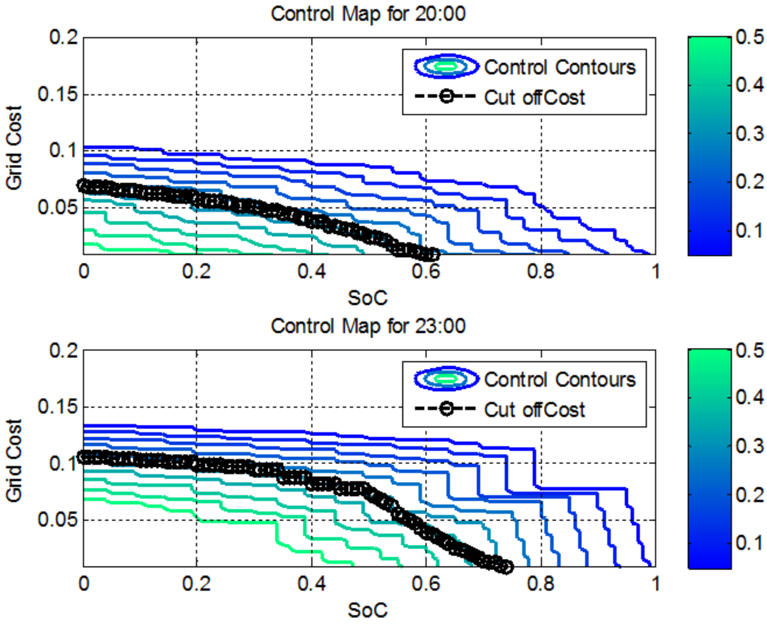


Figure 50: Controller Map for Losses

Figure 50 shows the optimal controller map for this scenario which differs significantly from the base shown in Figure 10-13. The price range at low SoC is much wider in this case, which is forced by lower net power due to losses. This does indicate that the gain of the optimal control law is significantly lower than without the penalty, exactly as expected. The map also shows how the controller compensate for the reduced charge power by considering charging earlier.

This is an interesting result, because it shows that both the square penalty term and the stochastic element of the grid model have a very similar effect: both make the controller charge earlier, before the minimum expected grid has been reached. The main difference is that the square penalty also leads to the charging spreading out over a longer period, while the stochastic element does not have any effect on the charge duration.

6.4.5 Scenario using Slow Charging

The final scenario looks at the effect of more limited available power for charging. Unlike in the previous case, where high power consumption was disincentivized by a penalty term, here it is prevented by a firm limit. Instead of a full charge in $2h$, it is assumed that it takes $5h$ to achieve the same at the reduced available power. The obvious implication is that with **lower charge power, the battery needs more time to charge**, so the timing of the control strategy will be affected.

Table 9: Input Parameters for Scenario 3

	Current Parameters
Time Step	1
Number of Hours	14
Alpha	0.8
Charge Power	0.2
Resistive Penalty	0
Event Probability	0.1
Event Time	3, 6, 12

Figure 51 shows the middle event case. The trajectory actually looks very similar to the previous case with a square penalty term, in that charging is spread out over a longer period of time. The time 21:00 has a lower cost and just like in the cases the controller decides to charge but encounters an event at 23:00. As the charging stops, the SoC achieved is lower compared to fast-charge scenarios.

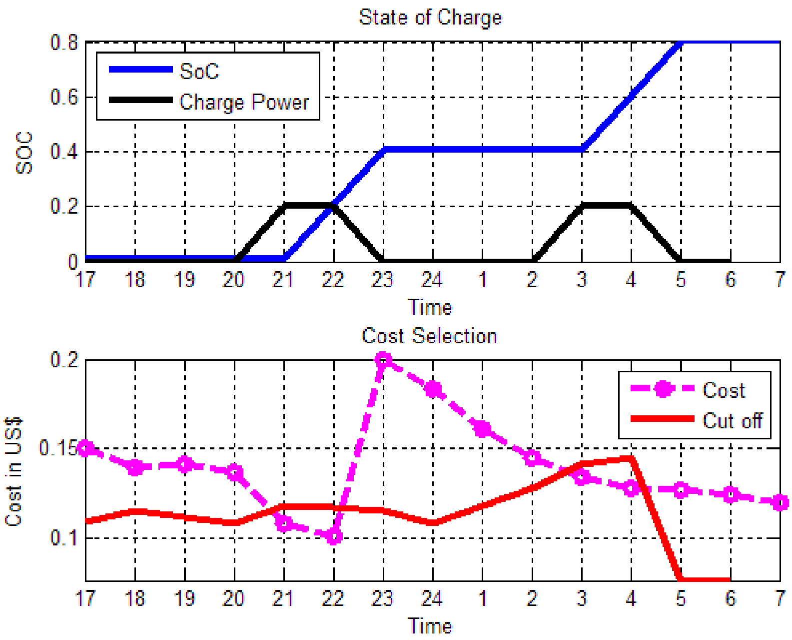


Figure 51: Controller output for slow charge

Figure 51 and Figure *52* both show the cut-off cost trajectories which describe the risk selections are much higher to compensate. The controller waits until 2:00 but takes the penalty of the higher costs (compared to 4:00) to be safe and provide charge.

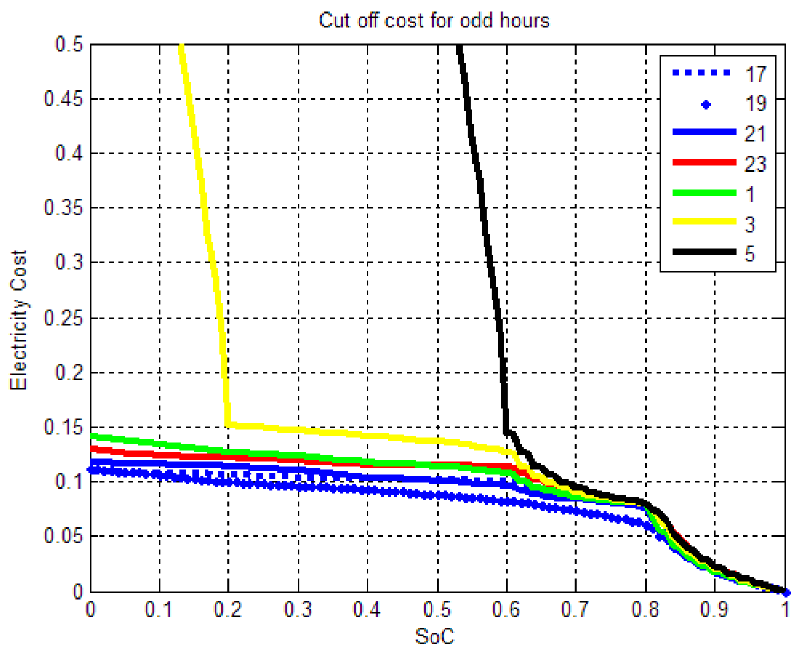


Figure 52: Cut-off Trajectories for slow charge

Figure 52 shows a very different plot compared to other cases; here at 3:00 as well as 5:00 the cut –off values are much higher for the range of SoC, indicating the willingness to accept penalties if the SoC is too low.

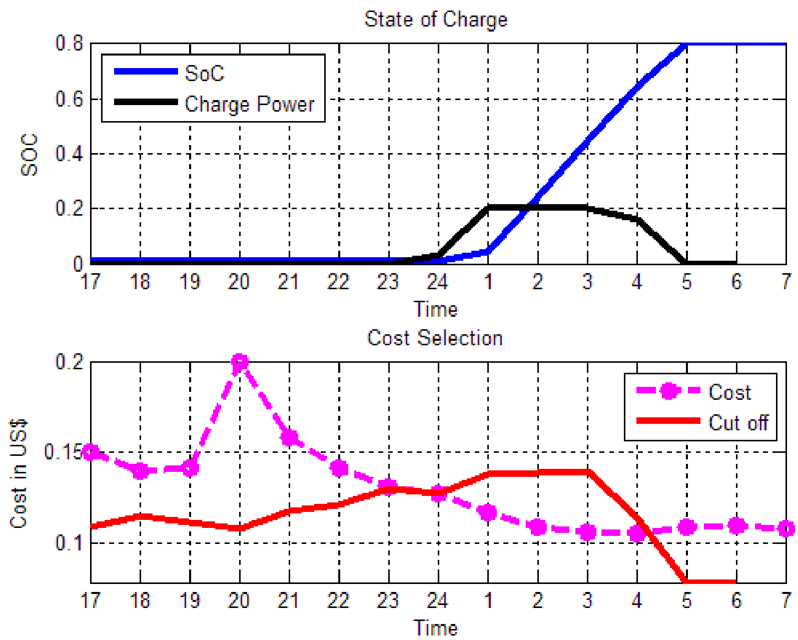


Figure 53: Slow Charge with early event

Figure 53 shows the strategy for the case with an early event. As the event causes a high price increase in the early hours, the controller completely avoids charging and waits for a really low cost. At 00:00, it picks a low cost but predicts a possible event later and for safety begins charging. It continues to provide power as the costs only fall over the period and provide SoC by 5:00. The cut-off cost penalties increase all the way through 20:00 to 3:00 (where the battery achieves 50% SoC).

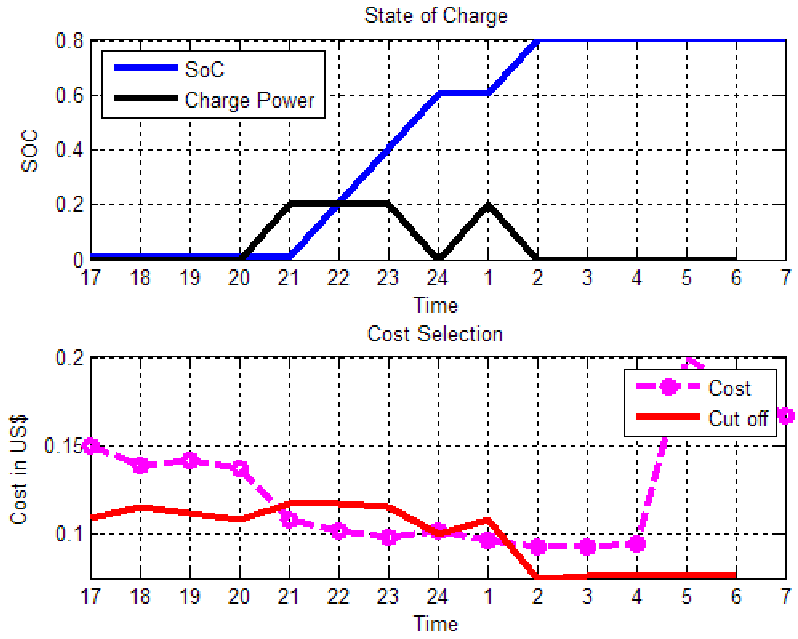


Figure 54: Slow charge with late event

Figure 54 shows a late event case; here the controller predicts an event because of the positive Pev. So, it picks a low enough cost at 20:00 and provides full power at 21:00 when the cost is low. It charges till 00:00, when the cost jumps a little; it stops charging and waits. The next hour, the cost falls again and the controller charges at full power and achieves SoC quickly. The event that occurs at 4:00, is therefore avoided completely by the controller's strategy.

6.5 Conclusions

The optimal charging controller is generated using dynamic programming, which solves a time-discrete stochastic optimization problem. It is implemented in MATLAB, which requires a custom function for this problem of medium complexity. The controller considers the required SoC and ToC (user-defined) and controls charging over the provided time period (and time period data). It successfully predicts possible price-events ('spikes') based on probability and compensates by picking hours with lower price. The main goal of the control is to provide required SoC in time, which it attempts to achieve by selecting the lowest possible costs. It is intelligent enough to accept penalty in case the charge required cannot be provided in time. The SDPM controller is a large improvement over the simple optimal controller presented before.

The prediction ability of the controller has been tested using several simulations over a 14-hour charging period of a typical night. Testing scenarios included positive and no event probabilities, effect of resistive losses and a lower charge power. In each case, the controller behaves as expected and not only predicts possible price jumps but also reacts to them in a 'safe' manner thus being able to provide SoC and not leaving the user stranded. In all cases it picks lower costs than otherwise if the charging was performed arbitrarily (for example: as soon as the EV is plugged in). The results prove that this as an optimal solution, where the EV would charge automatically, reducing charging cost and in turn offsetting high demand.

7 Case Study

The previous two chapters, 5 and 6, explain two charging strategies with the goal to minimise charging cost over a given period of time using simulation. Chapter 5 successfully showed a deterministic optimal controller which worked by picking the cheapest electricity price hours and offsetting the charging to those hours. However, it relied on the fore-knowledge of electricity prices, realistically it would only work if it used day ahead predictions (DAP) provided by the electricity provider. In practice, we have seen that DAP are not 100% accurate and the real time prices (RTP) would provide the best solution when used with the controller.

Therefore, to achieve a better solution, chapter 6 explained the use of a stochastic dynamic programming-based strategy (SDPM). It uses a time-discrete stochastic optimisation within a dynamic program, to achieve on average, the lowest optimal charging price. This was shown to be successful using simulations. The controller considers price variability via a simple grid model that allows of unexpected price rises and a gradual return to a normal grid price. The DP algorithm has two variables: the state of charge (SoC) and the current electricity cost. It traces the expected total cost based on the stochastic model and decides ‘to charge or not’ to minimize the expected (average) total cost.

Both these controllers are proven over a single day’s snapshot during which the charging had to occur, using arbitrary but representative electricity price values. This chapter uses real data from the Illinois electricity grid, provided by ComEd, to perform a case study and prove the successful functioning of the programmed strategies explained chapters 5 and 6. The data used is the same as the data used for analysis and to create the linear predictor in chapter 4. A baseline scenario is created to test the controllers over a year, to calculate the price of charging an electric vehicle (EV). The effectiveness of the linear regression predictor (introduced in chapter 4), is also analysed alongside. Furthermore, the effect of varying the important parameters in the baseline is also explained to try and cover different situations.

Section 7.2 revisits the discussion on the data used and discusses typical user driving behaviour on which a scenario has been based. Section 7.3 explains the baseline scenario created from the above discussion. Section 7.4 explains the results obtained in all the different cases. Section 7.5 presents an analysis and conclusion of the results of the case study. The case study to prove the advantage of SDPM has been presented at the SAE World Congress [16] and has been refined and presented here.

7.1 Price Data & Baseline Scenario

The case study is based on electricity markets in which the consumers are encouraged to shift their electricity usage to off-peak hours through high price alerts provided daily or hourly. This allows elastic behaviour from consumers, helping them to reduce costs. The strategies discussed in this research have been created to automate this behaviour with respect to EV charging on the basis of such electricity price information. Typically, the cost of electricity charged changes every hour or half an hour, and the electricity company communicates either the current tariff or the expected tariff development for the next day to the customer. This means that customers can move electricity intensive activities into periods where electricity is plentiful, and therefore cheap.

In USA, *'PJM'* is a neutral and regulated organisation which directs the operation for different generators. Illinois is one of the states in which *PJM* regulates the spot-electricity market and *'ComEd'* is one of the electricity resellers to the retail market. The reseller, *ComEd*, charges the end users based on the 'real time price' (RTP) from *PJM* which is determined by the average of twelve 5-minute prices from that hour, without any mark-up. The 'day-ahead price' (DAP) is also provided by *PJM,* and is the prediction based on weather, capacity, generation factors and other variables [95]. Figure 55 shows the RTP data and Figure 56 shows the DAP data, for 2013 as a colourmap.

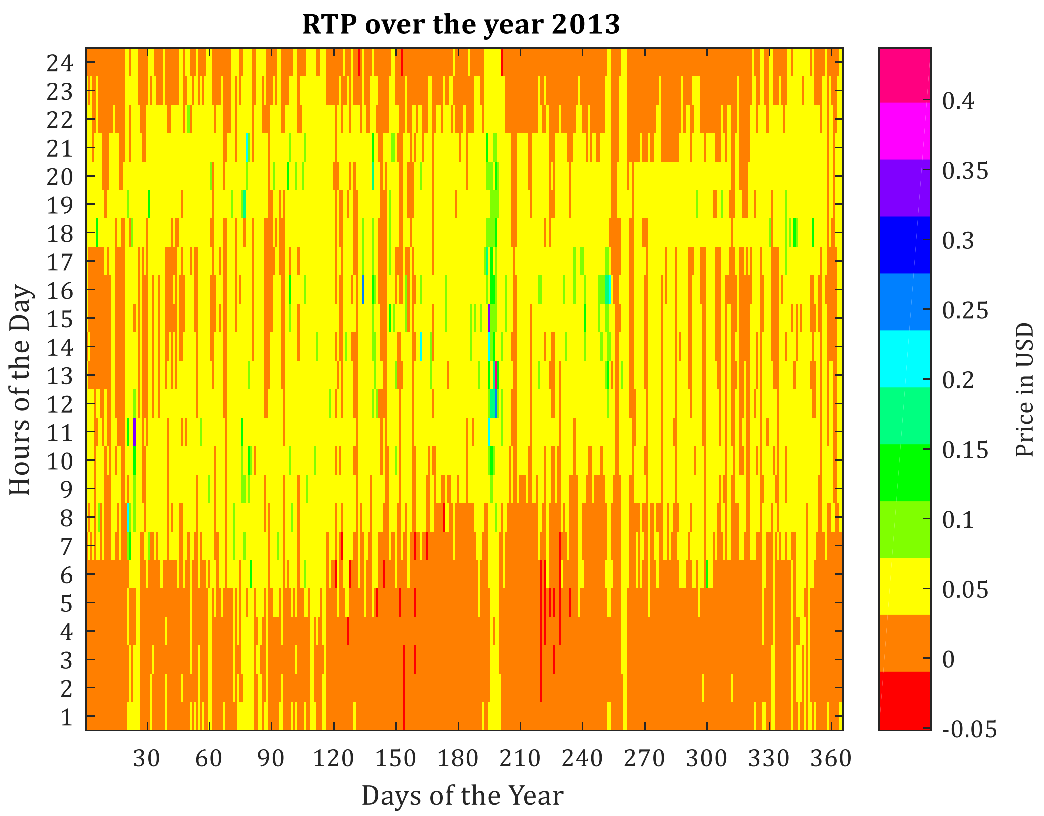


Figure 55: Real Time Price Data for the year 2013

For the case study, both tariffs are considered: DAP, where prices are set at 4:30 pm for the following day and RTP, available the hourly on the day. Retrospectively, the history of these prices is available. We have five years of price data but for the purposes of discussing the results, one year (2013) is selected (Figure 55 and Figure 56). The colourmap shows how

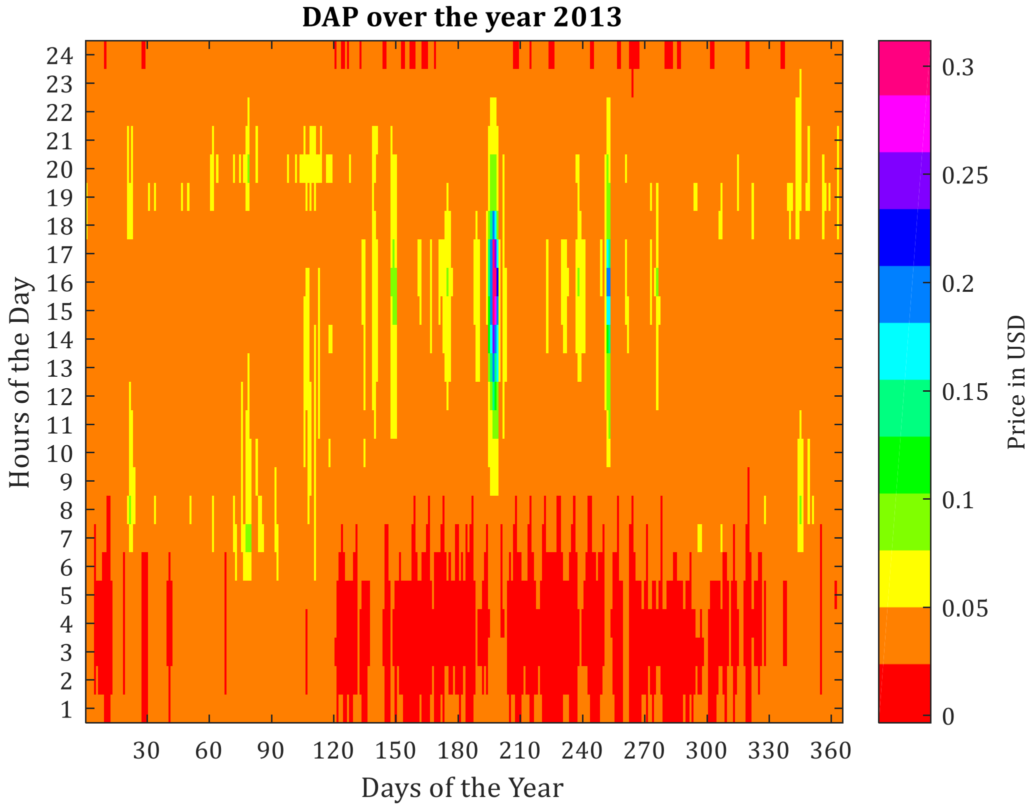


Figure 56: Day Ahead Price Data for the year 2013

the prices vary over the hours of the day, for a whole year. Clearly, there is a lot of load shifting and cost saving potential, if the right times to charge are picked. There is far more variability in the RTP than the DAP. This annual data is used to calculate the results of the case study.

Basic Situation

The charging behaviour of users is difficult to predict, although it can be derived from their usage profiles. To be able to derive representative drive and charging cycles specific to EVs would require large amount of studies in the area of EV driver behaviour, which is not in the scope of this research. Charging locations and times will vary on user type, infrastructure and geography. For instance, a company fleet user may opt to do most charging at work, where infrastructure may be provided. However, an individual user would choose to charge both at home and public infrastructure. It is likely though; human behaviour will propel users to plug-in their chargeable appliance as soon as they are home so ‘it remains charged’ by the time they need it. Most studies conclude that the tendency to charge vehicles as soon as users reach home is extremely high. There is a brief review of driver behaviour in the literature review chapter, where some of these studies have been mentioned. Studies like [35], [38], [36] and [39] conclude that the spike in charging requirements is pronounced between 5:00 pm and 7 am based on the time majority vehicles arrive home. What is more interesting is that these studies are done in different geographical regions of the world, and yet this time is of concern for peaks (regardless of whether users might charge in the morning during work hours).

The basic case assumes the use of a typical electric vehicle, the base model Nissan LEAF (24 kWh battery), being used for typical work commute during the week and in-city driving during the weekend (for example, to visit the departmental store). The car is driven to work at 7 am and back home at 5 pm with an assumed total mileage of 50 miles, with each journey taking one hour. Whilst the stated NEDC[6] mileage of the vehicle is up to 84 miles, a maximum of 70 possible miles are considered for a real-world situation, whilst using air-conditioning and other amenities in the vehicle (from the EPA, United States Environmental Protection Agency website). The user plugs in the vehicle for charging at home at 5 pm, and charging is possible till 7 am next morning when the vehicle is needed for commute to work. The charging station is a level 2 AC charging station with a typical output between 3.6kW and 7.2kW, as defined in *SAE J1772 (2012 revision) standards*[110].

Charge profile and time horizon

Figure 57 shows 24 hours as the case study looks at them. The user arrives home at 17:00 on and plugs in the vehicle to charge. Charging is allowed until 7:00 which is when the user needs the vehicle to drive. Between 7:00 and 17:00, there is a 50-mile journey that takes the user to their workplace and back home. During this period, we assume no charging takes place. This makes 1 day (24 hours) of the case, starting at 17:00. It is logical to look at a 24-hour horizon to test both the strategies (Optimal and SDPM) over a year. However, it is more interesting to see a 31-hour horizon (i.e. till 00:00 the next day) to show the strength of the SDPM strategy alone, because it can plan ahead by delaying charging to make it cheaper if there is a chance to save money by doing so.

On the other hand, the strategy in chapter 5 uses a deterministic route and is concerned only with the electricity prices in the first 14 hours when charging is allowed. Therefore,

[6] The *New European Driving Cycle (NEDC)*, is a driving cycle designed to assess fuel economy and emissions of passenger cars in Europe. It represents typical usage of a car and is comprised of four repeated urban driving cycles and one extra-urban driving cycle.

annual tests with both horizons are run, and shown for strategy comparison, but for all other variations the baseline is 31 hours. We assume DAP information has been received by the controller via a smart-grid connection prior to or as soon as the vehicle is plugged in, and historical RTP information is available in the same way. New RTP information is available hourly on the day. All the prices used are exclusive of the distribution fees which matter to the user but are not significant for study purposes because they remain constant.

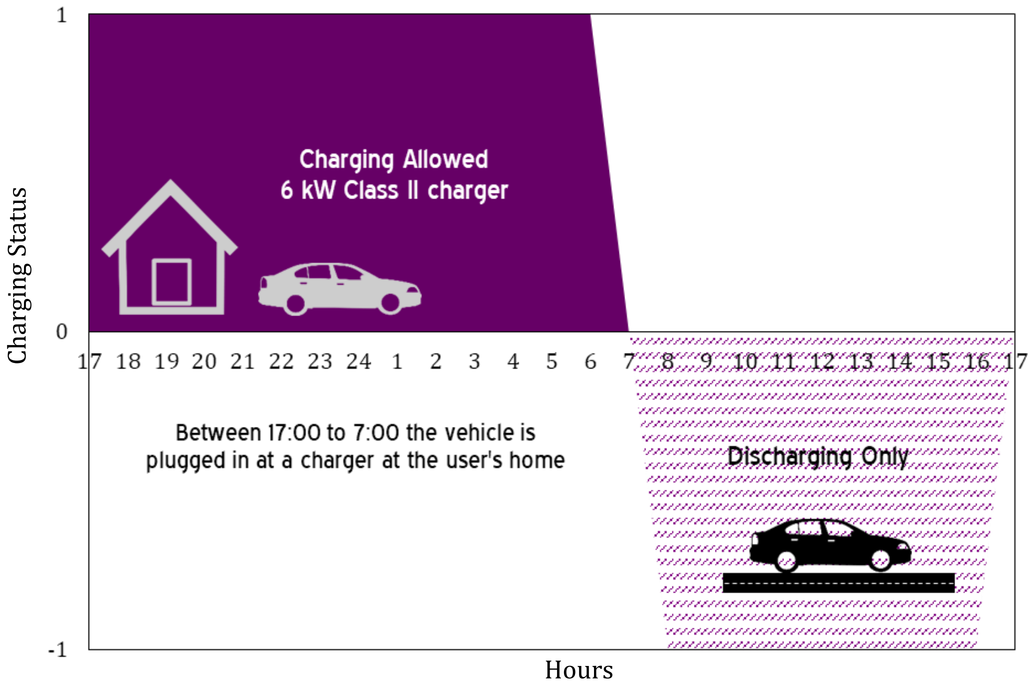


Figure 57: Charge profile of Baseline Scenario showing 24 hours

Baseline Parameters

Table 10 shows all parameters chosen for the baseline case. These have been selected on the basis of data analysis performed before in chapter 5 and assumptions made during the creation of the SDPM strategy. The idea is to try and achieve the minimum cost and the results of annual runs will be explained further in this chapter, also with respect to selection of baseline parameters and if they are a good selection. The allowable SoC range is 0-100%. The study initiates with the vehicle at 0%. The first charging period achieves either 100% or SoC required to meet the daily mileage, depending on the strategy. From then on, each day, the vehicle discharges according to fixed usage, and the initial SoC is updated before every charging period.

Table 10: Baseline Parameters

PARAMETER	VALUE
Vehicle	Nissan LEAF
Battery Size	24 kWh
Effective Range	70 mi
Journey Mileage	50 mi
Charger Power	6 kW
Decay Constant (α)	0.8
Time Horizon	31 hr
Price Event Size	5 cents
Probability of Event	10%
State of Charge Range	0 - 100%

Vehicle and Mileage: The Nissan LEAF base model was selected because it was the most widely available (and accessible) EV when the strategy was being developed. There is also a 30-kWh battery model available from Nissan, which has been explored in a case against other available pure EVs in the market, in section 7.2.2.5 of this chapter. The 24-kWh model has an 84-mile (claimed) range and fits the case of 50 mile a day travel. It is also the cheapest option for people to switch to EVs which makes it a good baseline to explore for test purposes.

Charge Power and Discharge Power: As discussed before, the charging station assumed is a class II charger based on SAE specifications which has a power output range in between 3.6 and 7.2 kW. The charge power assumed for the baseline case is a maximum of 6 kW from the charger. Discharge power is calculated as follows (normalised):

$$Discharge\ Power\ (normalised) = \frac{Mileage\ (per\ trip)}{Effective\ Range} = \frac{25}{70} = \mathbf{0.3571}$$

Similarly, the normalised charge power would be:

$$Charge\ Power\ (normalised) = \frac{Charge\ Power\ in\ kW}{Battery\ Size\ in\ kW} = \frac{6}{24} = \mathbf{0.25}$$

The following assumptions are made:

- There are no resistive losses considered in the charger or the vehicle battery
- Effective range of the vehicle is 70 miles
- EV battery temperature, ambient temperature effects are not considered

- Losses are represented by multiplying or adding penalties to the final cost (Especially for undercharging over the given time horizon)

Decay Constant: The decay constant (α) has been selected on the basis of the autocorrelation analysis performed in chapter 5. The values of the constant for RTP which are the actual prices lie between 0.6 and 0.8.

Probability: The probability of a price event occurring, is a calculation that can be made on the basis of deviation between both prices using historical data. This will of course be different on the basis of the year and time when the charging is occurring and is a varying parameter. As explained in chapter 5, data analysis for prediction purposes is not a perfect science yet, and many complicated strategies only get close to predicting such events. In the baseline, we use a 10% probability of a positive or negative event and further we test the effect of a varying probability to see the adaptability of the SDPM strategy.

7.2 Case Study Results

This section presents the results of the case-study. Sub-section 7.2.1 compares the cost reduction achieved by using the charging strategies discussed in chapters 5 and 6, versus direct charging. Sub-section 7.2.2 analyses the effect of varying different parameters (changing them from the baseline selected) on the annual charging cost, when using the *SDPM strategy.*

7.2.1 Annual Comparison of Optimal Strategies versus Direct Charging

A direct comparison of annual cost of charging is made using the baseline parameters as shown in Table 11. Each strategy is compared against charging the vehicle as soon as the user arrives home.

There is a possibility of using different combinations of prices for the purposes of optimisation and simulation, when running the optimal or SDPM strategy with a view to save on charging cost over the year. Table 11 shows the different combinations which are possible to use and combinations that are impossible (non-causal) but interesting to explore for comparison purposes.

Clearly the foreknowledge of electricity prices would give us the best result and lowest cost which would be the benchmark goal to achieve (RTP-RTP non-causal but ideal combination). Using DAP for both stages in the strategy means the deviation price would be zero. For the optimal strategy it is a simple case of it reacting to the lowest price predictions but for SDPM, it would still allow for the probability of price events or 'spikes' and adapt accordingly. When giving DAP for optimisation and RTP for simulation, the optimal strategy can better react to the hourly price time scale during the control decision stage and SDPM strategy should also have the advantage of reacting to price events.

Table 11: Price Combinations for Charging Strategies

	PRICE SELECTION FOR STAGE	
COMBINATIONS USED	**Optimisation**	**Simulation**
Deterministic Optimal Strategy	**DAP**	**DAP**
	DAP	**RTP**
	Linear Predictor	**RTP**
Stochastic Dynamic Programming (SDPM)	**DAP**	**DAP**
(Baseline)	**DAP**	**RTP**
	Linear Predictor	**RTP**
IDEAL (Fore-knowledge of real prices)		
Stochastic Dynamic Programming (SDPM)	**RTP**	**RTP**
Optimal (Deterministic)	**RTP**	**RTP**

Other combinations which would use RTP in the optimisation stage are also not possible but would not give us either a benchmark or important learning about the reaction of the strategies and are therefore ignored. As far as the linear predictor is concerned, the most important, first order predictor output is tested alongside the other combinations based on the conclusions of chapter 5.

Figure 58 shows the cost of charging over the year 2013 when the strategies are given a 24-hour horizon. There is a clear advantage of using either the optimal or the SDPM strategies over early charging.

When using the *Optimal Strategy* (light grey), there is already a significant advantage compared to charging directly. Using DAP for both optimisation and simulation saves ***$101.01***. In this situation, the strategy relies completely on the day ahead prediction, which is not always accurate as discussed in chapter 4. There is ***$6.65*** more saving when using RTP for simulation stage because this allows the strategy to react to the hourly price on the day during simulation, after having used the DAP for optimisation. When using the prediction provided by the linear predictor instead of DAP in the optimisation stage, the result is very similar to using DAP but is more expensive overall.

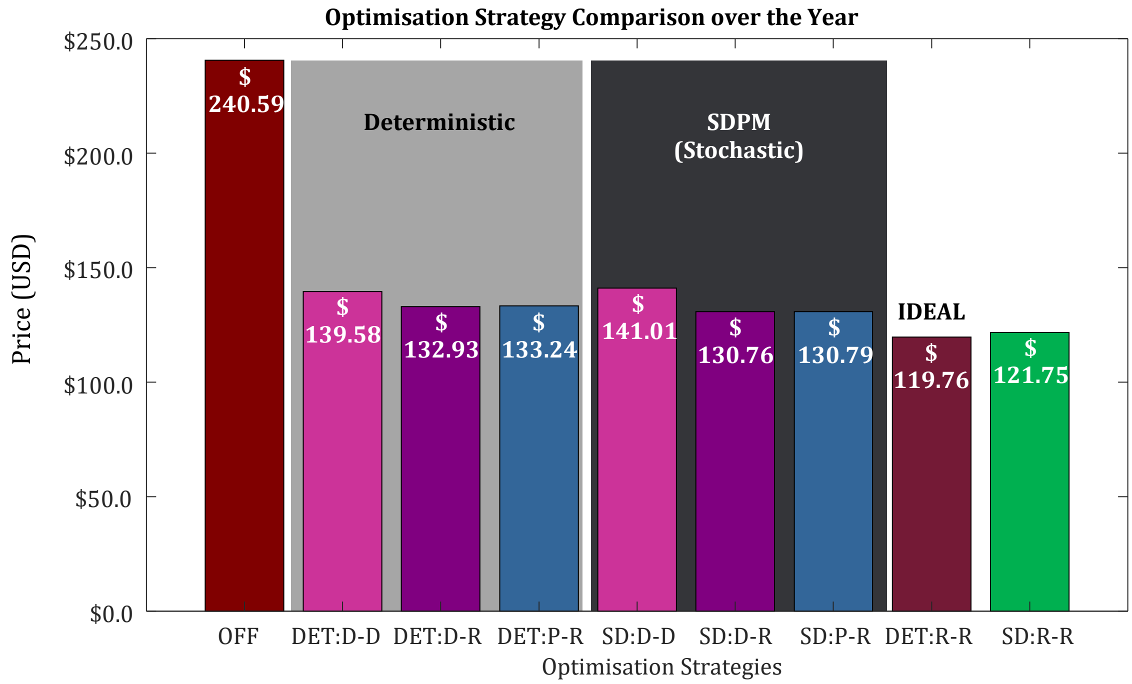


Figure 58: Case Study Result for year 2013 (24-hour horizon), comparing Optimal (light grey), SDPM (dark grey) versus using early charging (red) and ideal situation (green)

Looking at the results from the *SDPM strategy* (dark grey), using DAP for both stages, the cost savings are similar but slightly less at ***$99.58*** than the *Optimal Strategy*. This is due to the reaction of SDPM to a probability of an event spike which makes the strategy spread the charging times out, taking a small penalty at times to try and be safe in case of a spike. When using a 24-hour horizon, SDPM will always try and provide the *maximum possible* SoC by the end of the first charging cycle compared to deterministic strategy, which will charge at the cheapest predicted hours and provide the *maximum required* SoC.

Using RTP for the simulation stage, SDPM performs better and increases savings by ***$10.25.*** This is as expected because SDPM can make a better decision with the real prices and react hourly. Along with the ability to react to a price spike it can make a better control decision. When using the linear predictor's prediction instead of DAP, the saving is just a few cents lower.

Figure 59 shows the case study results when the strategies are given a ***31-hour horizon***. The results are consistent with the 24-hour horizon test but with improvements when using SDPM. A 31-hour horizon, allows SDPM to delay charging to the next charging cycle, if the user does not require the extra state of charge in the battery. The prices when using the optimal strategy are the same as a 24-hour horizon because it is only concerned with the first cycle of the charge profile, whereas SDPM has a notion of time. Of course, this is speculative on part of the strategy but it proves the advantage of using stochastic control to achieve a better prediction.

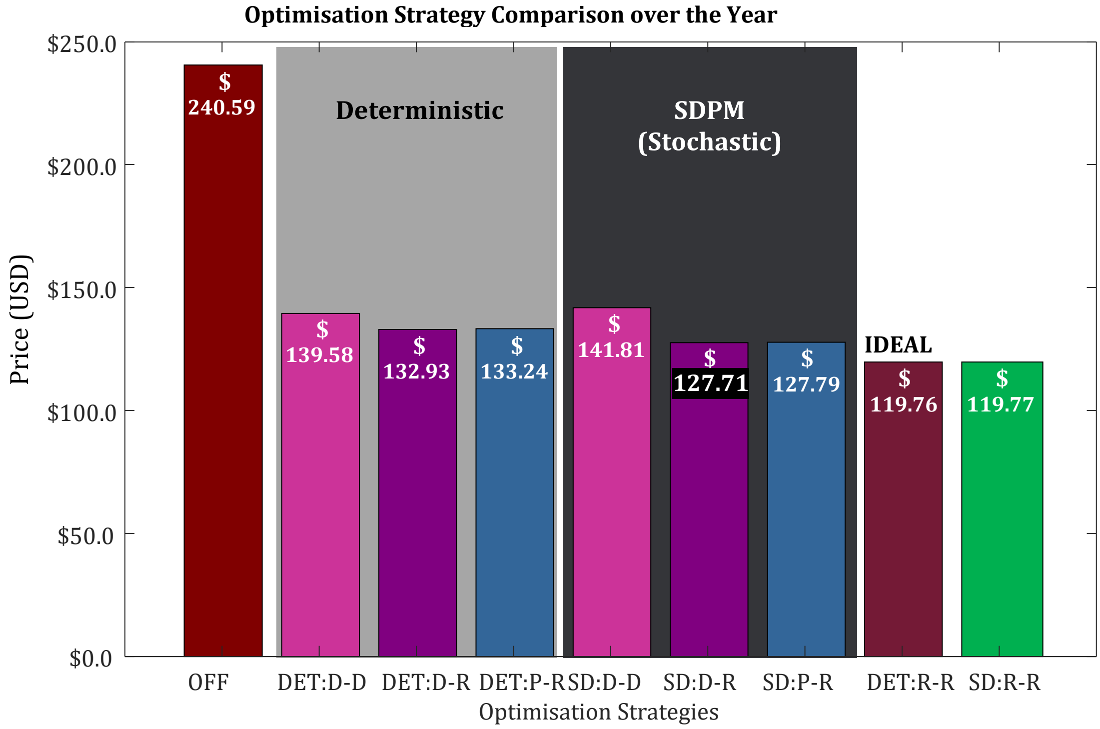


Figure 59: Case Study Result for year 2013 (31-hour horizon), comparing Optimal (light grey), SDPM (dark grey) versus using early charging (red) and ideal situation (green)

The green bars show the 'ideal' goal. Although non-causal, they show the benchmark of a maximum we can achieve. When using RTP in the optimisation stage with the *Optimal Strategy*, it is able to pick out the real cheapest prices and delay charging to those hours which is not possible without the prior knowledge of RTP i.e. the real prices on the day. SDPM does the same but with the probability of a price event, it spreads out the charging rather than delaying till late and in doing so, accepts a small penalty to be safe. Interestingly, the cheapest cost over the year is ***$ 119.76*** which is ***only $11.00 cheaper*** than using SDPM (DAP-RTP) combination when using the 24-hour horizon.

When using the longer horizon, the effect of the SDPM Strategy having a notion of time can be seen. The savings with *SDPM Strategy* using RTP are only a few cents lower (compared to $1.99) than the *Deterministic Optimal Strategy ideal*. It can be argued that the simpler solution provides a better result when only using DAP, which are available the day before (as a prediction). However, it becomes clear why the *deterministic optimal strategy* performs better with just DAP compared to *SDPM* when we consider that, *SDPM* defers charging above the maximum required charge, to the next charge cycle in the 31 hours. It awaits a better price but overall the time period in the next charge cycle between 17:00 and

00:00 is not likely to provide much lower prices. If the horizon was a full 48 hours, i.e. *SDPM* had until 7:00 to the end of the next cycle to fully charge, it provides a better price of ***$140.20***. In fact, with a 48-hour horizon, the DAP and RTP combination achieves an annual cost of ***$123.96***. In contrast, the *deterministic optimal strategy* only considers the first charge cycle and uses the DAP to provide the best possible result.

It is evident that using RTP together with DAP gives a ***$9-14*** advantage which is significant when we consider the theoretical optimum is only ***$119.76*** which is just ***$7.45*** cheaper than what the *SDPM strategy* can achieve. In the combined price case, *SDPM* is the clear winner because it accounts for the probability of price disturbance and can make use of the more up-to-date information of the RTP in both charging cycles. The fundamental difference is that it deals with risk better and manages it based on price versus time, assuming we have more accurate price information available on the day. This is proven by the result which shows that widening the horizon given to *SDPM* leads to much higher savings. In all cases, *SDPM* performs better than the *deterministic optimal strategy* which only works with the 24 horizon and achieves ***$132.93.*** Table 12 shows the percentage improvement in cost-saving, when increasing the charging horizon (and hence known prices) provided to the *SDPM controller.*

Table 12: SDPM Improvement with larger horizon

Horizon in Hours	***SDPM (DAP:RTP combo)***	***Percentage Improvement***
24	$130.76	Comparison Point
31	$127.71	2.3 %
48	$123.96	5.2%

SDPM effectively manages risk based on probability of events and has a notion of time. This means, if there was a better indication on the RTP, there could be more benefit and better results. During the times this research was done (2012-2016), RTP data was available hourly. This is still the case for billing customers i.e. they get billed on the hourly price but ComEd now provides the 5-minute prices that lead to the hourly price on which the customers are billed. The *PJM real-time hourly market* of Illinois provides hourly prices to different electricity companies like ComEd and they pass on these prices for billing without a mark-up. This hourly price is an average of the twelve '5 minute prices' of the hour [111]. In the last half year, the 5-minute price data has been made available, which means, the *SDPM strategy* can have a faster timescale prediction which it can use for optimisation, potentially leading to significantly better results.

The success of the more complex solution, *SDPM,* is based on having that accurate hourly price information for the simulation stage. The results show that the RTP lead to a significantly better cost saving and load shifting opportunity, and any good strategy will rely on having these prices to provide an optimal and non-trivial solution.

Therefore, we choose the baseline scenario result as the SDPM strategy, using the DAP-RTP combination with a 31-hour horizon, which achieves an annual charging cost of ***$ 127.71 (highlighted in black)*** to compare the effects of changing parameters further in the chapter. This is ***$112.88 cheaper*** than not using any charging control.

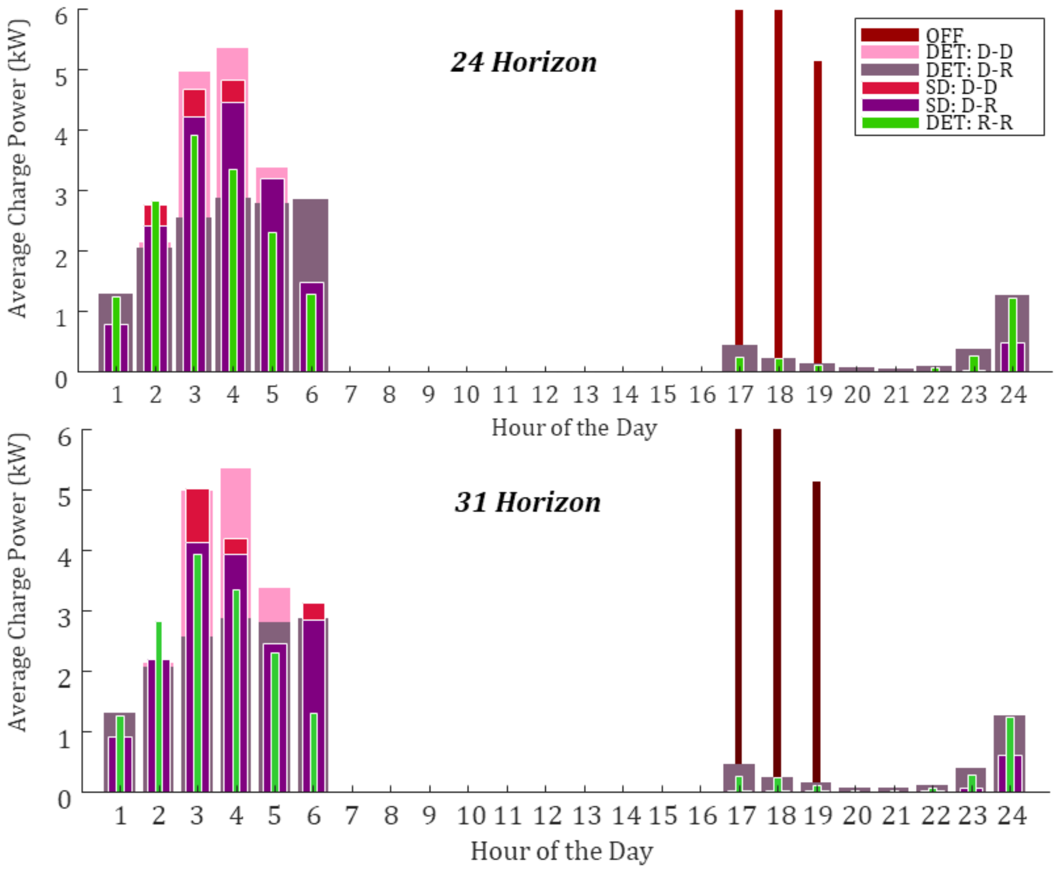


Figure 60: Application of Charging Power by hour of the charging cycle

Figure 60 shows the histogram of power usage during the different hours of the charging period, comparing the use of optimal charging strategies versus direct charging. It is evident that when charging directly, i.e. as soon as the vehicle is plugged in all the charging power is applied instantly and the vehicle is charged. The ideal case is shown in green which is the *Optimal Strategy Ideal.* The charging is shifted to the times of lowest cost because the knowledge of real prices is assumed*.* As can be seen, the baseline ***(SDPM D-R in dark purple)*** is fairly close to the ideal, whilst being a strategy and price combination that is possible to use.

Comparing the Optimal and SDPM strategies using the DAP-RTP combination, it can be seen that the SDPM spreads the charging over the later hours if required to allow for price events but the optimal strategy waits to find the lowest price and tries to charge as quickly at those times. As a result, SDPM doesn't always choose the expected hours because it accounts for the probability of a price change. When relying completely on DAP, both strategies are

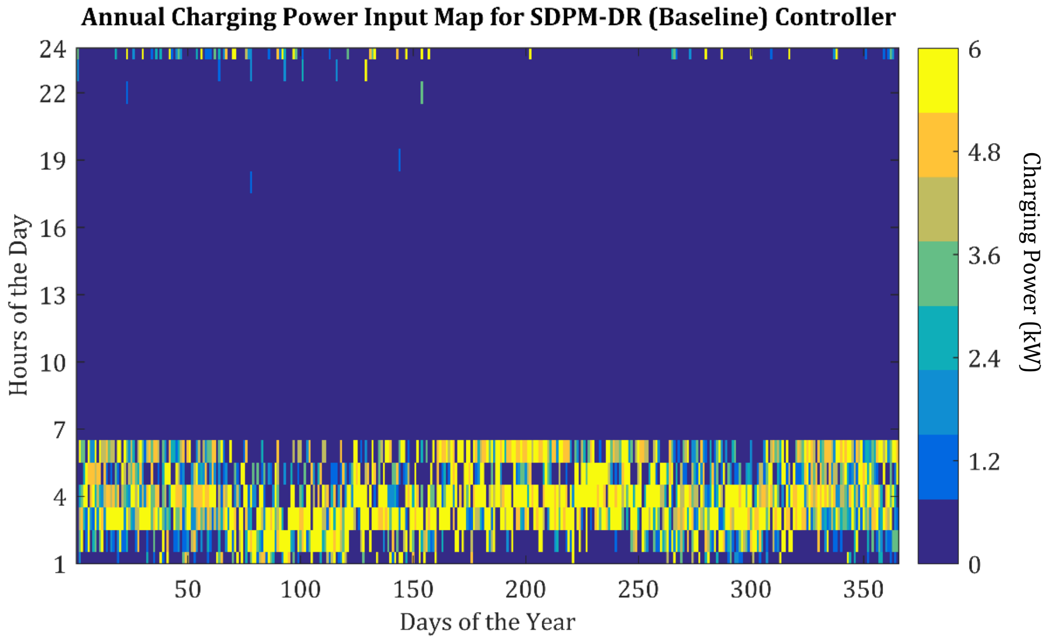


Figure 61: Application of Charging Power over the year, when using SDPM-DR Baseline Controller

further from the ideal hours, which is to be expected due to the deviation in between the DAP and RTP.

Figure 61 shows the annual charging power application pattern, when using the baseline controller (SDPM-DR). When comparing it to Figure 62, which shows the ComEd area power demand for the same year[7], it highlights the success of the SDPM controller with respect to load shifting.

The controller shifts the charging load away from the peak times, during the stipulated charging period (17:00 to 07:00). There is hardly any charging in the early time-period between 17:00 and 24:00, which is clearly the period with the highest power demand. The highest charging power seems to be applied during the morning hours of 01:00 to 06:00. Therefore, over the year, it successfully shifts the charging load to off-peak times.

[7] Available from the PJM website retrospectively [65].

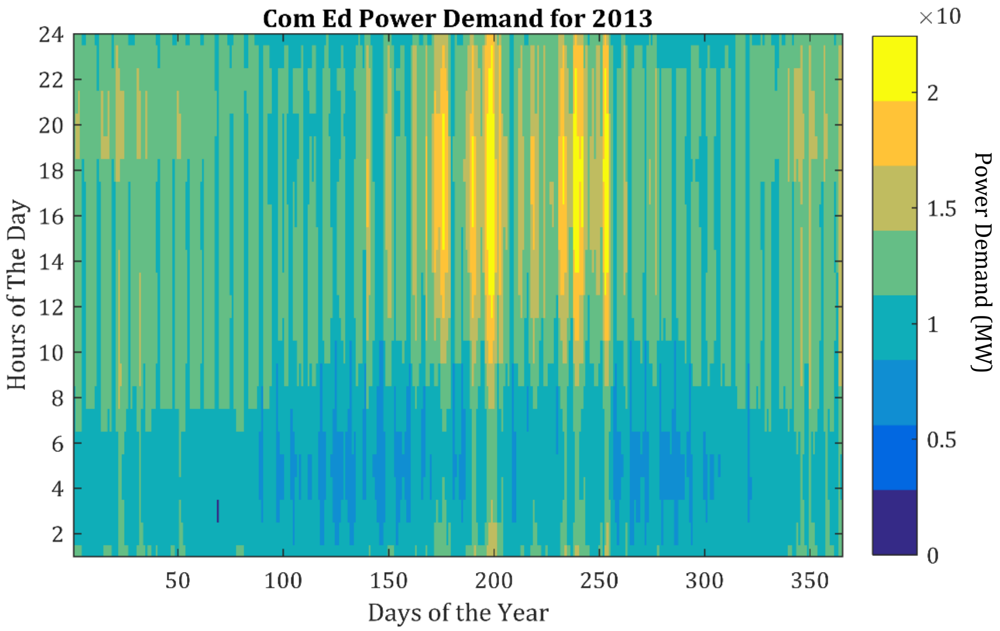


Figure 62: Power Demand for ComEd distribution area for 2013

Figure 63 shows the cumulative use of energy over the entire year, versus the price. When charging directly, the average cost of energy is naturally high. The green line shows the ideal line, using the *Optimal Strategy* and it is evident that the baseline ***(SDPM DAP-RTP)*** follows it closely. The ideal achieves lower costs throughout whereas the SDPM Strategy is more expensive to begin with and then reacts at lower prices overall until the end. This plot also shows the evolution of the two strategies explored, compared to direct charging.

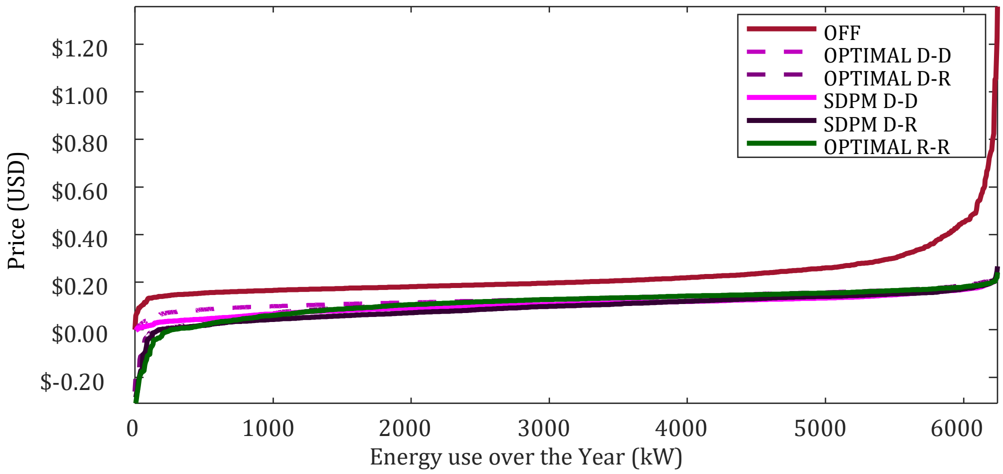


Figure 63: Cumulative Power Histogram against hourly cost over the year

7.2.2 Parameter Variations

7.2.2.1 Varying Price Event Size

The baseline uses a possible ***5 cent*** price variation (price highlighted in black) as the possible 'spike' based on the 10% probability given to the strategy. As explained in chapter 5 and 6, a price spike usually causes a parabolic delay before the prices return to normal and the grid model does mimic this behaviour.

Figure 64 shows the effect of varying this parameter whilst keeping all others constant including the probability. The shape of the curve, clearly has a minimum at ***5.5 cent*** before which the overall price increases again. Therefore, the selection of the baseline is good when looking at 1 cent intervals, but even with 0.5 cent intervals, the baseline is only ***$0.17*** higher than the minimum over the whole year.

With the probability of an event constant, when the possible event size is small, the SDPM strategy still takes measures to make sure required charge is provided and charges early depending on the prices. Ideally, the strategy would charge as late as possible but to account for the probability, it would charge a little earlier and take the small penalty by not choosing the lowest price. However, the smaller price event size means that the delay for prices to settle is shorter and the strategy could have charged later to save cost. It is safer to charge earlier though, as the penalty for not being able to provide full charge is much higher.

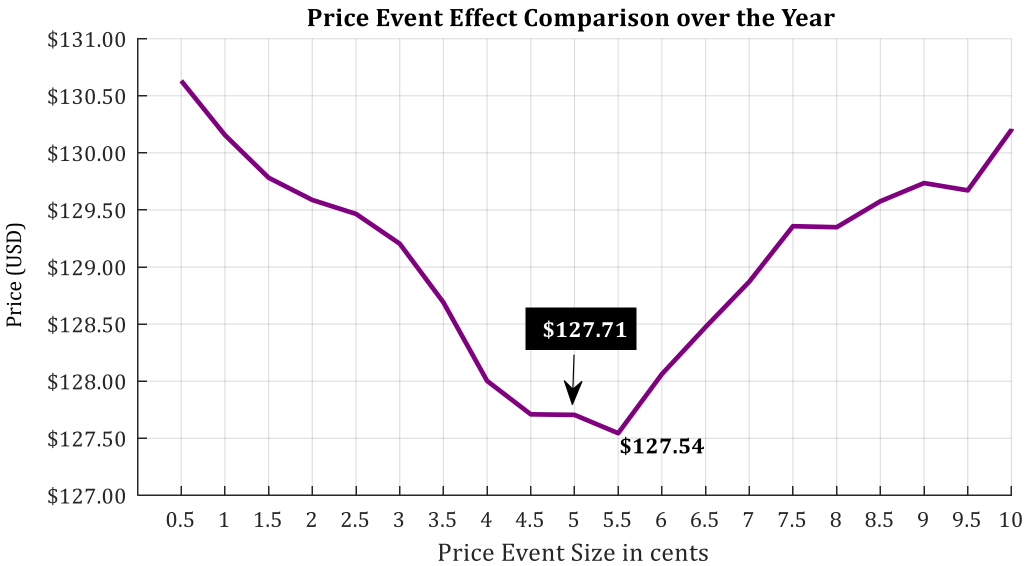


Figure 64: Effect on annual charging price with different Price Event Sizes

As the event size increases, the saving by reacting to the event probability is higher. After the minimum possible at 5.5 cents, the overall price shoots back up. The jump at 6 cents is sudden before it plateaus and jumps again. Larger event sizes mean the strategy goes into 'panic' mode. It charges the vehicle first early to avoid the event but the remaining charge as late as possible. At the end it must charge in panic during a higher cost (before the parabolic delay has levelled out) and takes a penalty which would be lower than not being able to provide the required state of charge (SoC) by 07:00. These are the points at which the penalty applied for not providing the charge is higher than the penalty of charging at a slightly higher cost.

7.2.2.2 Varying Decay Constant (α)

Figure 65 shows the effect of varying α whilst keeping all other parameters constant. The decay constant is selected based on the autocorrelation analysis of the electricity price data. The coefficients of the exponential curve for the autocorrelation lag sit in between 0.6 and 0.8. The figure here shows that the selection of α is optimum at ***0.8***, where the minimum overall cost is when doing the annual case study.

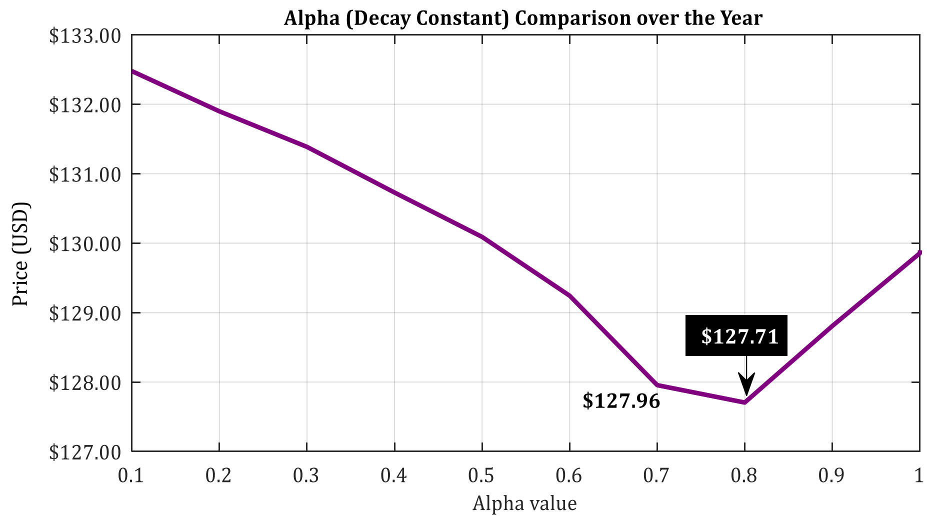


Figure 65: Effect on annual charging price with different Decay Constants (α)

The decay constant is the constant that defines the rate of decay between the costs of one hour to the next. The grid model (explained in chapter 6) is as follows and helps to see the effect of α.

$$x_{2_{k+1}} = \alpha x_{2_k} + d_k \tag{49}$$

As the decay constant gets higher, the cost of the next time step increases as well. It fulfils the decay trend as seen in the data analysis (chapter 4), and at the correct range of the price data's decay constant, it leads to the best possible result for the strategy.

7.2.2.3 Varying Probability of Price Event

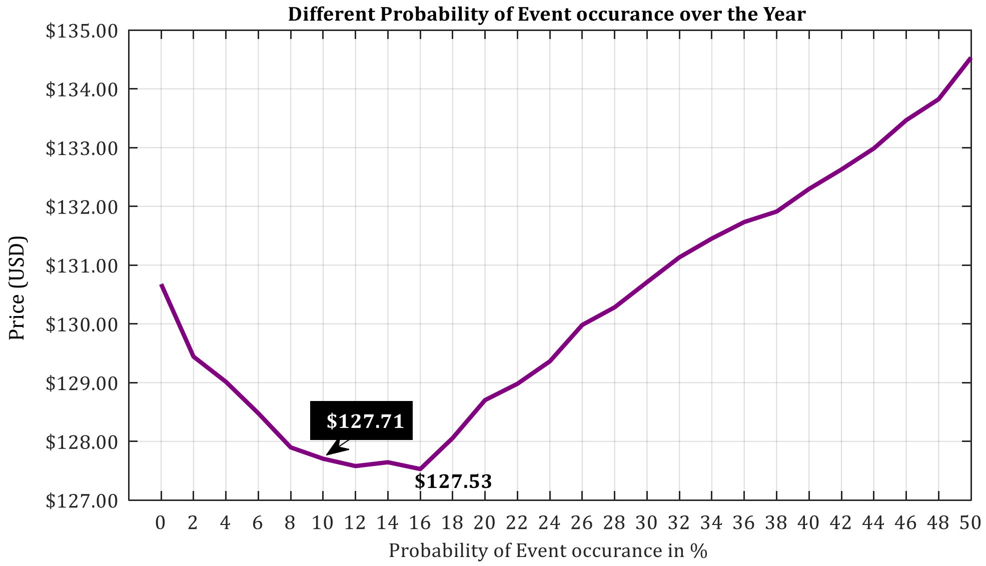


Figure 66: Effect on annual charging price with increasing probability of price event (spike)

Figure 66 shows the effect of increasing event probability on the overall annual cost of charging when using SDPM. The baseline is marked in black, at 10% event probability which is very close to the minimum possible at 16%. The annual price in between 10-16% is very close and as Table 13 shows, the baseline is only ***$0.18*** more than the minimum. At 0% probability, the price is higher, then it reaches the minimum and as the percentage probability of event (or spike) increases, the price increases again.

Table 13: Baseline versus Minimum Probability choice

Probability	Price	Difference
10%	$127.71	
12%	$127.58	$0.13
14%	$127.65	$0.06
16%	$127.53	$0.18

This is as expected because the SDPM strategy reacts earlier and earlier to a higher probability of event, taking the penalty of charging at a higher cost to avoid the penalty of not being able to provide a required charge when the price jumps. At 0% SDPM effectively acts in a similar way to the Optimal Strategy, thus not achieving a lower cost.

7.2.2.4 Effect of Varying Charge-power

Fast charging is becoming the norm as EVs become more mainstream, but SAE standards J1772, also mention a level 1 AC charging station with a typical output between 1.4kW and 4kW. This is closer to a normal output from a home plug using 120V, and it is interesting to consider whether a strategy like SDPM could save much over the year. At the same time, there is potential to save more when using faster chargers. Level III chargers can provide nearly 12kW and super-fast chargers can provide higher than 20kW.

Figure 67 shows the effect on overall charging cost when using different chargers. There is clearly an advantage of using fast charging but this advantage levels out, giving less and less advantage as it goes higher, proving that the law of diminishing return applies. When the charger is slow (3kW), the SDPM strategy is capable of throttling charging power where possible at higher prices but it has to charge at these moments regardless because it takes longer to charge overall. This achieves a higher annual cost compared to the baseline 6kW charger.

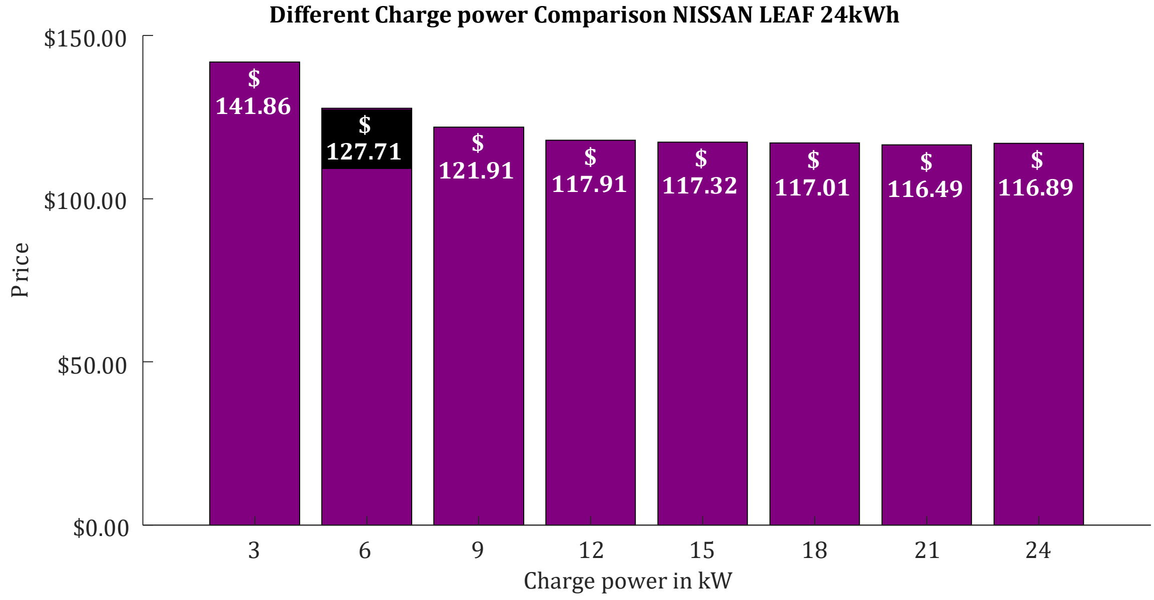


Figure 67: Effect on annual charging cost when using different chargers

The higher 9 and 12kW chargers show a saving of ***$5.80 and $9.80*** over the baseline, respectively. The higher the power from then on, the lower the saving following a decaying curve. The saving between 12 and 15kW, is ***$0.59*** and 15 and 18kW, is ***$0.31***.

7.2.2.5 Comparing Different Vehicles

Table 14 shows a list of electric vehicles with their specifications, which are compared in this section with the SDPM strategy applied to them over the 2013 year when charging.

Table 14: Electric Vehicles which are compared

Vehicle	Battery Size	Range (Effective EPA)
Nissan LEAF	24 kWh	70 mi
Nissan LEAF	30 kWh	107 mi
BMW i3	33 kWh	114 mi
TESLA 75	75 kWh	230 mi
TESLA 100	100 kWh	300 mi

Figure 68 shows the results of the different EVs charged using SDPM versus the baseline LEAF. The baseline scenario is used, where the user does 50 miles in the vehicle per day and uses a 6kW peak charger at home. The LEAF 30 shows a good improvement in overall cost of charging over the 24-kWh version. As expected, the BMW is slightly more expensive to use but it has 168bhp, using the battery quickly compared to the 108bhp power LEAF 30. (The normalised discharge power considered is still, $(Mileage\ per\ trip/Effective\ Range)$). Comparing the higher specifications in terms of power, battery size and range, the TESLA models give encouraging yearly charging costs for the baseline scenario which are cheaper than the LEAF 24.

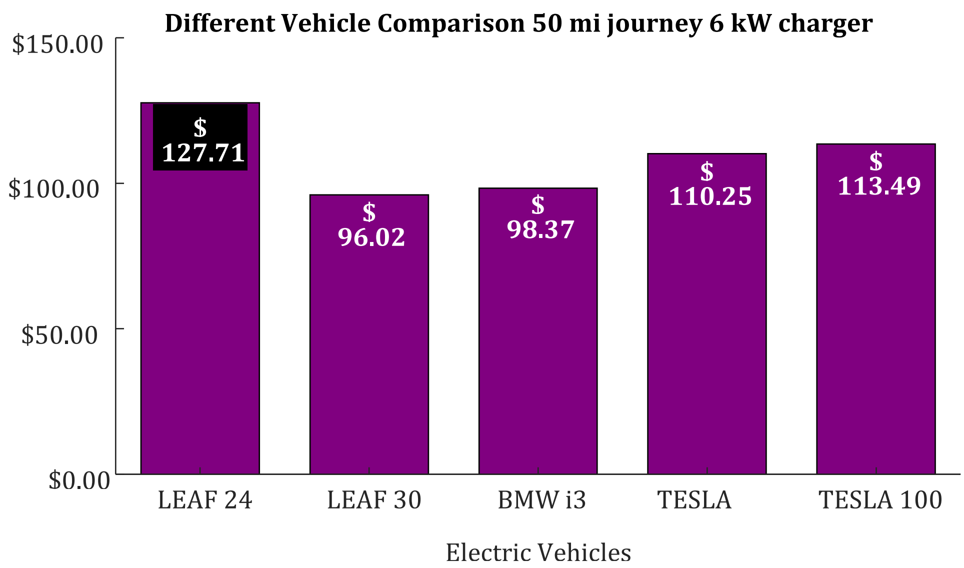


Figure 68: Charging different EVs using SDPM over the year 2013

Figure 69 shows the comparison of the other EVs in a slightly different scenario where the user does 100 miles a day and uses a more powerful 12 kW peak charger at home. The baseline LEAF 24 cannot be compared because it has a maximum range of 84 miles. As expected, the results are consistent with the above scenario but with the total cost of charging higher as more mileage is done over the year. It is interesting to see that the TESLA vehicles do not cost much more than the LEAF 30 and BMW i3 in terms of charging cost when using SDPM.

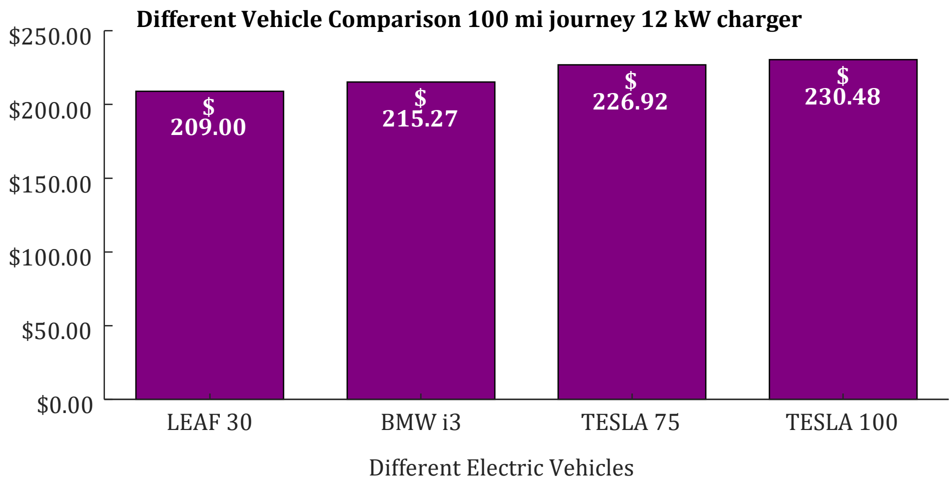


Figure 69: Charging different EVs using SDPM over 2013, 100-mile journey with 12kW charger

Calculating the energy cost per mile from the above for the different vehicles, gives us the following results.

$$Energy\ cost\ per\ mile = \frac{Energy\ cost\ per\ day}{number\ of\ days}$$

Where,

$$Energy\ cost\ per\ day = \frac{Cost\ of\ Charging\ per\ day}{Daily\ Mileage}$$

Figure 70 shows the energy cost per mile for each EV based on the baseline scenario for this case study and using the SDPM strategy to charge the vehicle each day. It is worth noting that these prices don't include distribution costs of the electricity provider and thus the value for each EV is very low. However, it gives us a good idea about the difference of energy cost of the vehicles compared when using SDPM to charge them. Therefore, the LEAF 30 has the cheapest energy cost per mile but as noted above, the TESLA EVs have a surprisingly low energy cost.

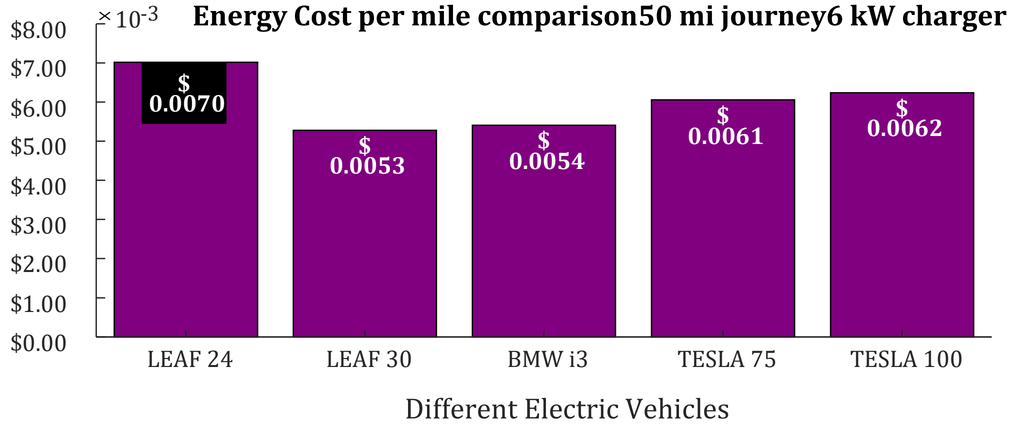


Figure 70: Energy Cost per Mile for different EVs

7.3 Analysis & Conclusions

This chapter has shown a successful case study result in favour of the two strategies that have been developed and discussed in this research project and thesis. There is a clear evolution and improvement going from direct charging to Optimal and SDPM strategies. Using the example of the year 2013 with the baseline case explained in section 7.2.1, the *deterministic optimal strategy* saves ***$107.66*** and the *SDPM strategy saves* ***$112.88.*** The maximum possible saving is ***$119.76*** using the ideal and non-causal combination which is only ***$7.95*** better over the year than the achieved optimum using *SDPM*. Moreover, the baseline parameters used for the *SDPM strategy* have been proved to be the minimum or very close to minimum possible, by varying the parameters one at a time.

The *Optimal strategy* is computationally simpler and is still ***only 11% higher*** than the benchmark. The *SDPM strategy* can predict price spikes and has a notion of time. It can react better than the *Optimal Strategy*, over a longer time horizon and therefore performs better. The optimum is ***only 6.64% lower*** than the SDPM result and therefore, *SDPM* is a small but clear improvement over the *Optimal Strategy.* It can be argued that the gains achieved follow the law of diminishing returns and the question arises: Would it be worth creating a more computationally complex strategy (and therefore requiring more processing power and cost) to achieve the extra 6.64%?

The results also highlight the success of the strategies in shifting the charging load to off-peak times. Figure 60, Figure 61 and Figure 62 show this well, by showing the times of charging power application when using these strategies, versus charging directly. *SDPM* successfully offsets the charging to the hours when the cost is low and so is the demand for electricity. The *deterministic optimal strategy* is successful in doing this as well but *SDPM* reacts better to unknown deviations between the RTP and DAP. It can be concluded from the difference in price and power usage (versus time), that it is essential to use such a strategy to charge an EV, but it is worth noting that this case study considers only one vehicle. There would be other concerns when there is many EVs to consider because if the number is high enough and all vehicles charge at the lower demand times, this in turn may increase the demand and therefore price at those hours.

Despite *SDPM* being more complex than the *deterministic optimal strategy*, it has a fairly low computational cost compared to other control methods like machine learning, fuzzy logic, or a combination of control methods. As a MATLAB code running the control strategy over annual price data, *SDPM* takes ***2.49 minutes*** and the *Deterministic Optimal Strategy* takes ***2.05 minutes*** to run on a 3rd generation Intel core i5 processor[8]. Although, the computation cost in terms of processing power and time is not high, SDPM is overall more effort, both in coding and performance. For example, the *deterministic optimal strategy* can be written in approximately ten lines of code, whereas, the core coding of *SDPM* is more than a hundred lines (split into a few functions).

[8] A Microsoft Windows 10 personal computer, with 8 gigabytes of random-access memory, 1.8Ghz with available turbo-boost up to 2.10Ghz quad-core Intel core i5-3337U. The programs run on MATLAB 2016-2017 (64-bit) versions.

8 Summary and Outlook

8.1 Review

This research has shown that smart EV charging offers a joint opportunity for saving the user money and electricity load shifting. Two strategies have been discussed in detail to achieve charging cost minimisation, using different approaches. The results show that both the savings and the load shifting effect are very significant, and the algorithm achieves a good part of the theoretical optimum.

Key Findings

Firstly, a *linear predictor* is proposed based on a linear regression analysis of historical price data, to try and achieve an improved prediction compared to DAP. The predictor is based on the available and derived factors from within the price data sets. The findings show that this predictor achieves a slightly better prediction for real prices, for the data set used. The improvement is so slight that over different years it is possible for the prediction to not be reliable. This means that the provided DAP are a fair indication of the RTP in the data set.

The next section discusses the *deterministic optimal strategy*, and simulation results proving the working of it. This achieves a solution to the optimal charging problem, by following a deterministic route to achieve the goal. Technically, the solution is sub-optimal because of the assumption that the provided price prediction is accurate and considering these as the real prices, the strategy optimally picks out the cheapest times for charging. In turn it provides the cheapest overall cost and a fully charged vehicle at the deadline.

Followed by this, the more complex solution which uses *stochastic dynamic programming* is explained. It performs better than the first controller and achieves the optimal solution which is proved by simulation. The SDPM controller takes into account the stochastic nature of the electricity prices, which is responsible for the RTP being different at times to the DAP. It uses a probability to account for price spikes and a grid model to simulate the sudden increase in price and the gradual fall back to normal prices. It successfully avoids the high price hours and in addition is able to avoid potential periods of sudden price changes to come. It spreads the charging cost across the time horizon it has and provides the lowest possible cost whilst providing the required SoC in time.

Finally, a case study based on real price data from the Illinois electricity grid is discussed. It shows the possible savings which can be achieved by using the strategies versus charging instantly. Working within the assumptions, there is a clear advantage in using the strategies presented in this thesis which show a saving of up to ***$112.88*** compared to charging the vehicle directly.

Summary

In a spot-electricity market like that of Illinois, USA, price information is available at the hourly timescale. One set of prices (DAP) are available as a prediction the day before. The second set of prices are available hourly on the day (RTP). Naturally, the fore-knowledge of prices is the ideal situation to know when to charge. The attempt to develop a *linear regression-based predictor* from available factors provided by the electricity distributors pricing tariffs showed that there is a strong correlation in between the provided DAP and the RTP, but with significant differences over the year. The linear predictor based on the DAP as a main factor itself shows to be a good approximation but not comprehensively better than the DAP itself.

Testing the predictor over different years shows that it cannot always be relied upon to give a better result. Some of the higher order predictors show slight improvements in prediction, but there is also evidence of overfitting for the more complex predictors, resulting in worse performance on the verification data. Overall, the prediction provided by the electricity company (the DAP) is good but not always accurate. The *case study* results (section 7.2) reflect that, the price saving achieved using the predictor (instead of DAP) is only marginally better or worse. The results of verification in chapter 4 also show that over different years, there is no real trend to show that using this predictor gives an advantage.

The first step of the research looked at using these DAP to decide the times to charge the EV. It successfully shifts the charging from the higher priced times to the lower priced times. This is a deterministic route to an optimal solution to automate the delaying of EV charging. It assumes that the DAP are accurate; and if this were true, it provides a simple solution to implement. The problem of electricity price 'spiking' due to problems in the grid, high demand, and other effects lead to the real prices (RTP) being different from DAP. These are very difficult to predict. Fore-knowledge of these prices would be the ideal situation. Developing a solution that could take this randomness (which leads to unknown real prices) into account was the next logical step.

To account for the price-spikes, a strategy using dynamic programming with the addition of a stochastic disturbance was developed. This controller solves a time-discrete stochastic optimization problem, taking into account the required SoC and ToC (user-defined) to control charging over the provided time period. It successfully adapts to price changes based on probability and compensates by picking hours with lower price. It uses the real price and price prediction more effectively to make a control decision when compared to the *deterministic optimal controller.* The main goal of the controller is to provide required SoC in time and minimize the expected cost at the final time-step. The controller has a built-in penalty in case the charge required is not provided by the end time. The *SDPM controller* provides a large improvement over the *deterministic optimal controller*.

Finally, a case study was developed and run with annual price data, to show the quantifiable benefit of using both these strategies versus charging directly. It is based on the Illinois real time tariff market, with price data provided by the electricity provider ComEd. The data is available retrospectively for all years and features both predicted and real prices, per hour and for each day. The study explores the use case of an EV user plugging in the vehicle at home at 17:00 after a daily 50-mile trip. The program uses a 6kW charger and is given the choice of using no strategy, *deterministic optimal strategy,* or *stochastic dynamic programming (SDPM) strategy.* The ideal case where having all fore-knowledge of electricity prices is considered as a benchmark. The achievements of this research as proofed in this thesis are presented in section 8.2.

8.2 Achievements

The *problem statement,* chapter 3 of this thesis, presents the question that this research was set out to answer. To summarize: In a future with EVs, there needs to be a way to make use of readily available electricity price data on smart-grids, to optimally charge the vehicle in order to achieve the lowest electricity cost and in turn offset the charging to off-peak times. The question posed is:

What would be the right kind of controller from the user's point of view, to charge an EV optimally, in order to minimise the charging cost while still achieving the required state of charge?

This research has shown that there is a sub-optimal and an optimal control solution to the problem. With the assumptions made in the case-study, both the control solutions show a significant saving in annual charging cost. Figure 71 (left) shows baseline case-study results[9], where the green bar shows the lowest achievable annual (2013) price[10] for charging the EV. This is based on using RTP for both optimisation and simulation and is non-causal because it requires fore-knowledge of the real prices. The cost-saving potential in this ideal case is ***50.22%***.

[9] Chapter 7 section 7.2 explains different time-horizons and their significance.
[10] Prices exclude distribution fee or standing charges.

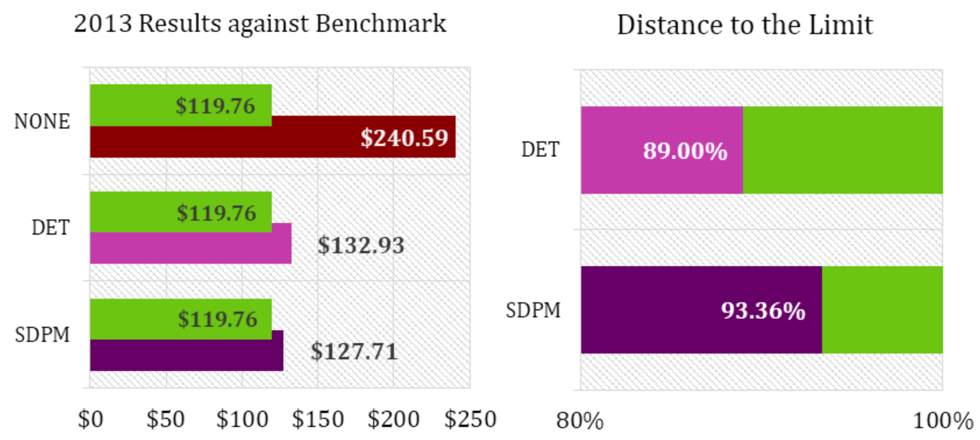


Figure 71: Achievement of Optimal and SDPM Strategies against theoretical limit

The achievements of this research can be summarised as follows:

- The *deterministic optimal controller* is the simpler solution, and it is still successful in providing significant cost-saving. Using this controller for charging versus using no charging control leads to a ***44.75 % lower*** annual charging cost.

- The *SDPM controller* is the more complex solution and it improves upon the *deterministic optimal controller*. When picking the right time to charge, it can account for the stochastic nature of the electricity prices, and avoid any potentially large deviation between the predicted and real prices. This controller reduces the annual charging cost further; providing a ***46.92 % lower*** annual charging cost compared to using no control.

- Figure 71 (right) shows how close the two solutions come to achieving the ideal case. Considering ***$119.76*** as the ***100%*** limit, the *deterministic optimal strategy* achieves ***89%*** success and *SDPM* gains a further 4.36% success bringing it to ***93.36%*** of the limit.

- Both the control strategies are also successful in shifting the charging load from peak to off-peak times. Figure 60, Figure 61 and Figure 62 (chapter 7 section 7.2) show that, when using the controllers versus not using any, the EV is charged during the times when power-demand is lower.

- The thesis has presented a novel approach to resolve the problem posed in the research question. The control solutions consider the user perspective first, and implicitly benefit the grid by filling the demand curve. The novelty is in the fact that the controller reacts to price data using a simple grid model, which is fast and uses a single optimisation stage. This simple and elegant grid model for real time price could be effective for scheduling demand response.

- The algorithms have a low complexity. Even the non-trivial *SDPM controller* has low enough complexity (as explained in chapter 7 section 7.3) and does not require high processing power. Either controller can be easily implemented on embedded systems and start benefitting users in areas with real time electricity tariffs.
- The solutions are also modular to a degree because they are purely mathematical, and are independent of any system standardisations. Either controller can be customised for any application which uses electricity, because the parameters provided to the programs can be changed. With some customisation, the optimisation stage could even include vehicle-to-grid and demand response signals.

Overall, there are several factors that determine the precise amount of money saved; for any specific application, it is worth tuning these in detail. The choice of the optimisation algorithm has some influence, with the stochastic optimisation outperforming the deterministic optimisation as expected but by a small amount. Given the increased complexity of the stochastic optimisation, it may not necessarily be the method of choice in a practical application. More complex approaches may be able to achieve better results, but clearly there is a law of diminishing return. More effort leads to less comparative reduction in charging cost, and it would be a matter of another study to assess the cost-effectiveness of working on a more complex solution.

The next logical step would be to perform an experiment by embedding the controllers on power electronics and testing their reaction to grid price changes. It would have to be performed along with an electricity provider who allows the reading and transmission of their historical and current price data to the system.

8.3 Outlook

It is evident that the use of strategies like these, applied as smart applications either in embedded systems or on-board, will be essential for future electricity demand management. The *Future Energy Scenario* report [3] from the National Grid in the UK, clearly states that the growth in EVs will have a significant impact in [electricity] demand. This impact must be managed carefully otherwise it will create challenges across all sections of the energy system, particularly at peak times. It also mentions that electricity requirement at peak time could increase by almost 1GW per year post 2030 but *smart applications* could lead to a reduction of this demand.

There are many research topics to explore, and questions to answer and address before any such strategy is integrated into the system. One main consideration is the infancy of the smart-grid infrastructure. Data problems were faced during this research: there were gaps in the price data of certain years, there were discrepancies in what time-interval the data is available at, and change in company policies meant consistent data was not available from the same source. The lack of standardisation in the area impacts the performance of real time tariffs and the real benefit that can be extracted out of them. This is conformed in a smart-grid progress review paper [96] by Cetin et al. The researchers conclude that the frequency and quality of data from smart-meters varies significantly and can be inconsistent.

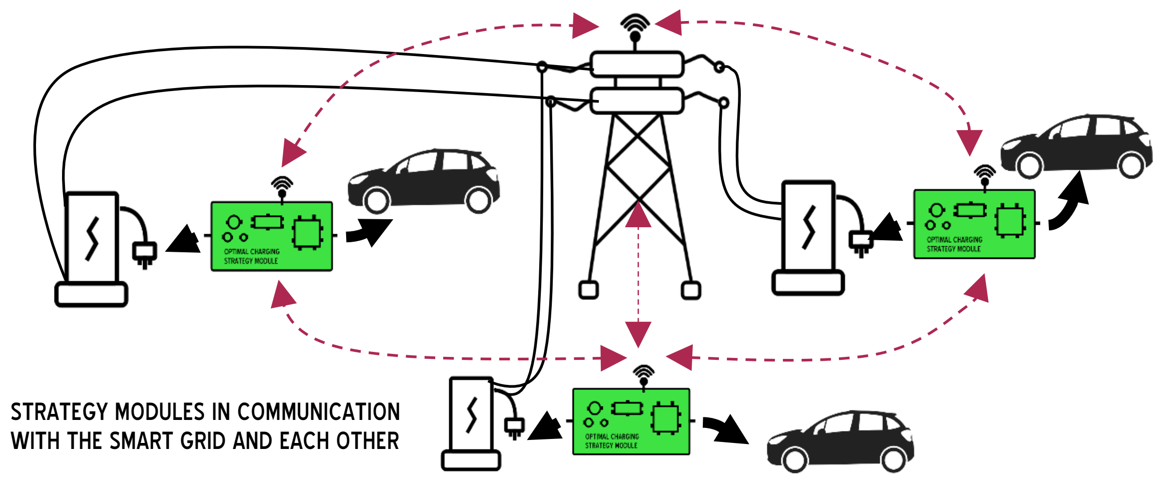


Figure 72: Optimal Charging Module communication and Placement

The next logical question is related to many EVs using strategies like *SDPM* at the same time. It would be a matter of a real-world study to see the effect this would have on the grid prices because all vehicles with the strategy would opt to charge during the off-peak, in turn causing a rise in demand. However, the electricity market has a distributed nature (the data supports this (section 4.1)), which should help achieve a co-operative result when many EVs use the strategy. Multiple EVs using such strategies also compels the question: will using strategies like these mean that the user plays against the electricity market? Approaching this problem from the user's perspective as this research has attempted, means this is certainly not the case. There is no intention of benefiting from the market or other user's losses. Also, this was never the intent of the development.

As Figure 72 suggests, it would certainly be beneficial if infrastructure would allow embedded modules or vehicles with this strategy to communicate with each other as well as the smart-grid. This would help in better decision making. An addition to the strategy or an additional code could aggregate the requirements of the EVs in the area and distribute the charging effectively.

The low computational complexity of this approach presents an opportunity to load it on embedded systems and be present on either side: the vehicle charging electronics or within the charging station itself. The strategy can be deployed onto a low-cost device like an Arduino or Raspberry Pi which is the next logical step recommended for this study. On the vehicle side, it could run on the vehicle's own electronic ICU or CPU since all modern vehicles have processors capable of handling a code like this, or it could be an additional device which communicates with the vehicle's computer. The vehicle or module itself would need a connection to the internet or the smart-grid to download price data periodically.

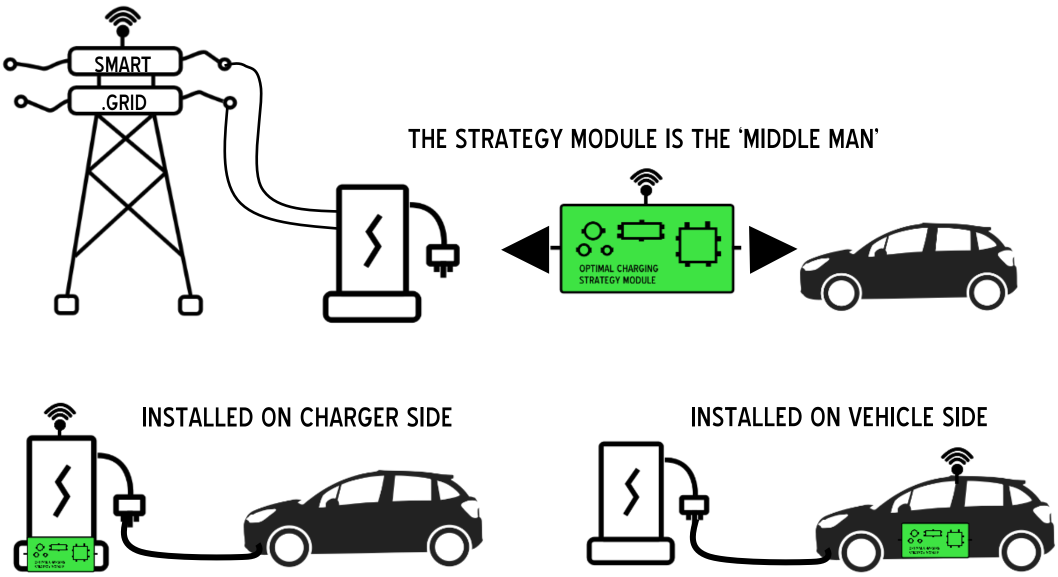


Figure 73: Placement of the Optimal Charging Strategy and Module

The module installed in a vehicle could benefit from various other information from the vehicle. For instance, if the vehicle has machine learning capabilities, it could effectively learn the user's driving pattern and this information could be used to alter charge and discharge profiles (times) used for the *SDPM strategy.* If the user would like to intervene and change any automation procedures, they could do so directly with the vehicle, rather than relying on another device. Essentially, the vehicle would function as a smart-device which uses information from the smart-grid infrastructure to benefit the customer. Figure 74 shows an example of how the consumer could benefit from a vehicle with an automation module using *SDPM* with machine learning.

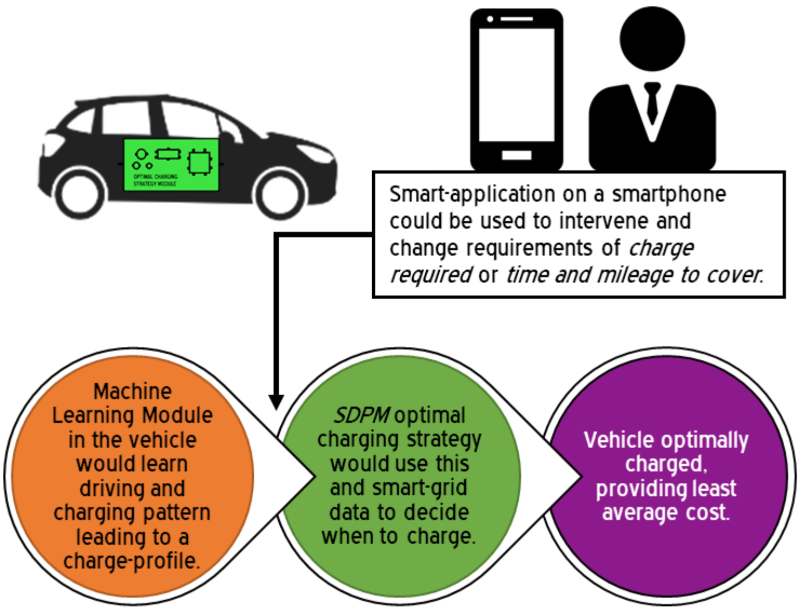


Figure 74: Example of process leading to automatic charging with SDPM

Closing Statement

While this research has shown the effectiveness of two strategies on optimal charging of EVs, there is a possibility of more complex solutions. These strategies are based on data from smart-meters and in turn on the clarity of this data. We find that the infancy of the smart-metering infrastructure needs to improve before price data can be completely relied upon. However, even if we just consider using the simpler *deterministic optimal strategy* with DAP, there is a ***$100*** advantage. The *SDPM strategy* achieves ***93.36%*** of the causal limit when using DAP and RTP, and is a fairly computationally simple. The more computationally complex solutions offer only a 6.64% improvement beyond *SDPM*. Therefore, *the SDPM strategy* is the recommended solution, which can easily be implemented on a simple embedded device in the charger or on-board on the control unit of the EV itself (as long as a data connection exists and the electricity providers can provide the data that is required). The return of the investment of this solution is expected within a few years for new hardware, and much less for a pure software change. Both are short compared to the lifetime of the vehicle and charger.

Abbreviations

BEV	Battery electric vehicle
EV	Electric vehicle
DAP	Day-ahead pricing
RTP	Real-time price
HEV	hybrid electric vehicle
ICE	internal combustion engine
MPC	model (based) predictive control
PHEV	plugin-in hybrid electric vehicle
SOC	(battery) state of charge
SD	standard deviation
V2G	vehicle to grid
DP	Dynamic programming
SDPM	Stochastic dynamic programming
ToC	Time of charge (required time)
SoC	State of charge (required)
TOUT	Time of use Tariff
EU	European Union
US DoE	United States Department of Energy
AMI	Advanced metering infrastructure

List of Symbols and Notations

$\boldsymbol{x_1}$	Battery SoC (state)
$\boldsymbol{x_2}$	Grid Cost (state)
$\boldsymbol{w}$	Known cost prediction (DAP)
$\boldsymbol{w'}$	Threshold Price
$\boldsymbol{u}$	Input (charge power)
$\boldsymbol{u'}$	Threshold charge power
$\boldsymbol{u_{max}}$	Maximum available Charge Power
$\boldsymbol{d}$	Random disturbance (stochastic variable)
$\boldsymbol{c}$	Cost
$\boldsymbol{c_0}$	Initial Cost
$\boldsymbol{\alpha}$	Decay constant (grid price decay)
$\boldsymbol{k}$	Current Time-Step
$\boldsymbol{N}$	Final Time-Step
$\boldsymbol{E}$	Expected Cost
$\boldsymbol{x_{1_0}}$	Initial SoC
$\boldsymbol{x_{1_N}}$	Required SoC
$\boldsymbol{\beta}$	Disturbance Scaling Factor
$\boldsymbol{T}$	Time Period (assumed 1 unit)
$\boldsymbol{T_s}$	Step-size

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