

A value-based approach to optimising long-term maintenance plans for a multi-asset k -out-of- N system

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ABSTRACT

Devising a long-term maintenance plan for a system of large infrastructure assets is an exacting task. Any maintenance activity that induces system downtime can incur a massive production or service loss. This problem becomes increasingly challenging for a system of which the performance is based on the collective output of assets. Current approaches that optimise each asset in isolation or consider a binary performance relationship insufficiently address this issue because the negligence of performance interactions among assets results in an inaccurate cost estimation. To overcome these hurdles, we formulate a mathematical model that explicitly demonstrates dynamic risk of production loss according to the system aggregate output. Further, we propose an integrated solution method that couples a finite loop search with a Genetic Algorithm. Application of our model to a real-world case study has proved to simultaneously strike the balance between cost and risk. Validated by Monte Carlo simulation, the proposed model has shown to outperform existing approaches. **By systematically scheduling maintenance actions over the planning horizon, the resultant strategy has demonstrated to offer considerable maintenance cost savings and significantly prolong the average asset life.** Sensitivity analyses also evince the robustness of the proposed model under the volatility in key parameters.

HIGHLIGHTS

- **A novel approach to modelling a k -out-of- N system with dependence is explored**
- Understanding dynamic natures of system criticality is essential to mitigate risks
- Inter-asset dependencies play a vital role in determining effective solutions
- A delicate balance between cost and risk is key to delivering maximum system values
- Integrated solution method can efficiently produce satisfactory results

1. Introduction

Among the reliability engineering and asset management problems addressed in the existing literature, those pertaining to multi-unit systems have particularly intrigued researchers because of their profound effects on the organisational performance. The term '*multi-unit system*' has been used to refer to either a single asset comprising of multiple components (a multi-component system) or a system of multiple assets (a multi-asset system) [1], [2]. In these systems, since multiple units operate collectively to produce an output or provide a service, devising a maintenance policy for each unit in isolation may not yield the maximum value from the system.

Dekker et al. [3] put forth three initial types of multi-component dependence: structural, stochastic, and economic dependence. Olde Keizer et al. [4] further classified structural dependence into technical and performance dependence. In addition, the authors also proposed the concept of resource dependence. In a recent review, Petchrompo and Parlikad [2] drew several types of multi-component dependence and extended their notions to accommodate multi-asset problems. Performance dependence is referred to a configuration in which the performance of one asset affects the performance of other assets and/or the overall system. Resource dependence occurs in a system in which common financial, human, and physical resources are shared among the assets in the system.

One of the most typical types of performance dependence found in multi-unit systems is k -out-of- N . In such a system that consists of N components, the system operability depends on a pre-specified number of components k . Pioneering studies on binary multi-component k -out-of- N systems (BMC) focused on two settings: G and F. A k -out-of- N :G system only functions if, out of N components, at least k components are operational [5], [6], while a k -out-of- N :F system fails if at least k components malfunction [7], [8]. The performance of these systems is expressed in binary terms; the entire system either works or fails according to a prescribed condition k . Later academics became aware that a component usually evolve over multiple states, and each state may contribute unequally to the system performance [9]. This characteristic has led to the advent of a general multi-state k -out-of- N :G system (GMS). Huang et al. [10] defines this as a system in which the component and the system can be in one of $M + 1$ possible states ($0, 1, \dots, M$). A system is in state j if at least k_j components are in state j or better.

Many studies considered multi-component k -out-of- N asset management problems with primary objectives to minimise intervention cost and/or maximise system reliability (minimise risk) [11], [12], [13], [14], [15]. To accommodate multiple objectives, previous researchers adopted two preference articulation modes: *a posteriori* and *a priori*. In the *a posteriori* mode, a user apply a multi-objective optimisation algorithm to generate a set of non-dominated solutions on a Pareto frontier [2]. Previous reliability engineering studies dealing with k -out-of- N systems employed several algorithms including Non-Dominated Sorting Genetic Algorithm-II (NSGA-II) [16] and ant colony optimisation [17]. The *a posteriori* mode possesses a major drawback in that the user is still required to make a trade-off from the Pareto frontier to derive the final solution.

Contrarily, in the *a priori* mode, multiple objectives are combined into a single objective. This mode requires the user to assign weights to multiple objectives before optimising the system using single-objective optimisation algorithms. In previous studies, GAs have been widely used to solve various optimisation models for k -out-of- N systems [18], [19], [20]. For short-term maintenance planning, Khordishi et al. [21] demonstrated that their GA achieved an effective solution within a reasonable amount of time for a four-period multi-state k -out-of- N system optimisation problem with six maintenance options. However, the authors mentioned that satisfactory solutions were obtained after the several runs of their GA [21]. Since GA is a meta-heuristic approach, there is a distinct possibility that the search process is terminated when a low-quality solution is found. This difficulty is expected to be more inherent for a problem with a longer decision horizon.

Existing k -out-of- N studies adopting the *a priori* mode typically converted risk into equivalent cost to make these objectives comparable. For instance, Doostparast et al. [11] proposed an optimisation model of which the single objective function consists of four terms. The first three terms considered in their study represent the actual

costs of maintenance actions (repair, replacement, and planned system downtime), whereas the other is the equivalent penalty cost derived from the expected monetary impact of random failures throughout the planning horizon. It is apparent that the conversion of risk into a monetary metric is straightforward in traditional multi-component k -out-of- N systems due to a distinct contribution of a component to the system performance.

However, there exists a case in which the system performance is more complicated. In this paper, we consider a k -out-of- N system in which the contribution of an asset to the system performance is expressed in binary, that is, an asset either fully contributes to the system performance if it is operational or has no contribution at all if it fails. Nonetheless, the system does not breakdown completely even if fewer than k assets are operational, but the system performance depends on the number of operational assets below k . In other words, the system can be in $k + 1$ states ($N - k, \dots, N$), while the assets can be in $M + 1$ states ($0, \dots, M$). This consideration and the corresponding modelling approach will be referred to as a multi-asset (MA) k -out-of- N system throughout this paper. Despite its binary expression of the asset contribution, the MA system may not be effectively addressed by BMC models due to the requirement of multiple asset states. GMS models, albeit involving multiple component and system states, may not be fully compatible with the MA system because of the differences in the number of system states and system performance threshold. The fundamental features of the MA k -out-of- N system compared with BMC and GMS systems are summarised in Table 1.

Table 1
Summary of fundamental features in three k -out-of- N systems.

System	Component/asset contribution	No. of component/asset states	No. of system states	Performance threshold
Binary k -out-of- N (BMC)	Binary	2	2	Pre-specified k
Multi-state k -out-of- N :G (GMS)	Component state dependent	$M + 1$	$M + 1$	Pre-specified k_j
Multi-asset k -out-of- N (MA)	Binary	$M + 1$	$k + 1$	No. of operational assets below k

The goal of this paper is to develop an optimisation model to address a long-term maintenance planning problem of an MA k -out-of- N system. We use a real-world industrial example of an effluent treatment system in an oil refinery to motivate the need for research on this aspect. The theoretical limitation indicated earlier and the investigation into real-world multi-asset system environments allowed us to identify the following contributions:

- **Modelling of a system with complex performance dependence:** We consider a system in which a failure to meet the required condition induces the system to disrupt other operations in the process and encounter a production loss. An investigation into a real-world problem indicates that the cost of production loss can be derived as a staircase function of which the step values are determined by the number of unavailable assets. Hence, the primary contribution of this paper is the novel approach to modelling inter-asset performance dependence to address a maintenance planning problem in a k -out-of- N system using the *a priori* preference articulation mode.
- **Incorporation of inter-asset resource constraint:** Since the impact of the system downtime is less significant in multi-component systems, it could be effective to perform group maintenance activities, so that this impact is minimised [22]. However, this may not lead to a cost-effective approach for a multi-asset system, especially that of large infrastructure assets. This is not only because of high cost of production loss, but also because undertaking many activities in the same time period can violate the budget constraint set by an organisation. This study incorporates this resource constraint into the optimisation model by adding a penalty cost when the maintenance cost exceeds the budget allowance.
- **Tailored solution method:** A genetic algorithm (GA) has been proven to reach effective solutions within a reasonable timeframe. This was validated using a case study of short-term maintenance planning for a

small system. Nonetheless, employing a GA solely does not produce effective solutions for a long-term planning for a larger system. Thus, in this paper, we develop a tailored finite loop search algorithm to help GA in searching for effective solutions.

To address these challenges, our value-driven model is set up to strike the balance between maintenance investments and risks of business disruption. The two goals are combined into a single objective by converting risk figures into monetary impacts in a meaningful way. That is, all possible scenarios are enumerated with their corresponding probabilities of occurrence and costs of production loss. Moreover, we also take into account the time value of money to determine the present value of costs incurred in the future [21].

The modelling approach used in this paper is comprised of four parts. Firstly, the deterioration of an asset over time is modelled as a non-stationary gamma process [23]. We determine the expected deterioration according to a convex gamma process. Secondly, we enumerate all possible scenarios and calculate corresponding probabilities of occurrence using the inverse of reliability function for non-identically distributed exact k -out-of- N :G systems [24], [25]. Thirdly, to solve the proposed deterministic optimisation model, we couple a finite loop search algorithm with a GA to perform a search for effective solutions. Lastly, to incorporate uncertainties, the results obtained from the model are validated using Monte Carlo simulation.

To determine the performance of the proposed solution method, the outputs produced by our integrated method are compared with those derived by a simple GA. Moreover, the results obtained from the proposed MA k -out-of- N formulation are also benchmarked against those achieved by the BMC. This model is chosen as the benchmark because of its applicability to our problem; that is, BMC also has a binary contribution of an asset on the system performance.

The remainder of this paper is organised as follows. **Section 2 provides the system description and the case study that lays the foundations for the system under our consideration.** The mathematical formulation of the problem and the numerical analyses are presented in Section 3 and 4 respectively. Finally, the conclusions and avenues for future research are provided in Section 5.

2. System description

2.1 Generic description

We consider a system with total N assets, among which at least k assets have to operate collectively to satisfy total demand. If the system capacity is higher than or equal to the demand, the workload is distributed equally to all available assets. In this scenario, the system can work at its full capacity. Contrarily, if the system capacity is lower than the demand, the system does not completely fail but suffers from production loss. The cost of production loss incurred depends on the number of available assets below k . This consideration leads the system to possess $k + 1$ system states ($s = N - k, \dots, N$) as previously mentioned. Each of the states $N - k + 1$ to N refers to the state in which there are $N - k + 1$ to N non-operational assets, whereas state $N - k$ refers to the state in which there are $N - k$ or fewer non-operational assets. Moreover, each asset can be in $M + 1$ states ($a = 0, \dots, M$) where $a = 0$ corresponds to the worst state (non-operational state) and $a = M$ corresponds to the healthiest state. To devise a long-term maintenance plan for this system, it is necessary to determine appropriate maintenance actions and time to perform them by making a trade-off between maintenance cost and cost of production loss.

The following assumptions are also associated with the system:

- The expected cost of production loss is expressed as a staircase function of which the step value is defined by the number of available assets below k .
- The evaluation of asset state is based solely on the remaining useful life (RUL).
- RUL is expressed as the condition of an asset and ranges from 0 (the worst condition) to 100 (the best condition).

- The deterioration of an asset cannot be visually detected from the outside and is assumed to follow a non-stationary convex gamma process.
- There are two possible maintenance actions: repair and replacement. Only one repair action is allowed throughout the life cycle of an asset.
- There are two asset types: the old type and the new type. At the beginning, all assets are in the old type. The old type turns into a new type when a replacement takes place.
- Maintenance cost is the same for each activity undertaken on similar asset types (different maintenance costs are applied to different asset types).
- The level of improvement on RUL as a result of a repair or a replacement activity is based on the asset state at the time the activity is undertaken.
- Asset downtime is fixed for each maintenance activity undertaken on similar asset types.
- Additional cost is incurred if the total maintenance cost exceeds the budget allowance in a specific time period. This is due to additional cost of borrowing that is required to address the shortfall.

2.2 Practical case

To provide illustrative examples along with the model formulation, we use an effluent treatment system in an oil refinery as a case study. The effluent treatment system (Fig. 1) treats dirty water from various refinery operations and produces a filtered water stream to be discharged to the sea. The aim of this support system is to ensure that the purity of the filtered water meets environmental standards set by the government. This means that if this system does not operate at a required capacity, it will directly disrupt other refinery processes. Since the system is used to filter water from multiple refinery systems, different numbers of asset breakdowns lead to different economic consequences for the plant. This is because the operations of different systems will be shut down.

The oil refinery is facing a major problem as the internal lining of the vessel corrodes over time and deteriorates to the point that it cannot protect the vessel wall. The lining deterioration cannot be detected from the outside. Moreover, an internal inspection requires a vessel to be uninstalled and incurs high downtime and labour cost. Currently, there are two available maintenance options: patch repair and vessel replacement. As with an internal inspection, both maintenance activities also induce high cost and downtime. A patch repair can take place only once throughout the lifetime of a vessel due to the internal structure of a vessel. The current policy is run-to-failure, which leads to a high risk of production loss, especially when a vessel is uninstalled for maintenance while the others are in poor conditions.

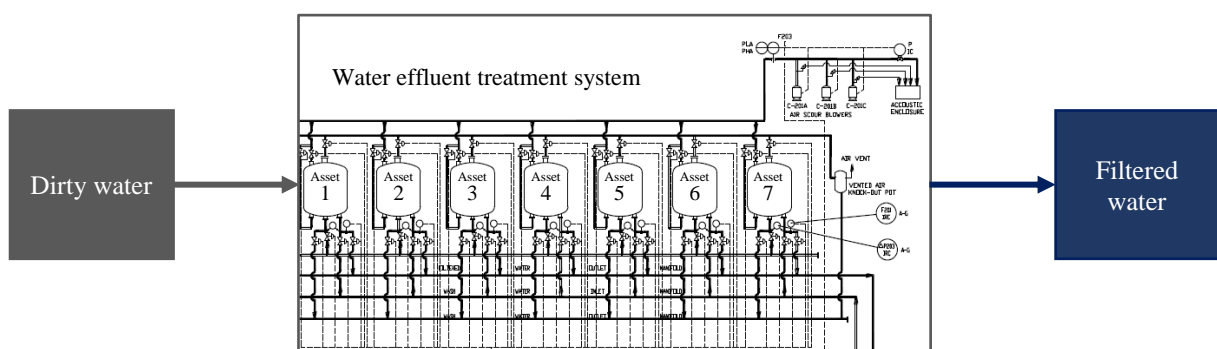


Fig. 1. The water effluent treatment system.

As shown in Fig. 1, the effluent treatment system consists of seven vessels. Although each vessel can work independently, five of them are required to operate together to meet the total demand of the plant. Moreover, the filter process necessitates that a vessel be carried out through a cleaning process at a fixed time period. In other words, the system can operate at full throughput only with one vessel out of service (failure or maintenance).

Thus, to help explain the mathematical formulation in Section 3 and illustrate the numerical experiments in Section 4, we will use this MA 6-out-of-7 system as an example where required. This example is based on six asset states ($a = 0, \dots, 5$), time interval in months, monetary unit in kGBP, and annual (fiscal year) budget constraint.

3. Model formulation

To devise an effective maintenance plan, we propose an optimisation model with an objective to minimise the total cost. Fig. 2 illustrates the total cost calculation process and demonstrates how different parts of the model are connected with relevant equations and algorithms appended to each part. The objective function to be optimised in our model is the sum of two main parts: maintenance cost and cost of production loss. Since the proposed model will be solved heuristically, we also include the penalisation of constraint violation to the final objective function. While the maintenance cost can be determined directly from maintenance decisions, the calculation of the cost of production loss is much more complex. As illustrated in Fig. 2, the total cost calculation entails identifying the system scenario and evaluating the risk of production loss at every time interval. A system scenario is a possible combination where each of the assets is assigned to an asset state. The complexity arises as the asset state identification requires the information on asset type, availability, and remaining useful life. While the first two components can be mathematically derived, the last component involves the application of a deterioration model. More importantly, the assessment of risk of production loss requires the enumeration of all possible system scenarios and the evaluation of corresponding probabilities of occurrence. These probabilities are evaluated by an inverse of an exact k -out-of- N :G reliability model. The remainder of this section will explain how the selected deterioration and risk models are integrated into the optimisation model formulation. The solution method that couples a finite loop search algorithm with a GA will also be described.

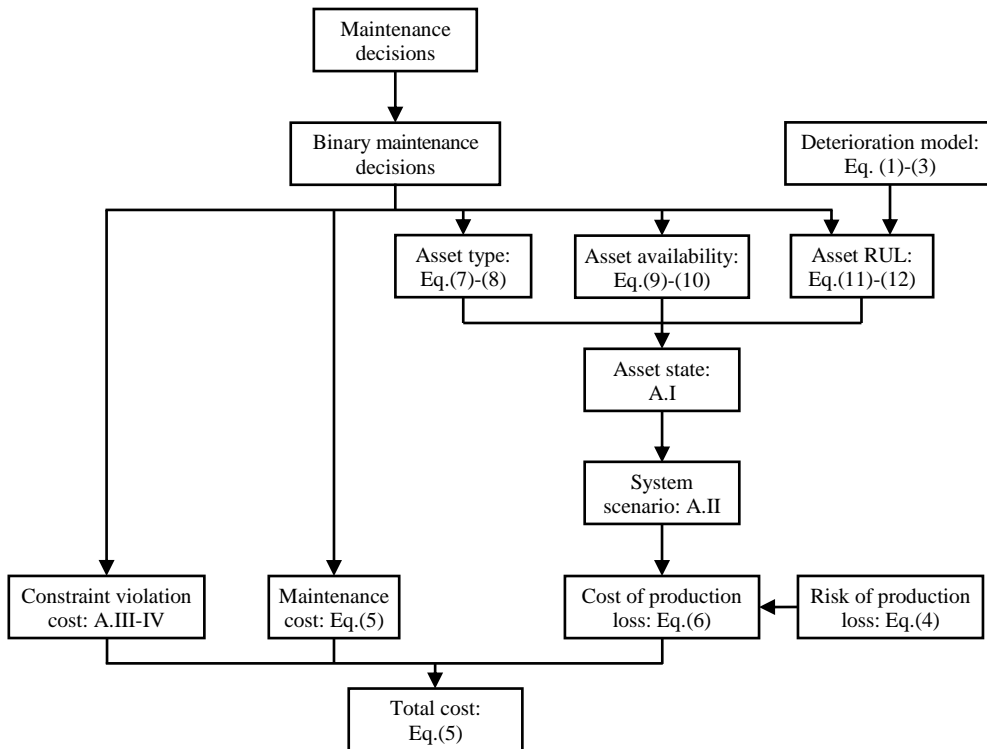


Fig. 2. Total cost calculation process.

3.1 Deterioration model

We model the deterioration as a non-stationary gamma process. The gamma process is chosen because it can effectively simulate the damage that monotonically accumulates over time due to corrosion [23], [26].

The gamma process is defined as follows. Let $X(t)$ denote the deterioration at time t ($t \geq 0$) and follow a gamma distribution with shape parameter $v(t)$ and scale parameter u . The time dependent probability density function of $X(t)$ can be written as:

$$f_{X(t)}(x) = \frac{u^{v(t)}}{\Gamma(v(t))} x^{v(t)-1} e^{-ux} \quad (1)$$

where $\Gamma(v(t)) = \int_0^{\infty} z^{v(t)-1} e^{-z} dz$ is the gamma function for $v(t) > 0$.

Hence, the expectation and the variance are given by:

$$E(X(t)) = \frac{v(t)}{u}, \quad Var(X(t)) = \frac{v(t)}{u^2} \quad (2)$$

Previous studies demonstrated that the expected deterioration at time t is proportional to the power law function [27] [28] [29]. Thus, the expected deterioration at time t , which in the remainder of this paper will be referred to as $g(t)$, is given by:

$$g(t) = \frac{v(t)}{u} = \frac{ct^b}{u} = at^b \quad (3)$$

The gamma process is stationary if the expected deterioration is a linear function of time ($b = 1$) and is non-stationary if the expected deterioration is a non-linear function of time ($b \neq 1$).

In our case study, historical information indicated that the internal lining of the vessel tends to deteriorate faster as time passes. Hence, according to this deterioration trend, the non-stationary convex gamma process ($b > 1$) is adopted. The values of related parameters are estimated by the expert judgment. More details will be provided in Section 4.

3.2 Risk of production loss

As aforementioned, the system incurs different levels of the expected cost of production loss in line with different numbers of operational assets below k . The expected cost of production loss is therefore the weighted average cost of production loss, with the probabilities of occurrence being weights. To determine the expected cost of production loss at any time interval, we enumerate all possible system scenarios and evaluate their corresponding probabilities of occurrence. A system scenario represents one of possible combinations where each of N assets is assigned to one of the asset states. **The number of all possible system scenarios is the combination with repetitions ${}_{(M+1)+N-1}C_N$ where N assets are assigned to one of the $M + 1$ asset states.**

As previously defined in Section 2.1, a MA k -out-of- N system can be in $k + 1$ system states in accordance with the number of non-operational assets. Thus, we can determine the risk of production loss in each system scenario by calculating probabilities that assets are in different states. Let $P(U_f = s)$ be the probability that the system is in state s ($N - k \leq s \leq N$) when the scenario f occurs. To determine this probability, we use the inverse of the reliability calculation for a non-identically distributed exact k -out-of- N :G system [24], [25]:

$$P(U_f = s) = \sum_{j=1}^{\binom{N}{s}} P_f^{\rho}(s, \rho_f)_j \cdot P_f^{\omega}(s, \rho_f)_{\binom{N}{s}-j+1}, \quad N - k < s \leq N \quad (4)$$

where ρ_f is the vector of asset probabilities of failure ($= \{r_{1,f}, r_{2,f}, \dots, r_{N,f}\}$), $P_f^{\rho}(s, \rho_f)$ is the set of products of the subsets of probabilities that exactly s out of total N assets are unavailable in scenario f , and $P_f^{\omega}(s, \rho_f)$ is the set of products of the subsets of probability that exactly $N - s$ assets are available when the scenario f occurs.

To illustrate this calculation, the enumeration of system scenarios and corresponding risk of production loss for our case study – the MA 6-out-of-7 system with six asset states and seven system states – is demonstrated in Table 2. The number of all possible system scenarios is determined by ${}_{6+7-1}C_7 = 792$.

Table 2
Enumeration of system scenarios and risk of production loss in the case study.

Scenario (f)	No. of assets in asset state a						$P(U_f = s)$						
	$a = 0$	$a = 1$	$a = 2$	$a = 3$	$a = 4$	$a = 5$	$s = 1$	$s = 2$	$s = 3$	$s = 4$	$s = 5$	$s = 6$	$s = 7$
1	0	7	0	0	0	0	0.3294	0.3177	0.2269	0.0972	0.0250	0.0036	0.0002
2	0	0	7	0	0	0	0.9829	0.0162	0.0008	0.0000	0.0000	0.0000	0.0000
3	0	0	0	7	0	0	0.9993	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000
4	0	0	0	0	7	0	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0	0	0	0	0	7	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	7	0	0	0	0	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
7	0	6	1	0	0	0	0.4111	0.3235	0.1894	0.0633	0.0117	0.0010	0.0000
8	0	6	0	1	0	0	0.4184	0.3240	0.1861	0.0603	0.0105	0.0008	0.0000
9	0	6	0	0	1	0	0.4199	0.3241	0.1854	0.0597	0.0103	0.0007	0.0000
10	0	6	0	0	0	1	0.4202	0.3241	0.1852	0.0595	0.0102	0.0007	0.0000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
791	1	1	1	1	1	2	0.6742	0.3146	0.0111	0.0001	0.0000	0.0000	0.0000
792	2	1	1	1	1	1	0.0000	0.6742	0.3146	0.0111	0.0001	0.0000	0.0000

3.3 Optimisation model

The optimisation model formulation shown in this sub-section is based on the following notations:

Sets

- V Set of assets = $\{1, \dots, N\}$
- D Set of asset types = $\{1, 2\}$ where 1 corresponds to the current type and 2 corresponds to the new type
- I Set of maintenance options = $\{1, 2\}$ where 1 corresponds to repair and 2 corresponds to replacement
- T Set of time (months)
- A Set of asset states = $\{0, \dots, M\}$
- S Set of system states = $\{0, \dots, k\}$
- F Set of possible system scenarios = $\{1, \dots, {}_{A+N-1}C_N\}$

Parameters

- N The number of assets in the system
- k Minimum number of operational assets required to operate at full capacity
- m_i Asset downtime due to maintenance option $i \in I$ performed on an asset
- $p_{1,a}$ Asset condition after a patch repair takes place on asset in asset state $a \in A$ (RUL)
- p_2 Asset condition of brand-new asset type 2 (RUL)
- q_a Equivalent asset operational age after a patch repair is performed on asset in asset state $a \in A$
- c_i^m Cost of maintenance activity $i \in I$ on an asset
- b_s Cost of production loss when the system is in state $s \in S$
- c_f^p Expected cost of production loss when the system is in system scenario $f \in F$
- $h_{a,d}$ Minimum RUL threshold of asset state $a \in A$ applied to asset type $d \in D$ (RUL)
- ζ Discount rate (nominal annual rate)

Decision variables

- $x_{v,i}$ Planned schedule for performing maintenance option $i \in I$ on asset $v \in V$

Auxiliary variables

$\delta_{v,i,t}$	A binary variable indicating whether maintenance option $i \in I$ is performed on asset $v \in V$ at time $t \in T$ on a rolling horizon basis (= 1 if it is, = 0 otherwise)
$\theta_{v,t}$	Asset operational age of asset $v \in V$ at time $t \in T$
$u_{v,t}$	Remaining useful life of asset $v \in V$ at time $t \in T$ (RUL)
$\lambda_{v,d,t}$	A binary variable indicating whether asset $v \in V$ is in asset type $d \in D$ at time $t \in T$ (= 1 if it is, = 0 otherwise)
$\alpha_{v,t}$	A binary variable indicating whether asset $v \in V$ is available at time $t \in T$ (= 1 if it is, = 0 otherwise)
$e_{v,t}$	Remaining unavailable time for asset $v \in V$ at time $t \in T$
$\gamma_{v,a,t}$	A binary variable indicating whether asset $v \in V$ is in asset state $a \in A$ at time $t \in T$ (= 1 if it is, = 0 otherwise)
$\eta_{a,t}$	Number of assets in asset state $a \in A$ at time $t \in T$
$\varphi_{f,t}$	A binary variable indicating whether the system is in system scenario $f \in F$ at time $t \in T$ (= 1 if it is, = 0 otherwise)
κ	Accumulated discounted constraint violation cost

The aim of this optimisation model is to identify the appropriate time to undertake one or both of the two maintenance options within a decision horizon. Therefore, the decision variables can be written as the following variable vector:

$$X = [x_{1,1} \quad x_{2,1} \quad \dots \quad x_{N,2}] \quad (5)$$

The decision variables are subsequently converted into a binary auxiliary variable $\delta_{v,i,t}$ by assigning the value 1 to $\delta_{v,i,t}$ when $x_{v,i} = t$.

3.3.1 Objective function

To back up the solution to the multi-objective optimisation problem with a value-based evidence, we propose converting risk of production loss and the penalty of constraint violation into commensurable values to the cost at each time period t . Hence, the objective function (6) is set to minimise the sum of the discounted maintenance cost (term 1), the discounted cost of production loss (term 2), and the penalty of constraint violation (term 3).

$$\text{Min} \sum_{v \in V} \sum_{i \in I} \sum_{t \in T} \frac{c_i^m \cdot \delta_{v,i,t}}{(1 + \zeta / 12)^t} + \sum_{f \in F} \sum_{t \in T} \frac{c_f^p \cdot \varphi_{f,t}}{(1 + \zeta / 12)^t} + \kappa \quad (6)$$

The expected cost of production loss for a system scenario f is shown in Equation (7). The expected cost is the weighted average cost of production loss, which is the sum of products of the probabilities that the system is in state s ($P(U_f = s)$) with corresponding costs incurred from the production loss (b_s). We enumerate all possible system scenarios and evaluate probabilities of occurrence using Equation (4).

$$c_f^p = \sum_{s \in S} b_s \cdot P(U_f = s), \quad \forall f \in F \quad (7)$$

As illustrated in Fig. 2, it is necessary to evaluate the condition states of all the assets before the system state can be determined. The condition state of each asset at any time interval depends on the asset type, asset availability, and RUL.

3.3.2 Asset type identification

We employ the auxiliary variable $\lambda_{v,d,t}$ to identify the asset type at time t . Equation (8) shows how the asset model is updated after the replacement takes place. The default value of $\lambda_{v,d,t}$ is 1, which means that all the assets are in the old model. If the replacement $\delta_{v,2,t}$ does not take place at time t , the value of $\lambda_{v,d,t}$ will remain 1. In

contrast, if an asset undergoes a replacement at time t , the value of $\lambda_{v,d,t}$ will become 0, which indicates that the asset is no longer in the old model. Equation (9) specifies that an asset can be in one asset model at a time.

$$\lambda_{v,1,t} = \lambda_{v,1,t-1}(1 - \delta_{v,2,t}), \quad \forall v \in V, t \in T \quad (8)$$

$$\lambda_{v,2,t} = 1 - \lambda_{v,1,t}, \quad \forall v \in V, t \in T \quad (9)$$

3.3.3 Asset availability

The asset availability at time t depends on whether the asset undergoes a maintenance activity in the considered time period. Hence, this necessitates the determination of the remaining unavailable time (RUT) denoted by $e_{v,t}$ to check whether an asset v is still under maintenance at that time t . Equation (10) demonstrates how the RUT is used to determine the amount of time that an asset will be unavailable as of a considered time period. For instance, if an asset has an RUT of four months in the current period, it means that there are four months left until an ongoing maintenance activity is completed. This equation shows that the RUT of the current period is calculated by adding the RUT of the previous period (term 1 on the RHS) by the amount of time required to take the asset off-line (term 2 on the RHS) if it undergoes a maintenance activity. If an asset is currently unavailable, the unavailable time will decrease by one every period (term 3 on the RHS becomes -1) until its RUT reaches zero and the asset becomes available again. Equation (11) explains the relationship between the asset availability and the RUT. An asset is considered available only if its RUT is 0.

$$e_{v,t} = e_{v,t-1} + \left(\sum_{d \in D} m_i \cdot \delta_{v,i,t} \right) + (\alpha_{v,t-1} - 1), \quad \forall v \in V, t \in T \quad (10)$$

$$\alpha_{v,t} = \begin{cases} 1 & \text{if } e_{v,t} = 0 \\ 0 & \text{otherwise} \end{cases}, \quad \forall v \in V, t \in T \quad (11)$$

3.3.4 Remaining useful life

Since the deterioration is a function of time that an asset is exposed to corrosion as defined in Equation (3), it is necessary to keep track of the asset operational age – the accumulated (or equivalent) time an asset has been operational – to determine the RUL. Equation (12) explains how the asset operational age is evaluated. Term 1 of the RHS is active if no maintenance takes place on asset v at time t . The operational age at time t increases by one month if the asset is operational in the previous period ($\alpha_{v,t-1} = 1$). Contrarily, if any of the maintenance activities is performed at time $t - 1$, term 1 becomes inactive. The operational age will either become q_d (the equivalent asset age) if a repair takes place (term 2 becomes active) or restart from 0 if a replacement takes place in the considered period. Equation (13) demonstrates how the RUL of asset v at time t is calculated. Term 1 of the RHS is active if asset v does not undergo any maintenance activity. If the asset is not operational in the previous period, the RUL of an asset decreases by $g_d(\theta_{v,t})$, where $g_d(\theta_{v,t})$ is the deterioration model of asset type d , as previously defined in Equation (3). Term 2 is active if an asset is repaired in that period; the level of RUL restored depends on the its asset state in the previous period ($\gamma_{v,a,t-1}$). For instance, if asset $v = 1$ is repaired in period $t = 10$ and its state in period $t - 1 = 9$ is $a = 4$, the level of RUL restored becomes $p_{1,4} \cdot \gamma_{1,4,9}$. Lastly, the third part is active if the replacement takes place. The RUL of the asset becomes p_2 .

$$\theta_{v,t} = \prod_{i \in I} (1 - \delta_{v,i,t}) (\theta_{v,t-1} + \alpha_{v,t-1}) + (\delta_{v,1,t} \sum_{a \in A} q_a \cdot \gamma_{v,a,t-1}), \quad \forall v \in V, t \in T \quad (12)$$

$$u_{v,t} = \prod_{i \in I} (1 - \delta_{v,i,t}) (u_{v,t-1} - \sum_{d \in D} \lambda_{v,d,t} \cdot \alpha_{v,t-1} \cdot g_d(\theta_{v,t})) + (\delta_{v,1,t} \sum_{a \in A} p_{1,a} \cdot \gamma_{v,a,t-1}) + (\delta_{v,2,t} \cdot p_2), \quad \forall v \in V, t \in T \quad (13)$$

3.3.5 Asset state identification

The pseudo code of Algorithm I (Fig. 3) demonstrates how an asset state is identified from the asset availability and RUL. If an asset is unavailable in a considered time period ($\alpha_{v,t} = 0$), the asset state becomes non-operational ($\gamma_{v,0,t} = 1$). If an asset is available, the asset state is defined by the RUL thresholds ($h_{a,d}$). The asset is in a considered condition state if the RUL of an asset is located between that minimum threshold of that state (inclusive) and the minimum threshold of the next state (exclusive).

```

Algorithm I: Asset state identification
Input:  $\alpha_{v,t}, u_{v,t}, \lambda_{v,d,t}$ 
Output:  $\gamma_{v,a,t}$ 
True for:  $\forall v \in V, t \in T$ 
IF  $\alpha_{v,t} = 0$ 
     $\gamma_{v,0,t} = 1$ 
ELSE
    IF  $\sum_{d \in D} \lambda_{v,d,t} \cdot h_{1,d} \leq u_{v,t} < \sum_{d \in D} \lambda_{v,d,t} \cdot h_{2,d}$ 
         $\gamma_{v,1,t} = 1$ 
        ⋮
    ELSE IF  $\sum_{d \in D} \lambda_{v,d,t} \cdot h_{a,d} \leq u_{v,t} < \sum_{d \in D} \lambda_{v,d,t} \cdot h_{a+1,d}$ 
         $\gamma_{v,a,t} = 1$ 
        ⋮
    ELSE IF  $\sum_{d \in D} \lambda_{v,d,t} \cdot h_{M-1,d} \leq u_{v,t} < \sum_{d \in D} \lambda_{v,d,t} \cdot h_{M,d}$ 
         $\gamma_{v,M-1,t} = 1$ 
    ELSE
         $\gamma_{v,M,t} = 1$ 
    END
END

```

Fig. 3. Pseudo code for Algorithm I (asset state identification).

3.3.6 System scenario identification

The system scenario is key to determining the expected cost of production loss. To identify the system scenario that occurs at any considered period, we match the asset state identified in Algorithm I with one of the possible system scenarios of which the enumeration was demonstrated in Section 3.2. We perform the matching operation between asset and system states as shown in Algorithm II. The pseudo code of the algorithm is illustrated in Fig. 4. The operation is performed by counting the number of assets that are in state a at time t ($\eta_{a,t}$) and find the system state that matches $\eta_{a,t}$ for all $a \in A$.

```

Algorithm II: System scenario
Input:  $\gamma_{v,a,t}$ 
Output:  $\eta_{a,t}, \varphi_{f,t}$ 
True for:  $\forall t \in T$ 
FOR  $v = 1$ : No. of assets
    IF  $\gamma_{v,a,t} = 1$ 
         $\eta_{a,t} = \eta_{a,t} + 1$ 
    END
END
FOR  $f = 1$ : Total system scenarios
    IF  $\eta_{a,t}$  matches the enumeration of system scenario  $f$ 
         $\varphi_{f,t} = 1$ 
    END
END

```

Fig. 4. Pseudo code for Algorithm II (system scenario identification).

3.3.7 Resource constraint

To incorporate resource constraint into the optimisation model, the constraint violation cost (κ) is included in the objective function (5). We consider the budget constraint that limits the number of maintenance activities allowed in a fiscal year. The calculation of the violation cost is based on the Algorithm III (Fig. 5). The algorithm determines the total cost incurred from repair and replacement activities and add the penalty cost if the total cost incurred exceeds the budget allowance in a specified time period. Each constrained time period is referred to as ‘period’ in the Psuedo code shown in Fig. 5.

```

Algorithm III: Resource constraint
Input:  $\delta_{v,i,t}$ 
Output:  $\kappa$ 
FOR  $y = 1$ : No. of budget constrained periods
    WHILE  $t$  is within the considered period
        FOR  $v = 1$ : No. of assets
            maintenance cost incurred in period  $y =$  maintenance cost
            incurred in period  $y + \sum_i c_i^m \delta_{v,i,t}$ 
        END
         $t = t + 1$ 
    END
    IF maintenance cost incurred in period  $y >$  budget limit
         $\kappa = \kappa +$  discounted budget violation cost
    END
END

```

Fig. 5. Pseudo code for Algorithm III (resource constraint).

3.4 Solution method

To overcome the possibility that a meta-heuristic algorithm terminates at a low-quality solution, we propose a two-step solution method. The proposed solution method is illustrated in Fig. 6. In the first step, a finite loop search algorithm is developed. This algorithm aims at identifying a combination of maintenance decisions that result in the lowest objective function by considering only decisions made within a pre-specified search space – the asset state thresholds. That is, the algorithm takes into account only the decisions to undertake maintenance activities right before assets deteriorate to their lower asset states. The motivation behind this concept is based on the assumption that the RUL improvement level depends on the asset state at the time a maintenance activity is undertaken. Hence, performing maintenance activities at the lower bound of an asset state will allow the user to reap maximum benefits from RUL extensions.

Nonetheless, it is possible that the outputs obtained from the first step can violate the resource constraint because of limited search space of the algorithm. Thus, in the second step, we employ a GA to finalise the maintenance decisions. The essence of GAs entails the creation of candidate solutions (the population), the evaluation of population fitness, and the alteration of the population to obtain better solutions. Solutions are encoded and decoded as arrays of bits or character strings to represent chromosomes. Throughout the search process, the population is evolved towards more satisfactory solutions. The usual GA evolution process begins with generation of population within a feasible solution area. In each generation, the fitness of every candidate solution in the population is evaluated. Potential solutions are stochastically selected from the current population are subsequently modified using crossover and mutation operations in order to create a new population. The new generation is applied in the following iteration. This population evolution process is repeated and terminates when pre-defined stopping criteria are met. The proposed GA will allow some of the solutions obtained from the first step to be slightly preponed or postponed to avoid the resource constraint violation. GA is chosen as a solution method in the second step because of two main reasons. Firstly, GA requires only objective function information, rather than derivatives, and thus is suitable for our problem with complex cost functions. Secondly, GA search is carried out

from one population to another, rather than from one individual to another. The parallelism enables GA to simultaneously diversify search in different directions [2], making it a pragmatic approach to tackling our problem that entails a high-dimensional variable vector.

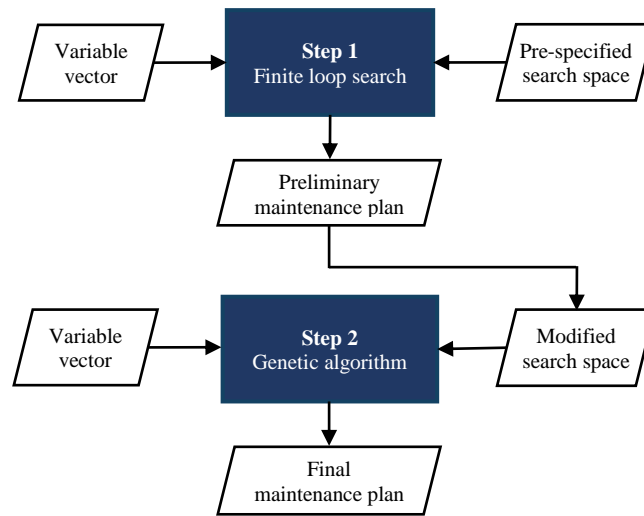


Fig. 6. Two-step solution method

The pseudo code of the proposed finite loop search algorithm applied in Step 1 is shown in Fig. 7. The aim of this algorithm is to determine preliminary repair and replacement plans for all the assets to guide the GA search. The loop search is performed by considering all possible asset state thresholds for both repair and replacement activities. As shown in the algorithm, the total cost is calculated for each loop. If the newly calculated cost is less than the previous lowest cost, the best total cost is updated and the corresponding solution $x_{v,i}$ is archived.

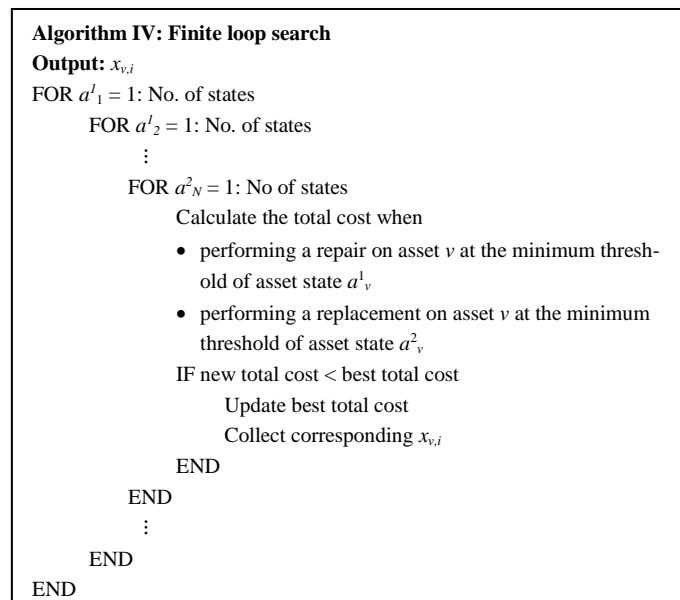


Fig. 7. Pseudo code for Algorithm IV (finite loop search).

As for the GA applied in Step 2, genes are decision variables for repair and replacement plans as stated in (5). We follow the general GA process as previously explained. The variables used to denote periods for undertaking maintenance actions can take integer values within the upper and lower bounds derived from the first step. The upper bound is the exact solution obtained from the loop search algorithm with an additional one-year slot (allow the solutions to be postponed), whereas the lower bound is the time at which the asset is in the same or a

better state (allow the solutions to be preponed). The fitness function is the objective function (6). The initial population for this GA search is generated by random sampling. Within the algorithm, the parent selection is based on a stochastic uniform pattern. A standard one-point crossover is employed with the crossover probability of 0.95, while the mutation probability of 0.01 is adopted. To cope with a large solution vector, we select the population size of 100 chromosomes. Two stopping criteria are employed; the search process is terminated either after 500th generation or when no new best solution is found for ten consecutive generations.

4. Numerical experiments

To validate the proposed model and demonstrate its capability, we conducted numerical experiments and sensitivity analyses based on a case study of the 6-out-of-7 system described in Section 2. Parameters related to asset deterioration explained in Section 3 are estimated by expert judgment. Expected deterioration behaviours expressed as RUL over time are illustrated in Fig. 8 along with thresholds for different asset states. Other parameters employed in these experiments are shown in Table 3. It is noteworthy that the parameters have been masked and scaled due to confidentiality issues. The time horizon considered in this study is 40 years. This time frame is chosen because all the assets will have been replaced in the run-to-failure scenario by this time. To solve the optimisation model proposed in Section 3.3, we employ the two-step solution method explained in Section 3.4. The results obtained from the proposed model are benchmarked against those derived from a general BMC function [30] and from a run-to-failure (RTF) policy. The effectiveness of the two-step solution method is also compared to that of the sole application of GA [20]. To incorporate uncertainties about asset failure, every results validation is carried out using Monte Carlo simulation. Last but not least, we perform two sensitivity analyses to assess the robustness of the proposed model.

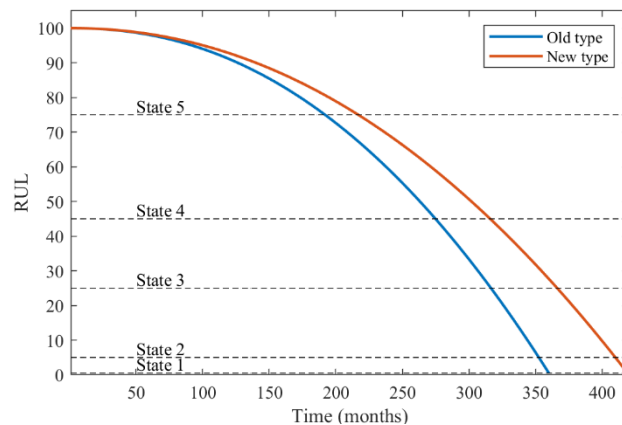


Fig. 8. Expected asset deterioration over time

Table 3
Summary of parameters.

Definition	Parameter	Value
Off-line time (months)	m_i	[1 6]
Improvement level (RUL)	$p_{1,a}$	[0 25.29 50.08 70.17 89.00]
Brand-new condition (RUL)	p_2	100
Equivalent age after repair (month)	q_a	[0 316 263 208 132 0]
Maintenance cost (kGBP)	c_i^m	[250 600]
Penalty cost (kGBP)	b_s	[0 4400 9000 18000 32000 38000 42000]
Asset state threshold (RUL)	$h_{a,d}$	[0 5 25 45 75; 0 5 25 45 75]
Beginning operational age (month)	$\theta_{v,0}$	[36 30 24 18 12 6 0]
Discount rate	ζ	0.06

4.1 Numerical results

We applied the two-step solution method described in Section 3.4 to solve the proposed MA model. As aforementioned, we also ran a sole GA with similar settings to provide a benchmark for measuring the effectiveness of the proposed solution method. All the experiments were performed on a PC with 2.4GHz Intel® Core™ i5 processor. As with the problem posed in the previous study, GA has to be run for several times until it produces a satisfactory solution [21]. This is because GA sometimes gets stuck at poor solutions even if one of the stopping criteria is met. On the other hand, the proposed two-step method can produce a comparable solution to the sole GA with shorter computational time. By running a finite loop search for 79 seconds, the GA in the second step can arrive at effective solutions within 192 generations on average.

The solutions obtained from the proposed MA model and the BMC model are exhibited in Table 4. The proposed MA model suggests that all repair activities are performed right before assets deteriorates from state 4 to state 3 (between month 238 and 274). Moreover, the replacements for assets 1-6 are brought in when assets are in state 3 and 4, with at least 12 months between each of the replacement activities (between month 300 and 378). The replacement for asset 7 is finally made when the asset is in state 2 (month 420). The model allows asset 7 to be replaced in its poor condition, but still results in low system risks. This is because all other assets are still in satisfactory conditions. On the other hand, the BMC model suggests to perform maintenance activities early when compared to the MA model. It can be seen that repair activities takes place right before all assets deteriorate to state 4 (between month 155 and 191). Moreover, it is also suggested that asset 1-6 are replaced in state 4 (between month 299 and 329) and that asset 7 are replaced in state 3 (month 377). The expected asset conditions over time by the MA, BMC, and RTF strategies are illustrated in Fig. 9.

Table 4

Long-term maintenance plans (recommended maintenance time in month) suggested by MA and BMC model.

Asset	Repair							Replacement						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
MA strategy	238	244	250	256	262	268	274	300	312	324	336	366	378	420
BMC strategy	155	161	167	173	179	185	191	299	305	311	317	323	329	377

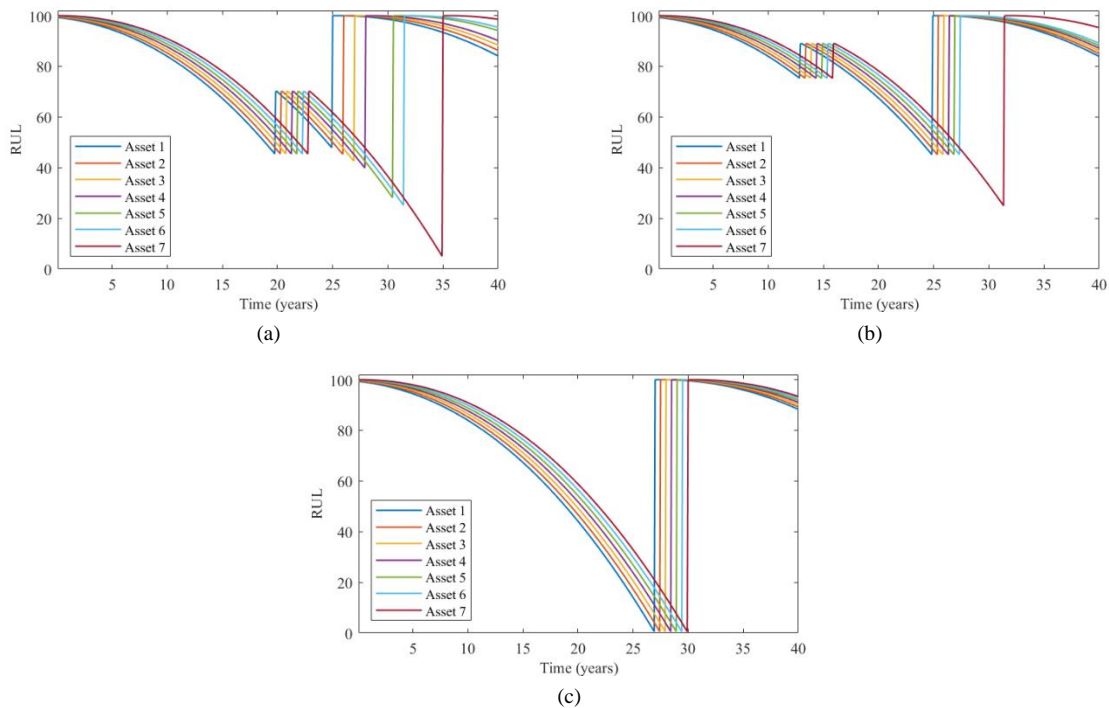


Fig. 9. Expected asset conditions over time estimated from (a) MA strategy, (b) BMC strategy, and (c) RTF strategy

After the Monte Carlo simulation is run for 10,000 iterations, expected numerical results derived from three strategies (MA, BMC, and RTF) are obtained. The convergence test for the proposed Monte Carlo simulation is performed. It can be seen from Fig. 10 that no significant variations of the intermediate mean value are obtained after 2,000 iterations. This means that the considered trial of 10,000 iterations provides a sufficiently accurate statistical analysis of the results.

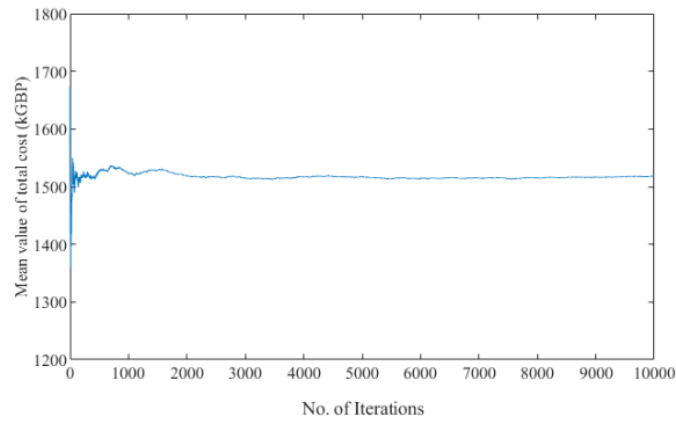


Fig. 10. Convergence plot of the Monte Carlo simulation for total cost calculation

The numerical results shown in Table 5 indicate that the MA strategy outperforms both BMC and RTF strategies in terms of the total cost. It is apparent that the RTF strategy leads to the most inferior results among the three strategies. Although the RTF strategy results in the lowest maintenance cost due to the absence of repair activities, this risky strategy could result in a considerable cost of production loss and a high penalty of budget constraint violation. Moreover, the RTF strategy also leaves the system in the least satisfactory condition with the lowest ending system RUL. Contrarily, the BMC strategy leads to the lowest risk as the cost of production loss is minimal. However, the total cost figures indicate that the BMC model puts forward an overprotective strategy because the maintenance cost – especially the repair cost – incurred by this strategy is significantly higher than that incurred by the MA strategy. Moreover, the negligence of the resource dependence in the BMC strategy also leads to a high constraint violation cost, compared to that of the MA strategy.

Since the risk figure can be accurately reflected in the MA total cost function, it can effectively strike a balance between cost and risk. Compared to the BMC strategy, the MA strategy puts off repair and replacement activities, albeit incurring a slightly higher level of risk. The deferral of maintenance activities in the MA strategy not only enables the system to derive benefits from the time value of money, but also results in the most favourable condition with the highest ending system RUL among the three strategies. From the viewpoint of an asset life cycle, we can see that the proposed MA strategy results in the longest average asset life. In the MA strategy, an asset undergoes a replacement activity on average when it is 349.15 months old (12.40 months longer than the BMC strategy and 26.42 months longer than the RTF strategy).

Table 5
Summary of expected figures obtained from Monte Carlo simulation (10,000 iterations).

Results	MA	BMC	RTF
Total cost (kGBP)	1,520.84	1,809.46	2,009.13
Replacement cost (kGBP)	842.73	870.09	947.30
Repair cost (kGBP)	449.50	738.69	-
Production loss (kGBP)	212.15	110.91	914.70
Constraint violation cost (kGBP)	16.47	89.77	147.13
Ending system RUL	615.64	608.78	588.43
Average asset life (months)	349.16	336.76	322.74

4.2 Sensitivity analyses

The main objectives of the sensitivity analyses are to advance the understanding of the relationships between key parameters and outputs and to determine the performance of the model should it be implemented under different environments. These analyses are performed on the discount rate (ζ) and the system state penalty cost (b_s). The experiments are conducted with respect to the expected total cost and ending system RUL, both derived from the Monte Carlo simulation. In these experiments, the annual discount rates are varied from 2% to 10% in increments of 2%, while the penalty cost vector is changed by -40%, -20%, +20%, and +40%. One parameter is taken at a time; other parameters remain unchanged. The effects of changes in the two parameters on the results suggested by the MA model throughout the 40-year horizon are discussed in the remainder of this sub-section.

4.2.1 Discount rate

Fig. 11 demonstrates how changes in the annual discount rate affect the key outputs derived from the three models. The double y-axis plot illustrates how the expected total cost (primary y axis) and the expected ending system RUL (secondary y axis) change as the discount rate varies. The MA model has been proven to be very robust as the solutions remain unchanged in every scenario. The BMC model is also relatively stable as it produces the same solution for the discount rate between 2% and 8%. The model starts to put forward a suggestion to postpone repair activities when the discount rate reaches 10%. It can be implied the model finds it worthwhile to have a slightly riskier strategy but allow the system derive benefits from the reduction in maintenance cost. As for the expected total cost, the plot shows that the MA strategy outperforms the other two strategies in every case, even though the difference among the three strategies becomes less evident at higher discount rates. It is noticeable that, at the discount rate of 8%, the expected total cost of the RTF strategy is almost as low as that of the BMC strategy because of high maintenance cost incurred in the latter strategy. The gap between RTF and BMC begins to widen after BMC starts to put off repair activities at the discount rate of 10%. That is, the BMC model tends to put forward suggestions towards the MA strategy at the high discount rate. Extrapolating from this trend, we could deduce that the total cost of the two strategies will converge to each other should the discount rate becomes extremely high. Expected ending RUL figures are relatively constant for the three strategies. It is apparent that the MA strategy also produces the most satisfactory results among the three strategies as the system are left in the healthiest condition regardless of the change in the discount rate.

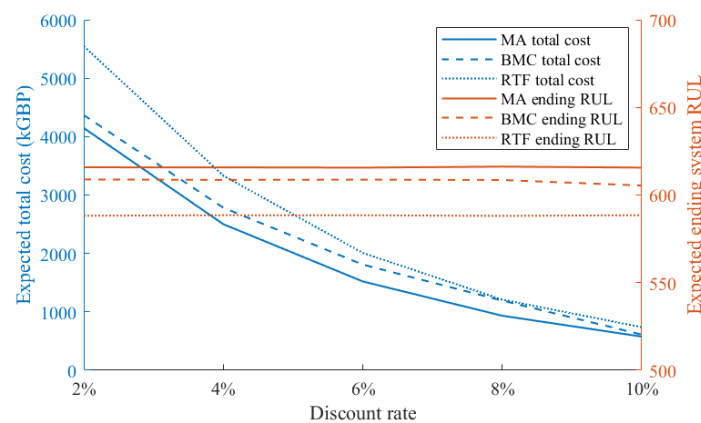


Fig. 11. Sensitivity analysis on discount rate (ζ)

4.2.2 System state penalty cost

The sensitivity analysis on penalty cost is illustrated in Fig. 12. The solutions obtained from the MA model indicates that the solution is very stable regardless of changes in the penalty cost, as the solutions remain unchanged in every scenario. The BMC model is also relatively robust; the model produces the same solution from the penalty cost of -20% to +40%. When the penalty cost vector is decreased by 40%, the BMC model starts to

postpone maintenance decisions. The sensitivity analysis shows that the expected total cost of every strategy increases as the penalty cost goes up. The expected total costs of MA and RTF strategies increase linearly with an increase in penalty cost. Nonetheless, the total cost of the RTF strategy rises at a more rapid rate than that of the MA strategy, because the effect produced by the cost of production loss is more profound. Contrarily, the total penalty cost of the BMC strategy increases sharply when the penalty cost vector changes from -40% to -20%. This is because the MA strategy is sensitive to the penalty cost vector and therefore suggests a highly preventive strategy to mitigate the risk of production loss when the penalty cost is higher. However, an investigation into cost breakdowns indicates that the MA model suggests an overprotective strategy and that more benefits can be derived from putting off maintenance activities. As for the system ending condition, it is apparent that the MA strategy outstrips the other two strategies. The ending system RUL figures are stable in every scenario; only a subtle change can be observed in BMC when the penalty cost vector increases from -40% to -20%.

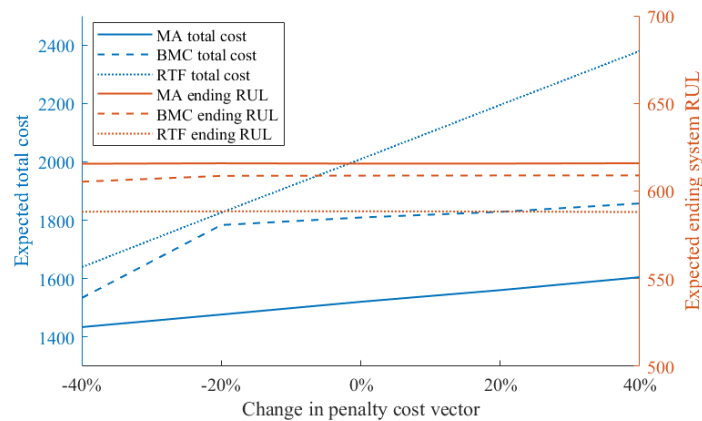


Fig. 12. Sensitivity analysis on system state penalty cost (b_s)

4.3 Discussion

The numerical results and sensitivity analyses demonstrated a vital role that performance and resource dependencies play in effective maintenance planning. The findings indicated that the RTF policy, which aims at minimising actual maintenance cost, could lead to a perilous strategy. This is because the system encounters a serious risk of production loss if an asset is replaced when other operational assets deteriorate to poor conditions. Likewise, the BMC model, which considers the performance dependence in a binary expression, also results in an overprotective strategy. This is due to an overestimation of production loss cost. With an accurate estimation of production loss impact, the MA model has shown to achieve the balance between the two value components. The model has also been proven to minimise the budget constraint violation by the inclusion of resource dependence. These characteristics make MA the most value-effective among the three strategies.

More importantly, using a case study of an effluent treatment system in an oil refinery, we also offer business insights through analyses of computational results. The model offered suggestions on maintenance actions and appropriate times to perform them on different assets. To put these suggestions into actual operation, an asset should be uninstalled and inspected at the scheduled time recommended by the model. If the asset is in the same state as or worse state than that expected by the deterioration model, the corresponding repair or replacement action should subsequently be performed. Otherwise, the asset should be reinstalled, and the asset manager should update deterioration parameters and rerun the model.

The business insights deciphered from our model outputs can be summarised as follows:

- **Prolonged asset life:** The numerical results showed that, among the three strategies, the proposed MA strategy left the system in the best condition at the end of the considered horizon. A further investigation revealed that the MA strategy enabled assets to be replaced at the latest possible period while maintaining

a satisfactory system risk level. The prolonged asset life allows the cost of equipment to be spread over a longer period. This leads to the lower annual depreciation expense, thereby offering great benefits for the company's income statement.

- **Discount rate:** The sensitivity analysis on discount rate indicated that the MA strategy excelled in terms of both total cost and ending system conditions. However, the analysis illustrated that the gap between the MA and BMC strategies becomes less significant as the discount rate increases. It is reasonable to deduce that the two strategies will converge to each other at a very high discount rate. Nonetheless, since the cost function of the MA model is more accurate, it is suggested that asset managers apply the MA model if the computational cost of the MA model is not significantly higher than that of the BMC model to ensure high-quality results.
- **System state penalty cost:** It was apparent from the sensitivity analysis that the MA model was very robust, as the resultant strategy remained unchanged regardless of changes in the system state penalty cost. This robustness could significantly bolster the business confidence. With a wide allowable margin of error of this parameter (-40% to +40%), we can put forward suggestion that expert judgement is sufficient for the estimation of this parameter.

5. Conclusions

In this paper, we explored a novel approach to modelling a multi-asset system with complex performance dependence. The proposed multi-asset (MA) k -out-of- N approach is significantly different from existing multi-component k -out-of- N systems with regards to component/asset states and their contributions to the system performance. The model aims at devising an effective long-term maintenance plan in which there are two available maintenance options: repair and replacement. Moreover, we proposed a tailored solution method that couples a finite search algorithm with a GA to work towards effective solutions. The results obtained from the model were validated via Monte Carlo simulation. To measure the performance of our model, the results derived by the MA model was benchmarked against those of the run-to-failure (RTF) and binary multi-component k -out-of- N (BMC) strategies.

The numerical results showed that the proposed value-driven approach could strike the balance between two value components: cost and risk. Our approach incorporates risk by estimating the cost of production loss should fewer than k assets are operational. Holistic viewpoints on all possible asset breakdown scenarios enable managers to quantitatively convert risk into a commensurable value to cost and make informed managerial decisions. The resultant strategy allows managers to benefit from lower maintenance costs and longer asset lives. Moreover, to offer penetrating insights into the model outputs, in-depth numerical experiments including two sensitivity analyses were conducted. Our analyses demonstrated the robustness of the proposed model under different choices of two key parameters: discount rate and system state penalty cost. This enables us to offer two additional suggestions. Firstly, expert judgment is sufficient for the estimation of these parameters. Secondly, the MA model should be implemented if it does not incur much higher computational cost than that of the BMC model since the MA strategy outperformed the other two strategies in every scenario in terms of total cost and system conditions.

Last but not least, we can put forward two main recommendations for further studies. Firstly, it could be beneficial for asset managers to incorporate the real-time condition data to obtain better results. In this paper, since we applied a gamma deterioration model to estimate the conditions of multiple assets over time, the estimation is unlikely to be perfectly accurate. If sensors for condition monitoring are installed, the variability in asset condition estimation can be significantly mitigated. This will certainly open up an opportunity for a dynamic condition-based optimisation model. Secondly, since our current study determines the cost and corollary of system conditions from a single stakeholder's point of view, it would be interesting to extend our model to accommodate a system with multiple stakeholders. Thus, future research that can work out an acceptable compromise among diverse entities will be invaluable for managing systems that provide direct services to customers.

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References

- [1] Cho DI, Parlar M. A survey of maintenance models for multi-unit systems. *European Journal of Operational Research* 1991;51:1–23. [https://doi.org/10.1016/0377-2217\(91\)90141-H](https://doi.org/10.1016/0377-2217(91)90141-H).
- [2] Petchrompo S, Parlikad AK. A review of asset management literature on multi-asset systems. *Reliability Engineering & System Safety* 2019;181:181–201. <https://doi.org/10.1016/j.res.2018.09.009>.
- [3] Dekker R, Wildeman RE, Van Der Duyn Schouten FA. A review of multi-component maintenance models with economic dependence. *Mathematical Methods of Operations Research* 1997;45:411–35.
- [4] Olde Keizer MCA, Flapper SDP, Teunter RH. Condition-based maintenance policies for systems with multiple dependent components: A review. *European Journal of Operational Research* 2017;261:405–20. <https://doi.org/10.1016/j.ejor.2017.02.044>.
- [5] Coit DW, Smith AE. Reliability optimization of series-parallel systems using a genetic algorithm. *IEEE Transactions on Reliability* 1996;45:254–60, 263. <https://doi.org/10.1109/24.510811>.
- [6] Liu H. Reliability of a load-sharing k -out-of- n : G system: Non-iid components with arbitrary distributions. *IEEE Transactions on Reliability* 1998;47:279–84. <https://doi.org/10.1109/24.740502>.
- [7] Chiang DT, Niu S-C. Reliability of Consecutive- k -out-of- n : F System. *IEEE Transactions on Reliability* 1981;R-30:87–9. <https://doi.org/10.1109/TR.1981.5220981>.
- [8] Chao MT, Fu JC, Koutras MV. Survey of Reliability Studies of Consecutive- k -out-of- n : F & Related Systems. *IEEE Transactions on Reliability* 1995;44:120–7. <https://doi.org/10.1109/24.376531>.
- [9] Li W, Zuo MJ. Reliability evaluation of multi-state weighted k -out-of- n systems. *Reliability Engineering and System Safety* 2008;93:161–8. <https://doi.org/10.1016/j.res.2006.11.009>.
- [10] Huang J, Zuo M, Wu Y. Generalized multi-state k -out-of- n : G systems. *IEEE Transactions on Reliability* 2000;49:105–11. <https://doi.org/10.1109/24.855543>.
- [11] Doostparast M, Kolahan F, Doostparast M. A reliability-based approach to optimize preventive maintenance scheduling for coherent systems. *Reliability Engineering and System Safety* 2014;126:98–106. <https://doi.org/10.1016/j.res.2014.01.010>.
- [12] Xie W, Liao H, Jin T. Maximizing system availability through joint decision on component redundancy and spares inventory. *European Journal of Operational Research* 2014;237:164–76. <https://doi.org/10.1016/j.ejor.2014.02.031>.
- [13] Faghih-Roohi S, Xie M, Ng KM, Yam RCM. Dynamic availability assessment and optimal component design of multi-state weighted k -out-of- n systems. *Reliability Engineering and System Safety* 2014;123:57–62. <https://doi.org/10.1016/j.res.2013.10.002>.
- [14] Taghipour S, Kassaei ML. Periodic Inspection Optimization of a k -Out-of- n Load-Sharing System. *IEEE Transactions on Reliability* 2015;64:1116–27. <https://doi.org/10.1109/TR.2015.2421819>.
- [15] Huynh KT, Barros A, Bérenguer C. Multi-level decision-making for the predictive maintenance of k -out-of- n : F deteriorating systems. *IEEE Transactions on Reliability* 2015;64:94–117. <https://doi.org/10.1109/TR.2014.2337791>.
- [16] Torres-Echeverría AC, Martorell S, Thompson HA. Multi-objective optimization of design and testing of safety instrumented systems with MooN voting architectures using a genetic algorithm. *Reliability Engineering and System Safety* 2012;106:45–60. <https://doi.org/10.1016/j.res.2012.03.010>.
- [17] Abdollahzadeh H, Atashgar K. Optimal design of a multi-state system with uncertainty in supplier selection. *Computers and Industrial Engineering* 2017;105:411–24. <https://doi.org/10.1016/j.cie.2017.01.019>.
- [18] Vu HC, Do P, Barros A, Bérenguer C. Maintenance grouping strategy for multi-component systems with dynamic contexts. *Reliability Engineering and System Safety* 2014;132:233–49. <https://doi.org/10.1016/j.res.2014.08.002>.
- [19] Babishin V, Taghipour S. Joint optimal maintenance and inspection for a k -out-of- n system. *International Journal of Advanced Manufacturing Technology* 2016;87:1739–49. <https://doi.org/10.1007/s00170-016-8570-z>.
- [20] Petchrompo S, Parlikad AK. Heuristic optimisation for multi-asset intervention planning in a petrochemical plant. 7th International Conference on Through-life Engineering Services, Cranfield: 2018.
- [21] Khorshidi HA, Gunawan I, Ibrahim MY. A value-driven approach for optimizing reliability-redundancy allocation problem in multi-state weighted k -out-of- n system. *Journal of Manufacturing Systems* 2016;40:54–62. <https://doi.org/10.1016/j.jmsy.2016.06.002>.

- [22] Wang H. A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research* 2002;139:469–89. [https://doi.org/10.1016/S0377-2217\(01\)00197-7](https://doi.org/10.1016/S0377-2217(01)00197-7).
- [23] van Noortwijk JM. A survey of the application of gamma processes in maintenance. *Reliability Engineering and System Safety* 2009;94:2–21. <https://doi.org/10.1016/j.ress.2007.03.019>.
- [24] Kuo W, Zuo MJ. *Optimal Reliability Modeling: Principles and Applications*. Wiley; 2003.
- [25] Myers A. *Complex System Reliability: Multichannel Systems with Imperfect Fault Coverage*. Springer London; 2010.
- [26] Cheng GQ, Zhou BH, Li L. Joint optimization of lot sizing and condition-based maintenance for multi-component production systems. *Computers and Industrial Engineering* 2017;110:538–49. <https://doi.org/10.1016/j.cie.2017.06.033>.
- [27] van Noortwijk JM, van der Weide JA, Kallen M-J, Pandey MD. Gamma processes and peaks-over-threshold distributions for time-dependent reliability. *Reliability Engineering & System Safety* 2007;92:1651–8.
- [28] Ling MH, Tsui KL, Balakrishnan N. Accelerated degradation analysis for the quality of a system based on the Gamma process. *IEEE Transactions on Reliability* 2014;64:463–72.
- [29] Strauss A, Wan-Wendner R, Vidovic A, Zambon I, Yu Q, Frangopol DM, et al. Gamma prediction models for long-term creep deformations of prestressed concrete bridges. *Journal of Civil Engineering and Management* 2017;23:681–98.
- [30] Huynh TK, Barros A, Bérenguer C. A reliability-based opportunistic predictive maintenance model for k-out-of-n deteriorating systems. *Chemical Engineering Transactions* 2013;33:493–8. <https://doi.org/10.3303/CET1333083>.