BLACK HOLES IN ECONOMICS

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Abstract. *The economics is one of the most complex systems in nature. There is still no united view on all processes happen there. Here we would like to present a geometrical approach to economics, where we introduce a new concept of economic metric space. We show that there may exist black holes, the objects, which may play a crucial role in the economics dynamics. We argue that the existence of black holes in such system from one side may stimulate economics while from other side may originate financial crisis.*

Keywords: *Geometro-economics, Geometro-dynamics, Riemann geometry, economic metric space, black holes, geodesics, financial crisis, demand-supply oscillations, gravitational rotations.*

Economics as a complex system

Social, economic and financial systems are considered as the most complex of all existing in nature. Its complexity is related to the nature main agents and players or actors in these systems – human beings. The matter is that these main players in these systems (humans) are very complex they-self and their behaviour is very unpredictable. The irony here is that humans are demanding the most logical behaviour from others while they-self behave in a most illogical way. From other side humans are advantageous and creative, and they use a lot of intellectual and virtual matter such as meanings, concepts, models, theories, and explanations. They generate ideas and make decisions. They regret, they suffer and have a lot of different fillings and can make very un-predictable crazy actions. They make a lot of sense of things, to understand. They predict, and want to control things. They are also the only creature that uses meanings to negate, contradict, and deceive itself. They misconceive, distort, and

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stereotype. They become dogmatic, prejudiced and narrow-minded. Humans are the only animal of which we know a lot and nothing. The humans they-self make the systems they created such as social, political, economic and financial, absolutely complex and un-describable in a conventional language of the single individual behaviour. Such complexity and unpredictability of human behaviour is convenient to describe in a framework of statistical mechanics, which provides a well description only for average characteristics of the system [1–4].

Here, we introduce the multi-dimensional space, which is a manifold or a multi-dimensional surface. Such manifold is an arena for the individual actions of humans. In the first instance the surface is a metric space, describing by the coordinates or variables associated with demand, supply and price of the individual products. For a first instance each our variable corresponds to a difference between a supply and demand of some individual product. We leave all other dependencies aside. Such coordinates may be introduced for any product [5]. So for n products we will have $n + 1$ dimensional metric space. Additional variable x^0 is related to the time, required to produce a product. Since all products can be produced during the same time we use one variable, x^0, for all products. Then, it is convenient to treat the space of all these products and their production times with a help of a metric tensor in a Riemann geometry.

Economic Spaces

In more general terms we may introduce a multi-dimensional space consisting of the coordinates $x^i = (x^0, x^1, x^2, x^3, \ldots, x^n)$, which are associated with n individual products. In fact x^i is a difference between a demand and a supply of the $i - th$ individual product and therefore it may take both positive and negative values. The sum of these variables may determine the GDPs of some individual countries, which form an united economical system such as EU or UK. So that each such economic space may consist of many small subspaces, that is, corresponding to any number of such countries having their individual economical systems as well. Such both contra-variant and one-form (covariant) coordinates may be introduced for any product including even services. For each product we also introduce *the consumption rate* defined as derivative of these products value over a time. $\upsilon^i = dx^i/dt$. In general, there is not only a consumption but also a production rate, but we include this in such general definition of the consumption rate, υ^i. Effectively, it is a difference between the consumption and production rates. Of course, such defined consumption or the production rates should have some maximal limiting value, which we define for each of these products as c_i, where the number i is an integer $1 \leq i \leq n$. Then, for these n products we will introduce $n + 1$ dimensional space: (x^0, x^1, x^2, x^3, ... , x^n). The first component, $x^0 = ct$, is defined here as a product received over a time t with the sum of the production rate squares of which value a square root was taken:

$$c = \sqrt{(c_1^2 + c_2^2 + c_3^2 ...... + c_n^2)}. \tag{1}$$

The same consideration is valid for a consumption rate. Below we assume that the consumption and the production rates are equal each other. That, however, excludes some cases from the consideration, but retains a main geometrical picture of economics, which is a main goal of the present paper.

Economic space is a metric space

For such as a space we may introduce the interval ds^2 defined as $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = g^{\mu\nu}\, dx_\mu dx_\nu$, that is

$$ds^2 = (c_1^2 + c_2^2 + c_3^2 + ... + c_n^2)dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - .... - dx_n^2. \tag{2}$$

with the metric tensor $g_{\mu\nu}$ that reminds a metric of the Minkovski space in Einstein's Special Theory of Relativity, which can be presented by a diagonal matrix: $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

It is instructive to consider here specific low-dimensional examples. In the lowest, two-dimensional space, there is only one product, x_1, e.g. a bread, which production is vital for the country existence. The country just spend all money available, Mc^2, say its "GDP" to buy or invest into the bread production. The parameter M stands for an inertia of the system. The efficiency of the economy is defined by the rate c which allows to get money available for next economic lap. The economic freedom of the country should be defined by inequality

$$x_1 < cT = M c^2, \tag{3}$$

where T of the period for the economic lap. As an example it could an economic year. If there are two products vital for an existence of the country, e.g. bred and cheese, then the economic freedom of the country will be defined by the following inequality:

$$\sqrt{x_1^2 + x_2^2 < c^2T^2 = (Mc^2)^2}. \tag{4}$$

That, means that each economy has a constraint defined by the available money or its ”energy” Mc^2, say by GDP or maximal amount of money which the country can spend. This money defined a radius of the sphere which limits all economic affairs for this country. Due to this fact the system reminds strongly the situation arising in general theory of relativity. That effectively means that the economic system can be described by a metric space and we have to introduce here a metric or a metric tensor [6], which may support the described constraint associated with limited money available to spend.

Then, to describe the constraint on limited money to spend the interval can be defined as

$$ds^2 = c^2dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - ...dx_n^2. \quad (5)$$

In this case the evolution of the system is defined by the “light” cone, $ct = r$, where $r = \sqrt{x_1^2 + x_2^2 + x_3^2 + ...x_n^2}$ is the radius of this economic sphere. There the real economic trajectory is oscillating around the vertical axis ct, which oscillations are limited by this “light” cone. In this form the economic system was presented as any physical system in a Minkovski space. Here the curvature of the space is zero, where we assume that the values c is a constant. In the present model, there is no gravitation interaction, neither a singularity.

The Dirac “light” cones define internal symmetry, associated with the symmetry elements, the hyperbolic rotations. The latter is defined or originated by the economy efficiency to produce an income. The parameter c is now playing a role of a speed of light in the theory of relativity [6].

In a more nontrivial and general case the economic system can be described by n-dimensional metric tensor $g_{\mu\upsilon}$, where the interval has a form:

$$ds^2 = g_{\mu\upsilon}\, dx^{\mu} dx^{\upsilon}, \quad (6)$$

here the metric tensor, $g_{\mu\upsilon}$, is a function of variables describing the amount of available or demanding products ($x^1, x^2, ..., x^n$), that is $g_{\mu\upsilon}$ ($x^1, x^2, ..., x^n$). The interval, ds, is a shortest distance in this space.

Riemann geometry in economic space

If there is a metric space describing an economy, in general, for such a space equivalent to some generalized surfaces, we may introduce a Riemann geometry. The optimal trajectories on these surfaces will determine all possible economic regimes. The optimal trajectories in such a Riemann geometry of the metric space are determined with geodesics lines obtained as a solution of the equation for null-geodesics. They are defined as

$$\frac{d^2x^{\mu}}{dt^2} - \Gamma^{\mu}_{\upsilon\delta}\frac{dx^{\upsilon}}{dt}\frac{dx^{\delta}}{dt} = 0 \qquad (7)$$

where $\Gamma^{\mu}_{\upsilon\delta}$ are Crystoffel symbols or connection coefficients. They may be explicitly presented with the help of the product of metric tensor and its derivatives [6]. For Minkovskii space the metric tensor has a diagonal form consisting of ±1. Therefore all symbols $\Gamma^{\mu}_{\upsilon\delta}$, are equal to zero. And dynamics in such economic space described by the simple equation:

$$\frac{d^2x^{\mu}}{dt^2} = 0 \qquad (8)$$

which solution corresponds to the constant consumption or production rates for each product, that is $\frac{dx^{\mu}}{dt} = \upsilon_{\mu} = const$. In reality, of course, the spending for different economic items is not defined by a constant rate [5]. These rates, of course, are arbitrary functions of the space variables. They are influenced by the geometry of the system and/or is defined by a geometry of the system. In this case, in general, any stationary state of an economic system with the metric space, $g_{\mu\upsilon}$, can be well described by the Einstein equations:

$$R_{\mu\upsilon} - \frac{g_{\mu\upsilon}}{2}R = 0 \qquad (9)$$

$R_{\mu\upsilon}$ is Ricci tensor and R is a scalar curvature. We dropped here the right hand side of the Einstein equations – the stress energy tensor [6] to consider a simplest but a nontrivial case here. The unknown variables of this equation are the parameters of $g_{\mu\upsilon}$. As an example we consider a particular case of economy with singularities, which is associated with the exact solutions of the Einstein equation 9. One of such nontrivial solutions describes a black hole. This solution of the Einstein equation is called a Schwarzshild metric. In this case, in any multi-dimensional space the interval can be presented in the following form:

$$ds^2 = c^2(1 - \frac{r_{SW}}{r})dt^2 - \frac{dr^2}{(1 - \frac{r_{SW}}{r})} - r^2 d\Omega^2. \qquad (10)$$

By $d\Omega$ we noted the angular variables of n-dimensional sphere. Here r_{SW} is a Schwarzshild radius, defining the size of the singularity. It is equal to $r_{SW} = 2GM/c^2$ and it is also known as a radius of the black hole horizon. Anything inside of the sphere defined by the black hole horizon is trapped by the black hole. If a particle is trapped by the black hole, it is never returned to a free space. And this does not depend on an energy the particle may have. For gravitating system the value of the Schwarzshild radius is determined by the mass of the black hole, M, the gravitational constant G, defining a relation between a Newton force and a mass M and it is inversely proportional to the speed of light, c. For economic system c is a summary production rate- the global characteristics of the economic system. Thus if the production rate c decreases to zero, e.g. it may happen during an economic crisis, then the black hole radius, r_{SW}, increases to infinity. This effectively means that the decrease in the production rates c_i can dramatically influence the dynamics and an evolution of any economic system creating a giant black hole or a singularity with a giant radius. Therefore, many particles which were existing outside of the black hole horizon will be trapped by the black. That phenomenon constitutes a creation of economic crisis.

Demand-supply oscillations as gravitational rotations

Therewith this approach we may put in a correspondence to any economic system some gravitational one. In such a correspondence the parameters such as a capital or wealth in economics, in the theory of general relativity are related to a mass of a black hole, M or the energy Mc^2. The radius of the black hole horizon r_{SW} corresponds to the minimal survival production of the systems. One may describe the following analogy between gravitational and economic systems: the gravity mass or the capital is attracting all other bodies around which have also some gravity masses m_i, where the number of these small bodies can be immense. The smaller masses are usually circulating around the big one, making lap by lap. For each of such body there is a set of the products used including their consumptions and productions. The majority of these bodies are just consumers and their economic space is defined by these products they are consuming. Their position in the economic space is effectively the difference between the supply and the demand of the product they are experiencing or consuming. Their economic space is the plane in which they are rotating around a black hole. Obviously, it is a subspace in the global economic space which covers the multi-dimensional black hole and all bodies rotating around it. In each lap for each product there is a time period when the value of x_i is positive. This corresponds to a situation when a supply is dominating the demand. And also there is another time interval existing within the period of the rotation, when the value of x_i is negative. This corresponds to a situation when a demand is dominating the supply. For economics analogy the circulation of smaller masses around the big one such as a black hole may be interpreted as the demand-supply oscillations. In each lap or the period of the rotation there is a time when the domination of the production (or availability for purchase) of some goods is dominating. And there is the other time interval, during the same lap where a domination of the demand over a supply for these particular goods does exist. This is in a good correspondence with the main concepts introduced by Adam Smith, which laid the foundations of classical free market economic theory [7, 8].

In such an analogy the good economics system can be compared with a solar system, which has, however, a very large space dimensions and much more planets. Or may be if we consider a Saturn Rings, then analogy will be more close. What happen when an economic crisis occurs [9]. In this case the production rates decrease and the radius of the black hole horizon increases to infinity. Because of that rotation of many small bodies, so-called small and medium businesses (SMB), has stopped since the radius of their orbits or their rotations becomes smaller than the radius of the black hole horizon. That may happen when the radius of the black hole (or Schwarzshild radius) will be larger than the radius of these orbiting small masses. That means that SMB are in most volnurable position when economic crisis occurs. On the other hand if they are trapped by other large bodies, which are still orbiting around black holes, they will survive, forming a large army of services. The appearance of large body in any "solar"-like system can stimulate the rotation of small bodies and so the local economics. On the other hand when the mass of this body increases it may reach some critical value and the star will be transformed into a black hole, which can limit the number of SMBs existing in the system.

In summary, we show that economic system having a largest grade of complexity among all existing systems may be described with the use of geometrical approach. There all complex interactions between individuals, participants or agents of the complex system are replaced by a geometry of some smooth manifold or hyper-surfaces. The economic systems may be self-organized in numerous forms dictated by their geometry. In some cases, the situation can be simplified and the economics may remind a system consisting of a black hole with accretion disk and orbiting gravitational bodies around. There, in the accretion disk, particles are in a process of trapping by black hole. The existence of such a disk may indicate a first signature of the coming economic crisis [9].

REFERENCES

[1] Kusmartsev, F. V., *Statistical mechanics of economics,* I. Physics Letters A, 375(6), pp. 966-973 (2011).

[2] Kürten, K. E. and Kusmartsev, F. V., Bose-*Einstein distribution of money in a free-market economy,* II. EPL (Europhysics Letters), 93(2), p. 28003 (2011).

[3] Spânulescu, I. and Gheorghiu, Anca, *A classification of the econophysics directions*, Hyperion International Journal of Econophysics & New Economy, 6(2) (2013).

[4] Gheorghiu, A., *Two decades of econophysics-theories, personalities. Romanian contributions*, Hyperion International Journal of Econophysics & New Economy, 10(2) (2017).

[5] Isohätälä, Jukka, Feodor Kusmartsev, Alistair Milne, and Donald Robertson. *Leverage con- straints and real interest rates*, The Manchester School 83, 83-109 (2015).

[6] Feodor V. Kusmartsev, *Introduction to General Relativity and Cosmology,* Lecture Notes, Loughborough University (2019).

[7] Adam Smith, The Wealth of Nations edited by R. H. Campbell and A. S. Skinner, *The Glasgow edition of the Works and Correspondence of Adam Smith*, Vol. 2a, p. 456 (1976).

[8] Adam Smith, *The Glasgow edition of the Works and Correspondence of Adam Smith,* Vol. 3, p. 49, edited by W. P. D. Wightman and J. C. Bryce, Oxford: Clarendon Press (1980).

[9] Kürten, K. E. and Kusmartsev, F. V., *When rich get richer there arises financial crisis and Bose-Einstein condensation in a wild economy*, Hyperion International Journal of Econophysics & New Economy, 5(1) (2012).

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