The Illusions of Calculating Total Factor Productivity and Testing Growth Models: From Cobb-Douglas to Solow and Romer

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Abstract: Growth models in the tradition of Solow and Romer are framed in terms of production functions. Consequently, they are equally subject to a criticism developed by, among others, Phelps Brown (1957), Simon (1979a), and Samuelson (1979). These authors argued that production function estimations are flawed exercises because output, labor and capital stock, are definitionally related through an accounting identity. The identity argument helps demystify two illusions in the literature: (i) *finding the Holy Grail*: total factor productivity is, by construction, a weighted average of dollars per worker and a pure number (the rate of profit or the rental rate of capital); and (ii) *the possibility of testing:* if estimated properly, production function regressions will yield: (a) a very high fit, potentially an R^2 of unity; and (b) estimated factor elasticities equal to the factor shares, hence they must always add up to 1. We illustrate these points through a series of well-known growth accounting exercises and models directly derived from production functions. They are futile exercises. We conclude that we know substantially less than we think about growth and that many of the discussions in the neoclassical growth literature are Kuhnian puzzles that only make sense within this paradigm.

Keywords: accounting identity, production function, Romer, Solow, Total Factor Productivity

JEL codes: E22, E23, E25, O11, O33, O47

> "It [the neoclassical production function] must have needed an even tougher hide to survive Phelps Brown's article on "The meaning of the Fitted Cobb-Douglas Function" than to ward off Cambridge Criticism of the marginal productivity theory of distribution." Joan Robinson (1970, p.317)

1. INTRODUCTION

It is surprising how only a few scholars have questioned the remarkable explanatory power of extremely simple growth models. These models relate a country's GDP growth (or in per capita terms) to only a few variables, namely capital and labor growth, or savings rate and population growth (and at times human capital or a few additional controls).[1] Common to these models is that they are derived from production functions with neoclassical properties. How is it possible that such simple models account for a significant share of the variation in growth rates across countries? Mankiw (1997, p.104), for example, expressed the view that: "I have always found the high R^2 reassuring when I teach the Solow growth model. Surely, a low R^2 in this regression would have shaken my faith that this model has much to teach us about international differences in income." The reason for the faith in these models probably lies in the fact that researchers believe, since Cobb and Douglas (1928), that such models reflect the "laws of production", and that they can be tested. We are not persuaded by this justification.[2]

This paper surveys the literature that questions the uncritical acceptance of the relevance of production functions. [3] It extends an argument put forward initially by Phelps Brown (1957) in a paper in the *Quarterly Journal of Economics* on the work of Cobb and Douglas. Phelps Brown showed that cross-sectional estimates of production functions are predetermined (hence the results are known *ex ante*) by an accounting identity that relates output, employment and capital stock. Production functions and this identity are "different sides of the same penny" as Phelps Brown put it. No wonder the good results, with the

[1] We acknowledge the large empirical literature of growth regressions that claim that the number of robust regressors "related" to growth is significant (Sala-i-Martin 1997).

[2] Much less in the light of the damaging conclusions of the Cambridge Capital Controversies and the aggregation problem. On these, see Felipe and Fisher (2003).

[3] Some of the points we make in this paper draw on Felipe and McCombie (2013).

consequence is that the estimation of production functions is a pointless exercise.[4] Ironically, this paper appeared the same year Solow (1957) published his seminal growth accounting exercise, which also entailed the estimation of several production functions.[5] Phelps Brown paper, however, was ignored.

A few years later, Simon and Levy (1963) and Shaikh (1974) also rehearsed essentially the same argument, the former in the context of cross-sectional data and the latter in the context of time series. Both papers passed unnoticed.[6] Simon (1979a) came back to it and thought that it was sufficiently important so as to mention it in his Nobel Prize lecture (Simon 1979b, p.497). Perhaps even more ironical is the fact that Samuelson (1979) used it to question Cobb and Douglas's (1928) work.[7] After going over the algebra of the argument, he commented: "I hope that someone skilled in econometrics and labor will audit and evaluate my critical findings" (Samuelson 1979, p.934). The message fell again on deaf ears, and still today, four decades later, most economists are unaware of it.

The logic of the argument is very simple. Let's start with the accounting identity $Y_t \equiv w_t L_t + r_t K_t$, where Y is constant-price , or real, value added (measured in dollars of a base year prices), w is the real wage rate (dollars per worker), L is employment (number of workers), r is the profit rate (a pure number), and K is the real value of the stock of capital (dollars of a base year). We show that this identity can be rewritten as particular forms (depending on the path of the data) of $Y_t \equiv A_t F(L_t, K_t)$ (e.g., Cobb-Douglas, CES, or translog), where A is a variable that picks up movements in w and r . As a consequence, regressions of output on capital, employment, and a trend or another variable (e.g., human capital), are

[4] Reder (1943), Bronfenbrenner (1944), and Marschak and Andrews (1944) had already made references to the problem in discussions of Cobb and Douglas' (1928) results, although not in the clearest way.

[5] Solow's (1957, p.312) warning that "it takes something more than the usual willing suspension of disbelief to talk seriously about aggregate production function" did not have any impact on the profession. Solow himself did not follow it.

[6] Actually, Shaikh's (1974) paper was criticized by Solow (1974). It is important to read Shaikh's (1980) rebuttal.

[7] Phelps Brown, Simon, and Samuelson, expressed the problem in slightly different ways. However, once the issue at hand is understood, it is clear that their expositions amount to the same.

problematical exercises because they cannot be refuted empirically. They will yield a very high fit and factor elasticities equal to the respective factor prices.

With his two path-breaking papers in the 1950s, Solow (1956, 1957) provided the basis for the neoclassical theory of growth and the empirical work on the sources of growth (i.e., growth accounting and the calculations of total factor productivity growth). His method to estimate total factor productivity (TFP) growth is still seen as iconic, and the idea that growth is about factor accumulation plus "something else," loosely designated as technical progress, or the "Solow" residual, still dominates the profession's thinking.

Romer (1986) opened the way for economists to deal with increasing returns to scale and imperfect competition. It led to a second wave of empirical work on growth (e.g., the determinants of growth and total factor productivity by positing an R&D production function, and the convergence literature). Both Solovian and endogenous growth models have in common that they model growth using a production function. While the growth debates of the 1990s have vanished (either because the profession thought they were settled and/or because researchers moved on), growth is still conceptualized in many quarters in terms of production functions. As we shall see, this is extremely problematic for empirical analyses.

In this paper, we elaborate upon the original Phelps Brown-Simon-Samuelson argument and show that it applies to *all* types of production functions, not just the Cobb-Douglas. In this sense, we accept Samuelson's (1979) invitation to assess his claims (arguments), which have generally been ignored in the literature, despite their severe consequences for much of standard growth analyses. We show that the arguments also apply to the production functions hypothesized by the endogenous growth literature pioneered by Romer (1987), and to the growth literature that estimates equations derived from production functions. The same equations can be derived from the identity, which questions their interpretation as refutable models. Finally, the argument also helps question the interpretation of total factor productivity in growth accounting exercises as a measure of technical change.

Our paper is not the first one to question both the concept of TFP and the methods used to calculate it. Solow (1957) himself, and after him authors such as Griliches (1987, 1994, 1995) or Nelson (1973, 1981), noted that the concept of TFP was problematic and that its

calculation was not problem-free. Today, there seems to be a general agreement that there is something problematic about the concept and measurement of TFP. Yet, it is still widely used in growth theory, sometimes with some modifications. This is even when authors provide definitions of TFP that are mutually incompatible, as Carlaw and Lipsey (2003, pp. 458-460) note. However, the arguments concerning the TFP problems are very different from ours.

We contend that the problems with the (production-function based) growth literature are much deeper than the measurement of capital (however important), the values of the factor shares used in growth accounting, or the endogeneity of the regressors. [8] The problems we discuss are insoluble. Hence, we do not think that offering a minor reinterpretation of what TFP putatively measures, or a methodological variant to calculate it, will advance our knowledge. The general view, however, is that the growth models we discuss (production-function based) are meaningful constructions. This is because they supposedly have a sound theoretical basis; and are useful because their assumptions and predictions can be tested with actual data. This procedure is what gives them a scientific basis (Friedman 1953), irrespective of the implausibility or otherwise of their assumptions. Moreover, many economists believe that the models are useful because empirical tests show that they indeed explain well the reality, i.e., the factors that determine the growth of nations. We dispute this position.

The rest of the paper is structured as follows. Section 2 states the logic of the problem that we discuss, namely why the accounting identity that underlies all empirical production function exercises undermines the interpretation of this approach. Sections 3, 4, 5, and 6, use a series of well-known papers as examples to develop the implications of the accounting identity argument. We develop the argument in two related ways. The first one involves the calculation and interpretation of standard estimates of TFP. TFP, in levels or growth rates, is central to many theoretical and applied growth models. Empirical calculations of TFP may be

[8] Growth accounting exercises are pervasive in the growth literature despite that most authors who undertake them are somewhat skeptical about what they do. This is partly the result of the problems measuring properly capital (Pritchett 2000). Moreover, it is generally accepted that the exercises do not prove causality (Aghion and Howitt 2007). Nevertheless, most growth theorists consider that performing a growth accounting exercise is a useful first step in looking at the data. Our view is that the concerns about TFP expressed by many authors are, at best, second order issues.

likened as the quest for the Holy Grail. The second argument concerns the possibility, or otherwise, of testing models, i.e., of statistically refuting a null hypothesis. We show that production function regressions can never be refuted statistically. This is simply because they are approximations (although potentially exact) to the accounting identity – there is no error term.

Section 3 revisits Cobb and Douglas's (1928) estimation and Solow's (1957) growth accounting exercise, from the point of view of the identity. Sections 4, 5, and 6, discuss a series of well-known papers as additional examples from the growth and macroeconomics literature to further develop implications of the accounting identity argument. Out of necessity we had to be selective in the choice of examples. All of them are well-known papers that have shaped the profession's knowledge and are still cited. We are aware that there are other papers and hope readers can discover the relationship with the accounting identity. In a way, one can liken this to the children's game of "where's Waldo?" The papers we discuss are: Young (1992, 1995), Hsieh (1999, 2002), Young (1994), Romer (1987), Mankiw et al. (1992), Jones (1997), Hall (1988, 1990), and Shapiro (1987). Section 7 offers some concluding remarks.

2. STATEMENT OF THE PROBLEM

This section starts by providing the general version of the Phelps Brown-Simon-Samuelson critique. Second, it provides a brief on the calculation of TFP as done in the literature. Third, it discusses the implications of the critique for the interpretation of the standard calculations of TFP as a measure of technical progress and model testing.

2.1 The argument

Let's start by writing how the data appear in the National Income and Product Accounts (NIPA) identity:

$$Y_t \equiv W_t + S_t \tag{1a}$$

where Y is real (i.e., deflated) GDP, or value added (e.g., dollars of a base year), W is the real total wage bill (dollars of a base year) and S is the operating surplus (dollars of a base year). It is very important to stress that identity (1a) (note the symbol ≡) is true at any level of

aggregation, including, at the firm level. NIPA statisticians construct the identity by arithmetic summation (aggregation) from individual firm-level data and government institutional data.[9] This aggregation is logically consistent, and unrelated to the problem of aggregating production functions (Felipe and Fisher 2003). Equation (1a) is theory-free (e.g., it does not depend on the zero profits assumption) and it is not related to or derived from production/cost theory.

We now dichotomize both the wage bill and operating surplus into the products of a *price* times a *quantity* as:

$$Y_t \equiv w_t L_t + r_t K_t \tag{1b}$$

or in per worker terms:

$$y_t \equiv w_t + r_t k_t \tag{1c}$$

where w is the average real wage rate (dollars of a base year per worker), L is total employment (number of workers), r is the ex post average profit rate (dollars of operating surplus per dollar of capital stock, a pure number), K is the stock of capital (dollars of a base year). Furthermore, $y_t = (Y_t / L_t)$ and $k_t = (K_t / L_t)$. Note that by construction $W_t = w_t L_t$ is the wage bill and $S_t = r_t K_t$ is total profits (operating surplus).[10] We discuss below an alternative way of writing equation (1b), splitting the surplus into the cost of capital and monopolistic profits.

One can simply now express the accounting identity (1b) in growth rates as

$$\hat{Y}_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t + a_t \hat{L}_t + (1-a_t)\hat{K}_t \tag{2a}$$

or,

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[9] The specifics of how national account statisticians construct equation (1a) in practice are not important for us.

[10] We just note that while it is self-evident that the wage bill (W_t) is split into the product of a price (w_t is measured in \$/worker) times a quantity (L_t is measured in no. workers), it is much less obvious that this is also the case of the operating surplus (S_t). This is because the units of r_t and K_t are a percentage and dollars of a base year, respectively. This does not mean that writing $S_t = r_t K_t$ is incorrect as the product still yields dollars. Also, it should be obvious that w_t and r_t may or may not be the marginal products of labor and capital, respectively, in the sense of being derived from a production function, even though this is what equation (1b) will always indicate, i.e., $(\partial Y / \partial L) \equiv w$ and $(\partial Y / \partial K) \equiv r$.

$$\hat{Y}_t \equiv \lambda_t + a_t \hat{L}_t + (1-a_t)\hat{K}_t \tag{2b}$$

where a_t and $(1-a_t)$ are the labor and capital shares in GDP. Rearranging terms yields:

$$TFP_t^1 \equiv \hat{Y}_t - a_t \hat{L}_t - (1-a_t)\hat{K}_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t \equiv TFP_t^{D1} \tag{2c}$$

where the numbers on the superscripts over TFP refer to the sequence that we will follow in the paper (i.e., 1, 2, 3, etc.) to distinguish the several and slightly different measures of TFP that we will derive. The superscript D is used to refer to the right-hand side of the identity (i.e., the weighted average of the growth rate of the wage and profit rates). We refer to the left and right-hand sides of equation (2c) as TFP and TFP^D, respectively, for reasons that will become clear in the discussion below.

If factor shares happen to be constant, then identity (2a) becomes:

$$\hat{Y}_t \equiv a\hat{w}_t + (1-a)\hat{r}_t + a\hat{L}_t + (1-a)\hat{K}_t \equiv \lambda_t^* + a\hat{L}_t + (1-a)\hat{K}_t \tag{2d}$$

which in levels is:

$$Y_t \equiv A_0 w_t^a r_t^{1-a} L_t^a K_t^{1-a} \tag{2e}$$

with $A_0 \equiv (a)^{-a}(1-a)^{-(1-a)}$.

Further, if the data show that $\lambda_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t \equiv \lambda$, i.e., λ_t happens to grow at a constant rate, then identity (2a) becomes:

$$\hat{Y}_t \equiv a\hat{w} + (1-a)\hat{r} + a\hat{L}_t + (1-a)\hat{K}_t \equiv \lambda + a\hat{L}_t + (1-a)\hat{K}_t \tag{2f}$$

which in levels is:

$$Y_t \equiv A_0 \exp(\lambda t) L_t^a K_t^{1-a} \tag{2g}$$

Recall that equations from (1a) to (2c) are identities; and consequently so are equations (2d)-(2e) (provided factor shares are constant) and (2f)-(2g) (provided factor shares and λ_t are constant). No assumption from production theory is needed to write them.

Before we discuss the implications of identities (1a) to (2g) in section 2.3, we summarize the essentials of the calculation of TFP growth in section 2.2.

2.2 A Brief on the Estimation of Total factor Productivity Growth

The essentials of production function estimation and TFP calculation are so well known and accepted in the literature that it may seem redundant to review them. However, since the theme of this article is the inadequacy of the framework, it is best to summarize it here.[11]

Since Solow (1957), the neoclassical approach starts with the assumption that there is a well-behaved aggregate production function $Y_t = A_t F(L_t, K_t)$. By totally differentiating it with respect to time, the growth rate of output is:

$$\hat{Y}_t = TFP_t + \alpha_t \hat{L}_t + \beta_t \hat{K}_t \qquad (3a)$$

where a circumflex hat over the variables denotes a growth rate; α_t and β_t denote the elasticities of output with respect to labor and capital, respectively; and TFP_t denotes what is often interpreted as the rate of technological progress (i.e., the growth rate of A_t). This is referred to as *total factor productivity growth*, or the *residual*, a variable that supposedly captures all output growth not due to increases in factor inputs. Growth accounting calculates TFP_t residually as $TFP_t = \hat{Y}_t - \alpha_t \hat{L}_t - \beta_t \hat{K}_t$, given values for the right-hand side variables.

The problem, however, is that there are very few reliable estimates of the output elasticities from econometric estimations because of the econometric issues that plague the estimation of production functions. To solve this problem, growth accounting exercises assume that: (i) production is subject to constant returns to scale, (ii) the objective function of the firms in the economy is to maximize profits, and (iii) labor and capital markets are perfectly competitive (wage and profit rates are given by the first-order optimizing conditions). Under these circumstances, the factor elasticities equal the shares of labor and capital in total output, i.e., $\alpha_t = a_t$ and $\beta_t = (1 - a_t)$, where a_t and $(1 - a_t)$ denote the labor and capital shares in output, respectively. Then output growth can be written as:

$$\hat{Y}_t = TFP_t + a_t \hat{L}_t + (1 - a_t)\hat{K}_t \qquad (3b)$$

and consequently, the growth rate of TFP is calculated as:

[11] Van Beveren (2012) provides an up-to-date survey of the methods to estimate TFP, especially of those that take account of the endogeneity problem that results from the fact that productivity and input choices are likely to be correlated; and of the selection bias that emerges if no allowance is made for entry and exit. There is another class of methods based on index number theory that we do not discuss.

$$TFP_t = \hat{Y}_t - a_t\hat{L}_t - (1-a_t)\hat{K}_t \qquad (3c)$$

given that data for all the right-hand side variables are now readily available (the shares of labor and capital in total output can be obtained from the national accounts). The residually measured TFP growth in equation (3c) is referred to as the *primal* measure of TFP growth. This is probably the most widely used method to calculate the growth rate of TFP. Since the calculation involves subtractions, it gives the impression that the resulting figure is sort of a mystery, a residual or measure of our ignorance, which is often how TFP growth is referred to. [12]

Solow (1957) is the seminal work on growth accounting, a paper still today widely cited, and which opened up a whole field of research and created the framework for thinking about economic growth that is now standard among economists. Although growth accounting predates Solow's work, this paper provided the theoretical foundation that growth accounting needed, explicitly deriving the concept from a production function. Using equation (3c), Solow concluded that TFP growth accounted for almost 90% of the overall non-farm sector growth of the United States during 1909-1949, while the remaining 10% was the result of factor accumulation. This result seemed "startling" to Solow.

Note that equation (3c) is a truism. In other words, the estimate of TFP growth is definitionally true and the equation is used to apportion the sources of growth. However, and this is crucial, it is based on an underlying theoretical model in that the output elasticities are assumed to be equal to the relevant factor shares. There is nothing in neoclassical production function theory that says that this has to be the case, hence it could be argued that one could potentially refute this assumption empirically (though erroneously as we shall see below). The fact that most authors today do not test this assumption prior to undertaking a growth accounting exercise does not mean that the point is not important.

[12] There is an important point to be made, which refers to the factor shares used in equation (3c). The derivation in the text assumes that technical progress is Hicks-neutral. In this case, technical progress does not affect the factor shares. If technical progress were biased, one should not use the observed factor shares in the National Accounts, to the extent that these incorporate the effect of technical progress. See Ferguson (1968).

The econometric estimation of the production function is a second method to obtain an estimate of the rate of TFP. This requires the specification of a particular functional form. The simplest case consists in estimating the Cobb–Douglas form

$$Y_t = A_0 \exp(\lambda t) L_t^{\alpha} K_t^{\beta} \exp(u_t) \tag{4a}$$

or in growth rates as

$$\hat{Y}_t = \lambda + \alpha \hat{L}_t + \beta \hat{K}_t + u_t \tag{4b}$$

where λ is the (constant) rate of TFP growth, although nothing in neoclassical production theory says that it needs to be a constant. This is simply done for convenience. Likewise, other functional forms can be estimated (e.g., translog). This method does not impose the assumption of perfectly competitive markets, i.e., the coefficients (factor elasticities) are estimated unrestricted. Barro (1999) claimed that the estimation of the production function is problematic as a method to calculate the rate of technical progress. He listed the following three problems: (i) the growth rates of capital and labor are not exogenous variables with respect to the growth of output; (ii) the growth of capital is usually measured with errors, which often leads to low estimates of the contribution of capital accumulation; and (iii) the regression framework must be extended to allow for variations in factor shares and the TFP growth rate. This is the rationale for the profession's general preference for the growth accounting procedure. We shall see below that Barro's concerns are misplaced.

The growth rate of TFP can also be calculated using the cost function $C_t = (\Omega_t, Y_t, t)$, where Ω denotes the vector of factor prices, that is, the wage rate (w) and the rental rate of capital (ρ). Technical progress here is equated with the rate of cost diminution, or the idea that technical progress lowers the cost of producing a given output. It is referred to as the *dual* measure of TFP growth. The cost function can also be used in a growth accounting exercise, or can be estimated econometrically. The dual is simply calculated by equating the rate of change in product prices with the rate of change in unit costs, and equals (denoted by the superscript *D*):

$$TFP_t^D = \vartheta_{Lt} \hat{w}_t + \vartheta_{Kt} \hat{\rho}_t \tag{5}$$

that is, a weighted average of the growth rates of the wage rate and of the rental rate of capital, where the weights ϑ_{it} are the respective cost shares (in general, different from the output shares. We discuss this below). The superscript D stands for dual.

2.3. What are the implications of the identity argument for empirical work?

We now discuss what the results in section 2.2 mean in the light of the derivation in section 2.2. Given the severe misinterpretations of the argument, we will examine it thoroughly.

There is an obvious point resulting from the above discussions. This is that the two sides of identity (2c) *resemble* equations (3c) and (5), derived from the production and cost functions, respectively.[13] From this, one is tempted to conclude that identity (2c) shows that primal and dual TFP growth rates can be derived from the accounting identity (Barro 1999; Hsieh 1999, 2002). This is incorrect, and the implications that this has for the interpretation of growth accounting exercises and for production function estimations are very serious. As they have been ignored or misunderstood, we elaborate upon them below:

Misinterpretations

(i) Researchers think of output Y as generated by the production function $F(K,L)$. This is the essence of the problem, namely the belief that output and inputs used in most actual production function estimations are observed *independently of the factor prices* (w and r), i.e., as if they were physical quantities, where inputs are transformed into an output through the function $F(\square)$. The reality, however, is that the output measure used (Y) is a monetary value (as is K), not a physical quantity, generated by the statisticians (and firm-level accountants) through the identity equation (1a), then written as equation (1b) (hence through the factor prices).[14] The series Y, L and K are the same in the identity and in the production function.

(ii) The neoclassical tradition acknowledges identity (1b) but argues that the production function, together with the usual neoclassical assumptions and Euler's theorem, provides a

[13] Note that while the left-hand side of equation (2c) is identical to (3c), the right-hand side of equation (2c) is not identical to equation (5). This difference will be discussed in section 4.

[14] GDP in most countries is generated through the demand side of the NIPA. This does not invalidate the claim that output Y comes from an identity, not from $Y = F(K,L,t)$.

theory of the income side of the NIPA. We consider that this line of reasoning is incorrect.[15] Identity (1b) holds by itself and is not dependent upon any conditions from production theory.

(iii) While the weights of the growth rates (the factor shares) in equations (3c) and (5) are theoretically derived by imposing the first-order conditions, the shares in the identity are simply the result of taking the derivative with respect to time. This means that they are the true weights whether factor markets are perfectly competitive or not. Identity (2c) is not a model.

(iv) Identity (2c) makes it clear that the residually calculated TFP growth $TFP_t^1 \equiv \hat{Y}_t - a_t\hat{L}_t - (1-a_t)\hat{K}_t$ is numerically equivalent to $TFP_t^{D1} \equiv a_t\hat{w}_t + (1-a_t)\hat{r}_t$, clearly not a "measure of our ignorance." This is the result of how identity (1a) was split into identity (1b), i.e., $W_t = w_t L_t$ and $S_t = r_t K_t$ (which is unrelated to a production function). This self-evident, yet important, point seems to have been missed by those who think of TFP_t^1 as derived from a production function because they do not see the immediate link with TFP_t^{D1}. They then seem to be surprised when it accounts for a high (or low) share of $\hat{Y}_t$. As the examples in the following sections will show, it is straightforward to rationalize why TFP_t^1 (or equivalently TFP_t^{D1}) takes a given value and then interpret it.

(v) Without loss of generality, suppose that the data for the economy in question show that factor shares are constant, i.e., $a_t \equiv W_t / Y_t = a$ and $(1-a_t) \equiv S_t / Y_t = (1-a)$. Equation (3a) becomes identity (2d), which in levels is (2e). This last expression shows that calculations of

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[15] This seems to be the view of, for example, Jorgenson and Griliches (1967, pp. 252-253) or Prescott (1998, p. 532). From $Y = F(K,L,t)$ one can write $Y = F_K K + F_L L$ (Euler's theorem), and from the first-order conditions, $F_K = r$ and $F_L = w$. Hence $Y = wL + rK$, taken to be identity (1b). That is, the neoclassical framework considers that the production function through Euler's theorem *implies* the identity. While this derivation is mathematically correct, it does not mean that the production function provides a theory of the accounting identity. See also Hulten (2009), who traces the history of growth accounting during the 1930s-1950s, with the identity as starting point. This formulation was "atheoretical" (Hulten 2009, p.4). Solow's (1957) contribution was to provide the economic structure that the approach lacked.

the level of TFP as ($Y / L^{a}K^{1-a}$) (e.g., Jones 1997, Hall and Jones 1999) amount to estimating ($w^{a}r^{1-a}$), and whichever way it is calculated, its units is ($/worker) (as the profit rate is a percent). Consequently, TFP_t^1 (TFP_t^{D1}) is a growth rate of dollars per worker. This also means that calculations of relative TFP levels of two countries will be the ratio of two figures measured in ($/worker); and given that r^{1-a} may not differ significantly between the two units compared, the overall ratio will mostly reflect the ratio of the two wage rates. This is all those who argue that in order to explain the observed large income differences across countries we need a theory of TFP, have discovered (e.g., Prescott 1998, Hall and Jones 1999). [16] It is a circular argument.

(vi) Identity (2a) can certainly be used to apportion growth in an accounting sense into the various components of the identity (the same way it is often done with the identity from the demand side. However, interpreting TFP_t^1 (TFP_t^{D1}) as a measure of the growth in efficiency or of the rate of technical progress (or rate of cost reduction) is problematic. Nothing in the identity identifies $TFP_t^{D1} \equiv a_t\hat{w}_t + (1-a_t)\hat{r}_t$ with the rate of technical progress. After all, identity (2a) is just $\hat{Y}_t \equiv a_t\hat{W}_t + (1-a_t)\hat{S}_t \equiv a_t(\hat{w}_t + \hat{L}_t) + (1-a_t)(\hat{r}_t + \hat{K}_t)$, at best a measure of distributional changes. It could be argued that the growth rate of the wage rate is the *consequence* of productivity growth, where both variables are related through the first-order condition ($w = \partial Y / \partial L$), hence the link with the production function (and similarly the profit rate and capital productivity); and this is what TFP_t^{D1} captures. The problem with this argument is that the relationship between the growth rate of the wage rate and labor productivity growth is definitional, and hence cannot be tested. Indeed, as the labor share is $a_t \equiv (w_t L_t) / Y_t$, in growth rates: $\hat{w}_t \equiv \hat{a}_t + \hat{y}_t$ (where $y \equiv Y / L$), and $\hat{w}_t \approx \hat{y}_t$ for short periods of time, as factor shares vary little and slowly. This relationship will always be true.

[16] See Lucas (1990) and Romer (1994), who argued that, under conventional assumptions about the extent of diminishing returns, the observed differences of over 30 times in labor productivity across countries cannot be explained by differences in capital intensity. Using the identity, it is straightforward to show that what accounts for most of the ratio of labor productivities between two countries is the ratio of wage rates, an insight resulting from circular reasoning that does not require a production function.

Econometric estimation

(vii) There is no statistical identification problem between the accounting identity and the production function. We have shown that they are just different ways of writing the same thing, the identity. Adding variables to the production function certainly does not identify it.

(viii) Identity (2b) is $\hat{Y}_t \equiv \lambda_t + a_t\hat{L}_t + (1-a_t)\hat{K}_t$, where $\lambda_t \equiv a_t\hat{w}_t + (1-a_t)\hat{r}_t$. Now recall the production function in growth rate form is $\hat{Y}_t = TFP_t + \alpha_t\hat{L}_t + \beta_t\hat{K}_t$ (equation [3a]). The latter is supposed to be a model that could be estimated unrestricted as $\hat{Y}_t = \varphi_t + \delta_t\hat{L}_t + \phi_t\hat{K}_t + u_t$, where u_t is the error term, and with time-varying coefficients (to be estimated). By comparing this regression with equation (2b), it should be self-evident that the only possible result is $\delta_t \equiv a_t$, $\phi_t \equiv (1-a_t)$, and $\varphi_t \equiv \lambda_t \equiv a_t\hat{w}_t + (1-a_t)\hat{r}_t$, with a perfect statistical fit (equation (2b) is an identity). Finding that $\delta_t \equiv a_t$ and $\phi_t \equiv (1-a_t)$ would seem to erroneously imply constant returns to scale and that factor markets are perfectly competitive. This interpretation is incorrect. It is simply the result of the accounting identity, which prevents any other statistical outcome.

(ix) These obvious results would not appear, however, if one assumed, for example, that α_t, β_t, and λ_t are constant (then estimate equation [4b]) when, in fact, they display variability. Under these circumstances, the fact that the statistical fit may not be perfect might lead to the misapprehension that a model (with a stochastic error term) is being estimated. This simply misunderstands the underlying logic. Any statistical estimation method that correctly picks up the variation in the factor shares and in λ_t (e.g., a time-varying-parameter estimation methodology), would show that what is being estimated is simply the identity.

(x) It will be appreciated that equation (2g) is simply the original accounting identity given by equations (1a) or (1b), rewritten in a different but equivalent form, which could be misinterpreted as a Cobb–Douglas production function.[17] What does the derivation of the identity in "Cobb–Douglas form" imply? Suppose someone estimates econometrically $Y_t = A_0\exp(\lambda t)L_t^{\alpha}K_t^{\beta}\exp(u_t)$ and the actual data show that factor shares are sufficiently constant (i.e., $a_t = a$ and consequently $(1-a_t) = (1-a)$), and also that $\lambda_t = \lambda$. It should be clear that the

[17] Some readers may think that the assumption that factor shares are constant implies a Cobb-Douglas production function. This is incorrect. See Fisher (1971).

expression estimated is the identity: as there is no error term, the fit will be perfect ($R^2=1$), and the estimated elasticities will equal the factor shares, i.e., $\alpha = a$ and $\beta = (1-a)$, i.e., $\alpha+\beta=1$.[18] This result does not mean that the underlying identity will "bias" the estimates toward constant returns to scale. Rather, it means that the identity completely undermines the justification for the estimation in the first place.

(xi) Naturally, if the data do not follow the paths above, i.e., if $a_t \neq a$ [and consequently $(1-a_t)\neq(1-a)$] and $\lambda_t \neq \lambda$ (this was Barro's 1999 third concern mentioned above), then $Y_t = A_0 \exp(\lambda t) L_t^{\alpha} K_t^{\beta} \exp(u_t)$ need not be a good approximation to the identity. Under these circumstances, results may show, for example, that $\alpha+\beta>1$. This, however, is simply the result of an incorrect approximation to the identity. All that is needed is to find the correct paths for the factor shares a_t and $(1-a_t)$ and for $\lambda_t \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t$ and substitute them back into the general form (2b). If the issue at hand is that factor shares vary, we will need a functional form that better approximates their path, e.g., CES (Felipe and McCombie 2001), translog (Felipe and McCombie 2003). If, on the other hand, the issue is that $\hat{\lambda}_t$ varies, we will also need a better approximation. Note, for example, that $\hat{\lambda}_t$ is a growth rate that varies cyclically and hence a constant term (λ) will not be, in general, a good proxy. Nothing in neoclassical production theory says that technical progress has to grow at a constant rate plus a random fluctuation. This is discussed below.

(xii) There are no estimation problems as discussed in the literature, e.g., regressors' endogeneity that call for instrumental variable, non-linear least squares, or GMM estimation (e.g., Olley and Pakes 1996, Blundell and Bond 2000, Levinsohn and Petrin 2003). Take, for example, regression (4b) and compare it with identity (2d). The likely deviation of the elasticities α and β from the factor shares in the regression will be the result of incorrectly approximating $\lambda_t^* \equiv a\hat{w}_t + (1-a)\hat{r}_t$ with a constant term (λ) (see identity [2d]). This is akin to omitted-variable bias. In this case, however, we know that what has been omitted is λ_t^* and that the true "model" is the accounting identity (2d) (which contains no error term). Hence,

[18] Strictly speaking, perfect multicollinearity will mean that the coefficients cannot be estimated.

the expected values of α and β will be the labor and capital shares a and $(1-a)$, respectively, plus a term (the "bias") that will capture the covariation between λ_t^* and $\hat{L}_t$ and between λ_t^* and $\hat{K}_t$, respectively.[19] To correct this problem all one needs is to better approximate λ_t^* through a variable highly correlated with it. This could be one of the variables one typically sees in growth regressions, e.g., human capital (it works at times). A second option is to construct a trigonometric function, or a high-order polynomial of a variable (or combination of variables), that accurately tracks λ_t. Finally, a third option would be to correct the capital stock for changes in capacity utilization (Lucas 1970). This will increase this variable's cyclical fluctuation and reduce that of r_t and hence of $\hat{r}_t$ (λ_t tends to be procyclical, mainly caused by fluctuations in $\hat{r}_t$). The goodness of fit will increase, and coefficients will approximate the elasticities. Barro's (1999) concerns about the estimation of production functions do not pose any econometric problem, although for reasons unrelated to his arguments.

3. THE COBB-DOUGLAS PRODUCTION FUNCTION AND SOLOW'S INGENIOUS ESTIMATION PROCEDURE

To understand the damaging implications of the accounting identity argument for the notion of TFP, we start by reviewing two seminal papers, Cobb and Douglas's (1928) and Solow's (1957). The purpose is to show that they seemed to be unaware of the fact that the series of output, labor, capital, and TFP were related through the identity. This led them to believe that their findings reflected the conditions of production theory.

Example 1: Cobb and Douglas's (1928) "A Theory of Production": Samuelson was right. Cobb and Douglas estimation exercise was received with great hostility (e.g.,

[19] Expected value of the elasticity of capital:

$E(\hat{\beta}_{OLS}) = (1-a) + E[\frac{Cov(\hat{K}_t, \lambda_t^*)Var(\hat{L}_t) - Cov(\hat{L}_t, \lambda_t^*)Cov(\hat{L}_t, \hat{K}_t)}{Var(\hat{L}_t)Var(\hat{K}_t) - [Cov(\hat{L}_t, \hat{K}_t)]^2}]$. The second part of this expression will be zero if: (i) λ_t^* is a constant, then $Cov(\hat{K}_t, \lambda_t^*) = Cov(\hat{L}_t, \lambda_t^*) = 0$; or if (ii) by coincidence the whole expression in the numerator is zero, then $Cov(\hat{K}_t, \lambda_t^*)Var(\hat{L}_t) = Cov(\hat{L}_t, \lambda_t^*)Cov(\hat{L}_t, \hat{K}_t)$.

Menderhausen 1938). It received attacks from both the conceptual and econometric points of view. At the time, many economists criticized any statistical work as futile because, it was argued, the neoclassical theory was not quantifiable. Others launched an econometric critique against this work, noticing problems of multicollinearity, the presence of outliers, the absence of technical progress, and the aggregation of physical capital (Samuelson 1979).

As is well known, Cobb and Douglas estimated the relationship between output and inputs (labor and capital) using data in index form for the US for 1899-1922. They estimated the famous functional form in per worker terms as $\ln(Y/L) = c + (\alpha + \beta - 1)\ln L + \beta \ln(K/L)$. When we reran their model, the coefficient of labor turned out to be zero (implying constant returns to scale), and the elasticity of capital (β) was 0.233 (statistically significant).

As Samuelson (1979, p.924) documents, Schumpeter was shocked to see that the Cobb-Douglas form did not allow for technical progress. The solution proposed was to add a time trend to proxy for it, that is: $Y_t = A_0 \exp(\lambda t) L_t^{\alpha} K_t^{\beta}$. The problem is that when we ran this regression using the original data set, the coefficient of the index of capital turned out negative (-0.526) and statistically insignificant. Moreover, when we tested for the stability of the coefficients, we found that if the regression without the time trend is estimated for 1899-1920, results are poor: the elasticity of labor turns out to be 1.21 and that of capital 0.08 (statistically insignificant). The recursive and rolling regression confirm this fragility. Only the regression with the complete period does yield sensible results.

What does the accounting identity tell us about this exercise? The intuition is that the functional forms discussed above are not good approximations to the identity. We know from the discussion in section (2) that $Y_t \equiv A_0 w_t^{a} r_t^{1-a} L_t^{a} K_t^{1-a}$ (equation [2e]) when factor shares are constant. We do not have the wage and profit rates corresponding to the original data set but we can approximate $w_t^{a} r_t^{1-a}$ as $A(t) = Y_t / L_t^{0.75} K_t^{0.25}$. When $A(t)$ is plotted, we can see that it fluctuates without trend (which explains the poor results found with the time trend). The identity tells us (section 2) that all we need to do is to find a variable that is correlated with $A(t)$. Through trial and error, we constructed $A(t) = \sin(t^5) + \cos(t^4) - \cos(t^2) - \sin(t^2)$. Recall that neoclassical economics does not say that "technological progress" has to be proxied through an exponential time trend. When we now estimate $\ln Y_t = A_0 + \lambda A(t) + \alpha \ln L_t + \beta \ln K_t$, results are

"perfect": the elasticity of labor is 0.726 and that of capital 0.274 (both statistically significant); and these values remain when the regression is estimated for 1899-1920, and in the recursive regressions (with the fit close to unity). We conclude that Samuelson (1979) was correct: all Cobb and Douglas (1928) did was to reproduce the income accounting identity that distributes output between wages and profits. Moreover, our analysis reveals that Cobb-Douglas's original results would have probably not passed today's econometric standards.

Example 2: Solow's (1957) "Technical Change and the Aggregate Production Function" and the estimation of TFP. Solow's (1957) main contribution was to derive the concept of total factor productivity from a production function, $Y_t = A_t F(L_t, K_t)$. Although TFP had been calculated before, this was the first time it had been explicitly linked to production theory. Solow derived the growth accounting equation from the aggregate production function and devised a method to calculate the contribution of total factor productivity growth to output growth. Equation (3c) above is known today as the primal measure of TFP growth.

Solow's approach consisted in estimating the production function in intensive form, i.e., as $(Y_t / L_t) = A_t F(K_t / L_t)$. In order to estimate this function, Solow argued that he needed to "deflate" the function in order to correct for its upward shift over time, supposedly due to the rate of technical progress. In other words, the general form of the function to be estimated was $(Y_t / L_t) / A_t = F(K_t / L_t)$.

Consequently, Solow first needed to construct an index of A_t. The fact that Solow (1957) was not aware that all he was doing was manipulating the accounting identity may be inferred from the fact that he estimated five separate regressions of this general functional form. This was a point made initially by Hogan (1958) and then by Shaikh (1974). [20] The latter is an extension of the Phelps Brown (1957) and Simon and Levy (1963) criticisms to time-series data (as the derivation in this paper). The five regressions Solow estimated were all variants of the specification $q = c + bk$, where $q = (y / A)$, $y = (Y / L)$ and $k = (K / L)$. In all cases,

[20] See also Shaikh (1987, 2005).

the correlation coefficient was 0.99 (not surprising as we show below). Solow, however, was surprised about the goodness of fit. As can be seen, Solow used A to 'deflate' the left-hand side of his regressions, the level of TFP, constructed as an index of $A = Y / F(K,L)$. In effect, he first calculated the annual growth of TFP as $\hat{A}_t = \hat{y}_t - a_t\hat{k}_t$.[21] He assigned a value of 1 to A in the first year and constructed the rest of the annual series of the index by using the subsequent growth rates of A.

To further understand Solow's procedure, recall that definitionally (equation [2e]) $(Y / L) \equiv y \equiv Ak^{1-a}$ where $A \equiv A_0 w^a r^{1-a}$. This implies $q \equiv (y / A) \equiv k^{1-a}$. Therefore, using the constructed index for A and then regressing $\ln q$ on $\ln k$ must yield a near perfect fit as the relationship is merely estimating a reformulation of the accounting identity. The procedure is self-evidently tautological.

Shaikh (1974) demonstrated this point by constructing a hypothetical data set with the property that a plot of output per worker against the capital-labor ratio would produce the word HUMBUG in the scatterplot. Since the factor shares were those used by Solow (roughly constant), the deflation procedure (y / A) produced, not surprisingly, a regression of $q = (y / A)$ on $k = (K / L)$ (with the HUMBUG data) with a correlation coefficient of 0.9964, and an estimate of the capital share of 0.34. Solow (1974) then used Shaikh's HUMBUG data set to estimate the implicit production function. He estimated a Cobb-Douglas form with a linear time trend. Results were very poor and Solow appeared to be vindicated. It is very interesting, however, that Solow (1974) did not estimate the production function using Solow's (1957) data set. When this is done, the results are so poor (coefficients that differ significantly from the factor shares) that one can hardly believe that what is being estimating a behavioral relationship. Finally, Shaikh (1980) explained clearly what happened to Solow (1974) when he fitted the Cobb-Douglas to the HUMBUG data set: the linear time trend was a very poor

[21] Solow actually used a slightly different approach for calculating growth rates, but it makes no significant difference to the argument.

proxy for the weighted average of the wage and profit rates. Shaikh instead added a trigonometric function that captured the weighted average. Back to the identity. [22]

We, consequently, find it remarkable that Solow's growth accounting method and the construction of TFP series as $A=Y/F(K,L)$ have uncritically survived the test of time.[23] Surprisingly, Solow (1958) did admit that his method was based on a tautology, but referred to it as a "good" tautology.[24] As we shall see later, the same method to obtain a series for A was used four decades later by Jones (1997) and Hall and Jones (1999), with the same problematical consequences.

Solow's other key result was that the residual measure of TFP growth through the primal equation (3c) accounted for almost 90% of the growth of the United States during the first half of the twentieth century, while the remaining 10% was the result of factor accumulation. Solow (1988) still labeled this result as "startling". However, in the light of the accounting identity and the "dual" $TFP_t^{D1} \equiv a_t\hat{w}_t+(1-a_t)\hat{r}_t$, Solow's result becomes self-evident. Given the stylized fact that the rate of return is roughly constant (as it was in the US during the period under consideration), then $TFP_t^{D1} \approx a_t\hat{w}_t$. As factor shares were roughly constant, this result implies that $\hat{w}_t \approx \hat{Y}_t-\hat{L}_t$ (i.e., approximately). It follows that $TFP_t \equiv TFP_t^{D1} \approx a(\hat{Y}_t-\hat{L}_t)$. Consequently, as a takes a value of about 0.70 to 0.75 in the national accounts, it follows that TFP growth *must* account for about three quarters of the growth of output per worker. This is merely due to the algebra of the accounting identity. Moreover, given that employment

[22] Hogan's (1958) comment on Solow (1957), the exchange between Shaikh (1974) and Solow (1974), as well as Shaikh's (1980) reply, are summarized and discussed in Felipe and McCombie (2013, pp.167-176).

[23] Solow (1957) was followed by a series of papers that tried to investigate what was behind such large residual, e.g., Denison (1962, 1967), Jorgenson and Griliches (1967). These set the pace for the TFP growth research program that has lasted until today.

[24] Solow (1958) is a reply to Hogan (1958), who pointed out the nature of Solow's tautological procedure. Hogan noted that it is obvious that the coefficient b in the regression $\ln q=c+b\ln k$ will be the capital share. Solow claimed that if the capital share had been exactly constant, then indeed the procedure would have been a bad tautology. However, insofar as the capital share showed some variation, he argued that it was a good tautology. We think this was not a particularly convincing argument.

growth is small compared with output growth, TFP growth will also explain a similar proportion of output growth. In light of this, it may be questioned whether this result is particularly surprising. In fact, it is surprising that anyone should find it startling.

The estimation of TFP growth following Solow's method, with some technical "improvements," has survived to this day. For example, Fernald (2015), using also the primal, discussed the decline in TFP growth in the US after the Global Financial Crisis by appealing to reasons such as the waning of the effects of information and general-purpose technologies. Our interpretation of the observed decline in TFP growth is different. Once again, $TFP_t^{D1} = a_t\hat{w}_t + (1-a_t)\hat{r}_t$. TFP growth has been low recently as a result of: (i) the fact that wage growth has been very low because a great deal of employment has been generated by non-tradable services, activities that pay relatively low wage rates and which experience low wage increases; and (ii) the well-documented decline in the US labor share (Dao et al. 2017, Stockhammer 2017).[25] Once again, this result follows directly from the accounting identity, and at best, it only says something about distributional changes.

4. RECENT GROWTH ACCOUNTING CONTROVERSIES: THE ILLUSION OF CALCULATING TOTAL FACTOR PRODUCTIVITY

Young's (1992, 1995) growth accounting (primal) exercises for Hong Kong, Korea, Singapore, and Taiwan, were among the most debated in the 1990s and 2000s. His finding that TFP growth accounted for very little of the phenomenal growth of these economies during 1965-1990 was very controversial. Young's estimates indicated that most of the growth of GDP of these fast-growing economies between 1965 and 1990 was accounted for by the growth of

[25] Fernald's analysis covered until 2011. Real wage growth in the US declined from an annual average of 1.5% during 2000-2002, to 0.2% during 2008-2009, and then slightly recovered to 0.4% during 2010-2011, and to 0.6% during 2010-2017. The labor share declined by 3 percentage points between 2000 and 2017. This implies that $a_t\hat{w}_t$ declined significantly between 2000-2007 (0.82%) and 2008-2009 (0.13%), and then increased in 2010-2011 (0.23%) and during 2010-2017 (0.33%). Data for the profit rate indicate that it actually increased after the financial crisis. This, together with the increase in the share of capital, implies that $(1-a_t)\hat{r}_t$ increased and compensated the decline in $a_t\hat{w}_t$, though the sum is still lower than during 2000-2007.

capital. Particularly interesting was the case of Singapore: Young concluded that the contribution of TFP growth to GDP growth was zero and argued that this was the result of Singapore's industrial and targeting policies. This is followed by the analysis of Young (1994), where instead he used regression analysis. Finally, we discuss Hsieh's (1999, 2002) use of the dual growth accounting to prove Young wrong. We stress the point we made in section 2, namely that the critique is not that the growth accounting exercises are incorrect. Rather, the argument questions their interpretation as if they had been derived from a production or cost function and the first-order conditions.

Example 3: Young's (1992, 1995) growth accounting of East Asian miracle economies. Young's (1992, 1995) primal estimates of TFP growth using (3c) can be recast in terms of the accounting identity (2c). [26] We have argued that the series in equations (2c) and (3c), are the same. As a consequence, it is impossible to interpret estimates of the primal $TFP_t^1 \equiv \hat{Y}_t - a_t\hat{L}_t - (1-a_t)\hat{K}_t$ as unequivocally implying anything about the rate of technical progress. The most interesting case in Young's analyses was that of Singapore, as his estimates indicated that the primal total factor productivity growth was approximately zero, i.e., $TFP_t^1 \approx 0$, during 1965-1990. What does the identity (2c) tell us about this result? Since factor shares did not vary significantly, then $\hat{w}_t \approx \hat{Y}_t - \hat{L}_t$ and $\hat{r}_t \approx \hat{Y}_t - \hat{K}_t$ (i.e., approximately), and the identity implies that $TFP_t^1 \approx 0.5[(\hat{Y}_t - \hat{L}_t) + (\hat{Y}_t - \hat{K}_t)] \approx 0$, that is, TFP growth *must* account for about half of the sum of the growth rates of labor and capital productivity. This is indeed the case if one looks at the figures in Young (1992, Tables 5, 6, 7) for Singapore. Why was TFP growth very small, even slightly negative? The important aspect to note is that $(\hat{Y}_t - \hat{L}_t) > 0$ and $(\hat{Y}_t - \hat{K}_t) < 0$ in Young's data (left-hand side columns in the three tables with the three growth rates), such that when added up and multiplied by 0.5, the resulting figure (TFP growth) was very small and even

[26] Equation (3c) is often derived by assuming a translog production function and the Tornqvist approximation (discrete approximation to a continuous Divisia index). This requires the use of the average factor shares at the start and end periods, i.e., $\bar{a}_t = (a_0 + a_T)/2$. This is what Young (1992, 1995) used.

negative given that the decline in $(\hat{Y}_t - \hat{K}_t)$ was larger than the increase in $(\hat{Y}_t - \hat{L}_t)$. As in Solow (1957), all this is simply the result of the algebra of the accounting identity.

Likewise, given equation (2c), we can easily interpret Young's results from the "dual's" point of view. If the growth accounting primal was approximately zero in Young's (1992, 1995) calculations, the identity implies that the right-hand side of (2c) must be $TFP_t^{D1} \equiv a_t \hat{w}_t + (1-a_t)\hat{r}_t \approx 0$. With a capital share approximately constant and taking on a value of about 0.5, this expression implies that $TFP_t^{D} \approx 0.5(\hat{w}_t + \hat{r}_t) \approx 0$, which implies $\hat{w}_t \approx -\hat{r}_t$ for Singapore. In other words, the wage rate grew during 1965-1990 at a rate that was approximately matched by the decline in the growth rate of the profit rate. This may be an interesting finding but recall that it has been derived directly from an accounting identity, and at best, it tells us something about distributional changes.

Example 4: Young's (1994) cross-country TFP-growth regression. We can further elaborate on why one does not learn anything from this literature by reviewing Young's (1994) growth accounting exercise, not undertaken by using equation (3c), but instead by estimating a growth regression based on equation (4b). Young estimated a cross-country production function using data for 118 countries for 1970–1985. This was the growth accounting regression $\hat{Y}_t = \lambda + \gamma_1 \hat{L}_t + \gamma_2 \hat{K}_t + u_t$, or $(\hat{Y}_t - \hat{L}_t) = \lambda + \gamma_2 (\hat{K}_t - \hat{L}_t) + u_t$, under the assumption that $\gamma_1 + \gamma_2 = 1$. As argued in section 2, this regression is erroneously considered to be a model in the sense that it can be tested (where the implicit null hypothesis is $H_0 : 0 < \gamma_2 < 1$) and potentially refuted. Each country (estimated) residual u_i^* measures the growth of country *i's* TFP less the estimate of the world average $\bar{\lambda}$. That is, the per-country TFP growth rate equals $\bar{\lambda} + u_i^*$. Young obtained the following result:

$$(\hat{Y}_i - \hat{L}_i) = -0.21 + 0.45(\hat{K}_i - \hat{L}_i) + u_i \qquad (6)$$

He noted that the residuals for the East Asian economies ($\bar{\lambda} + u_i^*$) were very close in value to his much more detailed analysis using the growth accounting methodology.

The question is: what does this regression tell us? Accounting identity equation (2a) can be written as $(\hat{Y}_i - \hat{L}_i) \equiv a_i \hat{w}_i + (1-a_i)\hat{r}_i + (1-a_i)(\hat{K}_i - \hat{L}_i)$, where the subscript *i* denotes the *ith* country. It will be recalled that, as argued earlier, this is not a model that the data can refute. This means that if one estimated econometrically equation (2a) as $(\hat{Y}_i - \hat{L}_i) \equiv \mu_1 \lambda_i + \mu_2 (\hat{K}_i - \hat{L}_i)^* + u_i$, where $\lambda_i = a_i \hat{w}_i + (1-a_i)\hat{r}_i$ and $(\hat{K}_i - \hat{L}_i)^* = [(1-a_i)(\hat{K}_i - \hat{L}_i)]$, it may be seen that the result must be $\hat{\mu}_1 = \hat{\mu}_2 = 1$ and $R^2 = 1$ as there is no error term ($u_i = 0$ for all observations). Consequently, if one estimates:

$$(\hat{Y}_i - \hat{L}_i) = c + \eta(\hat{K}_i - \hat{L}_i) + u_i \tag{7}$$

as Young did (where λ_i is now assumed to be a constant), it should be apparent that the estimate of η will measure (approximate) the average value of the share of capital, $(1-\bar{a}_i)$ in the sample. The sum of the error (as a result of constraining both λ_i and $(1-a_i)$ to be constants) plus the constant term c will, by definition, provide an estimate of λ_i, the weighted average of the growth rates of the wage and profit rates. Of course, the estimates may be subject to some bias if λ_i is not orthogonal to $(\hat{K}_i - \hat{L}_i)$. To stress the point, equation (18) contains the error term u_i because λ_i is proxied by the constant term c, and $(1-a_i)$ by the coefficient η (also constant). To the extent that these two variables are not constant (as the identity indicates), left- and right-hand sides of equation (7) will not be identical. It should be clear, nevertheless, that the nature of this error term is different from that in a true econometric model, i.e., a random variable that results from factors not considered in the estimated equation. Young's estimates of TFP growth of the East Asian economies from the regression exercise *must* be virtually identical to those from the accounting identity. The latter shows that it cannot be otherwise.

Example 5: Hsieh's (1999, 2002) dual growth accounting approach "without production function." Hsieh (1999, 2002) disputed Young's results. In particular, he disagreed with the implausibly rapid decline of Singapore's profit rate (r) implicit in Young's exercise (and with the overall extremely low TFP growth rate). Hsieh suggested that it was caused by the very high and possibly incorrect estimates of the capital stock. Hsieh argued that

Young's calculations were problematic because the latter had used the primal measure of TFP growth, which required information about capital stocks, difficult to construct. Hsieh's point, in particular for Singapore, was that with a more or less constant share of capital in GDP and an increasing capital–output ratio, the implied rate of return should have fallen dramatically. The data Young had used overstated investment and hence the estimated stock of capital (and its growth rate) was too high. However, different measures of the marginal product of capital showed no decline. Hsieh then concluded that Singapore's national accounts overstated the amount of investment spending, the data used to construct the capital stock.

To solve this problem, Hsieh proposed to calculate the dual measure of TFP growth. The use of factor prices required for the dual avoids the problems of the primal. Hsieh (1999, 2002), however, did not derive the dual from the cost function. Like Barro (1999), Hsieh claimed that one can derive the dual by expressing the national income accounting identity, equation (1b), in growth rates such as equation (2c). Then, he noted that the left-hand side resembles the primal derived from the production function, and the right-hand side is similar to the dual derived from the cost function.[27]

While Hsieh's algebra is correct, we question his claim (and Barro's) that growth accounting can be performed directly by using the accounting identity that relates factor payments to gross domestic product (GDP), without direct reference to the underlying production and cost theories. [28] We argued in section 2 that the identity can be decomposed

[27] This seems to be Harberger's (1998) approach, who wrote the identity in growth rates but gave it a completely neoclassical interpretation by assuming that each factor is rewarded according to its marginal product. He interpreted the residual as the rate of "real cost reduction."

[28] It is worth quoting them on this. Barro (1999, p.123) noted that: "the dual approach can be derived readily from the equality between output and factor income." He continued: "it is important to recognize that the derivation of equation (8) [the growth accounting equation in Barro's paper] uses only the condition $Y_t \equiv w_t L_t + r_t K_t$. No assumptions were made about the relations of factor prices to social marginal products or *about the form of the production function*" (Barro 1999, p.123; emphasis added). And: "If the condition $Y_t \equiv w_t L_t + r_t K_t$ holds, then the primal and dual estimates of TFP growth inevitably coincide […] If the condition $Y_t \equiv w_t L_t + r_t K_t$ holds, then the discrepancy between the primal and dual estimates of TFP has to reflect the use of different data in the two calculations" (Barro 1999, p.123–24).[28] To show this, he writes the income accounting identity, differentiates it, and expresses it in terms of growth rates (Barro 1999, equations [7] and [8], p.123). It is noticeable

into *w*, *r, L* and *K*. Yet, the only way to interpret the decomposition as Hsieh did (e.g., that factor shares reflect the marginal products; and that the result provides estimates of the rate of technical progress) is by equating it to the derivation from the cost function with the corresponding neoclassical assumptions. As we discussed in section 2, all the identity can do is to provide a disaggregation into distributional changes. In what follows, we make three related points about Hsieh's arguments.

First, Hsieh did not calculate $\hat{r}$ residually from the identity, so he did not use TFP^{D1} (equation [2c]). Instead, he used equation (5), which requires an estimate of the *rental rate of capital* (ρ), calculated independently as $\rho = (i + \delta - \hat{P}_K)(P_K / P)$ (i is the nominal cost of borrowing in the financial markets (some market rate, e.g., 5%); δ is the depreciation rate (a pure number), $\hat{P}_K$ is the growth rate (a pure number) of the price (a deflator) of capital (P_K), and P is the GDP deflator (Hall and Jorgenson 1967).[29] Essentially, Hsieh obtained a higher value of TFP growth with the dual (in particular for Singapore) than with the primal because while the wage rate increased (i.e., exhibited a positive growth rate), his estimates of the rental rate of capital did not show a marked decline (i.e., it was more or less constant, hence a growth rate of about zero).

Second, it is important to note that the accounting identity that is consistent with Hsieh's (1999, 2002) calculations is, conceptually, different from equation (1b). Given that both the rental rate of capital ρ and the constant-price value of the stock of K are calculated

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from the above quotation that Barro assumed that there exists an underlying production function, in spite of any impression to the contrary. We think that Barro is using Euler's theorem to connect the production function to the identity. We already disputed this in section 2.

Hsieh (2002, p.505), likewise, concurred that "with only the *condition* that output equals factor incomes, we have the result that the primal and dual measures of the Solow residual are equal. No other assumptions are needed for this result: we do not need any assumption about t*he form of the production function,* bias of technological change, or relationship between factor prices and their social marginal products" (emphasis added).

[29] It should be noted that in the textbook definition, "nominal" ρ (let us call it ρ^n) is actually a price, whose unit is $ per unit of physical capital (e.g., "leets"). Consequently, in "real" terms $\rho = (\rho^n / p)$, where p is the dollar price of a unit of output (e.g., dollars per widget). It is measured in units of physical output (i.e., widgets) per unit of physical capital (e.g., per square meter of office, per computer).

independently, the product ρK need not be equal to the operating surplus (unlike $r_t K_t$ in equation [1b]). Therefore, it is true that, in general, $Y_t \neq w_t L_t + \rho_t K_t$. The presumption is that $Y_t > w_t L_t + \rho_t K_t$, with the difference being pure profits. What this means is that the accounting identity *effectively* used by Hsieh may not be consistent with the NIPA because there is no guarantee that Y_t equals $w_t L_t + \rho_t K_t$. This means that under these circumstances, and to be consistent, Hsieh's income accounting identity should be written as:

$$\bar{Y}_t \equiv w_t L_t + \rho_t K_t \tag{8}$$

where now $\bar{Y}_t$ is constructed using the four series on the right-hand side. This could be defined as the *GDP consistent with competitive markets*, given that wage rate and rental rate of capital are considered to be the competitively determined factor prices. It can also be referred to as the *cost identity* ($\bar{Y} = C$). Naturally, whether the estimate of $\bar{Y}$ is correct or not depends on whether the series used to calculate it, in particular ρ and K, are accurate or not. This identity gives us the labor and capital cost shares $\vartheta_L = (wL/C)$ and $\vartheta_K = (\rho K/C)$, respectively, with $\vartheta_L + \vartheta_K = 1$. One could perform a growth accounting exercise and calculate TFP growth as:

$$TFP_t^2 \equiv \hat{\bar{Y}}_t - \vartheta_{Lt}\hat{L}_t - \vartheta_{Kt}\hat{K}_t \equiv \vartheta_{Lt}\hat{w}_t + \vartheta_{Kt}\hat{\rho}_t \equiv TFP_t^{D2} \tag{9}$$

Third, Hsieh (2002, p.505) made an additional point about the identity. In general, $(\bar{Y} = C)$ will differ from Y (actual GDP) as the latter appears in the NIPA. The difference is attributed to the existence of pure profits, although given how ρ and K are calculated, it is virtually impossible to know whether this is indeed the case.[30] Given this, the actual GDP identity could also be written as:

$$Y_t \equiv C_t + Z_t \equiv w_t L_t + \rho_t K_t + Z_t \tag{10}$$

where Z denotes pure profits and, for consistency, $S_t \equiv r_t K_t \equiv \rho_t K_t + Z_t$.[31] In this case, the GDP shares are (avoiding the subscript t for simplicity) $\theta_L = a = (wL/Y)$, $\theta_K = (\rho K/Y)$ and

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[30] Empirically, it might well be that one finds that $\bar{Y} > Y$.

[31] Note that equation (10) may be written as:
$Y_t \equiv w_t L_t + \rho_t K_t + Z_t \equiv w_t L_t + \rho_t K_t + \rho_t^* K_t \equiv w_t L_t + (\rho_t + \rho_t^*) K_t = w_t L_t + r_t K_t$, where ρ^* can be defined as the "monopolistic" rate of profit.

$\theta_Z = (Z/Y)$, with $\theta_L + \theta_K + \theta_Z = 1$. One can also express identity (10) in growth rates to undertake a growth accounting exercise as:

$$TFP_t^3 \equiv \hat{Y}_t - \theta_{Lt}\hat{L}_t - \theta_{Kt}\hat{K}_t \equiv \theta_{Lt}\hat{w}_t + \theta_{Kt}\hat{\rho}_t + \theta_{Zt}\hat{Z}_t \equiv TFP_t^{D3} \qquad (11)$$

Now the dual of TFP growth splits the contribution of the growth rate of the operating surplus into that of the cost of capital and that of pure profits.

None of this alters the argument. The slight differences between identities (2c), (9), and (11) are obvious (different shares, α, θ, ϑ; and in general $\hat{r} \neq \hat{\rho}$). It should also be self-evident that they would yield different results; and that if one calculates the primal using one of them and the dual using another one, the two will be different.

5. WHAT DO PRODUCTION FUNCTION REGRESSIONS TELL US? THE ILLUSION OF TESTING (I)

In this section and in the next, we discuss regressions of growth models. The analysis makes it clear that the authors believe that their regressions are testable models (we have already pointed out that this was the case of Solow 1957 and Young 1994). The discussion in section 2 made it clear that the production function approach is very problematic for the empirical literature on endogenous growth, increasing returns to scale, and imperfect markets. The latter two might exist but this method will always reject these hypotheses (once again, if the regression is specified correctly); and as we already pointed out, the result should not be interpreted as corroboration of constant returns to scale and competitive markets. It is simply the result of the underlying identity.

The late 1980s and 1990s saw a resurgence of the growth literature as researchers began to test growth models, especially the key insights of the old Solow model against those of the new endogenous growth models. This was partly triggered by the availability of large data bases. Moreover, there was growing dissatisfaction with the predictions of Solow's (1956) model. Given that countries are assumed to have access to the same technologies, the model predicts that the steady-state rate of growth of productivity will be equal to the common rate of technical progress. Any differences in growth rates can only be transitory, the results of

countries not being at their steady-state capital-labor ratio. The growth rate of productivity of those countries where their actual capital-labor ratios are below the steady-state value will temporarily exceed the rate of technical progress. If all countries invest the same proportion of their GDP, then there should be an inverse correlation between the growth of labor productivity and the initial level of productivity. The empirical evidence showed that this convergence (referred to as absolute) existed at the regional level within advanced countries but not at the country-level, i.e., countries were not converging.

The literature then split into two paths. The first path was to endogenize growth by abandoning the assumption that the growth rate of technology (a pure exogenous public good) was the same across countries. One way to do this was by introducing the idea that capital is special in that its contribution to growth is higher than that implicit in the capital share in the national accounts. In other words, capital accumulation has positive externalities, and this gave rise to the so-called *AK* model (Example 6). An alternative approach was to introduce a second production function for "ideas" (i.e., the R&D sector) and to allow for the diffusion of ideas from the high to low income countries. The second path of the literature is discussed in Example 7.

Example 6: Romer's (1987) tests for increasing returns to scale. Romer (1987) was one of the first papers on endogenous growth that included empirical work. His objective was to drop the notion of technical change as a separate argument of the production function. Romer developed two models, one of which included physical capital with an output elasticity of unity, and with overall increasing returns to scale. In the second model, Romer assumed that the greater the degree of specialization the greater, *ceteris paribus*, the level of output. To capture the degree of specialization, output was specified as a function of the number of specialized capital inputs as well as of labor. He also assumed that there is a fixed cost in producing the specialized capital goods. The production function takes the form $Y_t = AL_t^{\alpha} K_t^{\beta}$ with β assumed to equal 1 (Romer's equation [11]). Given our arguments in section 2, it should come as no surprise that the empirical evidence did not support a production function with the elasticities consistent with Romer's model.

What empirical evidence did Romer find in support of his hypothesized production function? He fitted Cobb-Douglas production functions to different data sets and with slightly different specifications, including and excluding the constant term, as well as including and excluding a time trend. The results were poor, with coefficients often either statistically insignificant or negative. This led Romer (1987, p.186) to conclude that "it should not be surprising that production function regressions using annual data yield estimates that are ambiguous." In a final attempt using long-run growth rates data for the G7 countries, Romer obtained a coefficient of 0.87 for capital (not statistically different from 1) and a statistically insignificant coefficient on the growth rate of hours worked.

All in all, the empirical support for Romer's model was pretty weak, and his rationalization of the results was not convincing. He argued that "the tentative conclusion that I draw from this exercise is that the appropriate growth accounting equation is $\hat{Y}_t = \alpha \hat{L}_t + \beta \hat{K}_t$, with values of β likely to fall in the range 0.7 to 1.0 and the values of α likely to fall in the range 0.1 to 0.5" (Romer 1987, p.198). As α was considerably below labor's observed share in national income, Romer was forced to provide an implausible *ex post* justification, namely, that there must be a significant negative externality associated with labor (Romer 1987, p.166). In retrospective, the evidence that the factor elasticities take upon the values hypothesized by the endogenous growth production functions has never been sound. Consequently, most empirical work today, in particular growth accounting, assumes elasticities equal to the factor shares and, therefore, constant returns to scale.

The identity allows us to determine when the regression $\hat{Y}_t = \alpha \hat{L}_t + \beta \hat{K}_t + u_t$ would yield a capital elasticity close to unity and a low labor elasticity, i.e., $\beta \to 1$ and $\alpha \to 0$. If the rate of profit does not display secular growth (i.e., $\hat{r}_t \to 0$) and factor shares are roughly constant, then identity (2a) can be written as $\hat{Y}_t \equiv a\hat{w}_t + a\hat{L}_t + (1-a)\hat{K}_t$. This equation becomes $\hat{Y}_t \equiv a(\hat{K}_t - \hat{L}_t) + a\hat{L}_t + (1-a)\hat{K}_t$, given the relationship between the wage rate growth and labor productivity growth $\hat{w}_t = (\hat{Y}_t - \hat{L}_t)$, and if $\hat{Y}_t = \hat{K}_t$ (i.e., Kaldor's stylized fact, which Romer assumed). It is straightforward to see now that if the term $(\hat{K}_t - \hat{L}_t)$ is omitted, this expression is simply:

$$\hat{Y}_t \equiv 0\hat{L}_t + 1.0\hat{K}_t \equiv \hat{K}_t \tag{12}$$

with the coefficients of labor and capital growth taking the orders of magnitude found by Romer (in his regression with long-run data), with the elasticities adding up to unity. This, however, merely reflects the "bias" in the estimation of the accounting identity and tells nothing about the contribution that capital and labor make to economic growth in a technological sense, that is, determined by an underlying aggregate production function.

Example 7: Mankiw, Romer and Weil's (1992) revival of the neoclassical growth model. The second path followed by the growth literature was to revive Solow's (1956) model, on the grounds that it had been misinterpreted. Mankiw et al. (1992) (MRW hereafter) claimed to take Robert Solow "seriously" by properly testing his canonical model (Solow 1956); in particular, the assumption of constant returns to scale (with the values of the factor elasticities about the size of the factor shares), and the prediction of convergence.[32] They argued that Solow's model did not predict absolute convergence but conditional convergence, the idea that correcting for the fact that countries invest different proportions of their output in physical and human capital, then they indeed converge to their own steady states.

Since this paper is well known, we only sketch its key aspects here. MRW started with a production function with constant returns to scale, namely $Y_t = (A_t L_t)^{\alpha} K_t^{1-\alpha}$. The authors assumed $L_t = L_0 e^{nt}$ and $A_t = A_0 e^{gt}$. They derived the steady-state solution of the model, which in logarithmic form is

$$\ln y = \ln A_0 + gt + \frac{1-\alpha}{\alpha}\ln s - \frac{1-\alpha}{\alpha}\ln(n+\delta+g) \tag{13a}$$

where $y = (Y/L)$, s is the savings rate, δ is the depreciation rate, n $(n = \hat{L})$ is the growth rate of employment, and g is the growth rate of technology. The model predicts that countries with greater savings and investment rates will have higher per capita income levels. These countries accumulate more capital per worker and, consequently, have more output per worker. Likewise, countries that have higher population growth rates will tend to be poorer. The model also predicts the magnitudes of the coefficients of these variables.

[32] See Solow (1994) and Romer (2001) for critiques of Mankiw et al. (1992).

For estimation purposes, MRW set $g+\delta=0.05$ and hypothesized that A_0 reflects not only the initial level of technology but also resource endowments, climate, institutions, and so on. As such, it may differ across countries, and it was assumed that $\ln A_0 = b_0 + u$, where u is an error term (this assumption is very important). They used cross-country data and fitted the regression

$$\ln y = b_0 + \frac{1-\alpha}{\alpha}\ln s - \frac{1-\alpha}{\alpha}\ln(n+0.05) + u \qquad (13b)$$

For our purposes, it is unnecessary to discuss the results save to note that these were mixed, in terms of both fit (very low in some of the samples) and estimated coefficients (how close these were to the factor shares in the national accounts). This led MRW to extend Solow's model by introducing human capital. Results improved in general.[33]

However, what underlies MRW's regression is, again, the accounting identity. In what follows, we show how equations (13a) and (13b) can be derived from the accounting identity (equation [2e]) and the definition of the increase in the capital stock, that is, $\Delta K_t \equiv I_t - \delta K_t$, where I is gross investment and δ is the depreciation rate, through a series of simple algebraic steps, and with no reference to neoclassical production theory:

(i) The growth rate of the stock of capital can be written as:

$$(\Delta K_t / K_t) \equiv \hat{K}_t \equiv (I_t / K_t) - \delta \equiv (sY_t / K_t) - \delta \qquad (14a)$$

where s is the investment-output ratio, assumed constant.

(ii) Assuming equal growth rates of output and capital, i.e., $\hat{Y}_t = \hat{K}_t$ (one of Kaldor's stylized facts), the accounting identity (2d) becomes:

$$\hat{Y}_t = \hat{K}_t \equiv a\hat{w}_t + (1-a)\hat{r}_t + a\hat{L}_t - (1-a)\hat{K}_t \equiv g_t + a\hat{L}_t - (1-a)\hat{K}_t \qquad (14b)$$

with $g_t \equiv a\hat{w}_t + (1-a)\hat{r}_t$, which implies:

$$\hat{K}_t - \hat{L}_t - \tilde{g}_t \equiv 0 \qquad (14c)$$

[33] They concluded that the production function consistent with their results is $Y = AK^{1/3}H^{1/3}L^{1/3}$, where H denotes human capital. As can be seen, the elasticity of physical capital is not different from its share in income and there are no externalities to the accumulation of physical capital.

where $\tilde{g}_t = (g_t / a)$.[34]

(iii) Substituting the definition of $\hat{K}_t$ given by equation (14a) into the identity (14c) yields $(sY_t / K_t) - \delta - n - \tilde{g}_t \equiv 0$, or

$$k_t \equiv \left(\frac{s}{n + \delta + \tilde{g}_t} \right) y_t \tag{14d}$$

where $k = (K / L)$. This definition (identity) is correct, we stress, if $\hat{Y}_t = \hat{K}_t$. In any case, this expression, given how it has been derived, is certainly not a testable 'model'. We just note the resemblance between this expression and the steady-state value of the capital-labor ratio in Solow's growth model.

(iv) Recall the identity can be rewritten assuming constant factor shares equation (2e). Substitute the definition of k in (14d) into the identity (2e). This yields:

$$y_t \equiv C_0 w_t r_t^{\frac{1-a}{a}} \left(\frac{s}{n + \delta + \tilde{g}_t} \right)^{\frac{1-a}{a}} \tag{14e}$$

Again, note the similarity between this expression and that for the steady-state income per capita obtained in the standard derivation by MRW. However, expression (14e) is just the income accounting identity *if* factor shares are constant and $\hat{Y}_t = \hat{K}_t$, and it has been derived without reference to a production function.

(v) It can be written in logarithmic form as:

$$\ln y_t \equiv c + 1.0 \ln w_t + \frac{1-a}{a} \ln r_t + \frac{1-a}{a} \ln s_t - \frac{1-a}{a} \ln[n + \delta + \frac{a\hat{w}_t + (1-a)\hat{r}_t}{a}] \tag{14f}$$

If expression (14f) is estimated econometrically with the coefficients unrestricted, it will produce a high fit (provided that factor shares are relatively constant and if output and capital grow at similar rates), and one will be able to recover the factor shares from the estimated coefficients. Moreover, it will confirm that indeed countries with higher savings rates are richer, and that countries that have higher population growth rates are poorer. This,

[34] Assuming that factor shares are constant, and that output and capital grow at the same rates, and "imposing" them on the identity does not make the latter a model. This simply means that MRW's model will work when both assumptions are approximately true in the data; and it will not work otherwise.

however, will be true simply by construction of the variables involved in the regression. No data set will reject these hypotheses. These regression results say nothing that is not already known before the regression is run. To be precise, we are not claiming that savings and population growth do not matter for growth. What we claim is that regression (14f) does not provide a test of this hypothesis.

(vi) Now compare (14f) to the equation estimated by MRW (13b). Equation (14f), an identity, encompasses (13b) and a comparison of the variables included in both equations sheds light as to why the latter gave relatively poor results in terms of goodness of fit (although savings rate and population growth had the correct signs). The reason is that MRW's regression was not a good approximation to the identity. Essentially, MRW constrained $\ln[n+\delta+\frac{a\hat{w}_t+(1-a)\hat{r}_t}{a}]$ to be $\ln(n+0.05)$, and $1.0\ln w+\frac{1-a}{a}\ln r$ to be a constant b_0. This is akin to omitted-variable bias. We emphasize that the signs were correct because of the identity. The poor goodness of fit was partially solved through the inclusion of human capital in their extended model (with better results). In our view, it simply means that the extended model provided a better approximation to the identity to the extent that the human capital variable partially picked the path of $(1.0\ln w+\frac{1-a}{a}\ln r)$. Again, we are not claiming that savings and population growth do not matter. What we have shown is that regression (13b) cannot be used to test this proposition.

MRW also tested what they saw as the other important prediction of Solow's model, namely that an economy's per capita income converges to its own steady-state value, which provides an explanation for the observed differences across countries. An economy that begins with a stock of capital per worker below its steady-state value will experience faster growth in per capita output along the transition path than a country that has already reached its steady-state per capita output. The convergence regression is

$$(\ln y_t-\ln y_0)=gT+(1-e^{-\xi T})\ln A_0+(1-e^{-\xi T})\frac{\alpha}{1-\alpha}\ln s-(1-e^{-\xi T})\frac{\alpha}{1-\alpha}\ln(n+0.05)+\tau\ln y_0+u_t \qquad (15a)$$

where ξ is the speed of convergence, i.e., how quickly a deviation from the steady-state growth rate is corrected over time (i.e., percentage of the deviation from steady state growth that is

eliminated each year), and T is the length of the period under consideration. In the neoclassical model $\xi = (n+\delta+g)\alpha$; and $\tau = -(1-e^{-\xi T})$. MRW assumed $gT + (1-e^{-\xi T})\ln A_0$ to be constant across countries. The speed of convergence was estimated at about 2% a year from $\tau = -(1-e^{-\xi T})$.

What do our arguments imply about the convergence regression and the speed of convergence? Simply subtract the logarithm of income per capita in the initial period from both sides of equation (14f). This yields:

$$(\ln y_t - \ln y_0) \equiv c + 1.0\ln w + \frac{1-a}{a}\ln r + \frac{1-a}{a}\ln s - \frac{1-a}{a}\ln[n+\delta+\frac{a\hat{w}_t + (1-a)\hat{r}_t}{a}] + \tau \ln y_0 \qquad (15b)$$

Equation (15b) continues being the accounting identity (all we did was to subtract $\ln y_0$ from both sides of the equation). Therefore, the coefficient of $\ln y_0$ has to be $\tau = -1$ by construction. If this equation were to be interpreted as being the neoclassical growth model, results would imply $\tau = -(1-e^{-\xi T}) = -1$, or $\xi = \infty$, a questionable result. Our argument is that, like the previous one, this regression tells us nothing. Why did MRW (1992, Table IV) obtain a much lower coefficient for τ (implying a rate of convergence of about 2% a year)? For the same reason as before: their regression assumed constant two terms that are not. To be precise, we are not claiming that the idea of convergence is nonsensical. What we claim is that regression (15a) does not provide a test.[35]

The empirical literature that succeeded MRW (Islam 1995, 1998; Knowles and Owen 1995; Nonneman and Vanhoudt 1996; Lee et al. 1998; Maddala and Wu 2000) re-estimated the convergence equation (different versions) and got into a conundrum. This was that as the estimated regressions improved (e.g., through additional variables, or heterogeneous intercepts), the speed of convergence increased significantly to values that authors thought were implausible. It is just that they were getting closer to the identity.

Example 8: Jones's (1997) test of the neoclassical growth model. Jones (1997) showed to his satisfaction that the neoclassical model describes fairly well the distribution of per capita income across countries. He reached this conclusion by calculating the steady-state

[35] Surely a regression of the growth rate of GDP on initial income per capita tells us something about absolute convergence. This is not the MRW hypothesis and test.

levels of labor productivity of a sample of countries, relative to that of the US. These values were then compared with the actual ratios of labor productivity, relative to that of US: the correspondence between actual and steady-state relative productivity levels was very high, with a slope of about 1. Jones concluded that the Solow model is extremely successful in explaining the wide variation in per capita income across nations. Jones also compared TFP levels across countries and showed that these are higher in advanced economies. As we show below, Jones's findings were the result of the fact that he had no independent measure of TFP, and the way he calculated it made it inevitable that relative actual and steady-state levels of productivity had to closely correspond. More simply, what explains this close relationship is, again, the accounting identity.

Jones (1997) hypothesized the production function $y=(Ah)^{\alpha}k^{1-\alpha}$, where y, A, h and k are output per worker ($y=Y/L$), the level of technology, human capital, and capital per worker ($k=K/L$), respectively. Jones assumed that $h=\exp[\phi S(t)]$, where S is the time devoted to skill accumulation. The steady-state level of productivity for country *i* (assuming that output and capital grow at the same rates, and that the growth rate of technical progress is the same across countries, $A=A_0\exp(gt)$) is:

$$y_i^*=\left(\frac{s}{n_i+g+\delta}\right)^{(1-\alpha)/\alpha}A_ih_i \qquad (16a)$$

where s is the share of physical investment in output, $n=\hat{L}$ denotes the growth rate of employment, and δ is the depreciation rate. Following Solow (1957), Jones defined A as:

$$A_i=(y_i/k_i)^{(1-\alpha)/\alpha}(y_i/h_i) \qquad (16b)$$

Then he substituted (16b) into the equation for the ratio of the steady-state level of productivity with respect to that of the US (y_i^*/y_{US}^*). When he compared $\ln(y_i^*/y_{US}^*)$ with $\ln(y_i/y_{US})$, he found a very close relationship between the two variables, with a slope of about 1.

However, the closeness between the two series was simply the result of how Jones calculated A, using Solow's (1957) tautological procedure (section 3).[36] To see this, we can

[36] See also Hall and Jones (1999).

disregard human capital h for expositional convenience because including it makes no difference to Jones's procedure. Then through the following steps, we show the tautological nature of Jones's procedure, and why, by construction, actual and steady-state levels of productivity have to be closely related:

(i) Equation (16b) without human capital becomes:

$$A_i = (y_i / k_i)^{(1-\alpha)/\alpha} y_i \tag{16c}$$

(ii) Taking the ratio of the steady-state level of productivity (16a) with respect to that of the US (also excluding human capital) yields the following expression: [37]

$$\frac{y_i^*}{y_{US}^*} = \frac{\left(\frac{s_i}{n_i + g + \delta}\right)^{(1-\alpha)/\alpha}}{\left(\frac{s_{US}}{n_{US} + g + \delta}\right)^{(1-\alpha)/\alpha}} \frac{A_i}{A_{US}} \tag{16d}$$

(iii) Consider the expression $s_i / (n_i + g + \delta)$. Given that $s \equiv \hat{K}(K/Y)$, it follows that:

$$\left(\frac{s_i}{n_i + g + \delta}\right)\left(\frac{Y_i}{K_i}\right) \equiv \left(\frac{\hat{K}_i}{n_i + g + \delta}\right) \tag{16e}$$

(iv) Substitute (16e) into (16d), together with (A_i / A_{US}) from (16c). We obtain:

$$\frac{y_i^*}{y_{US}^*} = \frac{\left(\frac{\hat{K}_i}{n_i + g + \delta}\right)^{(1-\alpha)/\alpha}}{\left(\frac{\hat{K}_{US}}{n_{US} + g + \delta}\right)^{(1-\alpha)/\alpha}} \frac{y_i}{y_{US}} \tag{16f}$$

or in logarithmic form:

$$\ln(y_i^* / y_{US}^*) = \frac{(1-\alpha)}{\alpha} \ln\left(\frac{x_i}{x_{US}}\right) + 1.0 \ln(y_i / y_{US}) \tag{16g}$$

[37] It is straightforward to see that the introduction of human capital makes no difference whatsoever because it disappears if we substitute instead $A_i = (y_i / k_i)^{(1-\alpha)/\alpha}(y_i / h_i)$ into the ratio of steady-state level of productivity with respect to that of the US. We obtain equation (16d) again.

where $x_i = \hat{K}_i / (n_i + g + \delta) = 1$ and correspondingly for x_{US}.[38] Consequently, if one were to regress $\ln(y_i^* / y_{US}^*)$ on $\ln(y_i / y_{US})$, it is self-evident that the statistical fit should be very good, given that $\ln(y_i / y_{US})$ is, by definition and construction, a component of $\ln(y_i^* / y_{US}^*)$, and given the stylized fact that the capital-output ratio does not vary greatly across countries (see Jones 1997, Figure 5).

Finally, we do not need to elaborate again upon what lies behind Jones's (1997) procedure and results. The procedure gives not only the very high fit between steady-state and actual labor productivity ratios, but also the slope of 1. This is, of course, the accounting identity.

6. THE ILLUSION OF TESTING (II)

We complete our tour of the identity by discussing three additional examples that highlight the fundamental problems of the approaches used and the discussions around them. They are an extension of the growth and macroeconomic discussions of the late 1980s and 1990s derived from the endogenous growth literature. These are: Hall (1988), Hall (1990), Shapiro (1987), and Wong and Gan (1994). The analysis in this section will allow us to appreciate in detail why the suspicions of Samuelson (1979) and Simon (1979a) were well founded, and why Barro's (1999) concerns (see section 2) about the regressions were erroneous.

Example 9: Hall's (1988, 1990) method to estimate the markup. Hall (1988, 1990) argued that the procyclicality of productivity was evidence that firms behave monopolistically and have excess capacity. He reasoned that, if a demand shock can lead to an increase in output with a small increase in input (vis-à-vis the price increase), then marginal cost must be low (i.e., the markup must be high). Competitive firms with the ability to increase output with little increase in input would cut price. Demand would increase and hence attenuate the procyclicality of measured productivity. Consequently, Hall interpreted the procyclicality of productivity as evidence that firms behave monopolistically. In Hall's model, cyclical

[38] Combining identity (14d) and the definition $s \equiv \hat{K}(K/Y)$, it follows that $\hat{K}_t \equiv (n + \delta + \tilde{g}_t)$.

fluctuations in productivity are the result to shocks in aggregate demand rather than shocks to productivity, as in the real business cycle model.

Here we show that his well-known procedure to estimate markups (ratio of price to marginal cost) by comparing movements in output and inputs, is equally invalidated by the accounting identity. Hall showed that when the assumptions of perfect competition and constant returns to scale are violated, the growth rate of the primal measure of TFP growth does not reflect the true productivity growth. the His model led to the following regression equation:

$$(\hat{Y}_t - \hat{K}_t) \equiv \lambda + \mu(a_t \hat{k}_t^*) + u_t \qquad (17a)$$

where $\mu = (p/x)$ is the markup (ratio of price of marginal cost), λ denotes the rate of technological change (assumed to be constant), a_t is the labor share in total revenue, $\hat{k}_t^* = \hat{L}_t - \hat{K}_t$, and u_t is the disturbance term. The original Solow's (1957) model implicitly assumed $\mu = 1$, the null hypothesis to test in equation (17a).[39]

What is the problem with the above method? Note that the identity equation (2b) can be written as $(\hat{Y}_t - \hat{K}_t) \equiv \lambda_t + a_t \hat{k}_t^*$, where $\lambda_t \equiv a_t \hat{w}_t + (1 - a_t)\hat{r}_t$. Clearly, if one estimates

$$(\hat{Y}_t - \hat{K}_t) = \tau\lambda_t + \mu^*(a_t \hat{k}_t^*) + u_t \qquad (17b)$$

the result will be $\hat{\tau} = \hat{\mu}^* = 1$ with a perfect fit, as there is no error term (i.e., $u_t = 0$ for every observation). Hall found putative large markups ($\mu > 1$). How is this possible?

It is obvious that if one estimates (17a), i.e., by effectively constraining λ_t to be a constant (and effectively omitting its variation in [17b]), then it is self-evident that the estimate of μ is likely to differ from unity. The problem is akin to one of omitted-variable bias, though in this case, we know exactly what is being omitted, namely λ_t. This explains why Hall found putative large markups. [40]

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[39] For estimation purposes, Hall ran the inverse of this regression, that is, $a_t \hat{k}_t^* = c + d(\hat{Y}_t - \hat{K}_t) + u_t$, where now the estimate of the markup is $\hat{\mu} = (1/\hat{d})$.

[40] The estimated μ can be interpreted as a biased estimate of $\mu^* = 1$ due to the misspecification of λ_t. The expected value of μ will be 1 plus a term that depends on the covariance of the omitted

Summing up: Hall's procedure is, in effect, a tautology and his results cannot be taken as evidence of large markups. If he had allowed the intercept to vary, the closeness of fit (in terms of the R-squared and the standard errors) would have increased, and μ would have necessarily approached unity.

Example 10: Hall's (1990) method to estimate the degree of returns to scale. Hall (1990) extended Hall's (1988) method to also test for the presence of increasing returns to scale. His method is different from Romer's (1987). Hall (1988) had assumed constant returns to scale. The mark-up (μ) equals the ratio of price to marginal cost, i.e., $\mu = (p/x)$. It is straightforward to show that μa (where a is the labor share in revenue) is the labor's output elasticity (α) when value-added data are used. It then follows that the degree of returns to scale ($\nu = \alpha + \beta$) is equal to $\nu = \mu a + \beta$, where β is the output elasticity of capital. Then, using revenue shares, Hall's equation to estimate the degree of returns to scale becomes $(\hat{Y}_t - \hat{K}_t) = \lambda + \mu a \hat{k}_t^* + (\nu - 1)\hat{K}_t$. Hall used cost rather than revenue shares, although he admitted that there was little difference between cost and revenue shares. From equation (5), the cost shares are given by $\vartheta_L = wL/(wL + \rho K)$ and $\vartheta_K = \rho K/(wL + \rho K)$, and shares, degree of returns to scale, and markup are related through $\nu = (a\mu)/\vartheta_L$. Hall's estimating equation of the degree of returns to scale then becomes:

$$\hat{Y}_t = \lambda + \nu[\vartheta_L \hat{L}_t + (1 - \vartheta_L)\hat{K}_t] + u_t \qquad (18)$$

It should be obvious by now that regression (18) may lead to estimates of ν significantly greater than 1, as Hall found, although this is simply the result of mis-specifying the accounting identity equation (2a). As indeed revenue and cost shares are very close, the identity can be written as $\hat{Y}_t \approx \vartheta_{Lt}\hat{w}_t + (1 - \vartheta_{Lt})\hat{r}_t + \vartheta_{Lt}\hat{L}_t + (1 - \vartheta_{Lt})\hat{K}_t$, or equivalently as $\hat{Y}_t \approx \lambda_t + \nu[\vartheta_{Lt}\hat{L}_t + (1 - \vartheta_{Lt})\hat{K}_t]$ (i.e., approximately). It is straightforward to see that $\nu \approx 1$ unless λ_t is erroneously proxied by

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and included term and the variance of the latter: $E[\hat{\mu}_{OLS}] = 1 + \dfrac{Cov(\lambda_t, a_t\hat{k}_t^*)}{Var(a_t\hat{k}_t^*)}$. $\mu = 1$ if $Cov(\lambda_t, a_t\hat{k}_t^*) = 0$, which in the context of the identity does not imply that markets are perfectly competitive; and $\mu > 1$ if $Cov(\lambda_t, a_t\hat{k}_t^*) > 0$.

a constant as Hall did. Once again, this procedure is not a test of an empirical hypothesis.

Example 11: Shapiro's (1987) test of fluctuations in productivity. Shapiro (1987) proposed a test of whether observed fluctuations in productivity result from supply (real business cycle theory), or from demand (Keynesian theory). In other words, is the measured Solow residual a true shift in the production function, or does it have a demand component, as Keynesian theories suggest? Specifically, Shapiro tested whether the observed fluctuations in factor prices are consistent with the hypothesis that measured productivity shocks are true productivity shocks. He used product and factor price data and argued that the latter "should provide an independent indication of the source of the productivity fluctuation" (Shapiro 1987, p.119).

His test consisted in regressing primal TFP growth on dual TFP growth, that is:

$$TFP_t = c + \tau TFP_t^D + u_t \qquad (19)$$

both derived *independently,* from the production and cost functions, respectively, i.e., $TFP_t \equiv \hat{Y}_t - a_t\hat{L}_t - (1-a_t)\hat{K}_t$ (equation [2c]) and $TFP_t^D = \vartheta_{Lt}\hat{w}_t + \vartheta_{Kt}\hat{\rho}_t$ (equation [5]). He tested the null hypothesis $H_0 : \tau = 1$.

In a second exercise, Shapiro (1987) considered whether departures from the predicted joint movement of measured productivity and factor prices are consistent with the Keynesian alternative that movements in measured Solow residuals are accounted for by movements in demand, i.e., that movements in demand drive fluctuations in TFP growth. To do so, he added an additional explanatory variable to the same regression:

$$TFP_t = c + \tau TFP_t^D + \phi\hat{X}_t + u_t \qquad (20)$$

where $\hat{X}_t$ is the growth of GNP in Shapiro (1987, Table 2). The joint null hypothesis is $H_0 : \tau = 1;\ \phi = 0$.

Wong and Gan (1994) recently used the same procedure and test with Singaporean data. They estimated regression (19) for 27 manufacturing industries (Wong and Gan 1994,

Table 5).[41] To calculate the dual TFP_t^D, they calculated and used the rental rate of capital ρ (although see below).

What did Shapiro and Wong and Gan find? We focus on the latter since the results are more extreme and these authors provide more statistical information than Shapiro. In their first set of regressions they found, except in one case, that the estimates of the coefficients τ were virtually 1 and quite tightly estimated (with t-values above 100 and up to 454), and the regressions had almost perfect fits ($R^2 > 0.99$ in most cases). The authors concluded that their findings suggested that "the movements in TFP growth reflect true changes in productivity." In their second set, the extended regressions, their results were $\tau = 1$ and $\phi = 0$ across most industries. Wong and Gan interpreted the finding that $\phi = 0$ as a refutation of the Keynesian theory. These results are qualitatively similar to Shapiro's (1987, Table 2).

It is true that Shapiro (1987) (as well as Wong and Gan 1994) computed separately primal and dual measures of TFP growth, using production and cost functions, respectively. Shapiro derived the primal from a general production function (equation (2) in his paper), hence factor shares varied, as in equation (1b). However, he derived the dual from a Cobb-Douglas cost function (equation (12) in his paper), hence the factor shares were constant. Naturally, this induces an additional reason why primal and dual measures are not identical.

In view of our discussion, however, the results discussed above should not come as a surprise and are to be expected. Regressing TFP_t on TFP_t^D is meaningless because they are "essentially" the same variable. Hence this method is not an empirical test of the putative hypothesis. Likewise, their results that $\tau = 1$ and $\phi = 0$ using the extended regression are also known without the need to run the regression. Note that the regressor $\hat{X}_t$ does not add anything to their accounting identity regression, hence its coefficient must be zero. It is simply the inclusion of an irrelevant variable.

It is worth noting that Wong and Gan (1994) estimated, systematically and across most of the 27 industries, slopes equal to unity in the regressions of the growth of the primal TFP

[41] Their calculations of TFP_t and TFP_t^D involve two additional terms each, corresponding to energy and materials, because their measure of industry output is gross output.

on the dual measure of TFP growth (with, as we have noted, extremely high t-values and an almost perfect fit). This is despite the fact that they had calculated and used the rental rate of capital ρ, theoretically different from the ex post profit rate r (i.e., ρ not derived residually from the identity [1b]). If ρ and r, and revenue and cost shares, are sufficiently different, then primal and dual measures will consequently differ, as Shapiro (1987) found in some instances. The reality, however, is that to calculate the rental rate of capital, Wong and Gan used the accounting identity equation (1b). Shapiro (1987) calculated properly the rental rate of capital (ρ), so probably ρ and r are sufficiently different and the regression does not look tautological. Yet, it is obvious that the accounting identity (2c) explains the regression results: TFP_t will differ from TFP_t^D if and only if ρ differs from r. Given that revenue and cost shares do not differ greatly (i.e., $a \approx \vartheta_L$), regression (19) is, at best, a test of the not particularly informative null hypothesis $H_0 : \rho = r$.

Finally, Shapiro (1987, footnote 4), referring to the rental rate, argues as follows: "if this measure of profits were found in the national accounts, which of course it is not, equation (14) would be tautologous." His equation (14) is the same as our equation (19). While Shapiro had to be aware of the underlying accounting identity to make this statement, nevertheless all regression (19) does is to test the equality of rental rate and profit rate, and that the identity (i.e., the fact that all variables involved in the analysis are related through equation [2c]) underlies the procedure.

7. FINAL THOUGHTS: IS THIS SCIENCE?

This is by its nature a somewhat nihilistic paper and we do not attempt to provide any alternative approach to orthodox growth theory. As John Locke (1998) [1689] put it "It is ambition enough to be employed as an under-laborer in clearing the ground a little and removing some of the rubbish which lies in the way to knowledge."

We believe the clarifications in this paper are necessary to explain the opening paragraph of this paper, namely, why extremely simple growth models (production-function based) appear to have a high explanatory power. It is simply a figment of how the variables in

these models are related. This argument implies that neoclassical growth theory has used for decades models that, if interpreted correctly, would not pass any serious scientific scrutiny. TFP growth calculations from growth accounting exercises as well as direct estimations of production functions, or equations derived from the latter, are not very informative as they ultimately merely reflect an underlying identity. The results of the production-function-based growth empirics literature that derive from Solow (1956, 1957) and continue until today with Romer's (1986, 1987) endogenous growth models, are fatally compromised by the fact that all they are capturing is an accounting identity.

The research programs underlying these models have created the illusion that we have the beginnings of a sound understanding of the growth process, because they produce results that appear, *a priori*, to be plausible. For example, estimates of the output elasticities are often close to the factor shares and the R^2 is close to unity, reflecting production theory and the competitive process. In some cases, the results seem putatively to have advanced our knowledge, e.g., the slow rate of convergence to steady-state, the fact that firms seem to have market power and set prices above marginal costs, and the very low TFP growth rates in the East Asian high-performing economies.

However, all these findings can be best understood by the fact that the functional forms used (derived from production functions) are simply transformations of an accounting identity, if not the accounting identity itself. In doing so, this research program has led to what may be seen as irrelevant discussions; puzzles that, by force, one has to answer within the paradigm (e.g., why has TFP growth declined?). The consequence is that much less is known about economic growth than many seem to think. The growth literature has to evolve and abandon the conceptualization of growth through a production function and through the TFP research program.

It may be argued that some of the examples cited in this paper are "old" and that the field has progressed since then, e.g., the more recent work of Hsieh and Klenow (2009), Fernald and Neiman (2011), Bollard et al. (2013), or Fernald (2015). We are not convinced by this argument. First, all of the papers that we have reviewed are exemplars in the standard growth theory literature. Second, more recent exercises are still more-or-less sophisticated

growth accounting exercises (i.e., there is no testing involved) whose findings can be interpreted in terms of the identity.

The recent literature on growth accounting has supposedly progressed by, for example, using firm-level data. This does not mean that the problem is solved, as the accounting identity holds at any level of aggregation. Indeed, these firm-level studies use deflated sales or value added as a measure of output and total fixed assets for the capital stock. The accounting identity argument remains irrespective of whether country-level or firm-level data is used. This is the case, for example, of the new literature on the misallocation of resources. See, for example, Bollard *et al.* (2013) on India. Hsieh and Klenow (2009) distinguish between "revenue" and "quantity" TFP. It is not clear, however, that their quantity TFP corresponds to the true physical TFP. This is because they do not have physical quantities of output, and "real" output is derived indirectly. Moreover, their measure of the capital stock is not the number of homogeneous machines, but the book value of capital stock. Fernald and Neiman (2011) do acknowledge the accounting identity in their growth accounting exercise, but they do not appear to realize the isomorphism with the production function, which arises as a result of using value data.

It is surprising that the critique of Phelps Brown (1957), Simon (1979a) and Samuelson (1979) has not had any serious impact on the (production-function) growth research program. One can speculate about the reason for this neglect, despite Simon's (1979b) concerns. It may be due to the extent of the sunk intellectual capital that has occurred over the years in this research program; or to the fact that the criticism may be seen so trivial that it is prone to be misunderstood and dismissed. Perhaps it is time to take it seriously and just as Leamer (1983) advocated to take the con out of econometrics, we should also *take the con out of growth empirics.*

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