Blumenfeld *et al.* Reply We recently argued [1] that the volume function (VF), being insensitive to most of the degrees of freedom (DFs), is unsuitable as the granular "Hamiltonian" and proposed an alternative. The previous comment [2] contests with the following arguments.

(1) Changes in positions of bulk particles in granular systems (GSs) may lead to rearrangements that change the boundary position, making the latter implicitly sensitive to many configurational changes.

(2) The form of the Hamiltonian does not determine the number of microstates and the entropy.

(3) The ideal gas Hamiltonian, H_{ig}, for example, is also independent of some DFs, yet it is a good model in thermal statistical mechanics.

(4) The insensitivity of the VF to the bulk particle positions is in agreement with Edwards's hypothesis of equiprobable microstates, supporting the VF. The implication of this argument is that the equiprobability assumption, if it holds, obviates the need to depend on the internal DFs.

As detailed next, we agree with argument (2), refute (1), (3), and (4), and clarify why the VF is not a useful Hamiltionian analog for GSs.

Argument (1): The statement "...each particle may affect the microscopic VF if another particle displaced by it moves far enough to become a border particle (which may become experimentally visible only near the jamming point, where fluctuations get amplified). This is a latent dependency of the Hamiltonian on many DFs..." is incorrect in general. Almost all structural perturbations inside almost all GSs will not affect the positions of boundary particles and hence would not change the VF. This is independent of whether there are rattlers in the system, whose change of positions also has a negligible relevance to the positions of boundary particles.

Argument (2): Indeed, the statement in our paper, that the VF fails to account correctly for the entire entropy, was incorrect. Nevertheless, this is not the main reason for the failure of the VF, as we explain next.

Argument (3): This is the Comment's main argument. H_{ig}, which is a good model for gases, is an example of a Hamiltonian independent of the position DFs. So why should a function, which does not depend on all DFs, be a problem? This argument misses the main thrust of our paper, which is to construct a statistical mechanical formalism to describe usefully *all* GSs. The aim of statistical mechanics in general is to make possible derivation of relevant measurable macroscopic properties as expectation values over a partition function. To this end it needs to be able to describe *all* systems. The energy of a thermal system need not depend on all the DFs, but if it does, then the Hamiltonian will also depend on all of them. This is why it is good for *all* thermal systems. For example, if the ideal gas is put in a potential field, such as gravity, then H_{ig} would include a term that depends on the position DFs. In contrast, the VF cannot depend on the internal structural DFs for all GSs. Consequently, it does not allow us to derive expectation values of quantities that depends on the internal structure. If the Hamiltonian of the ideal gas in a gravitational field were independent of the position DFs then we would be unable to calculate the macroscopic pressure or gas density as a function of position (height). To illustrate the deficiency of the VF, the permeability of a GS to fluid flow through it depends strongly on the distribution of its pore-to-pore openings. The VF cannot provide any way to calculate this distribution or its moments. Nor can it provide a way to calculate any property that depends on the internal structure.

Argument (4): Edwards's hypothesis of equiprobable microstates was shown to fail for a range of GSs [3–5], away from the jamming point. Thus, the nonuniform probabilities of microstates further undermines the applicability of the VF, as well as the basis for this argument in the comment.

In conclusion, while, for any specific thermal system, the Hamiltonian depends on all the relevant DFs, making it a good basis for thermal statistical mechanics, the VF lacks this feature—it always depends only on a very small subset of the DFs and, as such, it is not useful as an analog of the Hamiltonian for GSs. The argument that the boundary particles positions would change in response to any change in the internal structure is wrong because such changes would only propagate as far as the stress response correlation length, which is short is most systems, except at the jamming point. The argument that the equiprobability of microstates obviates the need for dependence on the internal DFs is also misconceived because we start to discover that the microstates of GSs are not equiprobable.

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